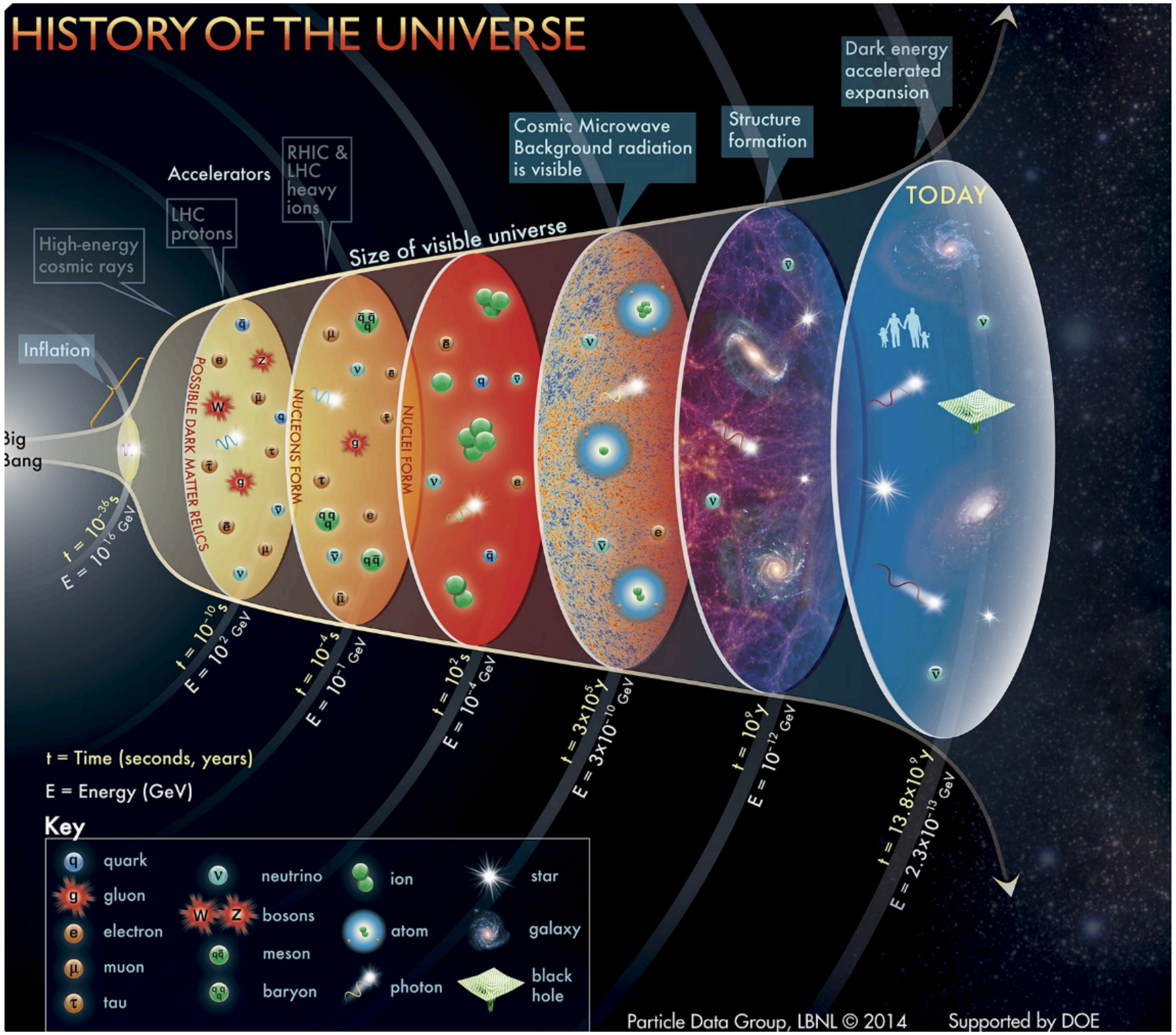
GWs for cosmology



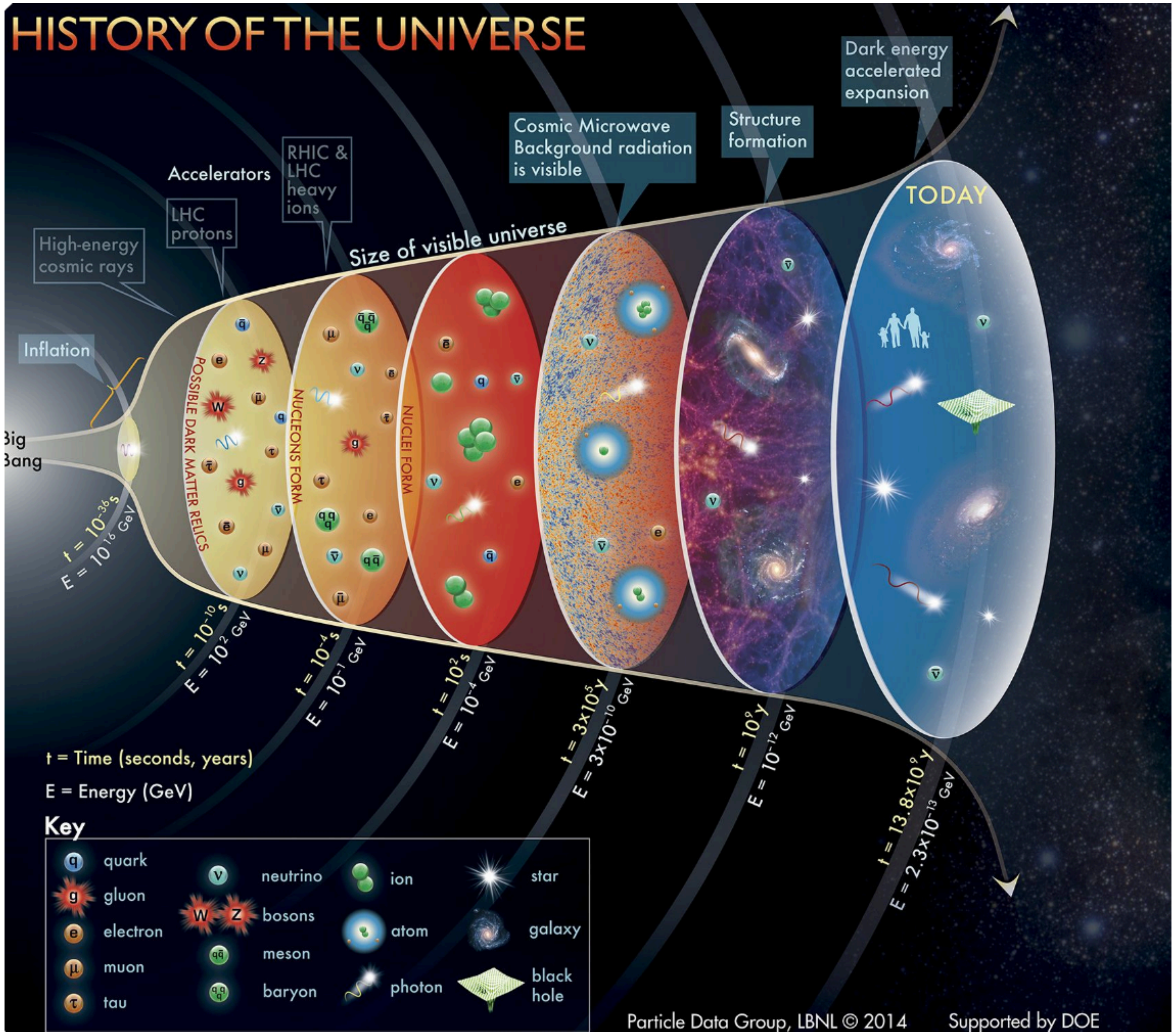
Danièle Steer

LPENS
LABORATOIRE DE PHYSIQUE
DE L'ÉCOLE NORMALE SUPÉRIEURE

Université
Paris Cité

VIRGO

GWs for cosmology



Particle Data Group, LBNL © 2014 Supported by DOE

Aims:

- how can probe cosmology and the expansion history of the universe at early and late times with GWs
- in practice how one tries to do it
- current status of results
- prospects

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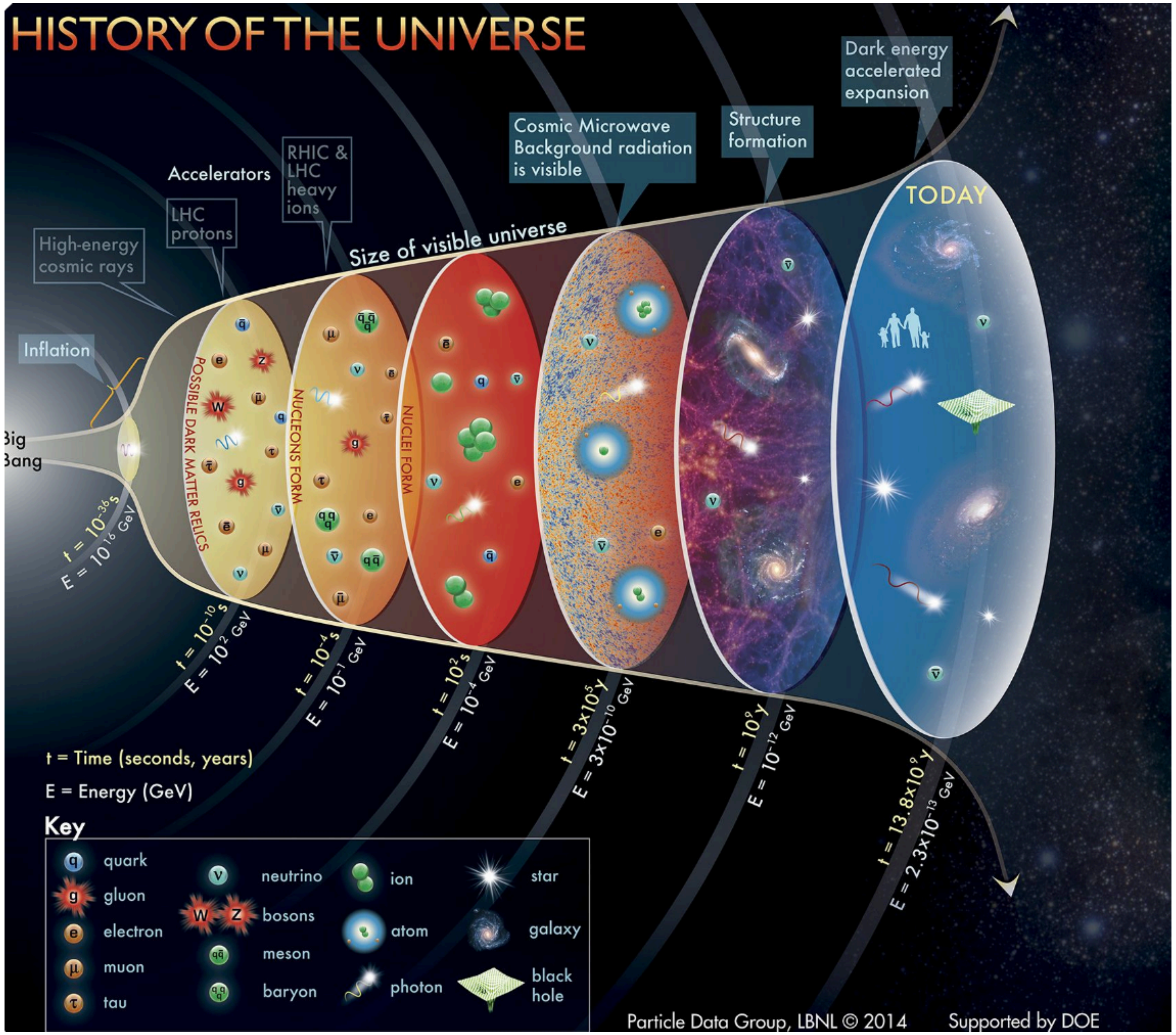
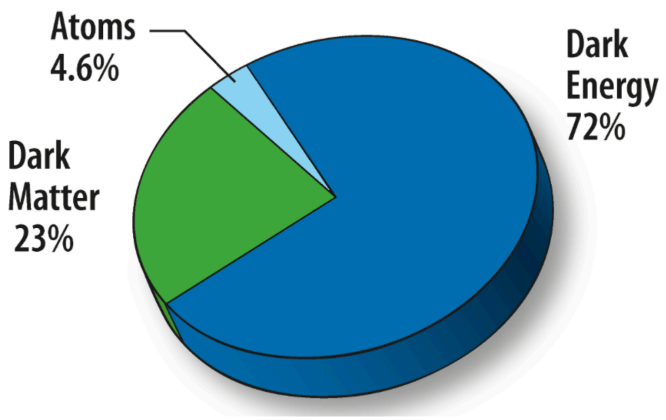
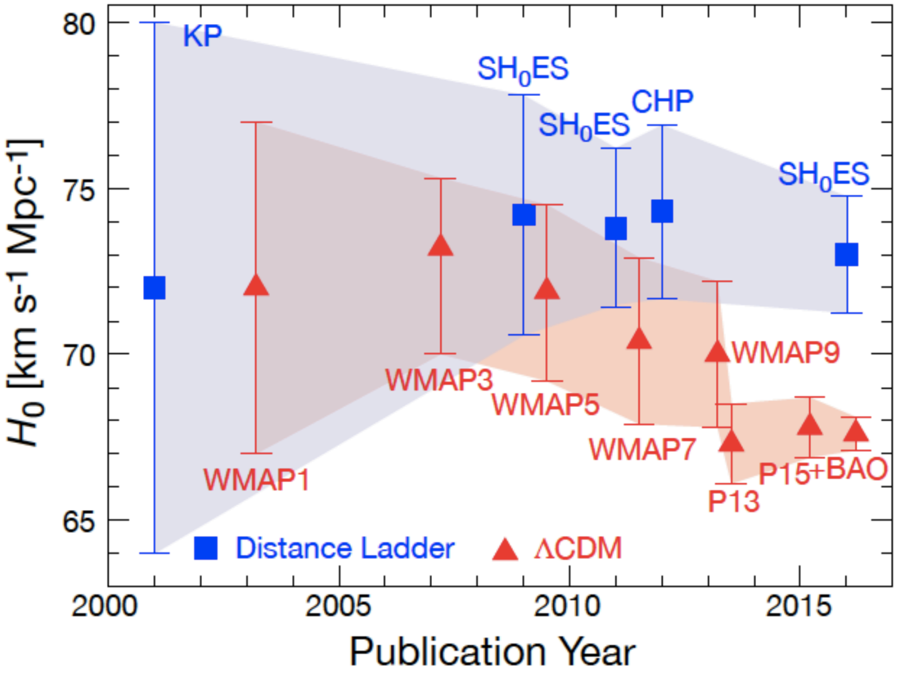




Mean expansion rate of universe today:
Hubble constant H_0 tension



$4 - 6\sigma$ tension between *early* (assuming cosmological model),
and *late-time* (local) measurements

- Pedro’s lectures with EM sources:

Redshifts $z \Rightarrow$ easy!
Distances $d_L(z) \Rightarrow$ hard! ⟶ *Distance ladder*: parallax, cepheids, SN…
With many difficulties

- GW sources:

Redshifts $z \Rightarrow$ hard!
Distances $d_L(z) \Rightarrow$ ~~easy!~~ ⟶ *No need for a distance ladder*: $d_L(z)$ comes directly from the observed signal
more straightforward

- Pedro’s lectures focused on scalar modes:

Linear and non-linear perturbation theory: breaks down quickly

- GWs: transverse and traceless tensor modes

Linear perturbation theory fine

- Lecture 1: – **Overview** on early- and late-time cosmology with GWs; current and future experiments,
  – **orders of magnitude**

- Lecture 2: – Late-time cosmology: GWs and $d_L(z)$
  – GWs in theories beyond GR, $d_L^{GW}(z)$
  – **standard sirens I:** Measuring H_0 with GWs and O3 results of LVK
  – Back to early-time universe: an example of what physics we can probe.

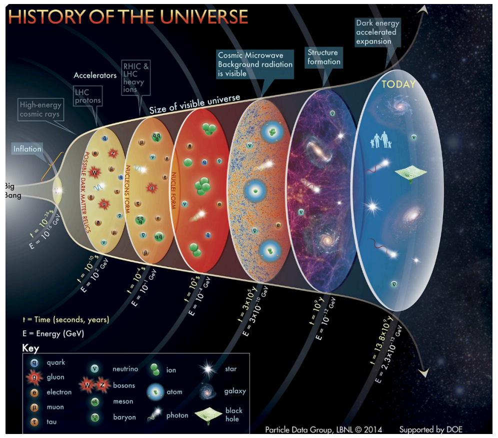
- Lecture 3 (Chiara Caprini):
  – *cosmological* stochastic GW background: **early-universe cosmology with GWs**
  Solutions of the GW propagation equation in FLRW; its calculation for different sources (inflation, topological defects, first order phase transitions)

- Lecture 4 (Nicola Tamanini):
  – **Standard sirens II**: more details, statistical methods, future prospects

- Lecture 5 (Tania Regimbau):
  – ***astrophysical* stochastic GW background**: Definition/statistical properties, pulsar timing arrays and background from supermassive BH binaries, LVK results, prospects for the future.

Overview

Gravitational waves for cosmology

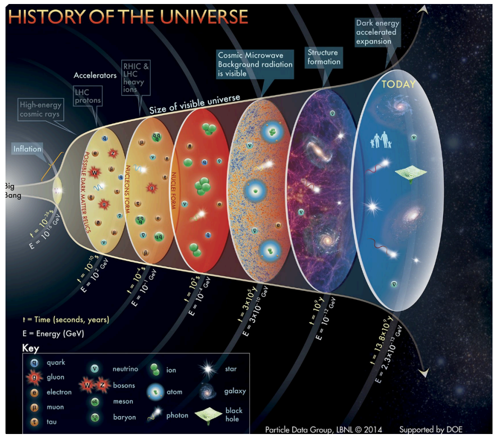


late-time universe

Individual resolvable
astrophysical sources
and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star-black-hole binary (NS-BH)
Rotating asymmetric neutron stars
supernova explosions…

– Expansion rate $H(z)$
– Hubble constant H_0
– Ω_m
– beyond ΛCDM, dark energy $w(z)$
– late-time modified gravity (modified GW propagation)
– astrophysics; eg populations of BBHs
….

Gravitational waves for cosmology



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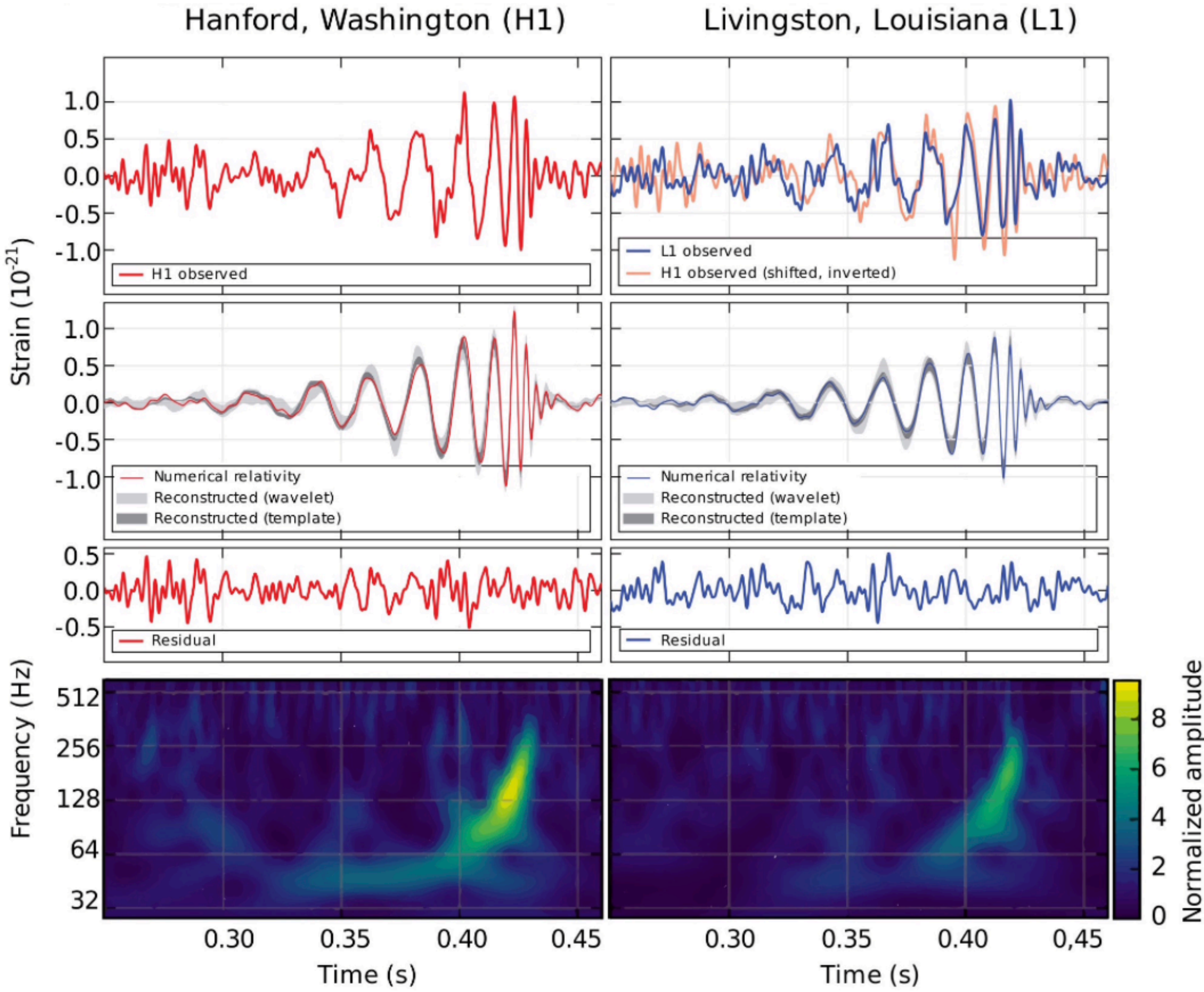
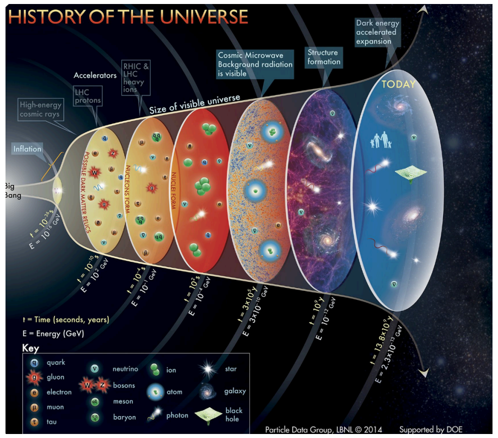
GW150914



Transient deterministic signal

Gravitational waves for cosmology



late-time universe

↕

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….

Very early universe until today

↕ $t \gtrsim t_{Pl}$

Stochastic GW background
astrophysical and cosmological origin

↕

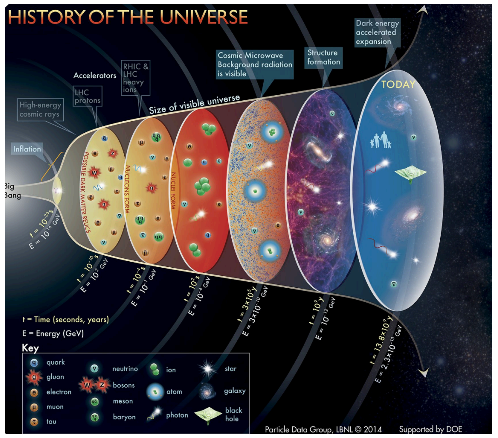
$$\Omega_{\rm gw}(t_0, f) = \frac{f}{\rho_c}\frac{d\rho_{\rm gw}}{df}(t_0, f)$$

↓

- population of BH, white dwarfs..
- inflationary GWs
- 1st order Phase transitions
- topological defects
- scalar induced GWs
- primordial black holes
- axions
- early modified gravity…

More speculative. Early universe sources beyond standard model of particle physics!

