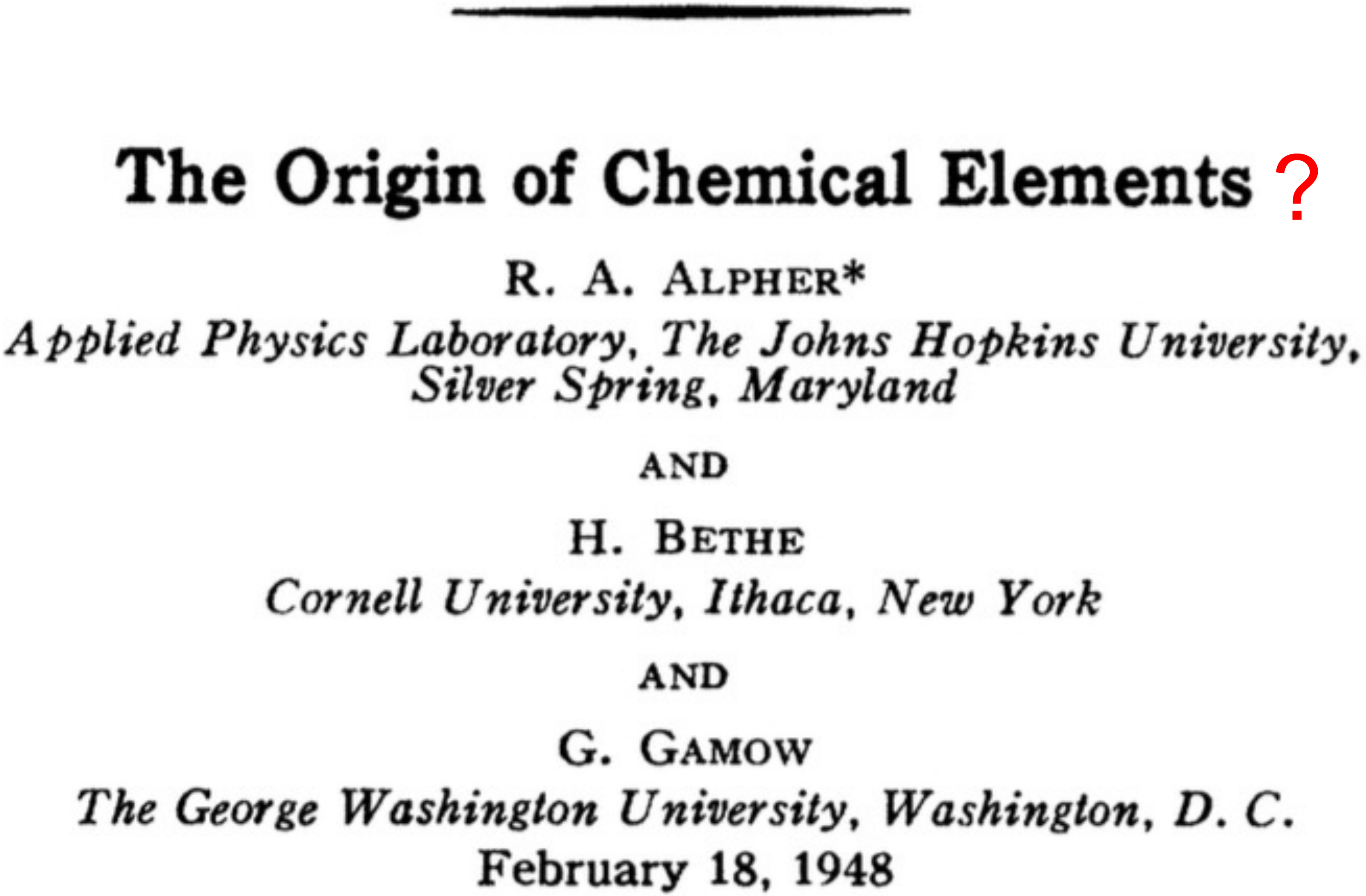
An overview of big-bang nucleosynthesis

Cyril Pitrou (24th July 2025, les Houches summer school)

Books and articles

- Peter&Uzan, *Cosmologie Primordiale* (Belin).
- Kolb&Turner, *The Early Universe*
- *C. Pitrou, A. Coc, J.-P. Uzan, E. Vangioni,*
  Precision big bang nucleosynthesis with improved He-4 production, PRIMAT code
  [arXiv:1801.08023], Physics Reports, **04**, (2018) 005.
- Bernstein, Brown, Feinberg,
  Cosmological helium production simplified
  Rev. Mod. Phys **61**, 1 (1989).
- Froustey, Julien
  The Universe at the MeV era : neutrino evolution and cosmological observables
  [arXiv:2209.06672]



The Origin of Chemical Elements ?

R. A. ALPHER*
Applied Physics Laboratory, The Johns Hopkins University,
Silver Spring, Maryland

AND

H. BETHE
Cornell University, Ithaca, New York

AND

G. GAMOW
The George Washington University, Washington, D. C.

February 18, 1948

PHYSICAL REVIEW VOLUME 73, NUMBER 7 APRIL 1, 1948

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length.

The Origin of Chemical Elements

R. A. Alpher*
Applied Physics Laboratory, The Johns Hopkins University, Silver Spring, Maryland
AND
H. Bethe
Cornell University, Ithaca, New York
AND
G. Gamow
The George Washington University, Washington, D. C.
February 18, 1948

As pointed out by one of us,[1] various nuclear species must have originated not as the result of an equilibrium corresponding to a certain temperature and density, but rather as a consequence of a continuous building-up process arrested by a rapid expansion and cooling of the primordial matter. According to this picture, we must imagine the early stage of matter as a highly compressed neutron gas (overheated neutral nuclear fluid) which started decaying into protons and electrons when the gas pressure fell down as the result of universal expansion. The radiative capture of the still remaining neutrons by the newly formed protons must have led first to the formation of deuterium nuclei, and the subsequent neutron captures resulted in the building up of heavier and heavier nuclei. It must be remembered that, due to the comparatively short time allowed for this process,[1] the building up of heavier nuclei must have proceeded just above the upper fringe of the stable elements (short-lived Fermi elements), and the present frequency distribution of various atomic species was attained only somewhat later as the result of adjustment of their electric charges by β-decay.

Thus the observed slope of the abundance curve must not be related to the temperature of the original neutron gas, but rather to the time period permitted by the expansion process. Also, the individual abundances of various nuclear species must depend not so much on their intrinsic stabilities (mass defects) as on the values of their neutron capture cross sections. The equations governing such a building-up process apparently can be written in the form:

$$\frac{dn_i}{dt}=f(t)(\sigma_{i-1}n_{i-1}-\sigma_i n_i) \quad i=1,2,\cdots 238, \qquad (1)$$

where n_i and σ_i are the relative numbers and capture cross sections for the nuclei of atomic weight i, and where $f(t)$ is a factor characterizing the decrease of the density with time.

We may remark at first that the building-up process was apparently completed when the temperature of the neutron gas was still rather high, since otherwise the observed abundances would have been strongly affected by the resonances in the region of the slow neutrons. According to Hughes,[2] the neutron capture cross sections of various elements (for neutron energies of about 1 Mev) increase exponentially with atomic number halfway up the periodic system, remaining approximately constant for heavier elements.

Using these cross sections, one finds by integrating Eqs. (1) as shown in Fig. 1 that the relative abundances of various nuclear species decrease rapidly for the lighter elements and remain approximately constant for the elements heavier than silver. In order to fit the calculated curve with the observed abundances[3] it is necessary to assume the integral of $\rho_n dt$ during the building-up period is equal to 5×10^4 g sec./cm^3.

On the other hand, according to the relativistic theory of the expanding universe[4] the density dependence on time is given by $\rho\leqq10^6/t^2$. Since the integral of this expression diverges at $t=0$, it is necessary to assume that the building-up process began at a certain time t_0, satisfying the relation:

$$\int_{t_0}^{\infty}(10^6/t^2)dt\leqq5\times10^4, \qquad (2)$$

which gives us $t_0\leqq20$ sec. and $\rho_0\leqq2.5\times10^5$ g sec./cm^3. This result may have two meanings: (a) for the higher densities existing prior to that time the temperature of the neutron gas was so high that no aggregation was taking place, (b) the density of the universe never exceeded the value 2.5×10^3 g sec./cm^3 which can possibly be understood if we

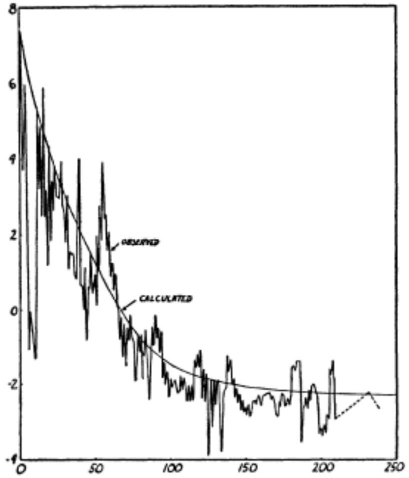


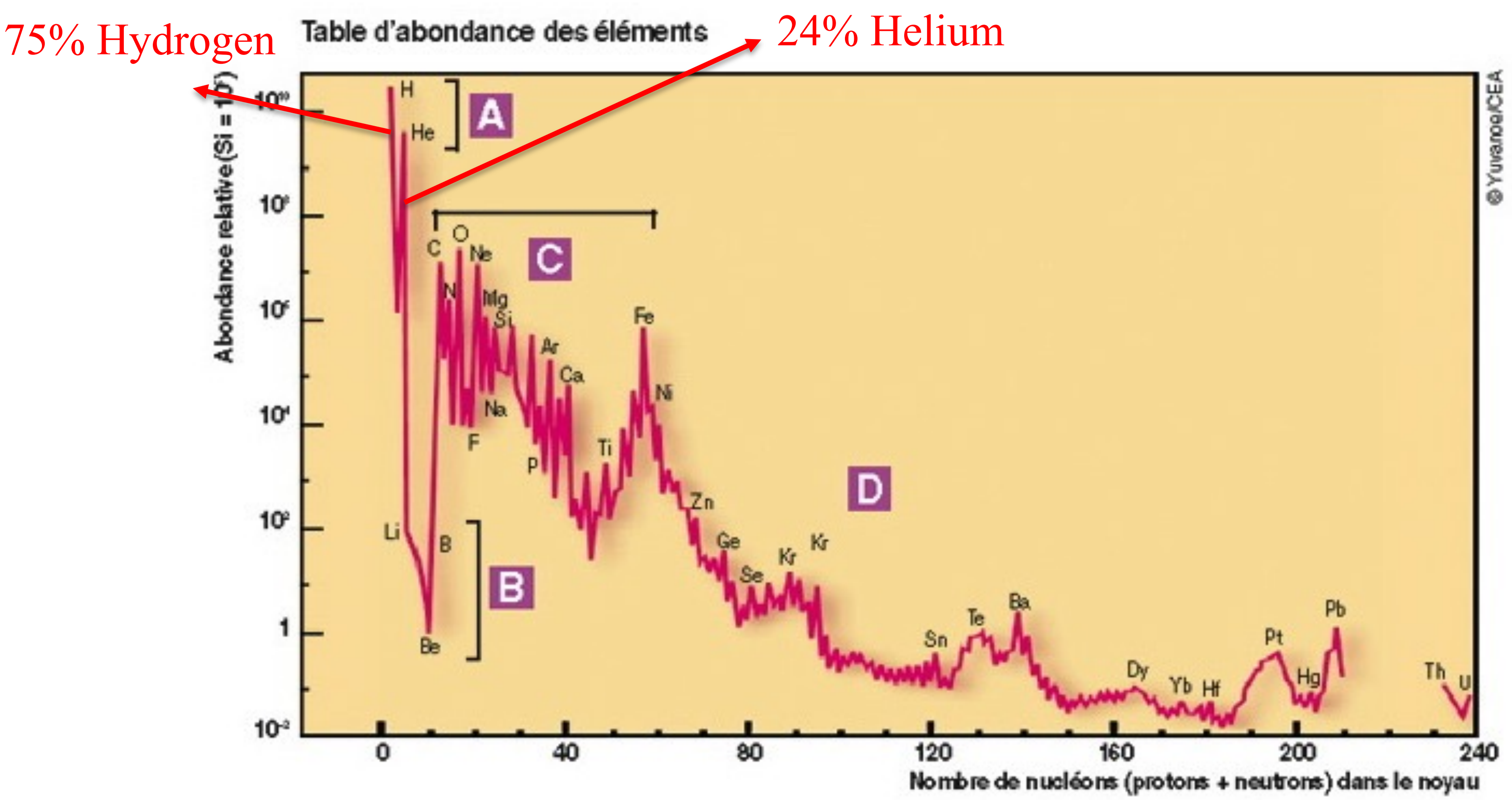
Fig. 1.

Log of relative abundance

Atomic weight

Origin of elements

1. Very light elements : primordial Universe
2. Some light elements : spallation from cosmic rays
3. Heavy elements : stars and star explosions

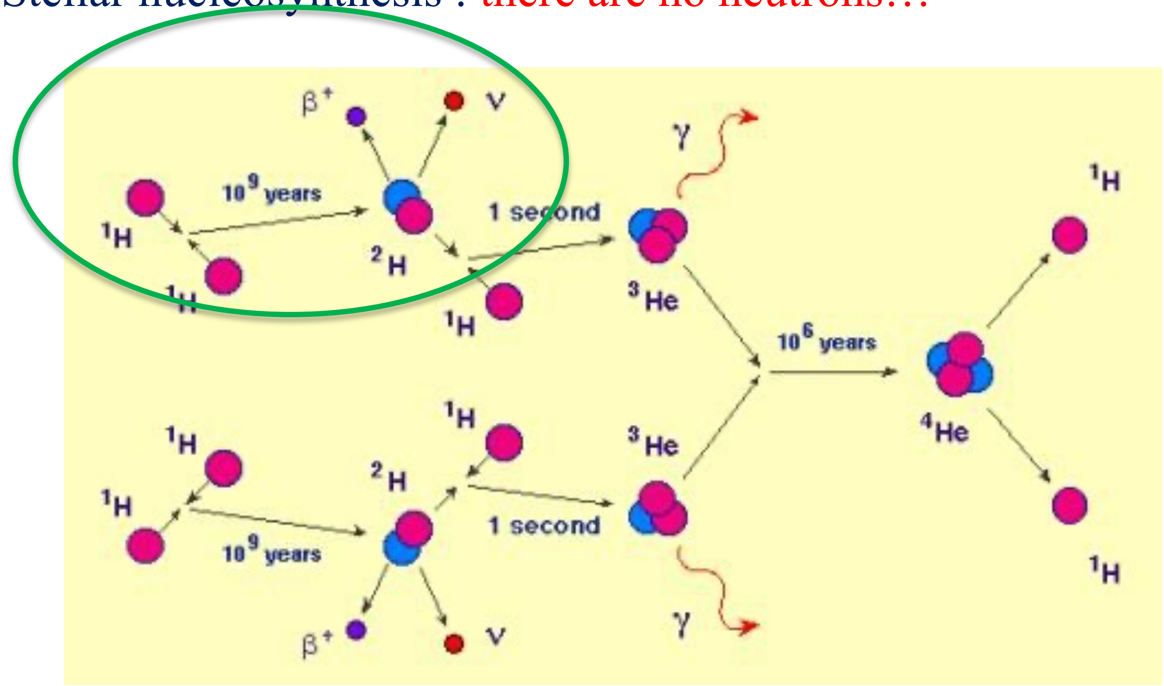


75% Hydrogen

24% Helium

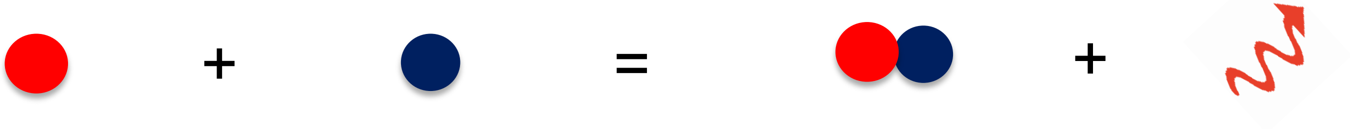
Why primordial synthesis ?

1. Stellar nucleosynthesis : there are no neutrons…



2. Primordial nucleosynthesis : there was plenty of neutrons !

$$p + n \leftrightarrow D + \gamma$$



2.2 MeV

When does it happen ? How many neutrons were available ?

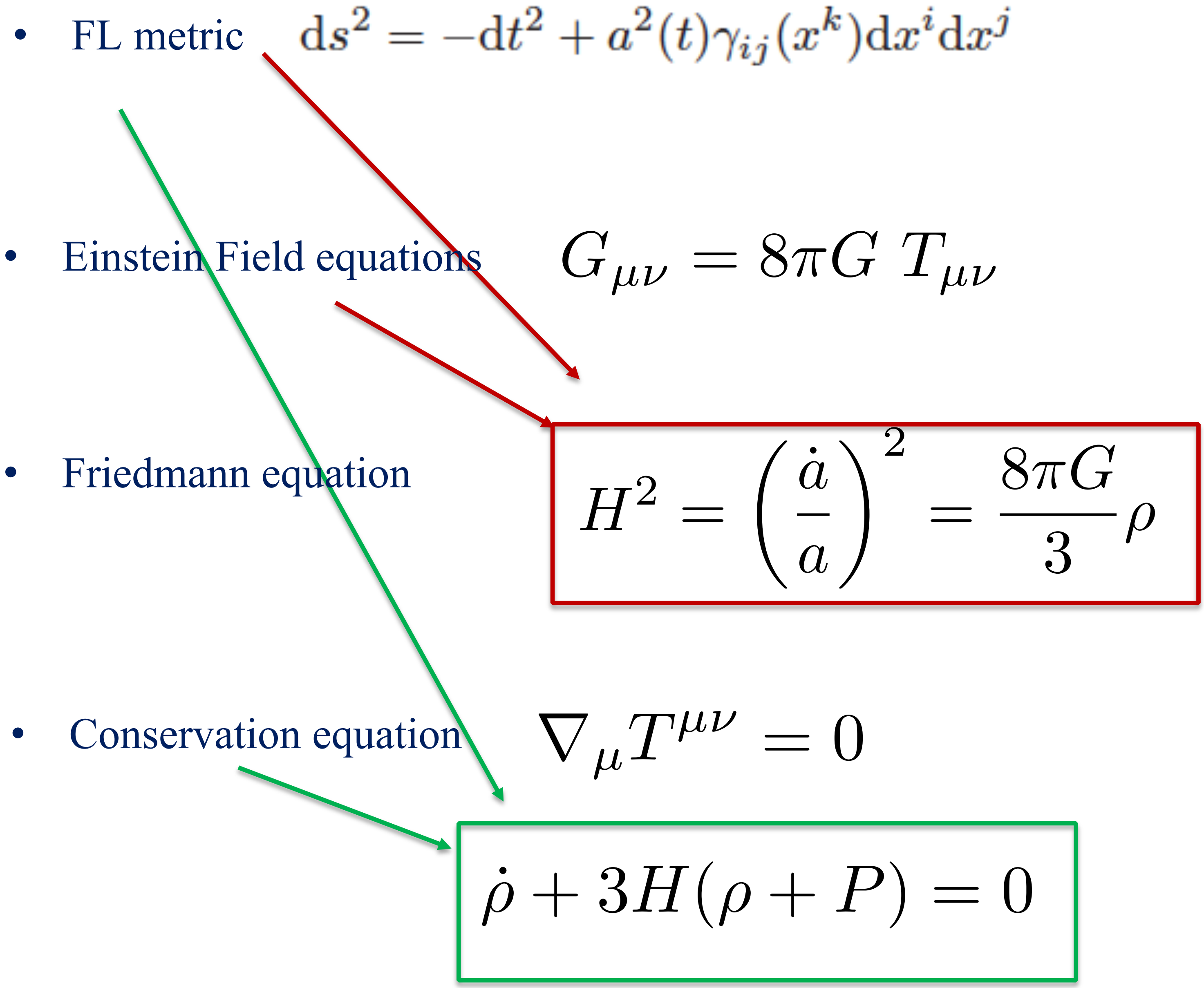
Outline :

1) Cosmology and plasma reheating
2) Weak interactions
3) Nuclear reactions
4) Observational constraints

Outline :

1) Cosmology and plasma reheating
2) Weak interactions
3) Nuclear reactions
4) Observational constraints

Summary of cosmology



- FL metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}(x^k)\mathrm{d}x^i\mathrm{d}x^j$
- Einstein Field equations $G_{\mu\nu} = 8\pi G\ T_{\mu\nu}$
- Friedmann equation

$$\boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho}$$

- Conservation equation $\nabla_\mu T^{\mu\nu} = 0$

$$\boxed{\dot{\rho} + 3H(\rho + P) = 0}$$

(perfect fluid)

Scaling of radiation and cold matter

1. Cold matter

$$P \ll \rho \qquad \dot{\rho} + 3H\rho = 0 \Rightarrow \rho = \rho_0/a^3$$

2. Radiation (relativistic matter)

$$P = \rho/3 \qquad \dot{\rho} + 4H\rho = 0 \Rightarrow \rho = \rho_0/a^4$$

Friedmann equation in radiation era

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$= \frac{8\pi G}{3}\left(\xcancel{\frac{\rho^{\text{today}}_{\text{matter}}}{a^3}} + \frac{\rho^{\text{today}}_{\text{rad}}}{a^4}\right)$$

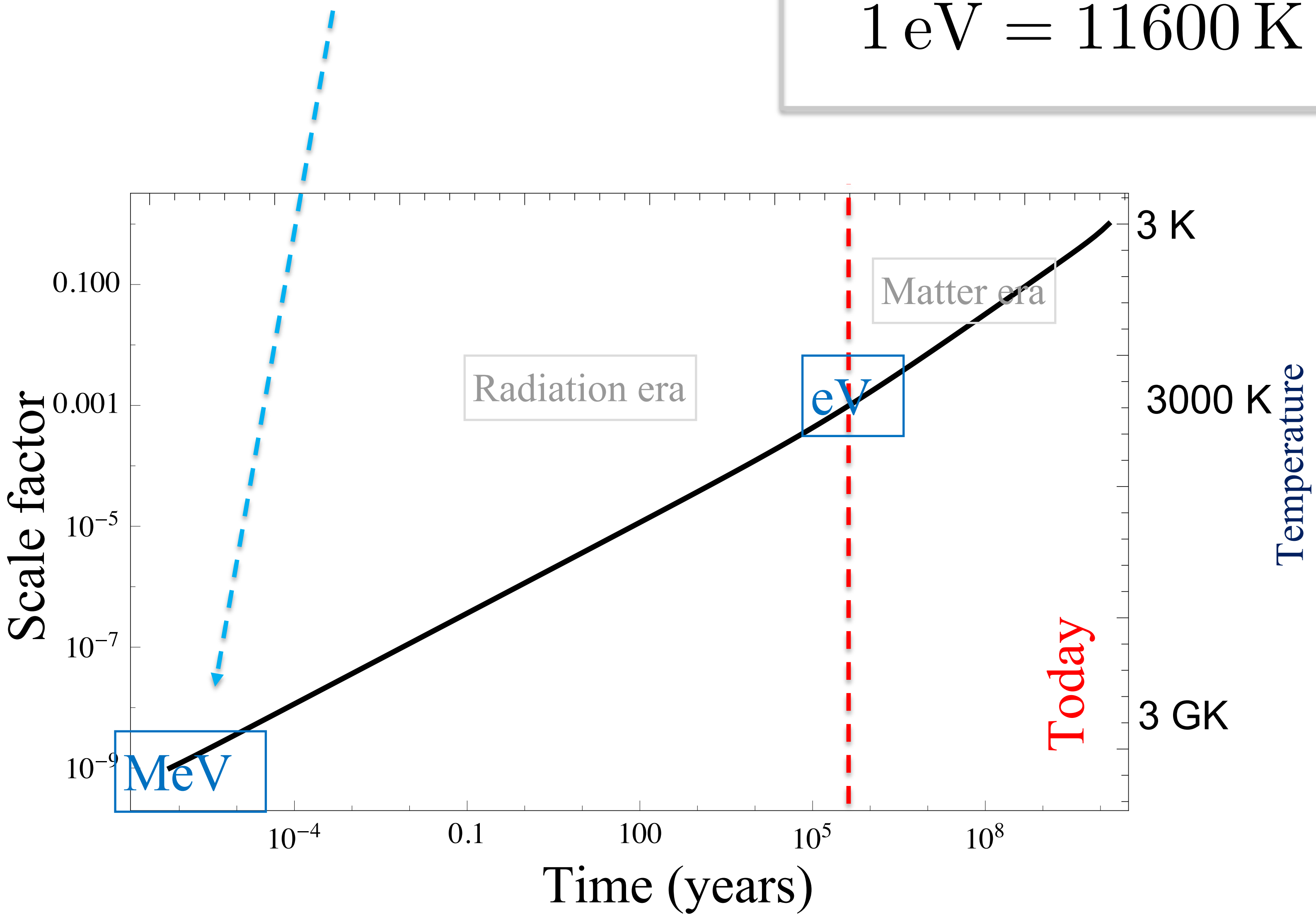
Radiation dominated universe

$$a \propto t^{1/2} \qquad \rho_{\text{rad}} \propto T^4 \propto a^{-4}$$

$$\boxed{T^2 \propto 1/t}$$

$$1\,\mathrm{MeV} = 11.6\,\mathrm{GK}$$
$$1\,\mathrm{eV} = 11600\,\mathrm{K}$$

The MeV scale



Individual particles

- Geodesic equation $p^\mu \nabla_\mu p^\nu = 0$
- Redshifting of momenta $\dot{p} + Hp = 0 \quad \Rightarrow p = p_0/a$
- Energy momentum relation $E^2 = p^2 + m^2$

