Cold light from the dark ages: the 21-cm signal

(and related physics)

B. Semelin

Les Houches, July 2025

Introduction

The dark ages: when? Where?

Physics of primordial galaxies SHAPES
physics of the dark ages

When? $5.5 < z < 1100$

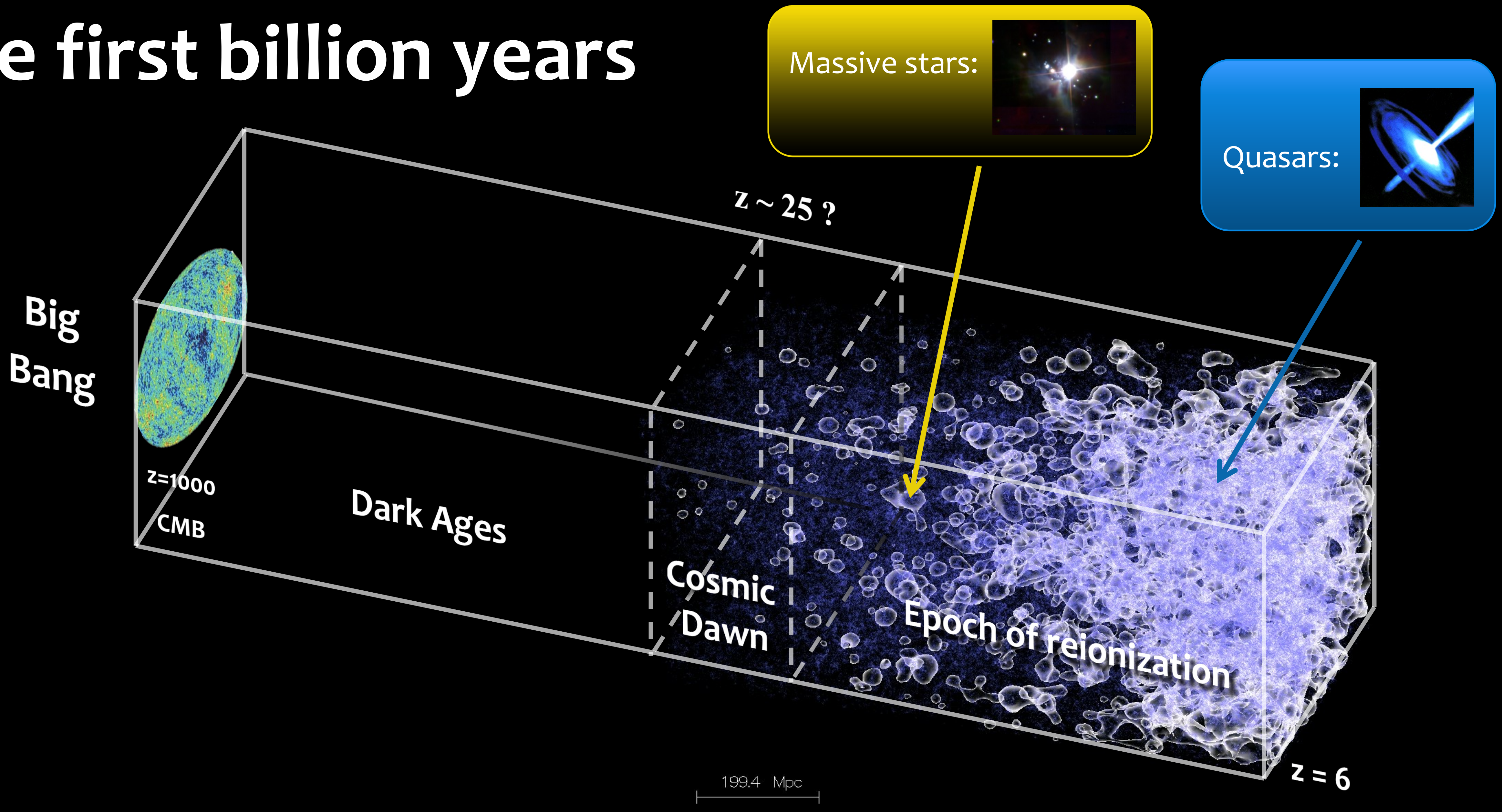
Where? Mainly in the IGM

Timing and topology of reionisation?

Evolution and fluctuation of temperature?

=> Constraints on cosmology and astrophysics

The first billion years



Probes of the first billion years

- Thomson scattering of CMB

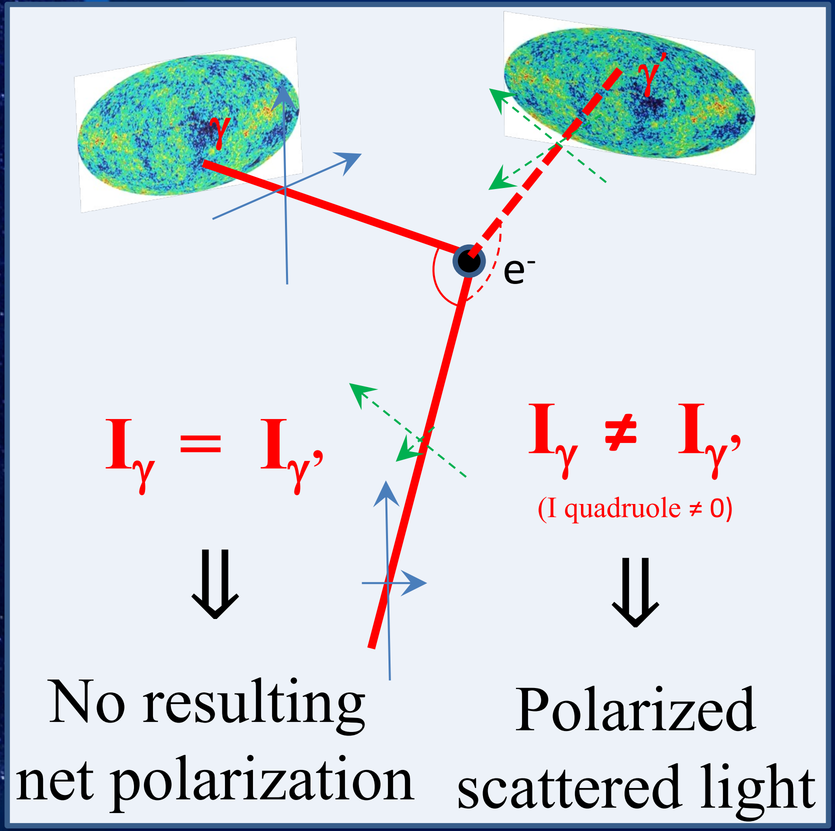
- Galaxies seen by HST, JWST, ALMA, etc.

- The Lyman-alpha line:
    - Gunn-Peterson effect
    - Lyman-alpha forest
    - Ly-alpha damping wing

- The 21-cm signal from the IGM

Thompson scattering of the CMB: the oldest probe?

CMB anisotropies + Thompson scattering = polarization



The CMB is sensitive to:

$$\tau = \int n_e \sigma_T \, dl$$

⇔ Integral contraint on reionization history

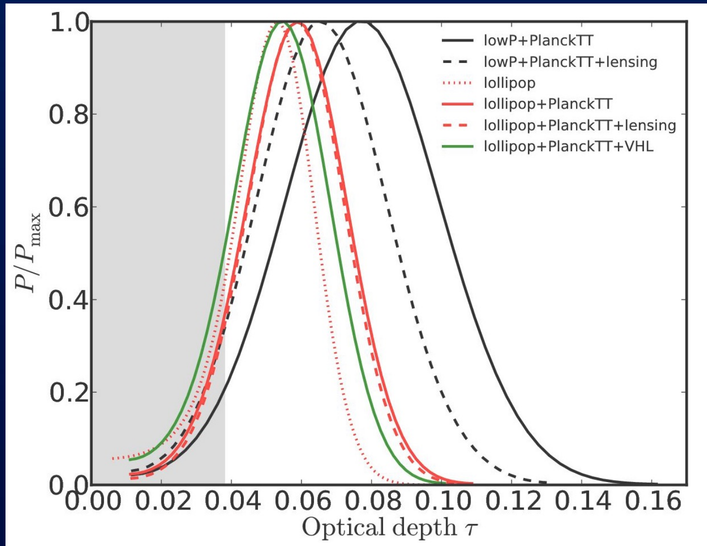
Result from Planck:

(Adam et al., 2016)

NB:

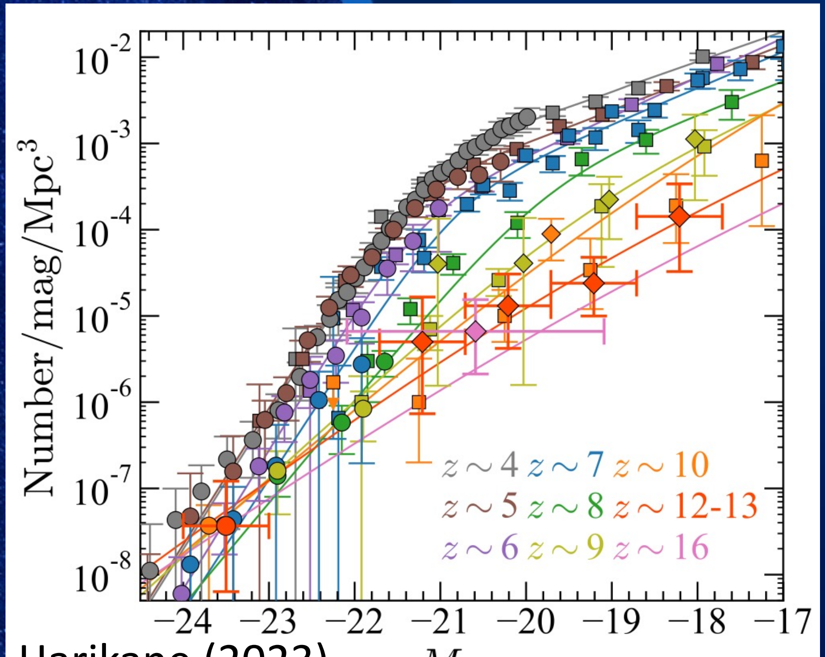
0.04 contributed by z<6

Simple reionization model

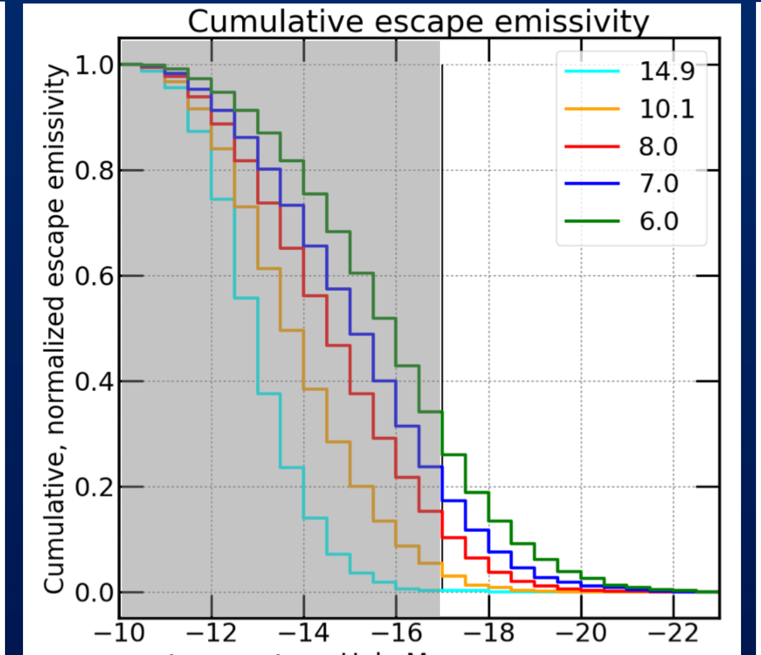


Probing the Epoch of Reionization with JWST

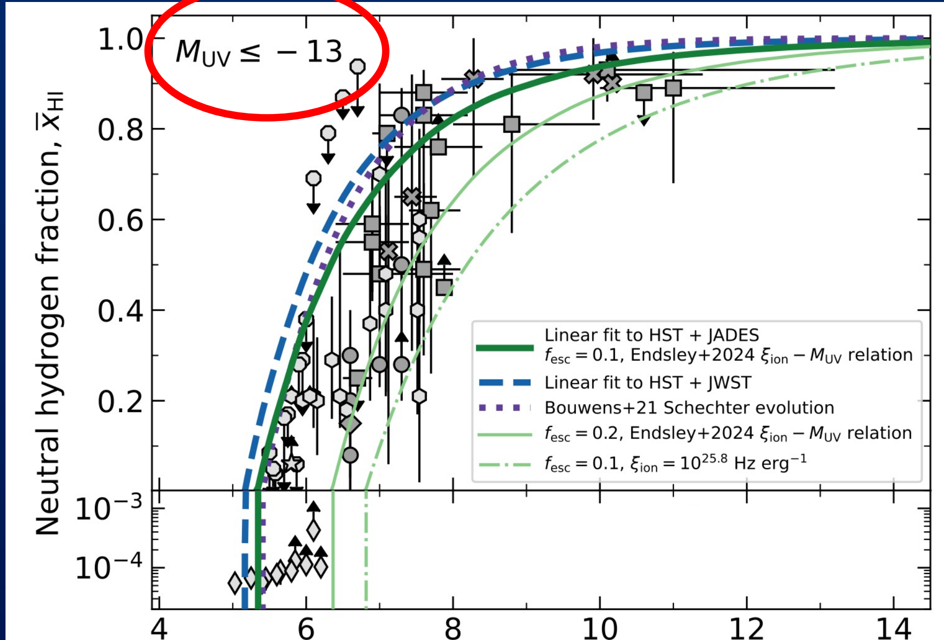
JWST sees only the tip of the iceberg



Harikane (2023)



Lewis (2020)



Whitler (2025)

At z=10 JWST misses 90%
of the ionizing photons (>99% at z=15)

Deduced reionization history

- Daring extrapolation
- Very model dependent
- Very uncertain

Probing the EoR with the Lyman-alpha line

Physics of the Lyman-alpha transition:

optical depth: $d\tau = n_{HI}\,\sigma(\nu)\,dl$

Neutral hydrogen density

1S -> 2P transition Cross section

$\sigma(\nu_\alpha)$ is very large. But ν changes along the path due to cosmological expansion.

The relation is: $dl = \frac{c}{H(z)}\frac{d\nu}{\nu}$

$\sigma_0 = \frac{f_{12}\sqrt{\pi}e^2}{m_e c}$

Voigt function

Considering a medium at temperature T: $\sigma(\nu) = \sigma_0\,H_V\left(\frac{\Delta\nu_L}{2\Delta\nu_D}, \frac{\nu-\nu_\alpha}{\Delta\nu_D}\right)$

The integrated optical depth is: $\tau = \int n_{HI}\sigma_0\,H_V\left(\frac{\Delta\nu_L}{2\Delta\nu_D}, \frac{\nu-\nu_\alpha}{\Delta\nu_D}\right)\frac{c}{H(z)}\frac{d\nu}{\nu}$

Lyman-alpha photons and the IGM

Integrated optical depth:

$$\tau = \int n_{HI}\sigma_0 \, H_V\left(\frac{\Delta\nu_L}{2\Delta\nu_D}, \frac{\nu-\nu_\alpha}{\Delta\nu_D}\right)\frac{c}{H(z)}\frac{d\nu}{\nu}$$

Integral of H_V is normalized… but $\nu, H(z), T, n_{HI}$ vary. Or not so much?

Typical width of the core of the line: $\Delta\nu_D = \sqrt{\frac{2k_BT}{m_p}} = 4\sqrt{\frac{T}{1000\ K}}$ km/s

$H(z{\sim}10) \sim 1000\ \mathrm{km.s^{-1}.Mpc^{-1}}$ => redshift trough the core in a few kpc and $\sim 10^2$ years.

=> $\nu, H(z), T, n_{HI}$ can be considered constant ($t_H{\sim}10^8$ years)

$$\tau = n_{HI}\frac{f_{12}\pi e^2}{m_e\nu_\alpha}\frac{1}{H(z)} \sim 10^6\left(\frac{1+z}{10}\right)^{\frac{3}{2}}$$

(in the IGM)

Lyman-alpha photons and the IGM

What happens to absorbed photons ? Two possibilities:

1) De-excitation to 1S ground state ⇔ **resonant scattering**

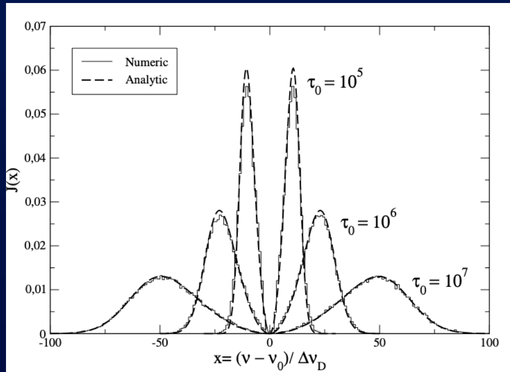
2) Reshuffling from 2P to 2S through collisions with p^+ or e^-, then 2-photon de-excitation:

proba $\sim 10^{-12}$ of reshuffling vs 1S de-excitation at IGM density

Lyman-alpha is a resonant line: many scatterings ($\sim 10^6$) in dust-free hydrogen

Incomplete picture!

- Thermal motion of atoms => frequency diffusion in gas rest-frame
- Atom recoil
- Effect of gas peculiar velocity (turbulence, inflows, outflows)



The Gunn-Peterson trough: Lyman-alpha absorption in QSO spectra

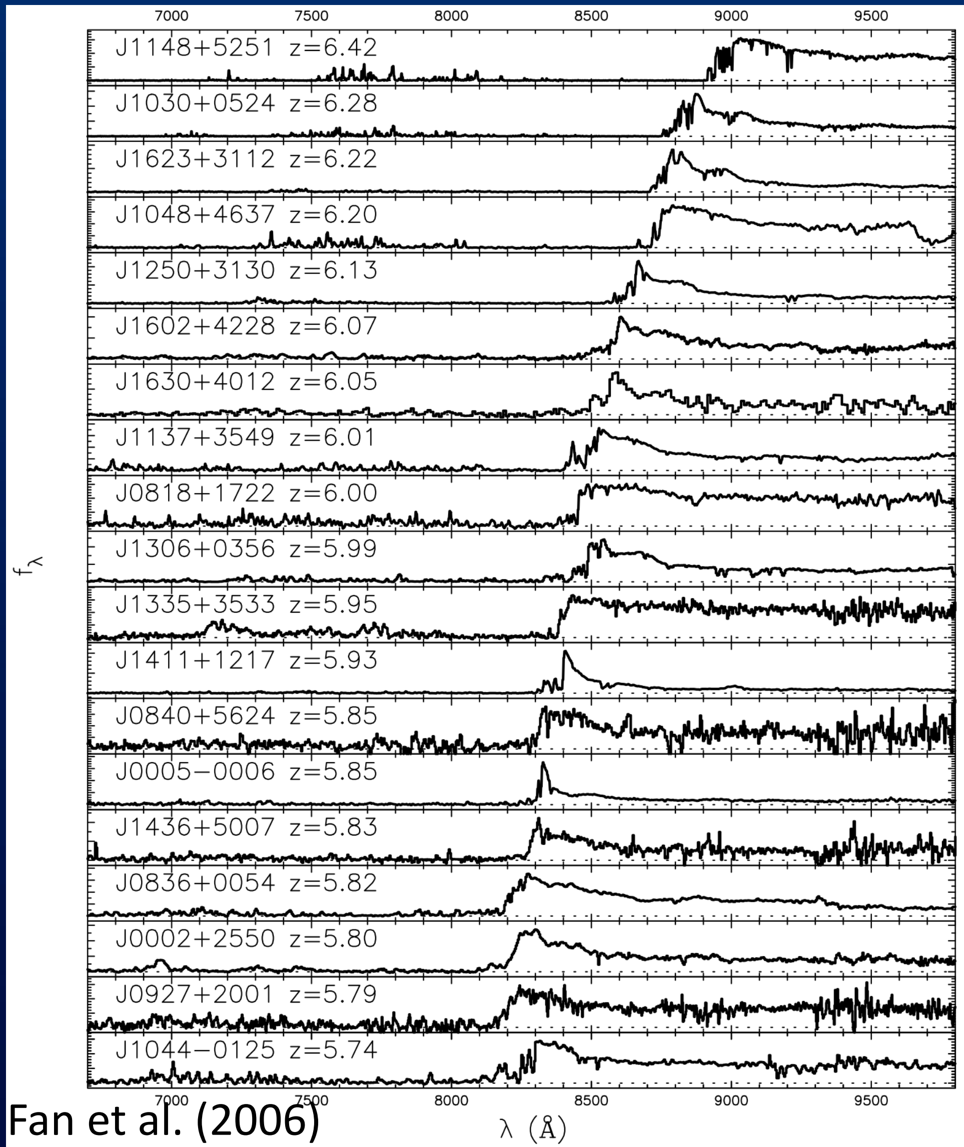
A redshifting photon enters the line in the IGM

If $x_{HI} \gtrsim 10^{-6}$ then $\tau > 1$.
=> scattered out of the line of sight.
=> absorption trough in the spectrum

A very sensitive probe of the end of reionization

Predicted in the 60's!

First observed in 2001 (Becker et al.)



Fan et al. (2006)

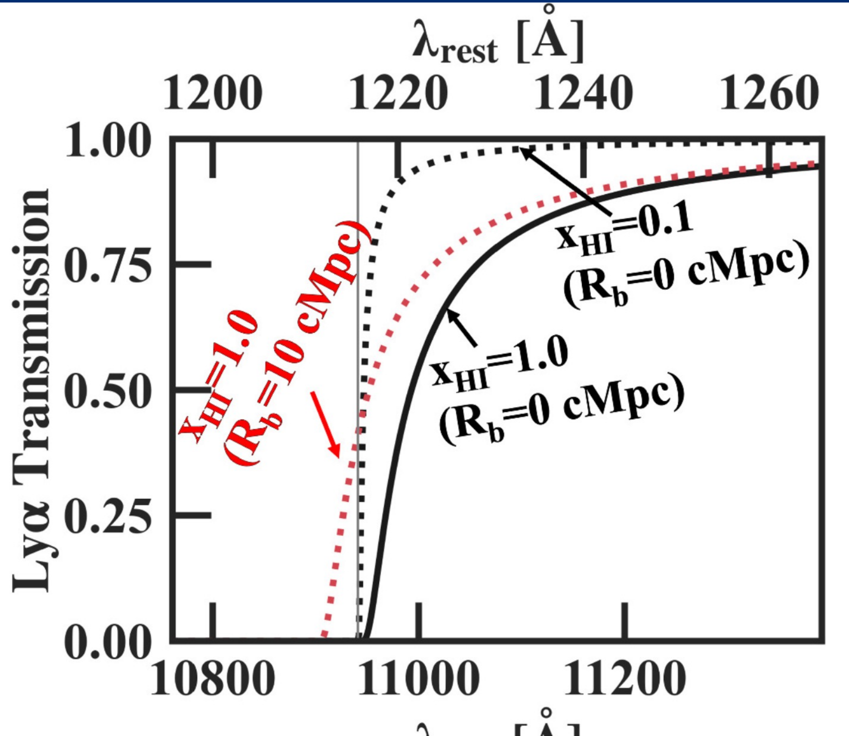
Probing the EoR with Ly-α damping wings

Idea by Miralde-Ecudé (1998)

Dirac line profile => Gunn Peterson trough

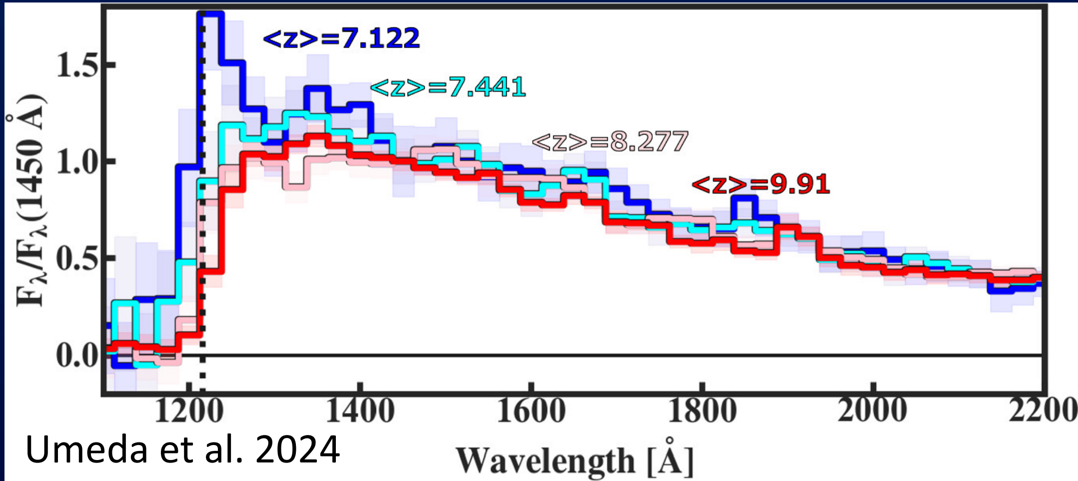
Strong line => absorption in the red wing

Unsaturated at high neutral fraction



Umeda et al. 2024

First observed in JWST spectra (Curtis-lake 2022)

