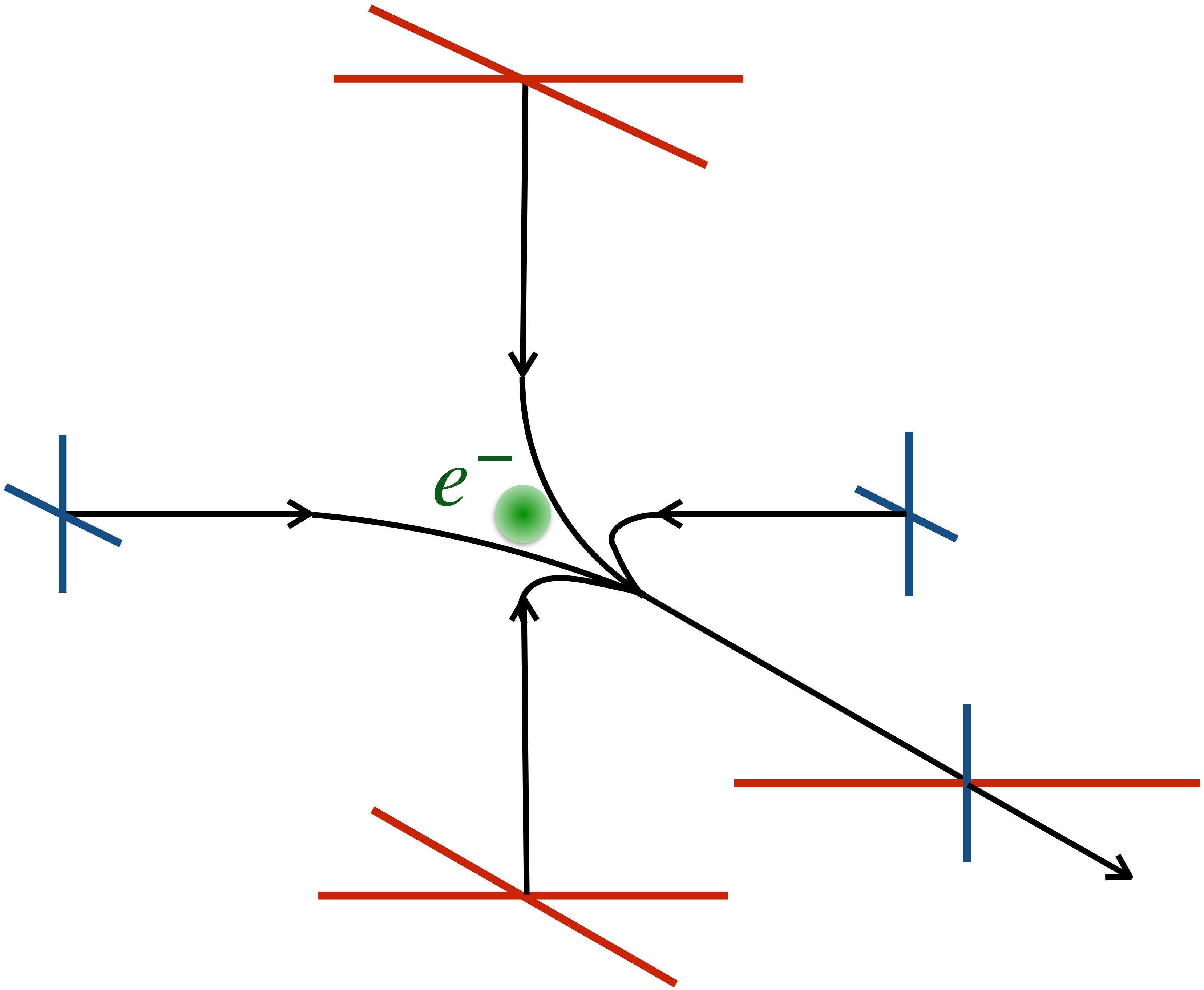
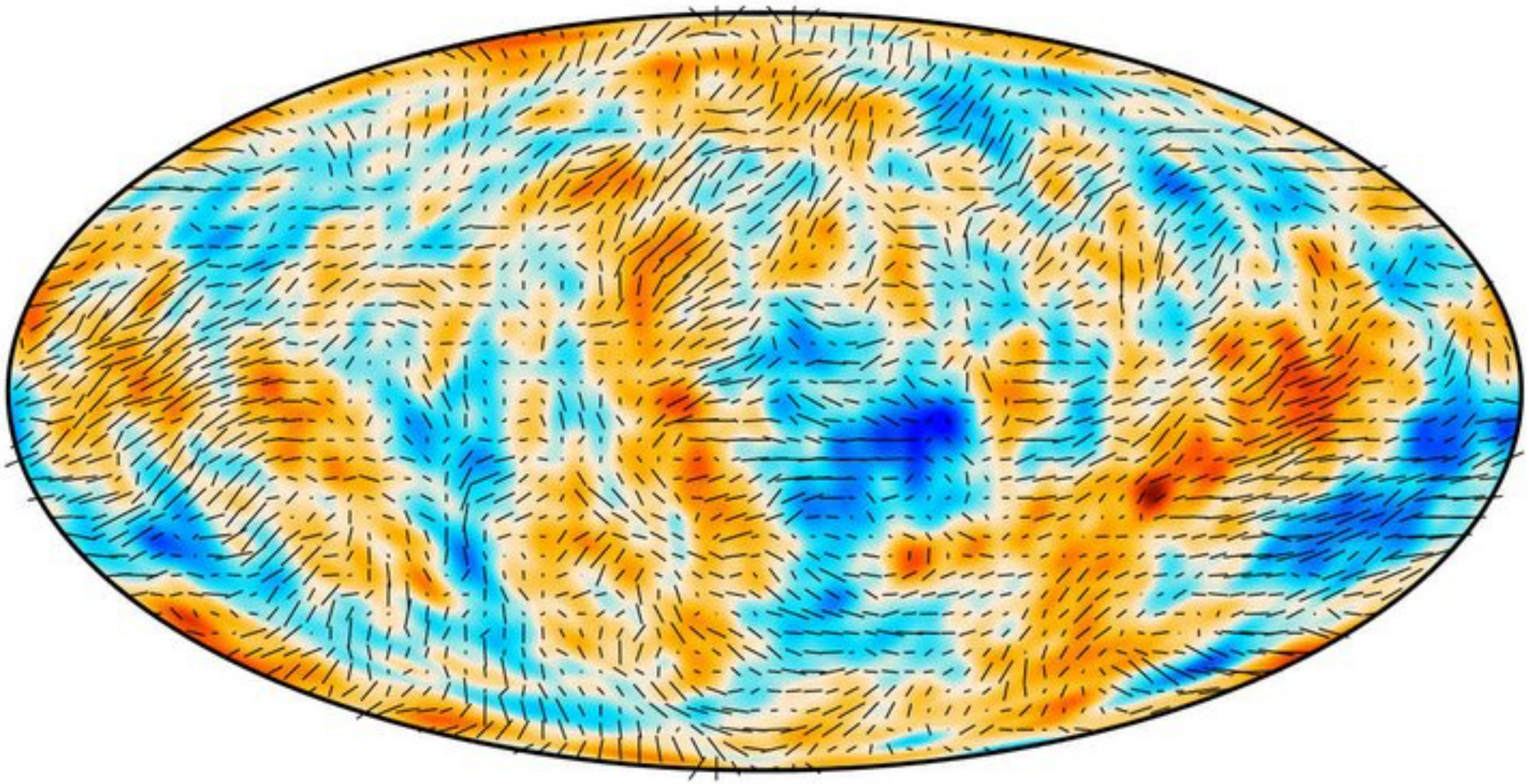
Summary of Lecture 4



Summary of Lecture 4



1 spin-two map ⇔ 2 scalar maps (E = gradient field, B = rotation field), but:

scalar modes → gradients → B-mode vanish

CMB polarisation

Temperature spectrum: $C_\ell^{TT} = \langle a_{lm}^T a_{lm}^{T*} \rangle = \frac{2}{\pi} \int dk\, k^2 [\Theta_\ell(\eta_0, k)]^2 \;\; P_{\mathcal{R}}(k)$

with transfer function
$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \, \{ g\,(\Theta_0 + \psi)\; j_l(k(\eta_0 - \eta))$$
$$+ \; g\, k^{-1} \theta_{\rm b}\; j_l'(k(\eta_0 - \eta))$$
$$+ \; e^{-\tau} (\phi' + \psi')\, j_l(k(\eta_0 - \eta)) \}$$

For polarisation:
Kosowsky 1996; Seljak & Zaldarriaga astro-ph/9609170; Hu & White astro-ph/9702170

CMB polarisation

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$+ \, g\, k^{-1} \theta_{\rm b}\; j_l'(k(\eta_0 - \eta))$

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E-mode polarisation spectrum: $C_\ell^{EE} = \langle a_{lm}^E a_{lm}^{E*} \rangle = \frac{2}{\pi} \int dk\, k^2 [\Delta_\ell^E(\eta_0, k)]^2 \;\; P_{\mathcal{R}}(k)$

E

with transfer function $\Delta_l^E(\eta_0, k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \;\; g \;\; \{\Theta_2 + \dots\} \; (\dots) \; j_l(k(\eta_0 - \eta))$

CMB polarisation

Temperature spectrum: $C_\ell^{TT} = \langle a_{lm}^T a_{lm}^{T*} \rangle = \frac{2}{\pi} \int dk\, k^2 [\Theta_\ell(\eta_0, k)]^2 \;\; P_{\mathcal{R}}(k)$

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$$+ \; g \, k^{-1} \theta_{\rm b} \; j_l'(k(\eta_0 - \eta))$$

$$+ \; e^{-\tau} (\phi' + \psi') \, j_l(k(\eta_0 - \eta))\} \;\; + \ldots$$

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Kosowsky 1996; Seljak & Zaldarriaga astro-ph/9609170; Hu & White astro-ph/9702170