Gravitational waves for cosmology



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Primordial cosmology

Individual resolvable cosmological sources
e.g. cosmic string GW bursts

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- – early modified gravity…

More speculative. Early universe sources beyond standard model of particle physics!

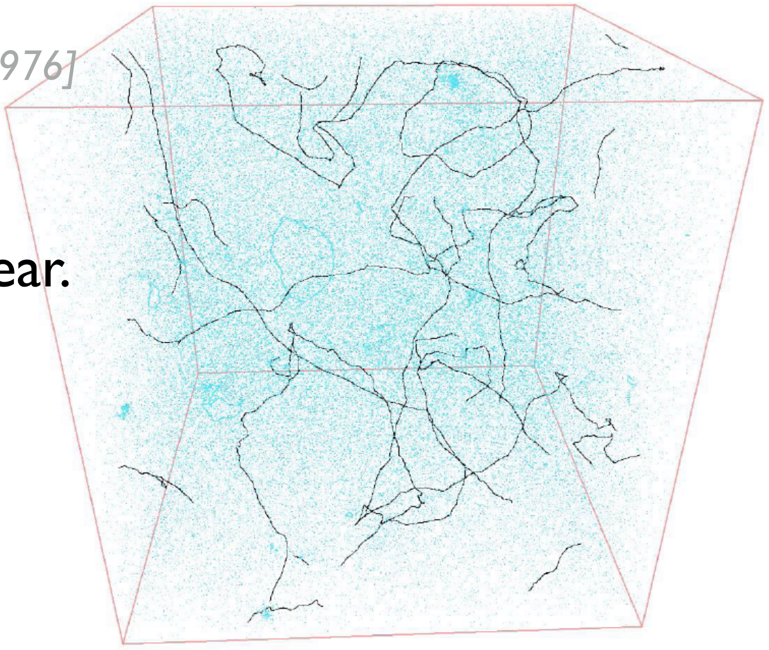
An example: Cosmic Strings

[T.Kibble 1976]

– Line-like topological defects, may be formed in a symmetry breaking phase transition, time t_i, temperature T_i. Stable, once formed cannot disappear.

– only one parameter describing physics of strings: their tension

$$G\mu \sim 10^{-6}\left(\frac{T_i}{10^{16}\ \text{GeV}}\right)^2$$

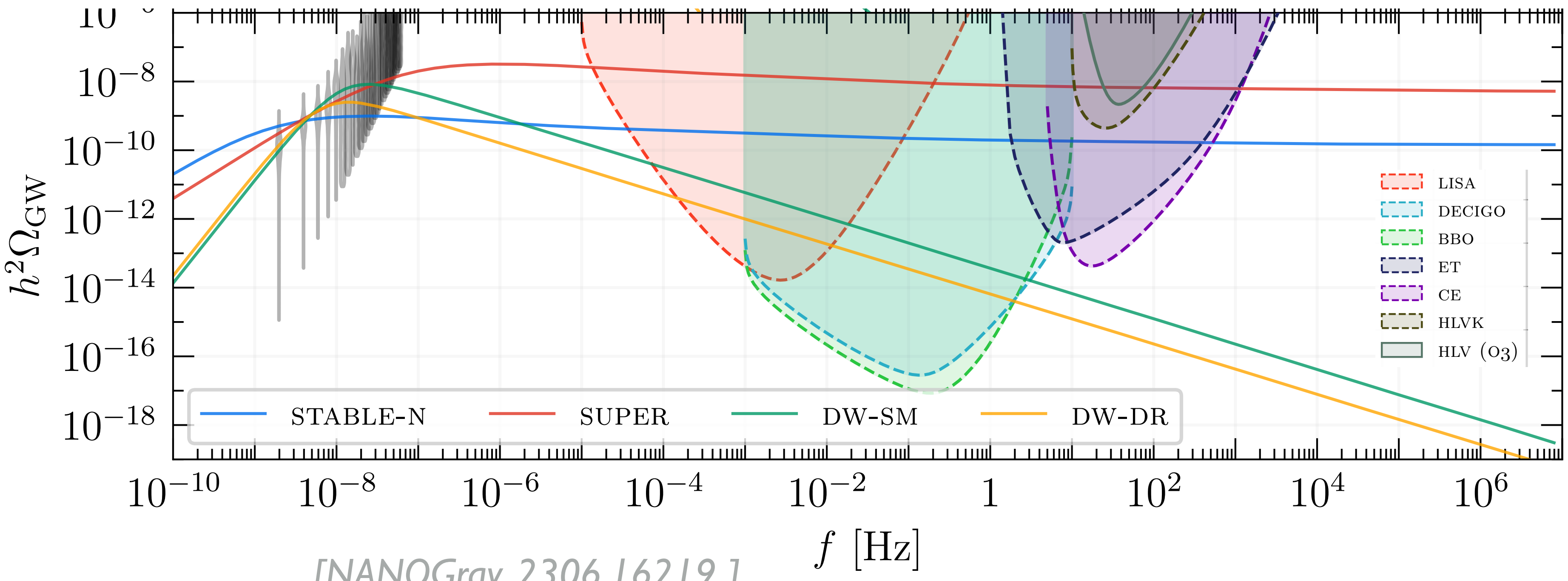


[C.Ringeval]

– loops are created for all times $t > t_i$, oscillate relativistically and emit GWs:

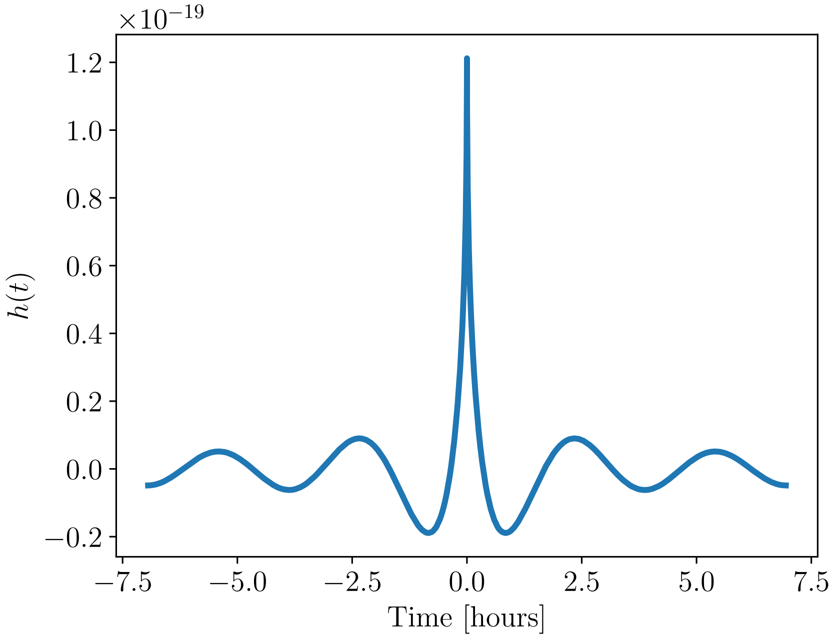
- individual loop, close by, emits a particular *short*, and periodically *repeating*, GW burst signal.
- effect of all loops is to generate a SGWB

Stochastic GW background



[NANOGrav, 2306.16219]

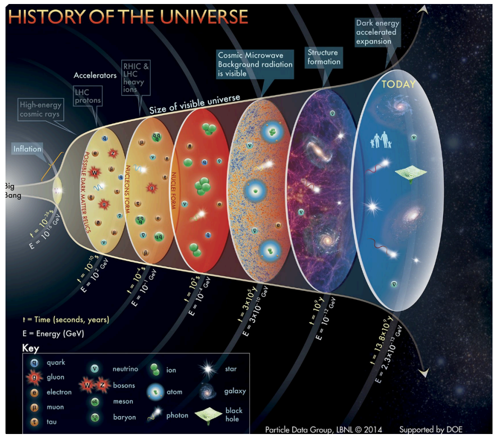
Repeating short burst



[Damour&Vilenkin, Auclair et al]

– Experiments, current and future, can either put constraints on, or measure $G\mu$. PTAs: $G\mu \lesssim 10^{-10}$

Gravitational waves for cosmology: detectors



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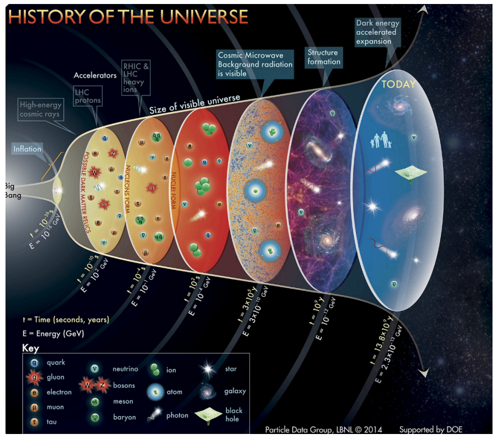
– astrophysics; eg populations of BBHs
….

– flat ΛCDM $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$

– Hubble parameter: $H(t) = \frac{\dot{a}(t)}{a(t)}$

– redshift: $1 + z = \frac{a(t_0)}{a(t)}$

Gravitational waves for cosmology: detectors



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↕

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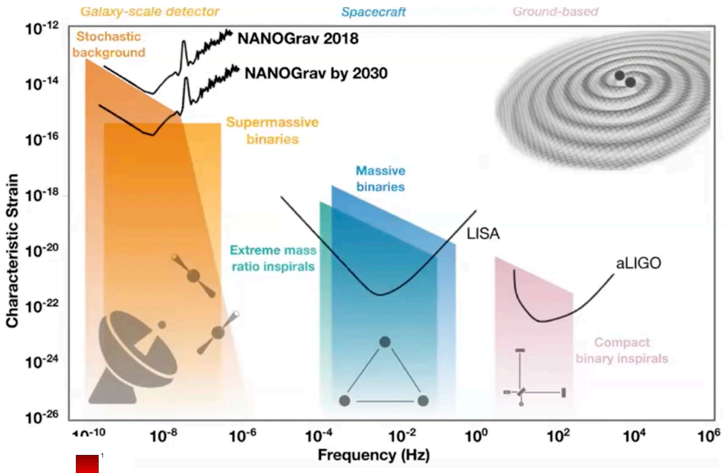
"Compact binary coalescences”
CBCs

Precisely what sources can be used depends on the detector(s): frequency band, noise/ sensitivity,…

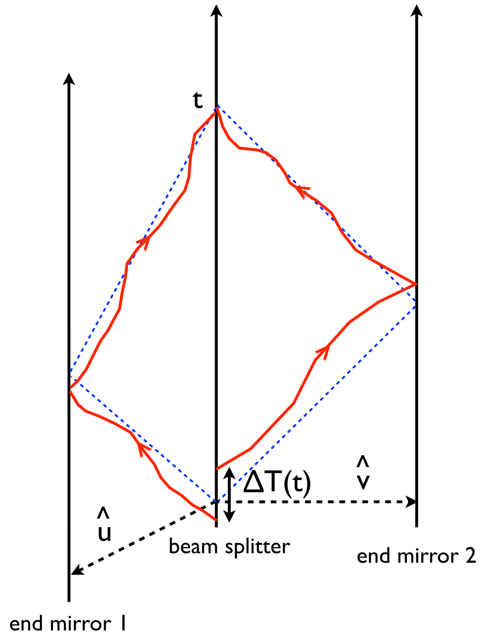
↓

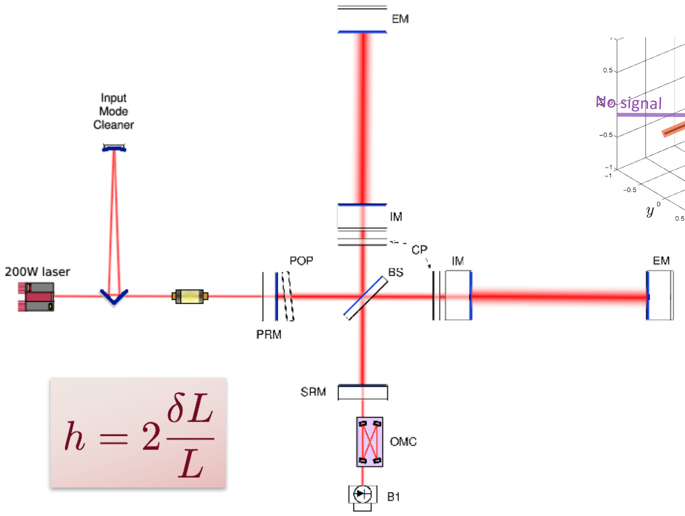
– Expansion rate $H(z)$
– Hubble constant H_0
– Ω_m
– beyond ΛCDM, dark energy $w(z)$
– modified gravity (modified GW propagation)
– astrophysics; eg populations of BBHs
….

- GW detectors: designed to be as sensitive as possible to time-varying changes in the separation between two freely-falling objects

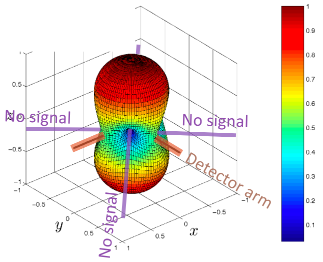


Laser interferometers.

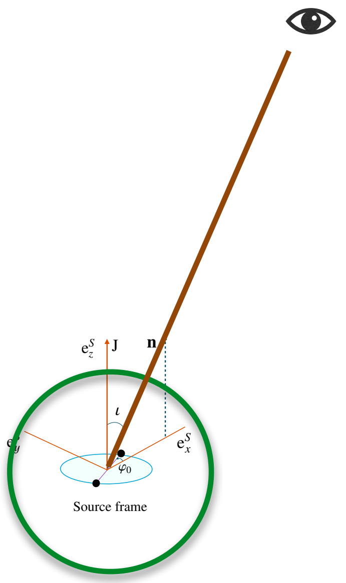


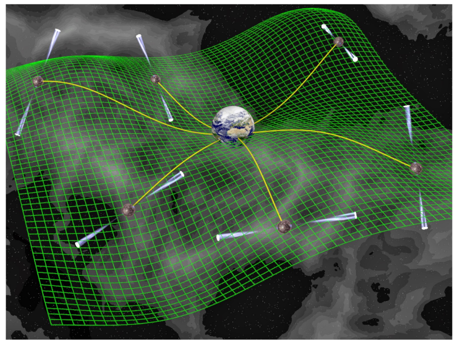


$$h = 2\frac{\delta L}{L}$$



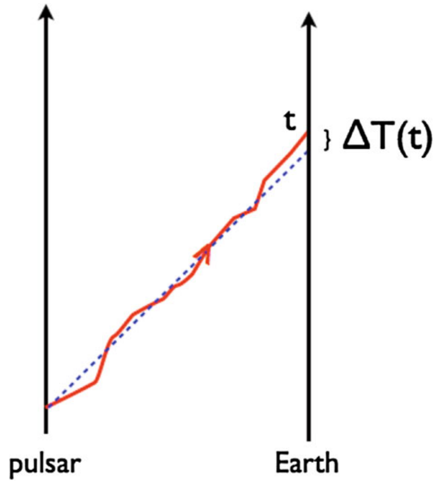
- in both cases, response depends on the orientation of the source wrt to detector





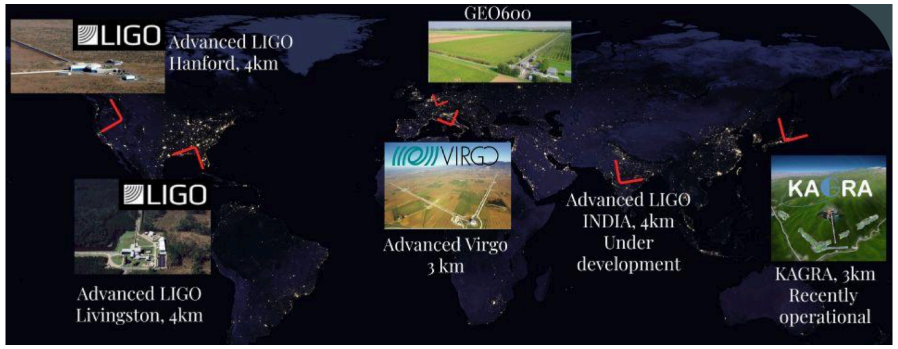
[Credit: D. Champion]

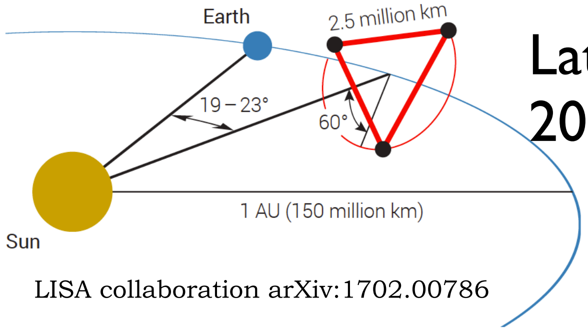
PTA:



$$h_{\text{timing}}(t) = \Delta T(t)$$

Ultra-stable millisecond pulsars used as beacons "clocks sending signals". In reality though messy astrophysical objects. … Measure TOA of pulse, and compare to expected TOA determined from detailed timing model for the pulsar





LISA collaboration arXiv:1702.00786

Late 2030s

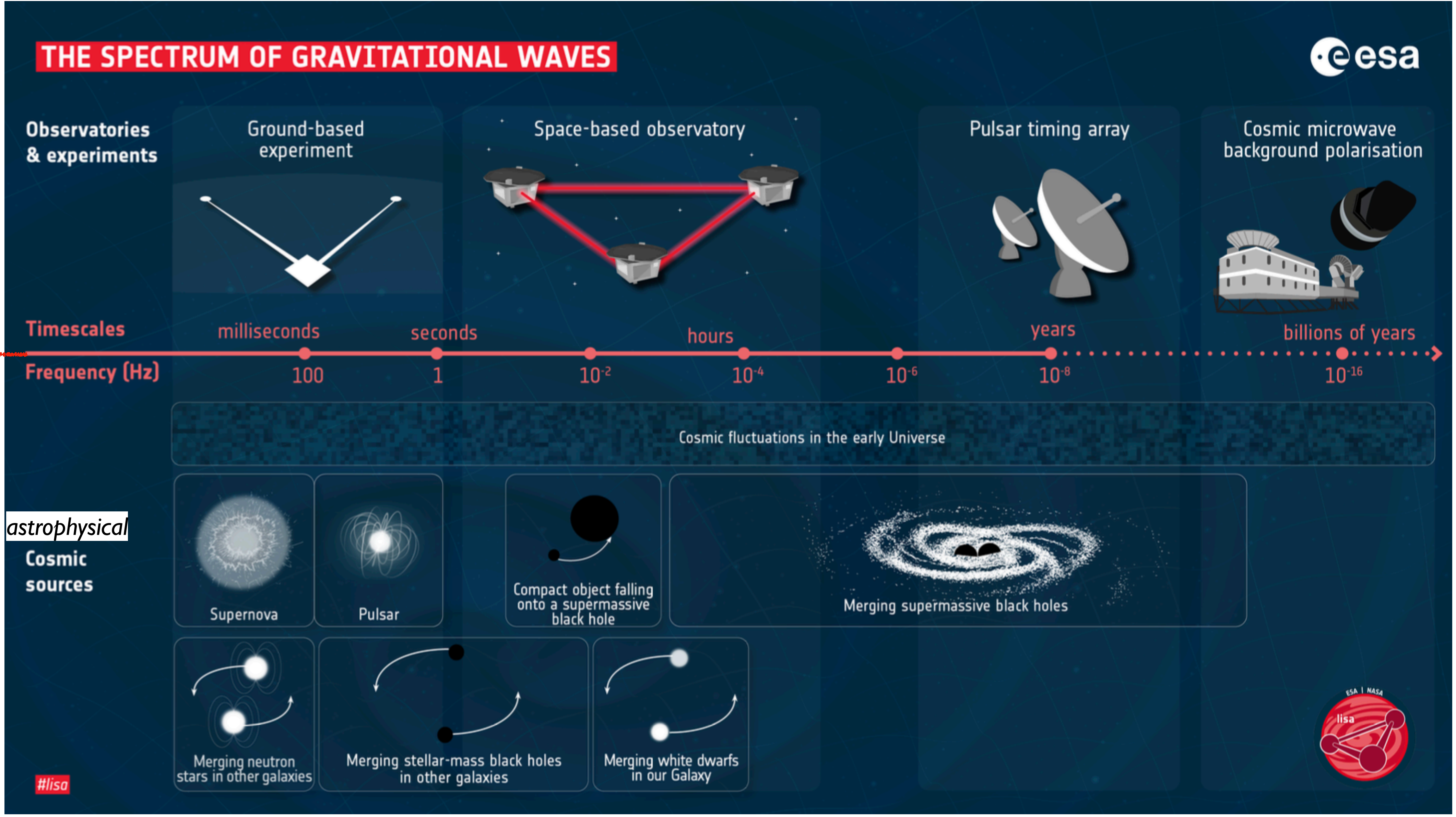
NANOGrav

EPTA

LVK: $10\,\text{Hz} \lesssim f_{\rm GW} \lesssim 5\,\text{kHz}$

LISA: $10^{-4}\,\text{Hz} \lesssim f_{\rm GW} \lesssim 1\,\text{Hz}$

$10^{-7}\text{Hz} \lesssim f_{\rm GW} \lesssim 10^{-9}\text{Hz}$



[LISA collaboration, LISA Definition Study Report, 2402.07571]

GHz

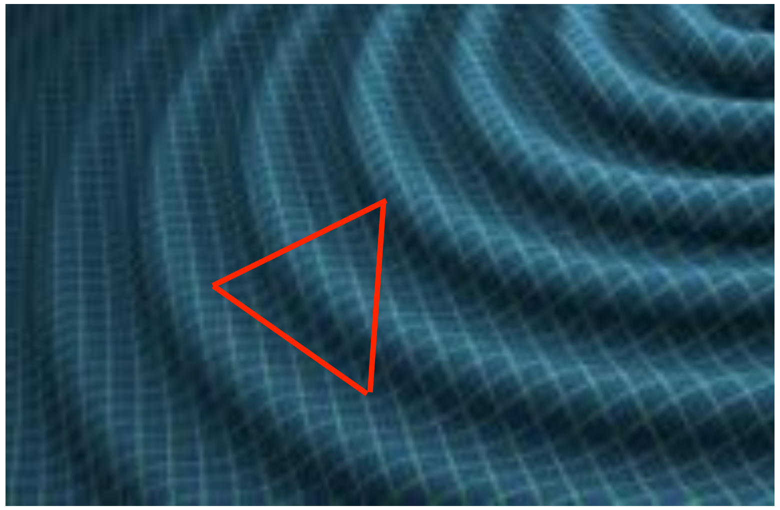
For a discussion of kHz-GHz detectors, see:

[Living Rev.Rel. 24 (2021) 1, 2011.12414]

