

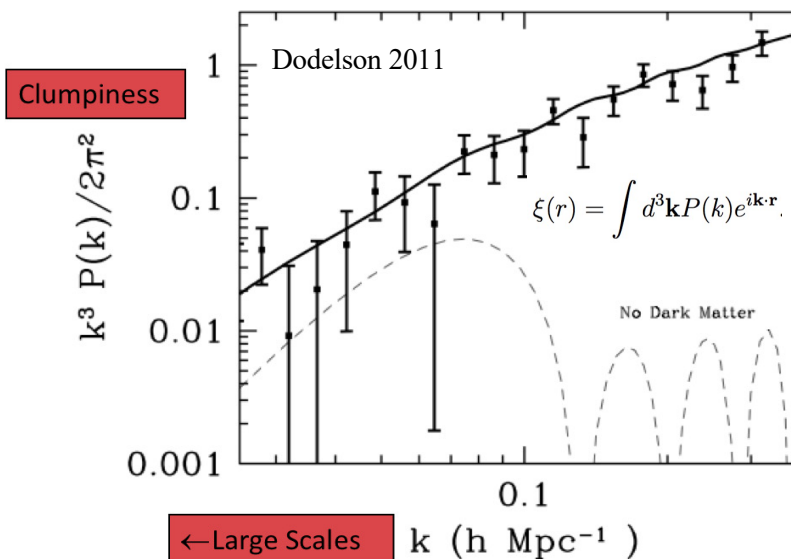
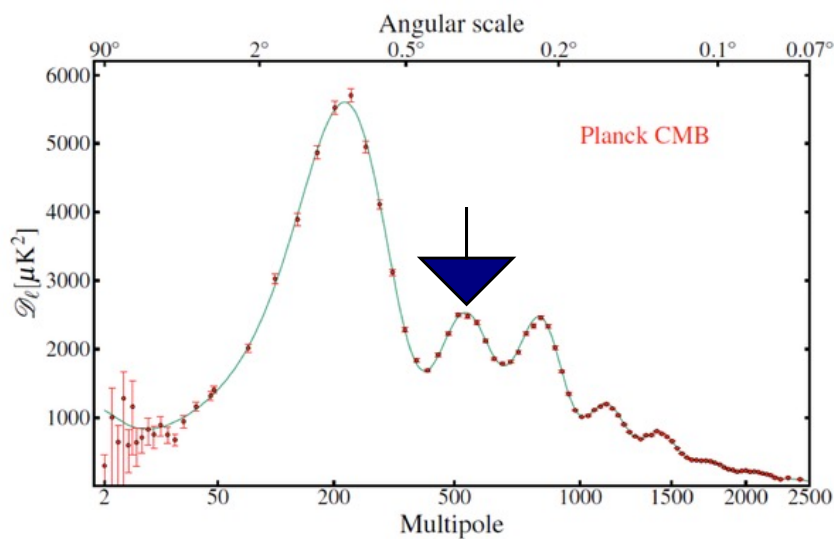
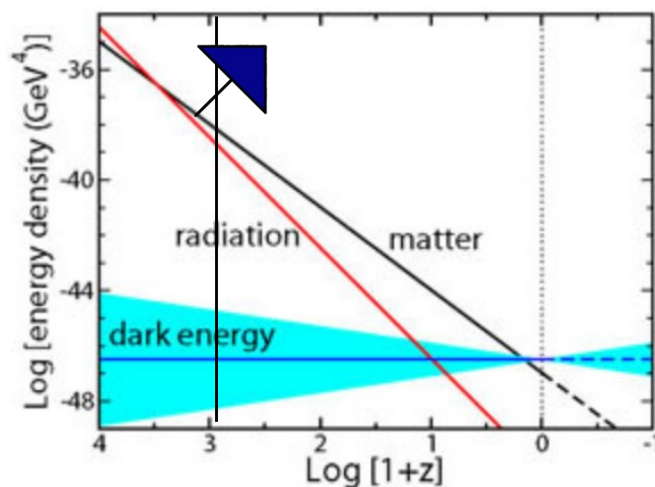


Short lecture: dark matter alternatives and alternatives to dark matter

Benoit Famaey

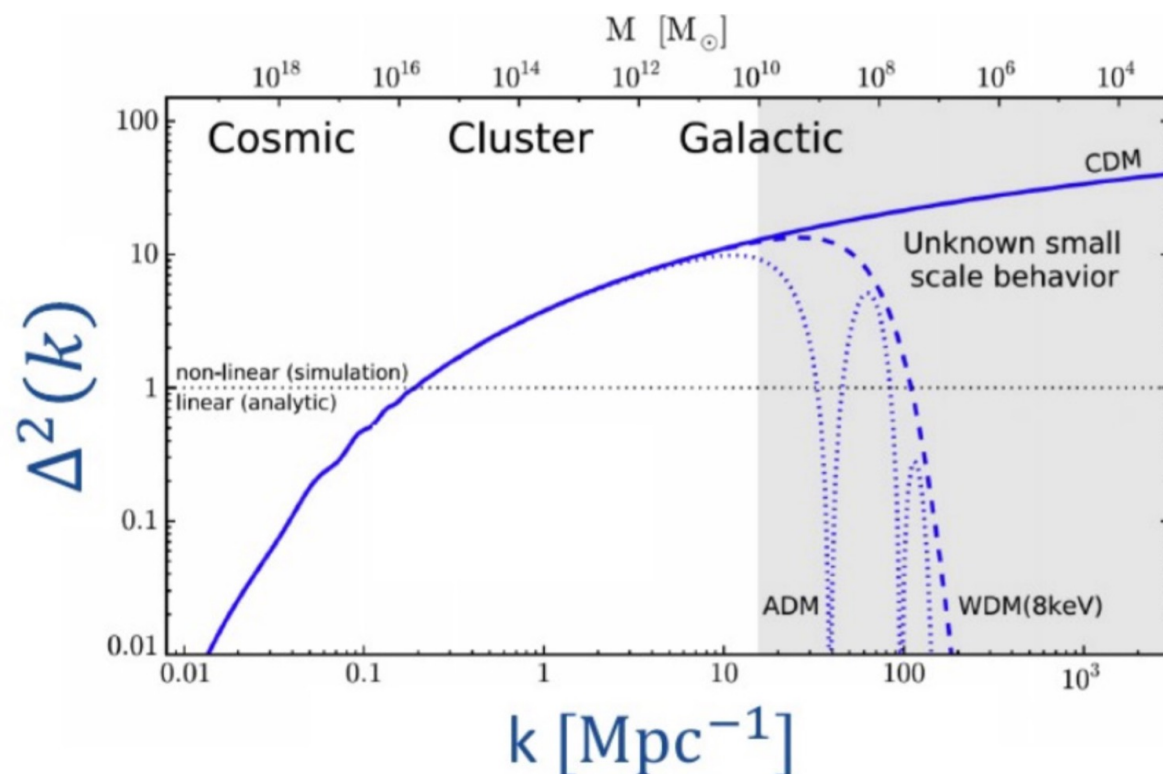
CNRS - Observatoire astronomique de Strasbourg

The need for something like dark « matter »



A plethora of alternatives to CDM

- All interesting in their own right: who knows what DM might be, how it is produced, etc. (see Ludovic's lecture)
- Most of them **mostly** affect the matter power spectrum (implying strong constraints from Lyman- α , see Eric's lecture)



M. Kuhlen et al. 2012

A plethora of alternatives to CDM

■ Generalized Dark Matter » (Hu 1998; Kopp et al. 2016):

consider a very general fluid with non-zero pressure and anisotropic stress, such that

$$w_g(a) = \bar{p}/\bar{\rho},$$

with a sound speed that can depend on scale $c_s^2(\mathbf{k}, a)$

and a « viscosity » $c_{vis}^2(\mathbf{k}, a)$

This can typically describe the evolution of a k-essence scalar field

$$\mathcal{L}_\phi \propto \sqrt{-g} f(X) \quad \text{where } X \propto \nabla^\nu \phi \nabla_\nu \phi$$

... describes a « dark matter » fluid, but what does one mean by « matter » ?

