The Dark Universe, Les Houches, 21-25.07.2025

CMB physics

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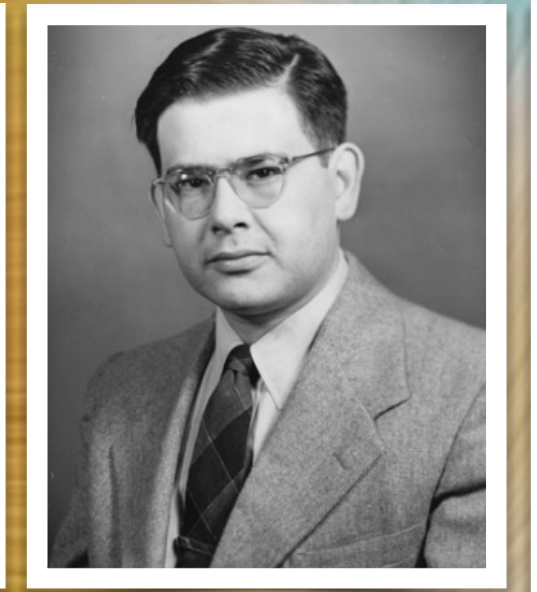
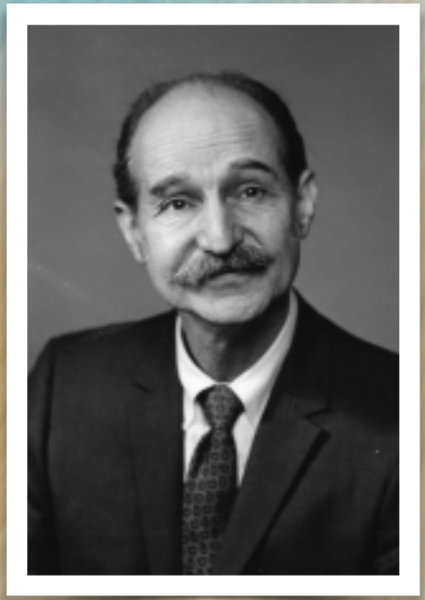
Cosmic Microwave Background: an early prediction!

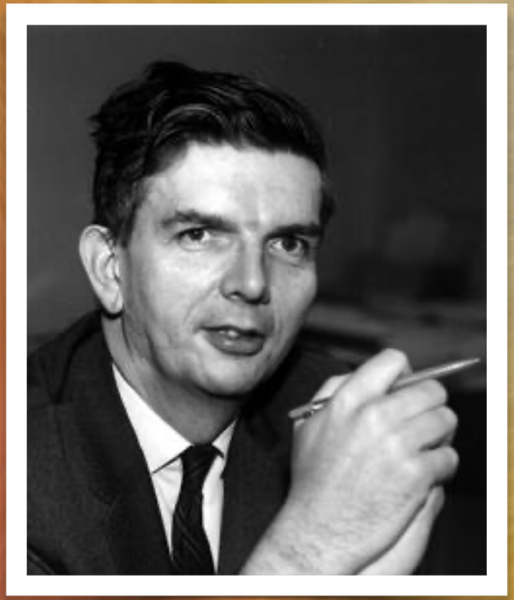
Cosmic photons with blackbody spectrum and $T \sim \mathcal{O}(10)K$ predicted on basis of models of Nucleosynthesis by

Gamow, Alpher, Hermann (40-50's),

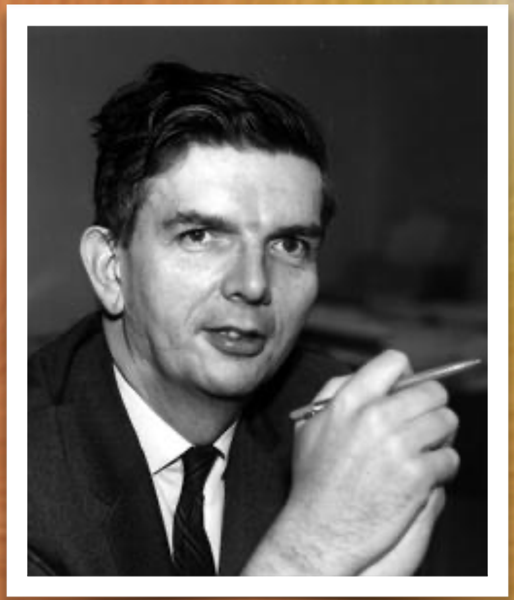
Zel'dovitch (60's),

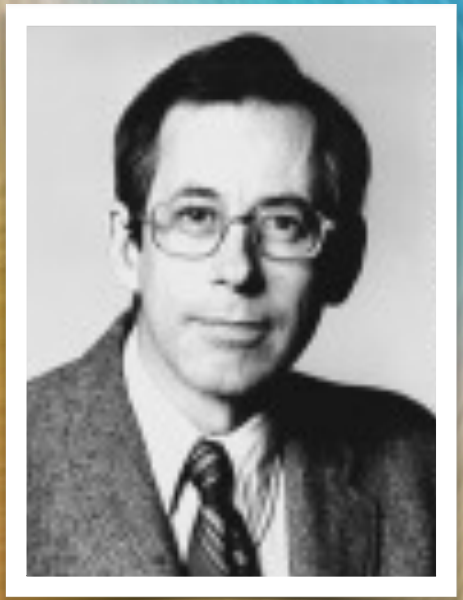
Dicke, Peebles (60's)…




The CMB is no pigeon shit!

Dicke, Peebles (60's)…





While discussing about dedicated experiment…

… contacted thanks to luck and coffee room discussions by…

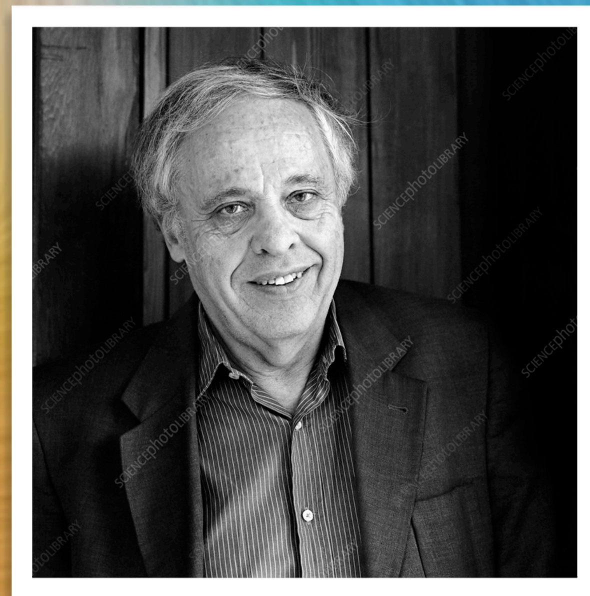
Penzias & Wilson (1964)