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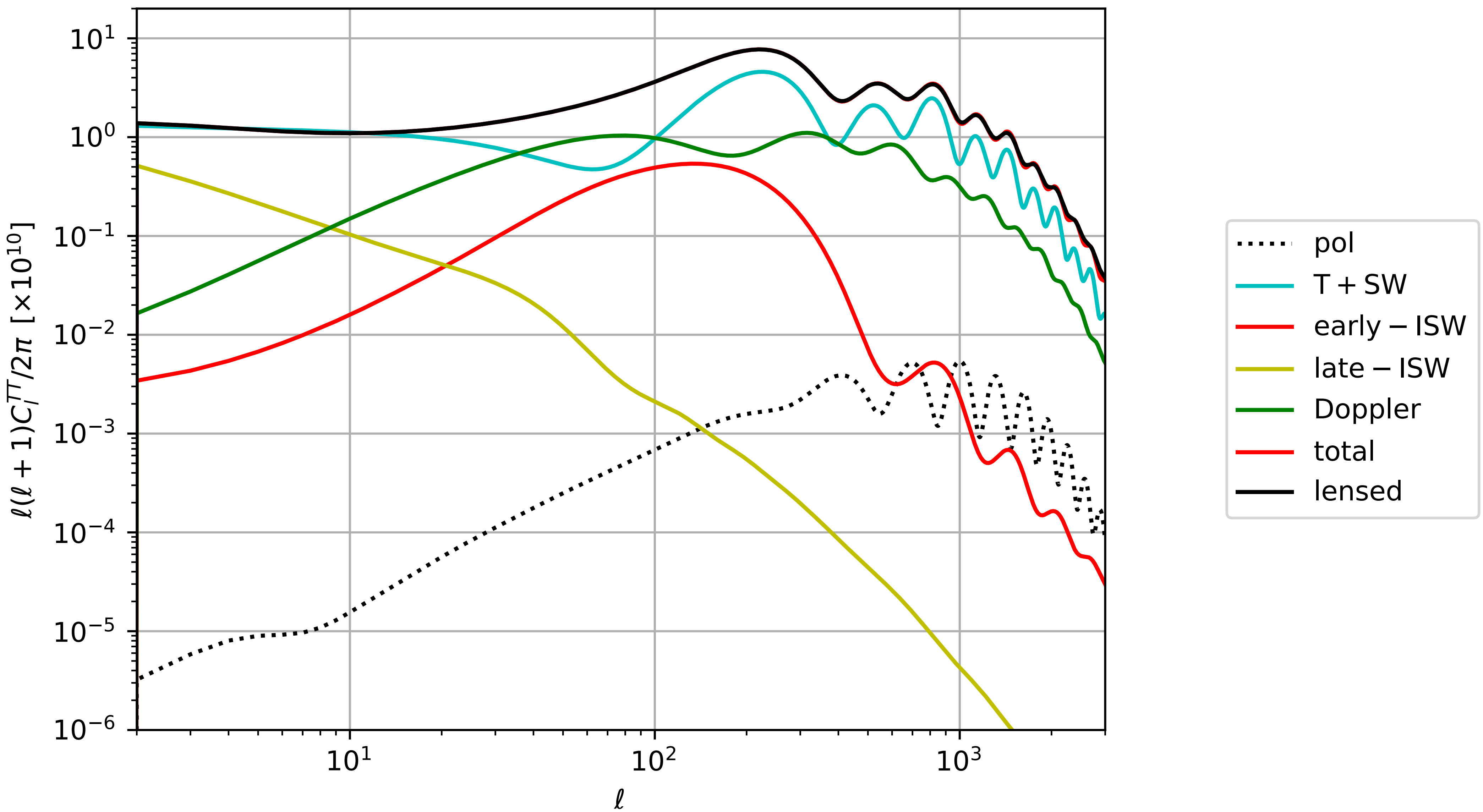
E

with transfer function $\Delta_l^E(\eta_0, k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \;\; g \;\; \{\Theta_2 + \ldots\} \; (\ldots) \, j_l(k(\eta_0 - \eta))$

CMB polarisation

Corrections to temperature spectrum taking into account polarisation anisotropies



notebooks/cltt_terms.ipynb + loglog + ‘temperature_contributions’:’pol’

CMB polarisation

Temperature spectrum: $C_\ell^{TT} = \langle a_{lm}^T a_{lm}^{T*} \rangle = \frac{2}{\pi} \int dk\, k^2 [\Theta_\ell(\eta_0, k)]^2 \;\; P_{\mathcal{R}}(k)$

with transfer function

$$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \,\{ g\, (\Theta_0 + \psi)\; j_l(k(\eta_0-\eta)) + g\, k^{-1}\theta_{\rm b}\; j_l'(k(\eta_0-\eta)) + e^{-\tau}(\phi' + \psi')\, j_l(k(\eta_0-\eta)) \} \; + \ldots$$

For polarisation:

Kosowsky 1996; Seljak & Zaldarriaga astro-ph/9609170; Hu & White astro-ph/9702170

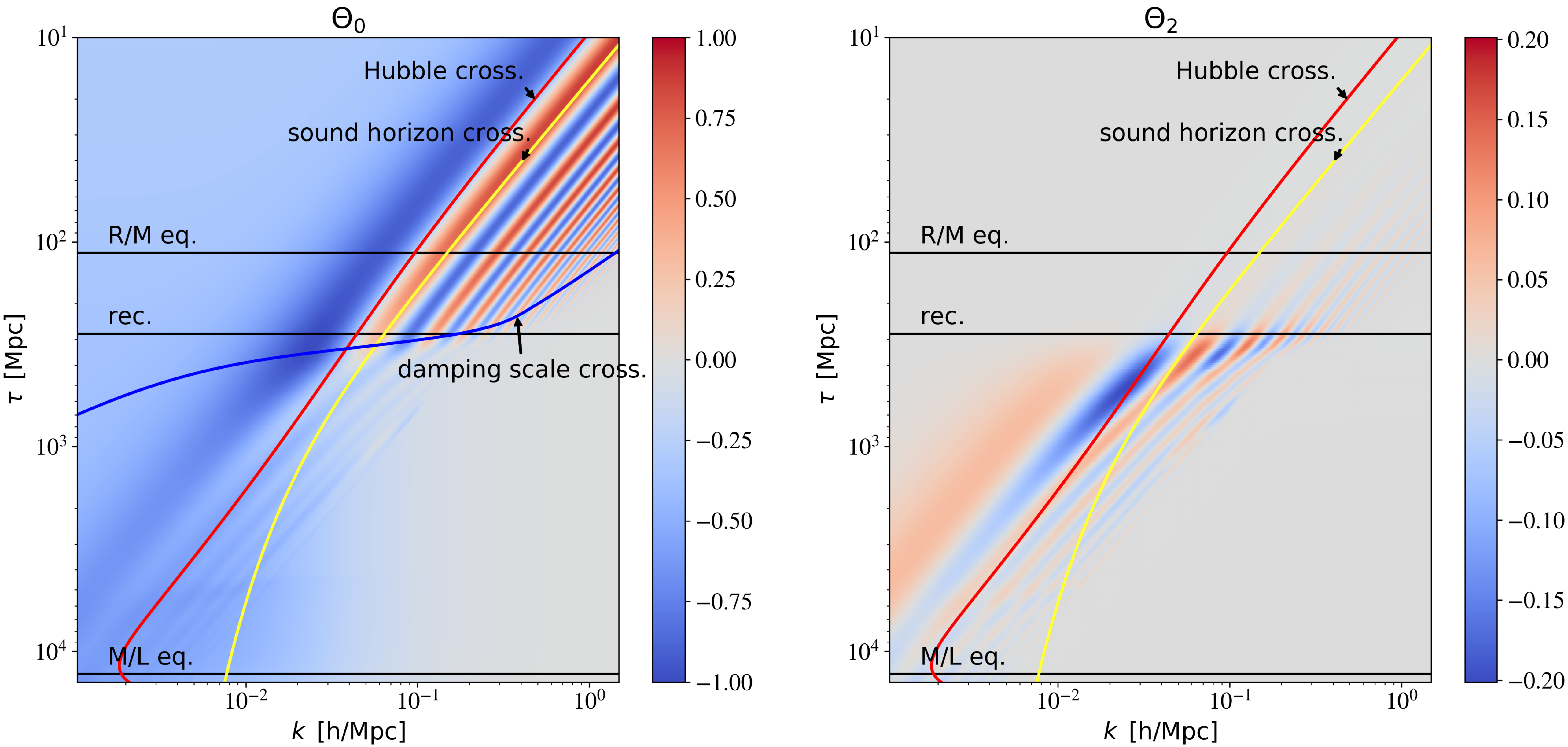
E-mode polarisation spectrum: $C_\ell^{EE} = \langle a_{lm}^E a_{lm}^{E*} \rangle = \frac{2}{\pi} \int dk\, k^2 [\Delta_\ell^E(\eta_0, k)]^2 \;\; P_{\mathcal{R}}(k)$

E

with transfer function $\Delta_l^E(\eta_0, k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \;\; g \;\; \{ \Theta_2 + \ldots \} \; (\ldots)\, j_l(k(\eta_0 - \eta))$

… no Doppler … no Sachs-Wolfe … no ISW …

CMB polarisation



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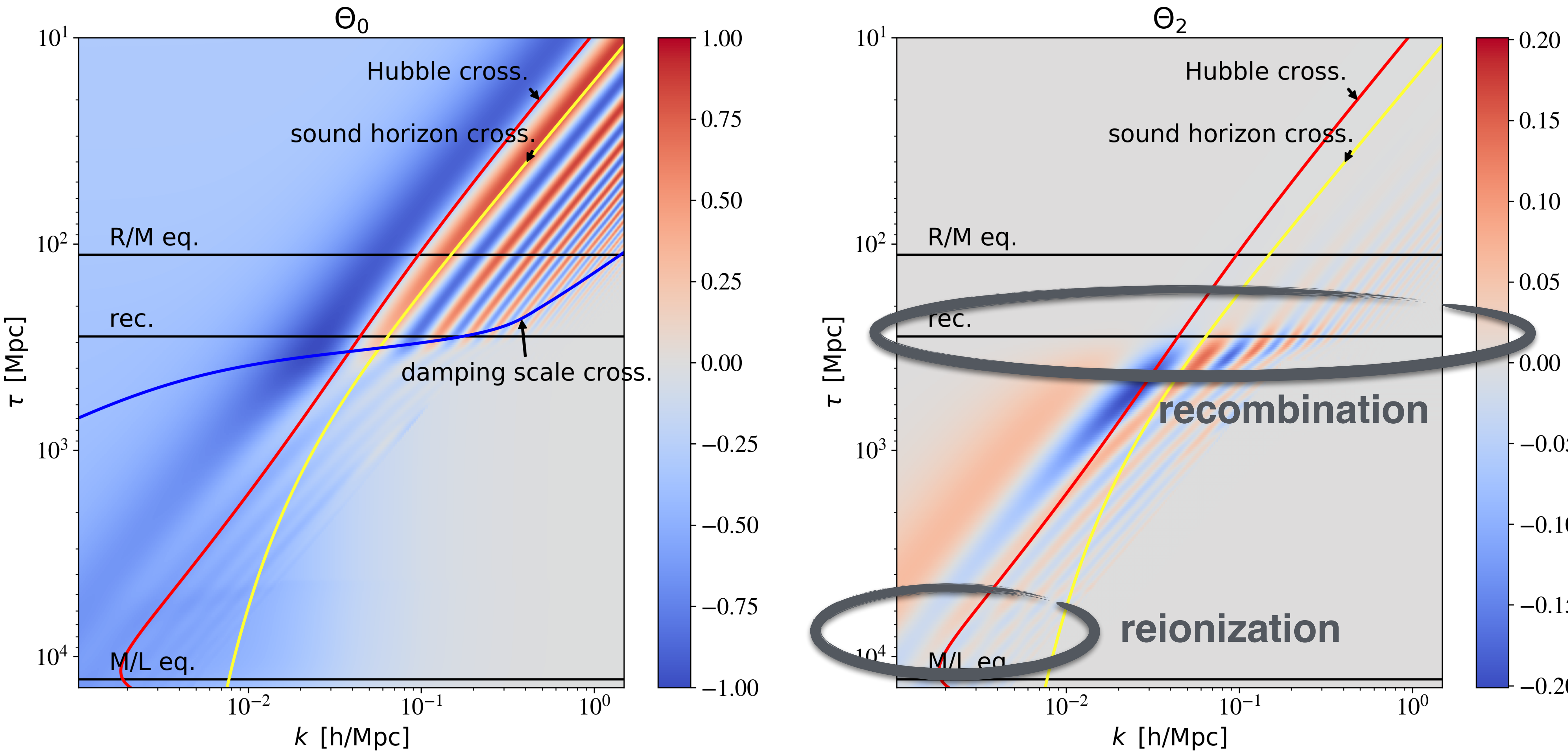
CMB polarisation



$$\Delta_l(\eta_0, k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta \;\; g \; \{\Theta_2 + \ldots\} \; (\ldots) \; j_l(k(\eta_0 - \eta))$$

