Beyond the Standard Dark Matter Paradigm?

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These lectures are not a review of alternative dark matter models!

REVIEWS & WHITE PAPERS:

"The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics" (**420 pages!**) E. Di Valentino, et al 2025

Relatively complete review on Dark Matter (**515 pages!**): Cirelli, Strum & Zupan 2024

Dark matter and the early Universe: a review (**72 pages**), A. Arbey and F. Mahmoudi 2021

"Probing the Fundamental Nature of Dark Matter with the Large Synoptic Survey Telescope" (**95 pages**), LSST Dark Matter group 2019

Lecture I: Known DM properties from observations. Known DM properties from theory. Main families of DM and their observational constraints.

Lecture II: Axion Quark Nuggets (QCD as a source of dark matter)

Lecture III: QCD as a source of dark energy. How to improve the efficiency of weak lensing to probe the dark matter power spectrum.

Key questions (why "beyond the standard DM paradigm?"):

The ultimate goal is to discover the nature of DM

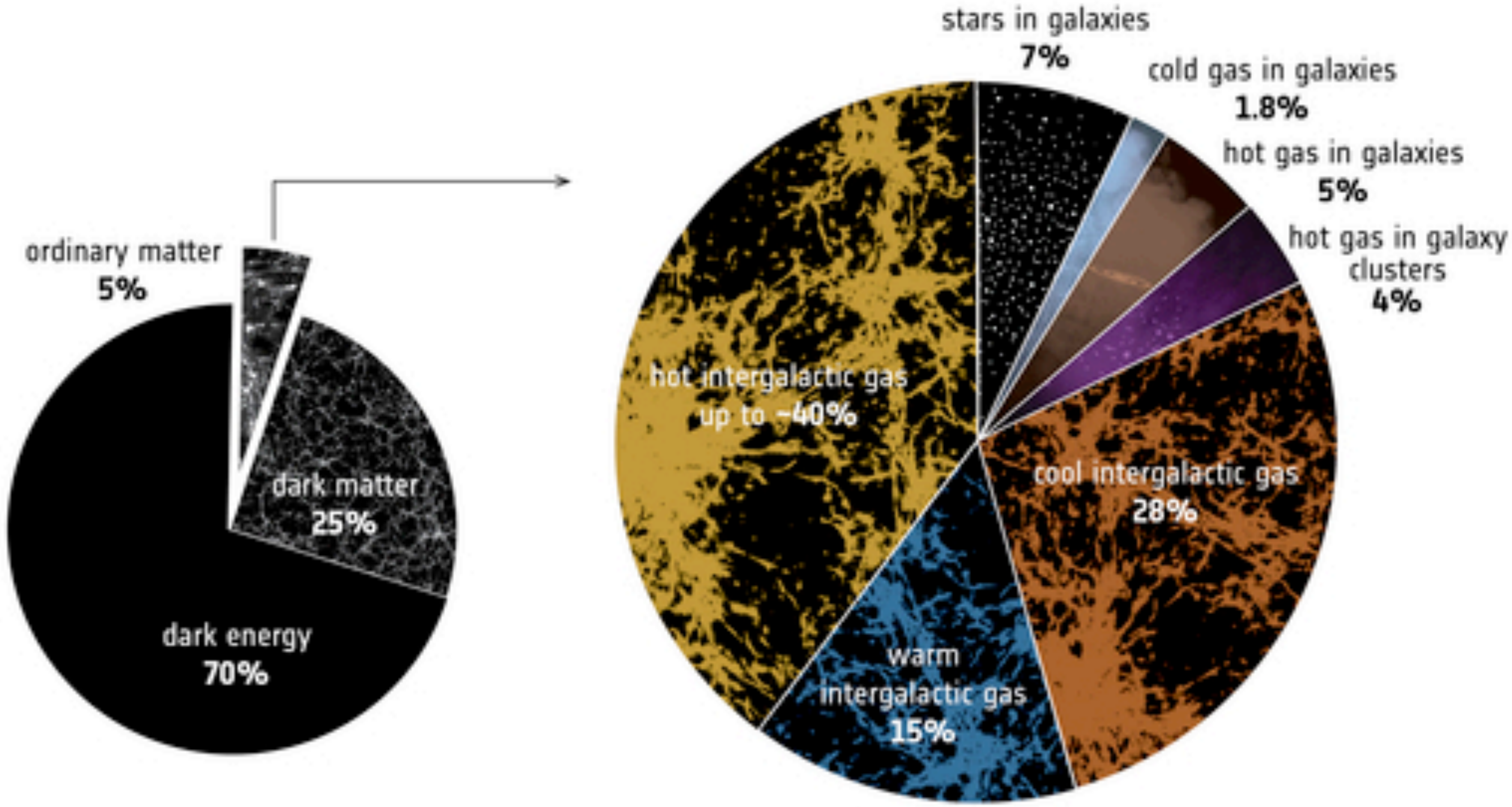
Why no discovery in 40 years of research?

Do we have the best strategy to search in this gigantic parameter space?

Is there a need for new dark matter paradigm?

Can we improve our approach to probe the nature of dark matter?

There is little doubt that dark matter exists…



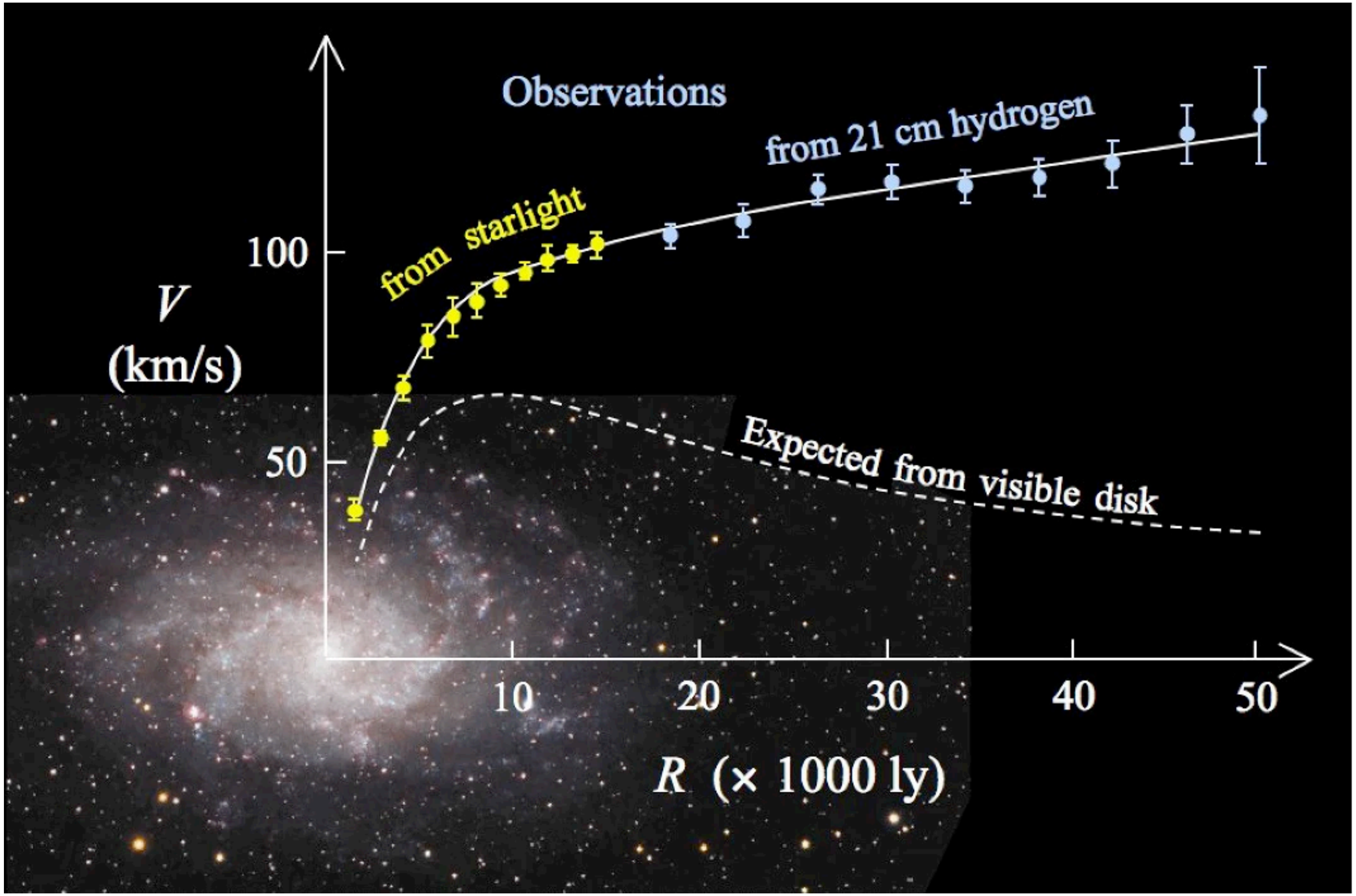
…and that it is abundant: $f_b \equiv \dfrac{\Omega_b}{\Omega_b + \Omega_{\rm DM}} \simeq 0.17$

Historical evidence for Dark Matter, which says nothing about its nature

Clusters of galaxies velocity dispersion

$$N\frac{mv^2}{2} = \frac{1}{2}\frac{N^2Gm^2}{R} \quad \Rightarrow \quad mN = \frac{2Rv^2}{G}$$

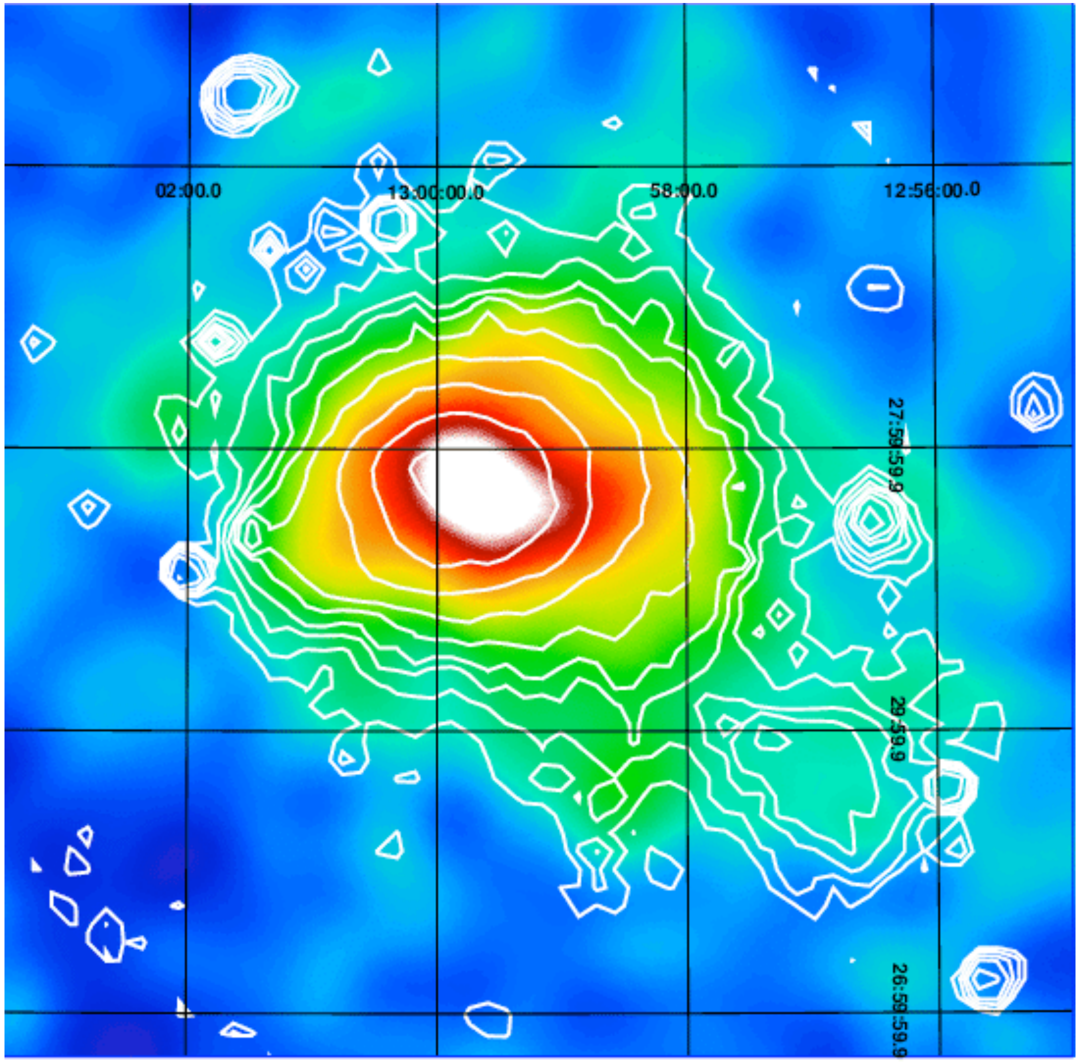
Galaxies rotation curves



https://link.springer.com/article/10.1007/s00159-018-0113-1

$$m\frac{v_{\rm circ}^2(r)}{r} = \frac{Gm\mathscr{M}(\leq r)}{r^2} \quad \Rightarrow \quad v_{\rm circ}(r) = \sqrt{\frac{G\mathscr{M}(\leq r)}{r}}$$

Hot gas in clusters of galaxies in hydrostatic equilibrium



https://ui.adsabs.harvard.edu/abs/2013arXiv1302.3355T/abstract

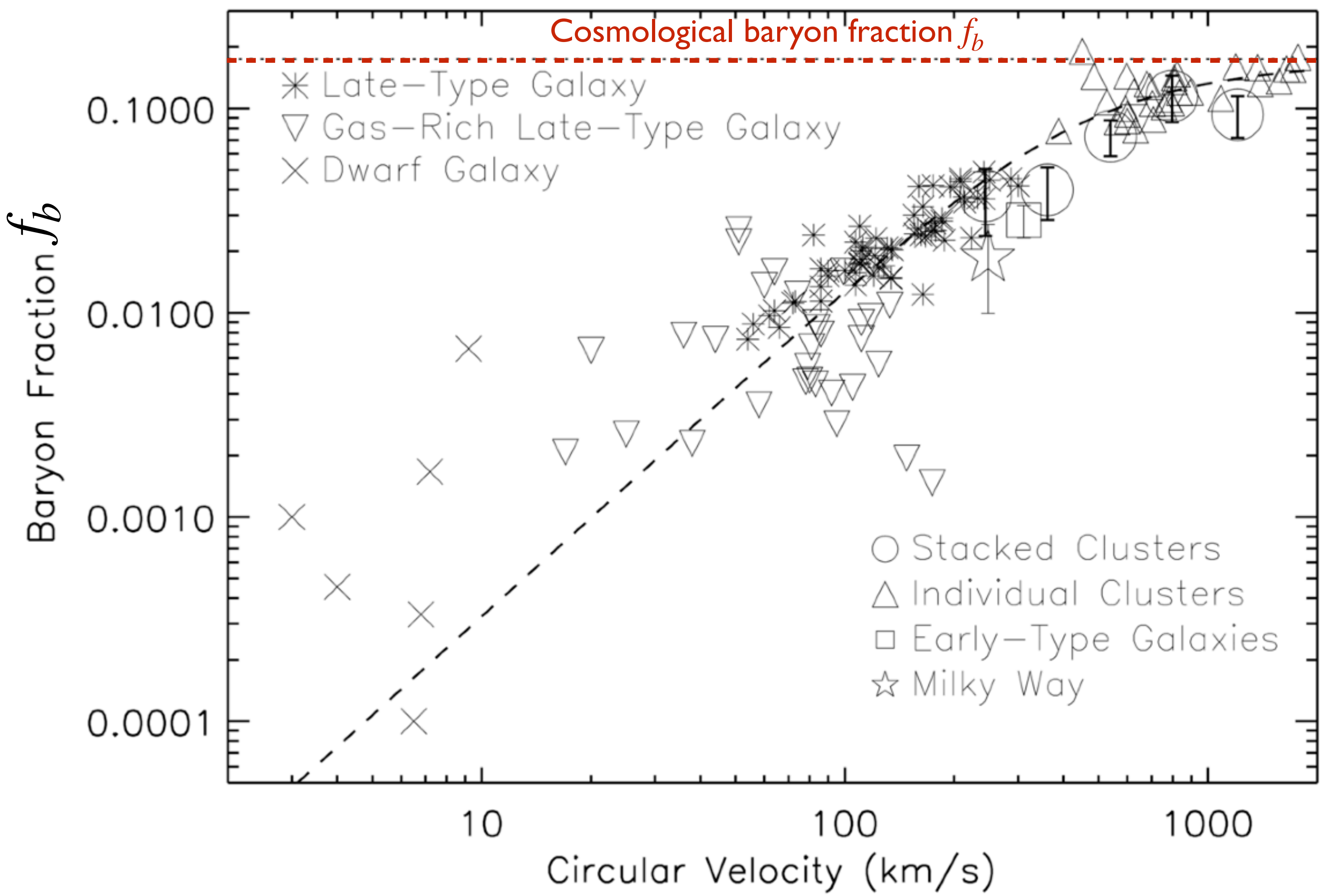
$$\frac{1}{\rho_{\mathrm{gas}}(r)}\frac{d\rho_{\mathrm{gas}}}{dr} = -\frac{d\phi}{dr} = -\frac{GM(r)}{r^2}$$

Strong gravitational lensing

Image credit: ESA/Euclid/Euclid Consortium/NASA, image processing by J.-C. Cuillandre, T. Li

$$M_E = \frac{c^2}{4G}\frac{D_L D_S}{D_{LS}}\theta_E^2$$

Where is dark matter?



(From Dai et al 2010)

$$f_{\mathrm{b}} = \frac{M_{\mathrm{b}}}{M_{\mathrm{b}} + M_{\mathrm{DM}}}$$

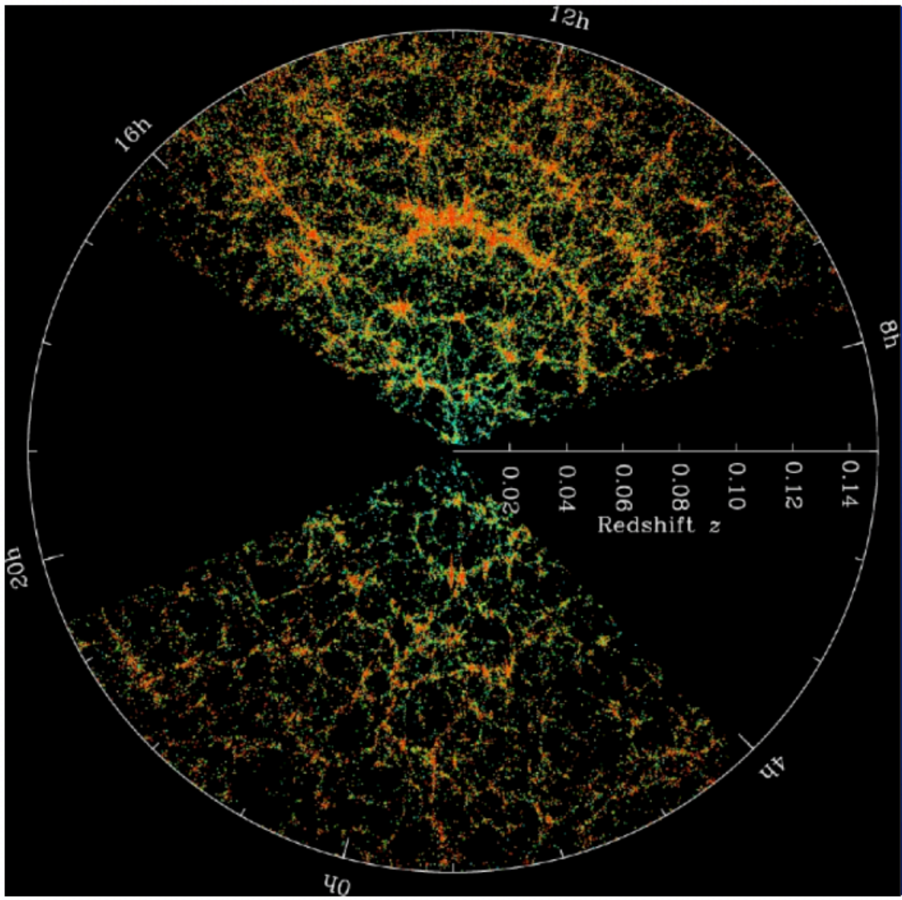
Difference between hot, warm and cold dark matter

The Nature of Dark Matter: Cold, Warm, Hot, or...?

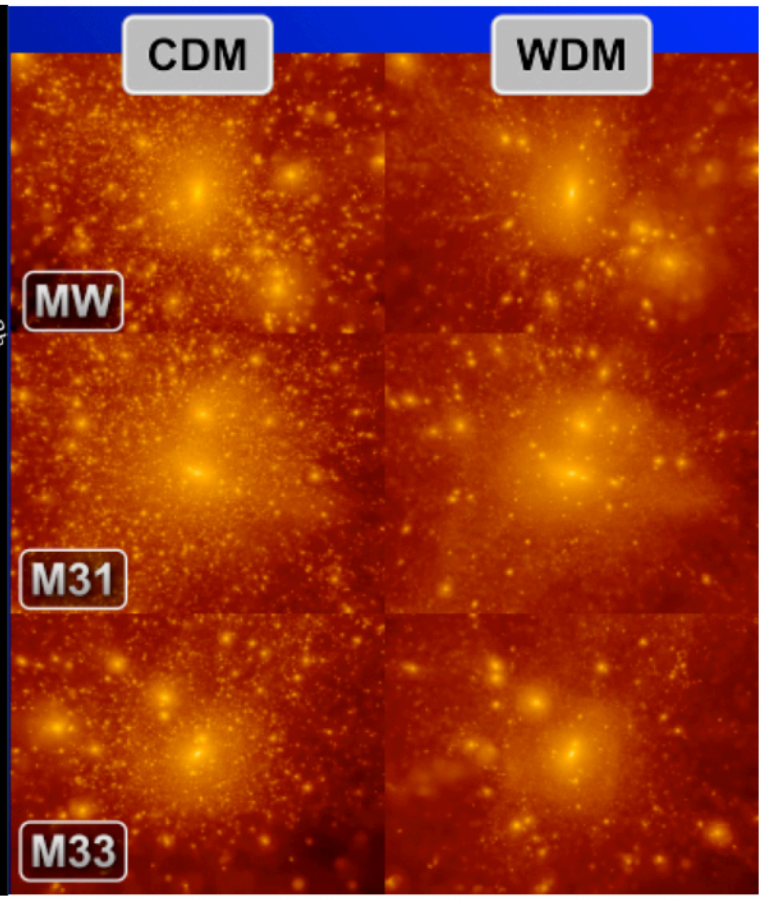
Dark matter important for large scale structure formation

Compare observation to simulated structure formation

Favor DM that is cold (slow) at time of decoupling → filaments, voids



Credit: SDSS



Credit: A. Khalatyan/CLUES team

What distinguish CDM from WDM and HDM?

Dark matter is classified based on whether it was relativistic (hot) or non-relativistic (cold) when cosmologically relevant density fluctuations (e.g., Milky Way–sized $\sim 10^{12}\ M_{\odot}$) entered the horizon.

$$M_H \sim 10^{12}\ M_{\odot} \text{ at } z \sim 2.5 \times 10^5\ (a \sim 4 \times 10^{-6})$$

Which is $t \sim 12\ \text{yrs}$. Temperature at this time is $kT \sim 60\ \text{eV}$

$\Longrightarrow$ DM particles with rest mass energy $mc^2 \sim 3kT < 180\ \text{eV}$ will be relativistic at that time

From the free streaming length:

$$\lambda_{\rm FS} = \int_0^{t_0} \frac{v(t)}{a(t)}\, dt \approx \int_0^{a_{\rm nr}} \frac{1}{a^2 H(a)}\, da + \int_{a_{\rm nr}}^{1} \frac{v(a)}{a^2 H(a)}\, da$$

a_{nr} is the scale factor when DM becomes non-relativist, and

$$v(a) = \frac{|\vec{p}\,|}{E} = \frac{p(a)}{\sqrt{p(a)^2 + m_\chi^2}} \implies \begin{array}{ll} v \approx 1 & \text{when } p \gg m_\chi \quad \text{(relativistic)}, \\ v \approx \dfrac{p}{m_\chi} & \text{when } p \ll m_\chi \quad \text{(non-relativistic)} \end{array}$$

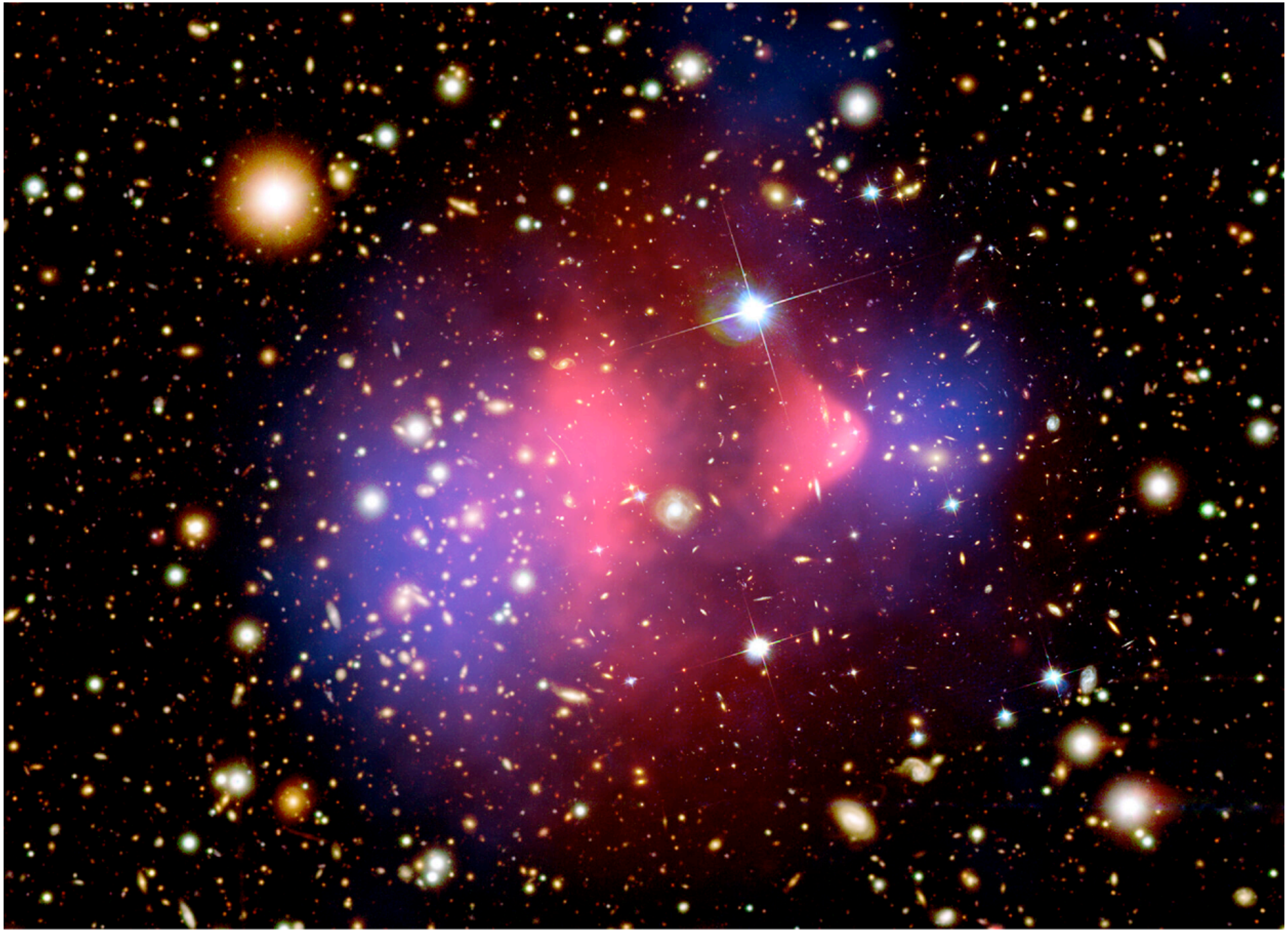
With $p(a_{nr}) \simeq m_\chi$ ($p \sim 3kT$ for thermal relics, e.g. BE or FD)

CDM $\lambda_{\rm fs} \ll 1$ kpc
WDM $\lambda_{\rm fs} \sim 10 - 10^2$ kpc
HDM $\lambda_{\rm fs} \sim 1$ Mpc

(Complete calculations with a mix: Boyanovsky 2008, Boyanovsky et al 2009)

Model independent constraints:
DM is collisionless and relatively cold

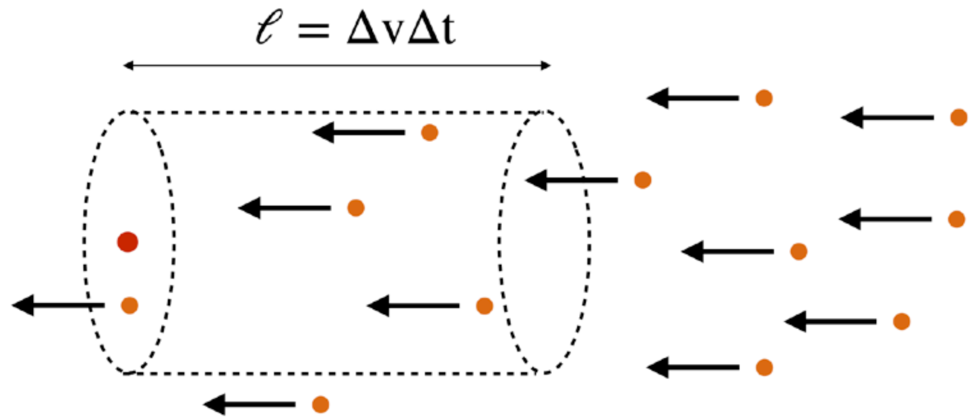
The bullet cluster: dark matter is collisionless



From Clowe et al 2006

Blue: mass from weak lensing.   Red: gas from X-ray

The bullet cluster: dark matter is collisionless



DM-DM scattering event when $\sigma_X n_X \ell \equiv 1$

$$\Longrightarrow \sigma_X n_X \ell = \left(\frac{\sigma_X}{m_X}\right) \rho_X \ell$$

Surface mass density $\Sigma = \rho\ell = n m_X \ell \Longrightarrow \left(\frac{\sigma_X}{m_X}\right)\Sigma = 1$

With $\Sigma_{obs} \sim 1\ \mathrm{g/cm^2} \Longrightarrow \boxed{\frac{\sigma}{M} \lesssim 1 \frac{\mathrm{cm^2}}{\mathrm{g}}}$ (DM-baryon has a similar bound)

Other constraints on σ/M

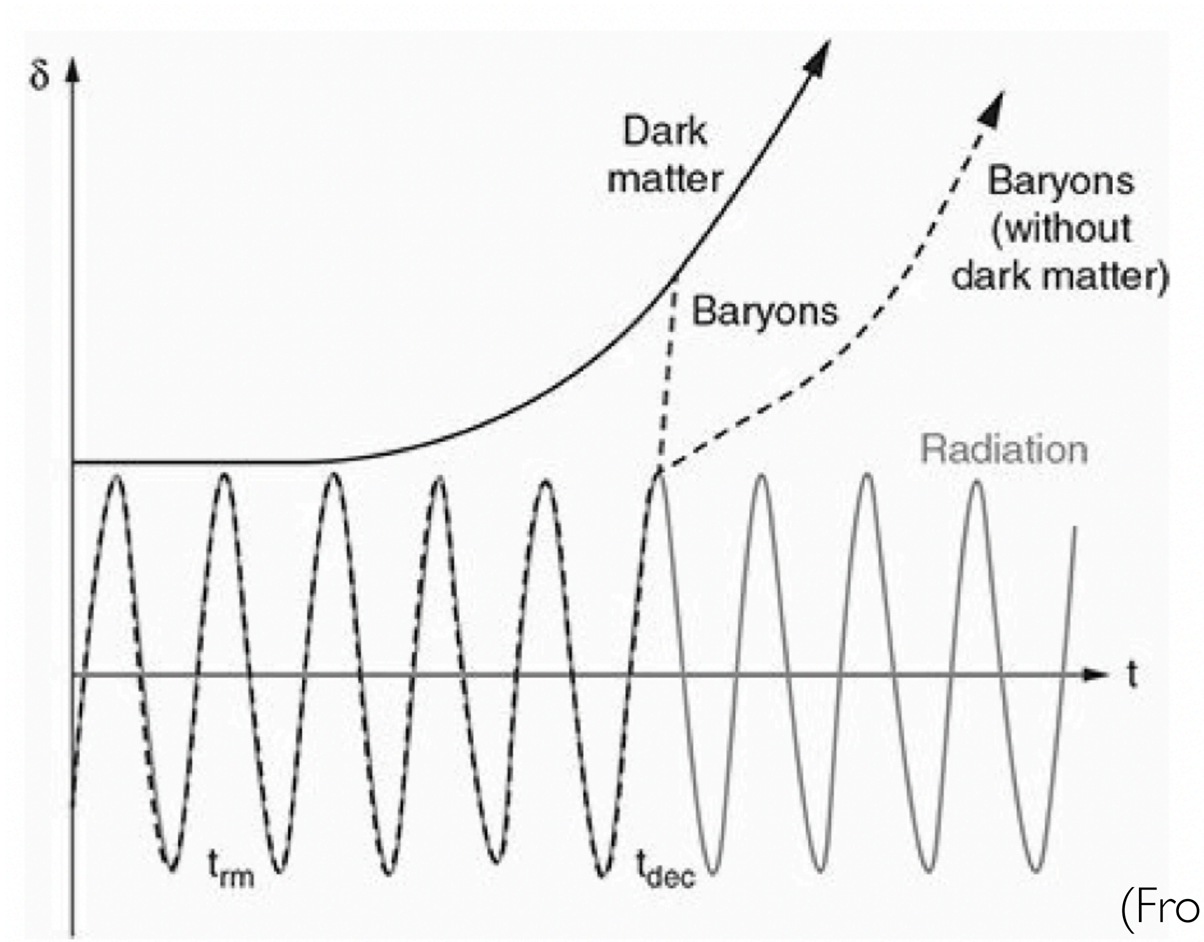
Halo core production?

Positive observations	σ/m	$v_{\rm rel}$	Observation	Refs.
Cores in spiral galaxies (dwarf/LSB galaxies)	$\gtrsim 1\ \mathrm{cm}^2/\mathrm{g}$	$30 - 200$ km/s	Rotation curves	[102, 116]
Too-big-to-fail problem				
Milky Way	$\gtrsim 0.6\ \mathrm{cm}^2/\mathrm{g}$	50 km/s	Stellar dispersion	[110]
Local Group	$\gtrsim 0.5\ \mathrm{cm}^2/\mathrm{g}$	50 km/s	Stellar dispersion	[111]
Cores in clusters	$\sim 0.1\ \mathrm{cm}^2/\mathrm{g}$	1500 km/s	Stellar dispersion, lensing	[116, 126]
Abell 3827 subhalo merger	$\sim 1.5\ \mathrm{cm}^2/\mathrm{g}$	1500 km/s	DM-galaxy offset	[127]
Abell 520 cluster merger	$\sim 1\ \mathrm{cm}^2/\mathrm{g}$	$2000 - 3000$ km/s	DM-galaxy offset	[128, 129, 130]
Constraints				
Halo shapes/ellipticity	$\lesssim 1\ \mathrm{cm}^2/\mathrm{g}$	1300 km/s	Cluster lensing surveys	[95]
Substructure mergers	$\lesssim 2\ \mathrm{cm}^2/\mathrm{g}$	$\sim 500 - 4000$ km/s	DM-galaxy offset	[115, 131]
Merging clusters	$\lesssim$ few cm^2/g	$2000 - 4000$ km/s	Post-merger halo survival (Scattering depth $\tau < 1$)	Table II
Bullet Cluster	$\lesssim 0.7\ \mathrm{cm}^2/\mathrm{g}$	4000 km/s	Mass-to-light ratio	[106]

TABLE I: Summary of positive observations and constraints on self-interaction cross section per DM mass. Italicized observations are based on *single individual systems*, while the rest are derived from sets of multiple systems. Limits quoted, which assume constant σ/m, may be interpreted as a function of collisional velocity $v_{\rm rel}$ provided σ/m is not steeply velocity-dependent. References noted here are limited to those containing quoted self-interaction cross section values. Further references, including original studies of observations, are cited in the corresponding sections below.

Tulin & Yu 2018

The CMB: dark matter is nearly cold



(From B. Ryden)

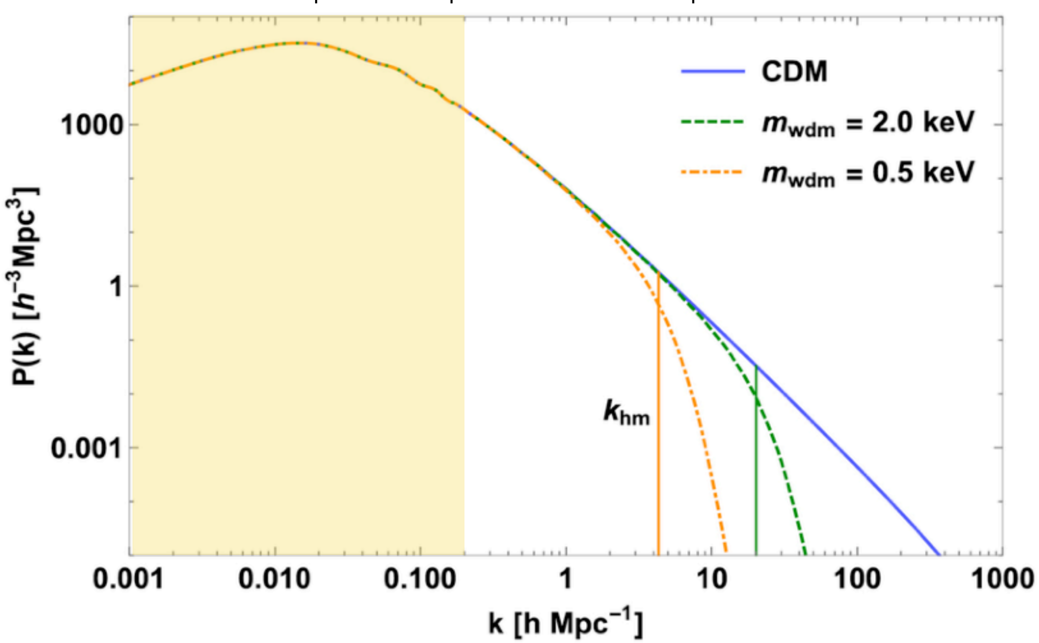
Without dark matter, structures would not have enough time to grow

$$\ddot{\delta}(t) + 2H(t)\dot{\delta}(t) = \frac{3}{2}H^2(t)\,\Omega_m(t)\,\delta(t)$$

The CMB: dark matter is nearly cold

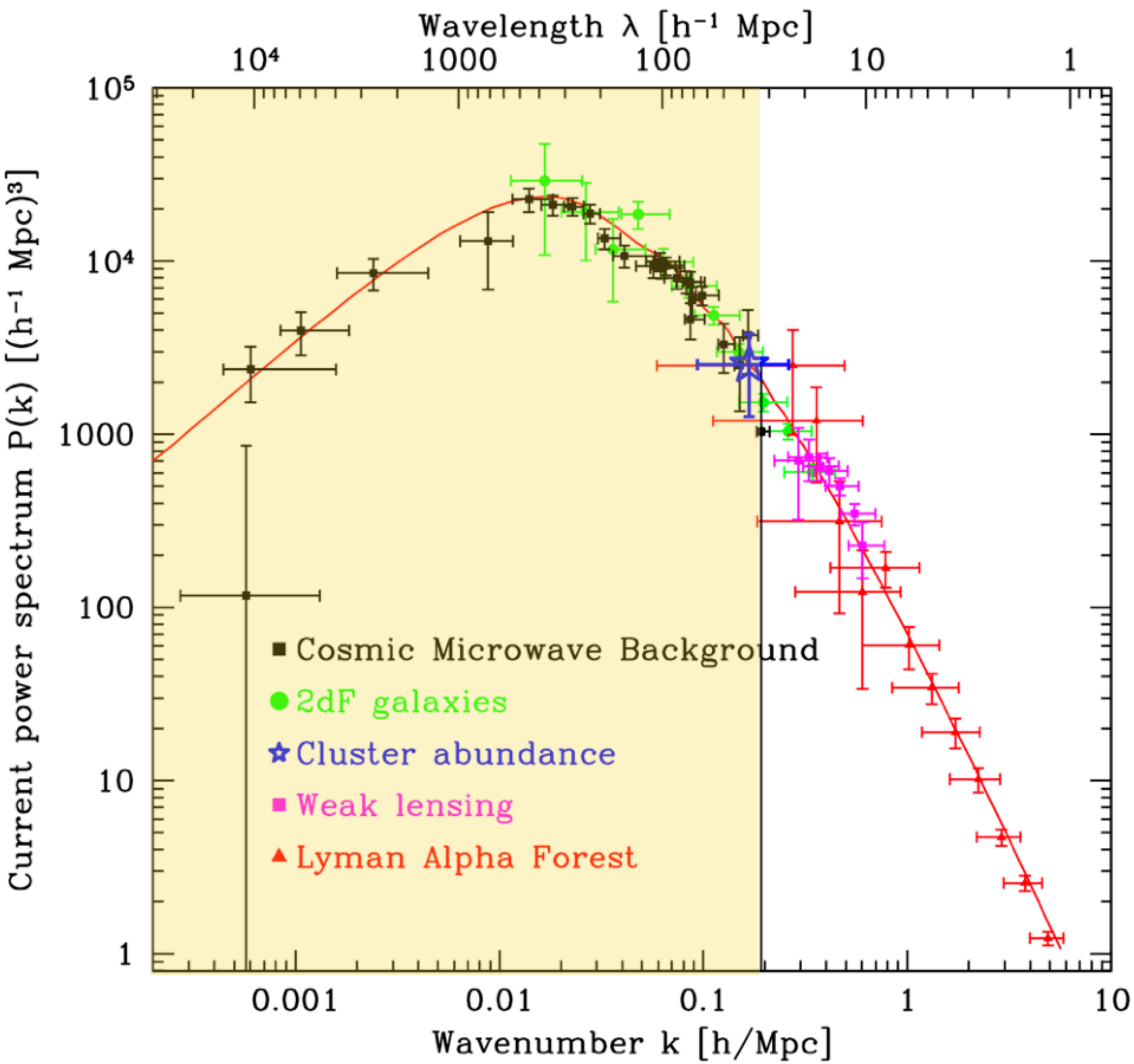
Observable	Scale probed	What it tells us
CMB (Planck, etc.)	$k \lesssim 0.2\ \mathrm{Mpc}^{-1}$	Dark matter was cold and clustering by recombination
Lyman-α forest	$k \sim 0.5\text{–}5\ \mathrm{Mpc}^{-1}$	Confirms presence of small-scale power → CDM
Low-z structure (galaxies, halos)	Even smaller scales in non-linear regime	Further rules out WDM, confirms CDM hierarchy

Linear power spectrum extrapolated at z=0



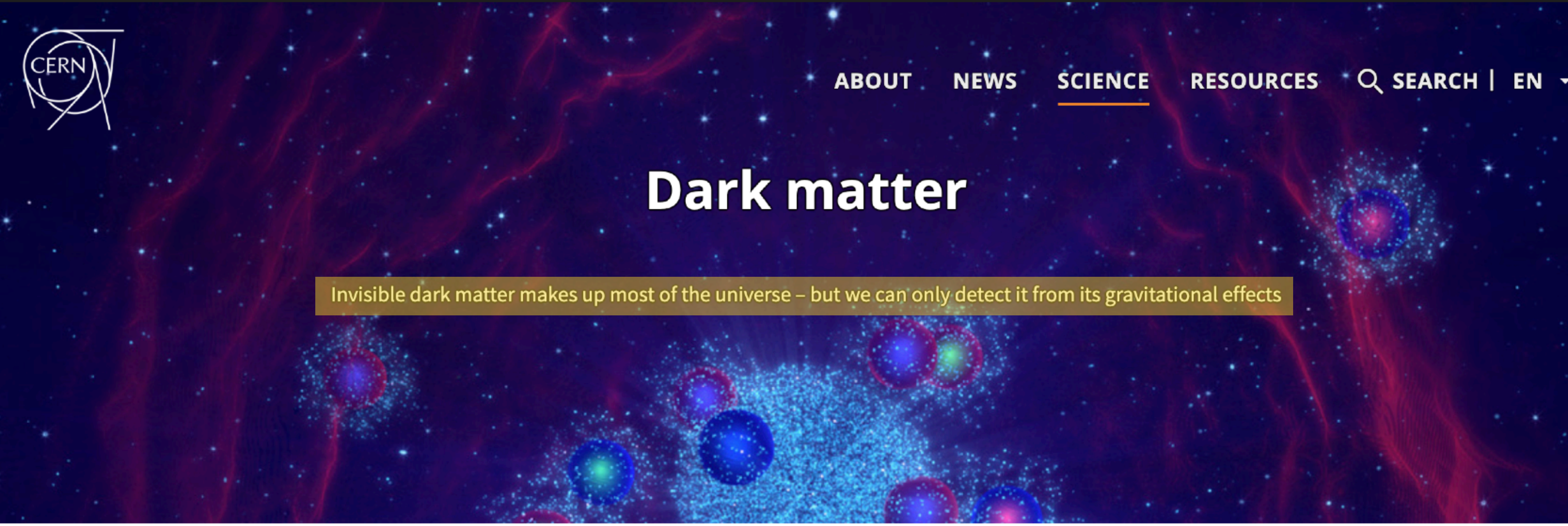
1. **High-k coverage**:
   - CDM maintains power even on **very small scales**
   - WDM (or HDM) **suppresses fluctuations** below k_{hm}
2. **CMB probes intermediate scales** (~$k \lesssim 0.2\,\mathrm{Mpc}^{-1}$):
   - CDM and WDM look almost identical → CMB **cannot distinguish** between them
3. **Low-redshift and small-scale probes** needed:
   - **Lyman-α forest** examines $k \sim 0.5\text{–}5$ h/Mpc
   - **Galaxy surveys** and **dwarf galaxy counts** probe even deeper ($k \gtrsim 10$ h/Mpc)
   - Observing **unattenuated power** at these scales is **strong evidence** for CDM

The CMB: dark matter is nearly cold



Tegmark & Zaldarriaga 2002

Do we know what DM is not?



Dark matter

Invisible dark matter makes up most of the universe – but we can only detect it from its gravitational effects

Galaxies in our universe seem to be achieving an impossible feat. They are rotating with such speed that the gravity generated by their observable matter could not possibly hold them together; they should have torn themselves apart long ago. The same is true of galaxies in clusters, which leads scientists to believe that something we cannot see is at work. They think something we have yet to detect directly is giving these galaxies extra mass, generating the extra gravity they need to stay intact. This strange and unknown matter was called "dark matter" since it is not visible.