↓

Nobel prize Penzias & Wilson (1978)

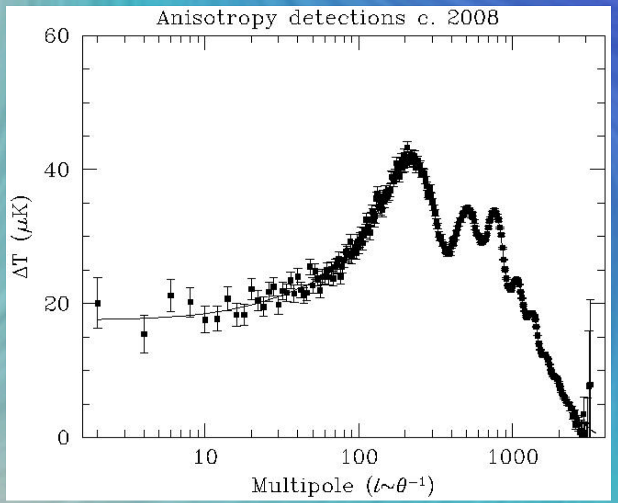
Prediction of non-trivial correlations in CMB anisotropies!

70-80's : Peebles, Silk, Sunyaev and respective collaborators (70-80's)…



…discuss information contain in CMB temperature spectrum and acoustic oscillations !

90's: precise prediction of CMB spectra:
Bertschinger, Hu, Kamionkowski, Ma, Seljak, Sujiyama, White, Zaldarriga + many others…

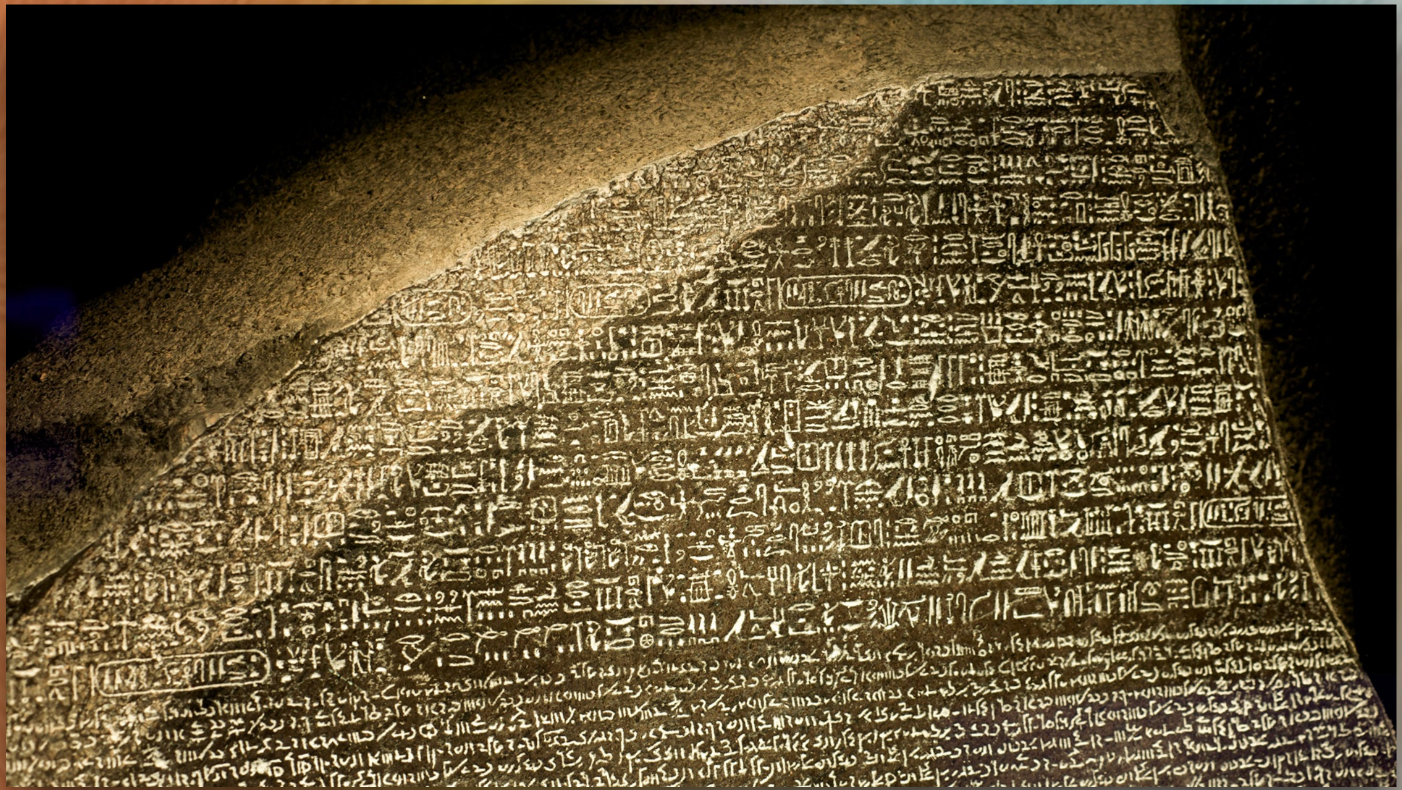


Confirmation by COBE, Boomerang, WMAP, Planck …

↓

Nobel prize Mather & Smoot 2006
Nobel prize Peebles 2019

The CMB is the Rosetta Stone of cosmology



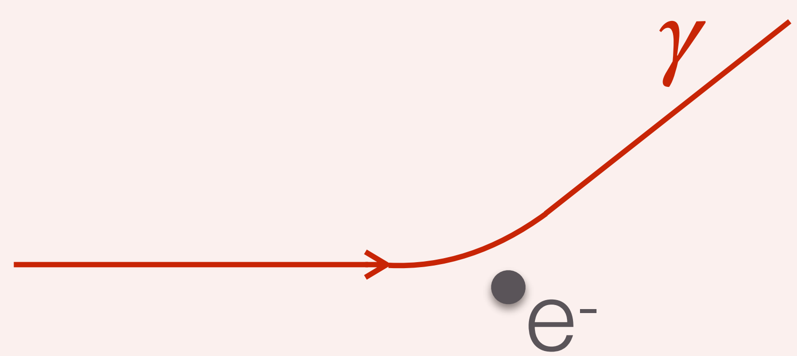
Plan

- Thomson scattering
- Linear theory of stochastic cosmological perturbations
- Spectrum of temperature anisotropies
- Why can we measure independently the ΛCDM parameters ?
- *Polarisation*
- *Tensors modes*
- *CMB lensing*
- *Spectral distortions*

