Recap: Eulerian Perturbation Theory

density $\delta(x,\eta)$: $\delta^{(1)} \sim D(\eta)$ $\quad \delta^{(2)} \sim D^2(\eta)\, \tilde{\delta}^{(1)\,2} \sim \delta_{lin}^2(\underline{k},\eta)$

$$\tilde{\delta}^{(2)}(\underline{k}) = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6} (2\pi)^3\, \delta_D(\underline{k}-\underline{k_1}-\underline{k_2})\, F_2(\underline{k_1},\underline{k_2})\, \tilde{\delta}_1(\underline{k_1})\, \tilde{\delta}_1(\underline{k_2})$$

$\boxed{\text{Clustering Statistics}}$

Two point correlations & power spectrum

statistical homoge. isotropy

$$\langle \delta(\underline{k},\eta)\, \delta(\underline{k}',\eta) \rangle = (2\pi)^3\, \delta_D(\underline{k}+\underline{k}')\, P(k,\eta)$$

comment originally $\langle \delta(\underline{k})\, \underbrace{\delta^*(\underline{k}')}_{\delta(-\underline{k}')} \rangle \sim \delta_D(\underline{k}-\underline{k}')$

Recap of Gaussian initial fields

1 point Gaussian: imagine $\delta_0(\underline{x}) \xrightarrow{\text{smoothing}} \delta_{R,0}(\underline{x})$

$$P(\delta_0(R), R) = \frac{1}{\sqrt{2\pi\, \sigma^2(R)}} \exp\left(-\frac{1}{2}\frac{\delta_0(R)^2}{\sigma^2(R)}\right)$$

$$\langle \delta_0(R)^{2n+1} \rangle = \int d\delta_0\, \delta_0^{2n+1}\, P(\delta_0) = 0 \qquad \forall n \in \mathbb{N}$$

$$\langle \delta_0(R)^{2n} \rangle = \int d\delta_0\, \delta_0^{2n}\, P(\delta_0) = \sigma_0^{2n}\, (2n-1)!! \qquad \forall n \in \mathbb{N}$$

$\langle \delta_0^2 \rangle = \sigma_0^2$ $\qquad$ double factorial

$\langle \delta_0^4 \rangle = 3\sigma_0^4$ $\qquad$ $n!! = n\cdot(n-2)\cdot\ldots$

n point Gaussian in Fourier

2point

$$\langle \delta_0(\underline{k},\eta)\, \delta_0(\underline{k}',\eta) \rangle = (2\pi)^3\, \delta_D(\underline{k}+\underline{k}')\, P_0(k,\eta)$$

3point

$$\langle \delta_0(\underline{k_1})\, \delta_0(\underline{k_2})\, \delta_0(\underline{k_3}) \rangle = 0$$

4point

$$\begin{aligned}
&\langle \delta_0(\underline{k_1})\, \delta_0(\underline{k_2})\, \delta_0(\underline{k_3}),\, \delta_0(\underline{k_4}) \rangle \\
&= (2\pi)^6 \big[\delta_D(\underline{k_1}+\underline{k_2})\, \delta_D(\underline{k_3}+\underline{k_4})\, P_0(k_1)\, P_0(k_3) \\
&\quad + \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k_2}+\underline{k_4})\, P_0(k_1)\, P_0(k_2) \\
&\quad + \delta_D(\underline{k_1}+\underline{k_4})\, \delta_D(\underline{k_2}+\underline{k_3})\, P_0(k_1)\, P_0(k_2) \big]
\end{aligned}$$

Power spectrum from SPT

PT kernel from mode coupling in fluid equations

$$F_2(\underline{k_1}, \underline{k_2}) = \frac{5}{7}\alpha(\underline{k_1}, \underline{k_2}) + \frac{2}{7}\beta(\underline{k_1}, \underline{k_2})$$

$$= \frac{5}{7} + \frac{k_1 \cdot k_2}{k_1 k_2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right) + \frac{2}{7}\frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2}$$

PT up to 2nd order

$$\langle \delta(\underline{k})\delta(\underline{k}')\rangle \approx \langle (\delta^{(1)} + \delta^{(2)})(\underline{k})(\delta^{(1)} + \delta^{(2)})(\underline{k}')\rangle$$

$$\approx \underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(1)}(\underline{k}')\rangle}_{\to P_L(\underline{k}, \eta)} + 2\underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle}_{\to P_{NLO}(\underline{k}, \eta)} + \ldots$$

next-to-leading order ?

