Numerical Cosmology

Lecture 4

Subgrid Models for Galaxy Formation

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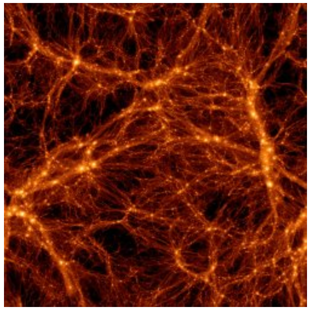
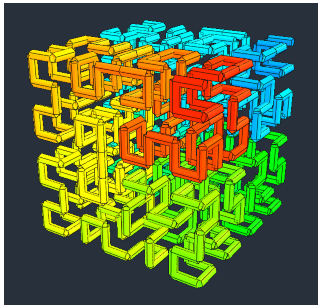
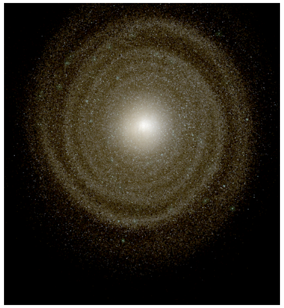
  

Subgrid Scales and Mean Flow

Current galaxy formation simulations reach a spatial resolution ranging from 1kpc to 10pc. Star forming cores are typically small than 0.1pc. We need to model scales $\ell < \Delta x$.

We introduce a smoothing scale Δx and describe unresolved density, velocity and pressure fluctuations below this scale as perturbations: $\rho = \overline{\rho} + \rho'$ and $P = \overline{P} + P'$ where $\overline{\rho}$ and $\overline{P}$ are volume-averaged at scale Δx.

For the velocity we use the Favre average: $\overline{\mathbf{u}} = \dfrac{\overline{\rho \mathbf{u}}}{\overline{\rho}}$ and $\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$.

Unresolved scales are called *mesoscopic* scales with enough scale separation:

$$\lambda_{\rm coll} \ll \ell < \Delta x < H_{\rm P}$$

$\overline{\rho}$, $\overline{\mathbf{u}}$ and $\overline{P}$ represents the mean flow (resolved by the simulation).

Mean Field Equations

We start from the mass conservation equation for the original flow:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho \mathbf{u} \right) = 0.$$

Injecting the perturbations and the mean variables $\rho = \overline{\rho} + \rho'$ and $\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \rho'}{\partial t} + \nabla \cdot \left(\rho \overline{\mathbf{u}} \right) + \nabla \cdot \left(\rho \mathbf{u}' \right) = 0.$$

We now average the entire equation to get the mean field mass conservation equation:

$$\frac{\partial \overline{\rho}}{\partial t} + \nabla \cdot \left(\overline{\rho}\, \overline{\mathbf{u}} \right) = 0$$ where we used $\overline{\rho'} = 0$ and $\overline{\rho \mathbf{u}'} = 0$.

Mean Field Equations

We then deal with the momentum conservation equation for the original flow:

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + P\mathbb{I} \right) = 0.$$

Injecting $\rho = \overline{\rho} + \rho'$, $P = \overline{P} + P'$ and $\mathbf{u} = \overline{\mathbf{u}} + \mathbf{u}'$:

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \overline{\mathbf{u}} \right) + \nabla \cdot \left(\rho \overline{\mathbf{u}} \otimes \mathbf{u}' \right) + \nabla \cdot \left(\rho \mathbf{u}' \otimes \mathbf{u}' \right) + \nabla \overline{P} + \nabla P' = 0.$$

We now average the entire equation to get the mean field momentum conservation equation:

$$\frac{\partial}{\partial t}(\overline{\rho}\,\overline{\mathbf{u}}) + \nabla \cdot \left(\overline{\rho}\,\overline{\mathbf{u}} \otimes \overline{\mathbf{u}} \right) + \nabla \overline{P} + \nabla \cdot (\overline{\rho \mathbf{u}' \otimes \mathbf{u}'}) = 0.$$

We introduce the turbulent pressure tensor:

$$\mathbb{P}_T = \overline{\rho \mathbf{u}' \otimes \mathbf{u}'}.$$

We finally get:

$$\frac{\partial}{\partial t}(\overline{\rho}\,\overline{\mathbf{u}}) + \nabla \cdot \left(\overline{\rho}\,\overline{\mathbf{u}} \otimes \overline{\mathbf{u}} + \overline{P}\mathbb{I} + \mathbb{P}_T \right) = 0$$

Turbulent Kinetic Energy Equation

We define the turbulent kinetic energy as $K_T = \overline{\frac{1}{2}\rho u'^2} = \frac{3}{2}\overline{\rho}\sigma_T^2$. Using the same methodology, we can derive the turbulent kinetic energy equation:

$$\frac{\partial}{\partial t}\left(K_T\right) + \nabla \cdot \left(K_T \overline{\mathbf{u}}\right) + \mathbb{P}_T : \mathbb{G} + \nabla \cdot \mathbf{Q}_T = -\,\epsilon_T.$$

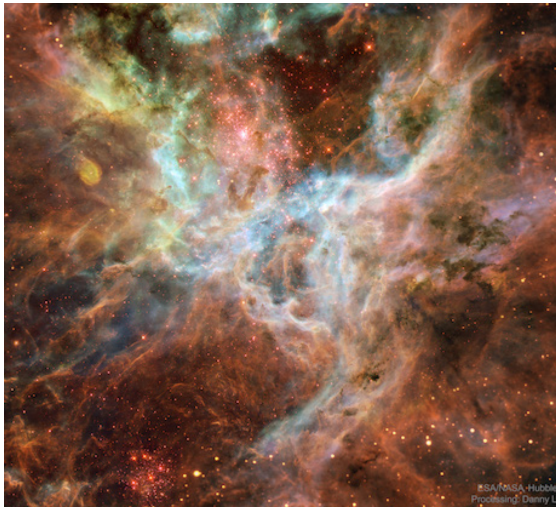
Here $\mathbb{G}$ is the mean-flow velocity gradient tensor, $\mathbf{Q}_T = \overline{\frac{1}{2}\rho u'^2 \mathbf{u}'}$ is the turbulent heat flux and ϵ_T is the turbulent energy dissipation rate.