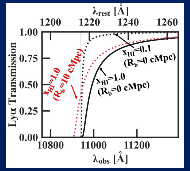


Umeda et al. 2024

Probing the EoR with Ly-α damping wings

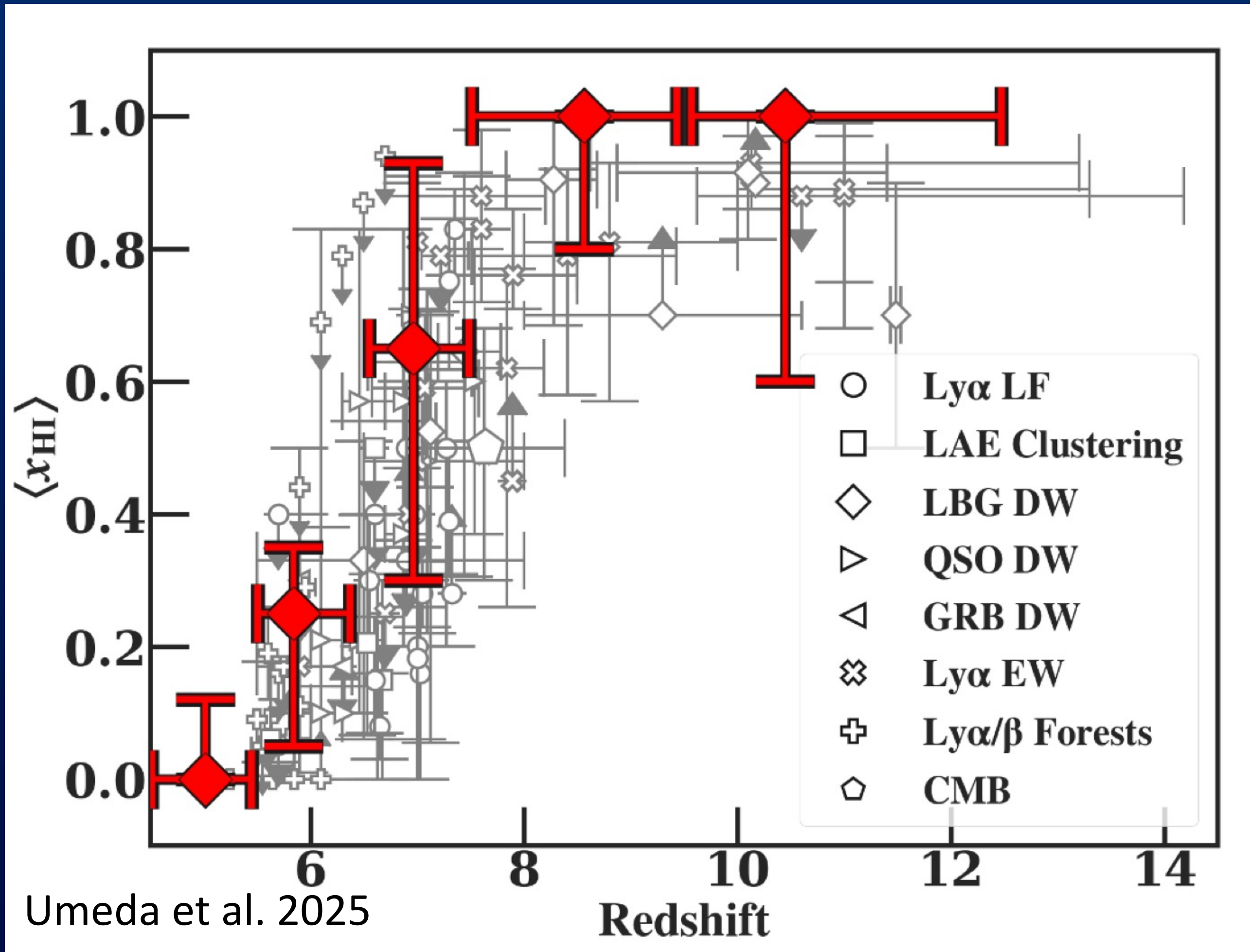
From DW observations to x_{HI} constraints:

- Model dependent: state of IGM



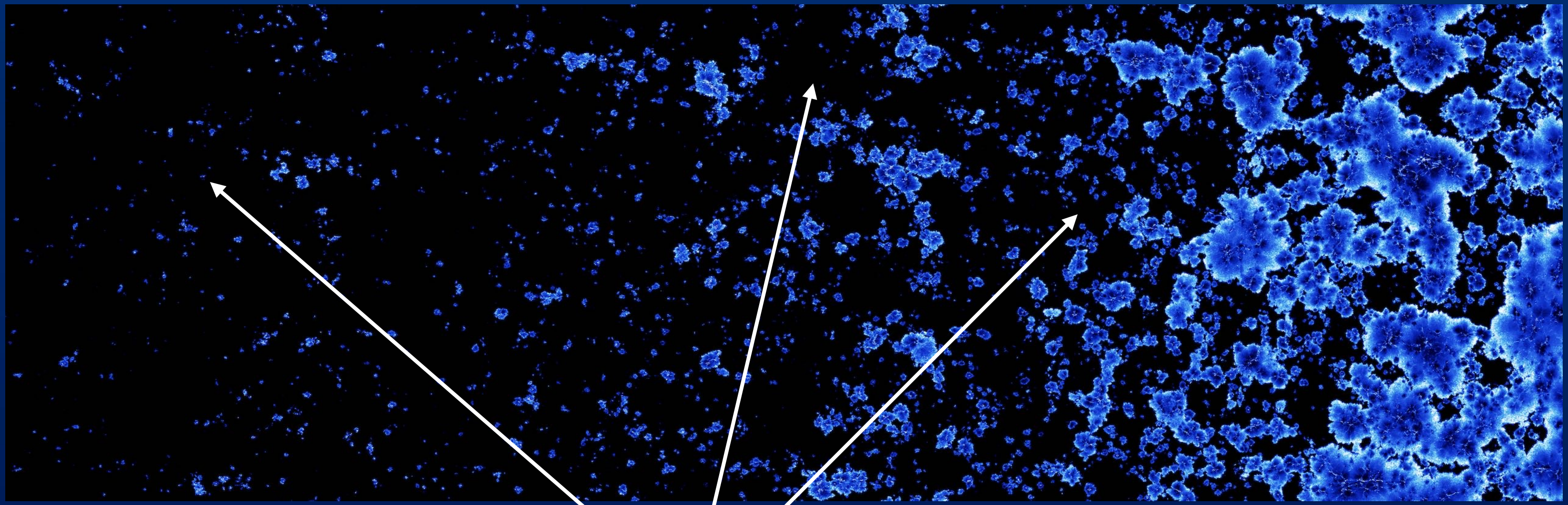
- High LoS variability
  => large statistics needed

A more direct probe is desirable.



Umeda et al. 2025

The 21-cm signal: the ultimate probe?



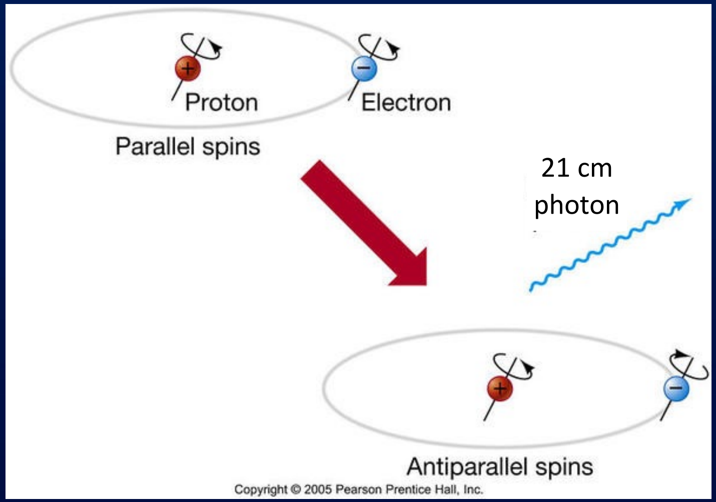
z ~ 20

z ~ 6

Neutral hydrogen:

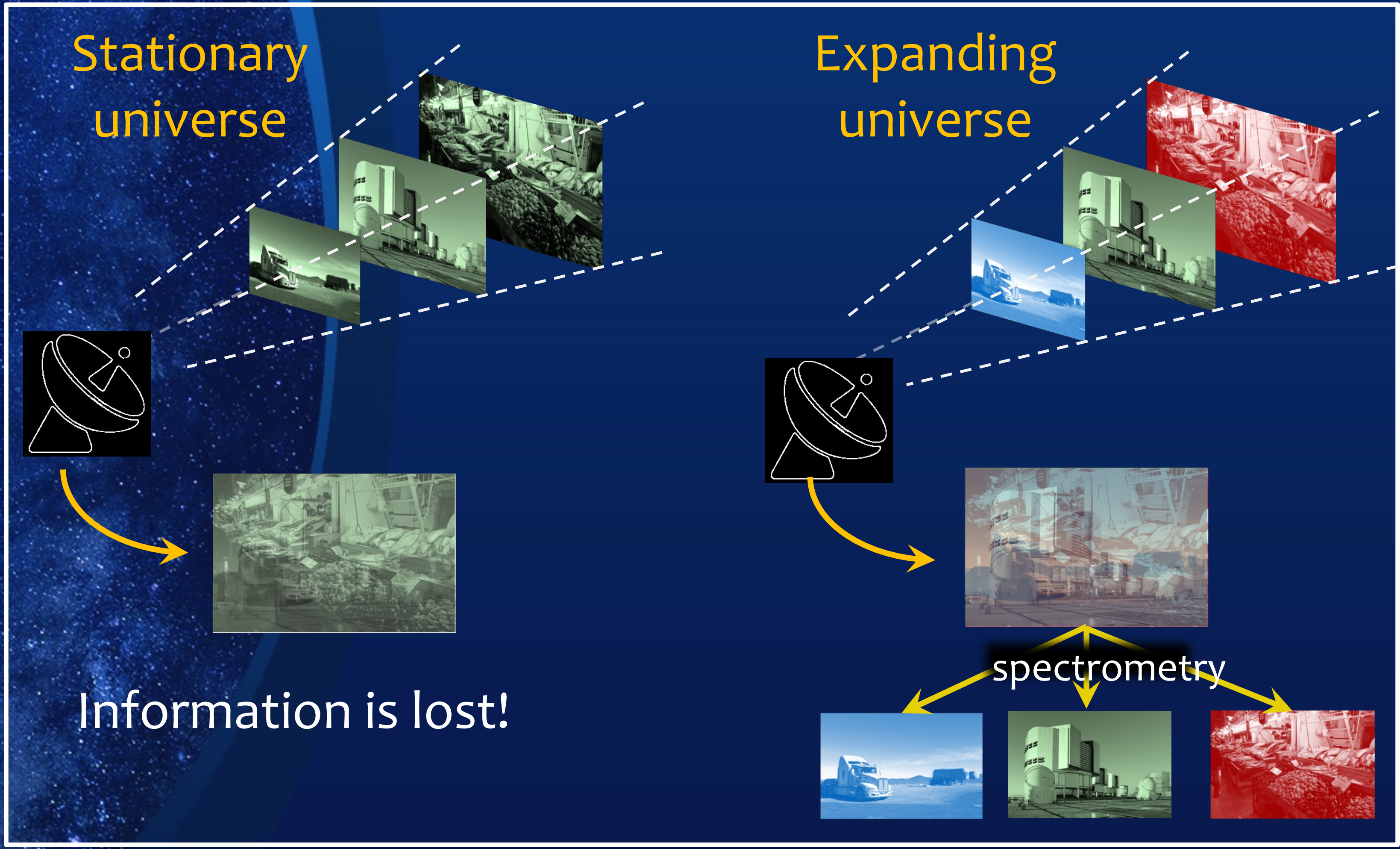
Hyperfine transition in ground state

=> 21 cm photon



The 21-cm signal: a huge potential

Expansion => observe a full 3D lightcone (tomography)



The physical limit:

Atoms thermal velocities
=> mix info between redshifts

Mixing scale: $v_{th} \sim H(z) L_{mix}$

At $z \sim 10$:
$H \sim 1000$ km/s/Mpc
$T \sim 10\ K \Rightarrow \ v_{th} \sim 300$ m/s

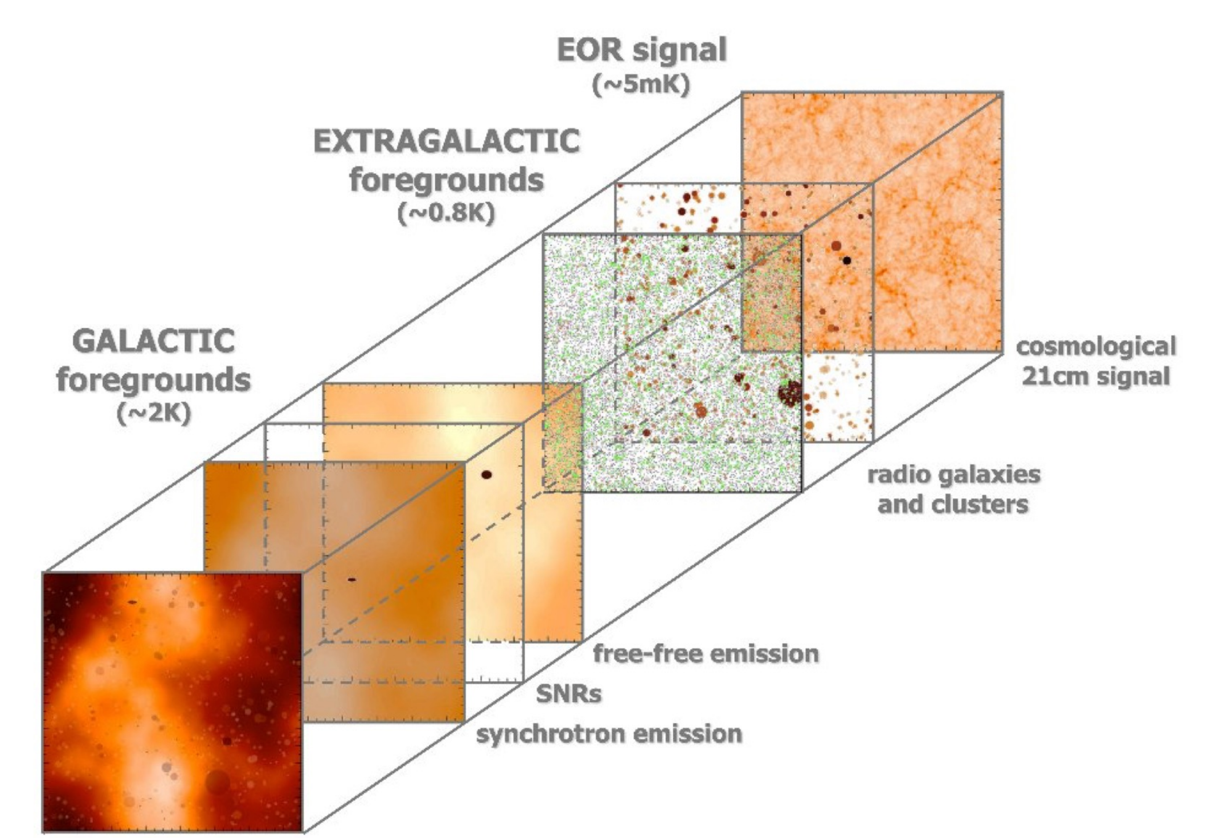
$\Rightarrow \ L_{mix} \sim$ a few ckpc

Also: $\tau \sim \ \sigma_{21} \overline{n_{HI}} L_{mix} \sim 10^{-2}$
=> no reabsorption

=> millions of indep images ?

The 21-cm signal: a difficult observation

Signal-to-nuisance ratio < 10^{-3}



Mellema et al. (2013)

Signal-to-noise ratio:

Depends on collecting area, integration time, angular scale, redshift, etc..

For SKA at z=10 , 1000h:

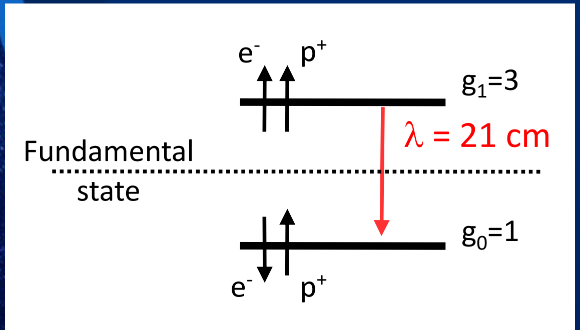
- Power spectrum: S/N=20 at k=0.1 h/cMpc
- Tomography: S/N ~ 1 at ~ 5' (~20 cMpc)

1) Beat thermal noise
2) Calibrate at better than 10^{-3}
3) Redo every 10 min obs (ionosphere)
4) Separate components with high dynamical range

Part 1
Theory of the 21-cm signal

Intensity of the 21-cm signal

The 21-cm transition



Between hyperfine level of hydrogen ground state

Forbidden emission line: $A_{10} = 2.85 \times 10^{-15}$ s^{-1}

But a lot of neutral hydrogen in the universe….

The specific intensity is determined by:

- Absorption
- Stimulated emission
- Spontaneous emission

Radiative transfer equation:

$$dI = -\underbrace{n_0\sigma_{01}dl\,I}_{\text{abs}} + \underbrace{n_1\sigma_{10}dl\,I}_{\text{stim}} + \underbrace{n_1A_{10}h\nu dl}_{\text{spont}} \quad (1)$$

I: specific intensity
dl: line element
n_0: number density in state 0
n_1: number density in state 1
σ_{01} : absorption cross section
σ_{10} : stimulated emission cross section
A_{10} : Einstein coef for 1-> 0 spont transition
ν : frequency

The 21-cm radiative transfer

Define the stimulated emission-corrected optical depth:

$$d\tau = (n_0\sigma_{01} - n_1\sigma_{10})\, dl$$

Radiative transfer equation changing variable from l to τ :

$$(1) \quad \Rightarrow \quad \frac{dI}{d\tau} = -I + \frac{A_{10} n_1 h\nu}{n_0\sigma_{01} - n_1\sigma_{10}}$$

Using the relations $B_{10} = \frac{A_{10}c^2}{2h\nu^3}$, $\sigma_{01} = h\nu B_{01}$(several defs!) and $B_{01} = 3B_{10}$

$$\frac{dI}{d\tau} = -I + \frac{2h\nu^3}{c^2} \frac{n_1}{(3n_0 - n_1)} \quad (2)$$

A local physical property of the medium.

The 21-cm radiative transfer

Introducing the spin temperature T_S : $\frac{n_1}{n_0} = 3\exp\left(-\frac{T_O}{T_S}\right)$

With excitation temperature $k_B T_0 = h\nu_{21cm} = 0.06\,K \ll T_S$ (approx 1)

Devel to first order in $\frac{T_0}{T_S}$: $(2) \Rightarrow \frac{dI}{d\tau} = -I + \frac{2k_B\nu^2}{c^2}T_S$

In the Rayleigh-Jeans regime $\mathrm{I} = \frac{T_B 2k_B\nu^2}{c^2}$. (approx 2) $\Rightarrow \frac{dT_B}{d\tau} = -T_B + T_S$

In expanding universe:
- ν is redshifting from blue to red side of the core of the line
- Core thermal width 300 m/s ⇔ redshift after only a few ckpc
- IGM properties constant across line profile => T_S=cst (approx 3)

=> simple solution:

$$T_B(\tau) = T_S(1 - e^{-\tau}) + T_B(0)e^{-\tau}$$

Computing the optical depth τ

(in an exanding universe)

From the expression of $d\tau$: : :

$$\tau = \int (n_0\sigma_{01} - n_1\sigma_{10})\, dl = \int_0^{\infty} \frac{h\nu}{4\pi} \phi(\nu_{gas})(n_0 B_{01} - n_1 B_{10})\, dl$$

Relation between dl, $d\nu$ and $d\nu_{gas}$:

$$dl = cdt \Rightarrow dl = c\frac{da}{\dot{a}}$$

$$\nu = \frac{\nu_{obs}}{a} \Rightarrow d\nu = -\frac{\nu_{obs}}{a^2} da$$

$$dl = -\mathrm{c}\frac{a^2}{\dot{a}}\frac{d\nu}{\nu_{obs}}$$

From Doppler:

$$\nu_{gas} = \nu\left(1 - \frac{\mathrm{v}_{||}}{c}\right)$$

$\phi(\nu)$: normalized line profile
ν: frequency in global cosmo rest frame
ν_{gas}: frequency in local gas rest frame
ν_{obs}: observed frequency
a: expansion factor

Computing the optical depth τ

(while redshifting across the core)

$$dl = -c\frac{a^2}{\dot{a}}\frac{d\nu}{\nu_{obs}} \quad \text{and} \quad \nu_{gas} = \nu\left(1 - \frac{v_{||}}{c}\right)$$

$$d\nu_{gas} = d\nu\left(1 - \frac{v_{||}}{c}\right) - \nu\frac{dv_{||}}{c}$$
$$= d\nu\left(1 - \frac{v_{||}}{c}\right) - \frac{\nu}{c}\frac{dv_{||}}{dl}dl$$
$$= d\nu\left(1 - \frac{v_{||}}{c}\right) - \frac{\nu}{c}\frac{dv_{||}}{dl}c\frac{a^2}{\dot{a}}\frac{d\nu}{\nu_{obs}}$$
$$d\nu_{gas} = d\nu\left(1 - \frac{v_{||}}{c} + \frac{1}{H}\frac{dv_{||}}{dl}\right)$$

Approx 4

$$dl = -c\frac{a}{H}\frac{1}{\left(1 + \frac{1}{H}\frac{dv_{||}}{dl}\right)}\frac{d\nu_{gas}}{\nu_{obs}}$$

Back to the optical depth:

Approx 3 again

$$\tau = \int_0^\infty \frac{h\nu}{4\pi}\phi(\nu_{gas})(n_0B_{01} - n_1B_{10})\,dl = \frac{h\nu}{4\pi}(n_0B_{01} - n_1B_{10})\int_0^\infty \phi(\nu_{gas})dl$$

$$= -\frac{h\nu}{4\pi}(n_0B_{01} - n_1B_{10})\,\frac{ca}{H\nu_{obs}}\,\frac{1}{\left(1+\frac{1}{H}\frac{dV_{||}}{dl}\right)}\int_\infty^{\nu_{obs}}\phi(\nu_{gas})d\nu_{gas}$$

= -1

Computing the optical depth τ

(while redshifting across the core)

$$\tau = -\frac{h\nu}{4\pi}(n_0 B_{01} - n_1 B_{10})\,\frac{ca}{H\nu_{obs}}\,\frac{1}{\left(1+\frac{1}{H}\frac{d\mathrm{v}_{||}}{dl}\right)}$$

Triplet/singlet hyperfine levels ⇒ $B_{01} = 3B_{10}$

$$T_0 = \frac{h\nu_{21cm}}{k_B} \ll T_S \Rightarrow \frac{n_1}{n_0} \simeq 3\left(1-\frac{T_0}{T_S}\right)$$

Approx 1 again and $n_{HI} = n_0 + n_1 = 4n_0$

Combining everything:

$$\tau = \frac{3}{32\pi}\,\frac{hc^3 A_{10}}{k_B \nu_{21cm}^2}\,\frac{n_{HI}}{H(z)T_S}\left(1+\frac{1}{H}\frac{d\mathrm{v}_{||}}{dl}\right)^{-1}$$

Pluging in usual cosmological notations and parameter values:

$$\tau = 0.0092\,\frac{(1+z)^{\frac{3}{2}}}{T_S}\,x_{HI}\,(1+\delta)\left(1+\frac{1}{H}\frac{d\mathrm{v}_{||}}{dl}\right)^{-1} \ll 1$$

Approx 5

21-cm differential Brightness Temperature

$$T_B(\tau) = T_S(1 - e^{-\tau}) + T_B(\tau = 0)e^{-\tau}$$

With $T_B(\tau = 0) = T_{CMB}(z)$, $\tau \ll 1$ and $\delta T_B = T_B - T_{CMB}(z)$

$\delta T_B = [\,T_S - T_{CMB}(z)]\,\tau$ at redshift z

At redshift 0:

$$\delta T_B = \frac{(T_S - T_{CMB}(z))}{1+z}\tau = \frac{3}{32\pi}\frac{hc^3 A_{10}}{k_B \nu_{21cm}^2}\frac{T_S - T_{CMB}}{T_S}\frac{n_{HI}}{H(z)(1+z)}\left(1 + \frac{1}{H}\frac{dv_{||}}{dl}\right)^{-1}$$

Plugging in numerical value or constants and cosmology:

$$\delta T_B = 28\,\mathrm{mK}\,x_{HI}(1+\delta)\left(\frac{1+z}{10}\right)^{1/2}\frac{T_S - T_{CMB}}{T_S}\left(1 + \frac{1}{H}\frac{dv_{||}}{dl}\right)^{-1}\left(\frac{\Omega_b}{0.042}\frac{h}{0.73}\right)\left(\frac{0.24}{\Omega_m}\right)^{1/2}\left(\frac{1 - Y_p}{1 - 0.248}\right)$$

21-cm differential Brightness Temperature

"Simplified" result"

$$\delta T_B = 28\,\mathrm{mK}\, x_{HI}(1+\delta)\left(\frac{1+z}{10}\right)^{1/2} \frac{T_S - T_{CMB}}{T_S}\left(1+\frac{1}{H}\frac{d\mathrm{v}_{||}}{dl}\right)^{-1}$$

Recap of approximations:

- $T_0 \ll T_S$
- Rayleigh-Jeans: $\mathrm{I} = \frac{T_B 2 k_B \nu^2}{c^2}$. Can be considered as change of variable ?
- IGM properties constant over a few ckpc
- $\mathrm{v} \ll c$ (0^{th} order devel)
- $\tau \ll 1$

Wrong in mini-haloes

21-cm signal: complex…or rich ?

