# PLAN

**Modelling dark matter dynamics**

- Phase space
- Eulerian Perturbation Theory
- Lagrangian Perturbation Theory
- Spherical Collapse

**Clustering statistics**

- Two-point statistics
- Three-point statistics
- One-point statistics + …

# Spherical Collapse

## Newtonian force law

$$\ddot{R} = -\frac{GM(R)}{R^2} = -\frac{4\pi G}{3}\rho(t)R \quad \Big| \cdot 2\dot{R}\,,$$

$$R^2 = -2GMR^{-1} - C$$

# SPHERICAL COLLAPSE

**Newtonian force law**

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Parametric solution: cycloid

$$R = GM(1 - \cos\varphi)/C$$

$$t = GM(\varphi - \sin\varphi)/C^{3/2}\,.$$

# SPHERICAL COLLAPSE

Parametric solution: cycloid

$$R = GM(1 - \cos\varphi)/C$$

$$t = GM(\varphi - \sin\varphi)/C^{3/2}\ .$$

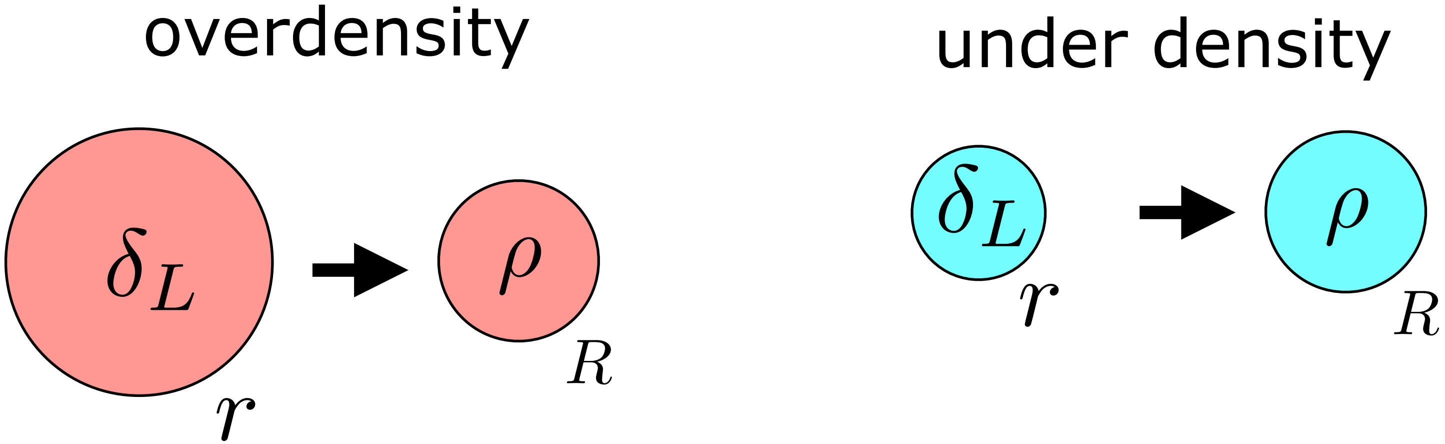
translate to densities, mass conserved

$$\delta_L = \frac{a(t)}{a_0} \propto \left(\frac{t_0}{t}\right)^{2/3} = \frac{3}{5}\left[\frac{3}{4}(\varphi - \sin\varphi)\right]^{2/3}$$

$$1 + \delta = \left(a(t)\frac{R_0}{R}\right)^3 \propto \left(\frac{t_0}{t}\right)^2 \left(\frac{R_0}{R}\right)^3 = \frac{9(\varphi - \sin\varphi)^2}{2(1 - \cos\varphi)^3}$$

