Summary of Lecture 3

CMB spectrum

$$C_l = \langle a_{lm} a^*_{lm} \rangle = \frac{2}{\pi} \int dk\, k^2 \Theta_l^2(\eta_0, k)\; P_{\mathscr{R}}(k)$$

photon transfer function

primordial spectrum

Boltzmann hierarchy

$$\delta'_\gamma + \frac{4}{3}\theta_\gamma - 4\phi' = 0$$

$$\theta'_\gamma + k^2 \left(-\frac{1}{4}\delta_\gamma + \sigma_\gamma \right) - k^2\psi = \tau'(\theta_\gamma - \theta_\mathrm{b})$$

$$\Theta'_l - \frac{kl}{2l+1}\Theta_{l-1} + \frac{k(l+1)}{2l+1}\Theta_{l+1} = \tau'\Theta_l \quad \forall l \geq 2$$

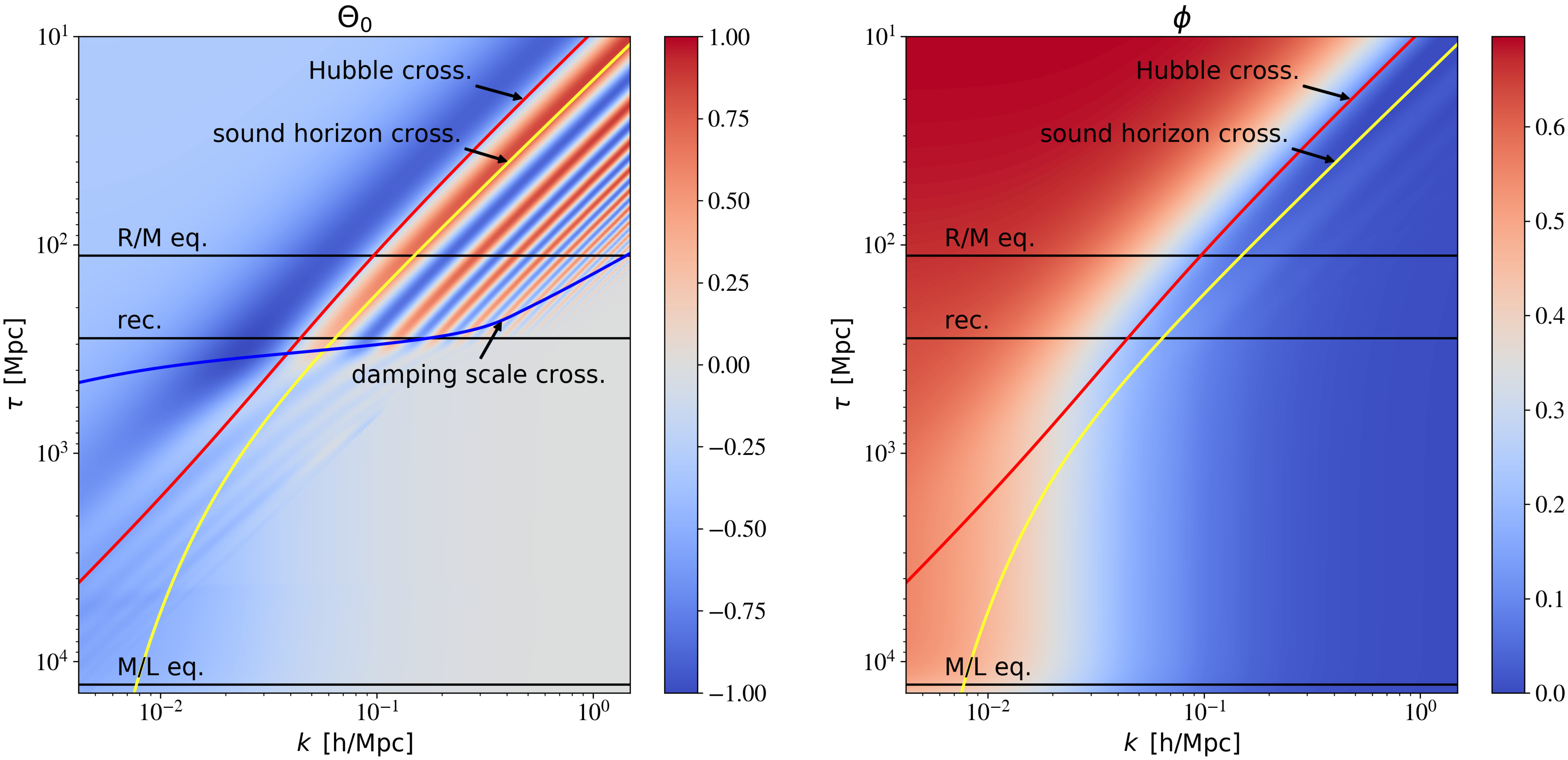
⇒ line-of-sight integral in Fourier space

$$\Theta_l(\eta_0, k) = \int_{\eta_\mathrm{ini}}^{\eta_0} d\eta \, \{ g\, (\Theta_0 + \psi)\; j_l(k(\eta_0 - \eta))$$

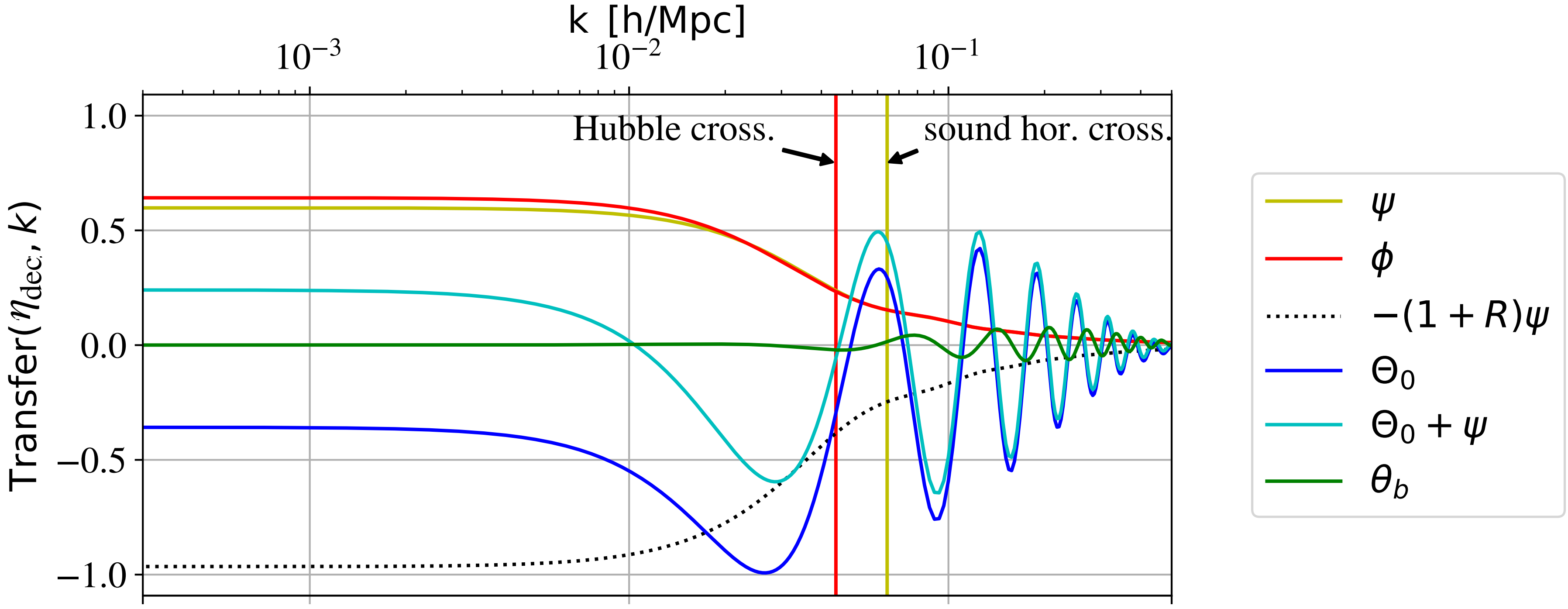
$$+ \, g\, k^{-1}\theta_\mathrm{b}\; j'_l(k(\eta_0 - \eta))$$

$$+ \, e^{-\tau} (\phi' + \psi')\; j_l(k(\eta_0 - \eta)) \}$$

Summary of Lecture 3



$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \left(\underbrace{g\,(\Theta_0+\psi)}_{\rm SW} + \underbrace{\left(g\,k^{-2}\theta_{\rm b}\right)'}_{\rm Doppler} + \underbrace{e^{-\tau}(\phi'+\psi')}_{\rm ISW} \right) j_l(k(\eta_0-\eta))$$



from transfer to C_ℓ :

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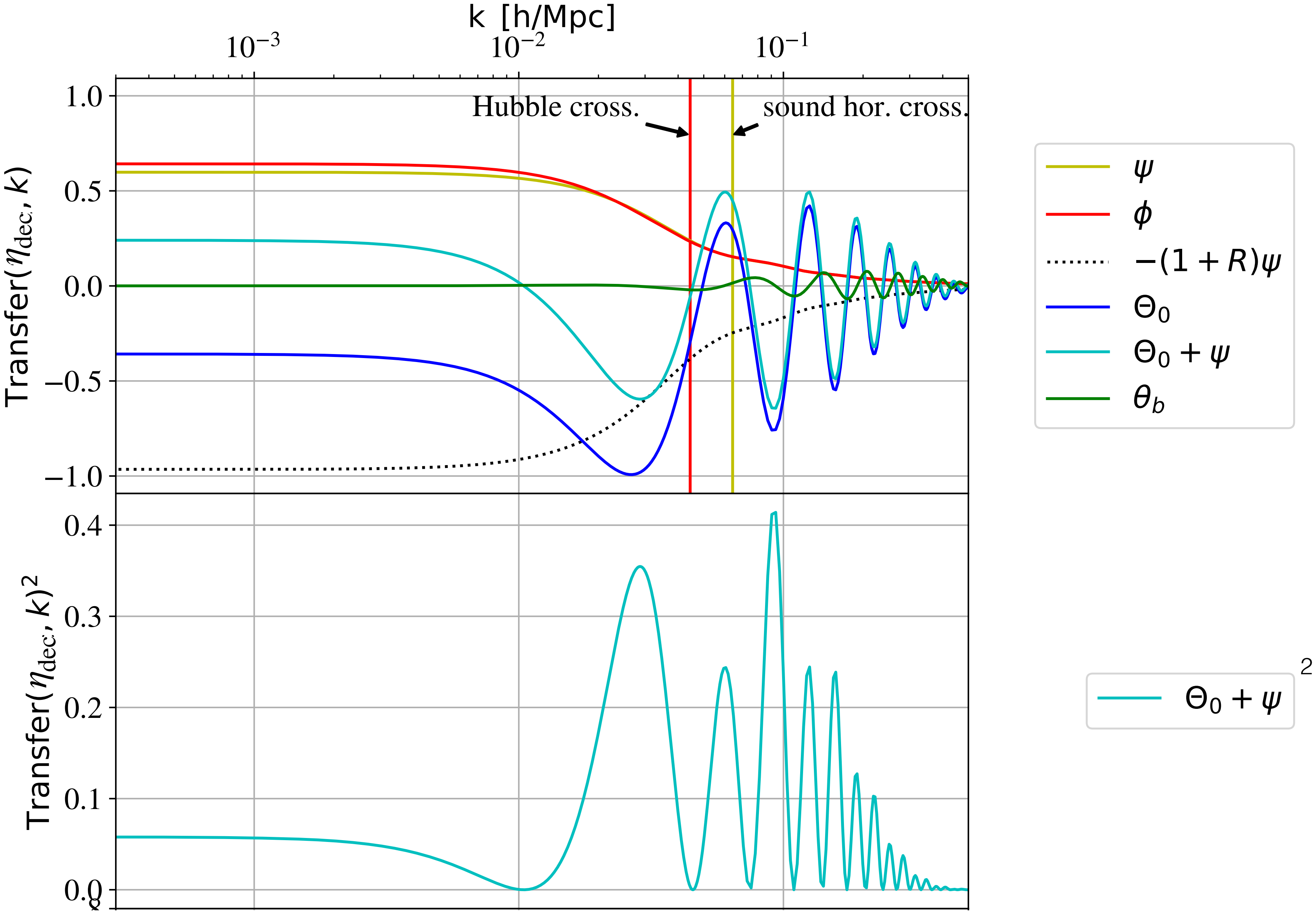
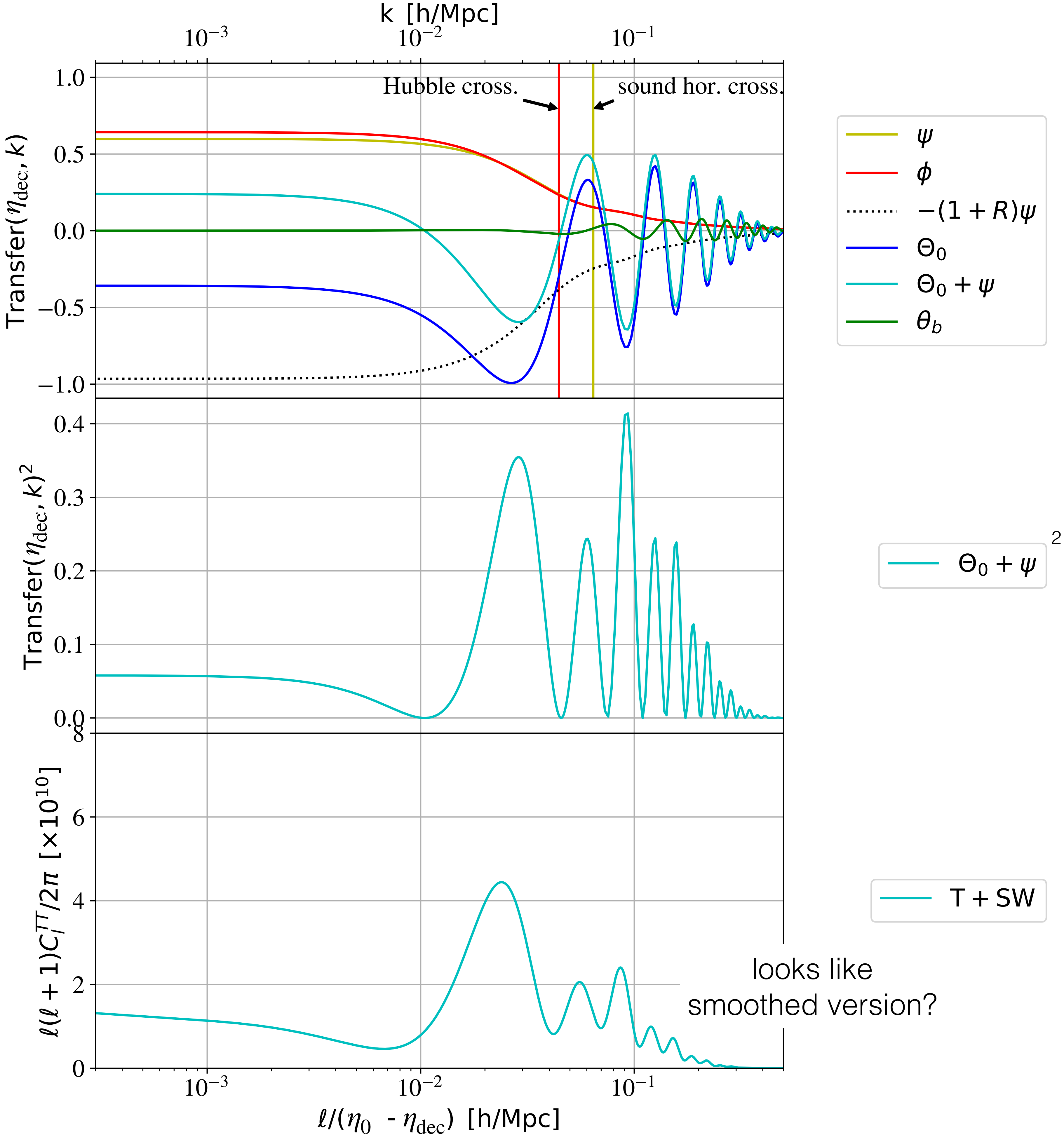
to C_ℓ :



from transfer to C_ℓ :

$$\Theta_0(\eta_{\rm dec}, k) + \psi(\eta_{\rm dec}, k)$$

independent of k would give $l(l+1)C_l =$ constant



looks like smoothed version?