Number of emitting atoms: $(1+\delta)x_{HI}$

Intensity « per » atom: $\left(\frac{T_S - T_{CMB}}{T_S}\right)$

$$\delta T_B \propto 28\,\mathrm{mK}\ (1+\delta)\,x_{HI}\left(\frac{T_S - T_{CMB}}{T_S}\right)\left(1+\frac{1}{H}\frac{dv_{\parallel}}{dl}\right)^{-1}$$

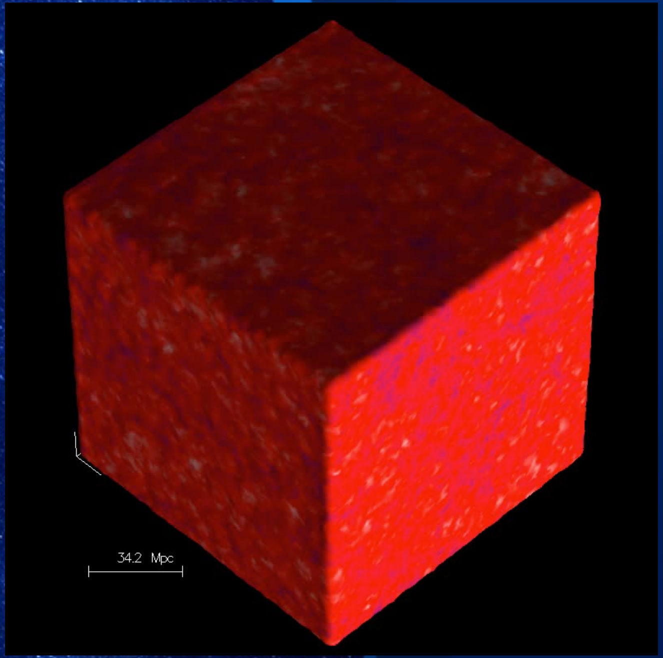
$$T_S^{-1} = \frac{T_{CMB}^{-1} + x_\alpha T_c^{-1} + x_c T_k^{-1}}{1 + x_\alpha + x_c}$$

(see next)

Local fields	Origin	Underlying physics
$\delta, \frac{dv}{dr}$	Cosmology	Gravity + hydro
x_{HI}	First stars	UV Rad Transf + hydro
T_K	X-ray bin, AGN, …	X-ray Rad Transf
x_α	First stars	Ly-alpha Rad Tranf

The process of reionization

The process of reionization



With universe expansion primordial radiation (CMB) loose energy

By z~1000 ionizing spectrum cannot balance recombination

=> universe is neutral until first stars start reionization

Reionization: 2 key quantities

Mean free path in avg IGM

$$l_{mfp} = \frac{1}{\sigma(\nu) n_{HI}} = 2\left(\frac{E}{E_{ion}}\right)^3 \left(\frac{10}{1+z}\right)^2 \text{ckpc}$$

2 ckpc => ionization “fronts” in IGM => bubbles

Recombination time in IGM

$$t_{rec} = \frac{1}{\alpha(T) n_H} \sim 240 \, \frac{\overline{\rho}(z)}{\rho} \left(\frac{10}{1+z}\right)^3 \text{Myr}$$

Only a few photons per atom are needed

Modelling reionization

Radiative transfer in an expanding universe:

$$\frac{\partial I_\nu}{\partial t} + \vec{\nabla}.\left(\frac{cI_\nu}{a(t)}\vec{n}\right) - \mathrm{H(t)}\left(\nu\frac{\partial I_\nu}{\partial \nu} - 3I_\nu\right) = -k_\nu I_\nu + S_\nu$$

$$\underbrace{\nu\frac{\partial I_\nu}{\partial \nu}}_{\text{redshifting}} \quad \underbrace{- 3I_\nu}_{\text{Flux dilution}} \quad \underbrace{-k_\nu I_\nu}_{\text{absorption}} \quad \underbrace{S_\nu}_{\text{Sources + recomb}}$$

Specific intensity I_ν is 7-dimentional $(\vec{x}, t, \vec{n}, \nu)$:

- Solve by taking moments (freq and/or direction) + closure relations

- Sample with (monte-carlo) ray-tracing

- Average on everything except t and solve analytically

Modelling reionization

- Compute local photo-ionization rate Γ from specific intensity (or moments)

- Solve time-dependent ionization-temperature state (see *Cen, R. 1992, ApJS, 78, 341*):

$$\frac{dx_i}{dt} = -\alpha(T)x_i n_e + \gamma(T)(1 - x_i)n_e + \Gamma(1 - x_i)$$

recombination — collisional ionisation — photo-ionization

Warning! "Stiff" system in EoR conditions

Feed back on hydro ⟵

$$\frac{dT}{dt} = \frac{T}{n}\frac{dn}{dt} + \frac{2}{3k_B n}\left[\,\mathcal{H}(T, x_i) - \Lambda(T, x_i)\,\right]$$

Heating:
Compton
Photo-ionization
Hydro
Adiabatic

Cooling:
Collisional ionization
Recombination
Collisional excitation
Bremsstrahlung
Adiabatic

=> Get x_i as a function from time and space.

Is ionization feedback on dynamics important?

If temperature increase creates pressure gradients comparable to gravity.

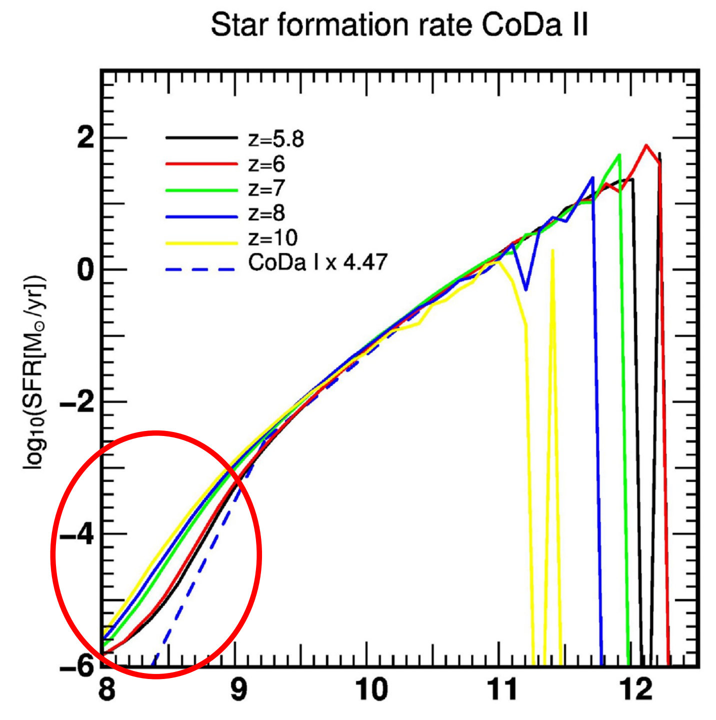
Not in the IGM. But in halos?

Condition: thermal energy similar to potential energy
⇔ gas temperature similar to virial temperature

Ionized gas is heated to ~10^4 K ⇔ T_{vir} of halos ~ 10^8 $M_\odot$

Feedback can quench star formation by photoevaporating small halos

Don't confuse with the effect of atomic cooling temperature floor.



Ocvirk et al. (2020)

The hydrogen spin temperature

Competing processes

T_s determined by populations of the hyperfine levels: $\frac{n_1}{n_0} = 3 \exp\left(-\frac{T_O}{T_S}\right)$

Spontaneous emission — Stimulated emission — Lyman-α pumping — Collisions — CMB absorption — Lyman-α pumping — Collisions

$$\begin{cases} \frac{dn_0}{dt} = n_1(A_{10} + I_{CMB}B_{10} + P_{10} + C_{10}) - n_0(I_{CMB}B_{01} + P_{01} + C_{01}) \\ n_{HI} = n_1 + n_0 \end{cases}$$

During EoR, timescale for CMB equilibrium $(I_{CMB}B_{01})^{-1} < 10^5$ years.

=> Assume equilibrium with secular evolution:

$$n_1(A_{10} + I_{CMB}B_{10} + P_{10} + C_{10}) = n_0(I_{CMB}B_{01} + P_{01} + C_{01})$$

T_S is driven by gas temperature and Ly-α flux

Plug in:

- Rayleigh-Jeans $I_{CMB} = \frac{T_{CMB} 2 k_B \nu^2}{c^2}$
- $B_{10} = \frac{A_{10} c^2}{2 h \nu^3}$
- $\frac{n_1}{n_0} \simeq 3\left(1 - \frac{T_0}{T_S}\right)$
- Detailed balance: $C_{01} = 3\left(1 - \frac{T_0}{T_K}\right) C_{10}$
- Def of color temp T_c: $P_{01} = 3\left(1 - \frac{T_0}{T_c}\right) P_{10}$

Get spin temperature relation:

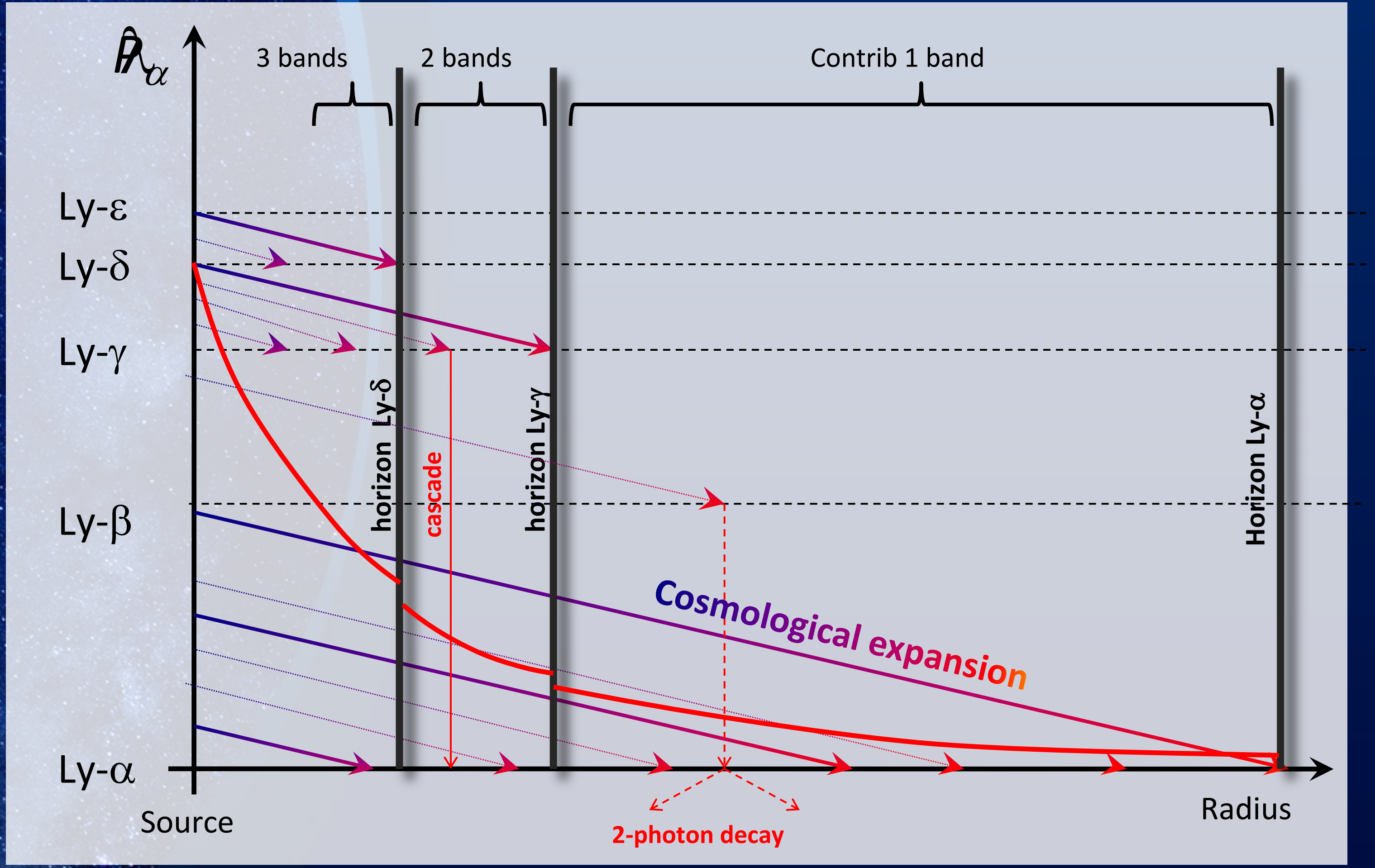
$$T_S^{-1} = \frac{T_{CMB}^{-1} + x_\alpha T_c^{-1} + x_c T_K^{-1}}{1 + x_\alpha + x_c}$$

$$x_\alpha = \frac{4 P_\alpha T_0}{27 A_{10} T_{CMB}}$$

- $x_c \ll 1$, except at z>40 or in minihalos (see Furlanetto et al. 2006 for formula and values)
- $T_c = T_K$ in EoR conditions
- P_α : number of Ly-α scattering per atom per second. Set by flux: J_α

The Ly-α flux (Wouthuysen-Field coupling)

The LOCAL Ly-α flux was redshifted from shorter wavelength at the source



Assuming free streaming and homogeneous universe:

$$J_\alpha(z) = \frac{(1+z)^2 c}{4\pi} \int_z^{z_\beta} \varepsilon\left(z_e, \frac{1+z_e}{1+z}\nu_\alpha\right) \frac{dz_e}{H(z_e)}$$

(e.g Furlanetto et al. 2006)

But:

1) Emissivity ε is NOT homogeneous

=> convolution of ε with $\frac{1}{r^2}$ kernel

(Mesinger et al. 2011)

2) Free streaming not valid

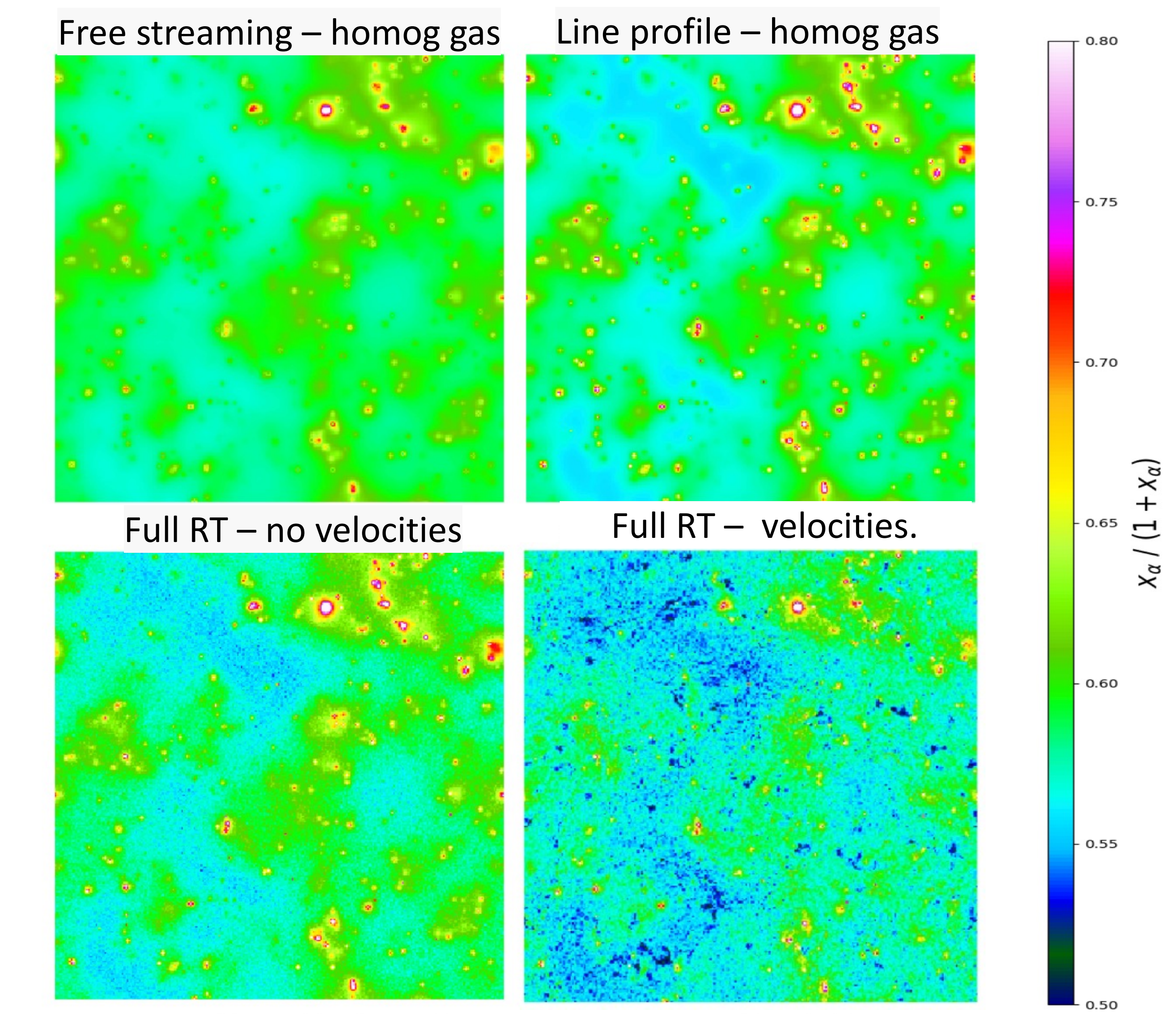
=> change kernel

(Reis et al. 2021, Semelin et al.2023)

3) Gas is not homogenous…

The Ly-α flux: assessing approximations

(Semelin et al. 2023)



Homogeneous ε not realistic.

Free streaming => reduced fluctuations

Inhomogenous gas density and temp
=> no big change

So full RT not necessary?

Gas peculiar velocities make a difference

Only full RT account for velocities (for now)

Kinetic temperature of the neutral gas

Until $z \sim 135$:

CMB energy dens >> gas thermal energy

Free e[-] couple T_{gas} to $T_{CMB} = 2.71\ \mathrm{K} \times (1+z)$

$z \sim 135$ to $z \sim 25$:

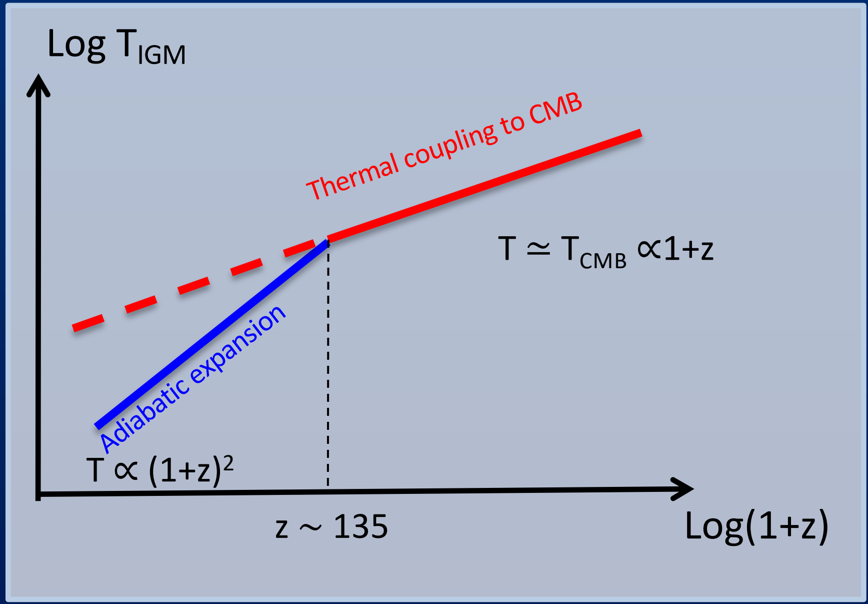
Adiabatic cooling $T\rho^{-\frac{2}{3}} = cst \Rightarrow T \propto (1+z)^2$

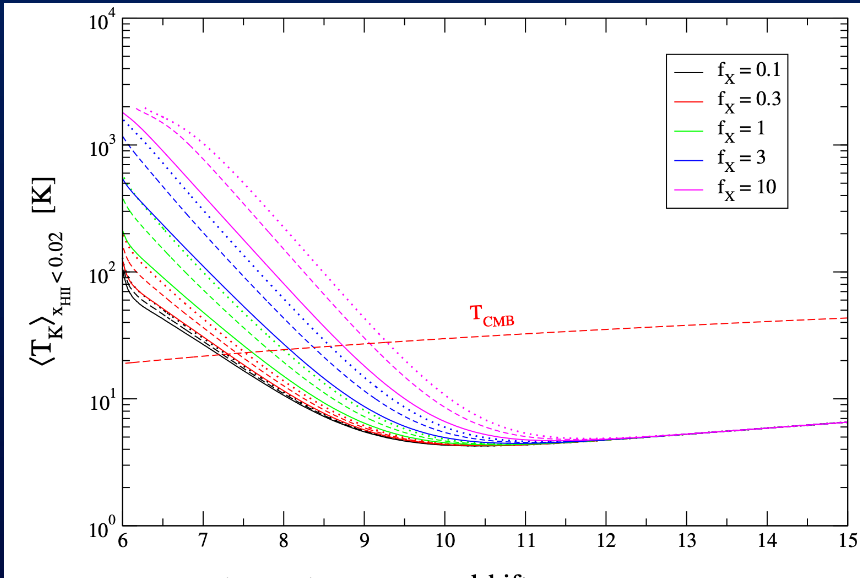
After $z \sim 25$:

Heating by X-rays from AGN, X-ray bin, SN

(Santos et al. 2008, Baek et al. 2010, etc.)

X-ray emissivity ⇔ model parameter



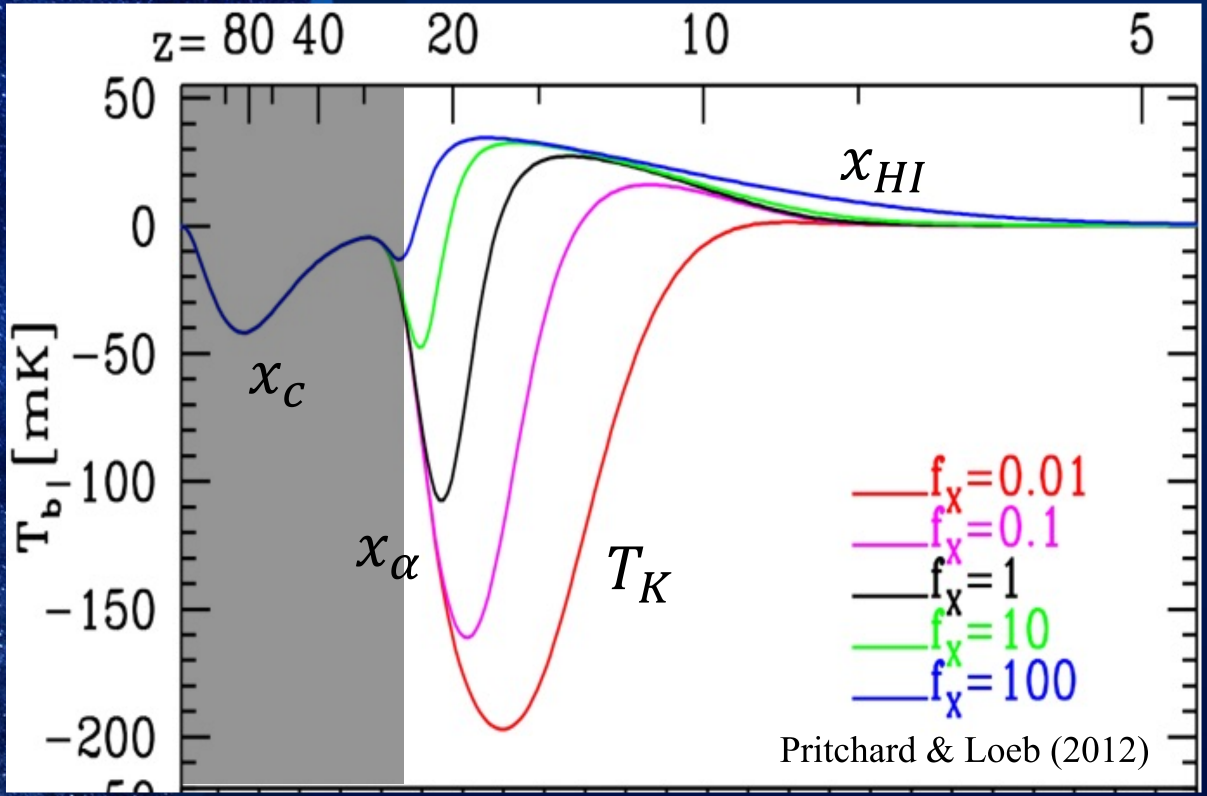


Semelin et al (2017)

Putting it all together

$$\delta T_B \propto 28\,\mathrm{mK}\,(1+\delta)\,x_{HI}\left(\frac{T_S - T_{\mathrm{CMB}}}{T_S}\right)\left(1+\frac{1}{H}\frac{dv_{\parallel}}{dl}\right)^{-1}$$

$$T_S^{-1} = \frac{T_{CMB}^{-1} + x_\alpha T_c^{-1} + x_c T_K^{-1}}{1 + x_\alpha + x_c}$$



Sky-averaged signal

δT_b is a 3D signal! But…

Sky averaged for a simple first analysis.

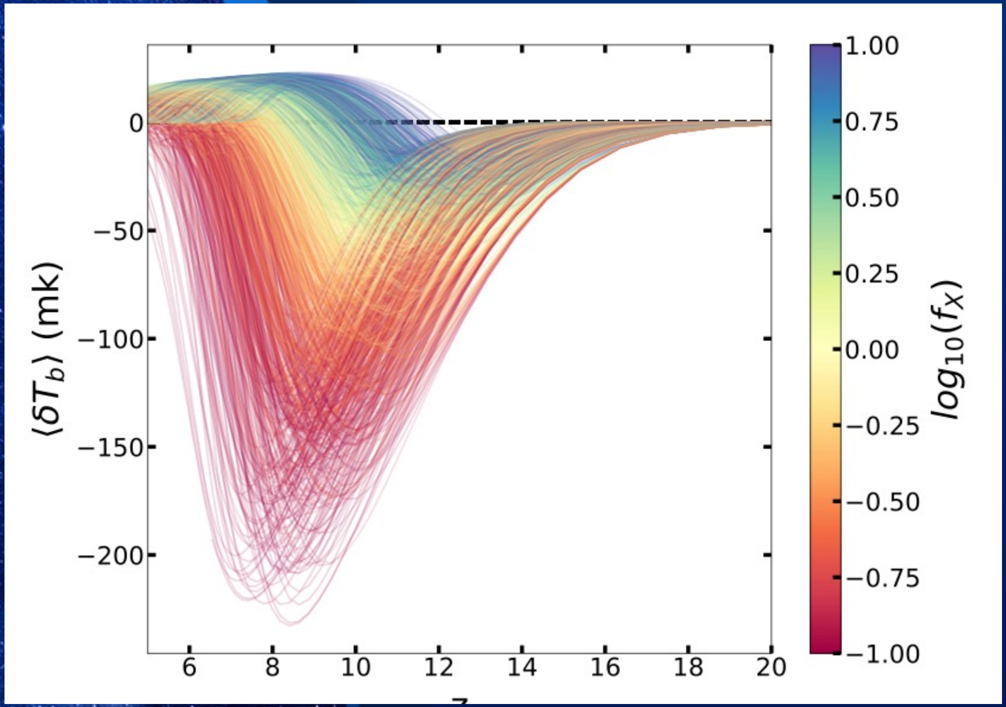
- $T_K < T_{CMB}$ => signal in absorption
  strong if late heating

- $T_K > T_{CMB}$ => signal in emission
  max ~ 20 mK

But foregrounds are at a few K…!