A plethora of alternatives to CDM

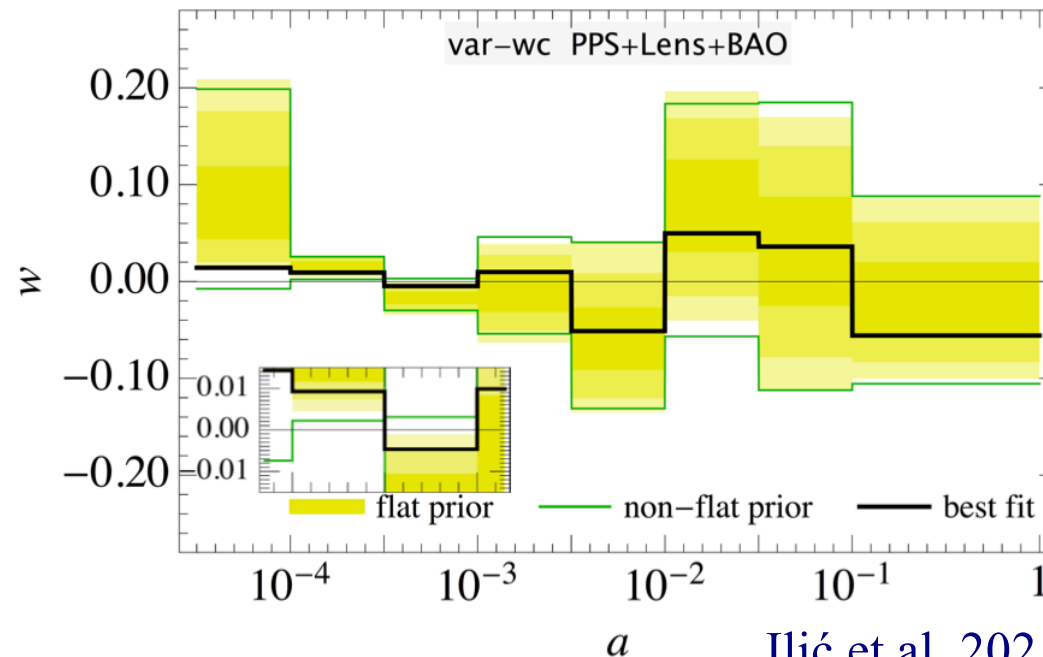
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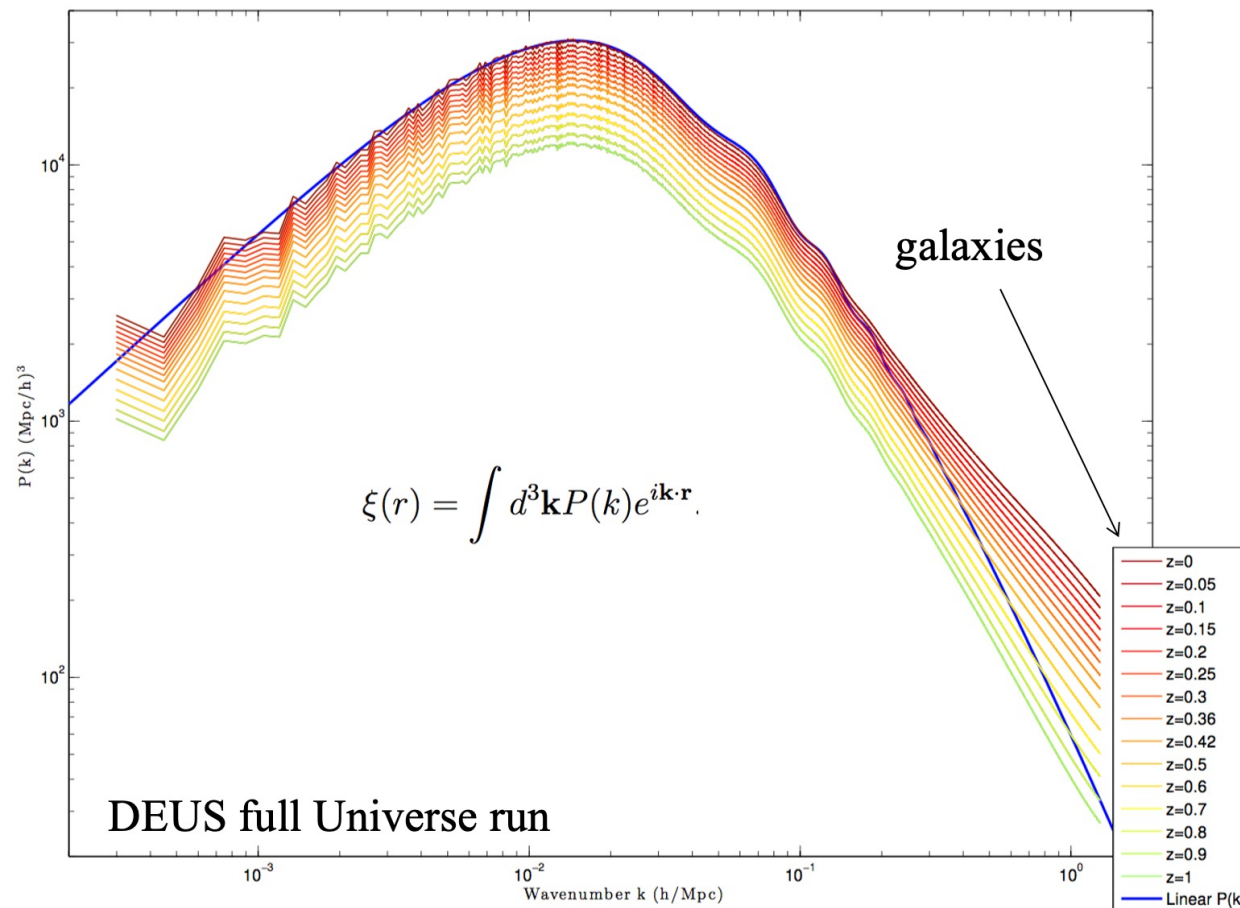
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Galaxy-scale tensions/anomalies and the nature of dark matter (or dark « matter »)

Galaxies in non-linear ($|\delta| \gg 1$) regime of structure formation





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Galaxies in non-linear ($|\delta| \gg 1$) regime of structure formation

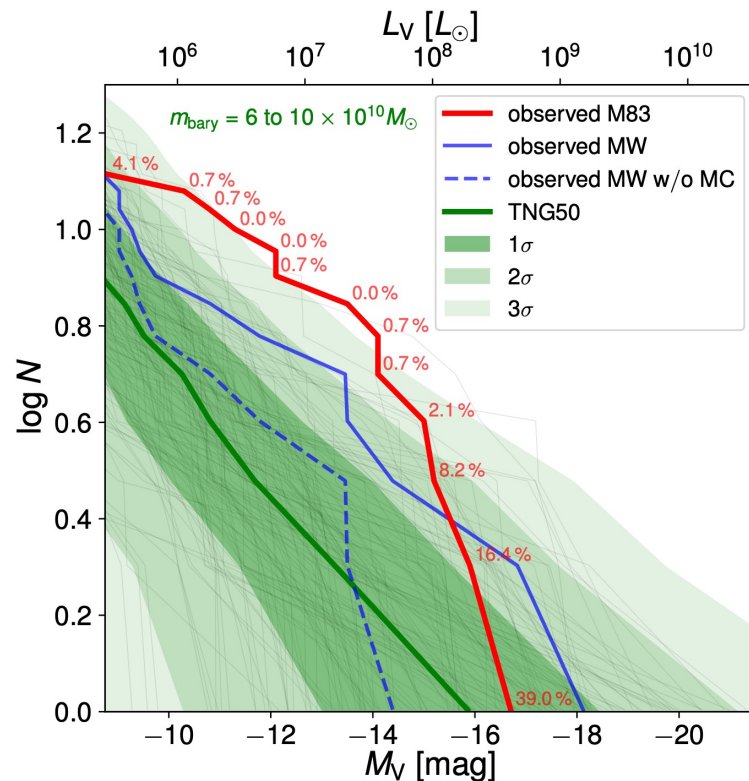
\Rightarrow it is **hard** because of the importance of baryonic physics (**feedback!**)

But simulations have made **huge improvements** at forming more realistic galaxies

But some tensions persist...

Missing satellite problem? (Moore 1999)

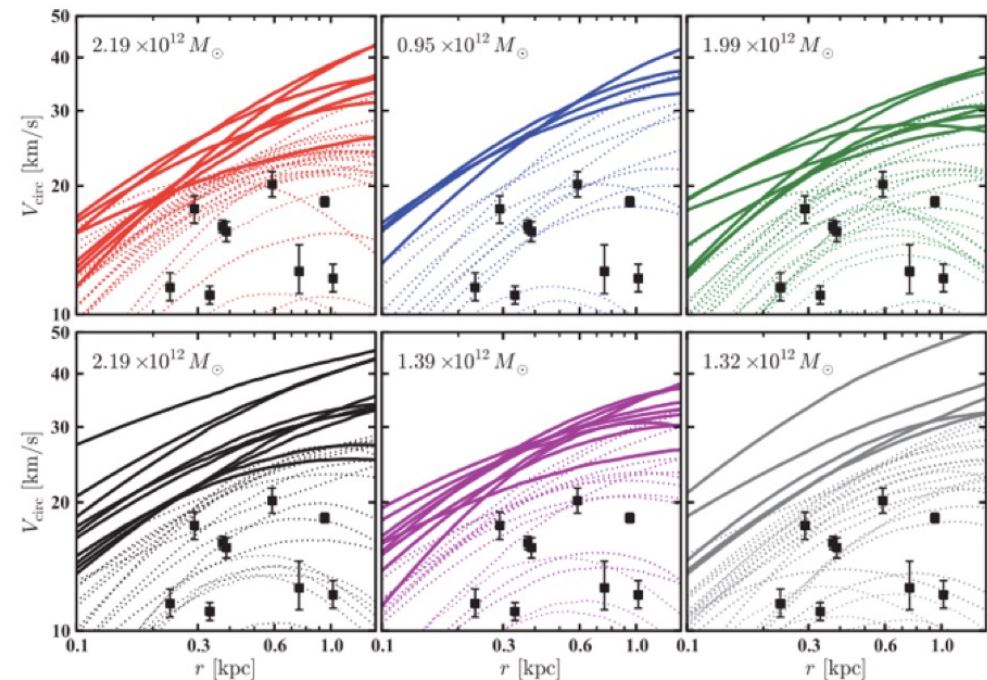
- ⇒ « DMO » problem
- ⇒ long been solved (reionization: halos lack sufficient dense gas to self-shield from UV background heating)
- ⇒ there is now sometimes a too-many-dwarf-galaxy-satellites problem (e.g., in M83, Müller et al. 2025)



Too big to fail? (Boylan-Kolchin 2011)

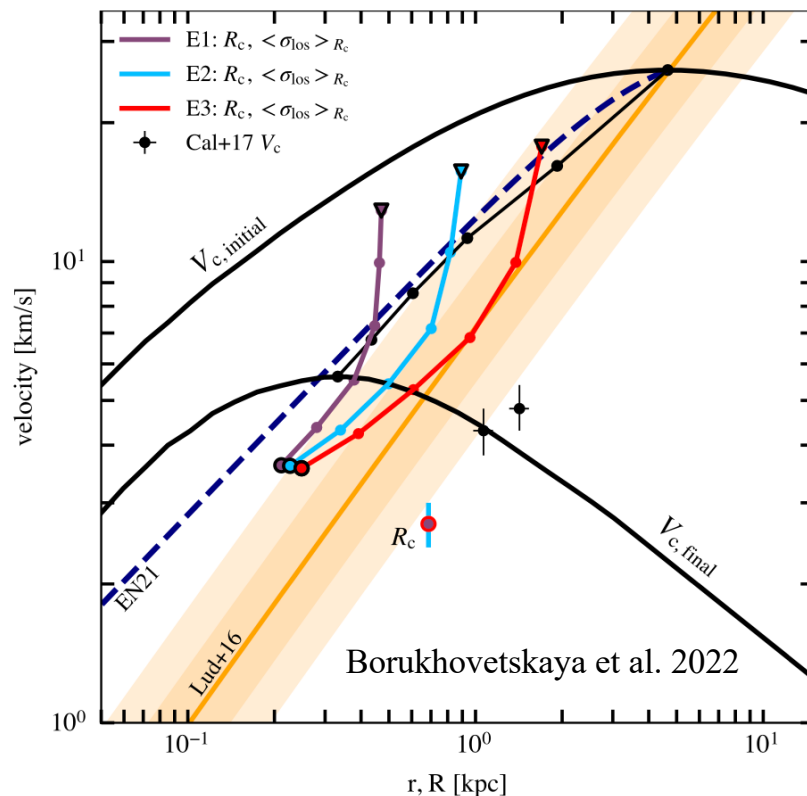
Given observed lum. function of sats,

- ⇒ Missing dense intermediate mass systems, both in the Local Group and in other nearby systems
- ⇒ solved by feedback (for the dense halo), tides, and because gravitational interactions within groups increase the mass of the most massive galaxies