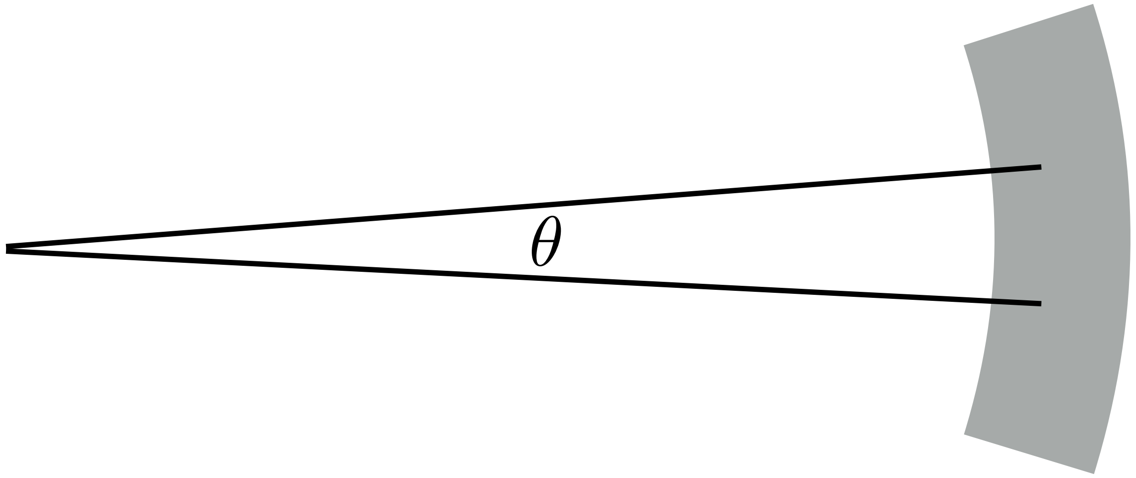
Projection effects

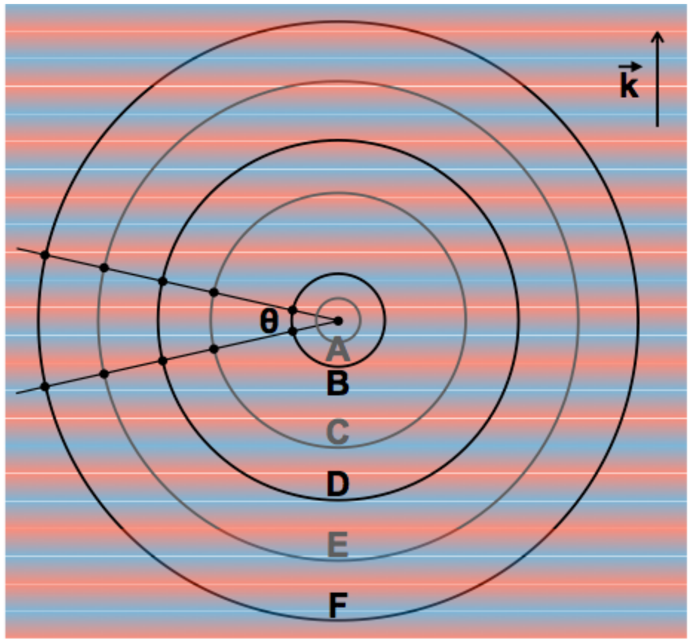
- two reasons for smoothing when going from k-space to l-space:

$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \left(g\,(\Theta_0+\psi) + \dots \right) j_l(k(\eta_0-\eta))$$

$$C_l \equiv \langle |a_{lm}|^2 \rangle = \frac{1}{2\pi^2}\int \frac{dk}{k}\Theta_l^2(\eta_0,k)\mathcal{P}_\mathcal{R}(k)$$

—> contribution of wide range of *times* and *wavenumber* to single C_l





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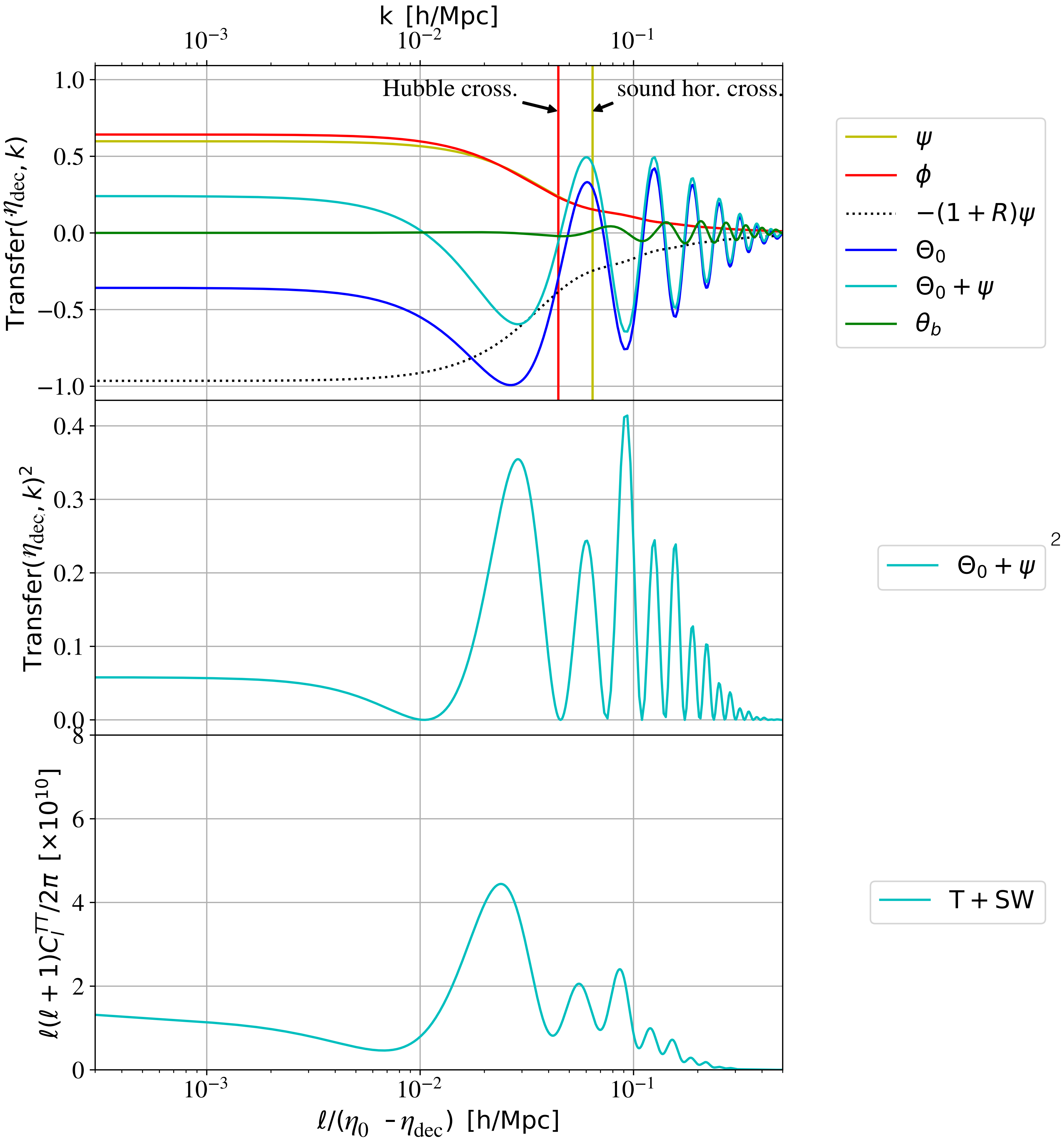
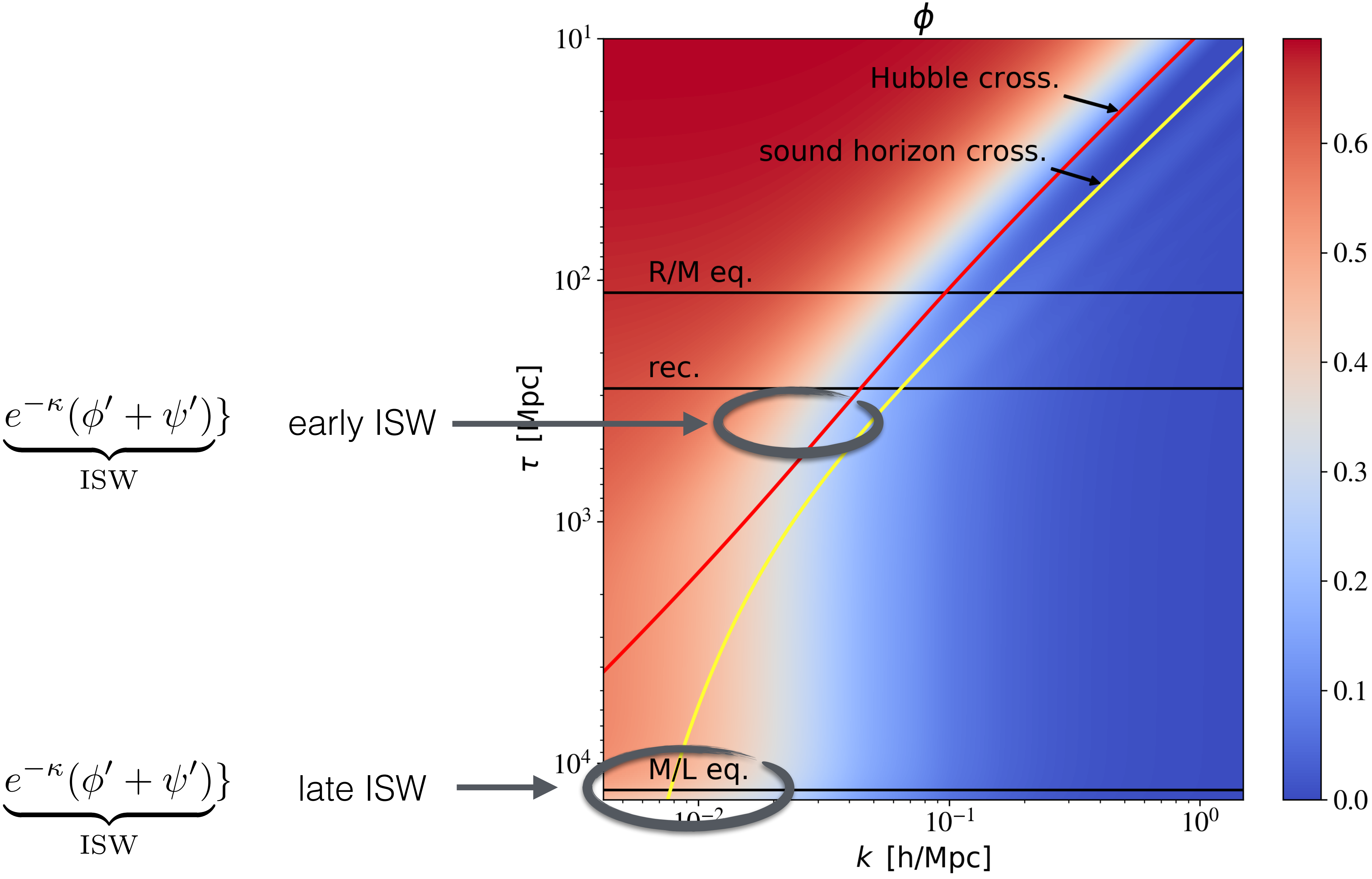


$$\Theta_0(\eta_{\rm dec}, k) + \psi(\eta_{\rm dec}, k)$$

independent of k would give $l(l+1)C_l =$ constant

ISW contribution



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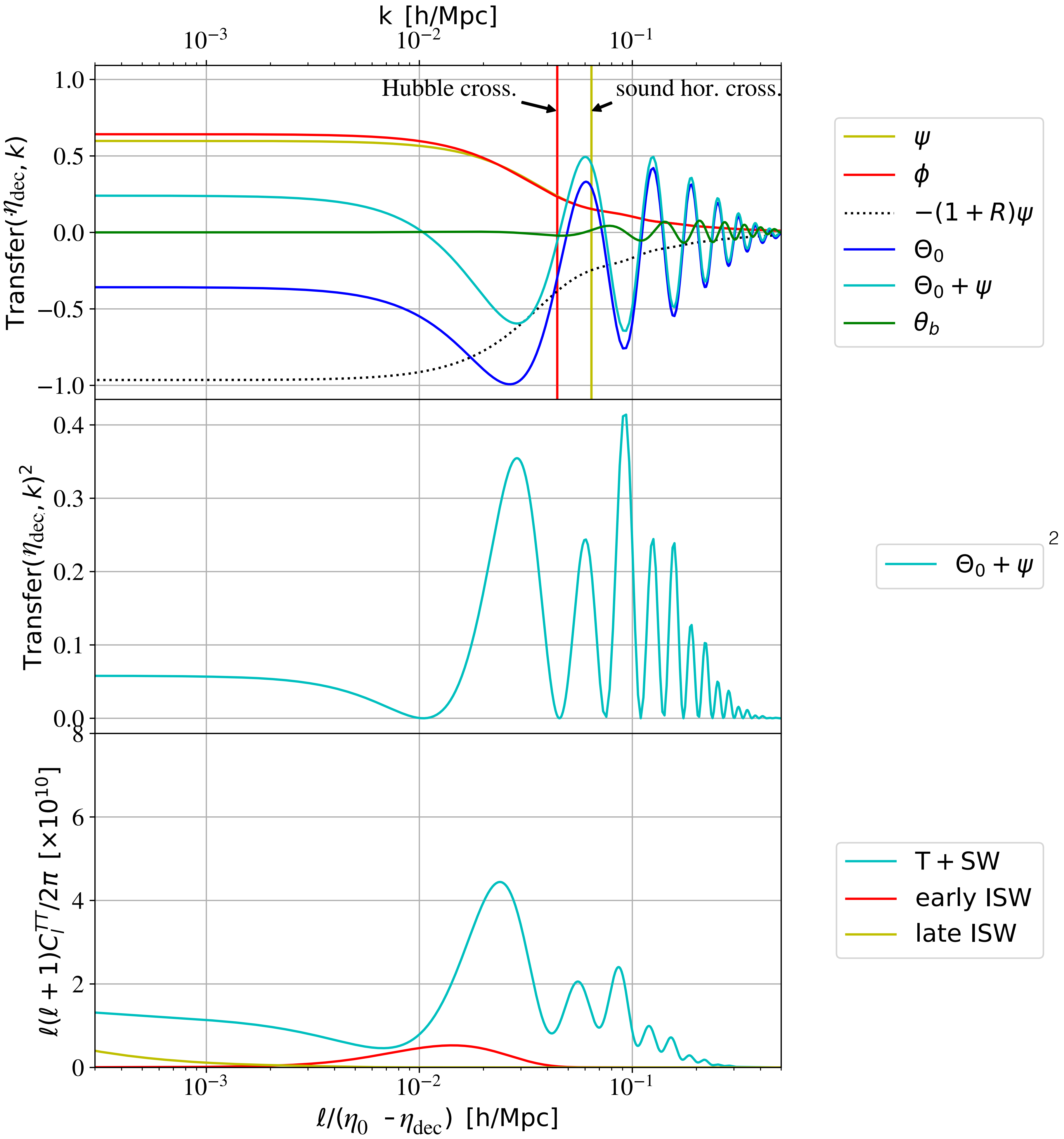

