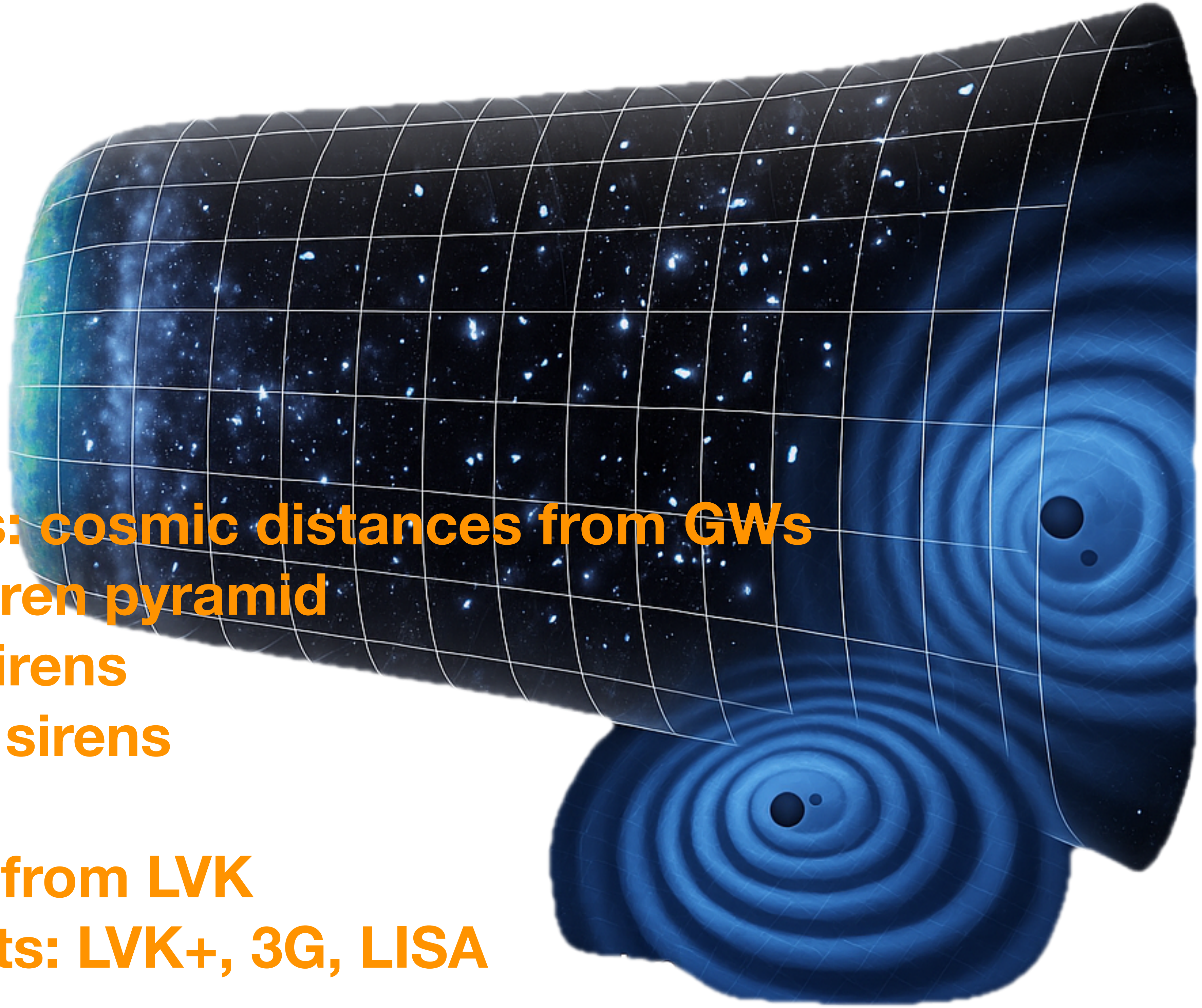
Gravitational wave cosmology: measurements and observations