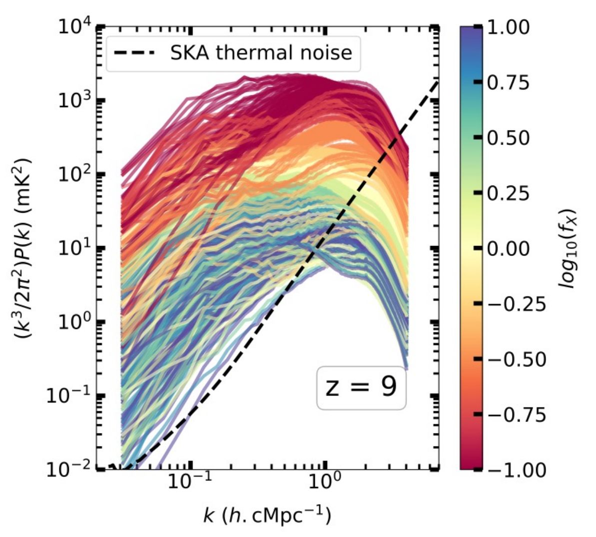
Information vs S/N tradeoff

Global signal: $\langle \delta T_B \rangle_{sky}(z) \propto \langle x_{HI} \rangle_{sky} \langle 1 - T_{cmb} / T_s \rangle_{sky}$



Power spectrum:

$P_{\delta T_B}(k,z)$



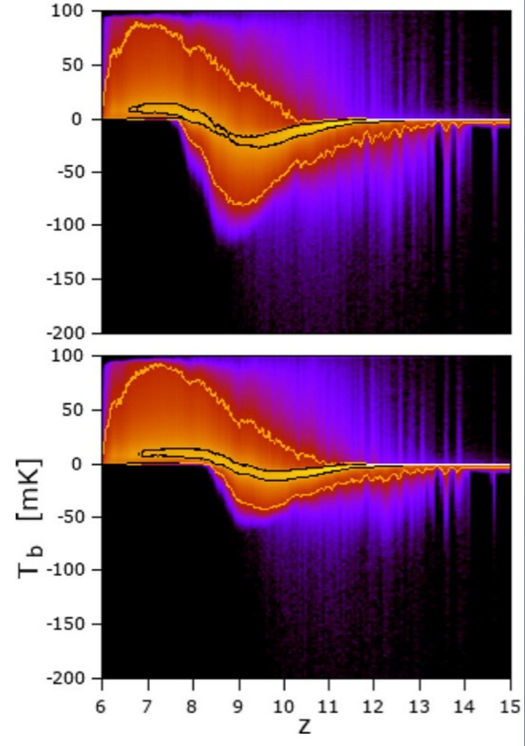
Non-gaussian Statistics:

ST,

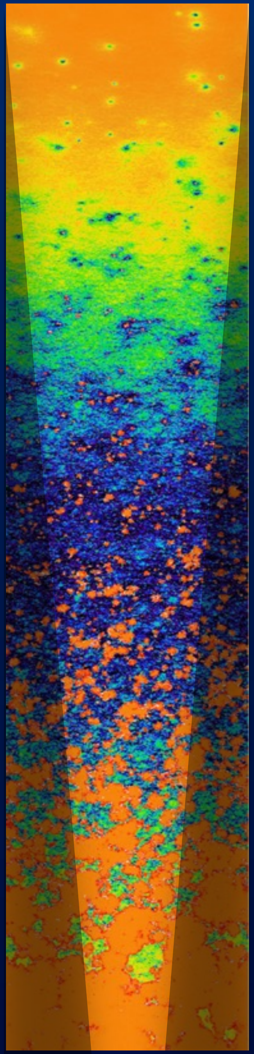
Bispectrum,

Minkovsky func

PDF



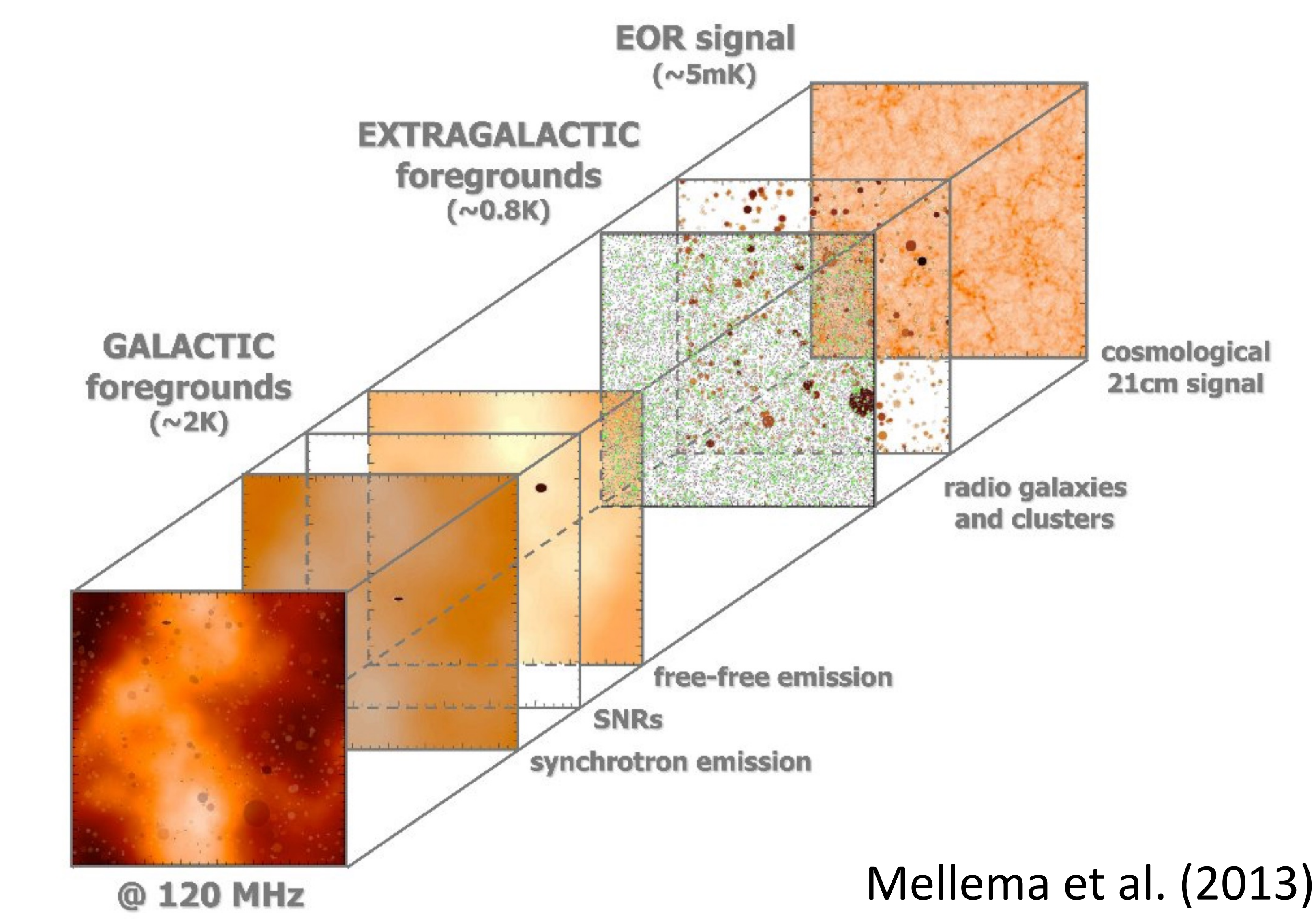
Imaging: $\delta T_B(\mathbf{x}, z)$



Full info + random phases

Part 2:
Observing the signal

The issue of foregrounds



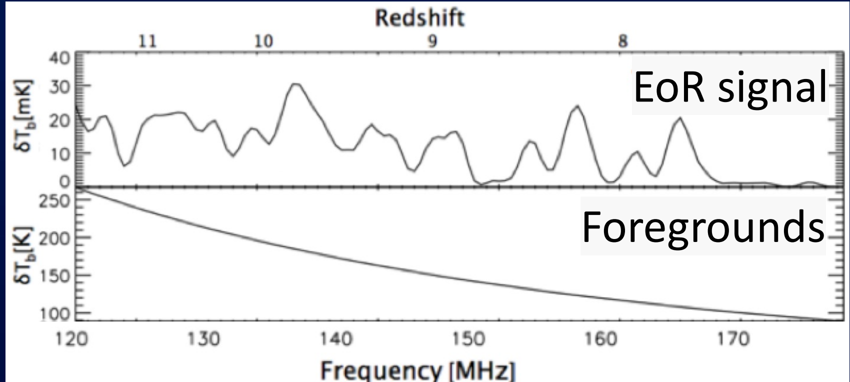
Mellema et al. (2013)

x10^3-10^4 brighter foregrounds

Different behaviour in freq

Various component separation methods

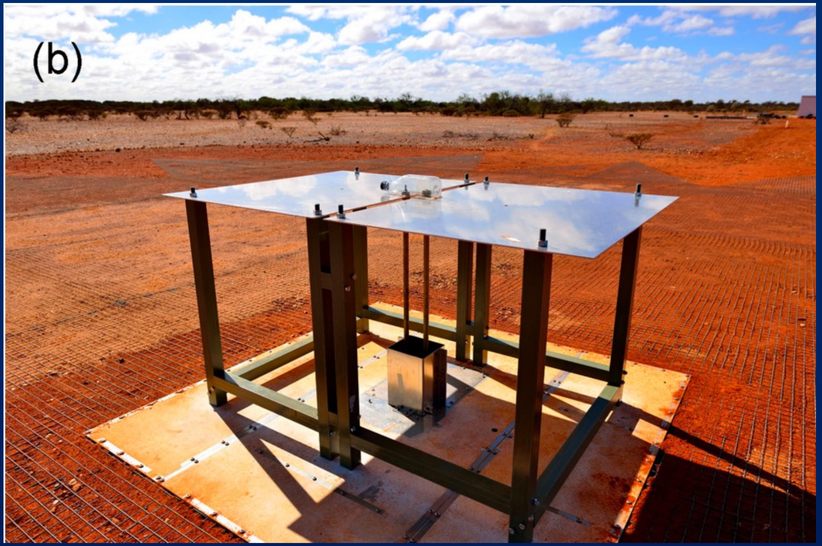
Signal can easily be absorbed in the foreground component



The EDGES « detection » (Bowman et al. 2018)

EDGES:

- single dipole experiment => sky-averaged
- At future SKA site, Australia
- > 100 h integration



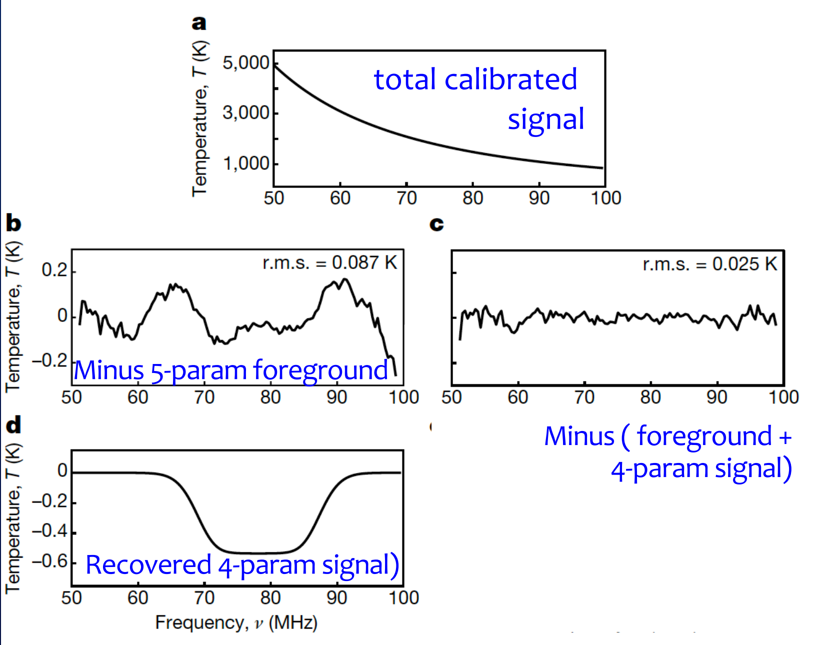
Detected signal: exotic!

Many Validations tests but…

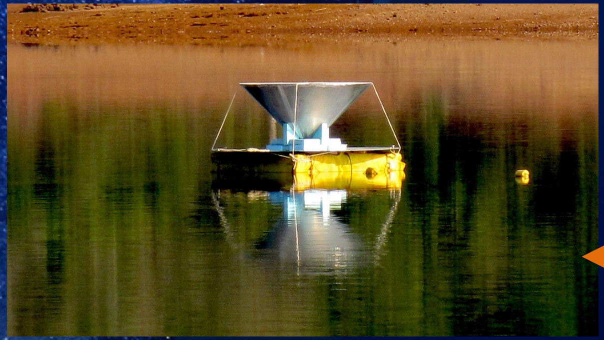
Some serious questions:

foreground modeling

instrumental effect (ground plane)



Hills et al. (2018), Badley et al. (2018)



← Detection rejected at 95% by SARAS-3 !

The EDGES signal, a theoretical challenge

$$dT_B = 28\text{ mK} \times x_{HI}(1+\delta)\left(\frac{1+z}{10}\right)^{1/2}\frac{T_S - T_{CMB}}{T_S}\left(1+\frac{1}{H}\frac{dv_{||}}{dl}\right)^{-1}$$

$$\langle dT_B\rangle \simeq 28\text{ mK}\left(\frac{1+z}{10}\right)^{1/2}\left\langle\frac{T_S - T_{CMB}}{T_S}\right\rangle$$

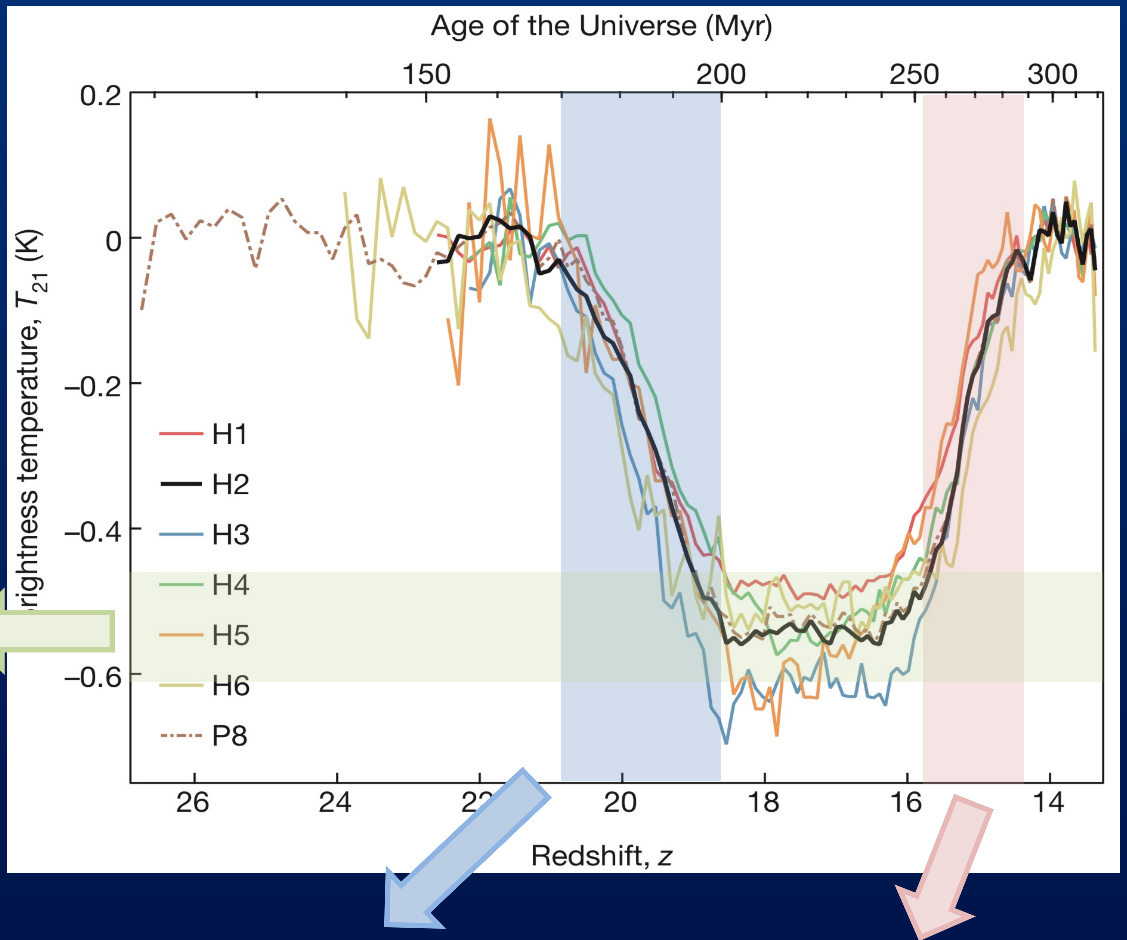
Maximum absorption at z=17:

- : $T_{CMB}(z=17) = 48.8$ K
- Pure adiab cooling + full Ly-alpha coupling: $T_S = T_K(\langle\rho\rangle) = 6.5$ K

=> $\langle dT_B\rangle \simeq -245$ mK ≠ -500 mK

Hypothetic solutions:

- Change background (ie T_{CMB}): add **strong** radio galaxies (e.g Skider et al. 2024)
- Cool the gas: interaction with exotic DM. (Barkana et al. 2018)



Ly-α coupling within 30 Myr

~200-300 Myr in simulations

X-ray heating within 30 Myr

> 100 Myr in simulations

The limits of the sky-averaged signal (a personal take)

Limited information: can be fitted with 5-6 param function

Not easily separated from foregrounds/instrumental artefacts

=> Degeneracy between model parameters and other components

We need more information!

=> Measure angular fluctuations.

Observing the power spectrum

The power spectrum is the variance of the complex amplitude of the Fourier modes:

$$\langle \delta T_B(k)\, \delta T_B^*(k') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P(k)$$

Limits in measuring the power spectrum:

- "Thermal" noise (McQuinn et al. 2006, Mellema et al. 2013):

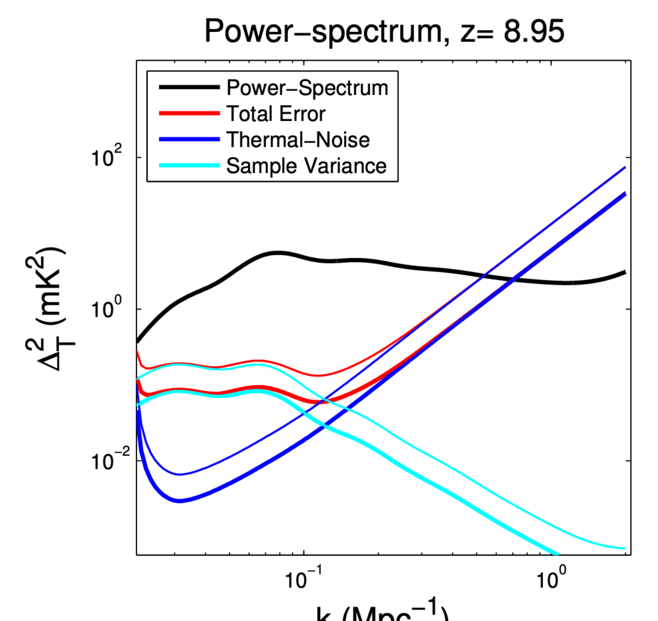
$$\Delta_{21}^{Noise} \propto \left(\frac{T_{sys}}{\sqrt{B t_{obs}}} \right)^2 \qquad \Delta_{21}^{Noise} \propto \frac{A_{core}}{A_{coll}^2}$$

- "Sample variance":

$$\Delta_{21}^{SV} \propto \frac{P(k)}{\sqrt{N_{bin}}}$$

Because ensemble avg ⟨ ⟩ is replaced by k-bin avg over N_{bin} modes

Typical SKA survey



Koopmans et al. (2015)

Interferometers searching for the signal

PAPER (South Africa)

GMRT (India)

LOFAR (Netherlands)

MWA (Australia)

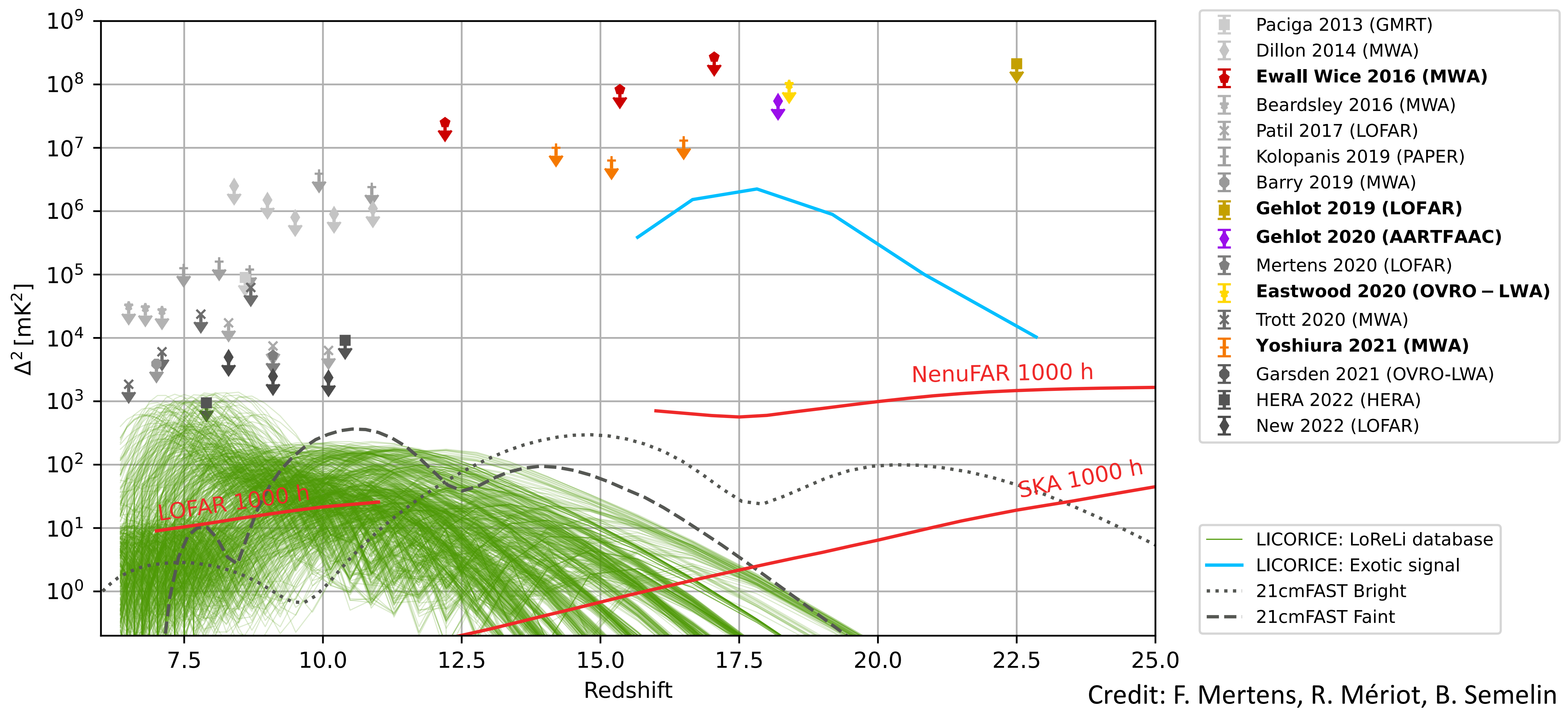
HERA (South Africa)

NenuFAR (France)

And of course SKA in 2027-2030!

