



(Machine)-learning (astro)-physical laws Wassim Tenachi

With Rodrigo Ibata & Foivos Diakogiannis



Nov 27 2023

ML IAP/CCA 2023 – Debating the Potential of Machine Learning in Astronomical Surveys





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Φ -SO : Motivations

Motivations

Empirical law





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Empirical law





Took him 4 years !

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Empirical law

General theory

 $mrac{doldsymbol{v}}{dt}=\mathbf{F}$





Took him 4 years !

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Empirical law

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 $mrac{doldsymbol{v}}{dt}=\mathbf{F}$





Took him 4 years !







Empirical law

General theory



Empirical law

General theory



Symbolic regression (SR)

(x , y	') (lata	;
x_1	<i>x</i> ₂	У	$f \cdot \mathbb{D}^n \setminus \mathbb{D}$ such that
0.75582	0.25850	0.02674	\rightarrow K Such that
0.36786	0.42401	0.06278	
0.69507	0.38057	0.74014	$((\cdot))$
0.96493	0.33398	0.81558	$V \equiv f(\mathbf{X})$
0.07139	0.16604	0.07735	y = f(x)
0.86413	0.41952	0.87872	
0.18012	0.40581	0.63637	·'

Symbolic regression (SR)

Φ -SO – SR



The virtues of obtaining symbolic models (1)

1. Interpretability



 \rightarrow Connecting with theory

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The virtues of obtaining symbolic models (1)

1. Interpretability



 $\boldsymbol{\rightarrow}$ Connecting with theory

2. Compactness

 $f(\boldsymbol{x}) = \cdots$

 \rightarrow Intellegible



The virtues of obtaining symbolic models (1)

1. Interpretability

3. Generalization



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Language processing for symbolic mathematics

(tree) (cat)	car •	
• •	•	

Language processing for symbolic mathematics



 Φ -SO : Embedding

Embedding (1): how to go from numbers to symbols?



Embedding (1): how to go from numbers to symbols ?



Embedding (1): how to go from numbers to symbols ?



Embedding (2): how to go from vector of symbols to expressions?



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Embedding (2): how to go from vector of symbols to expressions?



* 1:1 equivalence

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 Φ -SO : Embedding

Embedding (2): how to go from vector of symbols to expressions?



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Embedding (2): how to go from vector of symbols to expressions ?





* 1:1 equivalence

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Embedding (3): Encoding a whole expression



Embedding (4): Arity



Embedding (4): Arity



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Dimensional analysis constrains the search space



Dimensional analysis constrains the search space



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Dimensional analysis constrains the search space



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Search space reduction using physical units constraints

268 expressions



Search space

Prefix notation paths for expressing a velocity v using a library of symbols $\{+, /, \cos, v_0, x, t\}$ where v_0 is a velocity, x is a length, and t is a time (length < 5 for readability).

[Tenachi et al 2023] (this work)

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 Φ -SO : units constraints

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Search space reduction using physical units constraints

268 expressions 6 expressions der $\mathcal{F}^{\mathcal{O}}^{\mathcal$

Search space

9Å,



Prefix notation paths for expressing a velocity v using a library of symbols $\{+, /, \cos, v_0, x, t\}$ where v_0 is a velocity, x is a length, and t is a time (length < 6 for readability).

[Tenachi et al 2023] (this work)

In situ prior



In situ prior






Φ -SO : units constraints





Propagating units constraints (1)



 $v = \square$

https://youtu.be/clZsLj3oPy8

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[Tenachi et al 2023] (this work)

 Φ -SO : units constraints

Propagating units constraints (2)



$\rho(r) = \Box$

 $\underline{https://youtu.be/U \ hnFJuZ1dA}$

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[Tenachi et al 2023] (this work)





 Φ -SO : generation

 $v = \blacksquare$

Library of choosable tokens



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 Φ -SO : generation

 $v = \blacksquare$

Library of choosable tokens



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∎ + ■

Library of choosable tokens





 $v = \blacksquare + \blacksquare$ Expression tree + 1

 Φ -SO : generation

Library of choosable tokens



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 Φ -SO : generation

Library of choosable tokens



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 Φ -SO : generation

Library of choosable tokens



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"T⁻¹

L.T⁻¹

L.T⁻¹





 Φ -SO : generation

Library of choosable tokens



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Reinforcement learning in a nutshell (1)



https://youtu.be/spfpBrBjntg

Reinforcement learning in a nutshell (2)