$$\langle \delta^{(1)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle$$

$$= \langle \delta^{(1)}(\underline{k}) \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k}' - \underline{k_1} - \underline{k_2}) F_2(\underline{k_1}, \underline{k_2})\, \delta^{(1)}(\underline{k_1})\, \delta^{(1)}(\underline{k_2})\rangle$$

$$= \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k}' - \underline{k_1} - \underline{k_2}) F_2(\underline{k_1}, \underline{k_2}) \underbrace{\langle \delta^{(1)}(\underline{k})\, \delta^{(1)}(\underline{k_1})\, \delta^{(1)}(\underline{k_2})\rangle}_{=0}$$

$= 0$ not interesting

need to go to 3rd order for δ,
collect all terms up $(\delta^{(1)})^4$

$$\langle (\delta^{(1)} + \delta^{(2)} + \delta^{(3)})(\underline{k})(\delta^{(1)} + \delta^{(2)} + \delta^{(3)})(\underline{k}')\rangle$$

$$= \underbrace{\langle \delta^{(1)}(\underline{k})\delta^{(1)}(\underline{k}')\rangle}_{\to P_L(k)} + \langle \delta^{(2)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle + 2\langle \delta^{(1)}(\underline{k})\delta^{(3)}(\underline{k}')\rangle$$

$$\langle \delta^{(2)}(\underline{k})\delta^{(2)}(\underline{k}')\rangle = \int \frac{d^3k_1\, d^3k_2}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k} - \underline{k_1} - \underline{k_2}) F_2(\underline{k_1}, \underline{k_2})$$

$$\int \frac{d^3k_3\, d^3k_4}{(2\pi)^6}(2\pi)^3 \delta_D(\underline{k}' - \underline{k_3} - \underline{k_4}) F_2(\underline{k_3}, \underline{k_4})$$

$$\langle \delta^{(1)}(\underline{k_1})\delta^{(1)}(\underline{k_2})\, \delta^{(1)}(\underline{k_3})\, \delta^{(1)}(\underline{k_4})\rangle$$

$$= \int \frac{d^3k_1\, d^3k_3}{(2\pi)^6} F_2(\underline{k_1}, \underline{k}-\underline{k_1}) F_2(\underline{k_3}, \underline{k}'-\underline{k_3})$$

$$\langle \delta^{(1)}(\underline{k_1})\, \delta^{(1)}(\underline{k}-\underline{k_1})\, \delta^{(1)}(\underline{k_3})\, \delta^{(1)}(\underline{k}'-\underline{k_3}) \rangle$$

$$= (2\pi)^6 \Big[\delta_D(\underline{k_1}+\underline{k_2})\, \delta_D(\underline{k_3}+\underline{k_4})\, P_0(k_1)\, P_0(k_3)$$
$$+ \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k_2}+\underline{k_4})\, P_0(k_1)\, P_0(k_2)$$
$$+ \delta_D(\underline{k_1}+\underline{k_4})\, \delta_D(\underline{k_2}+\underline{k_3})\, P_0(k_1)\, P_0(k_2) \Big]$$

$$\delta_D(\underline{k})\, \delta_D(\underline{k}')\, P_0(k_1)\, P_0(k_3)$$
$$+ \delta_D(\underline{k_1}+\underline{k_3})\, \delta_D(\underline{k}-\cancel{\underline{k_1}}+\underline{k}'-\cancel{\underline{k_3}})\, P_0(k_1)\, P_0(|\underline{k}-\underline{k_1}|)$$
$$+ \delta_D(\underline{k_1}+\underline{k}'-\underline{k_3})\, \delta_D(\underline{k}-\underline{k_1}+\underline{k_3})\, P_0(k_1)\, P_0(|\underline{k}-\underline{k_1}|)$$

$\underline{k_1}-\underline{k_3} = -\underline{k}'$ $\quad\quad$ $\underline{k}+\underline{k}'$

$F_2(\underline{k_1}, \underline{k}-\underline{k_1})^2$

$$P_{22}(k) = 2 \int \frac{d^3k_1}{(2\pi)^3} F_2(\underline{k_1}, \underline{k}-\underline{k_1})\, F_2(+\underline{k_1}, +(\underline{k}-\underline{k_1}))\, P_L(k_1) P_L(|\underline{k}-\underline{k_1}|)$$

do the same for P_{13}

$$P_{13}(k) = 3 P_L(k) \int \frac{d^3k_2}{(2\pi)^3} F_3(-\underline{k}, \underline{k_2}, -\underline{k_2})\, P_L(k_2)$$