Upper limits on power spectrum

Limits at k around 0.1 h cMpc^{-1}



Credit: F. Mertens, R. Mériot, B. Semelin

An example: The Cosmic Dawn Program on NenuFAR

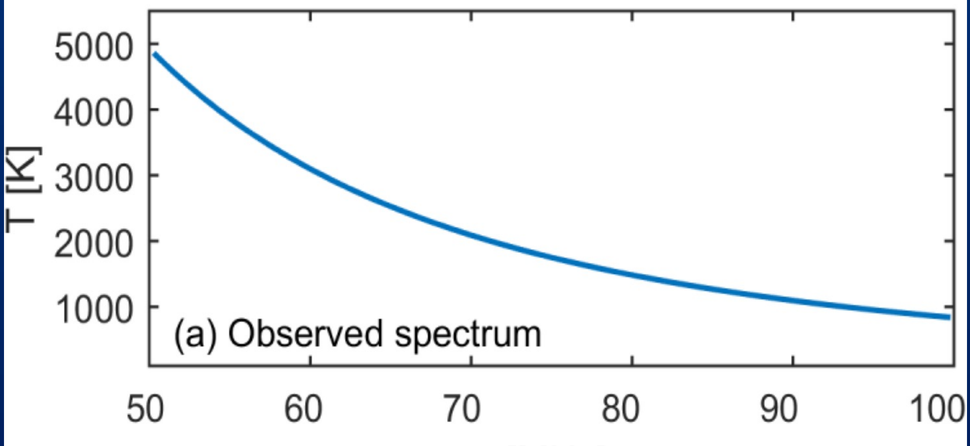
Mertens F. (PI)
Koopmans L. (PI)
Semelin B. (PI)
Aubert D.
Barkana R.
Bobin J.
Boulanger F.
Cecconi B.
Fialkov A.
Gan H.
Gehlot B.
Girard J.
Jelic V.
Levrier F.
Mériot R.
Mevius M.
Munshi S.
Ocvirk P.
Offringa A.
Pandey V.
Tasse C.
Vedantham H.
Yatawatta S.
Zarka P.
Zaroubi S.

https://vm-weblerma.obspm.fr/nenufar-cosmic-dawn/

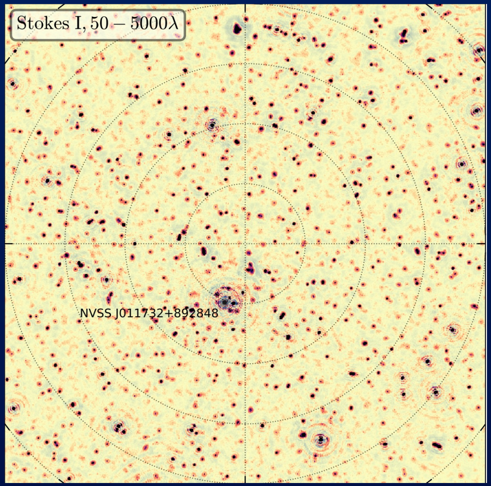
Observing the « Cosmic Dawn »

Difficulties compared to EoR:

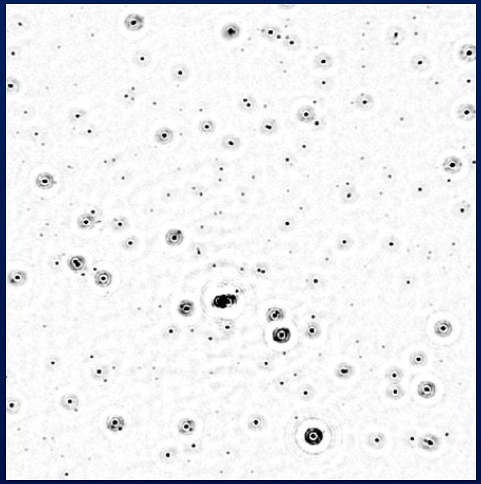
- Foregrounds:
  ~ x15 brighter at 50 MHz than at 150 MHz
- Instrumental resolution as ν^{-1}
  $\Rightarrow$ Confusion limit for brighter point sources
  $\Rightarrow$ Sky model with fewer sources
- Ionosphere
  Stronger at low freq
  Larger FoV => more DD calib
- Wider range of possible signals
  Impact on foregrounds removal (e.g. GRP prior)



Bowman et al. (2018)



NCP LOFAR, 140 MHz
(Mertens et al. 2019)



NCP LOFAR, 50 MHz
(Gehlot et al. 2019)

The NenuFAR interferometer

A SKA pathfinder in Nançay (France)

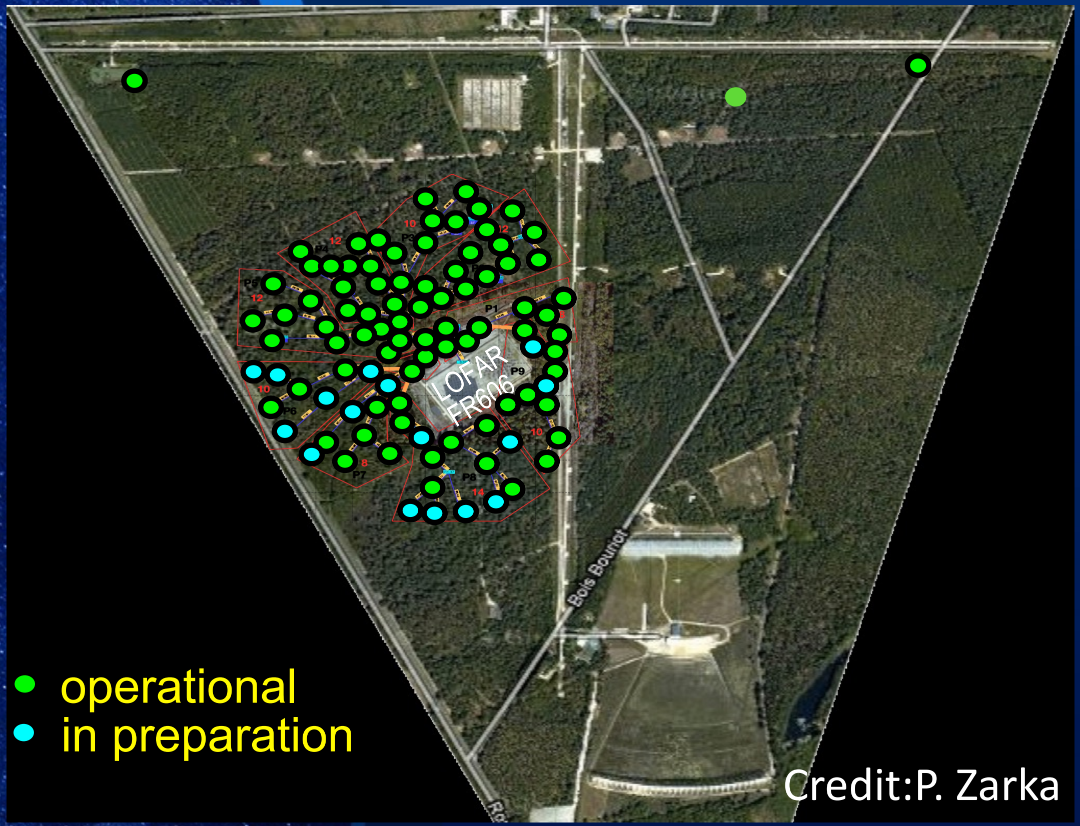
Standalone and LOFAR extension

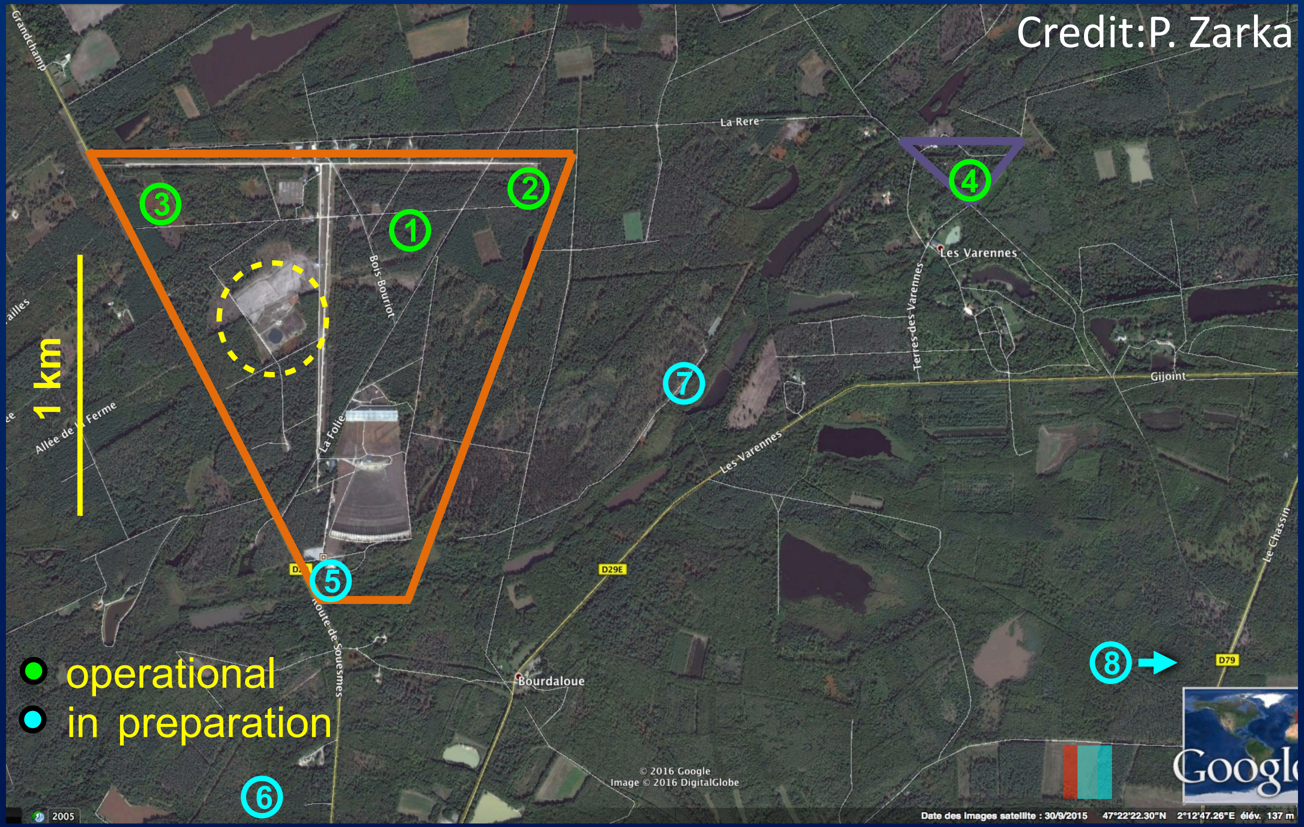
Inaugurated in 2019

- 10 to 85 MHz
- 1938 antennas (LWA design)
- 96+8 mini-arrays
- 400 m diameter core
- A few km maximum baseline
- Cobalt 2 corelator (LOFAR)
- Home-made preamplifier, software, etc…

} Dense core: good for 21cm

The NenuFAR interferometer





Currently operational:

80 core MA

4 distant MA

Upon (imminent) Completion :

96 core MA

8 distant MA (L_{max}= 5km)

First upper limits (Munshi et al. 2024)

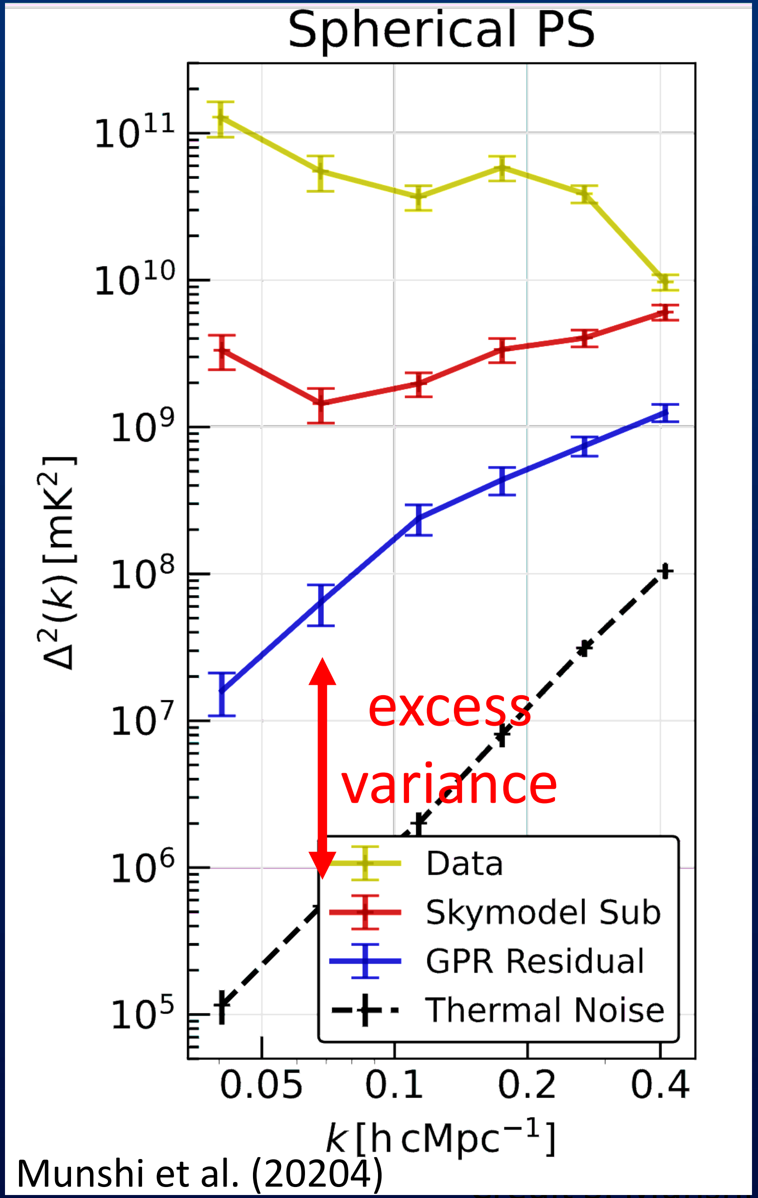
~ 1500 h of observation on NCP field

Single night of observation (11.4h)
76+3 Mini-Arrays
61-72 MHz band (best RFI)

Pipeline adapted from LOFAR-EoR project:

A running start!

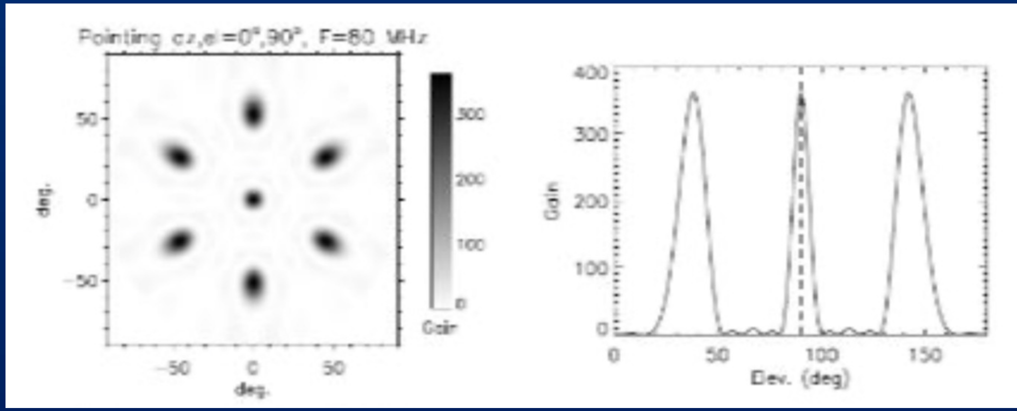
- Bandpass calibration
- RFI flagging
- Averaging
- Calibration-subtraction of A-team and 3c
- DD-calib of NPC sky model (7 clusters)
- ML-GPR foreground removal



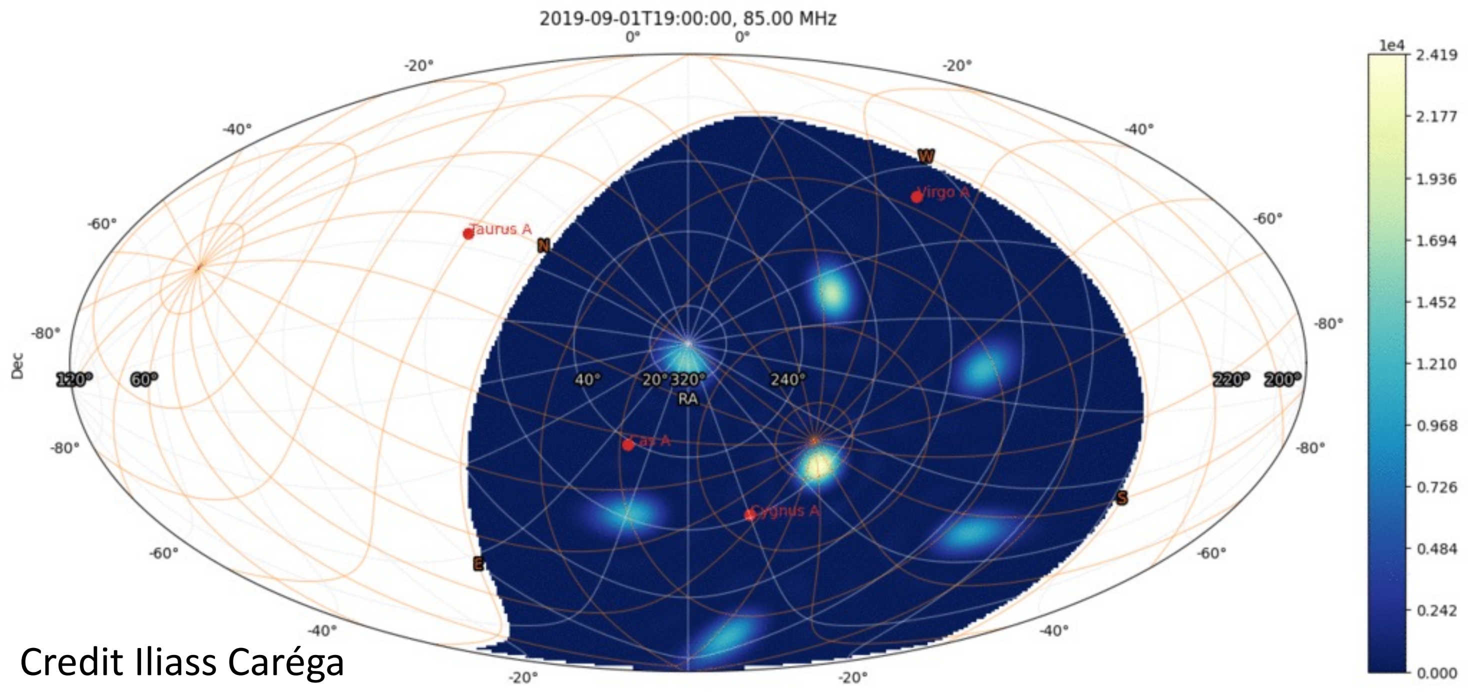
Causes for the excess variance: A-team

Dealing with the beam:

Strong grating lobes



Contamination of NCP by A-team residuals after sky-model subtraction

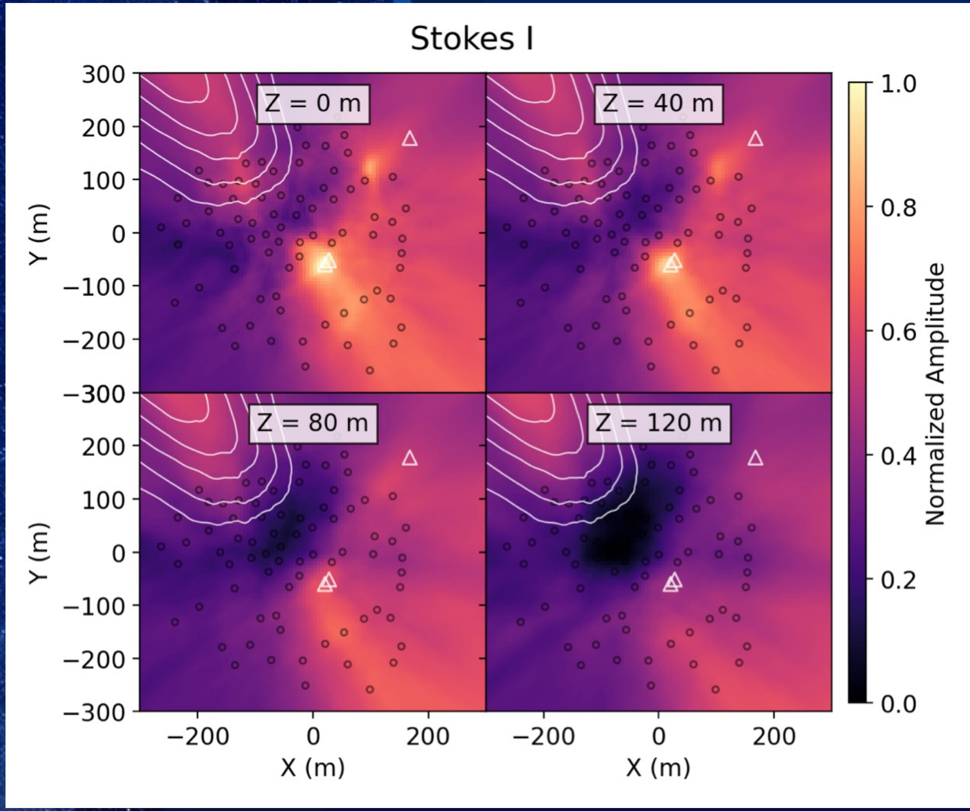


Credit Iliass Caréga

Causes for the excess variance: RFI

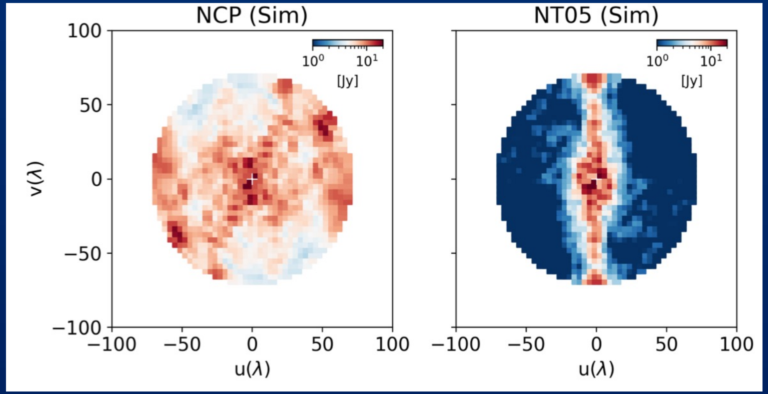
(Munshi et al. 2025)

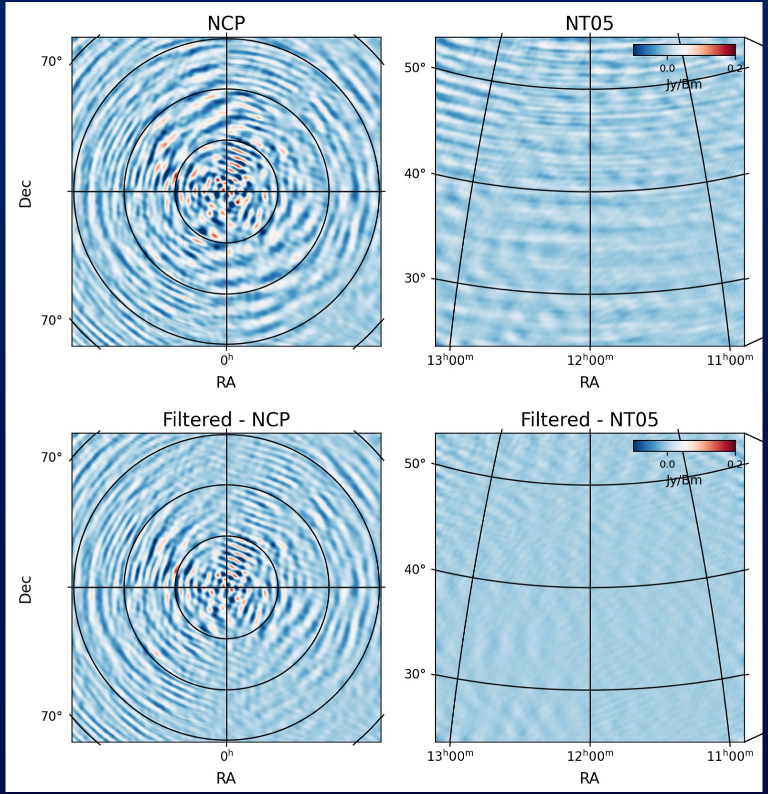
Local RFI idenfied trough near-field imaging

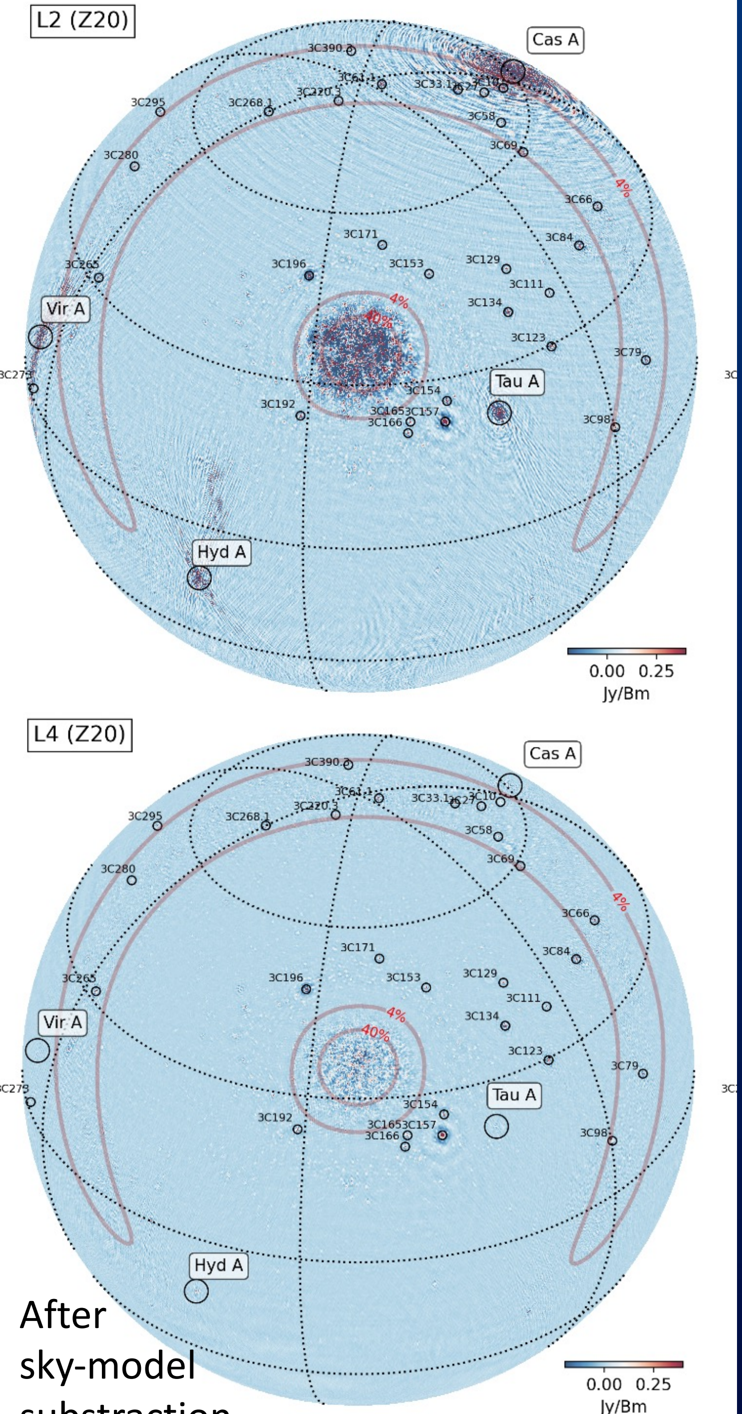


NCP does not allow for efficient treatment

Explore other fields







After sky-model substraction

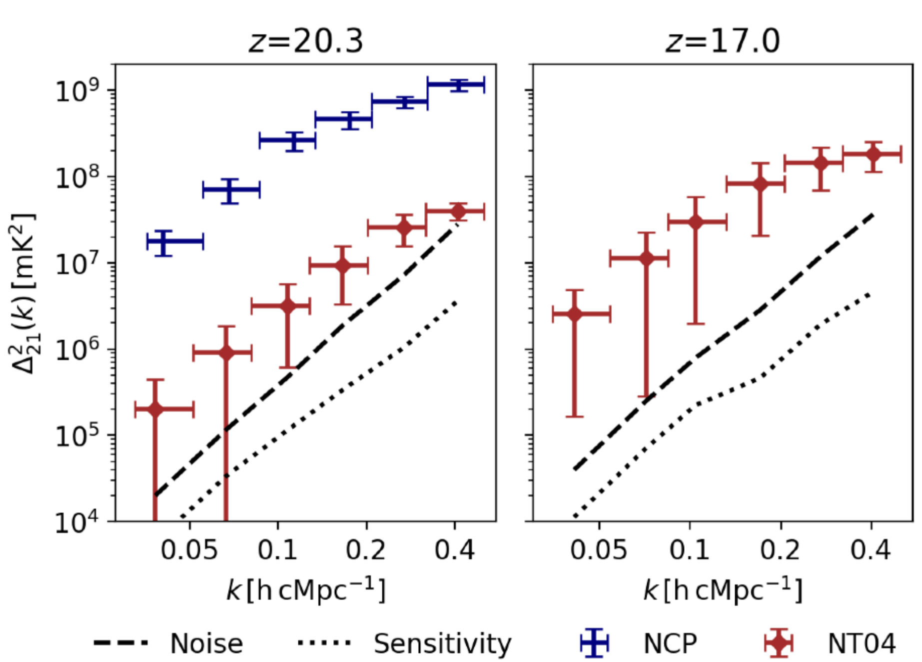
New upper limits

(Munshi et al. In prep)