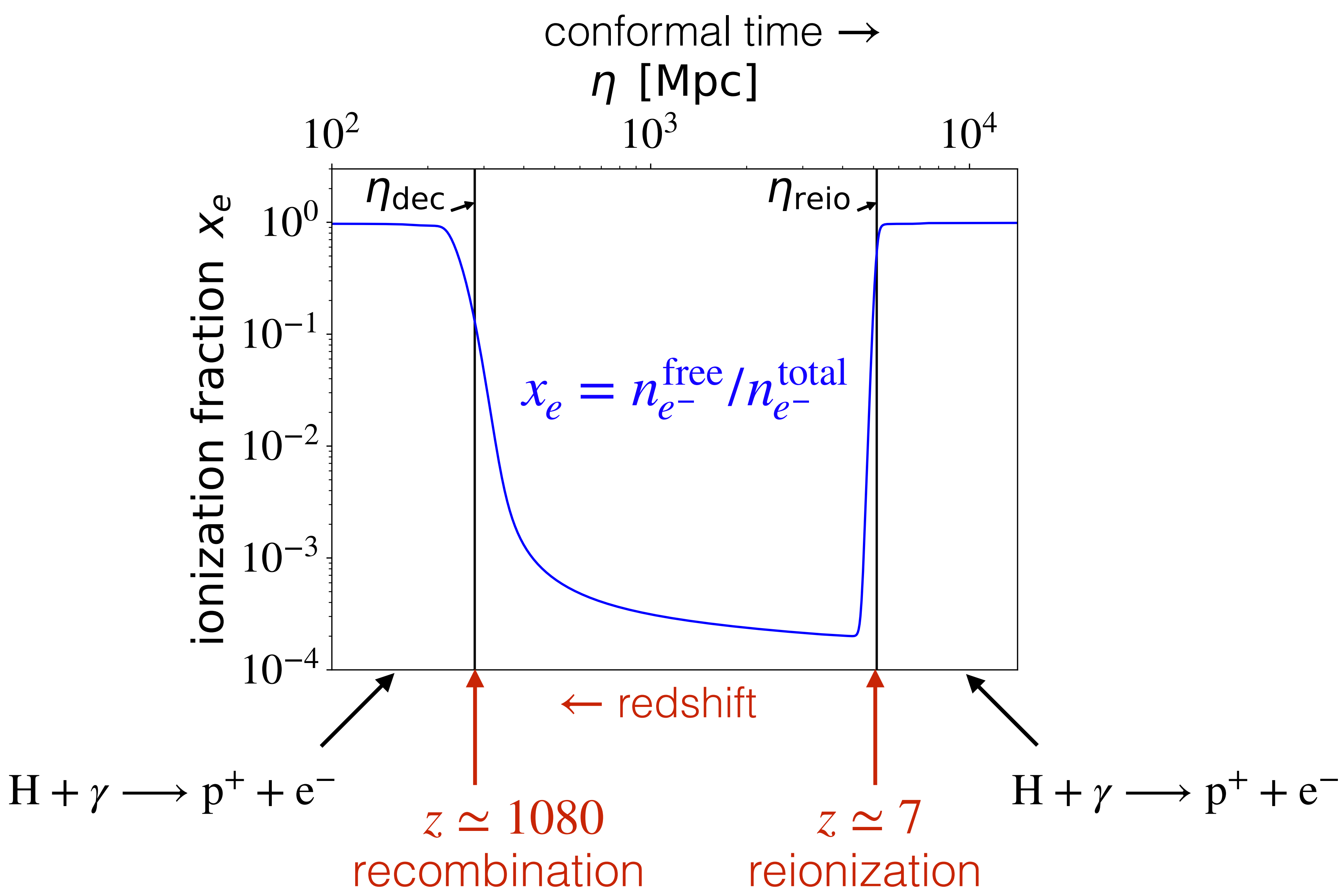
…following:

- The Young Universe: Primordial Cosmology, edited by R. Taillet (John Wiley & Sons, 2022) ISBN : 1789450322
  → Chapter 2: CMB, by JL
- Chapter 5 of: Neutrino Cosmology, JL et al., CUP 2013
  → Chapter 5: CMB
- The Ingredients of the Universe, Master course at RWTH Aachen U.
  Link on Indico