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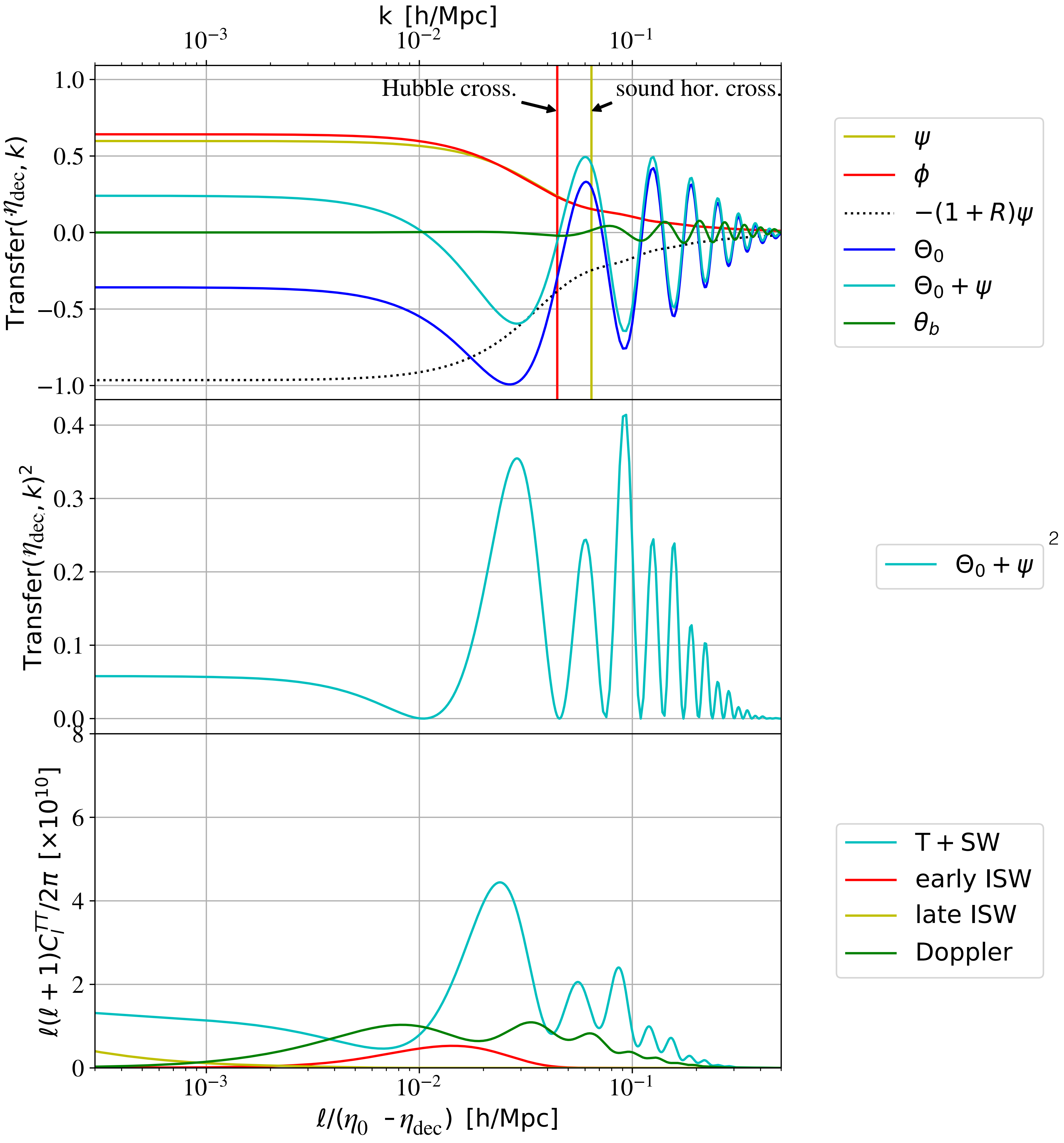

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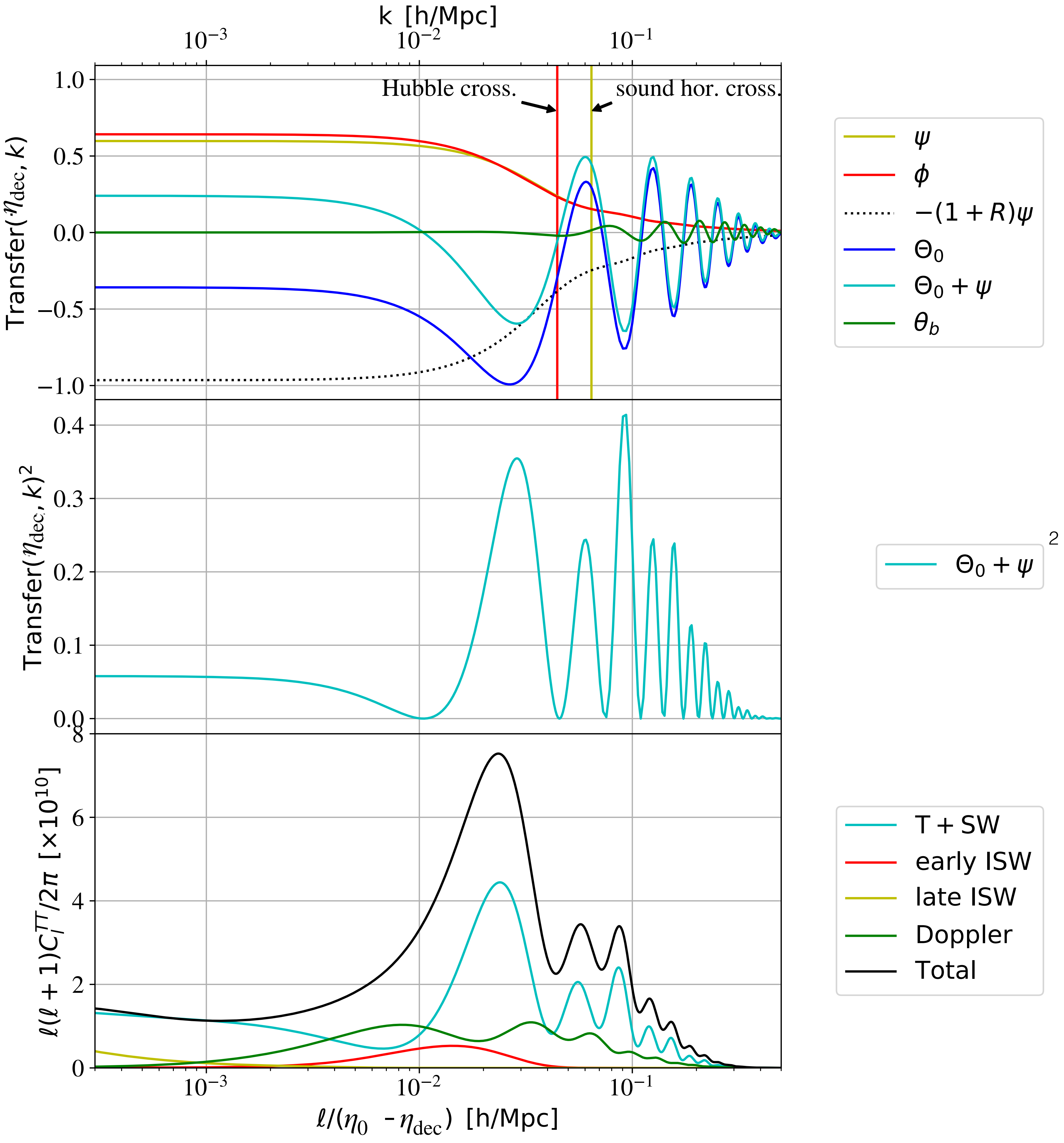


ΛCDM parameter effects on temperature spectrum

Why can we measure 6 ΛCDM parameters independently with CMB?

- Flat FLRW ($\Omega_k = 0$),
- Cosmological constant ($w = -1$),
- Plain decoupled / stable / cold dark matter,
- Neutrino mass neglected or fixed to minimal value,
- $N_{\rm eff} = 3.044$,
- Power-law primordial spectrum…

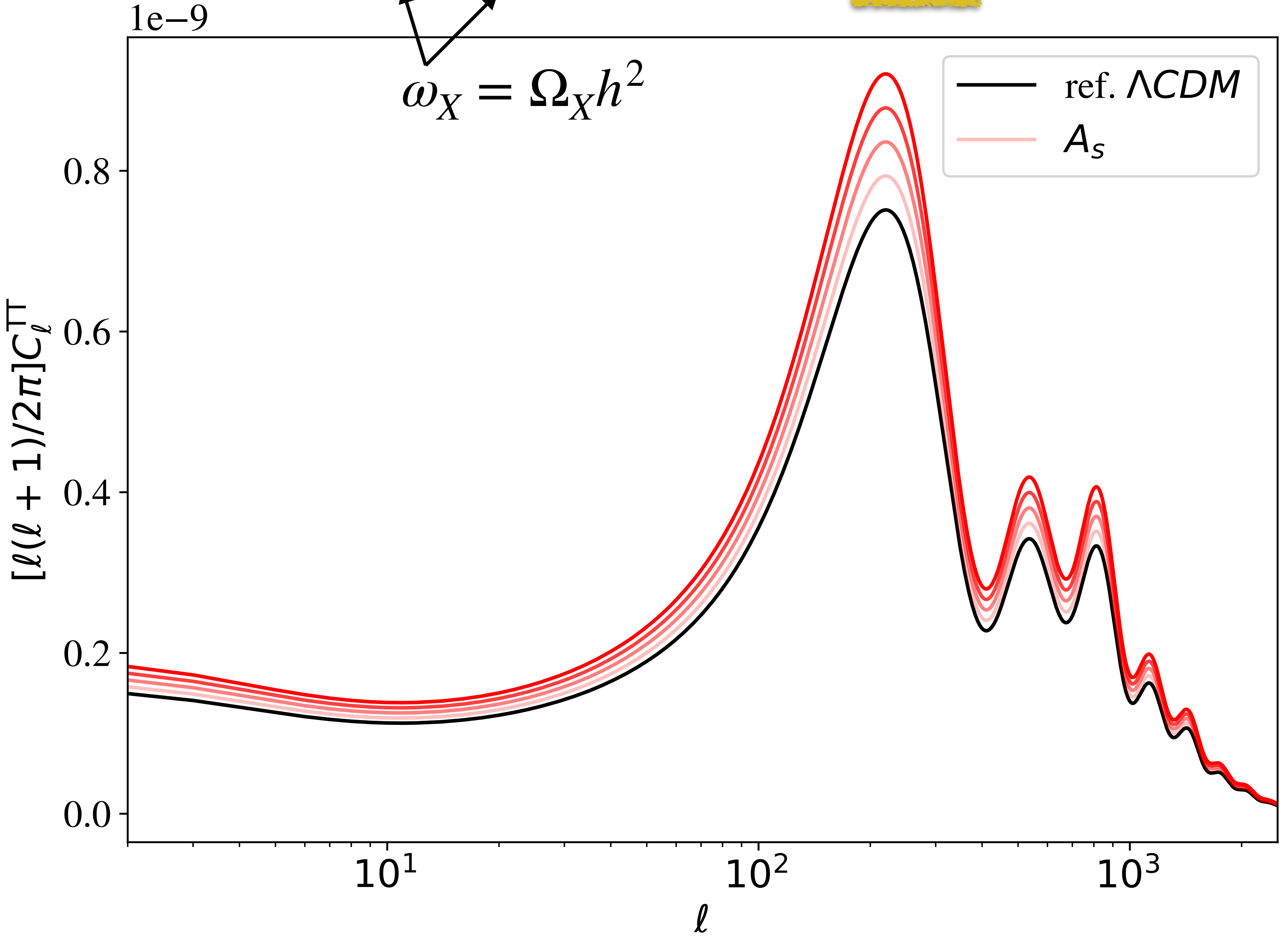
Possible basis: $\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$

$\omega_X = \Omega_X h^2$

parameter of CMB, not of LSS

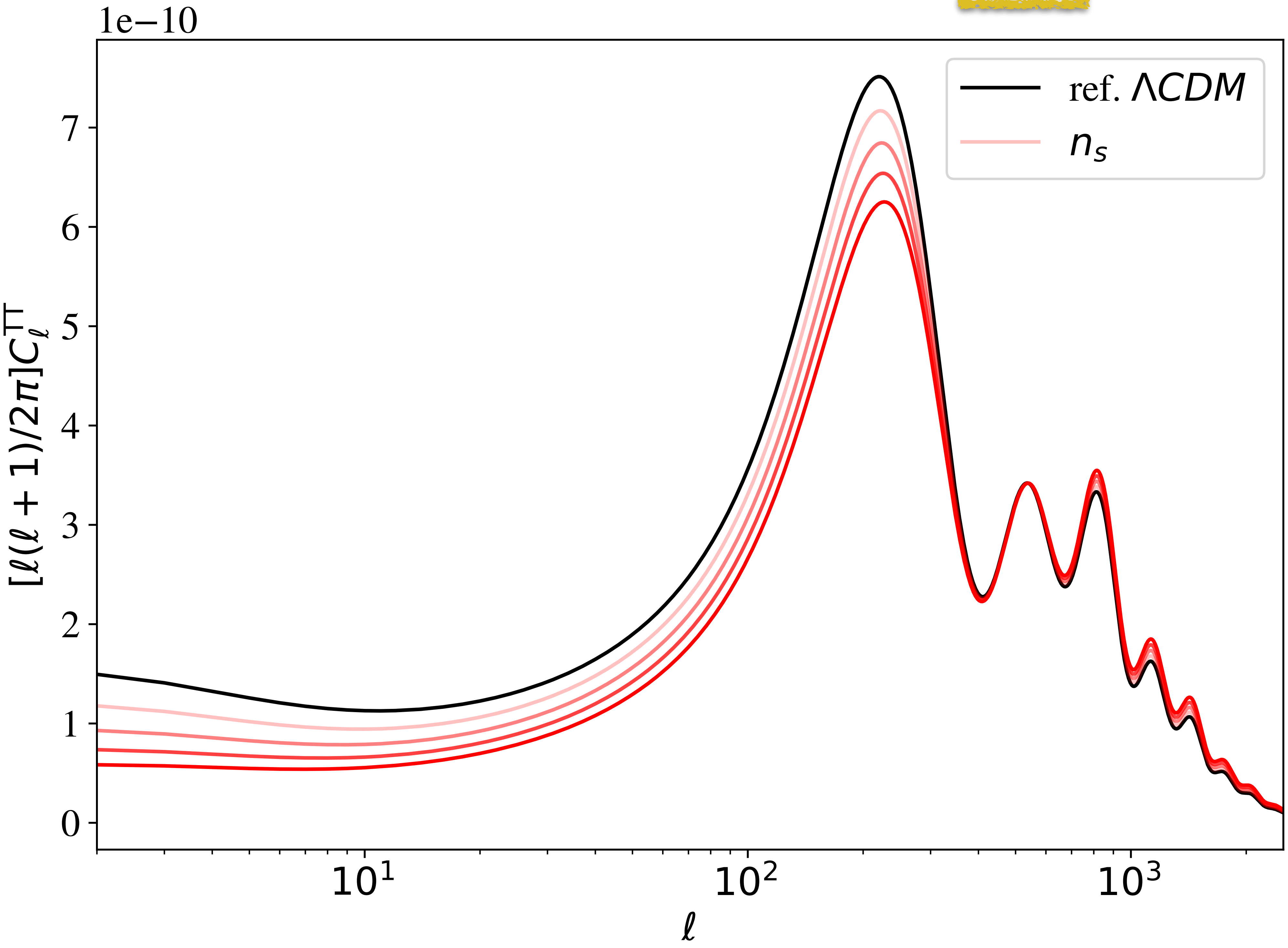
$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_\text{reio}, \boxed{A_s}, n_s\}$$

