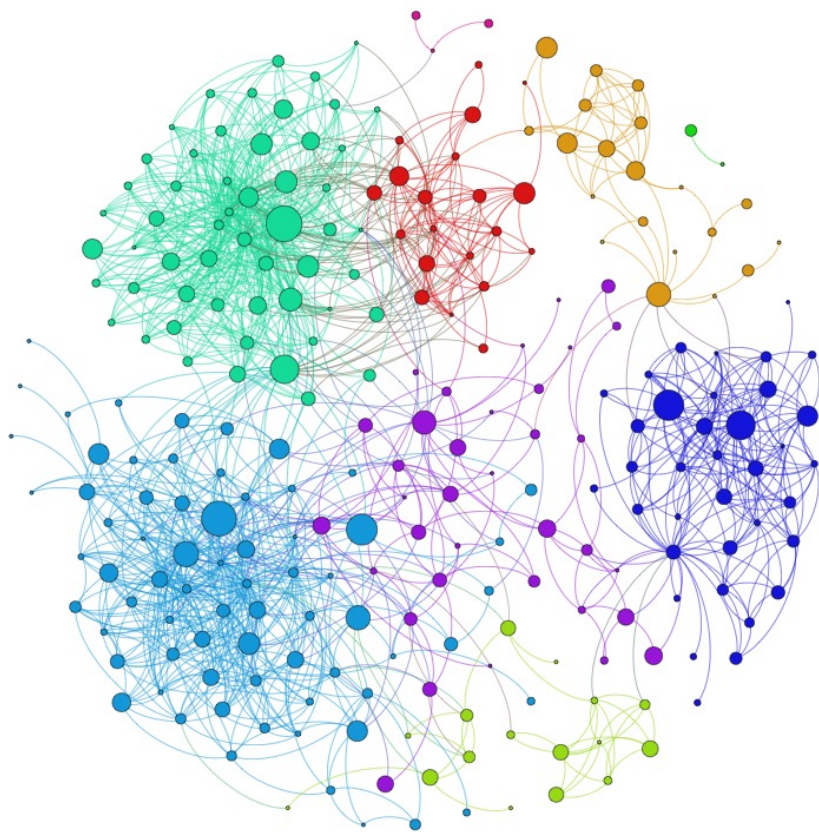


# FISHNETS

T. Lucas Makinen [Imperial College London]

ML x Cosmo Conference, IAP, November 2023

with Justin Alsing + Ben Wandelt

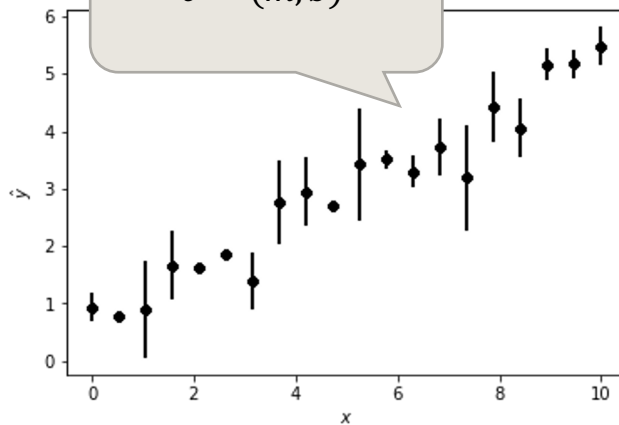




# FISHNETS

Given some data:  $\{\mathbf{d}\} = \{\mathbf{x}, \mathbf{y}, \boldsymbol{\sigma}\}$

$$\ln p(\{\mathbf{d}\}|\boldsymbol{\theta}) = \sum_{i=1}^{n_{\text{data}}} \ln p(d_i|\boldsymbol{\theta})$$





# FISHNETS

Given some data:  $\{\mathbf{d}\} = \{\mathbf{x}, \mathbf{y}, \boldsymbol{\sigma}\}$

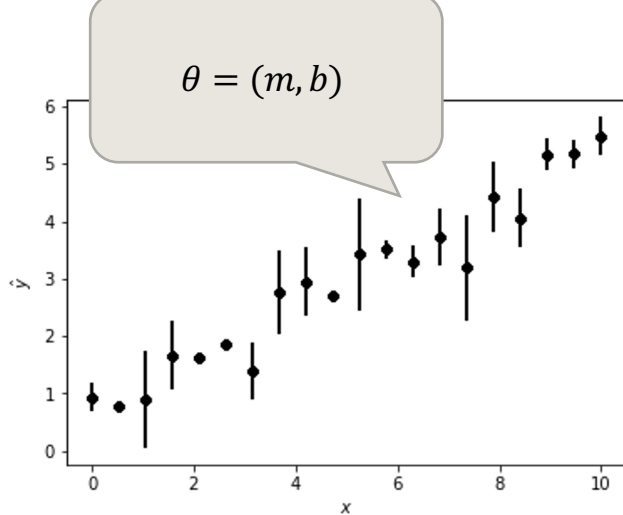
$$\ln p(\{\mathbf{d}\}|\boldsymbol{\theta}) = \sum_{i=1}^{n_{\text{data}}} \ln p(d_i|\boldsymbol{\theta})$$