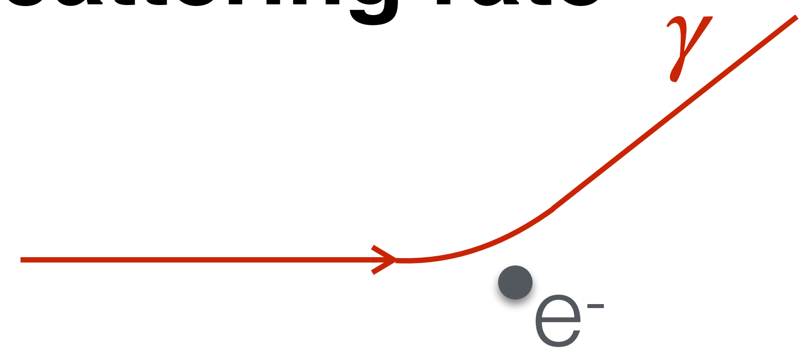
Thomson scattering

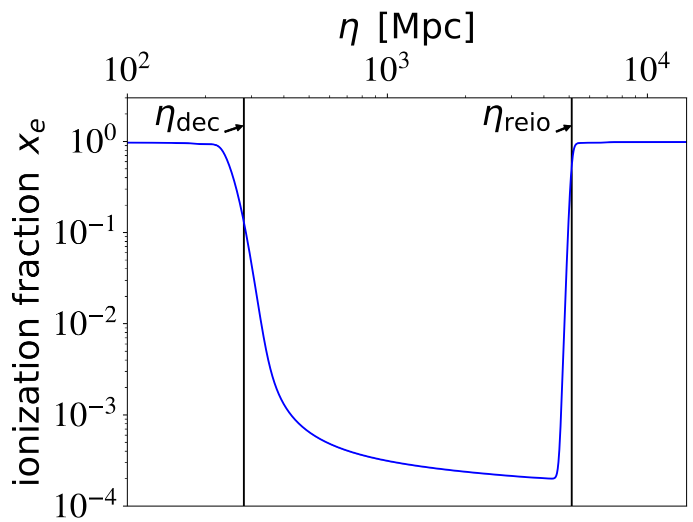


Ionisation fraction in the Universe

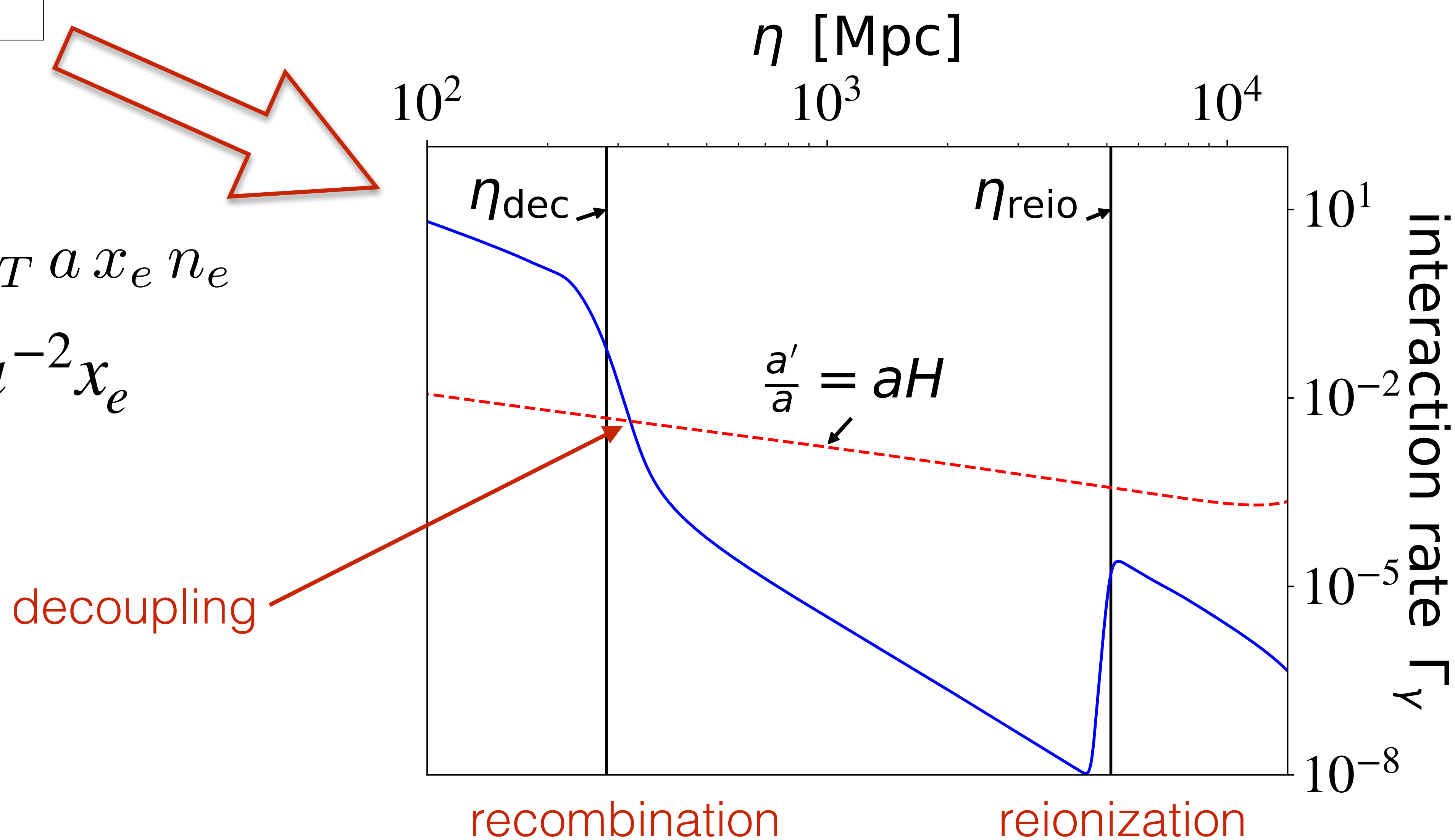


Thomson scattering rate





$$\Gamma_\gamma \equiv \frac{dN}{d\eta} = \sigma_T \, a \, x_e \, n_e$$
$$\propto a^{-2} x_e$$