 Φ -SO : RL



Categorical distribution	$\begin{array}{c} \text{utions} \\ \text{Candidates vs data} \\ \rightarrow \text{reward} \end{array}$
$v = v_0 + x/t$	\rightarrow R = 0.99
$v = v_0$	ightarrow R = 0.93
$\Phi(r) = \mathcal{V} = \frac{4\pi G\rho_0 R_s^3}{= \mathcal{V}_0} \left\{ \ln \left(\frac{r}{k_s} \right)_{\alpha=1}^{+} \arctan \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \right\}_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[1 + \frac{r}{R_s} \right]_{\alpha=1}^{+} \left[\operatorname{arctan} \left(\frac{r}{R_s} \right)_{\alpha=1}^{+} \left[\operatorname{arctan} \left$	$\rightarrow R = 0.73$ $\rightarrow R = 0.28$
$v = \frac{v_0^2}{x/t}$	ightarrow R = 0.64



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 Φ -SO : RL

constraints we want even if it is

not differentiable



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- \rightarrow We can apply any physical
 - We can apply any physical constraints we want even if it is not differentiable
- \rightarrow Complexity (Occam's razor)
- \rightarrow Symmetries
- \rightarrow Constraints on derivatives/primitives
- \rightarrow Symbolic computing using Sympy/Mathematica
- \rightarrow Fitness in ODEs, limits values
- \rightarrow Behavior in N-body simulation

 Φ -SO : RL





Wassim Tenachi, Physical Symbolic Optimization

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Learning curves





Learning to produce **physical** expressions



120 equations from the Feynman Lectures on Physics (+ other physics textbooks)