Dark matter

Unlike normal matter, dark matter does not interact with the electromagnetic force. This means it does not absorb, reflect or emit light, making it extremely hard to spot. In fact, researchers have been able to infer the existence of dark matter only from the gravitational effect it seems to have on visible matter. Dark matter seems to outweigh

The Dark Matter paradigm

Dark matter does **not emit/reflect/absorb light**

Dark matter is **not interacting strongly with baryons**

Dark matter is not made of "**baryonic**" material (i.e. it belongs to a "dark sector")

“DM as sub-atomic particles” $\Longrightarrow$ The Dark Matter paradigm

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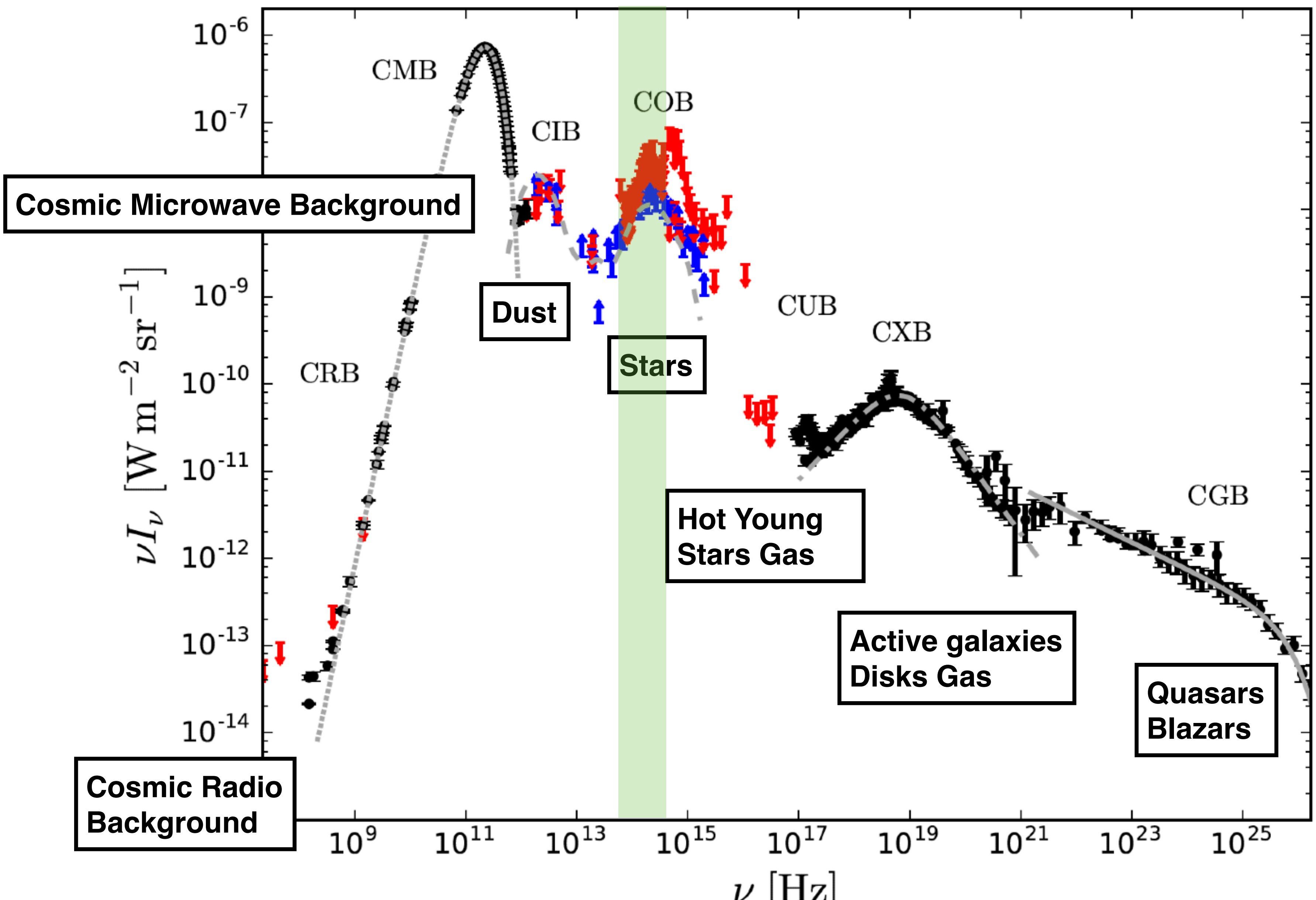
The Dark Matter paradigm $\Longrightarrow$ DM is sub-atomic particles

Myth no1: "Dark matter does not reflect, absorb or emit light"
It is dark relative to something that is not, within our instrumentation capability, not black

The monopole intensity of the sky as a function of frequency



Consider DM to be made of spherical clumps of radius R, emitting black body radiation at temperature T. Each clump emits luminosity:

$$L = 4\pi R^2 \sigma_{\rm SB} T^4$$

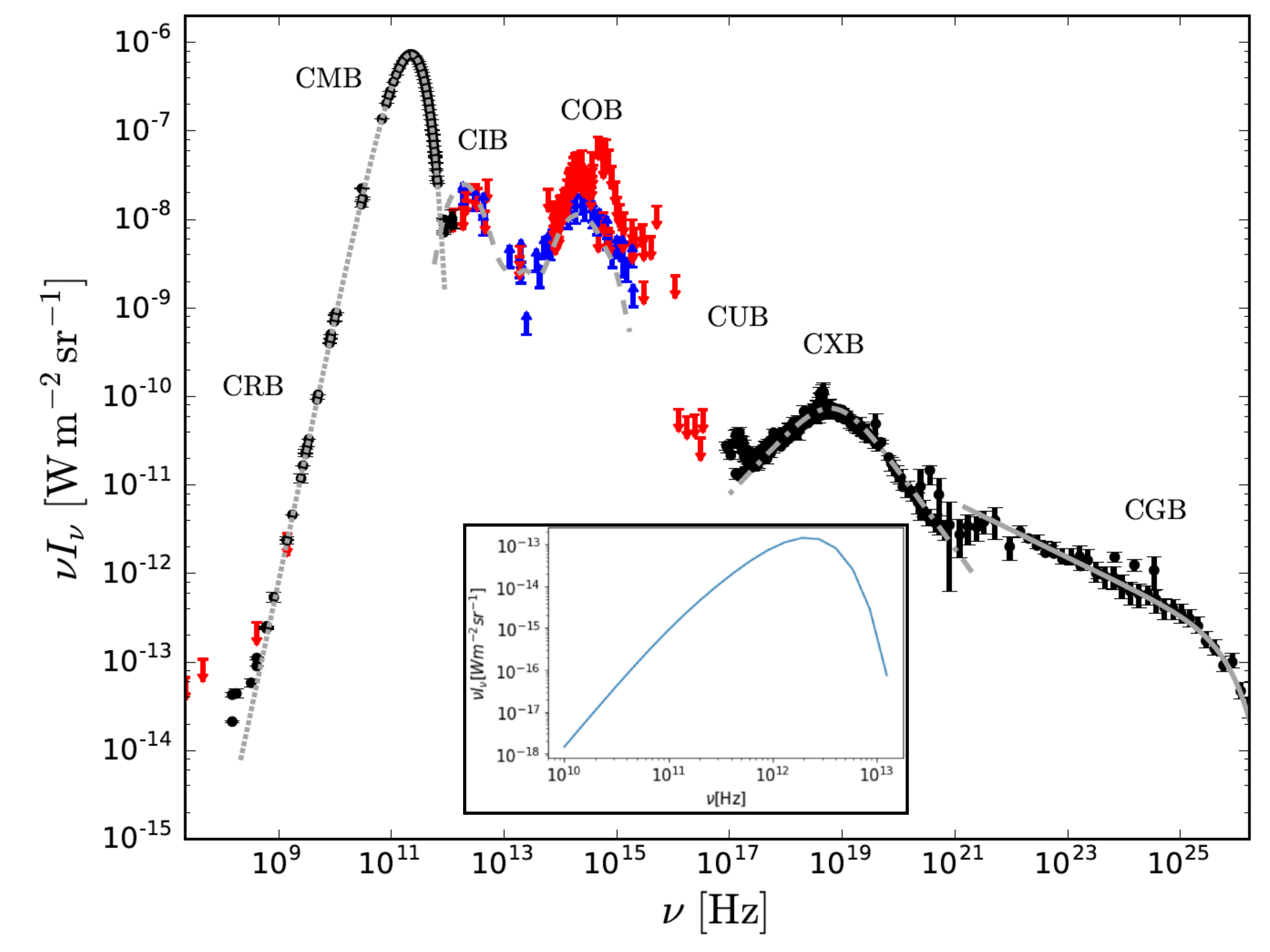
The total luminosity of dark matter scales as:

$$L_{\rm tot} \propto L n_{\rm DM} \propto R^{-1}$$

Where $n_{\rm DM} \propto R^{-3}$ is the number density of DM clumps.

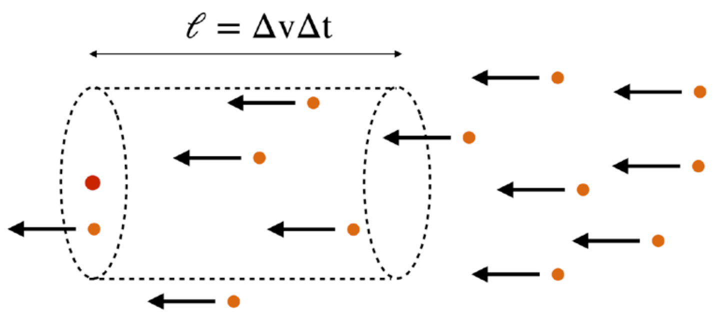
Calculate the observed monopole intensity νI_ν (in $W{\rm m}^{-2}{\rm sr}^{-1}$) with:

$$\rho_{\rm nucl} \simeq 2 \times 10^{14} \ {\rm g\ cm}^{-3}$$
$$R \simeq 10^{-6} \ {\rm cm}$$
$$n_{\rm DM} \simeq 10^{-9} \ {\rm km}^{-3}$$
$$T = 40 \ {\rm K}$$



CMB
CIB
COB
CUB
CXB
CRB
CGB
νI_ν [W m^{-2}sr^{-1}]
ν [Hz]
$\nu I_\nu [Wm^{-2}sr^{-1}]$
ν[Hz]

Myth no2: "Dark matter does not interact with baryons"
If DM is a sub-atomic particle $\Longrightarrow$ it is everywhere (high n)
if not seen $\Longrightarrow$ **very small cross section σ**



Dark matter collisionless over scale ℓ:

$$\sigma_X n_X \ell \equiv 1$$

Which can be fulfilled with: high n, low σ **or** low n, high σ

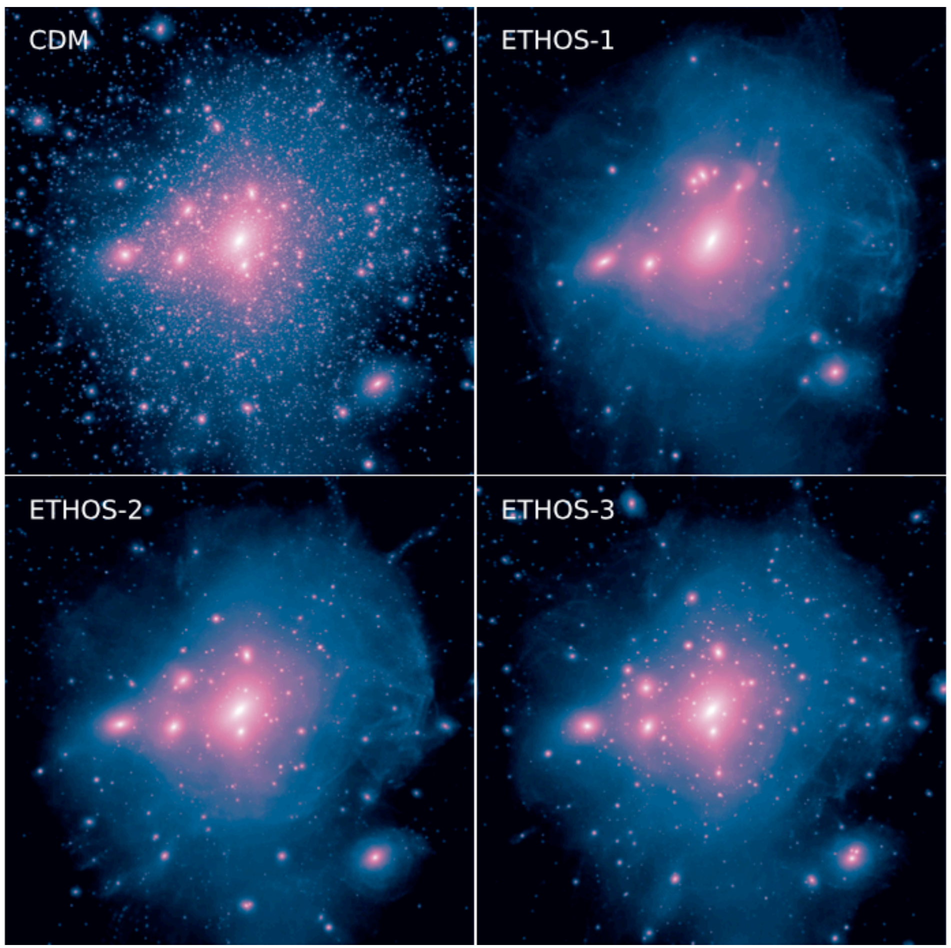
$$\sigma_X n_X \ell = \left(\frac{\sigma_X}{m_X}\right) \rho_X \ell$$

Observations say that DM is collisionless, not that it has a small cross-section

Constraints on σ_{XX} from DM-DM interaction:

-From Large scale structures:

Self-interacting (or strongly interacting) dark matter will change the mass power spectrum (Vogelsberger et al 2016)



$$\sigma_{XX}/m_X < 0.025 \ \mathrm{cm}^2/\mathrm{g}$$

Constraints on $\sigma_{X-\mathrm{bar}}$ from DM-baryons interaction:

-From Large scale structures:
a- DM-baryon interaction would cause extra damping of the acoustic oscillations in the baryon-photon plasma (Boehm et al, 2001, 2002, 2004)

b- Lyman-α observations (Dvorkin et al 2014)

c- Consistency of CMB and LSS power spectrum (Chen et al 2002)

$$\sigma_{X-\mathrm{bar}}/m_X < 0.003\ \mathrm{cm}^2/\mathrm{g}$$

-From gas heating
a- High m_x implies higher energy transfer to baryons in collisions, hence gas heating (Chuzhoy & Nusser, 2006)

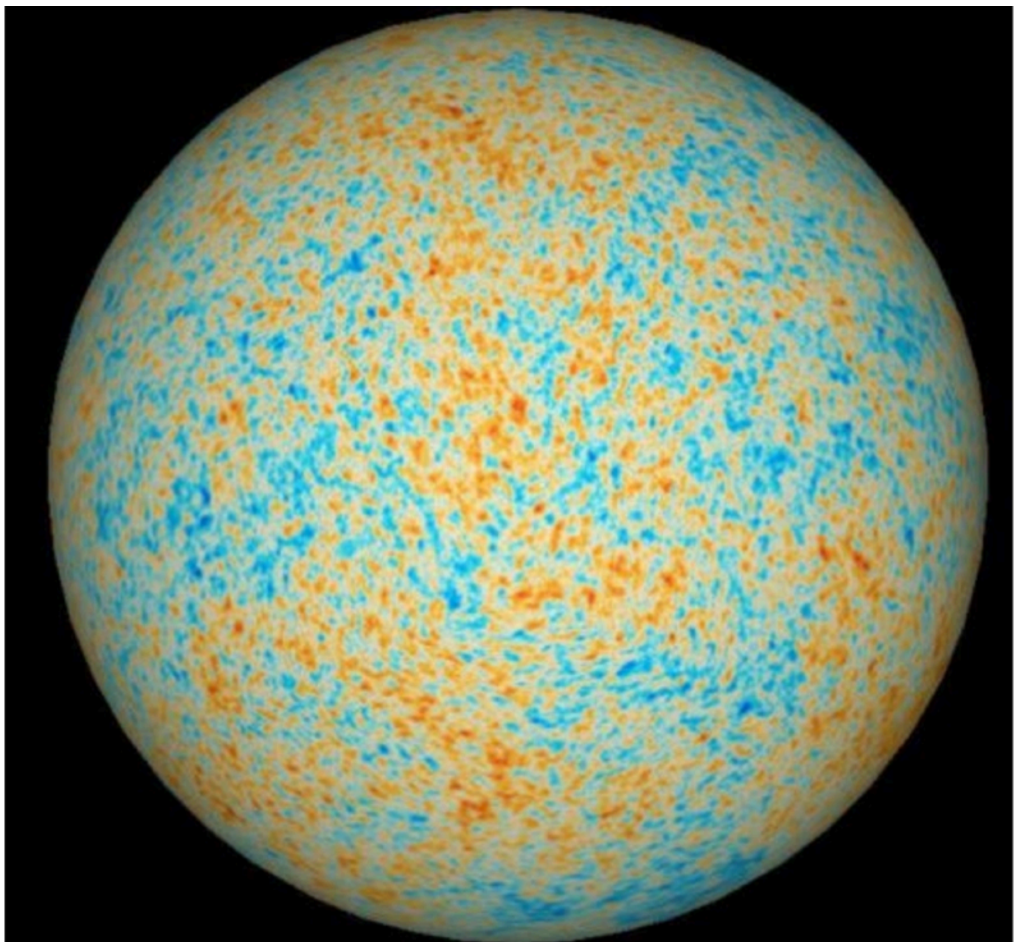
b- Gas should be hotter at the center of clusters (e.g. bullet cluster)

$$\sigma_{X-\mathrm{bar}}/m_X < 0.06\ \mathrm{cm}^2/\mathrm{g}$$

Constraints on $\sigma_{X-\text{photon}}$ from DM-photon interaction:

-From CMB

DM-photon interaction would cause extra damping of the acoustic peaks (Wilkinson et al 2014)



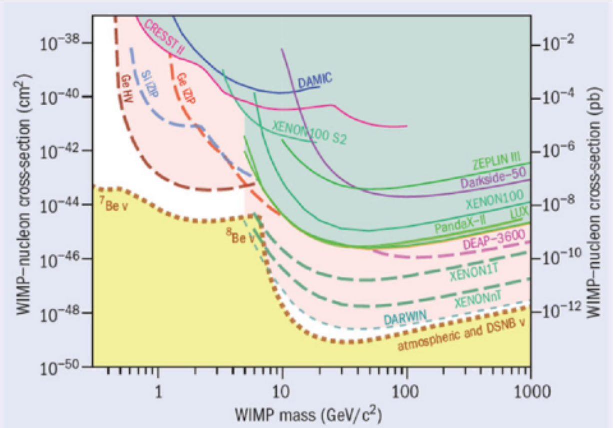
$$\sigma_{X-\text{photon}}/m_X < 4.5 \times 10^{-7}\ \text{cm}^2/\text{g}$$

Observational constraints on σ/m:

From Large scale structure: $\sigma_{XX}/m_X < 0.025\ \mathrm{cm}^2/\mathrm{g}$

From hot gas in clusters of galaxies, Ly-α: $\sigma_{X-\mathrm{bar}}/m_X < 0.003\ \mathrm{cm}^2/\mathrm{g}$

From the cosmic microwave background: $\sigma_{X-\mathrm{photon}}/m_X < 10^{-7}\ \mathrm{cm}^2/\mathrm{g}$

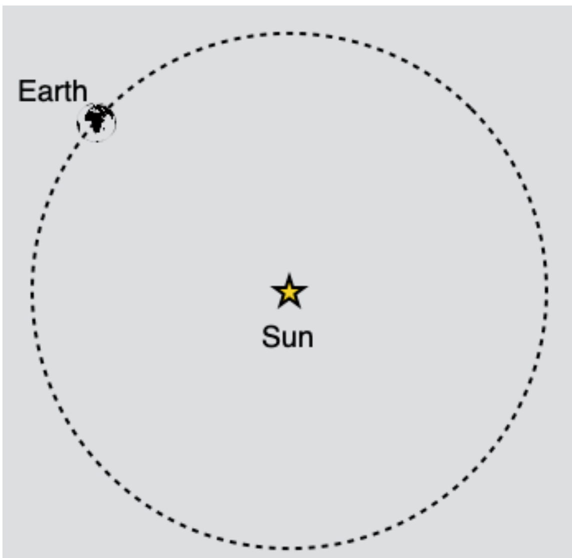


In comparison, $(\sigma_X/m_X)_{\mathrm{WIMPs}} \sim 10^{-20}\ \mathrm{cm}^2\mathrm{g}^{-1}$

There is more than 10 orders of magnitude of unexplored territory between The WIMPs and the observational limit.

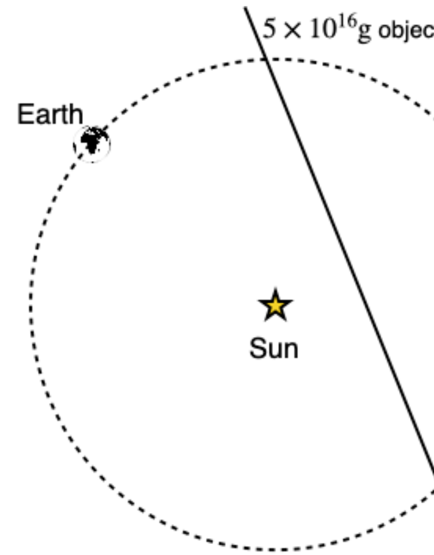
Low σ/m does not constrain σ nor n

DM as a sub-atomic particles



- Dark matter particles are everywhere $n \sim 1\ \mathrm{m}^{-3}$
- Very Weakly interacting with baryons
- Not emitting radiation

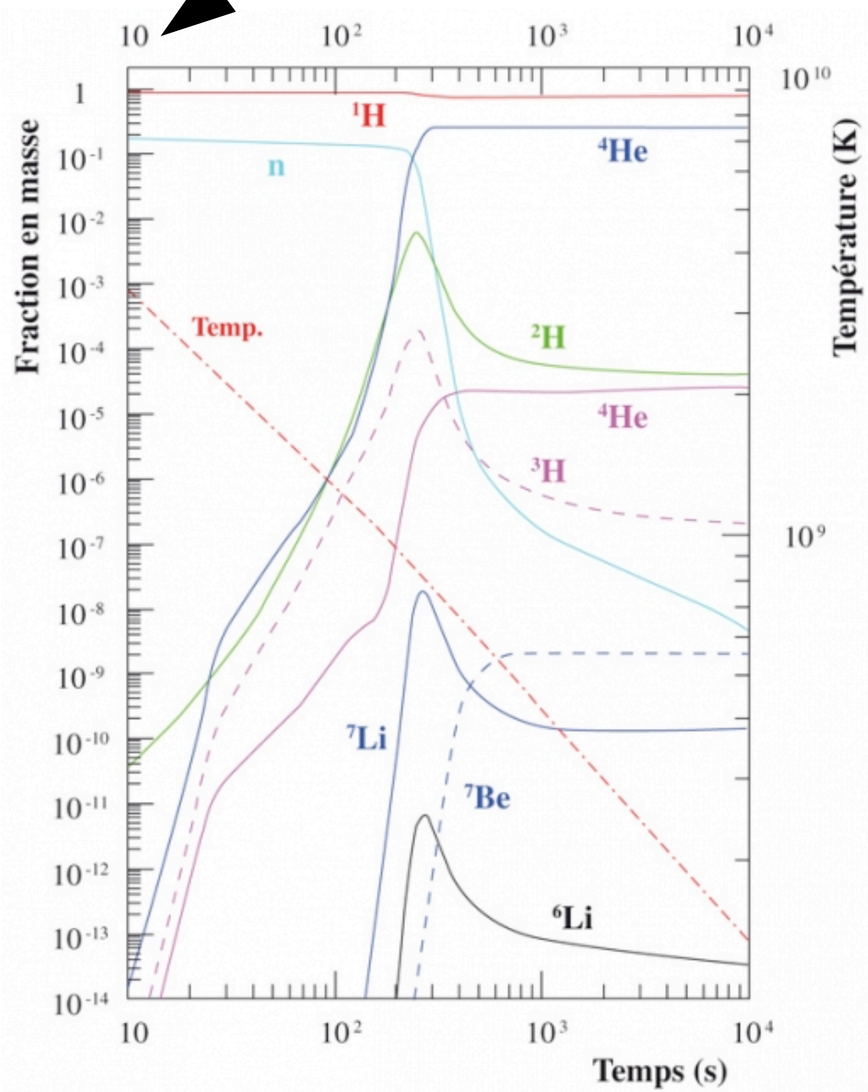
DM as Macros

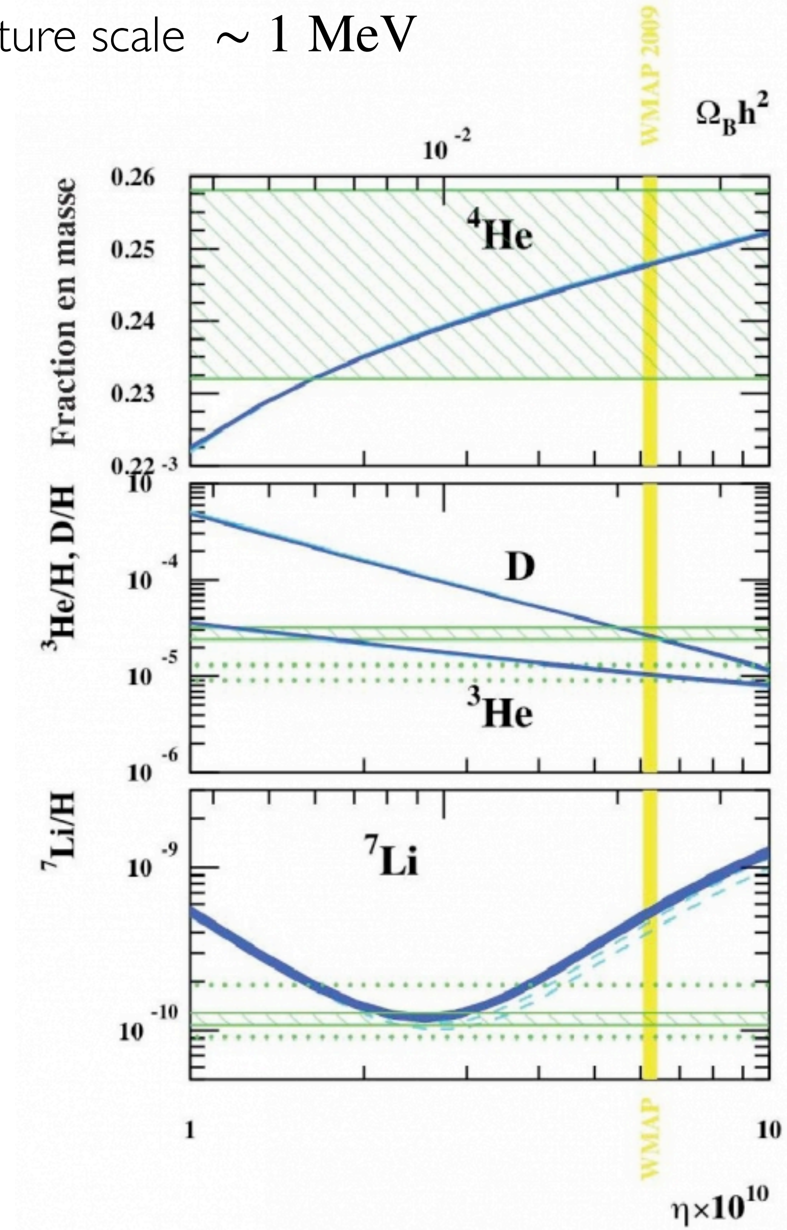


- Dark matter objects are rare $n \ll 1\ \mathrm{m}^{-3}$
- Could strongly interact with baryons
- Could emit radiation
- **No chance to detect anything on Earth**

Myth no3: Dark matter is made of "non-baryonic" matter
Big Bang Nucleosynthesis implies that DM cannot interfere with the formation of the lightest elements during the first three minute of the universe (D, He, Li)

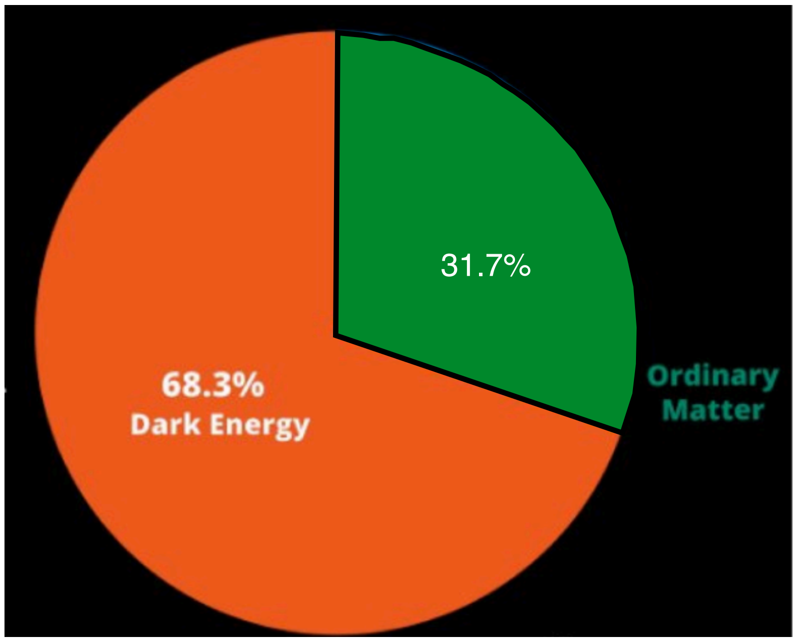
Primordial nucleosynthesis temperature scale ~ 1 MeV





C. Pitrou's lecture

Myth no3: Dark matter is made of "non-baryonic" matter
Big Bang Nucleosynthesis implies that DM cannot interfere with the formation of the lightest elements during the first three minute of the universe (D, He, Li)

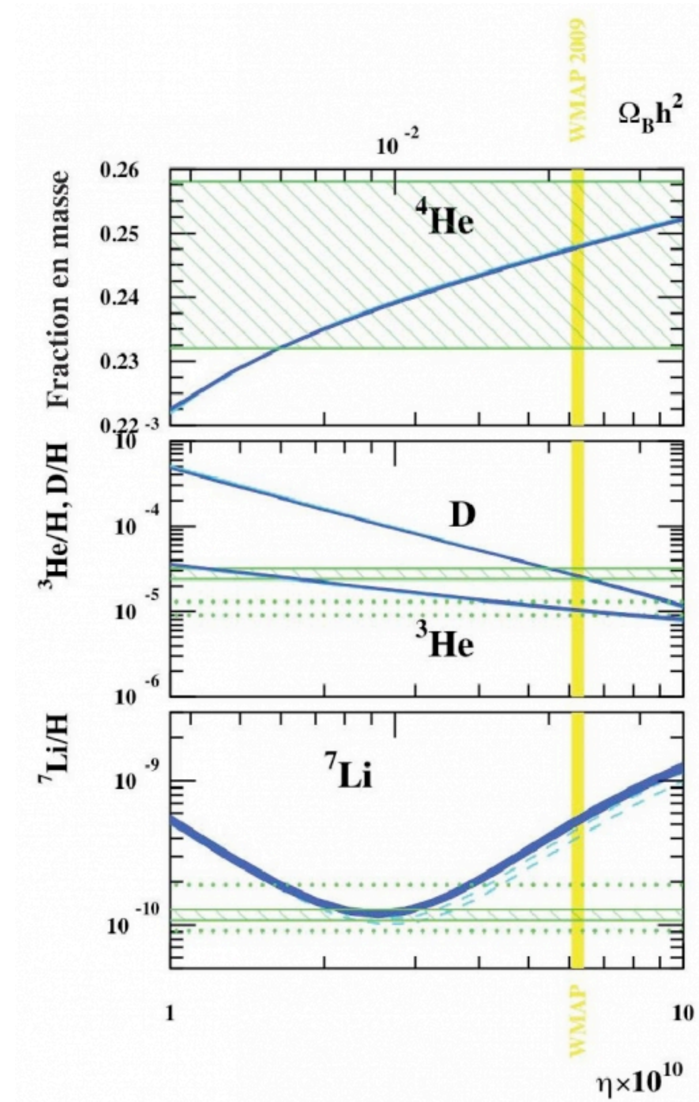


$\Omega_B \to 6\ \Omega_B$

$D \to D/20$

$He^3 \to He^3/2$

$Li \to 10\ Li$



There are only two options:

① If DM is baryonic:

- It has to be made before BBN
- It is likely a composite object with binding energy $\gg 1$ MeV
- It could easily explain why $\Omega_{DM} \sim \Omega_{bar}$
- It has to evade direct detection, death by DM, etc…

② If DM is non baryonic, elementary particle, it must:

- Small σ and high n
- Explain the factor ~5 in Ω's
- Be a testable extension of the SM
- Not be fined tuned

The Dark Matter Paradigm:

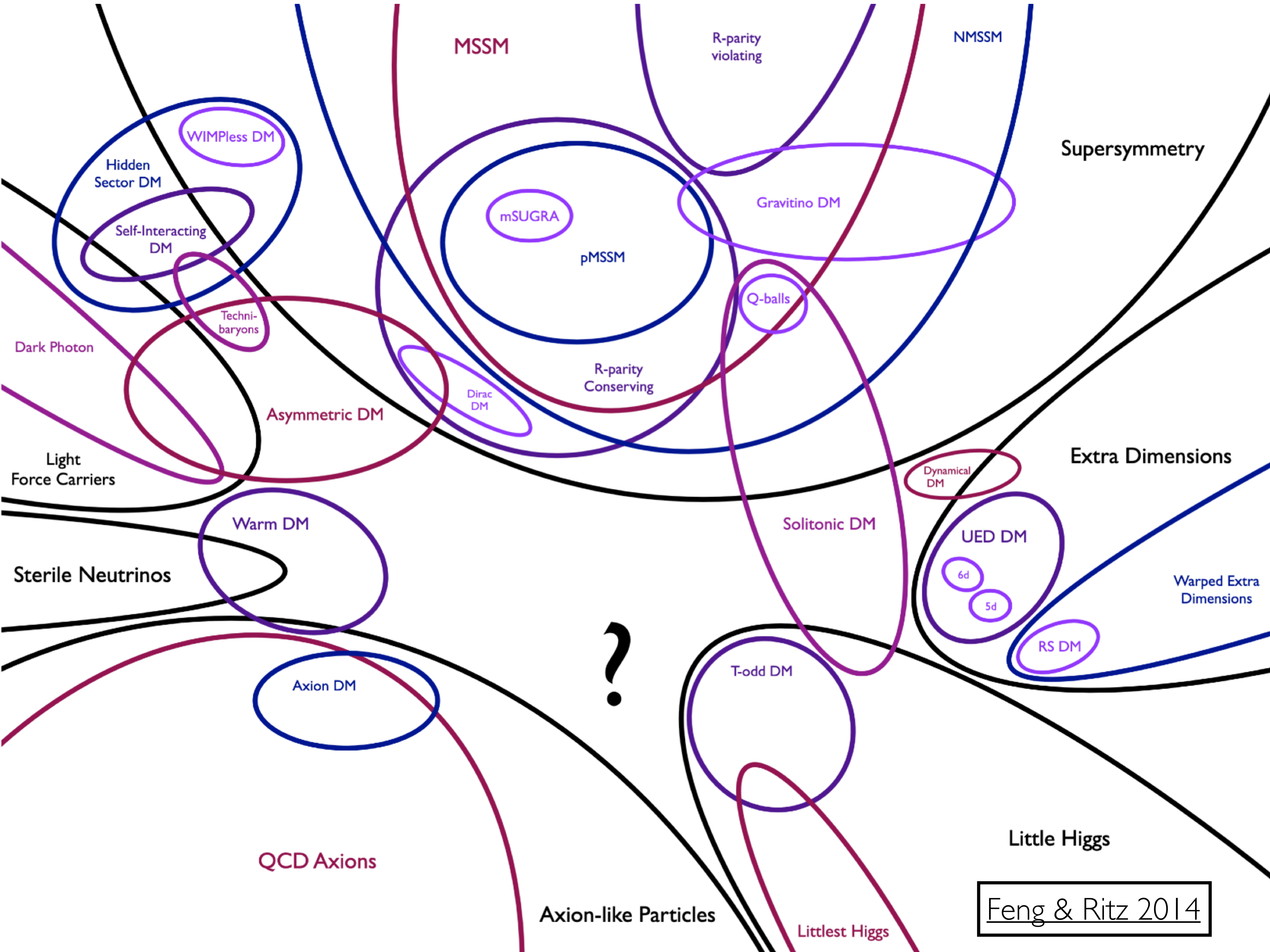
~~Dark matter is **not emitting light**~~

~~Dark matter is **very weakly interacting**~~

~~Dark matter is not made of **baryonic** material~~

There is no observational evidence to support this paradigm!

Overview of the most popular alternative DM models


MSSM
R-parity violating
NMSSM
Supersymmetry
WIMPless DM
Hidden Sector DM
Self-Interacting DM
Techni-baryons
mSUGRA
pMSSM
Gravitino DM
Q-balls
Dark Photon
R-parity Conserving
Dirac DM
Asymmetric DM
Light Force Carriers
Extra Dimensions
Dynamical DM
Warm DM
Solitonic DM
UED DM
Sterile Neutrinos
6d
5d
Warped Extra Dimensions
RS DM
?
T-odd DM
Axion DM
Little Higgs
QCD Axions
Axion-like Particles
Littlest Higgs
Feng & Ritz 2014

Signature	Dirac SUSY DM	WIMPless DM	Little Higgs DM	Extra-Dimensional DM	ALPs / Fuzzy / WDM
Nature / spin	Dirac fermion	Hidden-sector particle	Heavy gauge boson (vector)	KK photon / graviton (vector)	Light scalar/pseudoscalar (ALP), boson (fuzzy), fermion (WDM)
Mass scale	100 GeV–TeV	MeV–TeV (flexible)	100 GeV–1 TeV	∼TeV	neV–keV
Relic abundance	Thermal WIMP-like	Thermal (not weak-scale)	Thermal WIMP-like	Thermal WIMP-like	Non-thermal or suppressed thermal
Direct detection	SI, Higgs/Z mediated	Often suppressed (hidden sector)	Higgs portal coupling	Higgs/Z mediated	Very weak; ALP–photon coupling
Indirect detection	Annihilation to SM	Hidden-sector annihilation possible	Annihilation to bosons	Similar to WIMPs	ALP $\to \gamma\gamma$, fuzzy DM minimal
CMB / BBN effects	Like standard WIMPs	Possible energy injection	Mild to none	Standard WIMP scenario	ΔN_{eff} for ALPs, negligible for fuzzy
Structure formation	Standard CDM	CDM-like	Standard CDM	Standard CDM	Small-scale suppression (fuzzy/WDM)
Small-scale structure	No resolution to core/cusp	No unique feature	No resolution to core/cusp	No resolution to core/cusp	Fuzzy: cores; WDM: fewer subhalos
Gravitational lensing	Standard	Standard	Standard	Microlensing for compact KK states	ALP/fuzzy: weak lensing; boson-star features
Unique features	Inelastic transitions; Dirac interactions	Hidden-sector miracle; suppressed couplings	Vector DM; new resonances	KK tower; TeV-scale cutoff	Coherence, wave-like effects, Lyman-α cutoff

Theoretical limits independent of the model

Unitarity bound: (Griest & Kamionkowski 1990) you cannot have **thermal relic** (DM produced by s-wave annihilation) with arbitrarily large mass $\Longrightarrow$ overcooled universe:

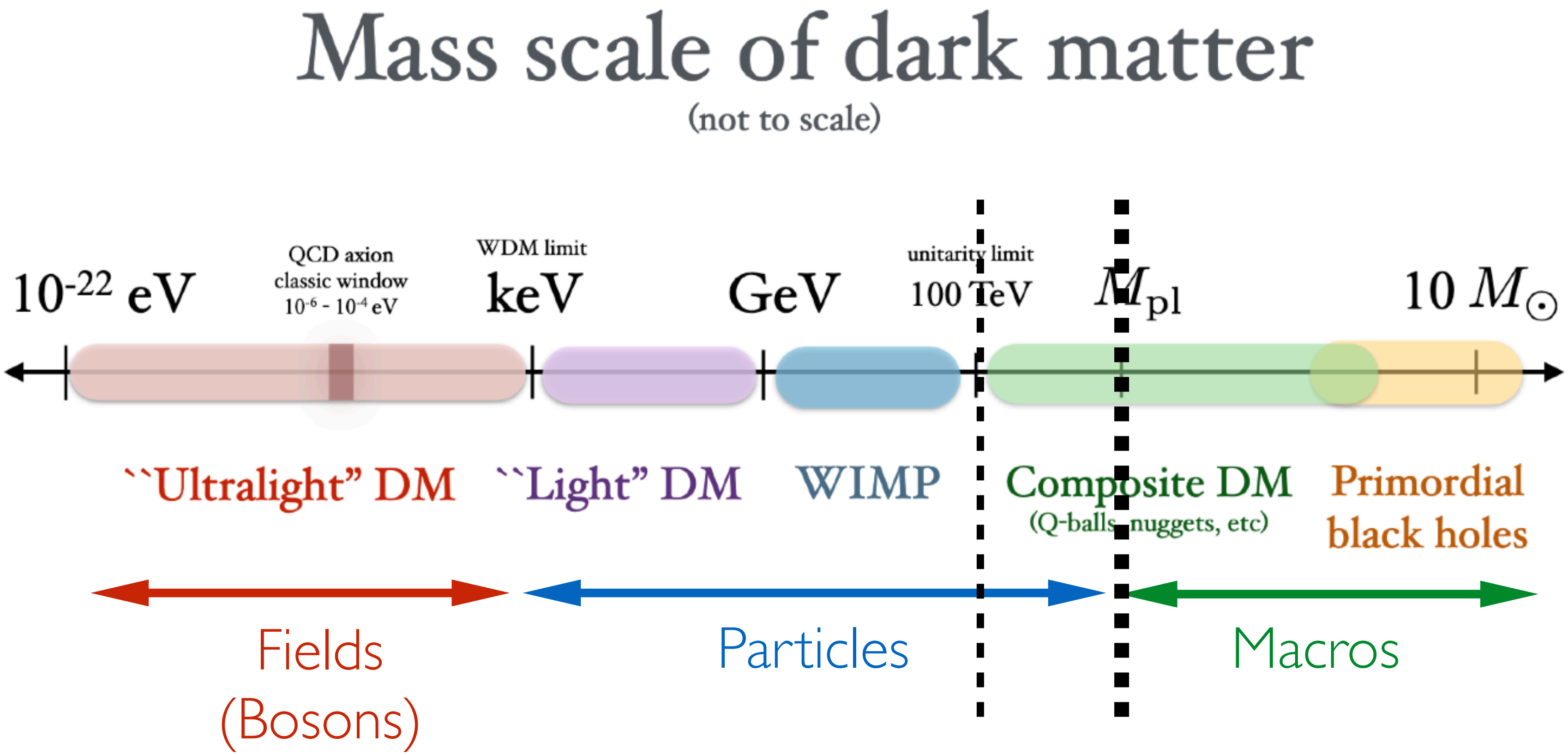
$$\langle \sigma v \rangle \lesssim \frac{1}{m_\chi^2 v} \text{ and } \Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle} \Longrightarrow M \lesssim 100 \text{ TeV}$$

Gunn & Tremaine bound: (Gunn & Tremaine 1979) **Fermionic** DM cannot occupy more than one quantum state (Pauli exclusion principle)

$$m_\chi \gtrsim \left(\frac{9\pi}{4}\right)^{1/4} \left(\frac{\hbar^3}{G^{3/2}}\right)^{1/4} \left(\frac{\rho_0}{\sigma^3}\right)^{1/4} \sim 1 - 2 \text{ keV}$$

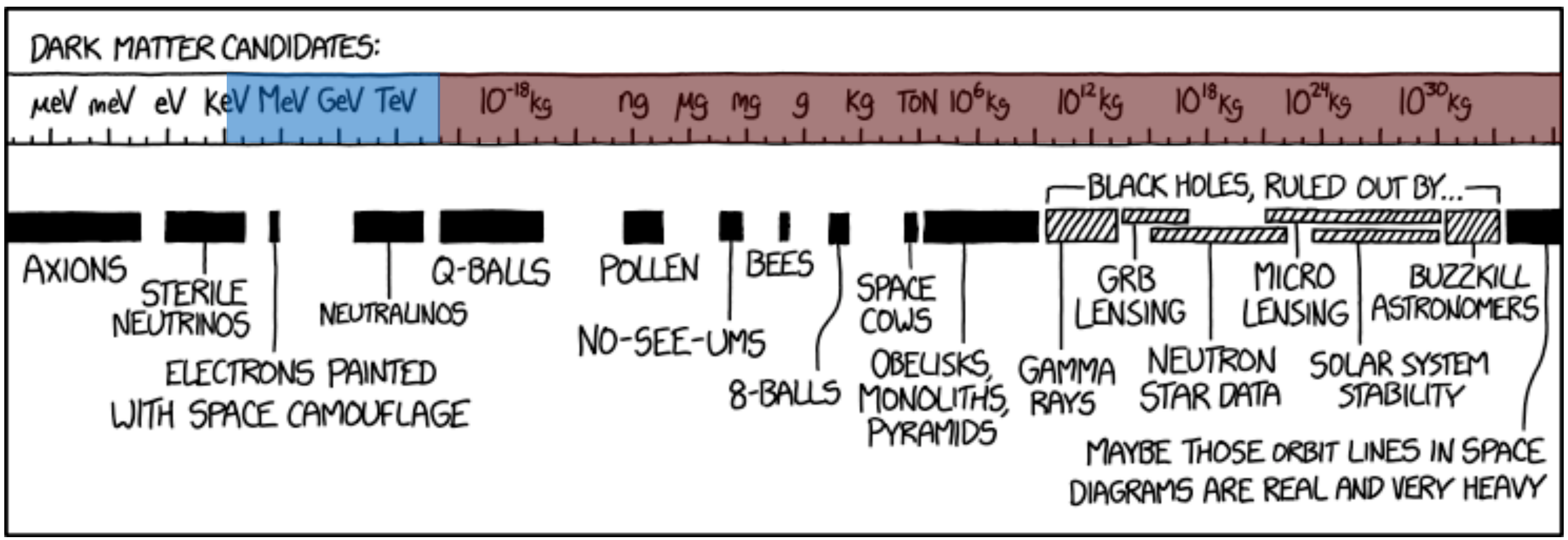
Planck mass bound: A particle with $m_\chi > M_{\rm Pl}$ would be a **black hole**, not a particle

The Nature of Dark Matter: mass range



Lin 2019

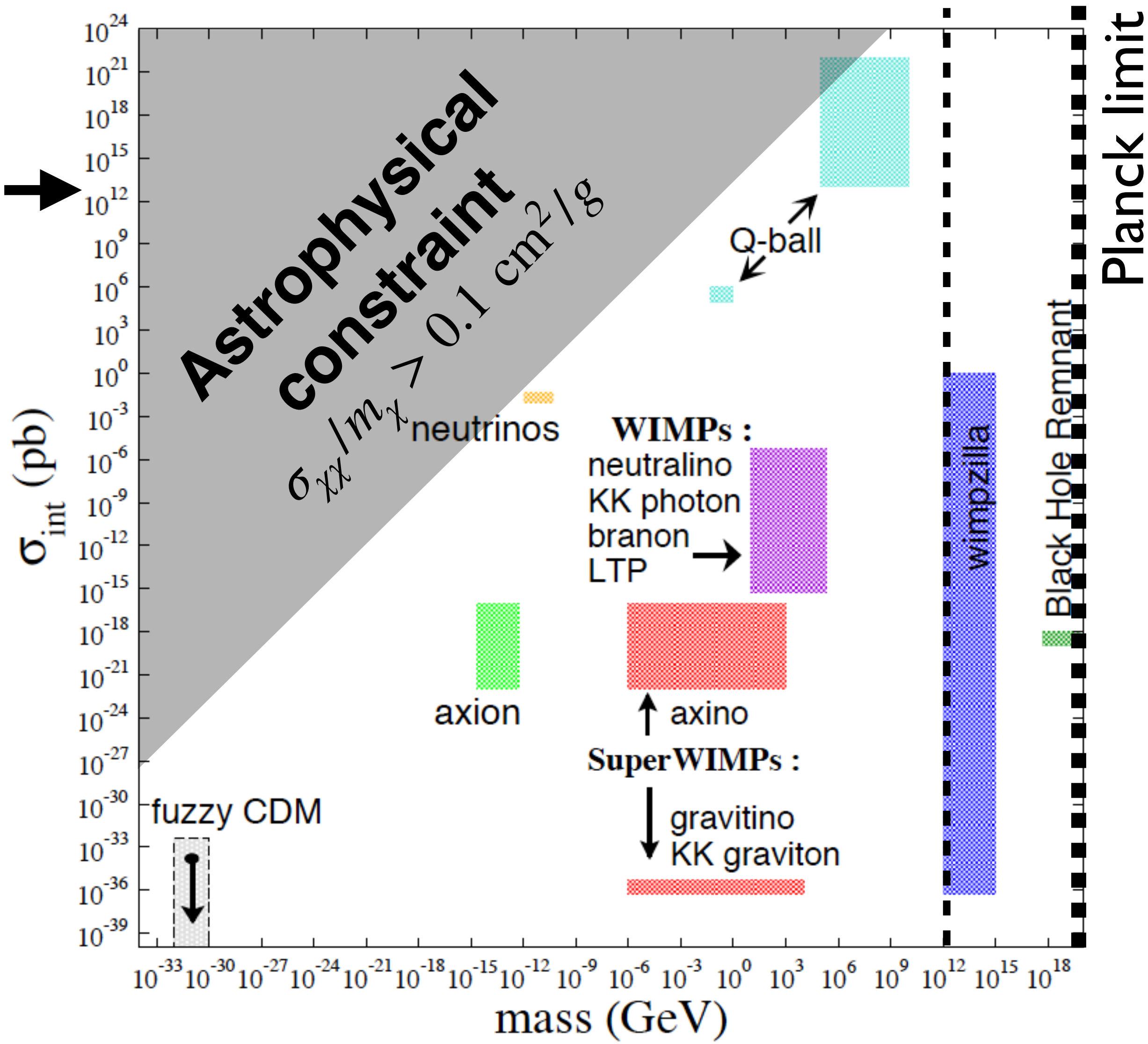
Mass range to scale



https://xkcd.com/2035/

The Nature of Dark Matter: cross-section range

Thomson σ_e
$R \sim 10^{-12}$ cm



Baer 2008

The standard paradigm: the WIMPs model

In the very early universe, DM is in thermal equilibrium with SM particles:

$$\chi + \bar{\chi} \leftrightarrow \text{SM} + \text{SM}$$

Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi,\text{eq}}^2\right)$$

Equilibrium is given by $n_{\chi,\text{eq}}(T) \approx g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} e^{-m_\chi/T}$

The annihilation rate becomes smaller than the Hubble rate:

$$\Gamma_\chi = n_\chi \langle\sigma v\rangle < H$$

Freeze-out occurs when $T_f \approx \frac{m_\chi}{20}$

Relic abundance:

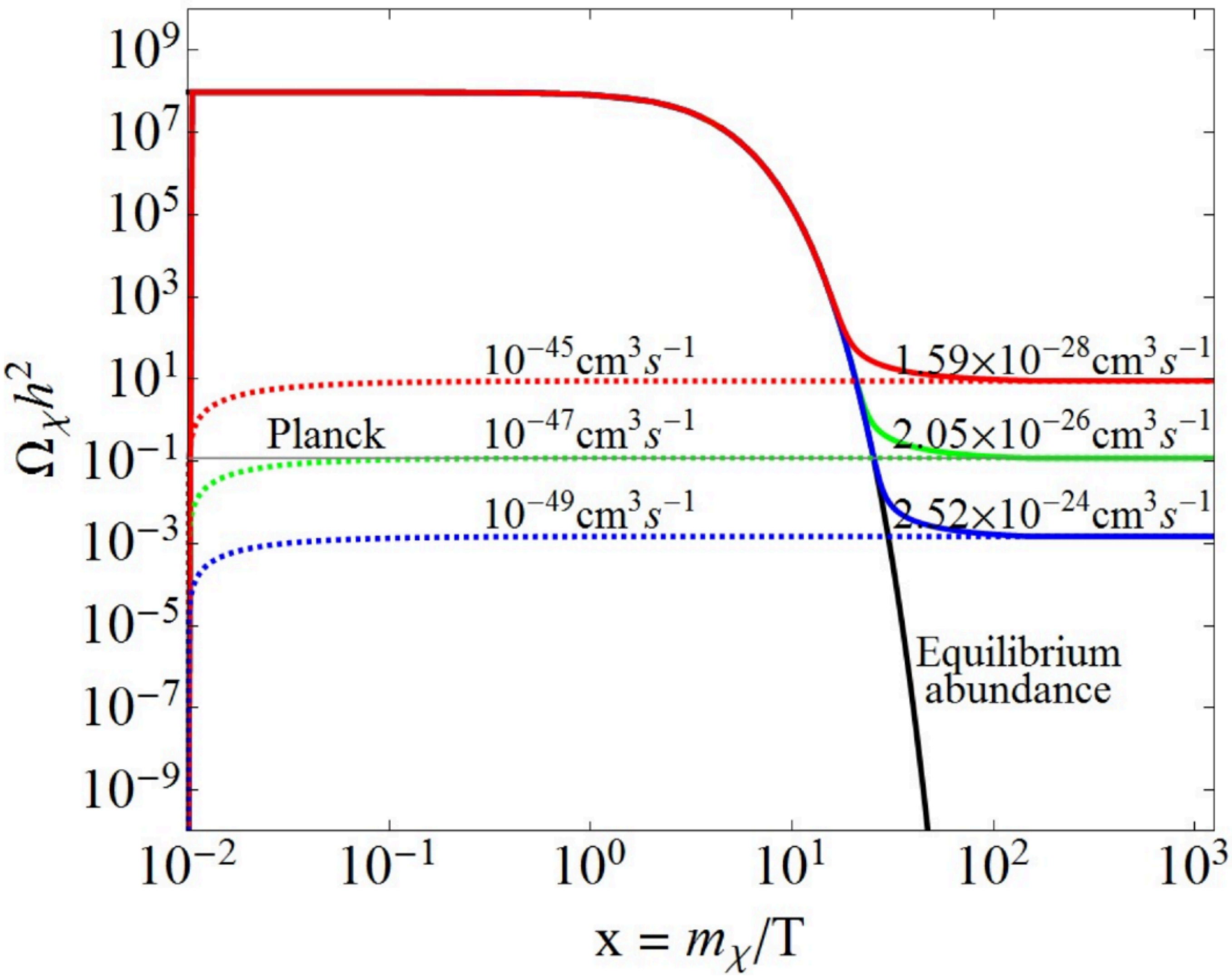
$$\Omega_\chi h^2 \approx \frac{1.07\times10^9\ \text{GeV}^{-1}}{g_*^{1/2} M_{\text{Pl}}\langle\sigma v\rangle} \approx \frac{3\times10^{-27}\ \text{cm}^3/\text{s}}{\langle\sigma v\rangle} \approx 0.12 \quad \Rightarrow \quad \langle\sigma v\rangle \approx 2-3\times10^{-26}\ \text{cm}^3/\text{s}$$

The miracle! $m_\chi \sim 100$ GeV and $\sigma v \sim \alpha^2/m_\chi^2$

$$\Longrightarrow \langle\sigma v\rangle \sim \frac{(10^{-2})^2}{(100\ \text{GeV})^2} \sim 10^{-26}\ \text{cm}^3/\text{s}$$

Weakly Interacting Massive Particles (WIMPs)

Lee & Weinberg 1977
Kolb & Turner "The Early Universe" 1994



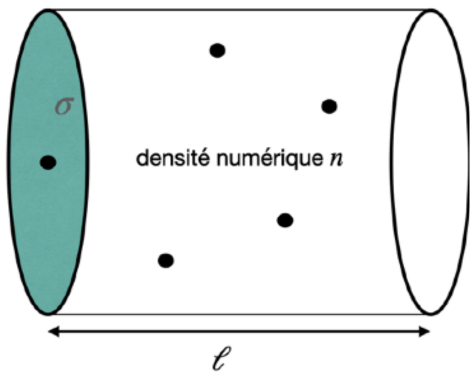
Dev et al 2013

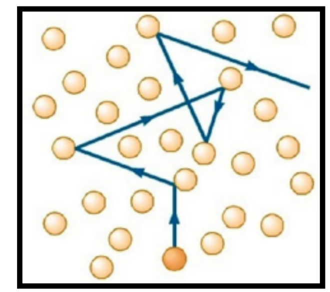
Signature Type	**Observable Effect**	**Key Experiments / Methods**
Direct Detection	Nuclear recoils in detectors, annual modulation due to Earth's motion	XENON, LUX, PandaX, DAMA/LIBRA, DarkSide
Indirect Detection	Excesses in γ-rays, neutrinos, positrons, antiprotons; annihilation signals from dense regions	Fermi-LAT, AMS-02, H.E.S.S., IceCube, Super-Kamiokande
Collider Searches	Missing transverse energy events in particle collisions	LHC (ATLAS, CMS), mono-jet/photon/Z analyses
CMB Constraints	Changes in ionization history due to annihilation, visible in CMB anisotropies	Planck satellite
Structure Formation	Cold dark matter behavior, small-scale suppression, halo formation patterns	SDSS, Euclid, Lyman-α forest

Overview of alternative Dark Matter models

Strongly Self-interacting Dark Matter (SIDM)

$\sigma_{\rm DM} n_{\rm DM} \ell = 1$ defines the mean free path ℓ





$$\sigma_{\rm DM} = 8.1 \times 10^{-25} {\rm cm}^2 \left(\frac{m_{\rm DM}}{GeV}\right) \left(\frac{\ell}{1{\rm Mpc}}\right)^{-1}$$

Using Thomson cross-section $\sigma \sim 6 \times 10^{-25}$ ${\rm cm}^2$, and a DM particle of mass $m_{\rm DM} \sim 1.6 \times 10^{-22}$ g, we get:

$$\sigma / m_{\rm DM} \sim 0.004 \ {\rm cm}^2/{\rm g}$$

Phenomenological model introduced by Spergel & Steinhardt 2000

$\Longrightarrow$ dark matter is collisionless

Strongly Self-interacting Dark Matter (SIDM)

More realistic SIDM needs a **mediator** and a **massive DM particle** (Feng et al 2010, Tulin et al 2013)

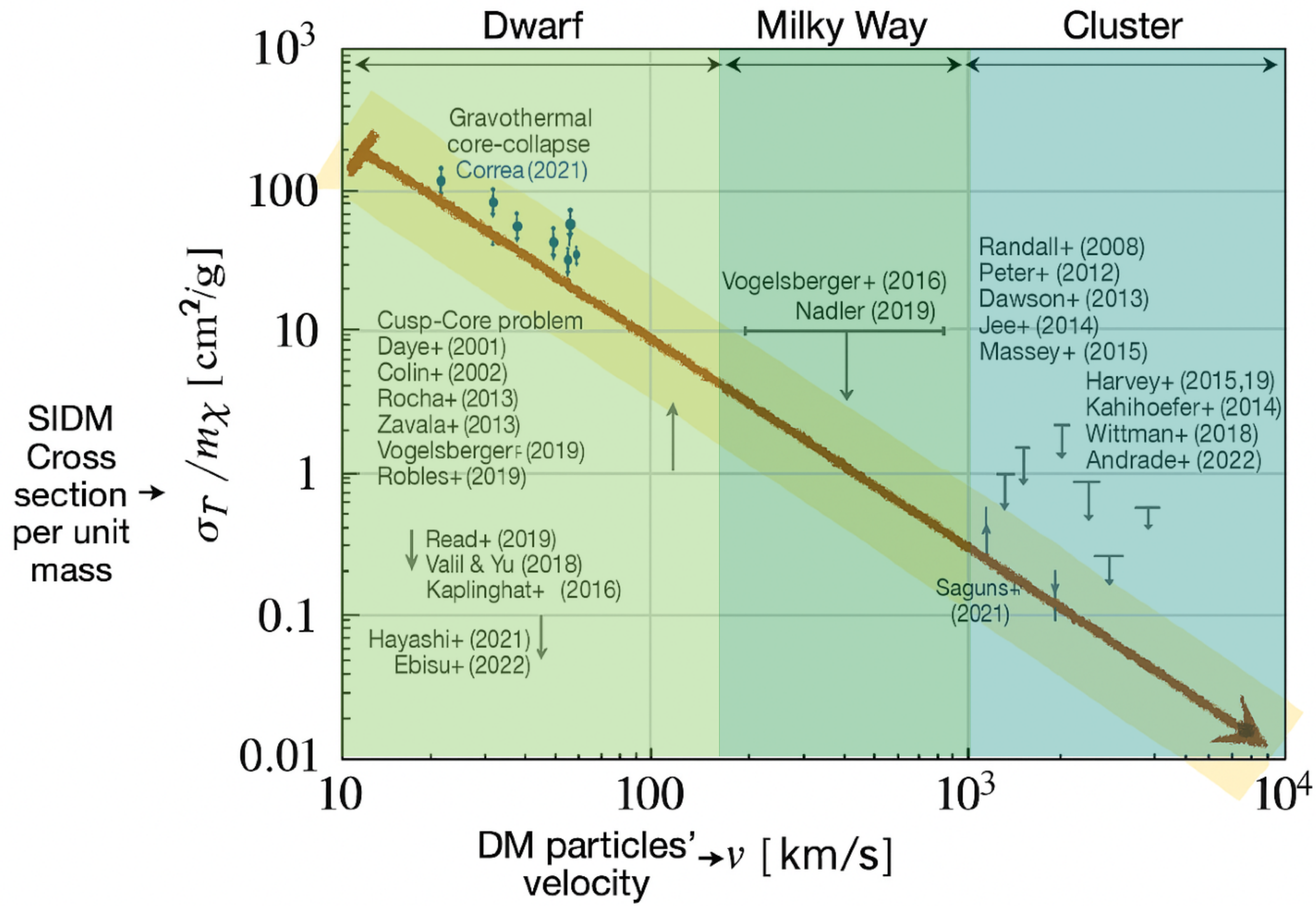
Viable SIDM models require two new particles:

- A dark matter particle χ
- Generally, a light mediator ϕ which is not part of the SM with a velocity dependent cross-section, e.g. from Yukawa potential (consistent with the dwarf vs cluster constraints):

$$\sigma_{\chi\chi} \propto 1/v^4$$

Heavy mediator possible but 1- velocity independent $\sigma_{\chi\chi}$ (problem with astrophysical constraints), 2- coupled to SM particle, so either secluded or strongly constrained by experiments.

Strongly Self-interacting Dark Matter (SIDM)



Indirect evidence for velocity dependent $\boldsymbol{\sigma}$?

Camila Correa 2013 (IAP colloquium)

QCD Axion Dark Matter

QCD allows a CP-violating term in the Lagrangian:

$$\mathscr{L}_\theta = \theta \, \frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Which is not observed in nature ($\theta < 10^{-10}$), this is the **strong CP problem**. The Peccei–Quinn mechanism promotes θ to a field which dynamically relaxes to zero after symmetry breaking (energy scale f_a).

Axion mass today $m_a \approx \frac{\sqrt{m_u m_d}}{m_u + m_d} \cdot \frac{f_\pi m_\pi}{f_a} \approx 5.7\,\mu\text{eV} \left(\frac{10^{12}\,\text{GeV}}{f_a} \right)$

Coupling to photons $\mathscr{L}_{a\gamma\gamma} = -\frac{1}{4} g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad g_{a\gamma\gamma} \simeq \frac{\alpha}{2\pi f_a}$

The misalignment mechanism:

In the early universe, the axion field $a(x)$ is "frozen" at some initial angle $\theta_i = a/f_a \sim 1$

When $H(t) \sim m_a$, the field begins oscillating and starts behaving like Cold Dark Matter with energy density:

$$\rho_a \sim \frac{1}{2} m_a^2 f_a^2 \theta_i^2$$

Leading to the present-day parameter density:

$$\Omega_a h^2 \approx 0.12\, \theta_i^2 \left(\frac{f_a}{10^{12}\,\text{GeV}} \right)^{7/6}$$

So that typically a lighter axion will over produce dark matter:

$$\Longrightarrow \quad m_a \gtrsim 20 - 30\ \mu\text{eV}$$

$$f_a \lesssim 4 \times 10^{11}\ \text{GeV}$$

Axion Like Particles

Axion-like particle (ALP) is an extension of the QCD axion:

$$m_a \propto \frac{\Lambda^2_{\rm QCD}}{f_a}, \qquad g_{a\gamma\gamma} \propto \frac{1}{f_a}$$

Such that mass and coupling are "independent" (Svrcek & Witten 2006):

$$m_a \propto \frac{\Lambda^2}{f_a} \qquad g_{a\gamma\gamma} \propto \frac{1}{f_a} \times C_\gamma$$

As a consequence, the decay rate of an axion-like particle (ALP) (e.g. in two photons) can be drastically different:

$$\Gamma(a \to \gamma\gamma) = \frac{g^2_{a\gamma\gamma} m_a^3}{64\pi}$$

The corresponding lifetime is:

$$\tau_a = \frac{1}{\Gamma(a \to \gamma\gamma)} = \frac{64\pi}{g^2_{a\gamma\gamma} m_a^3}$$

Axion Like Particles

For QCD axion, the allowed range is $f_a \sim [10^9 - 10^{12}]$ GeV

$$\tau_a > 10^{25}\ \text{s} \gg t_{\text{uni}} = 10^{17}\ \text{s}$$

QCD axions are stable.

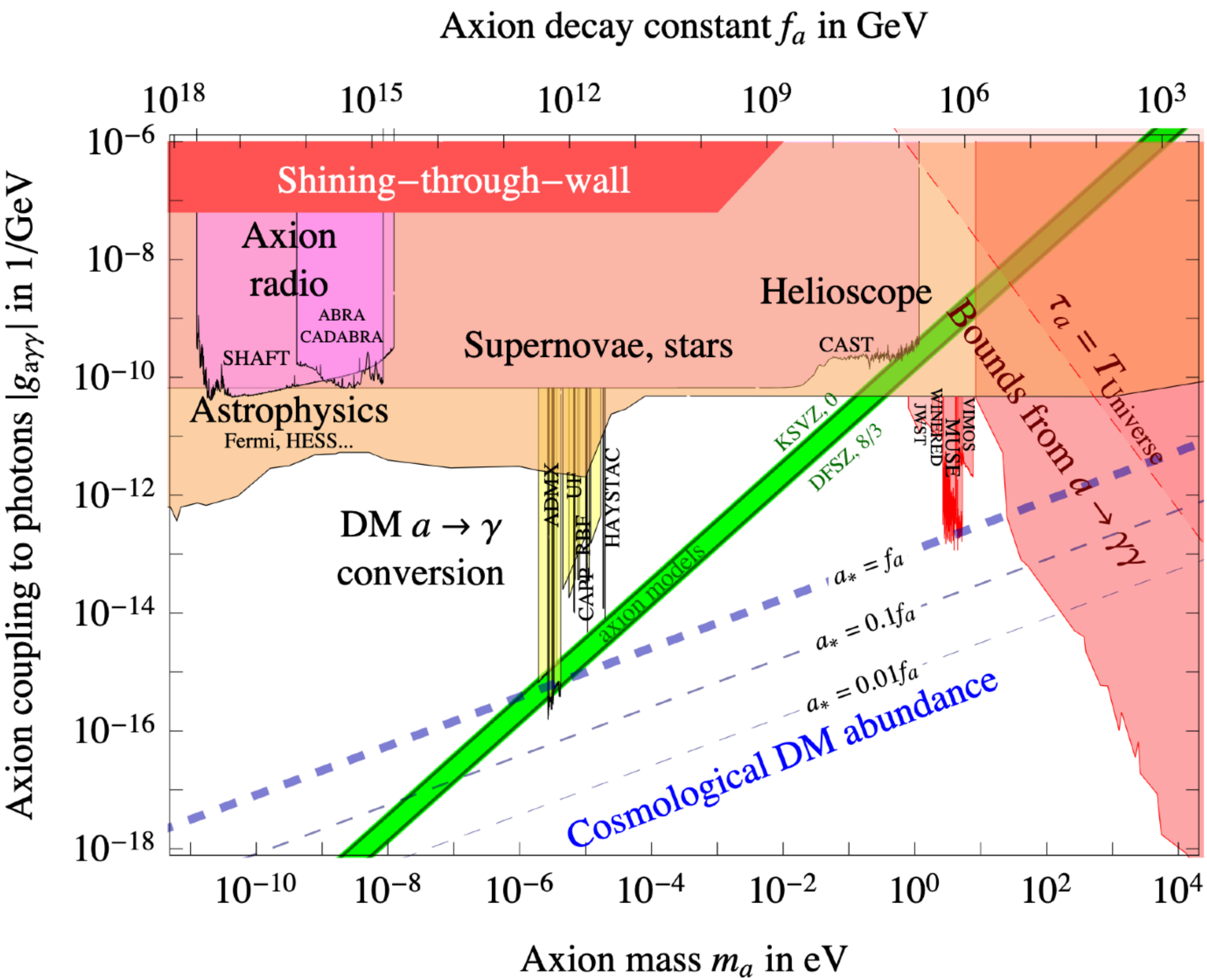
For ALPs:

$$m_a = 1\ \text{MeV} = 10^{-3}\ \text{GeV}, \quad g_{a\gamma\gamma} = 10^{-7}\ \text{GeV}^{-1}$$

We get $\tau_a \approx 2 \times 10^{25} \times 6.58 \times 10^{-25}\ \text{s} \approx 13.2\ \text{s}$

$$m_a = 10^{-6}\ \text{eV}, \quad g_{a\gamma\gamma} = 10^{-13}\ \text{GeV}^{-1}$$

We get $\tau_a \approx 10^{47}\ \text{s}$



From Cirelli et al 2024

Fuzzy Dark Matter

Fuzzy dark matter consist in ultralight ALPs (Hu et al 2000):

$$m_a \sim [10^{-23} - 10^{-20}]\ \mathrm{eV}$$

which leads to absurdly small coupling if we want fuzzy dark matter to account for all DM!

$$g_{a\gamma\gamma} < 10^{79}\ \mathrm{GeV}^{-1}$$

Leading to a quantum mechanical wavelength of astrophysical size:

$$\lambda_{\mathrm{dB}} \sim \frac{1}{m_a \mathrm{v}} \sim \mathrm{kpc}$$

As a result, **pressure is quantum mechanically supported** (at small enough scale)

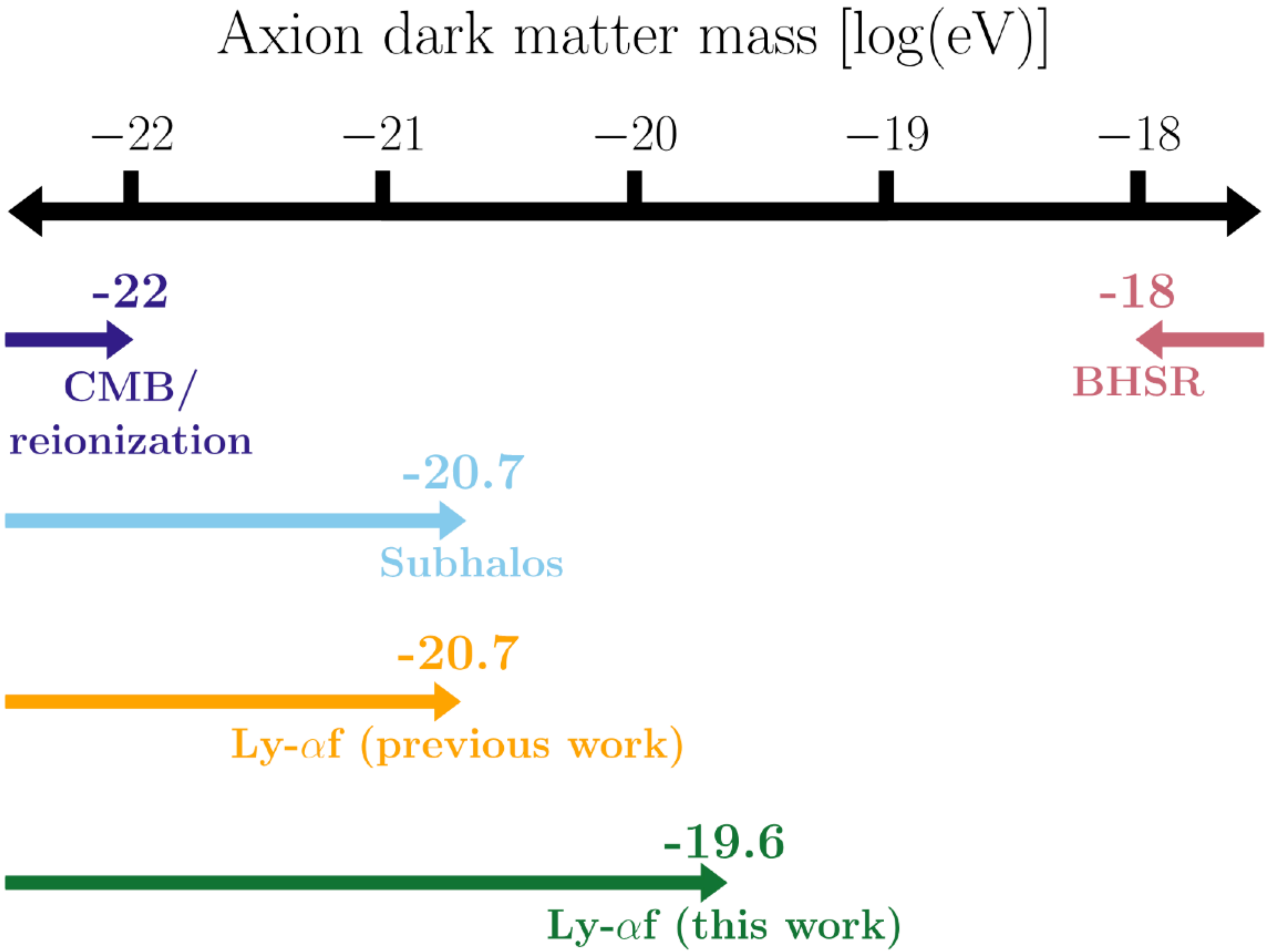
Fuzzy Dark Matter

Jeans length $\lambda_J = \dfrac{\pi\hbar}{\sqrt{G\rho m_a}}$

Jeans mass $M_J = \dfrac{4\pi}{3}\rho\left(\dfrac{\lambda_J}{2}\right)^3$

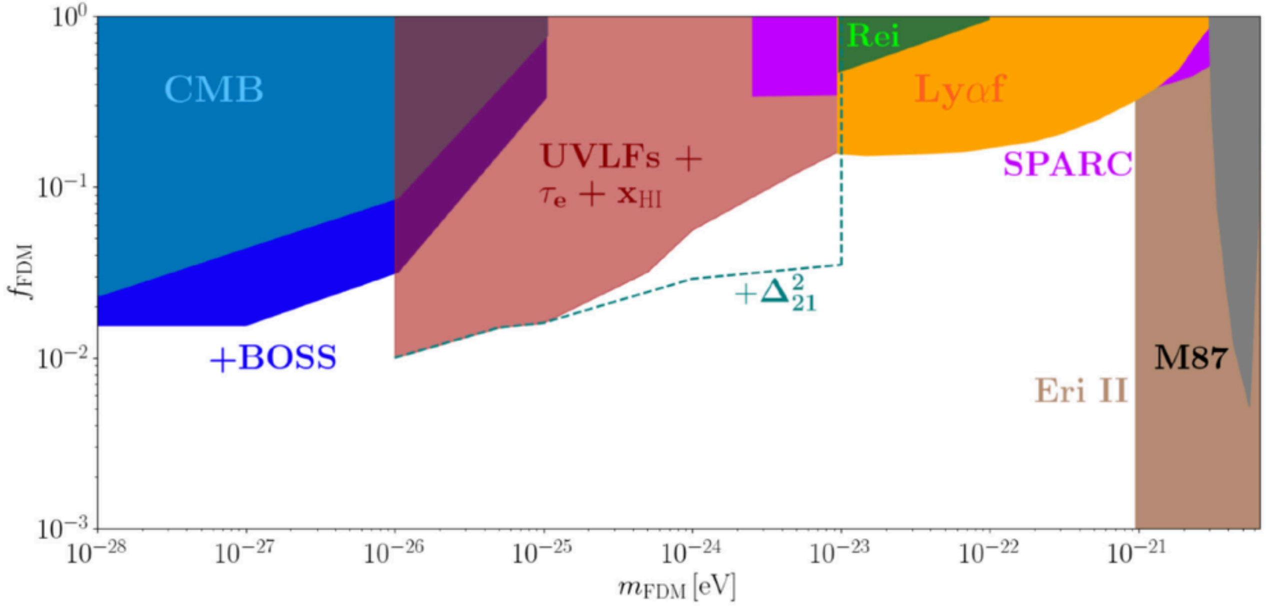
Jeans length $\lambda_J \approx 1.0\,\mathrm{kpc}\left(\dfrac{10^{-22}\,\mathrm{eV}}{m}\right)\left(\dfrac{10^{10}\,M_\odot/\mathrm{Mpc}^3}{\rho}\right)^{1/2}$

Jeans mass $M_J \approx 1.4\times10^8\,M_\odot\left(\dfrac{10^{-22}\,\mathrm{eV}}{m}\right)^3\left(\dfrac{10^{10}\,M_\odot/\mathrm{Mpc}^3}{\rho}\right)^{1/2}$

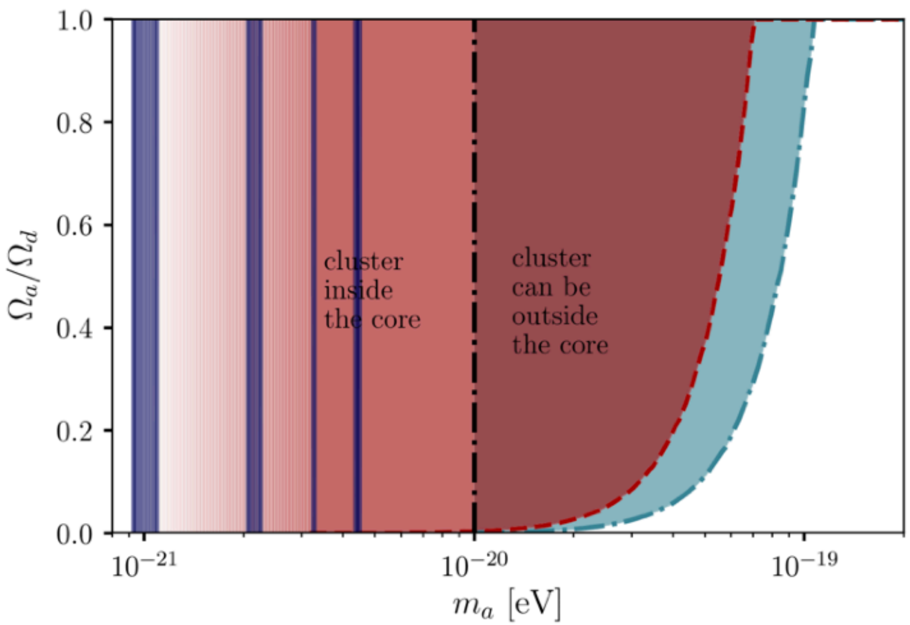


From Rogers & Peiris 2021

CMB, UV, 21cm and dynamic constraints:



Lazare et al 2024



Marsh & Niemeyer 2019

For ULA to be all DM, its mass must be $m_a \gtrsim 2 \times 10^{-20}$ eV

But… **it does not solve the strong CP problem**

Sterile neutrinos

Theoretical motivation (seesaw mechanism, from Dodelson & Widrow 1994):

$$\mathscr{L}_{\text{yukawa}} = -\frac{1}{2}\left(\overline{\nu_L} \quad \overline{N_R^c}\right)\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}$$

With

$$m_D = \frac{yv}{\sqrt{2}} \quad \langle H \rangle = \frac{v}{\sqrt{2}} \quad \text{(Higgs VEV)}$$

y is the Yukawa coupling (free parameter)
M is the Majorana mass (free parameter)
$\theta = \frac{m_D}{M}$ is the mixing angle

In this framework, the neutrino and sterile neutrino masses are:

$$m_\nu \approx \frac{m_D^2}{M} = \theta^2 M$$

$$m_{\nu_s} \approx M$$

Sterile neutrinos

Theoretical motivations:

-Lack of right handed neutrino is the Standard Model
-Original seesaw mechanism uses heavy ν_s and light ν
-Sterile neutrino can play a role in baryogenesis
-Light ν_s has cosmological consequences

$$\rho_{\rm rad} = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]$$

$N_{\rm eff} = 3.046 + \Delta N_{\rm eff}$ (see C. Pitrou's lecture)

Sterile neutrinos

Decay rate:

$$\Gamma_{\nu_s \to \nu_a \gamma} = \frac{9\,\alpha\,G_F^2}{256\,\pi^4}\,\sin^2(2\theta)\,m_{\nu_s}^5$$

Where:

$\Gamma_{\nu_s \to \nu_a \gamma}$: decay rate

α: fine structure constant

G_F: Fermi constant

θ: sterile-active neutrino mixing angle

m_{ν_s}: sterile neutrino mass

Lifetime is given by:

$$\tau \approx 2.3 \times 10^{15}\ \mathrm{Gyr} \left(\frac{1\ \mathrm{keV}}{m_s}\right)^5 \left(\frac{10^{-10}}{\sin^2(2\theta)}\right)$$

Sterile neutrinos

Sterile Neutrino Regime	Mass Range	Mixing Angle (θ)	Decay Rate	Stable	Production Mechanism	Observational Signatures
1. keV-scale (canonical)	1–100 keV	$\sin^2(2\theta) \sim 10^{-11}$–$10^{-7}$	Slow radiative decay: $\nu_s \to \nu_a + \gamma$	✓	Dodelson–Widrow, Shi–Fuller	X-ray line searches, Lyman-α forest constraints
5. Non-resonant production (Dodelson–Widrow)	~ 7 keV	$\sin^2(2\theta) \sim 10^{-10}$	Radiative decay to active neutrino + photon	✓	Mixing-induced thermal production	3.5 keV line candidate, warm DM structure suppression
6. Resonant production (Shi–Fuller)	~ 2–10 keV	Smaller mixing, enhanced by lepton asymmetry	Long-lived	✓	Resonant oscillations in early universe	Reduced suppression of small-scale power, X-ray searches difficult
3. Super-weakly interacting (freeze-in)	$\sim$ 10 keV–1 GeV	Very small mixing or feeble couplings	Lifetime $\gtrsim$ age of universe	✓	Freeze-in from decays of heavier states	Affects BBN if produced early; no N_{eff} contribution at CMB due to being non-relativistic
2. MeV–GeV sterile neutrinos	1 MeV $\lesssim m_s \lesssim$ few GeV	$\sin^2(2\theta) \gtrsim 10^{-6}$	Prompt decay to $\nu\ell^+\ell^-$, $\pi^0\nu$, etc.	✗	Seesaw, freeze-in	Early universe energy injection, BBN/CMB distortion constraints
4. Ultra-heavy sterile neutrinos	$m_s \gtrsim 10^9$ GeV	Model-dependent	Decay quickly unless protected	✗	Thermal or seesaw origin	No direct DM signal; relevant for baryogenesis

2

Lecture I: Known DM properties from observations. Known DM properties from theory. Main families of DM and their observational constraints.

Lecture II: Axion Quark Nuggets (QCD as a source of dark matter)

Lecture III: QCD as a source of dark energy. How to improve the efficiency of weak lensing to probe the dark matter power spectrum.

Condition for collapse:

The Schwarzschild radius for a mass M is:

$$R_S(M) = \frac{2GM}{c^2}$$

Horizon size and energy density during radiation domination:

$$R_H(t) \sim ct \implies M_H(t) \sim \rho(t) R_H(t)^3 \sim \frac{c^3 t}{G}$$

Then we have:

$$R_S(M_H) = \frac{2GM_{PBH}(t)}{c^2} = \frac{2G\gamma M_H(t)}{c^2} = 2\gamma ct = 2\gamma R_H(t)$$

With collapse efficiency $\gamma \sim 0.2$ (from simulations).

$\implies$ collapse can happen. Balance between gravity and pressure gradients.

Simulations show that collapse happen when $\frac{\delta\rho}{\rho} \gtrsim 0.3 - 0.5$

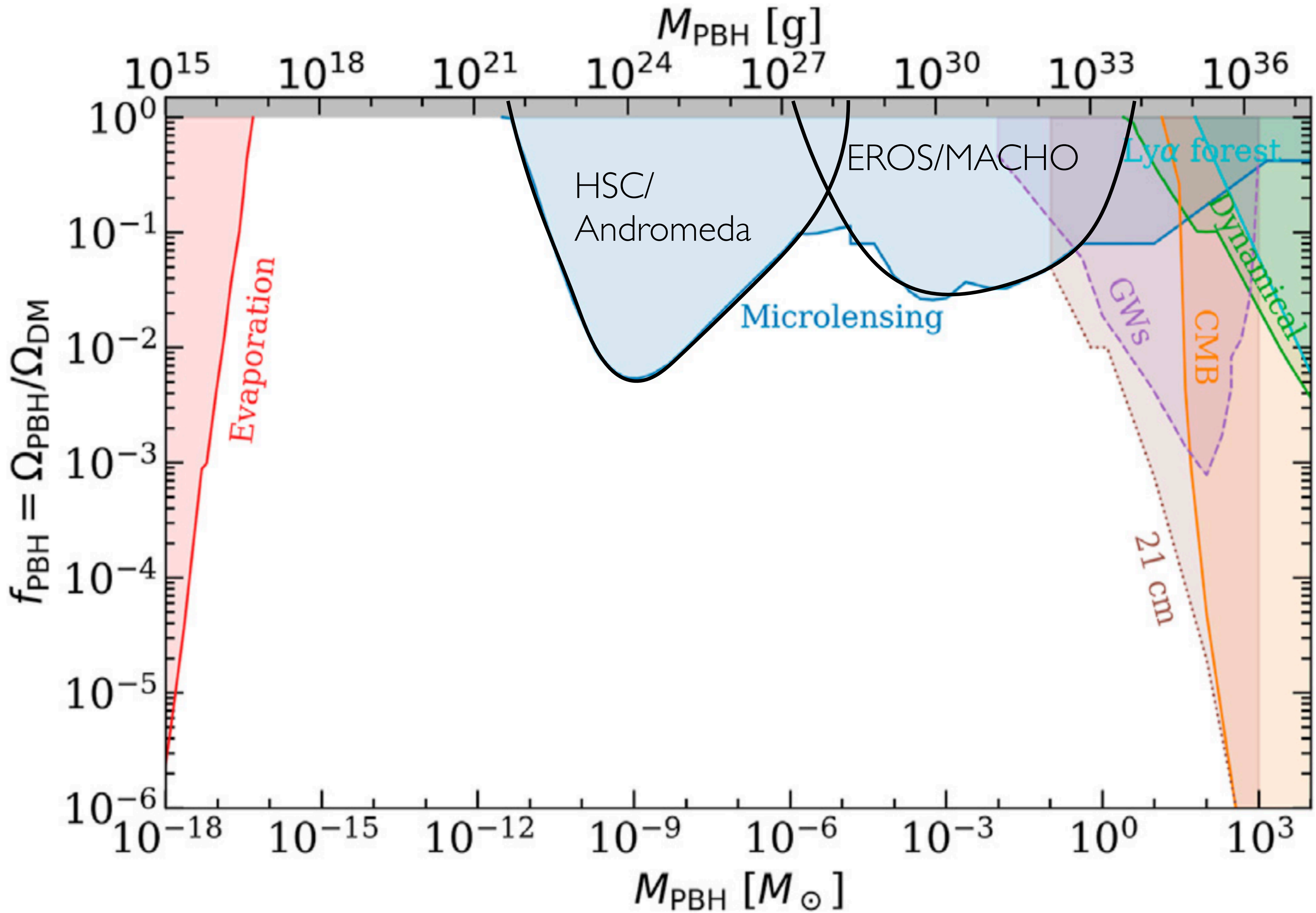
This process could "theoretically" happen until radiation-matter equality

Massive BH Dark Matter

$$t_{\text{form}} = \frac{M_{PBH}}{M_{Pl}}$$

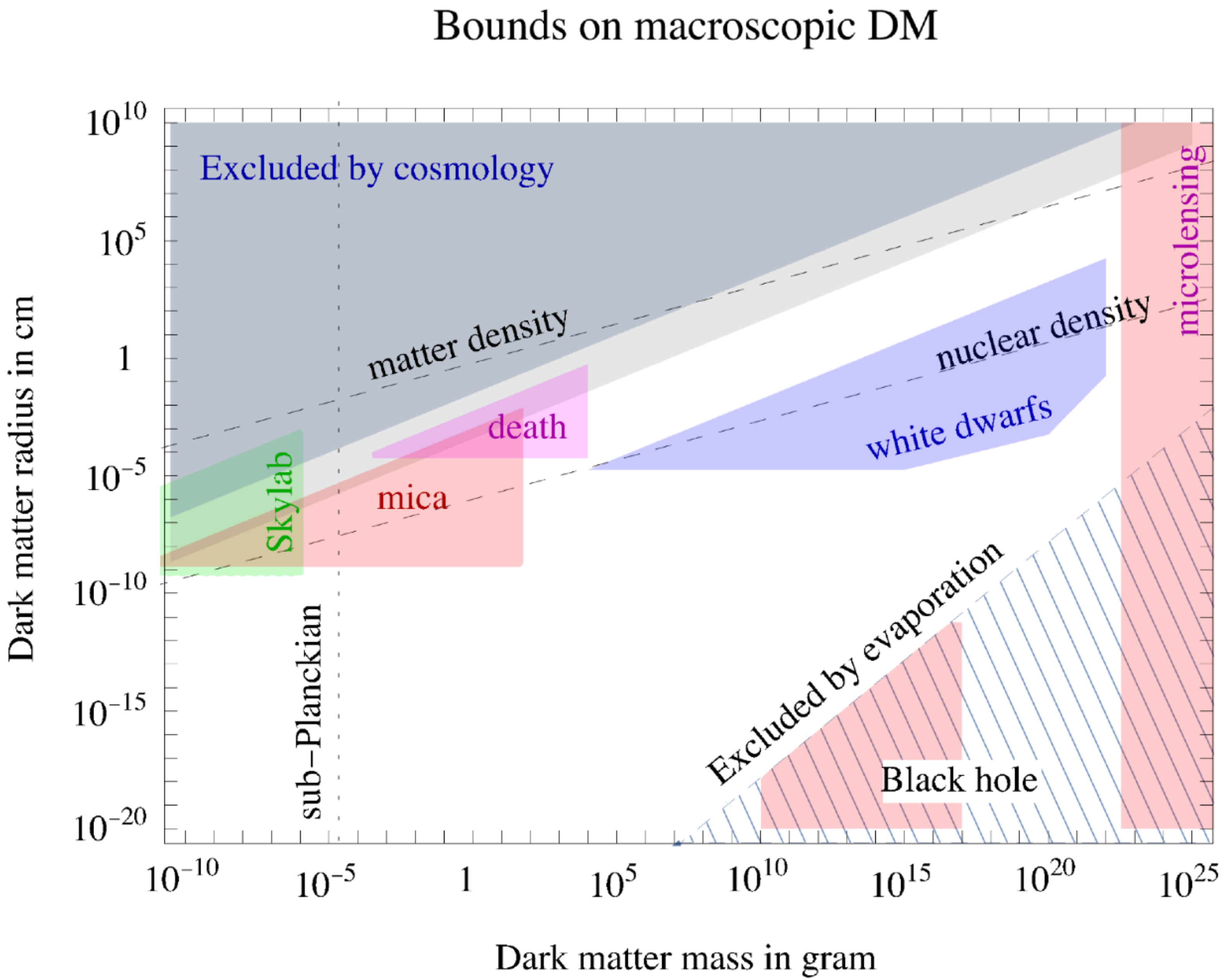
PBH Mass M_{PBH}	Formation Time t	Notes
10^{-5} g	10^{-43} s	Planck-scale mass
10^{15} g	10^{-23} s	Evaporates today via Hawking radiation
$10^{-10}\,M_\odot$	10^{-18} s	Asteroid-mass PBH
$1\,M_\odot$	10^{-5} s	Stellar-mass PBH (LIGO mass scale)
$10^{3}\,M_\odot$	1 s	Intermediate-mass PBH
$10^{6}\,M_\odot$	10^{3} s	SMBH seed mass scale

Mass Range	Constraint Type	Status
$M < 10^{15}$ g	Hawking radiation (gamma-rays, BBN)	Strongly ruled out
10^{17}–10^{23} g	Gamma-rays, femtolensing	Some open parameter space
10^{-10}–$10^{2}\,M_\odot$	Microlensing (EROS, OGLE, HSC)	Mostly ruled out
$\sim 30\,M_\odot$	LIGO/Virgo mergers	Viable as a subcomponent
$> 10^{3}\,M_\odot$	CMB accretion, LSS suppression	Strong constraints, not fully ruled out



Villanueva-Domingo et al 2021

Macroscopic DM



Possible twist with WD: Graham et al 2018

From Jacobs et al 2015, Cirelli et al 2024

Can we really probe the nature of DM?

“Extraordinary claims require extraordinary evidence” C. Sagan

1- Define precisely “nature of dark matter”

The difficulty lies in the fact that cosmology **is not an experimental science**, i.e. direct versus indirect detection.

2- What are currently the indirect detection probes:

Impact on structures ($P(k)$ cutoffs, suppression, fringes, C_ℓ), galaxy cores, mass profiles…

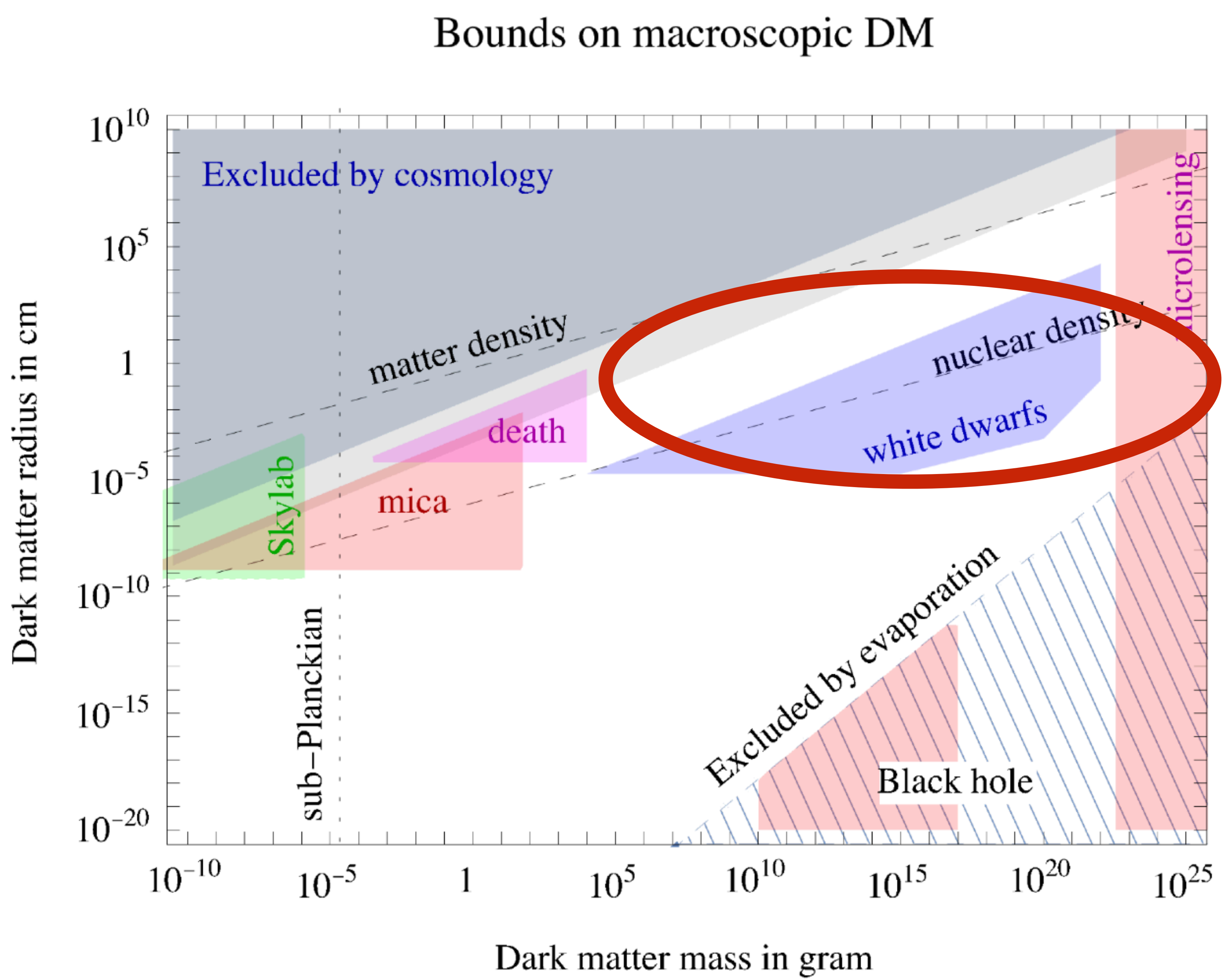
Energy/particles injection (e^+e^-, γ, lines and continuum, τ, $\Delta N_{\rm eff}$,…)

3- Redundancy is necessary!

e.g. considerate degeneracy between WDM, SIDM, ALPs:

Probe	Distinguishes?	Details
Lyman-α forest	WDM vs ALP	High-k cutoff; fringes in ALP
Strong lensing substructure	WDM/ALP vs SIDM	Subhalo abundance vs core profile
Galactic cores (rotation curves)	SIDM vs WDM/ALP	SIDM produces cored halos **without** linear suppression
Dwarf galaxies′ central densities	partial	Degenerate between SIDM and WDM with sufficient suppression
21cm signal	potential	Early small-scale structure sensitivity (less developed)

4- Check astrophysical consequences outside your field



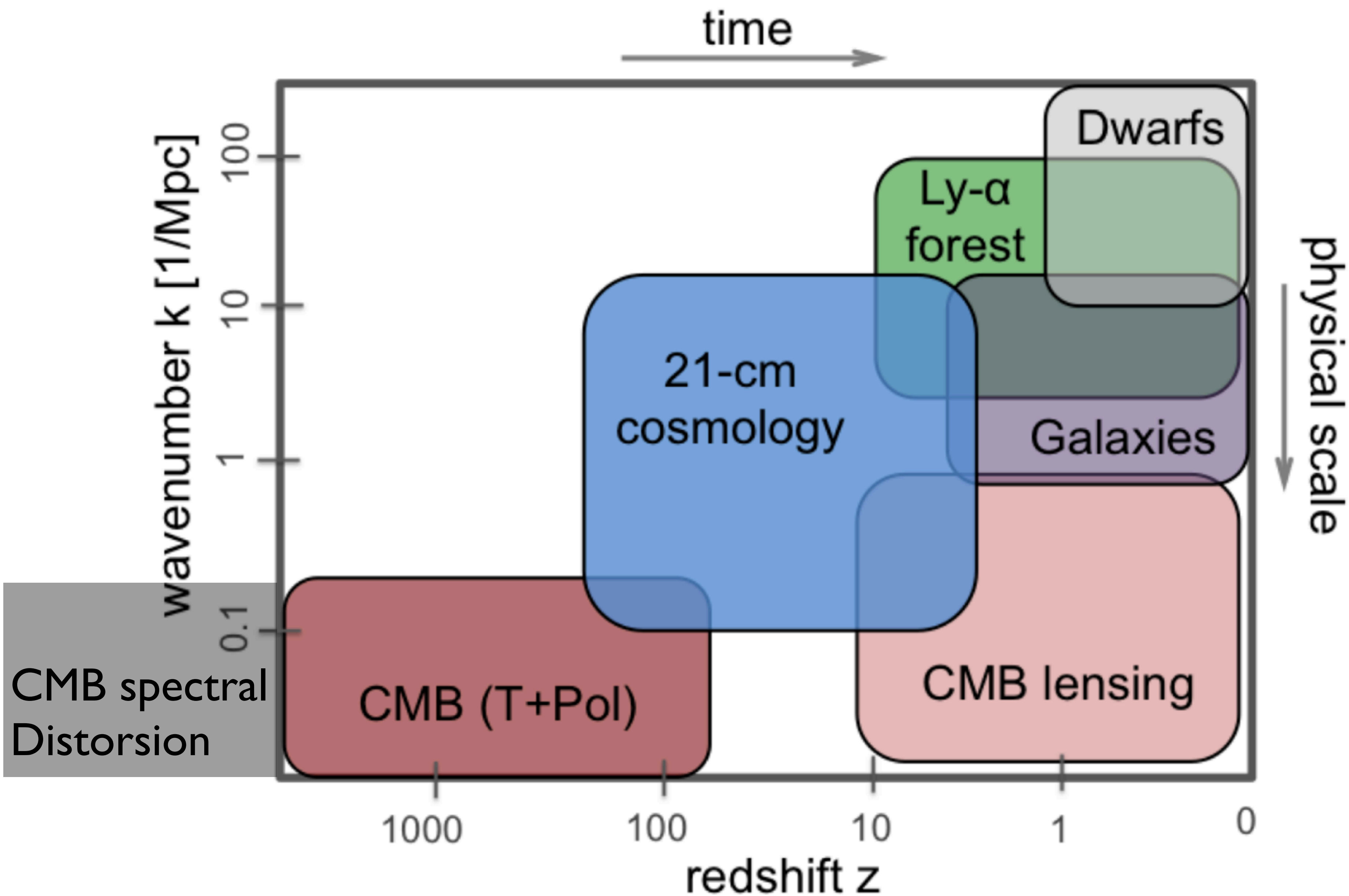
5- Consider inconsistencies

With the increasing size of data sets, cosmological observations seem to lead to more tensions and anomalies (Vivian Poulin lectures). **This might be a good thing** (i.e. more constraints than degrees of freedom)

Tensions refer to **quantitative, statistically significant discrepancies** between different measurements or between data and ΛCDM predictions. They're often robust, but not necessarily signs of new physics.