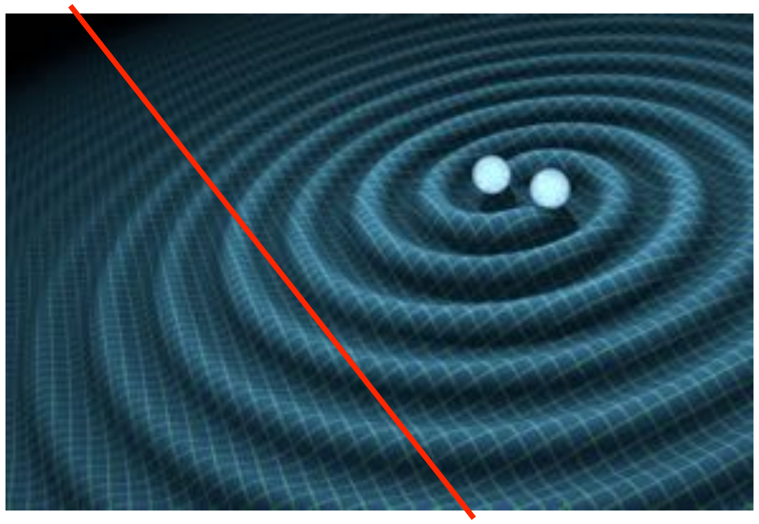
Cosmological sources

	Characteristic detector size (km)	GW frequency to which detector sensitive (Hz)	$f_{\rm GW} L$	L vs $\lambda_{\rm GW}$
LVK	~ 1	$10^1 - 10^4$	$f_{\rm GW} L \ll 1$	$L \ll \lambda_{\rm GW}$
LISA	$\sim 10^6$	$10^{-4} - 10^{-1}$	$f_{\rm GW} L \sim 1$	$L \sim \lambda_{\rm GW}$
PTA	$\sim 10^{17}$	$10^{-9} - 10^{-7}$	$f_{\rm GW} L \gg 1$	$L \gg \lambda_{\rm GW}$

LVK



LISA



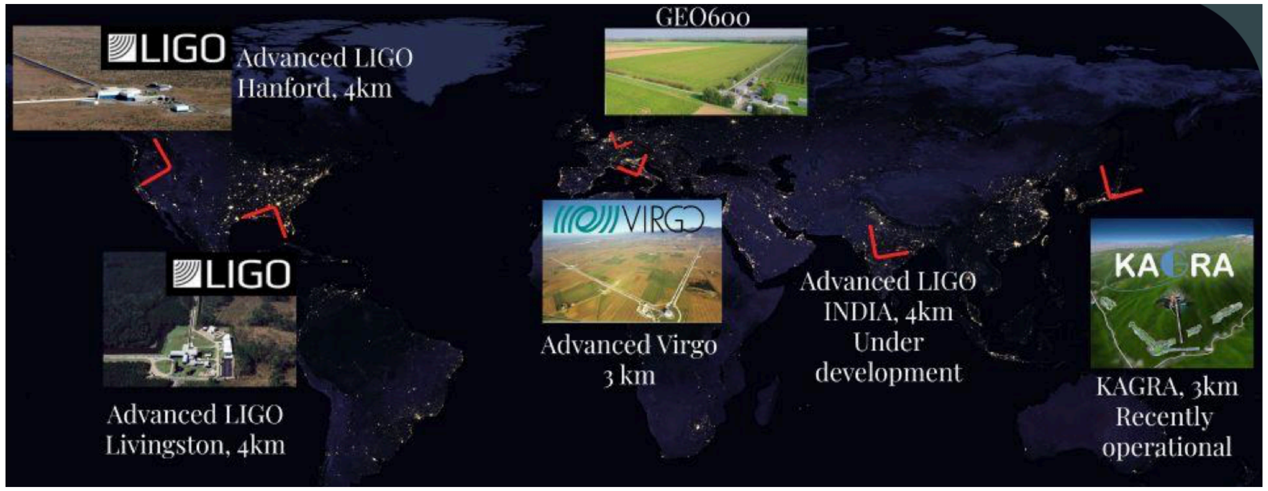
PTA

[Romano+Cornish]

- To detect higher GW frequencies → smaller experiments.

[Living Rev.Rel. 24 (2021) 1, 2011.12414]

LVK: $10\ \mathrm{Hz} \lesssim f_{GW} \lesssim 5\ \mathrm{kHz}$



- LVK is an interferometer **network**

1/ Localisation

Interferometers have bad angular resolution: *not pointing instruments*

For late-time GW cosmology accurate localisation useful as e.g.

- some of the GW sources may also emit EM radiation: to detect that with EM telescopes (which by their very nature are directional) need the localisation.
- useful to associate GW events with data from galaxy catalogues.

==> Network: localization determined through triangulation, using the observed time delays of the signal at several detectors.

2/ instrumental noise uncorrelated between detectors:

any correlated noise between detectors could be attributed to a SGWB.

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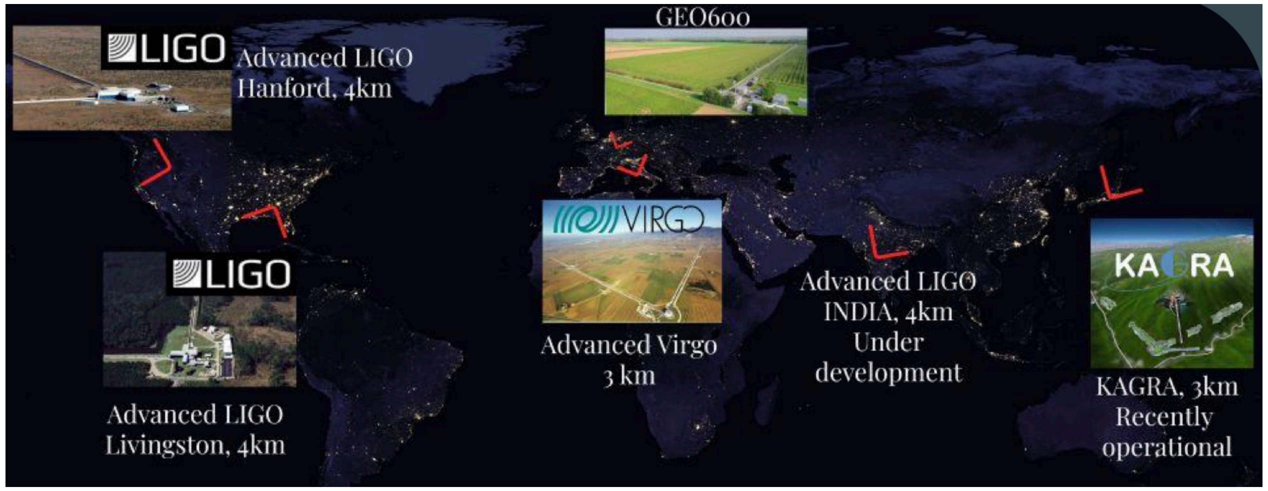

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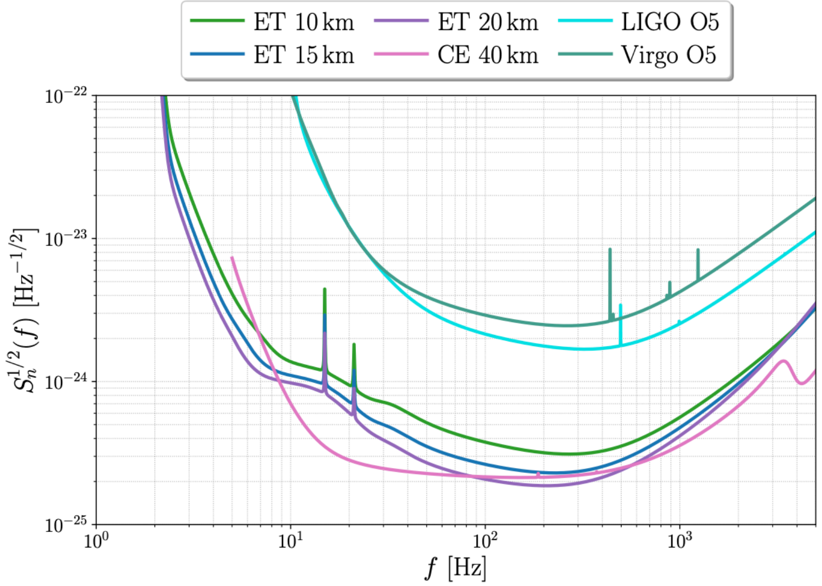
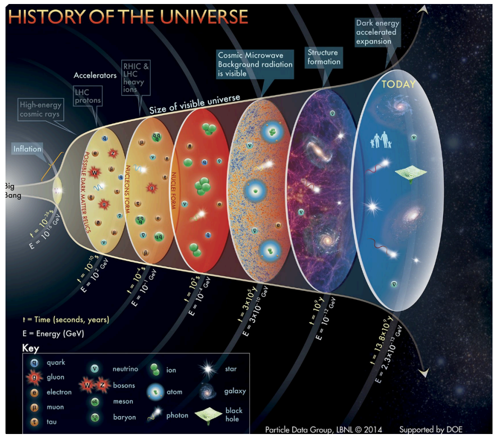
- Plans to build new interferometers on earth beyond LVK (late 2030s?)
  - Einstein Telescope in Europe & Cosmic Explorer in the USA, with $L \sim 10 - 40$ km

$\text{few Hz} \lesssim f_{\text{GW}} \lesssim 10^4 \text{Hz}$



Gravitational waves for cosmology: sources & observations



late-time universe

Individual resolvable astrophysical sources and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
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Rotating asymmetric neutron stars
supernova explosions…

– Expansion rate $H(z)$
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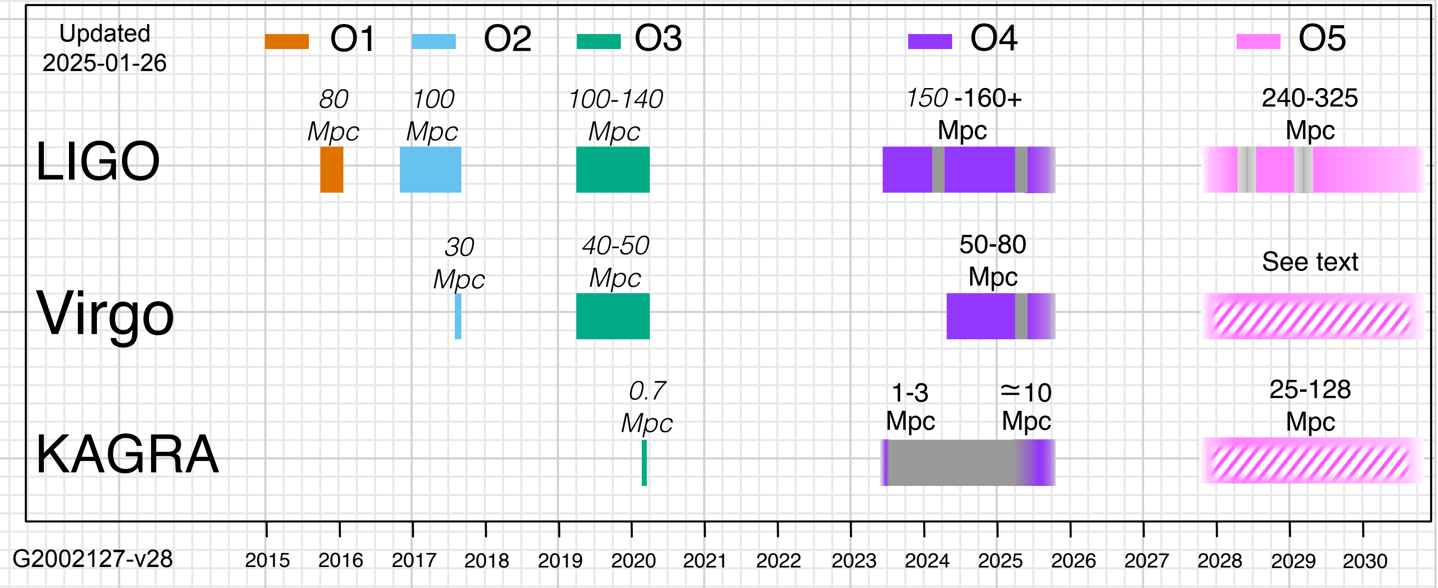
….

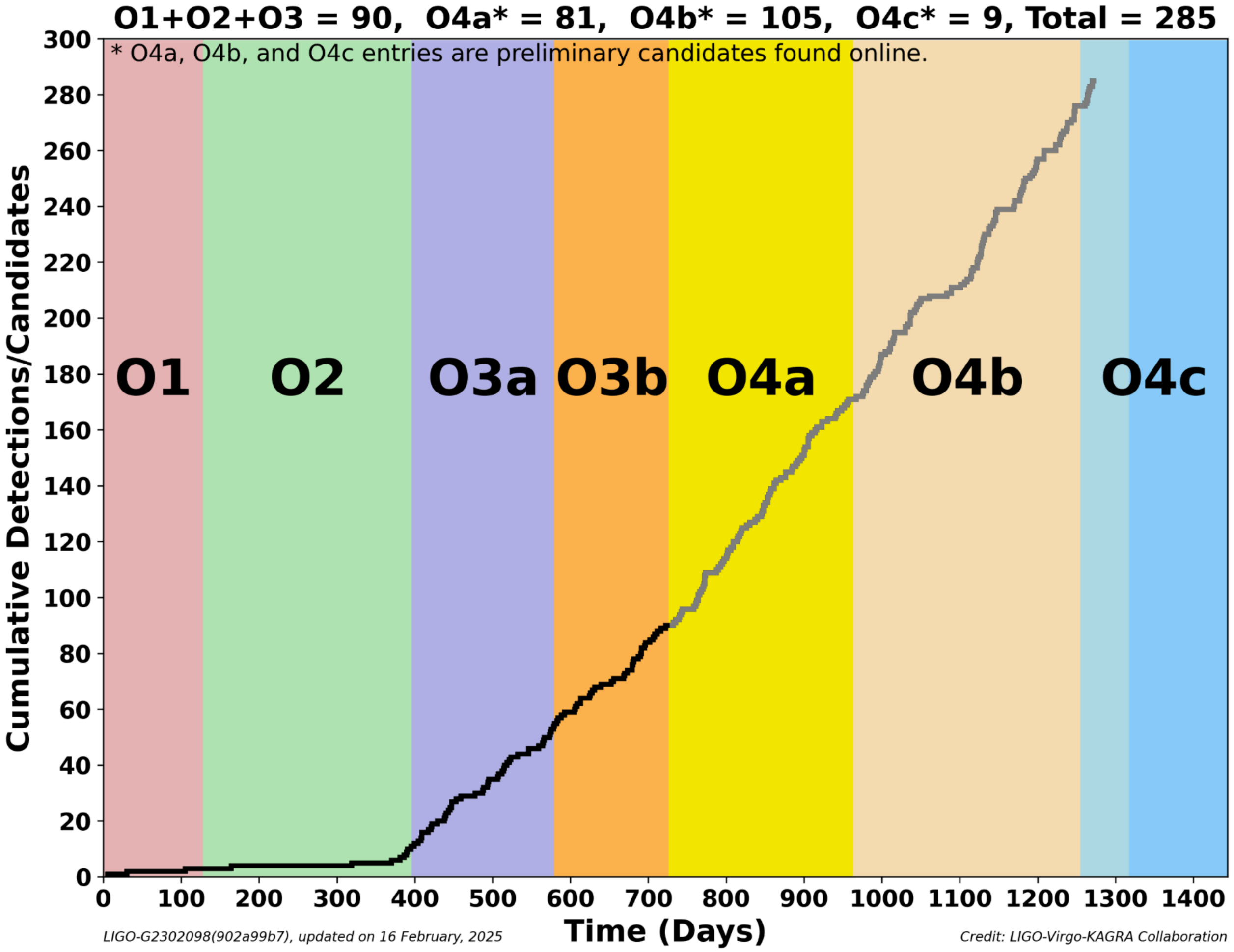
Primordial cosmology

Individual resolvable cosmological sources
e.g. cosmic string GW bursts

Current LVK observations: **only compact binary coalescences,** no cosmic strings, supernovae…!



- **O1** ○ 3 BBHs
- **O2** ○ 7 BBHs
  - ○ 1 BNS with EM counterpart GW170817
- **O3** ○ 4 events compatible with NSBH masses
  - ○ 2 events compatible with BNS masses
  - ○ ~80 BBHs.
- **O4a** ; **O4b** and since end January **O4c**



Public alerts: https://gracedb.ligo.org/
https://emfollow.docs.ligo.org/
https://gwosc.org/

For all of these have the SNR and posterior distributions for
masses,
distances,
sky localisation,
spins…
(at least 17 parameters describe the waveform)

Gravitational waves for cosmology

late-time universe

LVK O3 run, $z_{\max} \lesssim 0.9$

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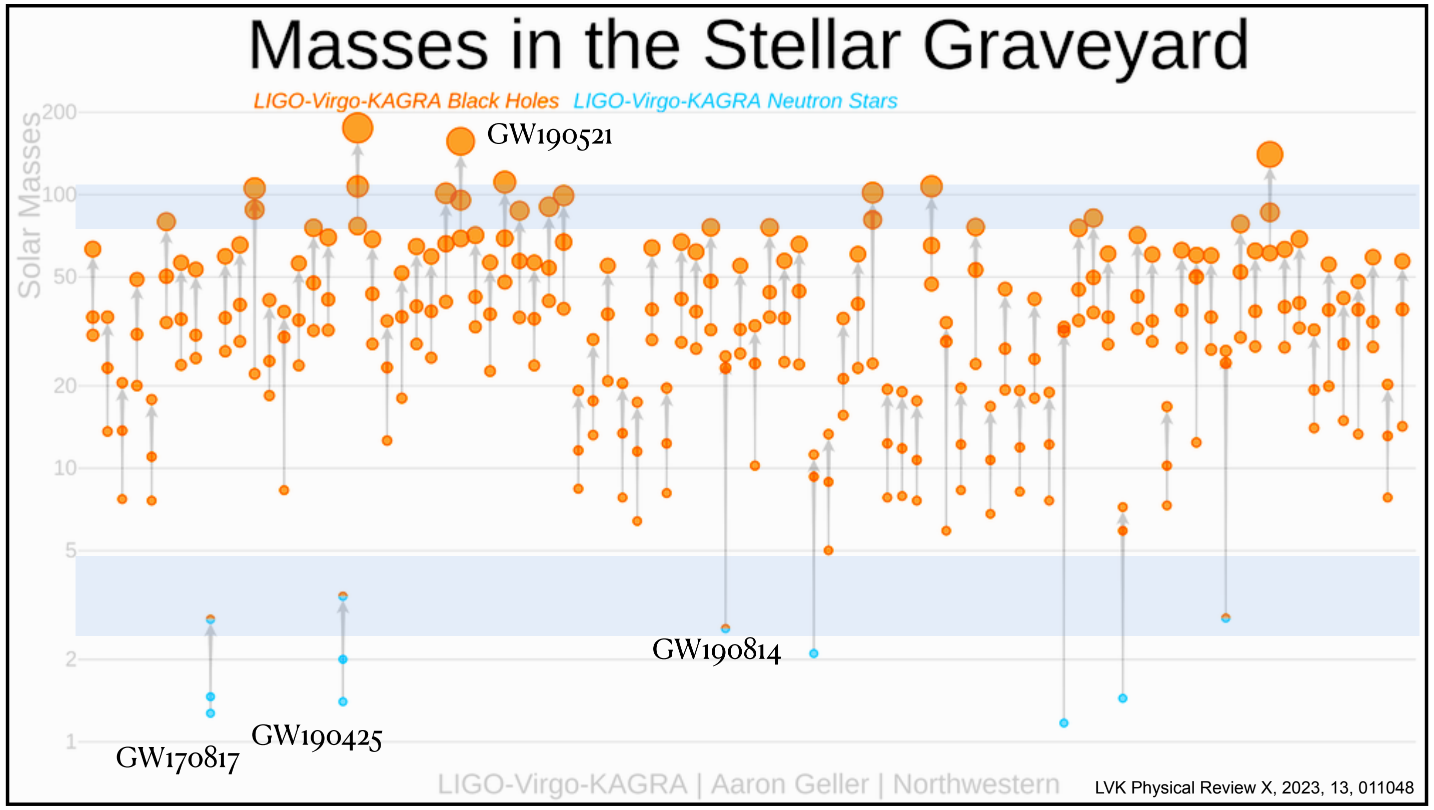
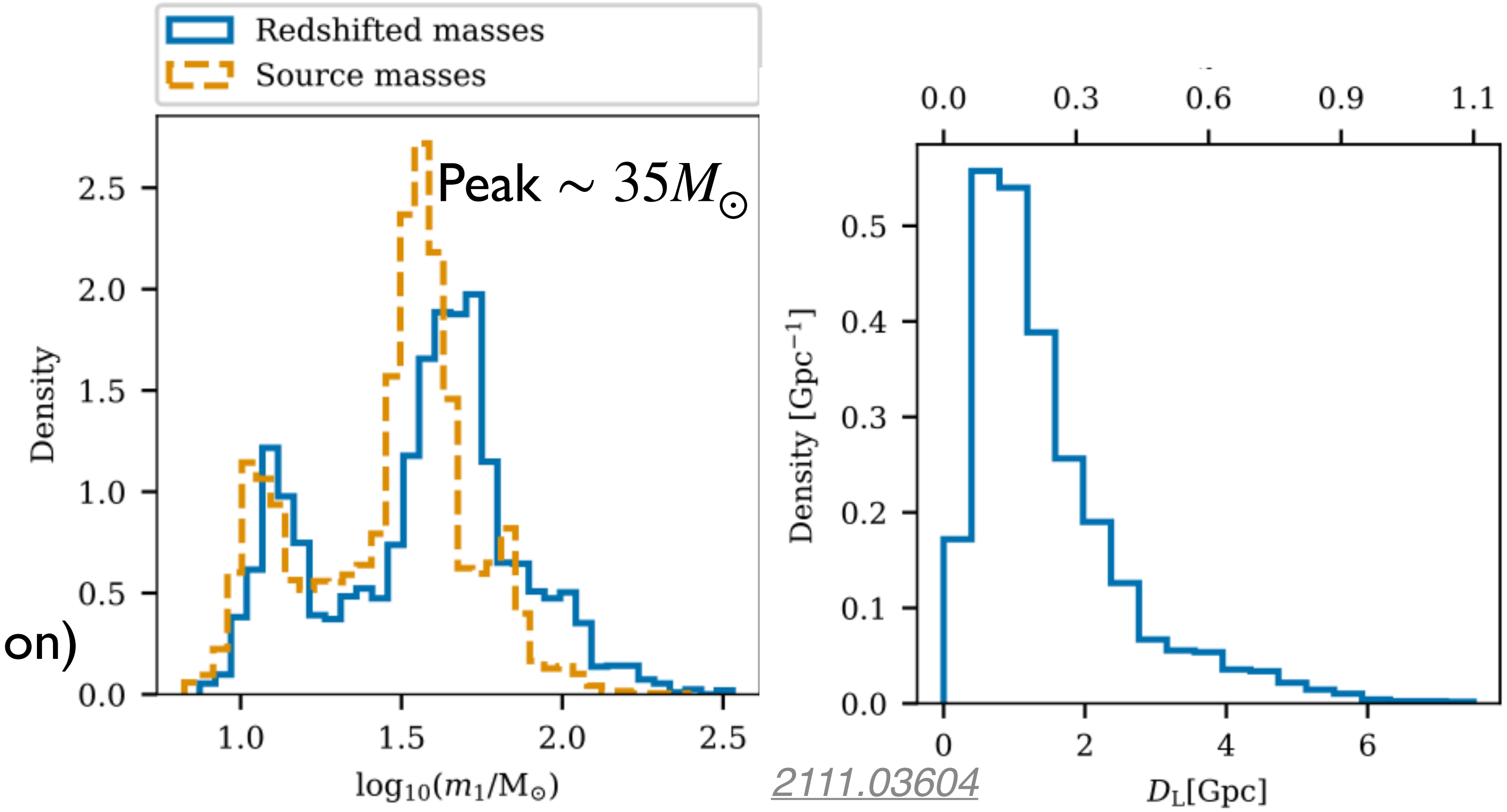


