Less is enough:
extending ΛCDM with representation learning

Davide Piras (and many others)

UNIVERSITÉ DE GENÈVE

UCL

But first… let me apologise

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- Title was not convincing

Less is enough:
extending ΛCDM with representation learning

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Less is enough:
extending ΛCDM with representation learning

arXiv > astro-ph > arXiv:2303.17059

Astrophysics > Instrumentation and Methods for Astrophysics

[Submitted on 29 Mar 2023]

As a matter of colon: I am NOT digging cheeky titles (no, but actually yes :>)

Joanne Tan, Tie Sien Suk

But first… let me apologise

- Title was not convincing

parameters

Less is enough:
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- So I did what any AI researcher would do…

But first… let me apologise

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parameters

Less is enough:
extending ΛCDM with representation learning

- So I did what any AI researcher would do…

- … I asked ChatGPT!

Enriching ΛCDM:
extending cosmological models with representation learning

Less is enough:

extending ΛCDM with representation learning

Less is enough:

extending ΛCDM with representation learning

ΛCDM extensions

- ΛCDM is good

ΛCDM extensions

- ΛCDM is good

Λ : cosmological constant
CDM: cold dark matter

[insert standard cosmological image here]

ΛCDM extensions

- ΛCDM is good... but not the entire story

ΛCDM extensions

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    - Tensions (H_0, S_8)

ΛCDM extensions

- ΛCDM is good… but not the entire story
  - Tensions ($\mathrm{H_0}$, $\mathrm{S_8}$)
  - What is dark matter?

ΛCDM extensions

- ΛCDM is good… but not the entire story
    - Tensions ($\mathrm{H_0}$, $\mathrm{S_8}$)
    - What is dark matter?
    - And dark energy?

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    - …

ΛCDM extensions

- ΛCDM is good... but not the entire story

- Beyond-ΛCDM models add extra parameters

ΛCDM extensions

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- Beyond-ΛCDM models add extra parameters

CDM

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}$$

ΛCDM extensions

- ΛCDM is good... but not the entire story

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CDM

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}$$

wCDM

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}\boxed{w_0 \ w_a}$$

ΛCDM extensions

- ΛCDM is good... but not the entire story

- Beyond-ΛCDM models add extra parameters

CDM

$$\boxed{\Omega_b \; \Omega_m \; h \; n_s \; A_s}$$

f(R)

$$\boxed{\Omega_b \; \Omega_m \; h \; n_s \; A_s}\boxed{f_{R0}}$$

ΛCDM extensions

- ΛCDM is good… but not the entire story

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CDM

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}$$

Dvali-Gabadadze-Porrati

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}\boxed{\Omega_{rc}}$$

ΛCDM extensions

- ΛCDM is good... but not the entire story

- Beyond-ΛCDM models add extra parameters

CDM

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s}$$

...

$$\boxed{\Omega_b \ \Omega_m \ h \ n_s \ A_s \ \boxed{\cdots}}$$

ΛCDM extensions

- ΛCDM is good… but not the entire story
- Beyond-ΛCDM models add extra parameters
- Find common parameterisation of all these models?

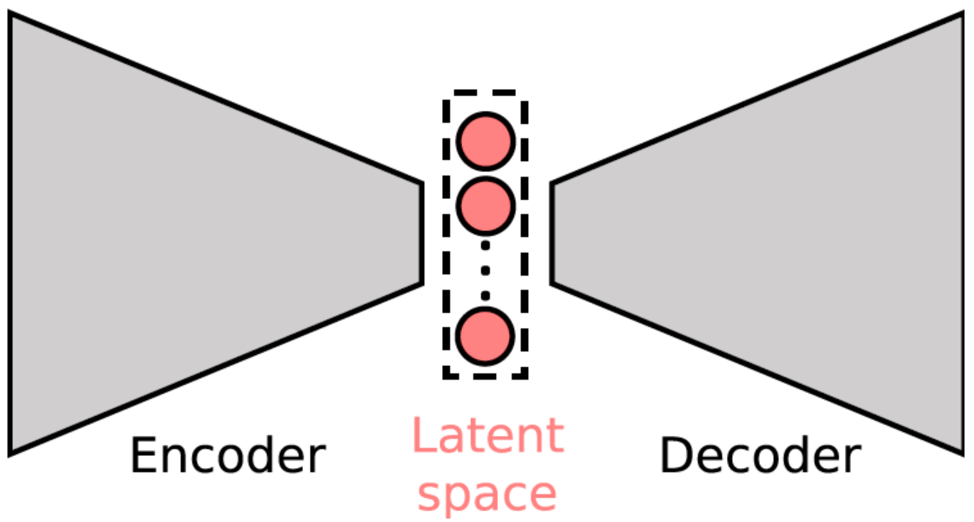
Less is enough:

extending ΛCDM with representation learning

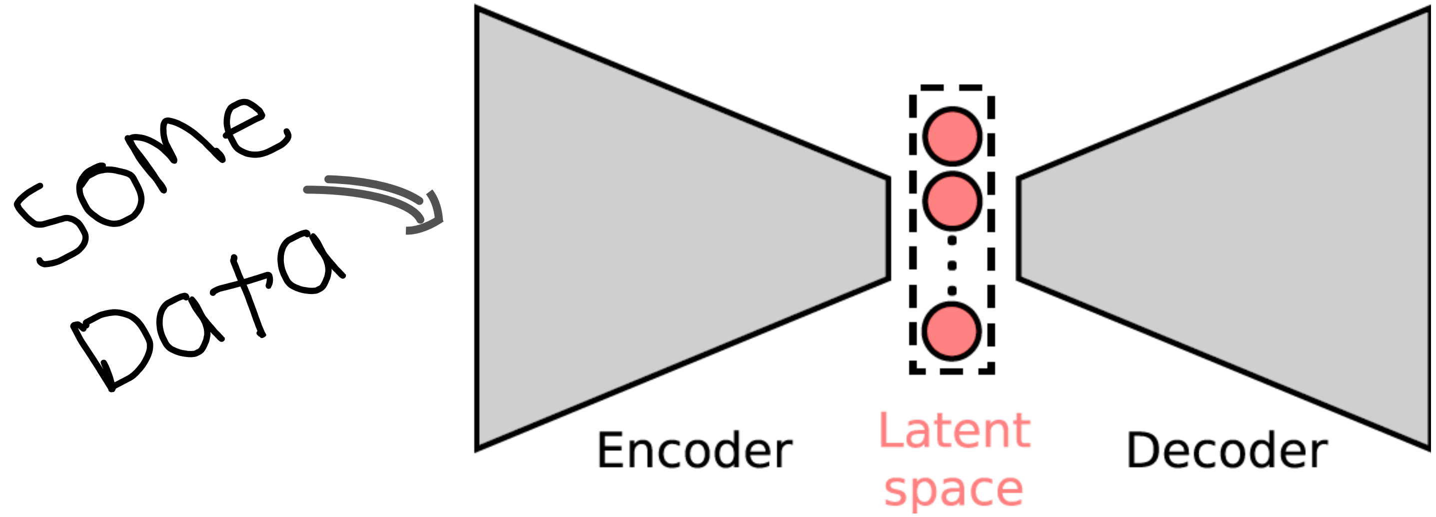
Less is enough:

extending ΛCDM **with representation learning**

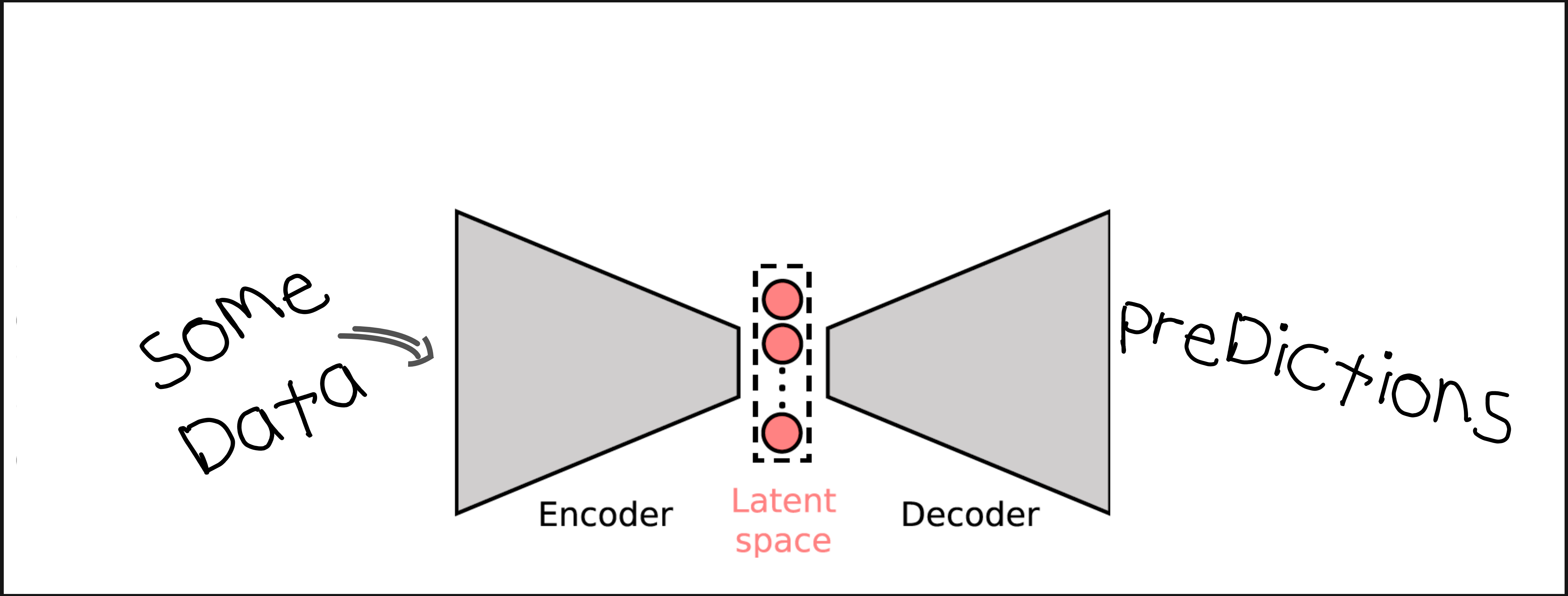
Representation learning



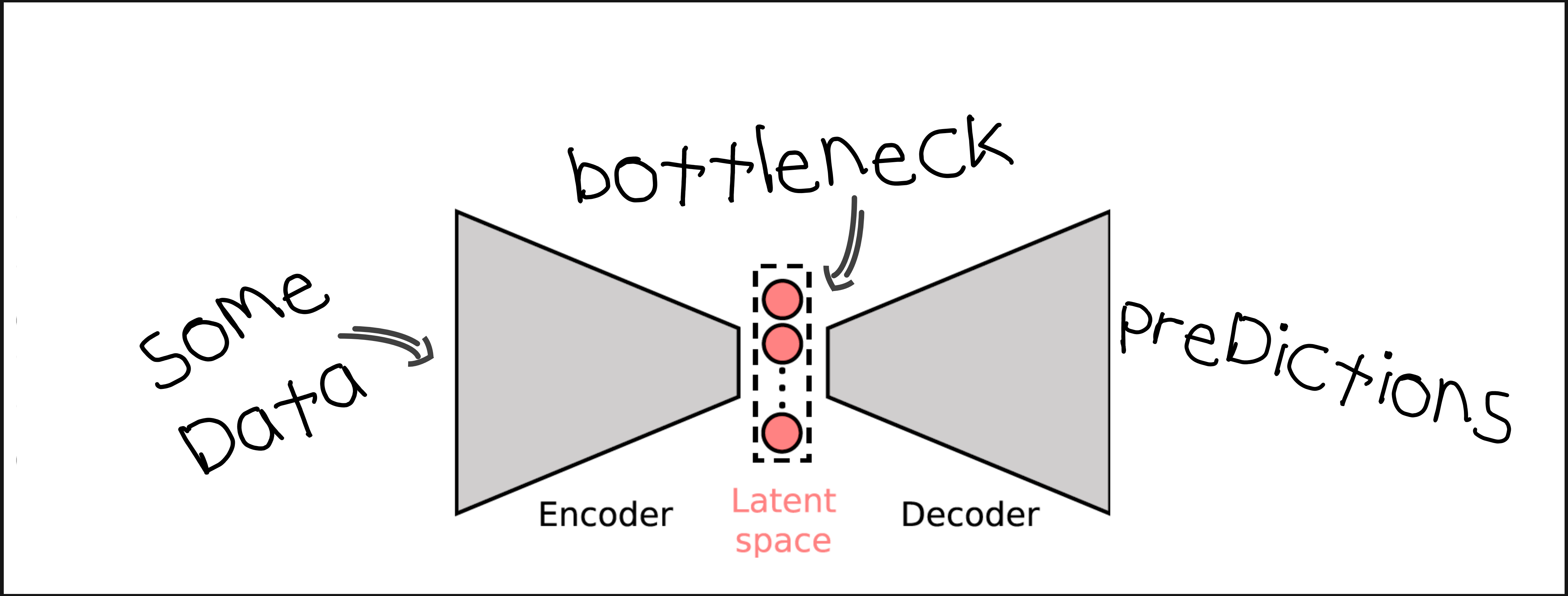
Representation learning



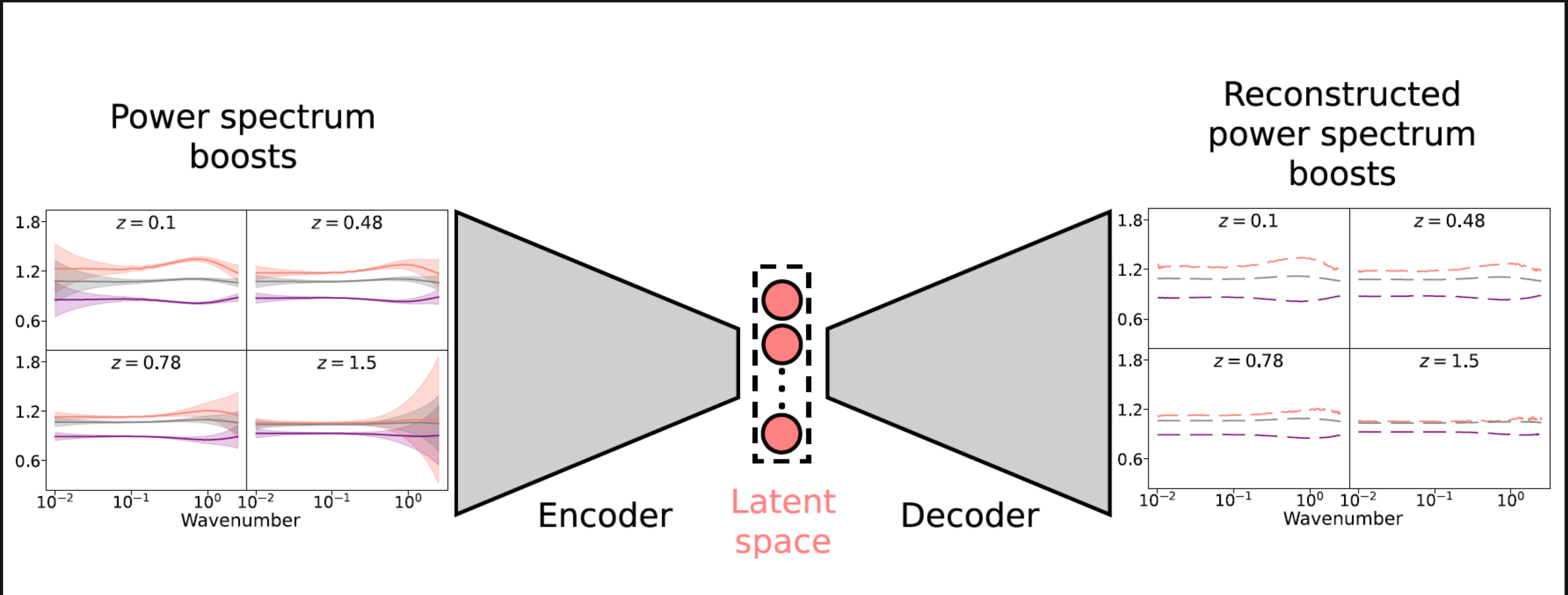
Representation learning



Representation learning



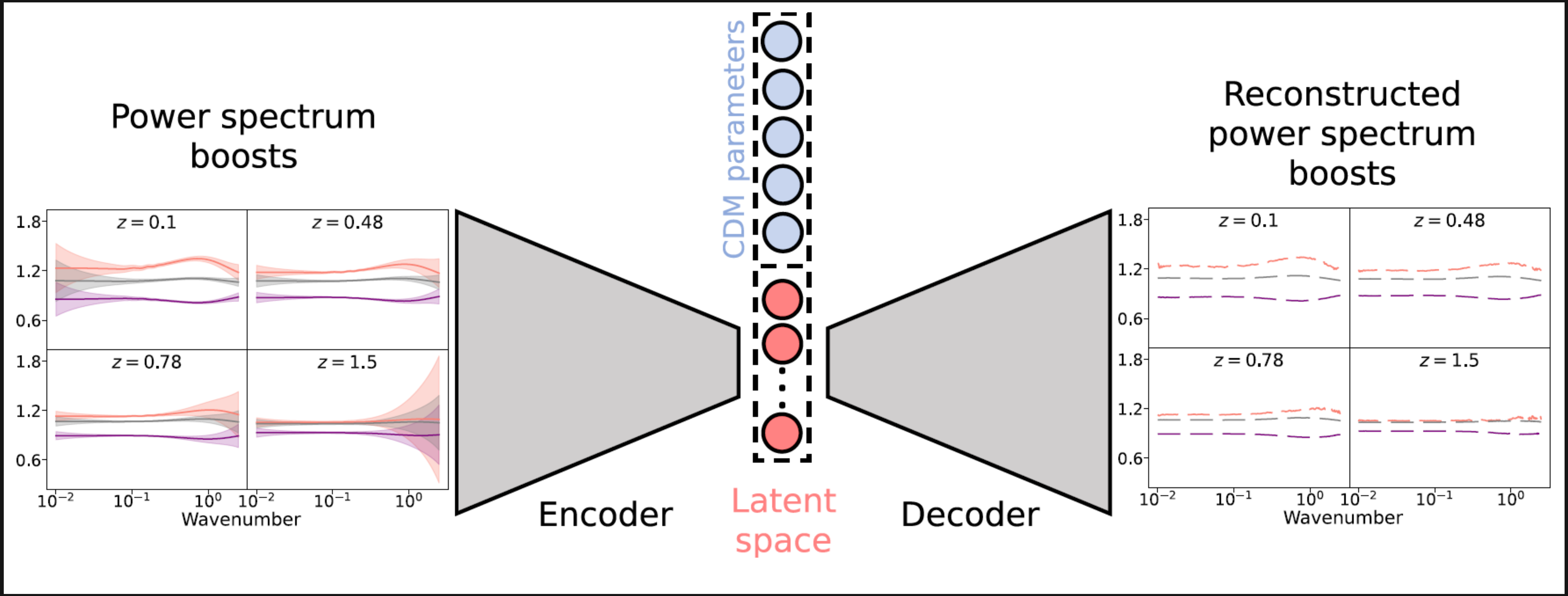
Representation learning



$$\text{Power spectrum boost} = \frac{\text{Power spectrum in extended model}}{\text{Power spectrum in }\Lambda\text{CDM model}}$$

Representation learning

Piras & Lombriser, arXiv 2310.10717



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Less is enough:

extending ΛCDM with representation learning

parameters

Less is enough:

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An application to dark energy

- Apply our framework to single extension: wCDM

An application to dark energy

- Apply our framework to single extension: wCDM

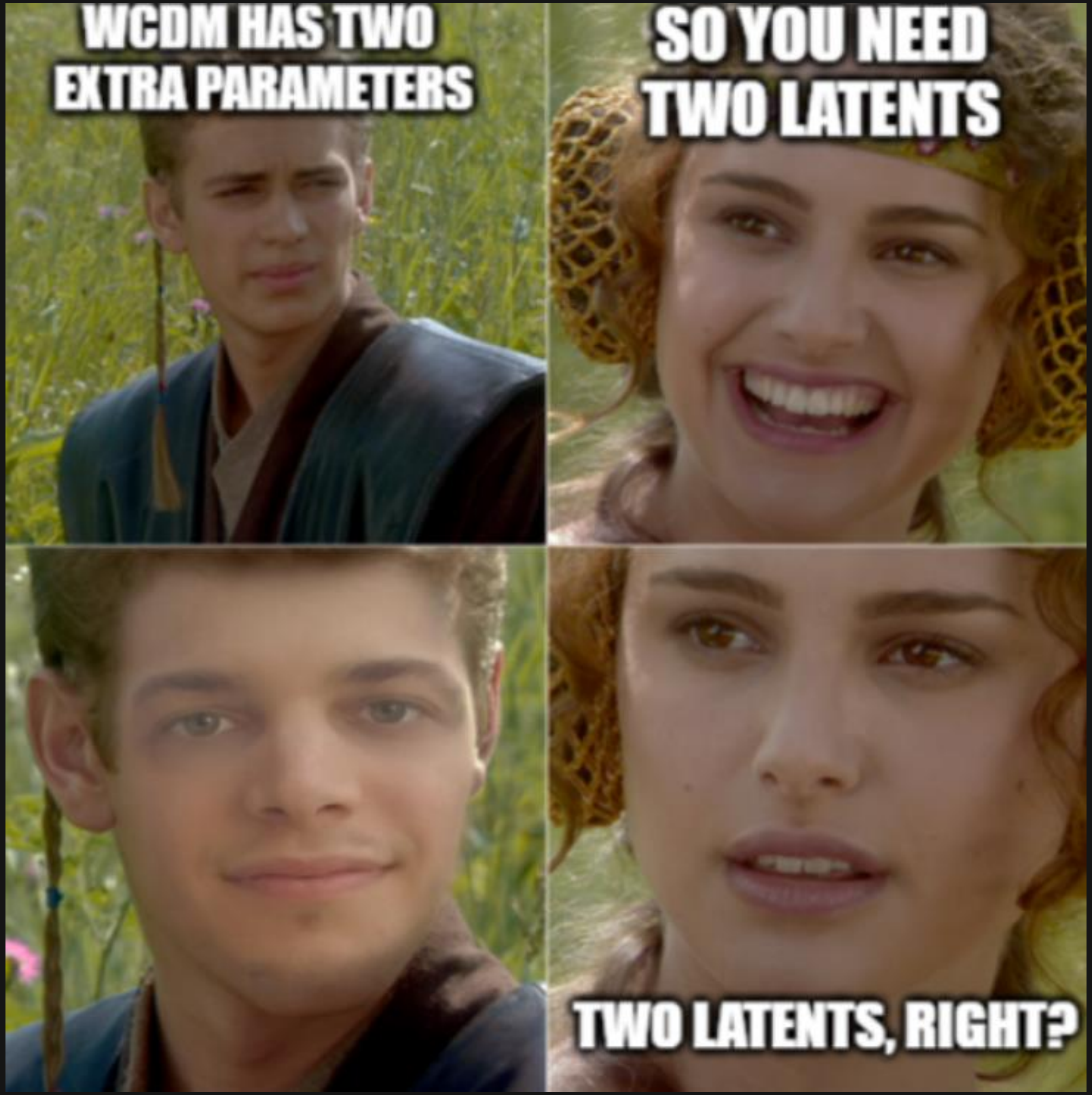
- Two extra parameters: w_0 and w_a

$$w(a) = w_0 + (1-a)w_a$$

An application to dark energy

- Apply our framework to single extension: wCDM
- Two extra parameters: w_0 and w_a
- Expect two latent variables are needed…?

