Recap: Eulerian Perturbation Theory

density $\delta(x,\eta)$: $\delta^{(1)} \sim D(\eta)$ $\quad \delta^{(2)} \sim D^2(\eta)\, \tilde{\delta}^{(1)\,2} \sim \delta_{lin}^{2}(\underline{k},\eta)$

$$\tilde{\delta}^{(2)}(\underline{k}) = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6} (2\pi)^3\, \delta_D(\underline{k}-\underline{k_1}-\underline{k_2})\, F_2(\underline{k_1},\underline{k_2})\, \tilde{\delta}_1(\underline{k_1})\, \tilde{\delta}_1(\underline{k_2})$$

$\boxed{\text{Clustering Statistics}}$

Two point correlations & power spectrum

statistical homog. isotropy

$$\langle \delta(\underline{k},\eta)\, \delta(\underline{k}',\eta) \rangle = (2\pi)^3\, \delta_D(\underline{k}+\underline{k}')\, P(k,\eta)$$

comment originally $\langle \delta(\underline{k})\, \underbrace{\delta^*(\underline{k}')}_{\delta(-\underline{k}')} \rangle \sim \delta_D(\underline{k}-\underline{k}')$

Recap of Gaussian initial fields

1point Gaussian: imagine $\delta_0(\underline{x}) \xrightarrow{\text{smoothing}} \delta_{R,0}(\underline{x})$

$$P(\delta_0(R), R) = \frac{1}{\sqrt{2\pi\, \sigma^2(R)}} \exp\left(-\frac{1}{2} \frac{\delta_0(R)^2}{\sigma^2(R)}\right)$$

$$\langle \delta_0(R)^{2n+1} \rangle = \int d\delta_0\, \delta_0^{2n+1} P(\delta_0) = 0 \qquad \forall n \in \mathbb{N}$$

$$\langle \delta_0(R)^{2n} \rangle = \int d\delta_0\, \delta_0^{2n} P(\delta_0) = \sigma_0^{2n} (2n-1)!! \qquad \forall n \in \mathbb{N}$$

double factorial

$$\langle \delta_0^2 \rangle = \sigma_0^2$$

$$\langle \delta_0^4 \rangle = 3\sigma_0^4$$

$n!! = n\cdot(n-2)\cdot \ldots$

n point Gaussian in Fourier

2point

$$\langle \delta_0(\underline{k},\eta)\, \delta_0(\underline{k}',\eta) \rangle = (2\pi)^3\, \delta_D(\underline{k}+\underline{k}')\, P_0(k,\eta)$$

3point

$$\langle \delta_0(\underline{k_1})\, \delta_0(\underline{k_2})\, \delta_0(\underline{k_3}) \rangle = 0$$

4point

$$\langle \delta_0(\underline{k_1})\, \delta_0(\underline{k_2})\, \delta_0(\underline{k_3}), \delta_0(\underline{k_4}) \rangle$$
$$= (2\pi)^6 \big[\delta_D(\underline{k_1}+\underline{k_2})\, \delta_D(\underline{k_3}+\underline{k_4})\, P_0(k_1)\, P_0(k_3)$$
$$+ \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k_2}+\underline{k_4})\, P_0(k_1)\, P_0(k_2)$$
$$+ \delta_D(\underline{k_1}+\underline{k_4})\, \delta_D(\underline{k_2}+\underline{k_3})\, P_0(k_1)\, P_0(k_2) \big]$$

Power spectrum from SPT

PT kernel from mode coupling in fluid equations

$$F_2(\underline{k}_1, \underline{k}_2) = \frac{5}{7}\alpha(\underline{k}_1, \underline{k}_2) + \frac{2}{7}\beta(\underline{k}_1, \underline{k}_2)$$

$$= \frac{5}{7} + \frac{\underline{k}_1 \cdot \underline{k}_2}{k_1 k_2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\frac{(\underline{k}_1 \cdot \underline{k}_2)^2}{k_1^2 k_2^2}$$

PT up to 2nd order

$$\langle \delta(\underline{k})\delta(\underline{k}')\rangle \approx \langle (\delta^{(1)} + \delta^{(2)})(\underline{k})(\delta^{(1)} + \delta^{(2)})(\underline{k}')\rangle$$

$$\approx \underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(1)}(\underline{k}')\rangle}_{\rightarrow P_L(k,\eta)} + 2\underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle}_{\rightarrow P_{NLO}(k,\eta)} + \ldots$$

next-to-leading order ?