Modified from https://ligo.northwestern.edu/media/mass-plot/index.html



2111.03604

Gravitational waves for cosmology

late-time universe

↕

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- beyond ΛCDM, dark energy $w(z)$ and dark matter
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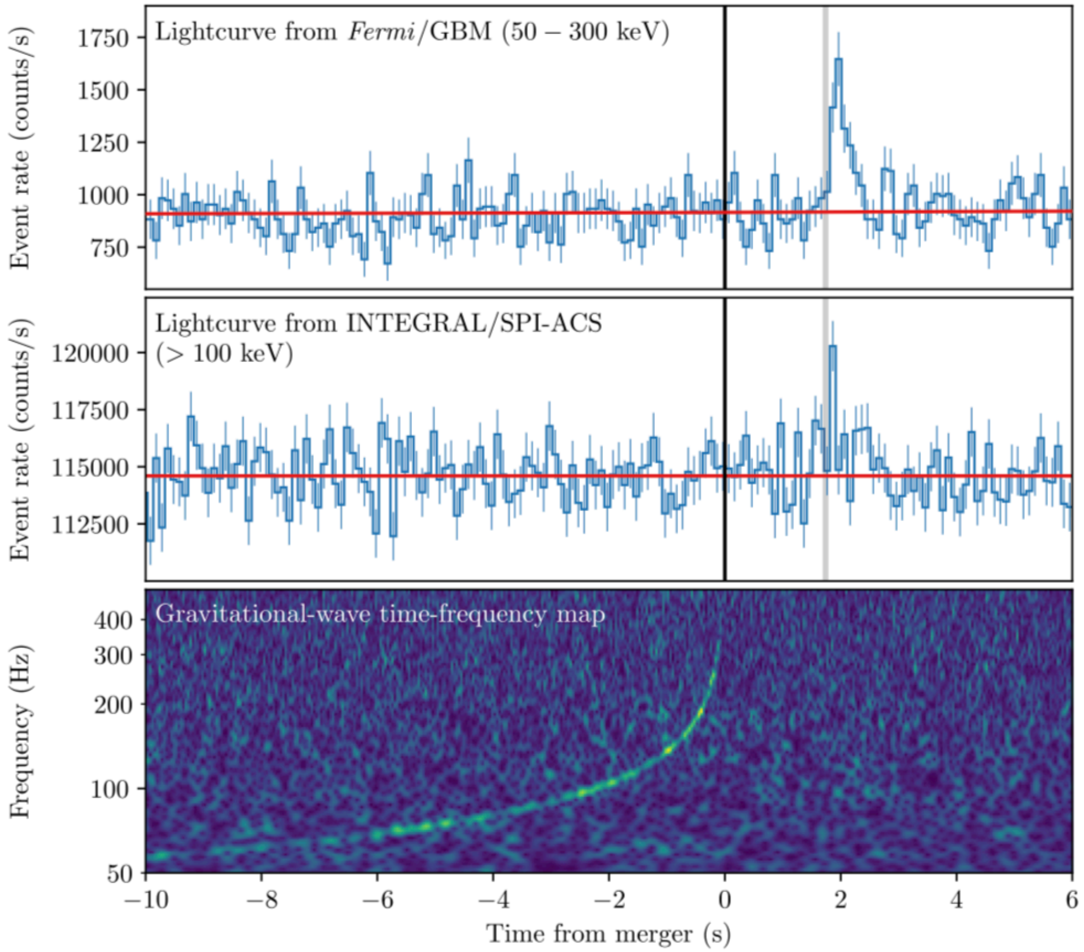
….

BNS-GW170817, $z \sim 0.01$

1.74 s



B. P. Abbott +, APJL, 848:L13 (2017)

$$-3 \times 10^{-15} \leq \frac{c_{GW} - c}{c} \leq +7 \times 10^{-16}$$

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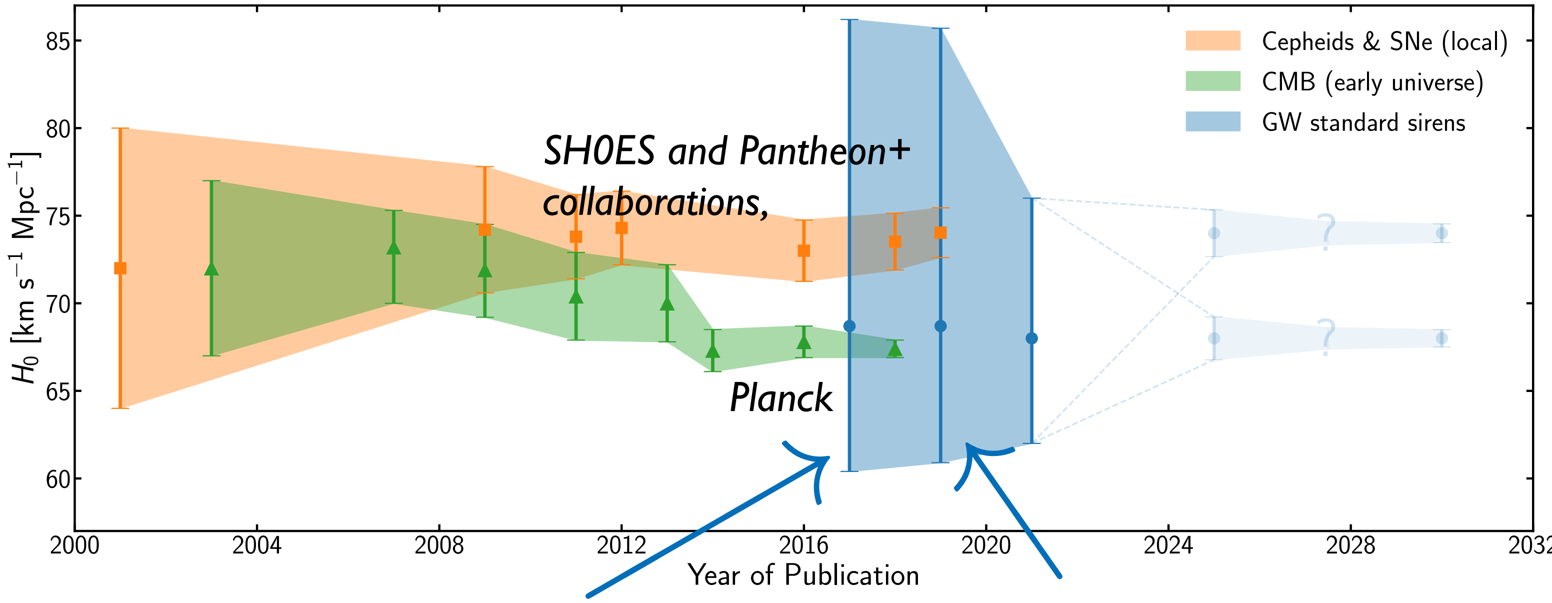


SH0ES and Pantheon+ collaborations,

Planck

dark siren contribution

GW170817 + EM counterpart

- Expansion rate $H(z)$
- Hubble constant H_0
- Ω_m
- beyond ΛCDM, dark energy $w(z)$ and dark matter
- modified gravity (modified GW propagation)
- astrophysics; eg populations of BBHs

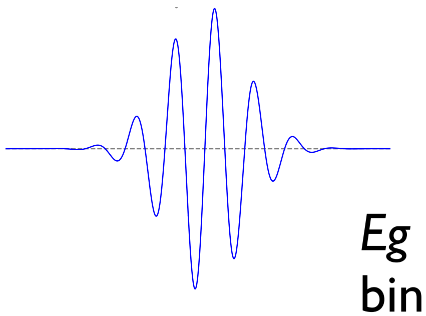
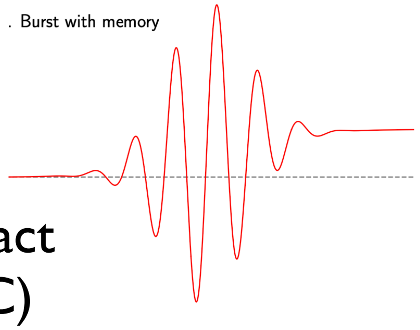
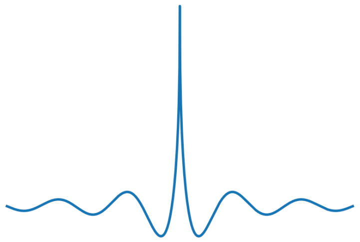
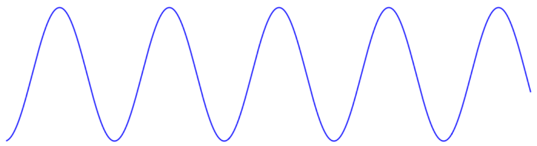
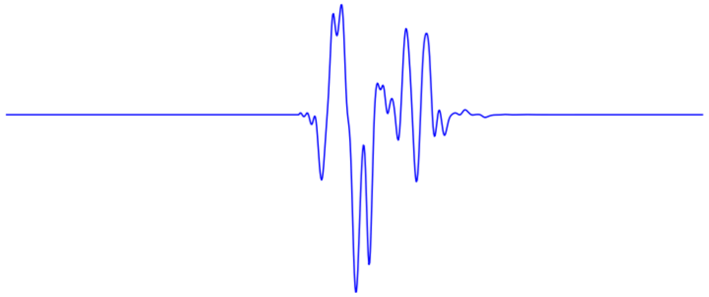
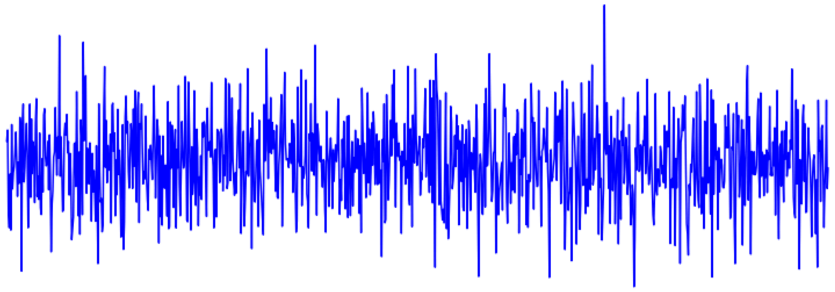
….

properties of GW signals

- *Transients* = short duration signals relative to the observation time-scale (~years)
- *Persistent* = long duration signals relative to the observation time-scale (~years)

- *Coherent/deterministic* = well defined phase evolution
- *Incoherent/Stochastic* = non-predictable/random phase evolution

[Figure inspired by A.Jenkins, PhD]

	Transients	*Persistent*
Coherent	  *Eg* Signal from compact binary mergers (CBC)  *Eg* Burst from cosmic strings	 *Eg* Spinning NS with a bump on it Early inspiral of a CBC Super radiance from BH
Incoherent	 *Eg* core collapse of a supernova	 *Persistent incoherent signals —> stochastic GW background*

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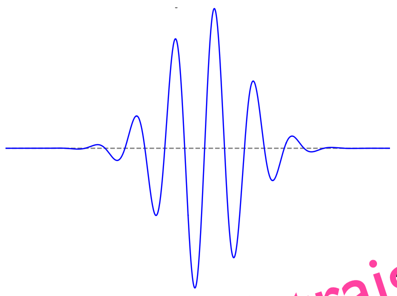
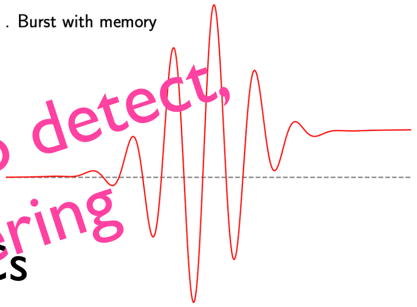
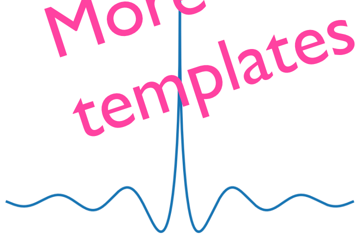
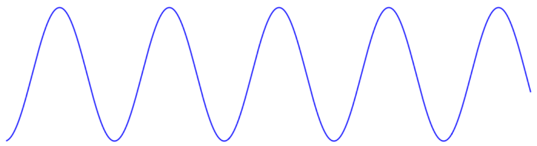
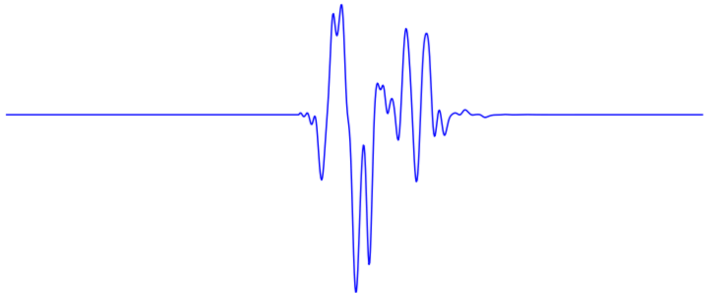
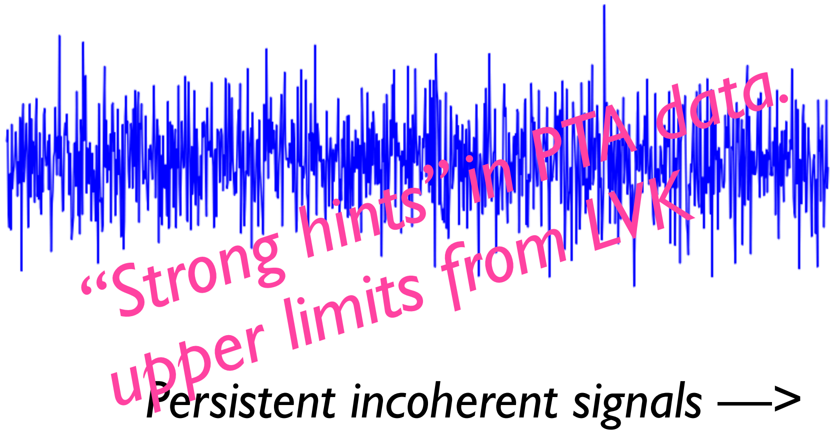
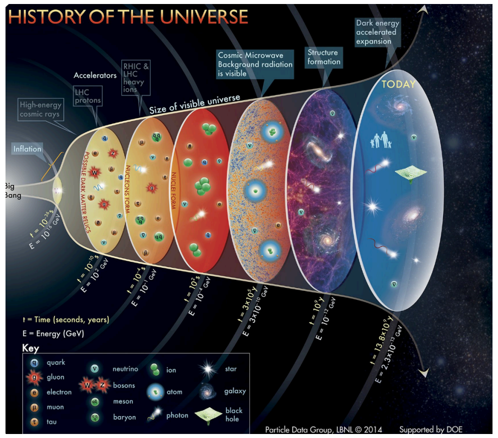
Coherent	 *Eg* Signal from CBCs  *Eg* Burst from cosmic strings More “straightforward” to detect, templates and match filtering	 *Eg* Spinning NS with a bump on it Early inspiral of a CBC Super radiance from BH
Incoherent	 *Eg* core collapse of a supernova	 “Strong hints” in PTA data, upper limits from LVK *Persistent incoherent signals —> stochastic GW background*

Gravitational waves for cosmology



late-time universe

Individual resolvable astrophysical sources and populations of sources
at cosmological distances
e.g. binary neutron stars (BNS),
binary black holes (BBH),
neutron star-black-hole binary (NS-BH)
Rotating asymmetric neutron stars
supernova explosions…

- Expansion rate $H(z)$
- Hubble constant H_0
- Ω_m
- beyond ΛCDM, dark energy $w(z)$ and dark matter
- modified gravity (modified GW propagation)
- astrophysics; eg populations of BBHs

….

Individual resolvable cosmological sources
e.g. cosmic string GW bursts

Primordial cosmology

Very early universe until today

$t \gtrsim t_{Pl}$

Stochastic GW background
astrophysical and/or cosmological origin

$$\Omega_{\rm gw}(t_0, f) = \frac{f}{\rho_c}\frac{d\rho_{\rm gw}}{df}(t_0, f)$$

- population of BH, white dwarfs.
- inflationary GWs
- 1st order Phase transitions
- topological defects
- scalar induced GWs
- primordial black holes
- ultra light dark matter
- axions…

More speculative. Early universe sources beyond standar
model of particle physics!

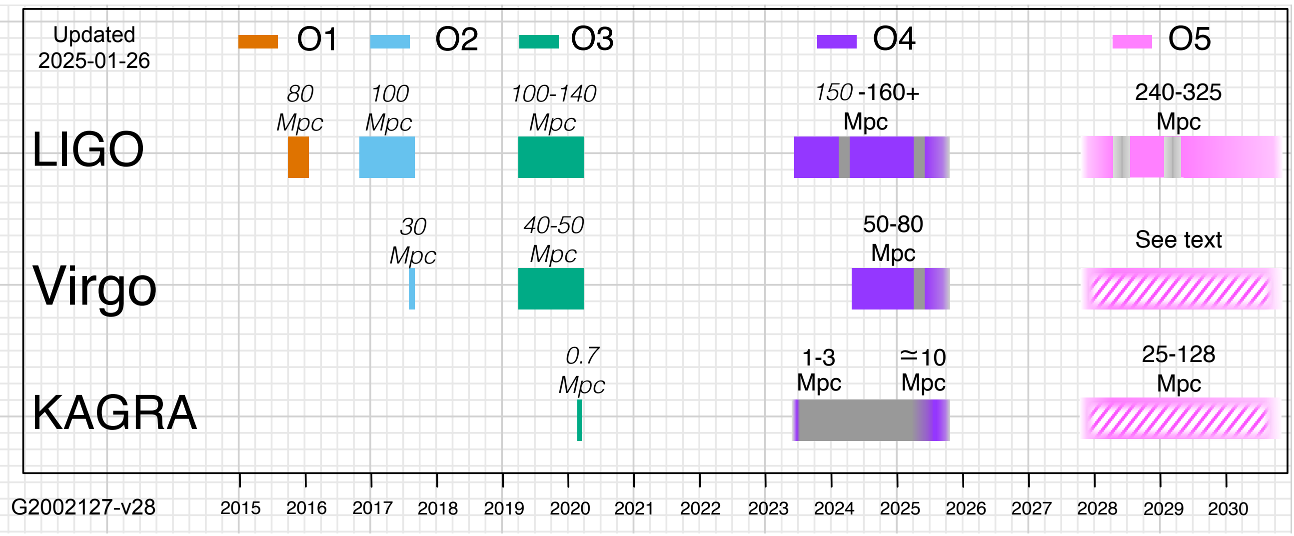
From individual signals to the stochastic GW background (SGWB)

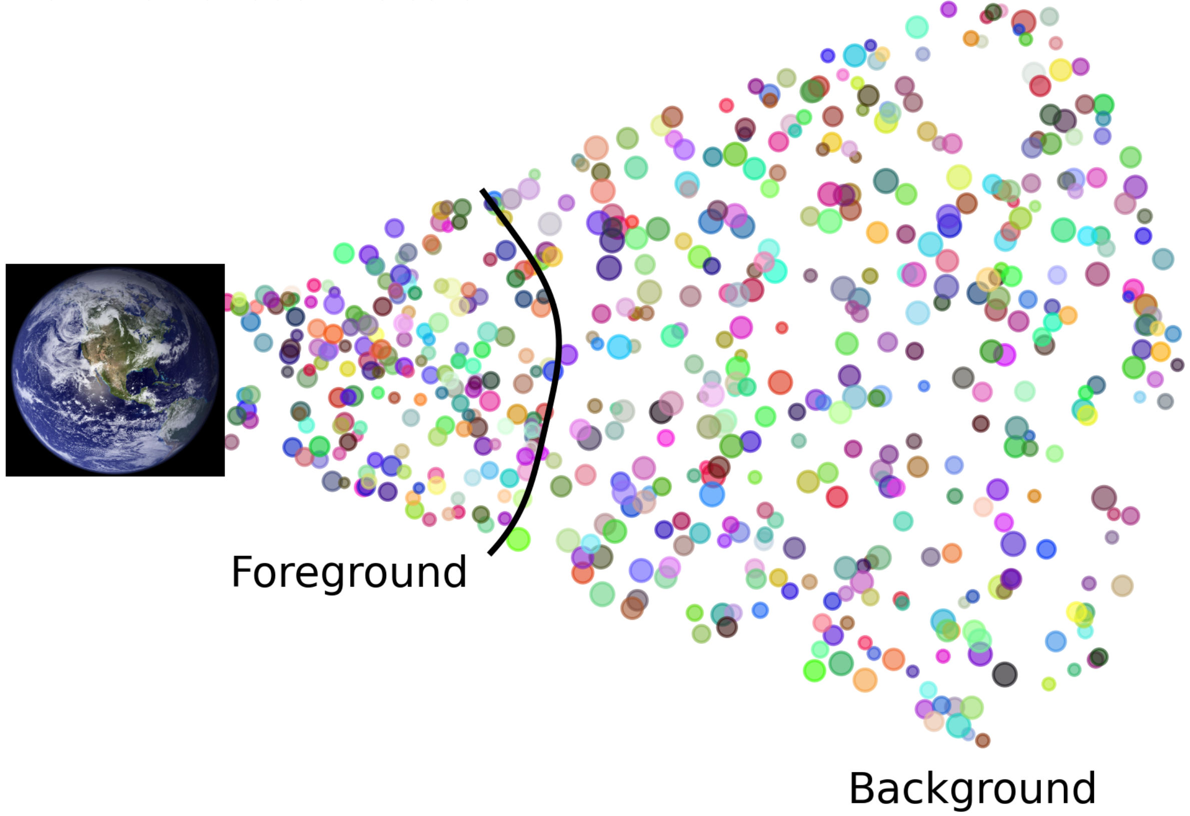
- Consider cosmic *population* of sources (astrophysical/cosmological) distributed in the universe

- For each source, amplitude GW signal $\propto d_L^{-1}$:
  - $\Rightarrow$ beyond some distance the signals will be too faint to distinguish from the noise in a detector
    - $\Rightarrow$ *detection horizon* (dependent on the source and detector);
  - – and even in the detection horizon, if the number of sources increases sufficiently, signals may overlap (in time and frequency domains) so can't be detected individually

- The combined GW signal of these is the **SGWB** — which can be of astrophysical or cosmological origin.