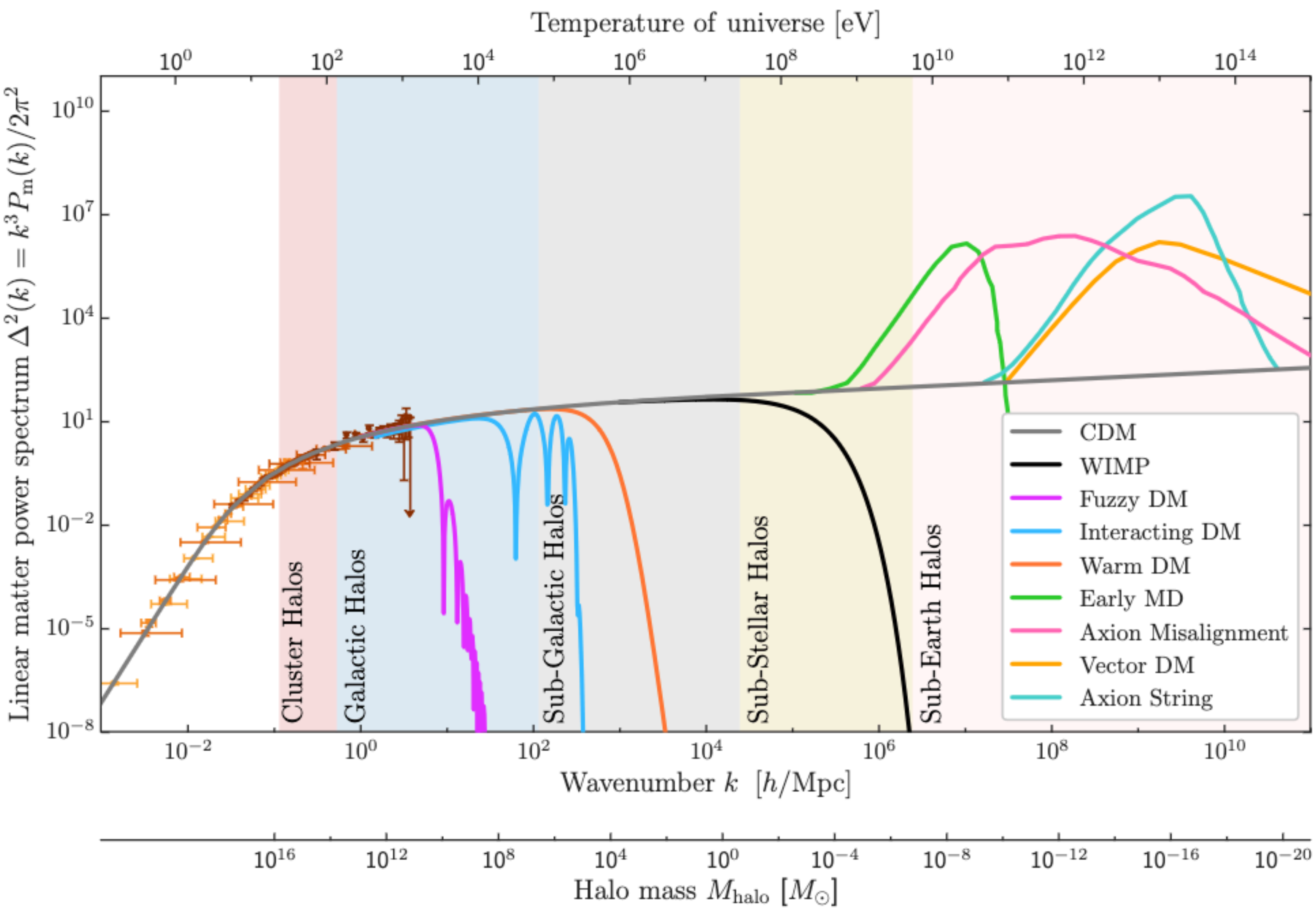
Anomalies refer to **unexpected features or patterns**, and might be due to systematics or astrophysical (modelling) uncertainties

"The CosmoVerse White Paper: Addressing observational tensions in cosmology with systematics and fundamental physics" (**420 pages!**) E. Di Valentino, et al 2025



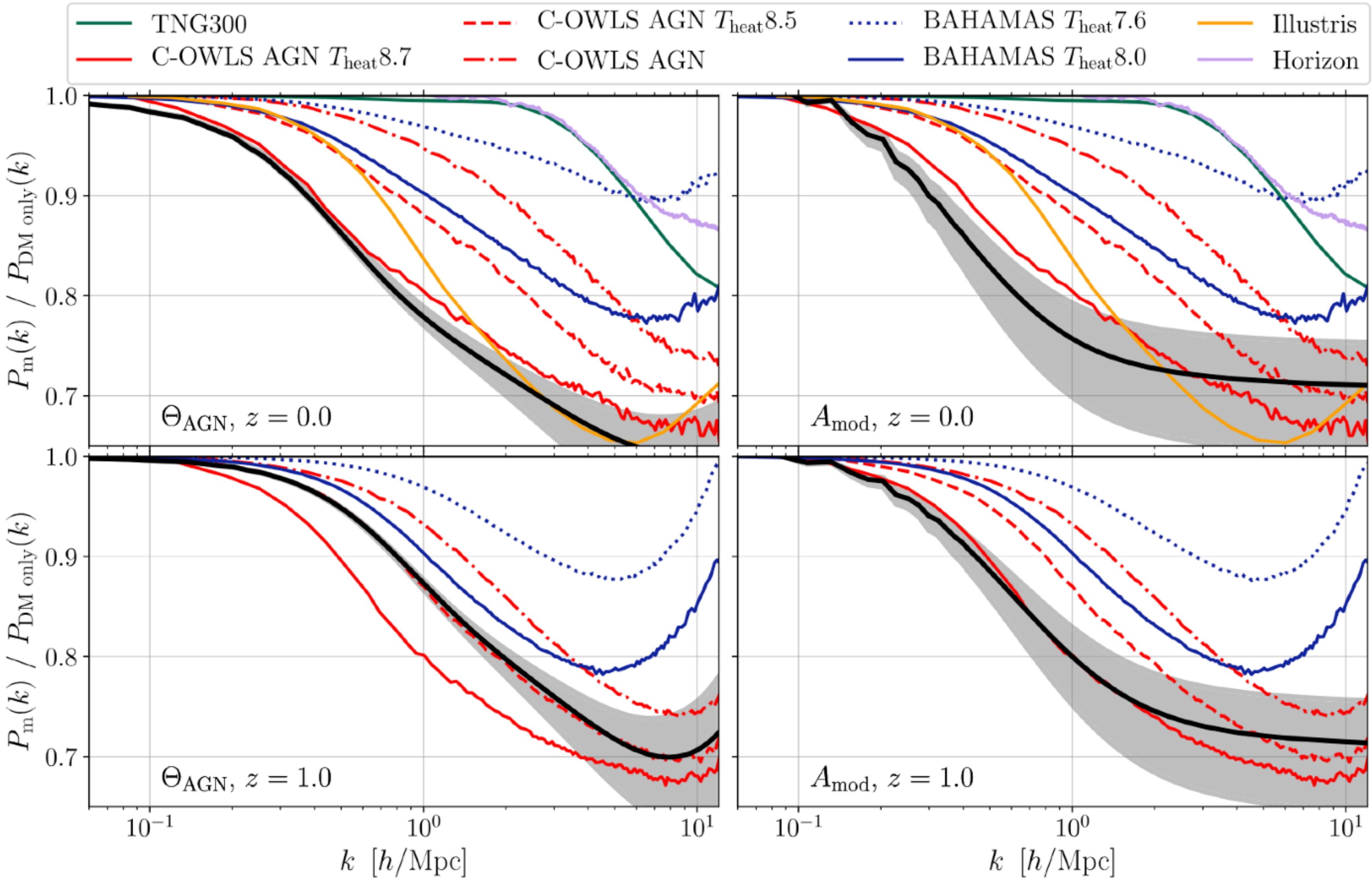
From Gluscevic et al 2019

Direct probe of the dark matter distribution (Power spectrum, halo mass profile)



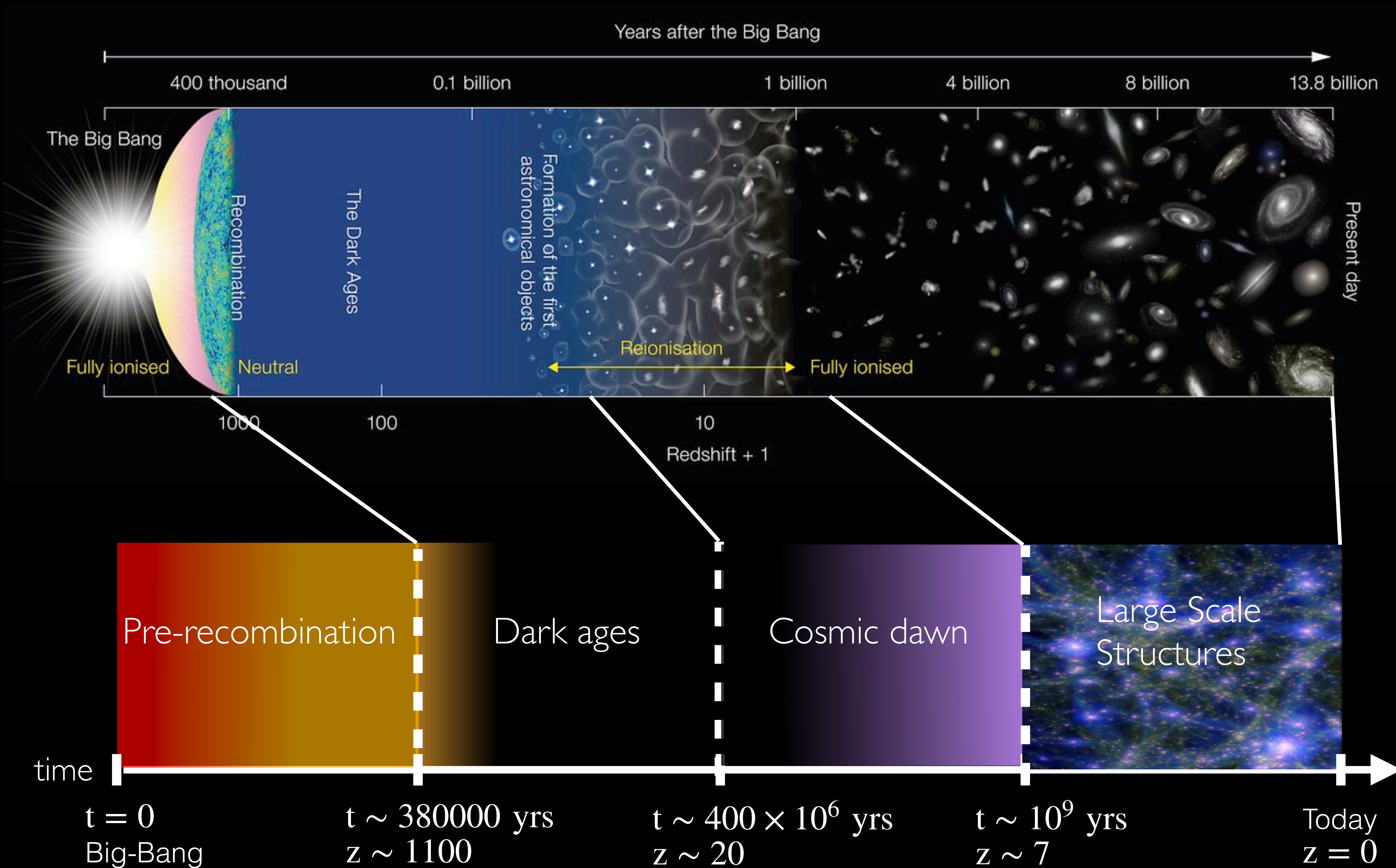
Bechtol et al 2023

Weak gravitational lensing can probe the matter spectrum

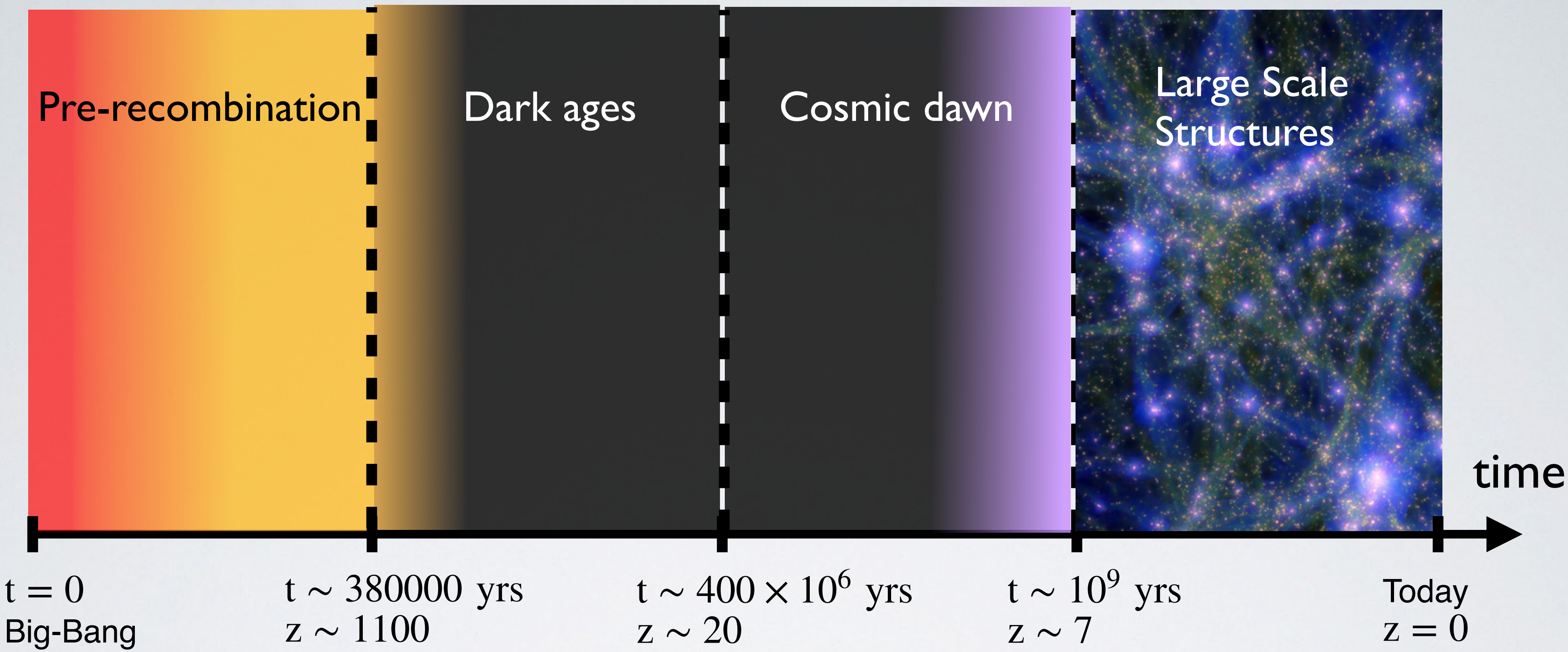


But the mass spectrum alone is not a clean probe of DM

Amon & Efstathiou 2022



Years after the Big Bang
400 thousand
0.1 billion
1 billion
4 billion
8 billion
13.8 billion
The Big Bang
Recombination
The Dark Ages
Formation of the first astronomical objects
Present day
Fully ionised
Neutral
Reionisation
Fully ionised
1000
100
10
Redshift + 1
Pre-recombination
Dark ages
Cosmic dawn
Large Scale Structures
time
t = 0
Big-Bang
t ~ 380000 yrs
z ~ 1100
t ~ 400 × 10^6 yrs
z ~ 20
t ~ 10^9 yrs
z ~ 7
Today
z = 0



Pre-recombination	Dark ages	Cosmic dawn	Large Scale Structures
High $n_{\rm bar}$	High $n_{\rm bar}$	Low $n_{\rm bar}$	Very low $n_{\rm bar}$
High $T_{\rm bar}$	Low $T_{\rm bar}$	Very low $T_{\rm bar}$	High $T_{\rm bar}$
$x_e \sim 1$	$x_e \sim 10^{-3}$	$x_e \gtrsim 10^{-3}$	$x_e \sim 1$

The annihilation is defined as:

$$\left.\frac{dE}{dV\,dt}\right|_{\mathrm{ann}} = \rho^2_{\mathrm{DM}}(z)\,\frac{\langle\sigma v\rangle}{m_\chi}\,f(z) = \rho_c^2\,\Omega^2_{\mathrm{DM}}\,(1+z)^6\,\frac{\langle\sigma v\rangle}{m_\chi}\,f(z)$$

Recast using the effective annihilation parameter p_{ann}:

$$\left.\frac{dE}{dV\,dt}\right|_{\mathrm{ann}} = p_{\mathrm{ann}}\cdot\rho_c^2\,\Omega^2_{\mathrm{DM}}\,(1+z)^6$$

DM Candidate	Mass Range	Predicted $\langle\sigma v\rangle$	Predicted $p_{\mathrm{ann}} = f(z)\,\langle\sigma v\rangle/m_\chi$
Thermal WIMP (s-wave, freeze-out)	10 GeV – TeV	$\sim 2.2\times10^{-26}$ cm^3 s^{-1}	$\sim 2.2\times10^{-28}$ cm^3 s^{-1} GeV^{-1}
Light WIMP ($\chi\chi\to e^+e^-$)	1 – 10 GeV	10^{-26} – 10^{-27} cm^3 s^{-1}	10^{-27} – 10^{-26} cm^3 s^{-1} GeV^{-1}
ALP (self-annihilation, loop-suppressed)	keV – MeV	$\ll 10^{-30}$ cm^3 s^{-1}	$\ll 10^{-32}$ cm^3 s^{-1} GeV^{-1}
Dark photon mediated DM	MeV – GeV	10^{-27} – 10^{-25} cm^3 s^{-1}	10^{-28} – 10^{-26} cm^3 s^{-1} GeV^{-1}
WIMPZILLA (superheavy freeze-in)	10^{10} – 10^{16} GeV	$\lesssim 10^{-38}$ cm^3 s^{-1}	$\lesssim 10^{-54}$ cm^3 s^{-1} GeV^{-1}
SIDM (light mediator, v-dependent)	1 – 1000 MeV	10^{-24} – 10^{-22} cm^3 s^{-1}	10^{-26} – 10^{-24} cm^3 s^{-1} GeV^{-1}

The decay is defined as:

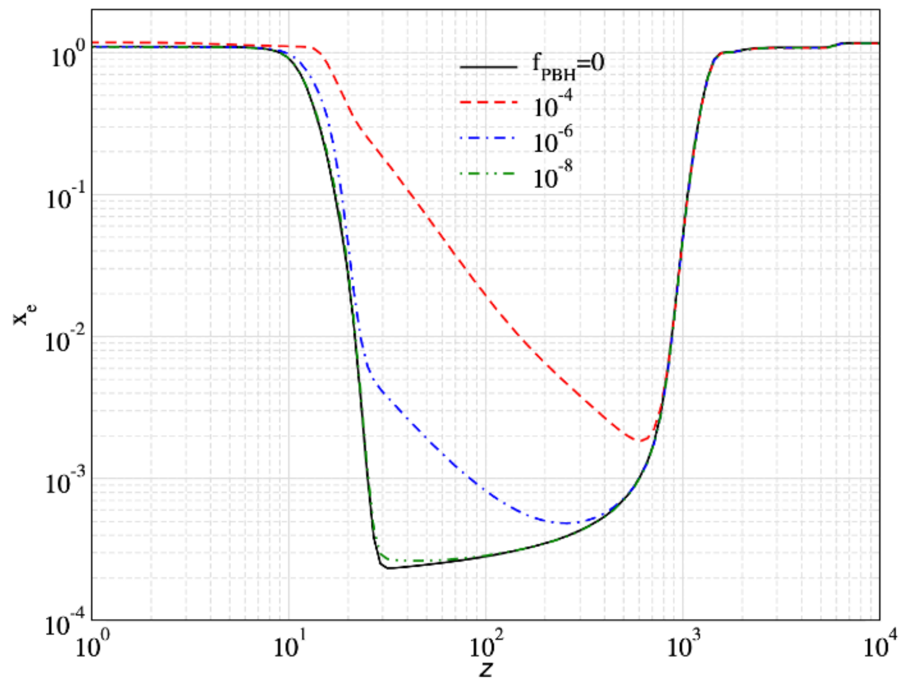
$$\left.\frac{dE}{dV\,dt}\right|_{\text{dec}} = \rho_{\text{DM}}(z)\,\frac{1}{\tau_\chi}\,f(z) = \rho_c\,\Omega_{\text{DM}}\,(1+z)^3\,\frac{1}{\tau_\chi}\,f(z)$$

Recast using the effective decay parameter p_{dec}:

$$\left.\frac{dE}{dV\,dt}\right|_{\text{dec}} = p_{\text{dec}} \cdot \rho_c\,\Omega_{\text{DM}}\,(1+z)^3$$

DM Candidate	Mass Range	Typical Theoretical Lifetime τ	Decay Channel or Notes
Sterile Neutrino	7 – 100 keV	$10^{26} - 10^{30}$ s	Decay via $\nu_s \to \nu + \gamma$; $\tau \propto \theta^{-2} m_s^{-5}$ with $\theta^2 \sim 10^{-10}$
Axion-like Particle (ALP)	MeV – GeV	$10^{20} - 10^{30}$ s	Decay to $\gamma\gamma$ via anomaly coupling; $\tau \propto f_a^2/m_a^3$
Dark Photon (massive A')	10 MeV – 10 GeV	$10^{15} - 10^{30}$ s	Lifetime depends on kinetic mixing $\epsilon \sim 10^{-10}$; decays to SM leptons if kinematically allowed
SIDM (with decaying mediator)	10 – 100 MeV (mediator)	$10^{5} - 10^{12}$ s	Mediator ϕ decays to e^+e^- or $\gamma\gamma$; controls timing of energy injection

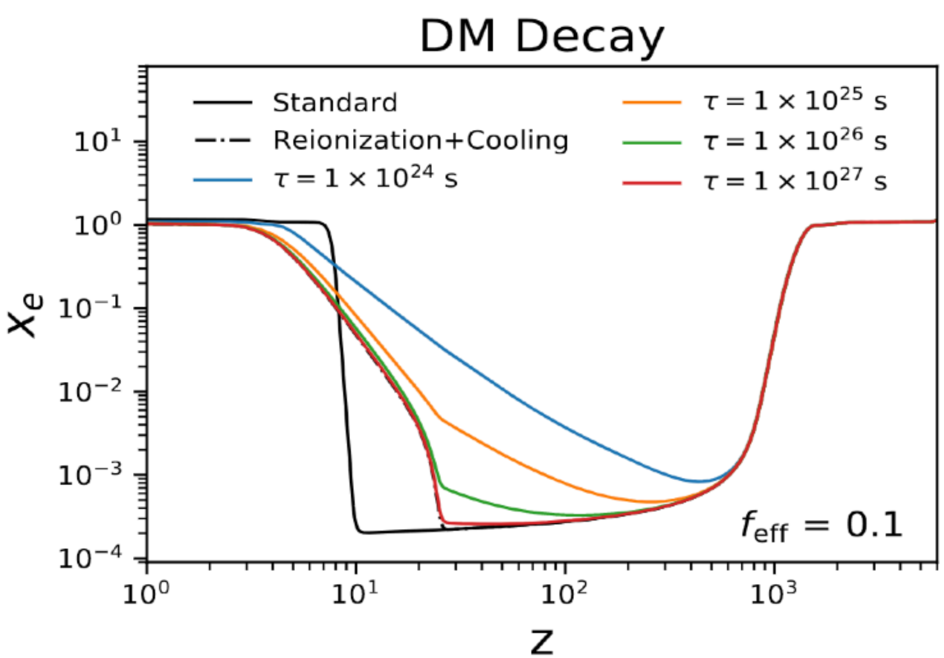
DM Candidate	Mass Range	Lifetime Constraint
Sterile Neutrino (decay to $\nu + \gamma$)	$\sim$ keV	$\tau \gtrsim 10^{26}$ s (X-ray, Lyman-α)
ALP (decay to $\gamma\gamma$)	μeV – GeV	$\tau \gtrsim 10^{24}$–10^{27} s (CMB, gamma rays, SN1987A)
WIMP (with decay via dimension-6 operator)	10 GeV – TeV	$\tau \gtrsim 10^{27}$ s (CMB + indirect detection)
Dark Photon (visible/invisible decay)	MeV – GeV	$\tau \gtrsim 10^{25}$–10^{28} s (CMB, beam dumps)
SIDM (decaying mediator scenarios)	MeV – GeV mediator mass	$\tau \gtrsim 10^{24}$ s (model dependent)

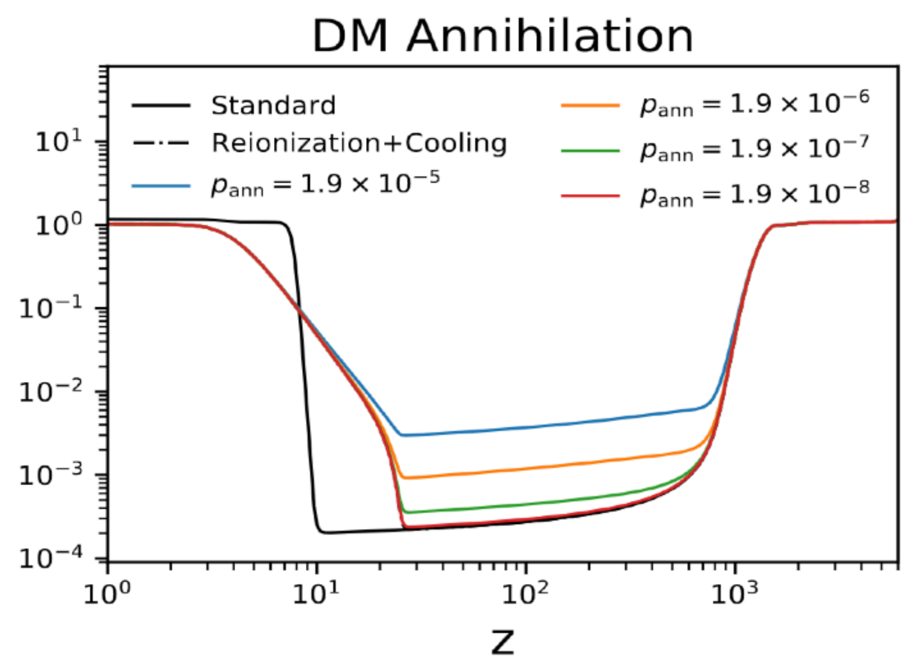


Super Massive Primordial Black Holes

$M = [10^5 - 12^{12}]\ M_\odot$

Acharya et al 2022

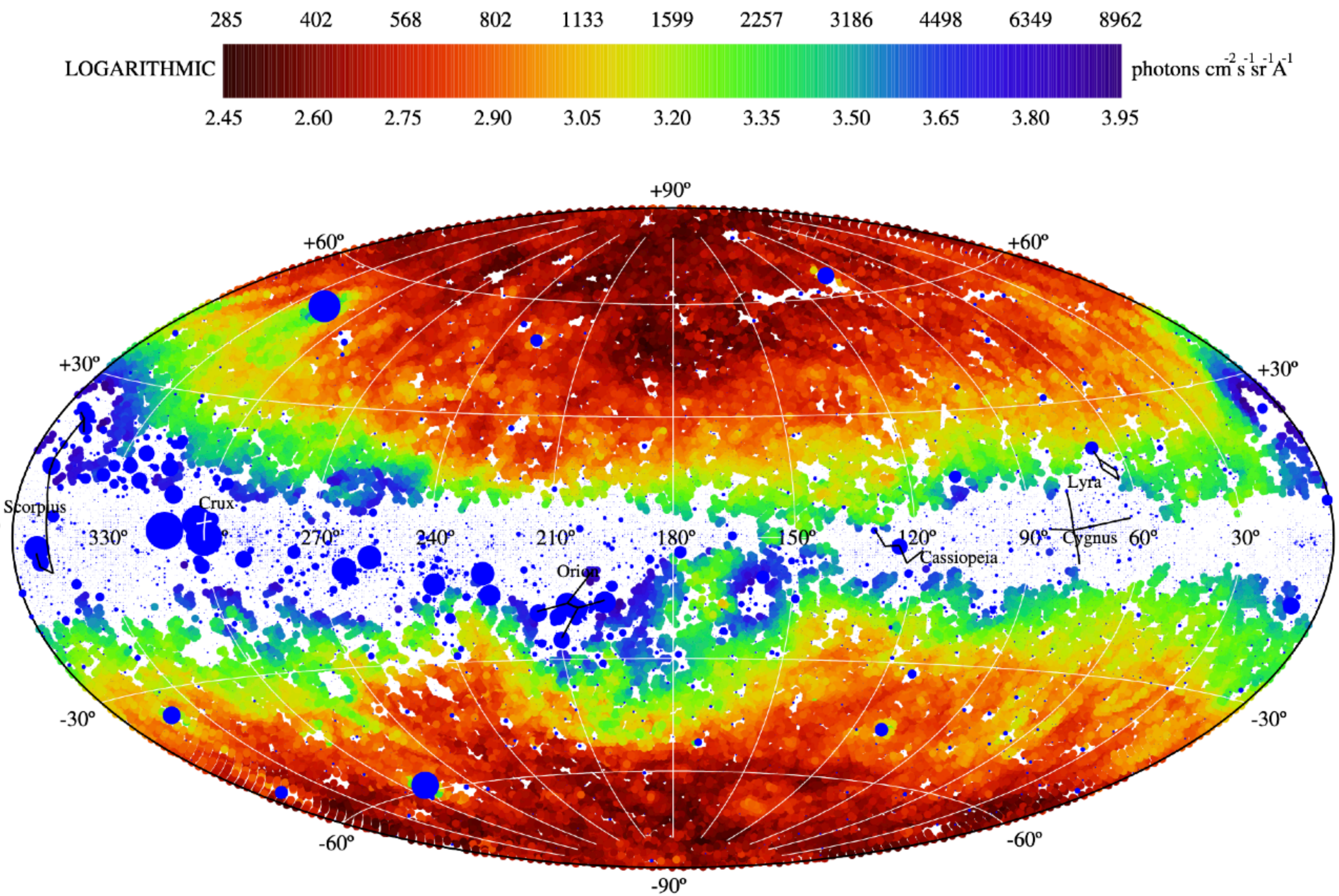




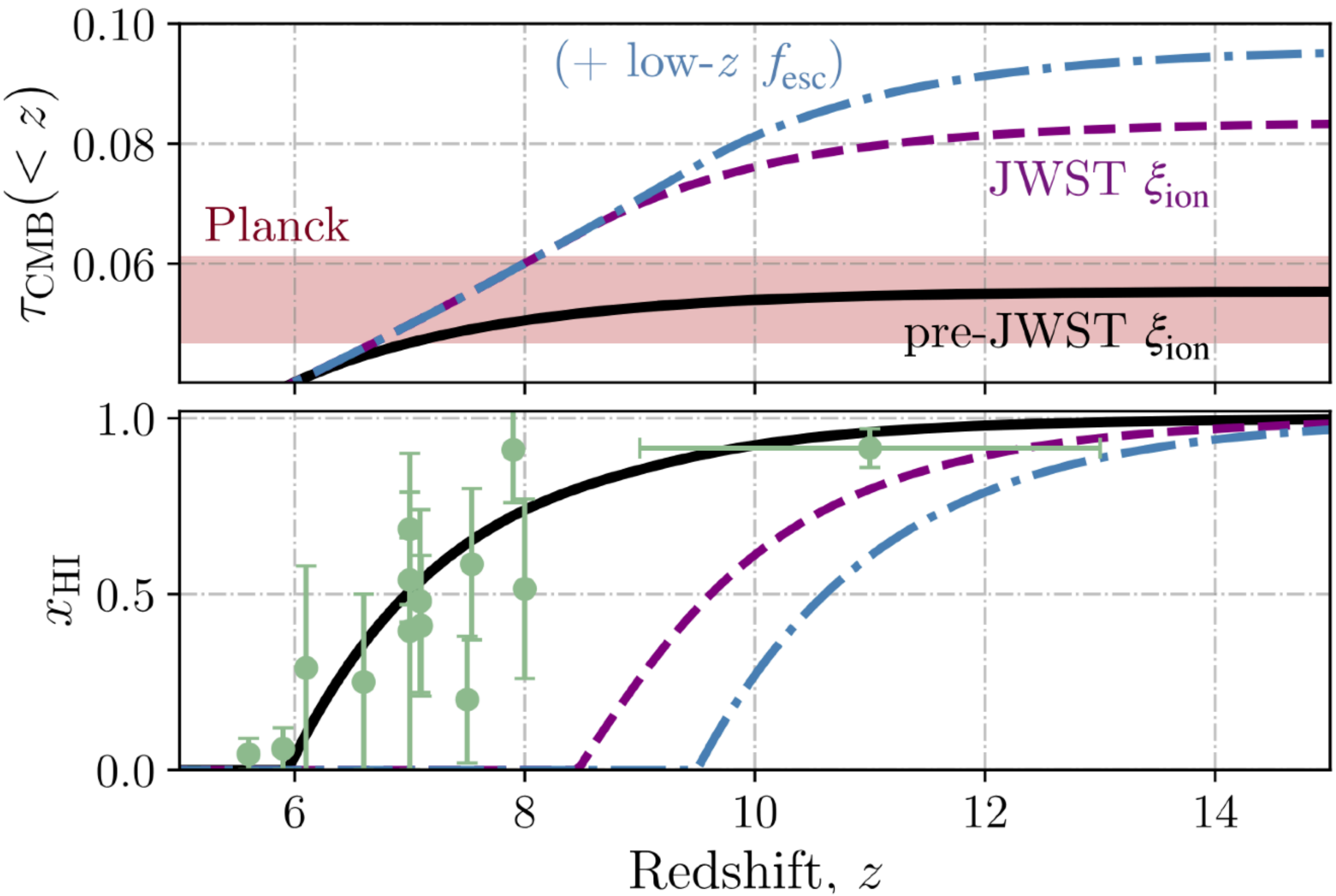
Short et al 2019

Milky-Way UV excess

Henry et al 2014



JWST: inconsistency with Planck's reionization? UV problem again?



Muñoz et al 2024

Family	Key Feature	Typical Mass	Main Motivation
Primordial Black Holes (PBHs)	Compact objects from early Universe	10^{-16}–$10^{5}\,M_\odot$	Non-particle DM, possible LIGO link
Composite DM	Macroscopic bound states (e.g., AQNs)	g–kg scale	QCD-scale physics, macroscopic effects
Cold Dark Matter (CDM)	Cold, non-relativistic particles	GeV–TeV	CMB + structure formation
Self-Interacting DM (SIDM)	Strong DM–DM interactions	GeV–TeV	Cores, rotation curve diversity
Asymmetric DM (ADM)	DM–baryon asymmetry link	∼5 GeV	Explains $\Omega_{\rm DM} \sim \Omega_b$
Decaying/Annihil DM	Unstable or actively emitting particles	GeV–TeV	Gamma-ray or positron excesses
Warm Dark Matter (WDM)	Semi-relativistic at early times	keV	Solves small-scale structure issues
Fuzzy/Ultralight DM (FDM)	Wave-like quantum pressure effects	$\sim 10^{-22}$ eV	Solitonic cores, small-scale suppression
Modified Gravity	No DM; modified gravity laws	N/A	Explains galactic dynamics (e.g., MOND, TeVeS)

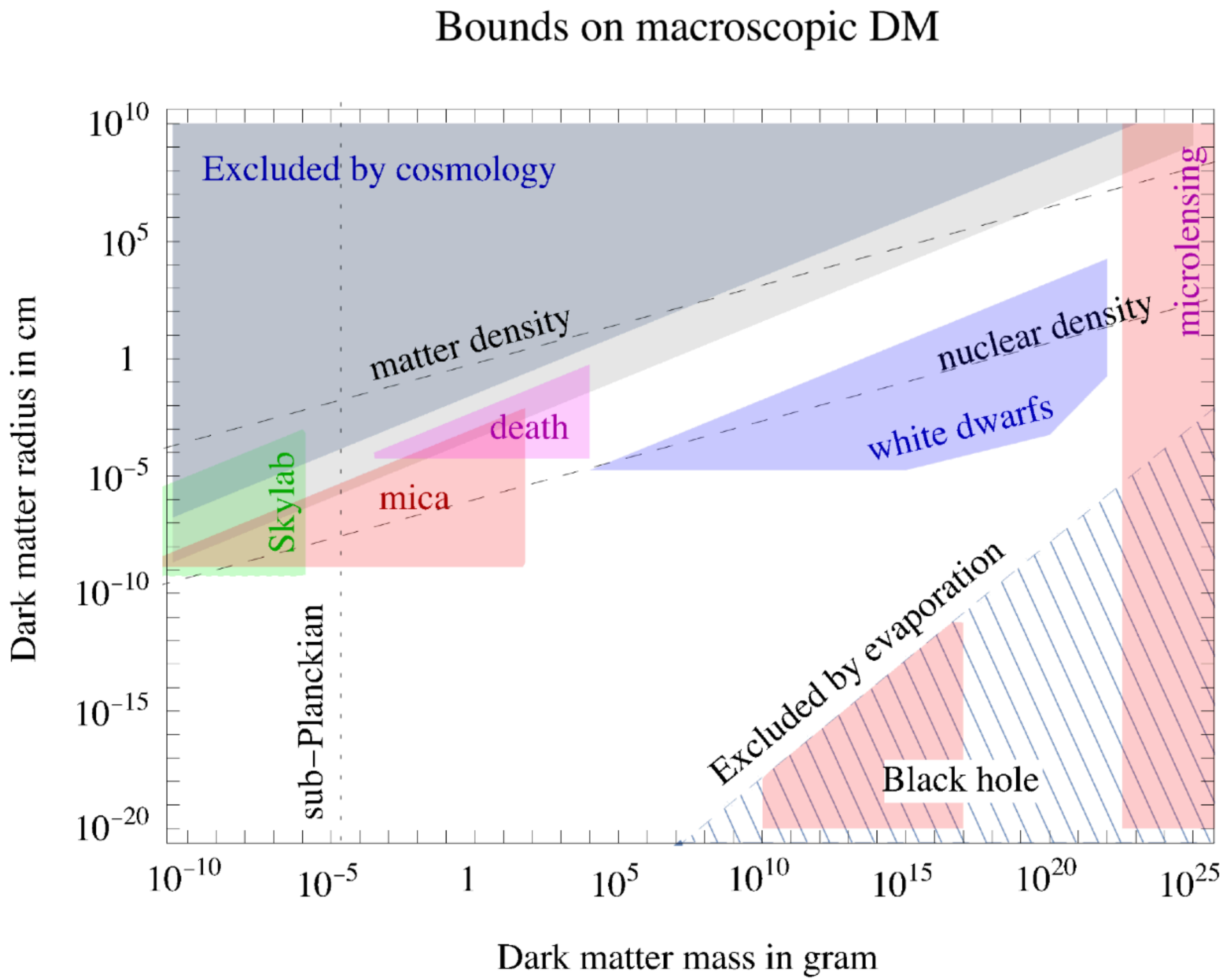
Family	Key Feature	Typical Mass	Unexplained Phenomena / Known Issues
Primordial Black Holes (PBHs)	Compact objects from early Universe	10^{-16}–$10^{5}\ M_{\odot}$	Strong observational constraints; doesn't explain galaxy scaling relations; evaporating PBHs constrained by gamma-rays and CMB
Composite DM	Macroscopic bound states (e.g., AQNs)	g–kg scale	No detection signature yet; requires exotic QCD physics; possible difficulty matching small-scale structure
Cold Dark Matter (CDM)	Cold, non-relativistic particles	GeV–TeV	Core/cusp problem; missing satellites; too-big-to-fail; does not explain BTFR naturally
Self-Interacting DM (SIDM)	Strong DM–DM interactions	GeV–TeV	Constrained by Bullet Cluster; needs tuning of cross-section to scale; limited predictive power for high-redshift data
Asymmetric DM (ADM)	DM–baryon asymmetry link	$\sim$5 GeV	No clear particle candidate yet; challenging to test experimentally; requires consistent baryogenesis scenario
Decaying/Annihilating DM	Unstable or actively emitting particles	GeV–TeV	Strong constraints from CMB and gamma-rays; hard to produce observed structure if lifetime is too short
Warm Dark Matter (WDM)	Semi-relativistic at early times	keV	Fails to form enough early galaxies; doesn't match Lyman-α forest constraints; may underpredict high-redshift structure
Fuzzy/Ultralight DM (FDM)	Wave-like quantum pressure effects	$\sim 10^{-22}$ eV	Hard to generate correct halo mass function; suppressed structure formation may be too strong; no clear production mechanism
Modified Gravity	No DM; modified gravity laws	N/A	Fails to explain CMB, gravitational lensing, or galaxy cluster dynamics without extra assumptions

Candidate Type	**Mass Range**	**Interaction Strength**	**Production Mechanism**	**WIMP?**
Neutralino (SUSY), Kaluza-Klein DM	GeV–TeV	Weak scale (electroweak)	Thermal freeze-out	✓
Asymmetric DM	GeV–TeV	Weak or feeble	Asymmetry-based (like baryons)	✗
FIMPs (Feebly Interacting Massive Particles)	keV–TeV	Extremely weak	Freeze-in (non-thermal)	✗
SIMPs (Strongly Interacting Massive Particles)	MeV–GeV	Strong self-interactions	$3 \to 2$ annihilation (thermal)	✗
Secluded/Hidden Sector DM	GeV–TeV	Interacts with dark sector only	Dark freeze-out or freeze-in	✗
Axions	10^{-6}–10^{-3} eV	Extremely weak (via EM couplings)	Vacuum misalignment	✗
Ultralight DM (FDM)	10^{-22} eV	Gravitational only	Non-thermal (field-based)	✗
Primordial Black Holes (PBHs)	10^{15} g – $10^{5}\,M_{\odot}$	Gravitational only	Collapse in early Universe	✗
WIMPzillas	$\gg$ TeV (up to 10^{13} GeV)	Very weak or gravitational	Non-thermal (e.g., inflationary)	✗

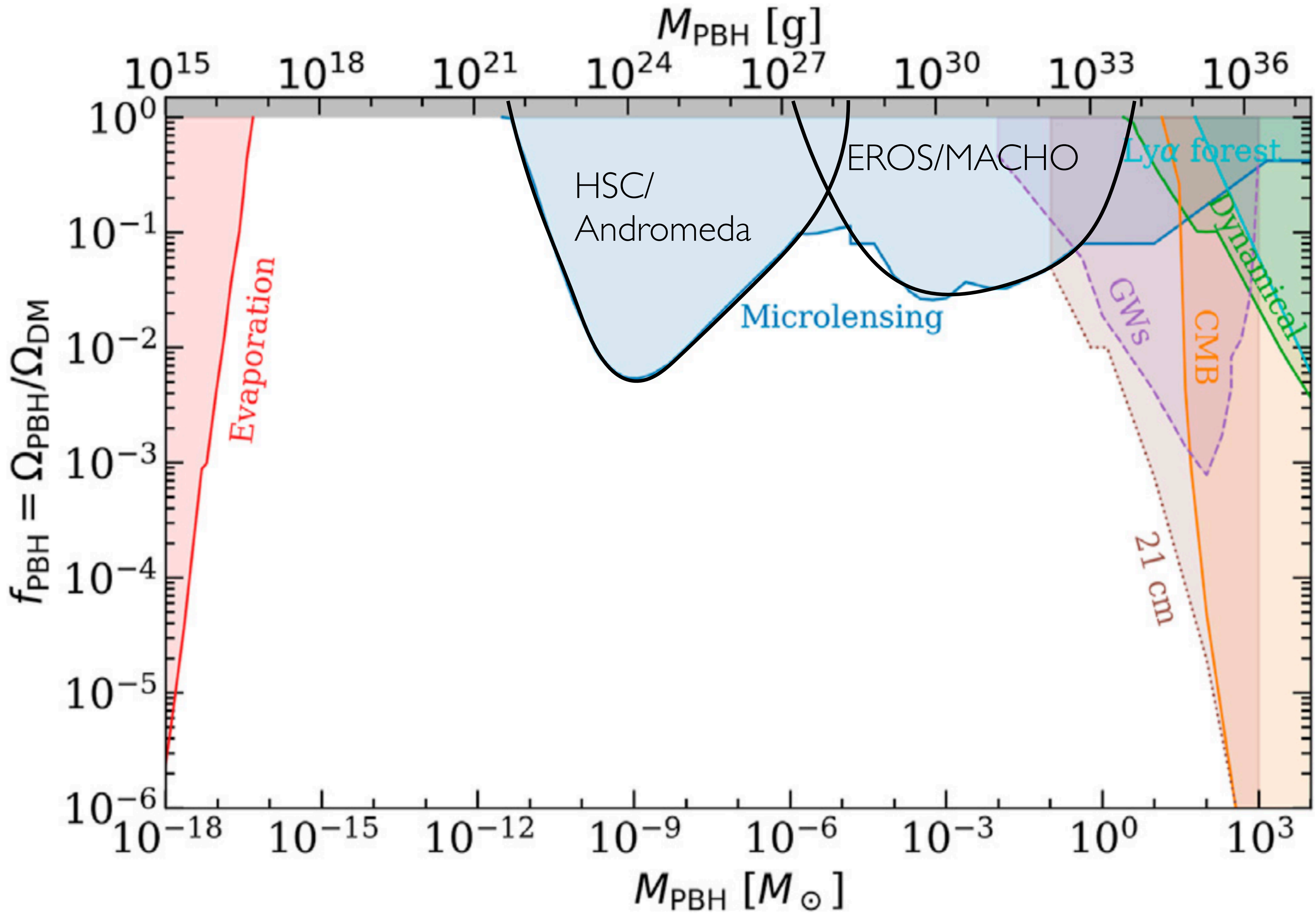
Candidate Type	Mass Range	Interaction Strength	Detection Methods
Neutralino (SUSY), Kaluza-Klein DM	GeV–TeV	Weak scale (electroweak)	Direct detection (nuclear recoil), indirect detection (gamma rays, antimatter), LHC
Asymmetric DM	GeV–TeV	Weak or feeble	Direct detection (nuclear recoil), cosmological constraints (CMB, BBN)
FIMPs (Feebly Interacting Massive Particles)	keV–TeV	Extremely weak	Cosmological imprints (CMB spectral distortions, Lyman-alpha, structure suppression)
SIMPs (Strongly Interacting Massive Particles)	MeV–GeV	Strong self-interactions	Indirect detection (gamma rays, cosmic-ray spectra), galaxy halo shapes and mergers
Secluded/Hidden Sector DM	GeV–TeV	Dark sector only	Indirect (missing energy signatures), cosmological signals (extra radiation, $N_{\rm eff}$)
Axions	10^{-6}–10^{-3} eV	Extremely weak (via EM couplings)	Haloscope (e.g., ADMX), helioscope (e.g., CAST), photon-axion conversion
Ultralight DM (FDM)	10^{-22} eV	Gravitational only	Wave interference in halos, soliton cores, suppression of small-scale structure
Primordial Black Holes (PBHs)	10^{15} g – 10^{5} $M_\odot$	Gravitational only	Microlensing (e.g., OGLE, MACHO), gravitational wave bursts, accretion constraints
WIMPzillas	$\gg$ TeV (up to 10^{13} GeV)	Very weak or gravitational	Ultra-high energy cosmic rays, CMB anisotropies, lack of thermal production signature

The Axion Quark Nugget Dark Matter model

Macroscopic DM



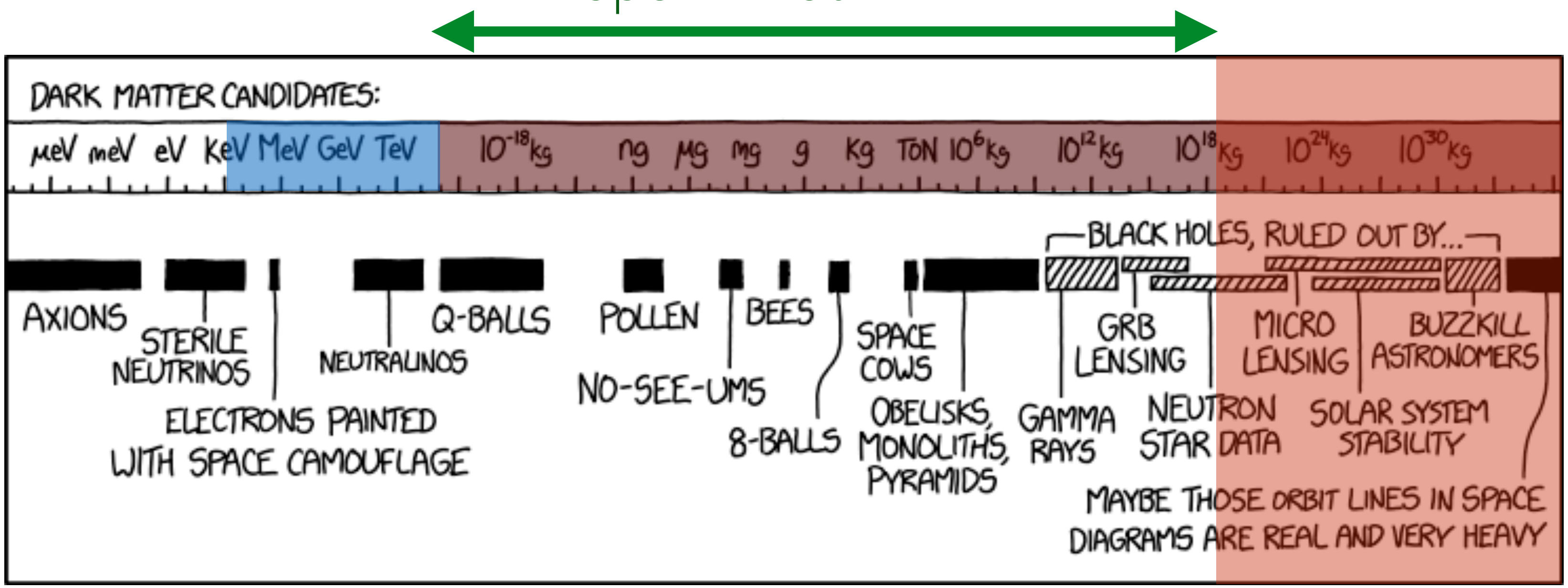
From Jacobs et al 2015, Cirelli et al 2024