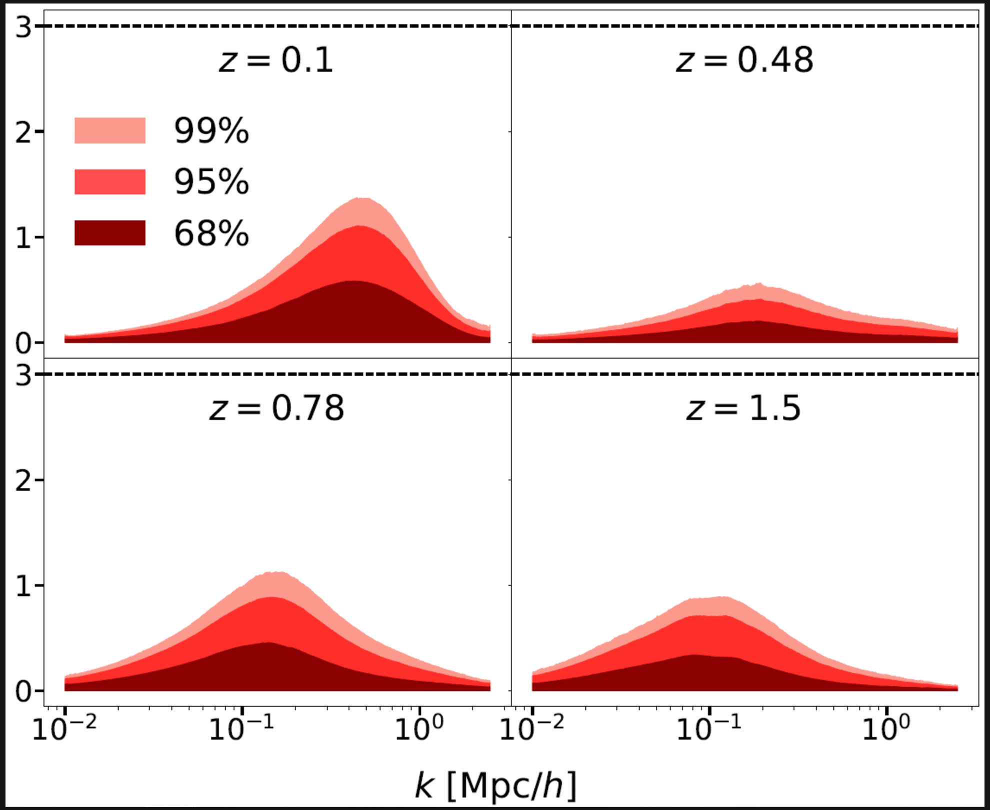
An application to dark energy



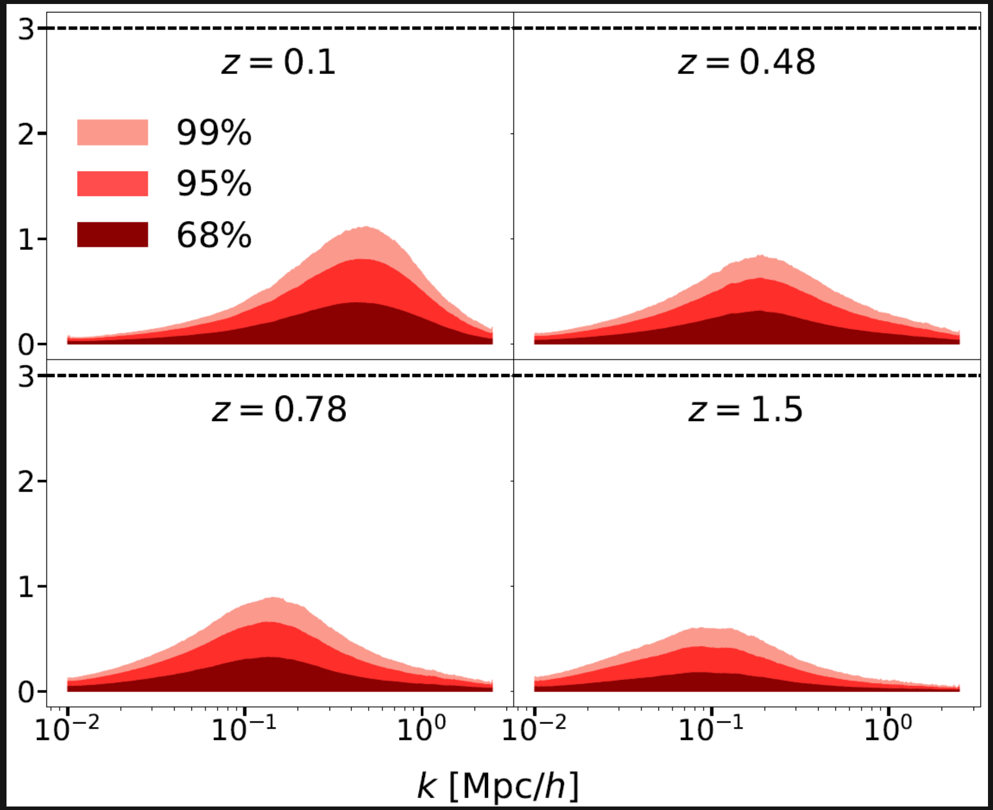
Results

Results

One latent variable

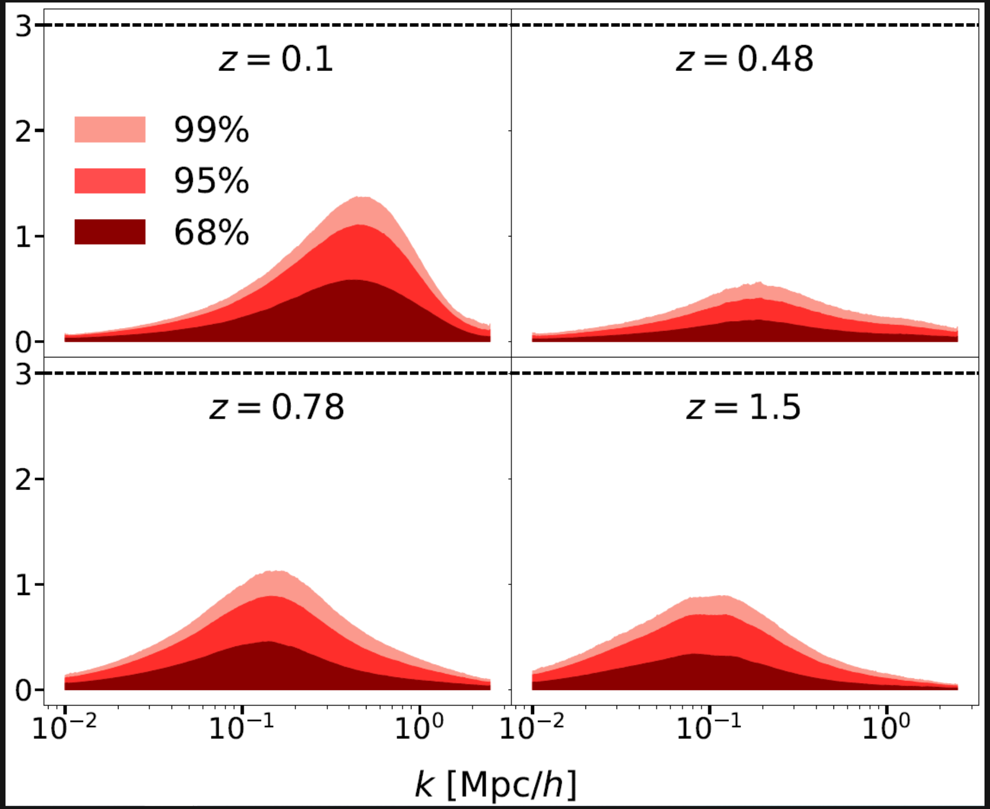


Two latent variables

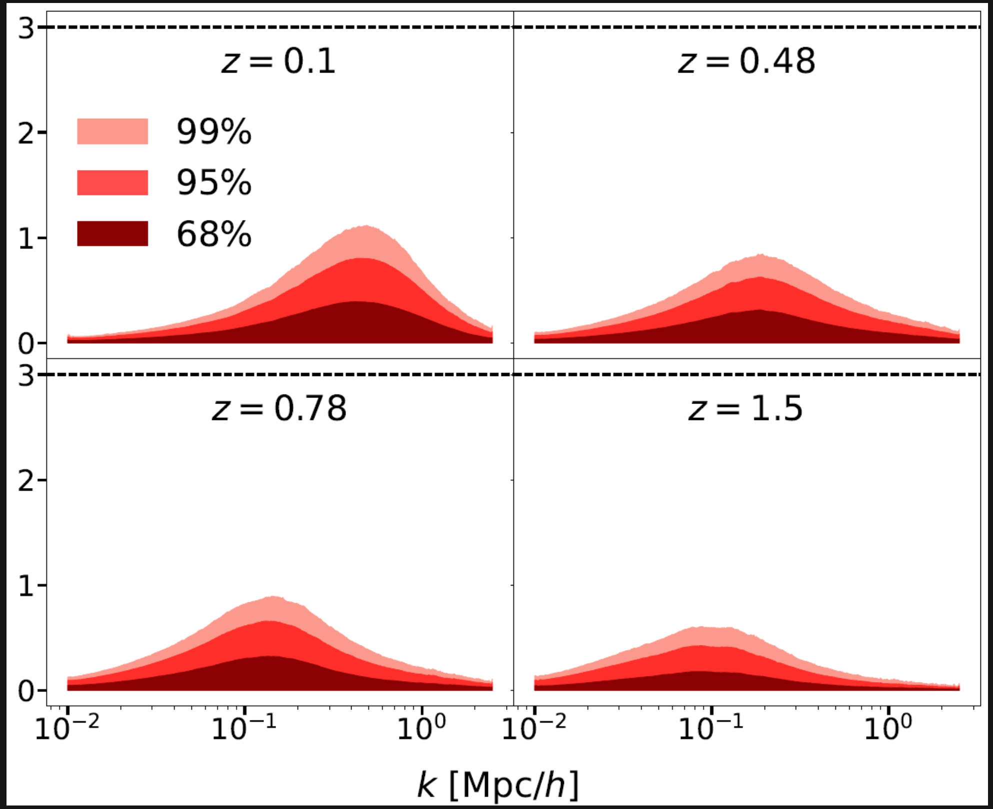


Results

One latent variable



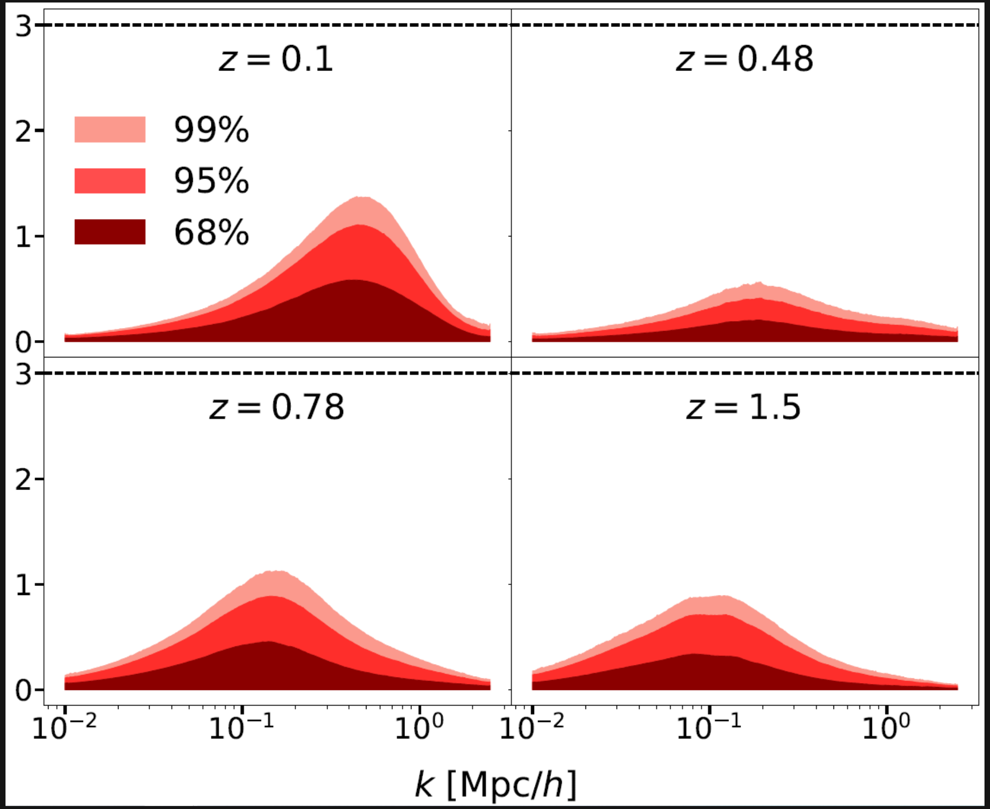
Two latent variables



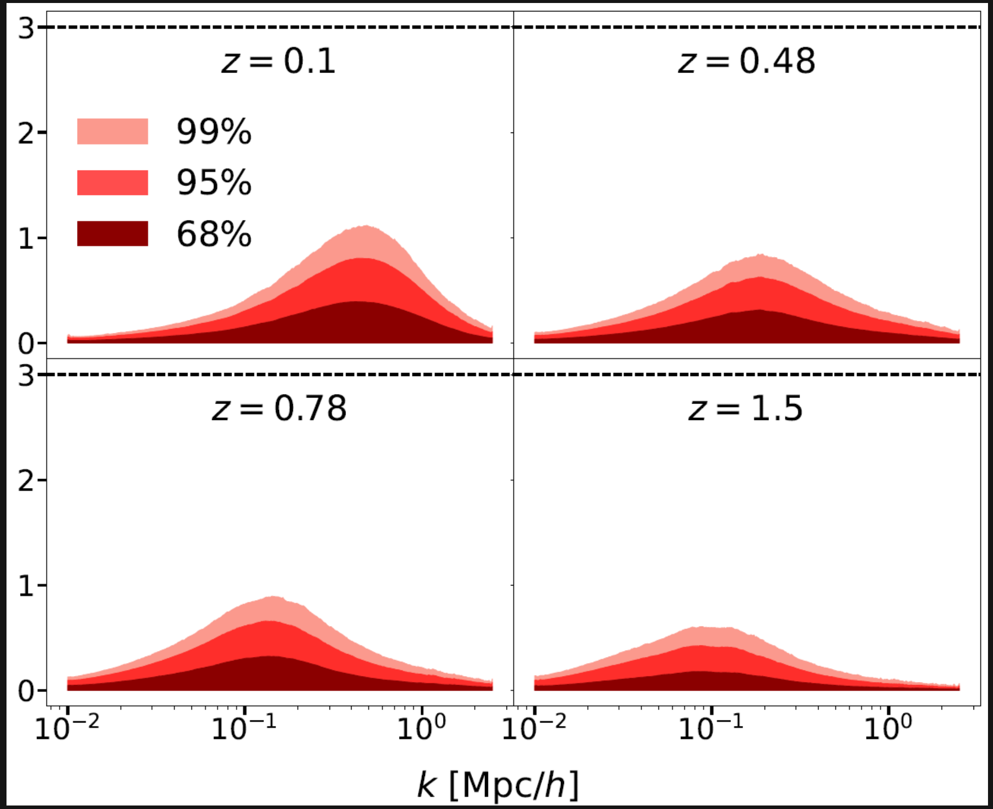
Vertical axis: how many error bars is the predicted spectrum away from the ground truth? (lower is better)