$\omega_X = \Omega_X h^2$



$$C_l^{XY} = 4\pi \int dk\ k^2 \Delta_l^X(k) \Delta_l^Y(k) \mathcal{P}_\mathcal{R}(k) \qquad \mathcal{P}_\mathcal{R}(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s-1}$$

$$\{\omega_b, \omega_m, \Omega_\Lambda, \tau_{\text{reio}}, A_s, \boxed{n_s}\}$$



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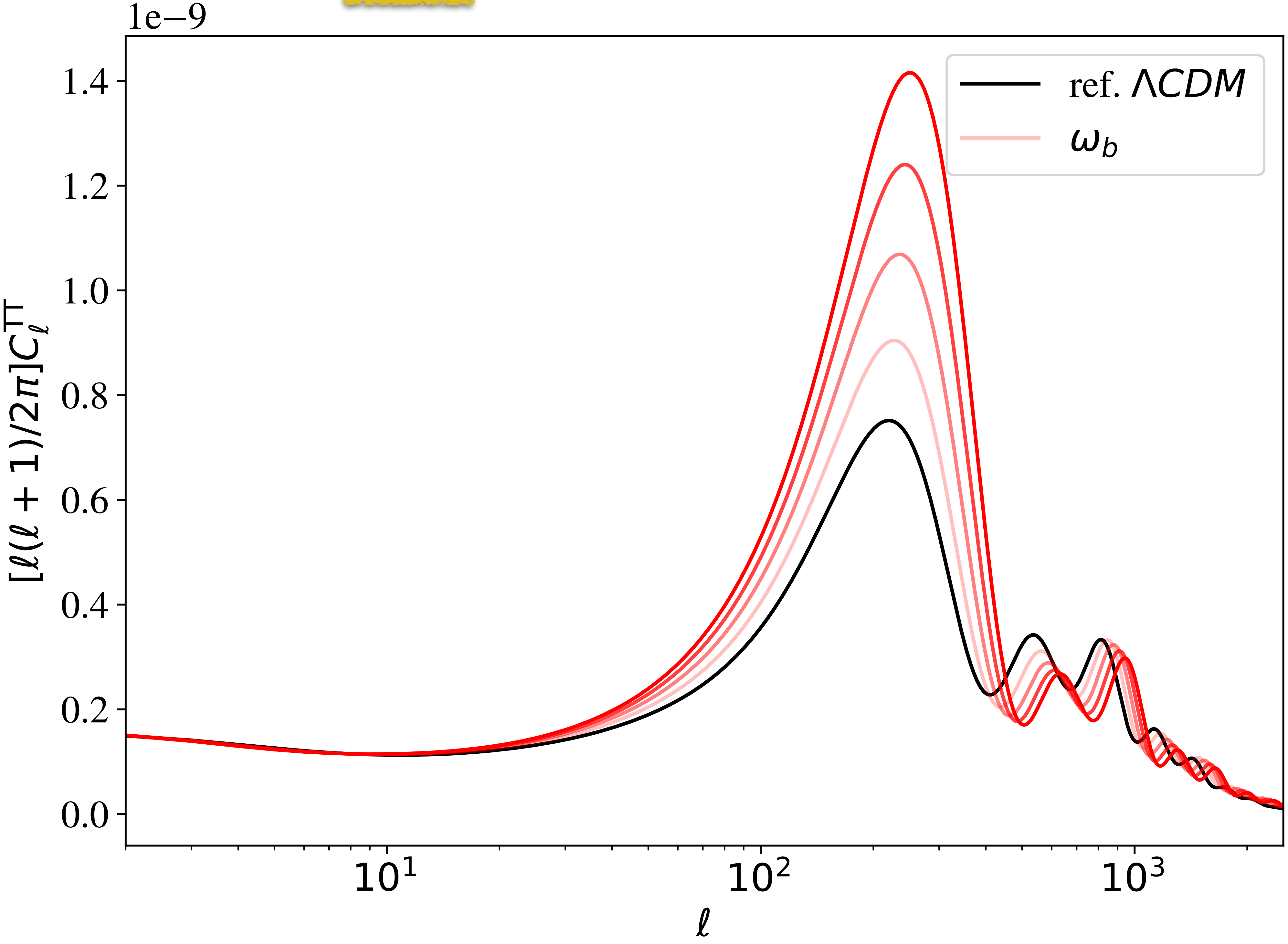
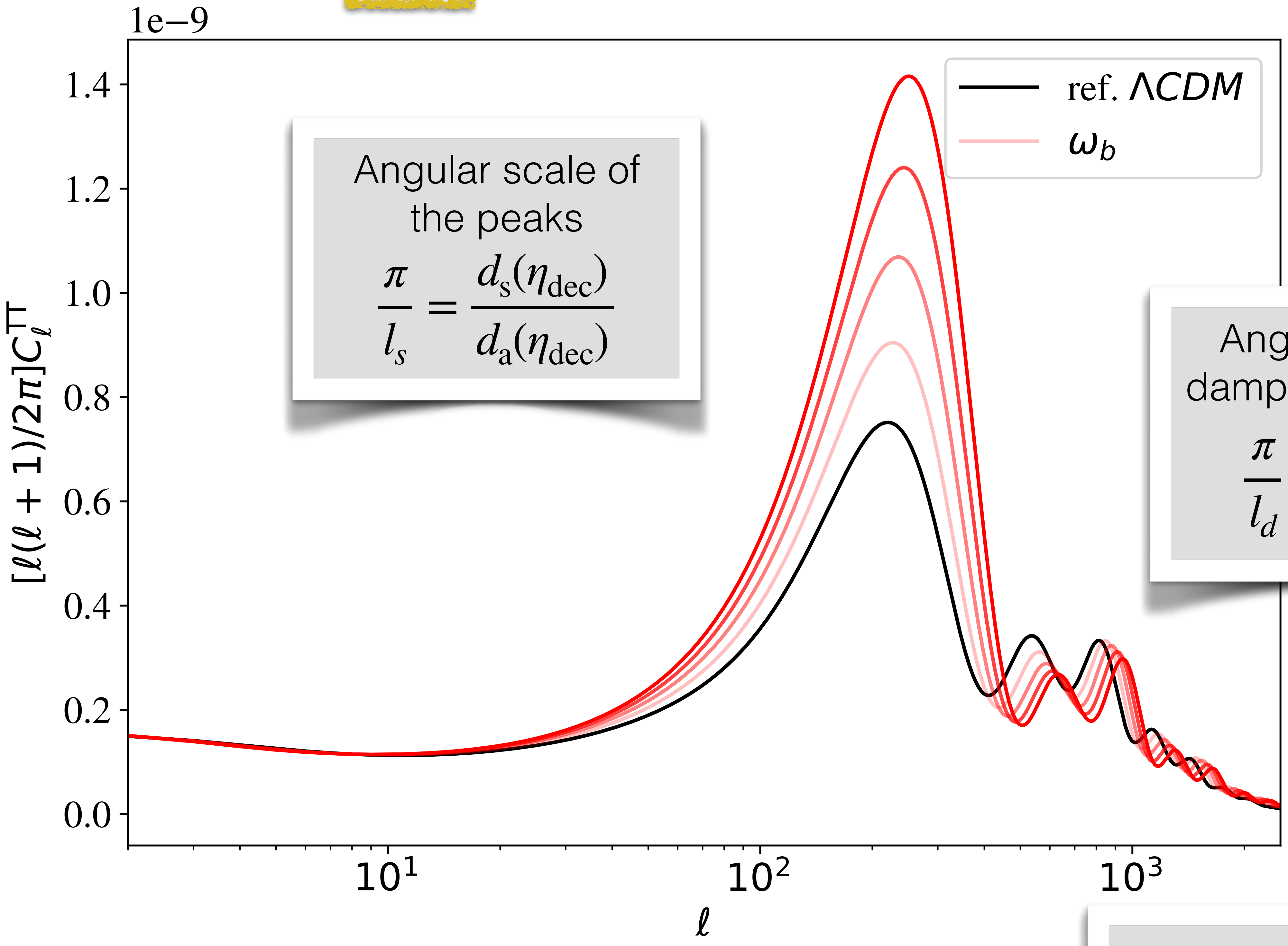
$$\{\boxed{\omega_b,}\,\omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



$$\{\boxed{\omega_b}, \omega_m, \Omega_\Lambda, \tau_{\rm reio}, A_s, n_s\}$$



Angular scale of the peaks

$$\frac{\pi}{l_s} = \frac{d_{\rm s}(\eta_{\rm dec})}{d_{\rm a}(\eta_{\rm dec})}$$

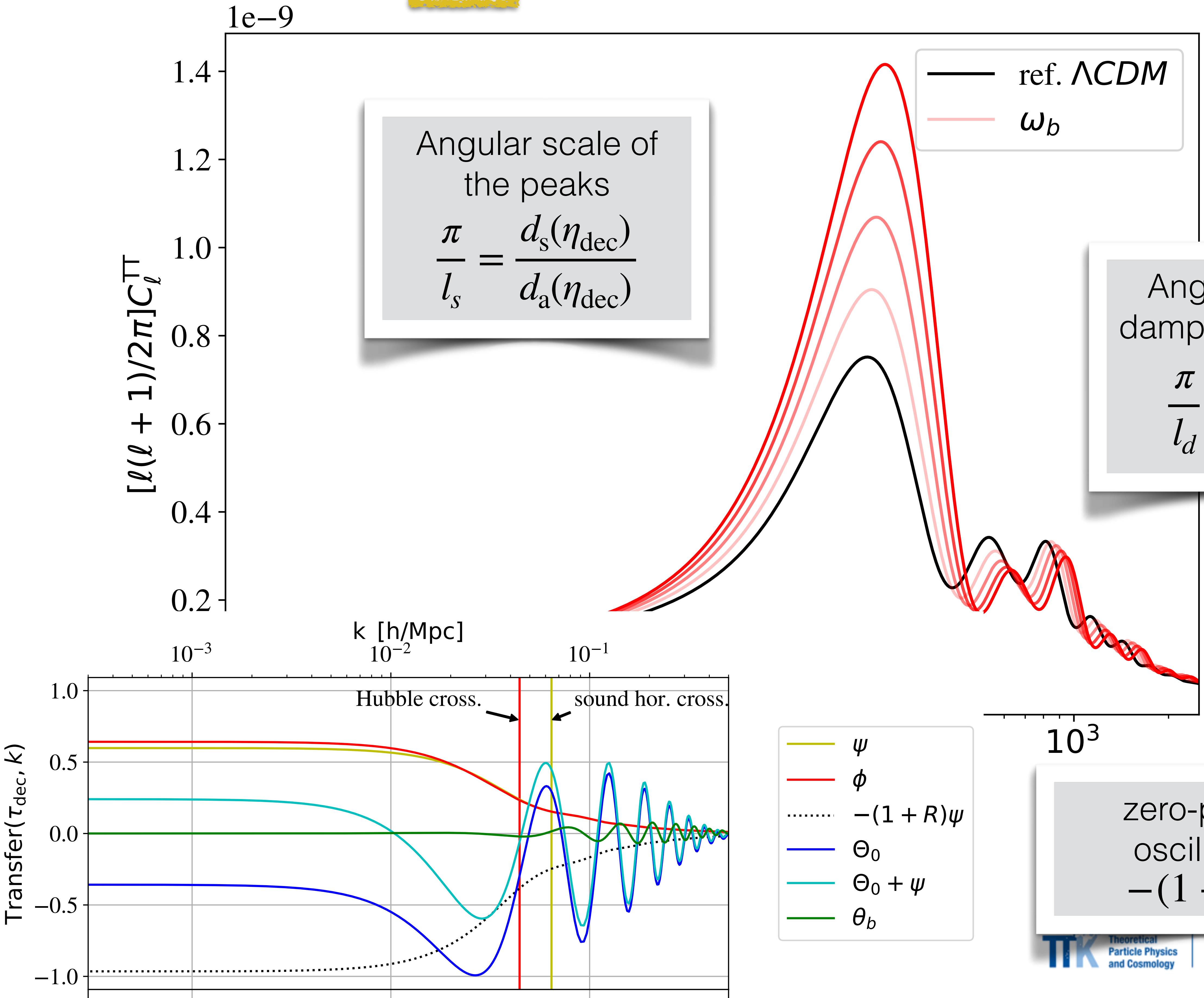
Angular scale of damping enveloppe

$$\frac{\pi}{l_d} = \frac{\lambda_{\rm d}(\eta_{\rm dec})}{d_{\rm a}(\eta_{\rm dec})}$$

zero-point of oscillations

$$-(1+R)\,\psi$$

$$\{\boxed{\omega_b,}\,\omega_m,\,\Omega_\Lambda,\,\tau_{\rm reio},\,A_s,\,n_s\}$$



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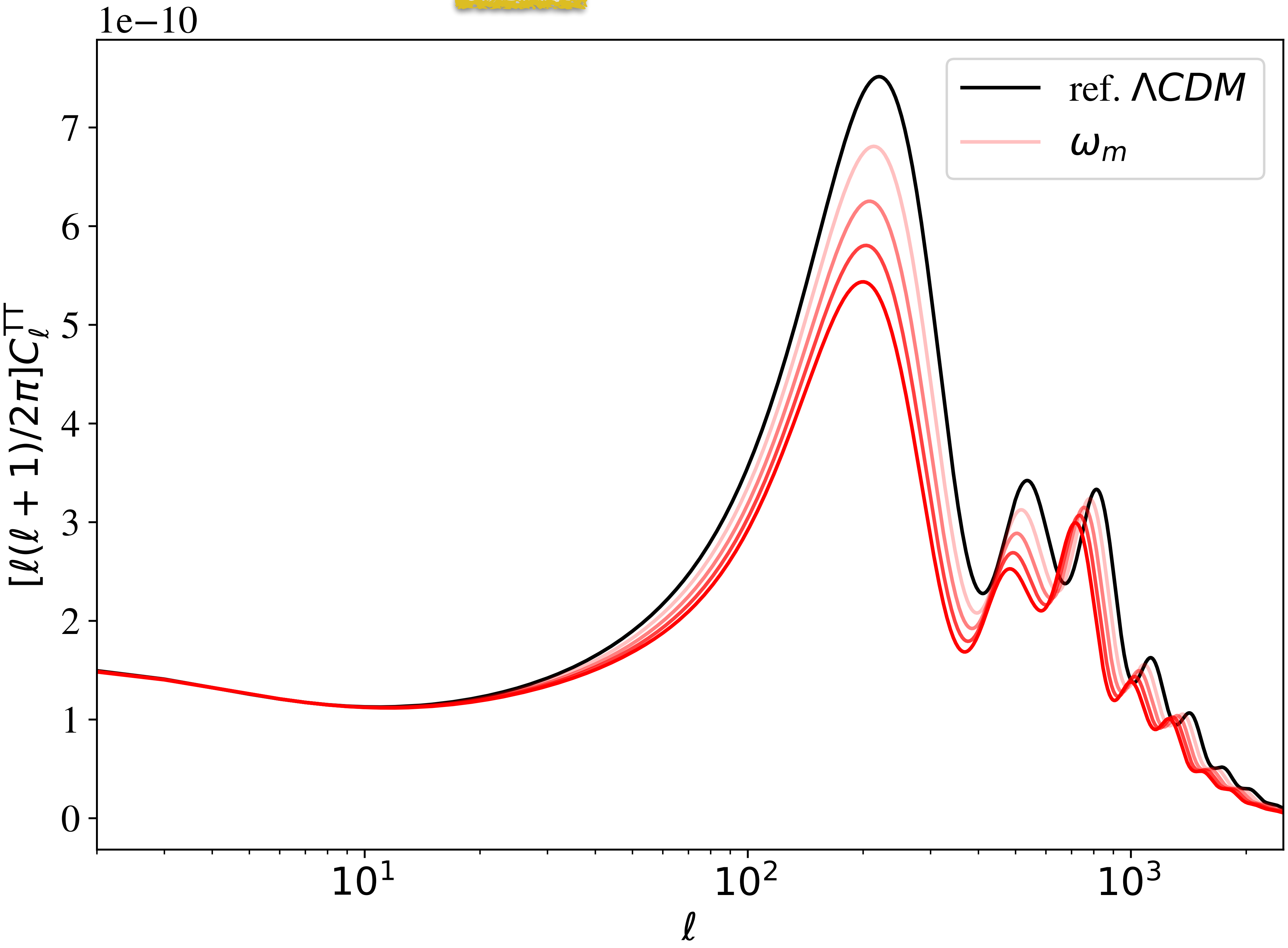