4 nights of observation on field NT04:

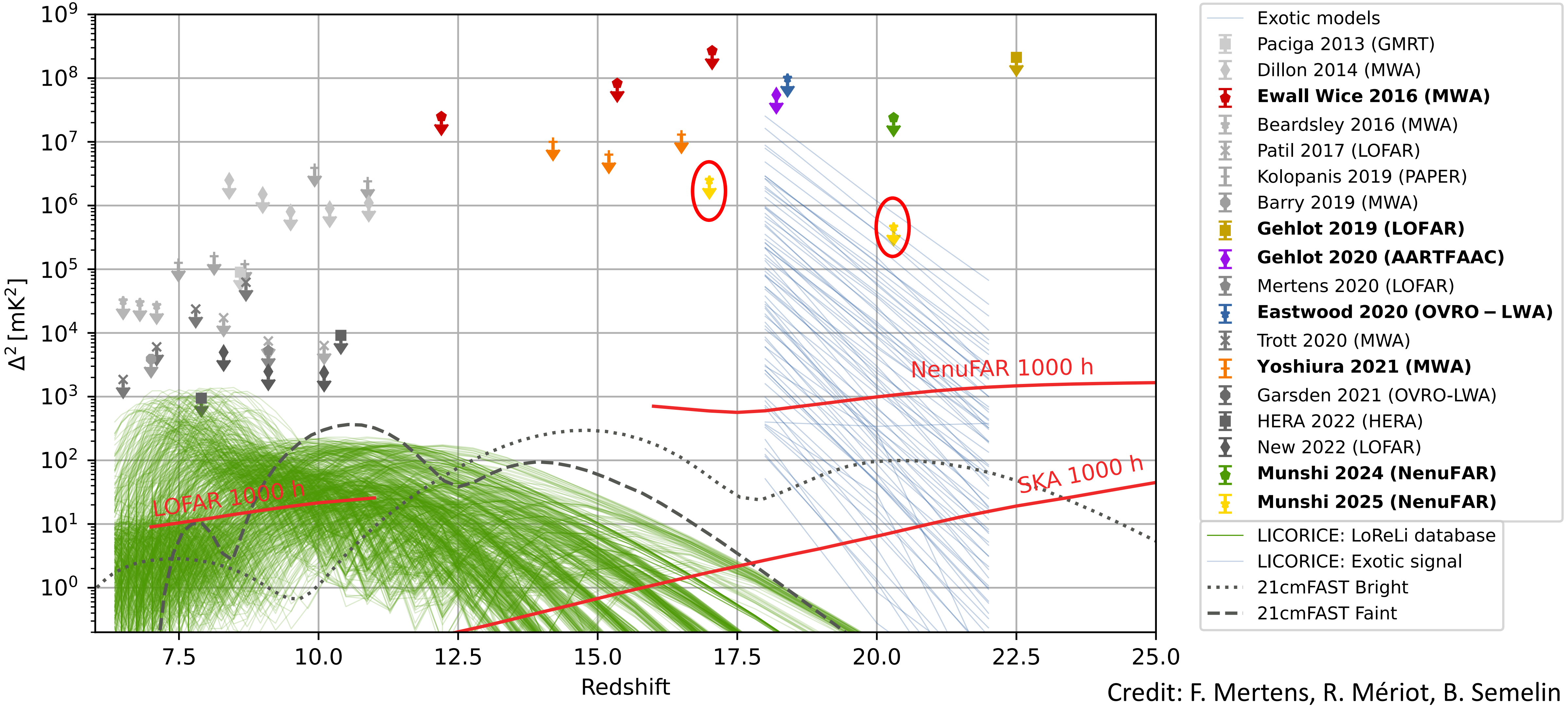
- RFI flagging, data time-averaging
- Sky-model subtraction / calibration
- Component separation (diffusion backg/signal/excess)

1) Reduced excess variance
2) Time-Incoherent => longer integration is possible.



A state of the art

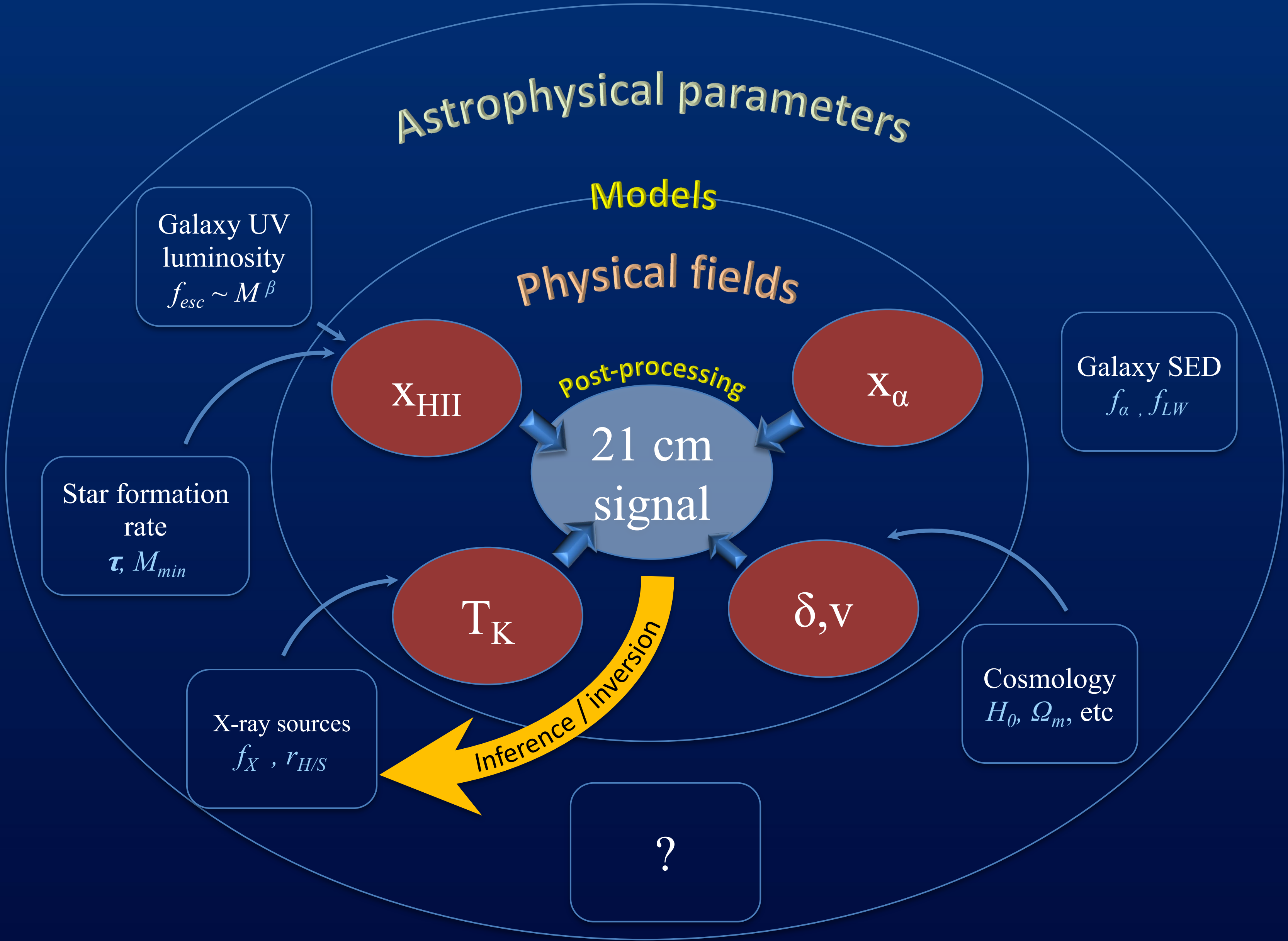
Limits at k around $0.1\ h\ \mathrm{cMpc}^{-1}$



Credit: F. Mertens, R. Mériot, B. Semelin

Part 3:
constraining astrophysics and cosmology with the 21-cm signal

Principle



Astrophysical parameters
Models
Physical fields
Galaxy UV luminosity
$f_{esc} \sim M^{\beta}$
Star formation rate
τ, M_{min}
X-ray sources
f_X , $r_{H/S}$
Galaxy SED
f_α , f_{LW}
Cosmology
H_0, Ω_m, etc
?
x_{HII}
x_α
T_K
δ,v
Post-processing
21 cm signal
Inference / inversion

The sources of stochasticity

If one-to-one mapping: model parameters ⇔ 21-cm signal

=> “simple” inversion problem.

But: sources of stochasticity

Instrumental:

- Separate from model
- Easily averaged (?)
- Possible complex likelihood

Physical:

- Instrinsic to model
  - ✓ Cosmic Var
  - ✓ Sub resol Var
- Averaging?
- Possibly simple likelihood

Building informative summary statistics

Learning from the full 3D signal:

- Recover model parameters + initial conditions (random density field)
=> infer millions of "parameters"
- Field level inference (using HMC) is possible…but very new/hard.

Build summary statistics sensitive mostly to (a few) model parameters:

Example: power spectrum

$$\langle \delta T_B(\vec{k})\, \delta T_B^*(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P(\vec{k})$$

No dimensionality reduction yet because $\langle \quad \rangle$ is an ensemble average.

Building informative summary statistics

Initial Gaussian random field:

- statistically invariant by translation and rotation
- Fully characterized (statistically) by $P(|\vec{k}|)$

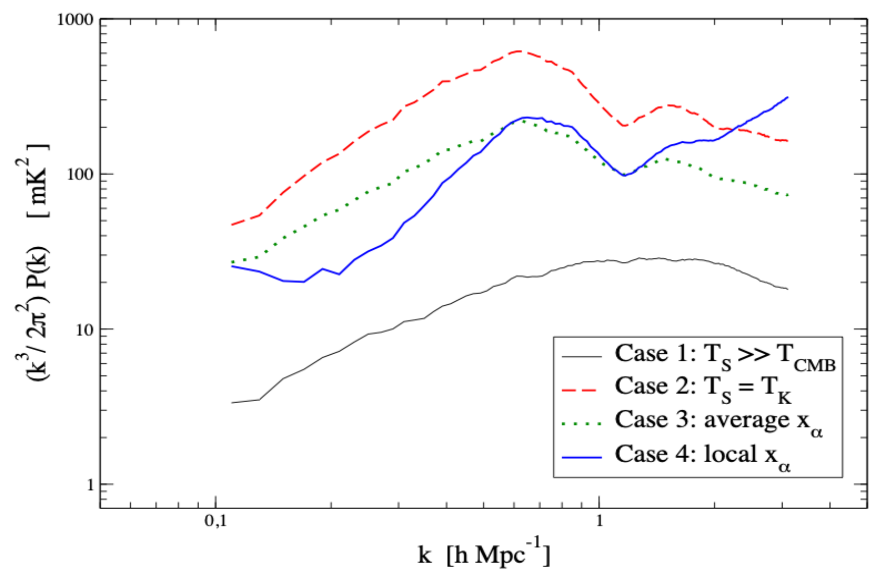
21-cm signal:

- Non-gaussian
- Line of sight (LOS) ≠ other directions (evolution, velocities, etc.)

$(2\pi)^3 I$

Ensem

Ca



Cylindrical powe

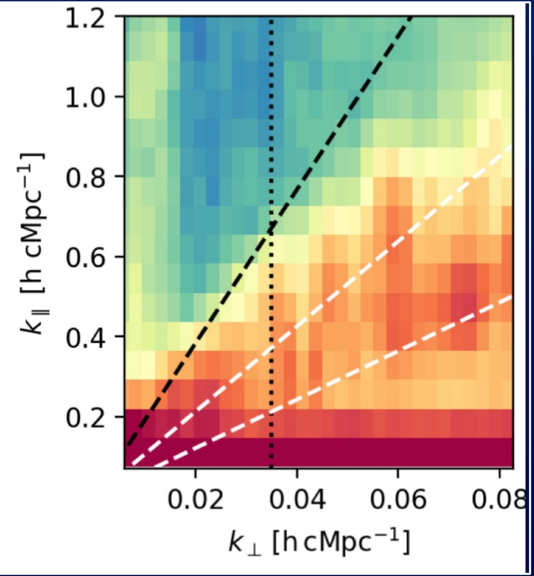
$(2\pi)^3 P(k_{||}, k_\perp, z) = \langle \delta T_B ($

What observers

More inforr

Good for compone

Still not all info



Building informative summary statistics

Non-gaussian summary statistics:

- Bispectrum: $\langle \delta T_B(\overrightarrow{k_1}), \delta T_B(\overrightarrow{k_2}), \delta T_B(\overrightarrow{k_2}) \rangle = V\ \delta(\overrightarrow{k_1} + \overrightarrow{k_2} + \overrightarrow{k_3})B(\overrightarrow{k_1}, \overrightarrow{k_2})$

  (e.g. Simabukuro (2016), Watkinson (2019), etc.)

- PDF: Pixel (probability) Distribution Function →

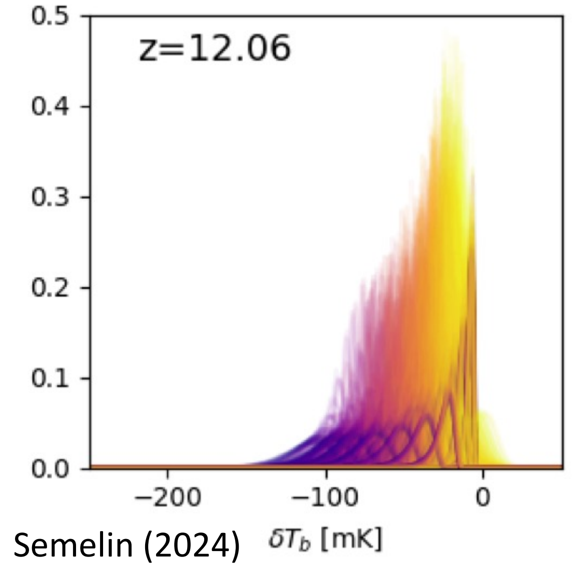
  (e.g. Ciardi (2023), Mellema (2006), Semelin (2017), Semelin (2025))

- Betty numbers, Minkovsky functionals

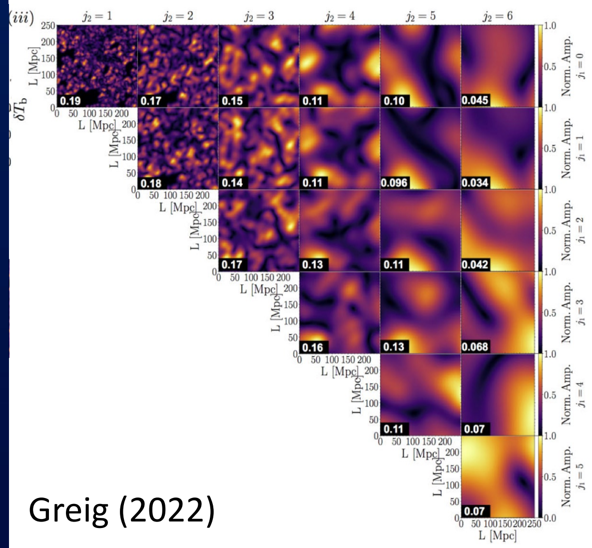
  (e.g. Yoshiura (2017), Giri (2021), etc.)

- Scattering transforms, wavelet based statistics →

  (Greig (2022), Hothi (2024))



Semelin (2024)

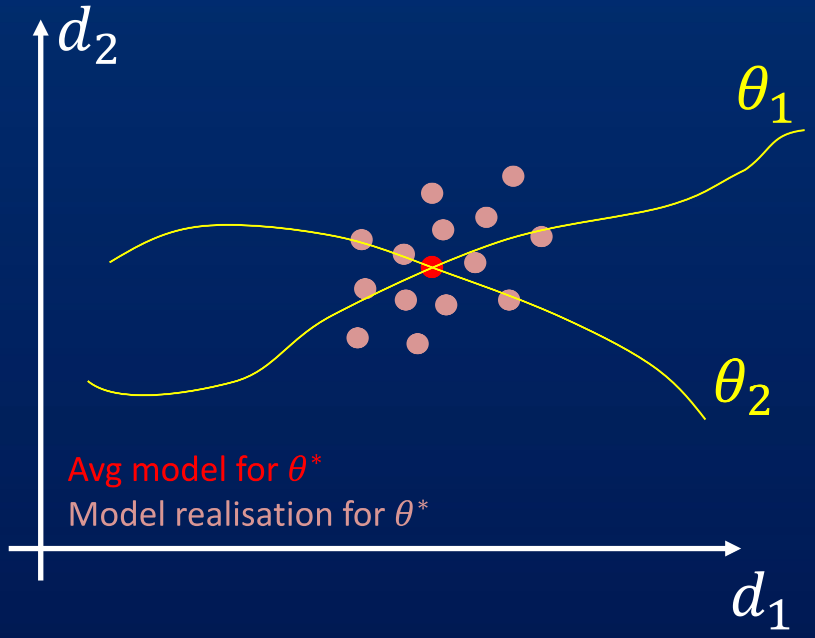


Greig (2022)

Quantifying information content: Fisher formalism

Definition of the Fisher information matrix:

$$\mathcal{F}_{ij}(\theta^*) = \mathbb{E}\left(\frac{\partial}{\partial\theta_i}\ln\mathcal{L}(d|\theta)\Big|_{\theta^*} \times \frac{\partial}{\partial\theta_j}\ln\mathcal{L}(d|\theta)\Big|_{\theta^*}\right)$$



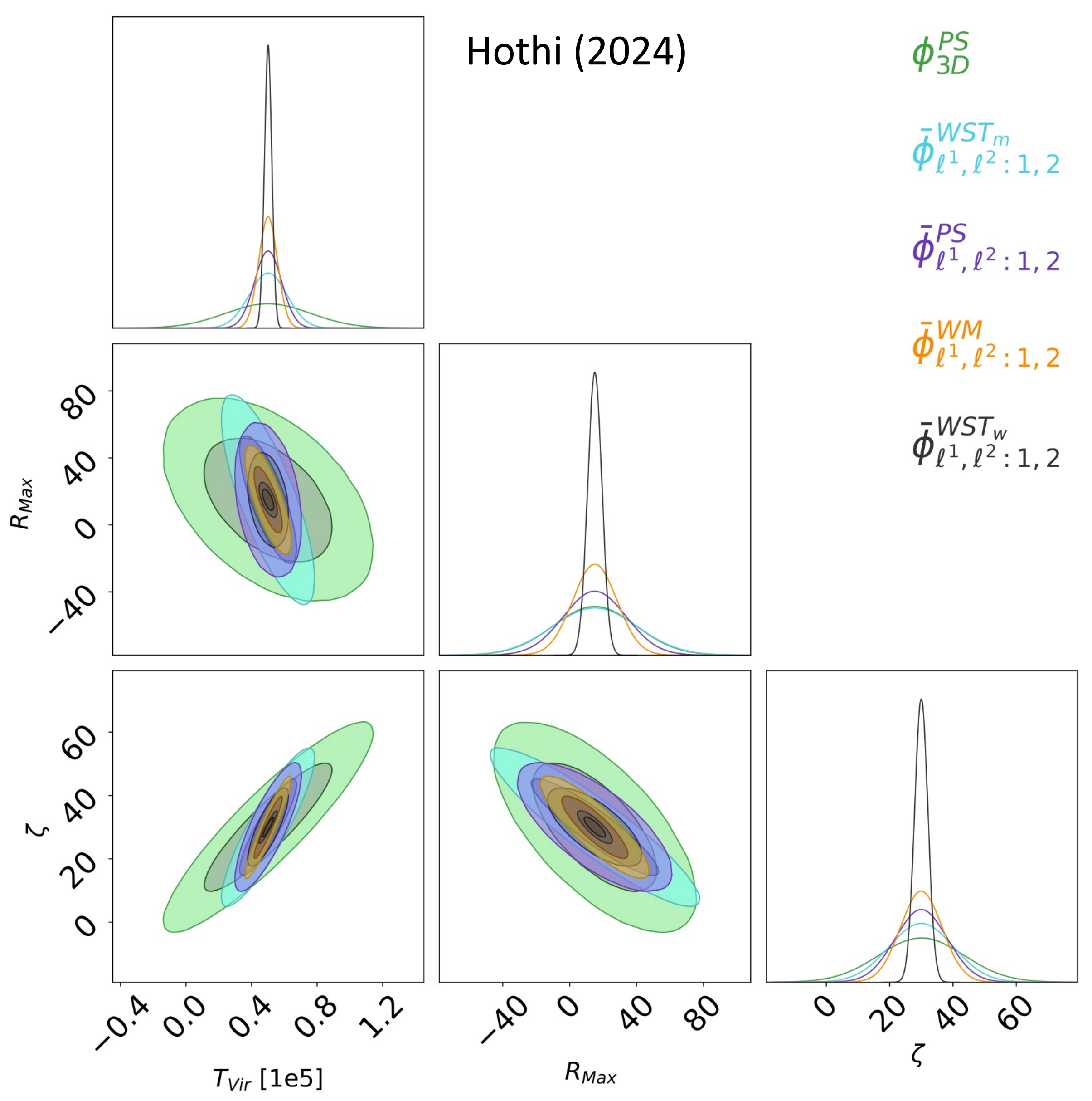
Very simple case: $\mathcal{L}$ gaussian
$d_1(\theta_1), d_2(\theta_2)$
variance σ_i indep of θ, mean μ_i

$$\mathcal{F}_{ii}(\theta^*) = \left(\frac{\partial\mu_i}{\partial\theta_i}\frac{1}{\sigma_i}\right)^2$$

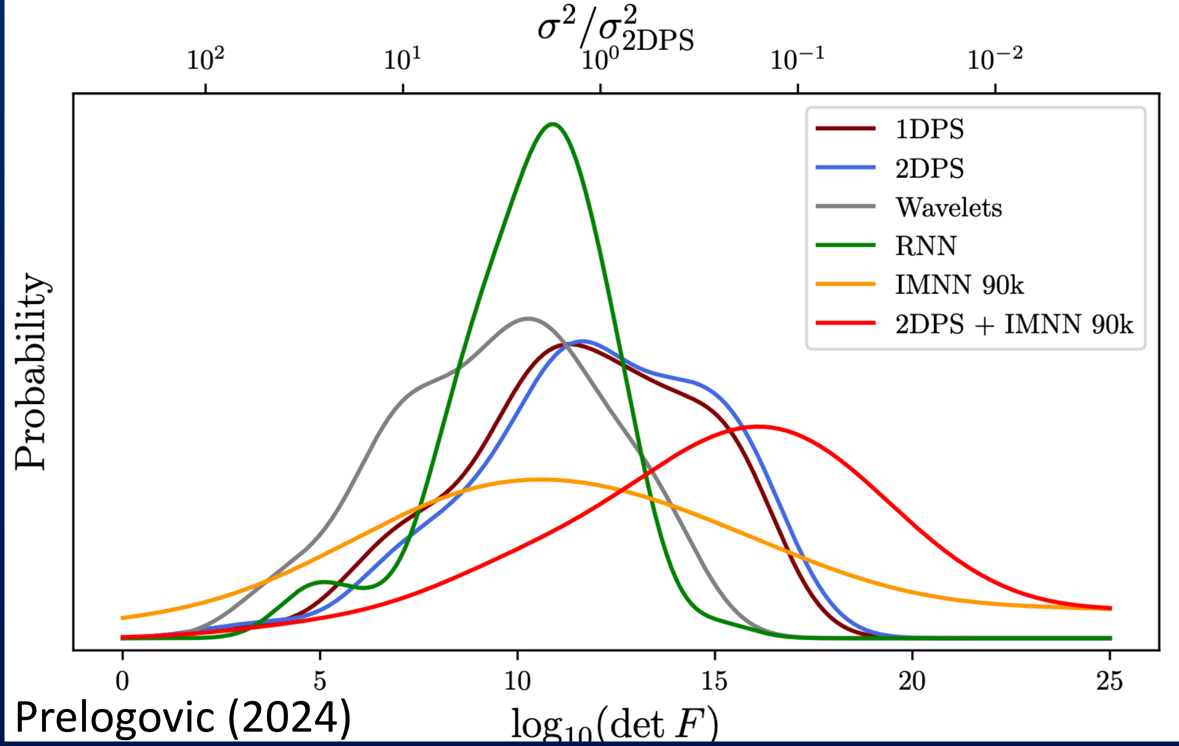
=> $\mathcal{F}_{ii}$ is the (square of) ratio of sensitivity of d to θ, to sentivity of d to stochastic noise

Large det($\mathcal{F}$) means good sensitity to θ => tight constraints

Application of Fisher to 21-cm summary statistics



Assessing det($\mathcal{F}$) over the whole prior :



New comparison expected in next SKA science book

Limitations of Fisher analysis

- An explicit likelihood $\mathcal{L}$ is required to compute $\mathcal{F}$

- Gaussian assumption is common

- Many samples are required to compute $\mathbb{E}$ (gradients sensitive to noise)

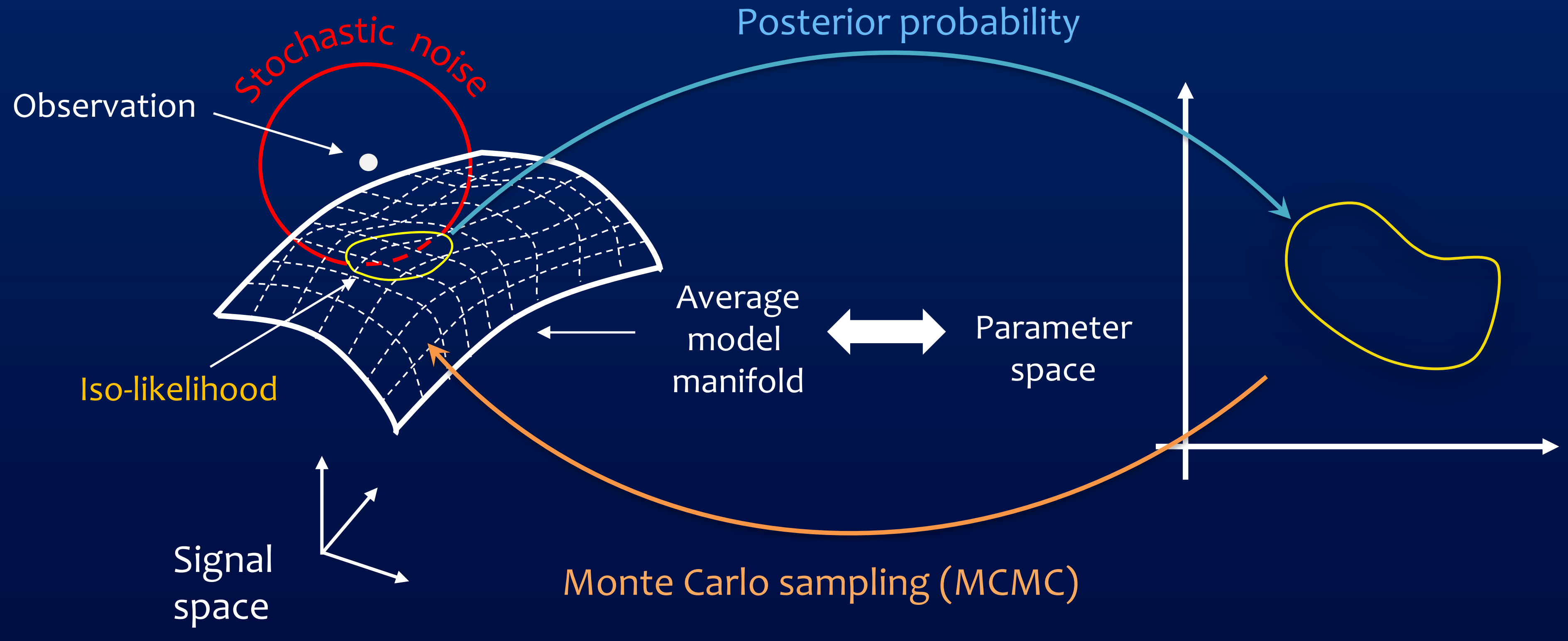
Fisher is NOT an inference:

- A fiducial AVERAGE model is required to compute $\mathcal{F}$
- Can only be provided with chosen fiducial parameters θ^*
- Real data is ONE stochastic realisation of the model (best case !)