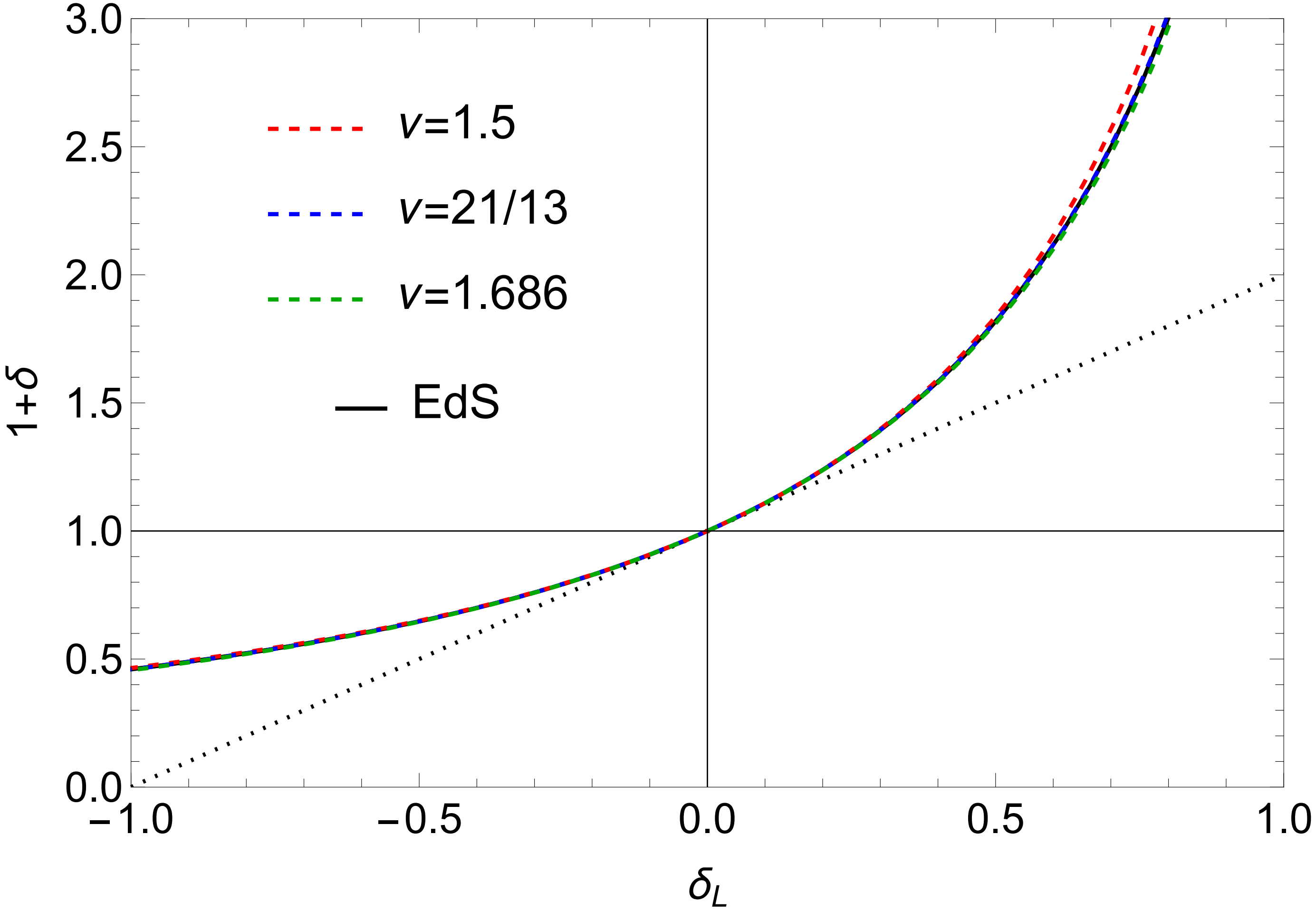
# SPHERICAL COLLAPSE

**mass conservation** $r = \rho^{1/3} R$



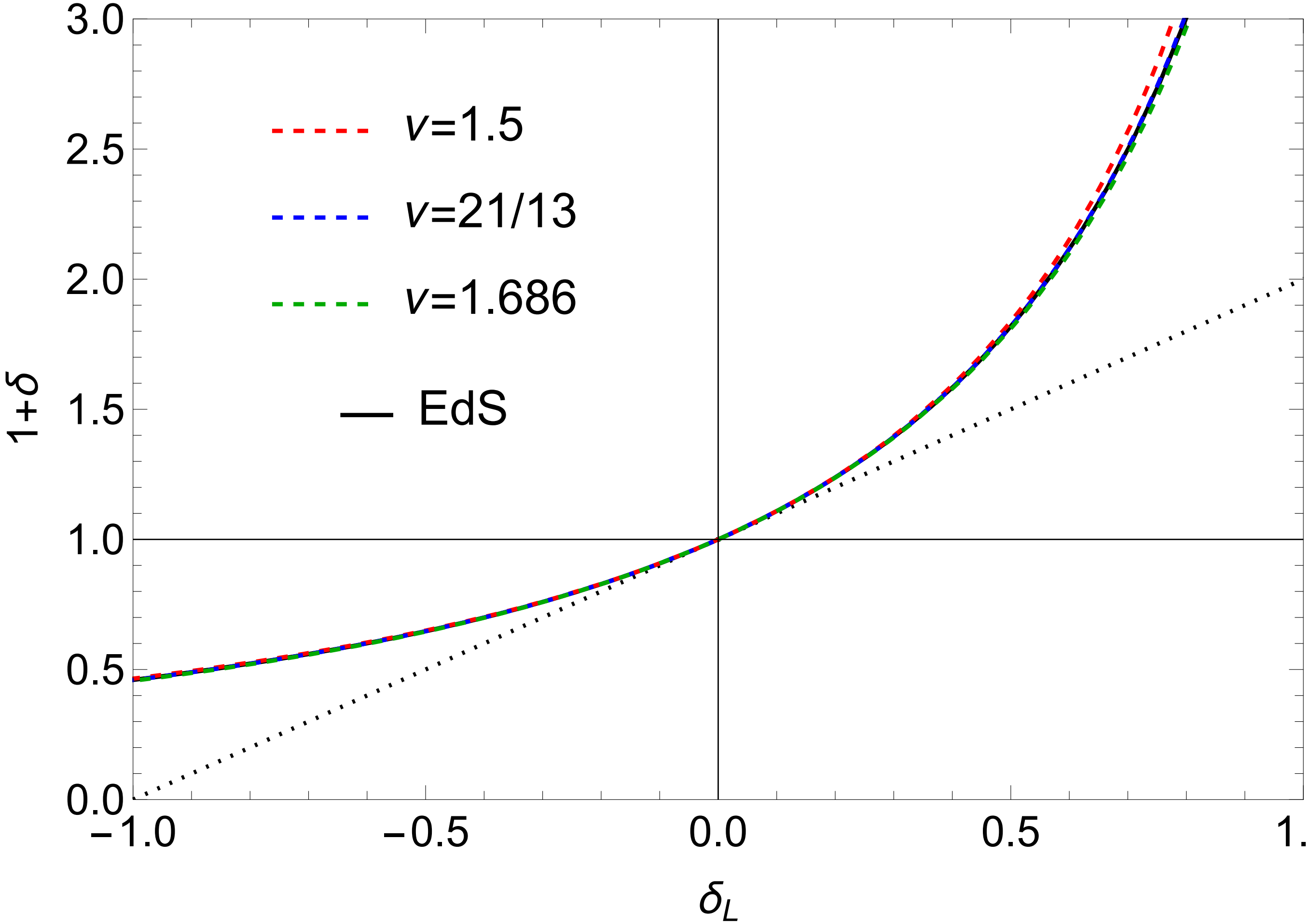
# SPHERICAL COLLAPSE

$$1+\delta = \frac{9(\varphi - \sin\varphi)^2}{2(1-\cos\varphi)^3} \qquad \delta_L = \frac{3}{5}\left[\frac{3}{4}(\varphi - \sin\varphi)\right]^{2/3}$$