Feeble giants (e.g., Crater II)
=> still a problem

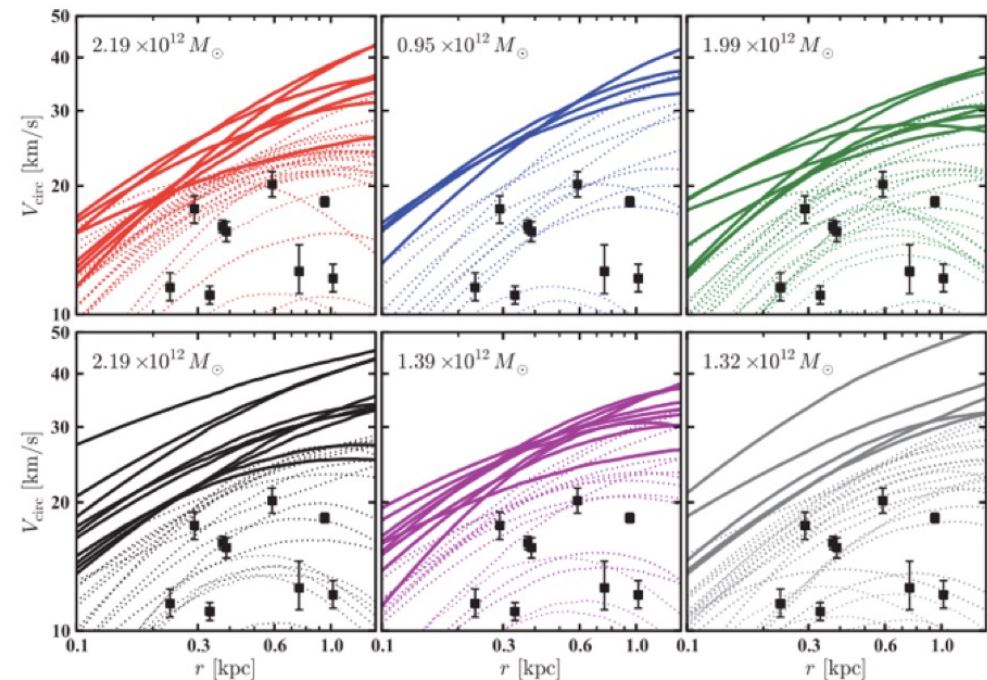
Half-light radius of ~ 1 kpc with
 velocity dispersion of only ~ 2 km/s



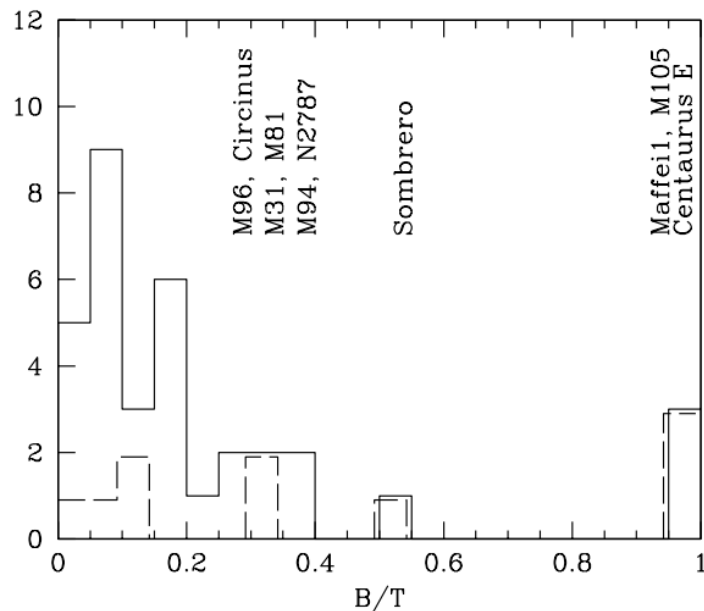
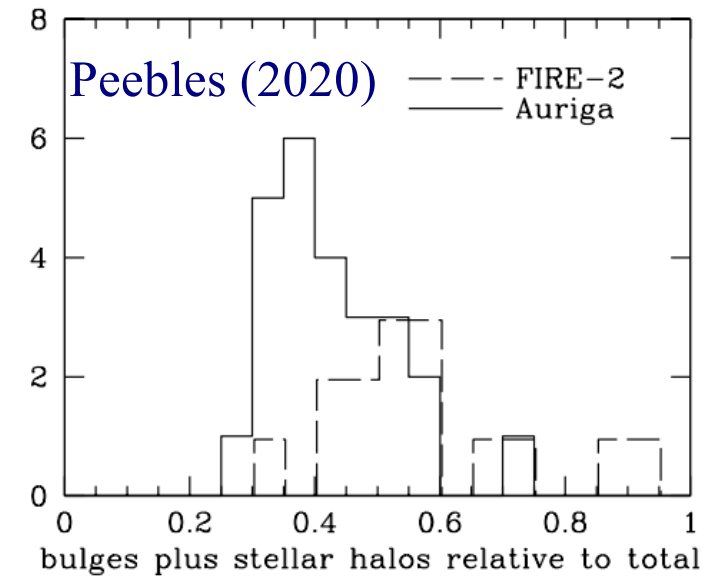
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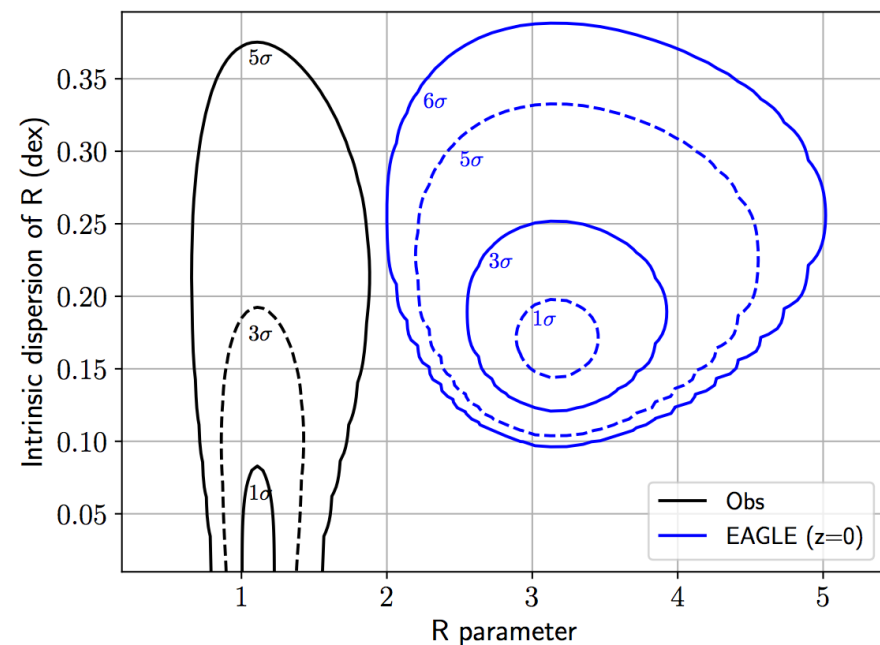


Hot orbits problem



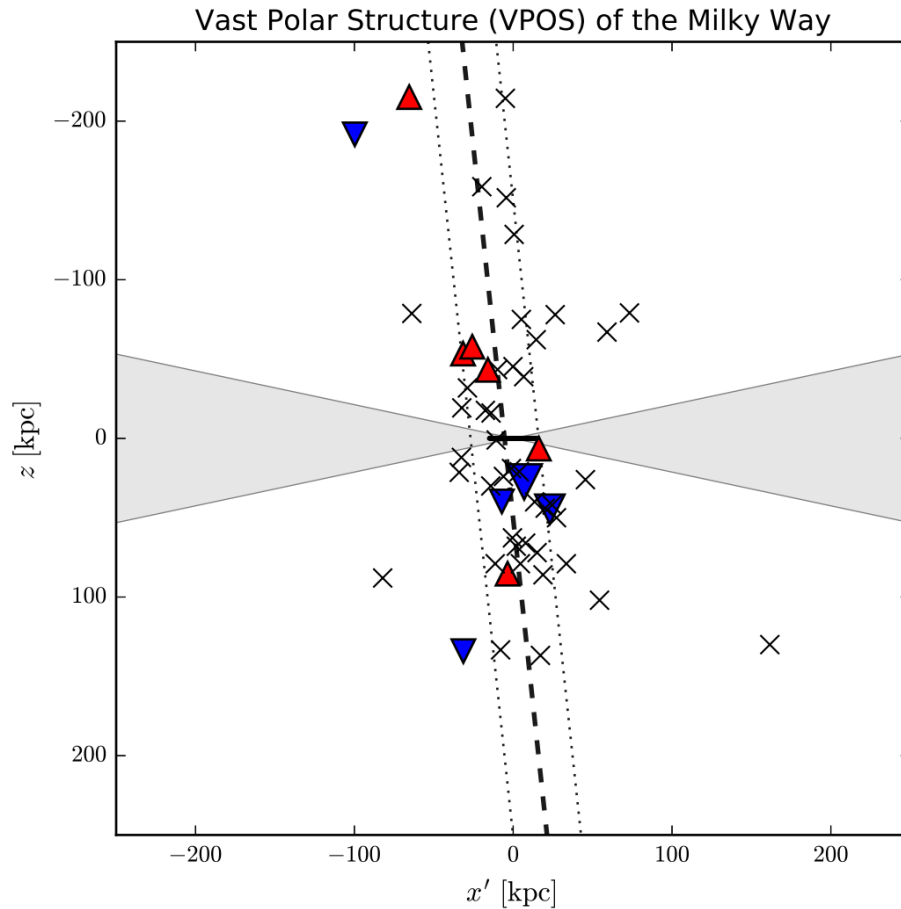
Bar formation problem

- Most local disk galaxies are nearly **bulgeless** with light stellar halos
- **70% are barred** at $M_* \sim 10^9 - 10^{10} M_{\text{sun}}$ (Erwin 2018) \Rightarrow not reproduced by any sim
- Bars are **fast** $R_{\text{CR}}/R_{\text{bar}} < 1.4$ (Aguerri et al. 2015)