- SGWB associated with distant sources, and its detection and characterisation can probe the high redshift primordial universe





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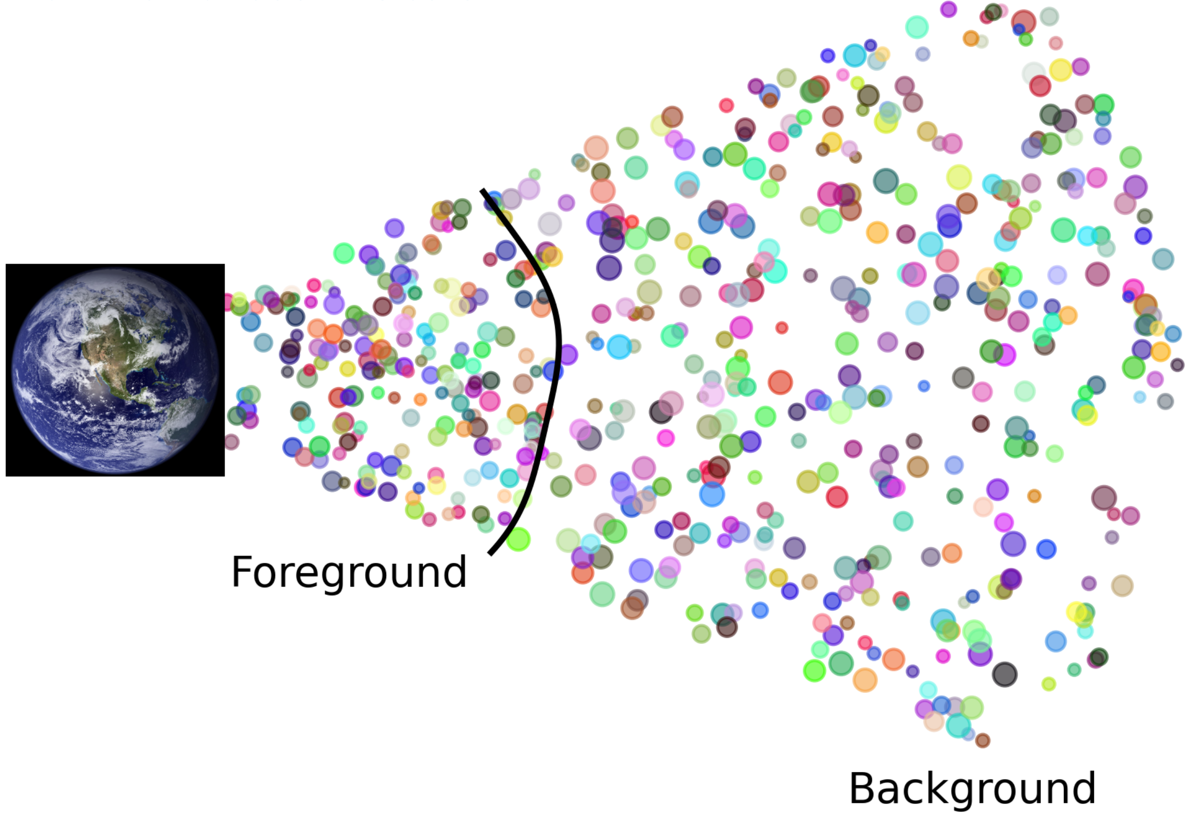
- SGWB associated with distant sources, and its detection and characterisation can probe the high redshift primordial universe

- To access the *cosmological* background, (C.Caprini) crucial to understand the *astrophysical* background (T.Regimbau) which will inevitably be present.

- Note: the cosmological SGWB is expected to be nearly *isotropic; unpolarised; gaussian.*
  the astrophysical one may be a*nisotropic*
  (galaxy distribution anisotropic up to ~100Mpc)



Photons decoupled from $T \sim 3000K$, Gravitons decoupled from t_{Pl}

- particles which decouple from primordial plasma at $t \sim t_{dec}$ or $T \sim T_{dec}$ give snapshot universe at that time.

$$\begin{array}{c} t < t_{dec} \\ T > T_{dec} \end{array}$$ they are coupled and interactions obliterate all information.

- In thermal equilibrium when

$$\Gamma \sim n\sigma|v| > H$$

rate of process maintaining thermal equilibrium

number density of particles

x-section interaction

typical velocity

For light/massless particles at temperature T

$$n \sim T^3$$
$$v \sim 1$$
$$H^2 \sim T^4 M_{\rm Pl}^{-2}$$

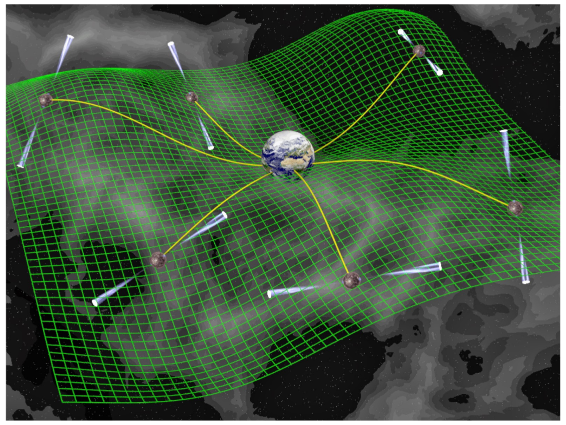
and drop out when $\Gamma \sim H$

Eg Neutrinos: $\sigma \sim G_F^2 T^2$ $\quad \left(\frac{\Gamma}{H}\right)_{\rm neutrino} \sim \left(\frac{T}{1{\rm MeV}}\right)^3$

Gravitons $\sigma \sim G_N^2 T^2 \sim \frac{T^2}{M_{\rm Pl}^4}$ $\quad \left(\frac{\Gamma}{H}\right)_{\rm graviton} \sim \left(\frac{T}{M_{\rm Pl}}\right)^3$

=> retain spectrum/shape/typical frequency & intensity of physics at corresponding high energy scales
=> but making predictions uncertain for such sources,
=> and need to deal with an astrophysical component

SGWB



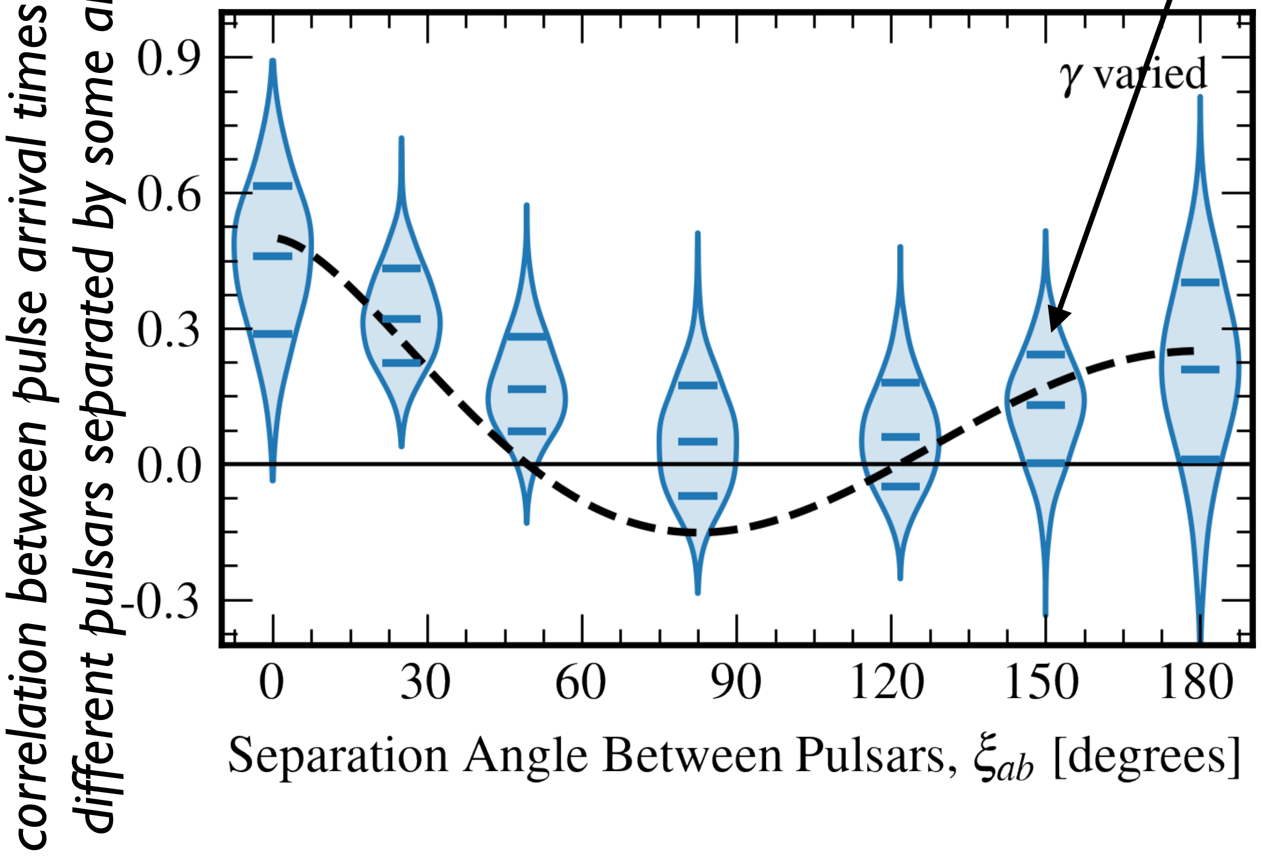
[Credit: D. Champion]

- Compelling **evidence** for a GW background in nHz frequency band

Assuming General relativity, and an unpolarised, stationary, isotropic and Gaussian SGWB (compatible with the cosmological principle), then the average correlation between pulse arrival times from 2 different pulsars separated by some angle should satisfy the Hellings&Downs curve [1983]

NANOGrav collaboration,
15 year data set; 68 pulsars

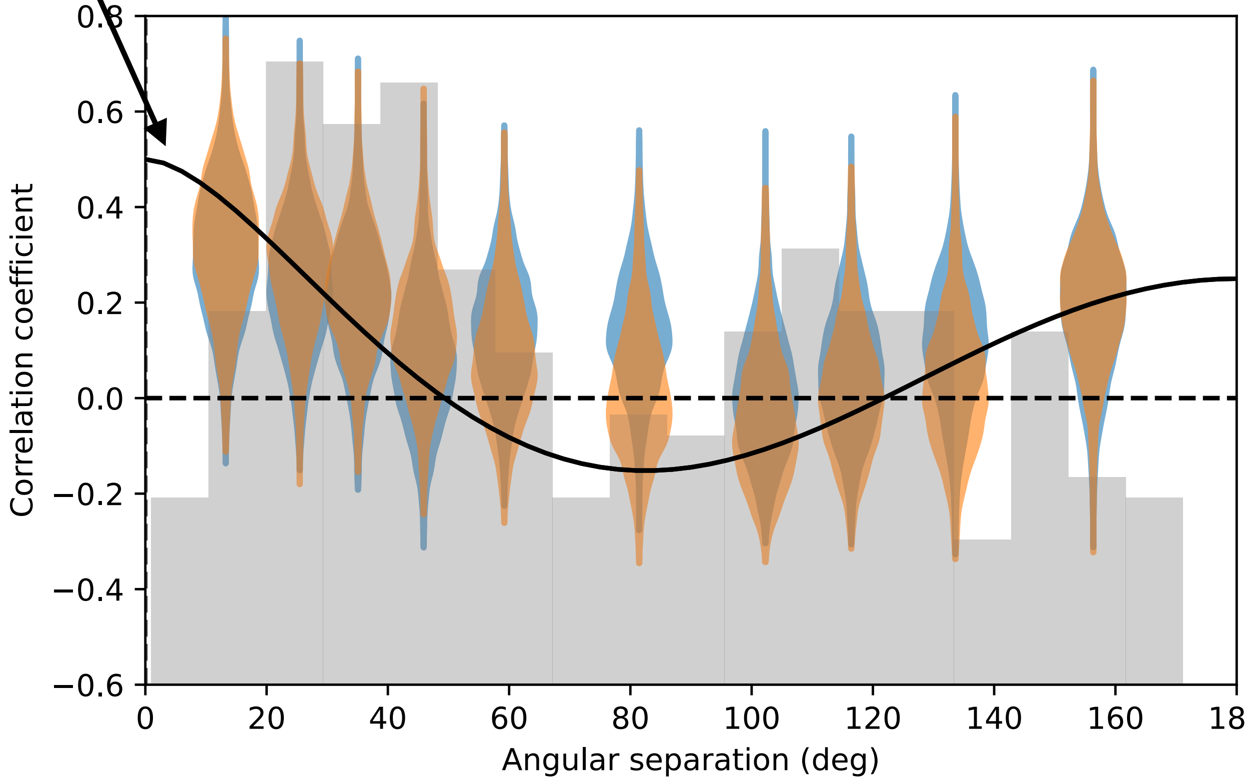
[NANOGrav, 2306.16213]



$3.5 - 4\sigma$ (NANOGrav)

Pulsar Timing Array
(EPTA); 25 pulsars

[EPTA III: search for GWs 2306.16214]

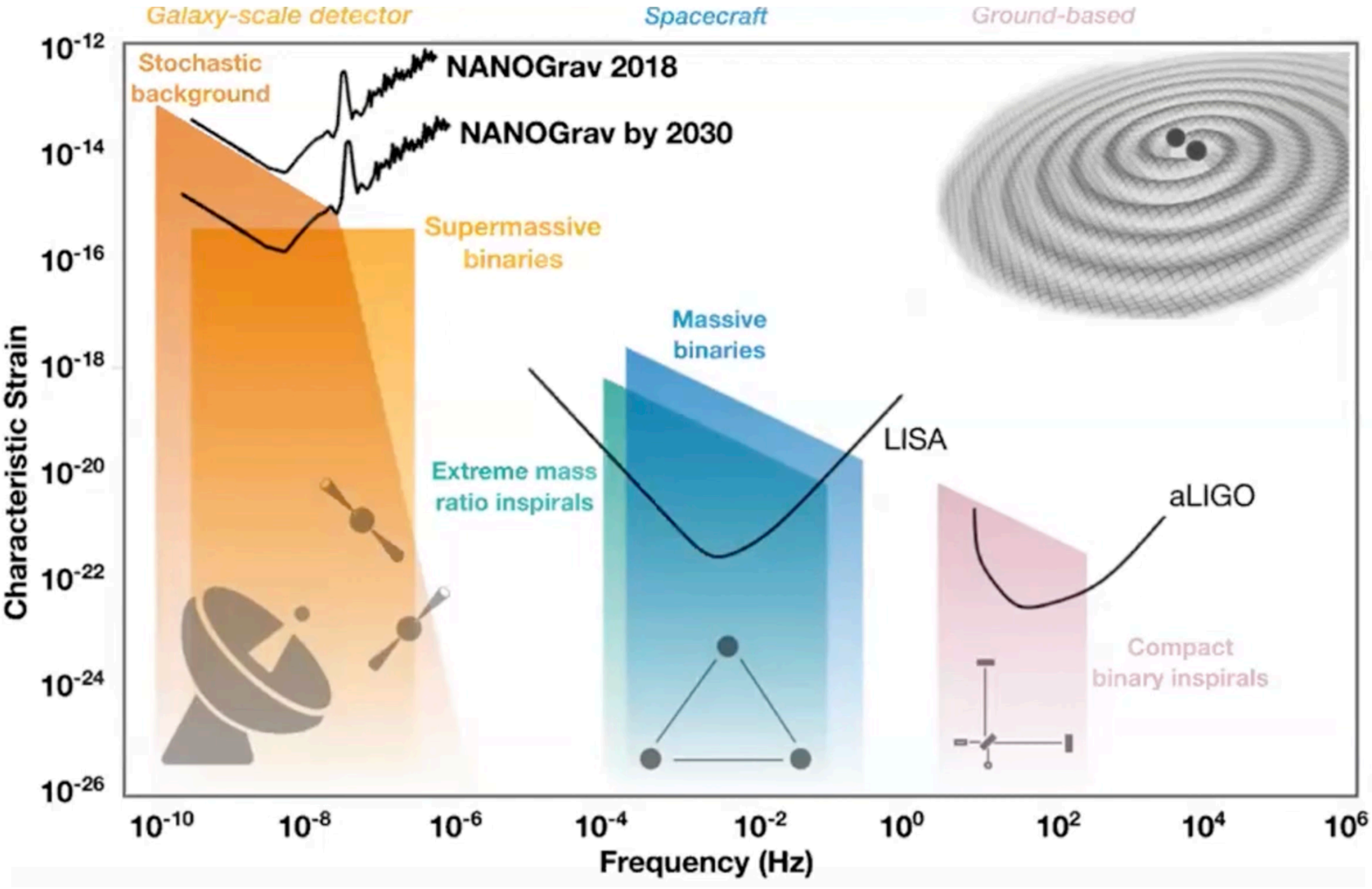


$\sim 3 - 3.5\sigma$ (EPTA+IPTA) $\sim 2\sigma$ (PPTA) (CPTA)

Detectors working in different frequency bands:

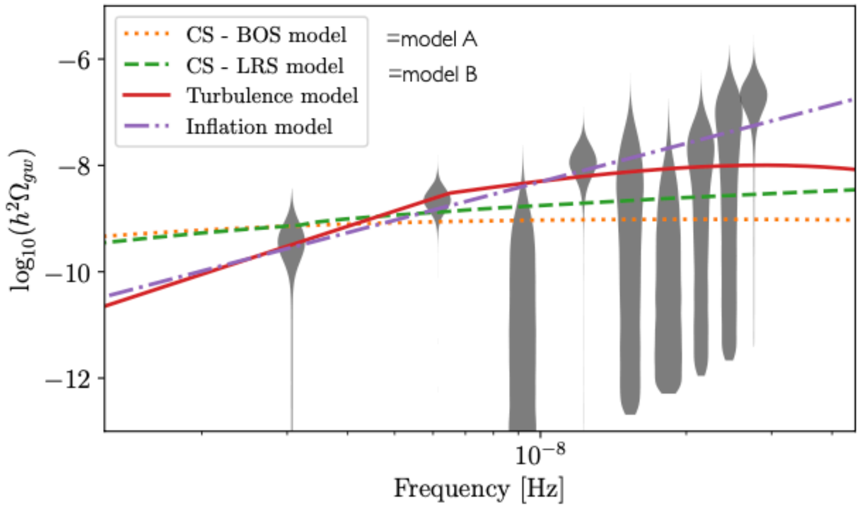
probe different GW sources with different characteristics.

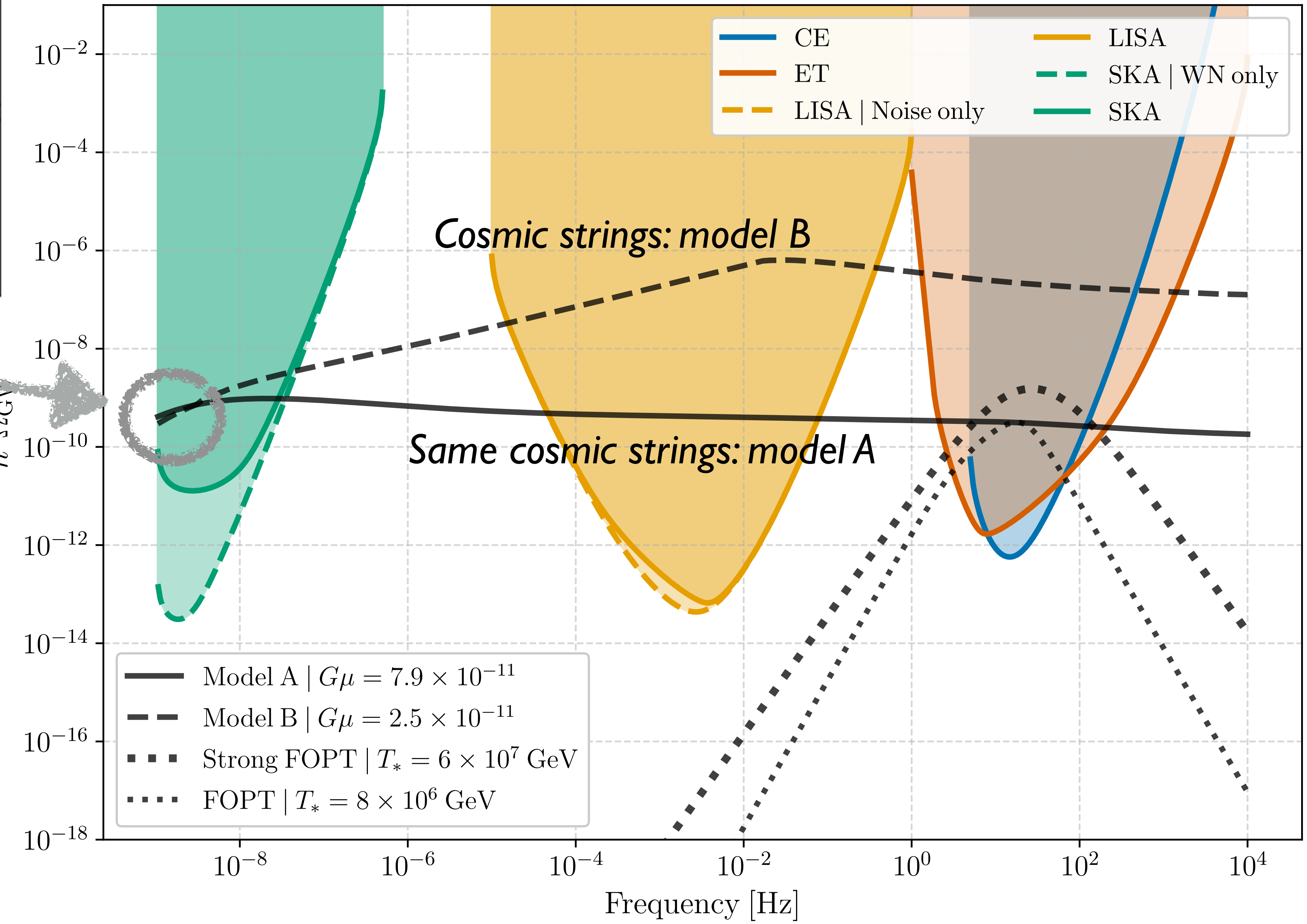
Work individually; as well as *together*



Examples of cosmological SGWB signals: Next generation detectors (SKA, LISA and ET/CE)

[Caprini et al, 2406.02359]





Model A: *[Blanco-Pillado, Olum, Shlaer, 2014]*

Model B: *[Lorentz, Ringeval, Sakellariadou, 2010]*
[Polchinski, Rocha et al]

- Models A and B are *meant to describe exactly the same physics!*
- If Model B is the unique source of the SGWB signal in PTA then LVK constraints actually already *exclude it!*

$\log_{10}(G\mu) = -10.63^{+0.24}_{-0.22}.$

$G\mu \gtrsim (4.0-6.3) \times 10^{-15}$

- Model A would lead to an extremely loud signal in ET, with SNR $\sim 10^3$
- Different spectral shapes, depending — amongst other things — on the properties of the source. Is the source producing GWs at t_* "short/long" duration relative to the Hubble time $H^{-1}(t_*)$?

Characteristic frequency today?

Consider a source of GWs operating at a time $t = t_*$, for order one Hubble time.