My view of Bayesian inference

Toy model

- 3-valued statistics of the signal
- 2-parameter **stochastic** model



Bayesian inference: refresher

Goal: compute **posterior probability** that data *d* result from parameters θ

Use Bayes theorem:

Prior Likelihood

$$P(\theta|d) = \frac{\pi(\theta)\mathcal{L}(d|\theta)}{P(d)}$$

Best case:

- $\mathcal{L}$ analytical function of *d* and the average model prediction $d_M(\theta)$
- But $d_M(\theta)$ is usually not known analytically (only numerically)

=> $P(\theta|d)$ easily sampled but no closed form

Realistic case:

- $\mathcal{L}$ is unknown, but hopefully not too far from Gaussian

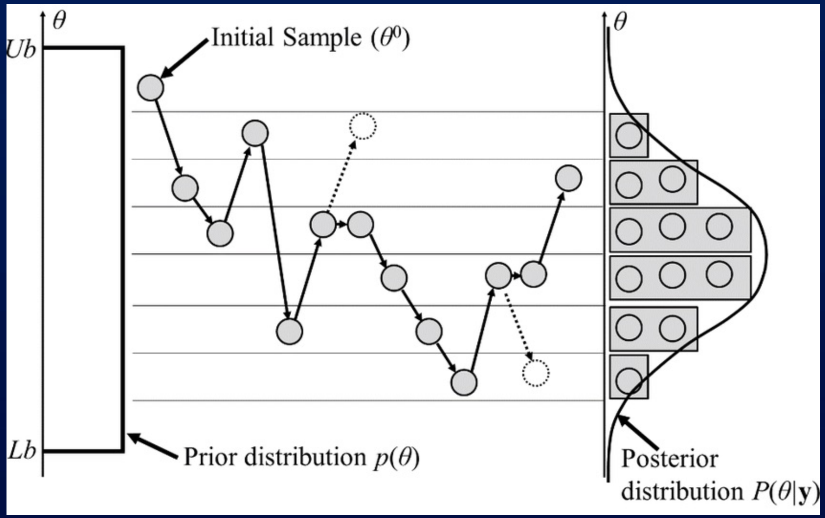
Usual approach: MCMC

We know or assume analytic form for the likelihood.

Algorithm: sampling such that parameter space sample density ∝ posterior

Example: Metropolis-Hasting

- Random walk in parameter space (step size from distribution)
- Accept new position with proba $\min\left(1, \frac{\mathcal{L}(d,\theta_{n+1})\pi(\theta_{n+1})}{\mathcal{L}(d,\theta_n)\pi(\theta_n)}\right)$



From wikipedia

Limitations:
- Slow convergence (nb steps)
- Multi-modal posteriors

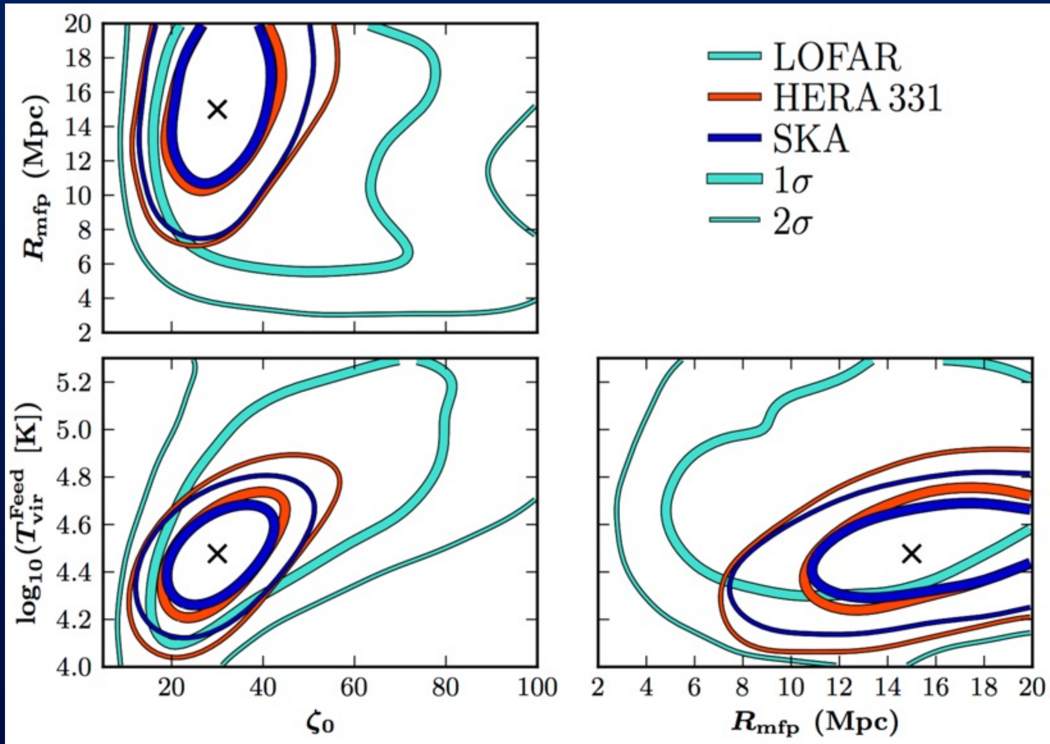
Alternatives:
- EMCEE, HMC, …
- Nested sampling

MCMC for 21-cm inference

10^5 to 10^6 steps for 5-parameters MCMC inference
One model computation for each step
Even with a few seconds/model run => high CPU cost

Exemple with 21cmFAST:
(Greig & Mesinger 2015)

- Perform inference on power spectrum
- Assume uncorrelated gaussian likelihood
- 128^3 resolution => 3 sec per step



Necessary improvements

Assumed likelihood is an approximation

Inference with more complex models

Inference with other summary statistics

Using machine learning to improve 21-cm signal inference

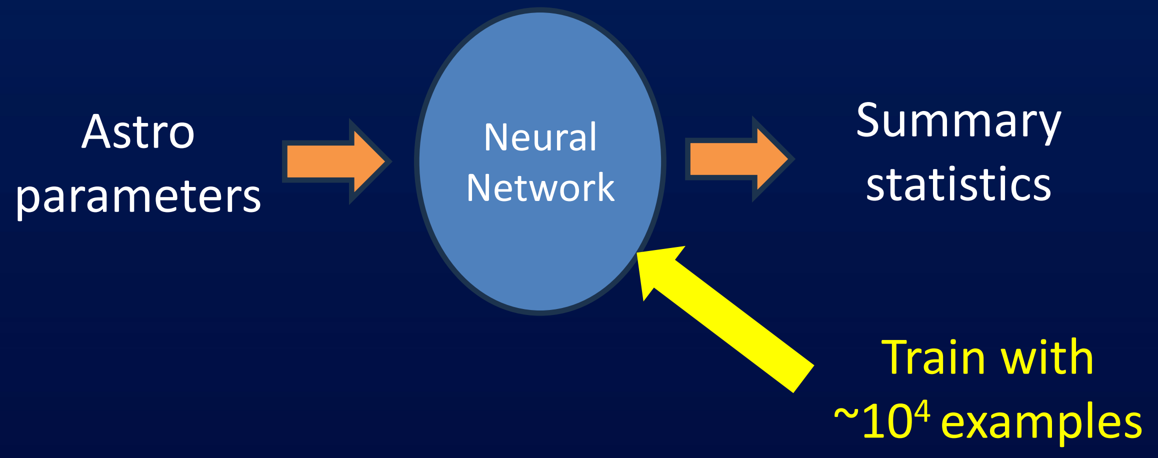
Training emulators

Typical 4-5 param MCMC inference: 10^5-10^6 steps

-> Slow with semi-analytical models (21cmFast -> few sec / step)

-> Impossible with full radiative transfer simulations

Replace model with fast emulator:



Training emulators

Fist employed to emulate semi-analytic models by Jennings et al. (2019)

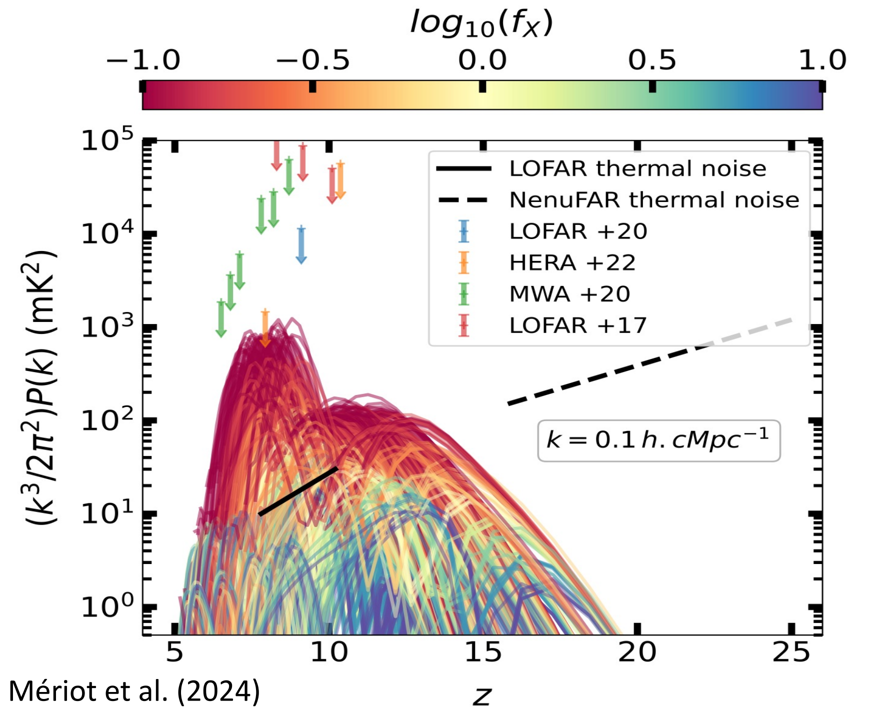
Application to full RT simulations?

Need a database of simulations to learn from:

Adap code to run realistic simu in ~500 CPUh

(instead of > 10^5 CPUh)

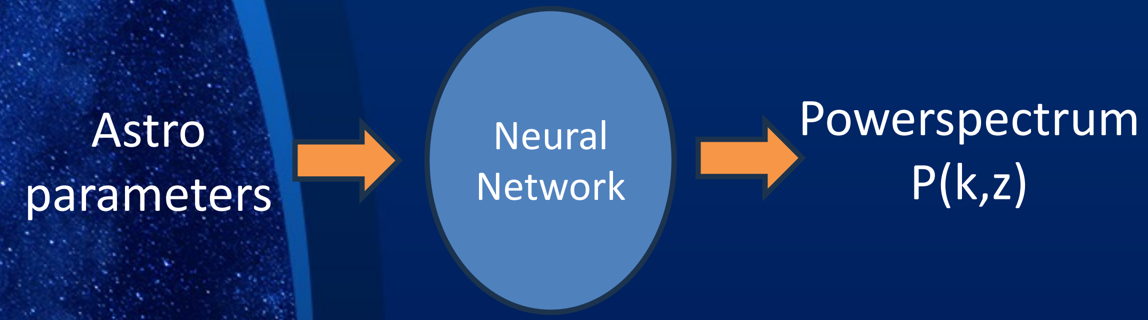
Loreli II, 10^4 simu, 5 param, 1.5 Po
(https://21ssd.obspm.fr)



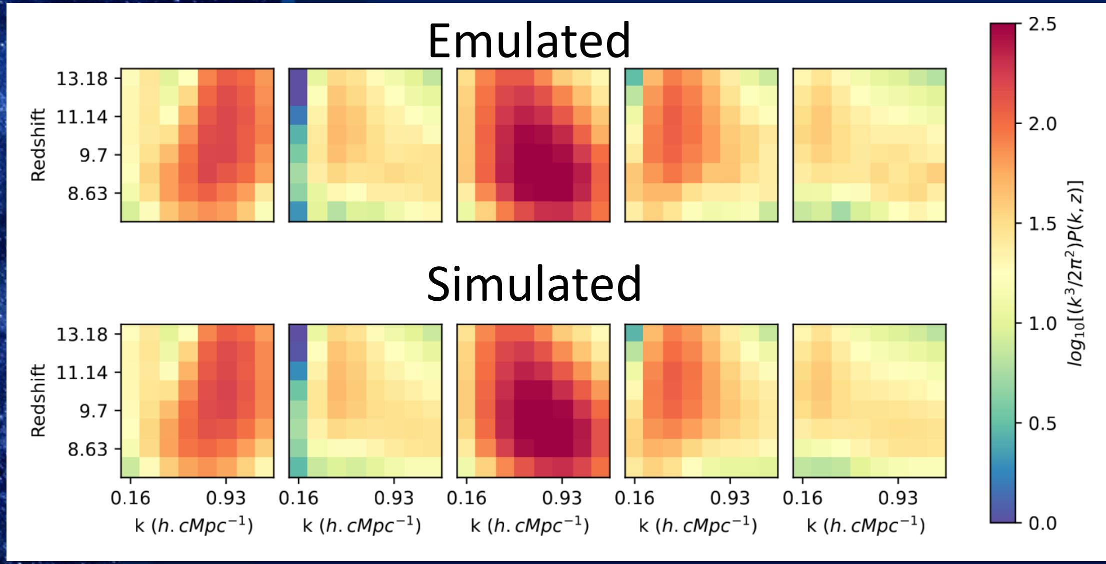
Mériot et al. (2024)

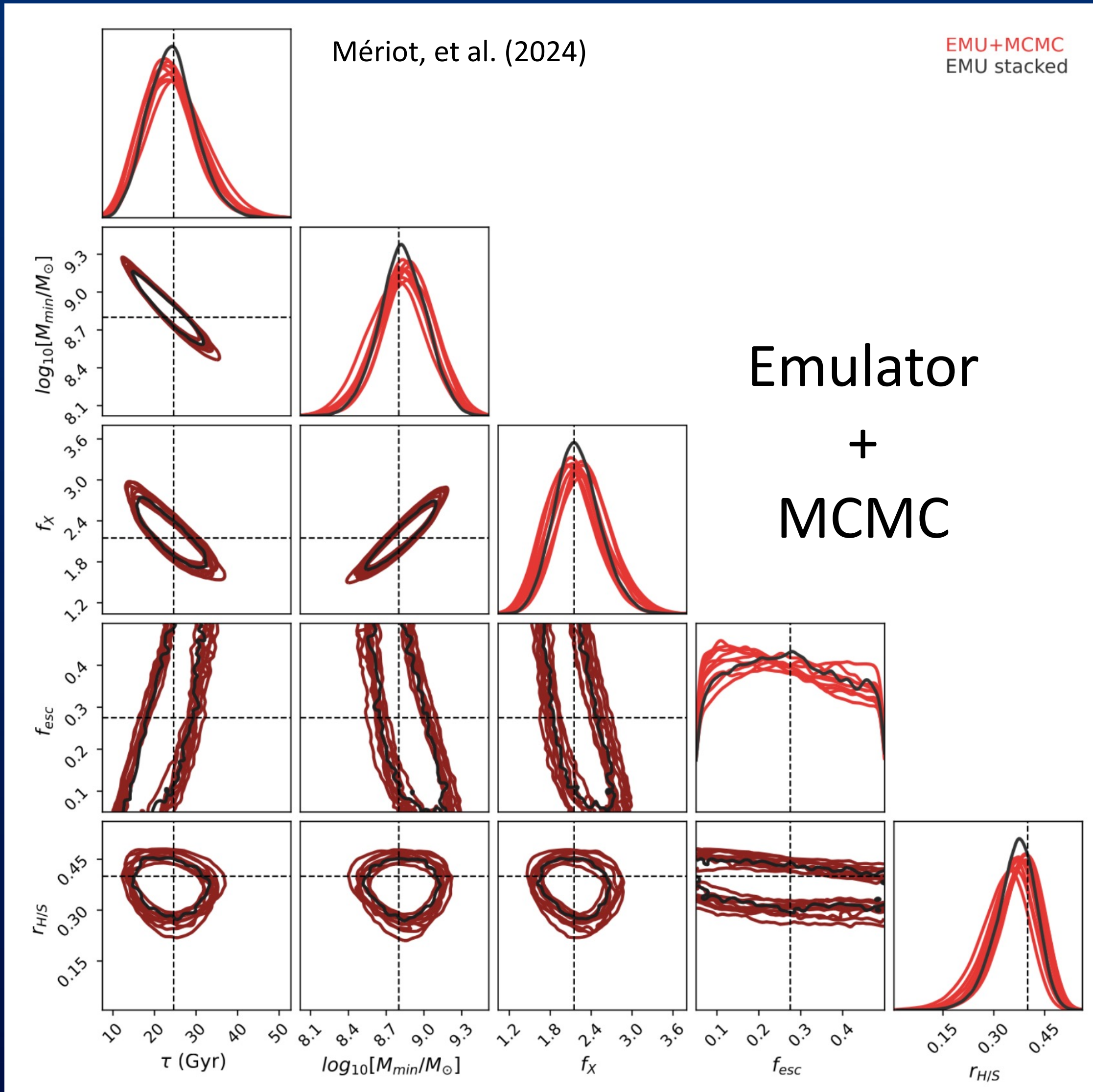
Training emulators

Train NN with Loreli II:



Percent level accuracy:





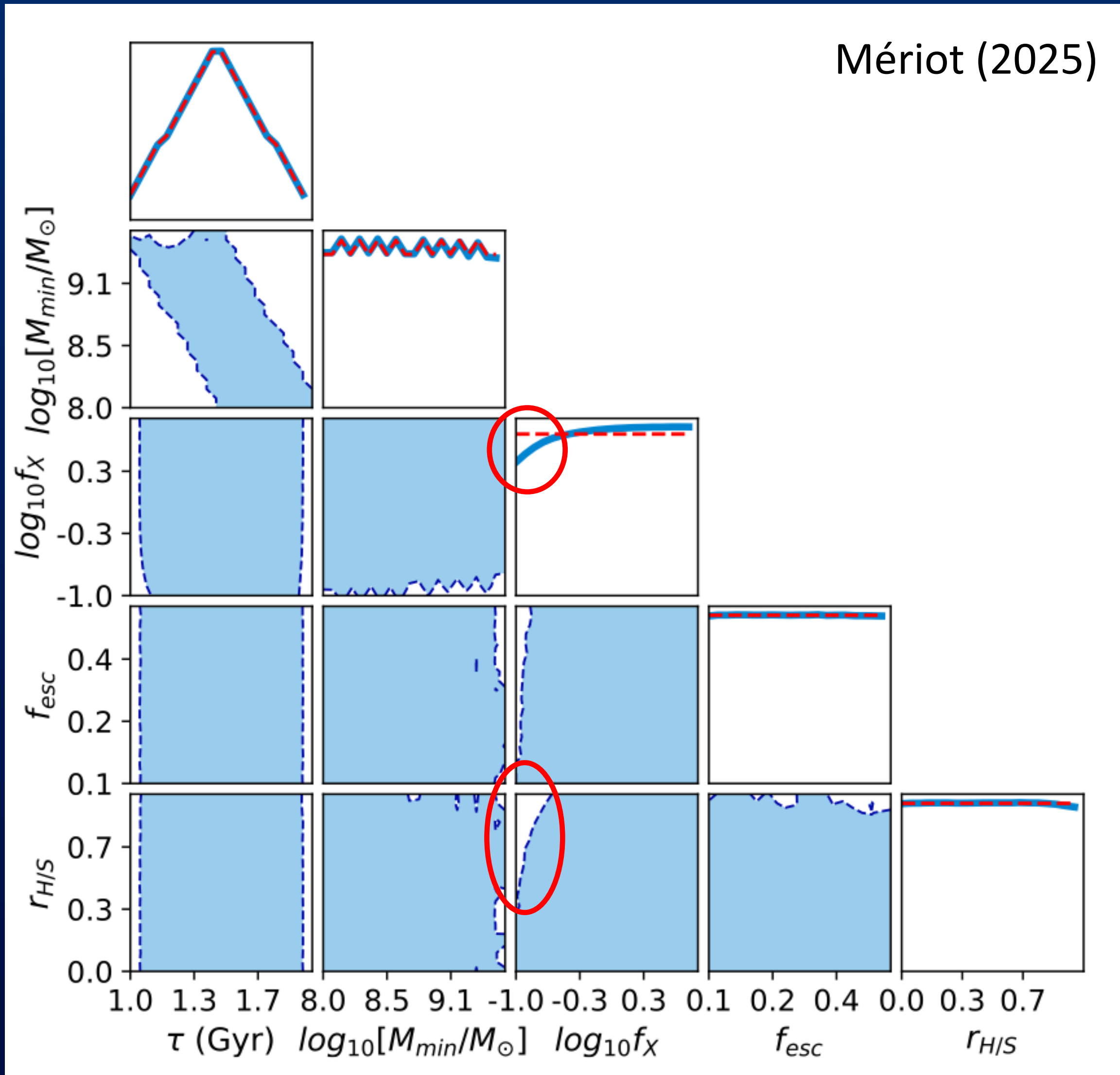
See also Breitman et al (2024): 21CMEMU

Application to HERA upper limits

Current HERA upper limit disfavor a zero Xray reinoization. (e.g. Abdurashidov 2022)

Same qualitative conclusion is reached with NN emulator + full RT simulations.

Quantitative agreement between methods yet to be reached.