Feynma	an Equation			1	
eq.	-				
II.2.42	$P = \frac{\kappa(T_2 - T_1)}{r_1}$	A			
II.3.24	$F_E = \frac{P^a}{1-2}$	-			
II.4.23	$V_e = \frac{4\pi r^2}{q}$	Feynman	Equation		
II.6.11	$V_e = \frac{4\pi er}{1} \frac{p_d c}{c}$	eq.	e = 0 ² /2 / /		
II.6.15a	$E_{\ell} = \frac{4\pi\epsilon}{3} \frac{r}{p_d}$	1.6.20a	$f = e^{-o^{-1/2}}/\sqrt{1-1}$	2π	
II 6 15b	$E_{\ell} = \frac{4\pi\epsilon}{3} \frac{r^5}{p_d}$	I.6.20	$f = e^{-\frac{1}{2\sigma^2}} / \sqrt{2}$	$2\pi\sigma^2$	
11.9.7	$F = 3 q^2$	1.6.20b	$f = e^{-\frac{(\theta-\theta_1)^2}{2\epsilon^2}}$	1/2772	
11.0.7	$L = \frac{1}{5} \frac{1}{4\pi\epsilon d} \frac{1}{\epsilon E^2}$	1.0.200	$d = \sqrt{(r_0 - r_0)}$	$(\sqrt{2\pi})^2 \pm (m_0 - m_1)^2$	
II.8.31	$E_{den} = \frac{dE_f}{2}$	1.9.18	$F = \frac{1}{\sqrt{x_2 - x_1}}$	Gm_1m_2 Gm_1m_2	
II.10.9	$E_f = \frac{\sigma_{den}}{\epsilon_n} \frac{1}{1}$	I 10 7	$m = \frac{(x_2 - x_1)^2}{m_0}$	$+(y_2-y_1)^2+(z_2-z_1)^2$	
II.11.3	$x = \frac{qE_f}{m(\omega^2 - \omega)}$	1.10.1	$\sqrt{1-\frac{v^2}{2}}$		
II 11 17	$n = n_0(1 + 1)$	I.11.19	$A = x_1 y_1 + x_2$	$x_{2}y_{2} + x_{3}y_{3}$	
	$n_0 p_1^2 E_1$	I.12.1	$F = \mu N_n$		
11.11.20	$P_* = \frac{1}{3k_bT}$	I.12.2	$F = \frac{q_1 q_2}{4\pi \epsilon r_1^2}$		
11.11.27	$P_* = \frac{n\alpha}{1 - n\alpha/3}$	1.12.4	$E_f = \frac{q_1}{4\pi\epsilon r^2}$		
II.11.28	$\theta = 1 + \frac{r}{1-(r)}$	1.12.5 1.19.11	$F = q_2 E_f$ $F = q(F_1 + P_1)$	augin (I)	
II.13.17	$B = \frac{1}{4\pi\epsilon c_s^2} \frac{2I}{r}$	1.13.4	$K = \frac{1}{2}m(v^2 + b)$	$u^2 + w^2$	
II.13.23	$\rho_c = \frac{\rho_{c_0}}{\sqrt{1-w^2}}$	I.13.12	$U = \hat{G}m_1m_2($	$\frac{1}{1} - \frac{1}{1}$	
II.13.34	$i = \frac{V_{\rho_{c_0}v}^{1-v}}{v_{\rho_{c_0}v}}$	I.14.3	U = mqz	r ₂ r ₁ '	
17.15.4	$\int \sqrt{1-v^2/c}$	L14.4	$U = \frac{k_{spring}x^2}{2}$		
11.15.4 11.15 E	$E = -\mu_M B$ $E = -\mu_M B$	I.15.3x	$x_1 = \frac{x^2 - ut}{x^2 - ut}$	-	
II.15.5 II.21.32	$E = -p_d E_f$		$\sqrt{1-u^2/c}$,2	
11.21.02	$v_e = \frac{4\pi \epsilon r (1 - 4\pi \epsilon r)}{4\pi \epsilon r (1 - 4\pi \epsilon r)}$	1.15.3t	$t_1 = \frac{t - u^2/c^2}{\sqrt{1 - u^2/c^2}}$	2	
II.24.17	$k = \sqrt{\frac{\omega^{2}}{c^{2}}} -$	I.15.10	$p = \frac{m_0 v}{\sqrt{1 - v^2/4}}$		
II.27.16	$F_E = \epsilon c E_f^2$	I.16.6	$v_1 = \frac{u+v'}{1+uv/c} S$	ource	Equation
II.27.18	$E_{den} = \epsilon E_f^2$	I.18.4	$r = \frac{m_1 r_1 + m_2}{m_1 + m_2}$		
II.34.2a	$I = \frac{qv}{2\pi r}$	I.18.12	$\tau = rF\sin\theta$ R	tutherford Scattering	$A = \left(\frac{Z_1 Z_2 \alpha \hbar c}{4E \sin^2(\frac{\theta}{2})}\right)^*$
11.34.2	$\mu_M = \frac{q_{er}}{2}$	I.18.16	$L = mrv \sin F$	riedman Equation	$H = \sqrt{\frac{8\pi G}{3}\rho - \frac{k_f c^2}{a_s^2}}$
11.34.11	$\omega = \frac{q_2 q_D}{2m}$	I.24.6	$E = \frac{1}{4}m(\omega^2)$	compton Scattering	$U = \frac{V}{E}$
11.34.29	$\mu_M = \frac{q_{\mu}}{4\pi m}$	1.25.13	$V_e = \frac{1}{C}$ $\theta_1 = \arcsin(1 R)$	adiated gravitational wave now	$P = -\frac{32}{mc^2} \frac{G^4}{(m_1m_2)^2(m_1+m_2)}$
11.34.29	$E = \frac{1}{h}$	1.27.6	$f_f = \frac{1}{1 + \pi}$	calativistic abovestion	$\begin{array}{c} 1 & -\frac{1}{5} \frac{1}{c^5} \frac{\tau^5}{\tau^5} \end{array}$
11.55.18	$n = \frac{1}{\exp(\mu_m E)}$	1.20.4	$L = \omega^{\frac{1}{d_1} + \frac{1}{d_2}}$	Left 100 abertation	$\begin{bmatrix} \sigma_1 & = \arccos\left(\frac{1-\pi}{c}\cos\theta_2\right) \\ r & = \left[\sin(\alpha/2)\sin(N\delta/2)\right]^2 \end{bmatrix}$