# SPHERICAL COLLAPSE

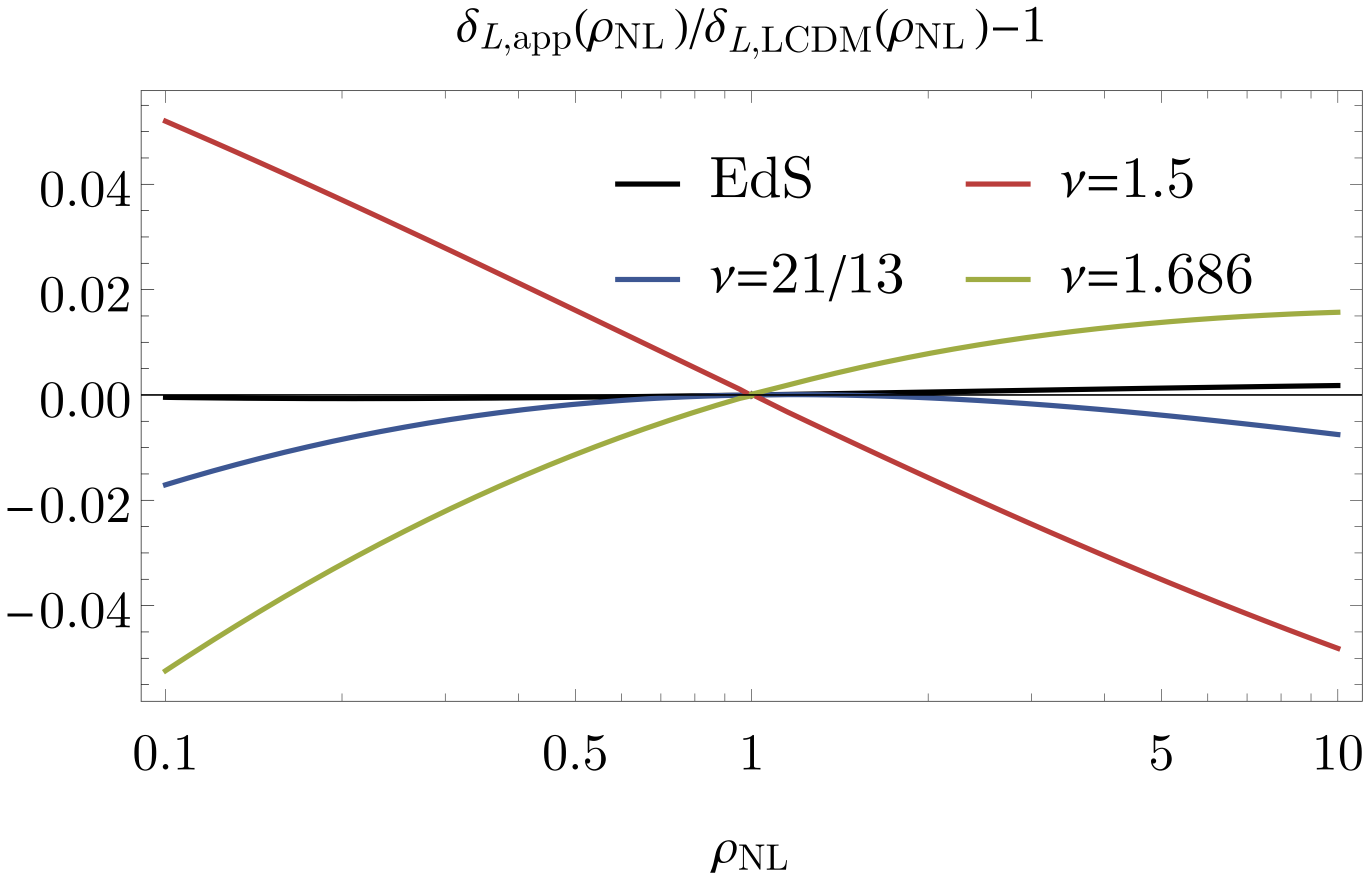
$$1+\delta = \left(1-\frac{\delta_L}{\nu}\right)^{-\nu} \approx 1+\delta_L+\frac{\nu+1}{\nu}\frac{\delta_L^2}{2}$$