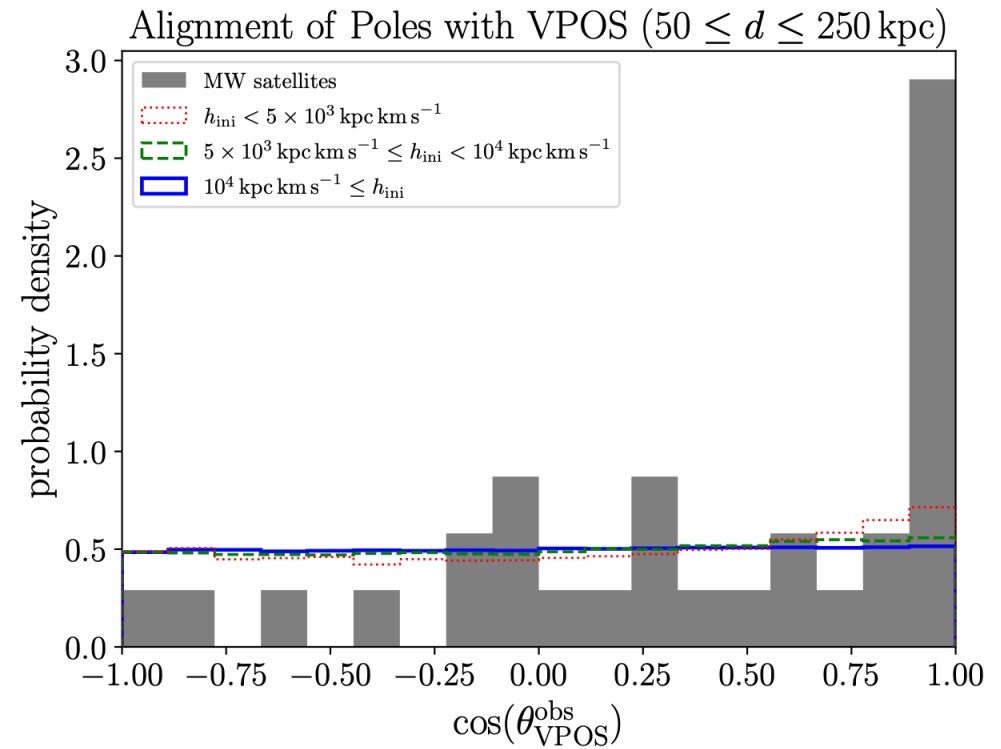


Roshan et al. (2021)

The satellites phase-space correlation problem



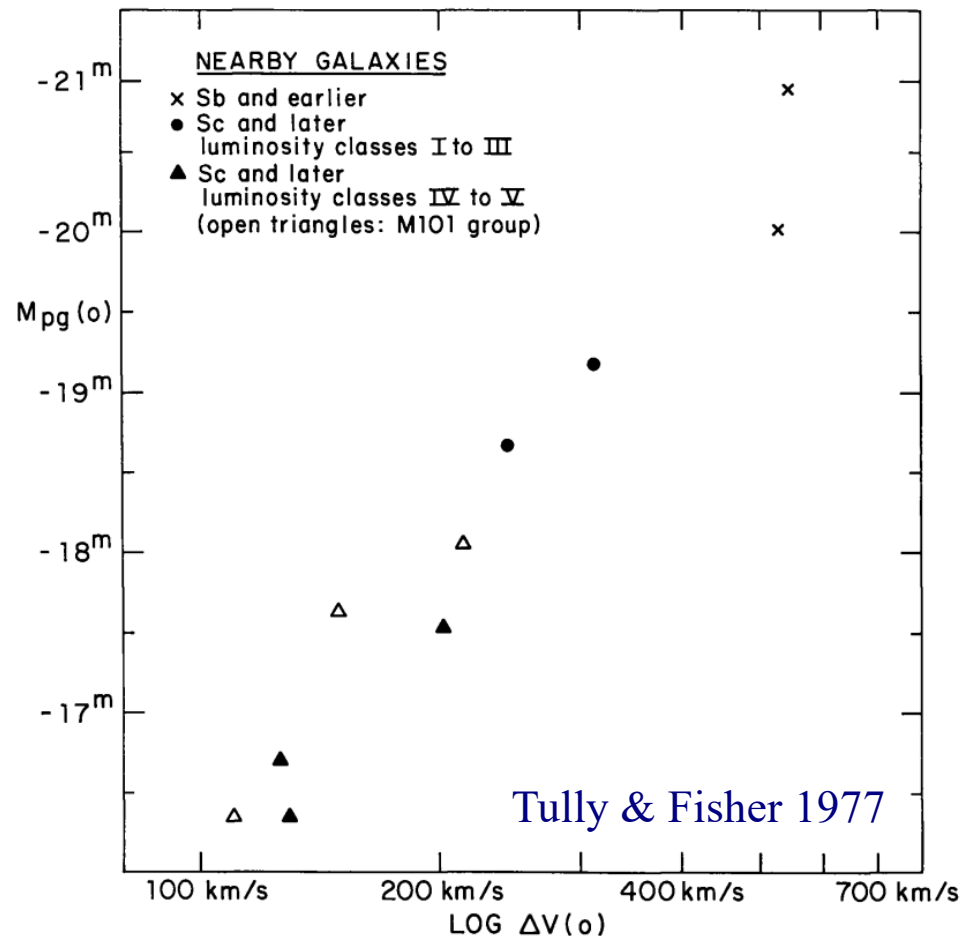
Pawlowski (2018)



Pawlowski et al. (2022)

1/202 in Sawala et al. 2023 \Rightarrow jury is still out on this one

Regularities in the dynamics of galaxies in HI



$$L \propto \Delta V^\alpha$$

$$\alpha = 2.5 - 4$$

(slope of 6.25-10
in mag)

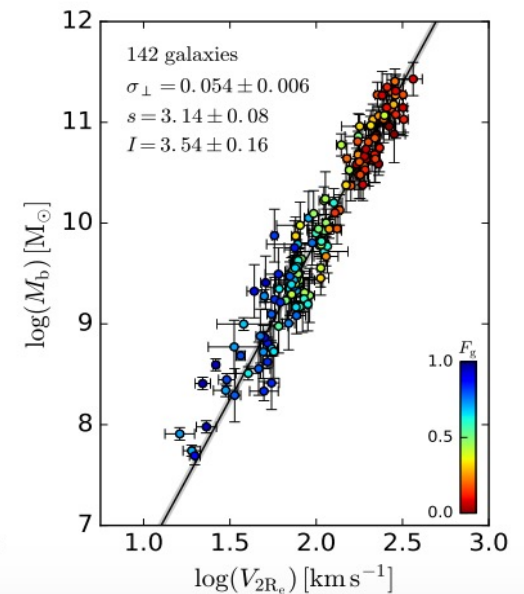
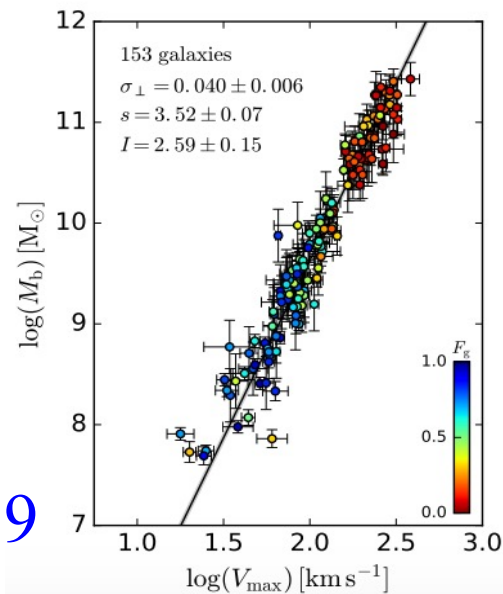
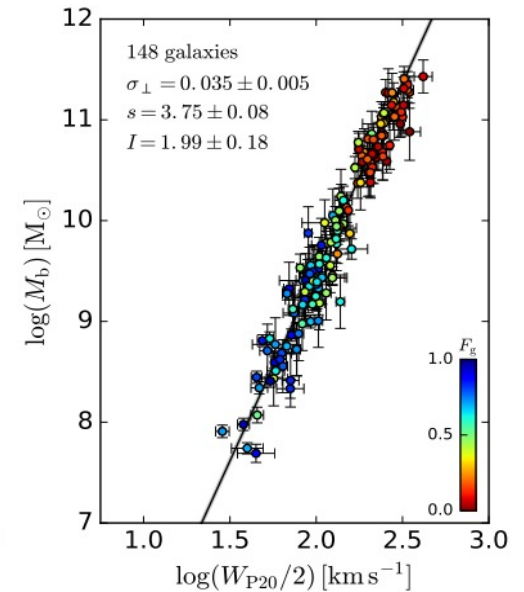
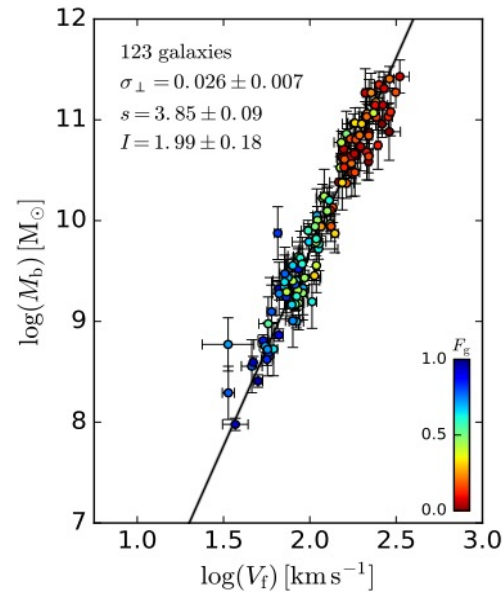
Half of the velocity width at 20% of the peak flux = proxy for rotational velocity