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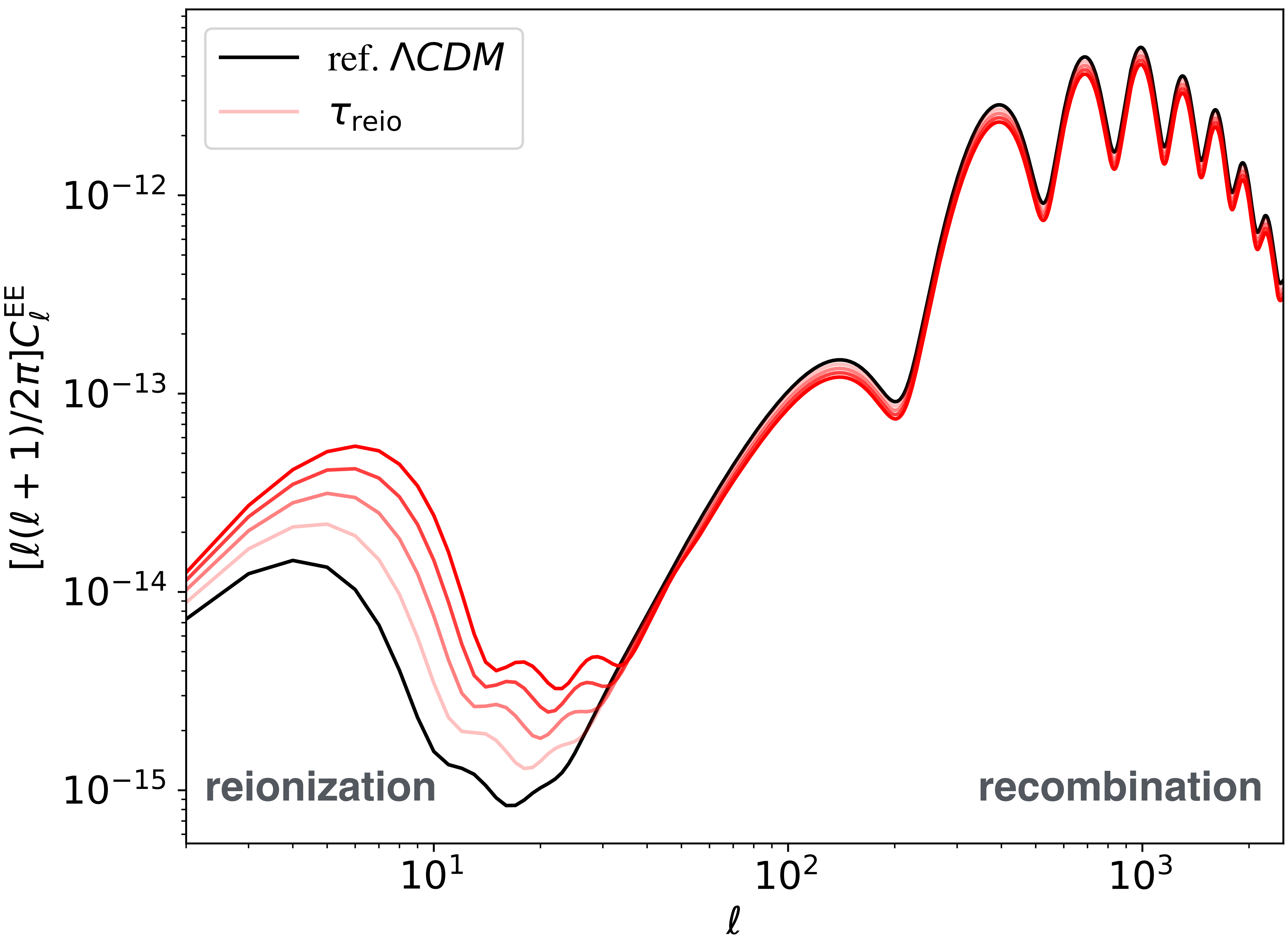
CMB polarisation



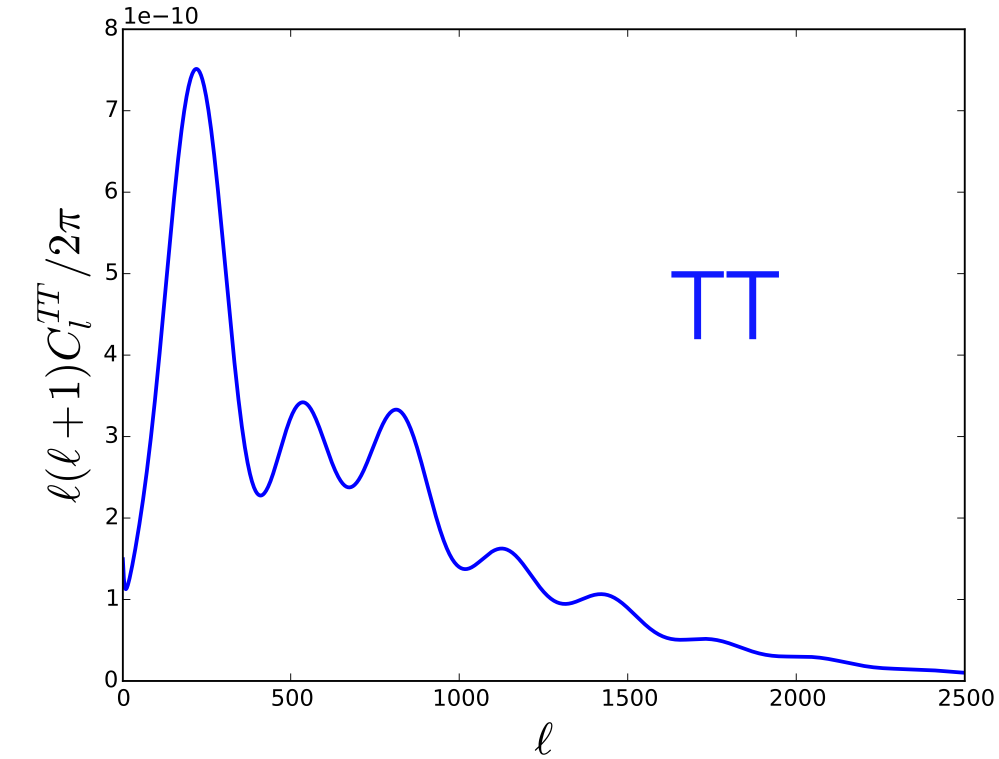
CMB polarisation

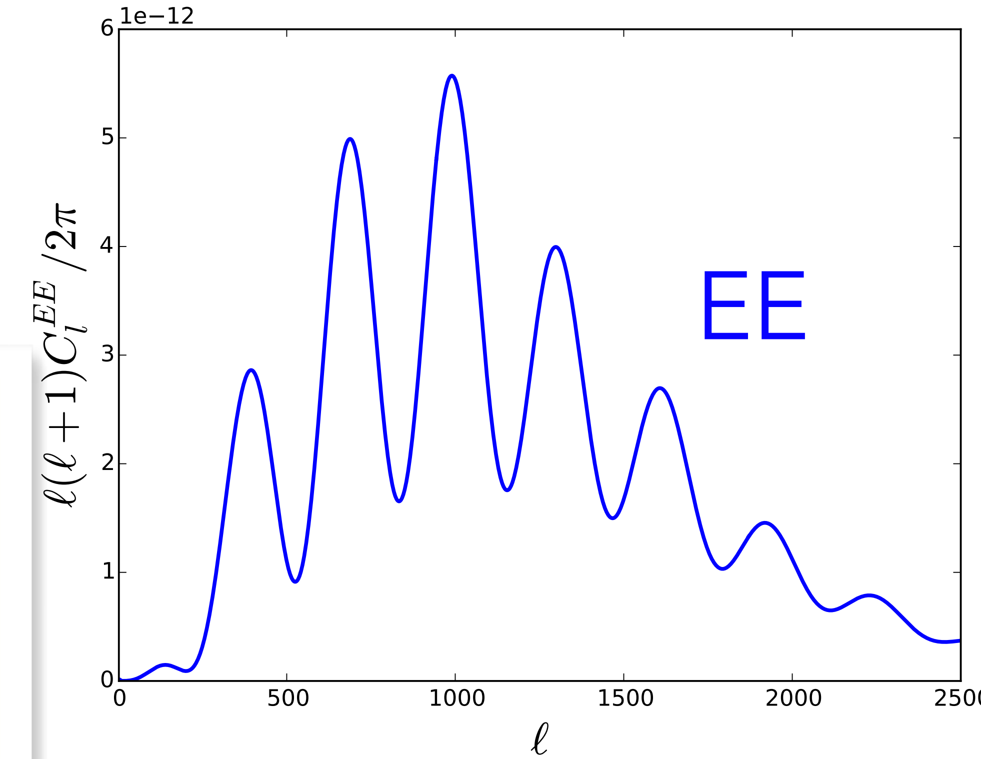
Cross spectrum: $$C_\ell^{TE} = \left\langle \frac{a_{lm}^T a_{lm}^{E*} + a_{lm}^E a_{lm}^{T*}}{2} \right\rangle = \frac{2}{\pi} \int dk\, k^2\, \Theta_\ell(\eta_0, k)\, \Delta_\ell^E(\eta_0, k)\;\; P_{\mathscr{R}}(k)$$

with transfer function $$\Theta_l(\eta_0,k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta\, \{g\,(\Theta_0 + \psi)\; j_l(k(\eta_0 - \eta)) + g\, k^{-1}\theta_{\rm b}\; j_l'(k(\eta_0 - \eta)) + e^{-\tau}(\phi' + \psi')\, j_l(k(\eta_0 - \eta))\} \;+\ldots$$

and $$\Delta_l^E(\eta_0, k) = \int_{\eta_{\rm ini}}^{\eta_0} d\eta\;\; g\;\{\Theta_2 + \ldots\}\;(\ldots)\; j_l(k(\eta_0 - \eta))$$