Results

One latent variable



Two latent variables



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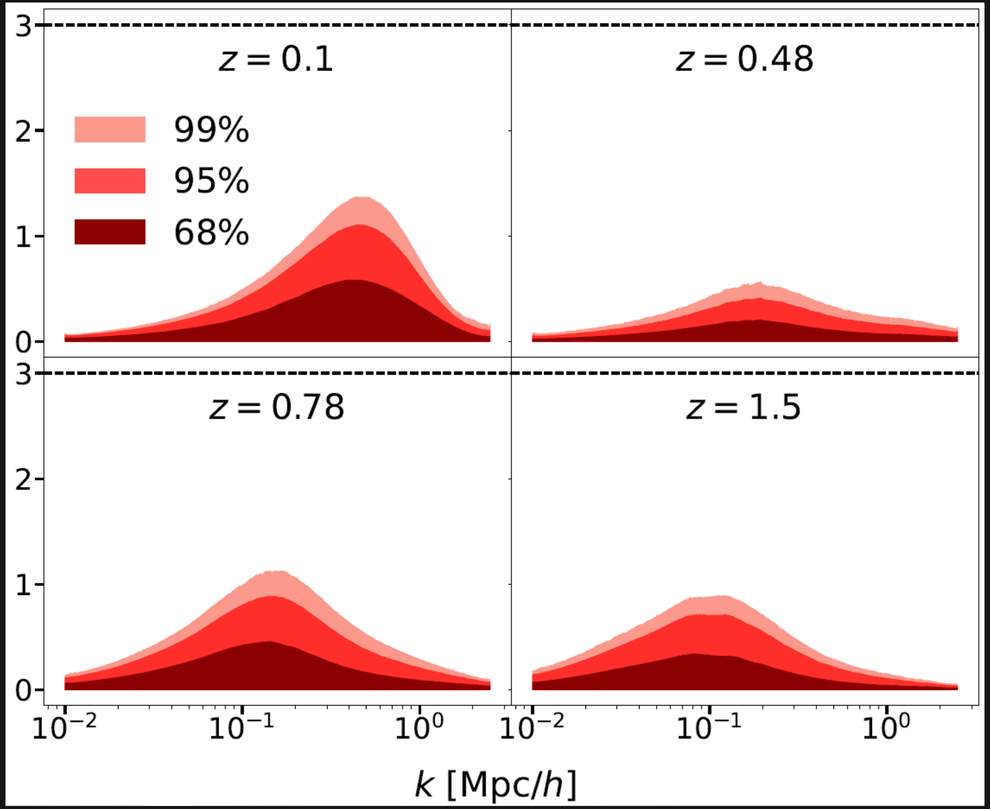
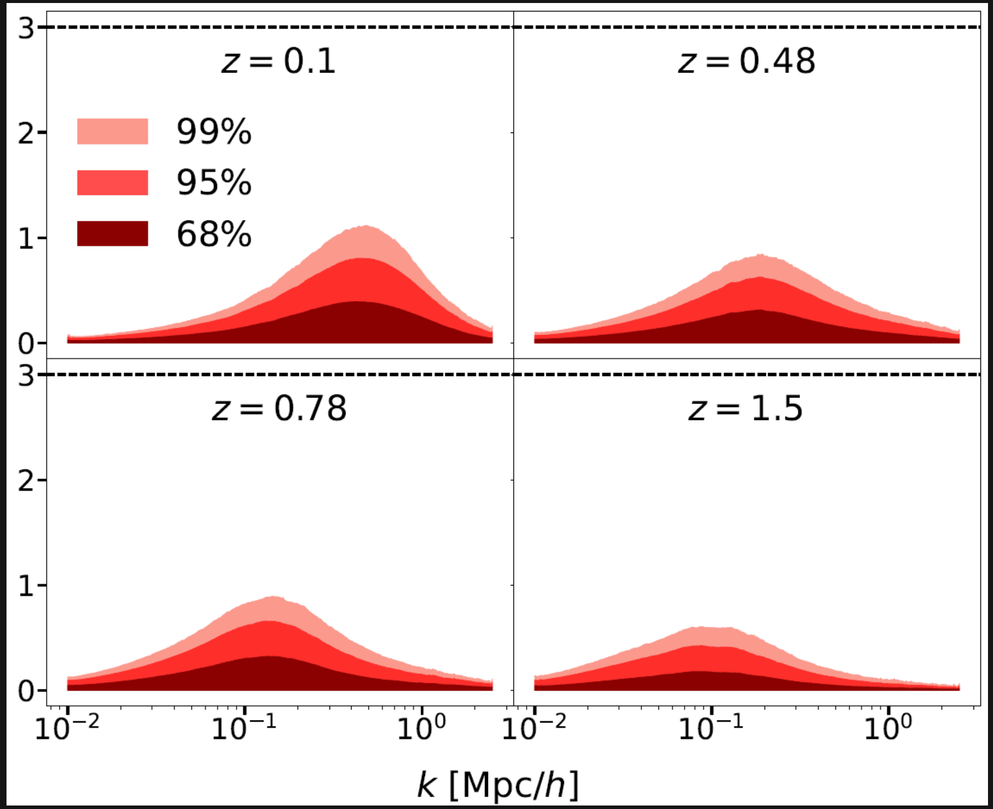
Results are pretty similar with one and two latents!

Results

One latent variable



Two latent variables



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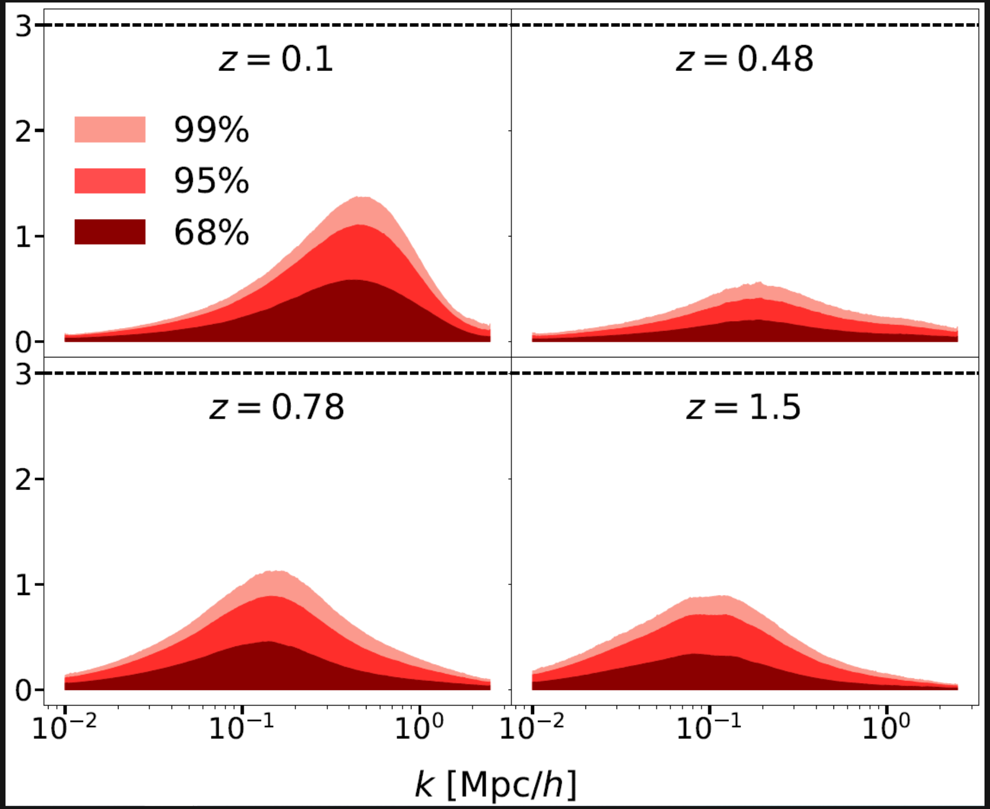
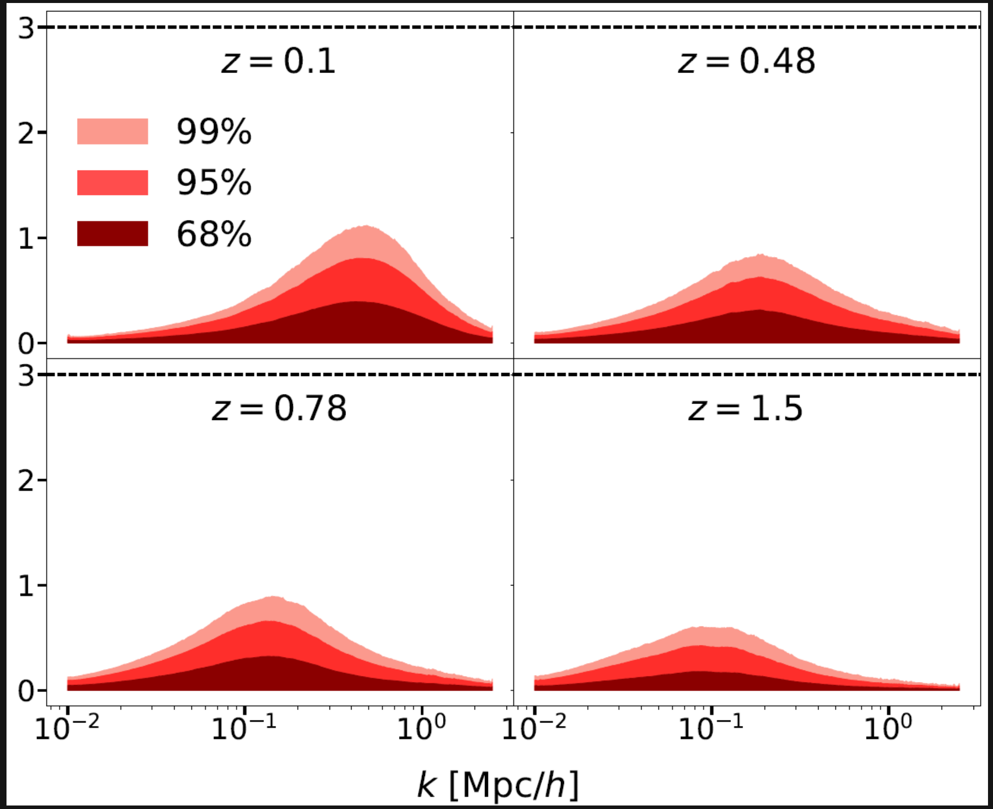
Third disentangled latent: no impact (not shown)

Results

One latent variable



Two latent variables



Vertical axis: how many error bars is the predicted spectrum away from the ground truth? (lower is better)

Results are pretty similar with one and two latents!

Third disentangled latent: no impact (not shown)

One variable is enough for w**CDM!**

How to analyse the latent space?

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- Mutual information

What is mutual information?

- Measures **dependence** between random variables (more general than Pearson, which measures correlation)

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- Well-established in information theory

What is mutual information?

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- Well-established in information theory

- Hard to estimate!

Estimating mutual information (MI)

- No available estimator returns uncertainty on MI

Estimating mutual information (MI)

- No available estimator returns uncertainty on MI

- **Solution:** density estimate with Gaussian mixture model

GMM-MI

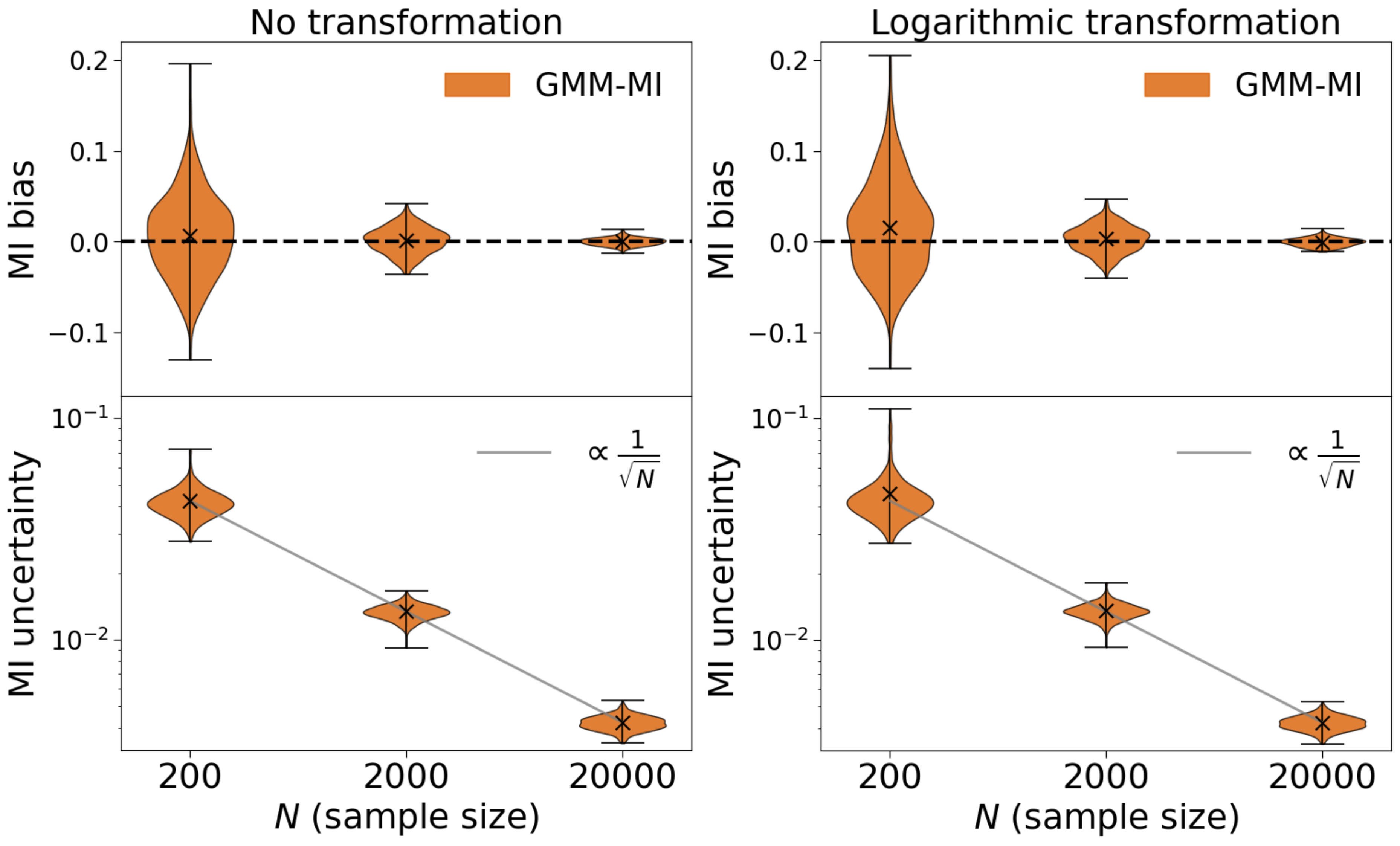
"Jimmie"

GMM-MI validation

Piras et al. (including Hiranya Peiris, Andrew Pontzen, Luisa Lucie-Smith, Lillian Guo, Brian Nord), MLST

Code

Ask me later!



How we use mutual information (MI)

- Calculate MI between latent variables (are they disentangled?)

Latent A ⟷ Latent B

How we use mutual information (MI)

- Calculate MI between latent variables (are they disentangled?)