By analogy with non-LTE perturbation theory, we use mixing length theory:

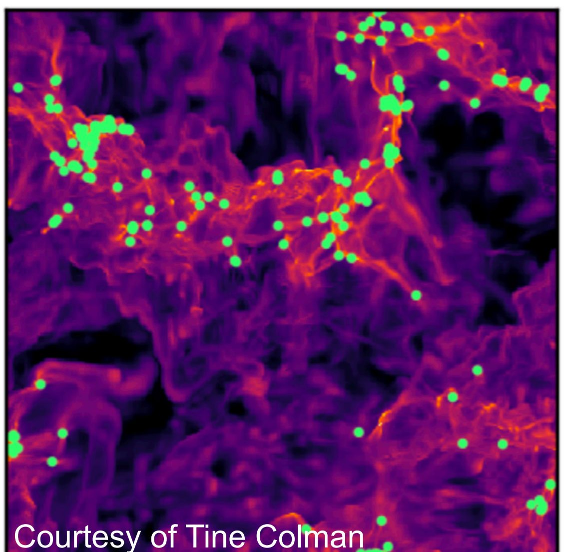
$$\mathbb{P}_T = P_T \mathbb{I} - \overline{\rho}\nu_T \left(\mathbb{G} + \mathbb{G}^T - \frac{2}{3}\left(\nabla \cdot \overline{\mathbf{u}}\right)\mathbb{I}\right) \text{ and } P_T = \overline{\rho}\sigma_T^2,$$

$$\mathbf{Q}_T = -\,\overline{\rho}\nu_T \nabla \left(\frac{K_T}{\overline{\rho}}\right) \text{ where } \nu_T = \ell\,\sigma_T \text{ and } \epsilon_T = \frac{\sigma_T}{\ell} K_T. \text{ Here } \ell = \Delta x.$$

Challenges in modelling star formation



Star formation in 30 Doradus (70 pc)

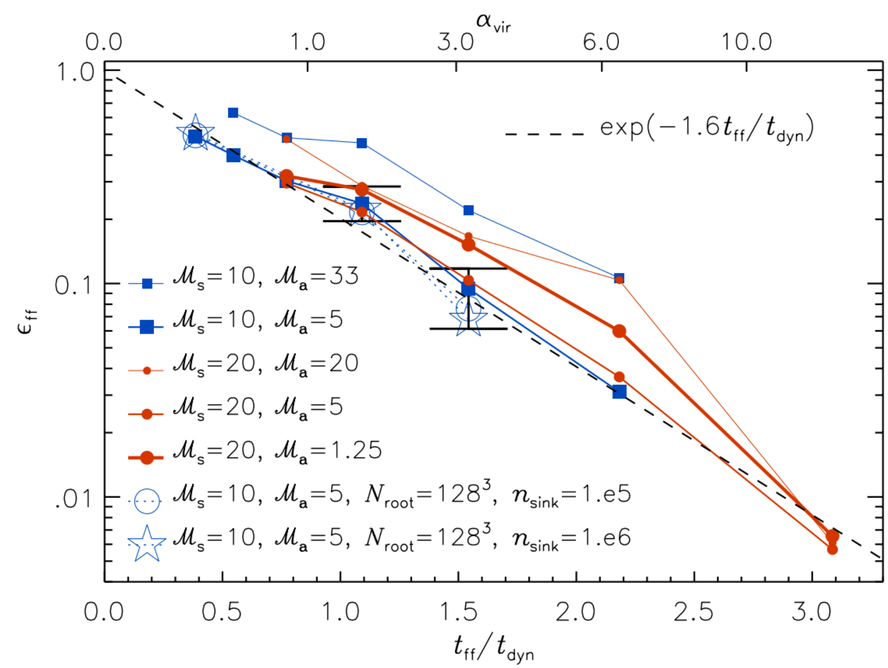


Star formation in 0.25 pc turbulent box

Star formation happens deep inside dense gas clouds with size between 1 and 10 pc.

Modern theories of star formation predict the local star formation efficiency to be a function of the local gas density and the local turbulence Mach number.

A subgrid approach is needed.



Padoan, Haugbolle, Nordlund 2012

Subgrid-scale turbulence and star formation model

Large Eddy Simulation (Boussinesq approximation or eddy viscosity model):

We introduce a new variable: the kinetic energy of subgrid turbulence

$$K_{\rm T} = \frac{1}{2}\rho\sigma_{\rm T}^2 \qquad \frac{\partial K_{\rm T}}{\partial t} + \nabla\cdot(K_{\rm T}\mathbf{u}) = -P_{\rm T}\nabla\cdot\mathbf{u} + \mathbb{R}_{\rm T} : \nabla\mathbf{u} - \frac{K_{\rm T}}{t_{\rm diss}}$$

Source term due to large-scale shear

$$\mathbb{R}_{\rm T} = \rho\nu_{\rm T}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^{\mathbf{T}} - \frac{2}{3}(\nabla\cdot\mathbf{u})\mathbb{I}\right)$$

Dissipation time-scale $t_{\rm diss} = \dfrac{\ell}{\sigma_{\rm T}}$

The mixing length parameter ℓ is set to the grid size Δx.

Turbulent viscosity $\nu_{\rm T} = \ell\sigma_{\rm T}$

Star formation subgrid model: $\dot{\rho}_* = \epsilon_{\rm ff}\dfrac{\rho}{t_{\rm ff}}$

We use the multi-free-fall model of Federrath & Klessen (2012).

The local SF efficiency $\epsilon_{\rm ff}(\alpha_{\rm vir}, \mathcal{M}_{\rm T})$ depends on

- the local Virial parameter $\alpha_{\rm vir} = \dfrac{\sigma_{\rm T}^2}{G\rho\ell^2}$
- the local sonic Mach number $\mathcal{M}_{\rm T} = \dfrac{\sigma_{\rm T}}{c_s}$

Gravo-Turbulent Star Formation Model

Details of the model (Hennebelle&Chabrier 2011, Federrath&Klessen 2012)

1- Supersonic turbulence with Burgers scaling $\sigma(\ell) \propto \sigma_T \left(\frac{\ell}{\Delta x}\right)^{1/2}$.