CMB polarisation





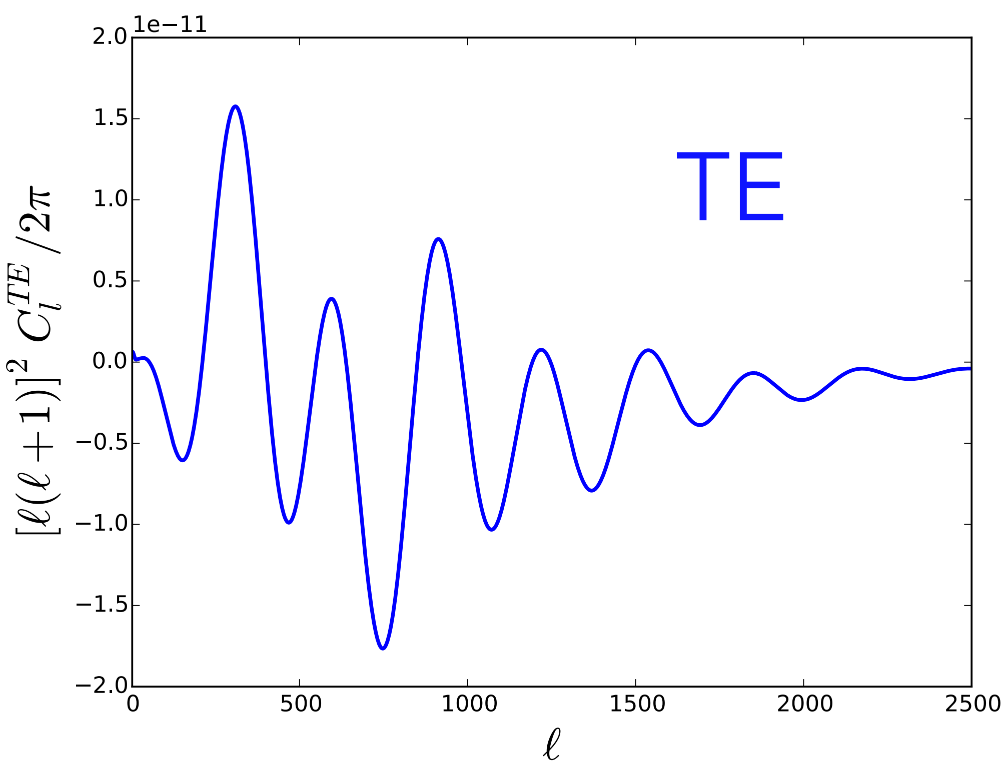
Bardeen scalars

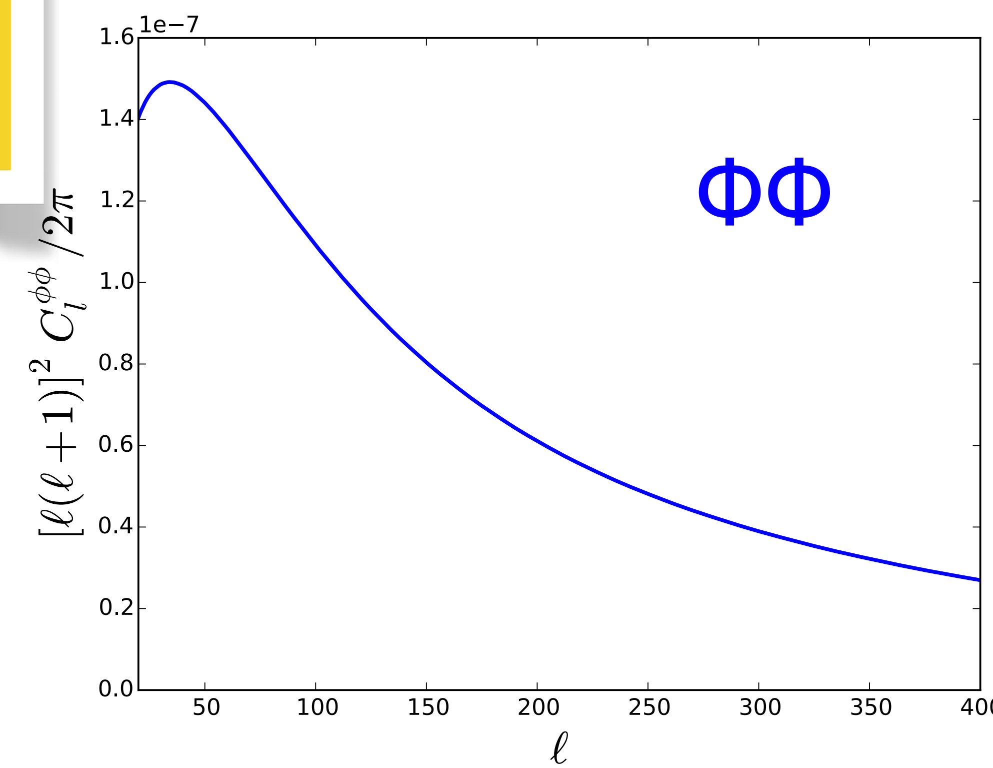
$\Downarrow$

CMB information

stored in 4 spectra

C_l^{XY}





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Tensor modes

Tensor modes

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu})$$

Bardeen scalars (spin-0)

$$h_{\mu\nu} = \begin{pmatrix} -2\psi & 0 & 0 & 0 \\ 0 & -2\phi & 0 & 0 \\ 0 & 0 & -2\phi & 0 \\ 0 & 0 & 0 & -2\phi \end{pmatrix}$$

(Newtonian gauge)

Bardeen tensors (spin-2)

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_1 & h_2 & 0 \\ 0 & h_2 & -h_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(for GWs along x^3)

Boltzmann with scalars: $\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = \hat{n}\cdot\vec{\nabla}\psi - \phi' + [\mathrm{Thomson}]$

grav. Dop. dilation

Boltzmann with tensors: $\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = -\frac{1}{2}h'_{ij}\hat{n}^i\hat{n}^j$ + [Thomson]

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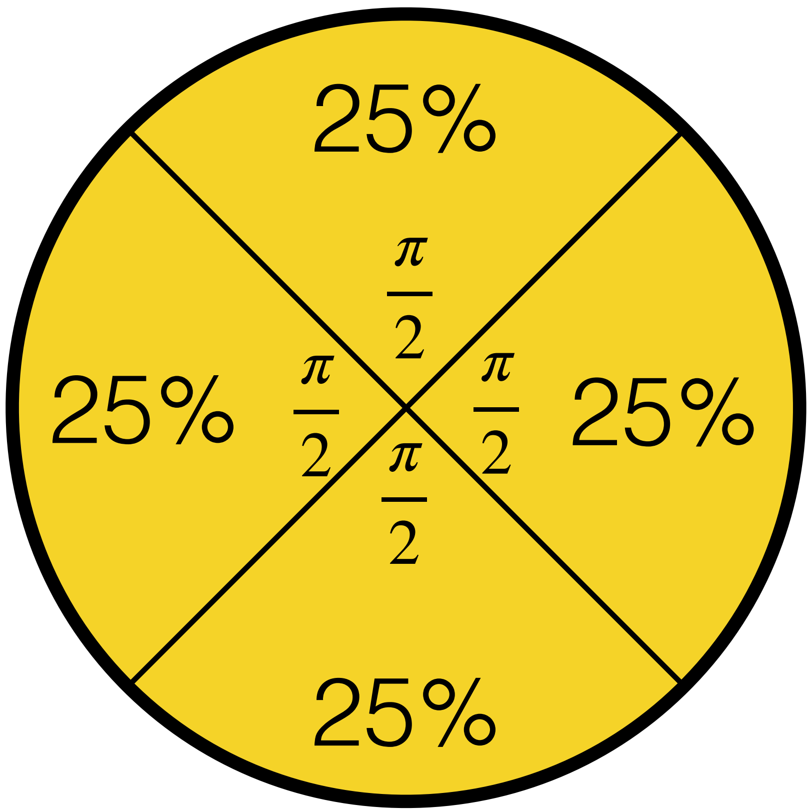
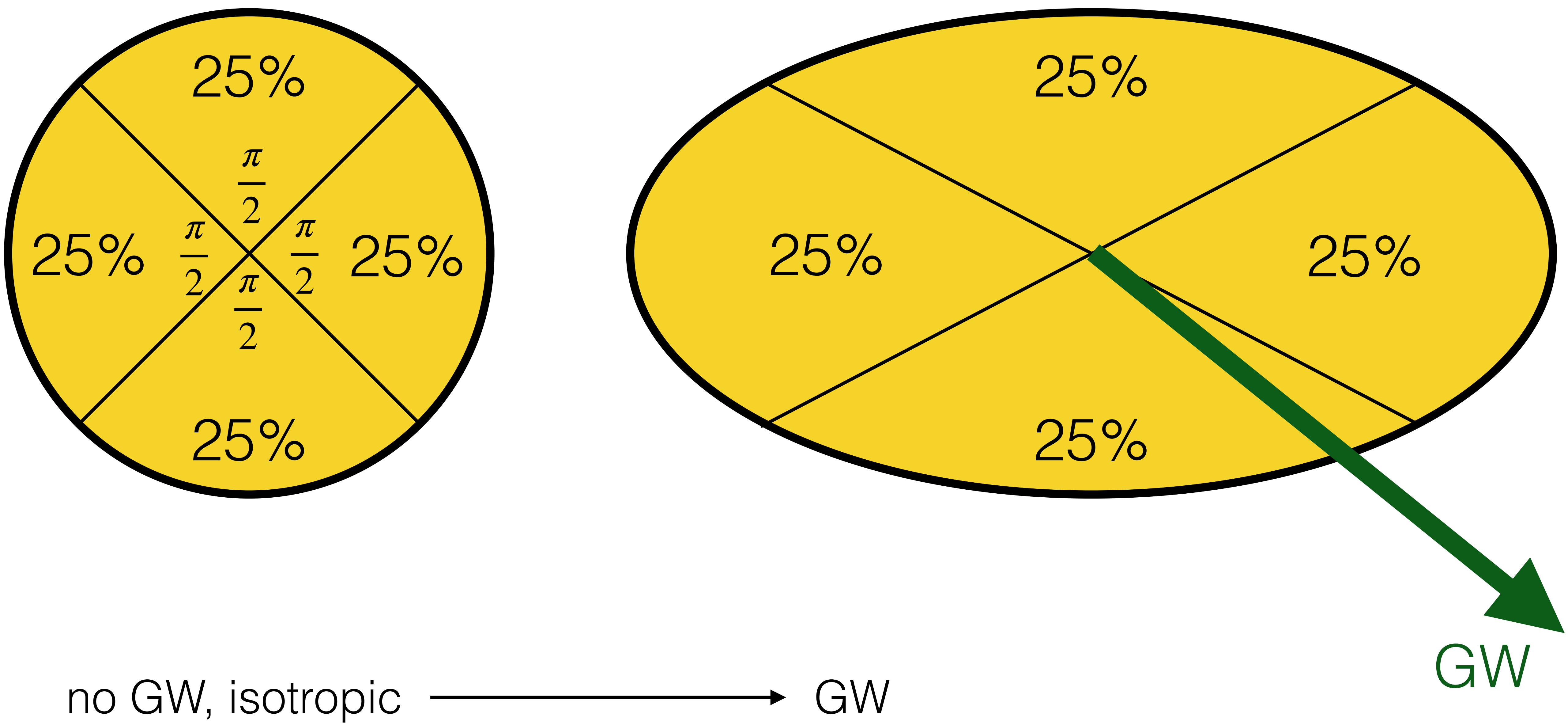
Tensor modes