# SPHERICAL COLLAPSE

**density mapping** $\delta_L(\rho) \simeq \frac{21}{13}\left(1 - \rho^{-\frac{13}{21}}\right)$

~ cosmology independent

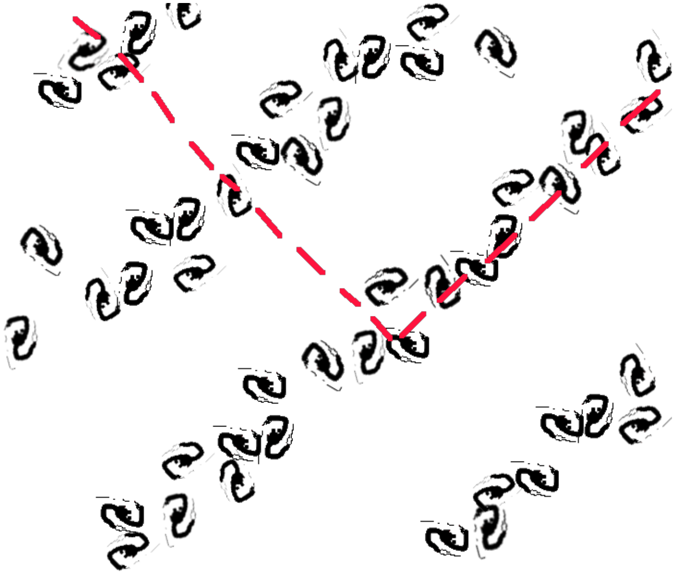


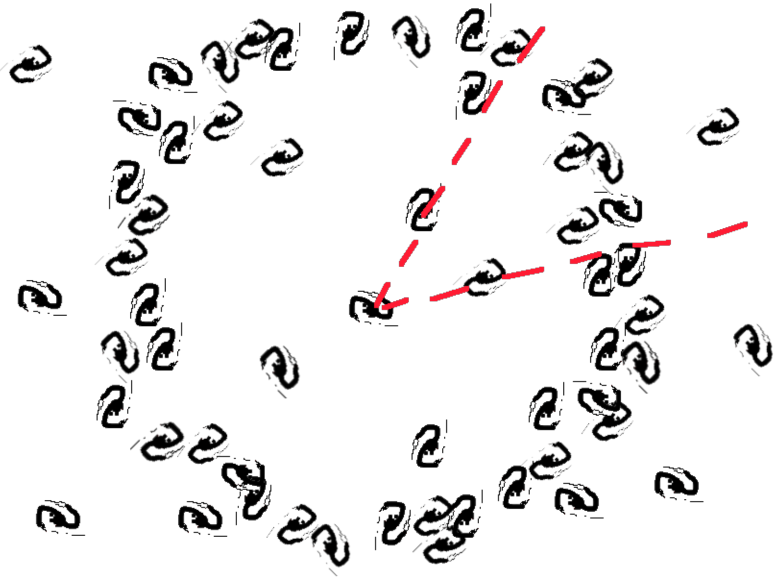
# GALAXY CORRELATION FUNCTION

on large scales: matter distribution statistically

homogenous

isotropic



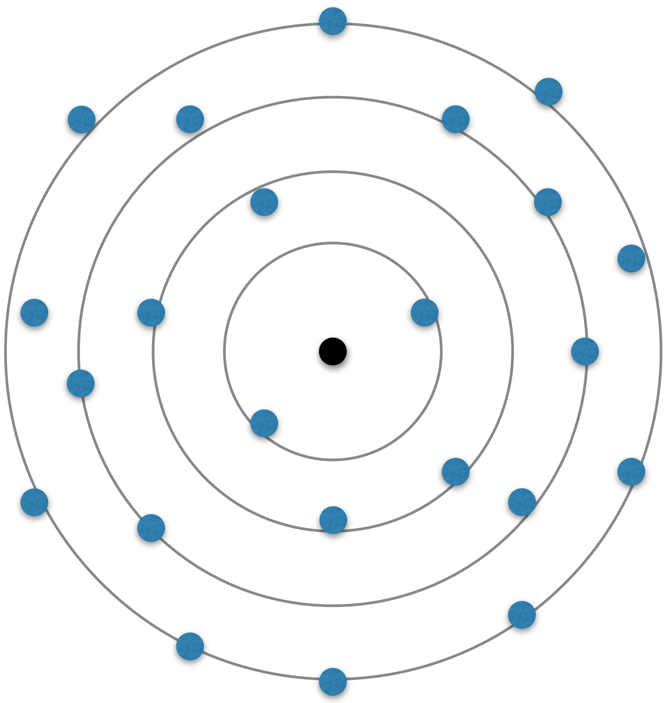


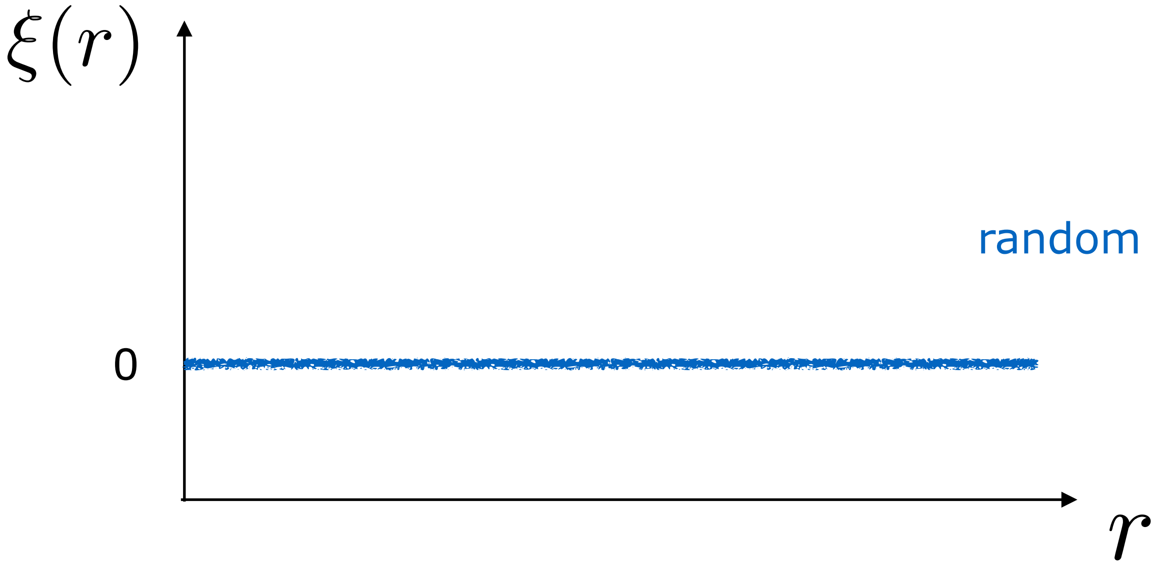
# GALAXY CORRELATION FUNCTION

excess correlation compared to random distribution

$$\xi(r) = \langle \delta(\boldsymbol{x}+\boldsymbol{r})\delta(\boldsymbol{x})\rangle$$