For massless particles

$$E = p \qquad \dot{E} + HE = 0 \quad \Leftrightarrow E = \frac{E_0}{a}$$

Set of particles

- Distribution function $f(t, x^i, p^\mu) \xRightarrow[\text{isotropy}]{\text{homogeneity}} f(t, E)$

- Liouville equation $L[f] = C[f]$

$$L[f] = \partial_t f + \dot{x}^i \partial_{x^i} f + \dot{p}^i \partial_{p^i} f \quad \Longrightarrow \quad L[f] = \partial_t f + \dot{p} \partial_p f$$

If C=0 (and relativistic) : ansatz $f(t, E) = g(E/T(t))$

$$L[f] = 0 \quad \Longrightarrow \quad \left\{ \begin{array}{l} \dot{T} + HT = 0 \\ T = \dfrac{T_0}{a} \end{array} \right.$$

Fluid description vs kinetic theory

Isotropy in phase space

$$n = g\int f(p)\frac{\boxed{4\pi p^2\mathrm{d}p}}{(2\pi)^3}$$

$$\rho = g\int f(p)E\frac{4\pi p^2\mathrm{d}p}{(2\pi)^3}$$

$$P = g\int f(p)\frac{p^2}{3E}\frac{4\pi p^2\mathrm{d}p}{(2\pi)^3}$$

g is spin degrees of freedom (2s+1)

These integrals on L[f] = C[f]

- Number density evolution :

$$\dot{n} + 3Hn = \mathcal{J}\,, \qquad \mathcal{J} \equiv \int C[f]\,\frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}$$

Particle source

Particular case of number density dilution : $\dot{n} + 3Hn = 0 \quad \Rightarrow \quad \frac{\mathrm{d}(na^3)}{\mathrm{d}t} = 0$

- Energy density evolution :

$$\dot{\rho} + 3H(\rho + P) = \dot{q}\,, \qquad \dot{q} \equiv \int C[f]\,\frac{4\pi E p^2 \mathrm{d}p}{(2\pi)^3}$$

Energy source

Particular case of relativistic species without interactions $\dot{\rho} + 4H\rho = 0$

Statistical equilibrium distributions

- Early universe relativistic particles : $\gamma, \underbrace{e^{\pm}, \nu_\alpha, \bar{\nu}_\alpha}$

Bose-Einstein

$$g(E) = \frac{1}{\mathrm{e}^{E/T} - 1}$$

Fermi-Dirac

$$g(E) = \frac{1}{\mathrm{e}^{(E-\mu)/T} + 1}$$

1. Relativistic regime T>>m

$$n = \frac{gT^3}{2\pi^2} \begin{cases} 2\zeta(3) & \text{if bosons} \\ 3\zeta(3)/2 & \text{if fermions} \end{cases}$$

$$\rho = \frac{gT^4}{2\pi^2}\frac{\pi^4}{15} \left\{ \boxed{\begin{array}{l} 1 \\ 7/8 \end{array}} \quad \begin{array}{l} \text{if bosons} \\ \text{if fermions} \end{array} \right.$$

$$P = \rho/3$$

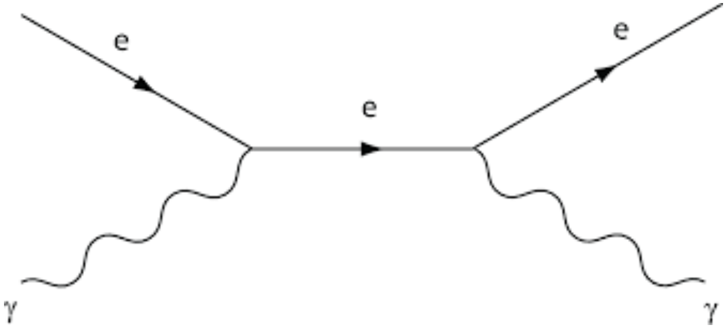
2. Non-relativistic regime T<<m

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} \mathrm{e}^{(\mu-m)/T}$$

$$\rho = (m + 3T/2)n$$

Statistical equilibrium enforced by strong collision term !

QED reactions :



- Momentum exchange :

$$\gamma + e^{\pm} \leftrightarrow \gamma + e^{\pm}$$

- Annihilations

$$\gamma + \gamma \leftrightarrow e^{\pm} + e^{\mp}$$

Collisions

$$\left\{ \begin{array}{l} \dot{\rho}_{e^{\pm}} + 3H(\rho_{e^{\pm}} + P_{e^{\pm}}) = \dot{q}_{e^{\pm}} \\ \dot{\rho}_{\gamma} + 4H(\rho_{\gamma}) = \dot{q}_{\gamma} \end{array} \right.$$

Temperature evolution, *method 1*

$$\dot{q}_\gamma + \dot{q}_{e^\pm} = 0$$

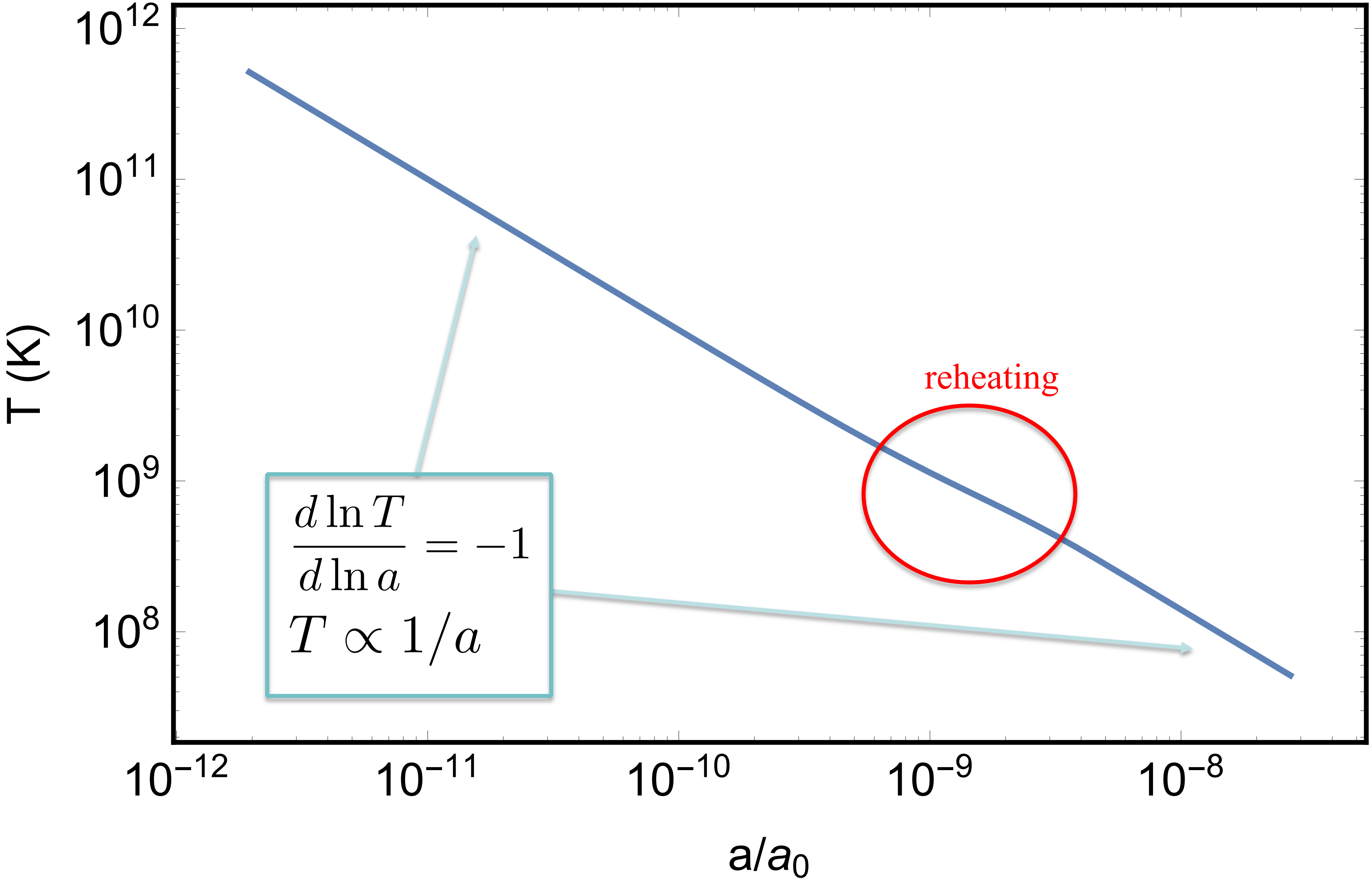
$$\boxed{\dot{\rho}_{e^\pm} + \dot{\rho}_\gamma + 3H(\rho_{e^\pm} + P_{e^\pm}) + 4H\rho_\gamma = 0}$$

1. Using that thermodynamic quantities are functions of temperature only

$$\dot{\rho} = \dot{T} d\rho(T)/dT$$

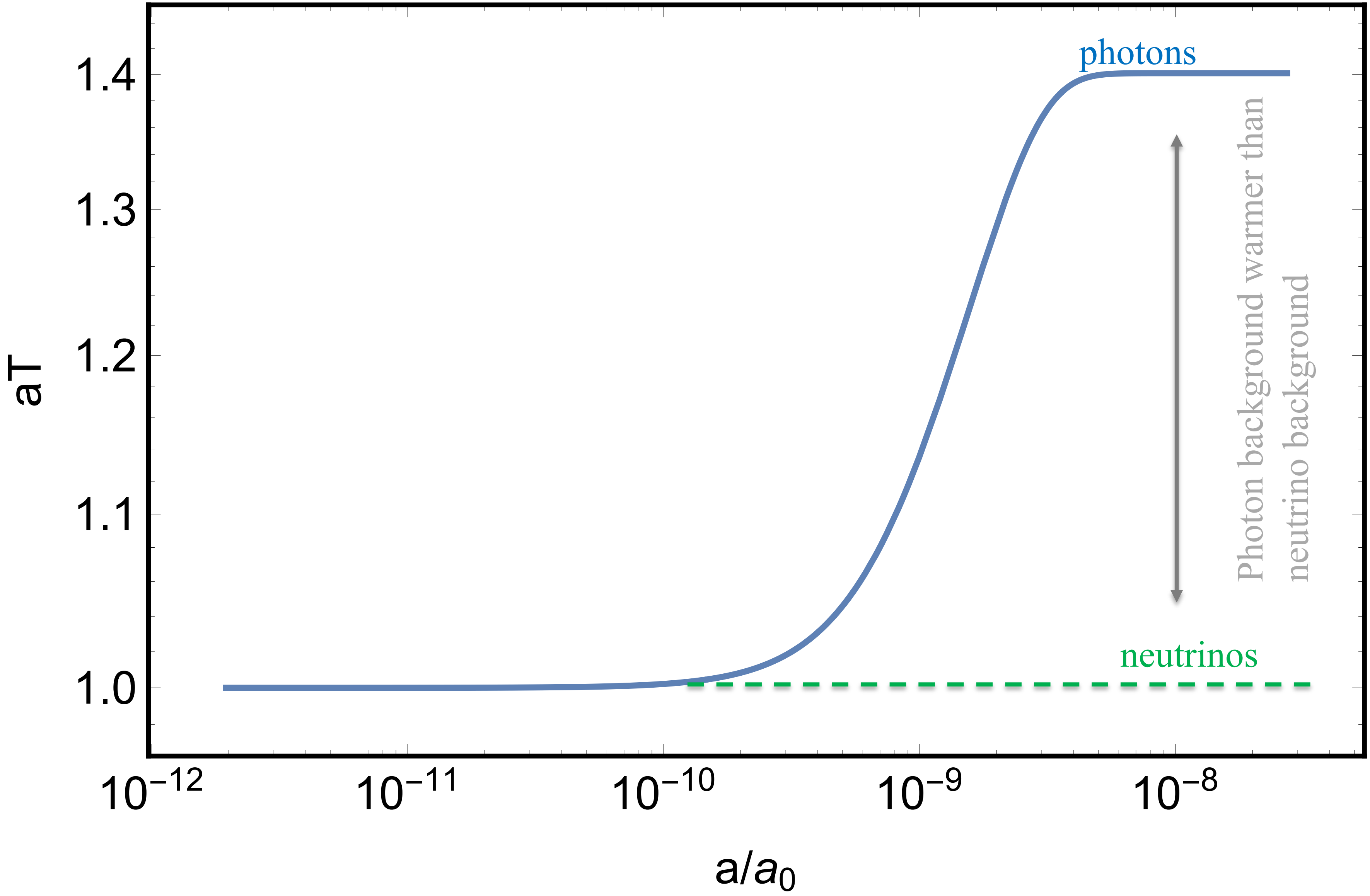
2. Trading time for scale factor with : $H = \frac{d\ln a}{dt}$

$$\boxed{\frac{d\ln T}{d\ln a} = -\frac{3[\rho_{e^\pm}(T) + P_{e^\pm}(T)] + 4\rho_\gamma(T)}{T[\rho'_{e^\pm}(T) + \rho'_\gamma(T)]}}$$



$$\frac{d \ln T}{d \ln a} = -1$$

$$T \propto 1/a$$


photons
Photon background warmer than
neutrino background
neutrinos
aT
1.4
1.3
1.2
1.1
1.0
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
a/a_0

Reheating, *method 2* (entropy conservation)

- Continuity equation

$$\mathrm{d}(\rho a^3) = -P\mathrm{d}a^3 + \delta q a^3 \quad \Longleftarrow \quad \mathrm{d}U = -P\mathrm{d}V + \delta Q$$

- Thermodynamic identity

$$\mathrm{d}(\rho a^3) = T\mathrm{d}(sa^3) - P\mathrm{d}(a^3) + \mu\mathrm{d}(na^3) \quad \Longleftarrow \quad \mathrm{d}U = T\mathrm{d}S - P\mathrm{d}V + \mu\mathrm{d}N$$

- Entropy evolution

$$T\mathrm{d}(sa^3) = -\mu\mathrm{d}(na^3) + \delta q a^3$$

- If no heating, and negligible chemical potential, then, rhs is 0 and we get entropy conservation :

$$\boxed{sa^3 = \mathrm{Cte}}$$

Entropy computation

- Gibbs free energy

$$G = U + PV - TS \Rightarrow \mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}P + \mu\mathrm{d}N \Rightarrow G = \mu(T,P)N$$

- Entropy

$$S = \frac{U + PV - \mu N}{T} \Rightarrow \boxed{s = \frac{\rho + P - \mu n}{T}}$$

- If negligible chemical potential and relativistic

$$\boxed{s = \frac{4}{3}\frac{\rho}{T}} \qquad s_{\mathrm{pl}} = \sum_{i=\gamma,e^+,e^-} s_i$$

Entropy dof

$$s_{\rm pl} = g \frac{2\pi^2}{45} T_\gamma^3$$

Early $g_{\rm early} = 2 + 2 \times \overset{e^{\pm}\text{ spins}}{2} \times \boxed{\frac{7}{8}}$ fermions

Today $g_0 = 2$

photon helicities

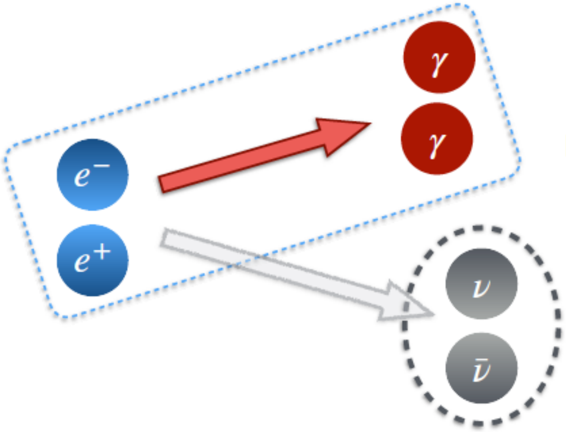
$$\left.\begin{array}{l} \dfrac{T^0_{\rm pl}}{T^{\rm early}_{\rm pl}} = \left(\dfrac{g_{\rm early}}{g_0}\right)^{1/3} \dfrac{a_{\rm early}}{a_0} = \left(\dfrac{11}{4}\right)^{1/3} \dfrac{a_{\rm early}}{a_0} \\[2ex] \dfrac{T^0_\nu}{T^{\rm early}_\nu} = \dfrac{a_{\rm early}}{a_0} \end{array}\right\} \boxed{\left(\frac{T_\gamma}{T_\nu}\right)_{\rm today} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.40102}$$

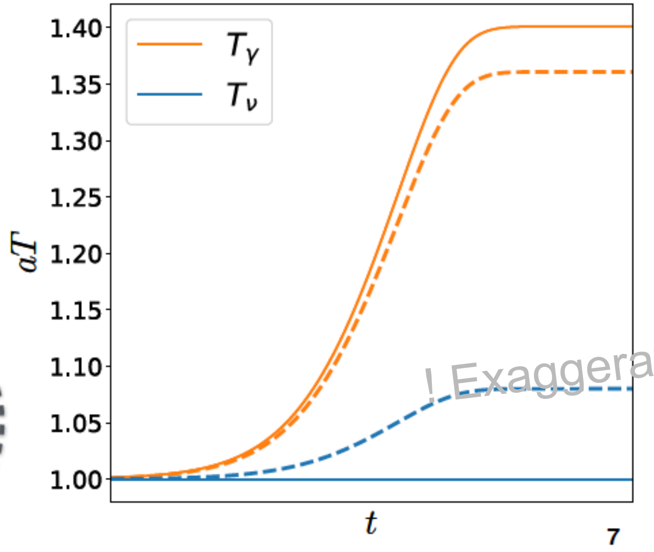
$$\boxed{\rho_{\rm rad} = \rho_\gamma \left[1 + 3 \times \frac{7}{8} \left(\frac{4}{11}\right)^{4/3}\right]}$$

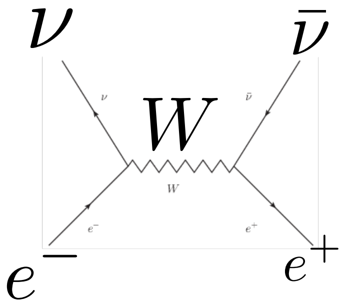
Beyond the instantaneous decoupling approximation

- Overlap between decoupling and $e^{\pm}$ annihilations

$\Longrightarrow$ smaller T_γ and increased T_ν







$$\rho_{\rm R} = \rho_\nu + \rho_\gamma = \rho_\gamma \left(1 + \frac{7}{8} N_{\rm eff} \left(\frac{4}{11}\right)^{4/3}\right)$$

definition

$$\boxed{N_{\rm eff} = 3.0440}$$

Outline :

1) Cosmology and plasma reheating
2) Weak interactions
3) Nuclear reactions
4) Observational constraints

- Early Universe ($T \gtrsim 1\mathrm{MeV}$):
  - Photons: γ
  - Leptons: $e^+, e^-, \nu_e, \nu_\mu, \nu_\tau, \overline{\nu}_e, \overline{\nu}_\mu, \overline{\nu}_\tau$
  - Baryons: n, p

$$\boxed{\eta = \frac{n_b}{n_\gamma} \sim 6 \times 10^{-10}}$$

baryon-to-photon ratio

When computing time, scale factor, and plasma temperature, we can ignore baryons

But we must look into baryon abundance for the nucleosynthesis.

How many protons and neutrons are available ?