Density fluctuations $s = \ln(\rho/\overline{\rho})$ with PDF $p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s-\overline{s})^2}{2\sigma_s^2}\right)$.

with $\sigma_s^2 = \ln(1 + b^2 \mathcal{M}_s^2)$ and $\mathcal{M}_s = \sigma_T / c_s$.

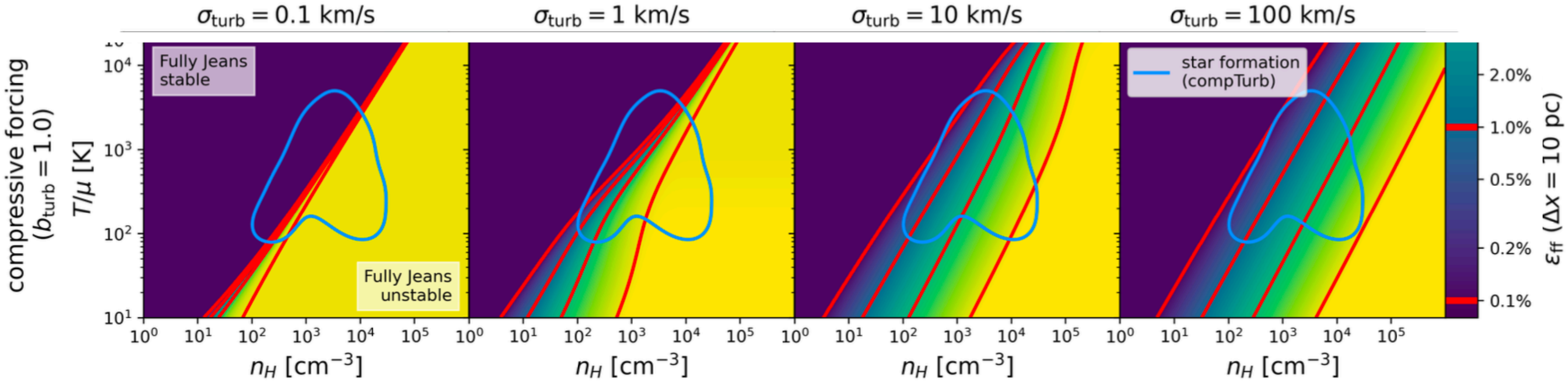
2- Collapse criterion from Krumholz & McKee (2005)

Molecular cores all have the same size equal to the sonic length $\ell_s = \frac{\Delta x}{\mathcal{M}_s^2}$.

They are gravitational unstable if $\rho > \rho_{\rm crit} \simeq \alpha_{\rm vir} \overline{\rho} \mathcal{M}_s^2$.

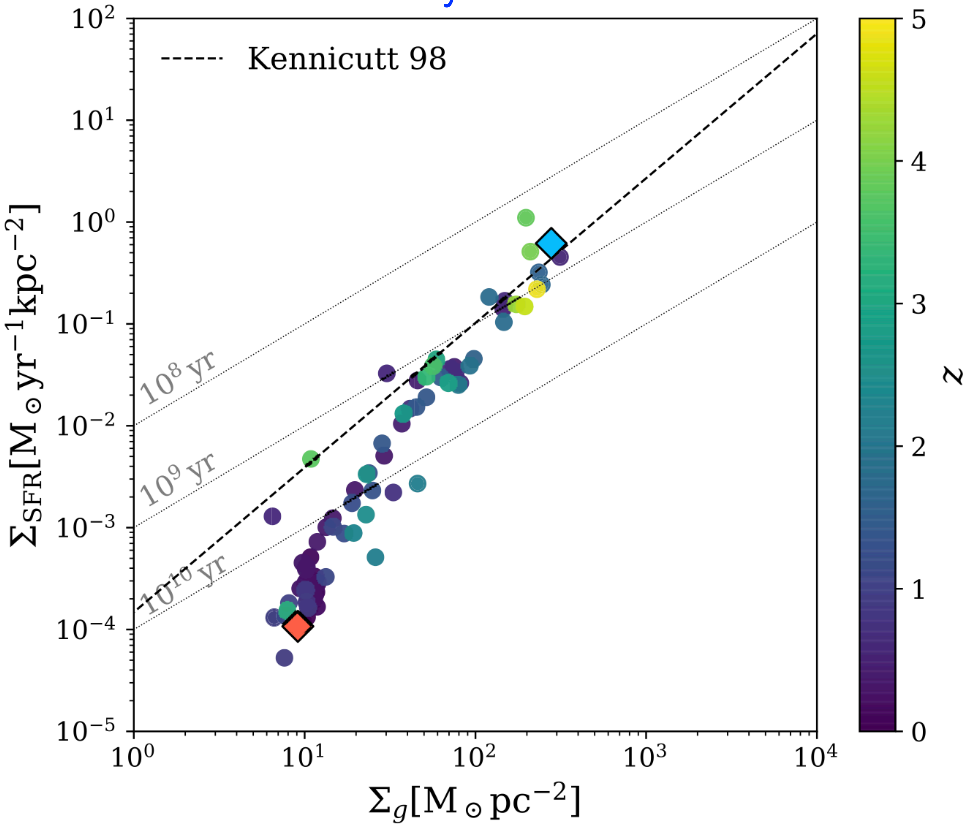
3- Star formation rate: $\dot{\rho}_* = \int_{s_{\rm crit}}^{+\infty} \frac{\rho}{t_{ff}(\rho)} p(s) {\rm d}s = \epsilon_{ff} \frac{\overline{\rho}}{t_{ff}(\overline{\rho})}$.