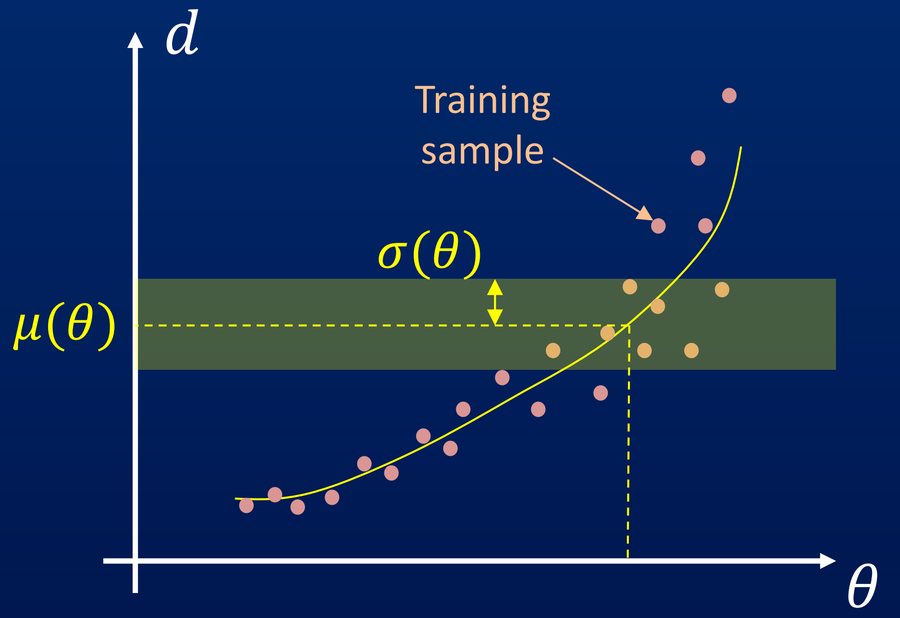


Simulation based inference (SBI)

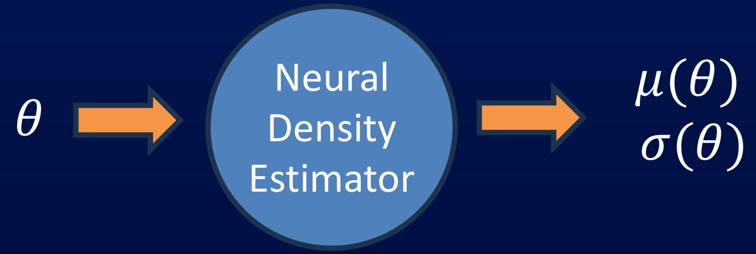
Sesto workshop 2025: “Who understands SBI ?” -> 4 raised hands…

Explanation with a toy model:

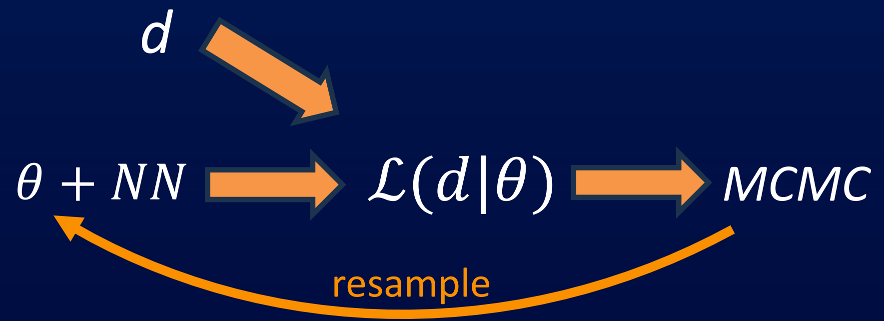
- 1 parameter θ
- 1 data *d*
- Gaussian likelihood with **unknown** $\mu(\theta)$ and $\sigma(\theta)$



Use a Neural Network + training sample to lean $\mu(\theta)$ and $\sigma(\theta)$



$$Loss = \min_{w_i} \sum_k -\log \mathcal{L}_{NDE}(d_k|\theta_k, w_i)$$



SBI for real applications

Data (summary statistics) vector of dim $\gtrsim 10$.

=> Full co-variance matrix needs to be learned (dim 100 to 1000)
… as a function of N parameters θ_i

NN training can become tricky.

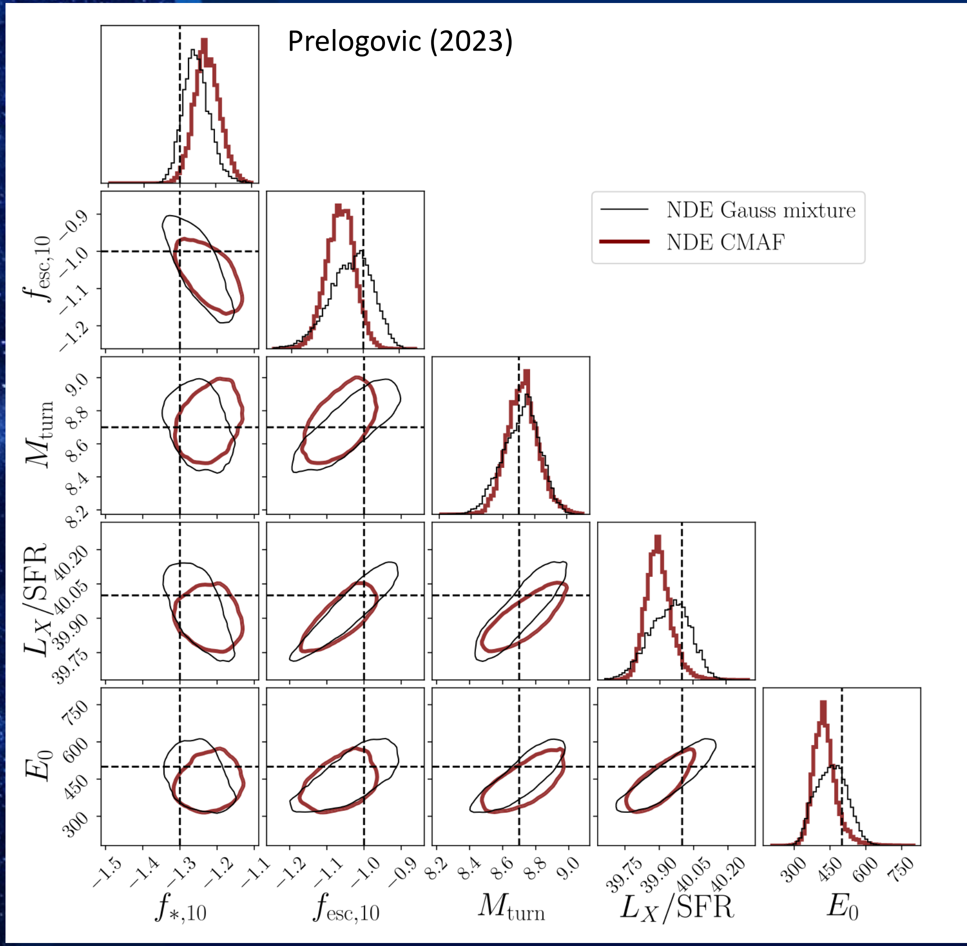
=> Learning the posterior (NPE) directly reduces the dimensionality.

What if likelihood is not gaussian?

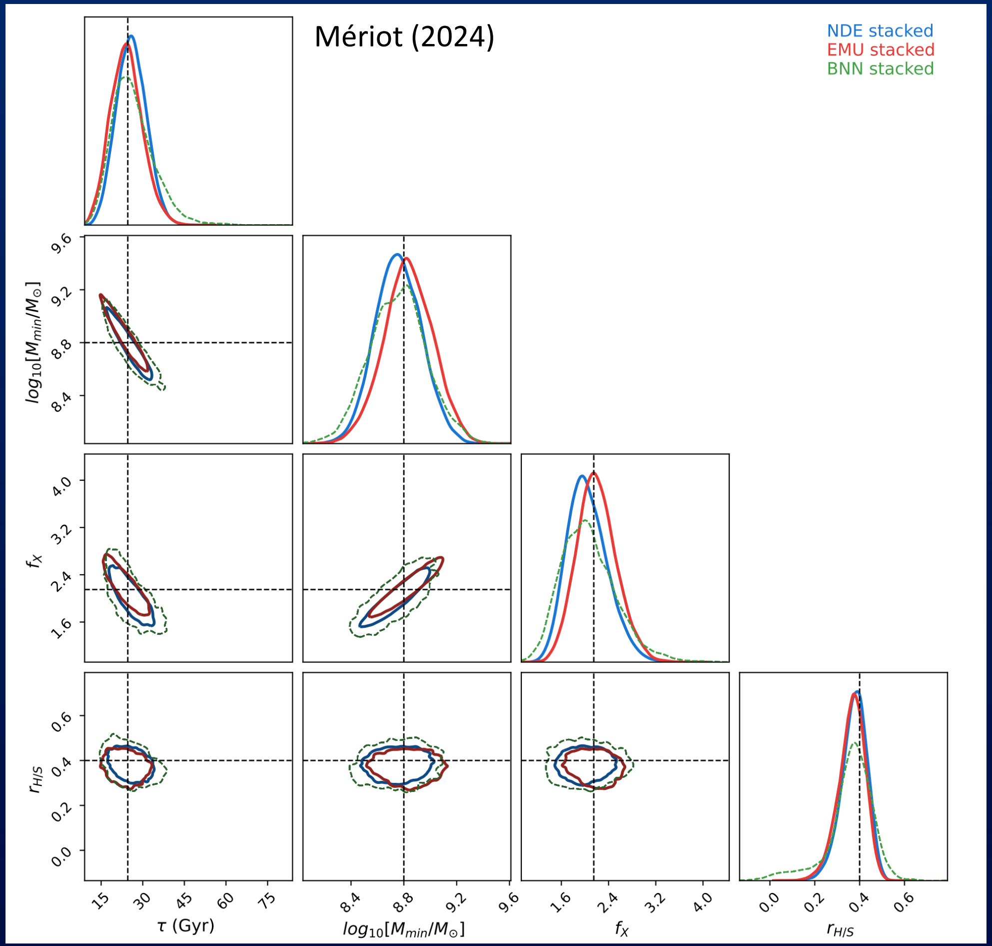
Approximate with Gaussian Mixture Models, MAF, normalizing flows, etc.

Application of SBI to 21cm Inference

With semi-numerical simulations



With full RT simulations



Validating SBI

Principle:

- θ_{true} is a draw from the « true » posterior
- Should be evenly distributed in each quantile

=> run many inferences with different signals

=> Build quantile histrogram

=> check flatness

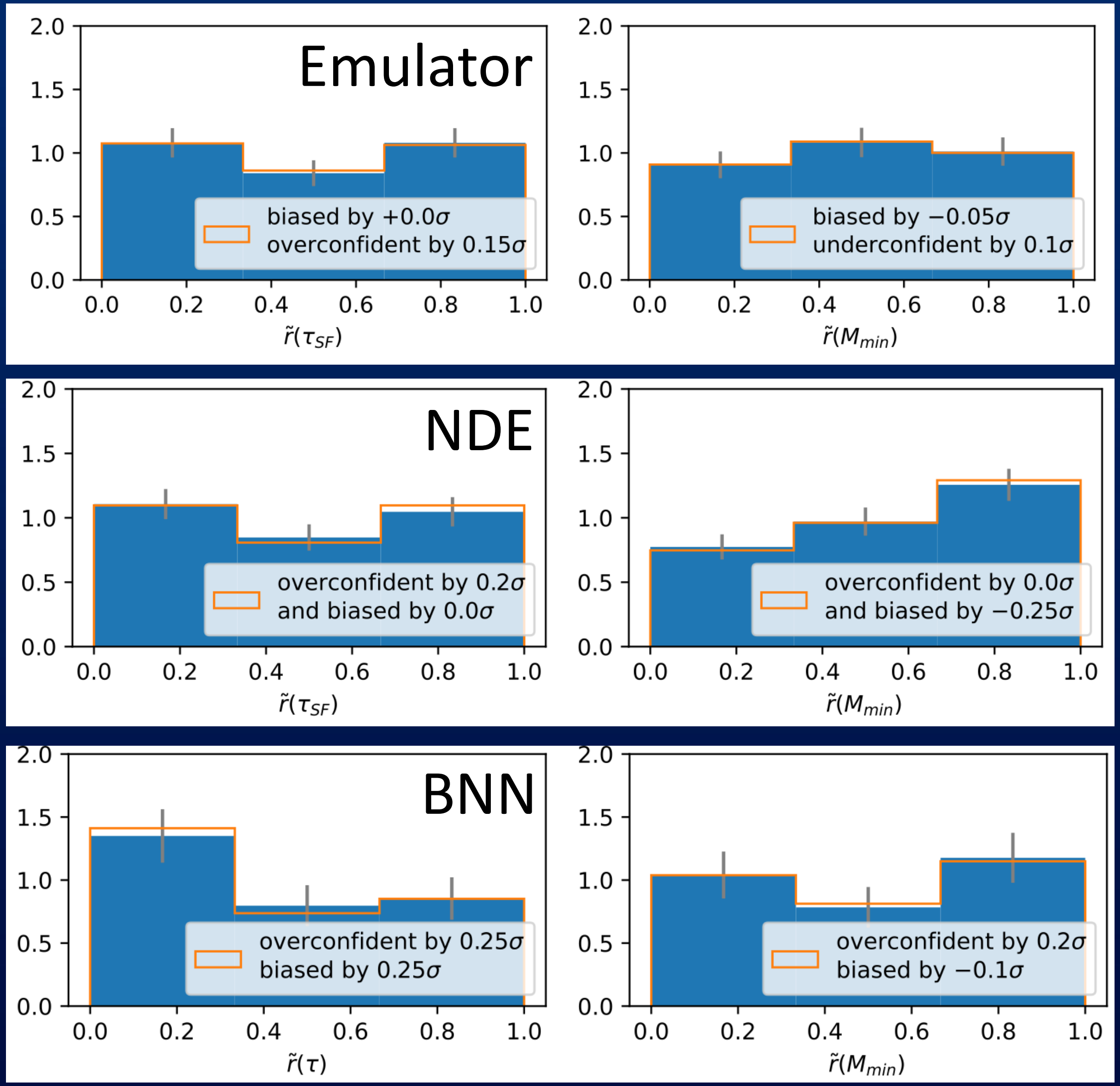
1000 inferences with Loreli II signals:

Compare with toy gaussian model

=> Bias ~ 0.2 σ

=> over/under confident by ~20%

100h noise + Loreli II => emulator performs best!



Increasing information with SBI

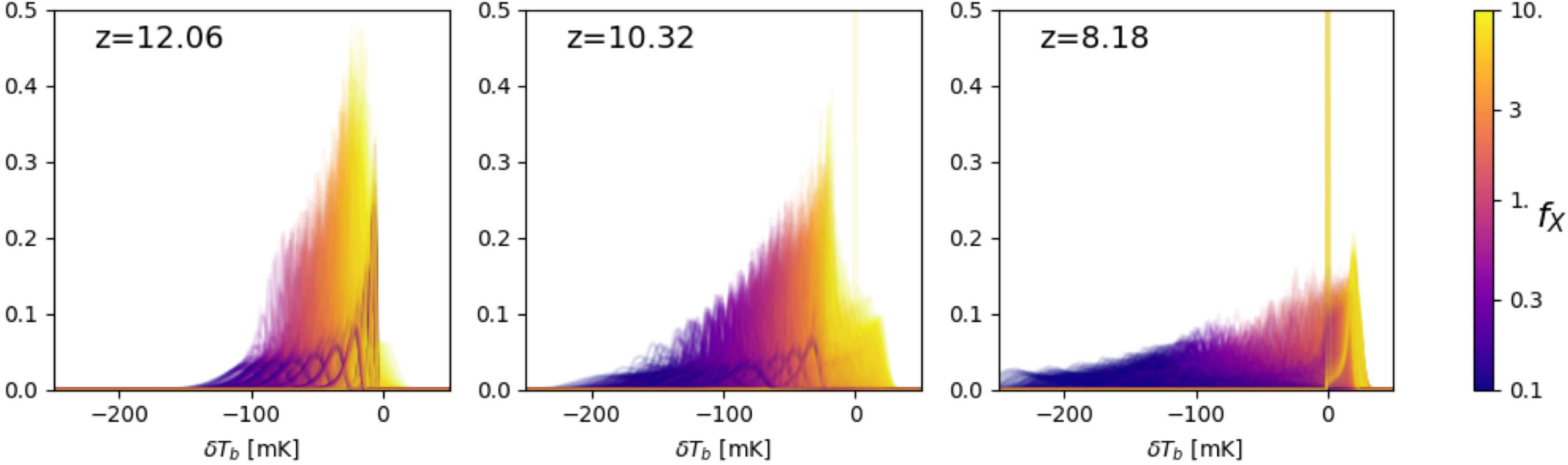
Combining **several** summary statistics:

- no analytic form for the likelihood
- not independent => correlations

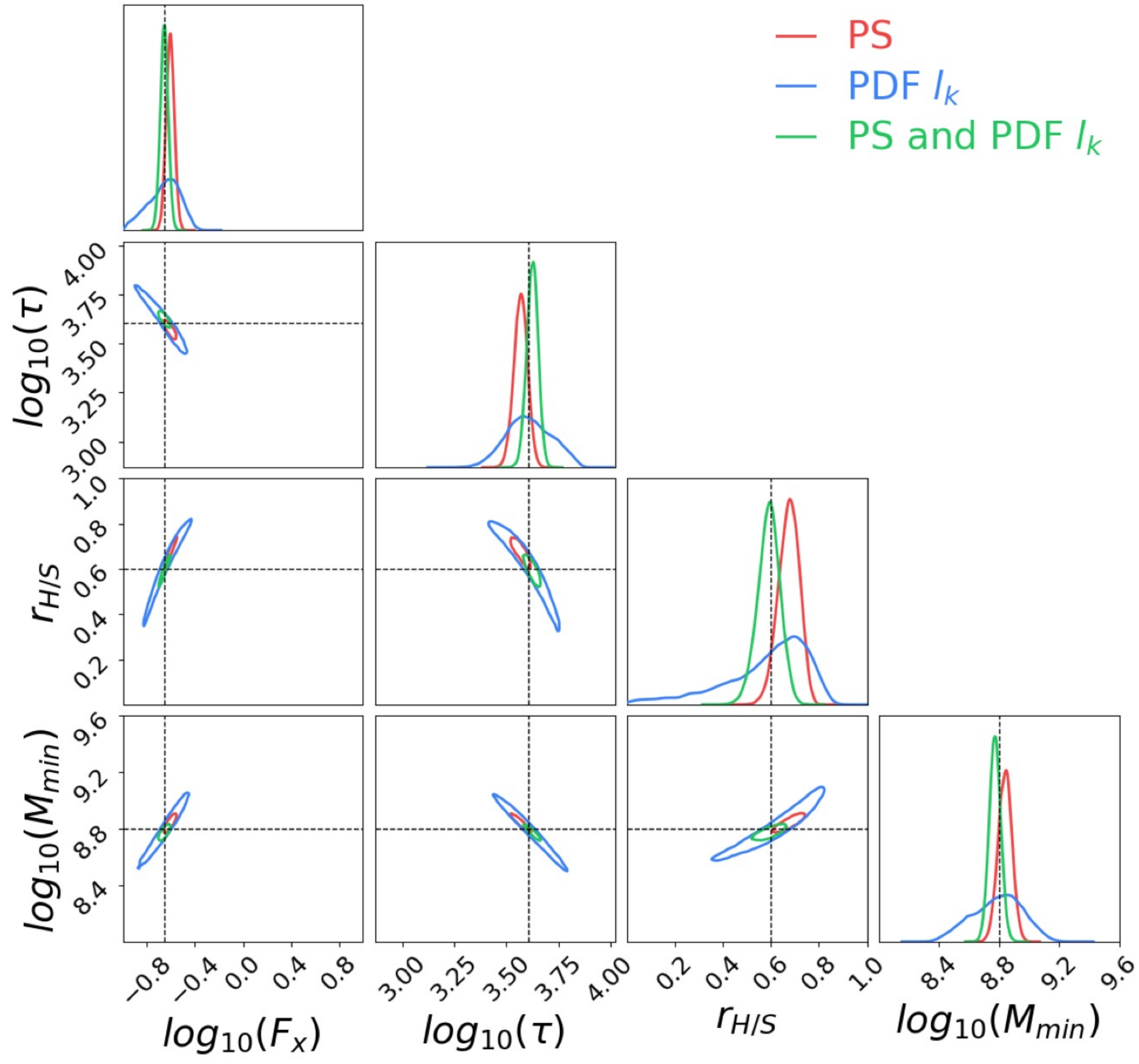
(see also Prelogovic et al. 2024)

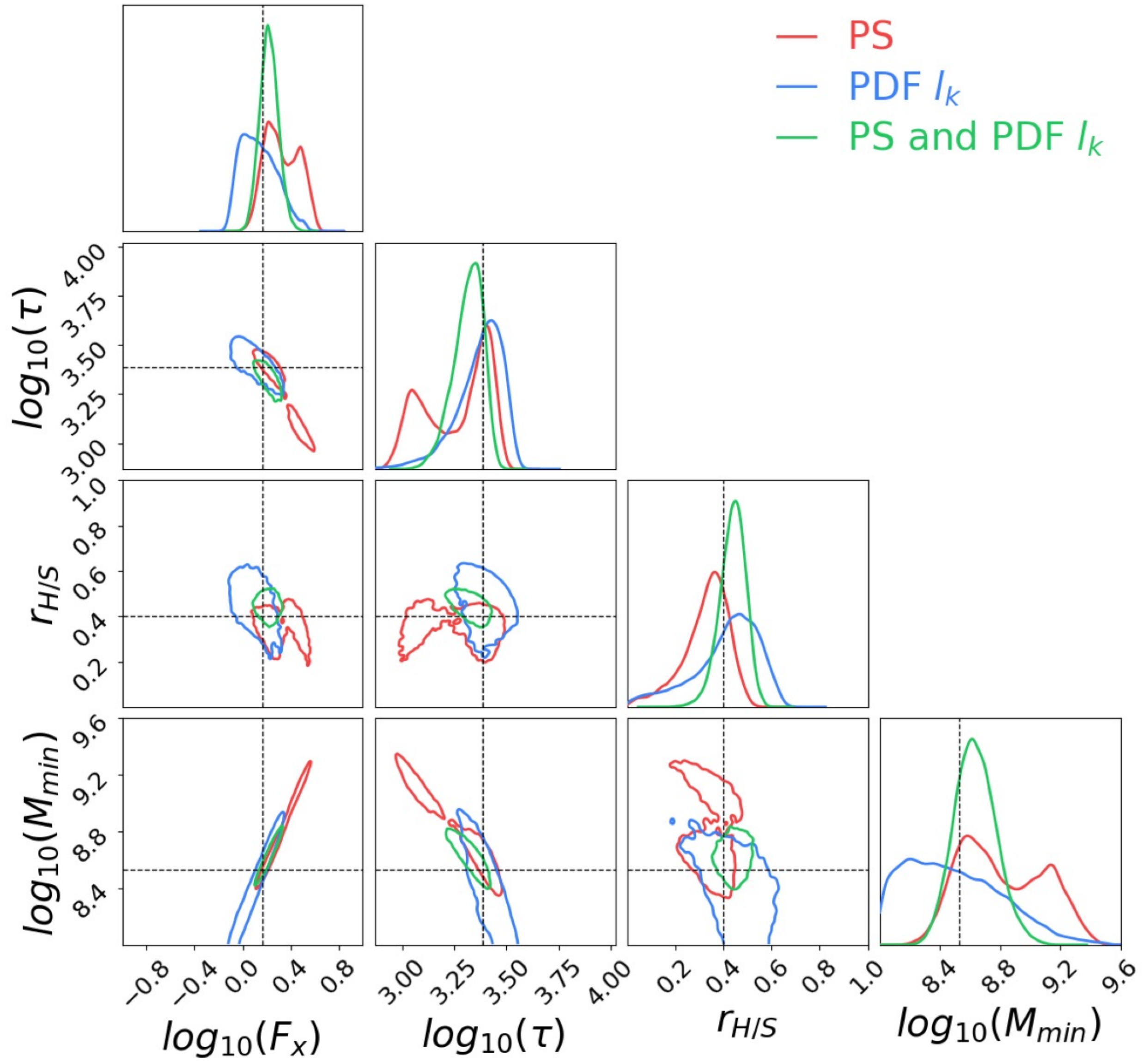
Example with 21-cm signal:

- Power spectrum + Pixel Distrib Func
- Summarize PDF with linear moments
- 5 summaries x 3 redshifts for each
- Fit joint likelihood with NDE



Examples of inferences





Information gain

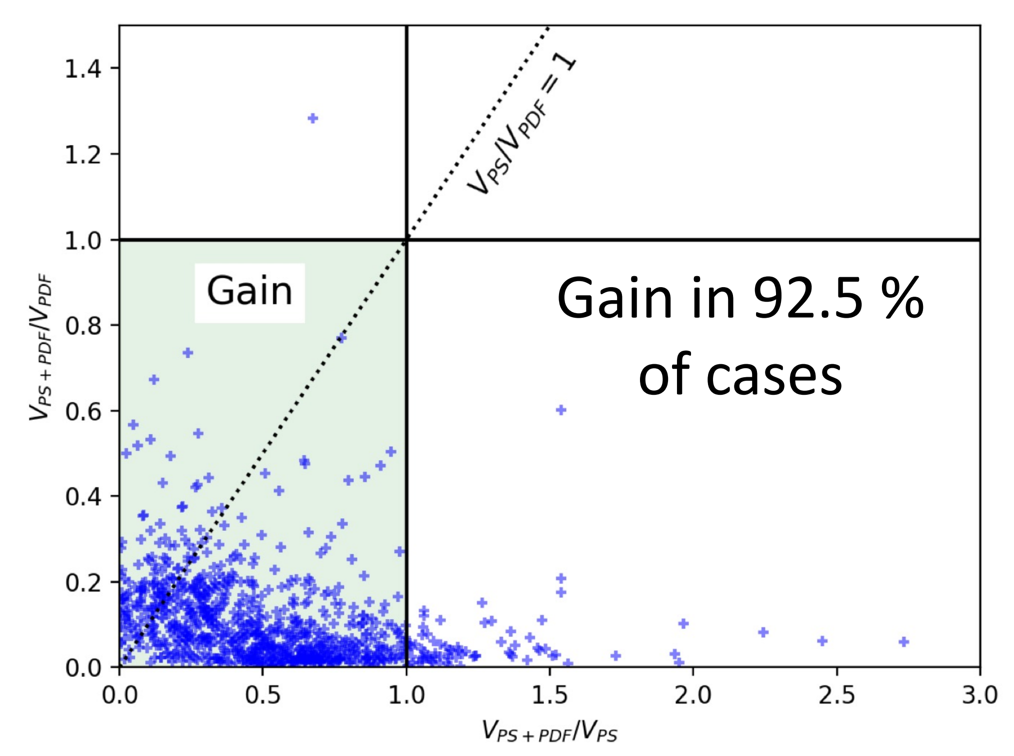
Quantifying the information gain:

- Average 1-sigma of marginalized posteriors
- Average generalized variance of 4D posterior: $\mathrm{V} = \det(\Sigma)^{1/2}$

Average over 908 inferences:

Statitics	PS + PDF l_k	PS	PDF l_k	PDF m_k
$\langle V \rangle \times 10^7$	0.48	1.33	7.53	8.58
$\langle \sigma_{\log_{10}(f_X)} \rangle$	0.019	0.024	0.0375	0.0369
$\langle \sigma_{\log_{10}(\tau)} \rangle$	0.052	0.075	0.080	0.087
$\langle \sigma_{r_{H/S}} \rangle$	0.035	0.049	0.060	0.067
$\langle \sigma_{\log_{10}(M_{min})} \rangle$	0.063	0.087	0.109	0.109

(smaller is better)



Qualitatively similar to Prologovic et al. (2024),
But difficult to compare quantitatively

Thank you!