Dataset score is additive !

$$\mathbf{t} = \nabla_{\boldsymbol{\theta}} \ln p(\{\mathbf{d}\}|\boldsymbol{\theta})$$

$$= \sum_{i=1}^{n_{\text{data}}} \nabla_{\boldsymbol{\theta}} \ln p(d_i|\boldsymbol{\theta})$$

$$= \sum_{i=1}^{n_{\text{data}}} \mathbf{t}_i(d_i)$$





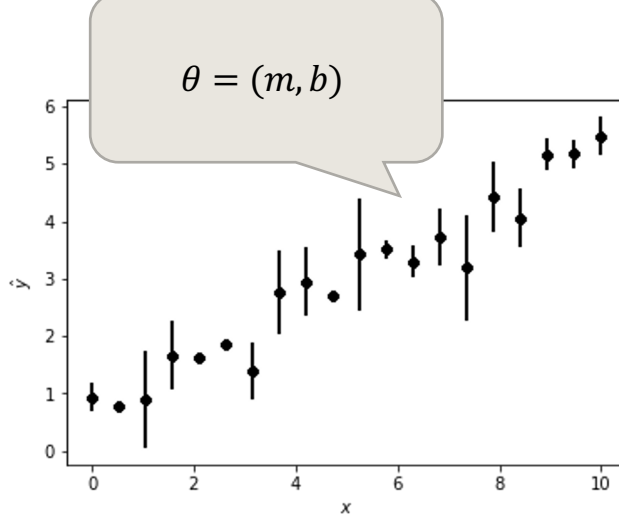
# FISHNETS

Given some data:  $\{\mathbf{d}\} = \{\mathbf{x}, \mathbf{y}, \boldsymbol{\sigma}\}$

$$\ln p(\{\mathbf{d}\}|\boldsymbol{\theta}) = \sum_{i=1}^{n_{\text{data}}} \ln p(d_i|\boldsymbol{\theta})$$

Dataset score is additive !

$$\begin{aligned} \mathbf{t} &= \nabla_{\boldsymbol{\theta}} \ln p(\{\mathbf{d}\}|\boldsymbol{\theta}) \\ &= \sum_{i=1}^{n_{\text{data}}} \nabla_{\boldsymbol{\theta}} \ln p(d_i|\boldsymbol{\theta}) \\ &= \sum_{i=1}^{n_{\text{data}}} \mathbf{t}_i(d_i) \end{aligned}$$



Fisher is also additive !

$$\mathbf{F}(\{\mathbf{d}\}) = \sum_{i=1}^{n_{\text{data}}} \mathbf{F}_i(x_i, \sigma_i)$$

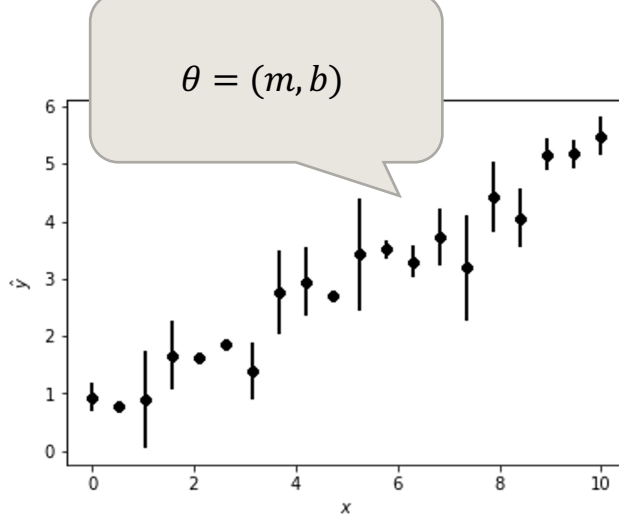
# FISHNETS

Given some data:  $\{\mathbf{d}\} = \{\mathbf{x}, \mathbf{y}, \boldsymbol{\sigma}\}$

$$\ln p(\{\mathbf{d}\}|\boldsymbol{\theta}) = \sum_{i=1}^{n_{\text{data}}} \ln p(d_i|\boldsymbol{\theta})$$