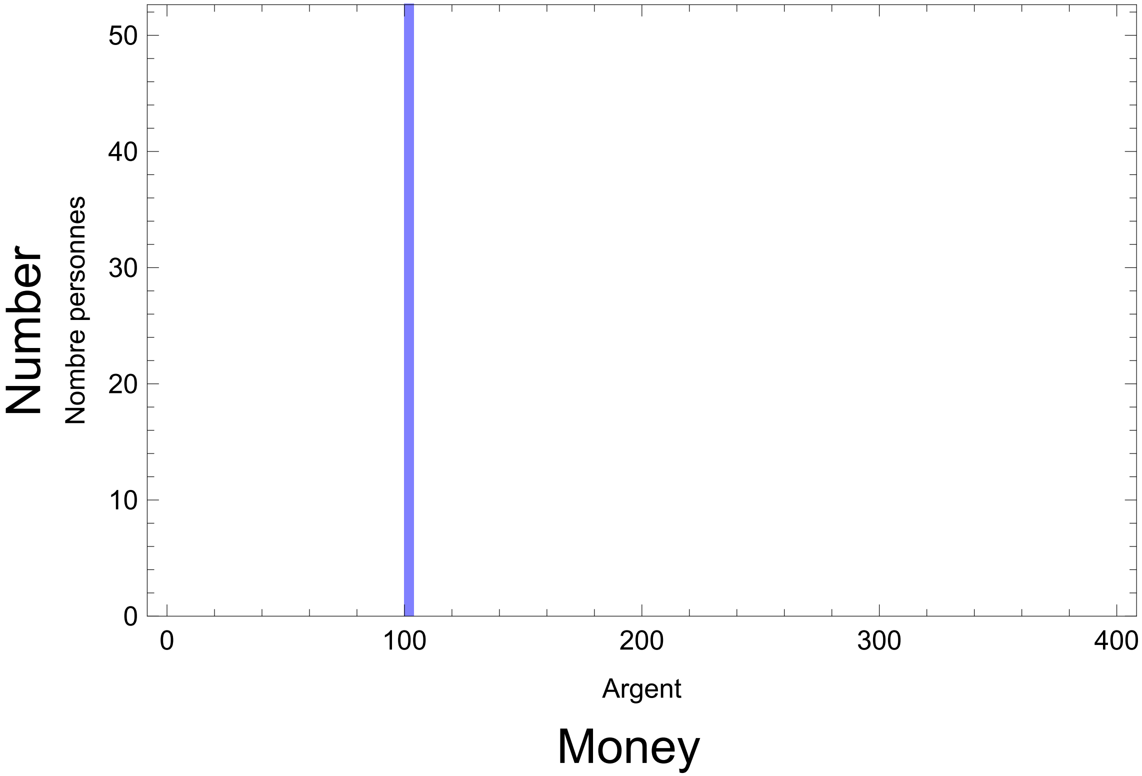
Statistical physics oversimplified

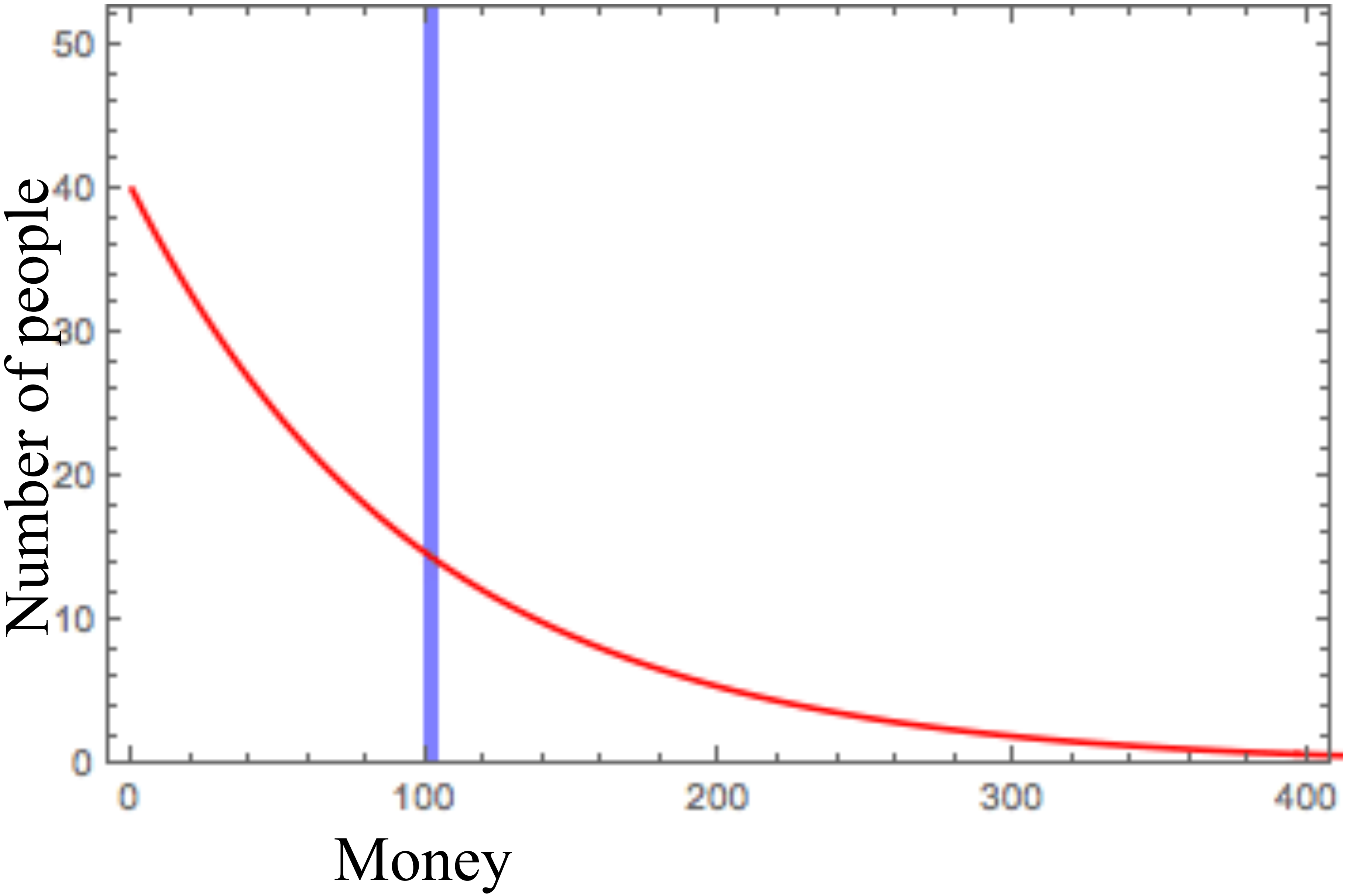
Social experience

1) Give 100 EUR to N persons

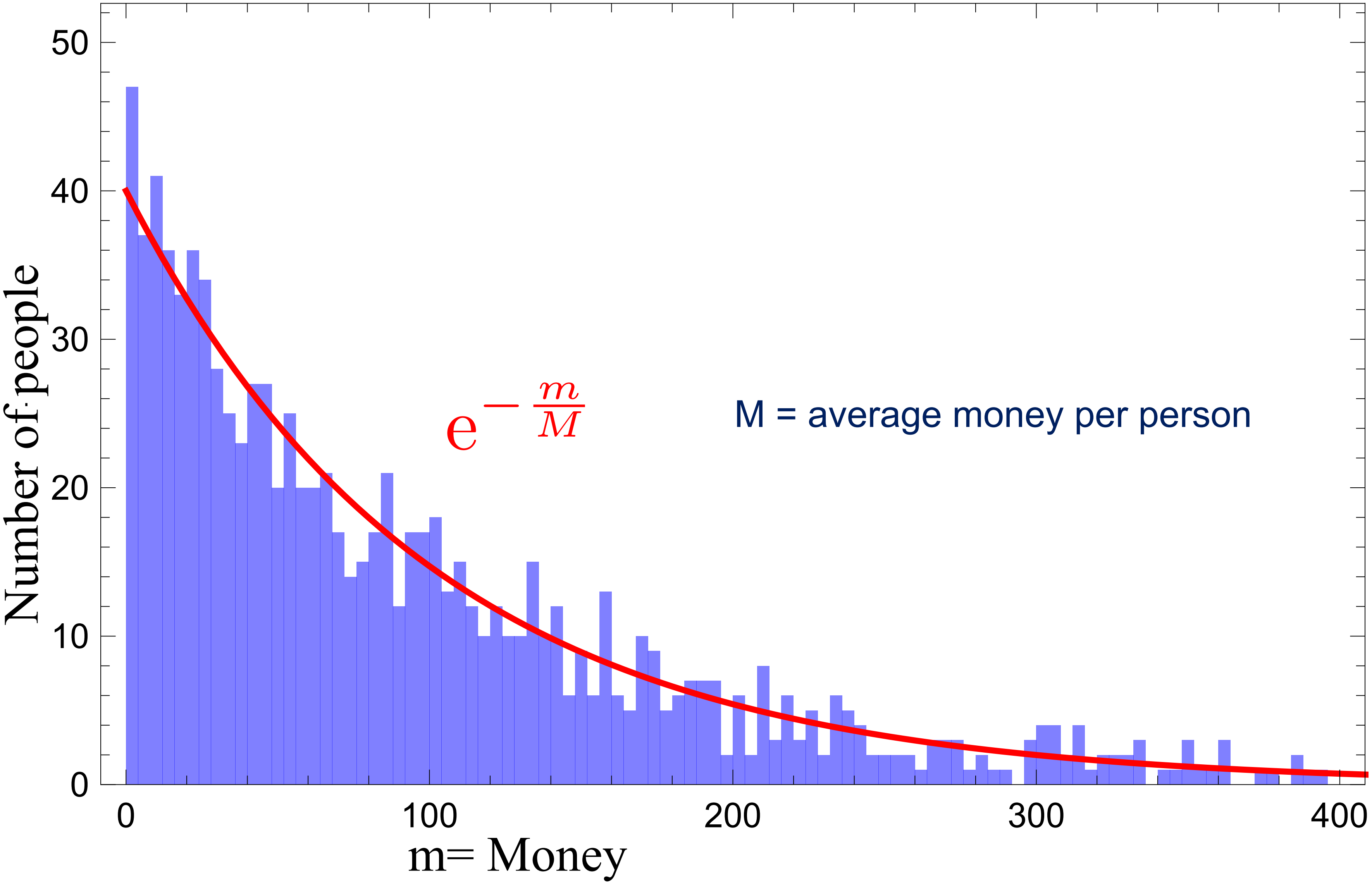
2) Let them exchange freely

3) Look at the results




Number of people
Money
0
10
20
30
40
50
0
100
200
300
400

Boltzmann factor

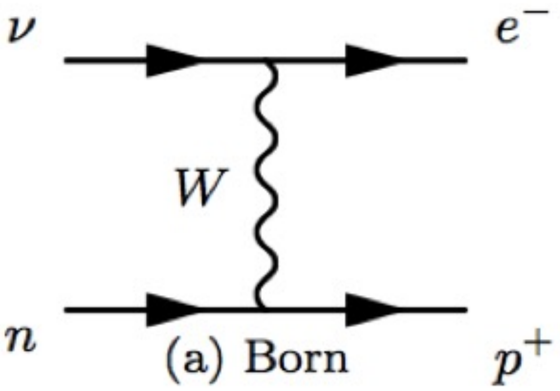


Weak interactions

$$n + \nu_e \longleftrightarrow p + e^-$$

$$n \longleftrightarrow p + e^- + \bar{\nu}_e$$

$$n + e^+ \longleftrightarrow p + \bar{\nu}_e$$



(a) Born

- If enough interactions, then statistical equilibrium

$$\boxed{n = \mathrm{e}^{-\frac{E}{k_B T}}}$$

Baryons are non-relativistic : $E \simeq m$

$$m_p = 938.2\,\mathrm{MeV}$$

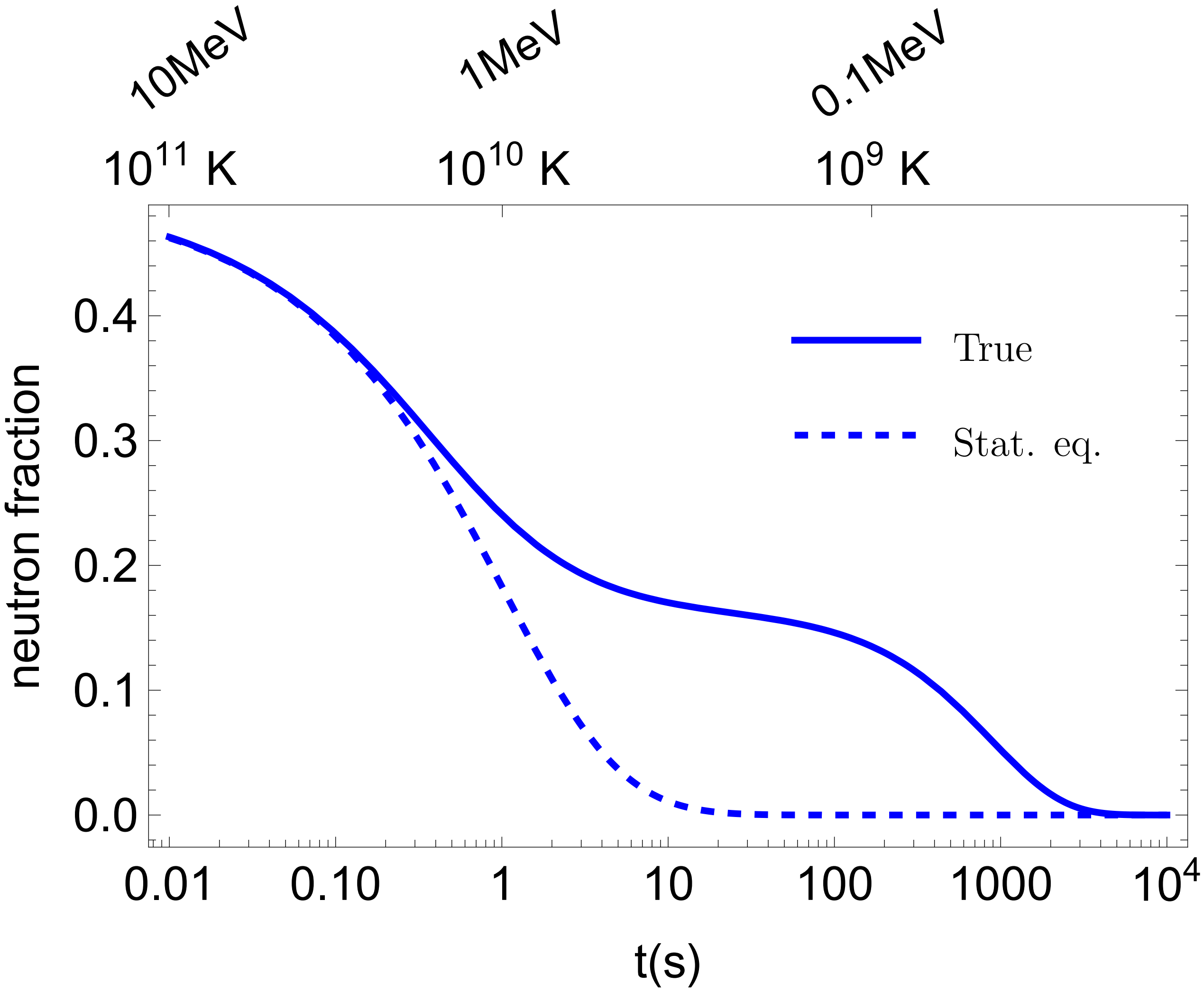
$$m_n = m_p + 1.3\,\mathrm{MeV}$$

Protons $$n_p = \mathrm{e}^{-m_p/T}$$

Neutrons $$n_n = \mathrm{e}^{-m_n/T} = n_p \boxed{\mathrm{e}^{-(m_n - m_p)/T}}$$

Evolution of neutron fraction

$$X_n = \frac{n_n}{n_n + n_p}$$



General expression of weak rates

$$\begin{aligned}\dot{n}_n + 3Hn_n &= -n_n\Gamma_{n\to p} + n_p\Gamma_{p\to n}\\ \dot{n}_p + 3Hn_p &= -n_p\Gamma_{p\to n} + n_n\Gamma_{n\to p}\end{aligned}$$

Weak interaction Matrix element

$$n_n\Gamma = \int \Pi_i[\mathrm{d}^3\mathbf{p}_i](2\pi)^4\delta^4\left(\underline{p}_n - \underline{p}_p + \alpha_\nu\underline{p}_\nu + \alpha_e\underline{p}_e\right)|M|^2 f_n(E_n)[1-f_p(E_p)]f_\nu(\alpha_\nu E_\nu)f_e(\alpha_e E_e)$$

momentum conservation

isotropy

$$[\mathrm{d}^3\mathbf{p}] \equiv \frac{\mathrm{d}^3\mathbf{p}}{2E(2\pi)^3} = \frac{4\pi p^2\mathrm{d}p}{2E(2\pi^3)}$$

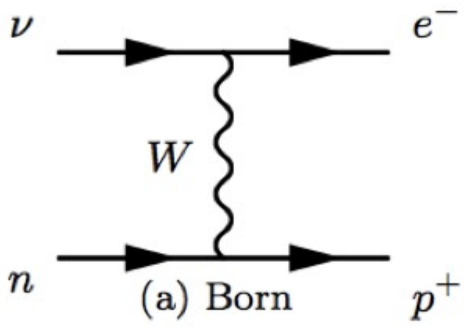
$$\begin{cases}\alpha_i = 1 & \text{if initial particle}\\ \alpha_i = -1 & \text{if final particle}\end{cases}$$

$$g(-E) = 1 - g(E)$$

Fermi-Dirac property

Interaction Hamiltonian

$$\boxed{\mathcal{H}_I = \frac{G_F}{\sqrt{2}} J^{\mu}_{e\nu} J_{pn,\,\mu}}$$



(a) Born

$$J^{\mu}_{e\nu} = \bar{\boldsymbol{\nu}}\gamma^{\mu}(1-\gamma^5)\mathbf{e}$$

$$J^{\mu}_{pn} = V_{ud}\bar{\mathbf{p}}\gamma^{\mu}(1-\boxed{g_A}\gamma^5)\mathbf{n}$$

CKM angle

Axial current coupling

Matrix element

$$\frac{|M|^2}{2^7 G_F^2} = c_{LL}\mathcal{M}_{LL} + c_{RR}\mathcal{M}_{RR} + c_{LR}\mathcal{M}_{LR}$$

$$\begin{aligned} c_{LL} &\equiv \frac{(1+g_A)^2}{4} \\ c_{RR} &\equiv \frac{(1-g_A)^2}{4} \\ c_{LR} &\equiv \frac{g_A^2-1}{4} \, . \end{aligned}$$

$$\boxed{\begin{aligned} \mathcal{M}_{LL} &= (\underline{p}_n \cdot \underline{p}_\nu)(\underline{p}_p \cdot \underline{p}_e) \\ \mathcal{M}_{RR} &= (\underline{p}_n \cdot \underline{p}_e)(\underline{p}_p \cdot \underline{p}_\nu) \\ \mathcal{M}_{LR} &= m_p m_n (\underline{p}_\nu \cdot \underline{p}_e) \, . \end{aligned}}$$

BORN approximation method

$$\Delta = m_n - m_p.$$

$$\boxed{E_n - E_p = \Delta + \underbrace{\delta Q_1 + \delta Q_2 + \delta Q_3}_{\text{Nucleon recoil corrections}}}$$

$$\begin{aligned}
\delta Q_1 &\equiv -\frac{\mathbf{p}_n \cdot \mathbf{q}}{m_N} \\
\delta Q_2 &\equiv -\frac{|\mathbf{q}|^2}{2m_N} \\
\delta Q_3 &\equiv \frac{|\mathbf{p}_n|^2}{2}\left(\frac{1}{m_n} - \frac{1}{m_p}\right) \simeq -\frac{|\mathbf{p}_n|^2 \Delta}{2m_N^2}
\end{aligned}$$

$$\mathbf{q} \equiv \mathbf{p}_p - \mathbf{p}_n = \alpha_\nu \mathbf{p}_\nu + \alpha_e \mathbf{p}_e$$

BORN approximation : Dirac expansion

$$n_n\Gamma = \int \frac{\mathrm{d}^3\mathbf{p}_n\mathrm{d}^3\mathbf{p}_e\mathrm{d}^3\mathbf{p}_\nu}{2^4(2\pi)^8}\,\boxed{\delta\left(E_n - E_p + \alpha_e E_e + \alpha_\nu E_\nu\right)}\,\boxed{\frac{|M|^2}{E_nE_pE_eE_\nu}}\,f_n(E_n)f_\nu(\alpha_\nu E_\nu)f_e(\alpha_e E_e)$$

Born order

Finite nucleon mass corrections

$$\delta\left(E_n - E_p + \alpha_e E_e + \alpha_\nu E_\nu\right) \simeq \boxed{\delta(\Sigma)} + \boxed{\delta'(\Sigma)\left(\sum_{i=1}^{3}\delta Q_i\right) + \frac{1}{2}\delta''(\Sigma)(\delta Q_1)^2}$$

$$\Sigma \equiv \Delta + \alpha_e E_e + \alpha_\nu E_\nu$$

Born order

Corrections

$$\begin{aligned}
\frac{\mathcal{M}_{LL}}{\Pi_i E_i} &\to 1 - \frac{\mathbf{p}_n}{m_N}\cdot\left(\frac{\mathbf{p}_e}{E_e} + \frac{\mathbf{p}_\nu}{E_\nu}\right) - \frac{\alpha_\nu|\mathbf{p}_\nu|^2}{m_N E_\nu} \\
\frac{\mathcal{M}_{RR}}{\Pi_i E_i} &\to 1 - \frac{\mathbf{p}_n}{m_N}\cdot\left(\frac{\mathbf{p}_e}{E_e} + \frac{\mathbf{p}_\nu}{E_\nu}\right) - \frac{\alpha_e|\mathbf{p}_e|^2}{m_N E_e} \\
\frac{\mathcal{M}_{LR}}{\Pi_i E_i} &\to \left(1 - \frac{|\mathbf{p}_n|^2}{m_N^2}\right)\left(1 - \frac{\mathbf{p}_e\cdot\mathbf{p}_\nu}{E_eE_\nu}\right).
\end{aligned}$$

BORN approximation

$$\boxed{\Gamma_{n\to p} = K \int_0^\infty p^2 \mathrm{d}p [\chi(E) + \chi(-E)]}$$

Product of Fermi-Dirac distributions

$$\chi(E) = E_\nu^2 g_\nu(E_\nu) g(-E)$$

Neutrino energy

$$E_\nu = E - \Delta$$

Preconstant

$$K = \frac{4G_F^2 V_{ud}^2 (1 + 3g_A^2)}{(2\pi)^3}$$

Neutron lifetime strikes again !

$$\frac{1}{\tau_n} = \Gamma_{n\to p}(T=0) = K \int_0^{\sqrt{\Delta^2 - m_e^2}} p^2 E_\nu^2 \mathrm{d}p$$

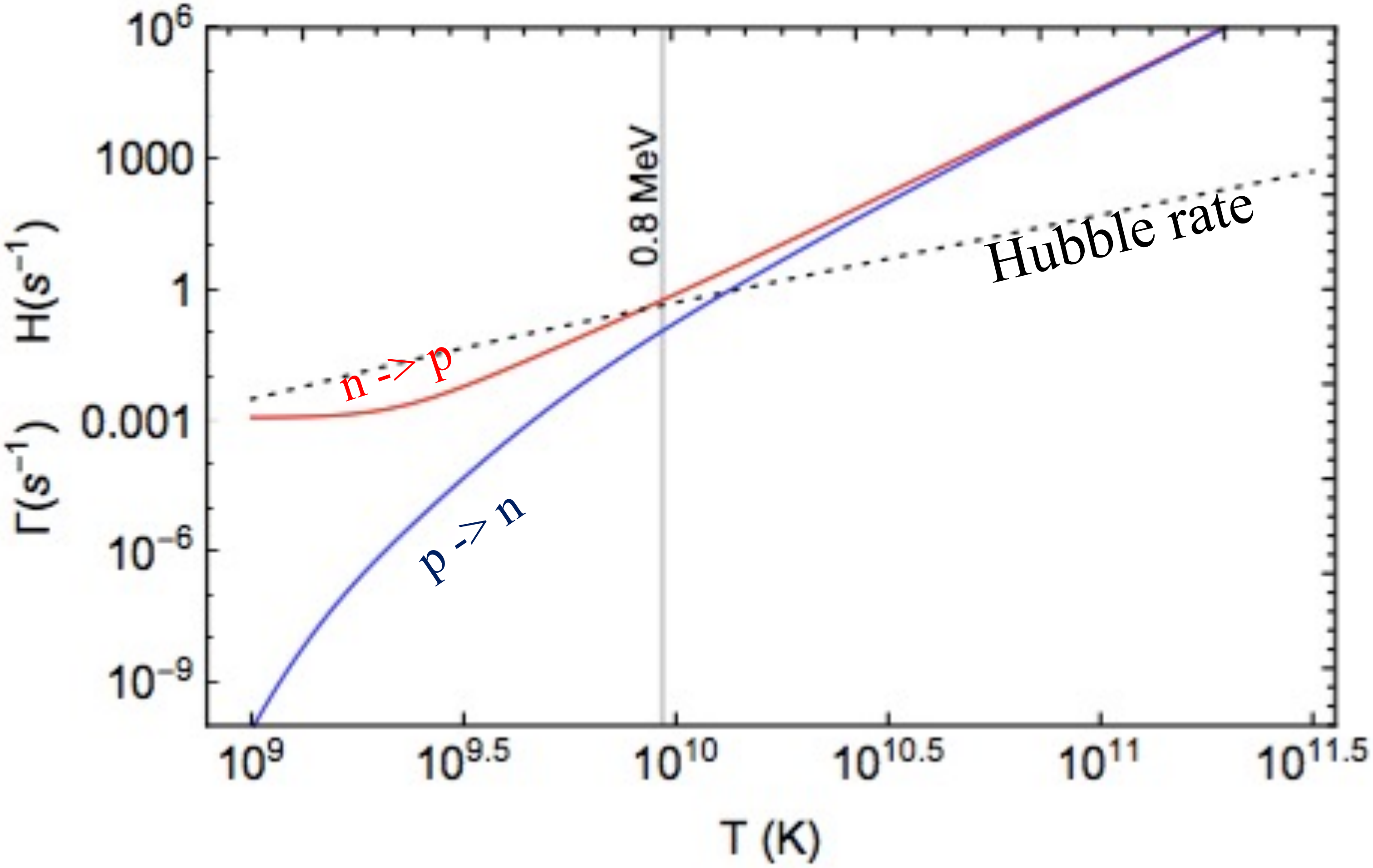
$$K = 1/(\tau_n \lambda_0 m_e^5)$$

$$\lambda_0 \simeq 1.75474$$

Instead of using the measured values of V_{ud} and g_A, we use neutron lifetime

$$\boxed{\tau_n = 878.4 \pm 0.5\,\mathrm{s}}$$

BORN approximation rates vs Hubble rate



Equilibrium abundances

$$\begin{aligned} \dot{n}_n + 3Hn_n &= \\ \dot{n}_p + 3Hn_p &= \end{aligned} \boxed{\begin{aligned} -n_n\Gamma_{n\to p} + n_p\Gamma_{p\to n} \\ -n_p\Gamma_{p\to n} + n_n\Gamma_{n\to p} \end{aligned}} = 0$$

$$\boxed{\frac{\Gamma_{p\to n}}{\Gamma_{p\to n}} = \mathrm{e}^{-\Delta/T}} = \frac{n_n}{n_p}$$