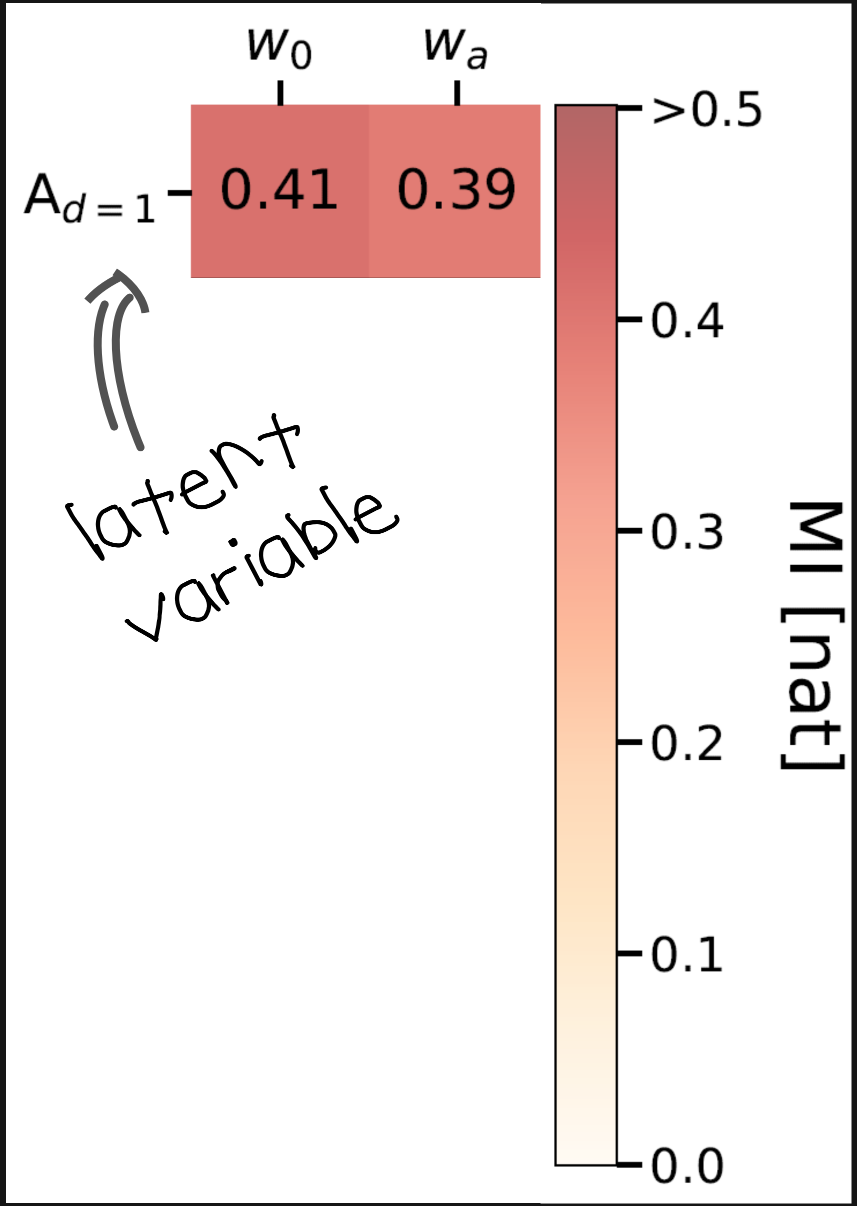
Latent A $\Longleftrightarrow$ Latent B

- Calculate MI between a latent variable and model parameters

Latent A $\Longleftrightarrow$ $\mathrm{w}_0, \mathrm{w}_\mathrm{a}$

Mutual information in latent space

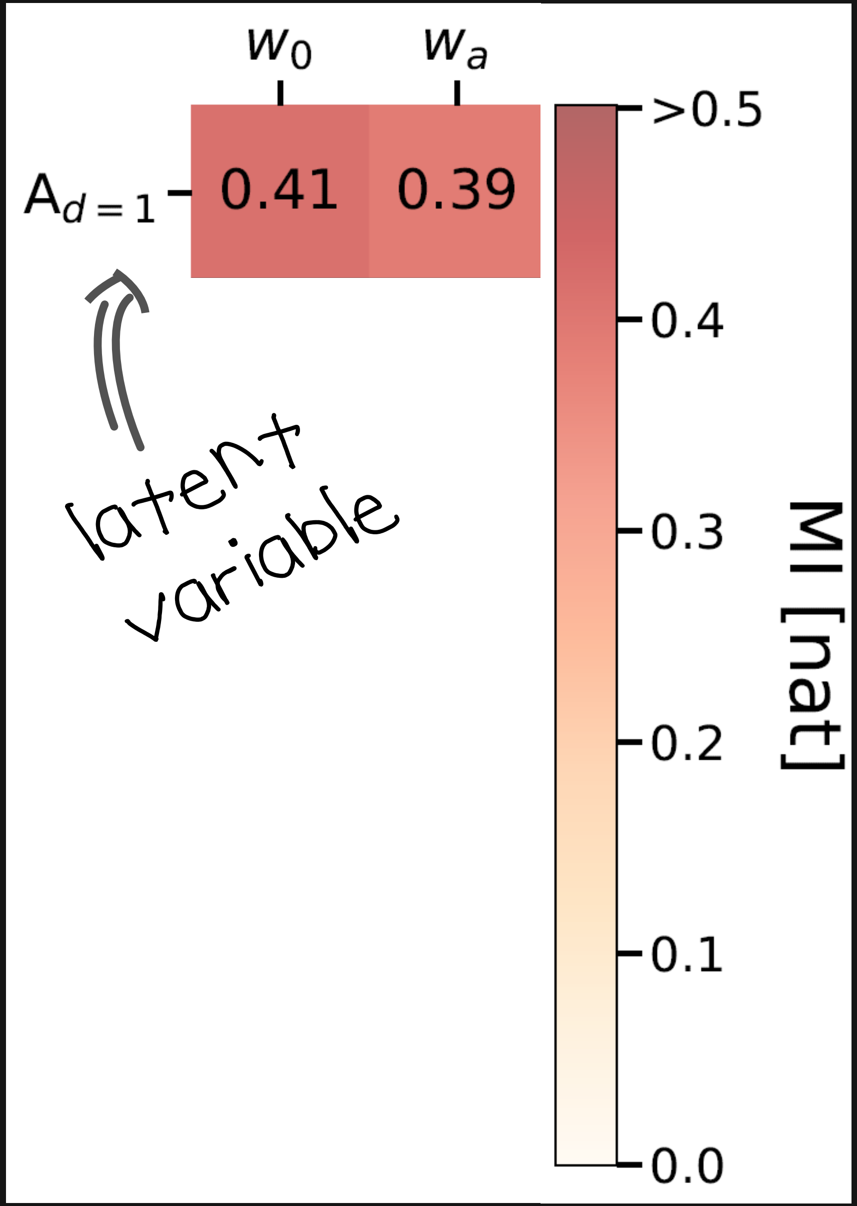
Latent variable has significant MI with wCDM parameters



Mutual information in latent space

Latent variable has significant MI with wCDM parameters

Little-to-no MI with other parameters (not shown)

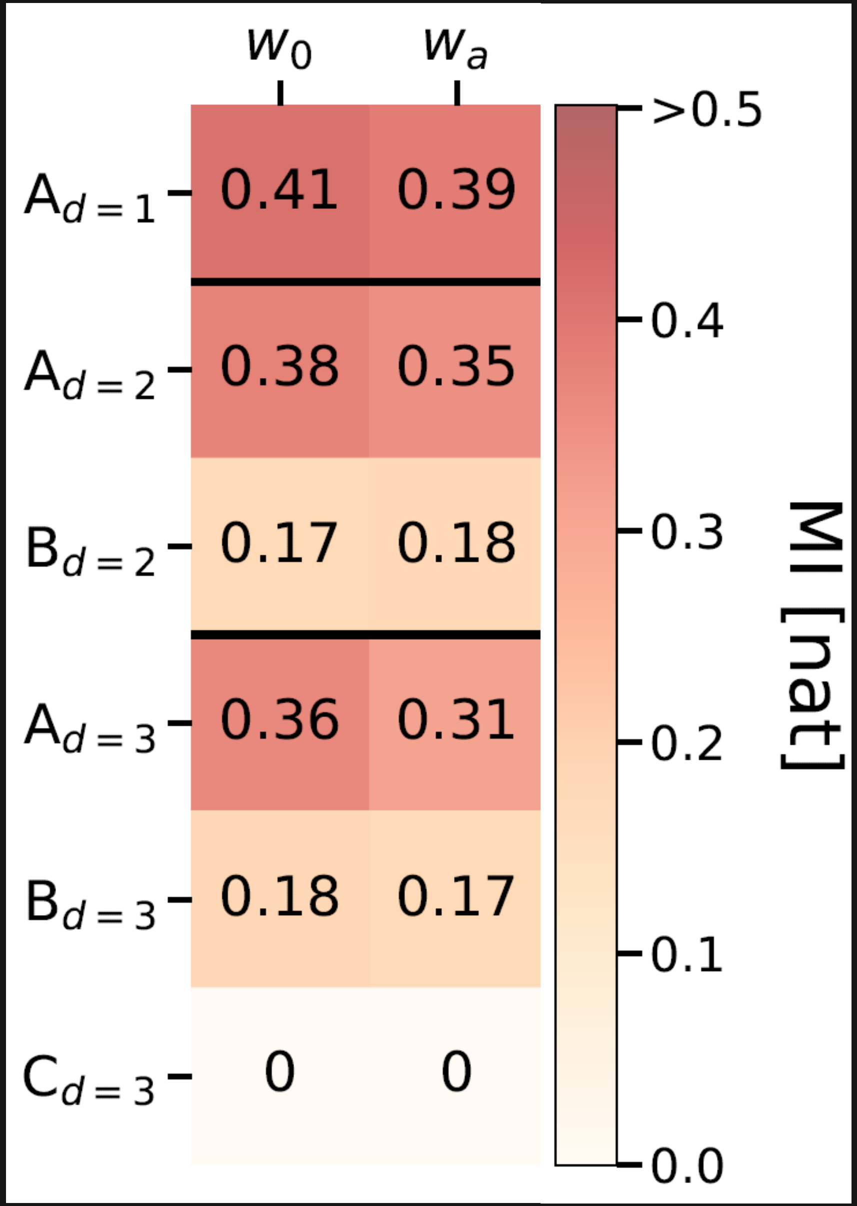


Mutual information in latent space

Latent variable has significant MI with wCDM parameters

Little-to-no MI with other parameters (not shown)

Third latent variable is unused



How to analyse the latent space?

- Mutual information
- Symbolic regression

What is symbolic regression?

Check out review on symbolic regression on Wednesday

Symbolic regression in latent space

- Link latent variable and wCDM parameters

Symbolic regression in latent space

- Link latent variable and wCDM parameters

$$\mathrm{A}_{d=1}(w_0, w_a) = w_0^2 + \frac{e^{w_a + \cos(w_0)}}{w_0} + e^{\cos(1)} - 1$$

latent variable

Symbolic regression in latent space

- Link latent variable and wCDM parameters

$$\mathrm{A}_{d=1}(w_0, w_a) = w_0^2 + \frac{e^{w_a + \cos(w_0)}}{w_0} + e^{\cos(1)} - 1$$

- Analogous to $\mathrm{S}_8 = \sigma_8(\Omega_\mathrm{m}/0.3)^{0.5}$ …?

Conclusions

- Only need one variable to describe wCDM matter power spectra

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- Can use mutual information and symbolic regression to interpret latent space

Conclusions

- Only need one variable to describe wCDM matter power spectra
- Can use mutual information and symbolic regression to interpret latent space
- Will apply our framework to multiple extensions and different summaries

Cheeky ad

Check out our poster
on accelerated Bayesian inference
with CosmoPower-JAX

Video

Code

Conclusions

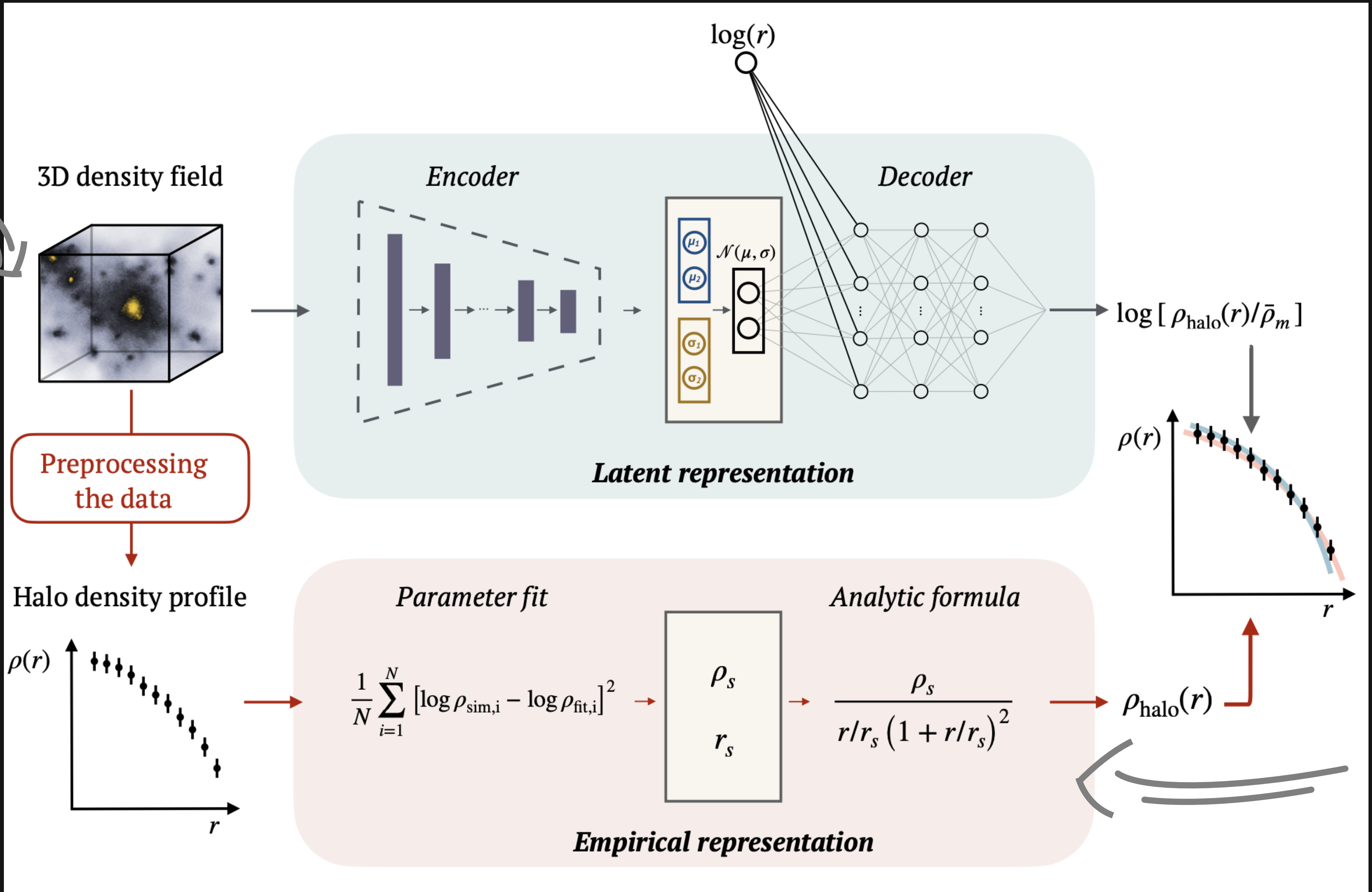
- Only need one variable to describe wCDM matter power spectra
- Can use mutual information and symbolic regression to interpret latent space
- Will apply our framework to multiple extensions and different summaries