Villanueva-Domingo et al 2021

Mass range to scale

Macros range still
open if not PBH



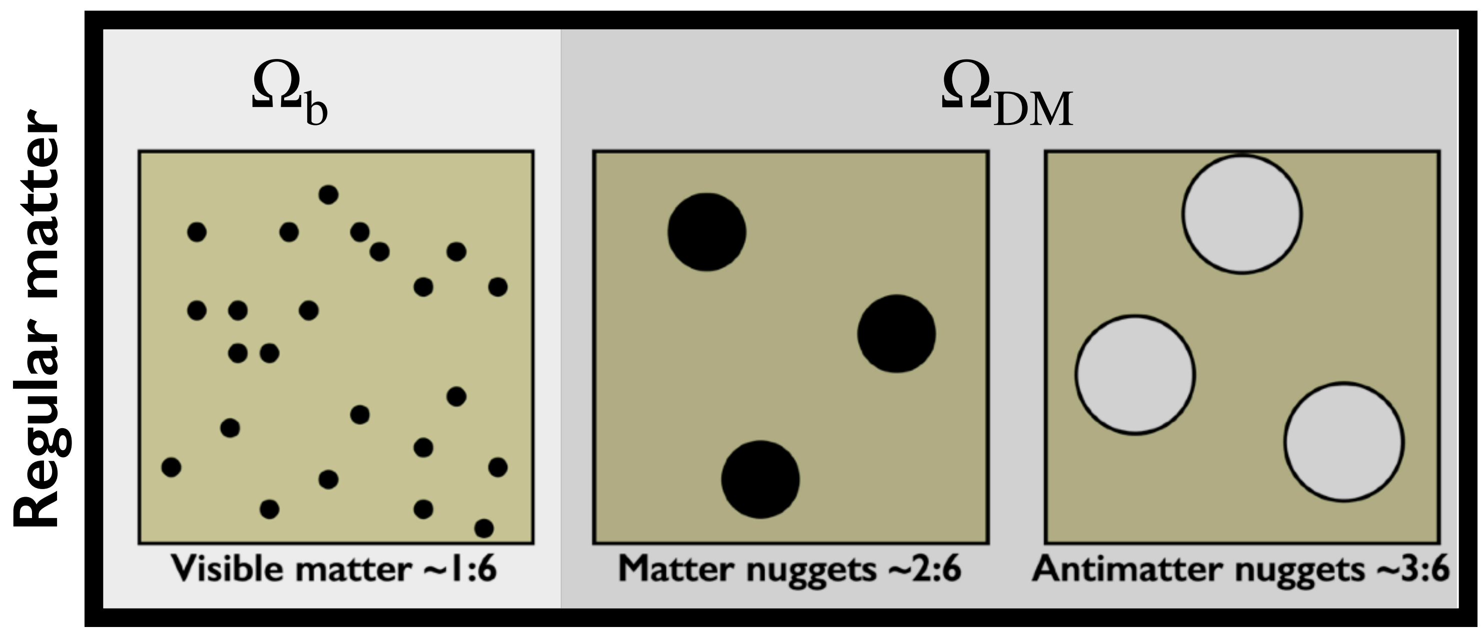
https://xkcd.com/2035/

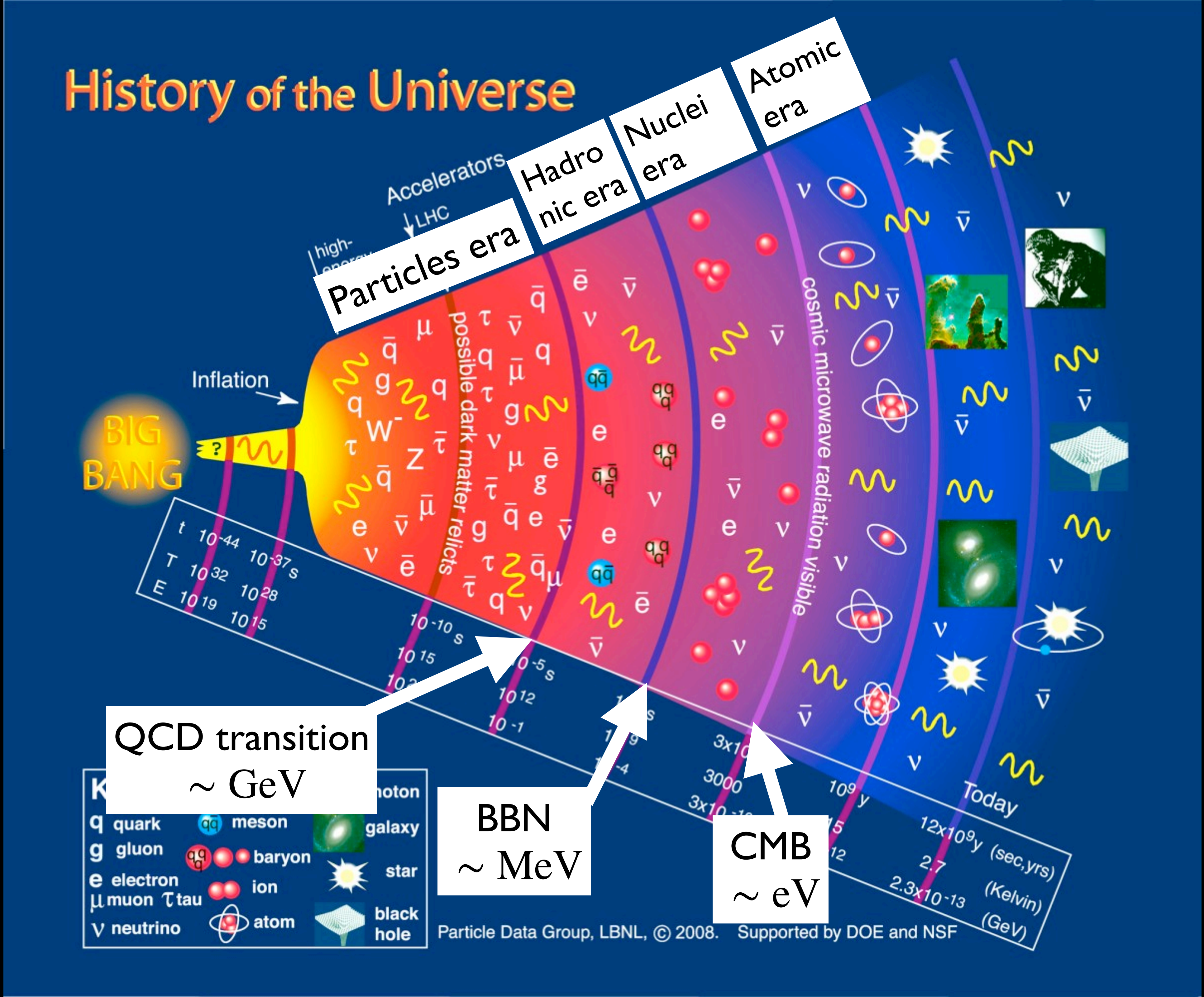
Axion Quark Nuggets: (Zhitnitsky 2003, Liang & Zhitnitsky 2016)

DM is a composite object made of baryons interacting strongly with baryons

Motivations: Naturally leads to $\Omega_{DM} \sim \Omega_b$
Has the potential to explain baryogenesis

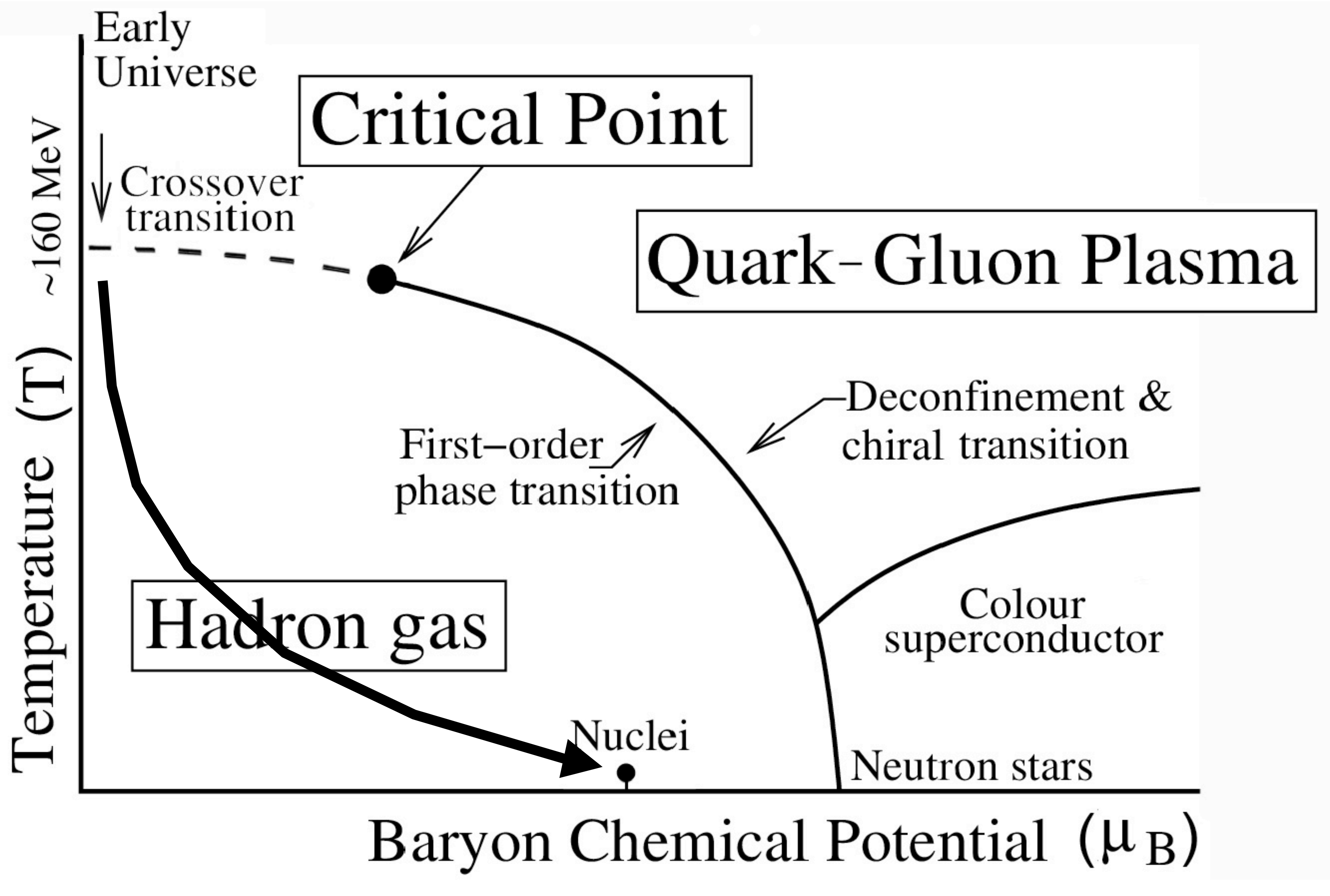
$$B_{\text{tot}} = 0 = B_{\text{nugget}} + B_{\text{visible}} - B_{\text{antinugget}}$$
$$B_{\text{DM}} = B_{\text{nugget}} + \bar{B}_{\text{antinugget}} \simeq 5B_{\text{visible}}$$





History of the Universe
Particles era
Hadronic era
Nuclei era
Atomic era
Accelerators
LHC
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
QCD transition
~ GeV
BBN
~ MeV
CMB
~ eV
Today
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
meson
baryon
ion
atom
galaxy
star
black hole
Particle Data Group, LBNL, © 2008.
Supported by DOE and NSF

The QCD transition happens at $\sim 1 - 0.1$ GeV



The physics of AQN formation is a **direct consequence** of Quantum Chromodynamics with an axion field (yet to be discovered).

It results from the non trivial topological structure of the QCD vacuum, which is a common feature of 3+1 dimensions non-abelian gauge field theory.

Summary of the main steps:

Remember that QCD allows a CP-violating term in the Lagrangian:

$$\mathscr{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Where θ is a free parameter. $\theta < 10^{-10}$ is required to solve the strong CP problem

Step 1: Introduce a global symmetry $U(1)_{\rm PQ}$. It is implemented via e.g. a scalar field ϕ:

$$\phi(x) \equiv (f_a + \rho(x))e^{i\alpha}$$

With $\langle\phi\rangle = f_a$ and α an unphysical global symmetry (not yet the axion field)

Step 2: Spontaneous symmetry breaking at scale f_a promotes α, the phase of ϕ, to a dynamical field a/f_a

Step 3: Axion field couples to gluons (and photons):

$$\mathscr{L}_{aG\tilde{G}} = \frac{a(x)}{f_a} \cdot \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu}$$

The effective angle of the CP violating term becomes:

$$\theta_{\text{eff}} \equiv \theta + \frac{a(x)}{f_a}$$

Step 4: At ~ 1 GeV, QCD non-perturbative effects generate a potential for the axion:

$$V(a) \propto \chi_{\text{QCD}}(T) \cdot \left(1 - \cos\left(\frac{a}{f_a} + \theta\right)\right)$$

The axion dynamically relaxes to cancel θ (i.e. $\theta_{\text{eff}} \to 0$)

QCD topological susceptibility:

$$\chi_{\text{QCD}}(T) = \begin{cases} \chi_0 \sim (75 \text{ MeV})^4, & T \ll \Lambda_{\text{QCD}} \\ \chi_0 \left(\frac{\Lambda_{\text{QCD}}}{T}\right)^n, & T \gg \Lambda_{\text{QCD}} \end{cases}$$

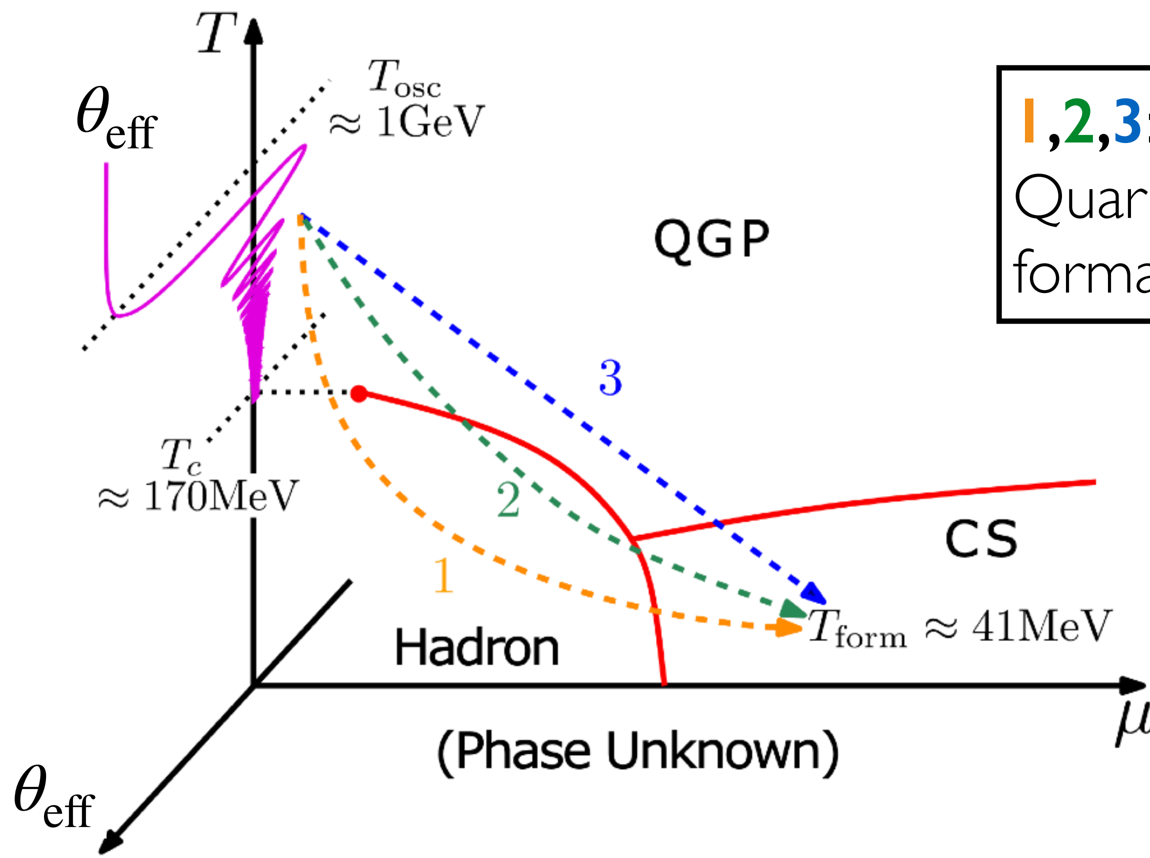
with $n \approx$ 7-8

Step 5: The axion acquires a small mass:

$$m_a^2 f_a^2 = \chi_{\rm QCD}(T)$$

The axion field oscillates around the CP-conserving minimum $\theta_{\rm eff} = 0$

CP violation in QCD is eliminated dynamically.



1,2,3: possible Axion Quark Nugget formation paths

Ge, Liang, Zhitnitsky 2018

The details are very complicated and far from being fully understood (progress In lattice QCD). But **QCD is potentially extremely important cosmologically** for dark matter and dark energy (see Friday lecture)

AQN formation steps:

- $N_{\rm DW} = 1$, i.e. 2π shift in a/f_a domain walls (bubbles) are copiously produced (even if PQ symmetry is broken after inflation)

- Matter and anti-matter populate these bubbles asymmetrically (because of the original global CP violation with $\theta_{\rm eff} \sim 10^{-2} - 10^{-3}$ at ~ 1 GeV)

- For a long time, these axion domain walls were thought to be short-lived….but at energy $T < 170$ MeV the η' meson which will act as a matter (antimatter) potential barrier and form **a stable axion-η' nugget** that squeezes matter (antimatter) in the Color Superconducting phase.

- At $T \sim 40$ MeV (CS gap scale), nugget formation stops and the remaining matter becomes the visible matter, approximately at the same time hadrons form $\Longrightarrow \Omega_b \sim \Omega_{\rm DM}$

Important points:

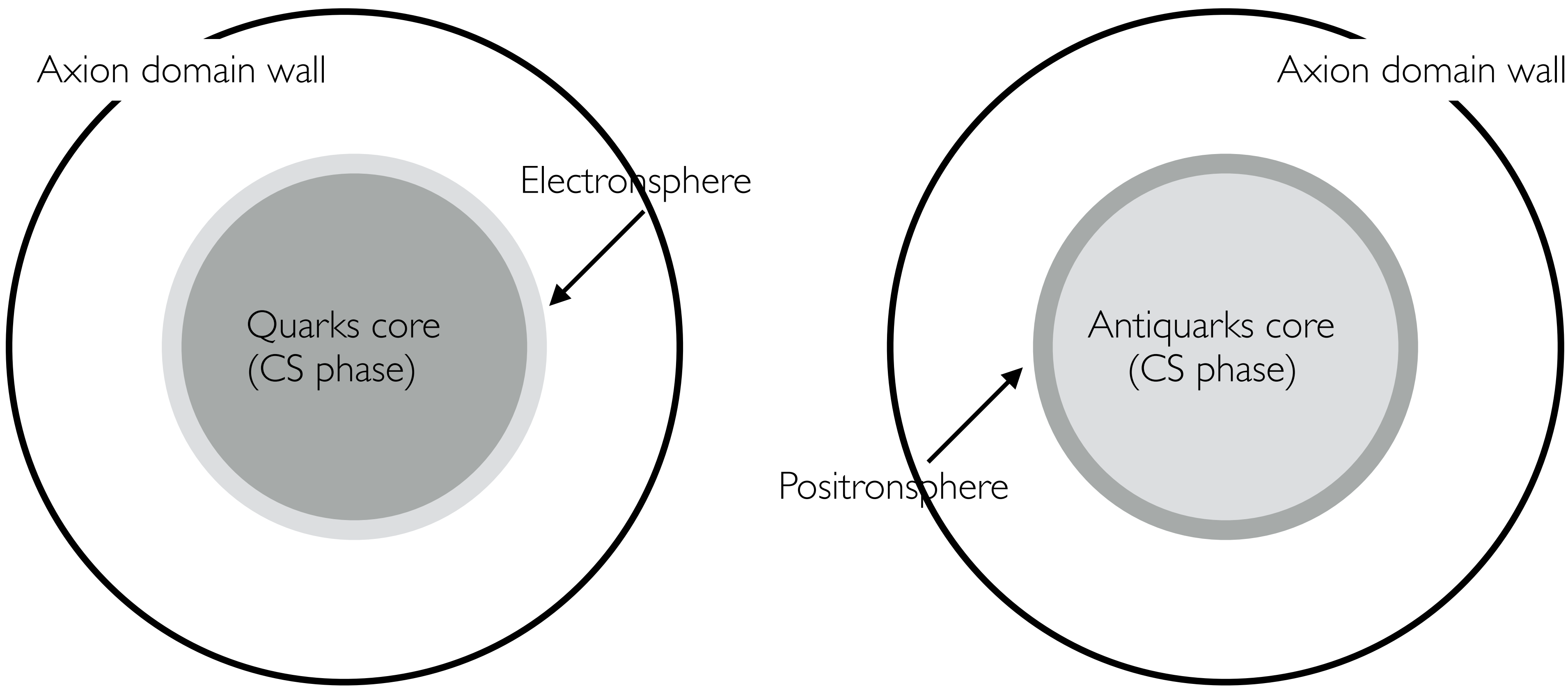
1- The baryonic ratio $\eta \sim e^{-m_p/T_{\rm form}}$, where $T_{\rm form}$ is the binding energy of the CS phase.

2- $T_{\rm form} \sim 40~\mathbf{MeV}$ is critical to obtain $\eta \sim 10^{-10}$. Without baryogenesis, conventional cosmology leads to $\eta \simeq 10^{-20}$ contradicting BBN and CMB measurements.

3- With $\theta_{\rm eff} \sim 10^{-3} - 10^{-2}$ at $\sim 1~\mathbf{GeV}$ it is the oscillations of the axion domain wall that amplifies the charge separation process and leads to a quark/antiquark ratio of order ~ 1 (Ge, Lawson, Zhitnitsky 2019)

4- ALPs do not couple to gluons, therefore cannot solve the strong CP problem: **ALPs are not needed by particle physics and ALPs do not form AQNs**

5- The binding energy of the AQN core is $\sim 40~\mathbf{MeV}$ per baryon, which protects the AQN from destruction during BBN



Quarks (antiquarks) core have nuclear density. They are like mini neutron (anti-neutron) stars (not held by gravity!).

Ge, Liang, Zhitnitsky 2018

Properties:

AQNs have a $\sigma_{\rm AQN}/m_{\rm AQN} \sim 10^{-10}\ {\rm cm}^2/{\rm g}$, fully consistent with CDM

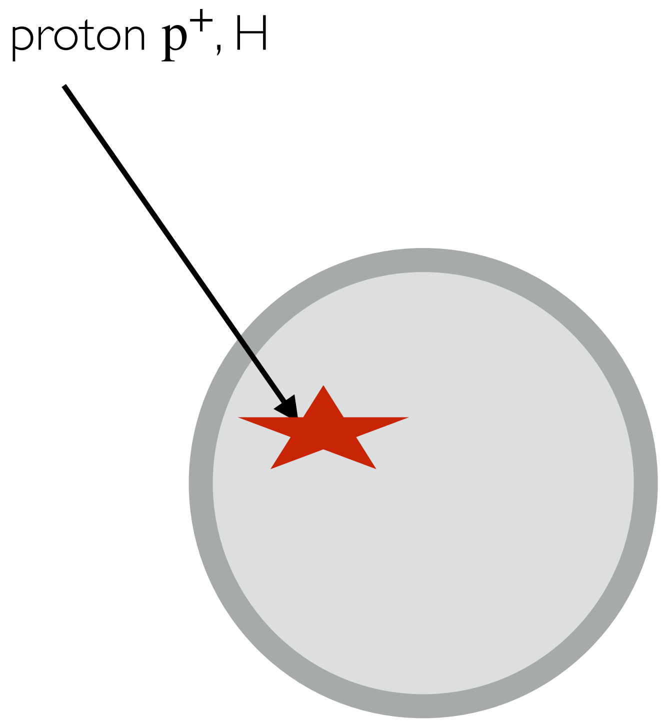
The AQN formation process hides baryons before BBN with an average binding energy per nucleon mass $\gg$ few MeV

AQN mass $m_{\rm AQN} > 10^{25}\ {\rm m_p} \sim 10\ g$, no upper bound
AQN mass density $\rho_{AQN} < \rho_{\rm nucl} = 1.5 \times 10^{14} {\rm g/cm}^3$
AQN size $R_{AQN} \sim 10^{-5}\ {\rm cm}$

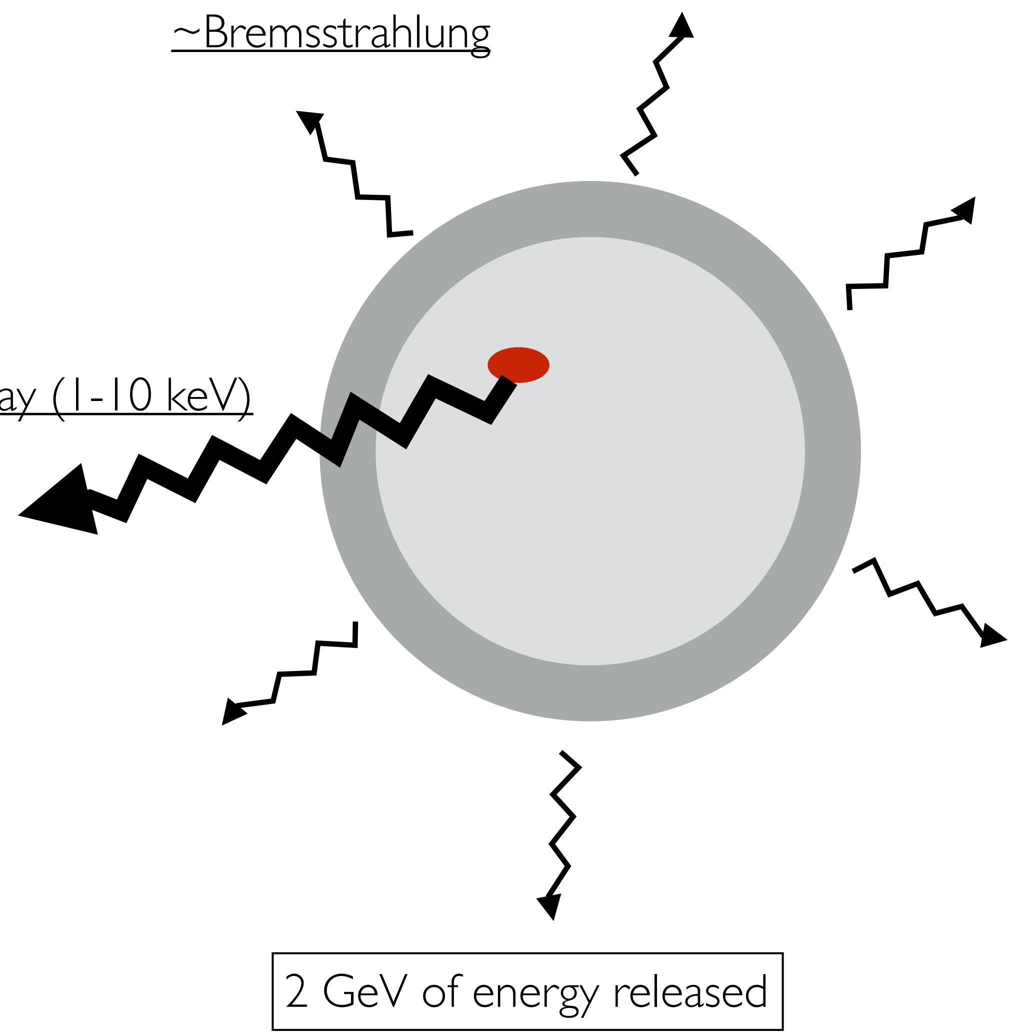
In the solar system, where $\rho_{\rm DM} \sim 0.3\ {\rm GeV\ cm}^{-3}$, the AQN number density is extremely small $(10^6\ {\rm km})^{-3} < {\rm n_{AQN}} < (10^3\ {\rm km})^{-3} \Longrightarrow$ impossible to detect in a DM particle detector!

Emission mechanisms (simplified version)

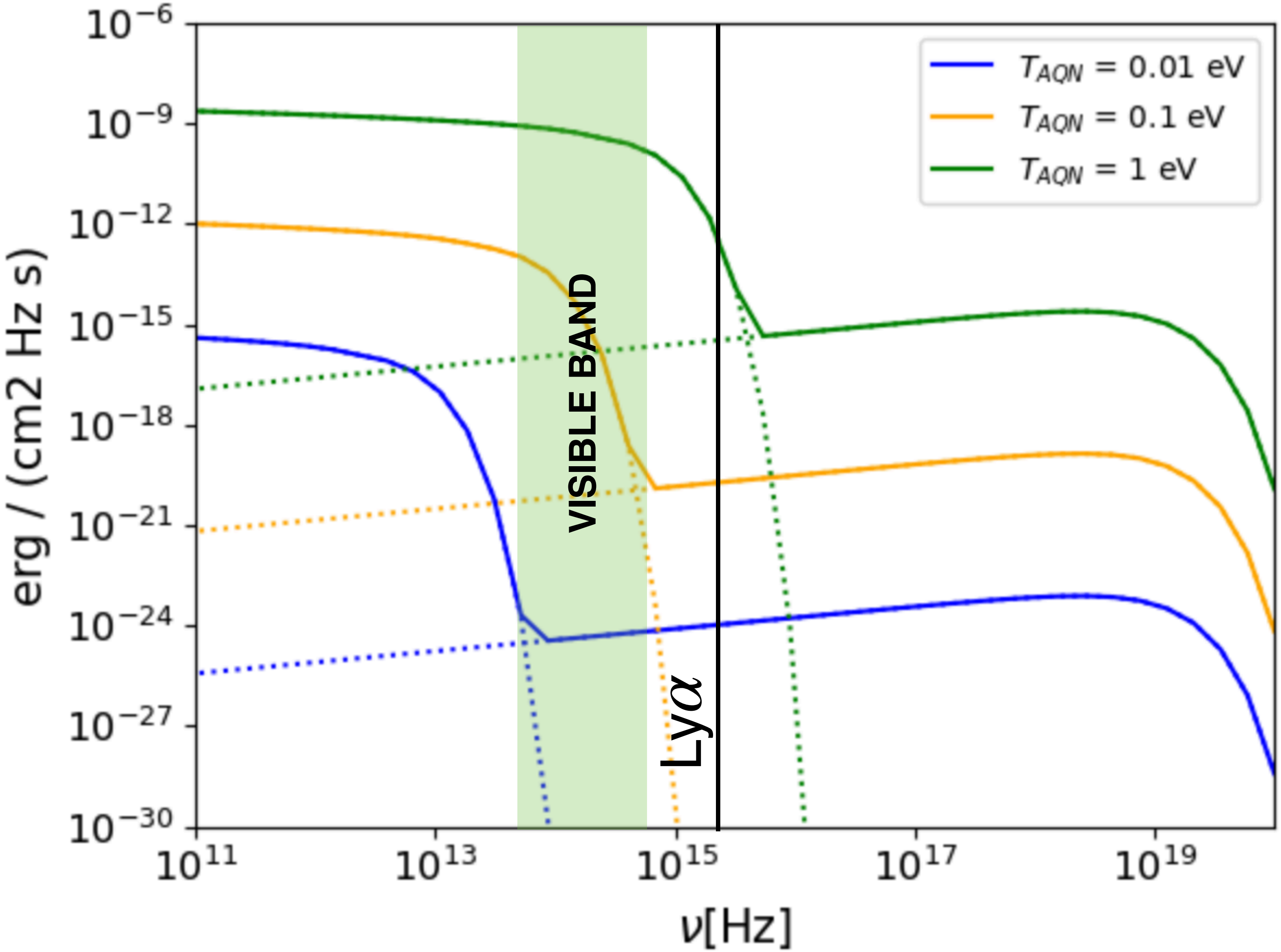
How can AQN be tested? what effects can be observed?
Antimatter AQNs interact with normal matter!



Forbes & Zhitnitsky 2008



Electromagnetic emission of an AQN

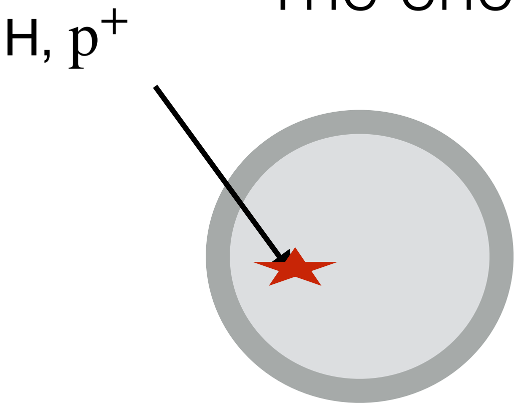


Thermal spectrum for a nugget at temperature $T_{\rm AQN}$:

$$\mathrm{dF}(\nu, T_{\rm AQN}) = \frac{4}{45}\frac{T_{\rm AQN}^3\alpha^{5/2}}{\pi}\left(\frac{T_{\rm AQN}}{m_e}\right)^{1/4}\left(1+\frac{\nu}{T_{\rm AQN}}\right)h\left(\frac{\nu}{T_{\rm AQN}}\right)\exp\left(-\frac{\nu}{T_{\rm AQN}}\right)$$

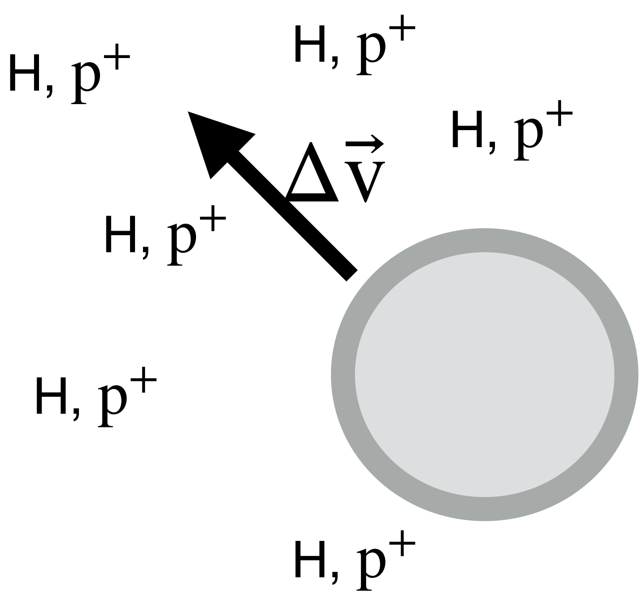
$$\mathrm{dF}_{\rm bol}(T_{\rm AQN}) = \int d\nu\ \mathrm{dF}_{\nu}(\nu, T_{\rm AQN})$$

The energy available per collision with a proton is:



$$dE_{\rm ann} = 2\ {\rm GeV} f(1-g)$$

Per unit of time dt, the energy injected in the nugget is:



$$\frac{dE_{\rm ann}}{dt} = 2\ {\rm GeV} f(1-g)\ \pi R_{\rm AQN}^2\ \Delta {\rm v}\ n_{\rm bar}$$

The nugget temperature is obtained from

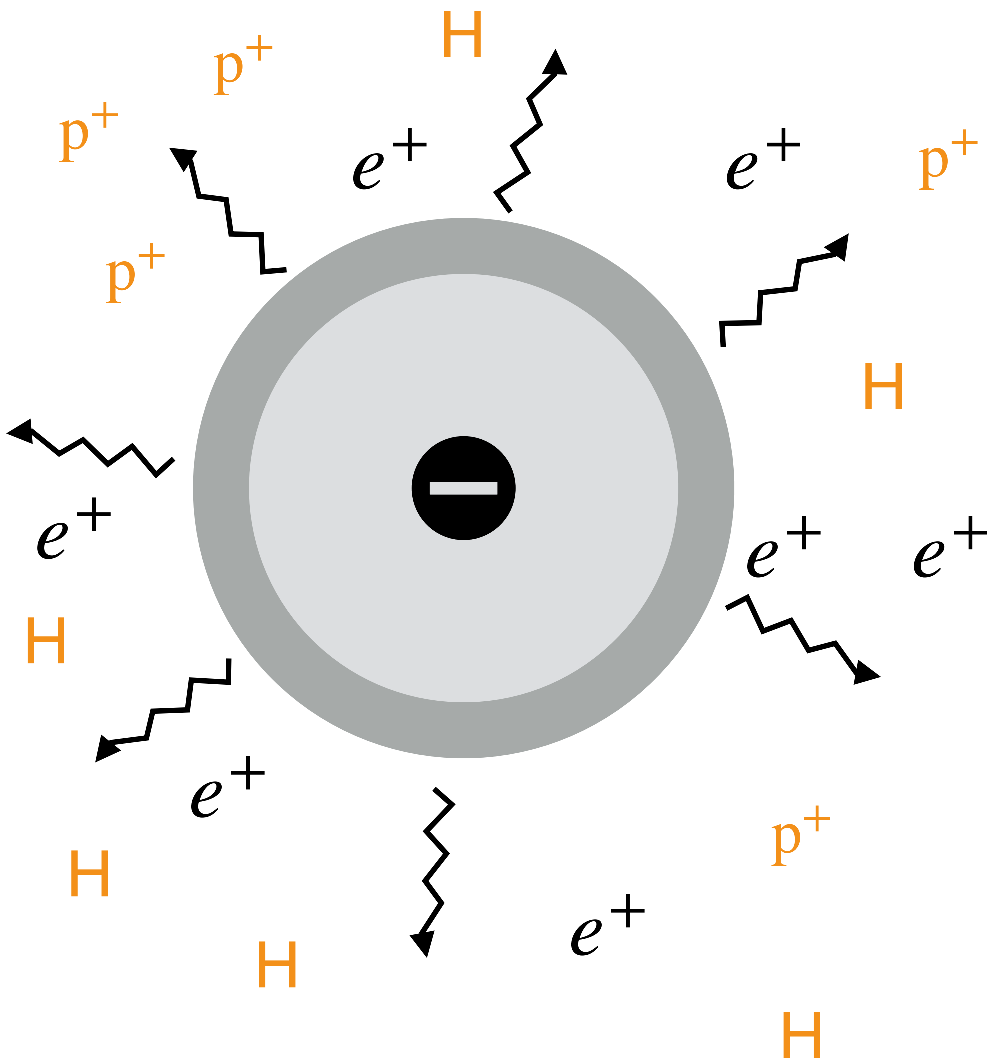
$$\frac{dE_{\rm ann}}{dt} = 4\pi R^2_{\rm AQN}\ {\rm dF_{bol}}(T_{\rm AQN})$$

The emissivity $d\epsilon(\nu, T_{\rm AQN})$ radiated away from the DM-baryon collision is then:

$$d\epsilon(\nu, T_{\rm AQN}) = 4\pi R^2_{\rm AQN}\ dF(\nu, T_{\rm AQN})\ n_{\rm AQN}$$

$d\epsilon(\nu, T_{\rm AQN}) \propto n_{\rm AQN}\ n_{\rm bar}^{\frac{13}{17}}$ which is not $\propto n^2_{\rm AQN}$

Complications when $T_{\text{AQN}} > 0$:



-Dark matter can be charged Q_{AQN}

-Q_{AQN}, T_{bar} and the baryon Ionization fraction control the Capture cross-section, hence T_{AQN}

-T_{AQN} controls Q_{AQN}

-Q_{AQN} controls the capture rate, i.e. T_{AQN}, etc….

Approximately 3/2 of dark matter would be made of antimatter! **Every collision with visible matter will generate energy.**

Imagine the number of observational windows suddenly opened with this physically motivated DM model:

1- What happens when AQN cross Earth or other planets?

2- What happens when it falls on the Sun, other stars, compact objects?

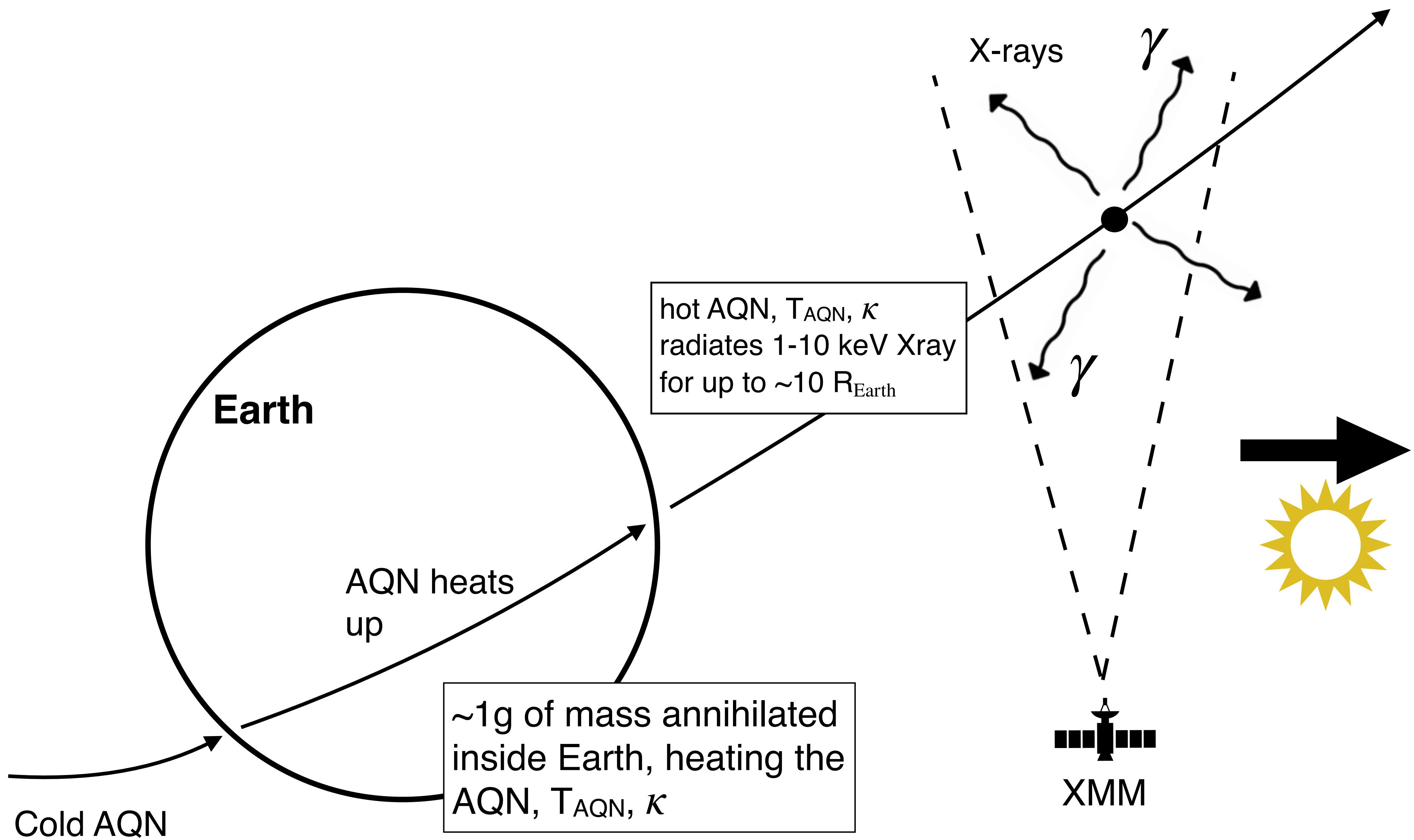
3- How does it survive BBN? How does it not completely mess with CMB physics? With reionization?

4- What happens in the Milky-Way, in ISM, galactic and cluster environments?

5- It is electrically charged, would it not mess with magnetic fields?

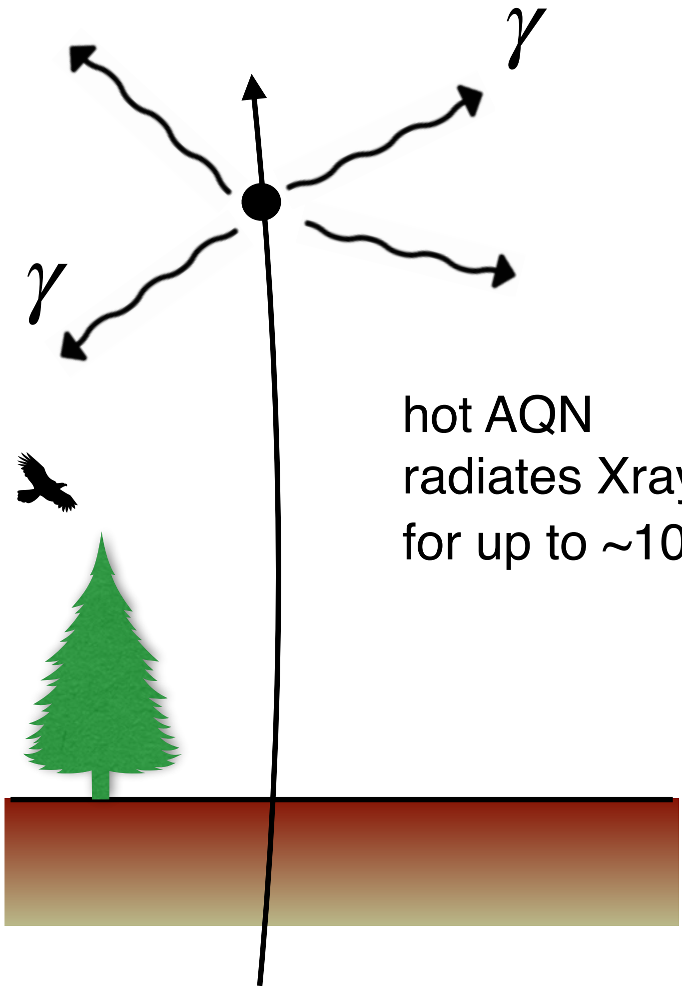
→This model is extremely vulnerable to observational tests, there are many ways it could fail a test.

AQN contribution to the Earth X-ray environment



Ge et al. 2022

X-ray background level on the ground

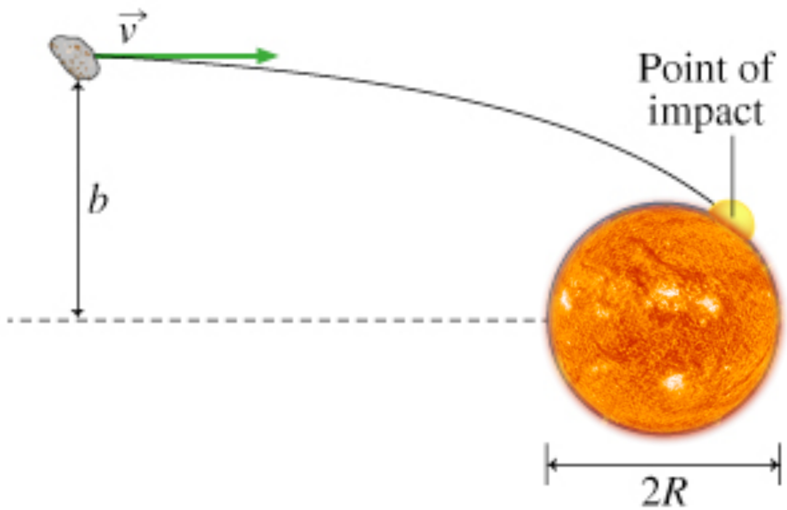


hot AQN
radiates Xray
for up to ~10 R_{Earth}

5,000 mSv	Half of people exposed to this level in a single dose will die within a month.
1,000 mSv	Causes acute radiation sickness in people exposed to this amount in a single dose.
100 mSv / year	Lowest level that causes a documented increase in cancer risk.
10-15 mSv	CT scan
9 mSv / year	Typical exposure by airline crew flying New York/Tokyo polar route.
2-3 mSv / year	Amount of background radiation people are generally exposed to each year.
.2 mSv	Chest x-ray
.01 mSv	Dental x-ray

Possible range of T_{AQN}, κ.