Evaluating the freeze-out temperature

- Proton and neutron fractions :

$$X_n = \frac{n_n}{n_n + n_p} \qquad X_p = \frac{n_p}{n_n + n_p}$$

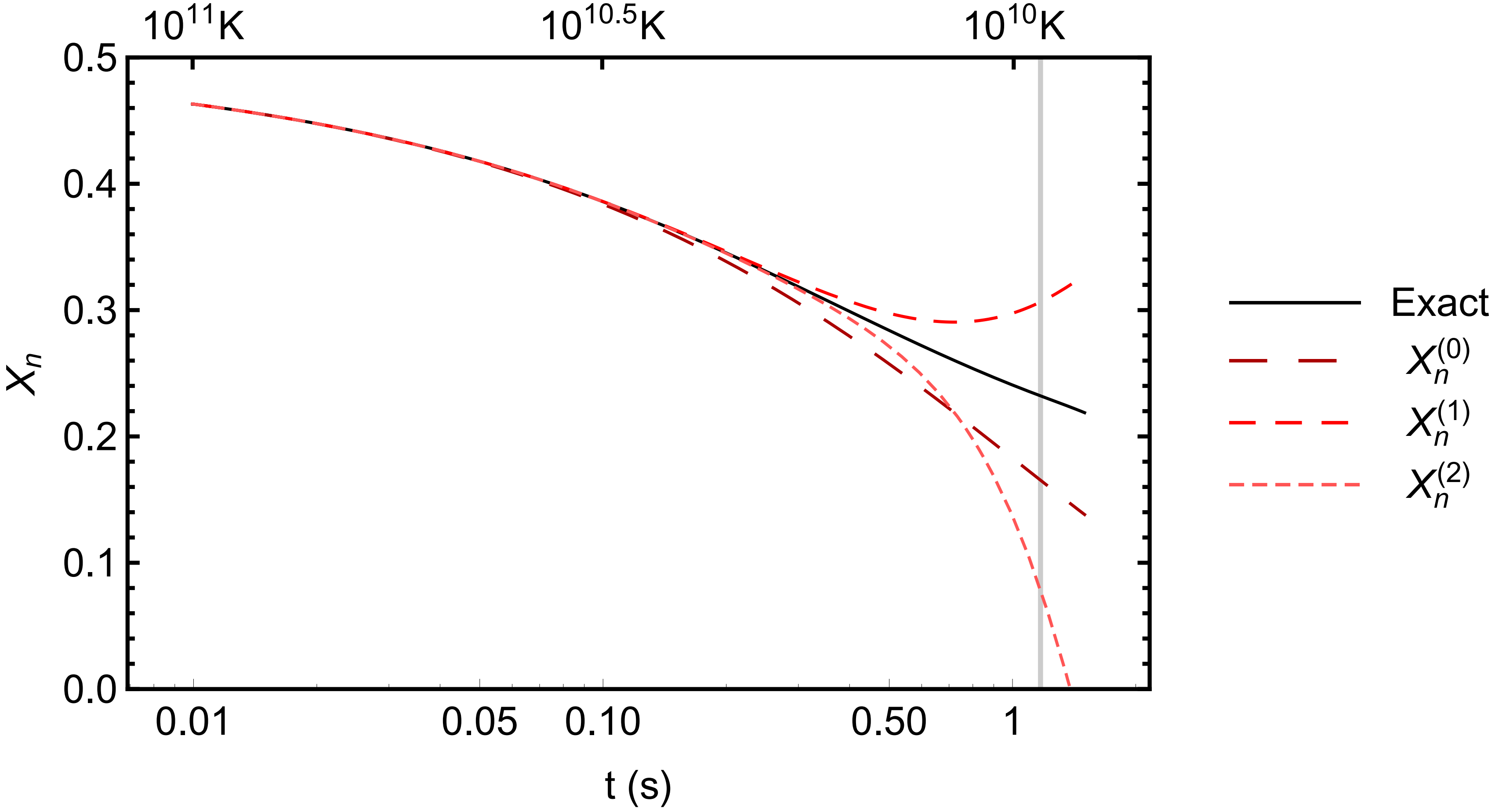
Neutron fraction evolution

$$\dot{X}_n = X_p \Gamma_{p\to n} - X_n \Gamma_{n\to p}$$

- Tight-coupling (aka large interaction) expansion

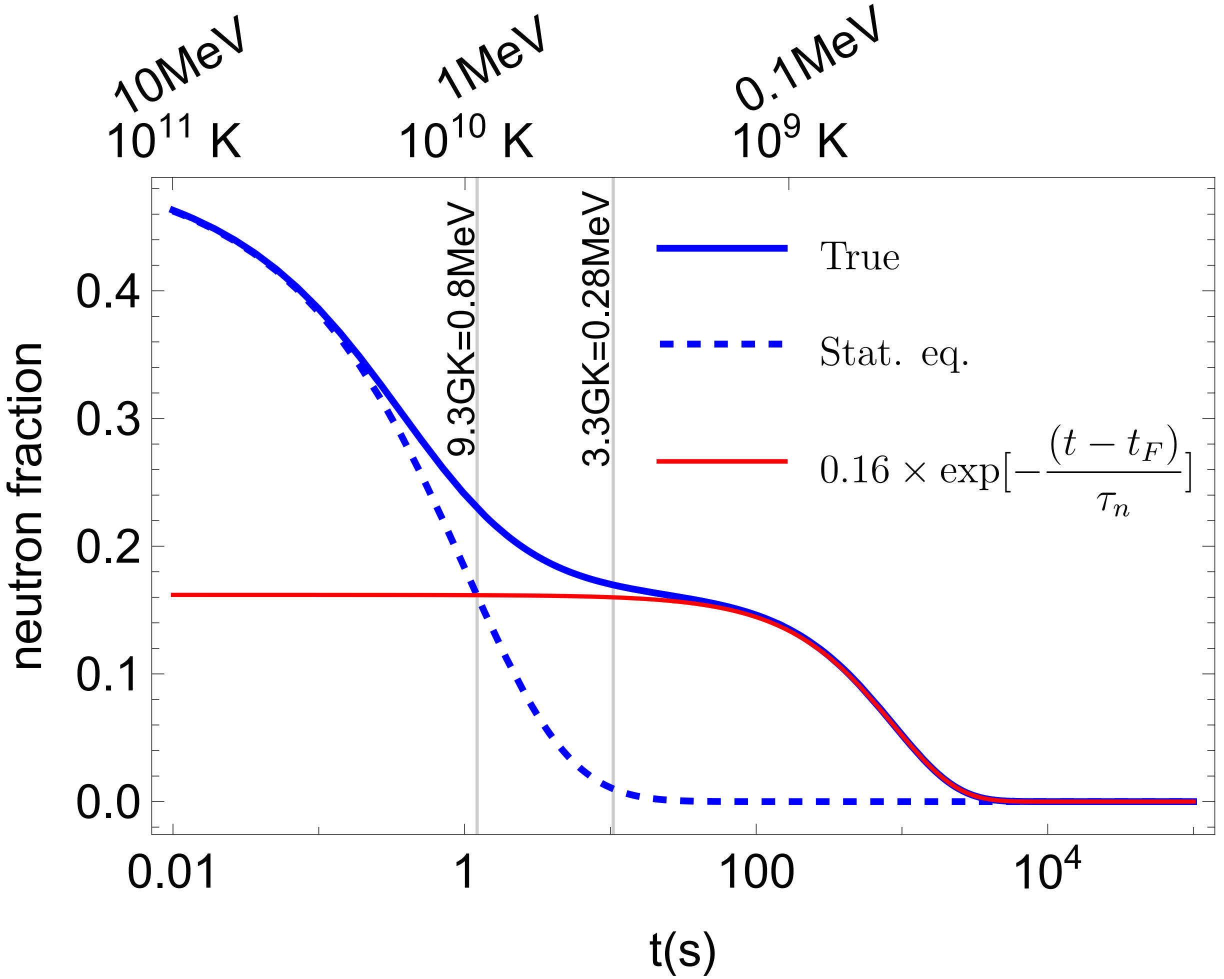
$$X_n = \frac{\Gamma_{p\to n}}{\Lambda} - \frac{1}{\Lambda}\dot{X}_n\,, \qquad \Lambda \equiv \Gamma_{p\to n} + \Gamma_{n\to p}$$

$$X_n^{(m)} = \sum_{k=0}^{m} \left(\frac{-1}{\Lambda}\frac{\mathrm{d}}{\mathrm{d}t}\right)^k \frac{\Gamma_{p\to n}}{\Lambda}$$



Gives freeze-out at T ~ 0.8 MeV

Evolution of neutrons



- When deuterium synthesis starts, we have around

$$X_n(T_{\rm Nuc}) = X_n^F \exp\left(-\frac{t_{\rm nuc} - t_F}{\tau_n}\right)$$

$$X_n^F = 0.16$$

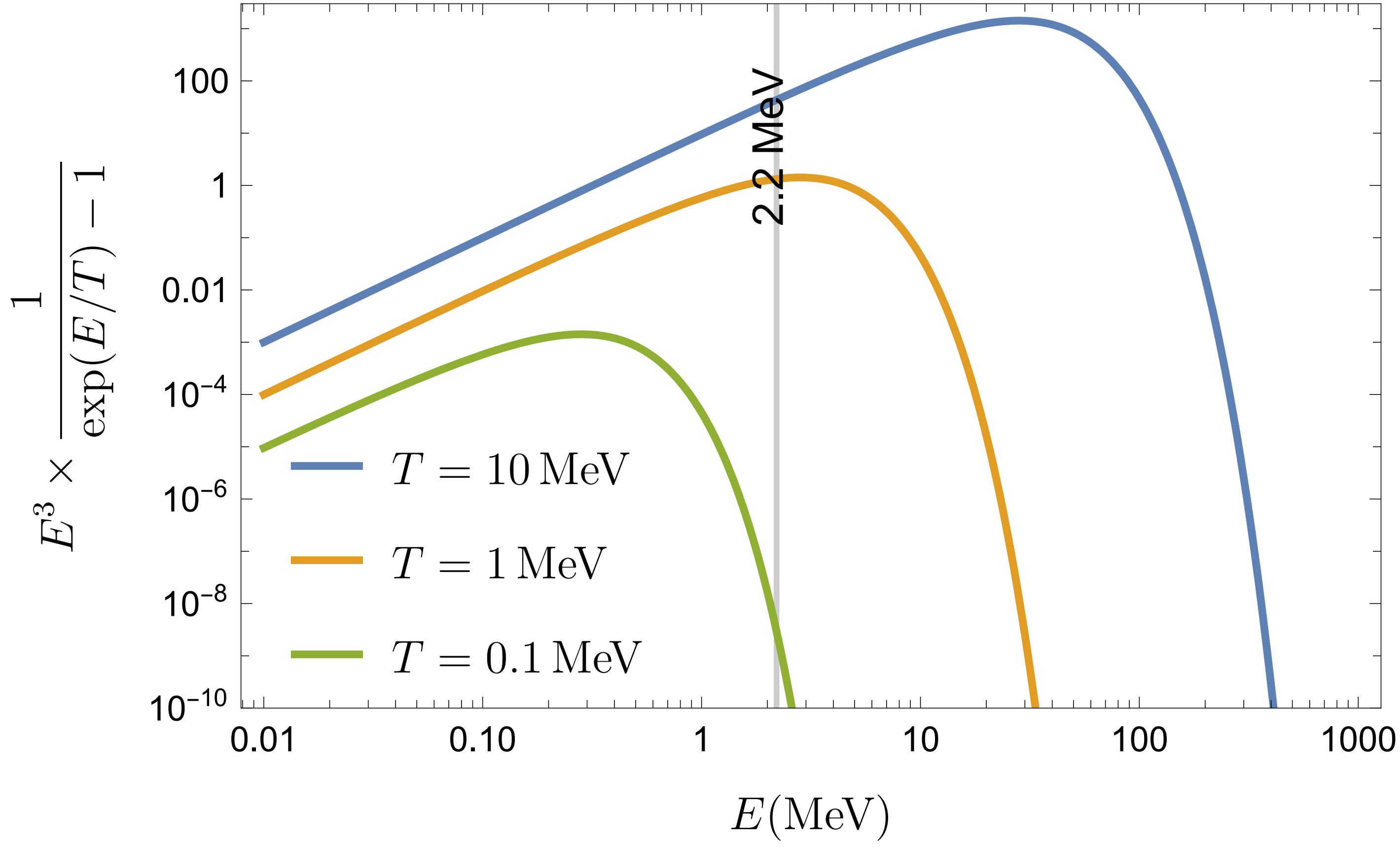
$$t_F = 10\,\mathrm{s}$$

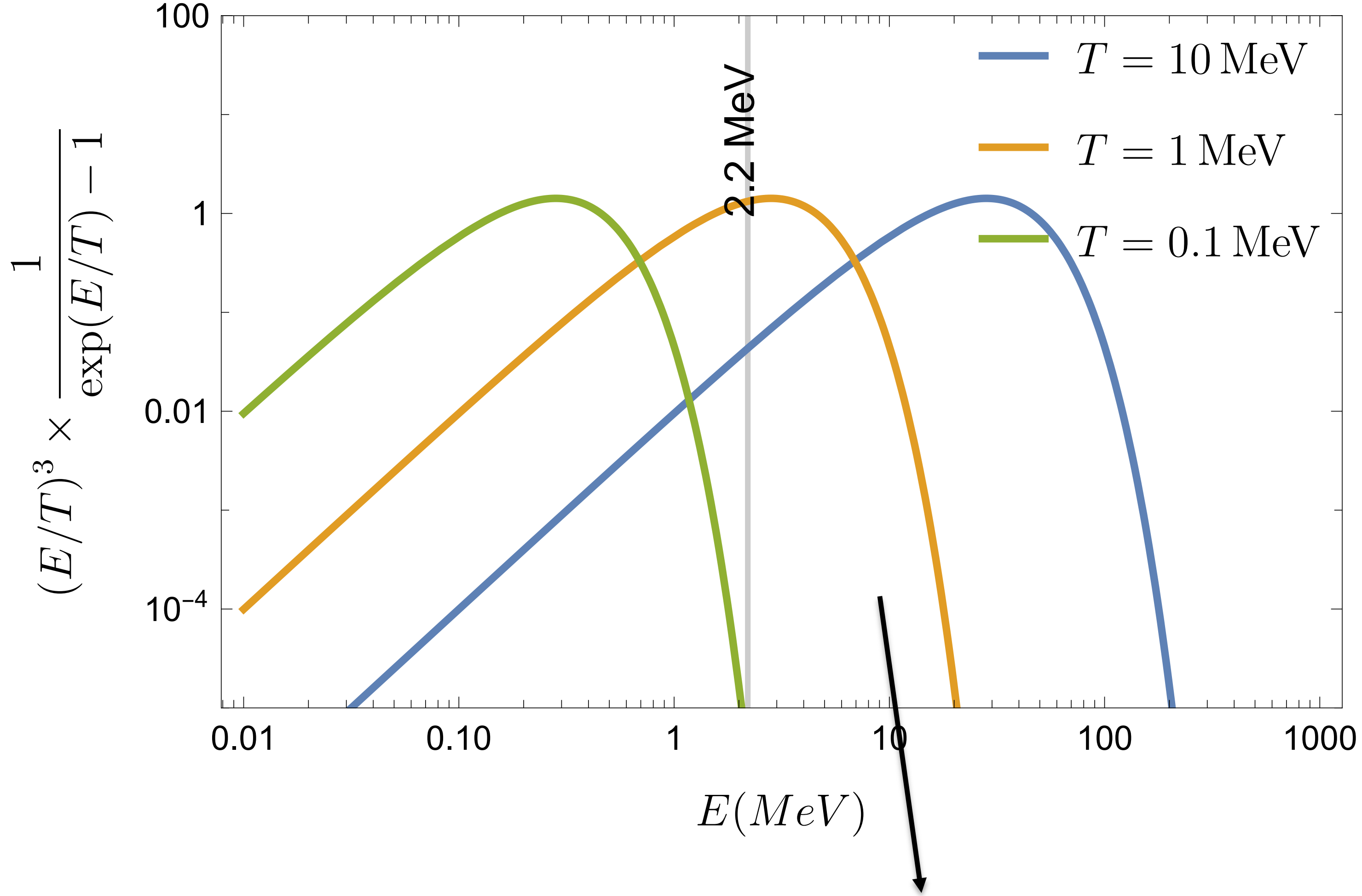
Outline :

1) Cosmology and plasma reheating
2) Weak interactions
3) Nuclear reactions
4) Observational constraints

Deuterium synthesis

$$p \quad + \quad n \quad \leftrightarrow \quad D \quad + \quad \overset{\boxed{2.2\text{ MeV}}}{\gamma}$$





Fraction above 2.2 MeV is ~ $\exp(-2.2\,\mathrm{MeV}/T)$

Hand-waving argument :

$$\boxed{B_D = 2.2\,\mathrm{MeV}}$$

Photons are super numerous

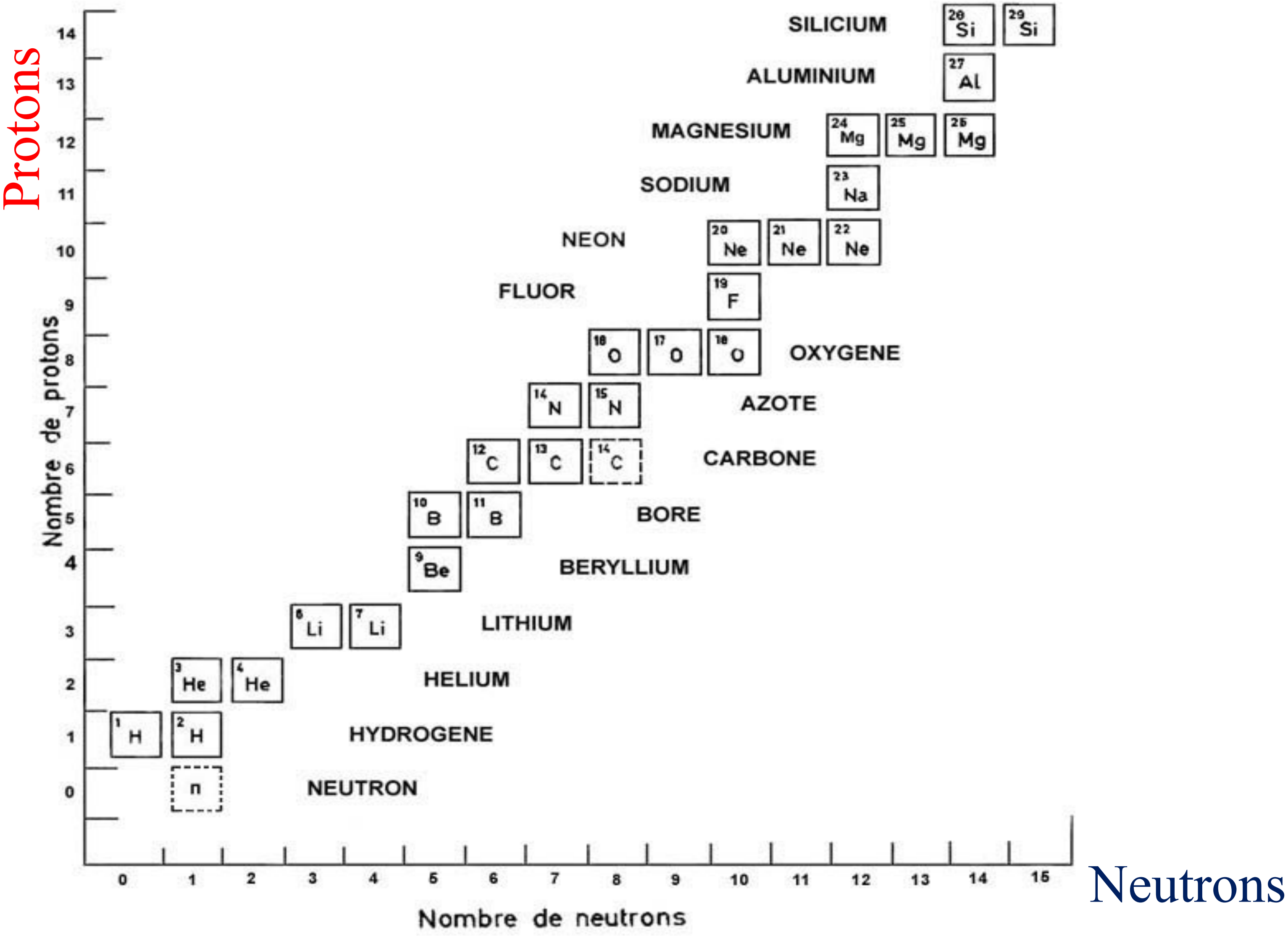
$$\eta = \frac{n_b}{n_\gamma} \simeq 6 \times 10^{-10} \simeq \mathrm{e}^{-21}$$

Synthesis should start when

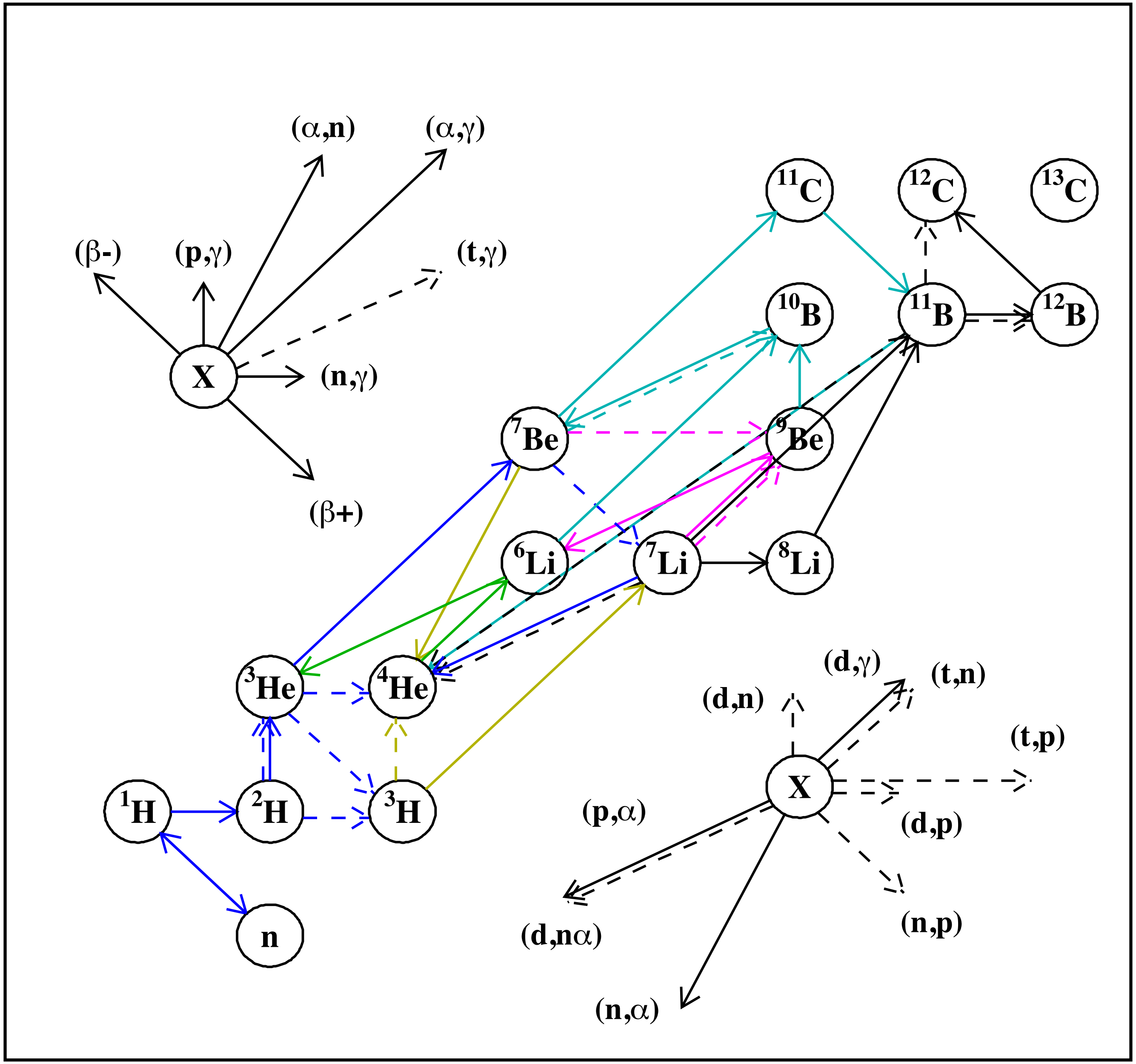
$$n_\gamma \mathrm{e}^{-B_D/T} \simeq n_b \quad \Rightarrow \quad \eta = \mathrm{e}^{-B_D/T_{\mathrm{nuc}}}$$

$$\boxed{T_{\mathrm{nuc}} = \frac{2.2\,\mathrm{MeV}}{21} \simeq 0.1\,\mathrm{MeV}}$$