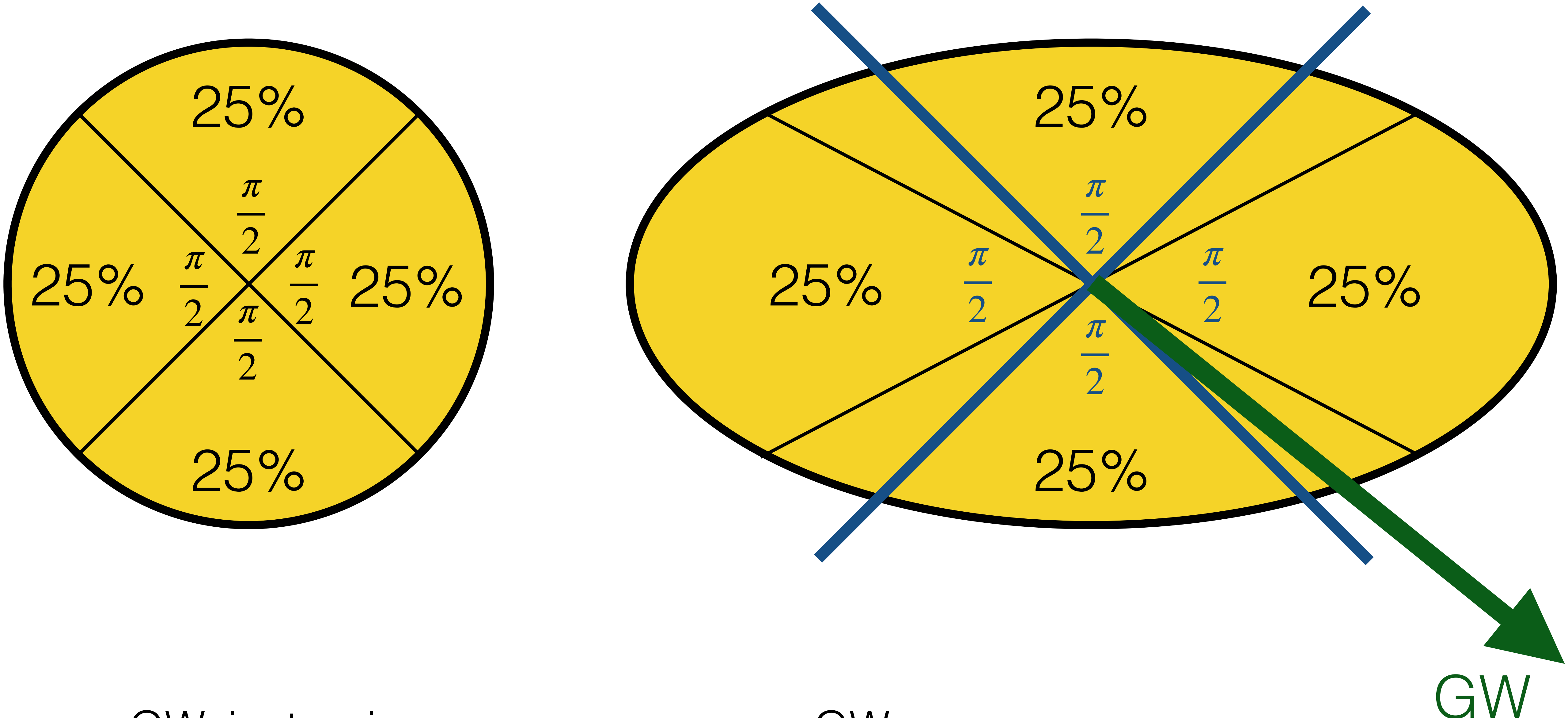


no GW, isotropic

Tensor modes

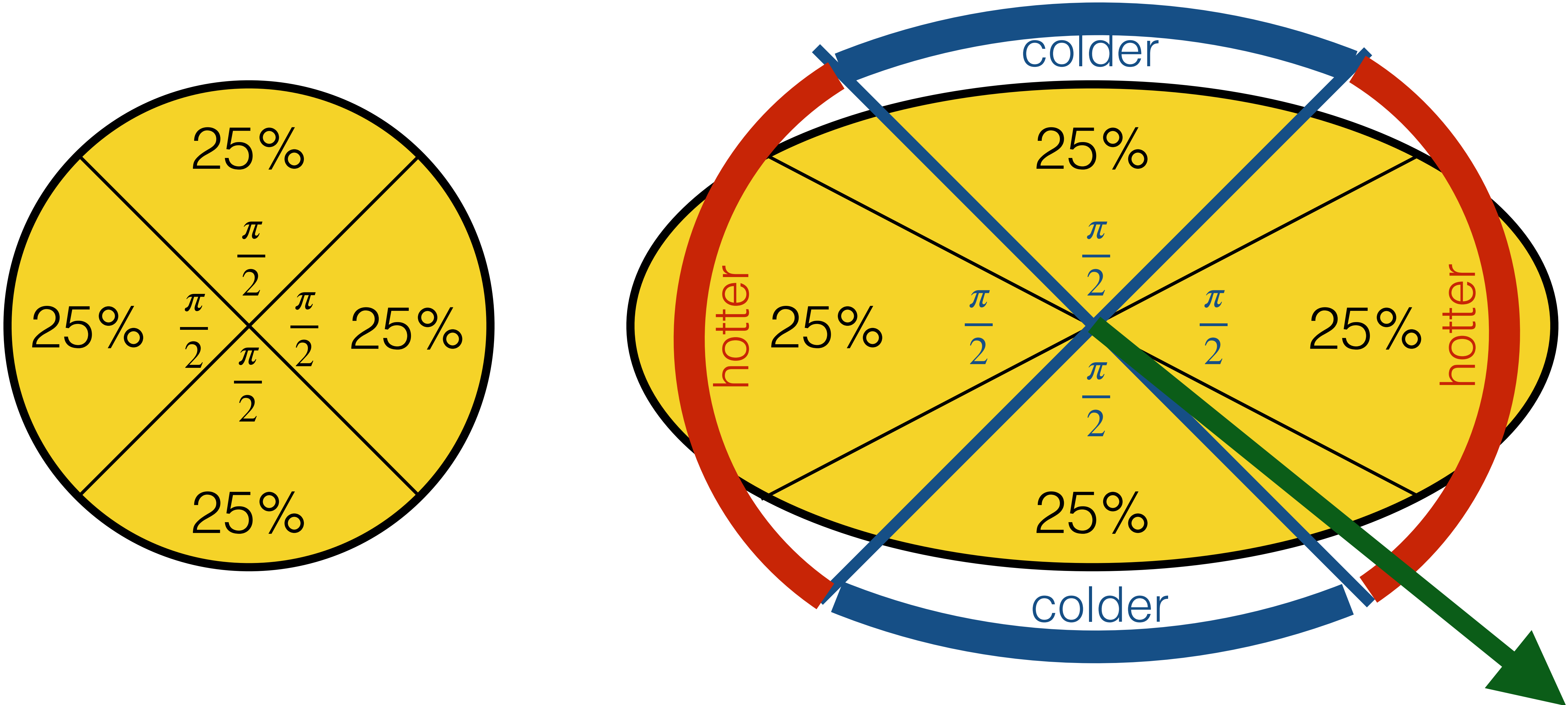


Tensor modes



no GW, isotropic → GW

Tensor modes



no GW, isotropic ⟶ GW, temperature quadruple

Tensor modes

Scalar Boltzmann: $\Theta' + \hat{n} \cdot \vec{\nabla}\Theta = \hat{n} \cdot \vec{\nabla}\psi - \phi' + [\mathrm{Thomson}]$