density contrast average



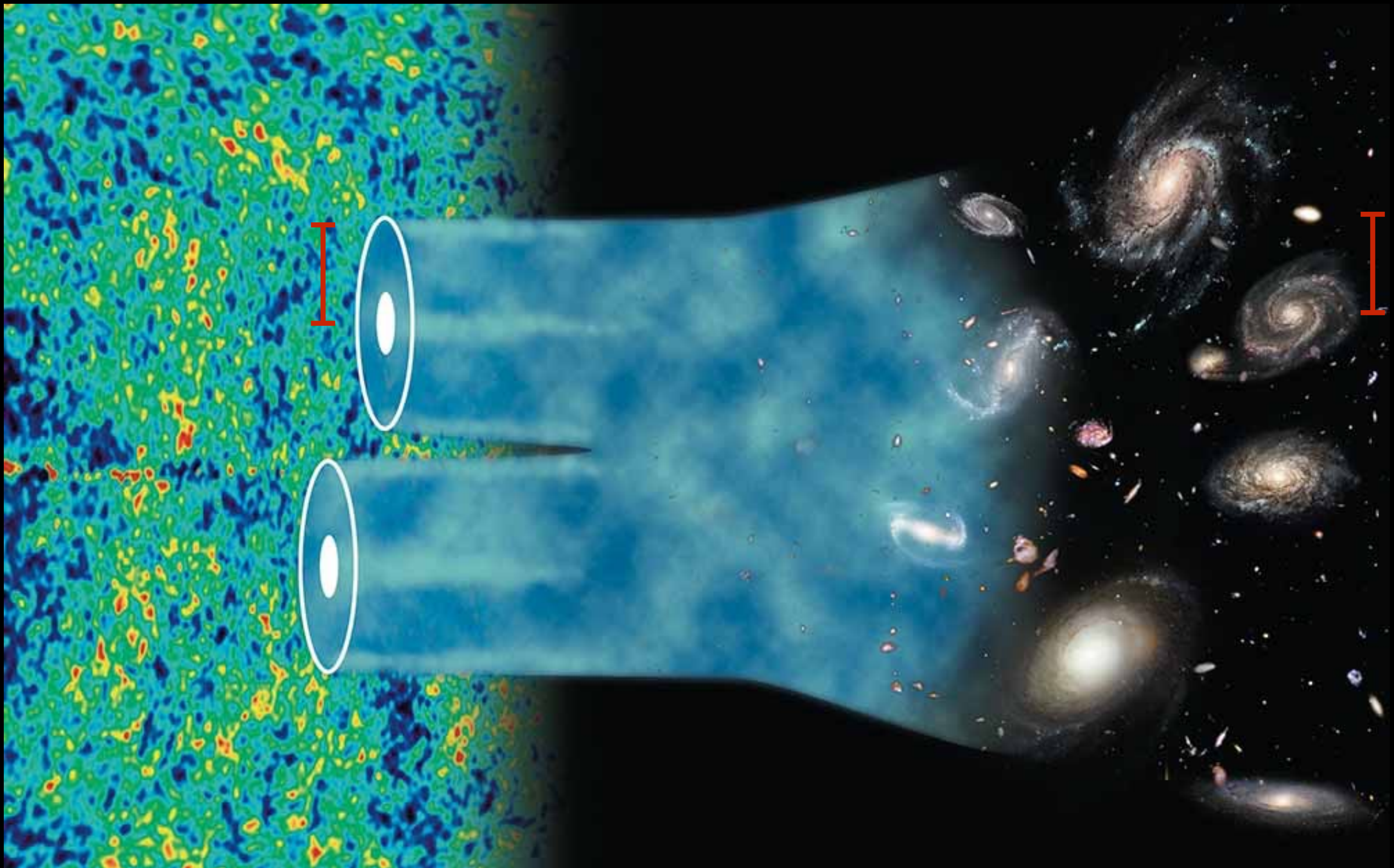


# CMB -> LARGE-SCALE STRUCTURE

**Baryon Acoustic Oscillations** survive all the way

Early Time
**CMB**

Late Time
**Galaxy Distribution**



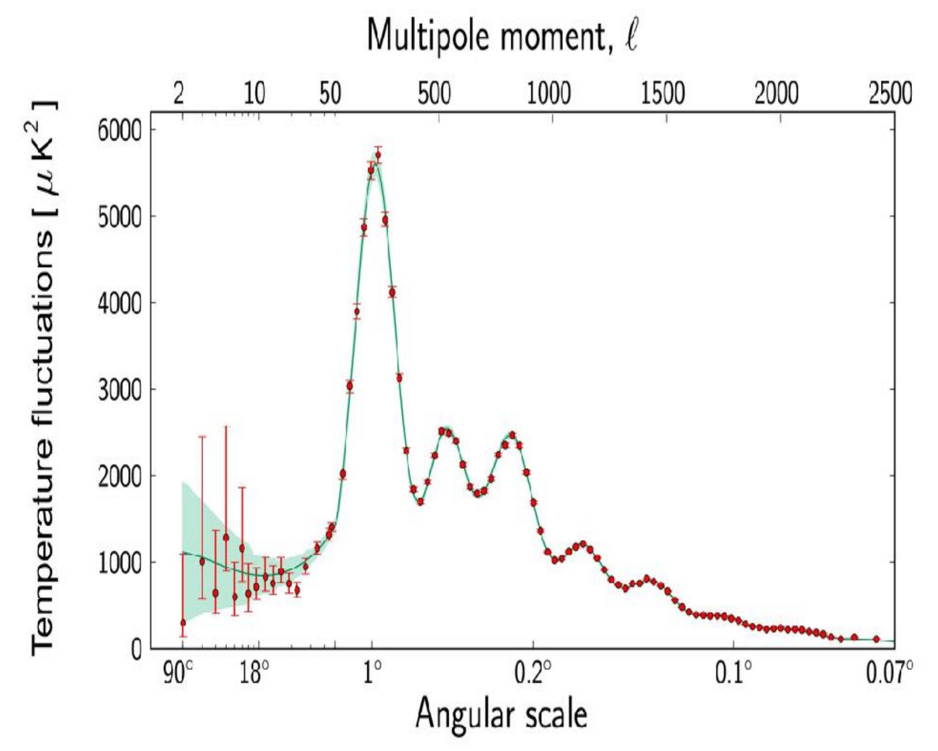
# A COHERENT PICTURE

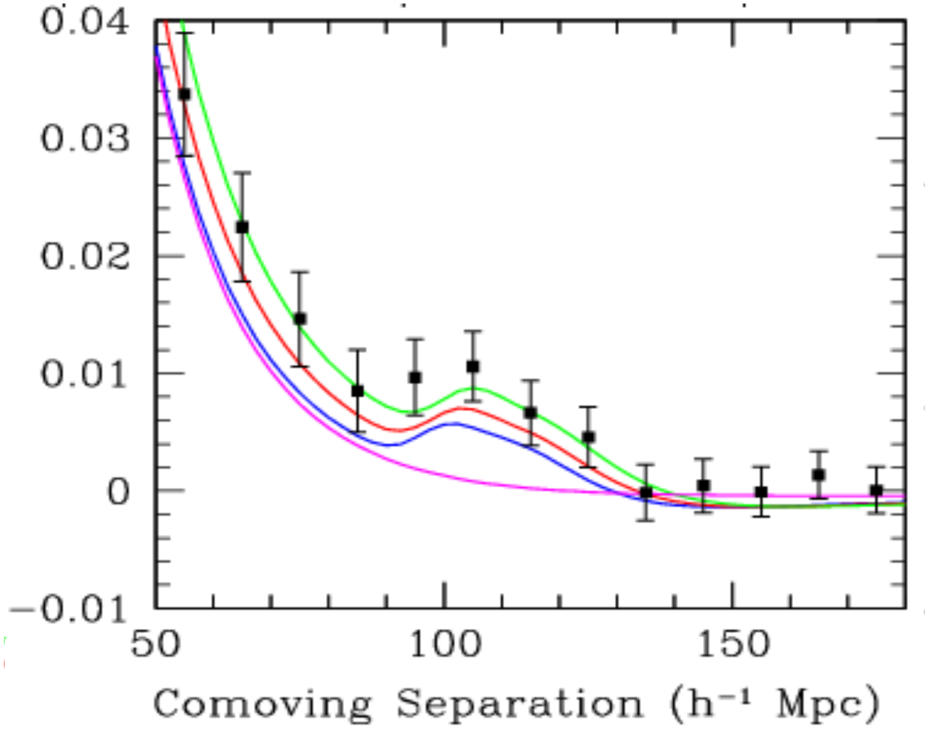
**Baryon Acoustic Oscillations** survive all the way