Representation Protons vs Neutrons



Nuclear network



Small network

$$\boxed{\begin{gathered} d + p \leftrightarrow {}^3\mathrm{He} + \gamma \\ d + d \leftrightarrow {}^3\mathrm{He} + n \\ d + d \leftrightarrow {}^3\mathrm{H} + p \\ {}^3\mathrm{H} + p \leftrightarrow {}^4\mathrm{He} + \gamma \\ {}^3\mathrm{H} + d \leftrightarrow {}^4\mathrm{He} + n \end{gathered}}$$

$${}^3\mathrm{H} + {}^4\mathrm{He} \leftrightarrow {}^7\mathrm{Li} + \gamma$$

$${}^3\mathrm{He} + {}^4\mathrm{He} \leftrightarrow {}^7\mathrm{Be} + \gamma$$

$${}^7\mathrm{Be} + n \leftrightarrow {}^7\mathrm{Li} + p$$

$${}^7\mathrm{Li} + p \leftrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

Evolution of abundances

$$\frac{\mathrm{d}n_i}{\mathrm{d}t}+3Hn_i=\mathcal{J}_i$$ → Source from nuclear reactions

$$\frac{\mathrm{d}n_\mathrm{b}}{\mathrm{d}t}+3Hn_\mathrm{b}=0,$$ Baryons are only diluted

$$X_i=n_i/n_b$$ → Removes dilution

- Two-body reactions of the type $i+j\leftrightarrow k+l$

$$\boxed{\dot{X}_i=\sum_{jkl}\Gamma_{kl\to ij}X_kX_l-\Gamma_{ij\to kl}X_iX_j}$$

Rates

$$\Gamma_{ij\to kl} = n_b\langle\sigma v\rangle_{ij\to kl}$$

Average of cross-section over
Maxwell-Boltzmann distribution

$$\langle\sigma v\rangle = \int_0^\infty \sigma(v)v\,p_{\mathrm{MB}}(v,T)\mathrm{d}v$$

$$\Big\downarrow$$

$$p_{\mathrm{MB}}(v,T) = \frac{1}{(2\pi T/m)^{3/2}}\mathrm{e}^{-\frac{mv^2}{2T}}4\pi v^2$$

Setting initial conditions

At nuclear statistical equilibrium (NSE), the densities of nuclei satisfy

$$n_i^{\mathrm{NSE}} = \frac{g_i m_i^{3/2}}{2^{A_i}} \left(\frac{n_p}{m_p^{3/2}}\right)^{Z_i} \left(\frac{n_n}{m_n^{3/2}}\right)^{A_i-Z_i} \left(\frac{2\pi}{T}\right)^{\frac{3(A_i-1)}{2}} \mathrm{e}^{B_i/T}$$

Number protons

Number neutrons

Binding energy

$$B_i \equiv Z_i m_p + (A_i - Z_i) m_n - m_i$$

Hints of demo :

- Non-relativistic number density $n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \mathrm{e}^{(\mu_i - m_i)/T}$
- Chemical equilibrium $\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n$

Improvement of hand-waving

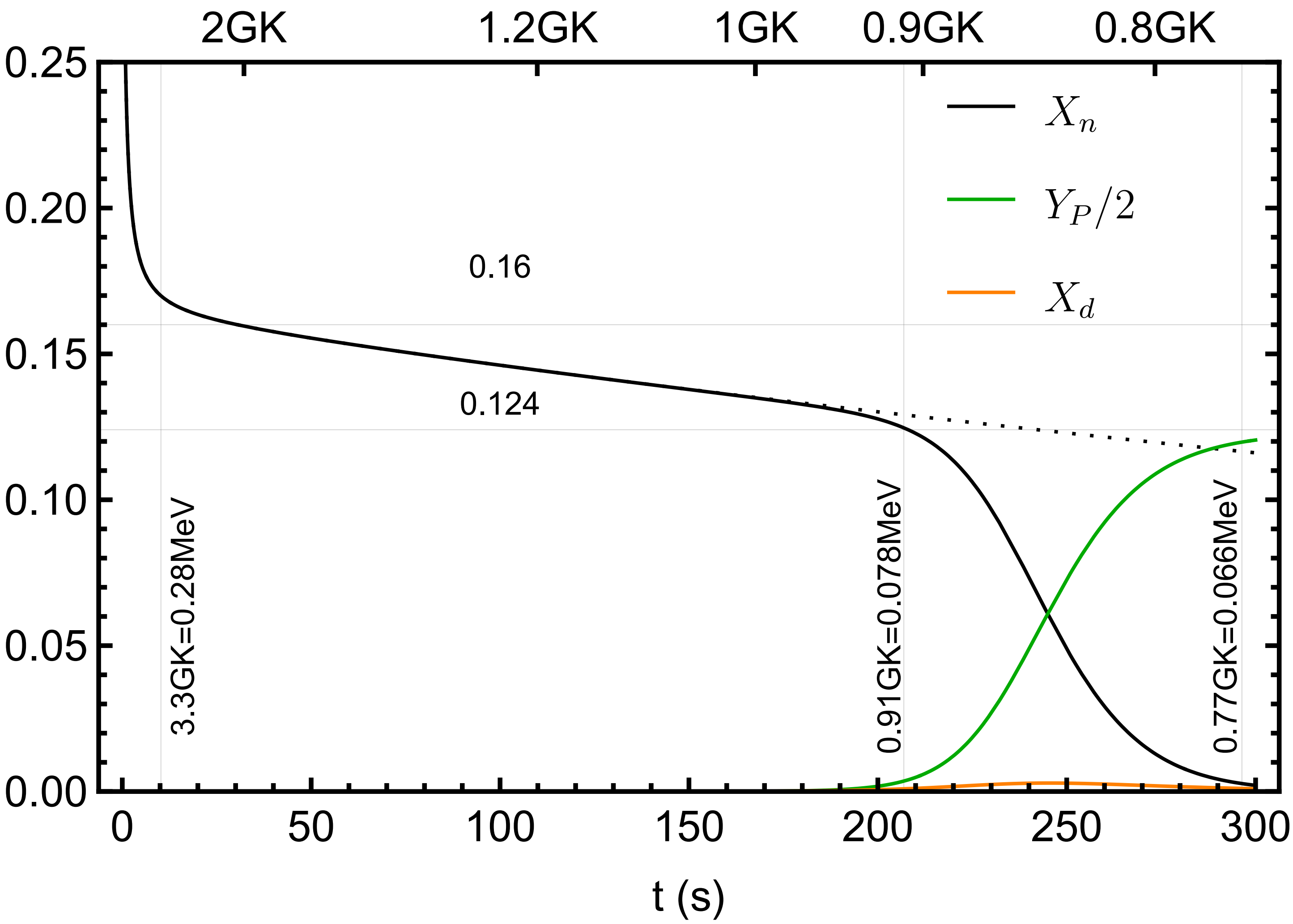
Saha equilibrium for deuterium $$n_d \simeq \frac{n_p n_n}{(m_b T)^{3/2}} \mathrm{e}^{B_D/T}$$

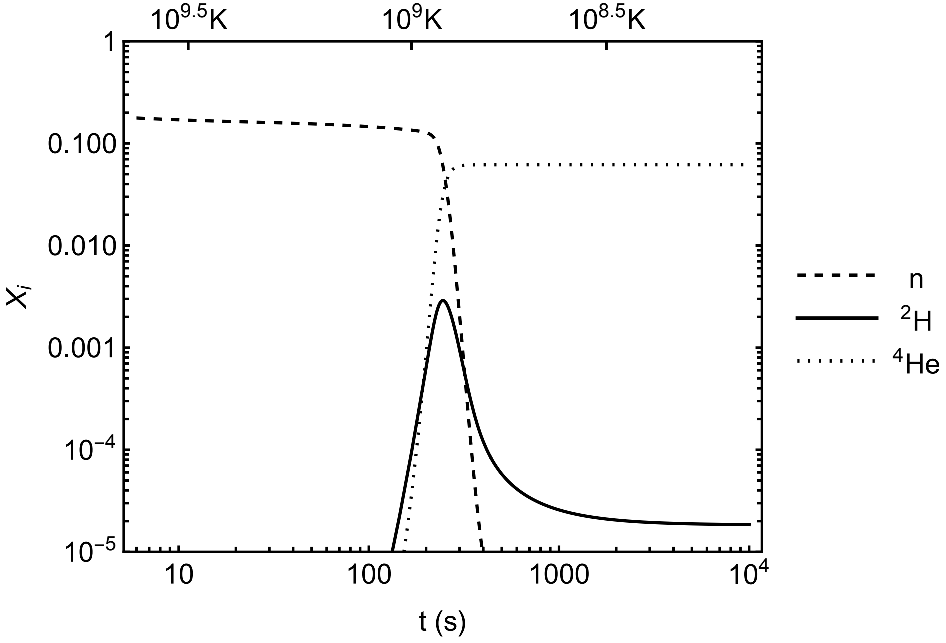
When produced substantially $$n_d \simeq n_n \simeq n_p \simeq \eta T^3$$

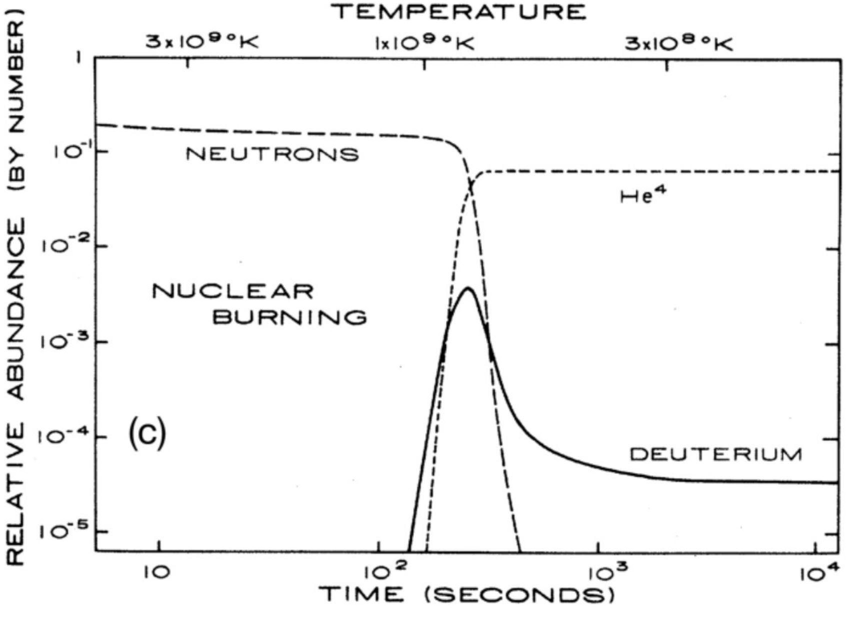
$$\mathrm{e}^{-B_D/T_{\rm nuc}} \simeq \eta (T_{\rm nuc}/m_b)^{3/2} \simeq 6 \times 10^{-10} \times 10^{-6}$$

⇨ $B_D/T_{\rm nuc} = 35$ ⇨ $T_{\rm nuc} = 0.065$ MeV

$$\boxed{Y_P = 4X_{\mathrm{He}}}$$







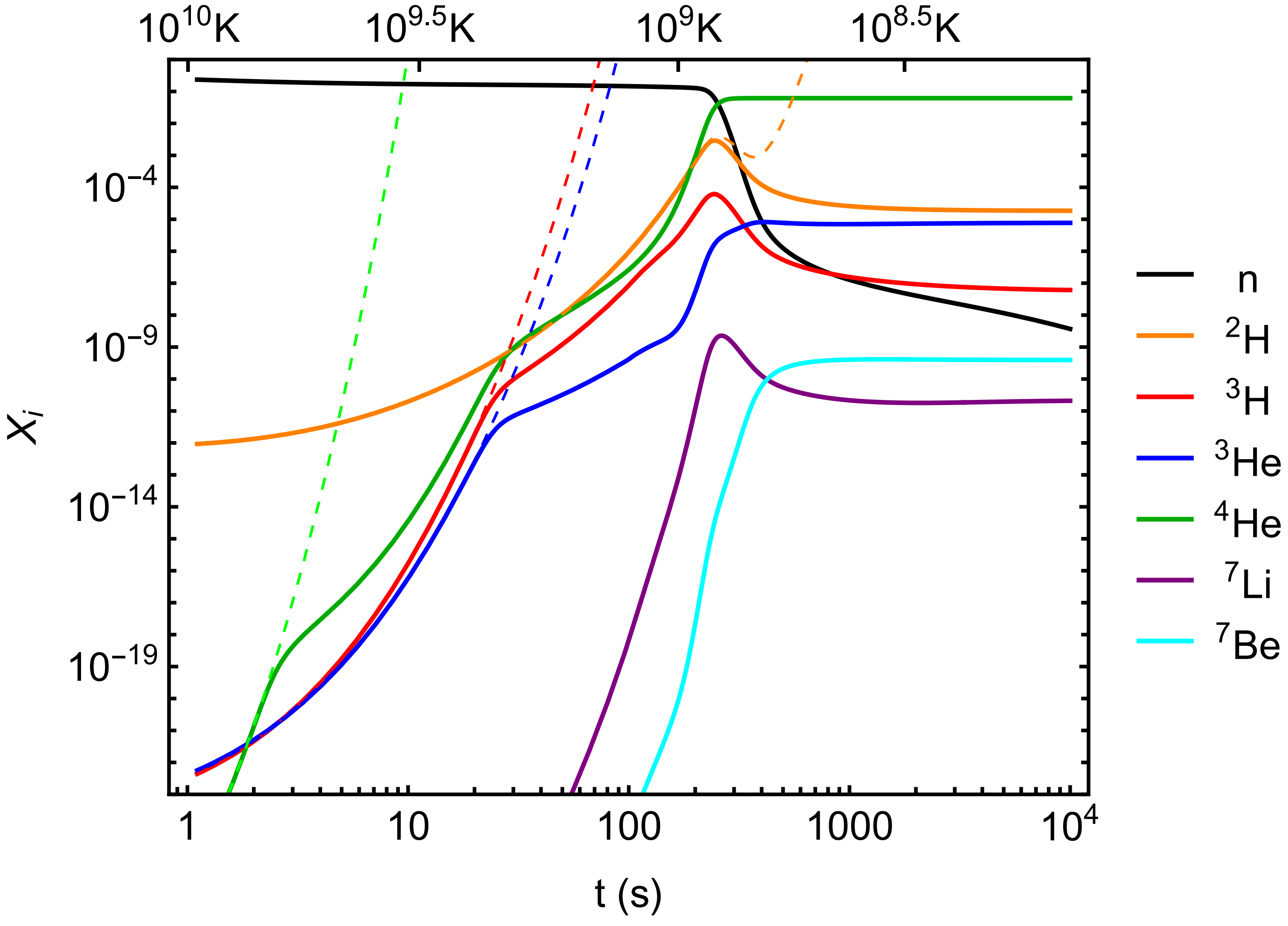
Obtained with public BBN code:

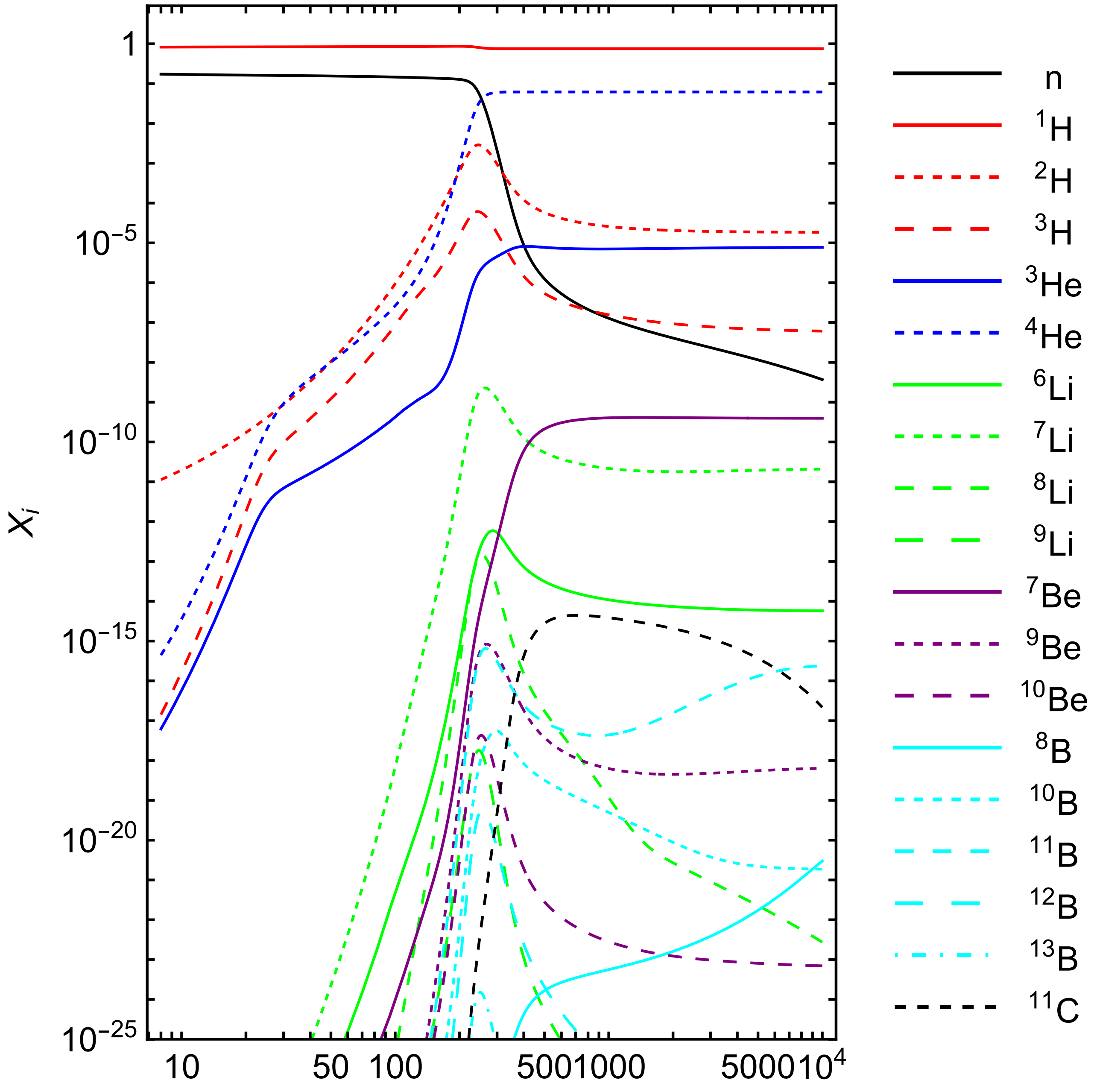
Peebles 1966

PRIMAT

Other codes are :

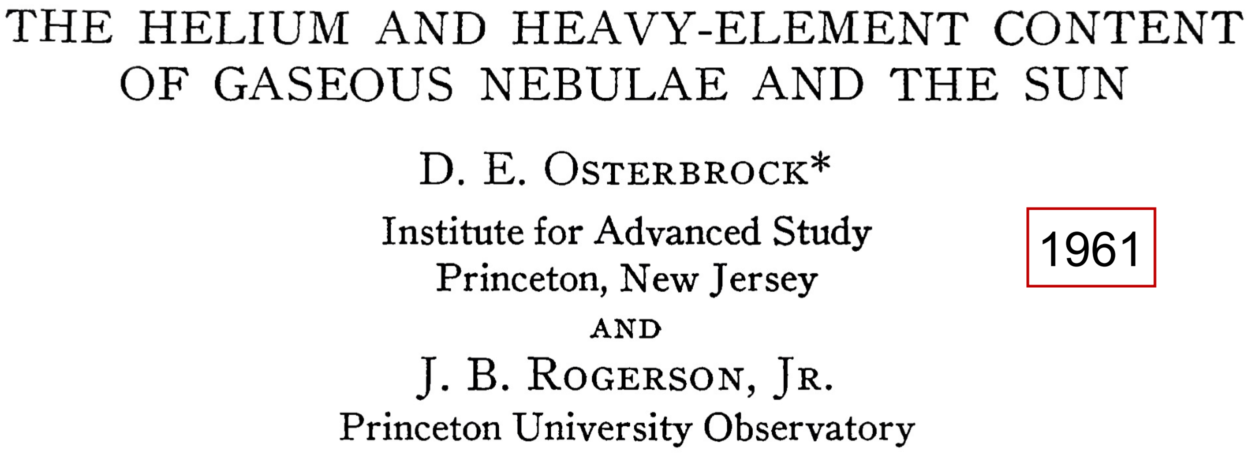
PArthENoPE **AlterBBN**


10^{10}K
$10^{9.5}$K
10^{9}K
$10^{8.5}$K
10^{-4}
10^{-9}
10^{-14}
10^{-19}
X_i
1
10
100
1000
10^4
t (s)
n
^{2}H
^{3}H
^{3}He
^{4}He
^{7}Li
^{7}Be


X_i
t (s)
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10
50
100
500
1000
5000
10^4
n
^{1}H
^{2}H
^{3}H
^{3}He
^{4}He
^{6}Li
^{7}Li
^{8}Li
^{9}Li
^{7}Be
^{9}Be
^{10}Be
^{8}B
^{10}B
^{11}B
^{12}B
^{13}B
^{11}C

Outline :

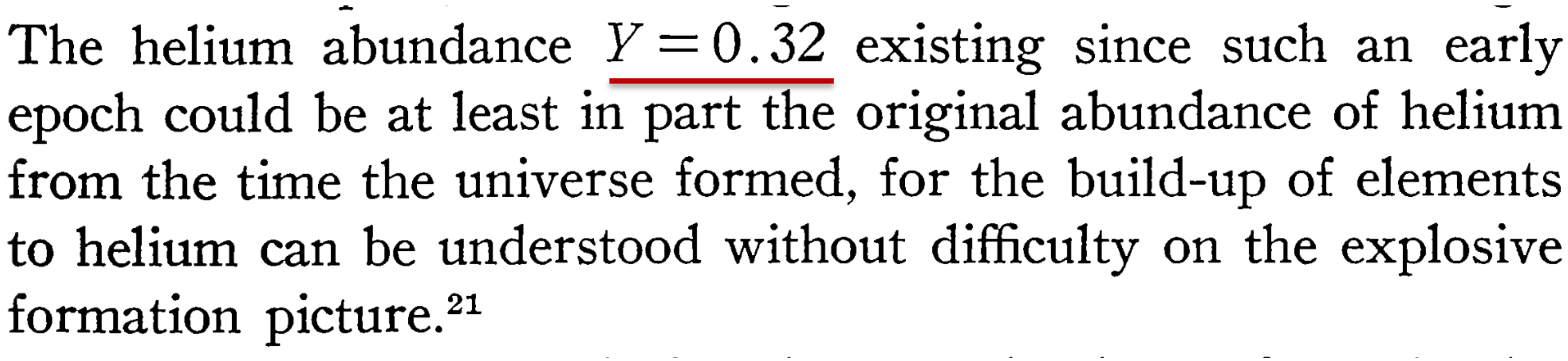
1) Cosmology and plasma reheating
2) Weak interactions
3) Nuclear reactions
4) Observational constraints



THE HELIUM AND HEAVY-ELEMENT CONTENT OF GASEOUS NEBULAE AND THE SUN

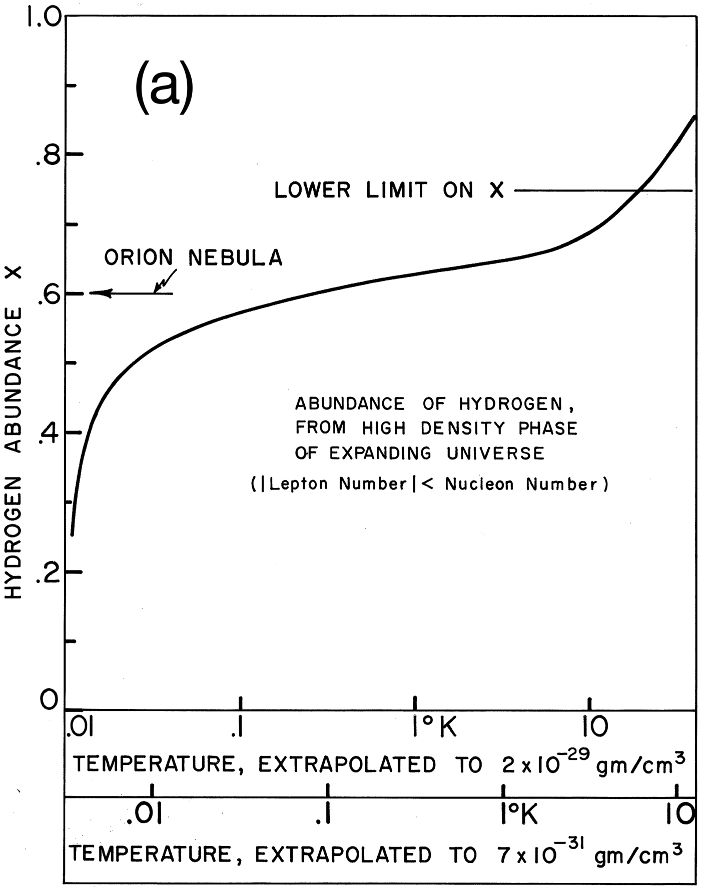
D. E. Osterbrock*
Institute for Advanced Study
Princeton, New Jersey
AND
J. B. Rogerson, Jr.
Princeton University Observatory

