# Eulerian Perturbation Theory

$$\partial_\eta \delta = -\underline{\nabla} \cdot \left[(1+\delta)\,\underline{v}\right]$$

$$\partial_\eta \underline{v} + \mathcal{H}\underline{v} + (\underline{v}\cdot\underline{\nabla})\underline{v} = -\underline{\nabla} V \qquad \Big| \; \underline{\nabla}\cdot$$

$$\nabla^2 V = \frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta \qquad\qquad \mathcal{H} = aH$$

## Eulerian Linear Perturbation Theory

assume $\delta$, $\underline{v}$ will be "small enough"

linearising $\quad \delta \to \delta^{(1)}$, $\underline{v} \to \underline{v}^{(1)} \qquad \theta = \underline{\nabla}\cdot\underline{v}$

$$\partial_\eta \delta^{(1)} = -\underline{\nabla}\cdot\underline{v}^{(1)} = -\theta^{(1)} \qquad \Big| \; \partial_\eta$$

$$\partial_\eta(\underbrace{\underline{\nabla}\cdot\underline{v}^{(1)}}_{\theta^{(1)}}) + \mathcal{H}(\underbrace{\underline{\nabla}\cdot\underline{v}^{(1)}}_{\theta^{(1)}}) = -\nabla^2 V^{(1)} = -\frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta^{(1)}$$

$$\partial_\eta^2 \delta^{(1)} = -\partial_\eta \theta^{(1)} = -\Big(-\mathcal{H}\underbrace{\theta^{(1)}}_{-\partial_\eta \delta^{(1)}} - \frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta^{(1)}\Big)$$

$$\mathrm{I}: \; \partial_\eta^2 \delta^{(1)} + \mathcal{H}\partial_\eta \delta^{(1)} = \frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta^{(1)}$$

$$\delta^{(1)}(\underline{x},\eta) = \underbrace{D(\eta)}_{\text{linear growth}}\,\overbrace{\tilde{\delta}_1(\underline{x})}^{\text{initial conditions}}$$

$$D''(\eta) + \mathcal{H}D'(\eta) = \frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,D(\eta)$$

$$\mathrm{II}: \; \theta^{(1)} = -\partial_\eta \delta^{(1)} = -D'(\eta)\,\tilde{\delta}_1(\underline{x})$$

$$D'(\eta) = D\frac{d\ln D}{d\eta} = D\mathcal{H}\underbrace{\frac{d\ln D}{d\ln a}}_{= f \text{ linear growth rate}}$$

$$\theta^{(1)} = -\mathcal{H}f\,\delta^{(1)}$$

## Eulerian Nonlinear PT

$$\partial_\eta \delta = -\underline{\nabla}\cdot\left[(1+\delta)\,\underline{v}\right] = -\theta - \delta\theta - \underline{\nabla}\delta\cdot\underline{v}$$
$$\partial_\eta \theta + \mathcal{H}\,\theta + \underline{\nabla}\left[(\underline{v}\cdot\underline{\nabla})\,\underline{v}\right] = -\frac{3}{2}\,\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta$$

switch to Fourier space

$$\underline{\nabla}\,\delta(\underline{x}) \to i\underline{k}\,\delta(\underline{k})$$

$$\theta(\underline{x}) = \underline{\nabla}\cdot v(\underline{x}) = i\underline{k}\cdot\underline{v}(\underline{k}) \qquad \underline{v}(\underline{k}) = -\frac{i\underline{k}}{k^2}\,\theta(\underline{k})$$

$$\delta(\underline{x})\,\theta(\underline{x}) \to \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}\,(2\pi)^3\,\delta_D(\underline{k}-\underline{k_1}-\underline{k_2})\,\delta(\underline{k_1})\,\theta(\underline{k_2})$$

$$\underline{\nabla}\delta\cdot\underline{v} \to \qquad \sim \qquad \frac{\underline{k_1}\cdot\underline{k_2}}{k_2^2}\,\delta(\underline{k_1})\,\theta(\underline{k_2})$$

$$\underline{\nabla}(\underline{v}\cdot\underline{\nabla})\cdot\underline{v} \to \qquad \sim \qquad \frac{(\underline{k_1}\cdot\underline{k_2})^2}{k_1^2 k_2^2}\,\theta(\underline{k_1})\,\theta(\underline{k_2})$$

$$(\underline{v}\cdot\underline{\nabla})\,\theta \to \qquad \sim \qquad \frac{\underline{k_1}\cdot\underline{k_2}}{k_1^2}\,\theta(\underline{k_1})\,\theta(\underline{k_2})$$

$$\partial_\eta\delta + \theta = - \qquad \sim \qquad \alpha(\underline{k_1},\underline{k_2})\,\delta(\underline{k_1})\,\theta(\underline{k_2})$$

$$\partial_\eta\theta + \mathcal{H}\theta + \frac{3}{2}\Omega_m(\eta)\,\mathcal{H}^2(\eta)\,\delta = - \qquad \sim \qquad \beta(\underline{k_1},\underline{k_2})\,\theta(\underline{k_1})\,\theta(\underline{k_2})$$

mode coupling

$$\alpha(\underline{k_1},\underline{k_2}) = \left(1 + \frac{\underline{k_1}\cdot\underline{k_2}}{k_2^2}\right) \qquad \beta(\underline{k_1},\underline{k_2}) = \frac{(\underline{k_1}\cdot\underline{k_2})^2}{k_1^2 k_2^2} + \frac{\underline{k_1}\cdot\underline{k_2}}{k_1^2}$$

$$\frac{1}{2}\,\underline{k_1}\cdot\underline{k_2}\left(\frac{1}{k_2^2}+\frac{1}{k_1^2}\right) = \frac{1}{2}\,\frac{\underline{k_1}\cdot\underline{k_2}}{k_1 k_2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)$$

## 2SPT

$\delta$, $\theta$ left $\to$ $\delta^{(2)}$, $\theta^{(2)}$ $\qquad$ right $\delta \to \delta^{(1)}$, $\theta \to \theta^{(\cancel{2})}$ typo 1

$$\delta^{(2)}(\underline{k},\eta) = D^2(\eta)\,\tilde{\delta}^{(2)}(\underline{k}) \qquad \text{in matter domination } f=1$$
$$\theta^{(2)} \quad \sim \quad = \mathcal{H}\,D^2(\eta)\,\tilde{\theta}^{(2)}(\underline{k})$$

$$\tilde{\delta}^{(2)}(\underline{k}) = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6} (2\pi)^3\, \delta_D(\underline{k} - \underline{k_1} - \underline{k_2}) \underbrace{\left(\frac{5}{7}\alpha(\underline{k_1},\underline{k_2}) + \frac{2}{7}\beta(\underline{k_1},\underline{k_2})\right)}_{F_2(\underline{k_1},\underline{k_2})} \tilde{\delta}_1(\underline{k_1})\, \tilde{\delta}_1(\underline{k_2})$$

$$\tilde{\theta}^{(2)}(\underline{k}) = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6} (2\pi)^3\, \delta_D(\underline{k} - \underline{k_1} - \underline{k_2}) \underbrace{\left(\frac{3}{7}\alpha(\underline{k_1},\underline{k_2}) + \frac{4}{7}\beta(\underline{k_1},\underline{k_2})\right)}_{G_2(\underline{k_1},\underline{k_2})} \tilde{\delta}_1(\underline{k_1})\, \tilde{\delta}_1(\underline{k_2})$$

happy, can generalise to arbitrary order