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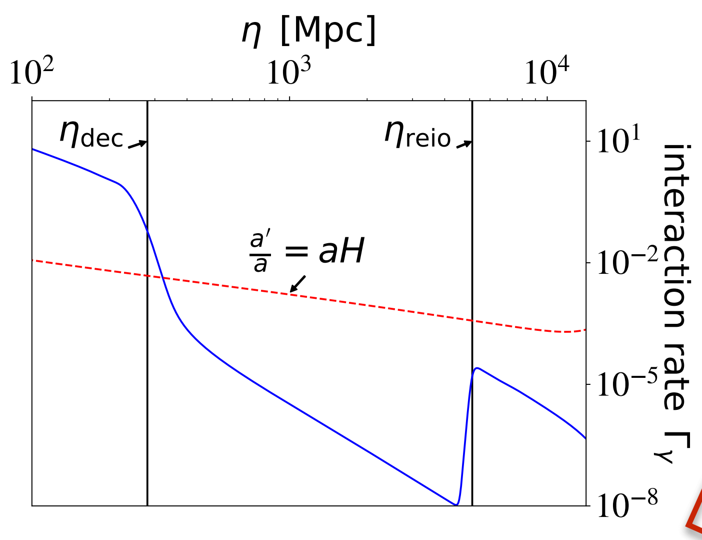
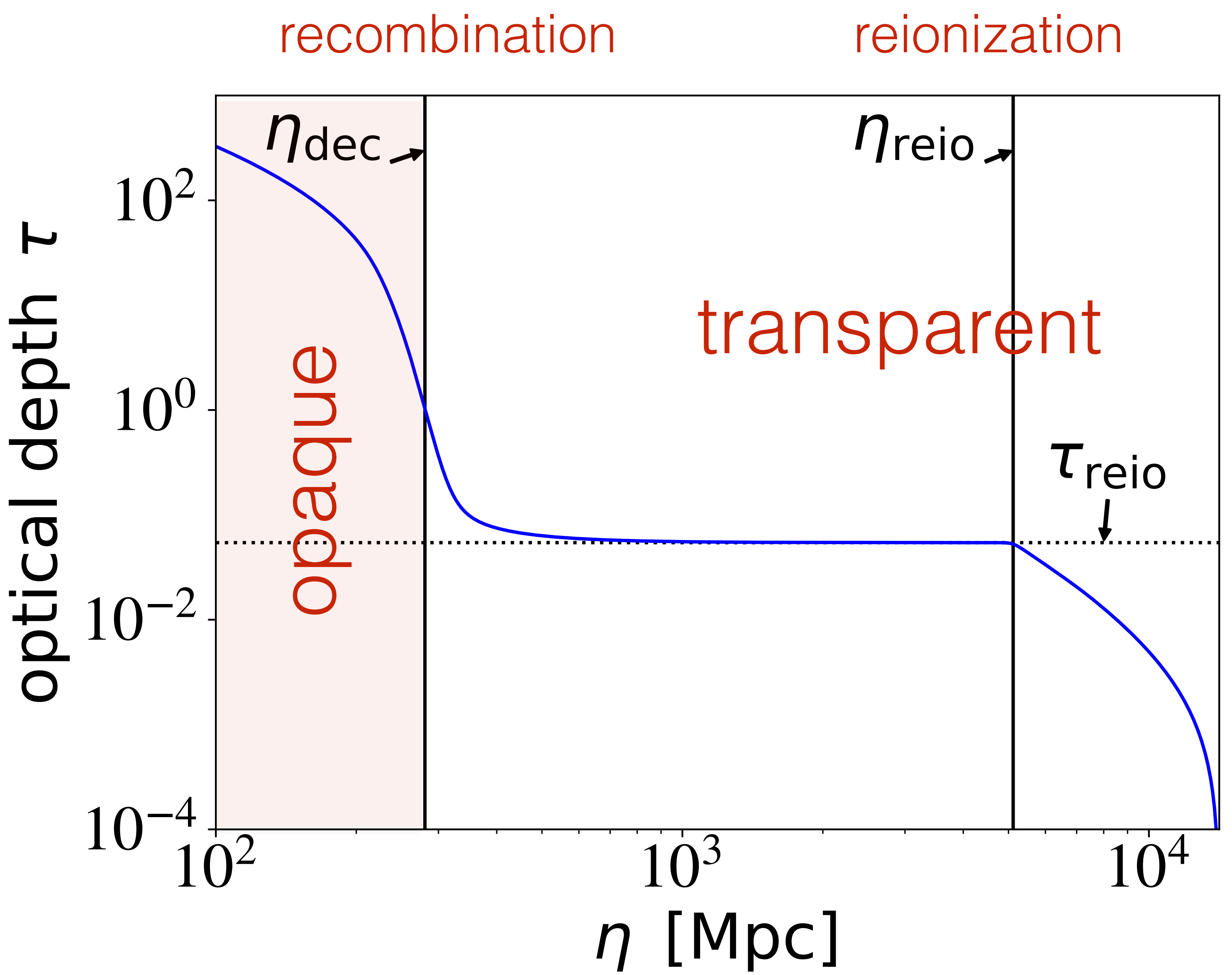
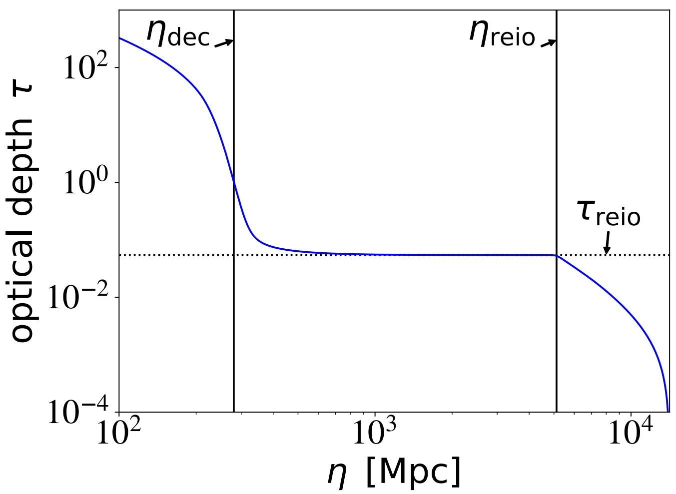
Optical depth of cosmic fog