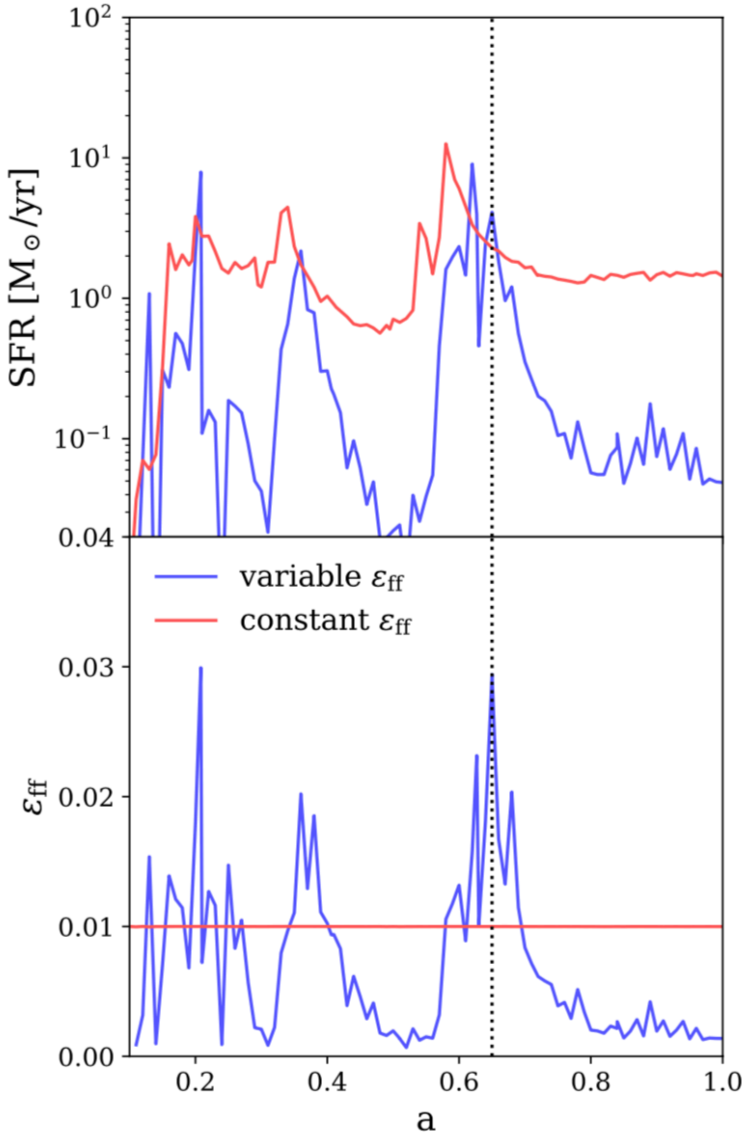
Gravo-Turbulent Star Formation Model



Varying SF efficiency in galaxy formation models

Kretschmer & Teyssier 2020



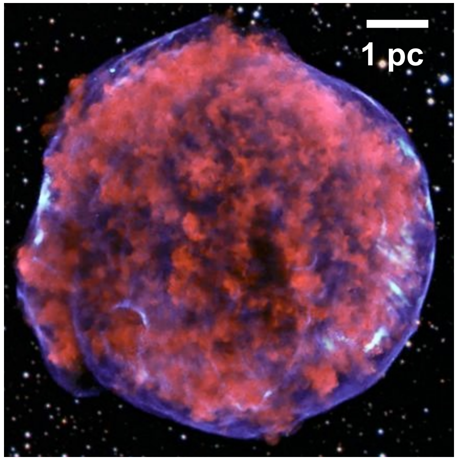


Challenges in modeling supernovae explosions

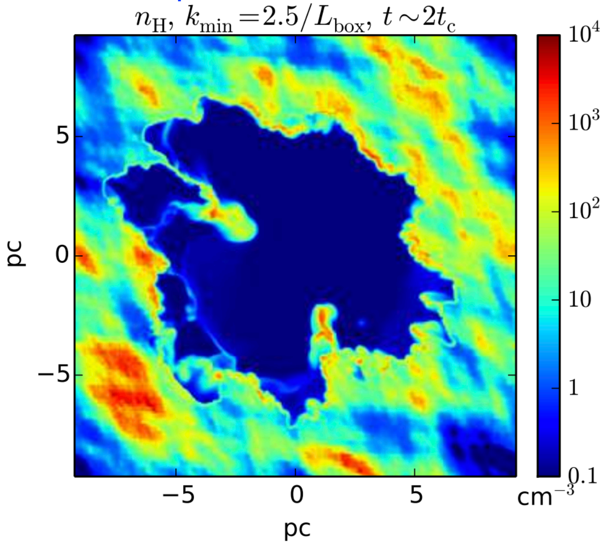
Massive stars deliver 10^{51} erg of energy in a turbulent/inhomogeneous environment.

First phase of the explosion is energy-conserving, dominated by thermal energy and progressively build-up momentum. Second phase is dominated by cooling. It is momentum-conserving and provides kinetic energy to the interstellar medium.