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Fisher is also additive !

$$\mathbf{F}(\{\mathbf{d}\}) = \sum_{i=1}^{n_{\text{data}}} \mathbf{F}_i(x_i, \sigma_i)$$

learn with neural networks !

create pseudo-MLE estimate

$$\hat{\boldsymbol{\theta}}^{\text{MLE}} = \mathbf{F}_{NN}^{-1} \mathbf{t}_{NN} + \boldsymbol{\theta}_{fid}$$



Train the networks using a “KL”-type loss:

$$\mathcal{L} = \frac{1}{2}(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{\text{MLE}})^T \mathbf{F}_{NN}^{-1} (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}^{\text{MLE}}) - \frac{1}{2} \ln \det \mathbf{F}_{NN}$$





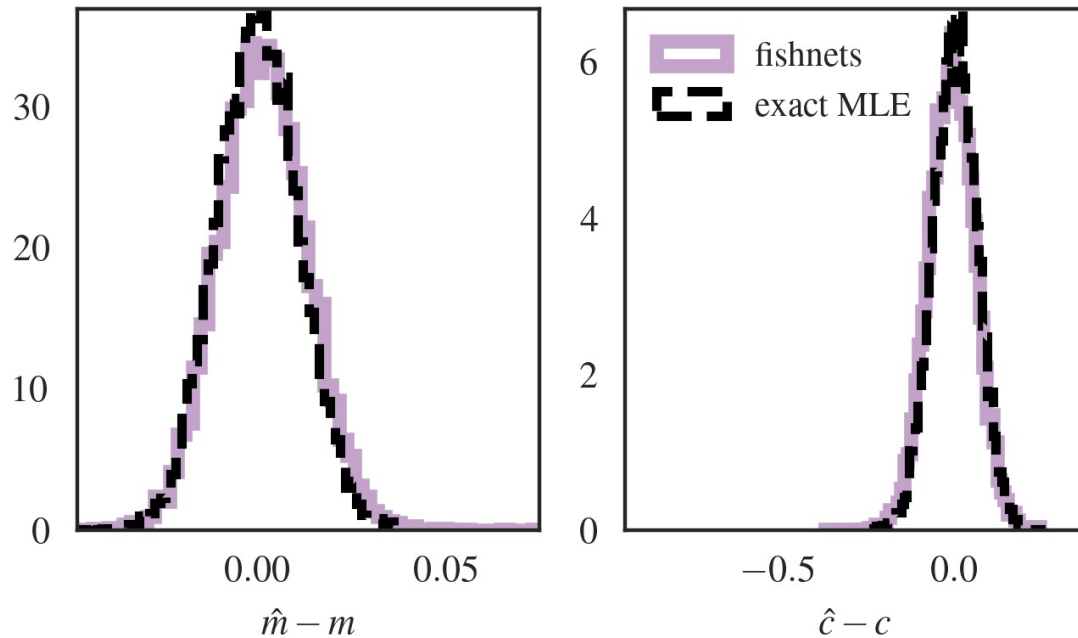
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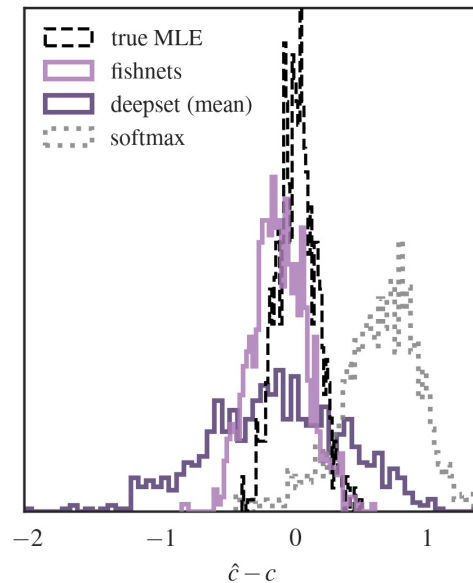
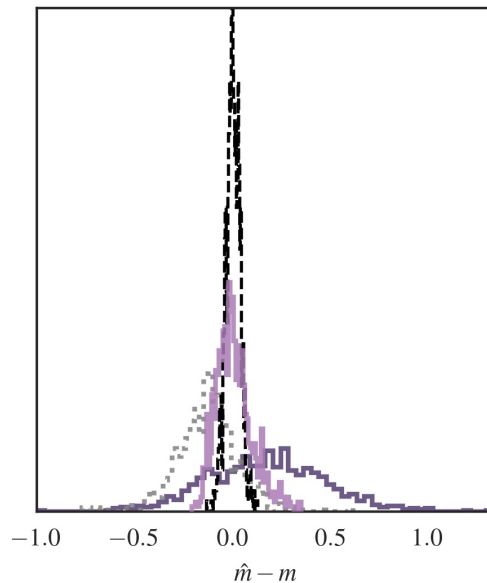
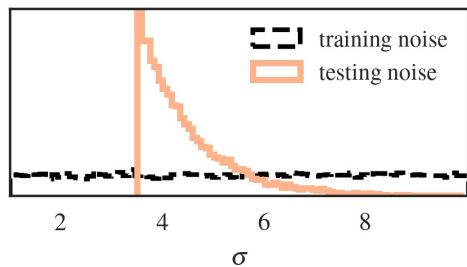
score network