Early Time
**CMB**

Late Time
**Galaxy Distribution**

peak series in frequency scale = 1 peak in spatial scales



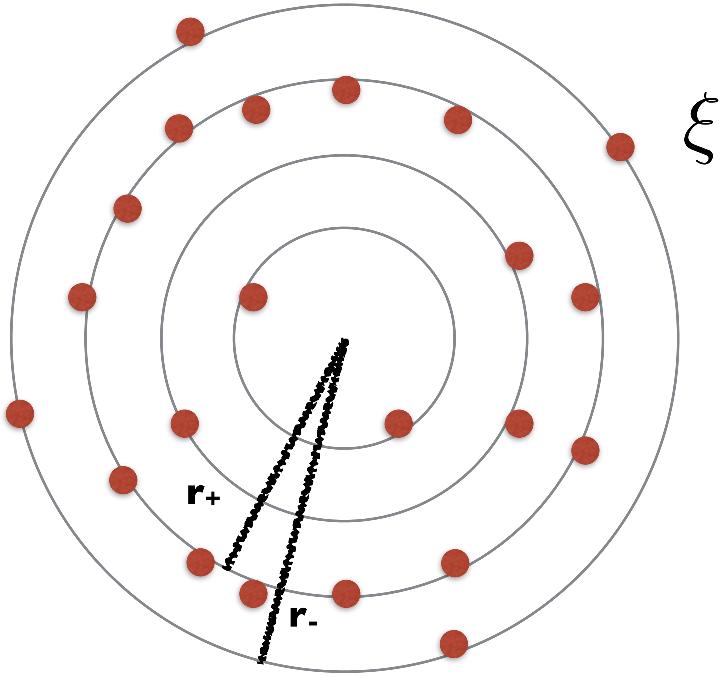


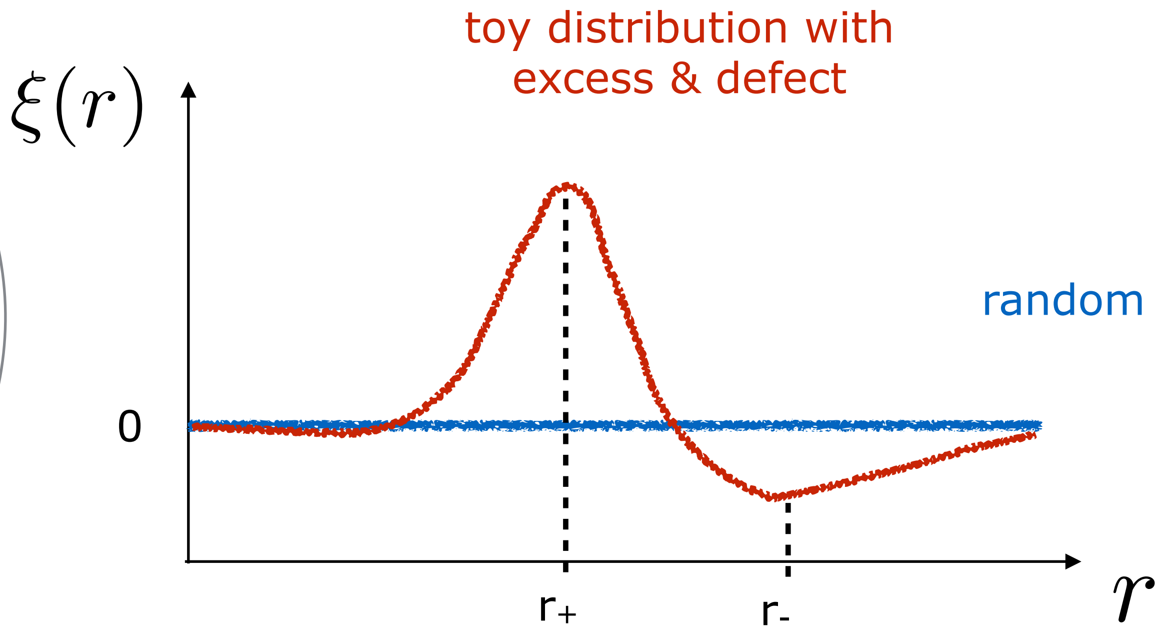
# GALAXY CORRELATION FUNCTION

excess correlation compared to random distribution