$$\tau(\eta) \equiv \int_{\eta}^{\eta_0} d\eta \, \Gamma_\gamma(\eta)$$



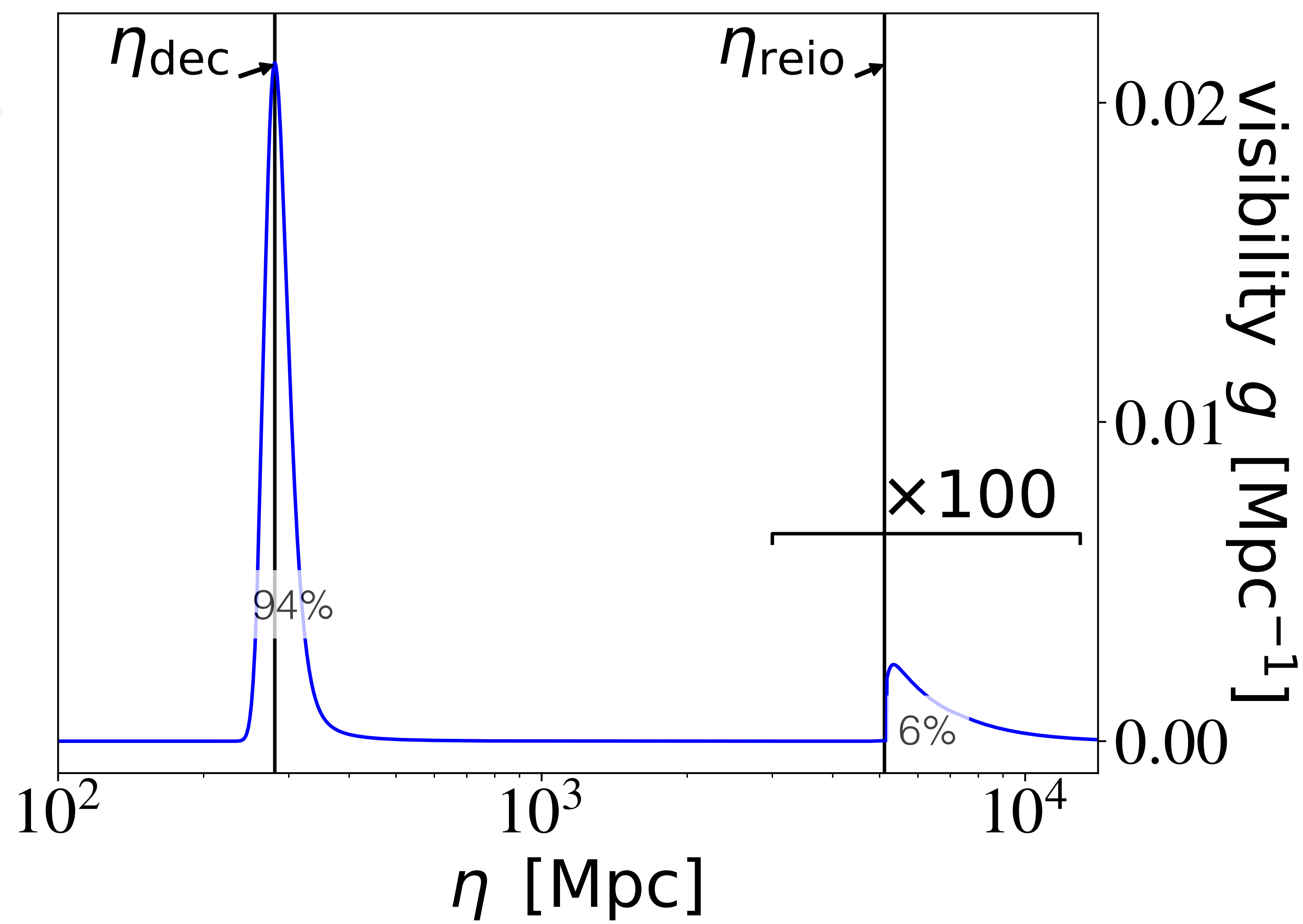
Visibility function



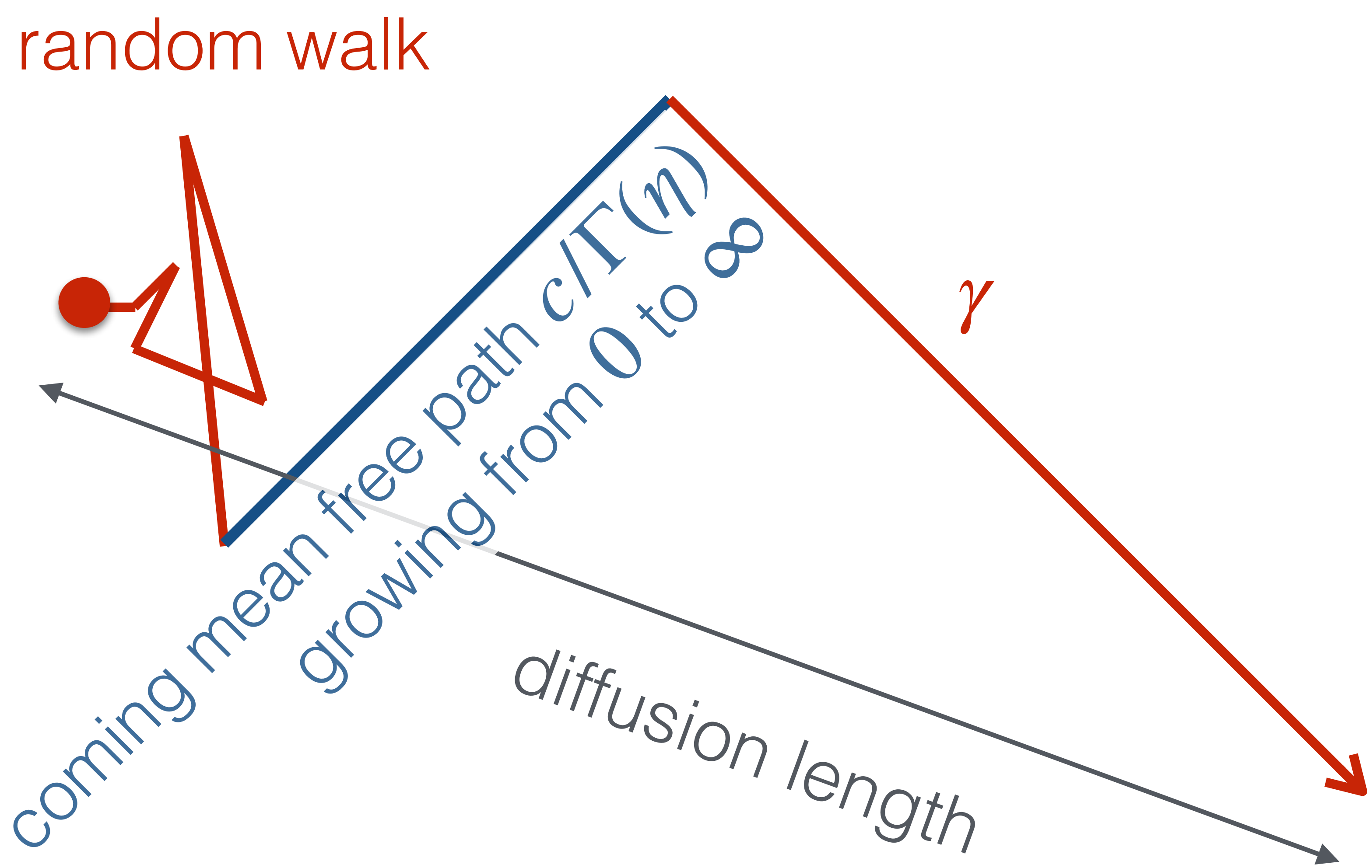
probability of last interaction at η
= probability of interaction at η
x (1-probability of interaction after η)

$$g(\eta) \equiv -\tau' e^{-\tau}$$

recombination reionization



Mean free path and diffusion length



$$\lambda_{\rm d}(\eta) = a(\eta)\, r_{\rm d}(\eta) \simeq a(\eta) \left[\int_{\eta_{\rm ini}}^{\eta} d\tilde{\eta}\, c^2 \Gamma_\gamma^{-1}(\tilde{\eta}) \right]^{1/2}$$

Linear cosmological perturbations

Bardeen decomposition

$$g_{\mu\nu}(\eta,\vec{x}) = \bar{g}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta,\vec{x}) \quad \text{and} \quad T^{\mu\nu}(\eta,\vec{x}) = \bar{T}^{\mu\nu}(\eta) + \delta T^{\mu\nu}(\eta,\vec{x})$$

background

perturbations
10 d.o.f

(for each species!)

James Bardeen 1986

FLRW background invariant under spatial rotations

⇒ irreducible representations of SO(3) → decoupled sectors

⇒ Bardeen | scalars (gravity forces) : 4 d.o.f.

vectors (gravito-magnetism) : 4 d.o.f.

tensors (gravitational waves) : 2 d.o.f.