Will not harm people



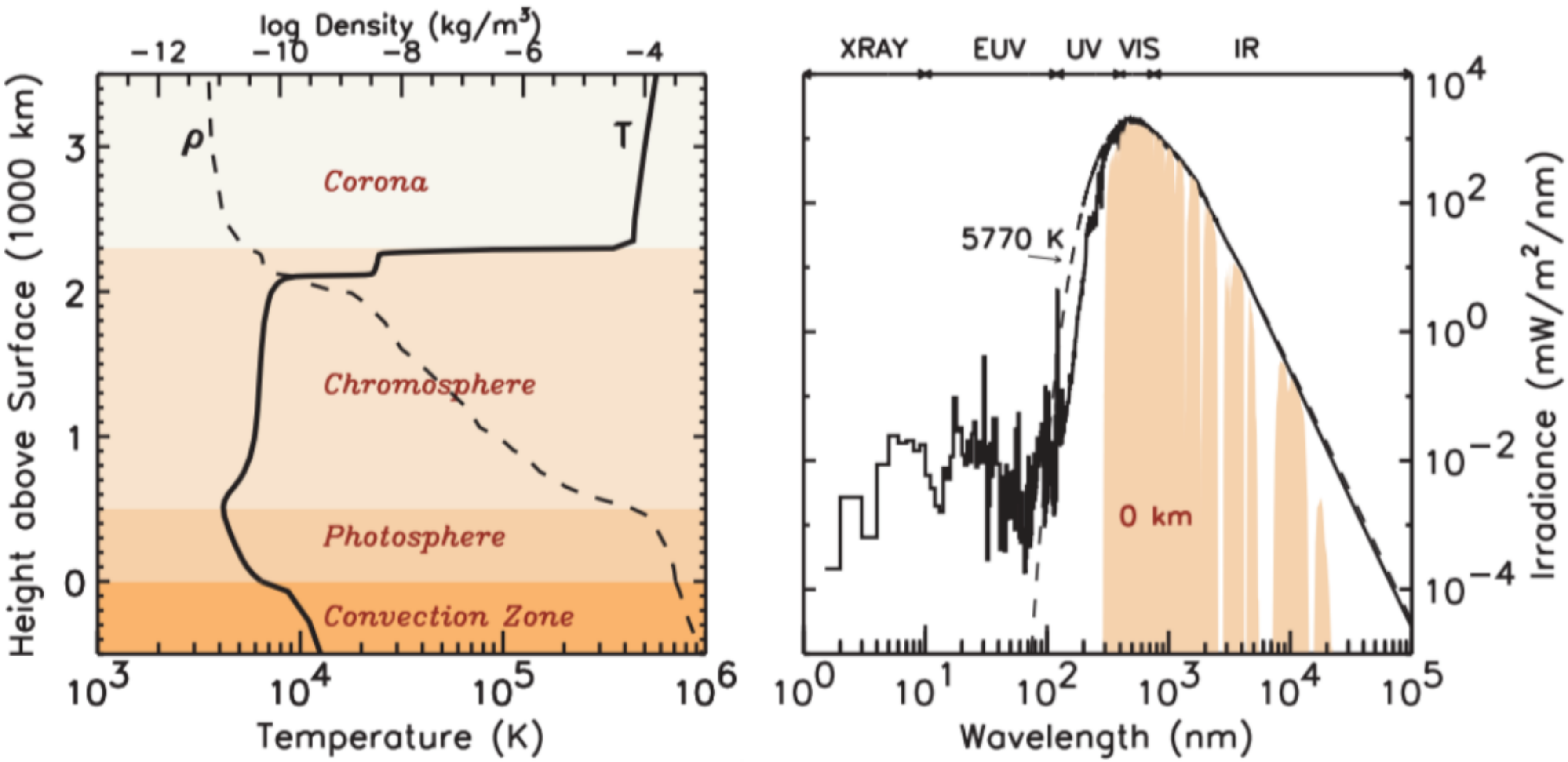
$$b_{\text{cap}} \simeq R_{\odot}\sqrt{1+\gamma_{\odot}}, \qquad \gamma_{\odot} \equiv \frac{2GM_{\odot}}{R_{\odot}v^2}$$

<u>Energy budget from DM environment:</u>

$$L_{\odot\ (\text{AQN})} \sim 4\pi b^2_{\text{cap}} \cdot v \cdot \rho_{\text{DM}} \simeq 3 \cdot 10^{30} \cdot \frac{\text{GeV}}{\text{s}} \simeq 4.8 \cdot 10^{27} \cdot \frac{\text{erg}}{\text{s}}$$

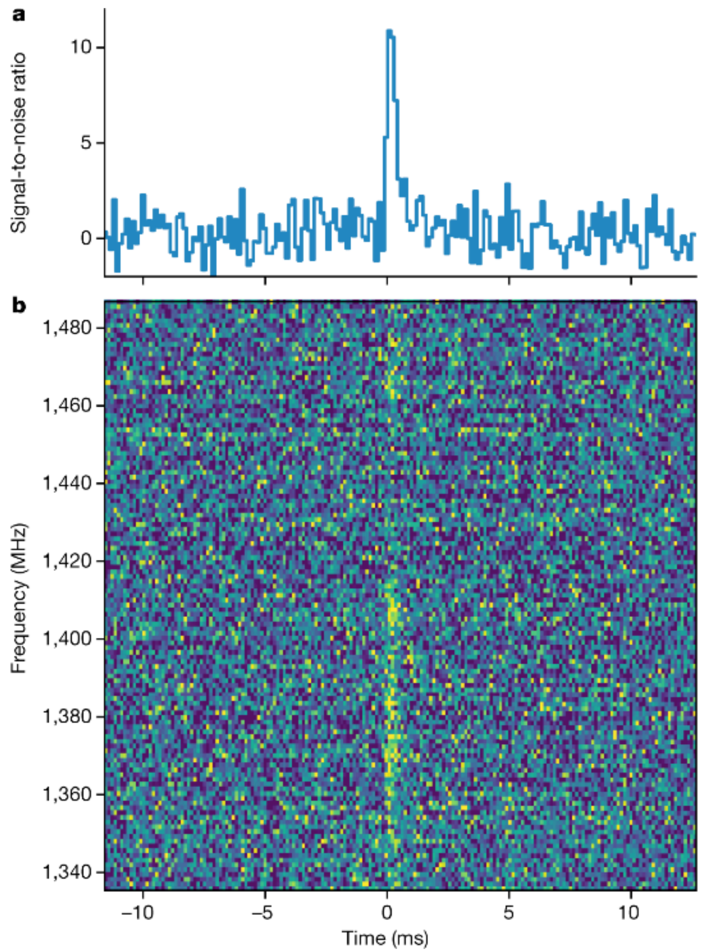
The solar corona problem: ~10^{27} erg/s missing

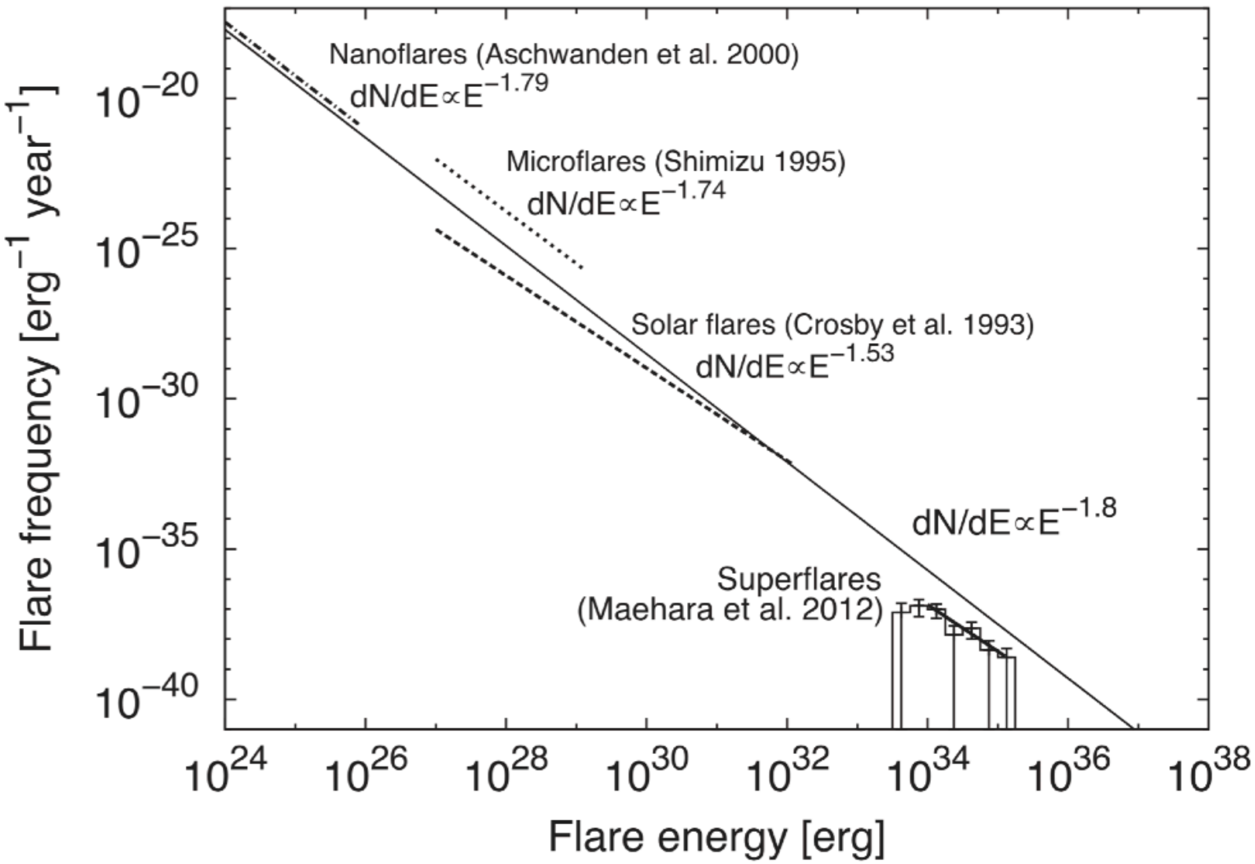
The solar corona problem: EUV/soft Xray from corona emits ~10^{27} erg/s not accounted for.



Raza et al. 2018

Fast Radio Bursts and AQNs





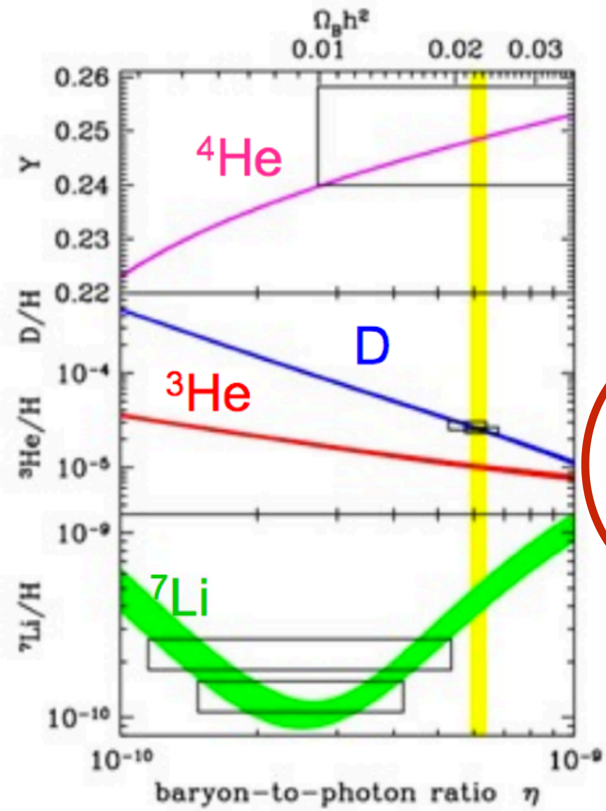
$E_{\rm FRB} \sim 10^{38} - 10^{41}$ erg

Magnetar+AQN infall $\Longrightarrow$ Magnetic reconnection consistent with FRB energy range/time scale and frequency

Van Waerbeke & Zhitnitsky 2019

The Li^7 problem

Flambaum & Zhitnitsky 2019



Primordial Lithium-7

Enhancement factor due to ion charge Z and AQN charge Q at distance r.

$$\frac{\delta n_Z}{n_Z} \simeq \frac{4\pi R_{\rm cap}^3}{3} \cdot n_{AQN} \cdot \exp\left[\frac{(Z-1)\alpha Q(r)}{rT}\right]$$

Geometrical capture volume due to the presence of AQN

Relative number of trapped and captured ions with atomic number Z

For Z=1, 2; $\frac{\delta n_{\rm D}}{n_{\rm D}}; \frac{\delta n_{\rm He}}{n_{\rm He}} \ll 1$

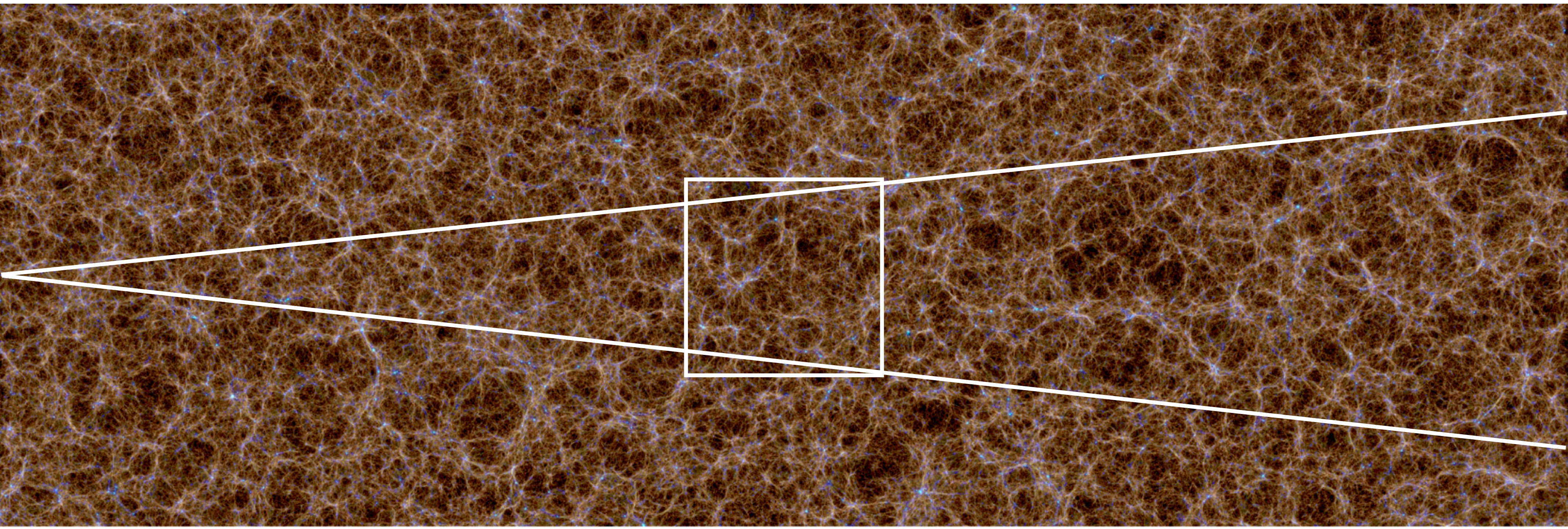
For Z=3 $\frac{\delta n_{\rm Li^7}}{n_{\rm Li^7}} \sim 1$

Contribution to the cosmological backgrounds (radio, IR, optical, UV)

The electromagnetic signature of AQNs is easy to calculate:

$$\left.\begin{matrix} n_{\rm bar}(\vec{r}, t) \\ \Delta \mathrm{v}(\vec{\mathrm{r}}, \mathrm{t}) \end{matrix}\right\} \longrightarrow \left.\begin{matrix} T_{\rm AQN}(\vec{r}, t) \\ n_{\rm AQN}(\vec{r}, t) \\ m_{\rm AQN} \end{matrix}\right\} \longrightarrow d\epsilon(\nu, T_{\rm AQN}, \vec{r}, t)$$

AQN Dark Matter glow: how bright for the large scale structures?

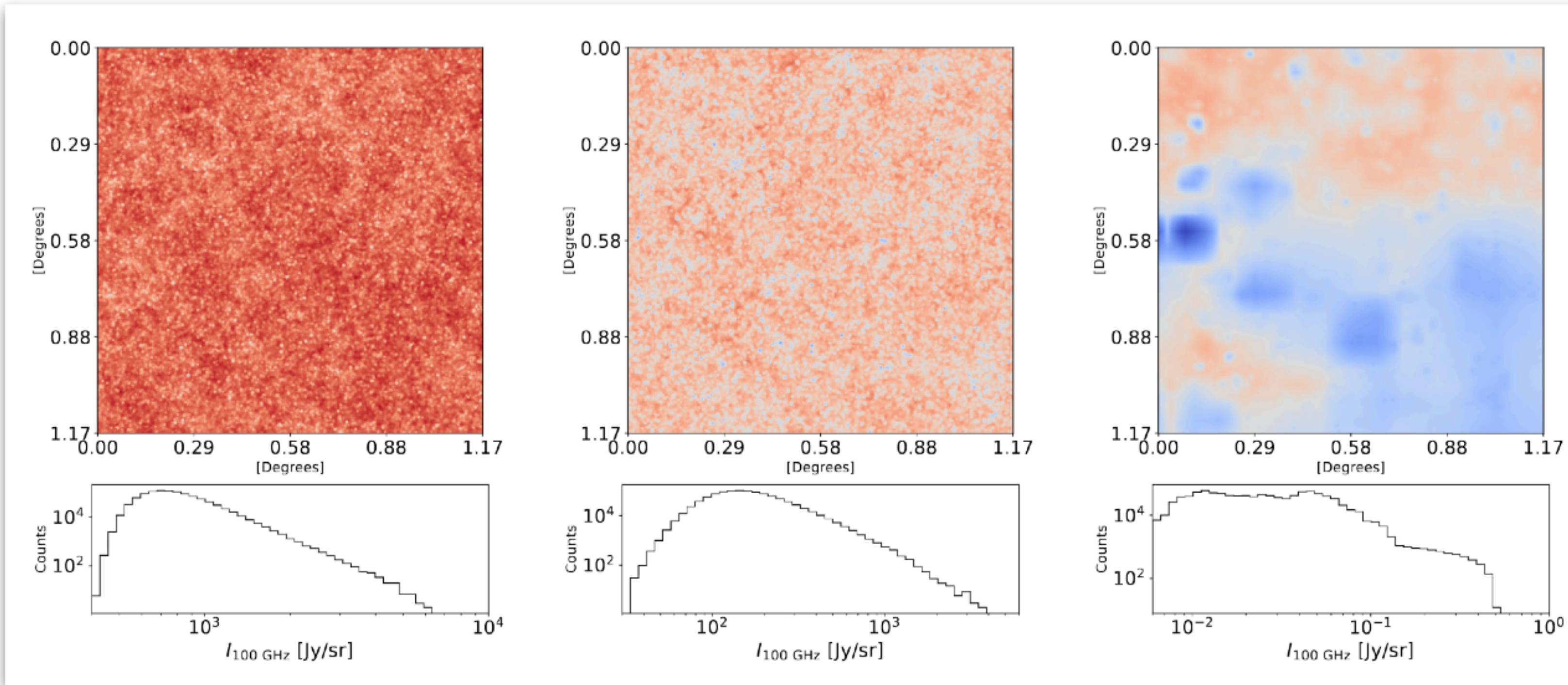


MAGNETICUM

Dolag et al. (LMU)

AQN Dark Matter glow: how bright for the large scale structures?

Sky intensity at 100 GHz from Magneticum simulation

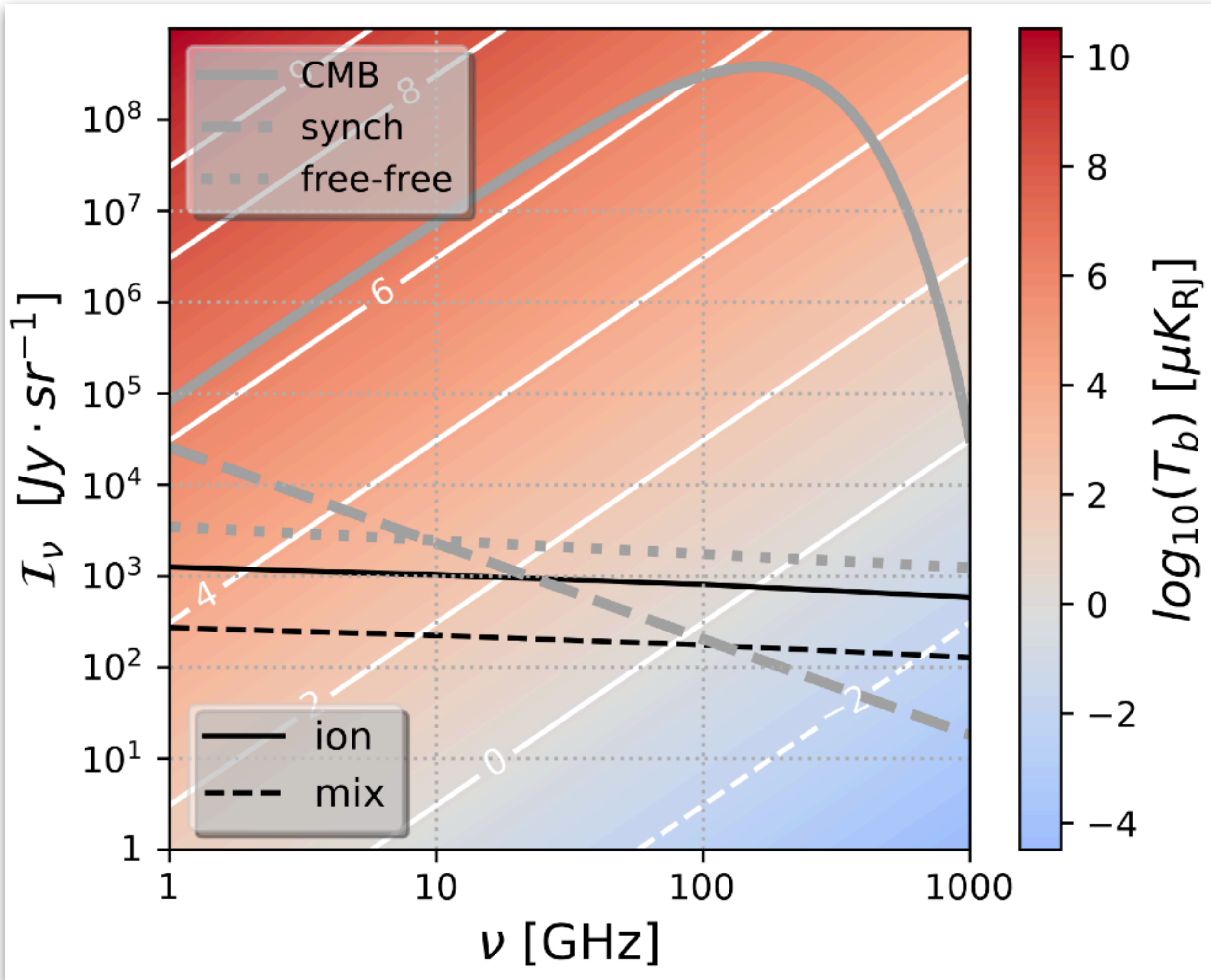


Ionized gas   Mixed gas   Neutral gas

Monopole + fluctuations dark matter glow

Majidi et al 2024

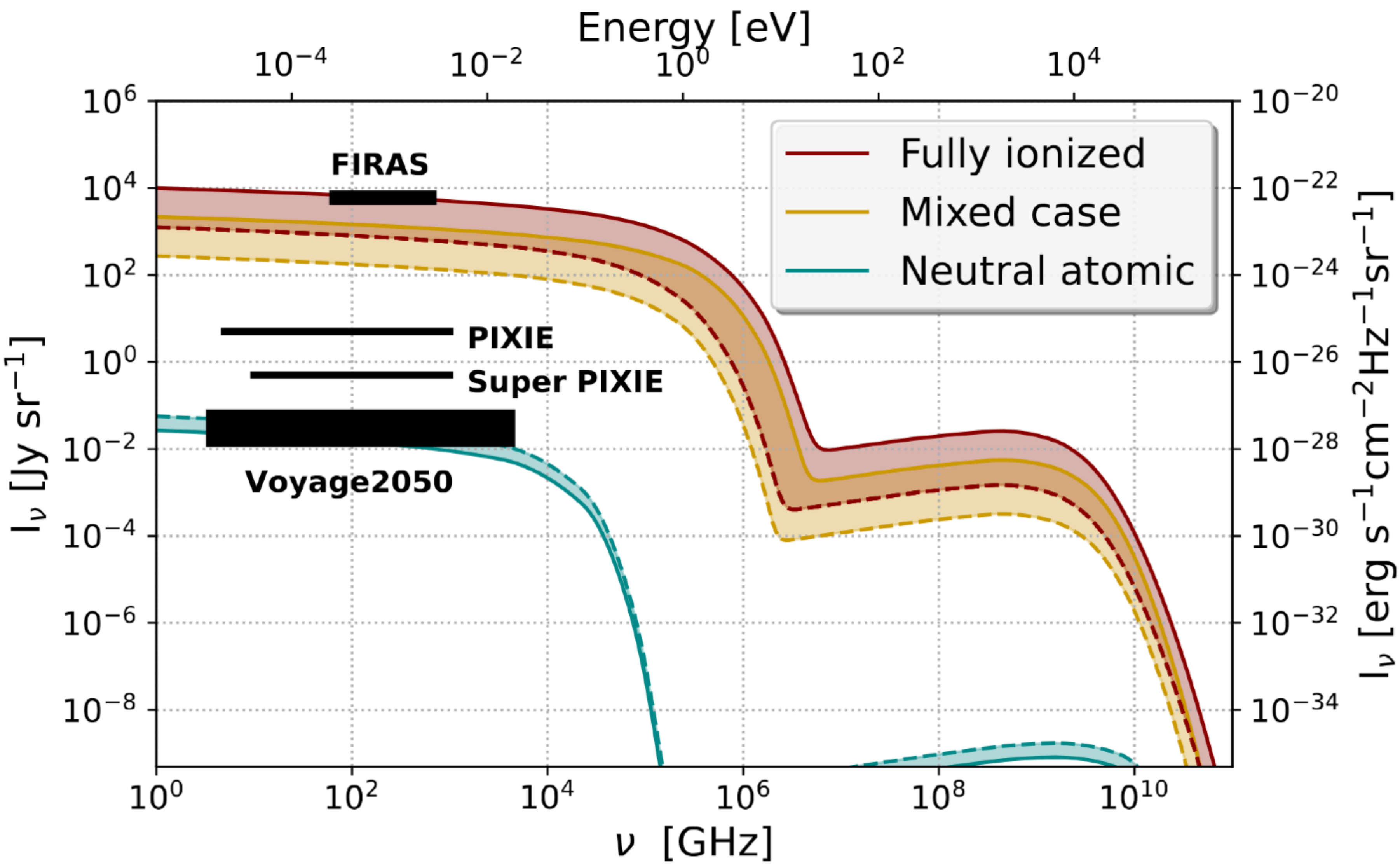
AQN Dark Matter glow: how bright in radio?



Dark matter glow monopole compared to the known monopole signals

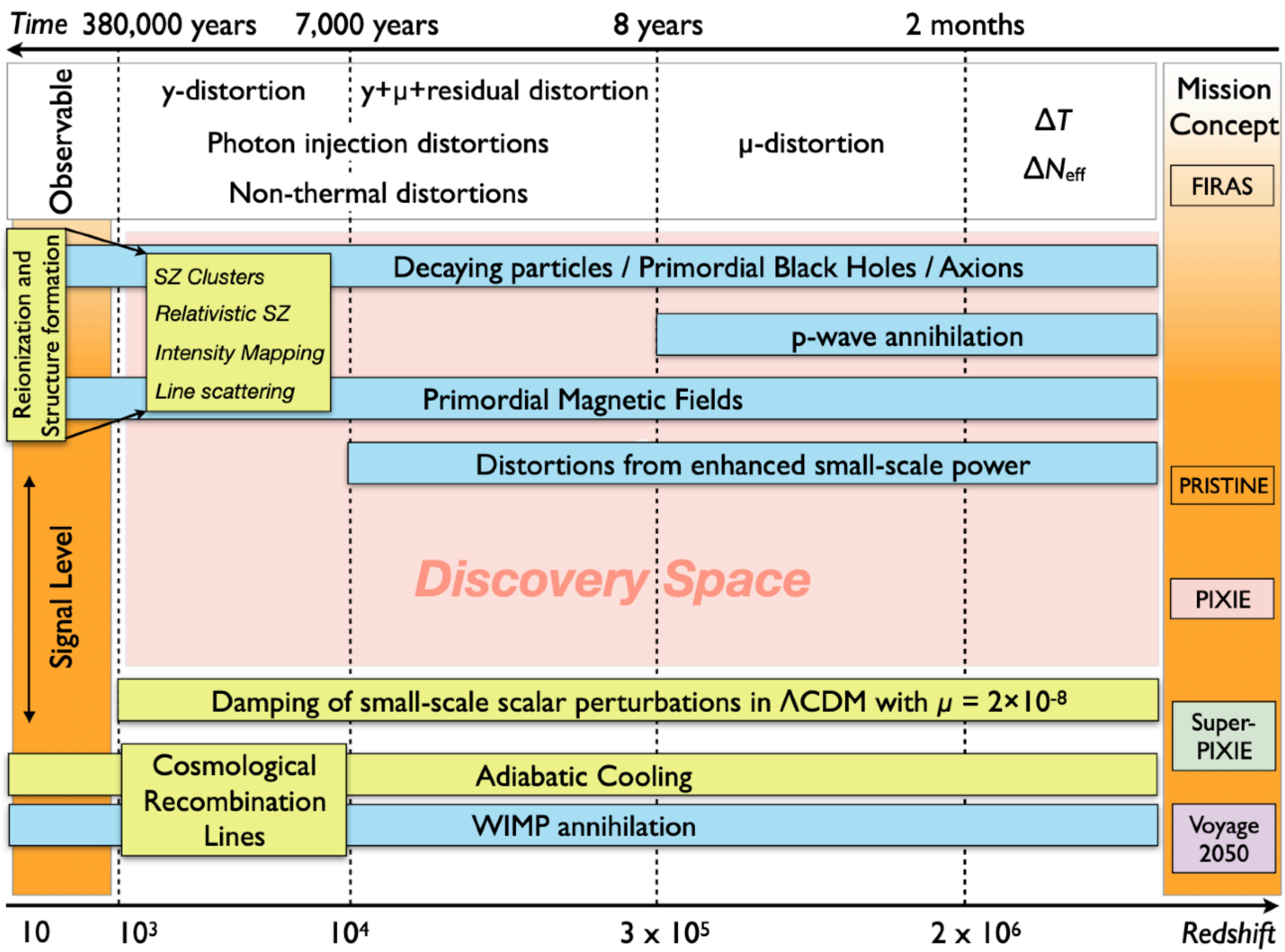
Majidi et al 2024

AQN Dark Matter glow: how faint can we probe?



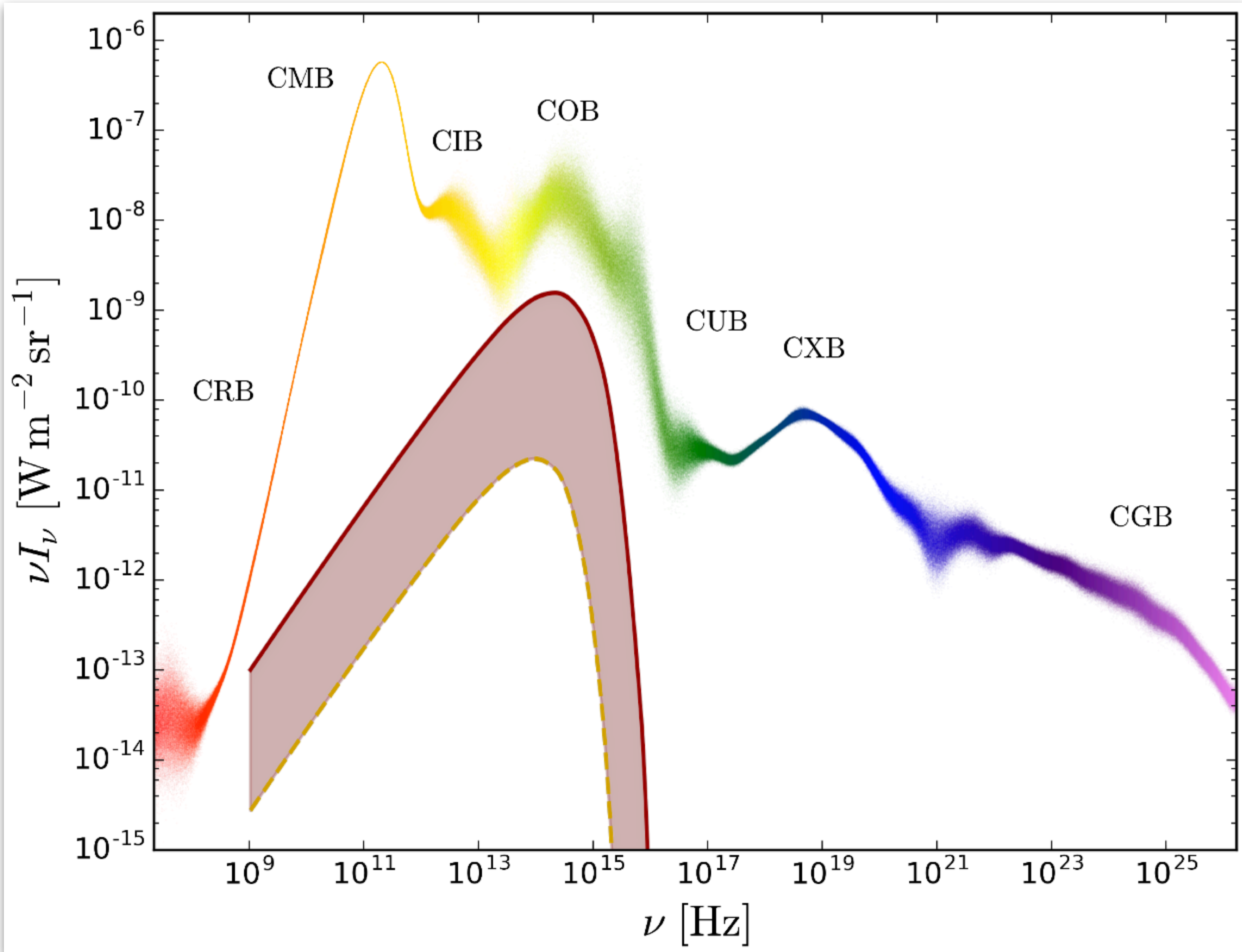
Majidi et al 2024

European Space Agency: Voyage 2050

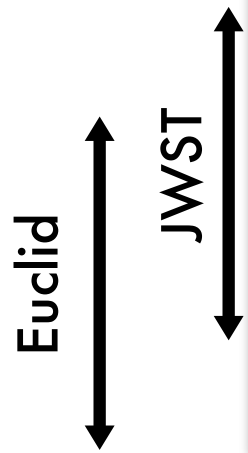


https://www.cosmos.esa.int/documents/1866264/3219248/ChlubaJ_Voyage-2050-SDWP-main.pdf/b91871ad-75c4-5b75-3300-049682255629?t=1565184628801

AQN Dark Matter glow: how bright is the sky monopole?



AQN Dark Matter glow: how bright in IR/optical?



ν [Hz]	λ [μm]	$I_\nu\|_{\max,\min}$ [m_{AB}/arcsec^2]
10^{13}	30	26.7, 30.7
10^{14}	3	27.6, 32.1
3×10^{14}	1.0	28.7, 33.8
4.3×10^{14}	0.7	29.3, 35.1
10^{15}	0.3	31.2, 39.0

Euclid: Early Release Observations – Programme overview and pipeline for compact- and diffuse-emission photometry ⋆

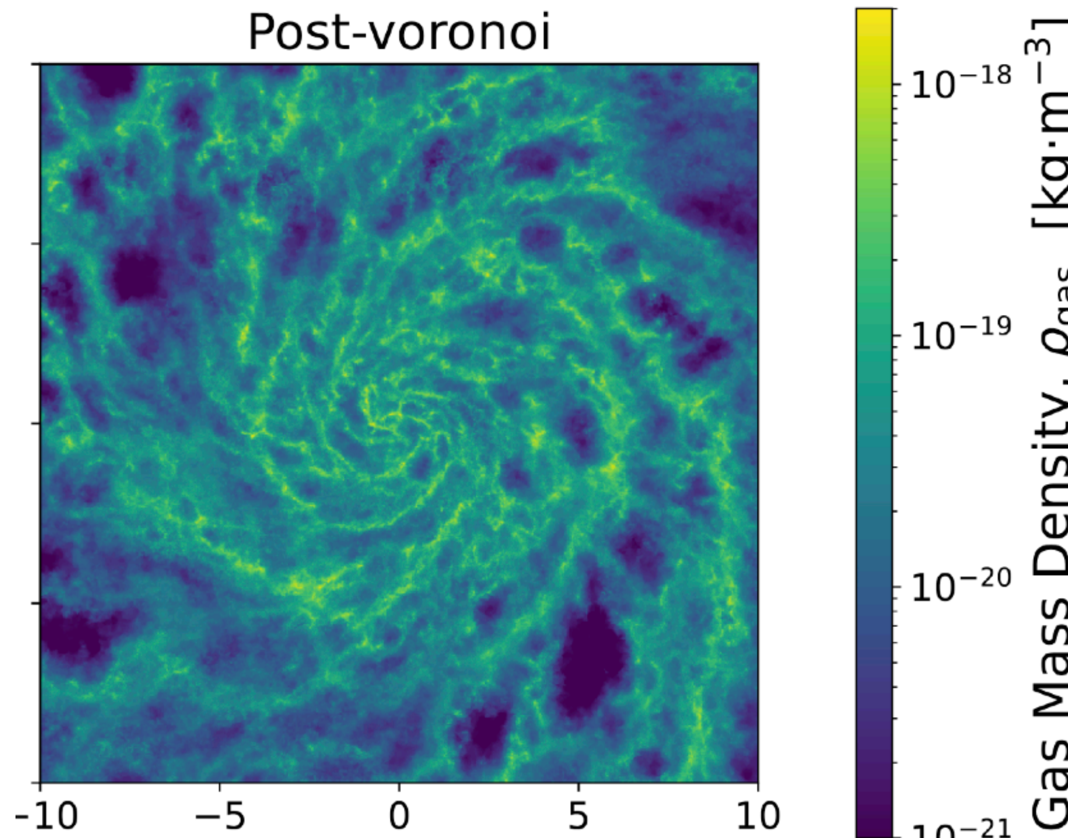
J.-C. Cuillandre[⋆⋆1], E. Bertin[1], M. Bolzonella[2], H. Bouy[3,4], S. Gwyn[5], S. Isani[6], M. Kluge[7], O. Lai[8],

arXiv:2405.13496

and opening a new observational window in the NIR. Median surface-brightness levels of 29.9 and 28.3, AB mag arcsec^{-2} are achieved for VIS and NISP, respectively, for detecting a $10'' \times 10''$ extended feature at the 1 σ level.

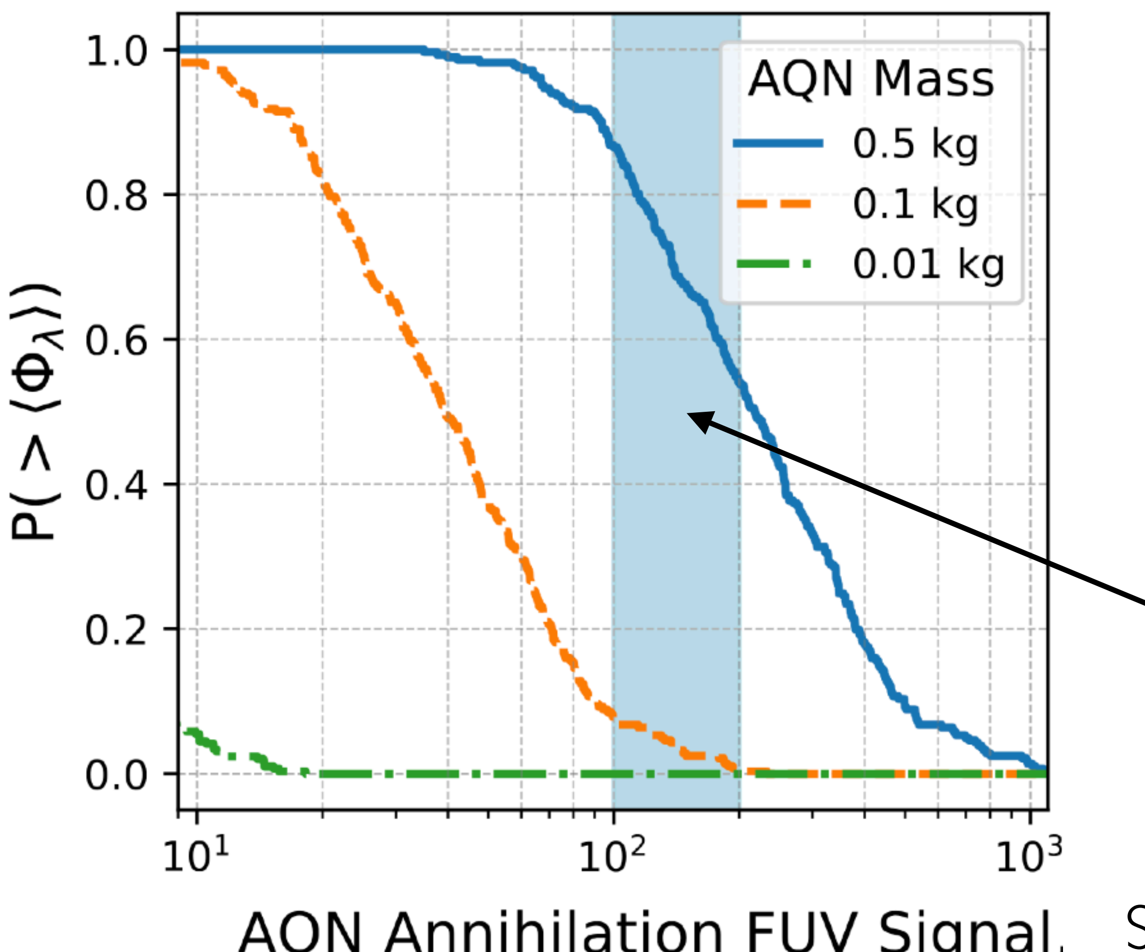
Contribution to the UV Galactic background

FIRE hydro simulations



Sekatchev et al 2025

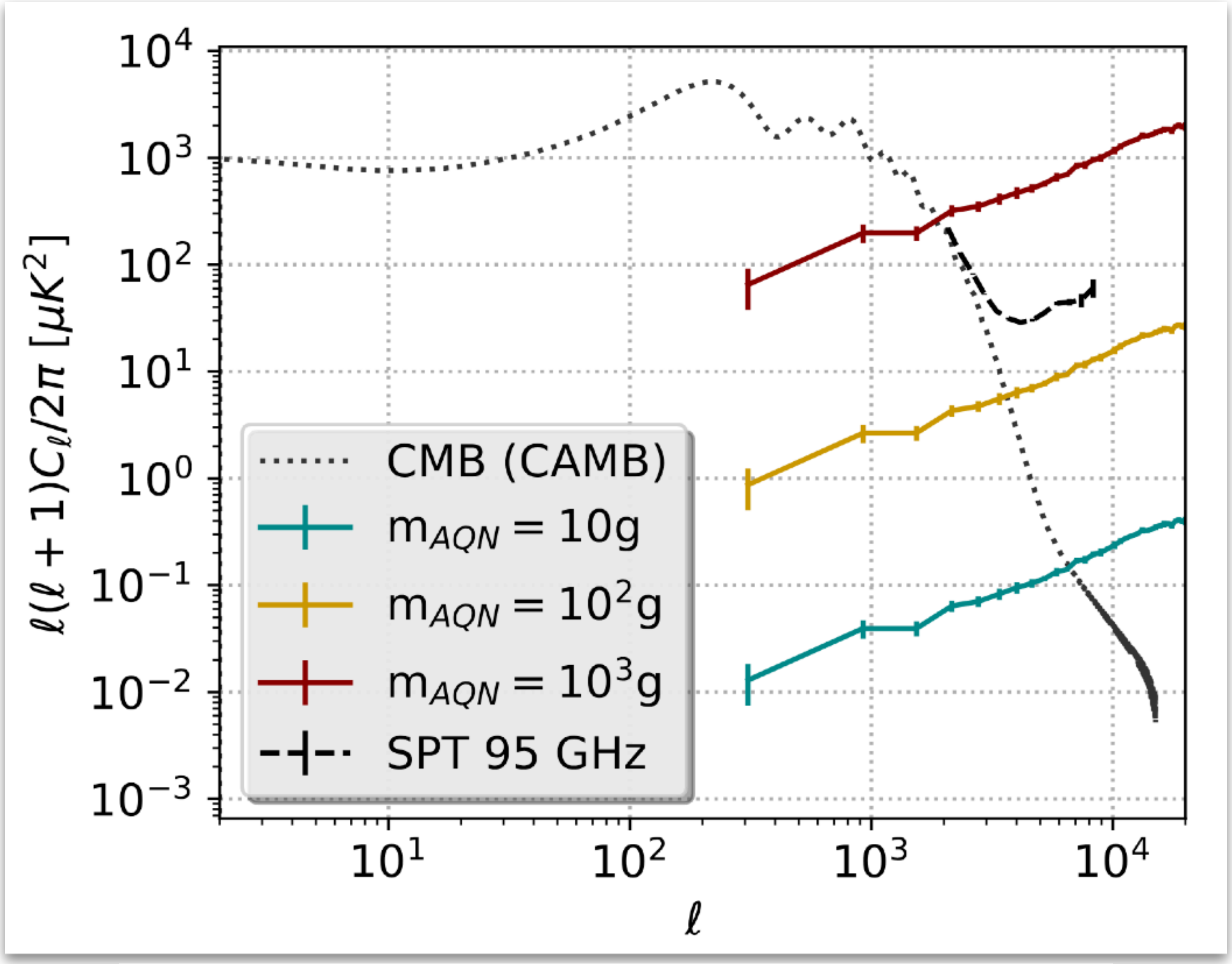
Parameter	Value	Range	Unit
Dark Matter Density (ρ_{DM})	0.42	0.06	GeV/cm^3
Ionized Gas Density (n_{ion})	0.018	0.002	cm^{-3}
Neutral Gas Density (n_{neut})	0.195	0.033	cm^{-3}
Radial distance of Sun ($R_\odot$)	8.20	0.09	kpc



Sekatchev et al 2025

Contribution to the radio cosmological background

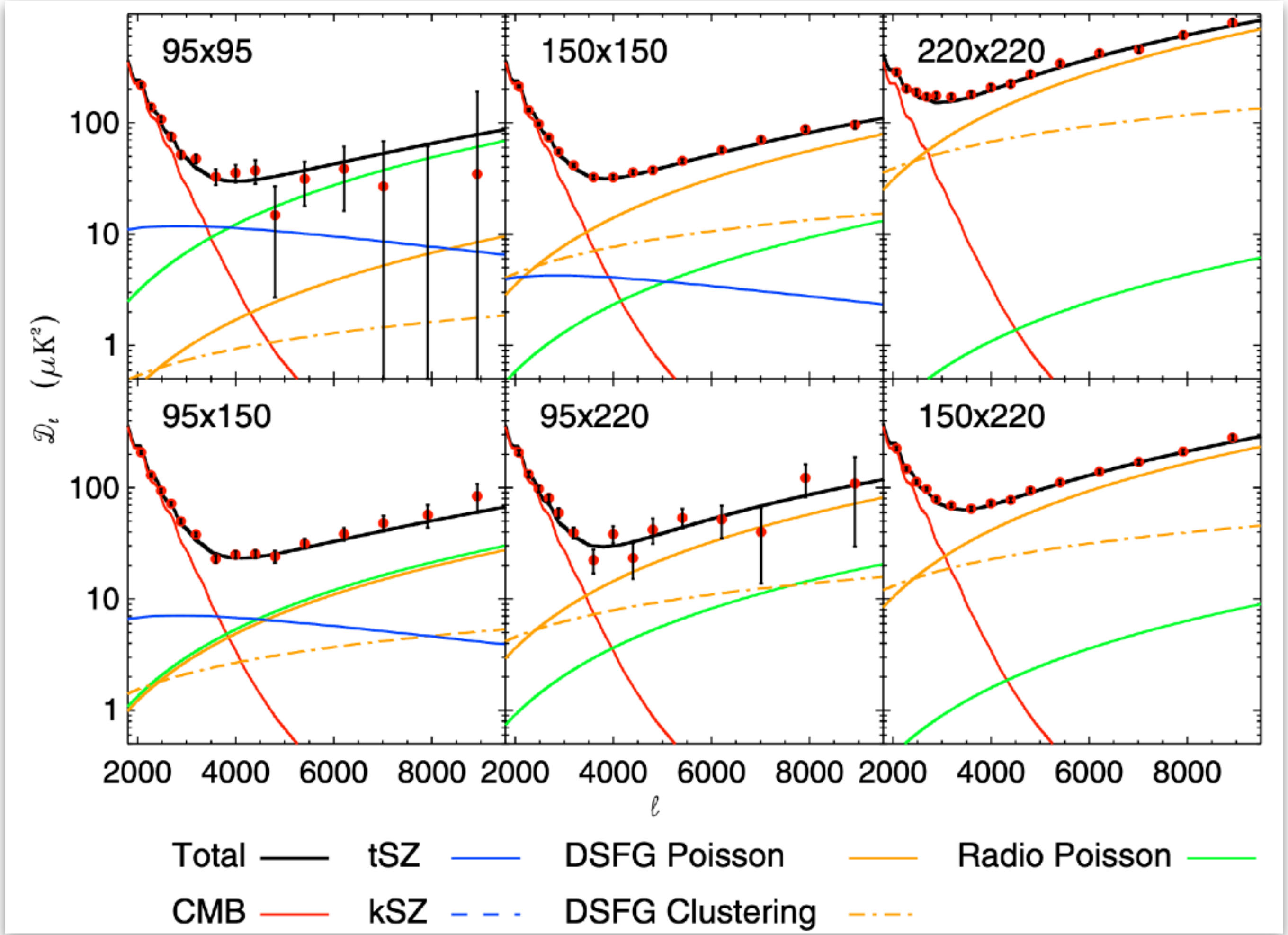
AQN Dark Matter glow: anisotropies in radio