11.35.21	$M = n_{\rho} \mu_M$	1.29.4	$\kappa = \frac{1}{c}$	-suit dimraction	$I \equiv I_0 \begin{bmatrix} \frac{1}{\alpha/2} & \frac{1}{\sin(\delta/2)} \end{bmatrix}$
11.36.38	$f = \frac{\mu m D}{k_b T} +$	1.29.16	$x = \sqrt{x_1^2 + x_2^2}$	foldstein 3.16	$v = \sqrt{\frac{1}{m}(E - U - \frac{1}{2mr^2})}$
II.37.1	$E = \mu_M (1 + V_A T)$	I.30.3	$I_* = I_{*0} \frac{\sin^2 G}{\sin^2}$	ioldstein 3.55	$k = \frac{1}{L^2} (1 + \sqrt{1 + \frac{mk_G^2}{mk_G^2}} \cos(\theta_1 - \theta_2))$
11.38.3	$F = \frac{1}{d} \frac{d}{Y}$	I.30.5	$\theta = \arcsin(\frac{1}{n})^{G}$	Goldstein 3.64 (ellipse)	$r = \frac{1}{1 + \alpha \cos(\theta_1 - \theta_2)}$
11.58.14	$\mu_S = \frac{1}{\frac{2(1+\sigma)}{1}}$	I.32.5	$P = \frac{q^2 a^2}{6\pi \epsilon c^3}$	Goldstein 3.74 (Kepler)	$t = \frac{2\pi a}{\sqrt{G(m_1 + m_2)}}$
111.4.32	$n = \frac{\hbar \omega}{e^{\frac{\hbar \omega}{k_h T}} - 1}$	I.32.17	$P = \left(\frac{1}{2}\epsilon c E_f^2\right) G$	oldstein 3.99	$\alpha = \sqrt{1 + \frac{2\epsilon^2 E L^2}{m(Z_1 Z_2 q^2)^2}}$
III.4.33	$E = \frac{\hbar \omega}{\hbar \omega}$	I.34.8	$\omega = \frac{qvB}{p}$ G	oldstein 8.56	$E = \sqrt{(p - qA_{vec})^2 c^2 + m^2 c^4} + qV_e$
111 7 20	$e^{\overline{k}_b T} - 1$ $2\mu_M B$	I.34.10	$\omega = \frac{\omega_0}{1 - v/c}$	oldstein 12.80	$E = \frac{1}{2m} [p^2 + m^2 \omega^2 x^2 (1 + \alpha \frac{\pi}{y})]$
111.7.38	$\omega = \frac{\mu m}{h}$	I.34.14	$\omega = \frac{1+v/c}{\sqrt{1-v^2}}$ J	ackson 2.11	$F = \frac{q}{4\pi\epsilon y^2} \left[4\pi\epsilon V_e d - \frac{qdy^3}{(y^2 - d^2)^2} \right]$
111.8.54	$p_{\gamma} = \sin(\frac{\pi}{\hbar})$	I.34.27	$E = \hbar \omega$ J	ackson 3.45	$V_e = \frac{q}{(r^2 + d^2 - 2dr \cos \alpha)^{\frac{1}{2}}}$
111.9.52	$p_{\gamma} = \frac{\mu - \gamma}{\hbar}$	I.37.4	$I_* = I_1 + I_2$	ackson 4.60	$V_e = E_f \cos \theta \left(\frac{\alpha - 1}{\alpha + 2} \frac{d^3}{r^2} - r \right)$
III.10.1	$9 E = \mu_M \sqrt{E}$	I.38.12	$r = \frac{4\pi\epsilon\hbar^2}{mq^2}$	1 11 00 (5 1)	$\sqrt{1-\frac{w^2}{c^2}}$
111.12.4	$3 L = n\hbar$	I.39.10	$E = \frac{3}{2}p_F V$	ackson 11.38 (Doppler)	$\omega_0 = \frac{1}{1 + \frac{\pi}{c} \cos \theta} \omega$
III.13.1	$8 v = \frac{2Ea^{-\kappa}}{\hbar}a^{V}$	I.39.11	$E = \frac{1}{\gamma - 1} p_F$	Veinberg 15.2.1	$\rho = \frac{3}{8\pi G} \left(\frac{c \kappa_f}{a_f^2} + H^2 \right)$
III.14.1	$4 I = I_0(e^{\frac{q+e}{k_bT}})$	I.39.22	$P_F = \frac{n\kappa_b T}{V_m}$	Veinberg 15.2.2	$p_f = -\frac{1}{8\pi G} \left[\frac{c^4 k_f}{a_f^2} + c^2 H^2 (1 - 2\alpha) \right]$
III.15.1	2 E = 2U(1 -	I.40.1	$n = n_0 e^{-\frac{m_s}{k_b}}$	chwarz 13.132 (Klein-Nishina)	$A = \frac{\pi \alpha^2 \hbar^2}{\pi \alpha^2} (\frac{\omega_0}{\omega_0})^2 \left[\frac{\omega_0}{\omega_0} + \frac{\omega}{\omega_0} - \sin^2 \theta \right]$
III.15.1	$4 m = \frac{\hbar^2}{2Ed^2}$	I.41.16	$L_{rad} =$	hω	m=c= (\overline v) \overline v \overline \overline v_0
III.15.2	$7 \mid k = \frac{2\pi \alpha}{nd}$		$\pi^2 c^2 (e$	$k_{b}T = 1$	
III.17.3	$7 \mid f = \beta(1 + \alpha)$	1.43.16 1.42.21	$v = \frac{r_{arift}q_{e}}{h}$		-
VI.19.5	$1 = \frac{-mq^4}{2(4\pi\epsilon)^{2h}}$	1.43.31	$D = \mu_e \kappa_b T$		Intr
11.2	$0 i = \frac{-\rho_{c_0} q A_v}{-\rho_{c_0} q A_v}$	1.43.43	$F = nh_{1}T\ln(3)$	2)	
	m	1.44.4	$L = n\kappa_b I \ln(\frac{1}{4})$	/1 /	For
		1.47.23	$c = \sqrt{\frac{p}{\rho}}$		
2023	}	I.48.20	$E = \frac{mc^2}{\sqrt{1-v^2/c^2}}$	2	
		I.50.26	$x = x_1 [\cos(\omega t)]$	$+ \alpha \cos(\omega t)^2$]	