$$\langle \delta^{(1)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle$$

$$= \langle \delta^{(1)}(\underline{k}) \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k}' - \underline{k}_1 - \underline{k}_2) F_2(\underline{k}_1, \underline{k}_2)\delta^{(1)}(\underline{k}_1)\delta^{(1)}(\underline{k}_2)\rangle$$

$$= \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k}' - \underline{k}_1 - \underline{k}_2) F_2(\underline{k}_1, \underline{k}_2)\underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(1)}(\underline{k}_1)\delta^{(1)}(\underline{k}_2)\rangle}_{=0}$$

$= 0$ not interesting

need to go to 3rd order for δ,
collect all terms up $(\delta^{(1)})^4$

$$\langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})(\underline{k})(\delta^{(1)} + \delta^{(2)} + \delta^{(3)})(\underline{k}')\rangle$$

$$= \underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(1)}(\underline{k}')\rangle}_{\rightarrow P_L(k)} + \langle \delta^{(2)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle + 2\langle \delta^{(1)}(\underline{k})\delta^{(3)}(\underline{k}')\rangle$$

$$\langle \delta^{(2)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3\delta_D(\underline{k} - \underline{k}_1 - \underline{k}_2) F_2(\underline{k}_1, \underline{k}_2)$$

$$\int \frac{d^3k_3\, d^3k_4}{(2\pi)^6}(2\pi)^3\delta_D(\underline{k}' - \underline{k}_3 - \underline{k}_4) F_2(\underline{k}_3, \underline{k}_4)$$

$$\langle \delta^{(1)}(\underline{k}_1)\delta^{(1)}(\underline{k}_2)\delta^{(1)}(\underline{k}_3)\delta^{(1)}(\underline{k}_4)\rangle$$

$$= \int \frac{d^3k_1\, d^3k_3}{(2\pi)^6} F_2(\underline{k_1}, \underline{k}-\underline{k_1}) F_2(\underline{k_3}, \underline{k}'-\underline{k_3})$$

$$\langle \delta^{(1)}(\underline{k_1})\, \delta^{(1)}(\underline{k}-\underline{k_1})\, \delta^{(1)}(\underline{k_3})\, \delta^{(1)}(\underline{k}'-\underline{k_3}) \rangle$$

$$= (2\pi)^6 \left[\delta_D(\underline{k_1}+\underline{k_2})\, \delta_D(\underline{k_3}+\underline{k_4})\, P_0(k_1) P_0(k_3) \right.$$
$$+ \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k_2}+\underline{k_4})\, P_0(k_1) P_0(k_2)$$
$$\left. + \delta_D(\underline{k_1}+\underline{k_4})\, \delta_D(\underline{k_2}+\underline{k_3})\, P_0(k_1) P_0(k_2) \right]$$

$$\delta_D(\underline{k})\, \delta_D(\underline{k}')\, P_0(k_1) P_0(k_3)$$
$$+ \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k} - \cancel{\underline{k_1}} + \underline{k}' - \cancel{\underline{k_3}})\, P_0(k_1) P_0(|\underline{k}-\underline{k_1}|)$$
$$+ \delta_D(\underline{k_1}+\underline{k}'-\underline{k_3})\, \delta_D(\underline{k}-\underline{k_1}+\underline{k_3})\, P_0(k_1) P_0(|\underline{k}-\underline{k_1}|)$$

$\underline{k_1}-\underline{k_3} = -\underline{k}'$ $\qquad$ $\underline{k}+\underline{k}'$

$F_2(\underline{k_1}, \underline{k}-\underline{k_1})^2$

$$P_{22}(k) = 2 \int \frac{d^3k_1}{(2\pi)^3} F_2(\underline{k_1}, \underline{k}-\underline{k_1})\, F_2(+\underline{k_1}, +(\underline{k}-\underline{k_1}))\, P_L(k_1) P_L(|\underline{k}-\underline{k_1}|)$$

do the same for P_{13}

$$P_{13}(k) = 3 P_L(k) \int \frac{d^3k_2}{(2\pi)^3} F_3(-\underline{k}, \underline{k_2}, -\underline{k_2})\, P_L(k_2)$$

<u>Range of applicability of PT</u>

need to consider smoothed / filtered density field

$$\delta_W(\underline{x}) = \int d^3\underline{x}'\, W_R(|x-x'|)\, \delta(\underline{x}') \iff \delta_W(\underline{k}) = \hat{W}_R(\underline{k})\, \delta(\underline{k})$$

↳ kernel

this can be used to make PT more rigorous

↳ extra terms in fluid equation

Effective Field Theory of LSS

rough idea from estimating nonlinear scale

$$k_{NL}^{-2} = \int \frac{d^3k}{(2\pi)^3} \, k^{-2} P(k)$$