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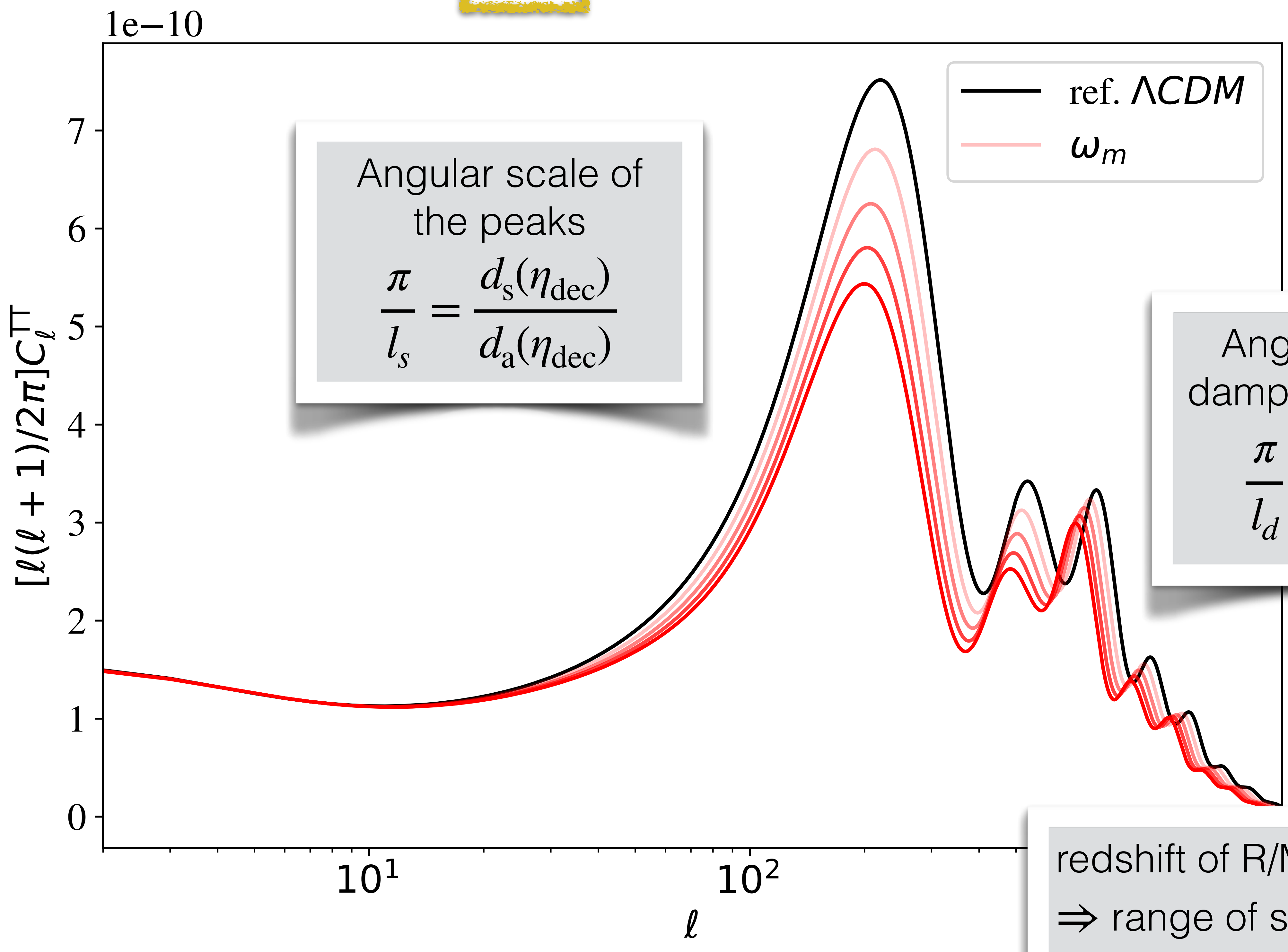
$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_{\mathrm{reio}}, A_s, n_s\}$$



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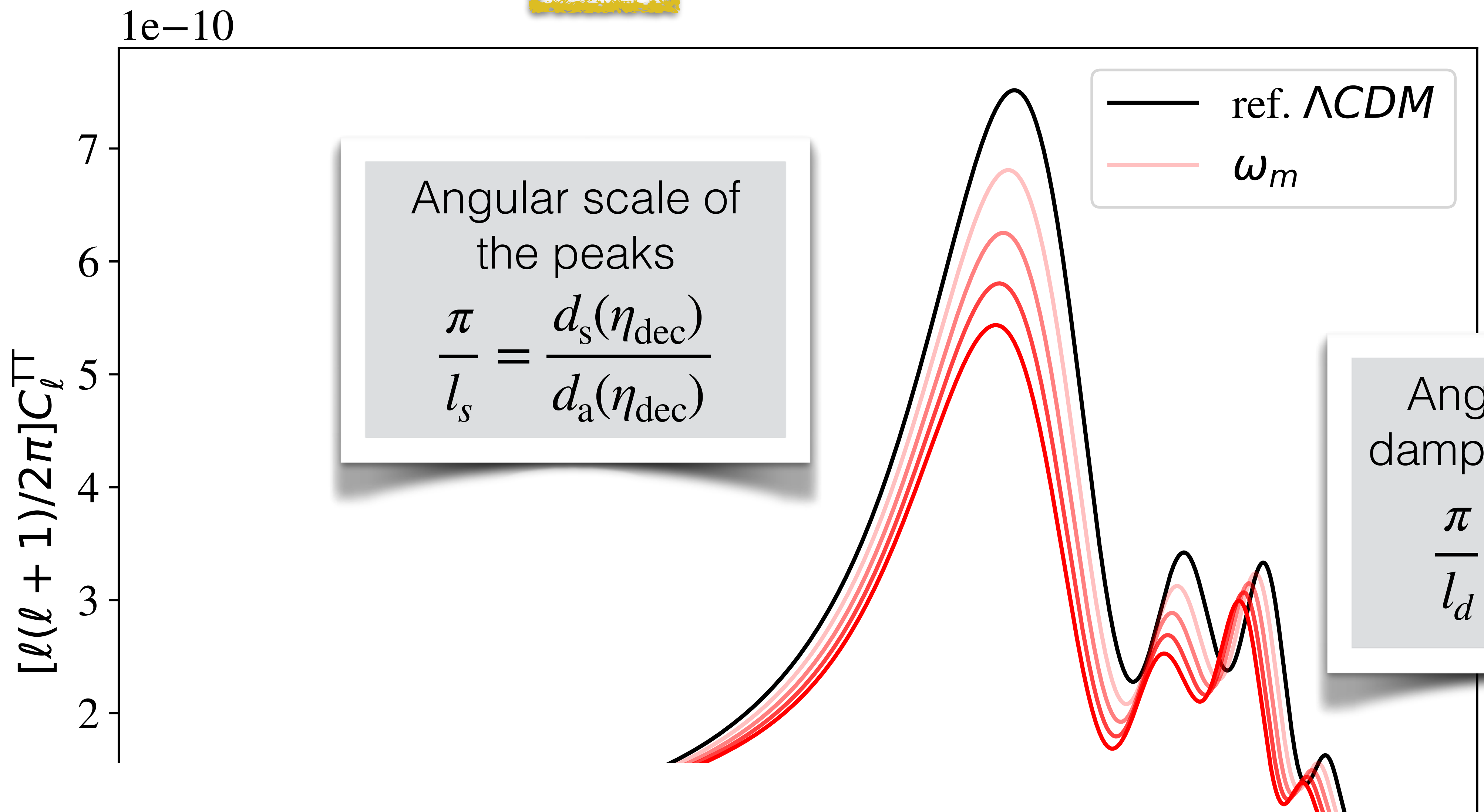
redshift of R/M equality z_{eq} :

⇒ range of scales affected by gravitational boost

⇒ early ISW

$$\{\omega_b, \boxed{\omega_m}, \Omega_\Lambda, \tau_\text{reio}, A_s, n_s\}$$



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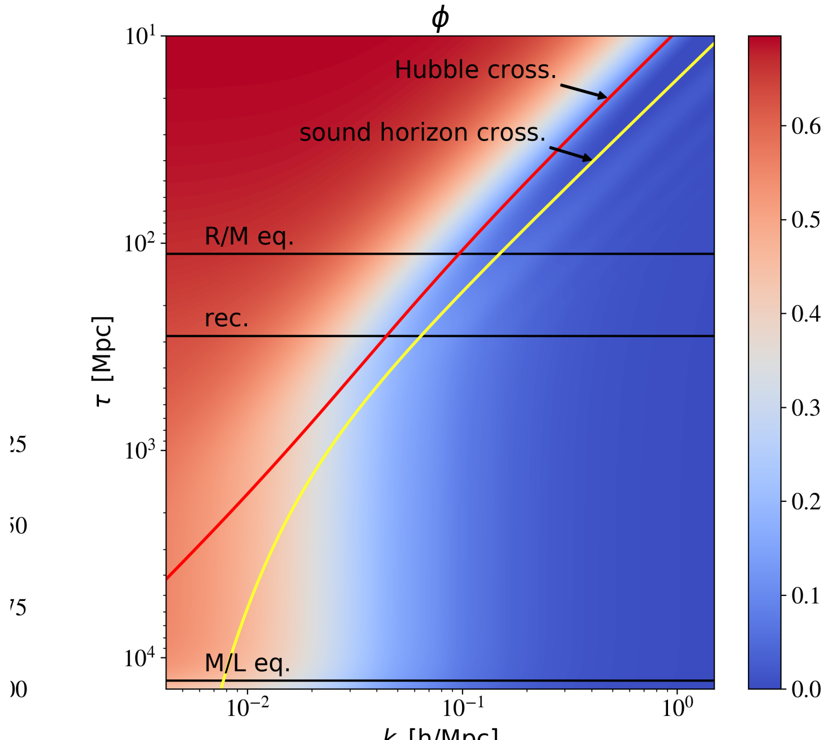

redshift of R/M equality z_eq :

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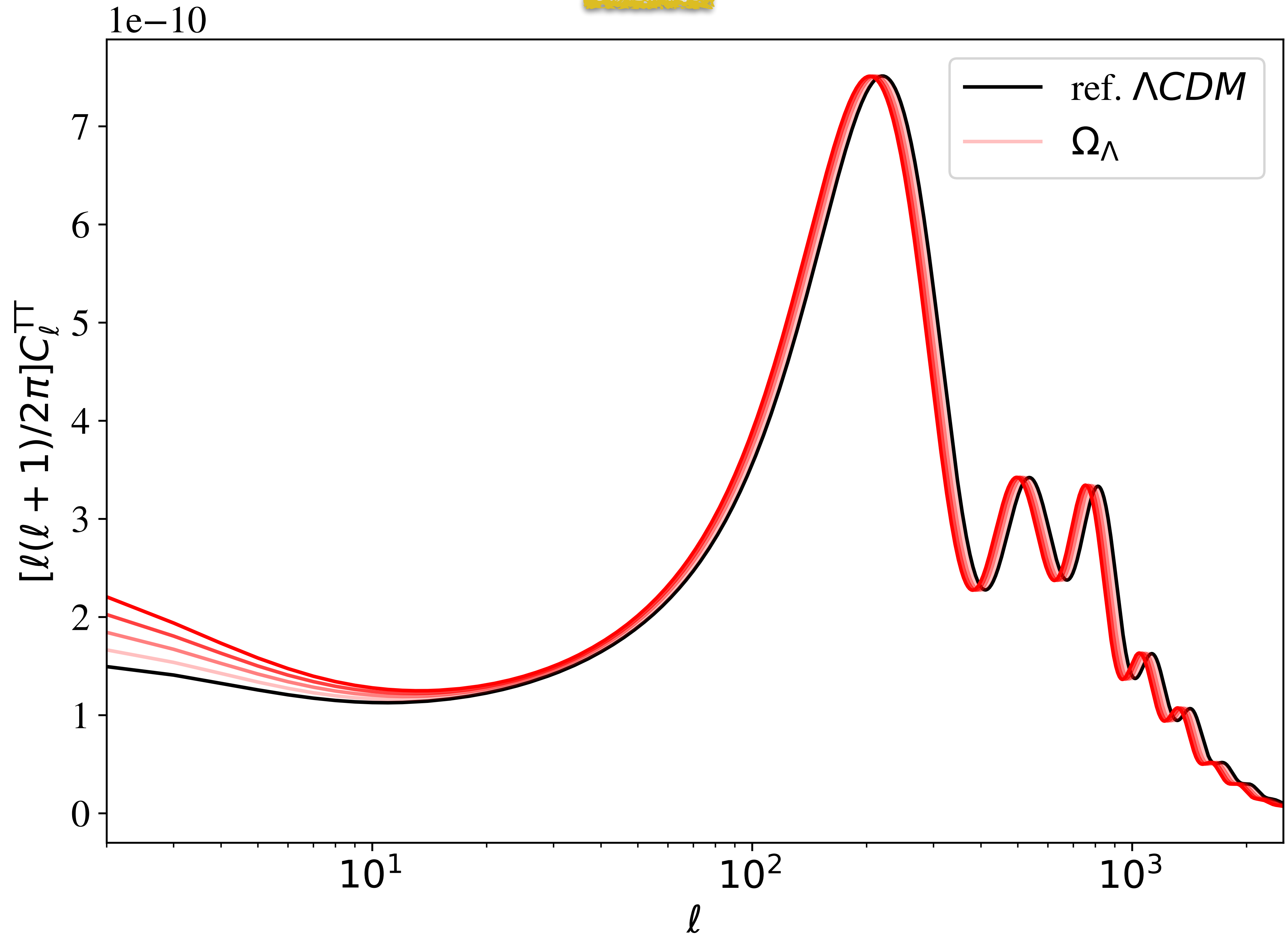