1961

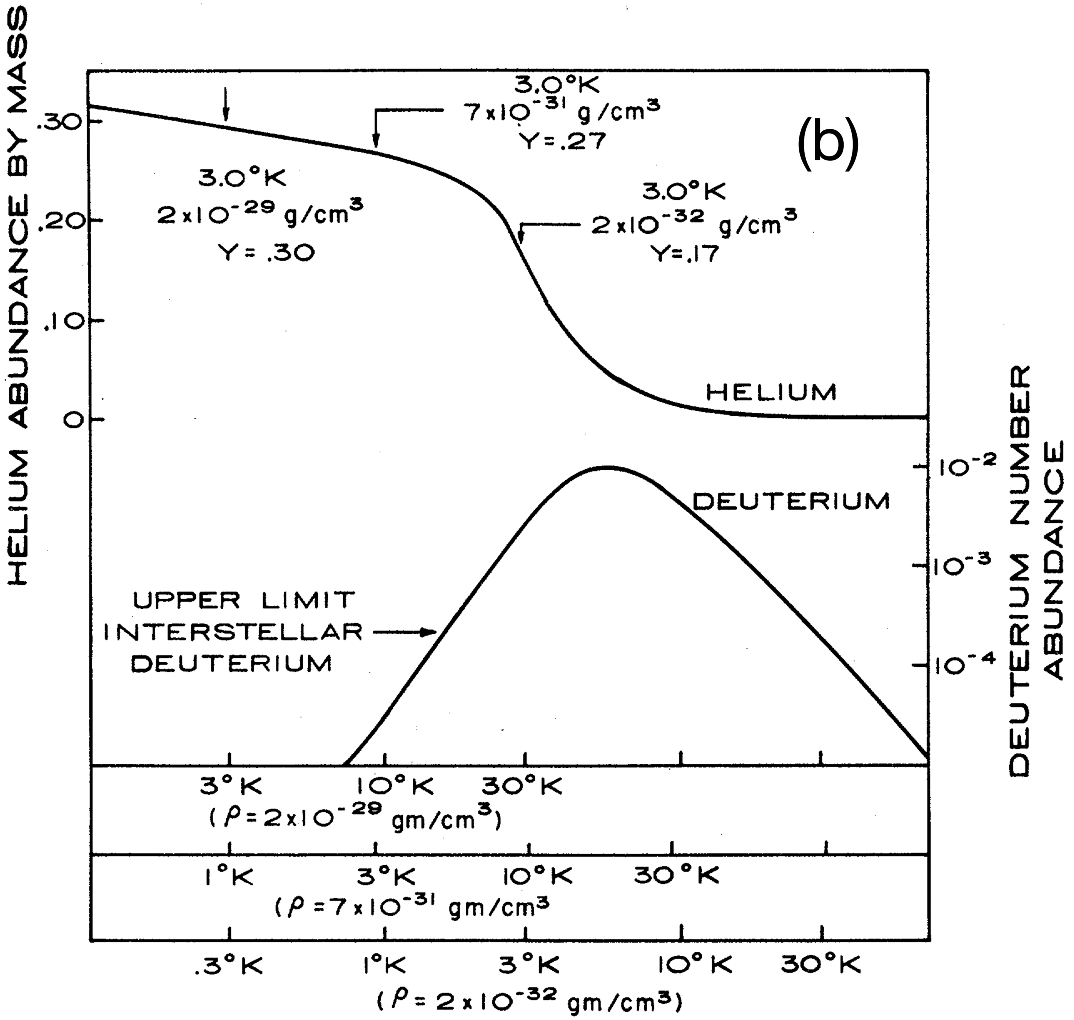


The helium abundance $\underline{Y = 0.32}$ existing since such an early epoch could be at least in part the original abundance of helium from the time the universe formed, for the build-up of elements to helium can be understood without difficulty on the explosive formation picture.[21]

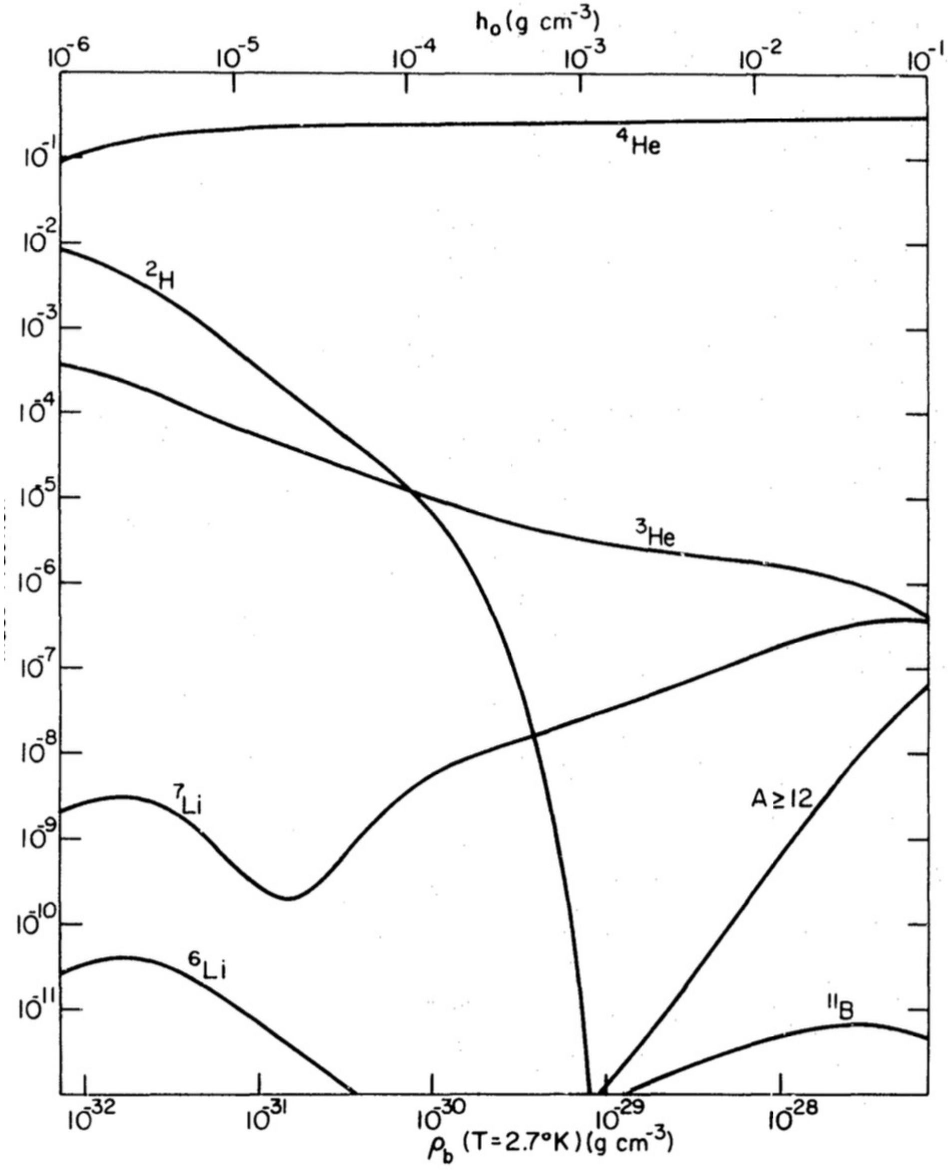
Peebles 1964



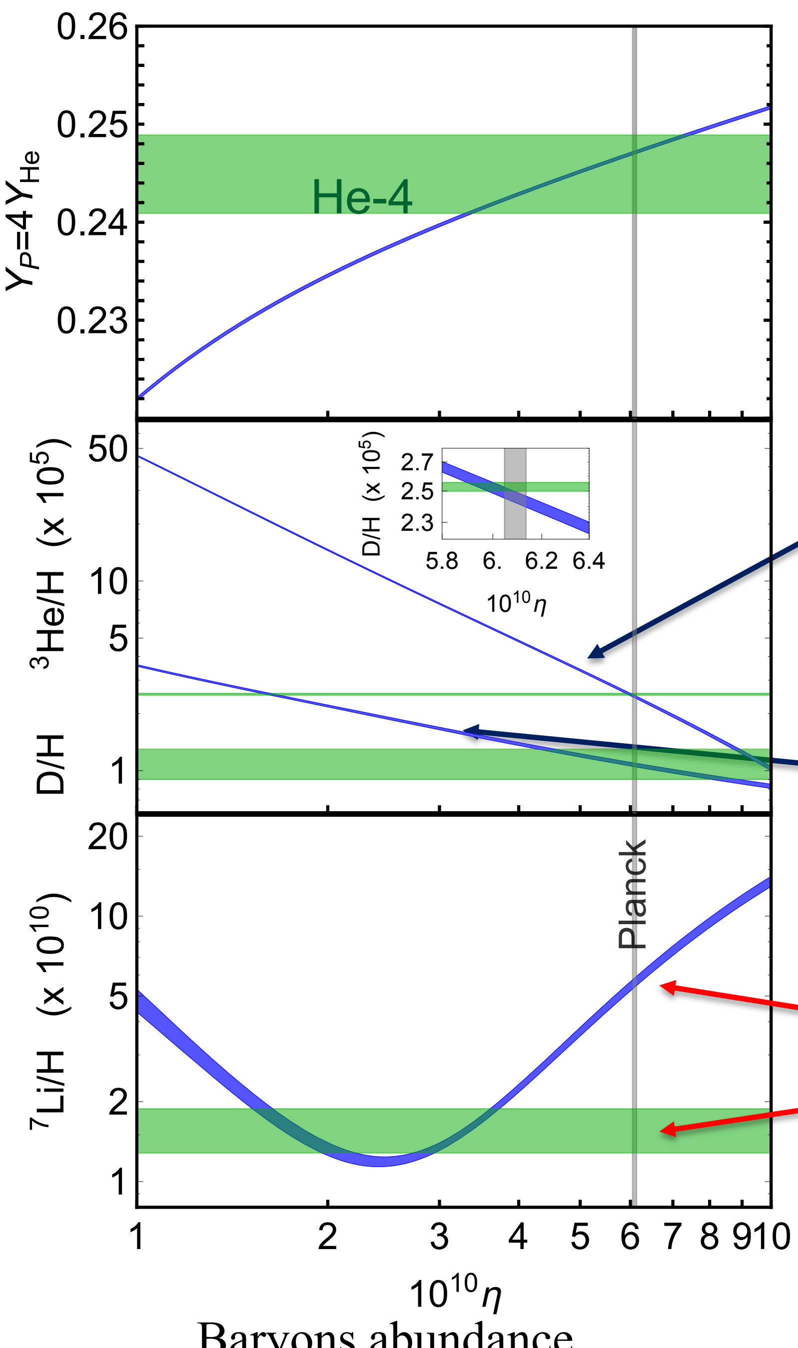
Peebles 1966



Wagoner, Fowler, Hoyle 1973



Baryon abundance completely unknown !



$Y_P = 0.2453 \pm 0.0034$ *Aver et al. 2021*

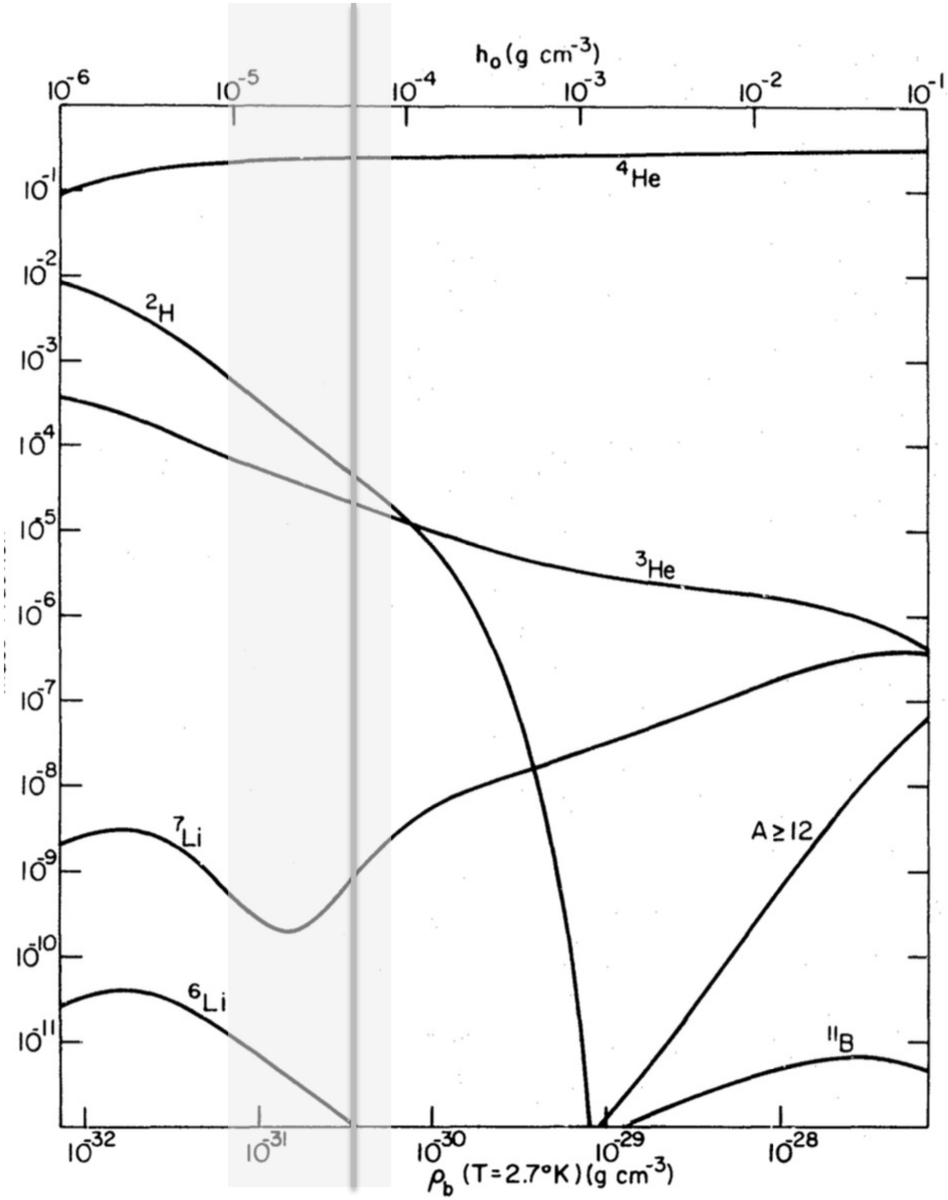
Deuterium (ratio of deuterium/hydrogen D/H)

$\mathrm{D/H} = (2.527 \pm 0.030) \times 10^{-5}$ *Cooke et al. 2018*

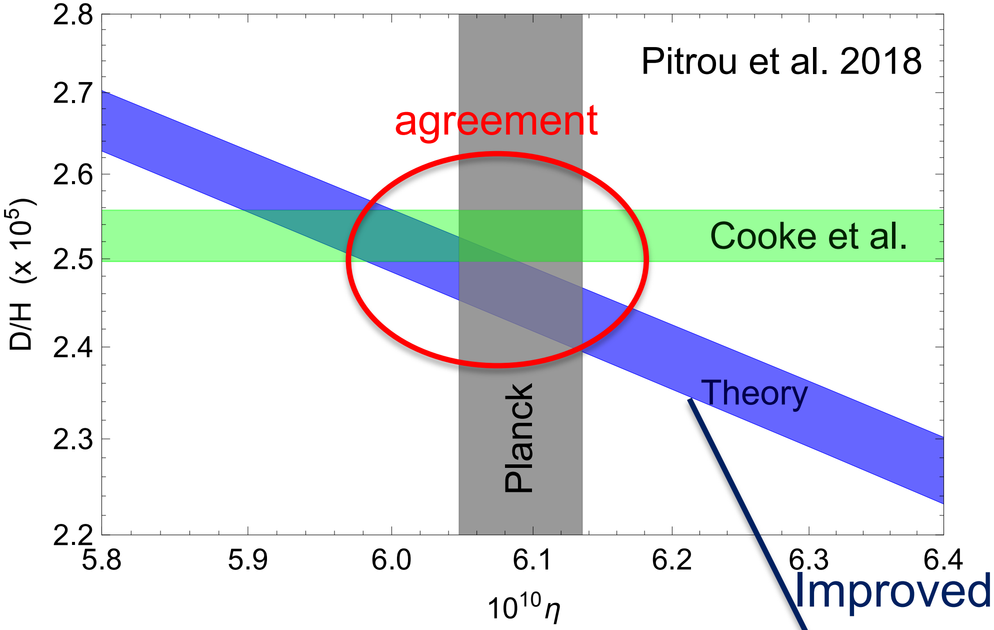
He-3

Lithium problem !