$$\xi(r) = \langle \delta(\boldsymbol{x} + \boldsymbol{r}) \delta(\boldsymbol{x}) \rangle$$

↑ density contrast ↑ average





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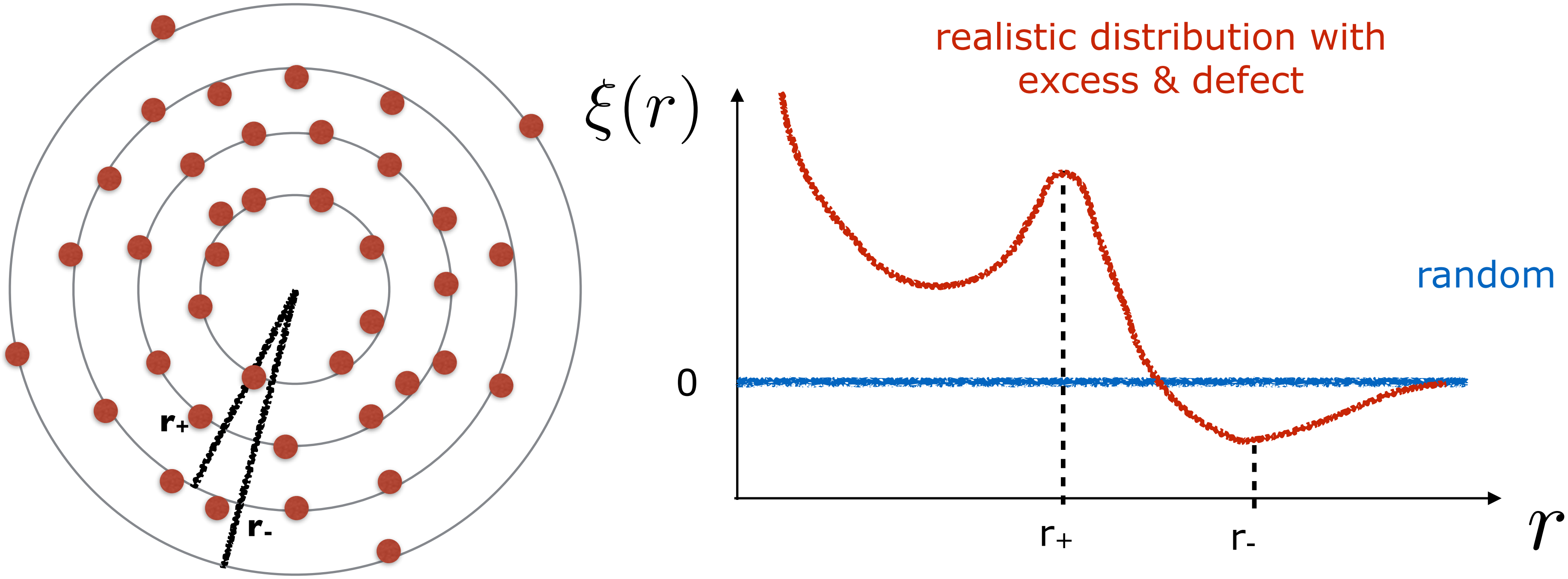
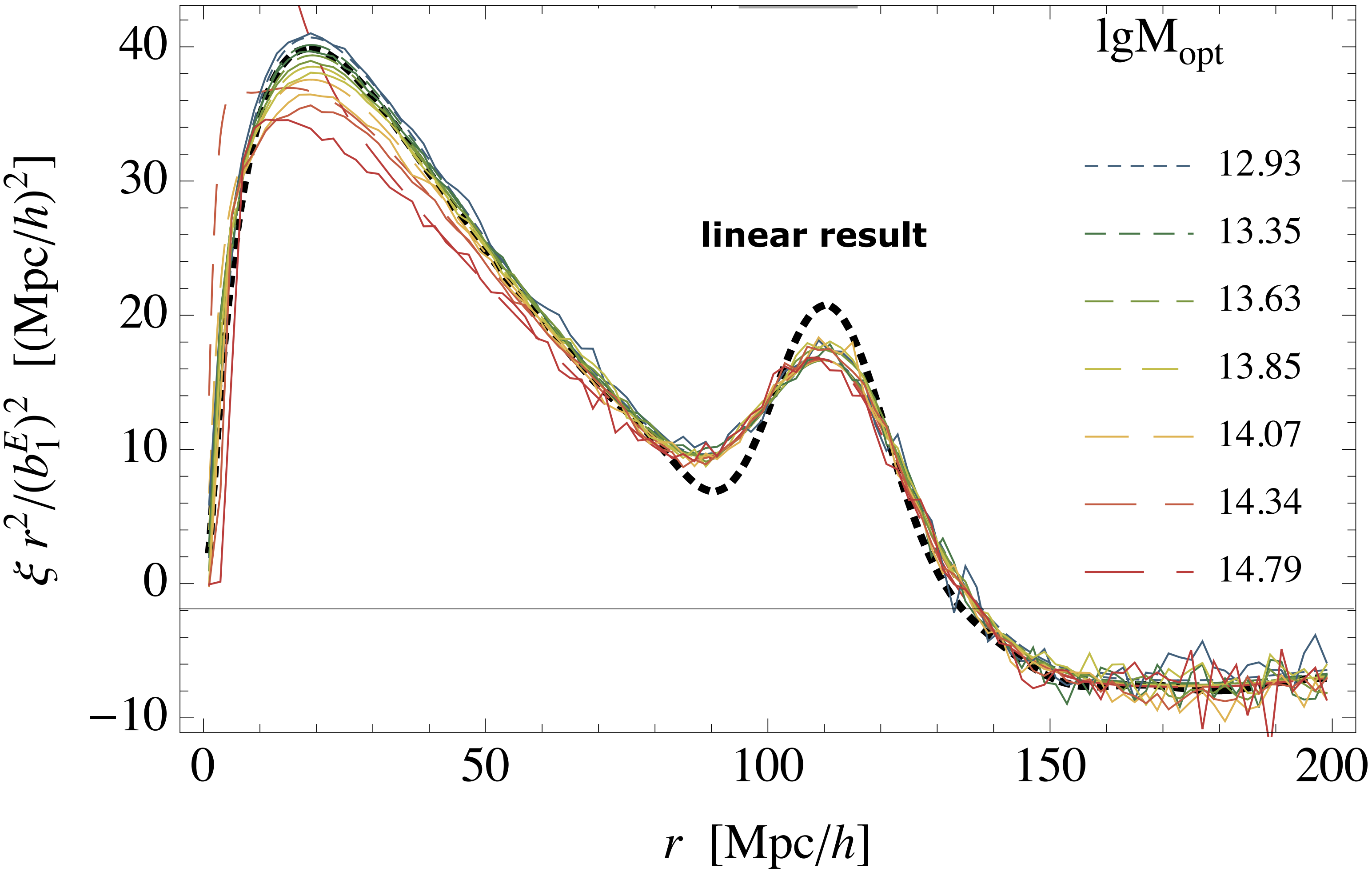