Newtonian gauge

Gauge freedom: perturbations depends on choice of coordinates

$\Downarrow$

Newtonian gauge: eliminate 2 scalar d.o.f. to stick to diagonal $\delta g_{\mu\nu}$

$$ds^2 = -(1+2\psi)dt^2 + (1-2\phi)a^2(t)d\vec{x}^{\,2}$$
$$= a^2(\eta)\left[-(1+2\psi)d\eta^2 + (1-2\phi)d\vec{x}^{\,2}\right]$$

Local distorsion of time
= generalised
gravitational potential

Local distorsion
of expansion rate

Matter perturbations

$\delta T_{\mu\nu,X}$: still 4 d.o.f. per species X

$\Downarrow$

δ_X : relative fluctuation of energy density

θ_X : divergence of bulk velocity

δp_X : fluctuation of (isotropic) pressure

σ_X : anisotropic stress = quadrupole of (anisotropic) pressure

Local pressure relates to local density (e.g. $\delta p_X = w_X \ \delta\rho_X$)

$\Downarrow$

only 3 d.o.f.

Perfect fluids (strong interactions)

$\Downarrow$

Pressure is isotropic ($\sigma_X = 0$)

$\Downarrow$

only 2 d.o.f.

Comoving Fourier space

Thereafter: only flat cosmologies for simplicity

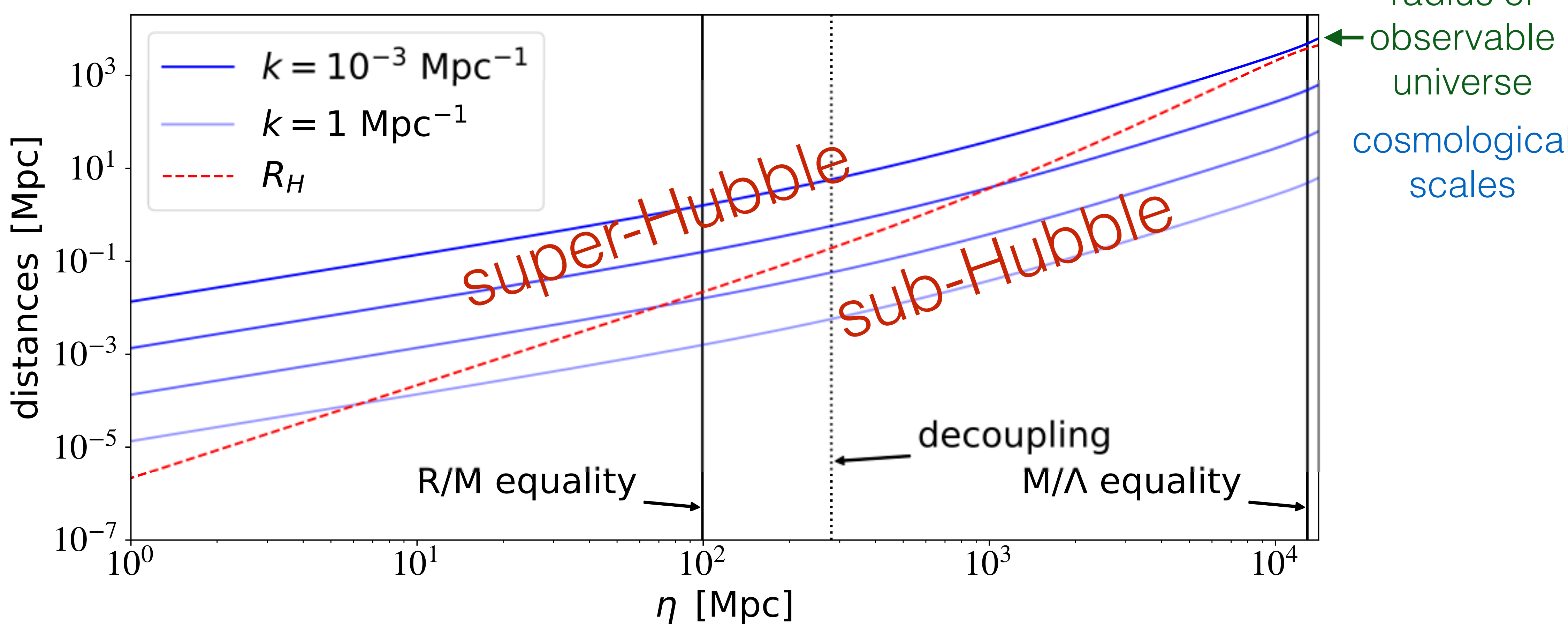
coming wavevector
comoving coordinate

$$\delta_X(\eta,\vec{k}) = \int \frac{d^3\vec{x}}{(2\pi)^{3/2}} \delta_X(\eta,\vec{x}) e^{-i\vec{k}\cdot\vec{x}}$$

Wavelengths $\lambda(\eta) = a(\eta)\ 2\pi/k$

$\left.\frac{da}{dt}\right|_\eta$

Decelerated expansion ⇒ grow slower than Hubble radius $R_H(\eta) = c\, a(\eta)/\dot{a}(\eta)$



Condition for Hubble crossing : $k \sim aH \sim 1/\eta$

Linearised Einstein Equations

- from $G^0_0 = 8\pi G\, T^0_0$:

$$\frac{2}{a^2}\left[k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right)\right] = -8\pi G \sum_X \bar{\rho}_X \delta_X$$

dominates on sub-Hubble
⇓
Poisson equation :

$$-\frac{k^2}{a^2}\phi = 4\pi G\, \bar{\rho}_{\rm total}\, \delta_{\rm total}$$

dominates on super-Hubble
⇓
Using Friedmann :

$$2\psi = -\,\delta_{\rm total}$$

- from $G^i_j = 8\pi G\, T^i_j$:

$$\frac{2}{3}\frac{k^2}{a^2}(\phi - \psi) = 8\pi G \sum_X (\bar{\rho}_X + \bar{p}_X)\sigma_X$$

perfect fluids $\Rightarrow \sigma_X = 0 \Rightarrow \phi = \psi$

- other scalar equations redundant with upcoming equations of motion (Bianchi identity)

Equations of motion

Without details: each decoupled species fulfils energy/momentum conservation:

✦ continuity equation: $$\delta'_X = -(1+w_X)(\theta_X - 3\phi') - 3\frac{a'}{a}(c^2_{s\,X} - w_X)\delta_X$$

✦ Euler equation: $$\theta'_X = -\frac{a'}{a}(1-3c^2_{a\,X})\theta_X + \frac{c^2_{s\,X}}{1+w_X}k^2\delta_X - k^2\sigma_X + k^2\psi$$