Thank you

Extra slides
(and memes)

Representation learning in cosmological structure formation

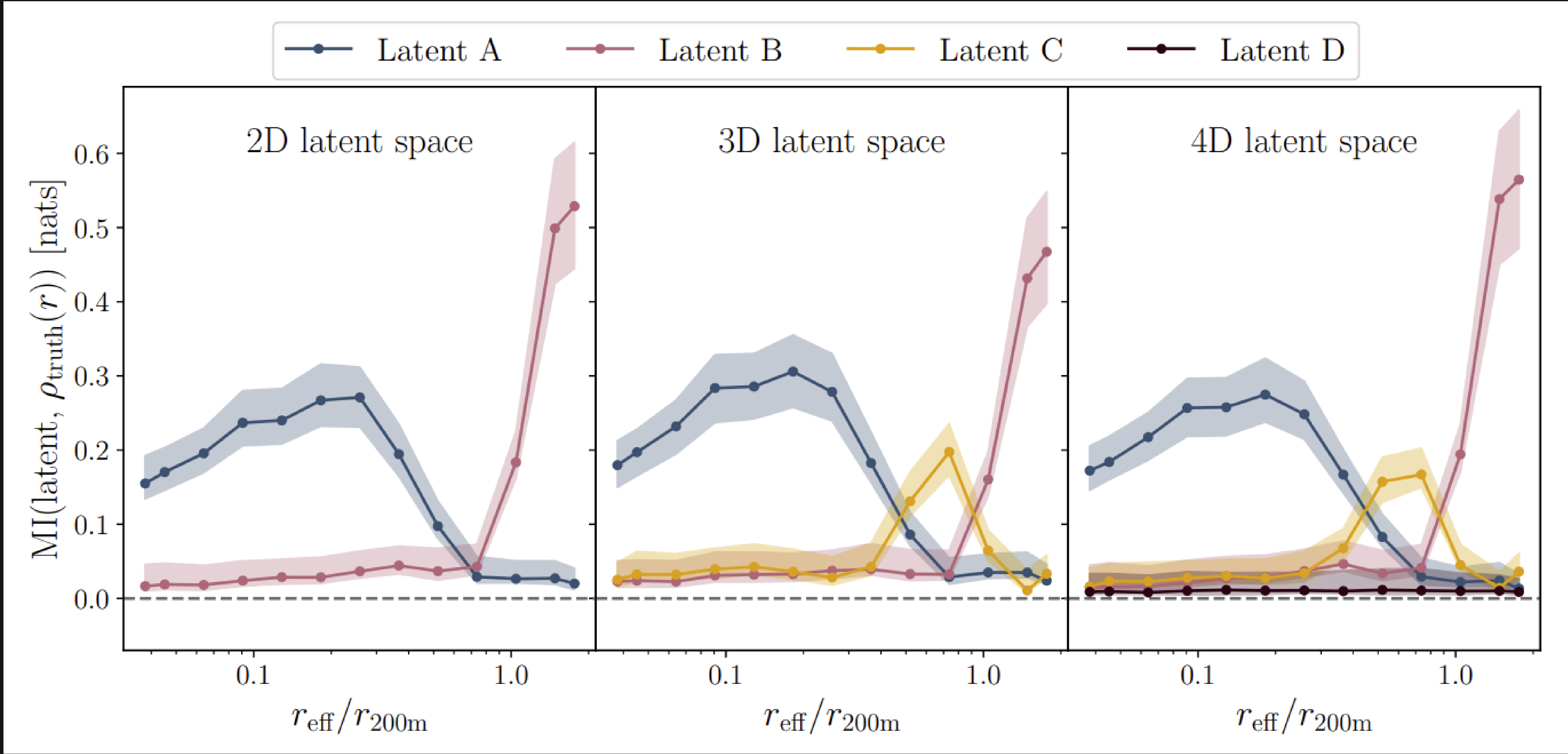
Single halo from N-body simulation



Navarro-Frenk-White profile

Lucie-Smith et al., **105**, 103533, PRD

Application to cosmological structure formation



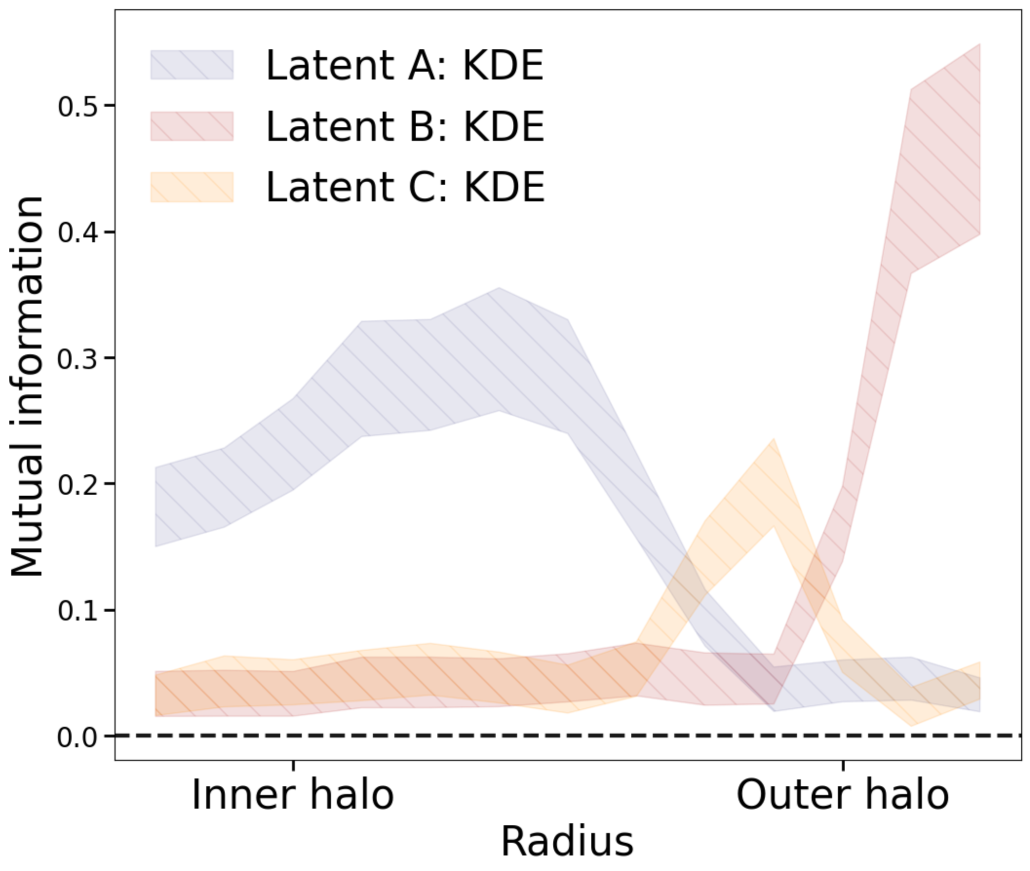
Lucie-Smith et al., **105**, 103533, PRD

Explore dependence of latent variables

Latent A
Latent B
Latent C

Mutual information

Density at each radius



KDE := kernel density estimation

An application to dark energy

- Expect two latent variables are needed…?

An application to dark energy

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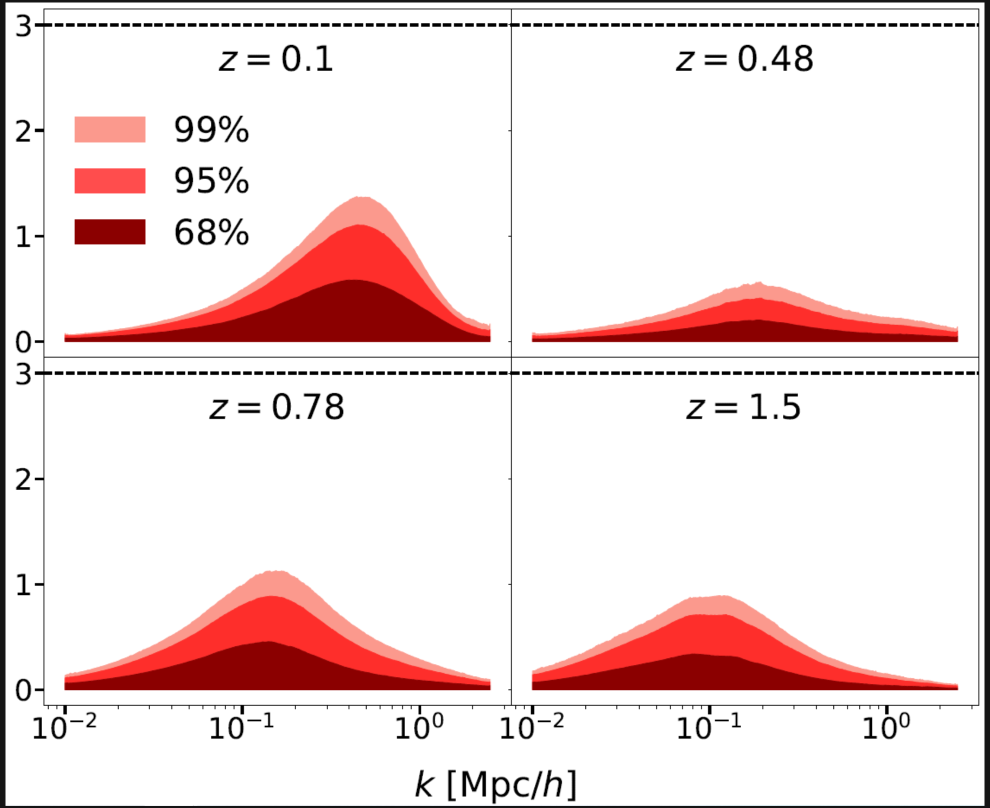
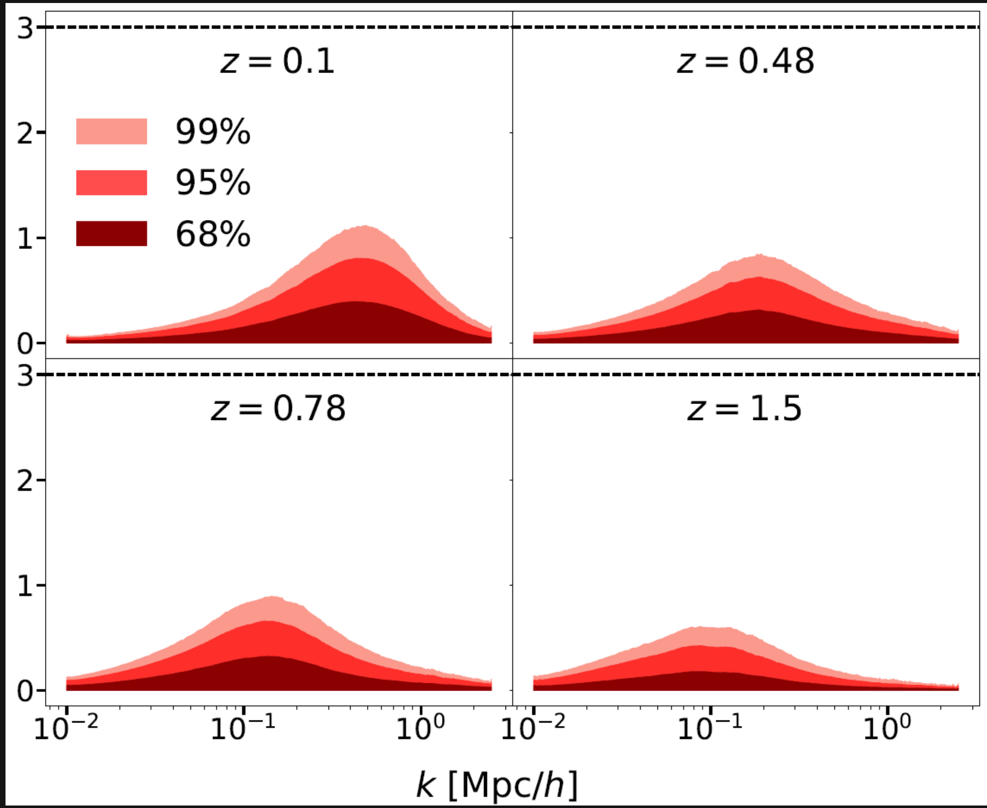
Results