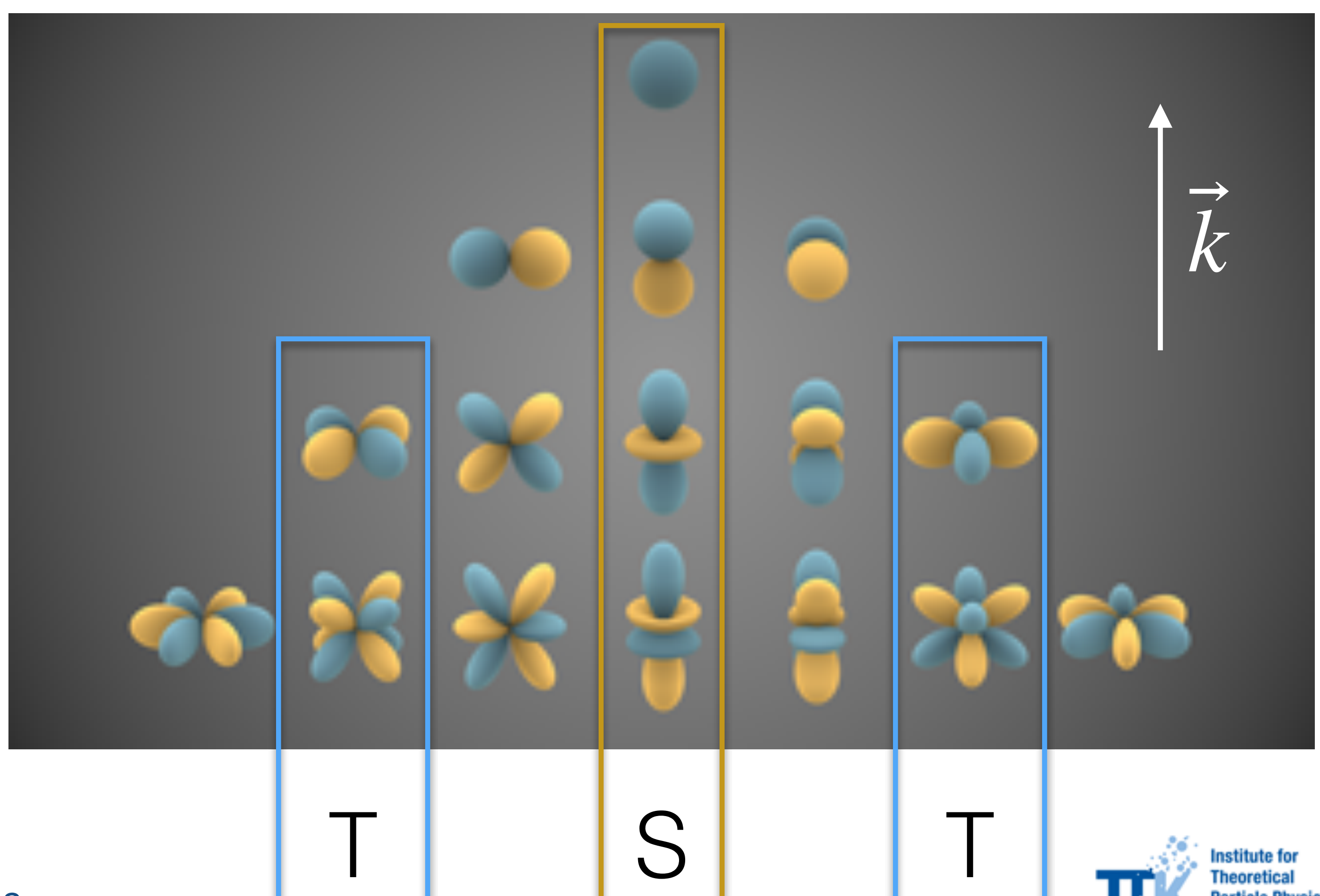
Tensor Boltzmann: $\Theta' + \hat{n} \cdot \vec{\nabla}\Theta = -\frac{1}{2} h'_{ij} \hat{n}^i \hat{n}^j$ +[Thomson]

Tensor modes

Scalar Boltzmann: $\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = \hat{n}\cdot\vec{\nabla}\psi - \phi' + [\mathrm{Thomson}]$

Tensor Boltzmann: $\Theta' + \hat{n}\cdot\vec{\nabla}\Theta = -\frac{1}{2}h'_{ij}\hat{n}^i\hat{n}^j$ +[Thomson]

General expansion: $\Theta(\eta,\vec{x},\hat{n}) \longrightarrow \Theta(\eta,\vec{k},\hat{n}) = \sum_{lm}\Theta_{lm}(\eta,\vec{k})Y_{lm}(\hat{n})$



Tensor modes

GW h'_{ij}

↓

Tensor temperature quadrupole

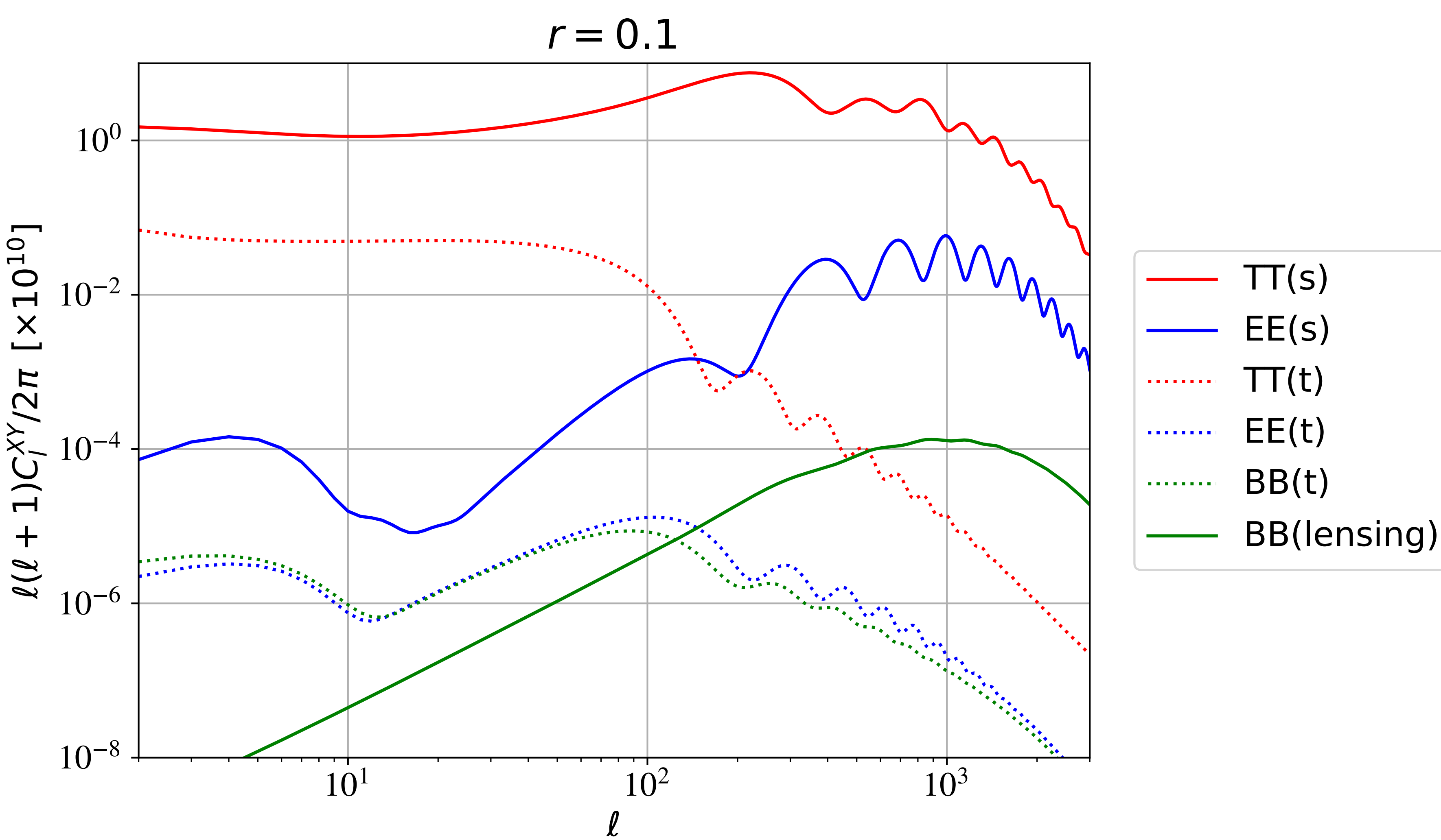
↙ ↘

Other tensor temperature multipoles → T-anisotropies

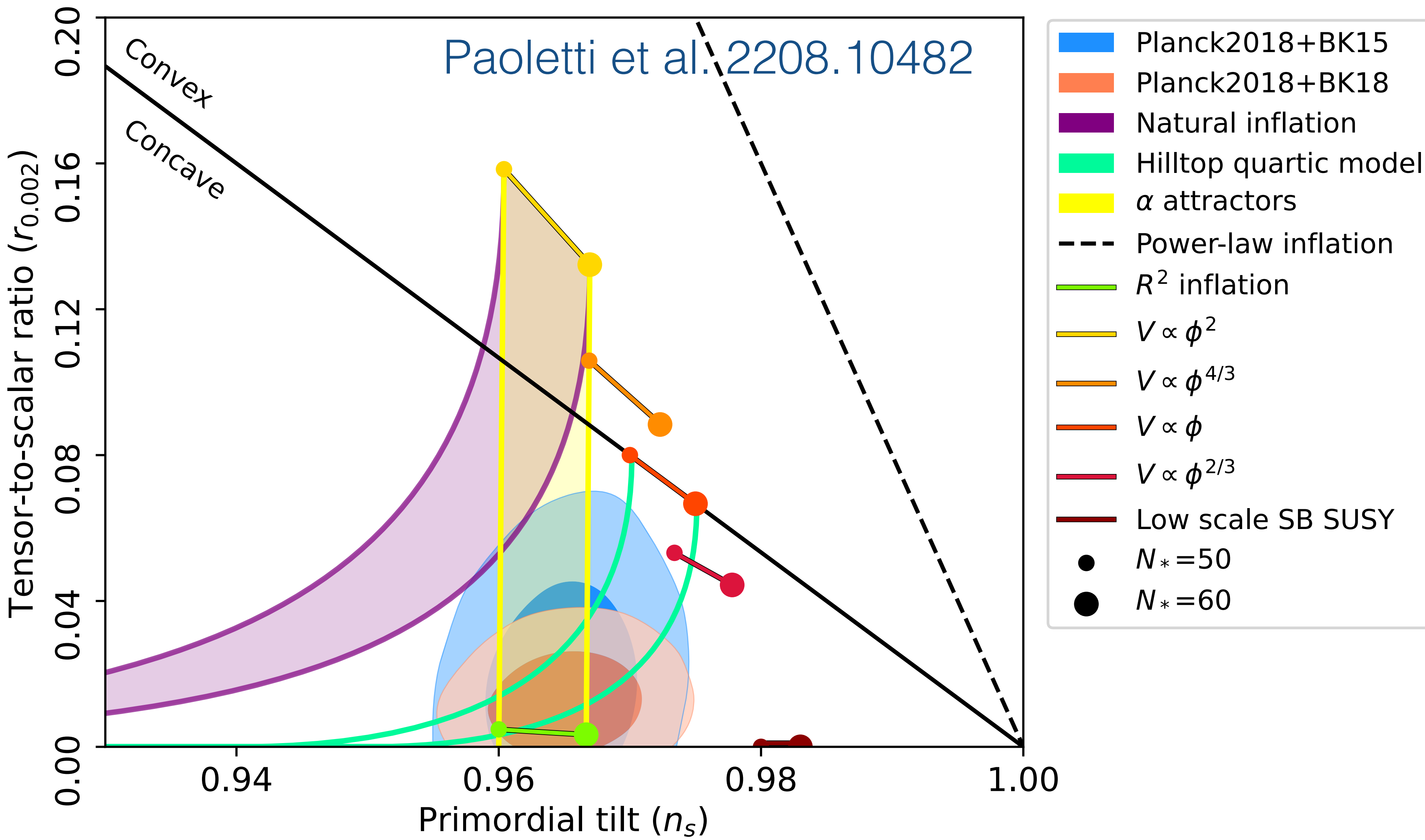
Tensor polarisation multipoles → E and B-anisotropies

Scalar and tensor sector statistically independent, solved independently,

$$C_l^{TT} = C_l^{TT(s)} + C_l^{TT(t)}$$
$$C_l^{EE} = C_l^{EE(s)} + C_l^{EE(t)}$$
$$C_l^{TE} = C_l^{TE(s)} + C_l^{TE(t)}$$
$$C_l^{BB} = 0 + C_l^{BB(t)}$$