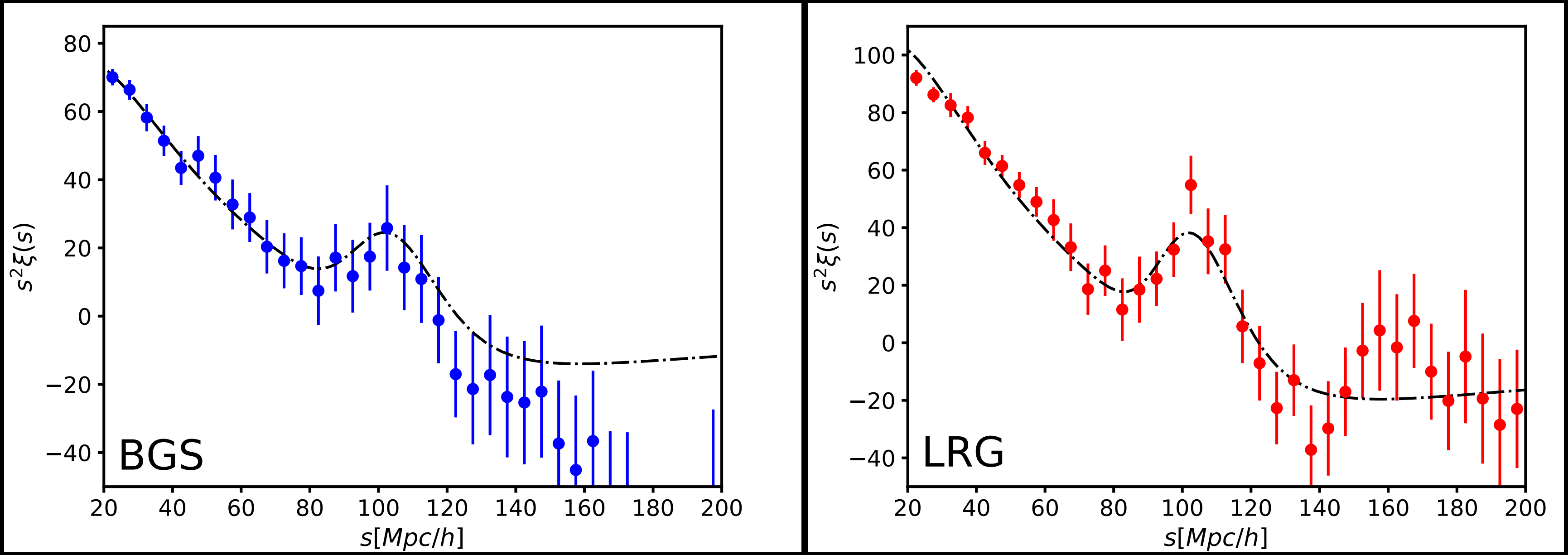
# CORRELATION FUNCTION

linear prediction reasonable on large scales



# GALAXY CORRELATION FUNCTION

- large scales: galaxies trace matter linearly



*Moon et al. 2023 Early DESI data*