Today: Baryonic TF relation

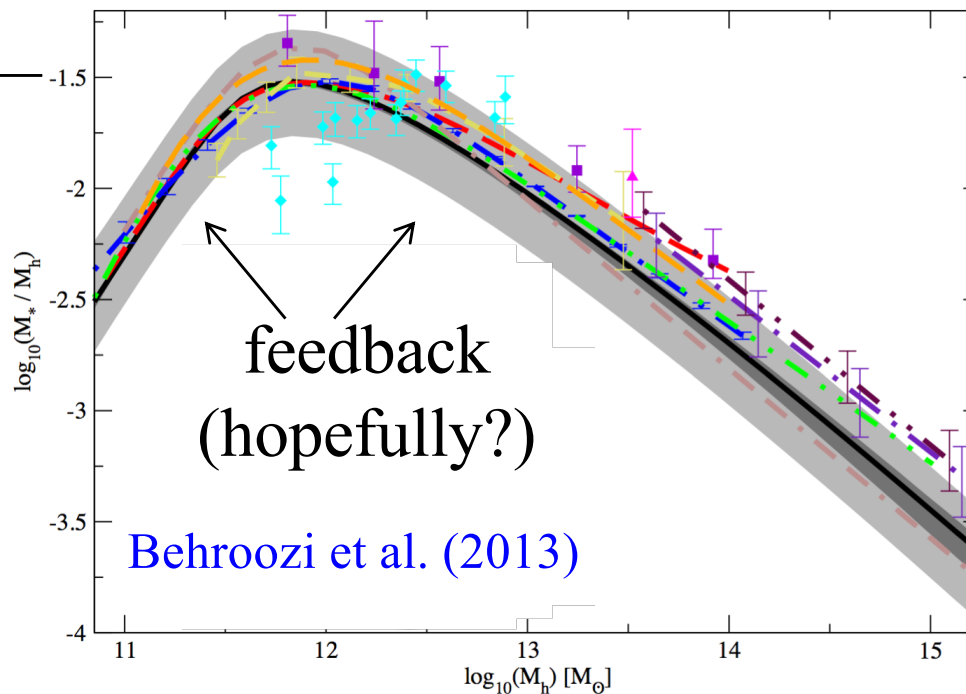
- $\log M_b = \alpha \log V_f - \log \beta$
- $\alpha \approx 4$

- Intrinsic scatter
~ 6%

Lelli et al. 2019



~20% of
cosmic
fraction



Typical scatter ~ 0.15 dex

\Rightarrow Adding the gas, the
intrinsic BTFR scatter
**cannot go below
0.05 dex**

Twice too high!

The scatter, residual correlations and curvature of the SPARC baryonic Tully–Fisher relation

Harry Desmond^{1,2*} (2017)

¹Kavli Institute for Particle Astrophysics and Cosmology, Physics Department, Stanford University, Stanford, CA 94305, USA

calculate the statistical significance of these results in the framework of halo abundance matching, which imposes a canonical galaxy–halo connection. Taking full account of sample variance among SPARC-like realisations of the parent halo population, we find the scatter in the predicted BTFR to be **3.6 σ too high**,

Dark matter halos are (almost) a one-parameter family (driven by mass)

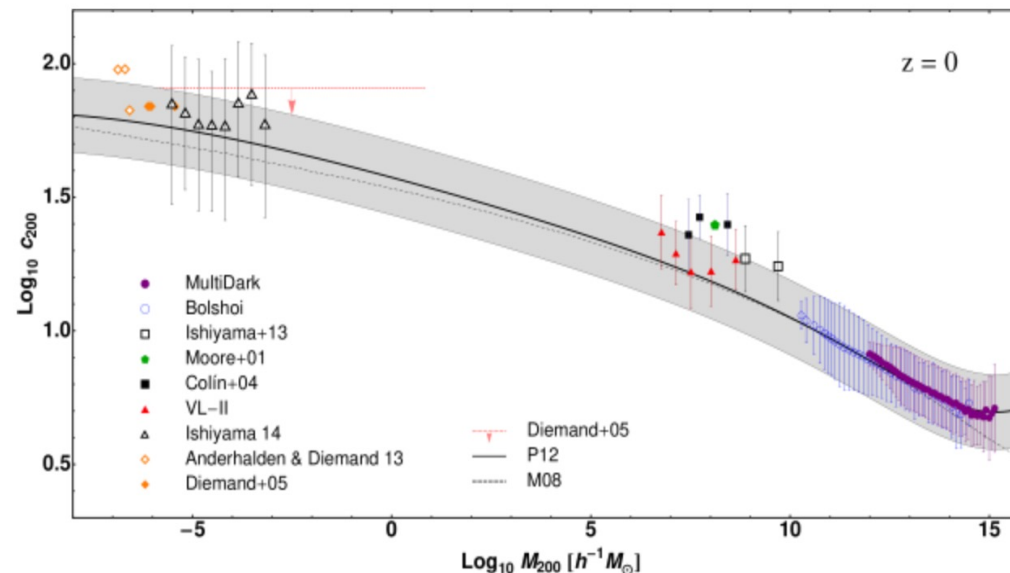
DMO simulations predict that, if we define the virial radius as

$$R_{200} = \left(\frac{M_{200}}{(4/3)\pi 200\rho_{\text{crit}}} \right)^{1/3}$$

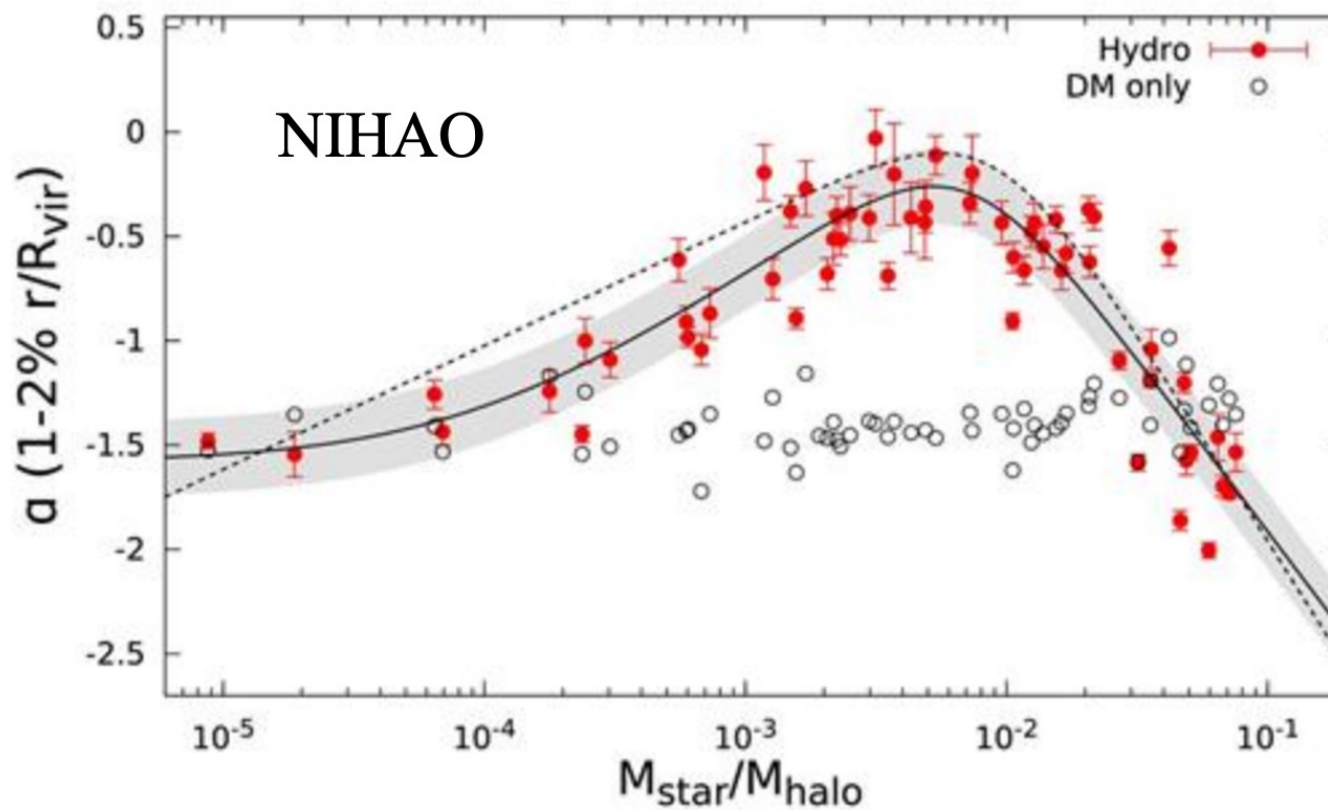
the universal profile of DM halos is the NFW profile :

$$\rho_{\text{DM}} = \frac{200\rho_{\text{crit}}R_{200}}{3r[c^{-1} + (r/R_{200})]^2[\ln(1+c) - c/(1+c)]}$$