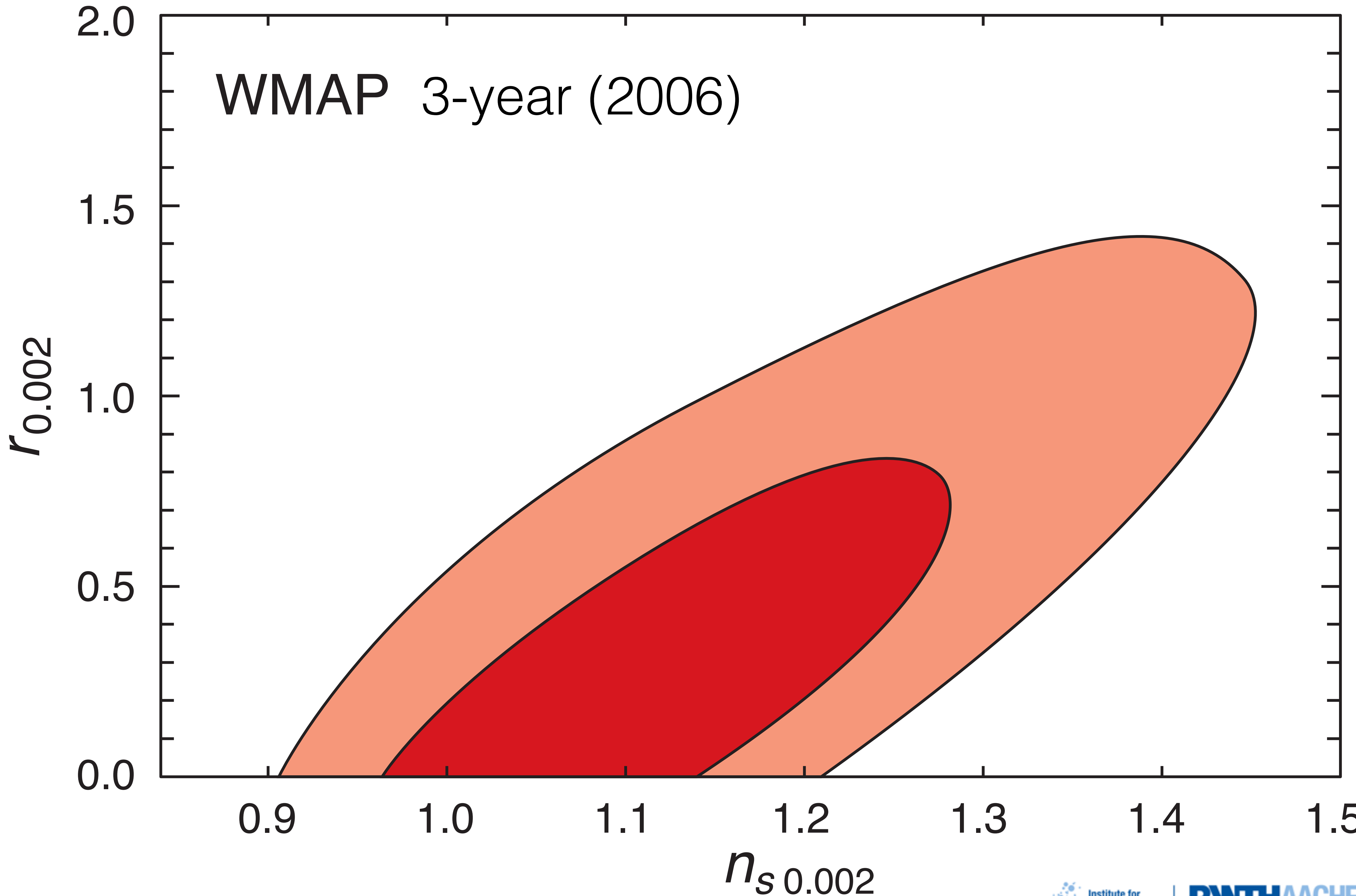


Observational constraints on ΛCDM + r

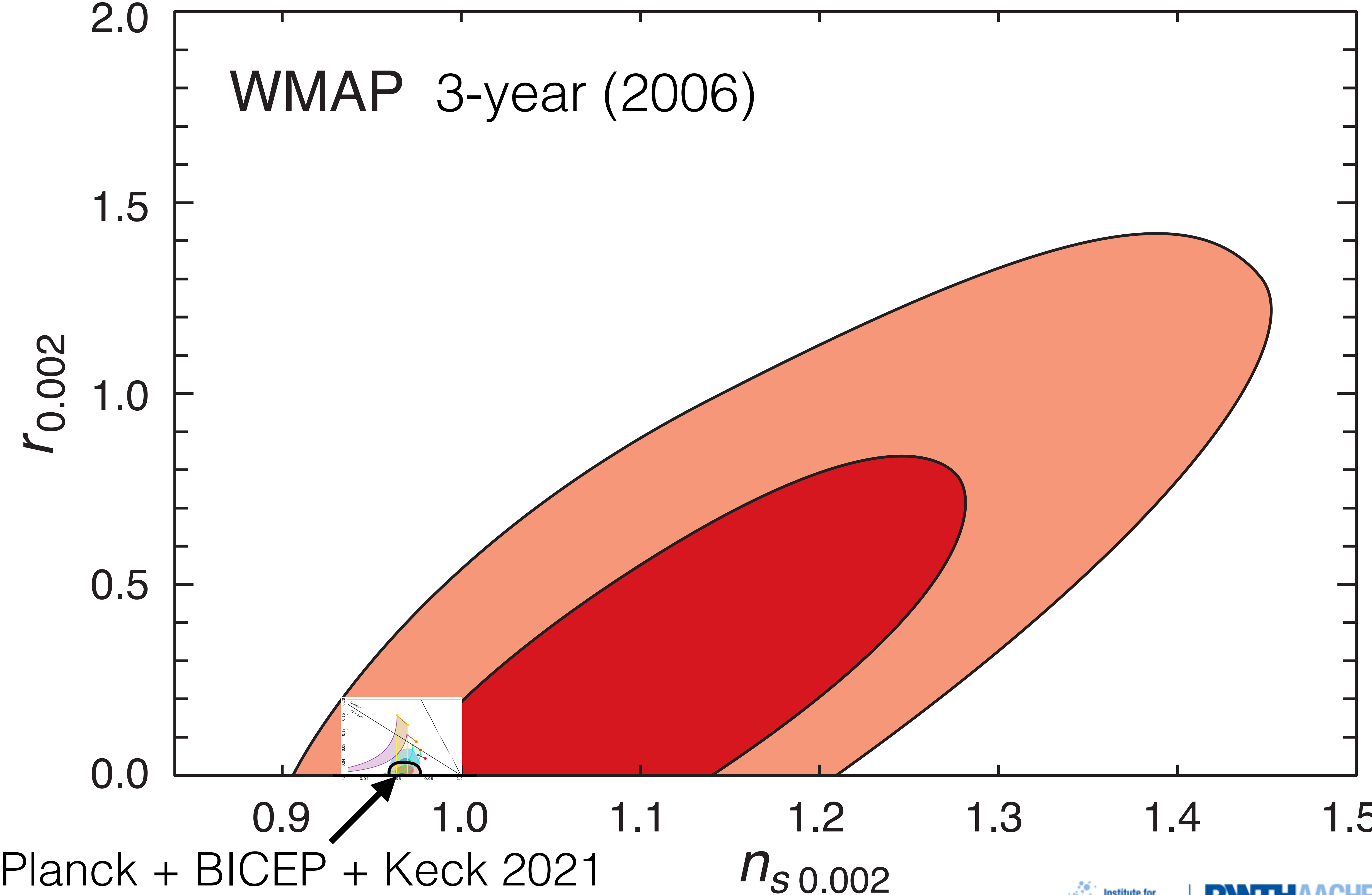


Energy scale of inflation V_* $$V_* = \frac{3\pi^2 A_s}{2}\, r\, M_{\rm Pl}^4 < (1.4\times 10^{16}\ {\rm GeV})^4 \quad (95\%\ {\rm CL})$$

Observational constraints on ΛCDM + r



Observational constraints on ΛCDM + r



CMB spectral distortions

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Blackbody radiation in early Universe

Elastic and inelastic scattering, $\Gamma > H$

$\downarrow$

Momentum exchange

$\downarrow$

Thermal/kinetic equilibrium

Bose-Einstein / Fermi-Dirac

$$f(p) = \frac{1}{e^{(E-\mu)/T} - 1}$$

$\downarrow$

for massless particles

$$f(p) = \frac{1}{e^{(p-\mu)/T} - 1}$$

Inelastic scattering, $\Gamma > H$

$\downarrow$

Chemical equilibrium

$$\Sigma\mu_i|_{\mathrm{left}} = \Sigma\mu_i|_{\mathrm{right}}$$

$\downarrow$

For particle without conserved numbers:

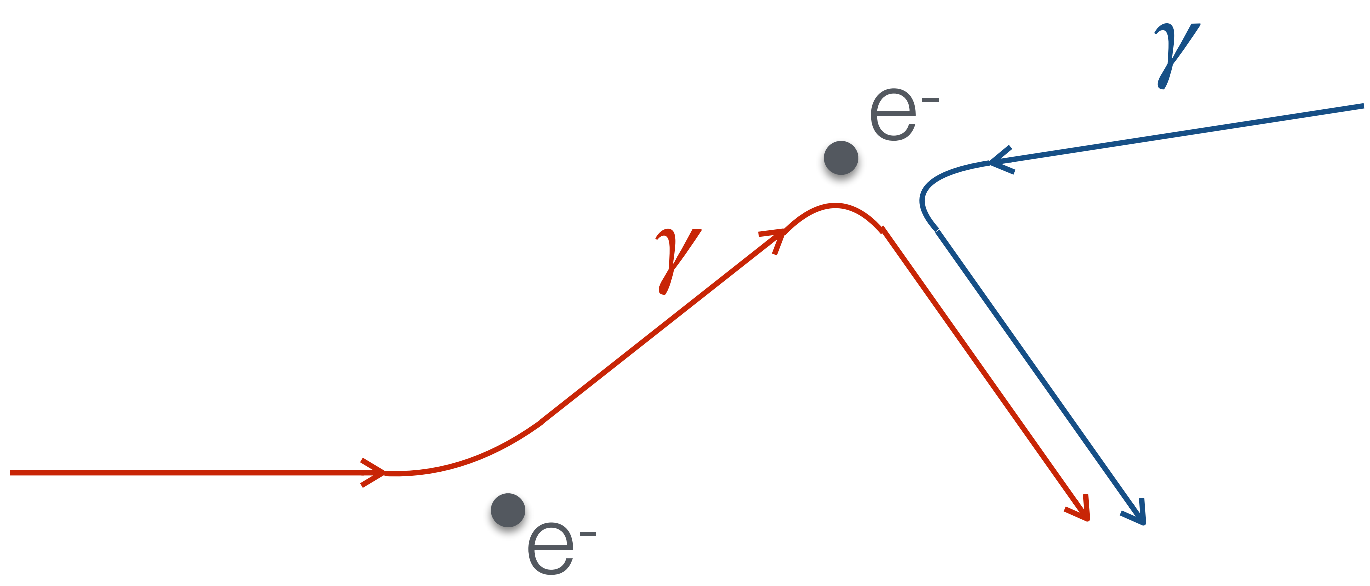
Number-changing reactions

$\downarrow$

$$\mu = 0$$

Photons: $f(p) = \dfrac{1}{e^{p/T} - 1}$ = blackbody/Planck spectrum

Blackbody radiation in early Universe



Redshifting along geodesics: $$\frac{d\ln(a\,p)}{d\eta} = \phi' - \hat{n}\cdot\vec{\nabla}\psi$$

Gravity preserves blackbody, but what about late interactions?