- Characteristic frequency today depends on:
  - production mechanism (model-dependent)
  - kinematical (depending on the redshift from the production era)

- GWs produced with frequency f_* at $t = t_*$ have characteristic frequency today of

$$f = \frac{a_*}{a_0} f_* = 1.65 \times 10^3 \text{ Hz} \left(\frac{T_*}{10^{10} \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \left[\frac{f_*}{H_*} \right]$$

(assuming standard thermal history and radiation era)

temperature (energy density) of the universe at the source time

- But expect by causality that $f_* \sim \ell_*^{-1} \geq H(t_*)$ so $\left[\frac{f_*}{H_*} \right] \geq 1$, with value depending on production mech.

$$f = \frac{a_*}{a_0} f_* \sim 1.65 \times 10^3 \text{ Hz} \left(\frac{T_*}{10^{10} \text{ GeV}} \right)$$

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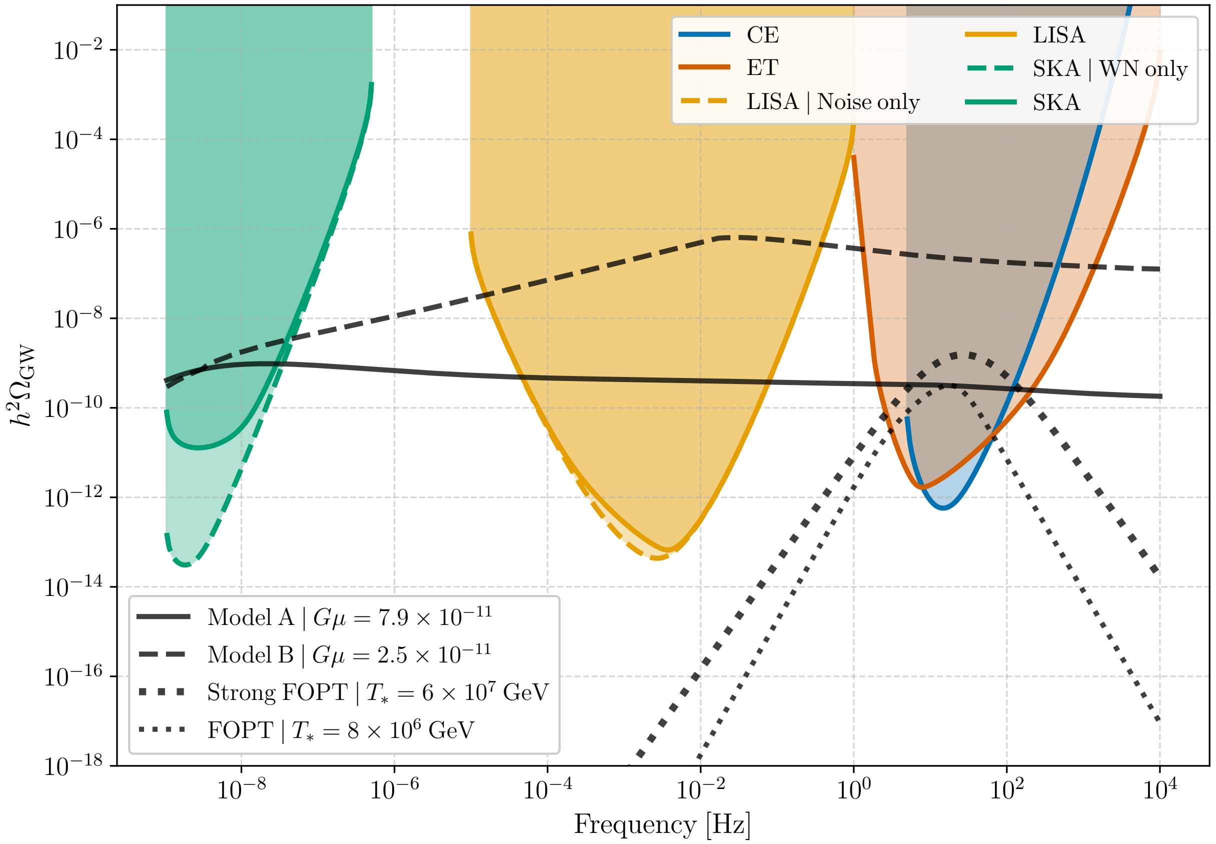


==> ground based interferometers (LVK, ET, CS..) correspond to scales $10^6 \text{ GeV} \lesssim T_* \lesssim 10^{10} \text{ GeV}$

==> GWs in the GHz band would correspond to GUT scales cosmological sources. (No known astrophysical sources are known)

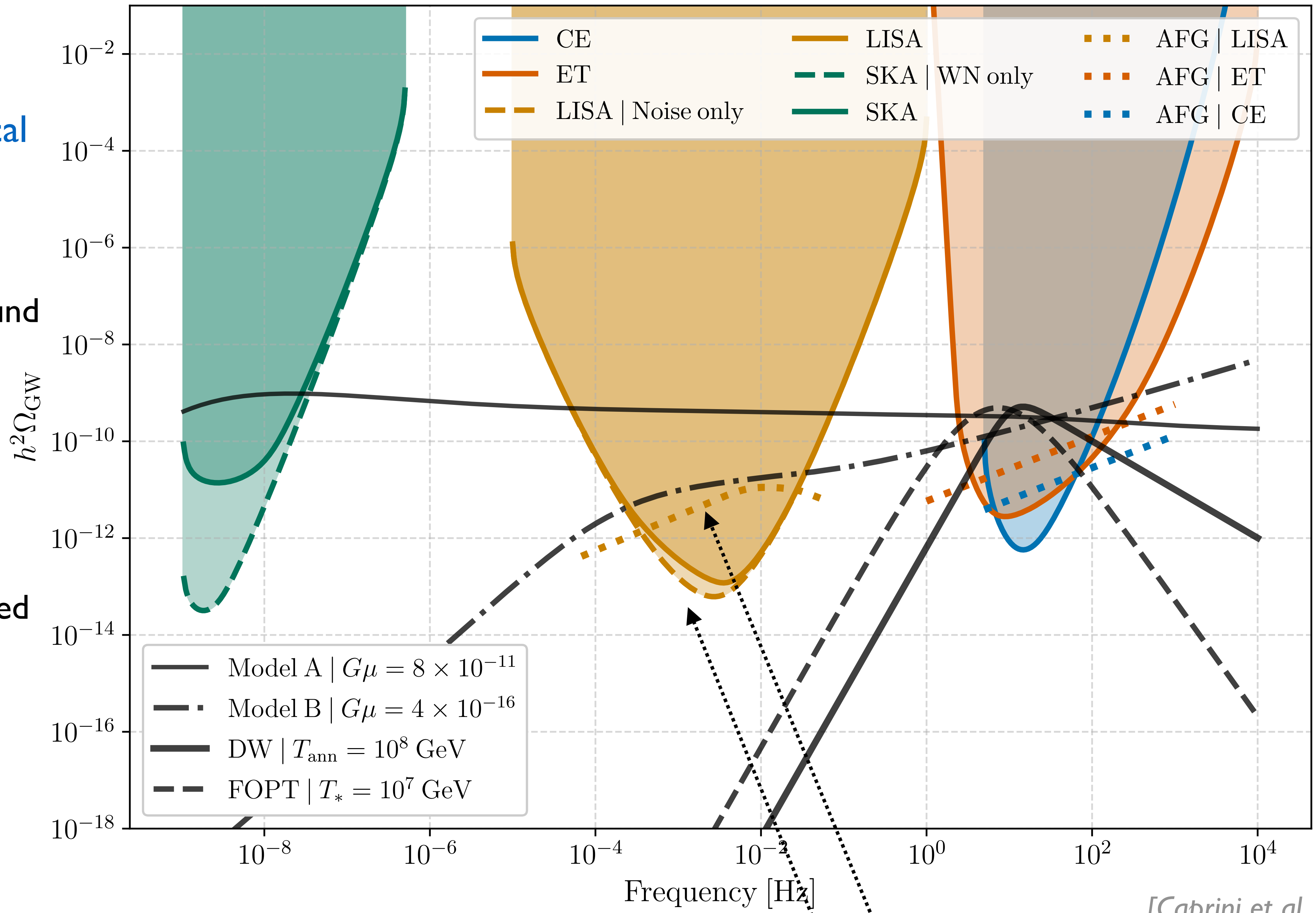
==> LISA frequencies included energy scale of EW symmetry breaking $T_* \sim 100 \text{ GeV}$

==> nHz frequencies of PTAs coincide with chiral symmetry breaking and quark-gluon confinement (QCDPT), $T_* \sim 150 \text{ MeV}$

Crucially important to understand the astrophysical foregrounds.

- Some astrophysical foreground are so loud that they must be considered as a noise in the detector.

- Others are weak relative to noise, and must be searched for just as the cosmological SGWB



[Caprini et al, 2406.02359]

LISA.
– galactic white dwarf binaries: loud
– extra galactic WDB and SMBHB: weak
Contribute a signal, SNR $\simeq 314$

SKA. (Assuming 50 ms pulsars, $T_{obs} = 15$yrs
– assumed a SMBHB background with amplitude and spectrum obtained from simulations

ET (CE).
– weak signals from unresolved compact Binary mergers, SNR $\simeq 50\ (72)$

- Lecture 1: – **Overview** on early- and late-time cosmology with GWs; current and future experiments,
  – **orders of magnitude**

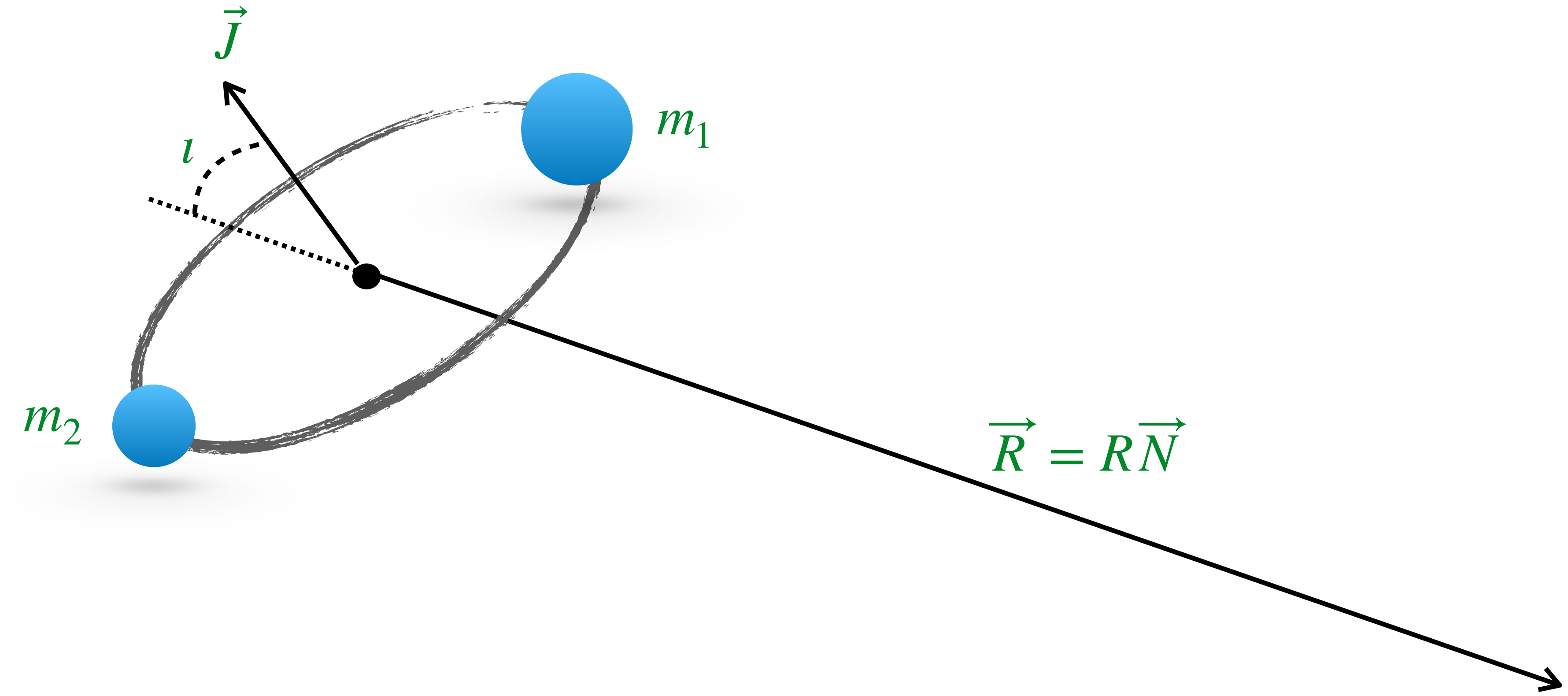
- Lecture 2: – Late-time cosmology: GWs and $d_L(z)$
  – GWs in theories beyond GR, $d_L^{GW}(z)$
  – **standard sirens I:** Measuring H_0 with GWs and O3 results of LVK
  – Back to early-time universe: an example of what physics we can probe.

- Lecture 3 (Chiara Caprini):
  – *cosmological* stochastic GW background: **early-universe cosmology with GWs**
  Solutions of the GW propagation equation in FLRW; its calculation for different sources (inflation, topological defects, first order phase transitions)

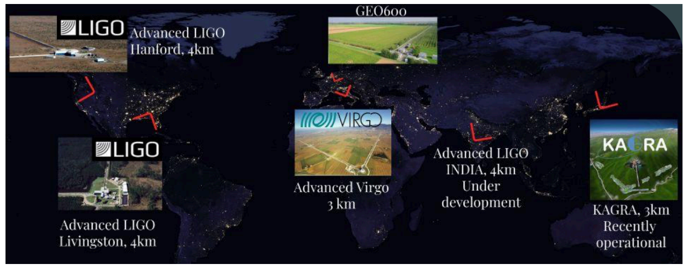
- Lecture 4 (Nicola Tamanini):
  – **Standard sirens II**: more details, statistical methods, future prospects

- Lecture 5 (Tania Regimbau):
  – ***astrophysical* stochastic GW background**: Definition/statistical properties, pulsar timing arrays and background from supermassive BH binaries, LVK results, prospects for the future.

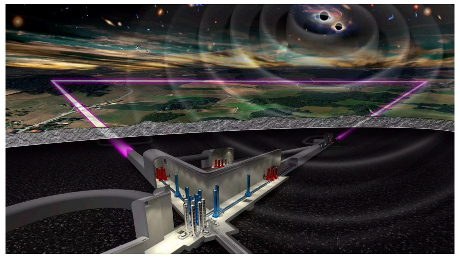
Late time cosmology with binaries:
characteristic scales and orders of magnitude.



LVK: $10 \text{ Hz} \lesssim f_{\mathrm{GW}} \lesssim 5 \text{ kHz}$



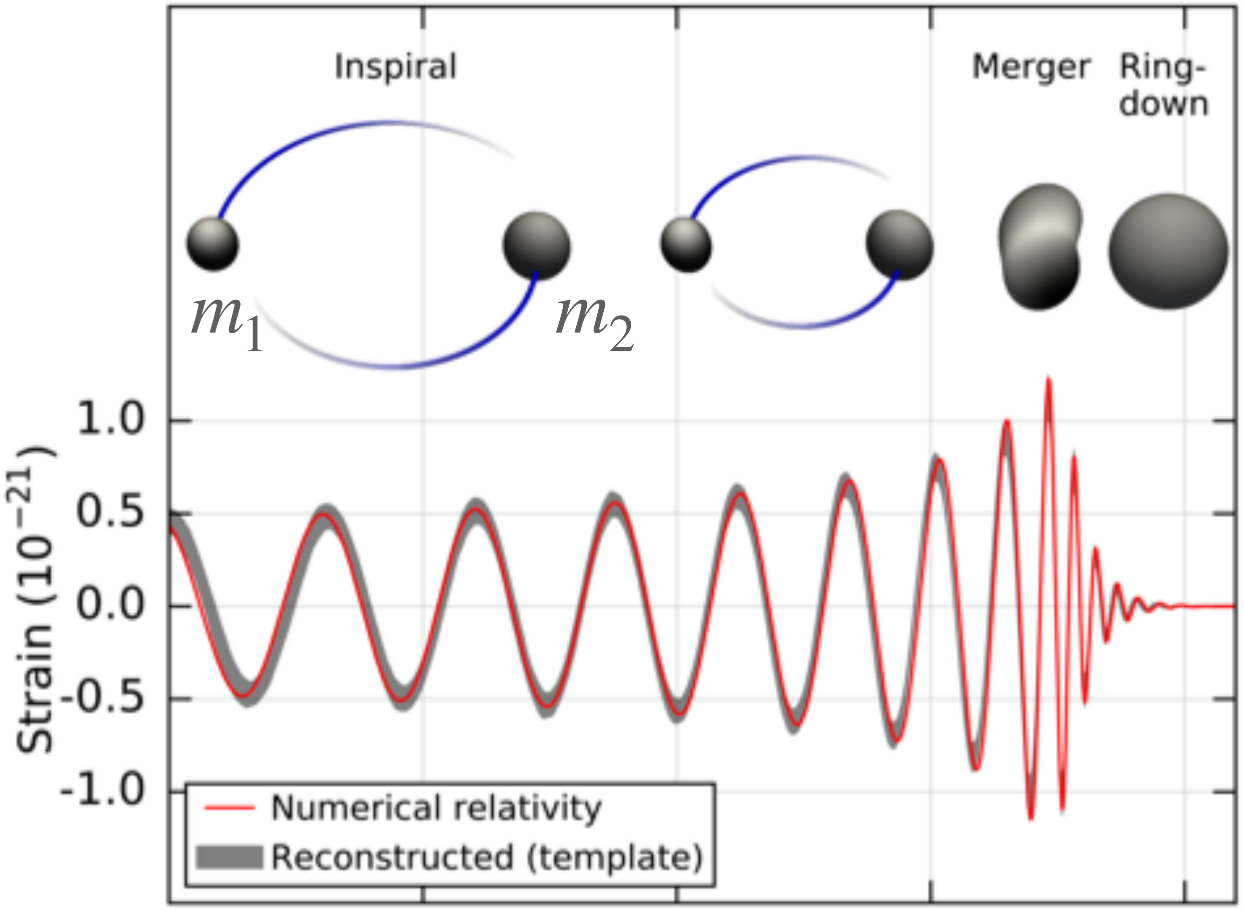
ET: $1 \text{ Hz} \lesssim f_{\mathrm{GW}} \lesssim 10^4 \text{ kHz}$



LISA: $10^{-4} \text{ Hz} \lesssim f_{\mathrm{GW}} \lesssim 1 \text{ Hz}$

ESA | NASA
lisa

$$R = d = d(z, H_0, \ldots)$$



$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \dots$

$v/c \ll 1$

- The *inspiral phase* can be understood with perturbation theory (the "post-Newtonian (PN) expansion" of the Einstein equations) presented below, more details in (Thorne 1980 ; Blanchet 2006 ; Poisson and Will 2014).
- The *merger phase* generally requires numerical relativity other other techniques such as effective one-body techniques, see e.g. (Deruelle and Uzan 2018) for an introduction.
- The *ringdown phase* can also be approached with perturbative methods, namely BH perturbation theory, see e.g. (Kokkotas and Schmidt 1999 ; Santoni 2024).

very basics on GW

[see Maggiore, Poisson and Will, Speziale and Steer…]

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

NEXT 6 SLIDES NOT DONE IN LECTURES

$\sim 10^{-43}\ \mathrm{kg}^{-1}\mathrm{m}^{-1}\mathrm{s}^{-1}$

space-time is elastic but very rigid (need massive energetic objects to produce detectible GWs)

- perturbative treatment of Einstein's equations

– background metric $\bar{g}_{\mu\nu}$ & perturb $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} + h^{(2)}_{\mu\nu} + \ldots$

Assumption: in some coordinate system $|h_{\mu\nu}| \ll 1$

– And then attempt to solve Einstein's equations order by order.

To first order $$G^{(1)}_{\mu\nu}(h) = \frac{8\pi G}{c^4} T^{(1)}_{\mu\nu} \,.$$

To 2nd order. $$G^{(1)}_{\mu\nu}(h^{(2)}) = \frac{8\pi G}{c^4}\left(T^{(2)}_{\mu\nu} + t^G_{\mu\nu}\right), \qquad t^G_{\mu\nu} := -\frac{c^4}{8\pi G} G^{(2)}_{\mu\nu}(h) \,.$$

first order solution feeds back as a source for the second order solution (standard from perturbatively solving non-linear equations)

Minkowski background (solar system or sub-Hubble scales) $\bar{g}_{\mu\nu} = \bar{\eta}_{\mu\nu}$

- **Weak field, *Post-Minkowskian* expansion:** $h^{(n)} \sim G^n$

- If further impose the non-relativistic approximation $v/c \ll 1$ ***Post-Newtonian*** expansion

– The 2 propagating d of f are obtained by (1) solving $\Box\, h_{ij} = -\dfrac{16\pi G}{c^4} T^{(1)}_{ij}$

(2) and imposing the transverse and traceless conditions: $h^i_i = 0 \ \partial_i h^{ij} = 0$

$$h^{TT}_{ij}(t, \vec{x}) = \Lambda_{ij,kl}(\hat{k}) h_{kl}(t, \vec{x})$$

$$\Lambda^{ab}_{cd}(\hat{k}) = = \delta^a_{(c}\delta^b_{d)} - \frac{1}{2}\delta^{ab}\delta_{cd} - \delta^a_{(c}\hat{k}^b\hat{k}_{d)} - \hat{k}^a\hat{k}_{(c}\delta^b_{d)} + \frac{1}{2}(\delta^{ab}\hat{k}_c\hat{k}_d + \hat{k}^a\hat{k}^b\delta_{cd} + \hat{k}^a\hat{k}^b\hat{k}_c\hat{k}_d)\,.$$

– Stress energy conservation reduces to $\partial^\mu T^{(1)}_{\mu\nu} = 0$:
matter does not interact with the gravitational field.
Sources follow geodesics in flat spacetime

Vacuum solutions: $\Box h_{ij} = 0$ $h^i_i = 0\ \partial_i h^{ij} = 0$

- Wave propagating in $\hat{z}$ direction

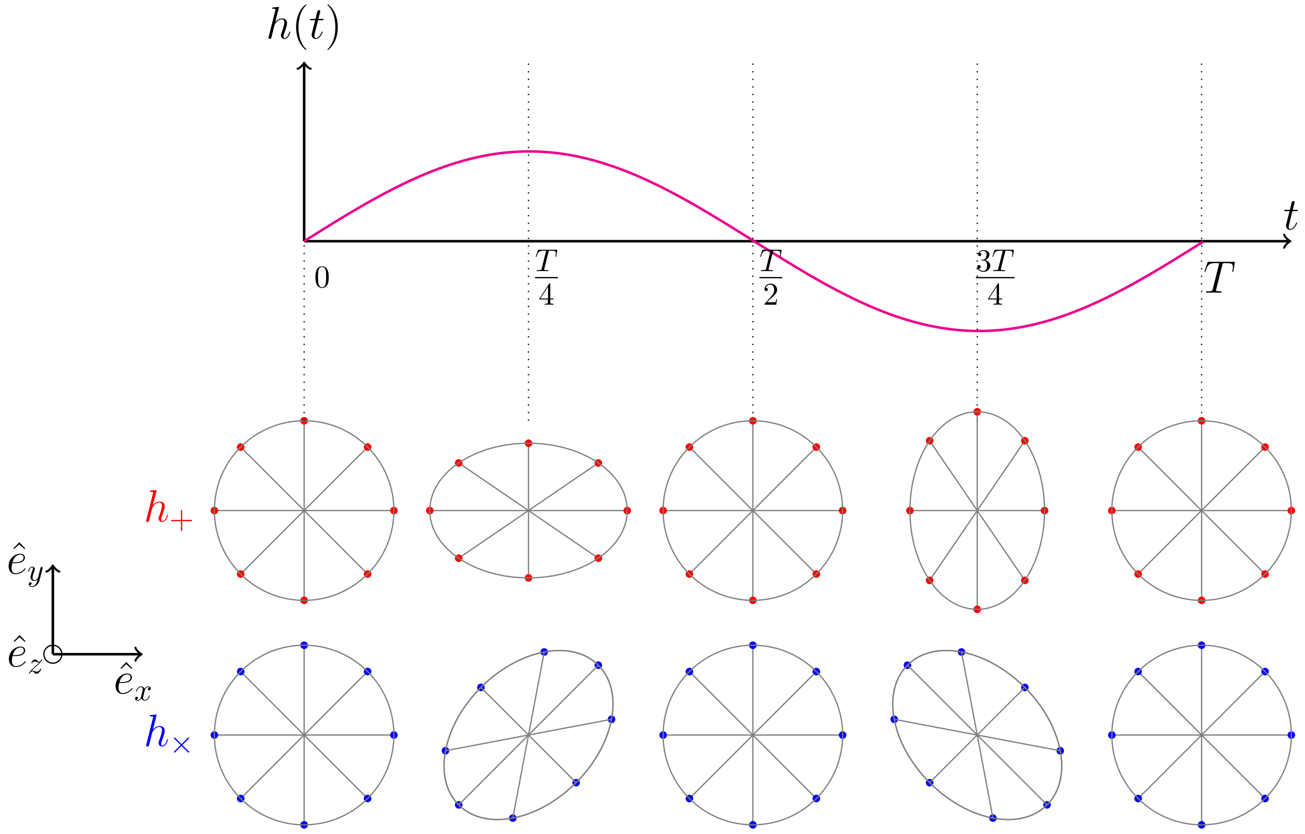
$$h_{ij}(t,z) = e^{2\pi i f(t-z)} \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} = \sum_{P=+,\times} e^{2\pi i f(t-z)} \epsilon^P_{ij} h_P,$$

polarisation tensors $e^+_{ij} = \hat{x}_i\hat{x}_j - \hat{y}_i\hat{y}_j,\ e^\times_{ij} = 2\hat{x}_{(i}\hat{y}_{j)},$