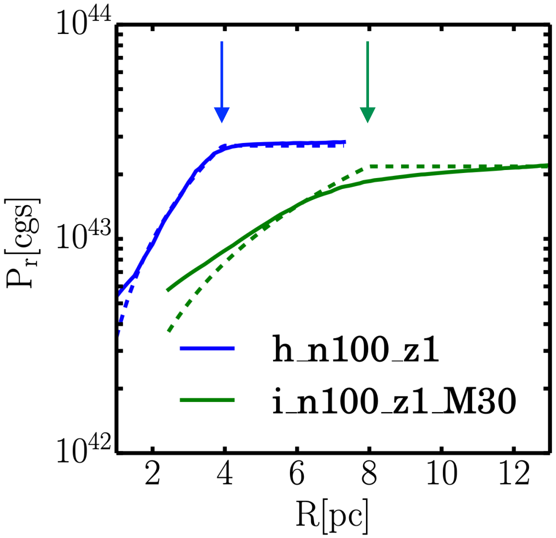
Chandra image of Tycho



SN explosion in a turbulent box



Position of the cooling radius



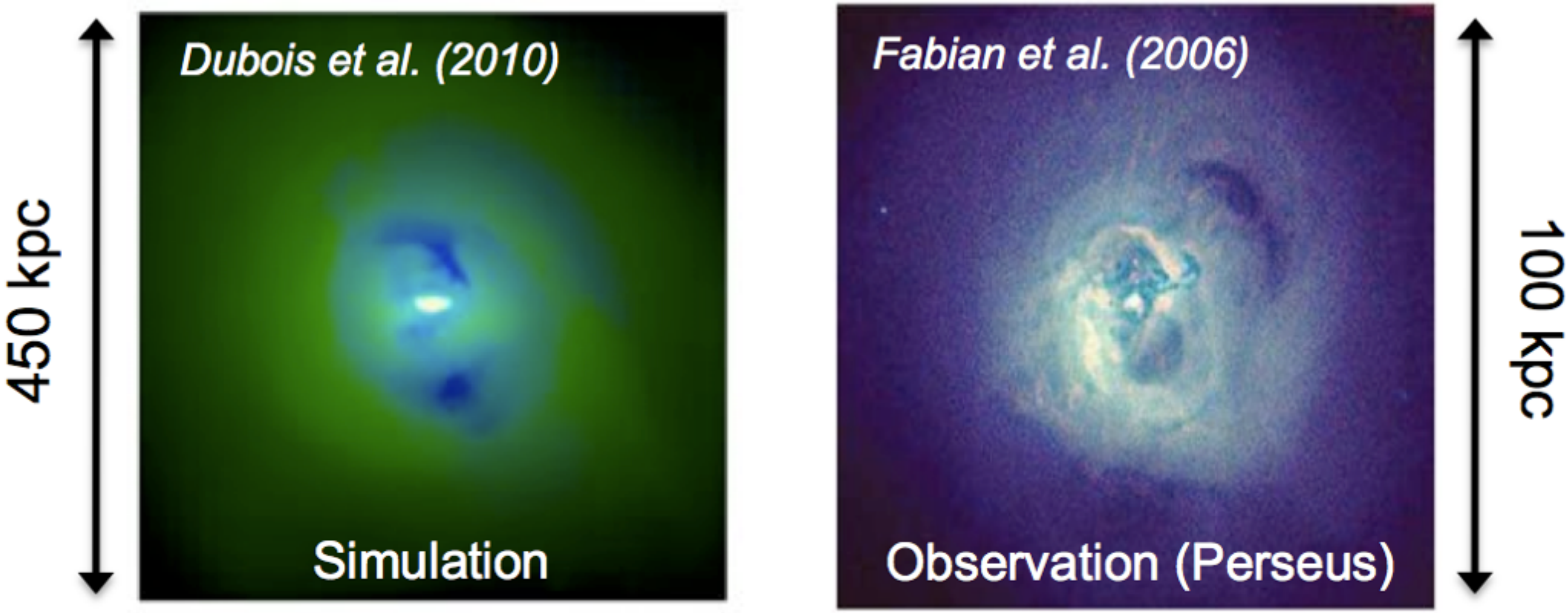
Subgrid model for supernova feedback: do we resolve the cooling radius?

Yes: inject thermal energy, No: inject momentum

Forming a galaxy in a computer (second try…)

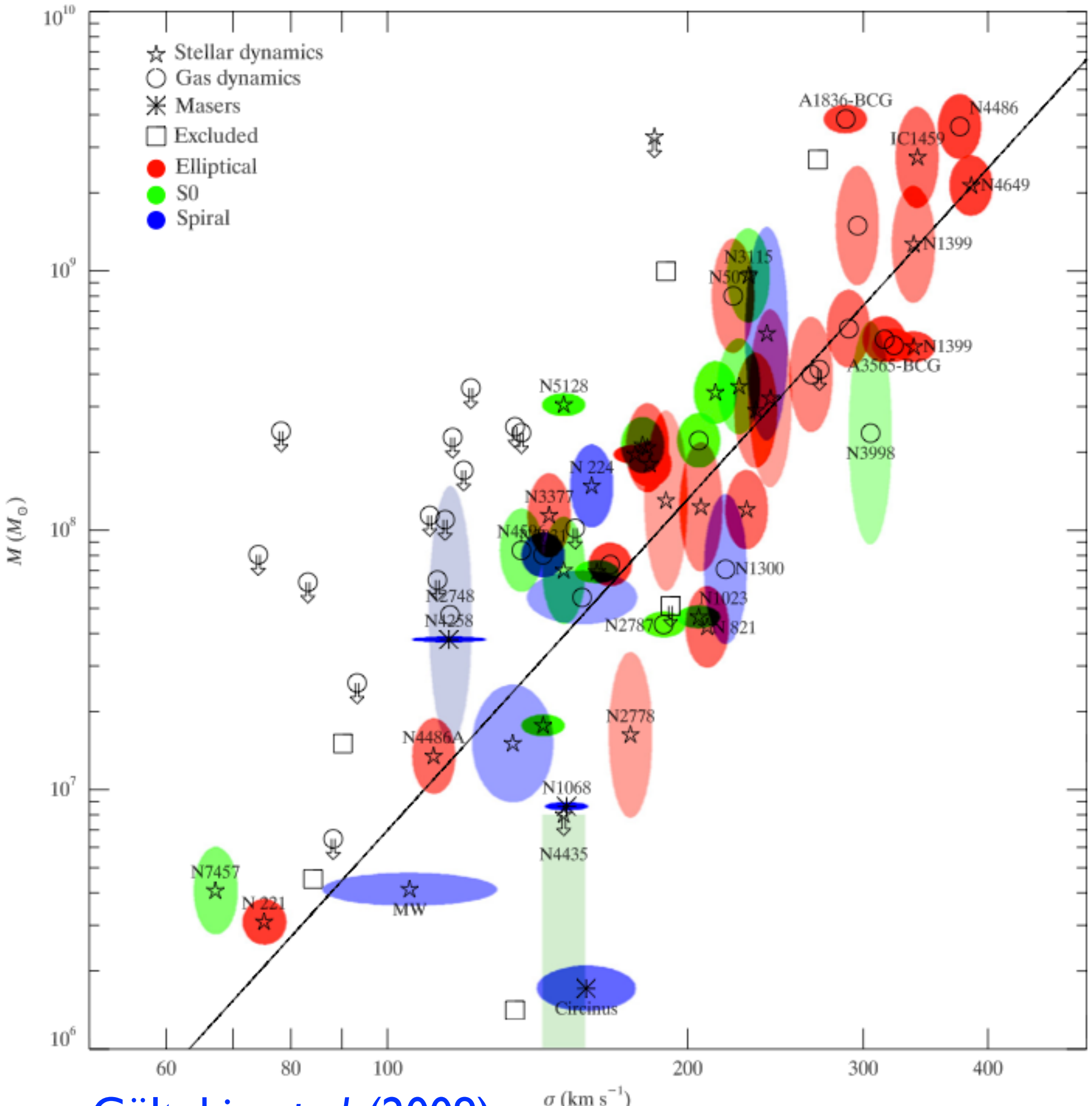
Simulation Michael Kretschmer

Feedback in massive galaxies: supermassive black holes



X-ray (3 bands)

Feedback in massive galaxies: supermassive black holes



Gültekin *et al.* (2009)

A simple model for SMBH growth and feedback

Numerical implementation in cosmological simulations: Sijacki et al. 2007; Booth & Schaye 2010 and many others. Constantly improving.