Against 17 other algorithms

Method	Technique(s)	Description	Reference
PhyS0	RL, DA	Physical Symbolic Optimization	This work
uDSR	RL, GP, Simp., Sup.	A Unified Framework for Deep Symbolic Regression	Landajuela et al. (2022)
AIFeynman 2.0	Simp., DA	Symbolic regression exploiting graph modularity	Udrescu et al. (2020)
AFP_FE	GP	AFP with co-evolved fitness estimates, Eureqa-esque	Schmidt & Lipson (2009)
DSR	RL	Deep Symbolic Regression	Petersen et al. (2021a)
AFP	GP	Age-fitness Pareto Optimization	Schmidt & Lipson (2011)
gplearn	GP	Koza-style symbolic regression in Python	Stephens (2015)
GP-GOMEA	GP	GP-Optimal Mixing Evolutionary Algorithm	Virgolin et al. (2021)
ITEA	GP	Interaction-Transformation EA	de Franca & Aldeia (2021)
EPLEX	GP	ϵ -lexicase selection	La Cava et al. (2019)
NeSymReS	Sup.	Neural Symbolic Regression that Scales	Biggio et al. (2021)
Operon	GP	SR with Non-linear least squares	Kommenda et al. (2020)
SINDy	NeuroSym	Sparse identification of non-linear dynamics	Brunton et al. (2016)
SBP-GP	GP	Semantic Back-propagation Genetic Programming	Virgolin et al. (2019)
BSR	MCMC	Bayesian Symbolic Regression	Jin et al. (2019)
FEAT	GP	Feature Engineering Automation Tool	Cava et al. (2019)
FFX	Rand.	Fast function extraction	McConaghy (2011)
MRGP	GP	Multiple Regression Genetic Programming	Arnaldo et al. (2014)

Table 3. Summary of baseline symbolic regression methods along with the the underlying techniques they rely on: reinforcement learning (RL), genetic programming (GP), problem simplification schemes (Simp.), end-to-end supervised learning (Sup.), dimensional analysis (DA), neuro-symbolic / auto-differentiation based sparse fitting techniques (NeuroSym), Markov chain Monte Carlo (MCMC) and random search (Rand.).