- Resulting perturbed metric:

$$ds^2 = -\,dt^2 + (1 + h_+ \cos k\cdot x)dx^2 + (1 - h_+ \cos k\cdot x)dy^2 + 2h_\times \cos k\cdot x\, dxdy + dz^2\,.$$

- Taking a ring of particles in the (x,y) plane and $\lambda_{GW} \gg L_0$ (ignore space dependence of h_{ij})



Vacuum solutions: $\Box h_{ij} = (\partial_t^2 - \nabla^2)h_{ij} = 0 \; h_i^i = 0 \; \partial_i h^{ij} = 0$

- Wave propagating in $\hat{z}$ direction

$$h_{ij}(t,z) = e^{2\pi i f(t-z)} \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} = \sum_{P=+,\times} e^{2\pi i f(t-z)} \epsilon_{ij}^P h_P,$$

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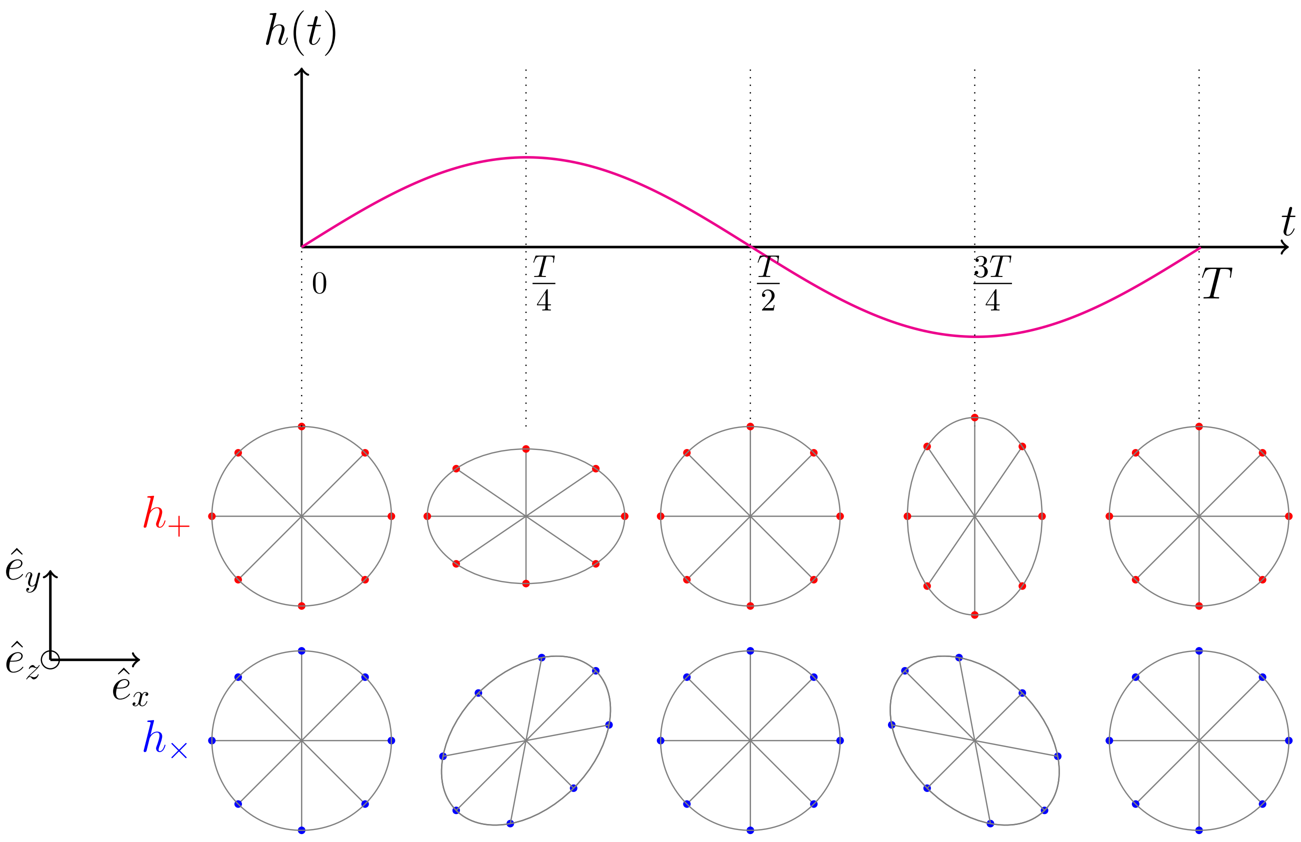
Taking a ring of particles in the (x,y) plane and $\lambda_{GW} \gg L_0$ (ignore space dependence of h_{ij})



$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = 0$$

$h_{ij} \sim a^{-1}$ on sub Hubble scales $\lambda \ll H^{-1}$

$$f = \frac{f_e}{1+z},\ \ dt = (1+z)dt_e$$

- For a source localised in space, of characteristic size d and at distance $|\vec{x}| = R \gg d$

$$h_{ij}(t,\vec{x}) = \frac{4G}{c^4}\int_{source} d^3y \, \frac{T_{ij}(t - \frac{1}{c}|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|} \, .$$

In general no analytic solution.

Approx 1: consider distances $R = |\vec{x}| \gg d$ large compared to the size of the source:

$$h_{ij}(t,\vec{x}) \sim \frac{4G}{c^4 R}\int_{source} d^3y \, T_{ij}(t - \frac{R}{c} - \frac{\vec{y}\cdot\overrightarrow{N}}{c},\vec{y})$$

$\vec{x} = R\overrightarrow{N}$

Approx 2: typical velocities $v/c \ll 1.$ On using $\partial_\mu T^{\mu\nu} = 0$ leads to

$$h_{ij}(t,\vec{x}) \sim \frac{2G}{c^4 R}\ddot{Q}_{ij}(t - R/c) \qquad Q_{ij} = \frac{1}{c^2}\int d^3y T^{00}(t,\vec{y})(y_i y_j - \frac{1}{3}y^2\delta_{ij})$$

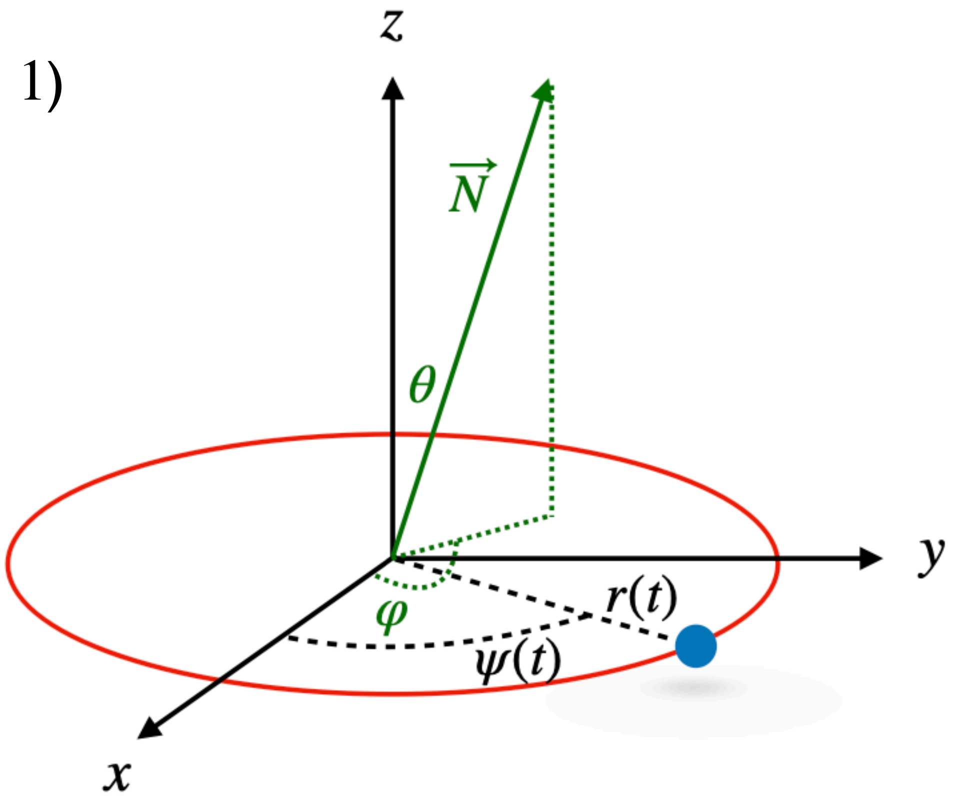
from which one can extract $h_{+,\times}$

- GWs carry energy momentum and angular momentum from the source

$$\frac{dE_{GW}}{dt} = \frac{c^3}{32\pi G}\oint_{S_2} \dot{h}^{TT}_{ab}\dot{h}^{ab}_{TT} dS \qquad \frac{J^a_{GW}}{dt} \overset{v \ll c}{=} \frac{2G}{5c^5}\epsilon^{abc}\ddot{Q}_{bd}\dddot{Q}_c{}^d|_{t_R} \, .$$

$$\overset{v \ll c}{=} -\frac{G}{8c^5}\dddot{Q}_{ab}\dddot{Q}^{ab}$$

Example: Binary systems



— assume source in the (x, y) plane satisfies Newtonian equations $(v/c \ll 1)$
ie. Keplers orbits, eccentricity e,
semi-latus rectum p
total energy $E \propto e^2 - 1$
angular momentum L

Circular orbits: $e = 0$, bound elliptical orbits: $e < 1$
unbound hyperbolic orbits: $e > 1$

— Straightforward to calculate Q_{ij} as well as GW energy and angular momentum radiation

— use conservation of energy and angular momentum

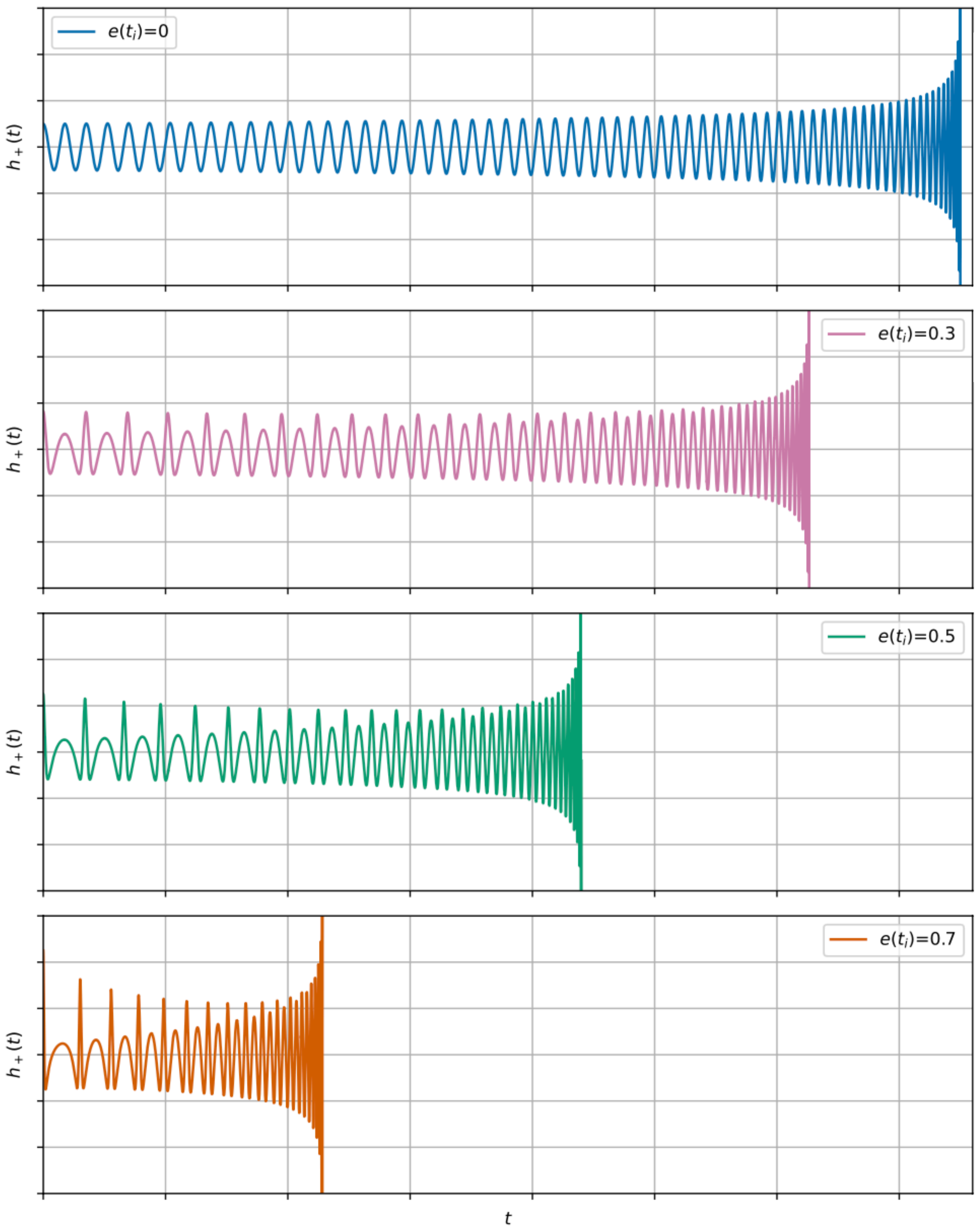
$$\frac{dE}{dt} = -\frac{dE_{GW}}{dt}, \qquad \frac{dL}{dt} = -\frac{dJ^z_{GW}}{dt}$$

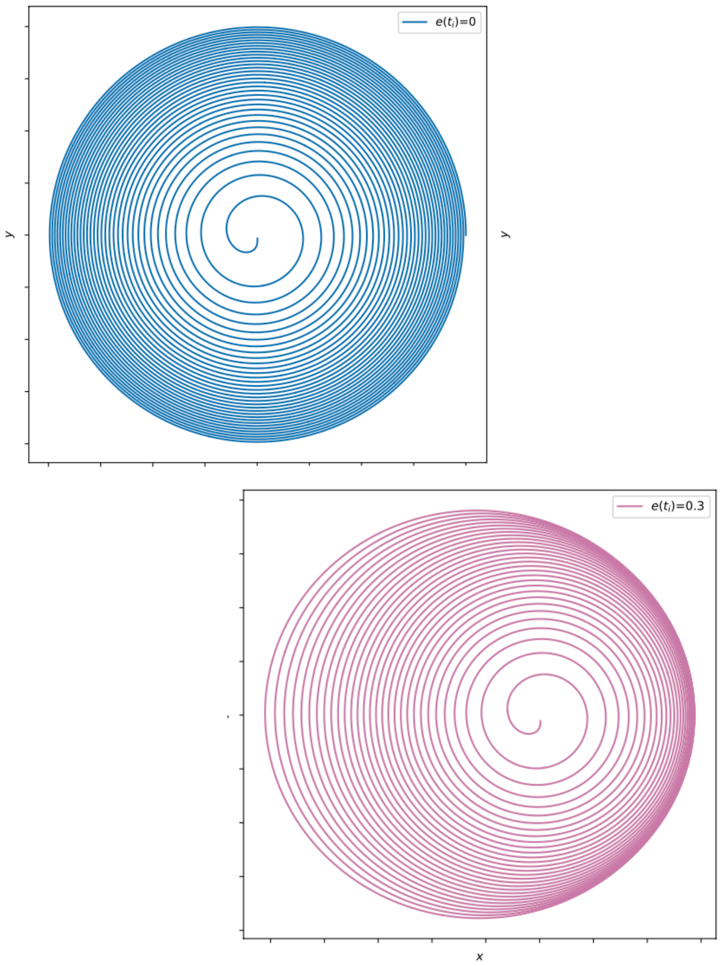
To determine $e(t), p(t)$ and $h_{+,\times}(t, \vec{N})$

$$h_{ij}(t, \vec{x}) \sim \frac{2G}{c^4 R}\ddot{Q}_{ij}(t - R/c)$$

$$Q_{ij} = \frac{1}{c^2}\int d^3y T^{00}(t, \vec{y})(y_i y_j - \frac{1}{3}y^2\delta_{ij})$$

$$\frac{J^a_{GW}}{dt} \overset{v \ll c}{=} \frac{2G}{5c^5}\epsilon^{abc}\ddot{Q}_{bd}\dddot{Q}_c{}^d|_{t_R}.$$



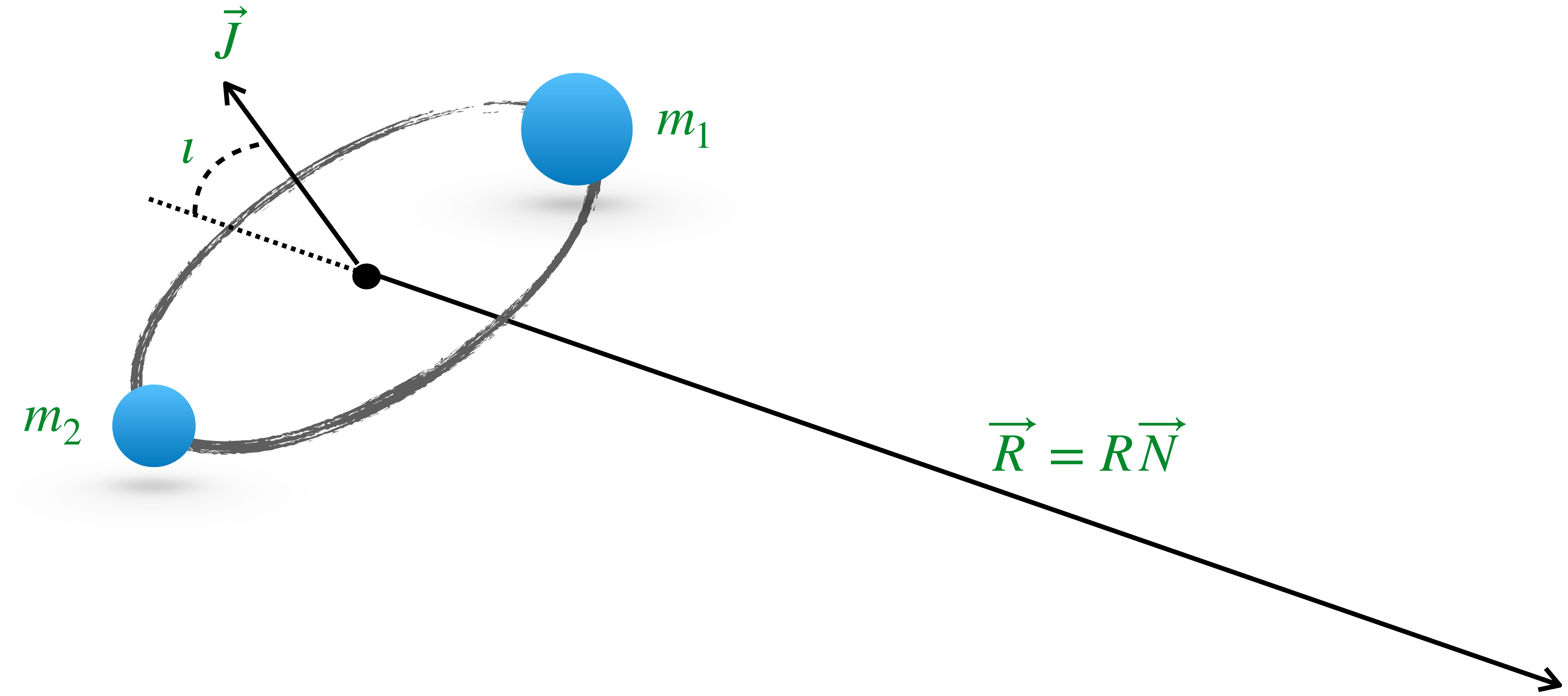


For circular orbits,
the solutions are analytical

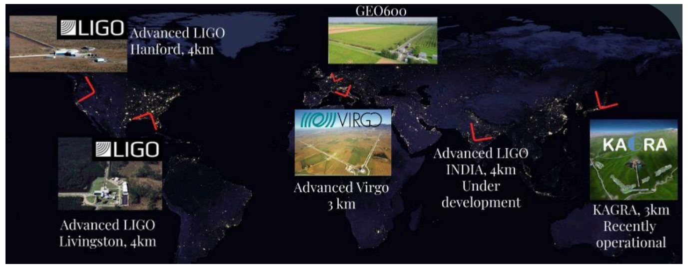
From [Speziale and Steer]

FIG. 14: *Four waveforms, in the lowest order PN expansion, with initial values of eccentricity given by* $e = 0, 0.3, 0.5$ *and* 0.7*. Most GW power is emitted near the pericenter where the orbital velocity is the largest. Also since more GW radiation is emitted as e increases, the merger occurs earlier.*

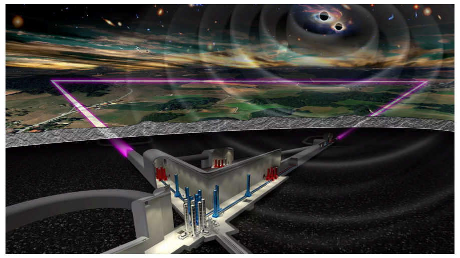
On characteristic scales for binary systems, and detector reach



LVK: $10\ \text{Hz} \lesssim f_{\text{GW}} \lesssim 5\ \text{kHz}$



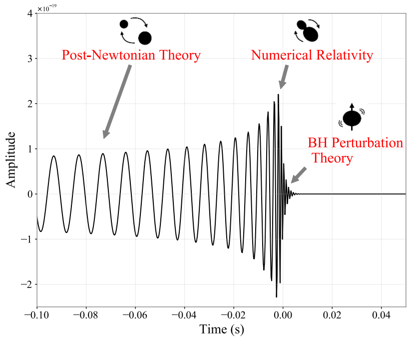
ET: $1\ \text{Hz} \lesssim f_{\text{GW}} \lesssim 10^4\ \text{kHz}$