In high density regions with stellar 3D velocity dispersion > 100 km/s, we create a seed BH of mass 10^5 M_{sol}.

Accretion is governed by 2 regimes:

Bondi-Hoyle regime $$\dot{M}_{\rm BH} = \alpha_{\rm boost} \frac{4\pi {\rm G}^2 M_{\rm BH}^2 \rho}{(c_{\rm s}^2 + u^2)^{3/2}}$$

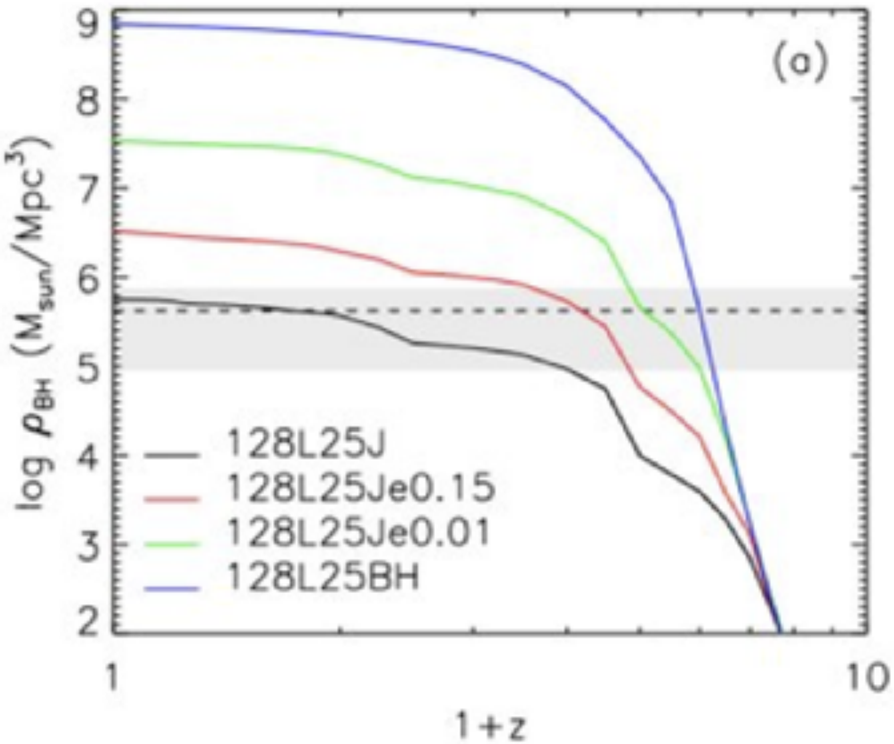
Eddington-limited $$\dot{M}_{\rm ED} = \frac{4\pi {\rm G} M_{\rm BH} m_{\rm p}}{\epsilon_r \sigma_{\rm T} c}$$

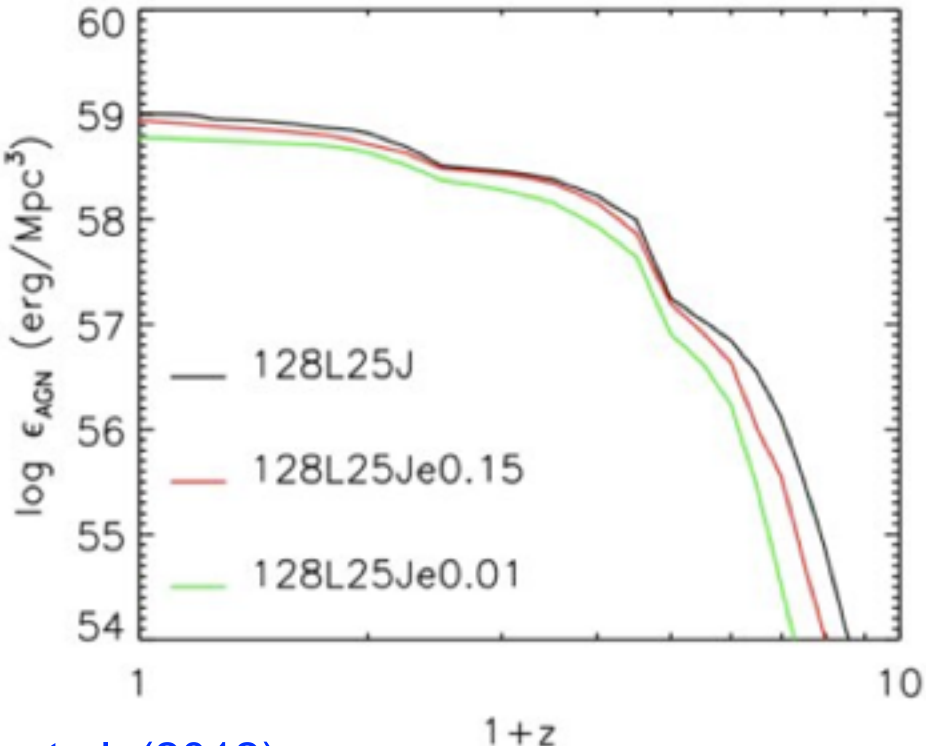
Feedback performed using a thermal dump $$\Delta E = \epsilon_{\rm c} \epsilon_{\rm r} \dot{M}_{\rm acc} c^2 \Delta t.$$

Free parameter epsilon_c and alpha_boost calibrated on the M_BH-sigma and M_star-M_halo relations.