with an obvious $\sim r^{-1}$ cusp at the center



**Dark matter halos are (almost)
a one-parameter family (driven by mass)**



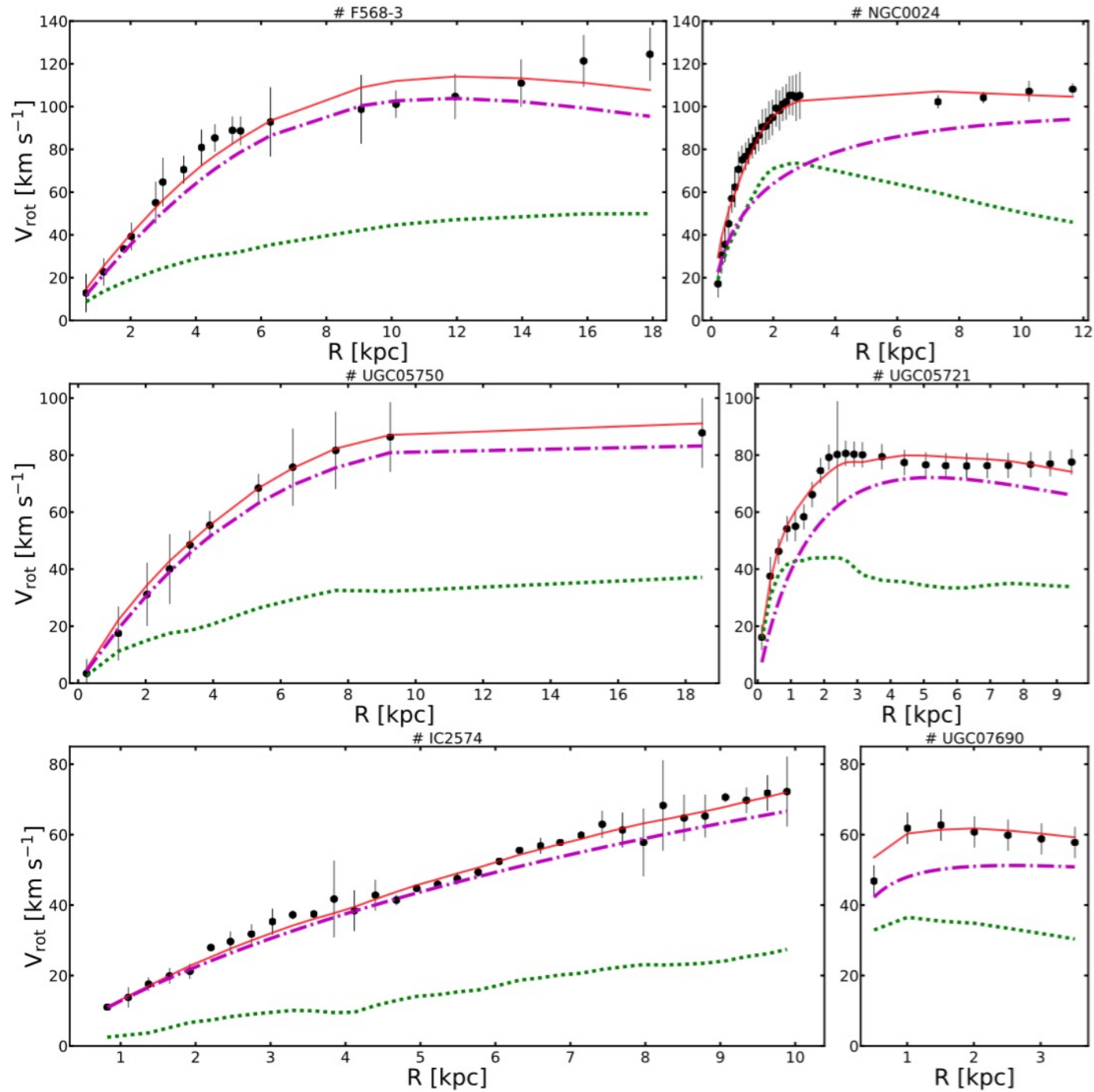
The rotation curves shapes of late-type dwarf galaxies

R. A. Swaters^{1,2,★}, R. Sancisi^{3,4}, T. S. van Albada³, and J. M. van der Hulst³ (2009)

HI observations for a sample of 62 galaxies [...] procedure takes the rotation curve shape, the HI distribution, the inclination, and the size of the beam into account, and makes it possible to correct for the effects of beam smearing.

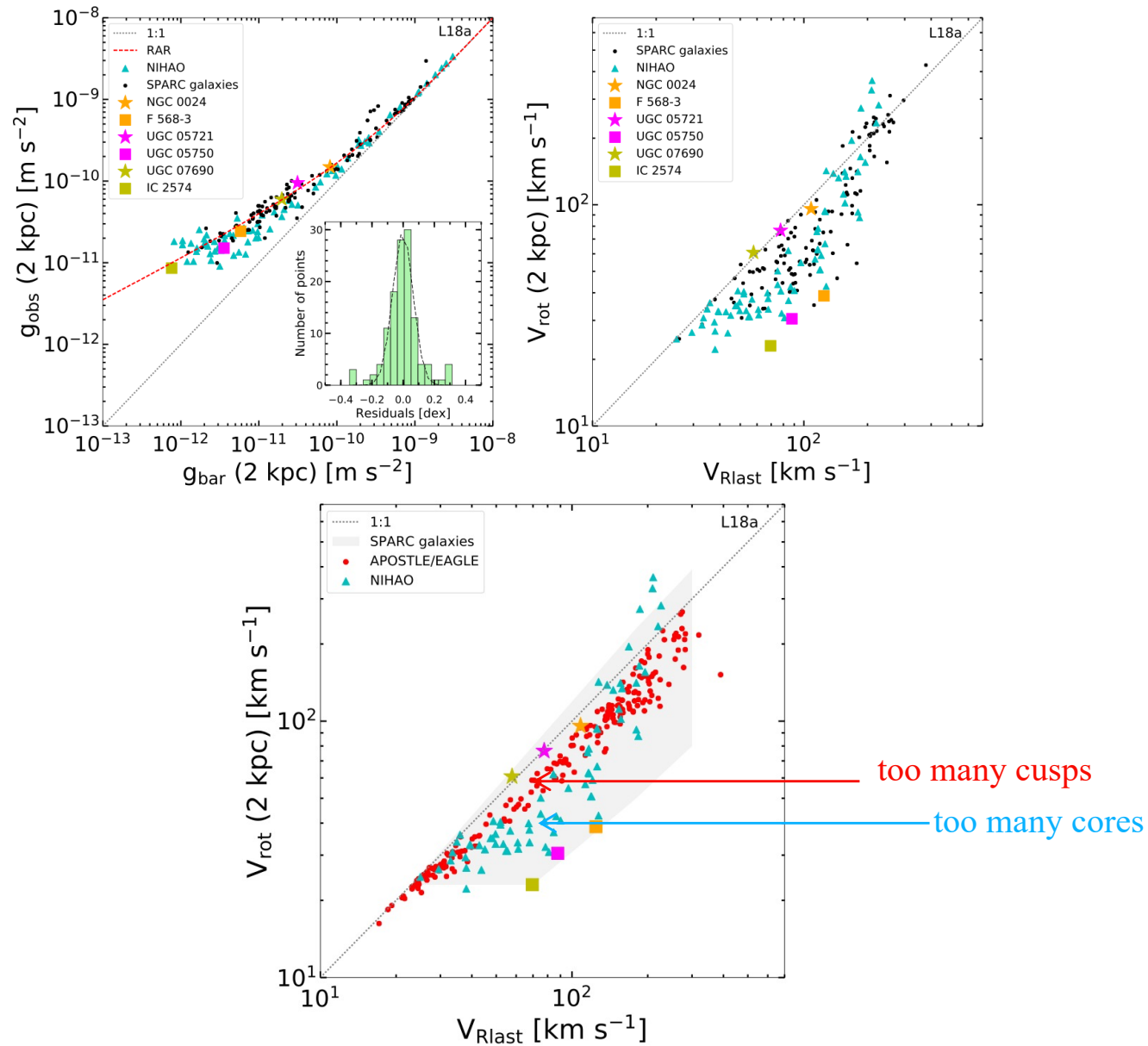
In spiral galaxies and even in the central regions of late-type dwarf galaxies, the shape of the central distribution of light and the inner rise of the rotation curve are related. This implies that galaxies with stronger central concentrations of light also have higher central mass densities, and it suggests that the luminous mass dominates the gravitational potential in the central regions, even in low surface brightness dwarf galaxies (NB: dominated by... dark matter?!)





Ghari, Famaey,
et al. (2019)

Diversity of rotation curves

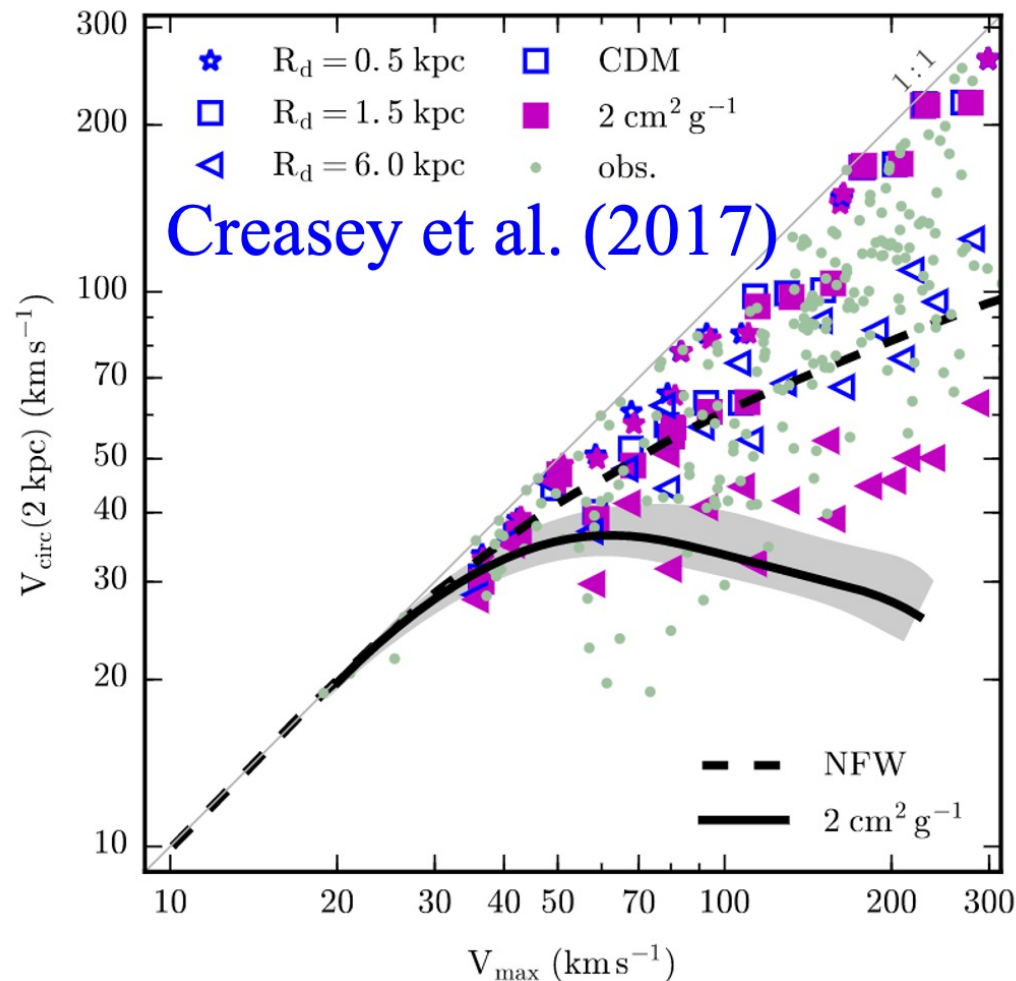


Does moving away from CDM help?