fisher network

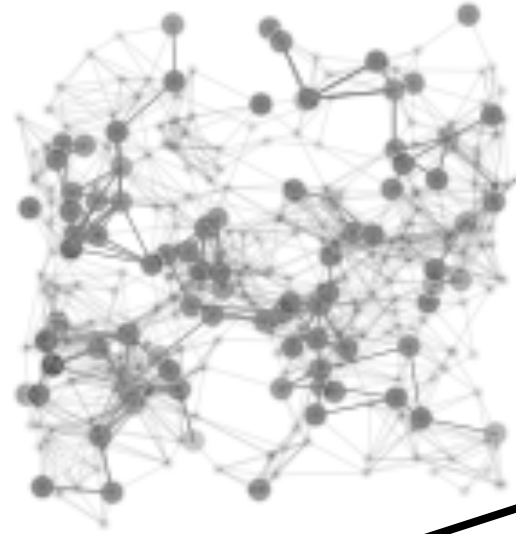
optimal for linear regression ...



... even when the global noise distribution changes !

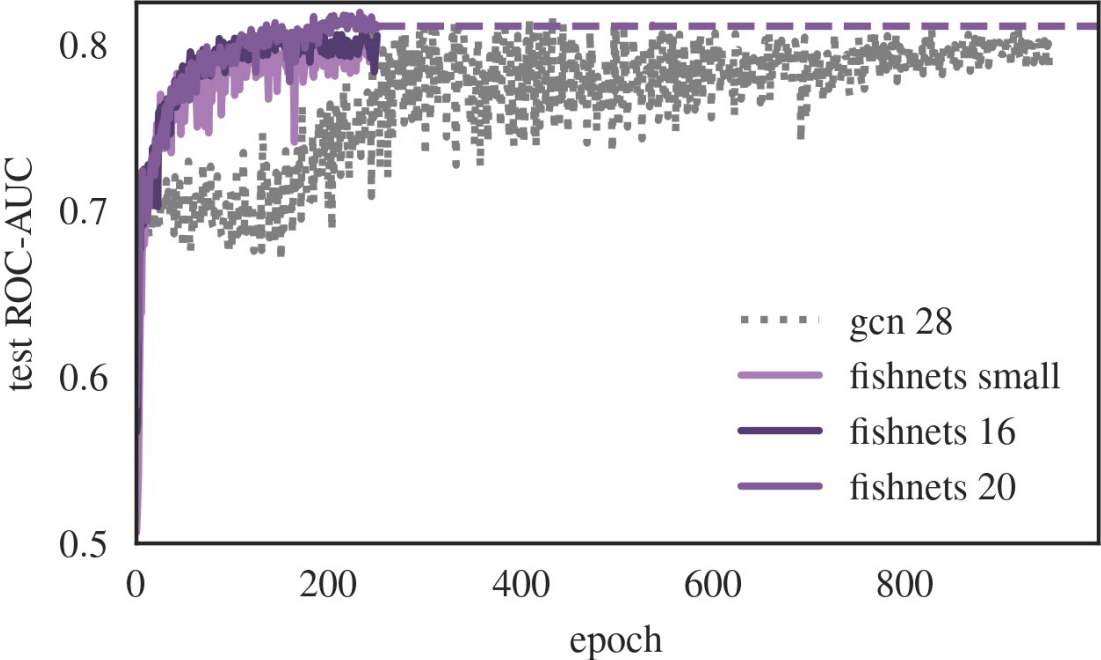


what about graphs ?



aggregate this information !

# Use as a drop-in replacement for graph network aggregation



# improve information embedding + extraction for common benchmarks