AGN feedback: calibrating the coupling efficiency

$$\Delta E = \epsilon_{\rm c}\epsilon_{\rm r}\dot{M}_{\rm acc}c^2\Delta t.$$

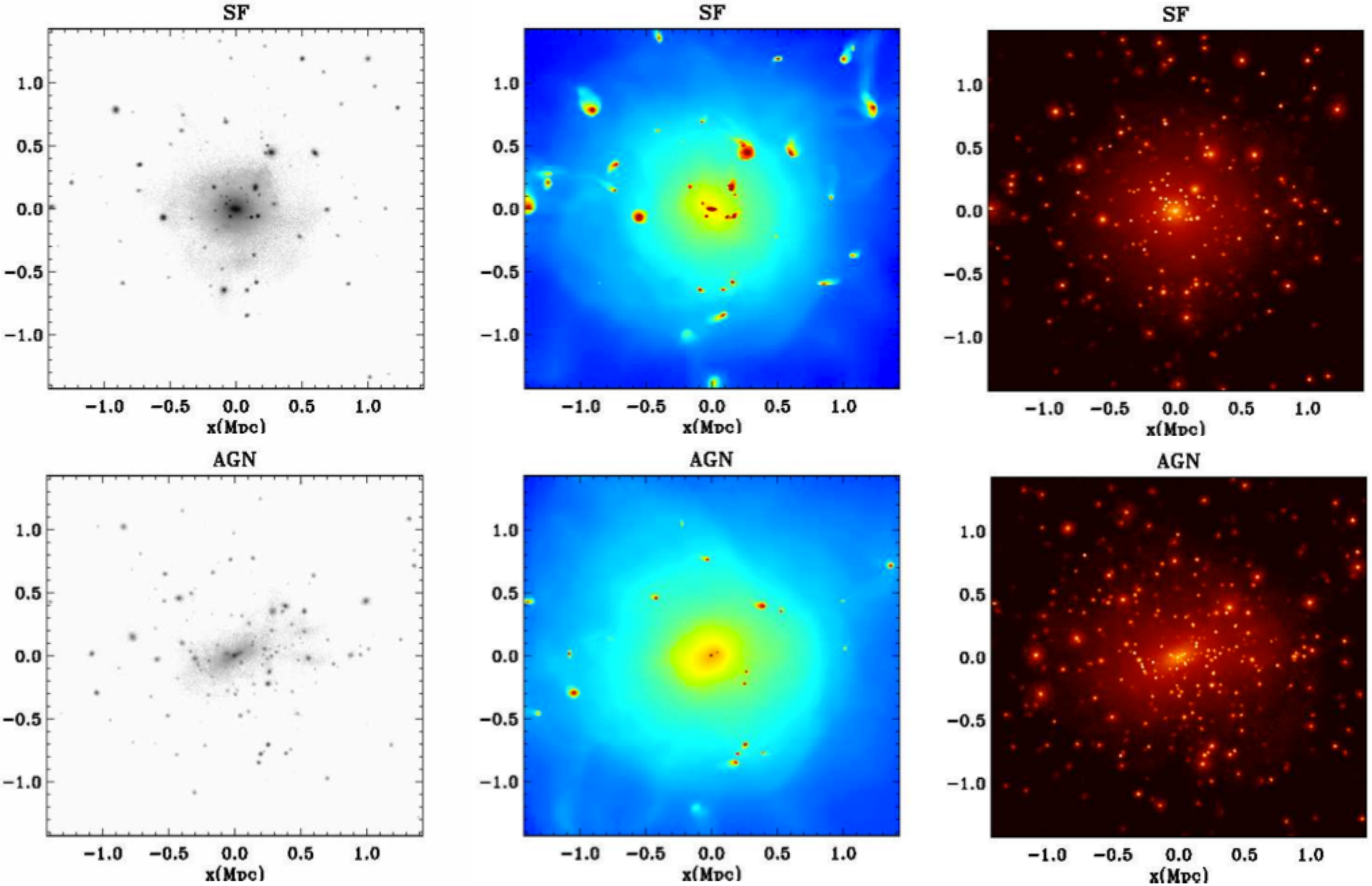




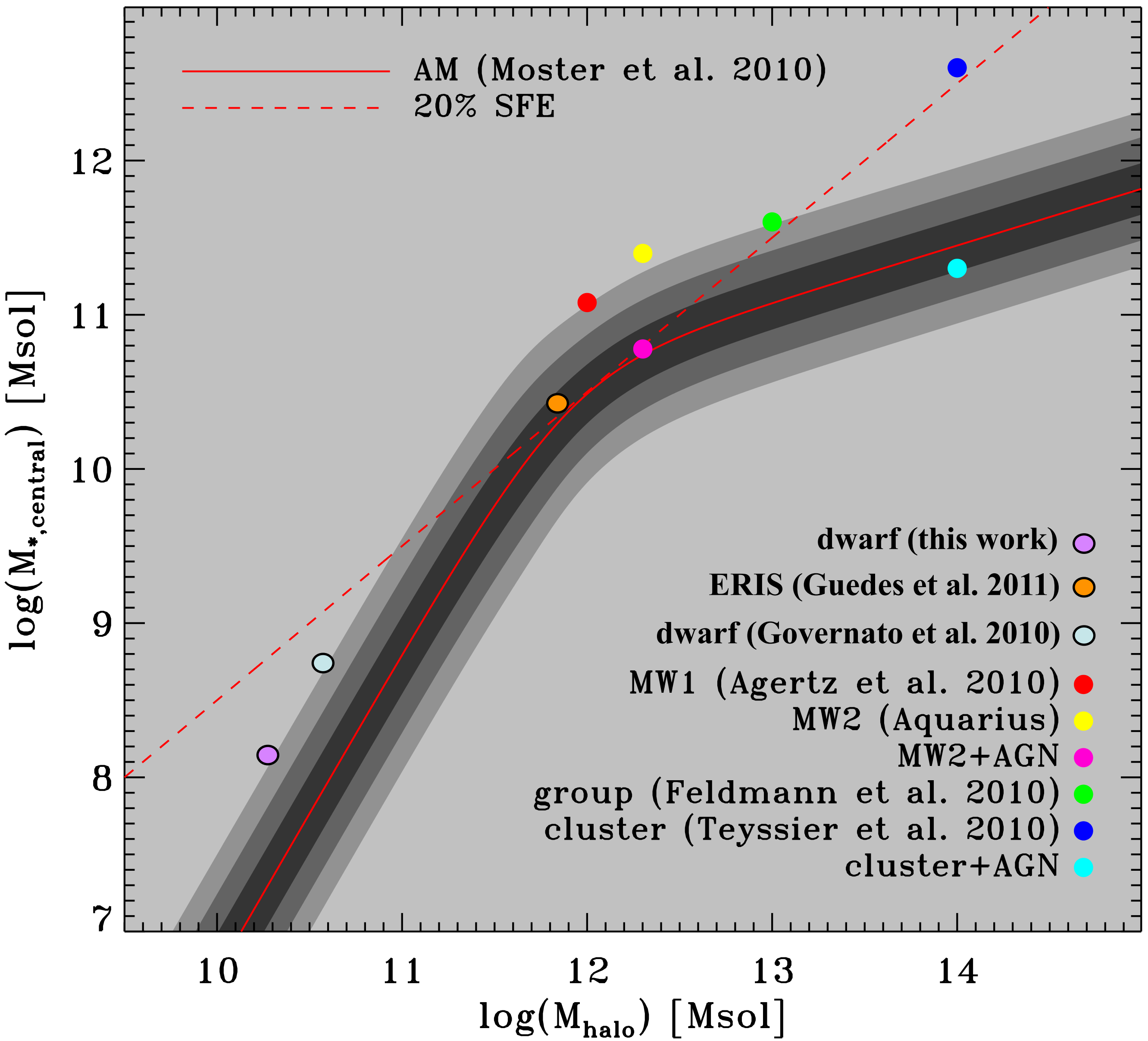
Dubois at al. (2012)