Dark matter glow fluctuations compared to the South Pole Telescope measurements

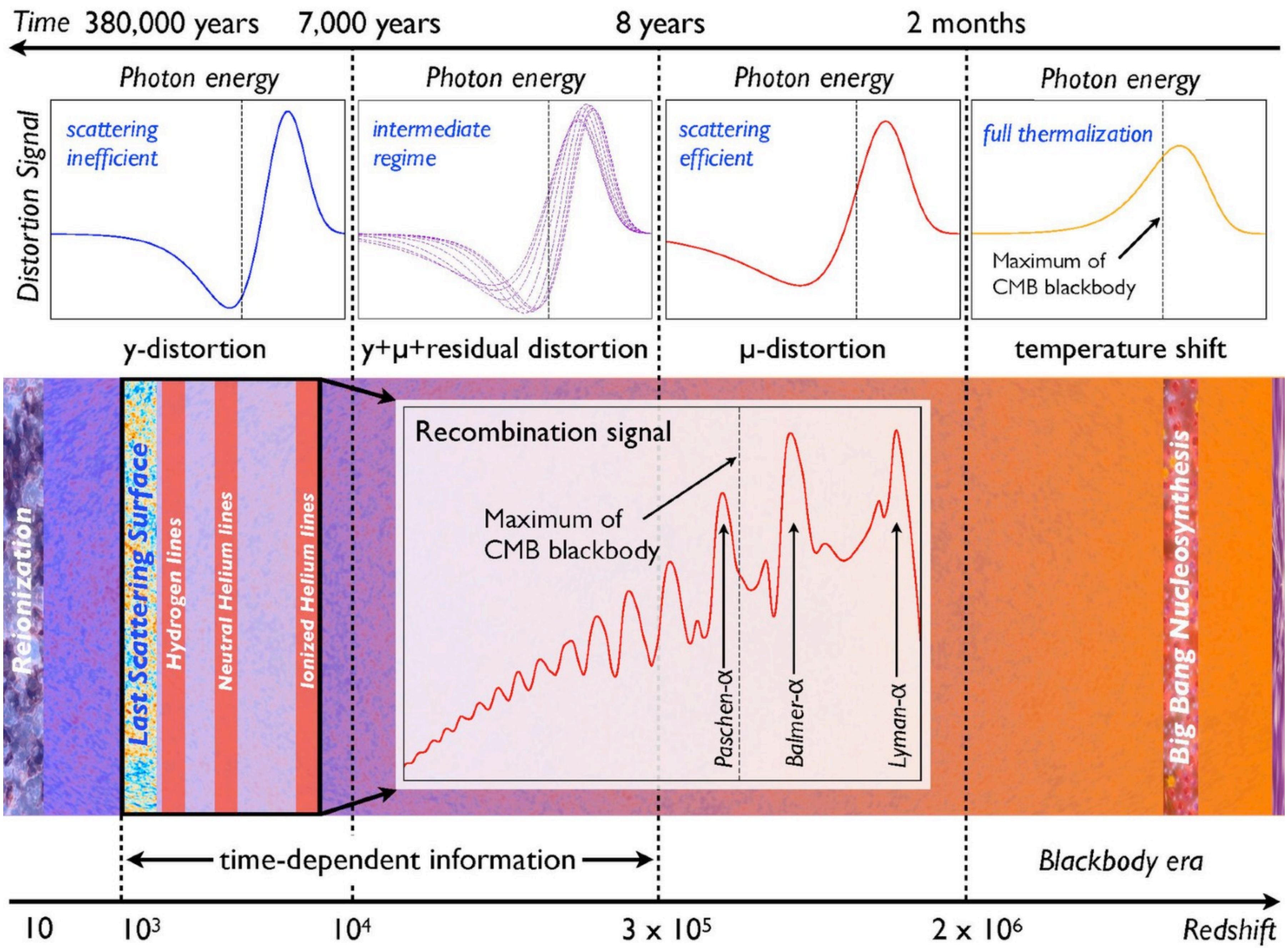
Majidi et al 2024

AQN Dark Matter glow: South Pole Telescope components separation



"A Measurement of Secondary Cosmic Microwave Background Anisotropies with Two Years of South Pole Telescope Observations" Reichardt et al. 2012, ApJ, 755, 1, arXiv:1111.0932

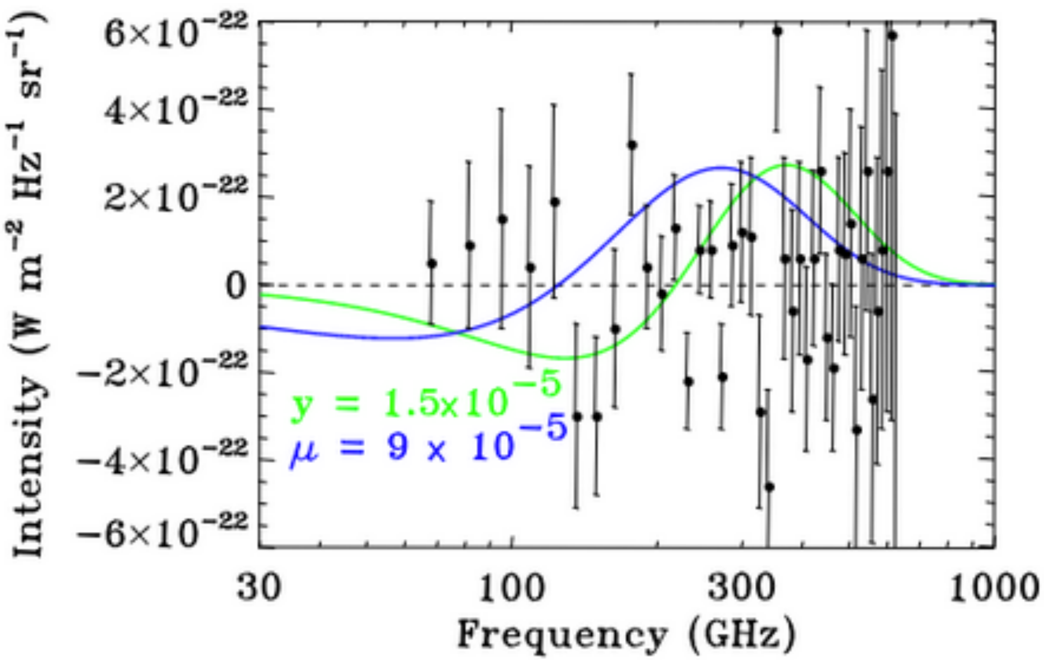
Contributions to the CMB spectral distortions



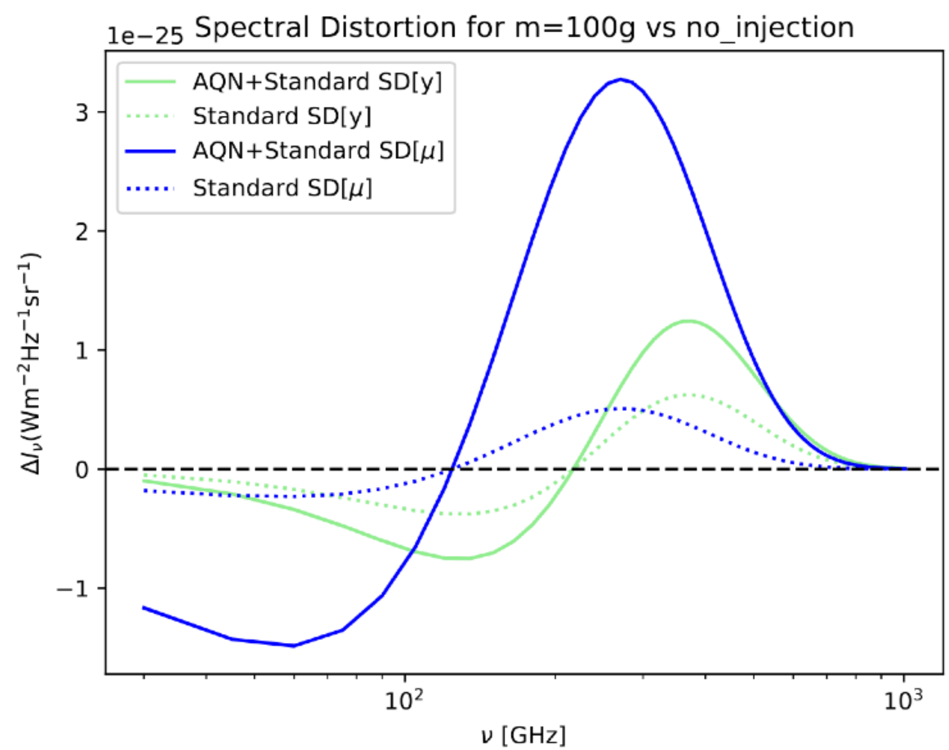
Spectral distortions (wikipedia)

Energy injection in the CMB

Kogut et al 2019 WP



μ and y distorsions with AQN



Majidi et al, in prep

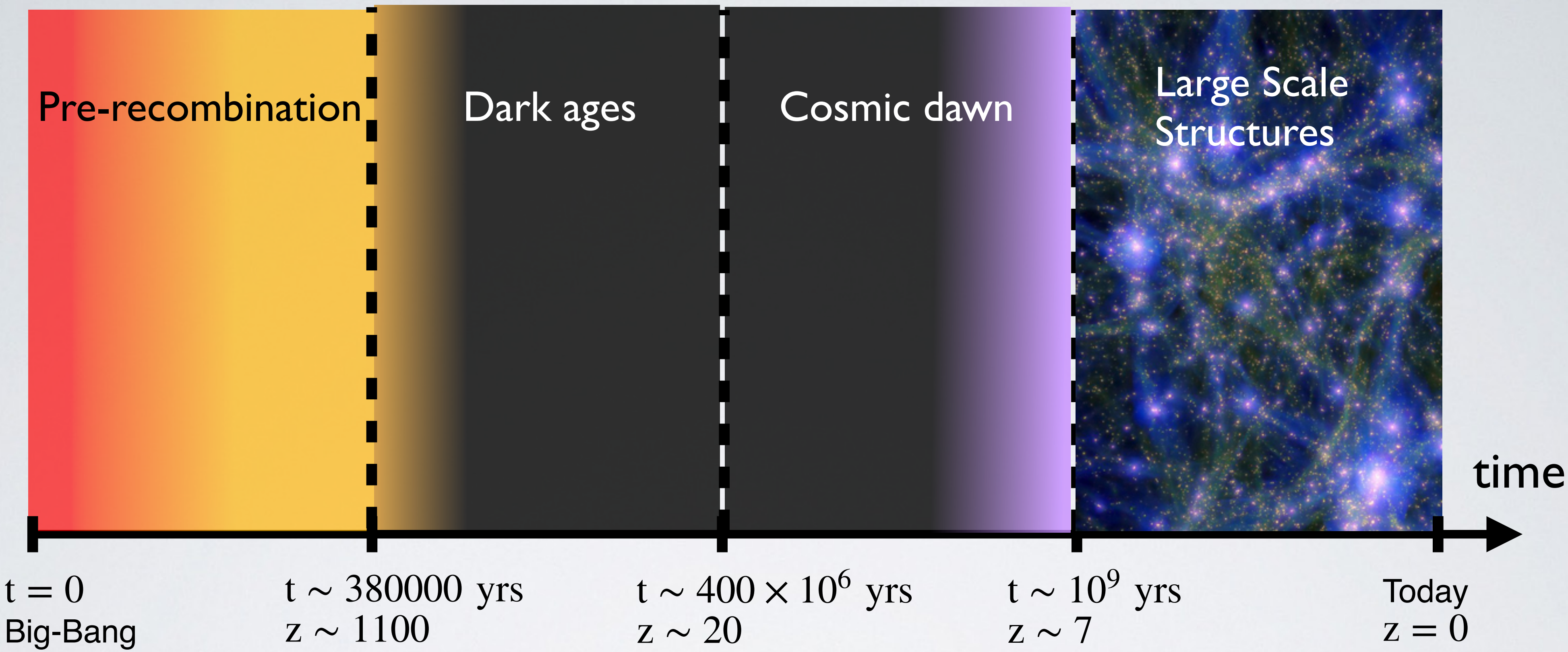
3

Lecture I: Known DM properties from observations. Known DM properties from theory. Main families of DM and their observational constraints.

Lecture II: Axion Quark Nuggets (QCD as a source of dark matter)

Lecture III: QCD as a source of dark energy. How to improve the efficiency of weak lensing to probe the dark matter power spectrum.

Contributions to the dark ages, cosmic dawn



High $n_{\rm bar}$
High $T_{\rm bar}$
$x_e \sim 1$

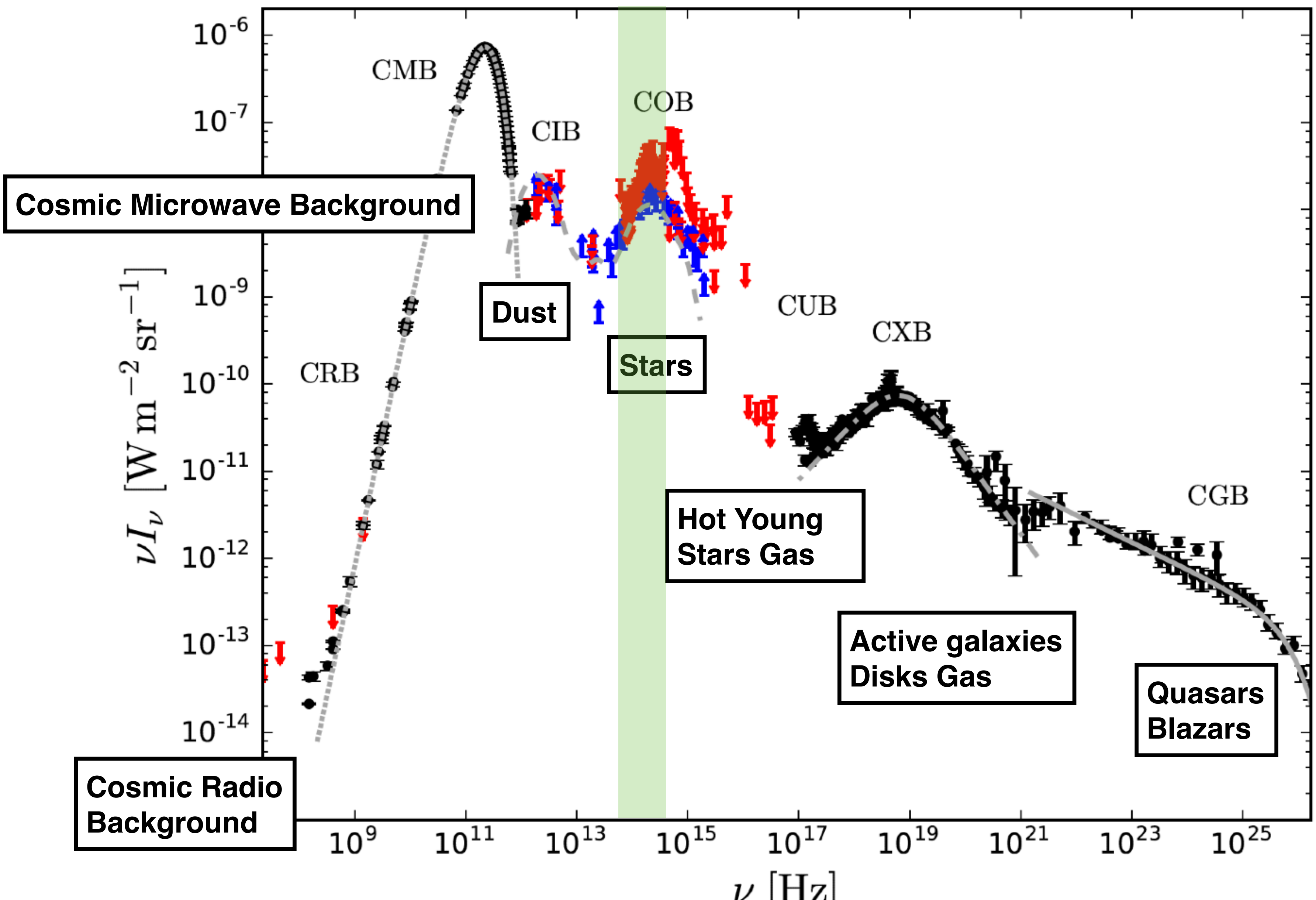
High $n_{\rm bar}$
Low $T_{\rm bar}$
$x_e \sim 10^{-3}$

Low $n_{\rm bar}$
Very low $T_{\rm bar}$
$x_e \gtrsim 10^{-3}$

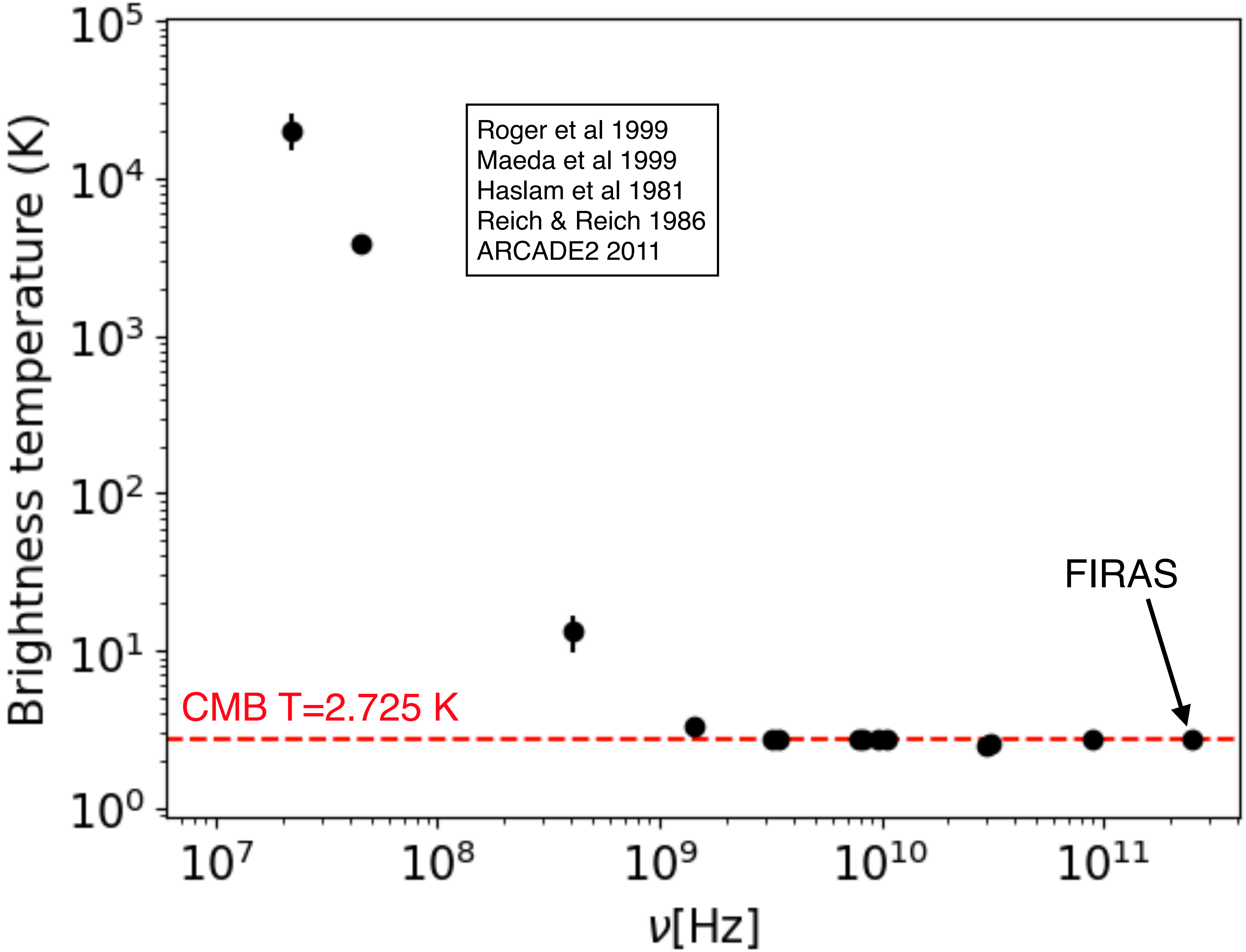
Very low $n_{\rm bar}$
High $T_{\rm bar}$
$x_e \sim 1$

Myth no1: "Dark matter does not reflect, absorb or emit light"
It is dark relative to something that is not, within our instrumentation capability, not black

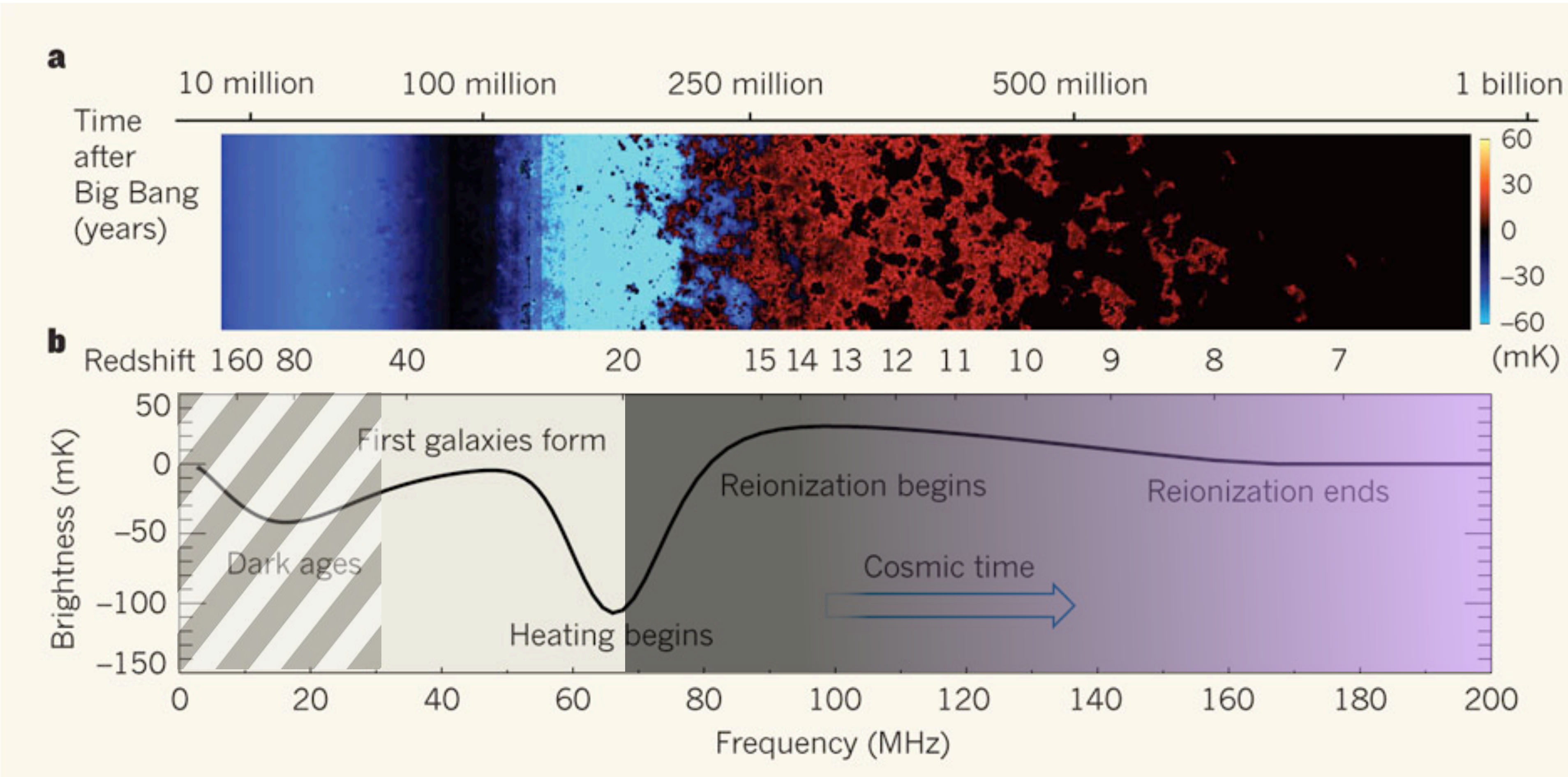
The monopole intensity of the sky as a function of frequency



The ARCADE excess



The 21cm signal during cosmic dawn



“DM as sub-atomic particles” $\Longrightarrow$ The Dark Matter paradigm

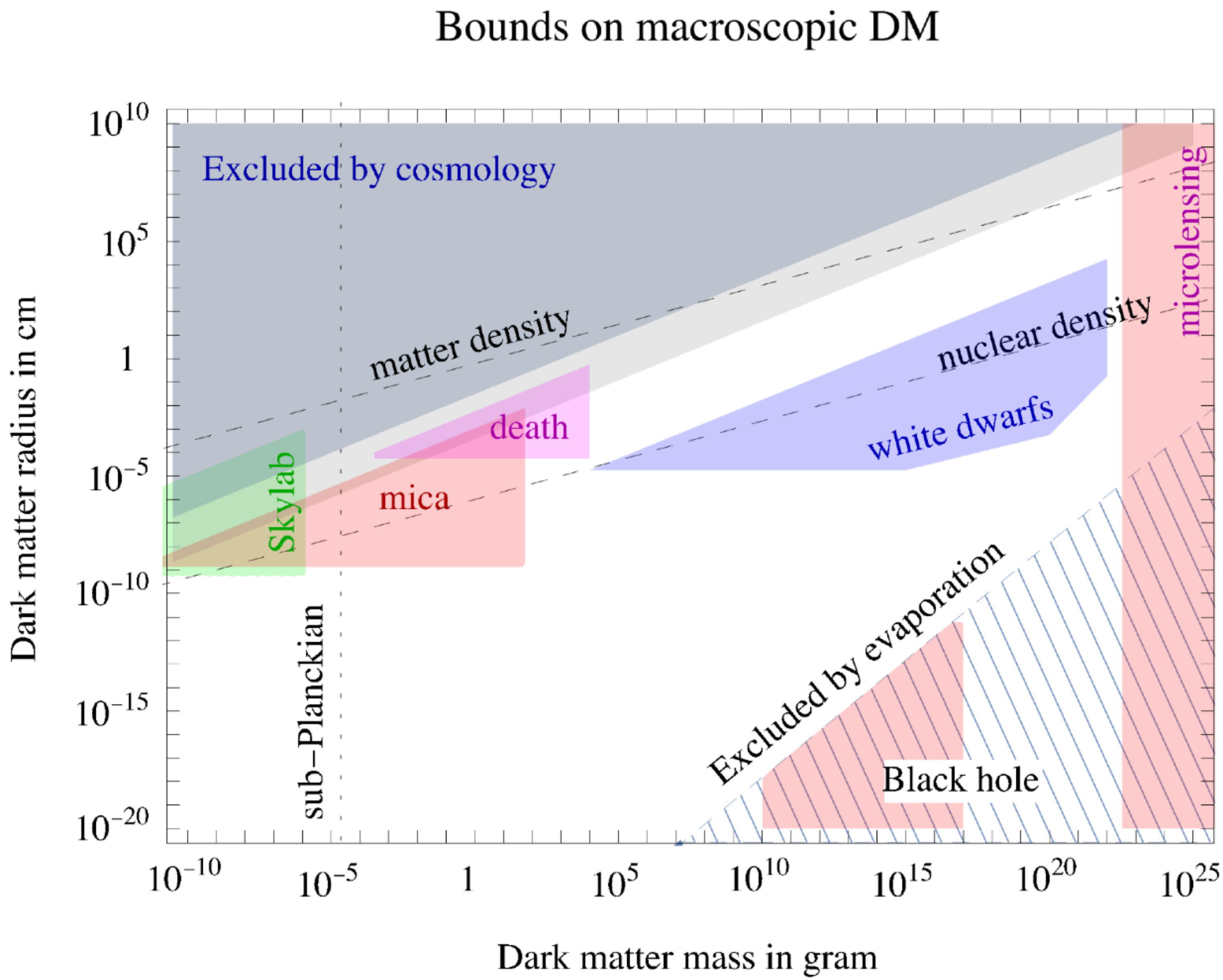
Dark matter does **not emit/reflect/absorb light**

Dark matter is **not interacting strongly with baryons**

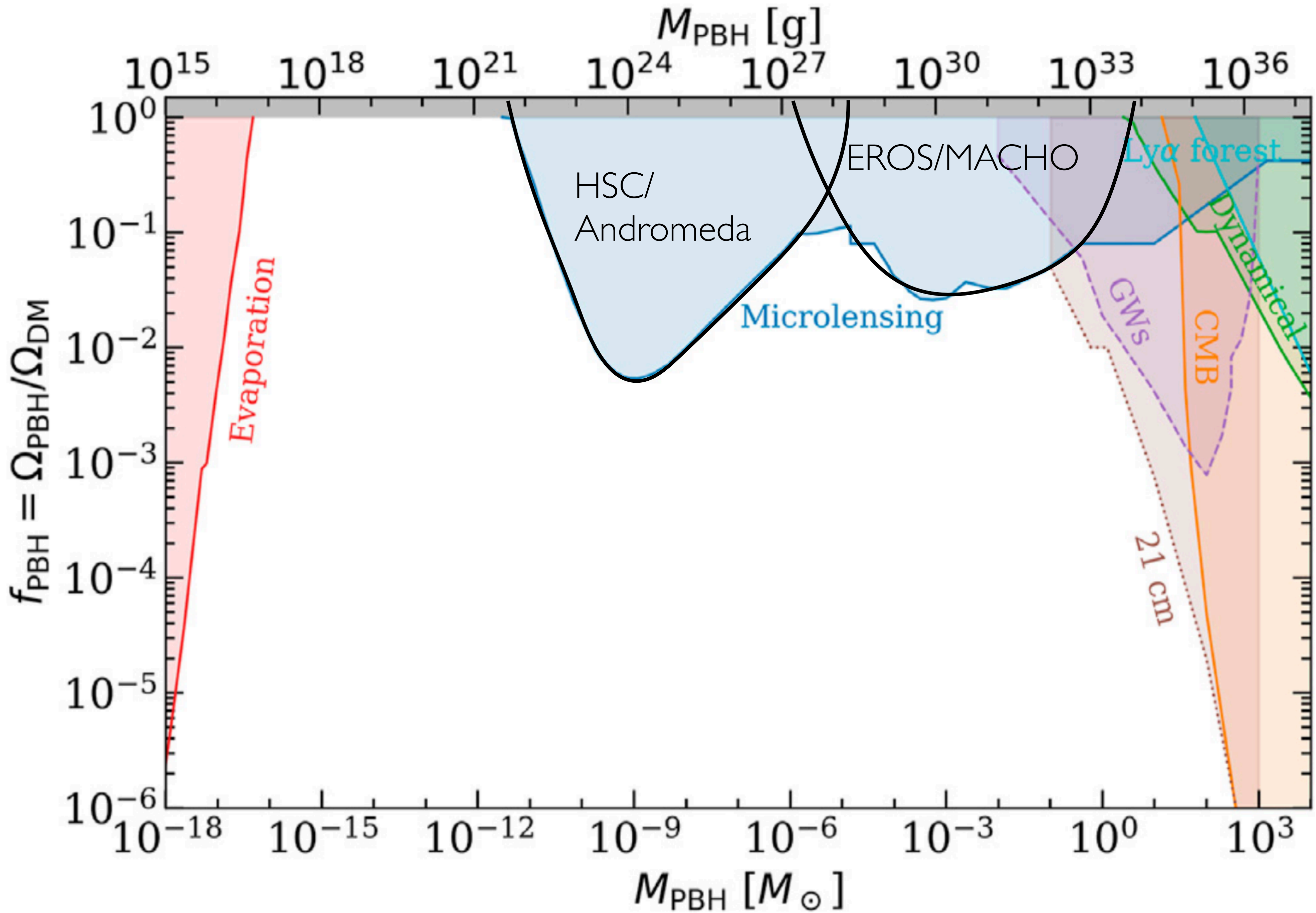
Dark matter is not made of “**baryonic**” material (i.e. it belongs to a “dark sector”)

The Dark Matter paradigm $\Longrightarrow$ DM is sub-atomic particles

Macroscopic DM



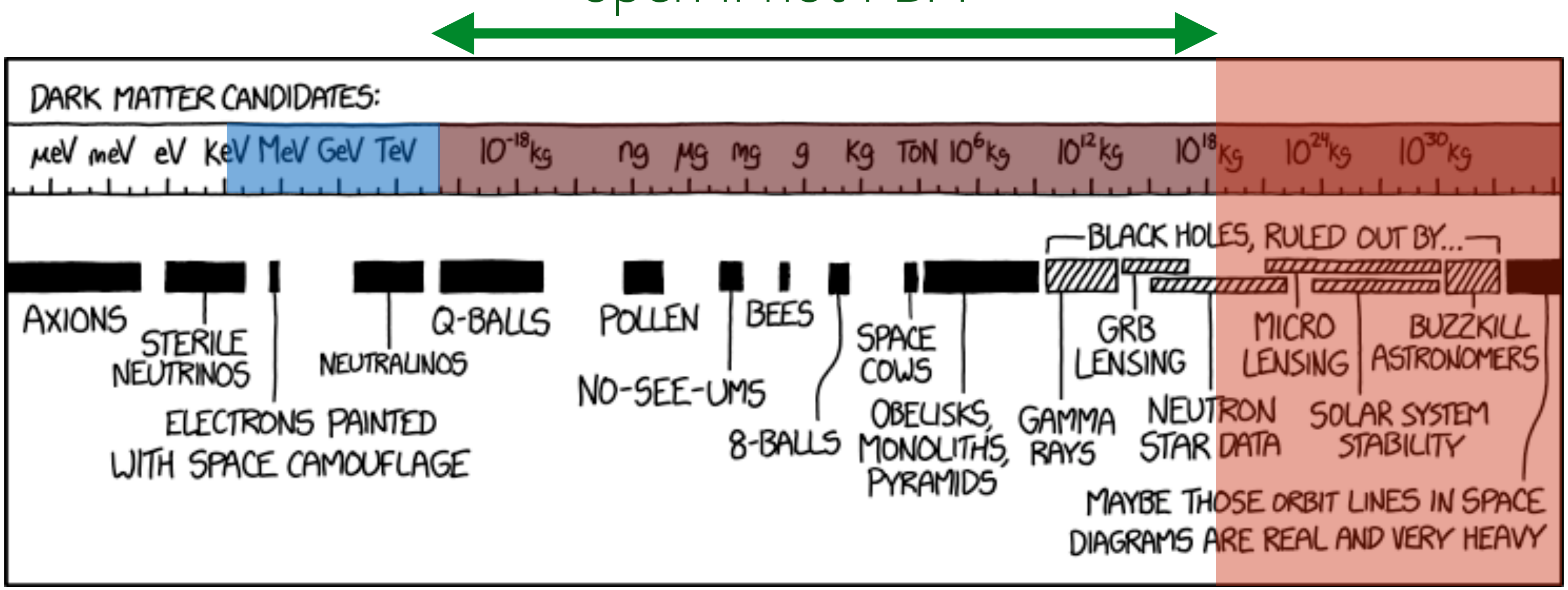
From Jacobs et al 2015, Cirelli et al 2024



Villanueva-Domingo et al 2021

Mass range to scale

Macros range still
open if not PBH



https://xkcd.com/2035/

Dark Energy from QCD topological sectors

Van Waerbeke & Zhitnitsky 2025

QCD has a complex vacuum topology which can lead to global (non-local) effect that can look like a dynamical dark energy.

How does it work? Can it be tested?

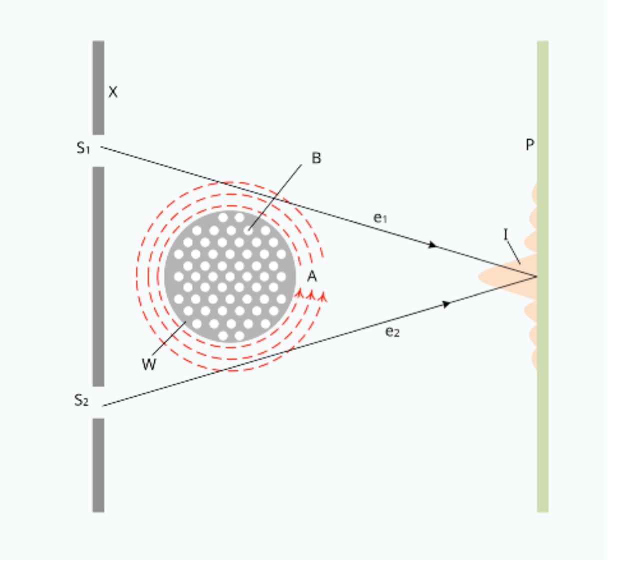
Analogy: **The Bohm-Aharonov effect**

In **classical electromagnetism**, particles respond only local fields $\overrightarrow{E}$ and $\overrightarrow{B}$, not to the vector potential $\overrightarrow{A}$ or scalar potential ϕ directly. If $\overrightarrow{E} = \overrightarrow{B} = \vec{0}$ along the path of a classical charged particle, no force acts on it.

In **quantum mechanics**, the wave function evolves according to the full potential via:

$$\psi(\vec{r}) \to \psi(\vec{r})\exp\left(\frac{iq}{\hbar}\int \overrightarrow{A} \cdot d\overrightarrow{\ell}\right)$$

Chambers 1960



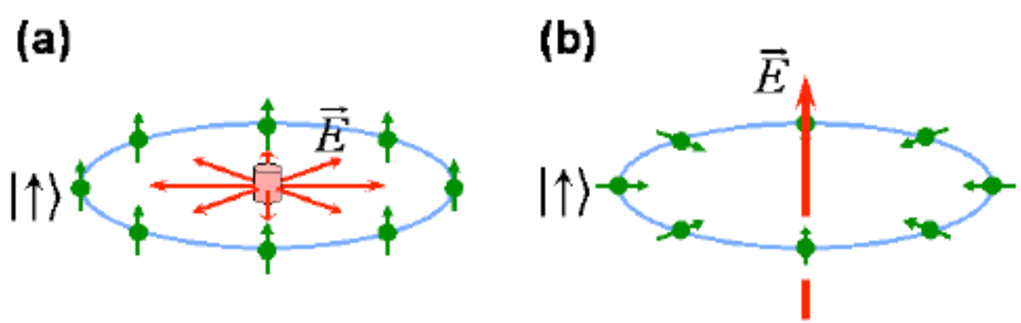
This is a non-local effect

Analogy: **The Aharonov-Casher effect**

A neutral particle (e.g. a neutron) with a magnetic dipole moment $\vec{\mu}$ moves around a line of electric charge (e.g. infinite charged wire).
There is no magnetic field, and the particle is electrically neutral, yet it acquires a **quantum phase shift**:

$$\Delta\phi = \frac{1}{\hbar c^2} \int \left(\vec{\mu} \times \vec{E} \right) \cdot d\vec{\ell}$$

Cimmino et al 1989



This is a non-local effect

Main ideas:

What happens if the background is now **time dependent**?

Modifications might happen, which are not described by a local (field-based) theory.

Zeldovich 1967 proposal: the vacuum energy $\Delta\rho$ entering the Friedmann equation is given by:

$$\Delta\rho \equiv \rho_{\rm FRW} - \rho_{\rm Mink} \simeq G m_p^6$$

In general $\Delta\rho$ cannot be calculated for arbitrary spacetime (Zhitnitsky 2015).

Main ideas:

Zhitnitsky 2015 calculated $\Delta\rho$ in the case of a relativistic hyperbolic spacetime $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$. The Minkowski spacetime is $\mathbb{R}^3 \times \mathbb{S}^1$.

In Minkowski, the QCD vacuum energy is $E_{\text{vac}}\left[\mathbb{R}^3 \times \mathbb{S}^1\right] \sim \Lambda^4_{\text{QCD}}$ (in natural units). The difference with the hyperbolic spacetime is:

$$\begin{aligned}\Delta\rho &\equiv E_{\text{vac}}\left[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}\right] - E_{\text{vac}}\left[\mathbb{R}^3 \times \mathbb{S}^1\right] \\ &\approx -\left[\Lambda^4_{\text{QCD}}\left(1 - c_\kappa \frac{\kappa}{\Lambda_{\text{QCD}}}\right) - \Lambda^4_{\text{QCD}}\right] \\ &\approx c_\kappa \kappa \cdot \Lambda^3_{\text{QCD}}\end{aligned}$$

Where $c_\kappa \sim 1$

Conjecture: for FRW spacetime, $\Delta\rho$ is also proportional to H, i.e $\kappa \to H$ and $c_\kappa \to c_H$ (Barvinsky & Zhitnitsky 2018)

In the de Sitter limit $H \to \overline{H}$, the conjecture implies:

$$\rho_{\rm DE} \sim c_H \Lambda_{\rm QCD}^3 \overline{H}$$

In natural units $G \equiv M_{\rm PL}^{-2}$, the relevant equations become:

$$\overline{H}^2 = \frac{8\pi G}{3}\rho_{\rm DE}$$

$$\overline{H} = c_H \frac{8\pi\Lambda_{\rm QCD}^3}{3M_{\rm PL}^2}$$

$$\rho_{\rm DE} \approx c_H^2 \frac{8\pi\Lambda_{\rm QCD}^6}{3M_{\rm PL}^2}$$

Plugging in numbers, we get $\rho_{\rm DE} \sim$ observed value today.
This is valid for the de Sitter limit ($z \to -1$).

Now, let's revisit Friedmann….

$$1 = \Omega_m + \Omega_r + \Omega_\Lambda$$

Friedmann equation anchored at a_i:

$$\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm crit}} = \frac{\rho_{\rm m,i}\left(a_i/a\right)^3}{\rho_{\rm crit,i}\left(H/H_i\right)^2} = \left(\frac{H_i}{H}\right)^2 \Omega_{\rm m,i}\left(\frac{a_i}{a}\right)^3$$

$$\Omega_{\rm r} = \frac{\rho_{\rm r}}{\rho_{\rm crit}} = \frac{\rho_{\rm r,i}\left(a_i/a\right)^4}{\rho_{\rm crit,i}\left(H/H_i\right)^2} = \left(\frac{H_i}{H}\right)^2 \Omega_{\rm r,i}\left(\frac{a_i}{a}\right)^4$$

$$\Omega_{\rm DE} = \frac{\rho_{\rm DE}}{\rho_{\rm crit}} = \frac{c_H \Lambda_{\rm QCD}^3 H}{\frac{3H^2}{8\pi G}} = \frac{8\pi G}{3}\Lambda_{\rm QCD}^3 \frac{c_H}{H} = \frac{\overline{H}}{H}$$

$$\Longrightarrow \quad H^2 - \overline{H}H - H_{\rm i}^2\left[\Omega_{\rm m,i}\left(\frac{a_i}{a}\right)^3 + \Omega_{\rm r,i}\left(\frac{a_i}{a}\right)^4\right] = 0$$

$$\boxed{H(a) = \frac{\overline{H}}{2}\left(1 + \sqrt{1 + B\left(\frac{a_i}{a}\right)^3 + C\left(\frac{a_i}{a}\right)^4}\right)}$$

With: $B \equiv 4\left(\frac{H_i}{\overline{H}}\right)^2 \Omega_{m,i}$ and $C \equiv 4\left(\frac{H_i}{\overline{H}}\right)^2 \Omega_{r,i}.$

Defining the DE equation of state:

$$P_{\mathrm{DE}} \equiv \omega \rho_{\mathrm{DE}}$$

The acceleration equation:

$$\dot{H} = -4\pi G\left(\rho_m + \rho_{\mathrm{DE}}(1+\omega)\right)$$

The de Sitter limit:

And defining:

$$x(z) \equiv \Omega_{m,0} \left(\frac{H_0^2}{\bar{H}^2} \right) (1+z)^3$$

We get:

$$w + 1 = \frac{\rho_{\mathrm{DE}} + P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} = \frac{\left[\frac{2x(t)}{\sqrt{1+4x(t)}} \right]}{\left[1 + \sqrt{1+4x(t)} \right]}$$

$$\Longrightarrow \quad \begin{array}{l} \omega \to -1 \text{ as } x(z) \to 0 \\ \omega \simeq -0.7 \text{ for } x \simeq 1 \end{array}$$

Solution for $z > 0$:

We define a "switch" function $\beta(t)$ such that:

$$\rho_{\text{DE}} \sim c_H \Lambda^3_{\text{QCD}} \overline{H} \rightarrow \beta(t) c_H \Lambda^3_{\text{QCD}} H$$

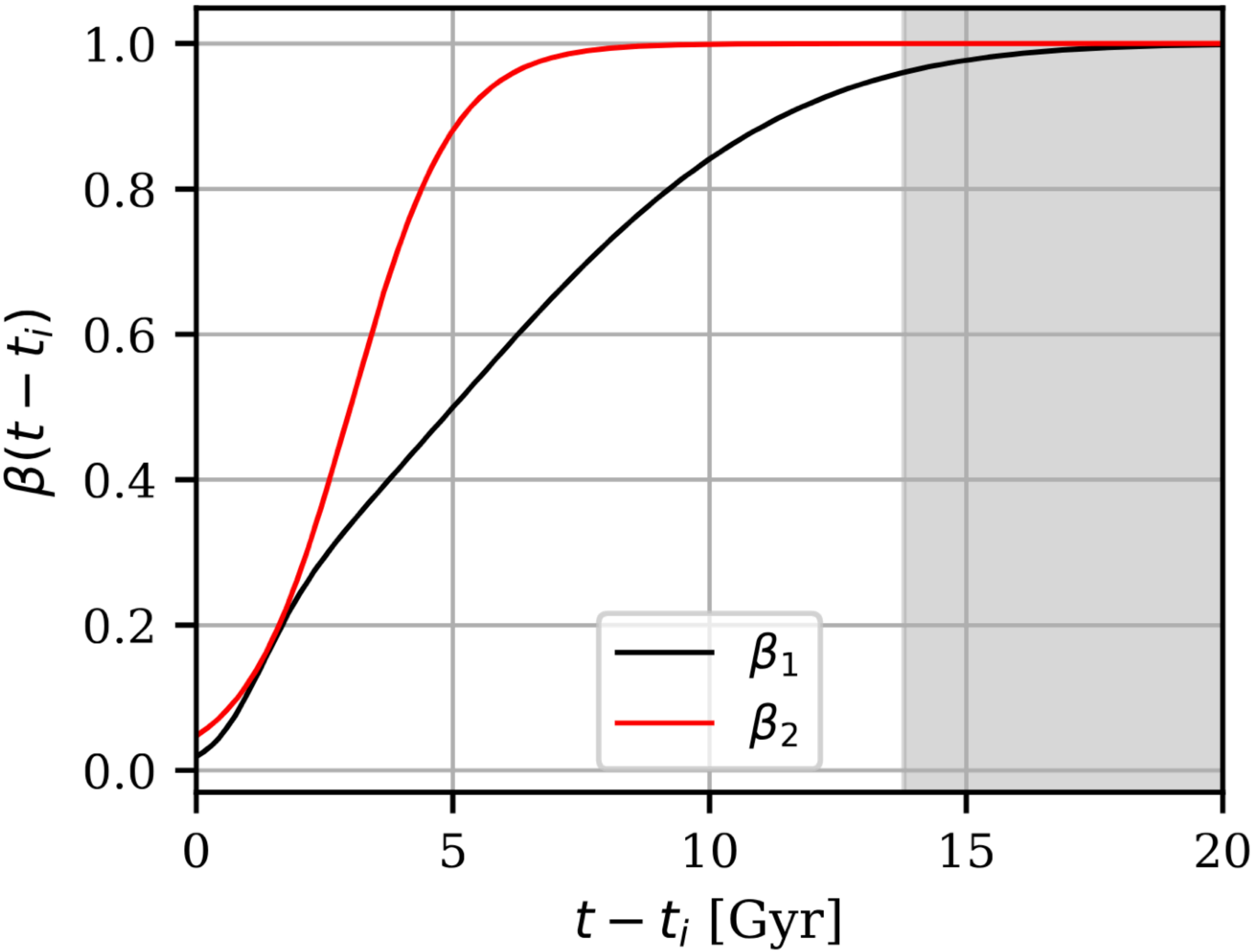
The role of the switch function $0 < \beta(t) < 1$ is to activate the QCD vacuum energy (DE) at a certain time with a certain rate.

The Friedmann equation becomes:

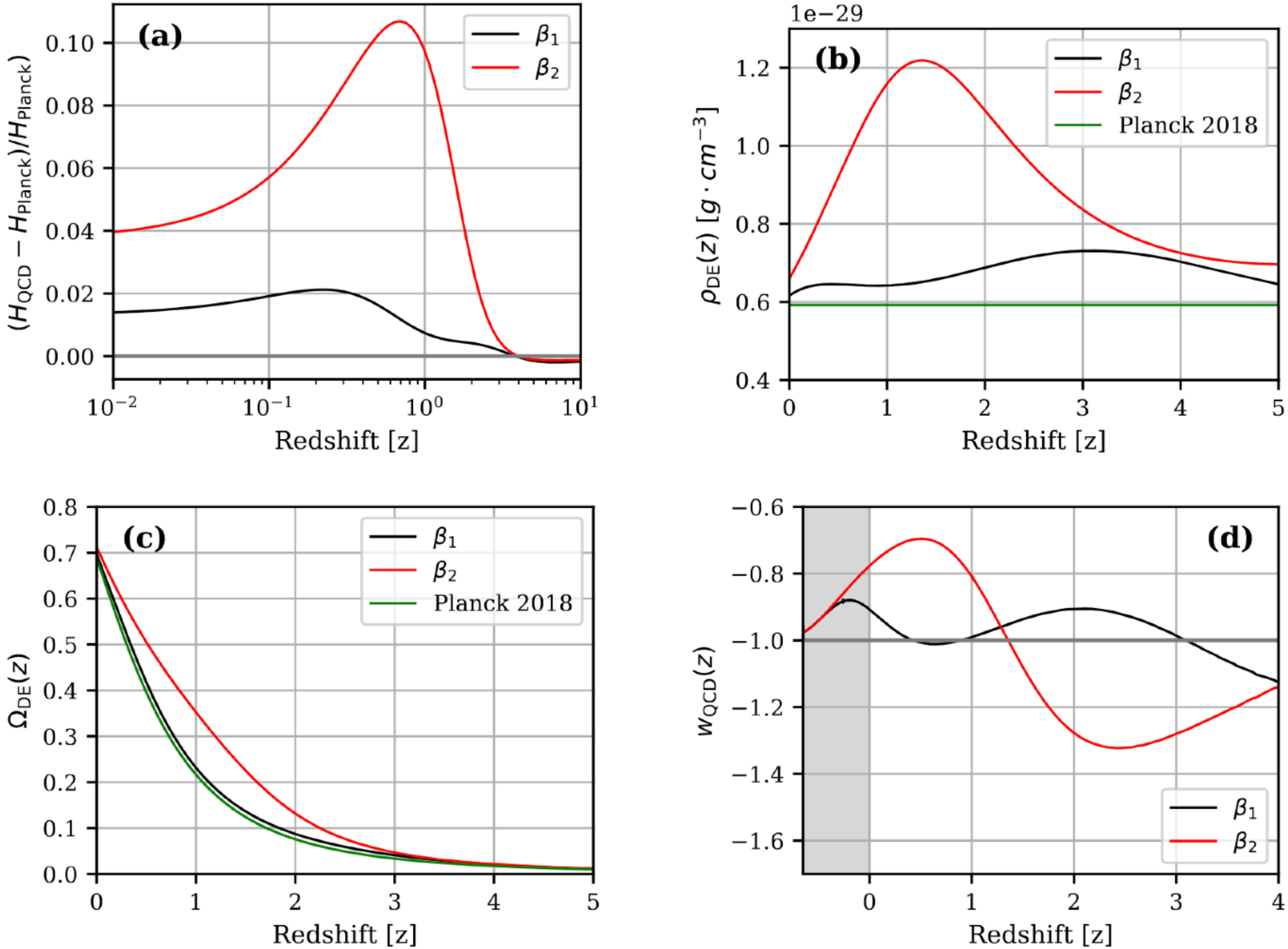
$$\boxed{\frac{da}{d\tau} = \beta(\tau)\frac{a}{2}\left(1+\sqrt{1+\frac{B}{\beta^2}\left(\frac{a_i}{a}\right)^3+\frac{C}{\beta^2}\left(\frac{a_i}{a}\right)^4}\right)}$$

With dimensionless time $\tau \equiv \dfrac{8\pi G}{3} \Lambda^3_{\text{QCD}} c_H t$

Note: with the right $\beta(t)$ the usual Friedmann solutions are always possible solutions.