Introduced in <u>[Udrescu & Tegmark 2020]</u> Formalized benchmark in SRBench <u>[LaCava et al 2021]</u>





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 $\Phi\text{-}\mathrm{SO}:\mathrm{showcases}$

Discovering new physics with SR



Conclusion

 Φ -SO is a RL framework that:

- 1. Can be used to resolve any symbolic optimization task
- 2. While taking full advantage of physical units constraints



Conclusion

- Φ -SO is a RL framework that:
- 1. Can be used to resolve any symbolic optimization task
- 2. While taking full advantage of physical units constraints



Big data from surveys & simulations



Perspectives

<u>Curse of accuracy guided SR :</u> "One can improve fit quality of candidates over learning iterations while getting further away from the correct solution in symbolic arrangement."



 \rightarrow Leverage data in a more meaningful way

Perspectives

"Book smart" or "street smart"?

<u>Trial and error</u>: "I try to resolve this specific problem by trial and error without benefiting from past experience."

<u>Supervised learning:</u> "I learn a lot beforehand and if what I learned does not help me guess the answer for this specific problem, then too bad there is nothing I can do."

Perspectives

"Book smart" or "street smart"?

<u>Trial and error</u>: "I try to resolve this specific problem by trial and error without benefiting from past experience."

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Physical Symbolic Optimization

An open source ...

ſ	7
F	∕

Github repository: WassimTenachi/PhySO

```
PhySO
                                                   Public
Physical Symbolic Optimization
● Python ☆ 1.5k 😵 205
```

```
Creating a physical free-form symbolic analytical function
f: \mathbb{R}^n \to \mathbb{R} fitting y = f(X) given (X, y) data:
```

```
expression, logs = physo.SR(X, y,
                           X_{units} = [ [1, 0, 0] , [1, -1, 0] ],
                           y_{units} = [2, -2, 1],
                           fixed consts
                                              = [ 1.
                                                       ],
                           fixed_consts_units = [ [0,0,0] ],
                           free_consts_units = [ [0, 0, 1] , [1, -2, 0] ],
```

... and documented package

Documentation: physo.readthedocs.io

# PhySO		itHub
Physical Symbolic Optimization.	Symbolic Regression	
latest	A simple symbolic regression task	
earch docs	Combalis repression /CD) equalsts in the inference of a free form symbolic and third function	
stallation	symbolic regression (sit) consists in the interference of a free-form symbolic analytical function $f: \mathbb{R}^n \longrightarrow \mathbb{R}$ that fits $y = f(x_0, \dots, x_n)$ given (x_0, \dots, x_n, y) data.	
tting Started	Given a dataset (x_0,\ldots,x_n,y) :	
nbolic Regression		
simple symbolic regression task dding custom functions Defining function token Behavior in dimensional analysis	<pre>import numpy as nom z = np.random.uniform(-10, 10, 50) v = np.random.uniform(-10, 10, 50) X = np.stack(12, v), axis=0 y = 1.224v.087xz + 1.224v+v2 </pre>	
Additional information Protected version (optional) Running the functions unit test (optional)	Where $X = (z, v)$, z being a length of dimension L^1, T^0, M^0, v a velocity of dimension $L^1, T^{-1}, M^0, y = E$ if an energy of dimension L^2, T^{-2}, M^1 .	
ustom symbolic optimization task	Given the units input variables (x_0,\ldots,x_n) (here (z,v)), the root variable y (here E) as well a	s
ut computational performances	free and fixed constants, you can run an SR task to recover f via:	
ults reproducibility	torest store	
ig tills work	expression, logs = physo.SR(X, Y, X_units = [[1, 0, 0] , (1, -1, 0]], y_units = [2, -2, 1], fixed_const_units = [[1, 0] , fixed_const_units = [[1, 0] , free_consts_units = [[0, 0, 1] , [1, -2, 0]],	
	(Allowing the use of a fixed constant 1 of dimension L^0,T^0,M^0 (le dimensionless) and free constants m of dimension L^0,T^0,M^1 and g of dimension $L^1,T^{-2},M^0.$)	
	It should be noted that here the units vector are of size 3 (eg: [1, 0, 0]) as in this example th variables have units dependent on length, time and mass only. However, units vectors can be c size ≤ 7 as long as it is consistent across X, y and constants, allowing the user to express any u (dependent on length, time, mass, temperature, electric current, amount of light, or amount of matter). In addition, dimensional analysis can be performed regardless of the order in which un are given, allowing the user to use any convention ([length, mass, time] or [mass, time, length] or as long as it is consistent across X, y and constants.	e fany nits its etc.)