BHs deposit the same energy / independant of the AGN efficiency

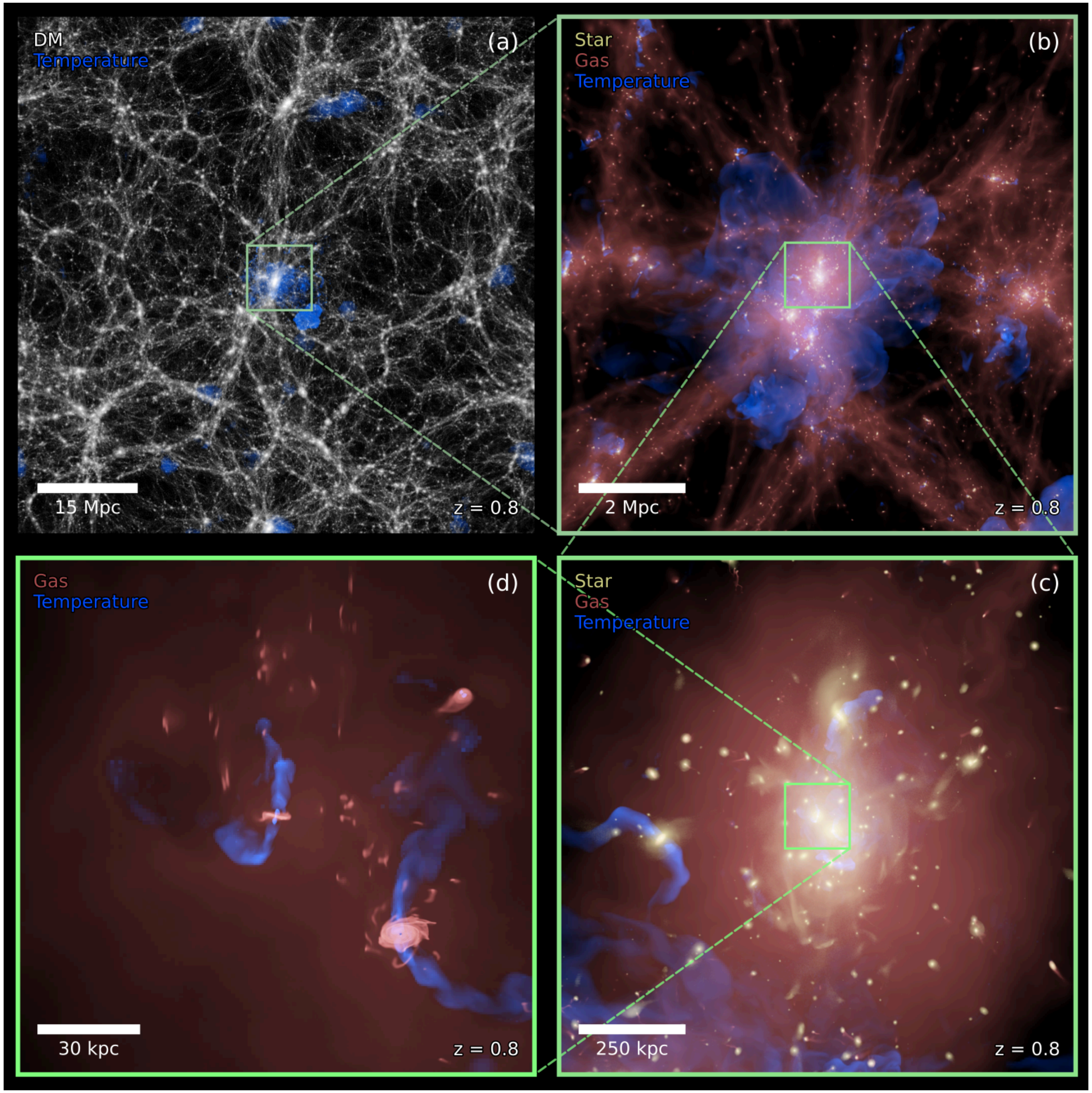
Galaxy formation on cluster scales



Constraints from abundance matching



New Cluster Simulation https://arxiv.org/abs/2507.06301



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