Blackbody radiation in early Universe

- Compton scattering (CS):

$$\gamma + e^- \longrightarrow \gamma + e^- \text{ (number conserving)}$$

$$\frac{\partial f}{\partial t} = \dot{\tau}\frac{T_e}{m_e}\frac{1}{x^2}\frac{\partial}{\partial x}\left(x^4\left[\frac{\partial f}{\partial x} + \frac{T_z}{T_e}f(1+f)\right]\right)$$

Kompaneet equation

(solution: BE with arbitrary μ)

- double Compton scattering (DC):

$$\gamma + e^- \longrightarrow \gamma + \gamma + e^- \text{ (non-number conserving)}$$

- Bremsstrahlung (BR):

$$e^- \longrightarrow e^- + \gamma \quad \text{(non-number conserving)}$$

Blackbody radiation in early Universe

- $z > 3 \times 10^6$: CS, DC, BR efficient: BE with $\mu = 0$ = blackbody

  energy injection-> no distortion

- $z > 4 \times 10^4$: only CS: BE with arbitrary mu, Kompaneet can only impose

$$f(p;T,\mu=0) \rightarrow f(p;T',\mu) \simeq f_{BE}(p;T,0)\left\{1+\mu\left[0.4561-\frac{T}{p}\right]\right\}$$

  energy injection-> μ-distortion

- $z > 10^3$: CS not efficient: Kompaneet at next-to-leading order in H/Γ can only impose

$$f(p;T,\mu=0) \rightarrow f_{BE}(p;T,0)\left\{1+y\left[\frac{p}{T}\frac{e^{p/T}+1}{e^{p/T}-1}-4\right]\right\}$$

  energy injection-> y-distortion

- $z \sim 10^3$: additional residuals

- Even later: CMB photons decoupled anyway
- Reionization: CS again, possible y-distortions (Sunyaev-Zel'dovitch 1970)

Source of distortions in standard cosmology

Lucca, Schöneberg, Hooper, JL, Chluba 1910.04619

- Adabatic cooling of electrons and photons:
  - UR particles in equilibrium with themselves: $T \propto a^{-1}$
  - NR particles in equilibrium with themselves: $T \propto a^{-2}$
  - Efficient CS: $T_e = T_b = T_\gamma \propto a^{-1}$
  - Inefficient CS: $T_e = T_b < T_\gamma$

    → energy extracted from photon, $\mu = -3 \times 10^{-9}, \; y = -5 \times 10^{-10}$

- Dissipation of acoustic waves:
  - Diffusion damping → superposition of BB with different temperature,

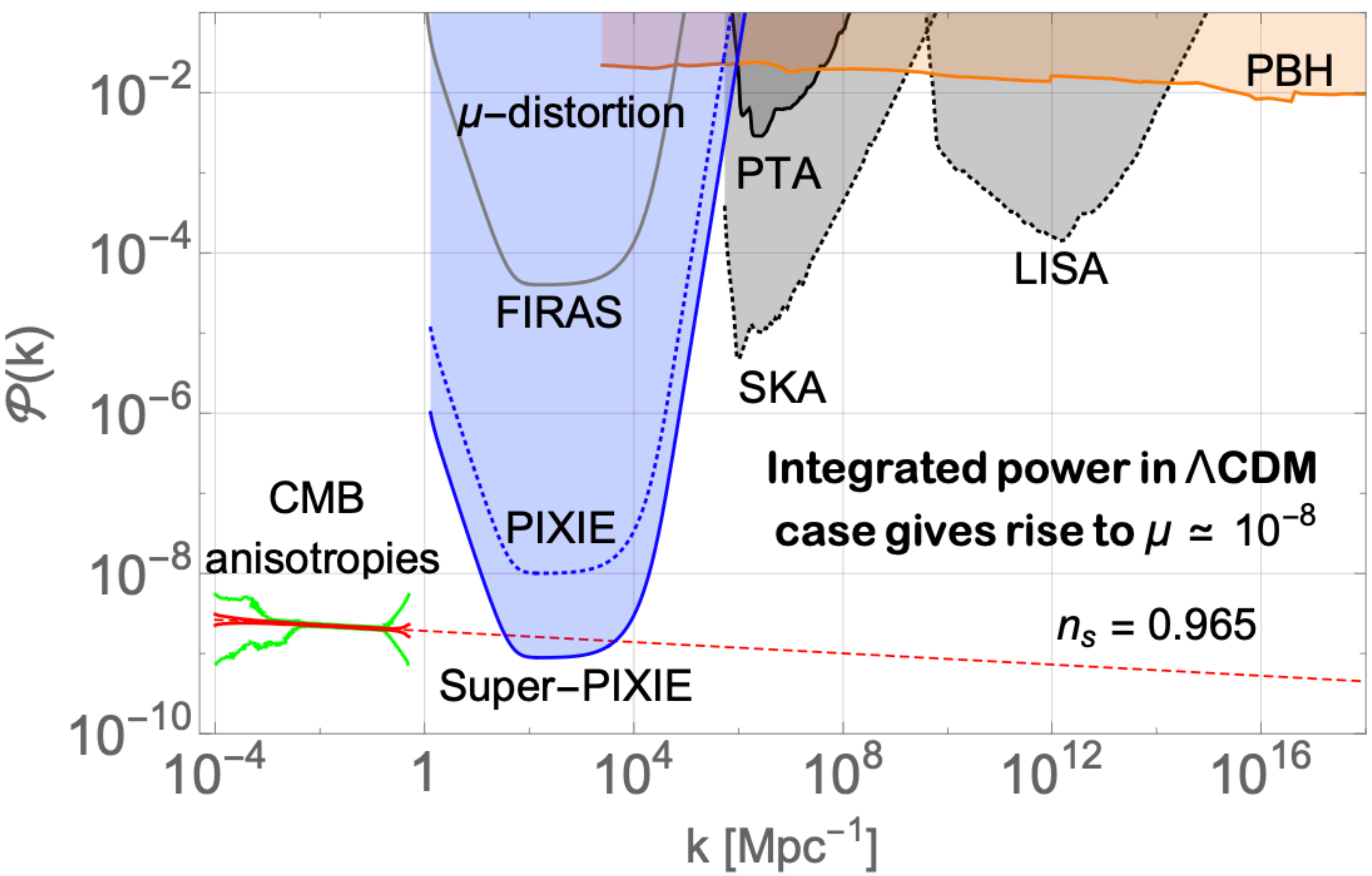
    → reprocessed as $\mu = 2 \times 10^{-8}, \; y = 4 \times 10^{-9}$
  - Transfer of energy from small-scale anisotropies to spectral distortions
  - Accurately computed by CLASS
  - Probe of $P_{\mathscr{R}}(k)$ on very small scales

- Emission/absorption lines during H and He recombination: y-distorsions + small residuals

- Sunyaev-Zel'dovitch effect from hot electrons during reionization → $y \sim 10^{-6}$

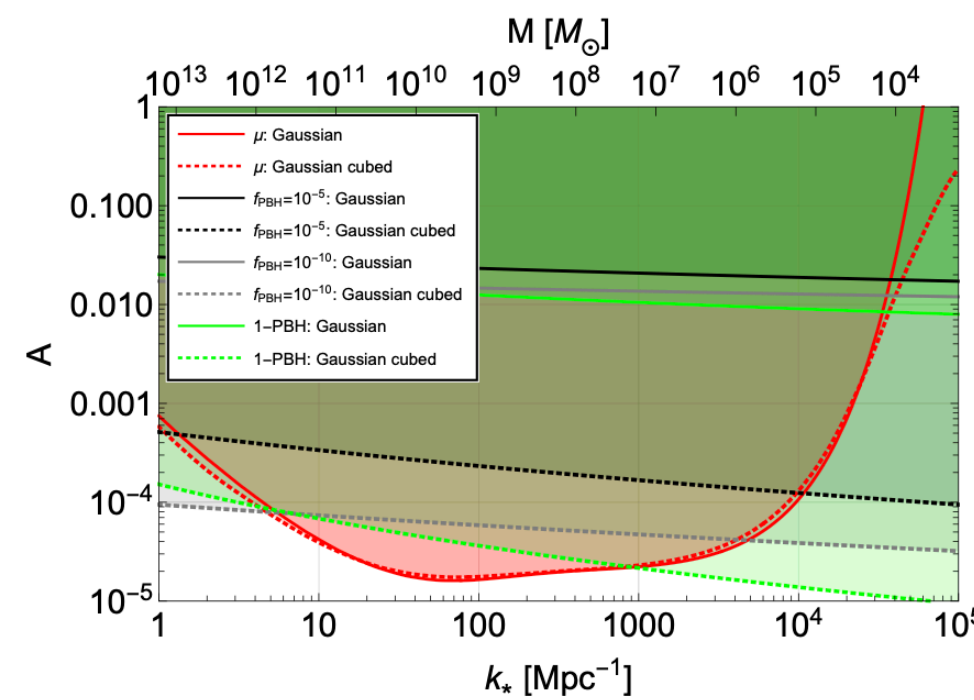
Source of distortions in non-minimal cosmology

- Extra power in small-scale $P_{\mathcal{R}}(k)$



J. Chluba et al., BAAS 51, 184 (2019), 1903.04218

Exclusion plots on peaks producing PBH



Pritchard, Byrnes, JL, Sharma 2505.08442

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Source of distortions in non-minimal cosmology

- DM annihilation or decay: products end up heating electrons
- PBH accretion or evaporation
- Other exotic energy injection mechanisms in dark sector

Lucca, Schöneberg, Hooper, JL, Chluba 1910.04619

- also produces change in recombination, and thus CMB anisotropies…

→ anisotropy/distortion synergy → distorsion module in CLASS, ExoCLASS branch