LISA: $10^{-4}\ \text{Hz} \lesssim f_{\text{GW}} \lesssim 1\ \text{Hz}$

$R = d = d(z, H_0, \dots)$

Binaries on Circular orbits: orders of magnitude



– Inspiral phase: $f_{\rm GW} = \frac{1}{\pi}\left(\frac{G\mathscr{M}}{c^3}\right)^{-5/8}\left(\frac{5}{256\tau}\right)^{3/8}$

with chirp mass $\mathscr{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

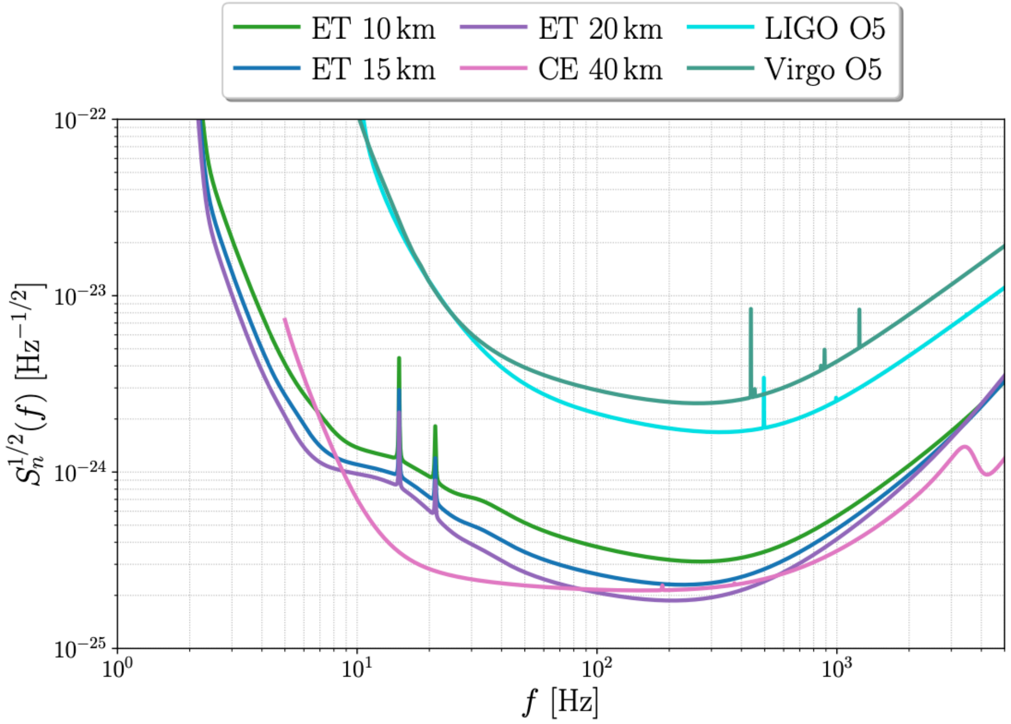
time to coalescence $\tau = t - t_c$

– Assuming merger at ISCO $a = \frac{6Gm}{c^2}$ with $m = m_1 + m_2$

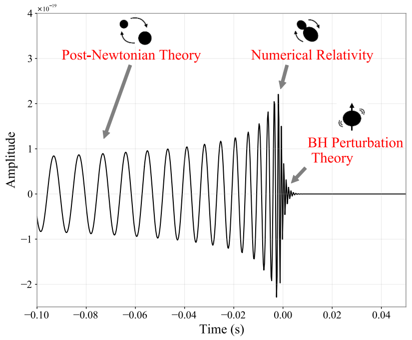
=> *Merger frequency*:

$$f_{\rm merger} = \frac{1}{6^{3/2}}\left(\frac{c^3}{Gm}\right)$$

- BNS, $m_{1,2} \sim 1.4 M_\odot$ $f_{\rm merger} \sim 1.5\,{\rm kHz}$
- stellar mass BHs, $m_{1,2} \sim 35 M_\odot$ $f_{\rm merger} \sim 60\,{\rm Hz}$



Binaries on Circular orbits: orders of magnitude



– Inspiral phase: $f_{\rm GW} = \frac{1}{\pi}\left(\frac{G\mathscr{M}}{c^3}\right)^{-5/8}\left(\frac{5}{256\tau}\right)^{3/8}$ with chirp mass $\mathscr{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

time to coalescence $\tau = t - t_c$

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=> <u>*Merger frequency*</u>:

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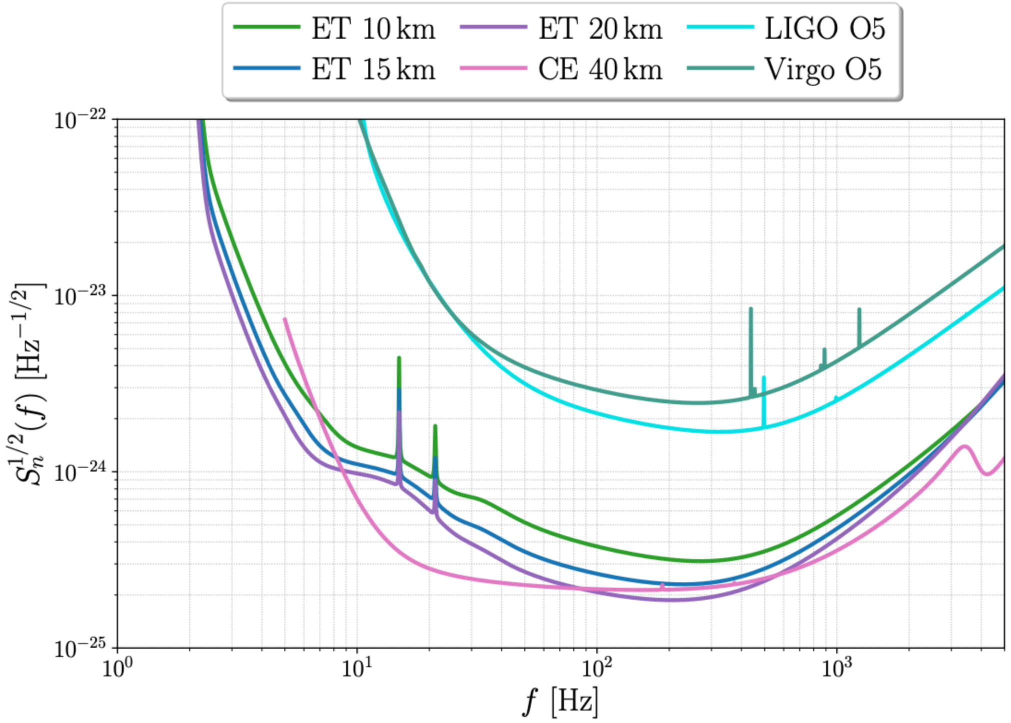
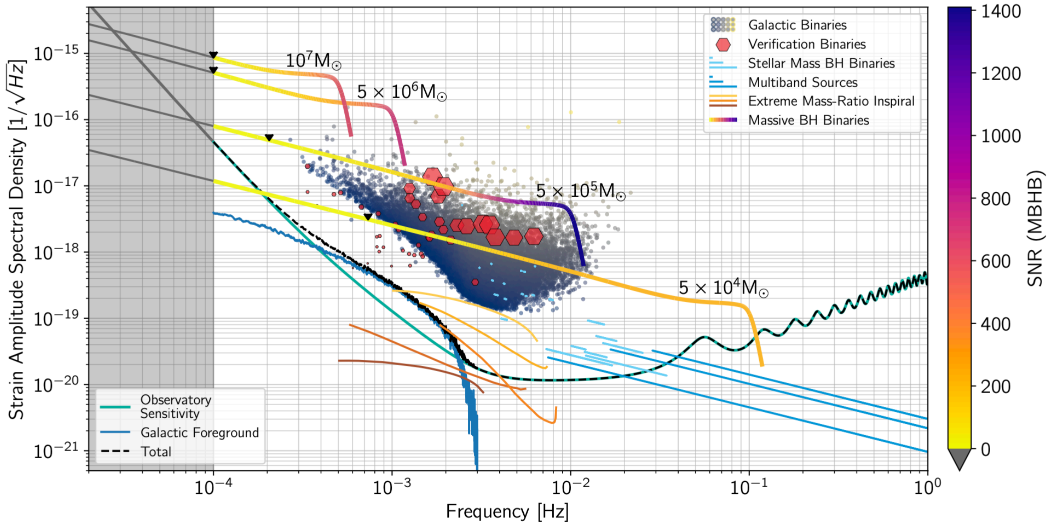
- stellar mass BHs, $m_{1,2} \sim 35 M_\odot$ $f_{\rm merger} \sim 60\,{\rm Hz}$
- Supermassive BBHs, $m_{1,2} \sim 10^6 M_\odot$ $f_{\rm merger} \sim 10^{-3}\,{\rm Hz}$

– *nHz frequencies (PTA) do* ***not*** *correspond to of SMBHB coalescence, but emitted by binaries with masses* $10^7 - 10^{10} M_\odot$*,on broad orbit (period ~ year(s))*





- *Time to merger*

If GWs enter frequency band of a detector at observed frequency f_{low}

$$T \sim 10^{-3} f_{\text{low}}^{-8/3} \left(\frac{c^3}{G\mathcal{M}}\right)^{5/3}$$

$$f_{\text{merger}} \gg f_{\text{low}}$$

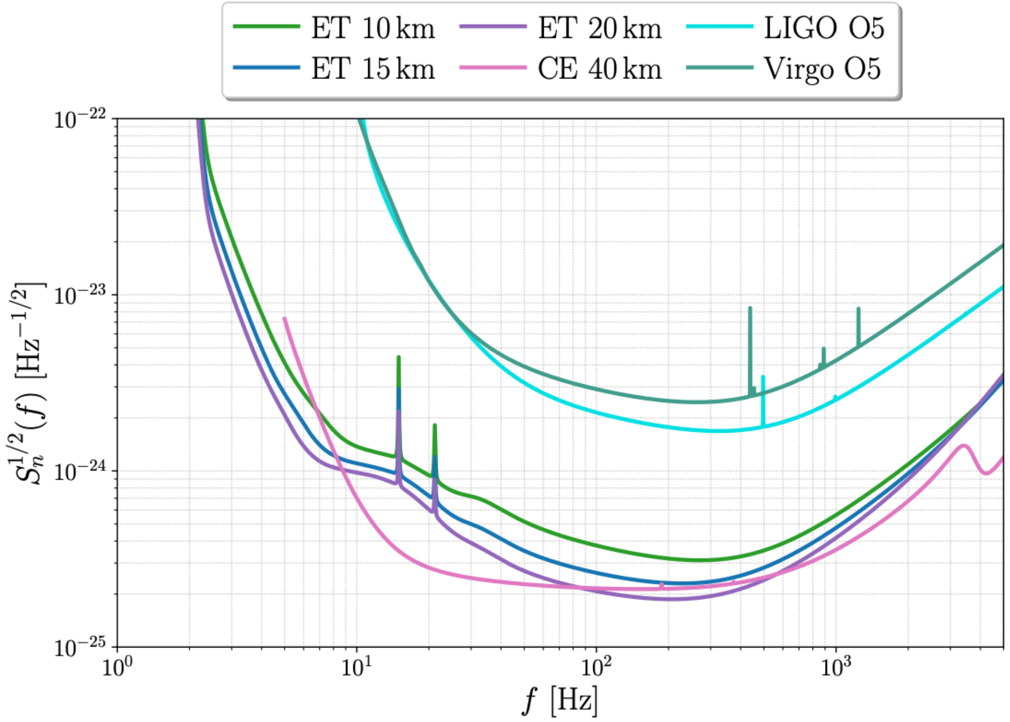
- BNS, entering LIGO-Virgo detector window at observed frequency $f_{\text{low}} \sim 20\,\text{Hz}$ $T \sim 4\,\text{min}$

  $m_{1,2} \sim 1.4 M_\odot$ $f_{\text{merger}} \sim 1.5\,\text{kHz}$

- BNS, entering ET detector window at observed frequency $f_{\text{low}} \sim 1\,\text{Hz}$ $T \sim 5\,\text{days}$

=> cannot neglect the rotation of the earth

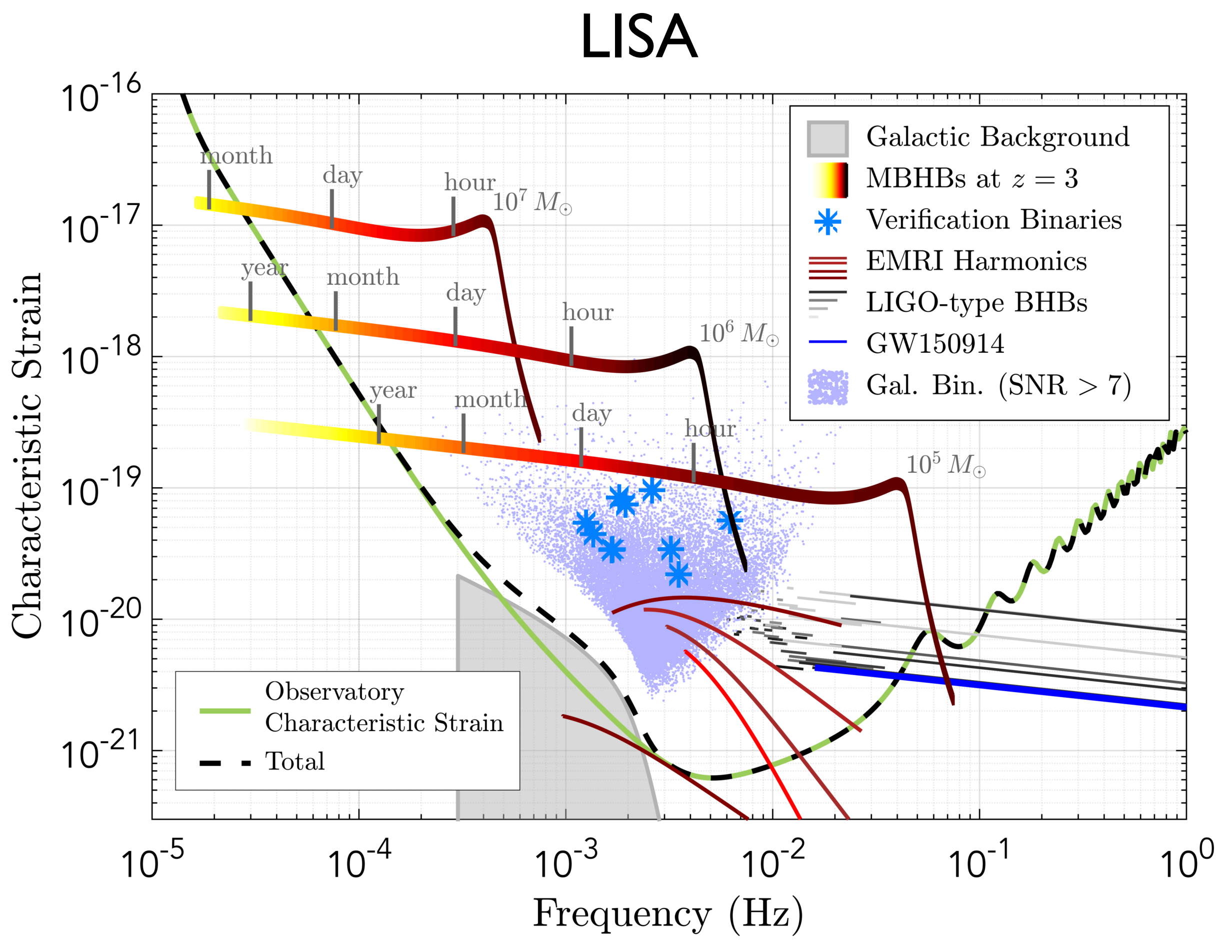
=>Given the merger rates for BNS, BBH and BH-NS, expect a typical BNS signal will be overlapped by a number of BBH signals, which may merge at similar times



- stellar mass BHs entering LIGO-Virgo detector window

  $m_{1,2} \sim 35 M_\odot$ $T \sim 0.1\,\text{s}$

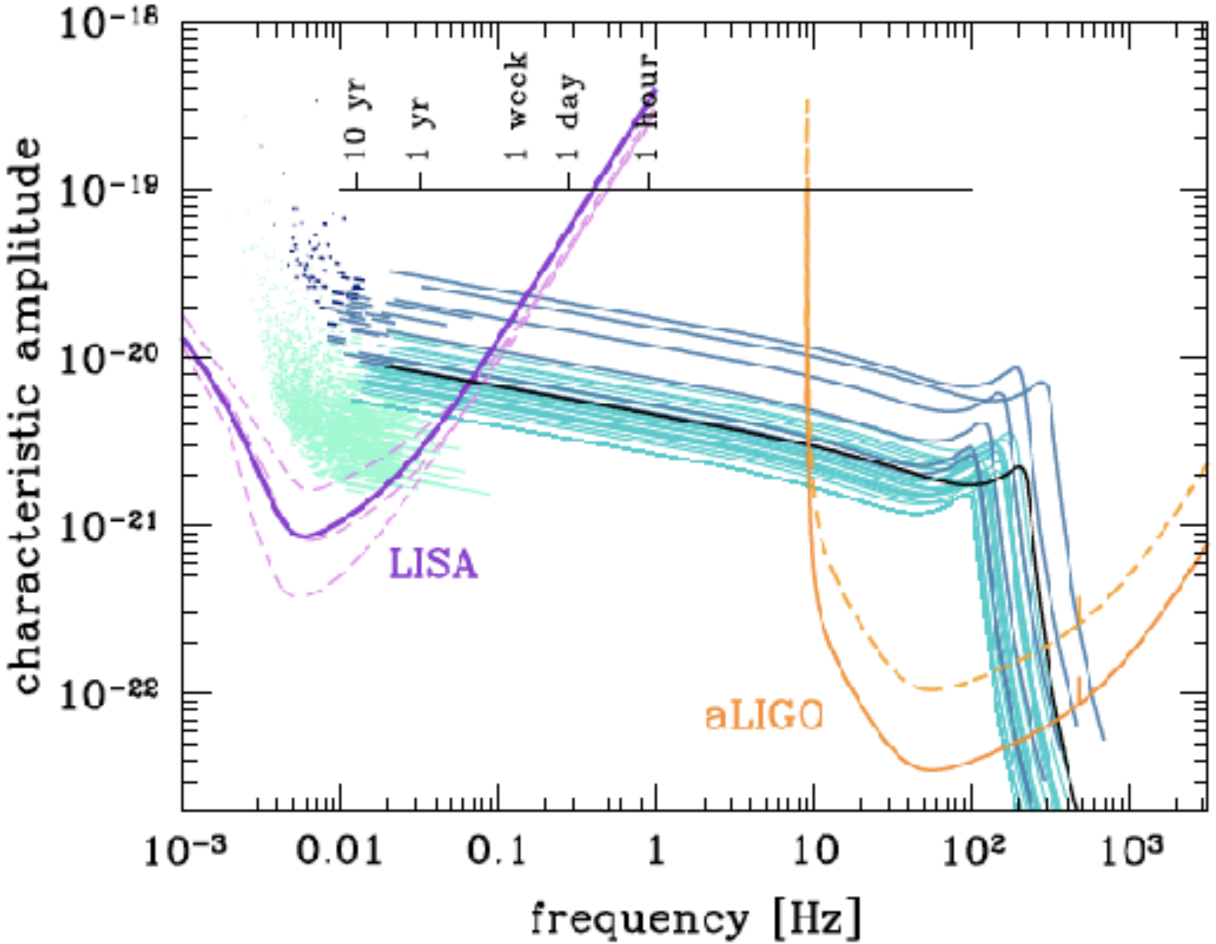
  $f_{\text{merger}} \sim 60\,\text{Hz}$



- Supermassive BBHs,

$$m_{1,2} \sim 10^6 M_\odot$$

$$T \sim 1\,\mathrm{month}$$



- stellar mass BHs entering LISA detector window

$$f_{\rm low} \sim 10^{-2}\,\mathrm{Hz}$$

$$T \sim 20\,\mathrm{yrs}$$

- ***Amplitude/distance***

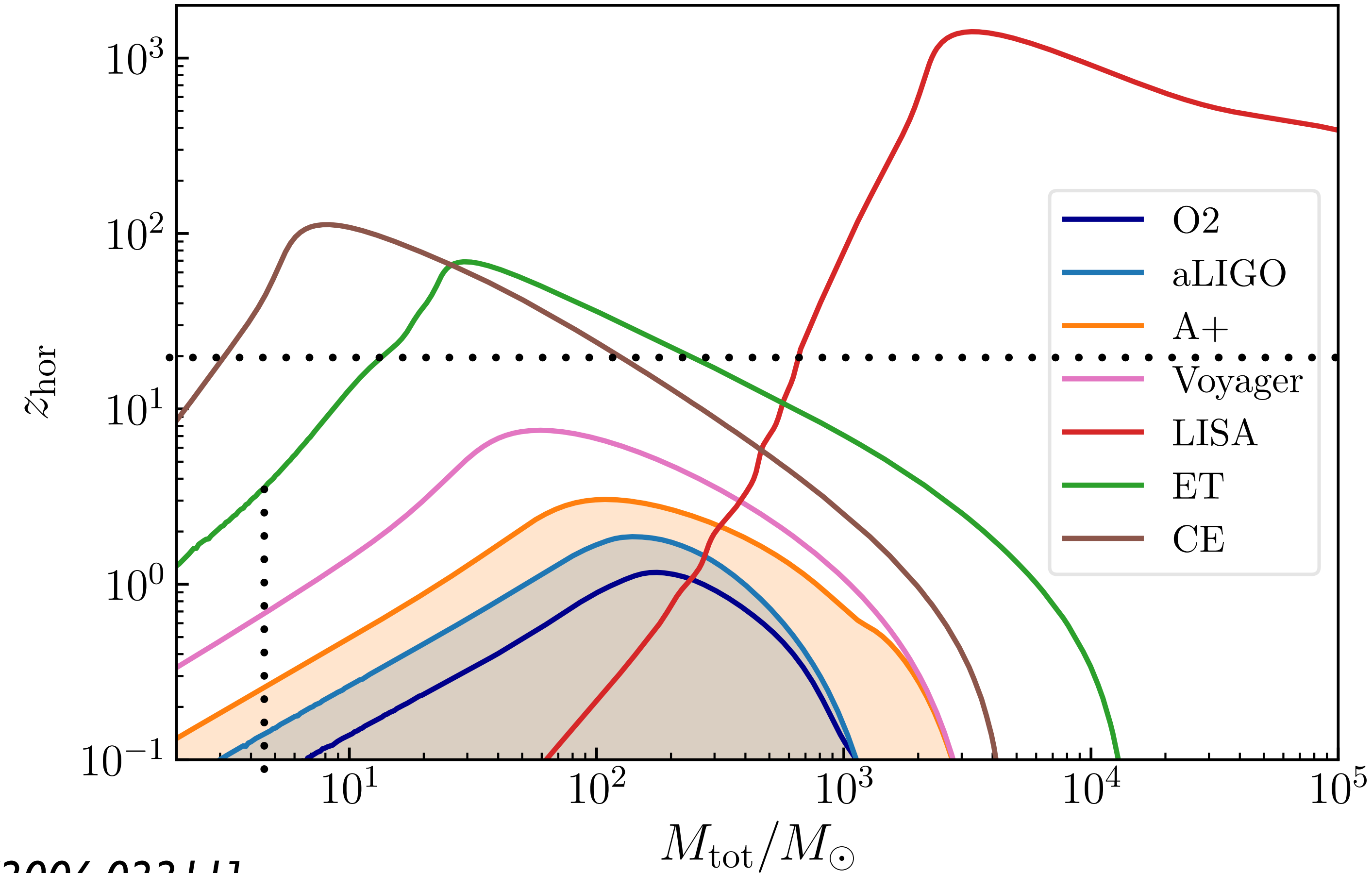
$$\boxed{h \sim \frac{4}{R}\left(\frac{G\mathcal{M}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm GW}}{c}\right)^{2/3}}$$

- stellar mass BHs in LIGO-Virgo

$m_{1,2} \sim 35 M_\odot$ $f_{\rm merger} \sim 60\,{\rm Hz}$ $h \sim 10^{-21}$ gives $R \sim 400\,{\rm Mpc}$

converted to a redshift *assuming* the Planck values of cosmological parameters $dH_0 \sim c\,z$

Horizon redshift as a function of total source frame mass for an SNR detection threshold of rho=8. For LISA assumes 4 yrs obsv.



z>20; dark era preceding birth of first stars: any detected BHs must be primordial

[2006.02211]

Conclusions:
1/ LVK,ET ==> BNS+ stellar mass and intermediate mass BHs
2/ LISA ==> merger of supermassive BHs
3/ cannot neglect expansion of the universe