- **Warm dark matter** (see Ludovic and Eric's lectures): creating cores without resorting to feedback? To create a core of ~ 1 kpc needs a thermal relic of about 0.1 keV, which **prevents the formation of the dwarf galaxies altogether** (and is of course excluded by Lyman- α)
- **Fuzzy dark matter** (see Ludovic and Eric's lectures): creates **central cores w/ reduced dynamical friction** (by one order of magnitude) + spike at the center + large-scale fluctuations. Constraints from cuspy ultra-faint dwarfs and from Lyman- α imply $m > 10^{-20}$ eV which does not solve any tension
- **Self-interacting dark matter**: very much changes the picture !
Matter power-spectrum constraints subdominant

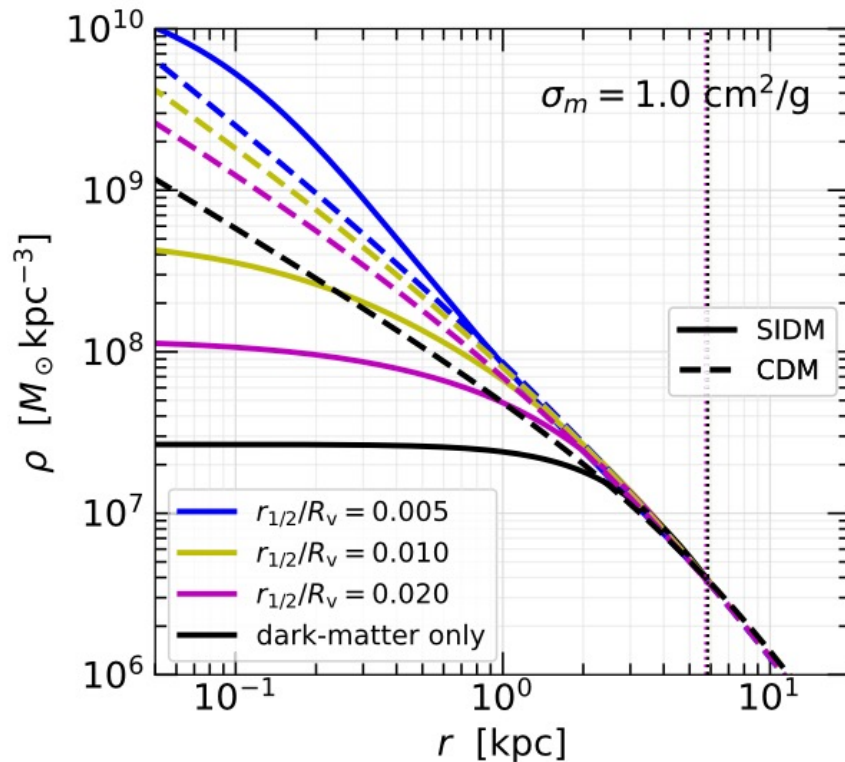
Self-interacting dark matter

Self-interacting cross-sections $\sigma/m = 1-10 \text{ cm}^2/\text{g}$ can have a drastic effect on halo profiles \Rightarrow needs a velocity-dependent cross-section to pass galaxy cluster constraints ($< 0.1 \text{ cm}^2/\text{g}$)



Self-interacting dark matter

Jiang et al. (2023) :



$$\frac{4}{\sqrt{\pi}} \rho_{\text{dm}}(r_1) v(r_1) \sigma_m = \frac{1}{t_{\text{age}}}$$

Stitch the isothermal core
to an adiabatically
contracted NFW

« Age » of gravothermal core-
collapse can also be estimated

Even **too** dense in MW-like galaxies? (e.g., [Correa et al. 2025](#))

And... **not cuspy enough** in ultrafaint dwarfs such as Tuc 3, Seg 1, Seg 2, Ret 2, Tri 2, and Wil 1, as these should have disrupted if accreted on to the Milky Way $\gtrsim 10 \text{ Gyr}$ ago ([Errani et al. 2023](#)) => but survivor bias ?

What about gravity?

$g = g_N$	if $g \gg a_0$
$g = (g_N a_0)^{1/2}$	if $g \ll a_0$

MOND
Milgrom 1983

$$a_0 \sim 10^{-10} \text{ m/s}^2 \sim c^2 \sqrt{\Lambda}$$

$$a_N = \mu(a/a_0) a$$

or

for $x \ll 1$, one should have $\mu(x) \rightarrow x$ or $v(x) \rightarrow 1/\sqrt{x}$.

$$a = \nu(a_N/a_0) a_N,$$

What about gravity?

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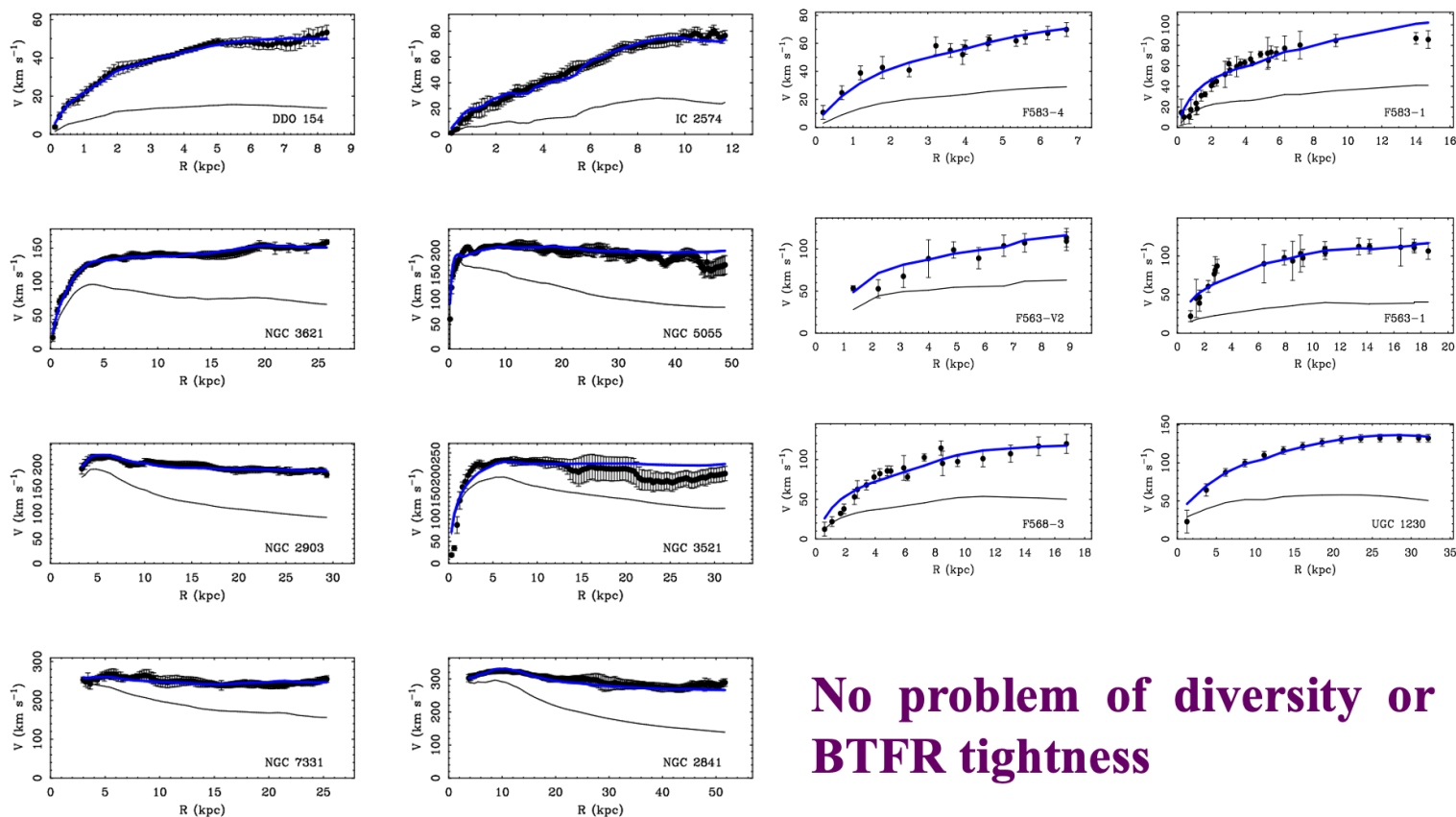
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**No problem of diversity or
BTFR tightness**

Famaey & McGaugh (2012)

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Newtonian Lagrangian density in the non-relativistic limit : $\mathcal{L} = -\rho \left(\Phi - \frac{1}{2} \vec{v} \cdot \vec{v} \right) - \frac{1}{8\pi G} \mathcal{L}_N$

$$\mathcal{L}_N = \nabla \Phi \cdot \nabla \Phi \equiv (\nabla \Phi)^2 \rightarrow \mathcal{L}_{\text{AQUAL}} = a_0^2 \mathcal{F}((\nabla \Phi)^2 / a_0^2).$$

$$\nabla \cdot \left(\frac{\partial \mathcal{L}_{\text{AQUAL}}}{\partial (\nabla \Phi)} \right) = \frac{\partial \mathcal{L}_{\text{AQUAL}}}{\partial \Phi}$$

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$$\mathcal{F}(Y) \rightarrow Y \text{ for } Y \gg 1 \text{ and } \mathcal{F}(Y) \rightarrow \frac{2}{3} Y^{3/2} \text{ for } Y \ll 1$$

$$\mathcal{F}'(Y) = \mu(\sqrt{Y}) \rightarrow \nabla \cdot (\mu(|\nabla \Phi|/a_0) \nabla \Phi) = 4\pi G \rho$$