[Tenachi et al 2023]

 Φ -SO

Physical Symbolic Optimization

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Thank you for your attention !





Evaluating a candidate function f

1. Fitting free constants (α)

$$f_{\alpha}(\mathbf{x}) = \hat{\mathbf{y}} \quad \text{vs} \quad \mathbf{y} \longrightarrow \alpha$$

 $(using the Broyden-Fletcher-Goldfarb-Shanno\ (BFGS)\ optimization\ algorithm)$
Evaluating a candidate function f

1. Fitting free constants (α)

$$f_{\alpha}(\mathbf{x}) = \hat{y} \text{ vs } y \longrightarrow \alpha$$

 $(using the Broyden-Fletcher-Goldfarb-Shanno\ (BFGS)\ optimization\ algorithm)$

2. Reward (R)



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Loss details (1)

• Expressions of a given batch have different sizes

→ Placeholder out of valid expression range are not taken into account

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Loss details (2)

• <u>Weighting is different along sequence dimension</u>: via a γ^t coefficient

→ We give more importance to first symbols via $\gamma = 0.7 < 1$ to avoid searching around the same initial nodes

 Φ -SO : RL

Loss details (3)

Weighting gradients accordingly with reward value ullet

• Weighting gradients accordingly with reward value

$$5\% \text{ best} \qquad (R - \text{baseline})$$

$$(R - \text{baseline})$$

[Petersen et al 2019]

 Φ -SO : RL

Feynman Benchmark



[Tenachi et al 2023]

- RNN : vanilla model \checkmark
- Φ -RNN : model can observe physical units context \checkmark
- Φ -prior : physical units prior

prior : physical units prior		Φ_{-RNNJ}					
	expression,	{\$\phi_prior,	~	, RNNJ		, RNGJ	
Expression	# Trial,	\$.SO	{\$P_RNN	{ \$-prior,	$\{R_{NN}\}$	{\$-Drior_	{RNG}
$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	10M	100~%	0 %	60~%	0 %	20~%	0 %
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(L + \frac{1}{2} \sqrt{L^2 - 4GMb} \right)$	4M	100~%	$0 \ \%$	80 %	$0 \ \%$	60~%	0 %
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	100~%
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100~%	$0 \ \%$	0 %	$0 \ \%$	0 %	0 %
$F = \frac{Gm_1m_2}{r^2}$	100K	$100 \ \%$	80~%	100~%	20~%	80~%	0 %
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	100~%	100~%	100~%	100~%	40~%	40 %

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables

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 \checkmark RNN : vanilla model

- $\checkmark~\Phi\text{-RNN}:$ model can observe physical units context
- $\mathbf{X} \quad \Phi$ -prior : physical units prior

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Expression	*	10 ¹	\$ }	\$~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	<i>{H</i>	{\$\$	$\{R_{i}\}$
$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	10M	$100 \ \%$	0 %	60~%	0 %	20~%	0 %
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(\mathbf{L} + \frac{1}{2} \sqrt{\mathbf{L}^2 - 4GMb} \right)$	$4\mathrm{M}$	100~%	0~%	80~%	$0 \ \%$	60~%	0 %
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	100~%
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100~%	$0 \ \%$	$0 \ \%$	0 %	0~%	$0 \ \%$
$F = \frac{Gm_1m_2}{r^2}$	100K	100~%	80~%	100 $\%$	20~%	80~%	0 %
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	100~%	100 %	$100 \ \%$	$100 \ \%$	40~%	40 %

RNN

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables

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 \checkmark RNN : vanilla model

- \times Φ -RNN : model can observe physical units context
- $\checkmark \Phi$ -prior : physical units prior

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	$T_{T_{i}}$	SO	<i>B</i>	JQ_{-}	VN	\tilde{Q}_{-}	NC NC
Expression	*	Por to	\$\$ }	\$ \$	Ĩ,	\$ \$	E S
$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	10M	$100 \ \%$	0~%	60~%	0~%	20~%	0 %
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(L + \frac{1}{2} \sqrt{L^2 - 4GMb} \right)$	$4\mathrm{M}$	100~%	$0 \ \%$	80~%	$0 \ \%$	60~%	$0 \ \%$
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	100~%
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100 $\%$	$0 \ \%$	0 %	$0 \ \%$	0 %	$0 \ \%$
$F = \frac{Gm_1m_2}{r^2}$	100K	100~%	80~%	100~%	20~%	80~%	$0 \ \%$
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	100 $\%$	100~%	100~%	100~%	40~%	40~%