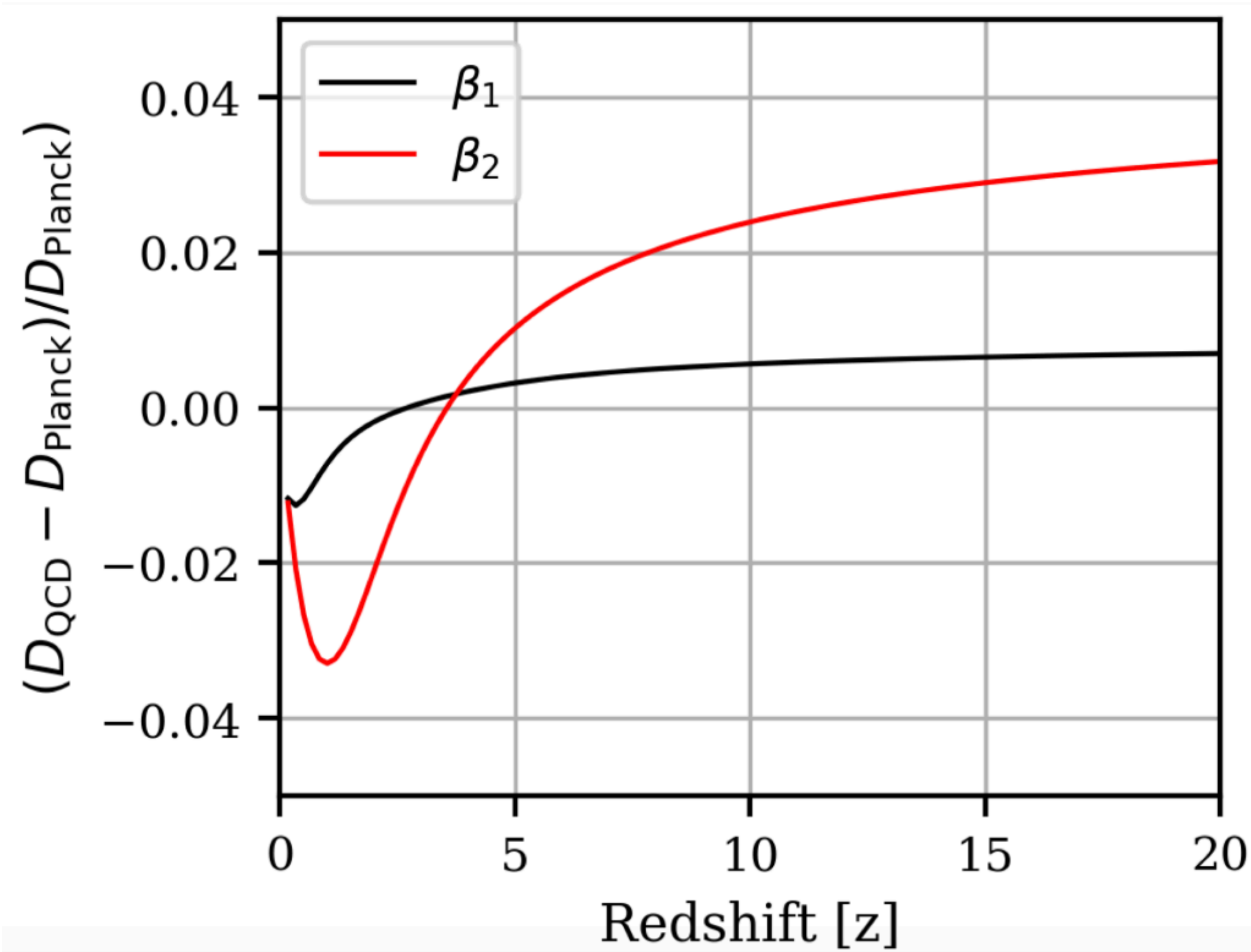


1.0
0.8
0.6
0.4
0.2
0.0
$\beta(t - t_i)$
0
5
10
15
20
$t - t_i$ [Gyr]
β_1
β_2



(a)
$(H_{\rm QCD} - H_{\rm Planck})/H_{\rm Planck}$
0.10
0.08
0.06
0.04
0.02
0.00
10^{-2}
10^{-1}
10^{0}
10^{1}
Redshift [z]
β_1
β_2
(b)
1e−29
$\rho_{\rm DE}(z)\ [g \cdot cm^{-3}]$
1.2
1.0
0.8
0.6
0.4
0
1
2
3
4
5
Redshift [z]
β_1
β_2
Planck 2018
(c)
$\Omega_{\rm DE}(z)$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
Redshift [z]
β_1
β_2
Planck 2018
(d)
$w_{\rm QCD}(z)$
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
0
1
2
3
4
Redshift [z]
β_1
β_2

Early and late universe can be affected

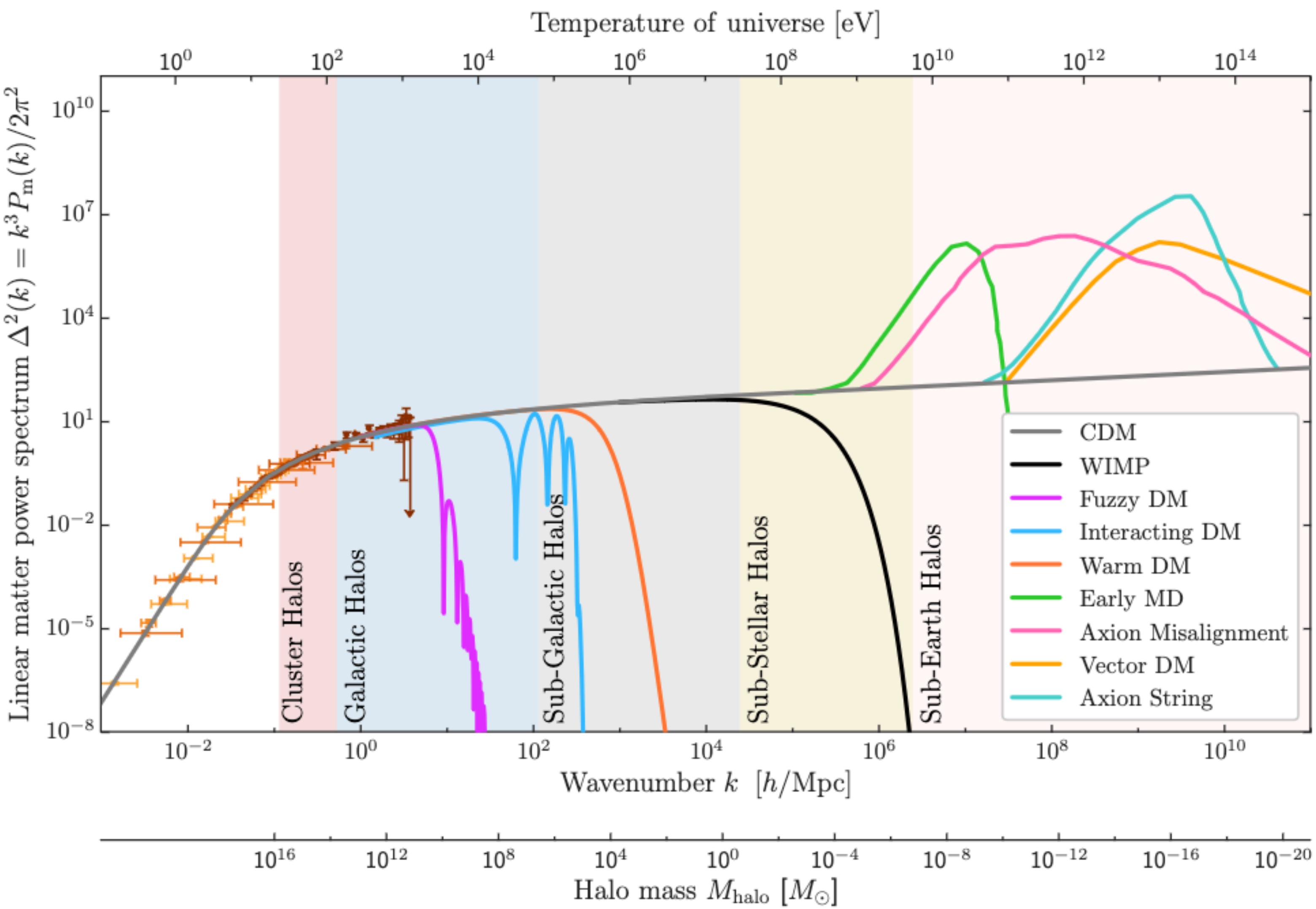


Making weak lensing as a 3D probe: the BNT* approach

*BNT: The Bernardeau-Nishimichi-Taruya 2014

Work by Gu et al. 2025

Direct probe of the dark matter distribution
(Power spectrum, halo mass profile)



Bechtol et al 2023

Shear correlation functions:

$$\xi_+^{ij}(\theta) = \frac{1}{2\pi}\int_0^\infty d\ell\, \ell\, C_\ell^{ij}\, J_0(\ell\theta) = \sum_\ell \frac{2\ell+1}{4\pi}\, C_\ell^{ij}\, P_\ell(\cos\theta)$$

$$\xi_-^{ij}(\theta) = \frac{1}{2\pi}\int_0^\infty d\ell\, \ell\, C_\ell^{ij}\, J_4(\ell\theta) = \sum_\ell \frac{2\ell+1}{4\pi}\, C_\ell^{ij}\, P_\ell^4(\cos\theta)$$

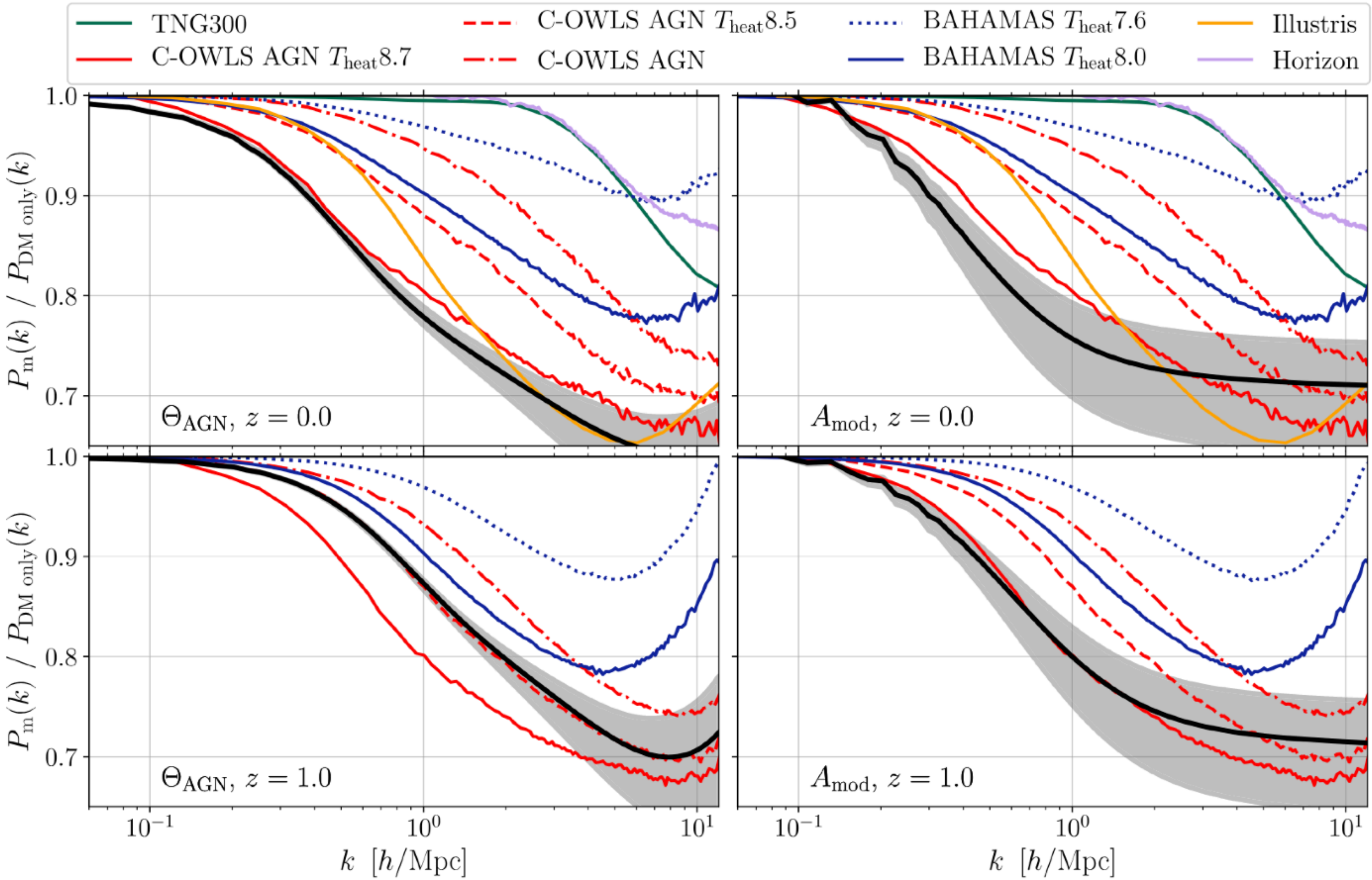
Angular power spectrum:

$$C^{i,j}(\ell) = \int \frac{d\chi}{\chi^2}\, W_\gamma^i(\chi)\, W_\gamma^j(\chi)\, P\left(k = \frac{\ell + 1/2}{\chi}; z(\chi)\right)$$

Kernel:

$$W_\gamma^i(\chi) = \frac{\Omega_m^2 H_0^4}{c^2}\int d\chi'\, \frac{n_i(\chi')}{a(\chi)}\, \frac{f_K(\chi'-\chi) f_K(\chi)}{f_K(\chi')}$$

Weak gravitational lensing can probe the matter spectrum



But the mass spectrum alone is not a clean probe of DM

Amon & Efstathiou 2022

Introduce weights $p_i \neq 1$:

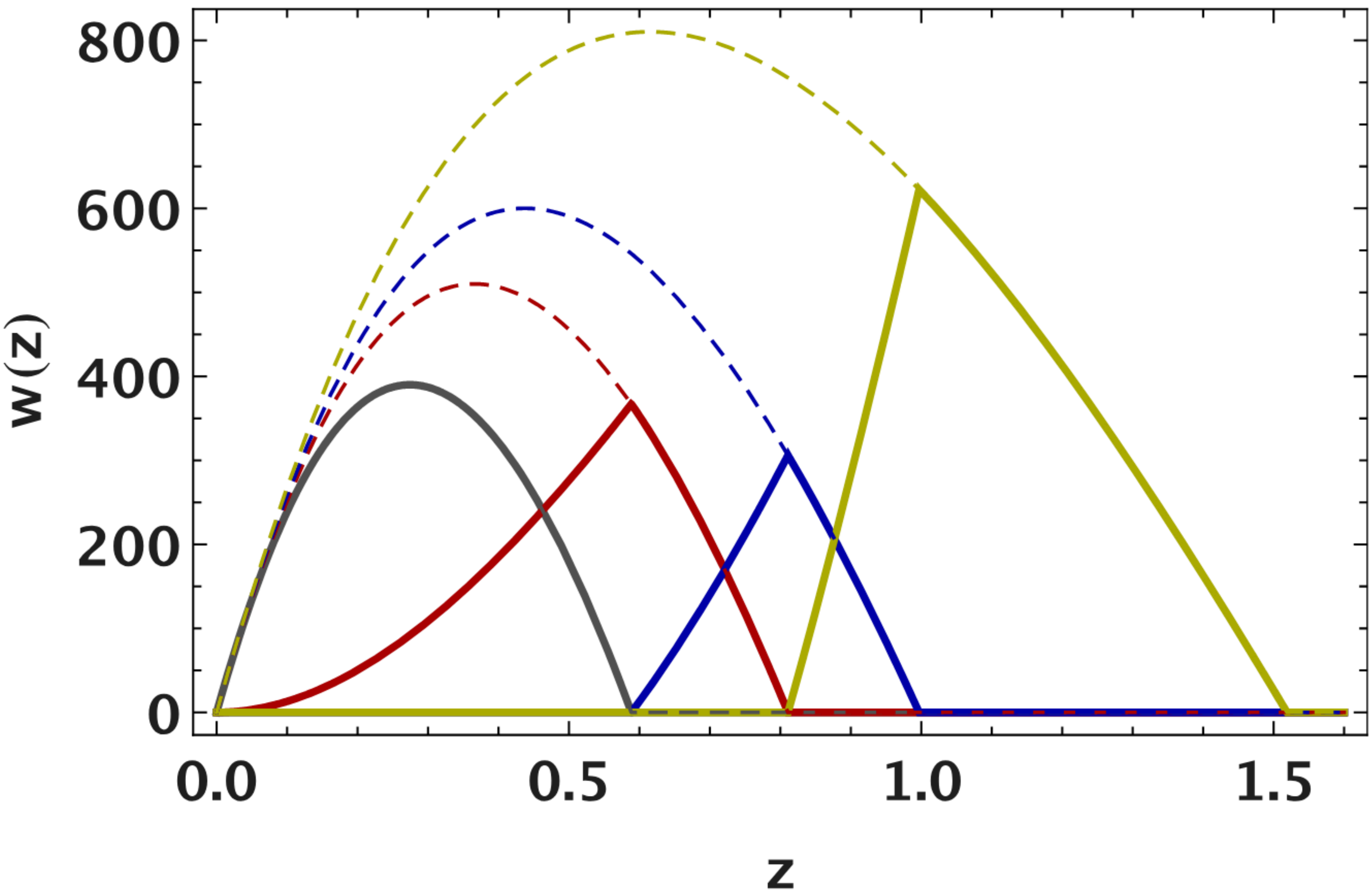
$$\kappa = \frac{3\Omega_0 H_0^2}{2c^2} \sum_i p_i \int_0^{\chi_i} \mathrm{d}\chi \; \frac{f_K(\chi_i - \chi) f_K(\chi)}{f_K(\chi_i)} \frac{\delta(\chi)}{a(\chi)}$$

$$w(\chi) = \sum_{i,\, \chi_i > \chi} p_i \; \frac{f_K(\chi_i - \chi) f_K(\chi)}{f_K(\chi_i)}$$

Such that they obey the relations:

$$\sum_{i=1}^{3} p_i = 0, \quad \sum_{i=1}^{3} \frac{p_i}{g_K(\chi_i)} = 0$$

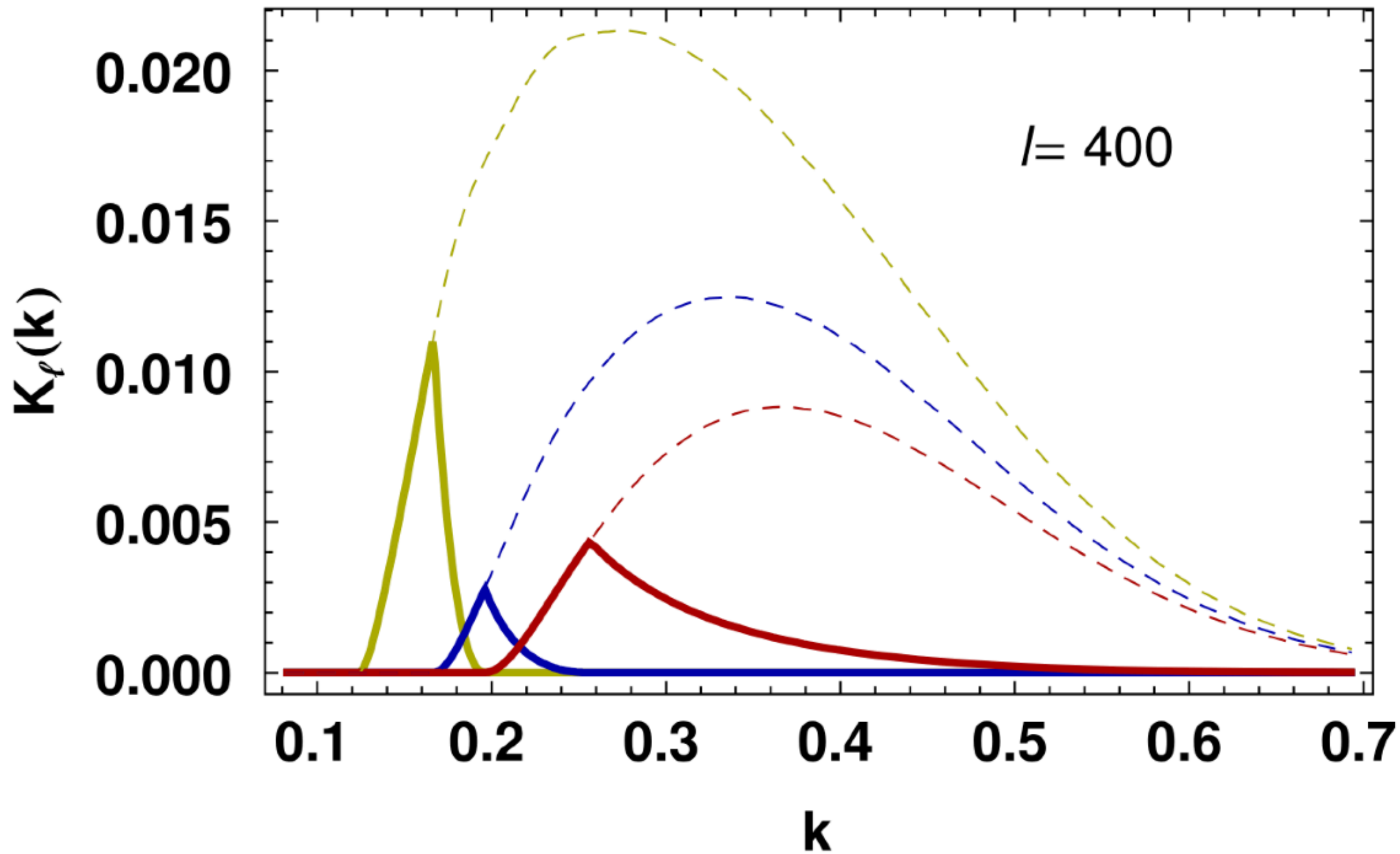
The BNT lensing kernels are more localized in redshift than the noBNT ones:

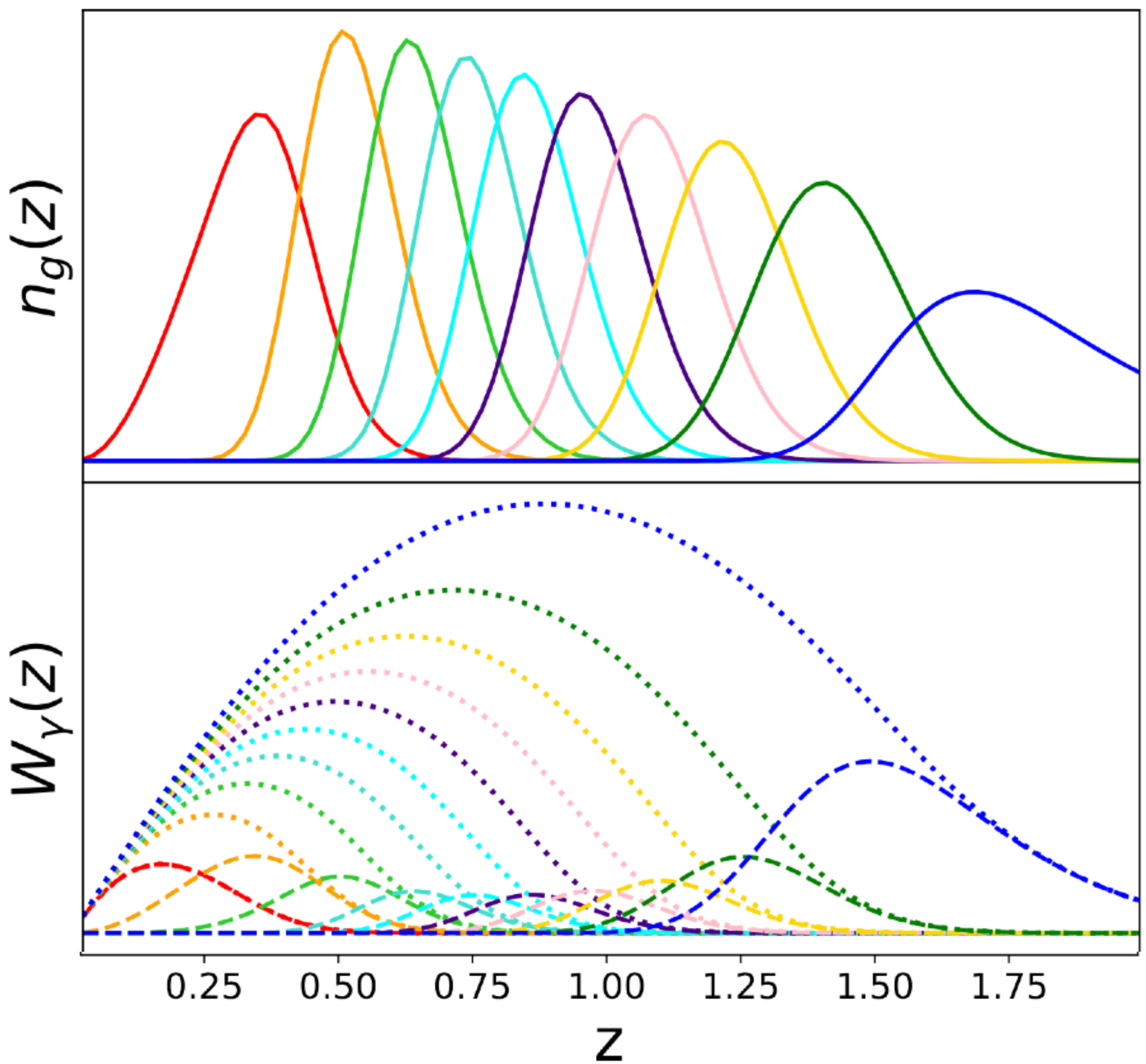


$$C_\ell^{ab} = \int \mathrm{d}\chi \; \mathcal{K}_\ell \left(\frac{\ell}{f_\mathrm{K}(\chi)} \right)$$

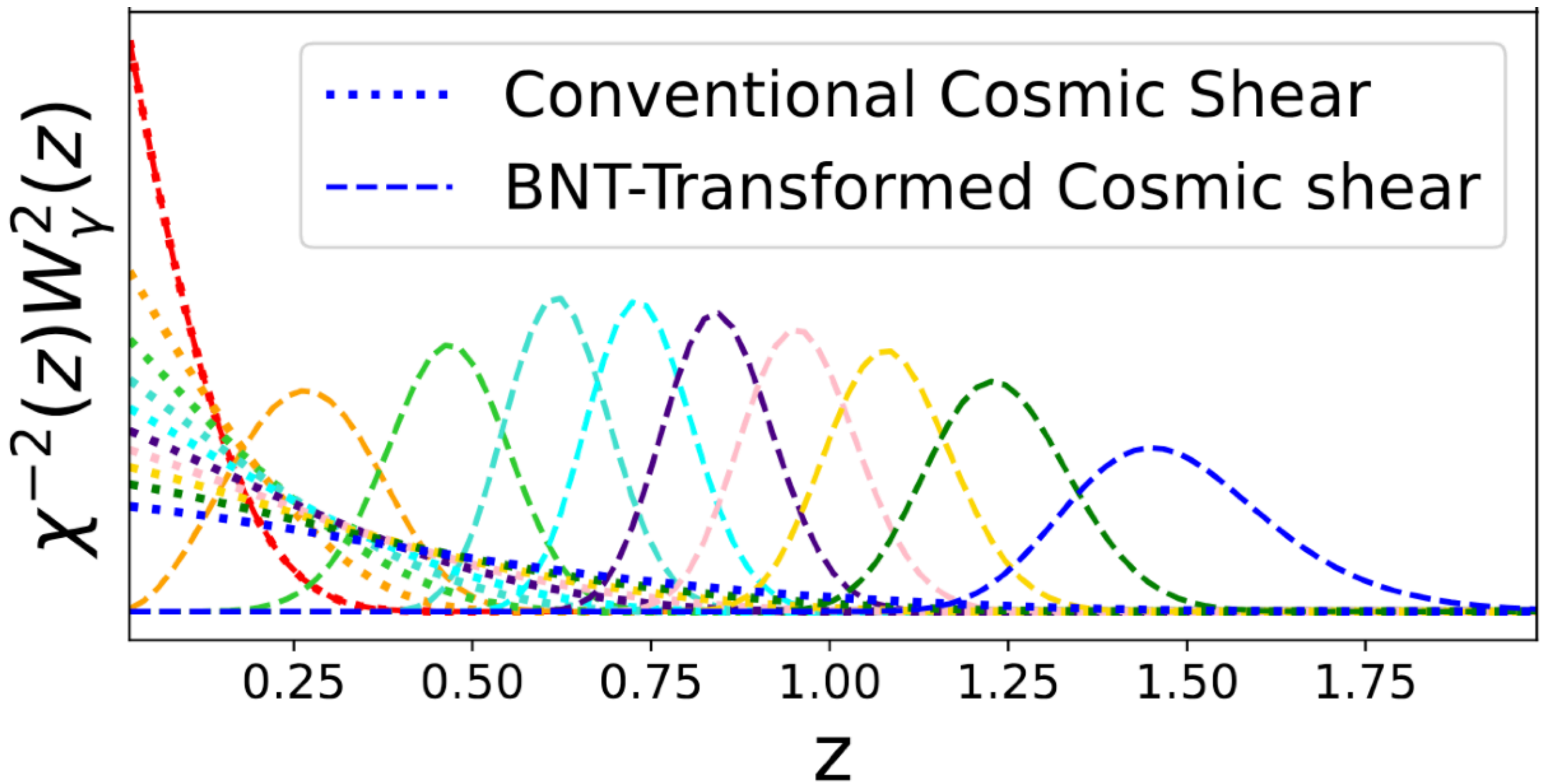
with

$$\mathcal{K}_\ell(k) = \frac{9\Omega_0^2 H_0^4}{4c^4} \; P\left(k, \eta(\chi)\right) \frac{w_a(\chi) w_b(\chi)}{a(\chi)^2 f_\mathrm{K}(\chi)^2}$$





$n_g(z)$
$W_\gamma(z)$
0.25
0.50
0.75
1.00
1.25
1.50
1.75
z



Conventional Cosmic Shear
BNT-Transformed Cosmic shear
$\chi^{-2}(z)W_{\gamma}^{2}(z)$
0.25
0.50
0.75
1.00
1.25
1.50
1.75
z

Definition of BNT:

$$n_i^0 = \int d\chi \, n_i(\chi)$$

$$n_i^1 = \int d\chi \, \frac{n_i(\chi)}{\chi}$$

The new kernel:

$$\widehat{W}_\gamma^a(\chi) = \sum_{i=1}^{n_T} p_i^a \, W_\gamma^i(\chi)$$

The system of equation to solve:

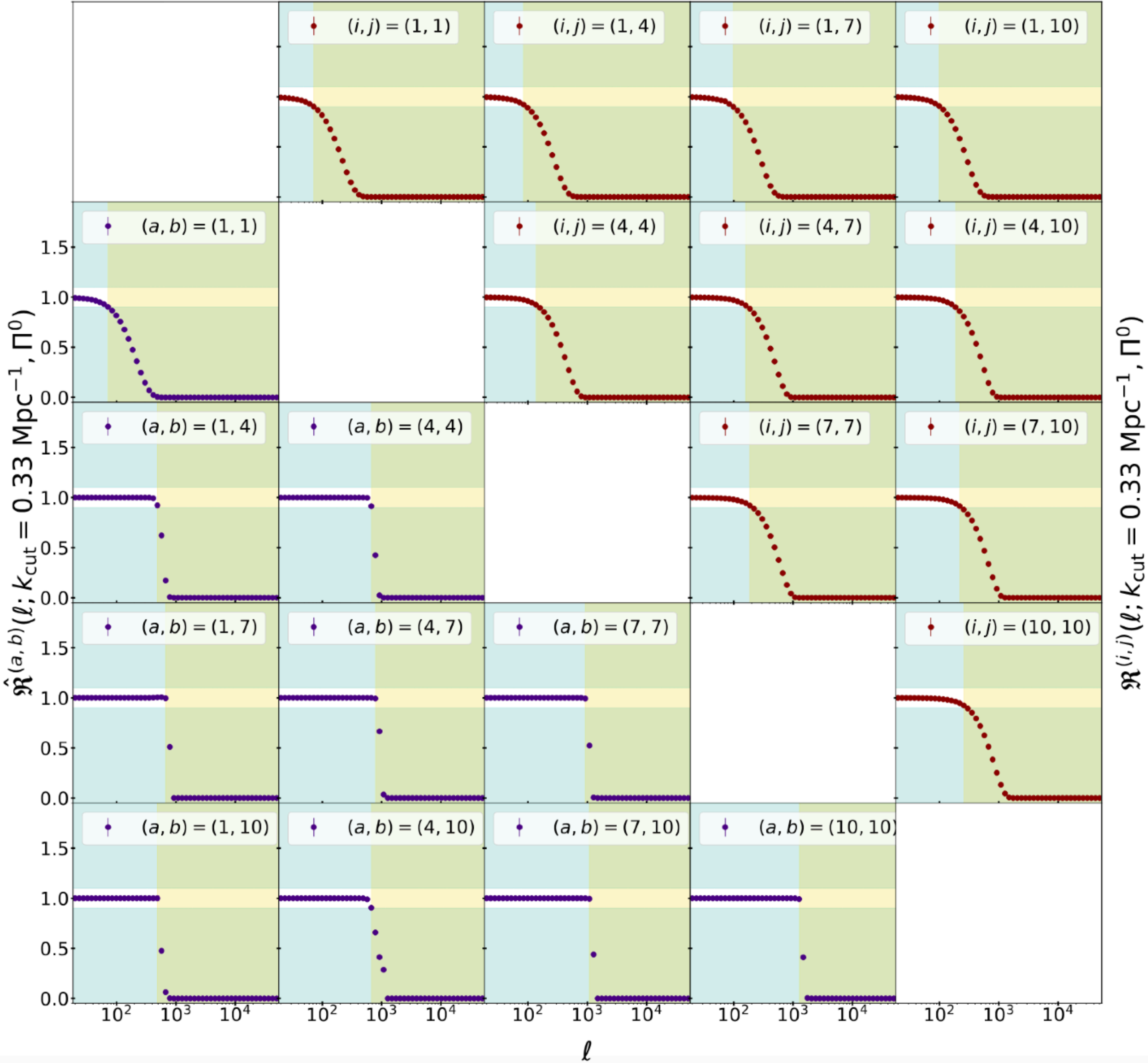
$$\sum_{i=a-2}^{a} p_i^a \, n_i^0 = 0$$

$$\sum_{i=a-2}^{a} p_i^a \, n_i^1 = 0$$

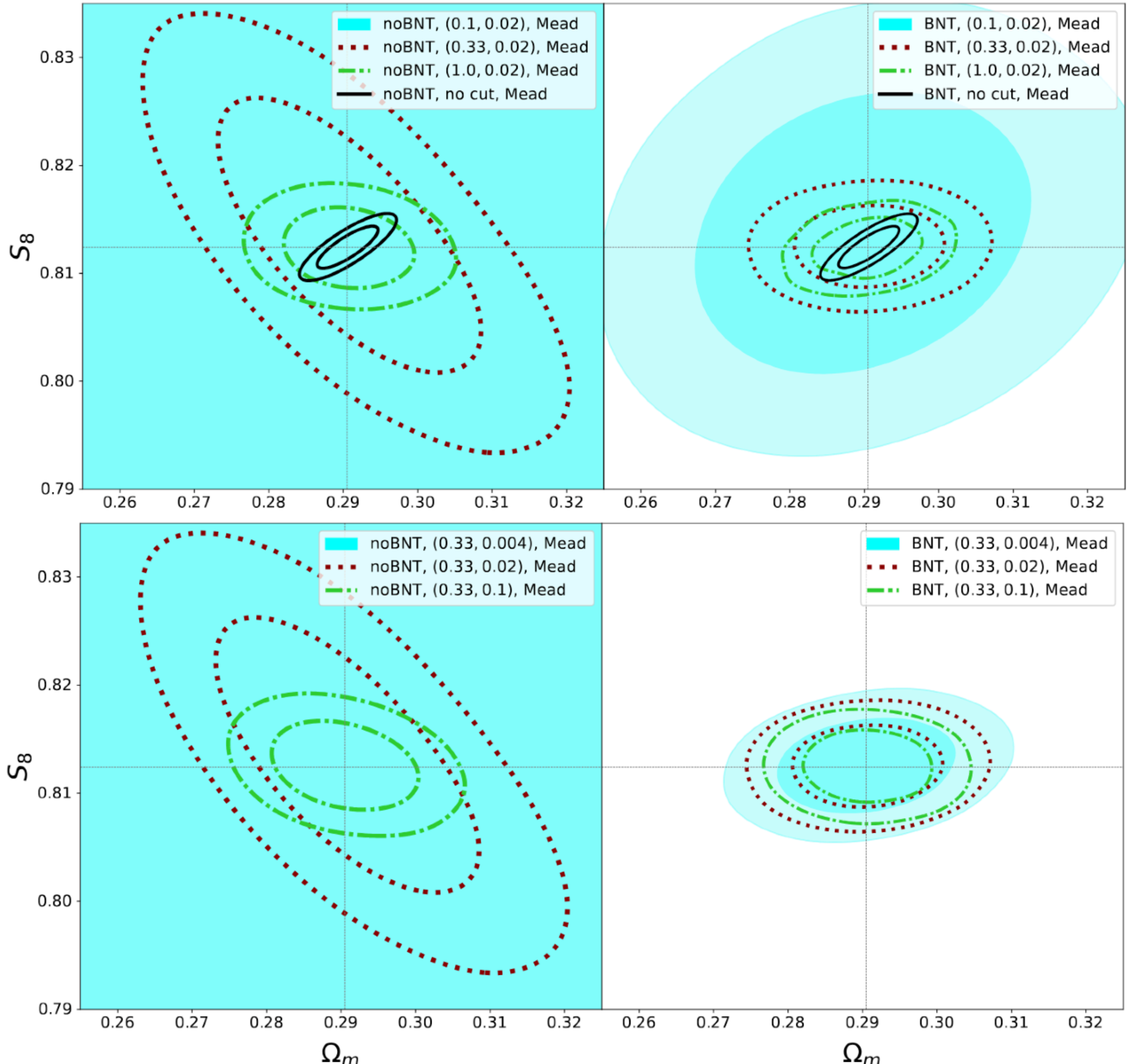
BNT spectrum:

$$\widehat{C}^{a,b}(\ell) \equiv \int \frac{d\chi}{\chi^2}\, \widehat{W}^a_\gamma(\chi)\, \widehat{W}^b_\gamma(\chi)\, P\left(\frac{\ell + 1/2}{\chi}; z(\chi)\right)$$
$$= p_i^a\, p_j^b\, C_\ell^{i,j}$$

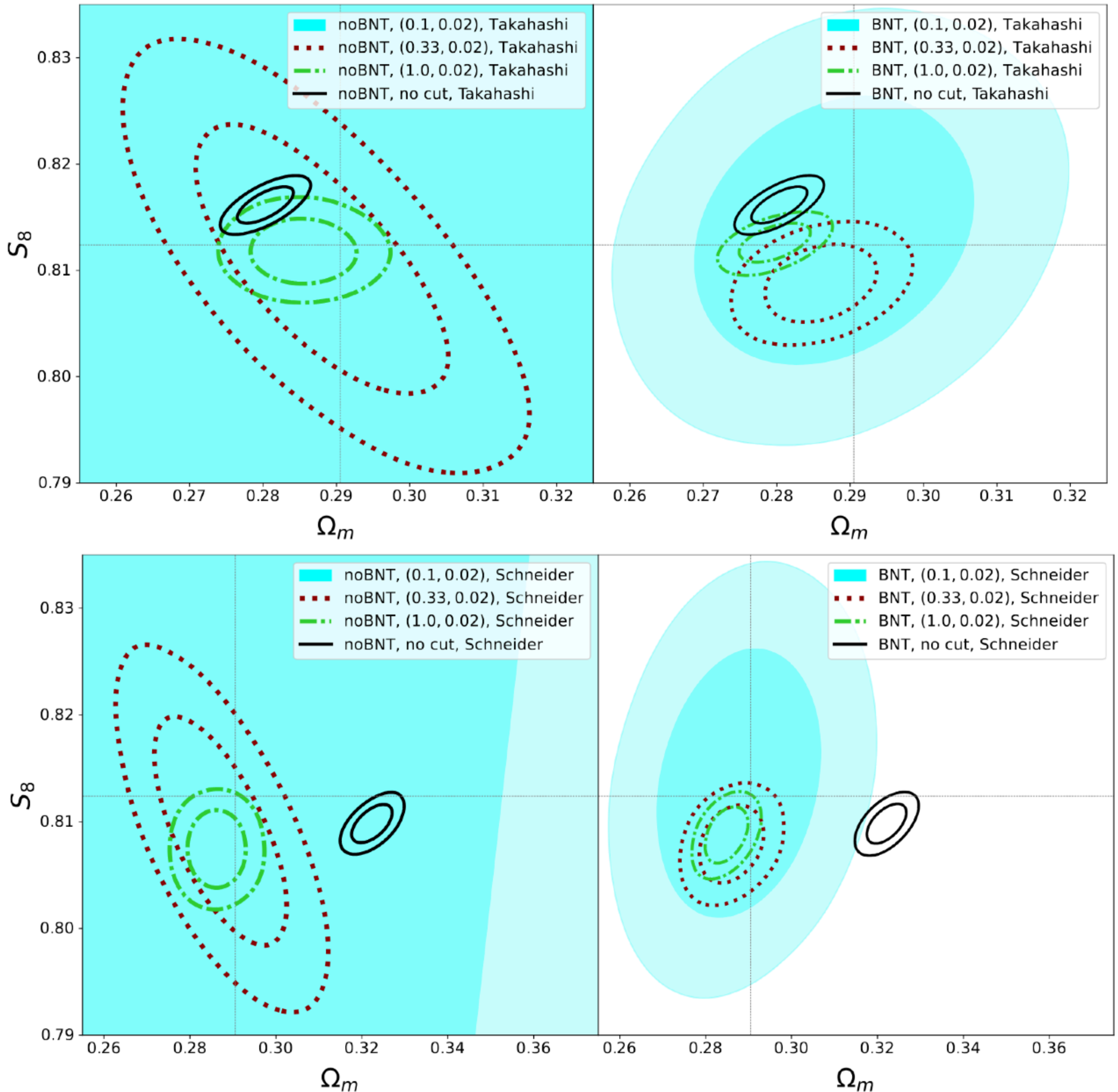
$$\begin{bmatrix}
1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.0 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.26 & -1.26 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.6 & -1.6 & 1.0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.72 & -1.72 & 1.0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.79 & -1.79 & 1.0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.86 & -1.86 & 1.0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.92 & -1.92 & 1.0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.01 & -2.01 & 1.0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.31 & -2.31 & 1.0
\end{bmatrix}$$



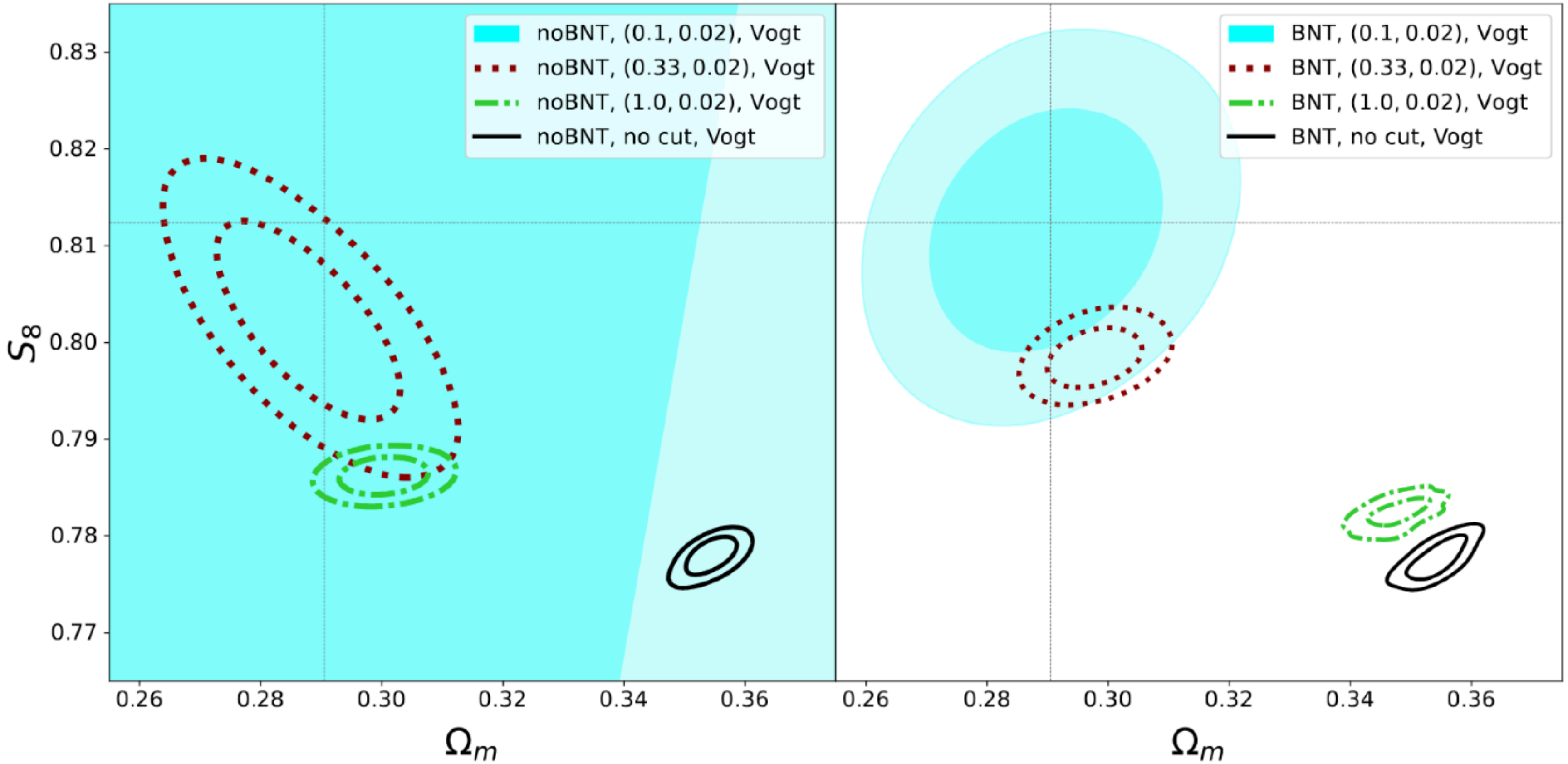
$(i,j) = (1,1)$
$(i,j) = (1,4)$
$(i,j) = (1,7)$
$(i,j) = (1,10)$
$(a,b) = (1,1)$
$(i,j) = (4,4)$
$(i,j) = (4,7)$
$(i,j) = (4,10)$
$(a,b) = (1,4)$
$(a,b) = (4,4)$
$(i,j) = (7,7)$
$(i,j) = (7,10)$
$(a,b) = (1,7)$
$(a,b) = (4,7)$
$(a,b) = (7,7)$
$(i,j) = (10,10)$
$(a,b) = (1,10)$
$(a,b) = (4,10)$
$(a,b) = (7,10)$
$(a,b) = (10,10)$
$\hat{\mathfrak{R}}^{(a,b)}(\ell; k_{\rm cut} = 0.33\ \mathrm{Mpc}^{-1}, \Pi^0)$
$\mathfrak{R}^{(i,j)}(\ell; k_{\rm cut} = 0.33\ \mathrm{Mpc}^{-1}, \Pi^0)$
ℓ
0.0
0.5
1.0
1.5
10^2
10^3
10^4



noBNT, (0.1, 0.02), Mead
noBNT, (0.33, 0.02), Mead
noBNT, (1.0, 0.02), Mead
noBNT, no cut, Mead
BNT, (0.1, 0.02), Mead
BNT, (0.33, 0.02), Mead
BNT, (1.0, 0.02), Mead
BNT, no cut, Mead
noBNT, (0.33, 0.004), Mead
noBNT, (0.33, 0.02), Mead
noBNT, (0.33, 0.1), Mead
BNT, (0.33, 0.004), Mead
BNT, (0.33, 0.02), Mead
BNT, (0.33, 0.1), Mead
S_8
Ω_m



noBNT, (0.1, 0.02), Takahashi
noBNT, (0.33, 0.02), Takahashi
noBNT, (1.0, 0.02), Takahashi
noBNT, no cut, Takahashi
BNT, (0.1, 0.02), Takahashi
BNT, (0.33, 0.02), Takahashi
BNT, (1.0, 0.02), Takahashi
BNT, no cut, Takahashi
noBNT, (0.1, 0.02), Schneider
noBNT, (0.33, 0.02), Schneider
noBNT, (1.0, 0.02), Schneider
noBNT, no cut, Schneider
BNT, (0.1, 0.02), Schneider
BNT, (0.33, 0.02), Schneider
BNT, (1.0, 0.02), Schneider
BNT, no cut, Schneider
S_8
Ω_m



noBNT, (0.1, 0.02), Vogt
noBNT, (0.33, 0.02), Vogt
noBNT, (1.0, 0.02), Vogt
noBNT, no cut, Vogt
BNT, (0.1, 0.02), Vogt
BNT, (0.33, 0.02), Vogt
BNT, (1.0, 0.02), Vogt
BNT, no cut, Vogt
S_8
Ω_m
Ω_m
0.83
0.82
0.81
0.80
0.79
0.78
0.77
0.26
0.28
0.30
0.32
0.34
0.36