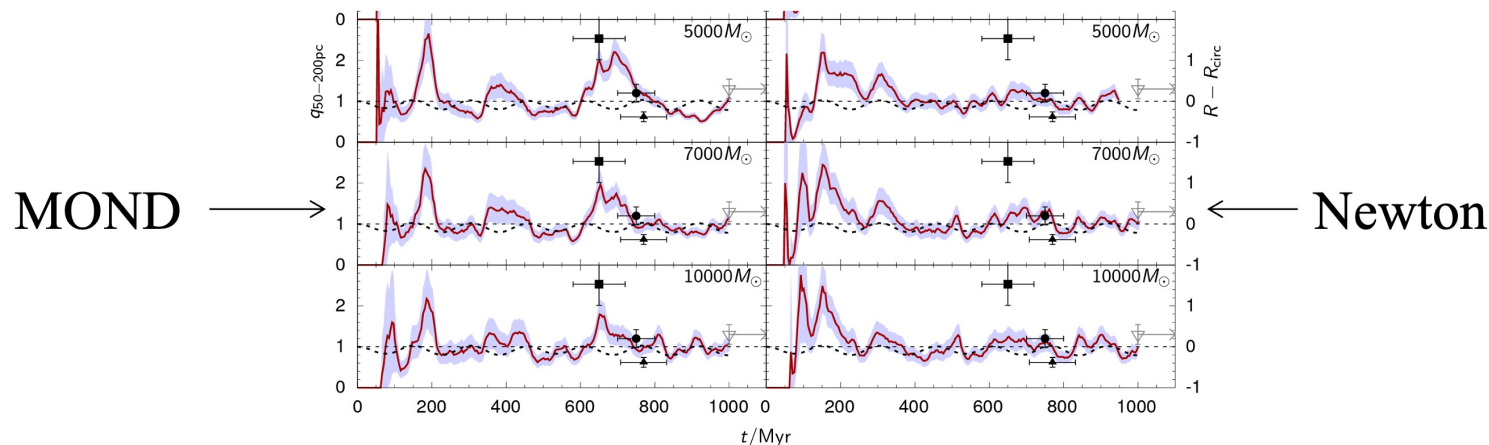
External field effect

$$\nabla \cdot (|\nabla\Phi_{\text{int}} - \vec{a}_{\text{ext}}| (\nabla\Phi_{\text{int}} - \vec{a}_{\text{ext}})) = 4\pi G a_0 \rho$$

when $a_{\text{int}} \ll a_{\text{ext}} \ll a_0$, back to Newton with renormalization of G (+ squashing of isopotentials)

$$G \rightarrow G/\mu \text{ and } r \rightarrow (1 + \log'(\mu) \sin^2(\theta))^{1/2} r$$

[Kroupa et al. \(2022\)](#) : **denser** leading tails (50-200 pc) of open clusters close to pericenter, e.g. Hyades, Coma Berenices, NGC 752



$$-\nabla \cdot [\mu(X)(\mathbf{g}_{\text{ext}} - \nabla\Phi_{\text{int}})] = 4\pi G \rho, \quad X = \frac{|\mathbf{g}_{\text{ext}} - \nabla\Phi_{\text{int}}|}{a_0}$$

Relativistic MOND

MOND action: $S_{\text{grav BM}} \equiv - \int \frac{a_0^2 F(|\nabla \Phi|^2 / a_0^2)}{8\pi G} d^3x dt,$

⇒ Add a k-essence-like scalar + a vector field for lensing (goes back to TeVeS, [Bekenstein 2004](#), now generalized), but can be recasted as a pure « khronon » scalar field theory as in [Blanchet & Skordis \(2024\)](#)

$$\begin{aligned} \mathcal{Q} = A^\mu \nabla_\mu \phi &\longrightarrow \dot{\phi} \\ \mathcal{Y} = \mathcal{Q}^2 + (\nabla \phi)^2 &\longrightarrow |\vec{\nabla} \phi|^2 \end{aligned}$$

$$\mathcal{F} = -2\mathcal{K}_2 (\mathcal{Q} - \mathcal{Q}_0)^2 + (2 - K_B) \mathcal{Y} + \frac{2(2 - K_B)}{3a_0} \mathcal{Y}^{3/2} + \dots$$

“dust” cosmology

Mixing

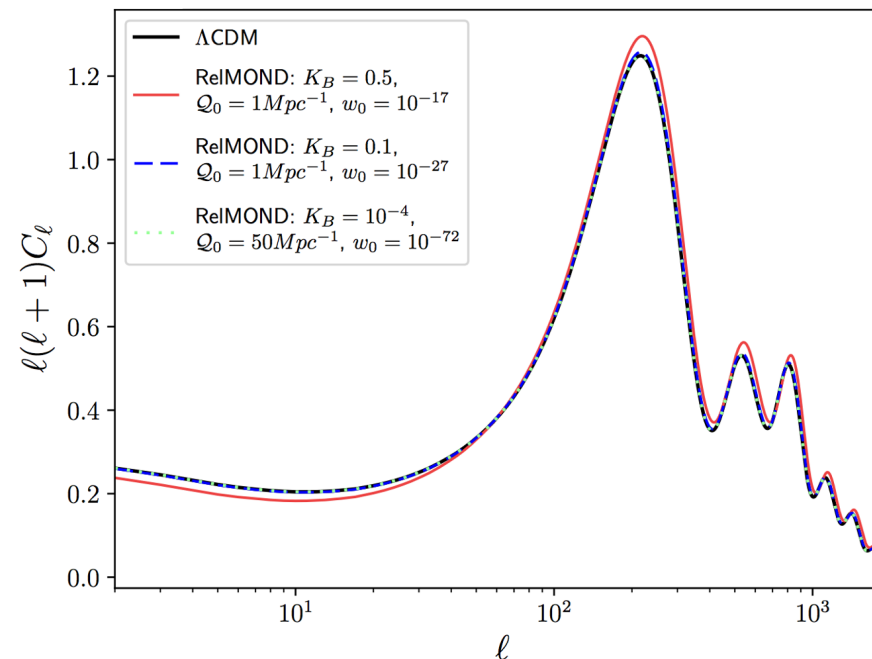
MOND

Bonus: GW and light speeds are equal

Relativistic MOND

Perturbations: $\phi(t, x) = \mathcal{Q}_0 t + \varphi(t, x)$

=> mimicks LCDM to linear order, but pressure contrast (GDM)



Skordis & Zlosnik

Static weak field limit: $\vec{\nabla} \cdot (\mathcal{M} \vec{\nabla} \Phi) + \tilde{\mu}^2 \Phi = 4\pi G_N \rho_b$

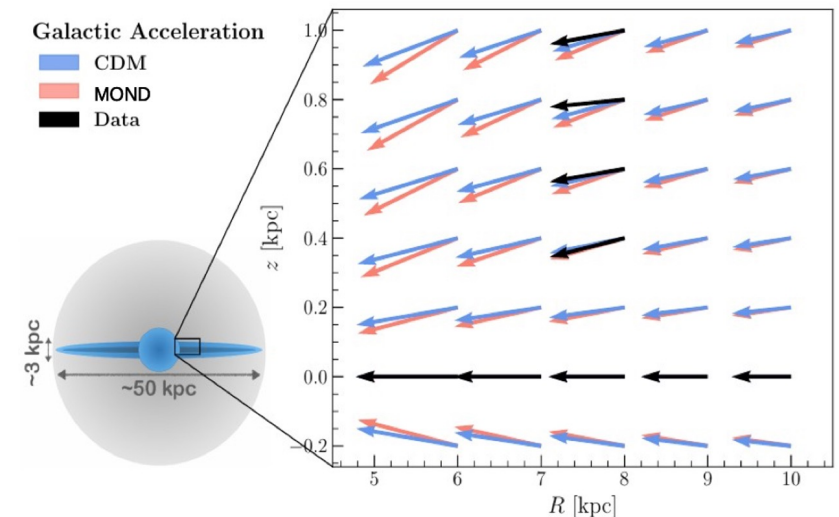
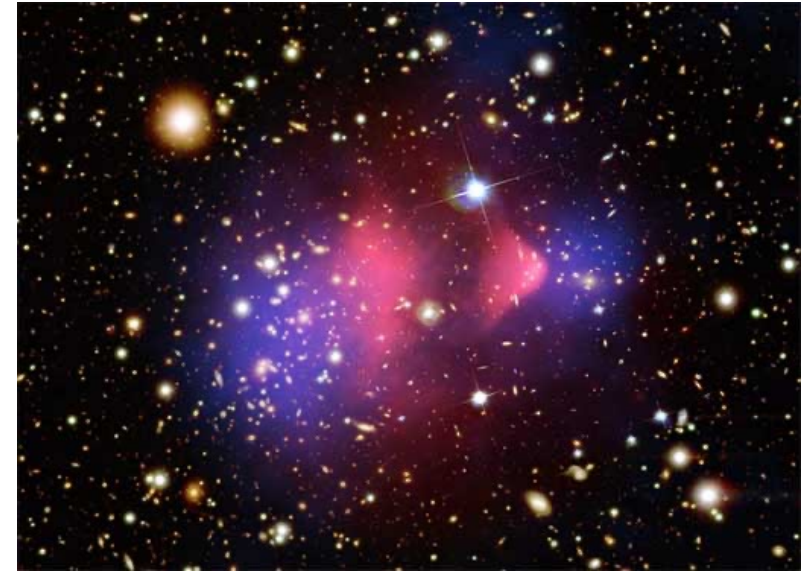
=> Almost MOND but oscillations of the potential at large radii (problem ?)

Relativistic MOND

MOND is rather successful at predicting the dynamics of **galaxies**, especially rotationally-supported ones: the question is **why** does it make successful predictions?

Main problems with the modified gravity approach:

- Clusters! MOND **does not work** (without unseen baryons). But see what happens with relativistic MOND? Bullet cluster always complicated to get...
- CMB ok, but at the price of oscillations around galaxies?
- Modified gravity MOND fails in Milky Way disk
- Also fails in the Solar System (quadrupole well constrained at Saturn by Cassini, needs screening, that should also screen any effect in wide binaries)



Possible generalizations with more dimensioned constants

$$\mathcal{L}_{\text{EMOND}} = \Lambda(\Phi) \mathcal{F}((\nabla\Phi)^2 / \Lambda(\Phi)),$$

$$\mathcal{L}_{\text{GQUMOND}} = 2\nabla\Phi \cdot \nabla\Phi_N - a_0^2 \mathcal{P}(\Phi_N, \nabla\Phi_N, \nabla^2\Phi_N, \dots, \nabla^n\Phi_N)$$

Fun to try out and see what we get...

After all, it's Thursday and we haven't detected dark matter particles yet