One latent variable



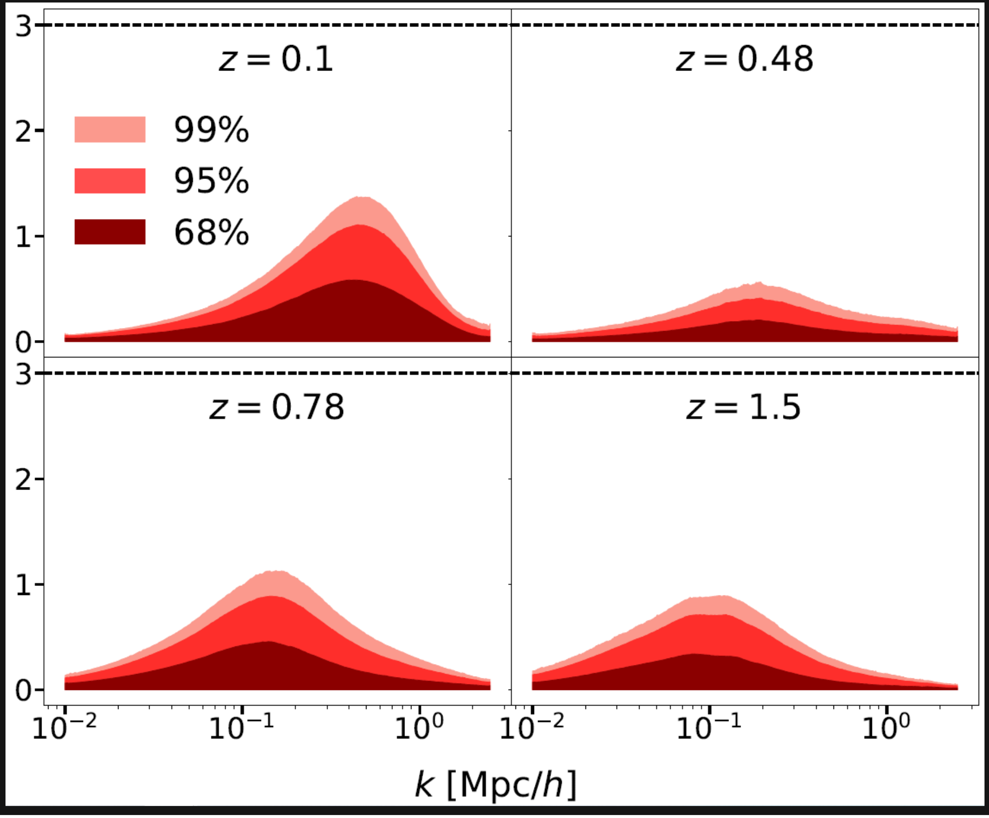
Two latent variables



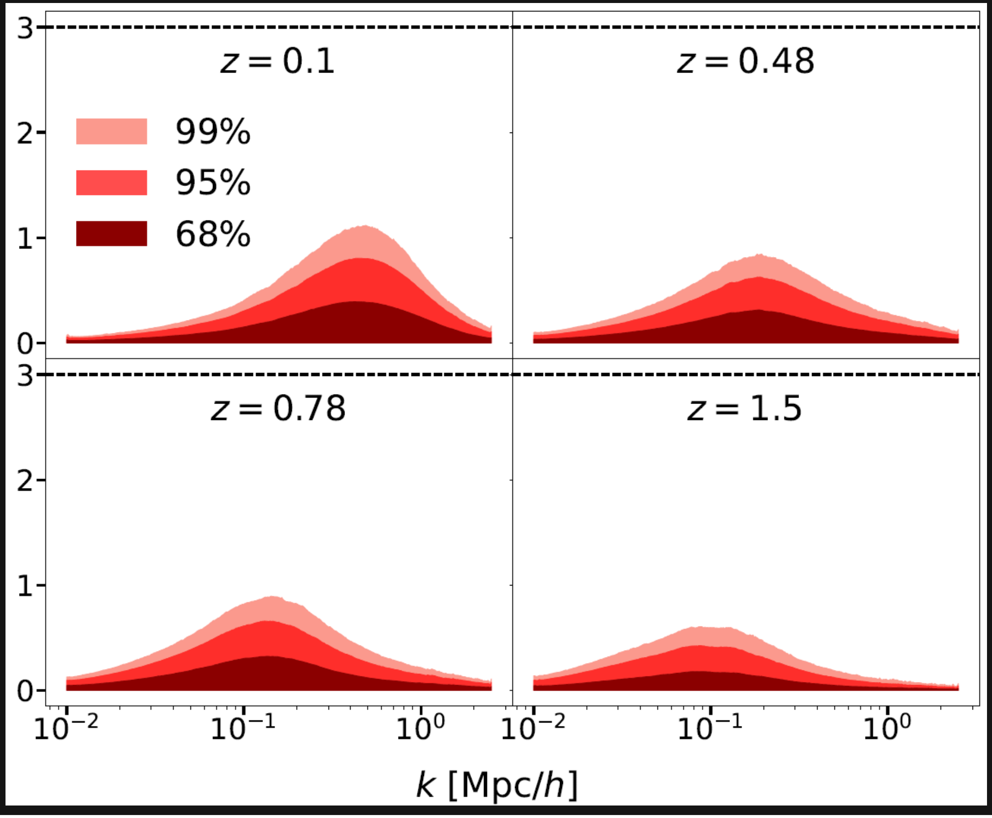
$$\sigma(k,z) = \sqrt{\frac{4\pi^2}{k^2 \Delta k V(z)} \left(P_{\delta\delta}(k,z) + \frac{1}{\bar{n}(z)} \right)^2 + \sigma_{\rm sys}^2}$$

Results

One latent variable



Two latent variables



arXiv > cs > arXiv:1506.02640

Computer Science > Computer Vision and Pattern Recognition

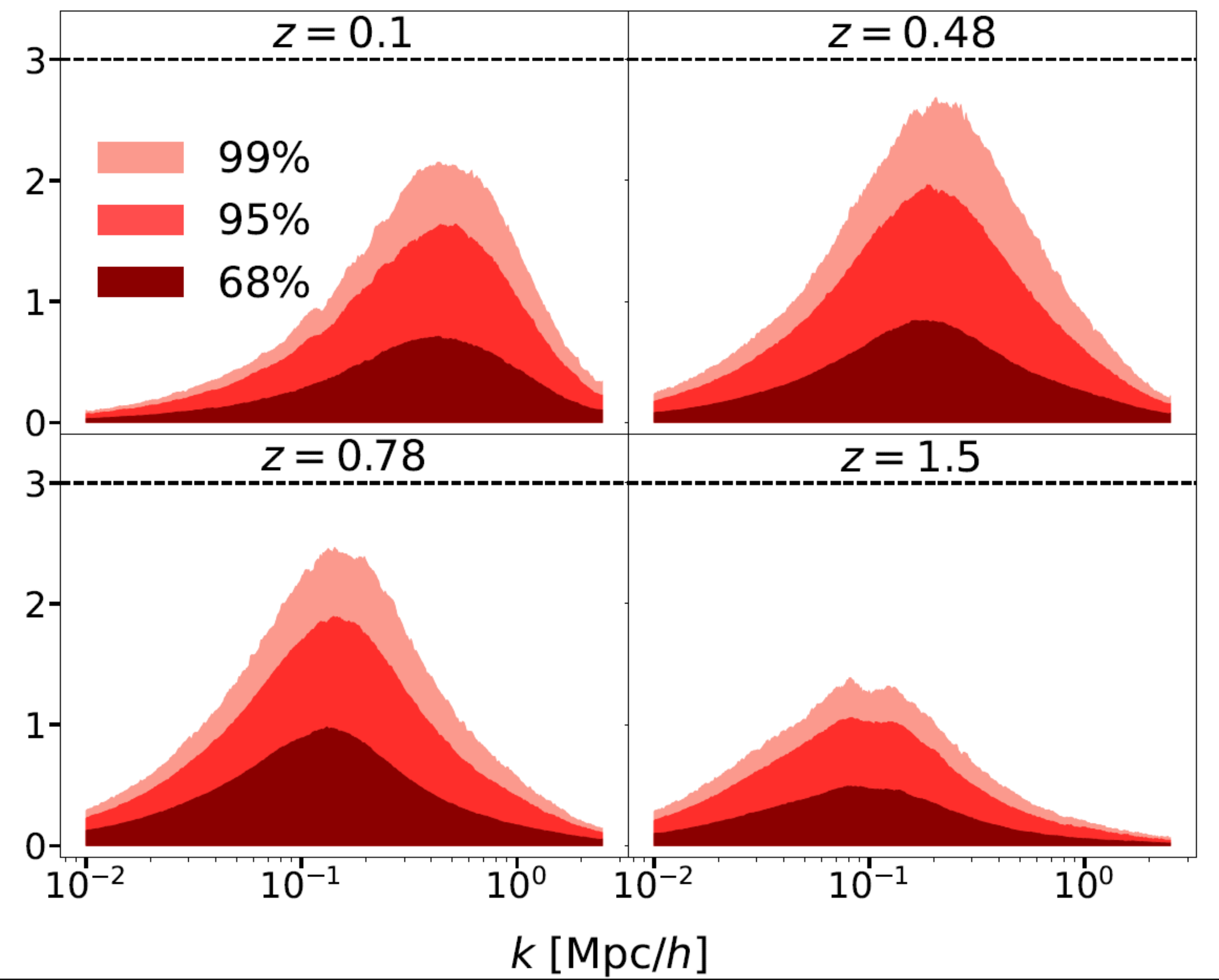
[Submitted on 8 Jun 2015 (v1), last revised 9 May 2016 (this version, v5)]

You Only Look Once: Unified, Real-Time Object Detection

Joseph Redmon, Santosh Divvala, Ross Girshick, Ali Farhadi

YONOV: You Only Need One Variable

Symbolic regression results



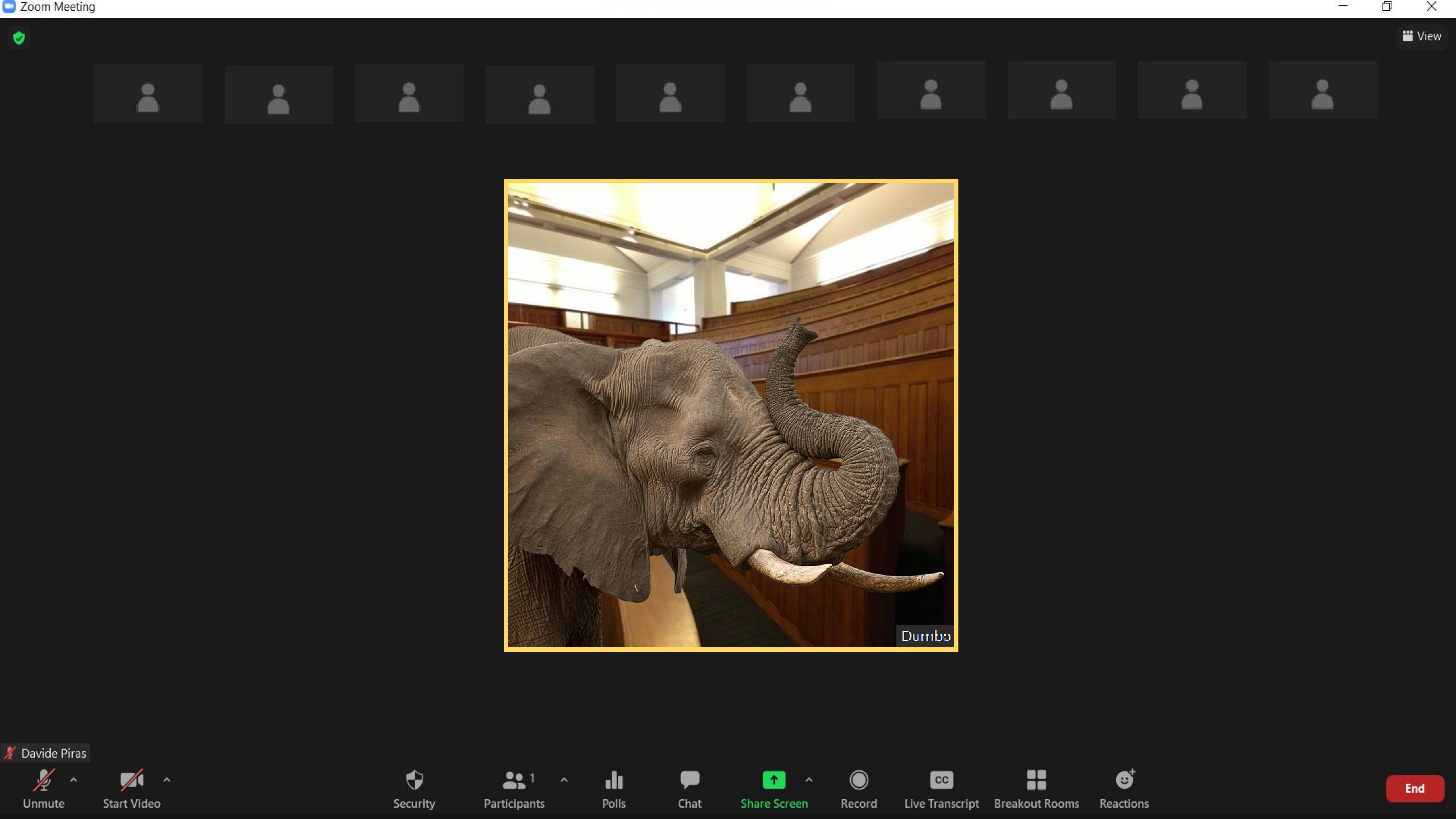
What is mutual information?