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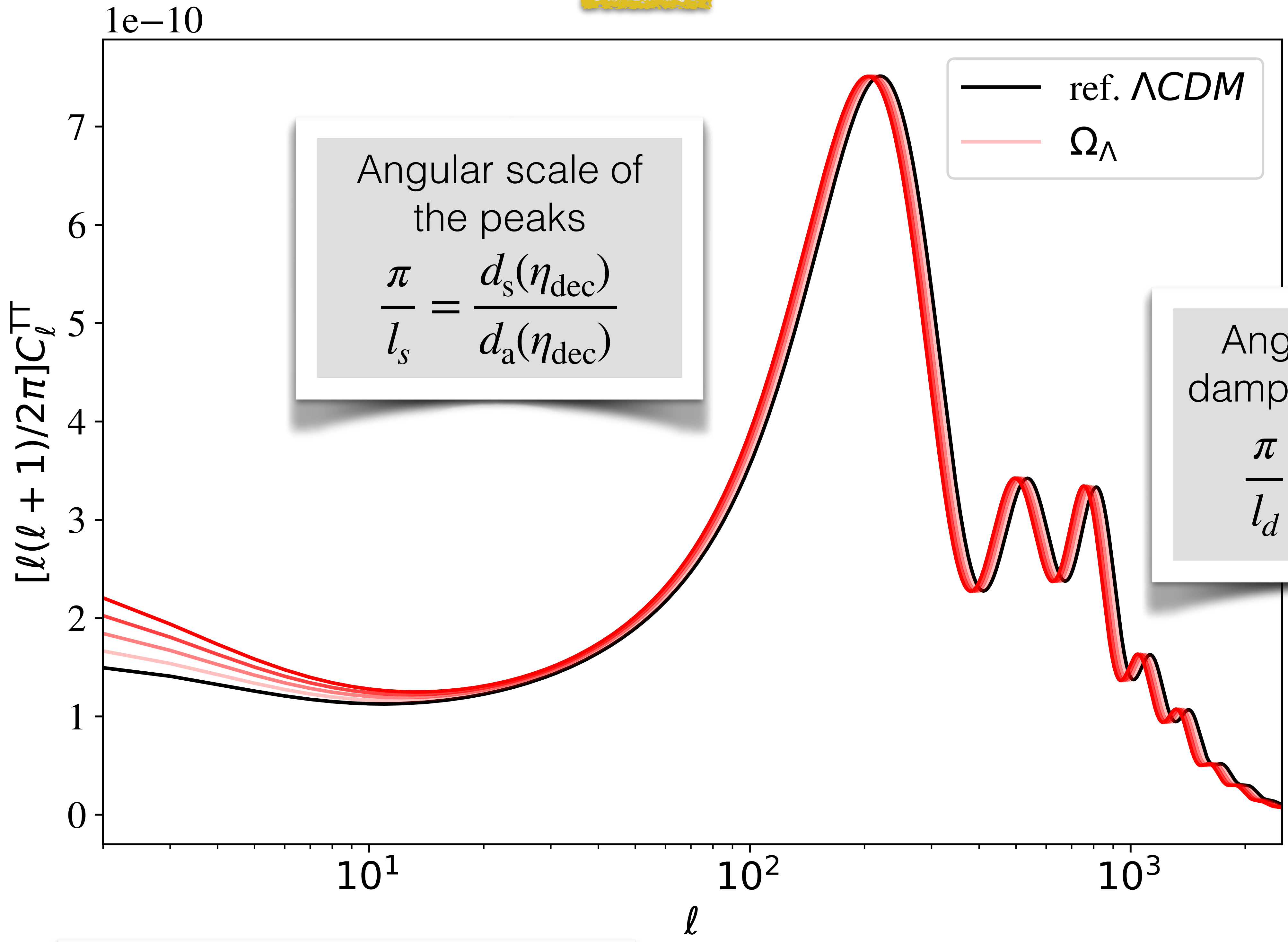

Angular scale of the peaks

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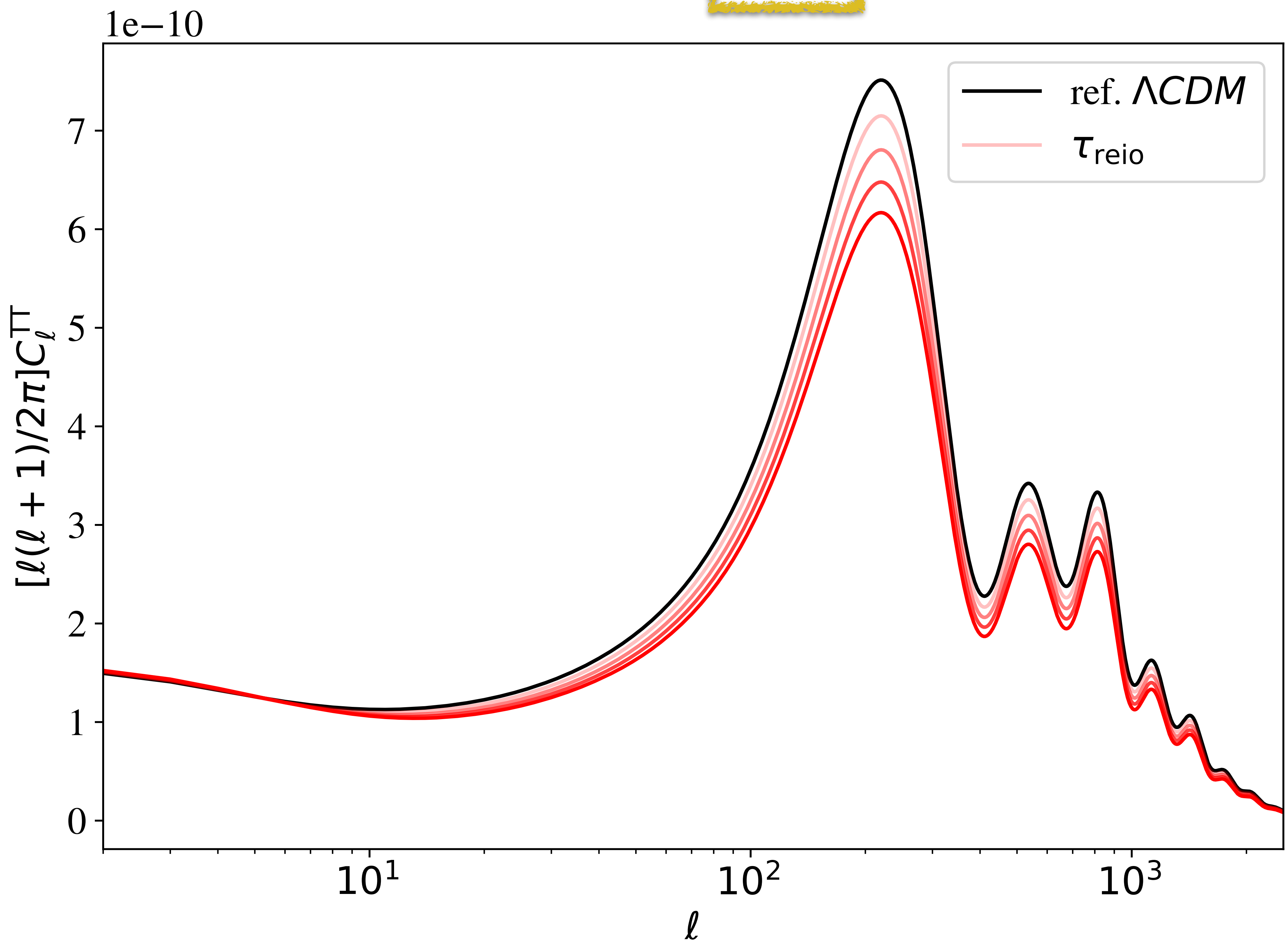
Angular scale of damping enveloppe

$$\frac{\pi}{l_d} = \frac{\lambda_d(\eta_{\rm dec})}{d_a(\eta_{\rm dec})}$$

redshift of M/Λ equality $z_{\rm eq}$:

⇒ late ISW

$$\{\omega_b, \omega_m, \Omega_\Lambda, \boxed{\tau_{\rm reio}}, A_s, n_s\}$$



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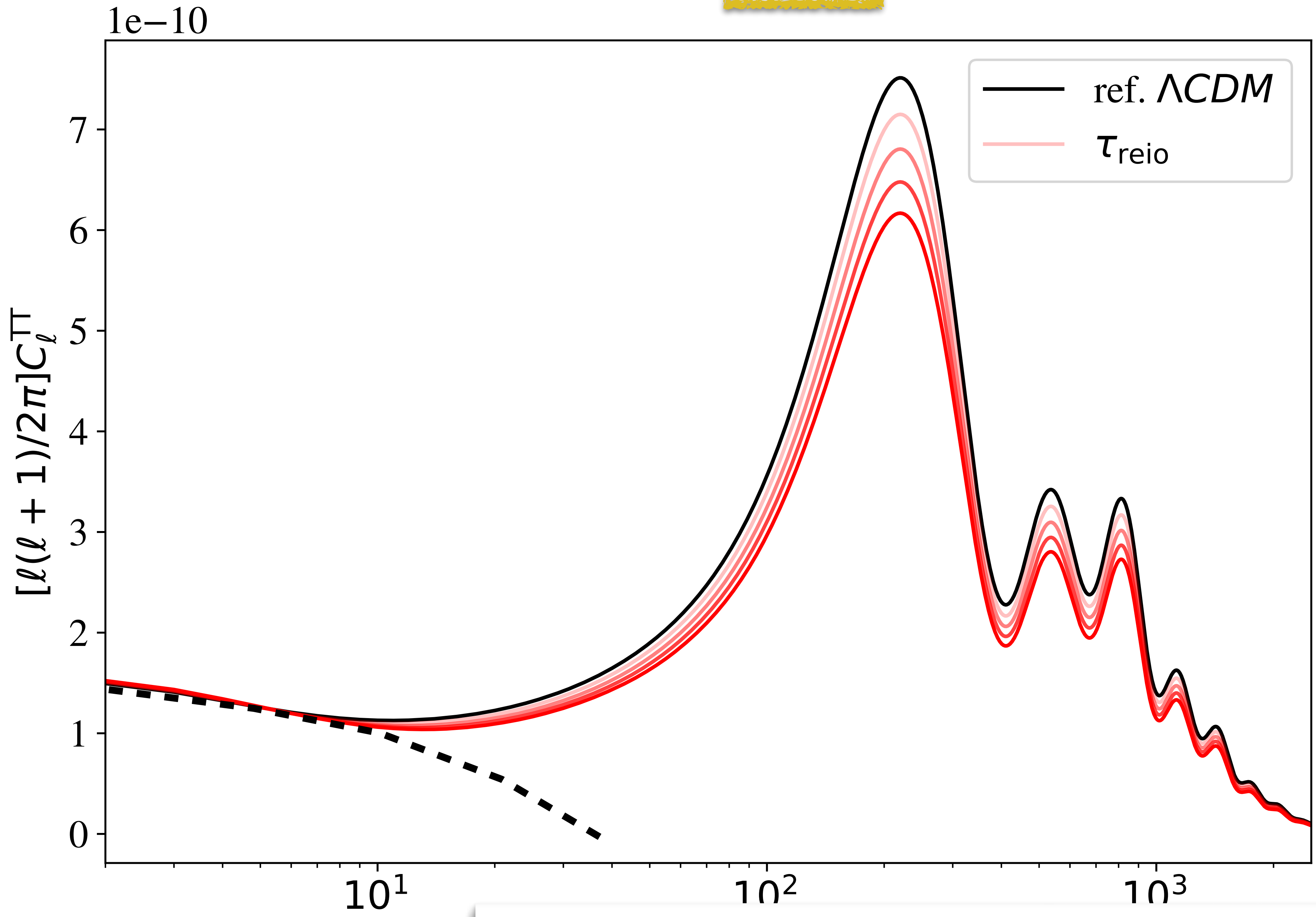
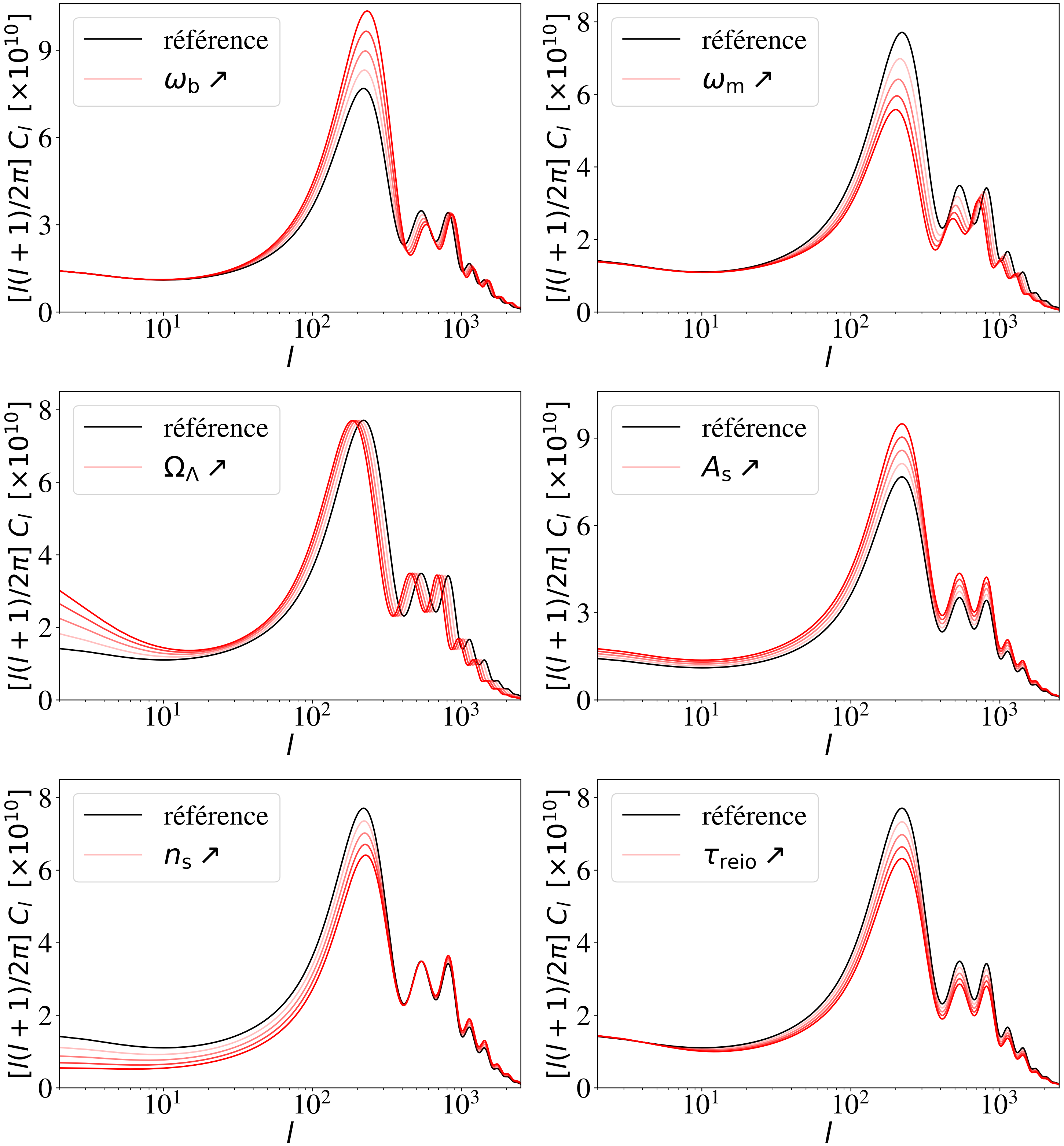