RNNA

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables

 \checkmark RNN : vanilla model

- $\boldsymbol{\times}~\Phi\text{-RNN}:$ model can observe physical units context
- $\mathbf{X} \quad \Phi$ -prior : physical units prior

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E-maggion	¥ Trial	^{₽,S} 0	$\phi_{-RN_{I}}$		RNNJ		RNGJ
$E = \frac{mc^2}{\sqrt{1 - w^2/c^2}}$	10M	100 %	$\frac{\sim}{0\%}$	~ 60 %	\sim 0 %	~ 20 %	$\frac{\sim}{0~\%}$
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(L + \frac{1}{2} \sqrt{L^2 - 4GMb} \right)$	$4\mathrm{M}$	100~%	$0 \ \%$	80~%	$0 \ \%$	60~%	$0 \ \%$
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	100~%
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100~%	$0 \ \%$	$0 \ \%$	0~%	$0 \ \%$	$0 \ \%$
$F = \frac{Gm_1m_2}{r^2}$	100K	100~%	80~%	100~%	20~%	80~%	$0 \ \%$
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	100~%	100~%	100~%	$100 \ \%$	40~%	40 %

RNN

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables

 \mathbf{X} RNN : vanilla model

- $\boldsymbol{\times}~\Phi\text{-RNN}:$ model can observe physical units context
- $\checkmark \Phi$ -prior : physical units prior

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	$T_{T_{1\hat{a}}}$	SO	5-BN	5-Drii		5-Drii	
Expression	*	Ŕ	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	20	\lesssim	2	
$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	10M	$100 \ \%$	0 %	60~%	0 %	20~%	0 %
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(L + \frac{1}{2} \sqrt{L^2 - 4GMb} \right)$	$4\mathrm{M}$	100~%	0 %	80~%	0 %	60~%	0 %
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	100~%
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100~%	$0 \ \%$	$0 \ \%$	0~%	0 %	0~%
$F = \frac{Gm_1m_2}{r^2}$	100K	100~%	80~%	100~%	20~%	80~%	0 %
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	$100 \ \%$	100~%	100~%	$100 \ \%$	40~%	40 %

-RNN3

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables

 \mathbf{X} RNN : vanilla model

- $\boldsymbol{\times}~\Phi\text{-RNN}:$ model can observe physical units context
- $\mathbf{X} \quad \Phi$ -prior : physical units prior

	2XDressious	{\$Prior, \$	~	R_{NNJ}		RNG_{j}	
Expression	# Thial	\$.SO	{\$P_R_N	{ \$ Drior,	{RNN}	{ \$ Drior,	{RNG}
$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$	10M	100~%	0 %	60~%	0 %	20~%	0 %
$J_r = \frac{GM}{\sqrt{-2E}} - \frac{1}{2} \left(L + \frac{1}{2} \sqrt{L^2 - 4GMb} \right)$	$4\mathrm{M}$	100~%	0 %	80~%	$0 \ \%$	60~%	0 %
$\rho = \rho_0 / \left(\frac{r}{R_s} (1 + \frac{r}{R_s})^2 \right)$	2M	100~%	100~%	40~%	100~%	20~%	$100 \ \%$
$y = e^{-\alpha t} \cos(ft + \Phi)$	$1\mathrm{M}$	100~%	$0 \ \%$	$0 \ \%$	0 %	0 %	0 %
$F = \frac{Gm_1m_2}{r^2}$	100K	100~%	80~%	100~%	20~%	80~%	0 %
$H^{2}(x \equiv 1+z) = H_{0}^{2}(\Omega_{m}x^{3} + (1-\Omega_{m}))$	100K	$100 \ \%$	100 %	100 $\%$	$100 \ \%$	40~%	40 %

-RNN3

Allowing: $\{+, -, \times, /, 1/\Box, \sqrt{\Box}, \Box^2, \exp, \log, \cos, \sin, 1\}$, free constants & input variables