- Measures dependence between random variables (more general than Pearson, which measures correlation)

- Well-established in information theory

- Defined by : $$\boxed{\mathrm{MI}(X,Y) = \int p(x,y)\log\frac{p(x,y)}{p(x)p(y)}\mathrm{d}x\mathrm{d}y}$$

$\mathrm{MI}(X,Y) = 0$ if and only if X and Y are independent


Zoom Meeting
View
Dumbo
Davide Piras
Unmute
Start Video
Security
Participants 1
Polls
Chat
Share Screen
Record
CC
Live Transcript
Breakout Rooms
Reactions
End

GMM-MI: a robust estimator of mutual information

- Cross-validation and multiple initialisations to optimise fit

GMM-MI

GMM-MI: a robust estimator of mutual information

- Cross-validation and multiple initialisations to optimise fit

- Works with continuous and discrete variables

GMM-MI: a robust estimator of mutual information

- Cross-validation and multiple initialisations to optimise fit
- Works with continuous and discrete variables
- GMM-MI returns **uncertainty** on MI through bootstrapping

GMM-MI at work

```
(gmm_mi) davide@crash:~$
```

GMM-MI at work

```
(gmm_mi) davide@crash:~$ pip install gmm-mi
```

```
In [1]:
   ...:
```

GMM-MI at work

```
(gmm_mi) davide@crash:~$ pip install gmm-mi
```

```
In [1]: import numpy as np
   ...: from gmm_mi.mi import EstimateMI
   ...:

In [2]: 
   ...:
   ...:
   ...:
   ...:
   ...:
```

GMM-MI at work

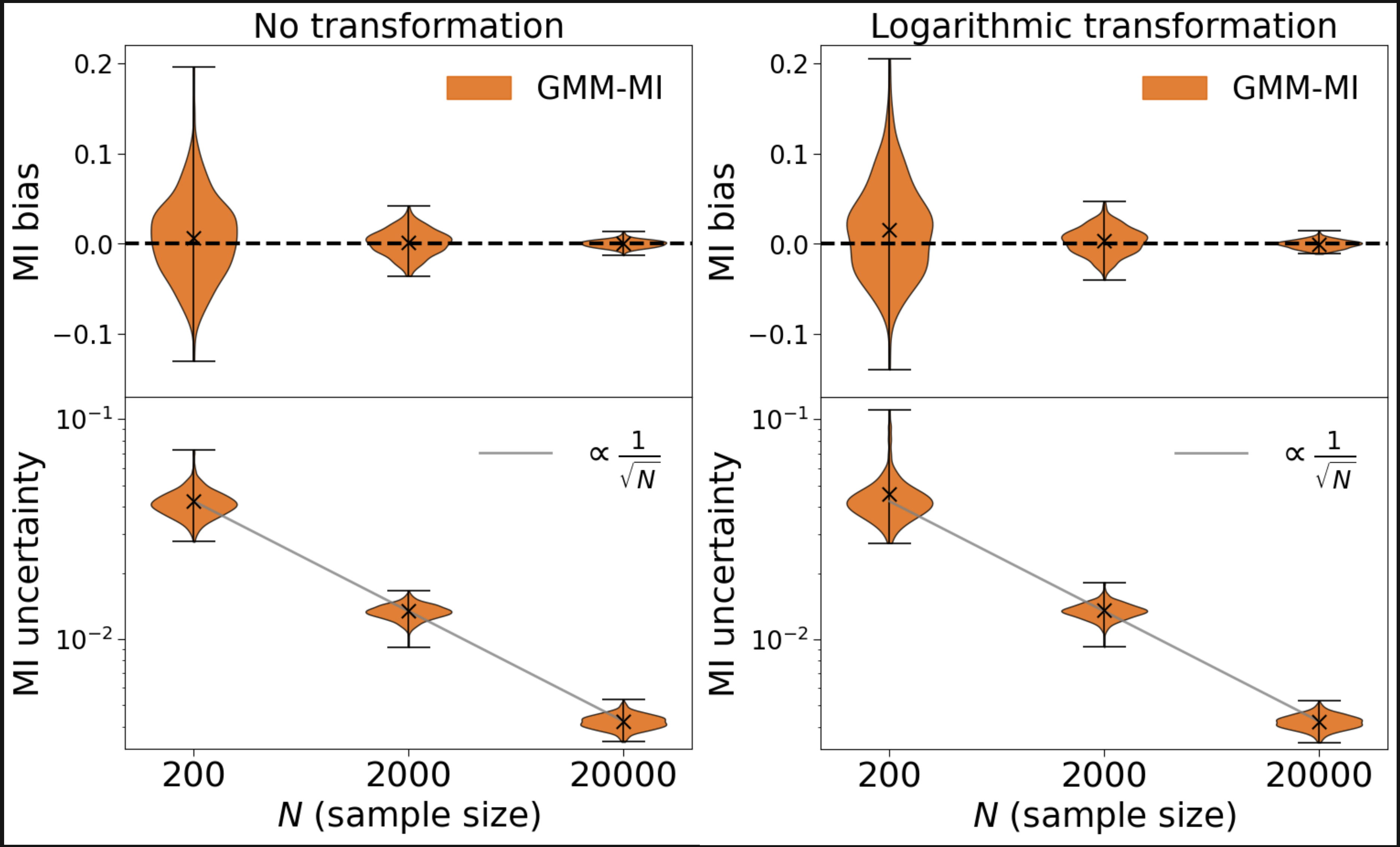
```
(gmm_mi) davide@crash:~$ pip install gmm-mi
```

```
In [1]: import numpy as np
   ...: from gmm_mi.mi import EstimateMI
   ...:

In [2]: # create bivariate Gaussian data
   ...: mean = np.array([0, 0])
   ...: cov = np.array([[1, 0.6], [0.6, 1]])
   ...: rng = np.random.default_rng(0)
   ...: X = rng.multivariate_normal(mean, cov, 200)
   ...:

In [3]:
   ...:
   ...:
```

GMM-MI validation

- GMM-MI is unbiased

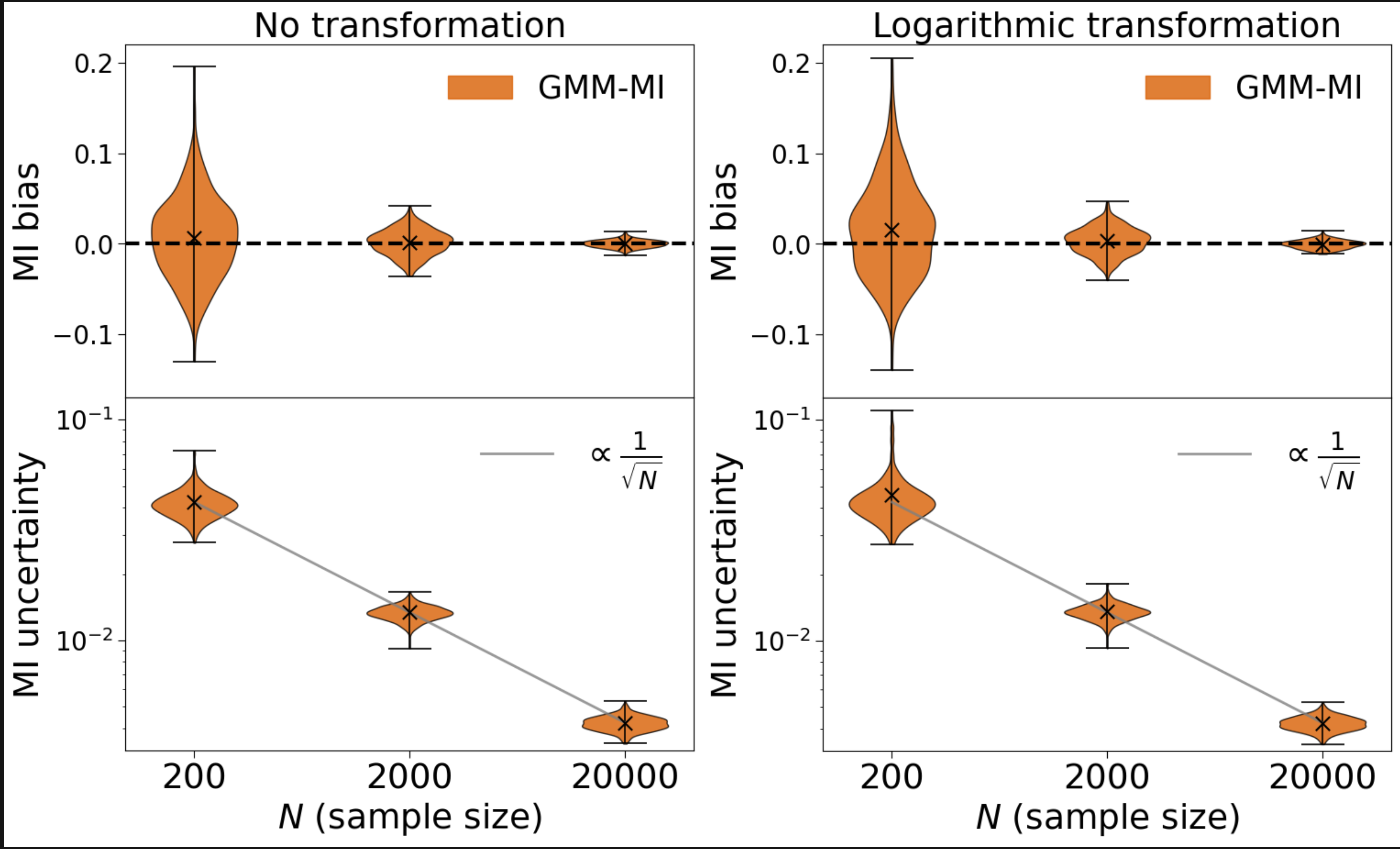
Piras et al., MLST



GMM-MI validation

- GMM-MI is unbiased
- GMM-MI respects MI invariance

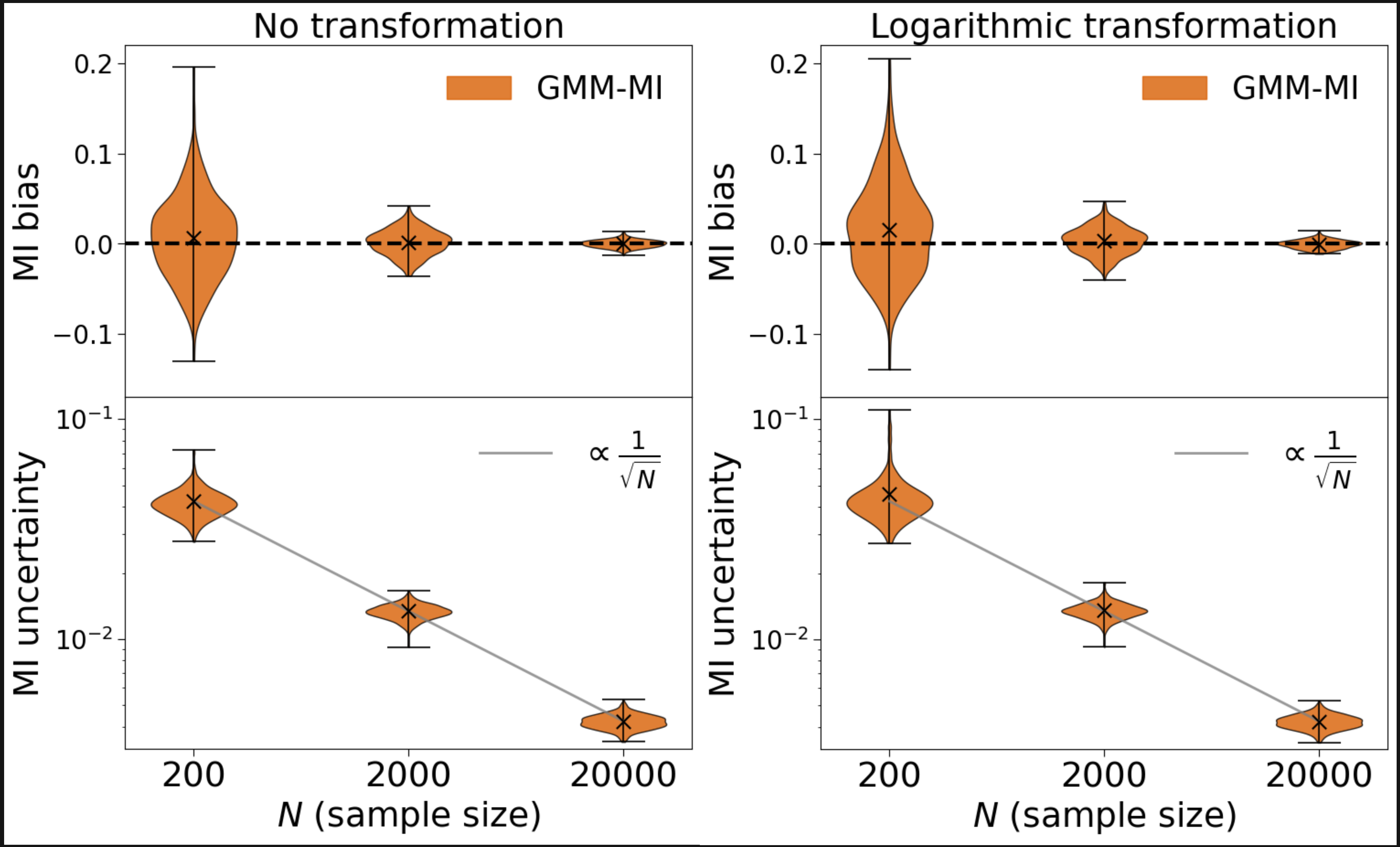
Piras et al., MLST



GMM-MI validation

- GMM-MI is unbiased
- GMM-MI respects MI invariance
- GMM-MI errors scale as expected

Piras et al., MLST



What is symbolic regression?

- Finds analytic equation linking variables

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- Less accurate, but more interpretable (?)

What is symbolic regression?

- Finds analytic equation linking variables
- Less accurate, but more interpretable (?)
- Many implementations available

Material

Representation learning