(featuring sound speed c_s and adiabatic sound speed c_a)

In ΛCDM:

- CDM : negligible pressure/stress: closed system
- e-/baryons: negligible pressure/stress but Thomson scattering: $+\frac{4}{3}\frac{\bar{\rho}_\gamma}{\bar{\rho}_\mathrm{b}}\tau'(\theta_\mathrm{b} - \theta_\gamma)$
- photons, neutrinos: when not strongly coupled, anisotropic stress

→ need Boltzmann equation

$$\frac{d}{d\eta}f_\gamma = C\left[f_\gamma, f_e\right] \qquad\qquad \frac{d}{d\eta}f_\nu = 0$$

Summary: perturbed degrees of freedom and equations of motion

Degrees of freedom	Equation of motion
Gravitational potential $\psi(\eta, \vec{x})$	Einstein 00: $\phi, \psi \leftrightarrow \delta$
Scale factor distortion $\phi(\eta, \vec{x})$	Einstein ij: $(\phi - \psi) \longrightarrow \sigma$
CDM density/velocity $\delta_{\rm c}(\eta, \vec{x})$, $\theta_{\rm c}(\eta, \vec{x})$	continuity $\delta_{\rm c}'$ + Euler $\theta_{\rm c}'$
Baryon density/velocity $\delta_{\rm b}(\eta, \vec{x})$, $\theta_{\rm b}(\eta, \vec{x})$	Continuity $\delta_{\rm b}'$ + Euler $\theta_{\rm b}'$ (incl. Thomson)
Photons $f_\gamma(\eta, \vec{x}, p, \hat{n})$	Boltzmann $\frac{d}{d\eta} f_\gamma =$ Thomson
[Neutrinos $f_\nu(\eta, \vec{x}, p, \hat{n})$]	[Boltzmann $\frac{d}{d\eta} f_\nu = 0$]

Photon phase-space distribution

Blackbody shape: $f_\gamma(\eta, \vec{x}, p, \hat{n}) = \frac{1}{e^{\frac{p}{T(\eta,\vec{x},\hat{n})}} - 1}$

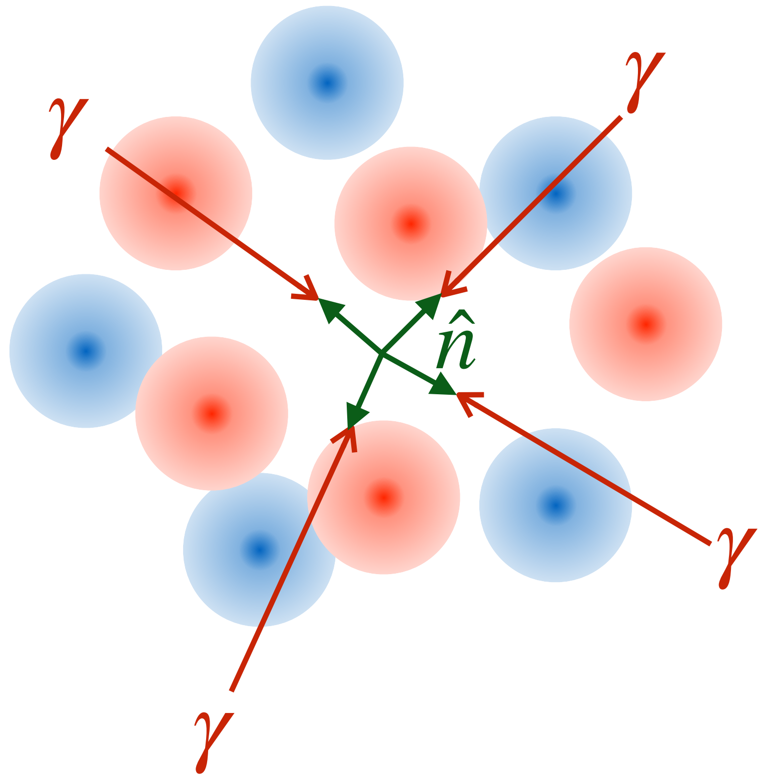
Up to very good approximation: preserved even when leaving thermal equilibrium, but becomes direction-dependent due to gravitational interactions:

redshifting along geodesics:

$$\frac{d \ln(a\,p)}{d\eta} = \phi' - \hat{n}\cdot\vec{\nabla}\psi$$

dilation

gravitational Doppler



Then: $T(\eta, \vec{x}, \hat{n}) = \bar{T}(\eta)\,\left(1 + \Theta(\eta, \vec{x}, \hat{n})\right)$

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Linearised Boltzmann equation

Temperature fluctuation: $T(\eta, \vec{x}, \hat{n}) = \bar{T}(\eta) \, \big(1 + \Theta(\eta, \vec{x}, \hat{n})\big)$

Monopole and dipole of Θ account for local density & bulk velocity:

$$\Theta(\eta, \vec{x}, \hat{n}) = \frac{1}{4}\delta_\gamma(\eta, \vec{x}) + \hat{n} \cdot \vec{v}_\gamma(\eta, \vec{x}) + \text{higher multipoles}$$

Linearised Boltzmann:

$$\Theta' + \hat{n} \cdot \overrightarrow{\nabla} \Theta - \phi' + \hat{n} \cdot \overrightarrow{\nabla} \psi = -\Gamma_\gamma \Big(\hat{n} \cdot (\vec{v}_\gamma - \vec{v}_\mathrm{b}) + \text{higher multipoles} \Big)$$

dilation

gravitational Doppler

Thomson scattering

Thomson scattering wants to align velocity of photons vs. electron/baryons, and to wash out higher multipoles!

Boltzmann hierarchy

Start from linearised Boltzmann and perform:

1. Fourier transformation
2. Legendre expansion $\Theta(\eta,\vec{k},\hat{n}) = \sum_l (-i)^l (2l+1)\,\Theta_l(\eta,\vec{k})\,P_l(\hat{k}\cdot\hat{n})$

$$\delta'_\gamma + \frac{4}{3}\theta_\gamma - 4\phi' = 0$$

$$\theta'_\gamma + k^2\left(-\frac{1}{4}\delta_\gamma + \sigma_\gamma\right) - k^2\psi = \tau'(\theta_\gamma - \theta_\mathrm{b})$$

relates to Θ_2

$$\Theta'_l - \frac{kl}{2l+1}\Theta_{l-1} + \frac{k(l+1)}{2l+1}\Theta_{l+1} = \tau'\Theta_l \quad \forall l \geq 2$$

⇒ Solved together with previous equations by Einstein-Boltzmann solvers (CMBFAST, CAMB, CLASS…)

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