Wagoner, Fowler, Hoyle 1973

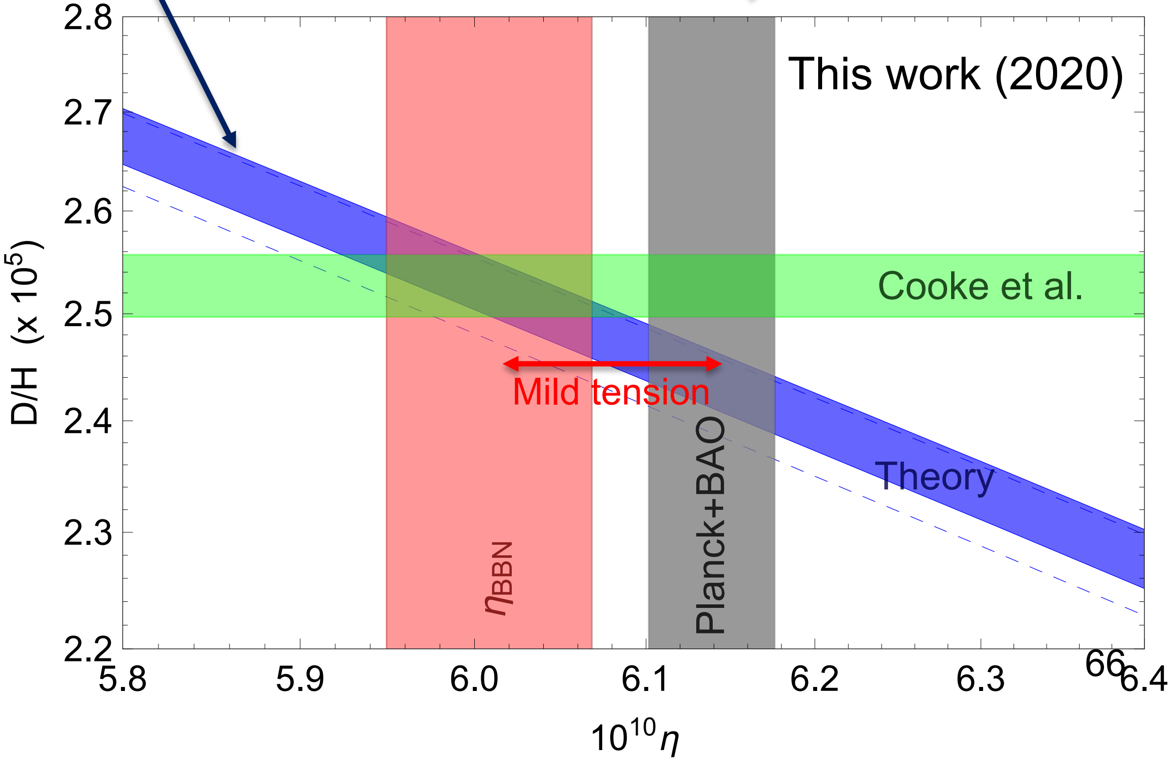


$$\rho_b \simeq 4 \times 10^{-31}\,\mathrm{g/cm^3}$$

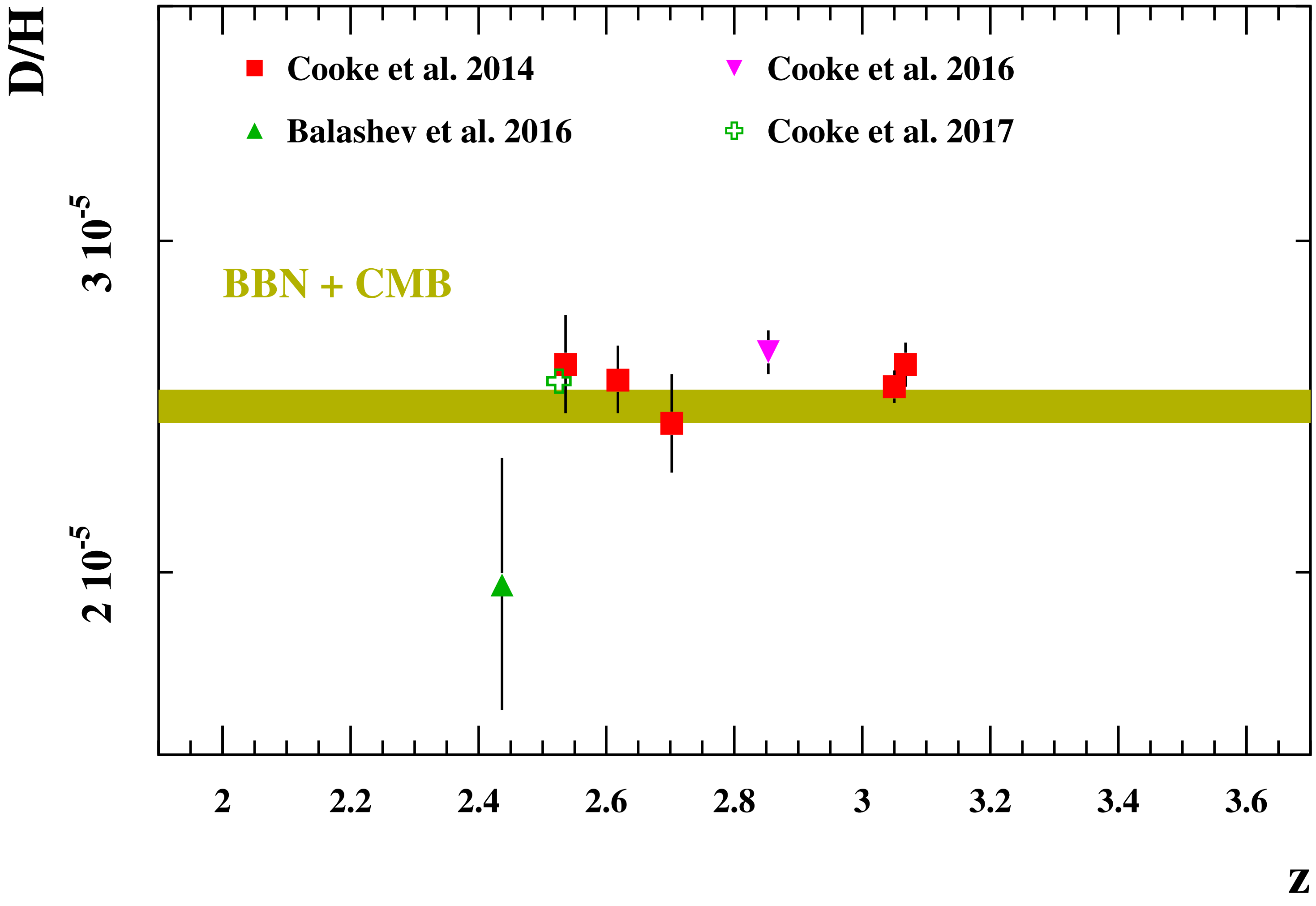


Improved nuclear rates

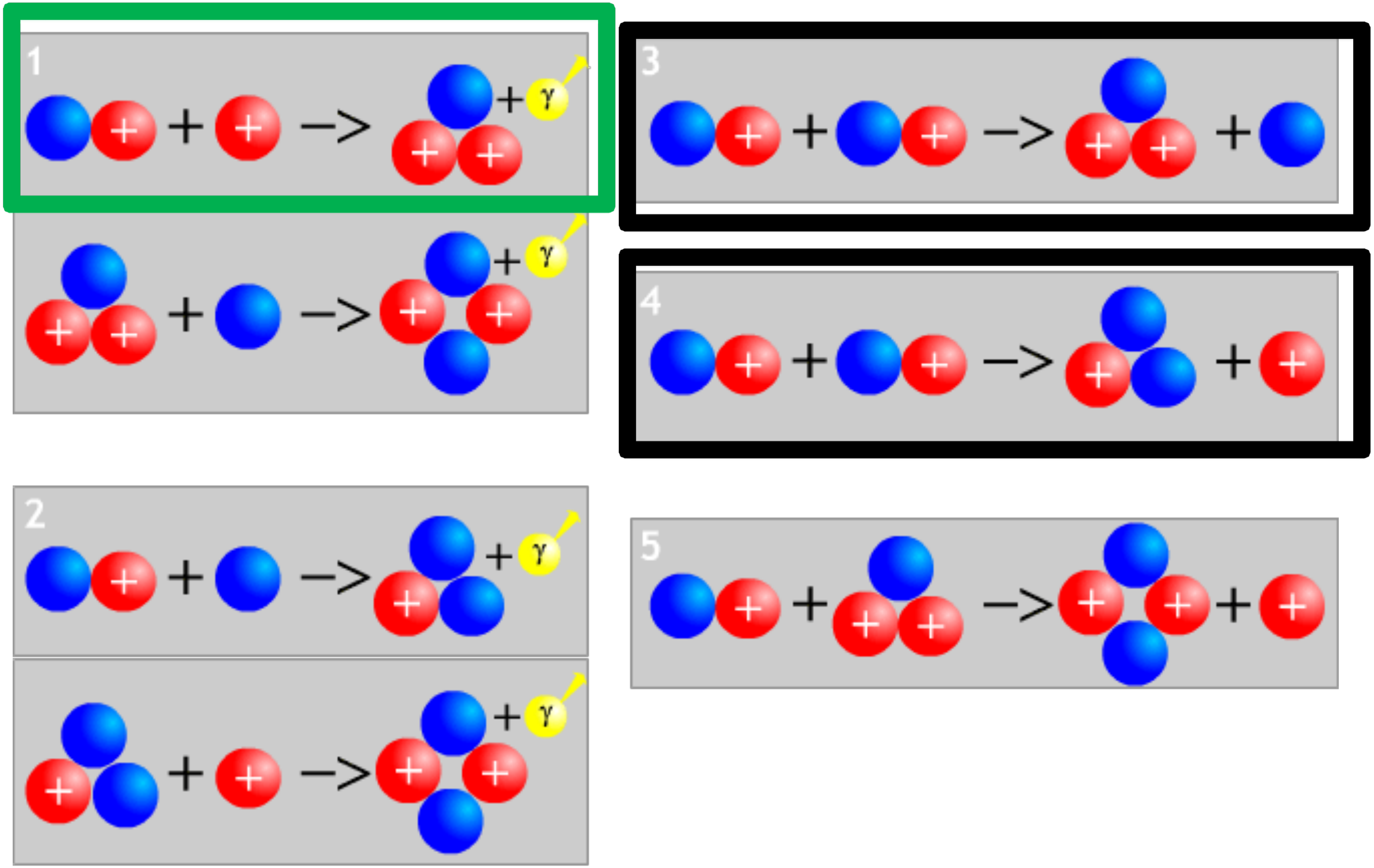
Slightly better constraints from CMB

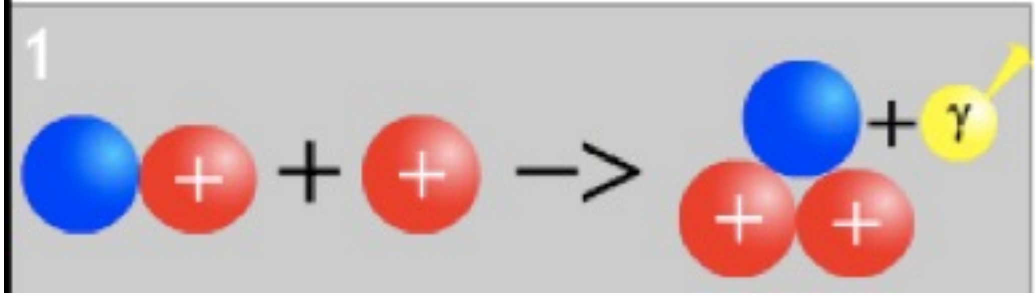


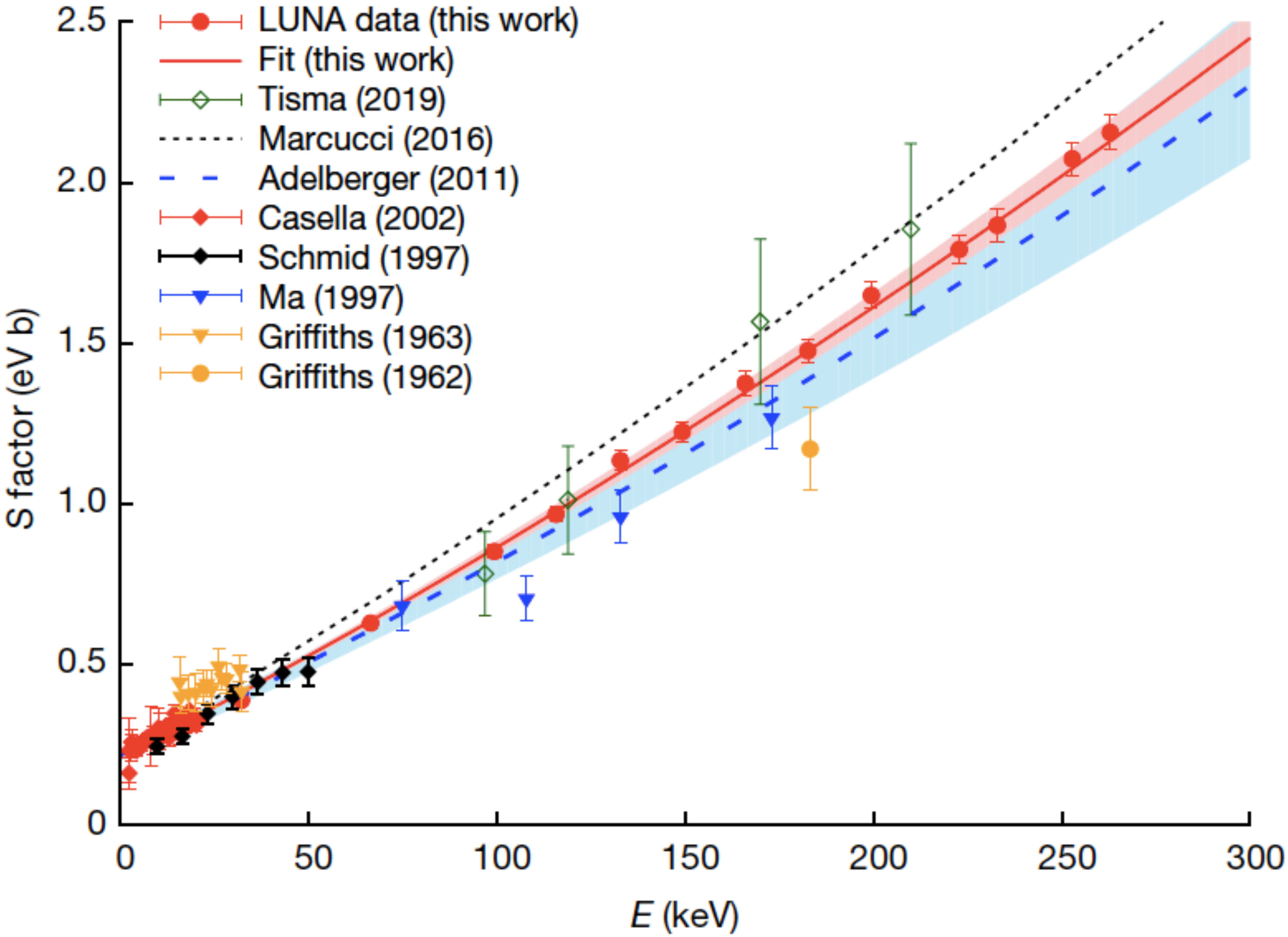
Quasar absorption lines

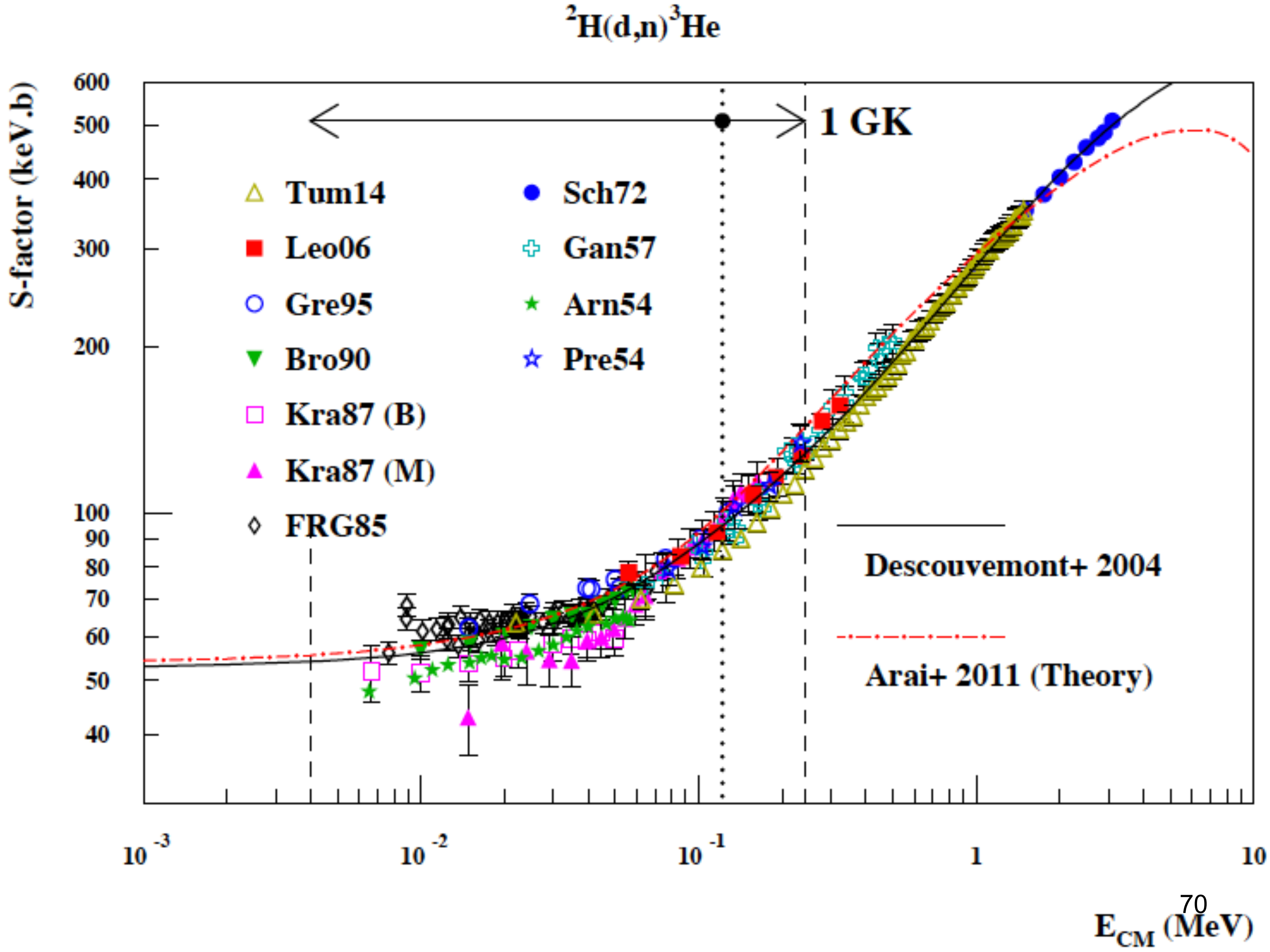


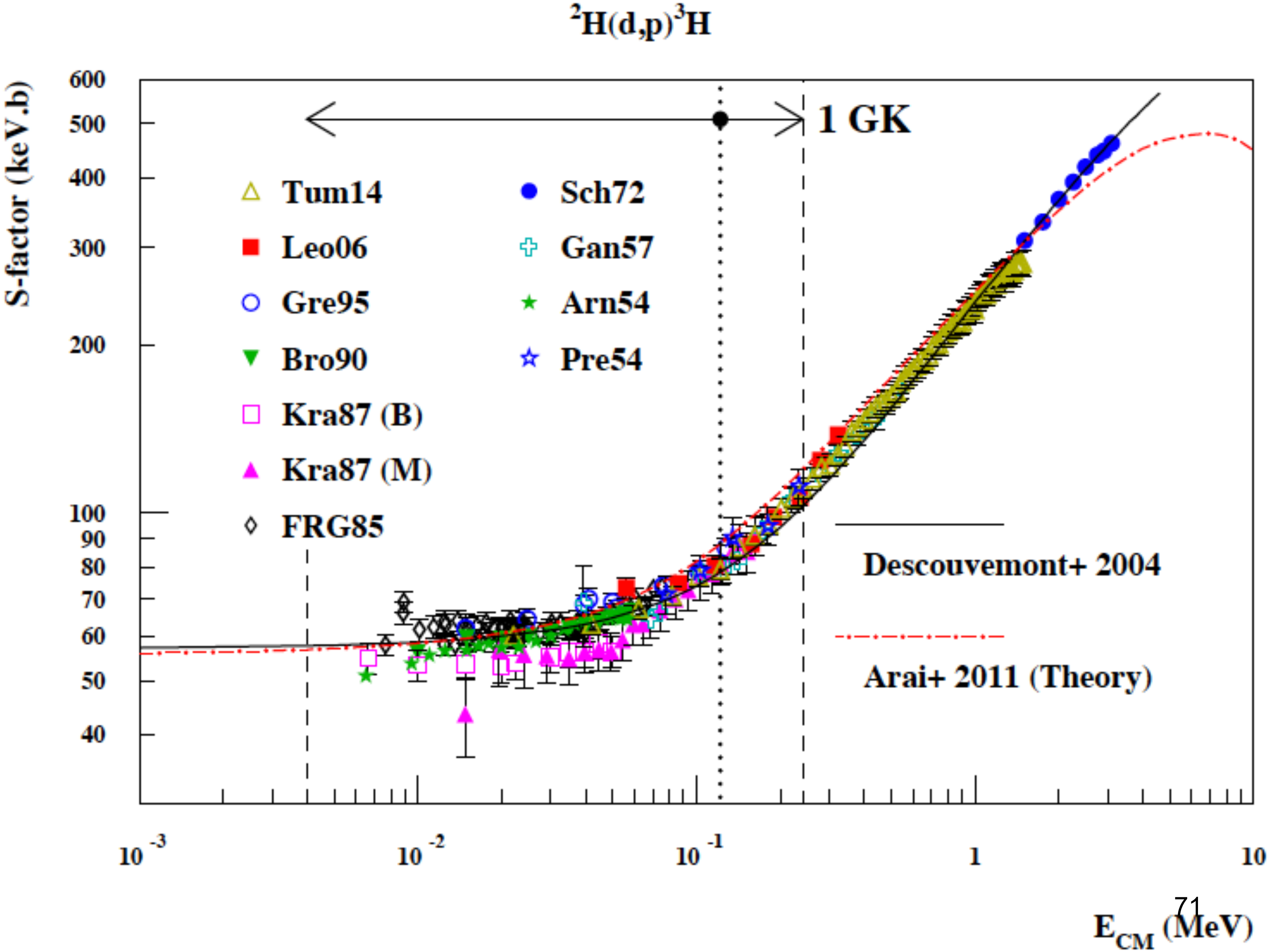
Main reactions for D destruction




1
+
->
+
γ

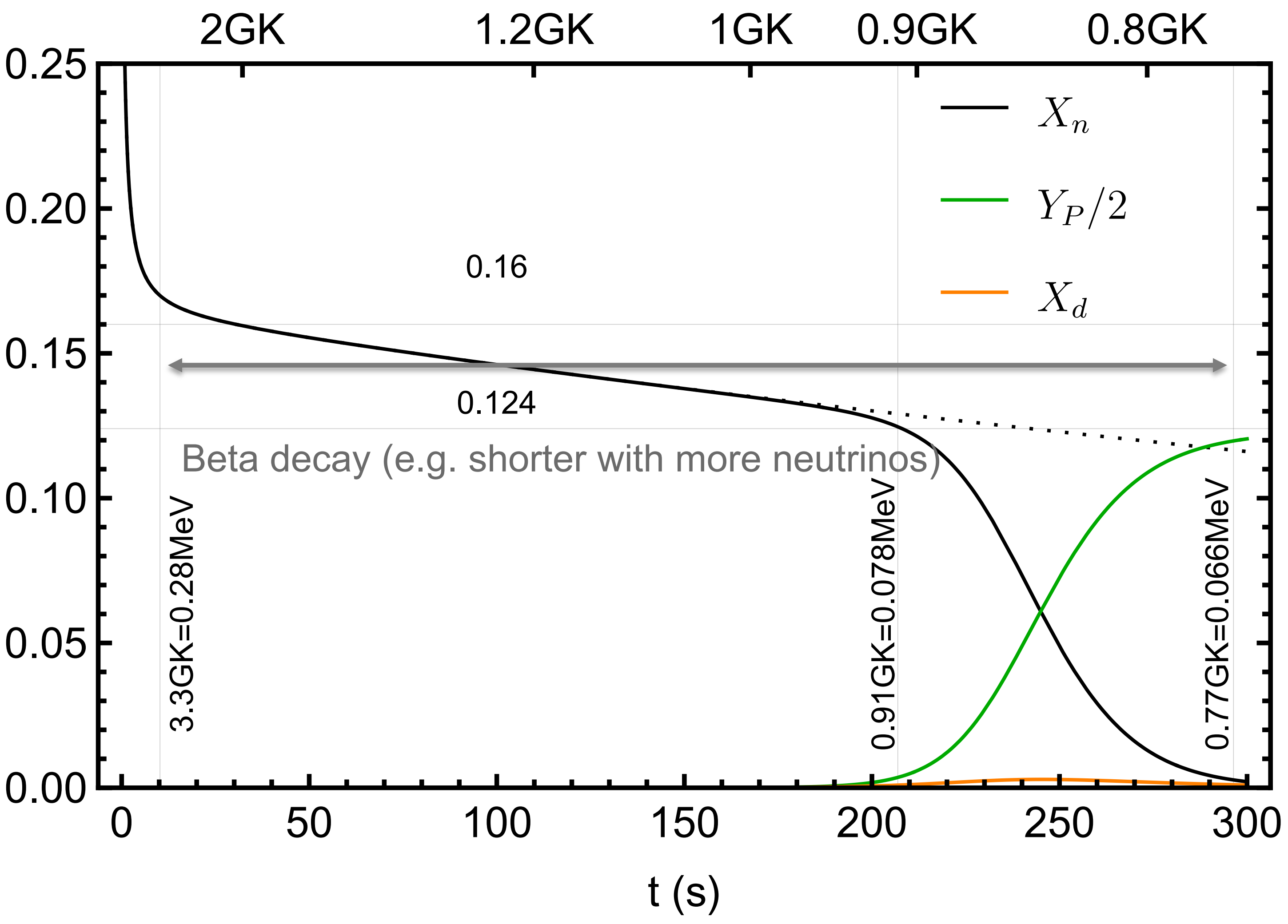

LUNA data (this work)
Fit (this work)
Tisma (2019)
Marcucci (2016)
Adelberger (2011)
Casella (2002)
Schmid (1997)
Ma (1997)
Griffiths (1963)
Griffiths (1962)
S factor (eV b)
2.5
2.0
1.5
1.0
0.5
0
0
50
100
150
200
250
300
E (keV)


$^2H(d,n)^3He$
S-factor (keV.b)
600
500
400
300
200
100
90
80
70
60
50
40
1 GK
△ Tum14
■ Leo06
○ Gre95
▼ Bro90
□ Kra87 (B)
▲ Kra87 (M)
◊ FRG85
● Sch72
✚ Gan57
★ Arn54
☆ Pre54
Descouvemont+ 2004
Arai+ 2011 (Theory)
10^{-3}
10^{-2}
10^{-1}
1
10
E_{CM} (MeV)

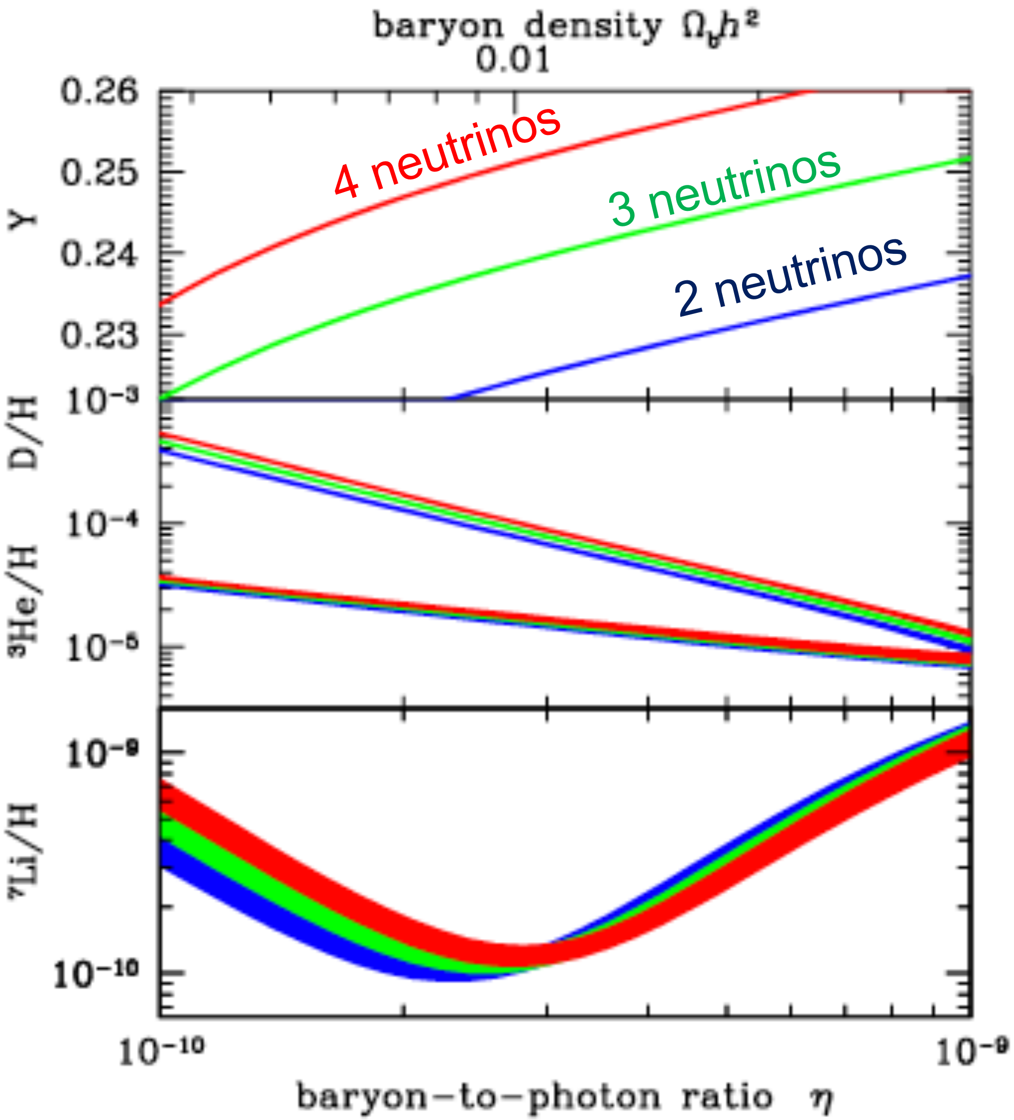

$^2H(d,p)^3H$
S-factor (keV.b)
600
500
400
300
200
100
90
80
70
60
50
40
1 GK
Tum14
Leo06
Gre95
Bro90
Kra87 (B)
Kra87 (M)
FRG85
Sch72
Gan57
Arn54
Pre54
Descouvemont+ 2004
Arai+ 2011 (Theory)
10^{-3}
10^{-2}
10^{-1}
1
10
E_{CM} (MeV)