Range of applicability of PT

need to consider smoothed / filtered density field

$$\delta_W(\underline{x}) = \int d^3\underline{x}'\, W_R(|\underline{x}-\underline{x}'|)\, \delta(\underline{x}') \Leftrightarrow \delta_W(\underline{k}) = \hat{W}_R(\underline{k})\, \delta(\underline{k})$$

↑ kernel

this can be used to make PT more rigorous

↳ extra terms in fluid equation

Effective Field Theory of LSS

rough idea from estimating nonlinear scale

$$k_{NL}^{-2} = \int \frac{d^3k}{(2\pi)^3} \, k^{-2} P(k)$$

Hands on

$$\langle \delta^3(\underline{x})\rangle = \langle (\delta^{(1)}+\delta^{(2)}+\cancel{\delta^{(3)}})^3(\underline{x})\rangle$$

$$= \underbrace{\langle \delta^{(1)}(\underline{x})^3\rangle}_{=0} + 3\langle \delta^{(1)}(\underline{x})^2\,\delta^{(2)}(\underline{x})\rangle$$

for Gaussian initial field

$$= 3\int \frac{d^3k_1\, d^3k_2\, d^3k_3}{(2\pi)^9} \exp[i\,\underline{x}\cdot(\underline{k_1}+\underline{k_2}+\underline{k_3})]\,\langle\delta^{(1)}(\underline{k_1})\,\delta^{(1)}(\underline{k_2})\,\delta^{(2)}(\underline{k_3})\rangle$$

redefine

plug in F_2

$$= 3\int \frac{d^3k_1\, d^3k_2\, d^3\tilde{k}_3\, d^3k_4}{(2\pi)^{12}} \exp[i\,\underline{x}\cdot(\underline{k_1}+\underline{k_2}+\underline{\tilde{k}_3}+\underline{k_4})]\, F_2(\underline{\tilde{k}_3},\underline{k_4})$$

$$\langle \delta^{(1)}(\underline{k_1})\,\delta^{(1)}(\underline{k_2})\,\delta^{(1)}(\underline{\tilde{k}_3})\,\delta^{(1)}(\underline{k_4})\rangle$$

$$(2\pi)^6\Big[\delta_D(\underline{k_1}+\underline{k_2})\,\cancel{\delta_D(\underline{\tilde{k}_3}+\underline{k_4})}\,P(k_1)\,P(\tilde{k}_3)$$

$$+\,\delta_D(\underline{k_1}+\underline{\tilde{k}_3})\,\delta_D(\underline{k_2}+\underline{k_4})\,P_L(k_1)\,P_L(k_2)$$

$$+\,\delta_D(\underline{k_1}+\underline{k_4})\,\delta_D(\underline{k_2}+\underline{\tilde{k}_3})\,P_L(k_1)\,P_L(k_2)\Big]$$

$$= 6\int\frac{d^3k_1\, d^3k_2}{(2\pi)^6}\,F_2(-\underline{k_1},-\underline{k_2})\,P_L(k_1)\,P_L(k_2)$$

$d^3k = dk\, k^2\, d\mu\, d\varphi$

$$= 6\cdot\left(\frac{5}{7}\cdot 1+\frac{2}{7}\cdot\frac{1}{3}\right)\langle\delta_L^2\rangle^2 = 6\cdot\frac{17}{21}\langle\delta_L^2\rangle^2 = \frac{34}{7}\sigma^4$$

$$\frac{\langle\delta^3\rangle}{\langle\delta^2\rangle^2} = \frac{34}{7}$$

$\alpha(\underline{k_1},\underline{k_2})$ angular average $\bar{\alpha} = 1$

$\beta(\underline{k_1},\underline{k_2})$ $\bar{\beta} = 1/3$

$$F_2 = \frac{5}{7}\alpha + \frac{2}{7}\beta$$

or bispectrum

$$\langle \delta^{(1)}(\underline{k_1})\,\delta^{(1)}(\underline{k_2})\,\delta^{(2)}(\underline{k_3})\rangle = \ldots$$

+cyclic

$$\Rightarrow B_{112}(k_1,k_2,k_3) = 2\left(F_2(\underline{k_1},\underline{k_2})\,P_L(k_1)\,P_L(k_2) + \text{cyclic}\right)$$