Visibility function

$$g(\eta) \simeq \delta(\eta - \eta_{\rm dec})$$

$$\downarrow$$

$$g(\eta) \simeq e^{-\tau_{\rm reio}}\,\delta(\eta - \eta_{\rm dec}) + (1 - e^{-\tau_{\rm reio}})\,\delta(\eta - \eta_{\rm reio})$$



8 physical governing C_l's shape

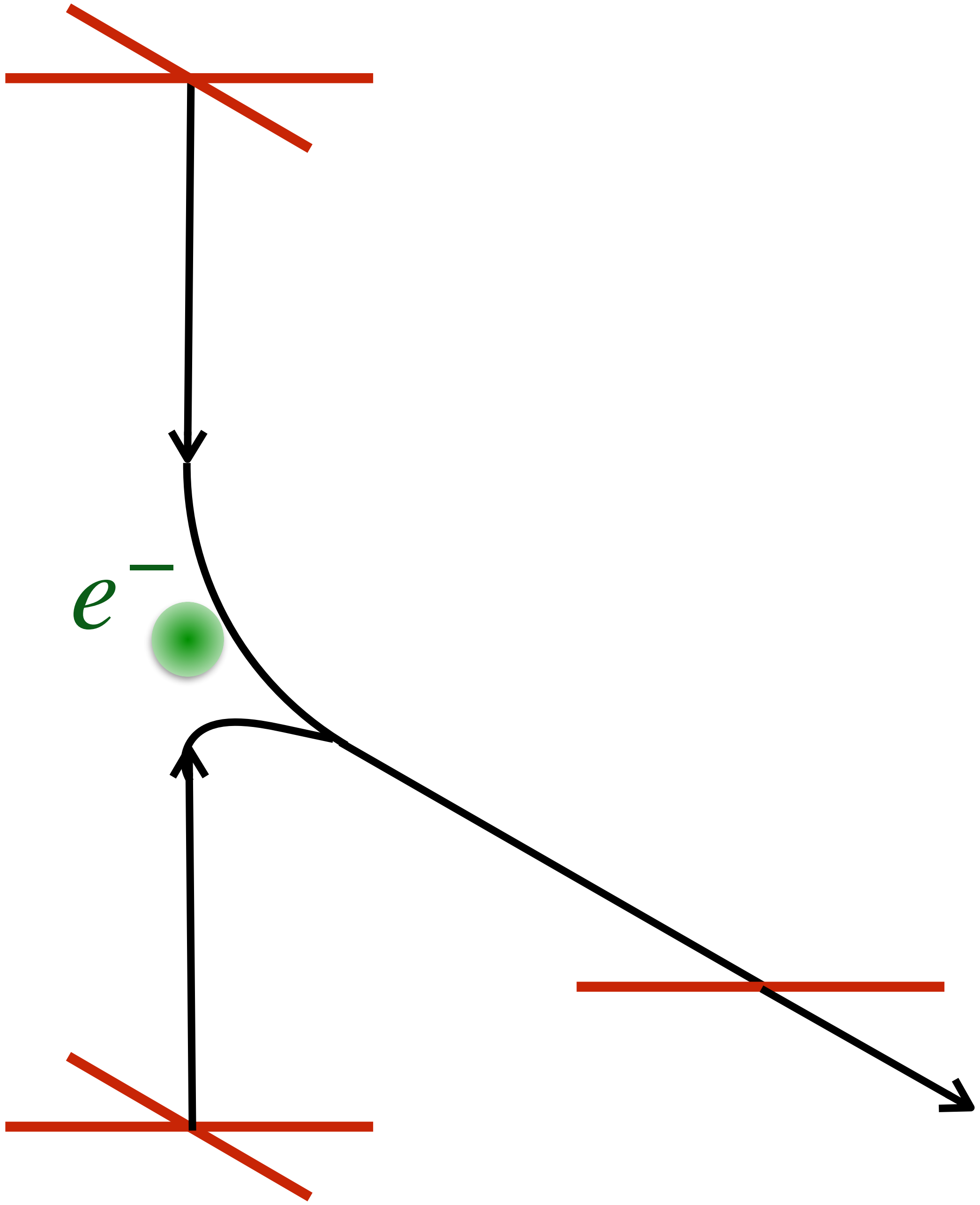
- C1: angular scale of the peaks, θ_s
- C2: gravity/pressure at rec., R_{rec}
- C3: interval between z_{eq} and z_{dec}
- C4: angular scale of damping, θ_d
- C5: global amplitude
- C6: global tilt
- C7: plateau tilting by late ISW
- C8: reionisation steplike suppression

but all tight to 6 parameters in ΛCDM

Extended cosmologies? … more parameters … but also more effects …

CMB polarisation

CMB polarisation



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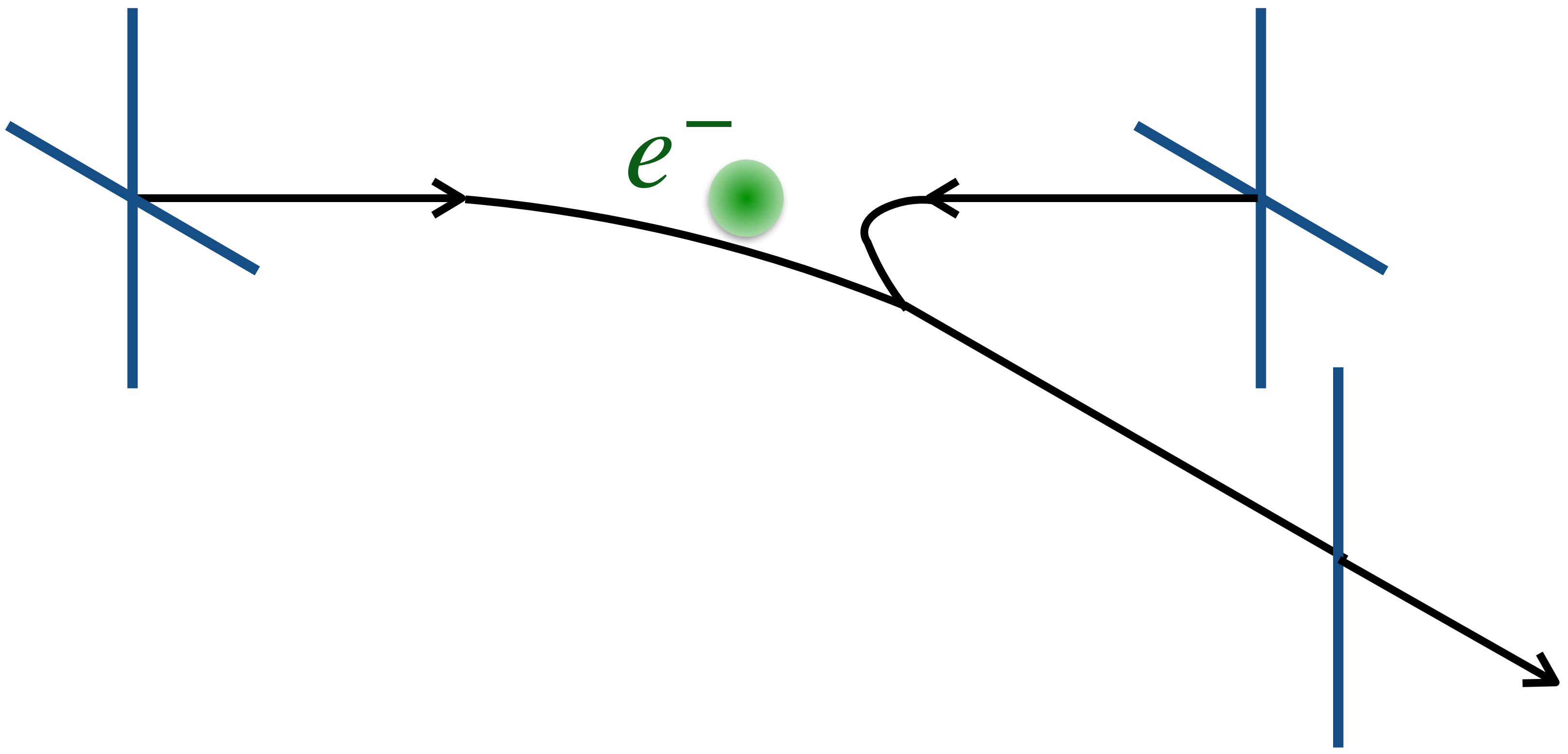

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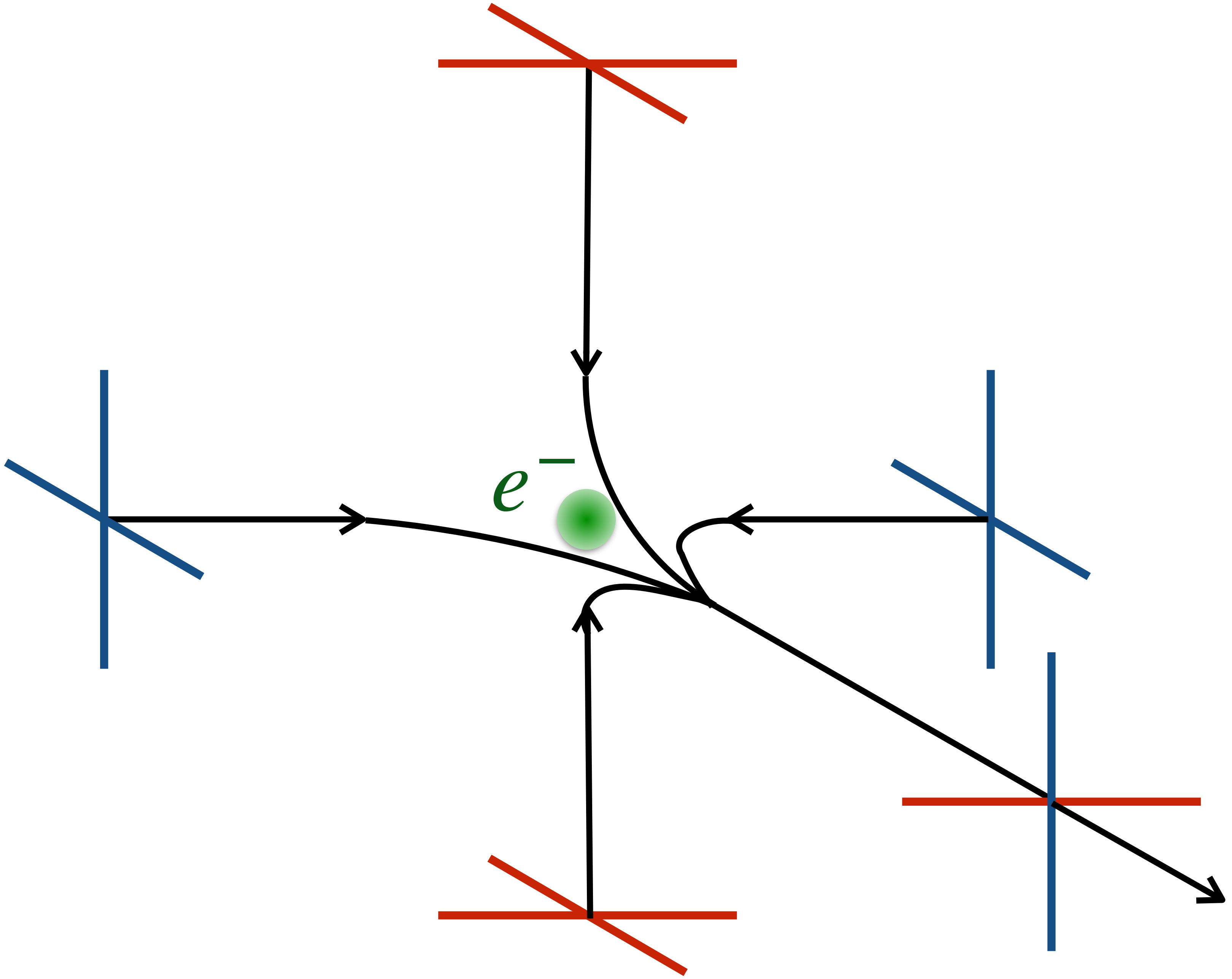

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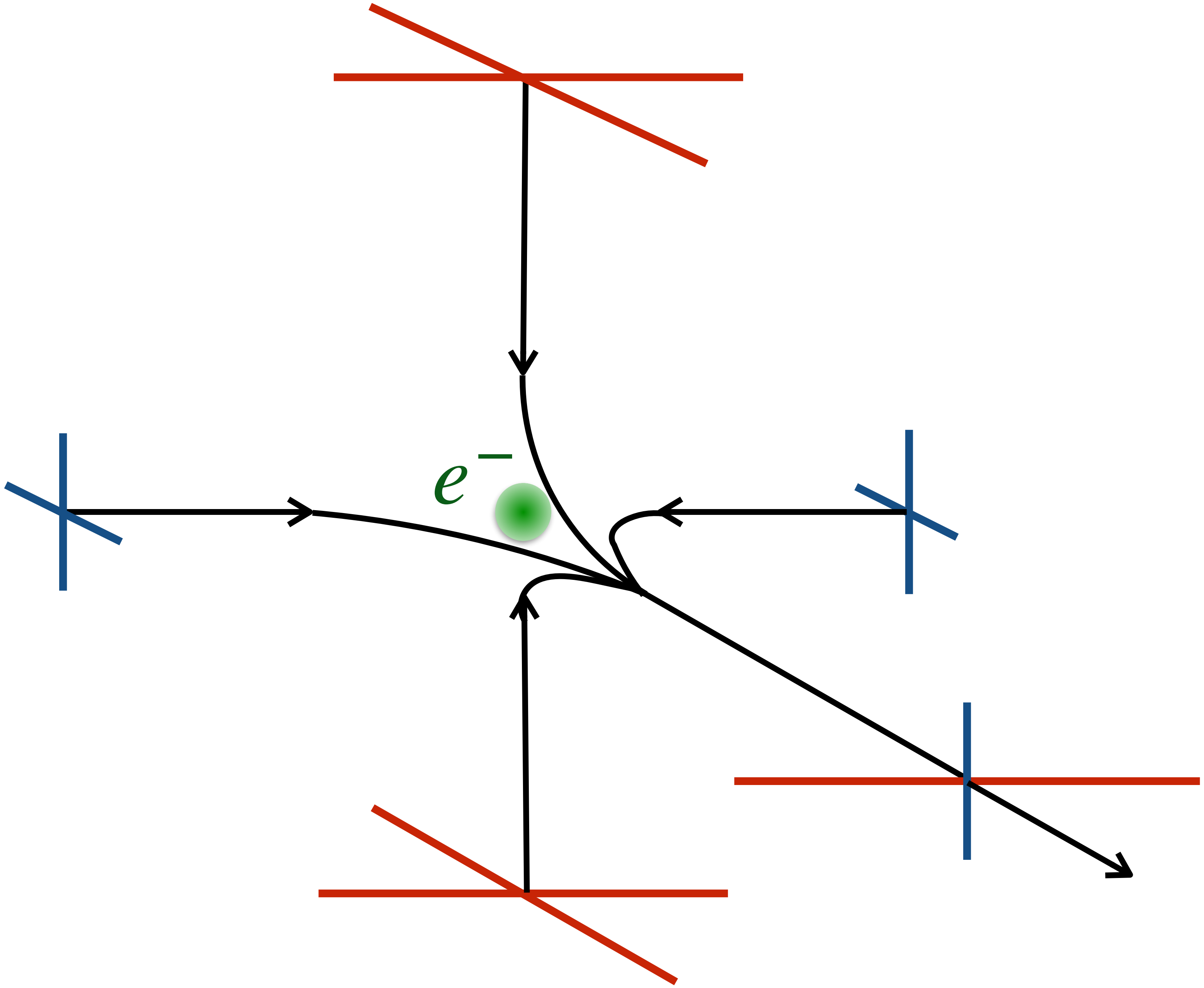