Effect of N_{eff} *on beta decay*

$$H \simeq \frac{d\ln T}{dt} \propto \sqrt{\rho}$$

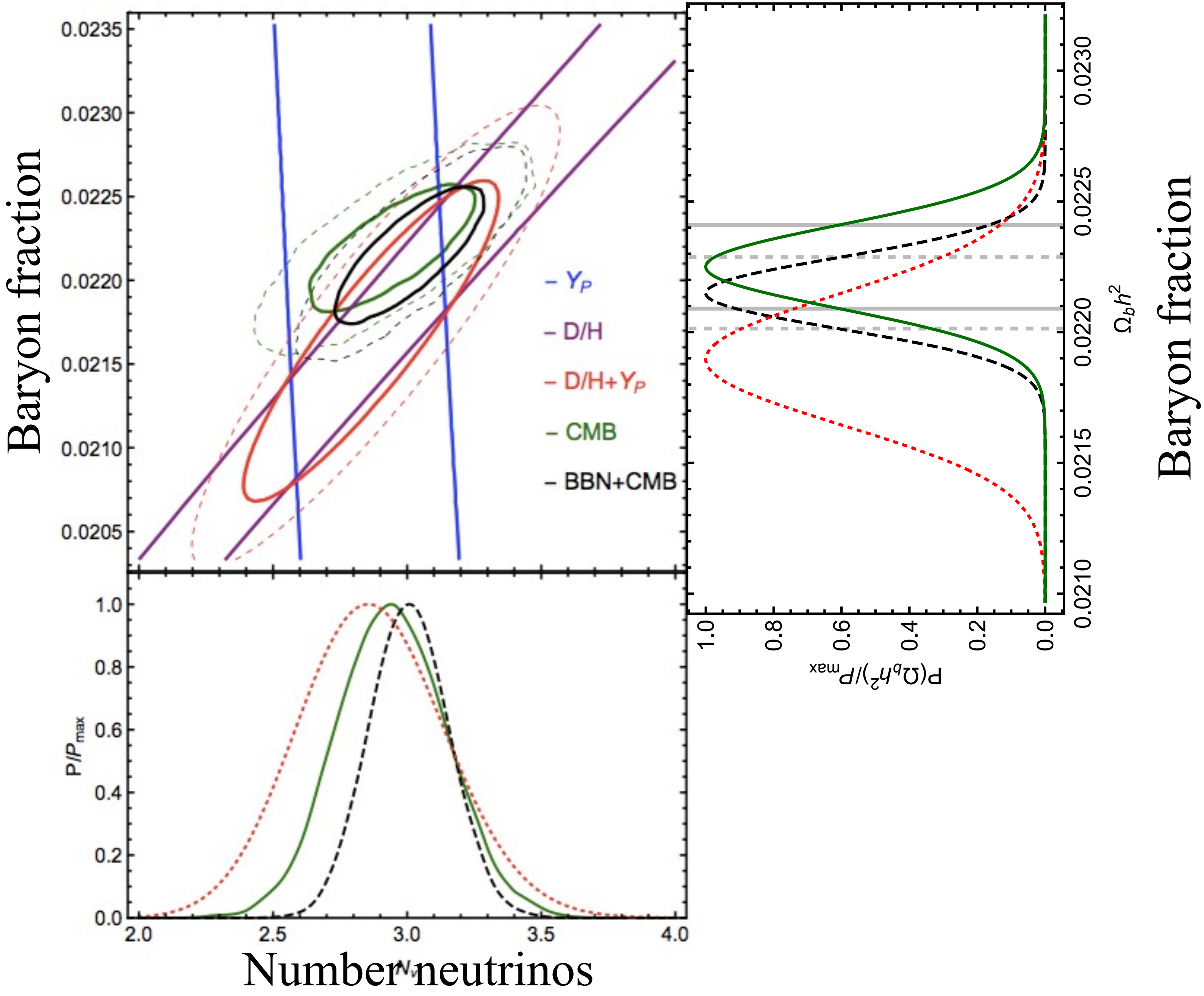


Dependence on number of neutrino species



Cyburt et al. 2015

Joint constraints on baryons and N_{eff}

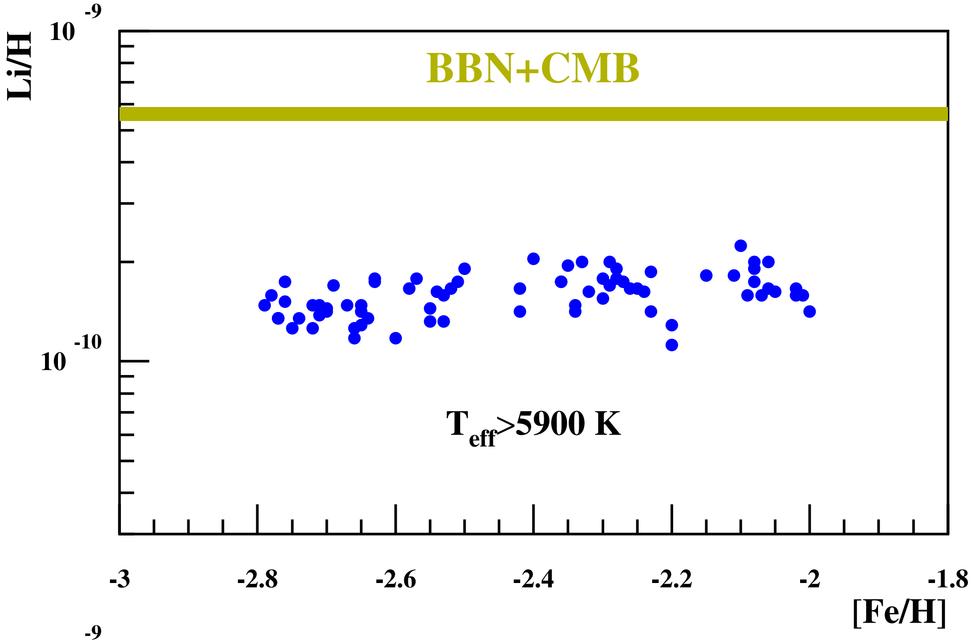


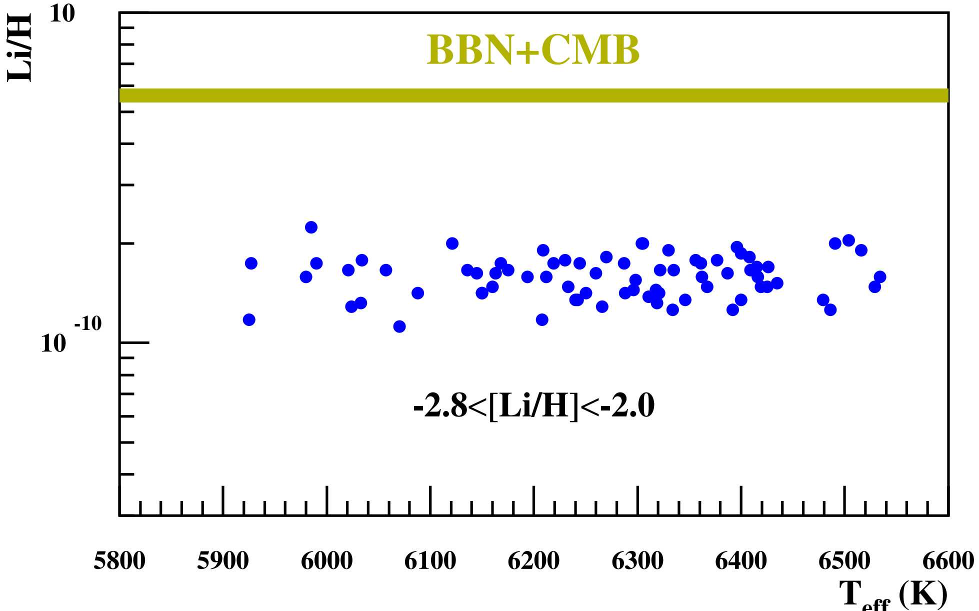
Conclusion

- BBN was paramount in establishing the big bang model
- It has now reached a precision era
- There is amazing consistency with CMB
- Future progress relies on improving nuclear rates
- Lithium problem is probably of astrophysical origin
- BBN is a powerful theory killer via N_{eff}

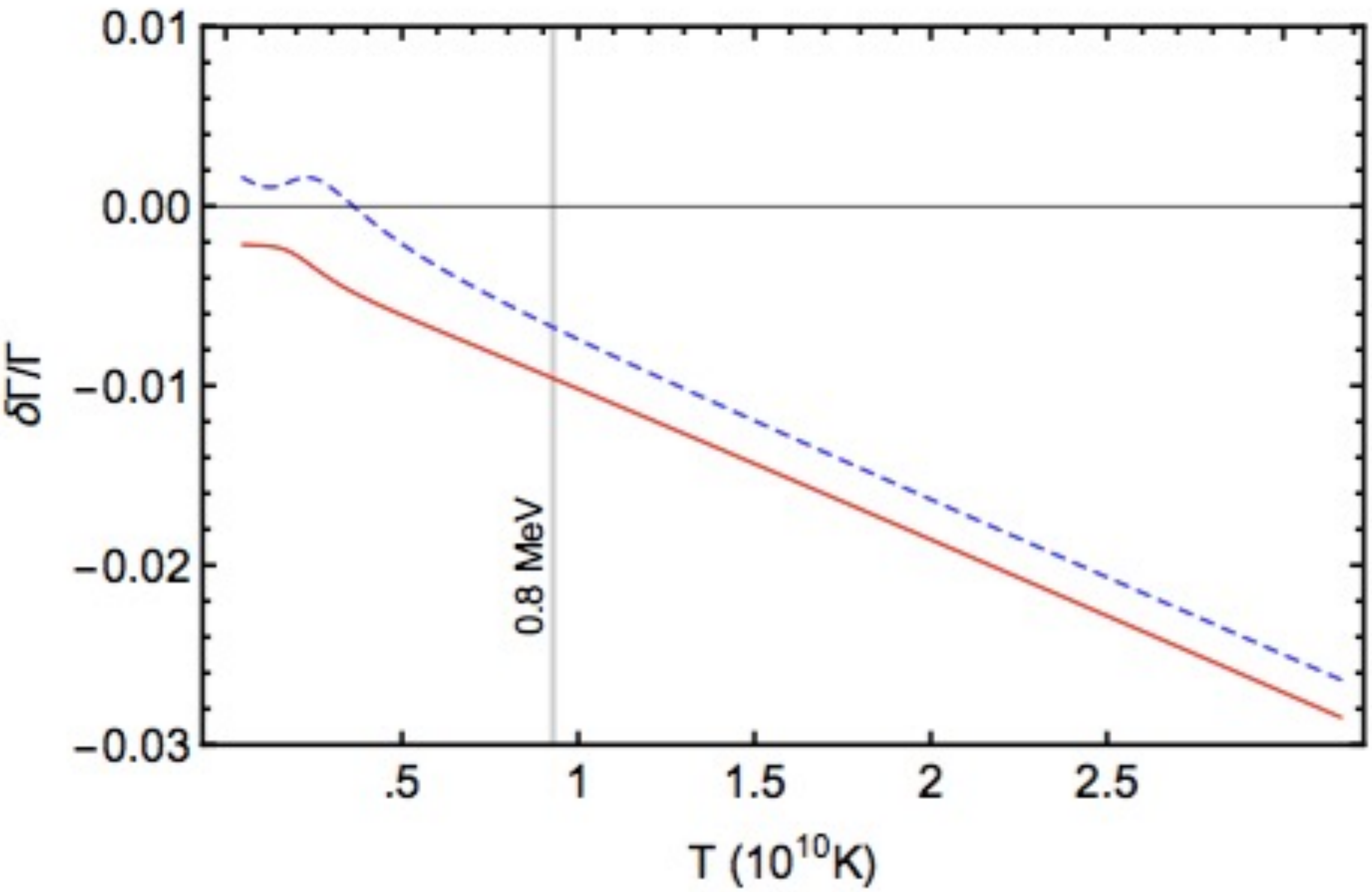
Supplementary material

Lithium measurement



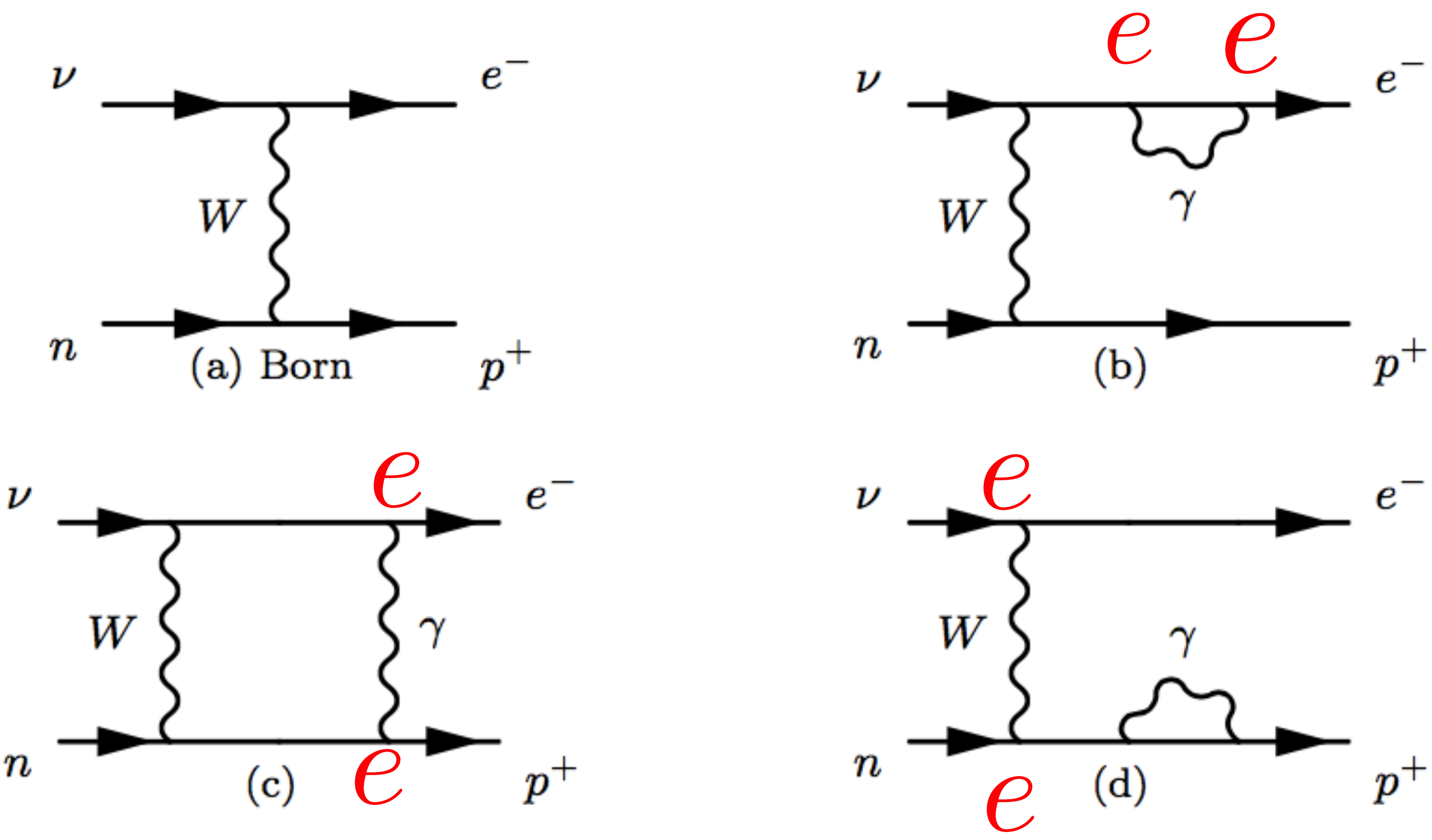


Finite nucleon mass corrections

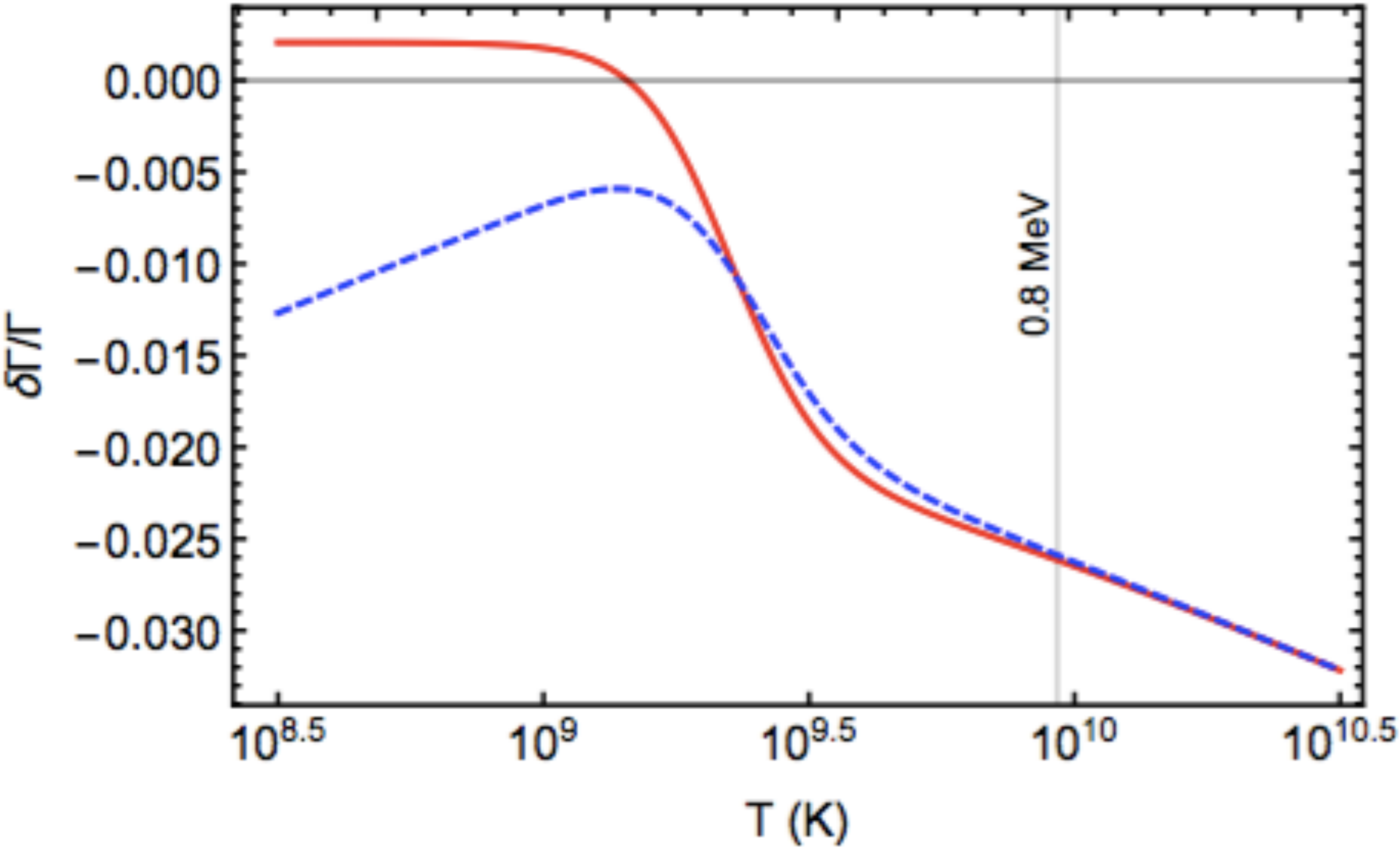


Radiative corrections

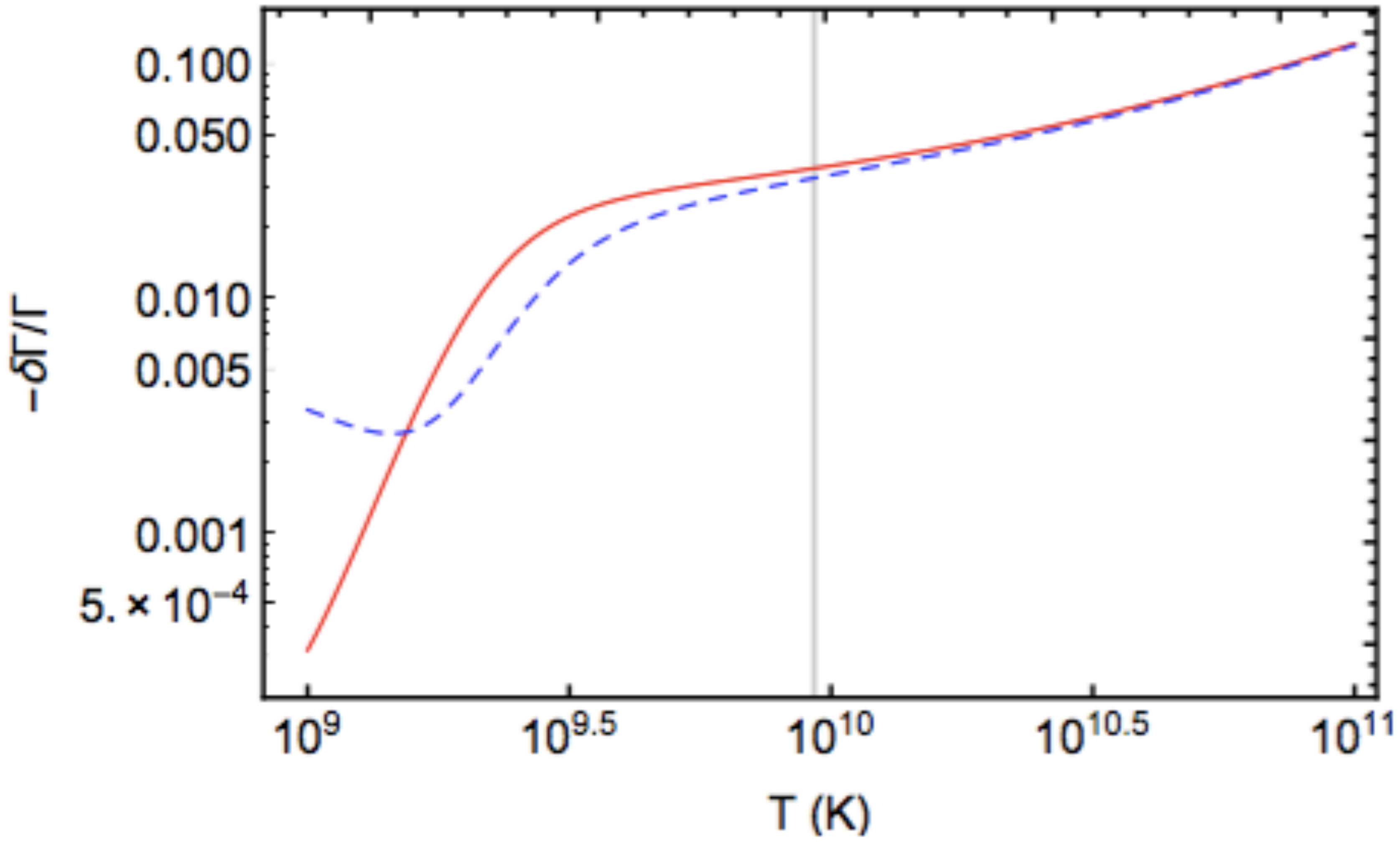
$$\frac{e^2}{4\pi} = \alpha_{\mathrm{FS}} \simeq \frac{1}{137}$$



Radiative corrections



Total corrections to weak rates



Chemical potential of neutrinos displaces freeze-out abundance

$$\frac{n_n}{n_p} = \left.\frac{n_n}{n_p}\right|_{\xi_\nu=0} \times \mathrm{e}^{-\xi_\nu}$$

$$\xi_\nu = \mu_\nu / T_\nu$$

$$\boxed{\xi_\nu = 0.001 \pm 0.016}$$

$$\begin{aligned} \frac{\Delta Y_{\mathrm{P}}}{Y_{\mathrm{P}}} &\simeq -0.96\,\xi_\nu \\ \frac{\Delta \mathrm{D/H}}{\mathrm{D/H}} &\simeq -0.53\,\xi_\nu \\ \frac{\Delta {}^3\mathrm{He/H}}{{}^3\mathrm{He/H}} &\simeq -0.18\,\xi_\nu \\ \frac{\Delta {}^7\mathrm{Li/H}}{{}^7\mathrm{Li/H}} &\simeq -0.62\,\xi_\nu \end{aligned}$$