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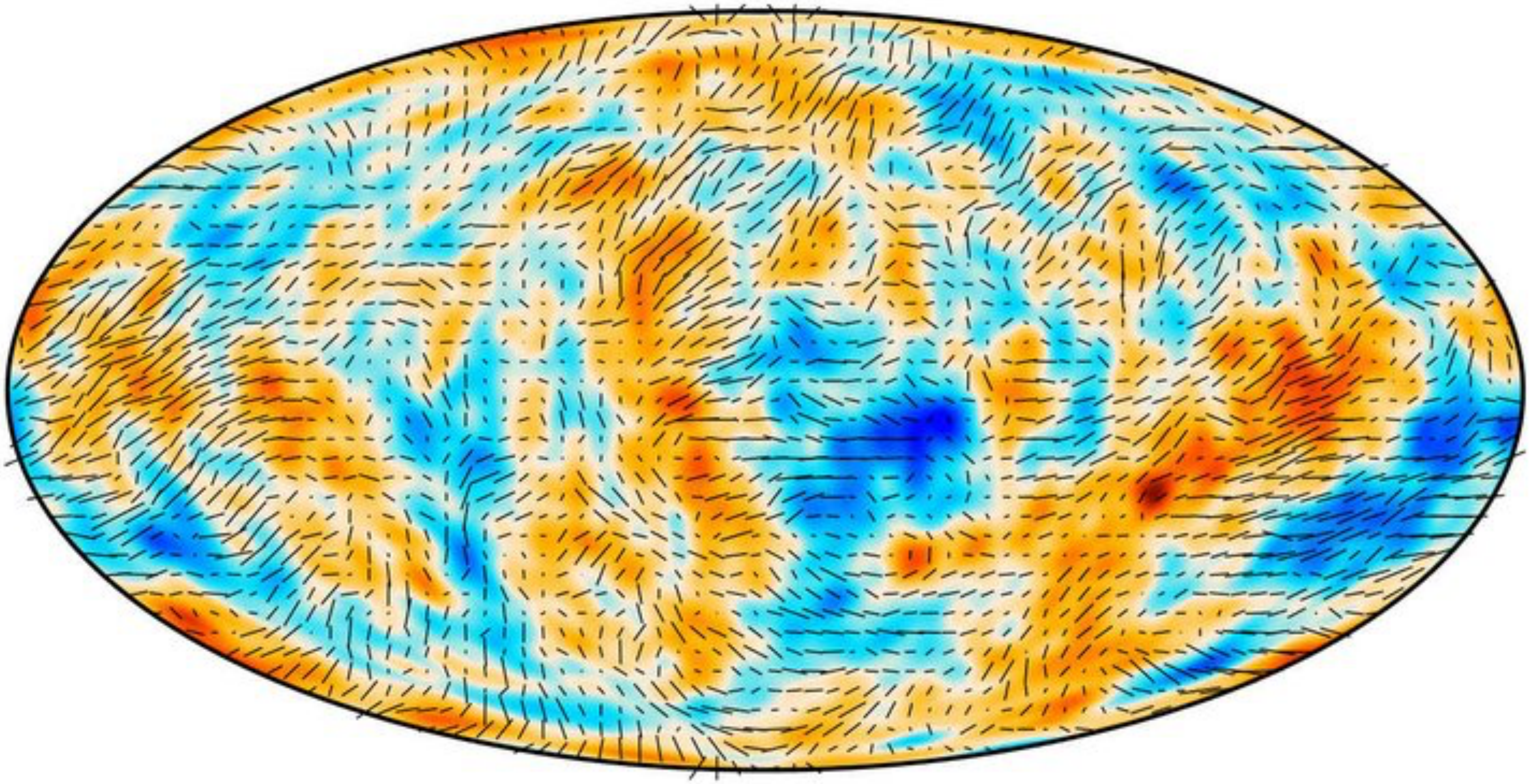

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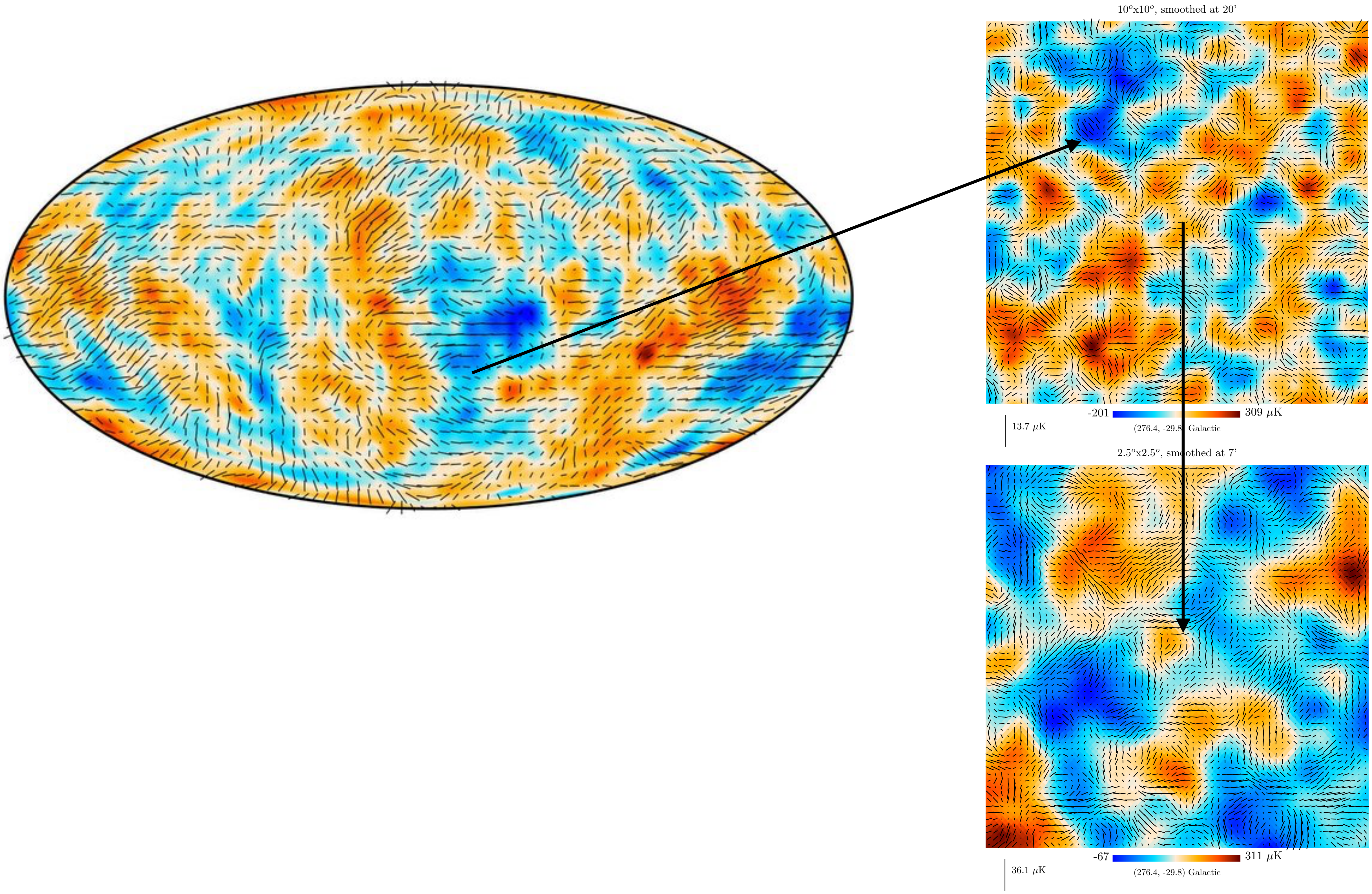

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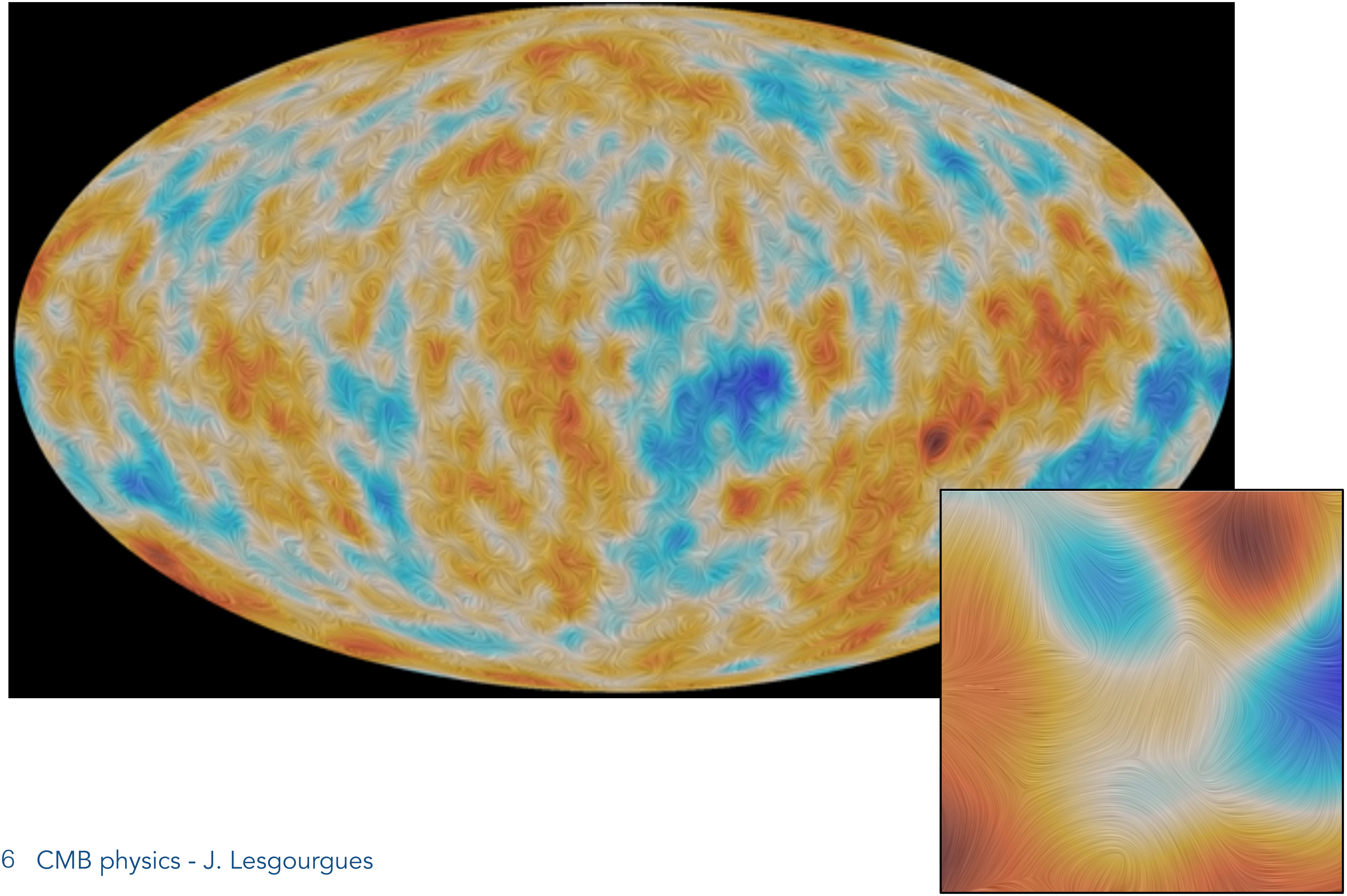

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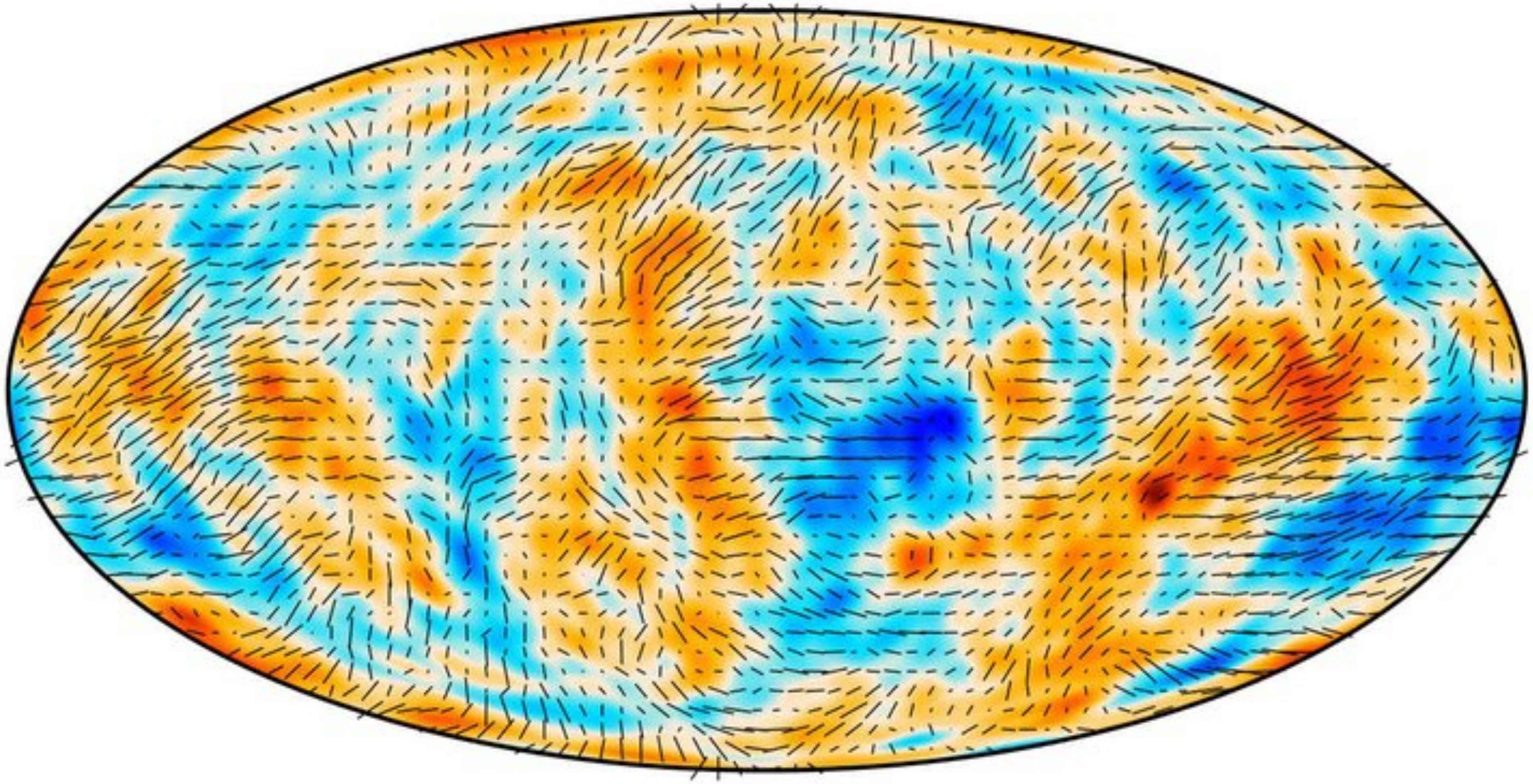

1 spin-two map ⇔ 2 scalar maps (E = gradient field, B = rotation field), but:

scalar modes → gradients → B-mode vanish

CMB polarisation