Nicola Tamanini

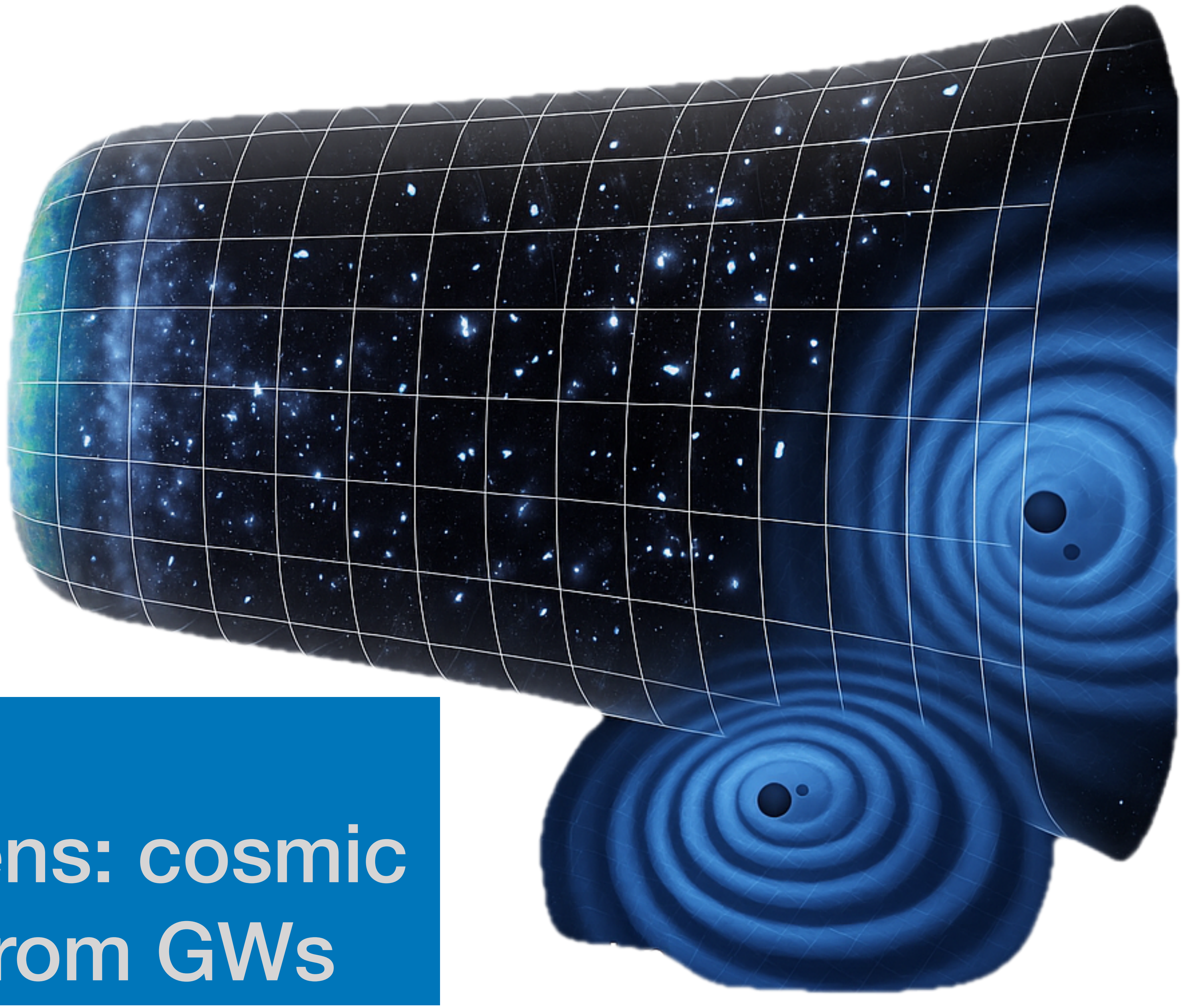
Laboratoire des 2 infinis - Toulouse
CNRS / IN2P3 / Univ. de Toulouse

2025

Outline



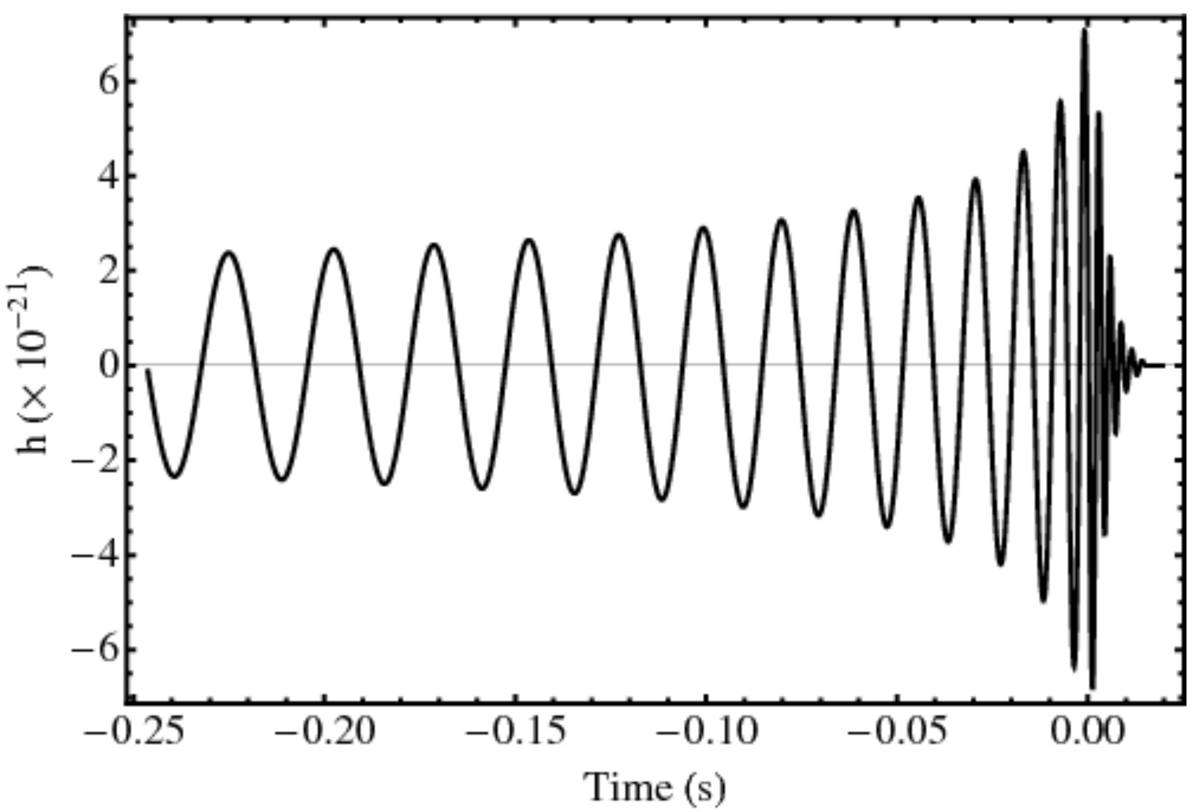
1. Standard sirens: cosmic distances from GWs
2. The standard siren pyramid
3. Dark spectral sirens
4. Dark statistical sirens
5. Brigth sirens
6. Current results from LVK
7. Future prospects: LVK+, 3G, LISA



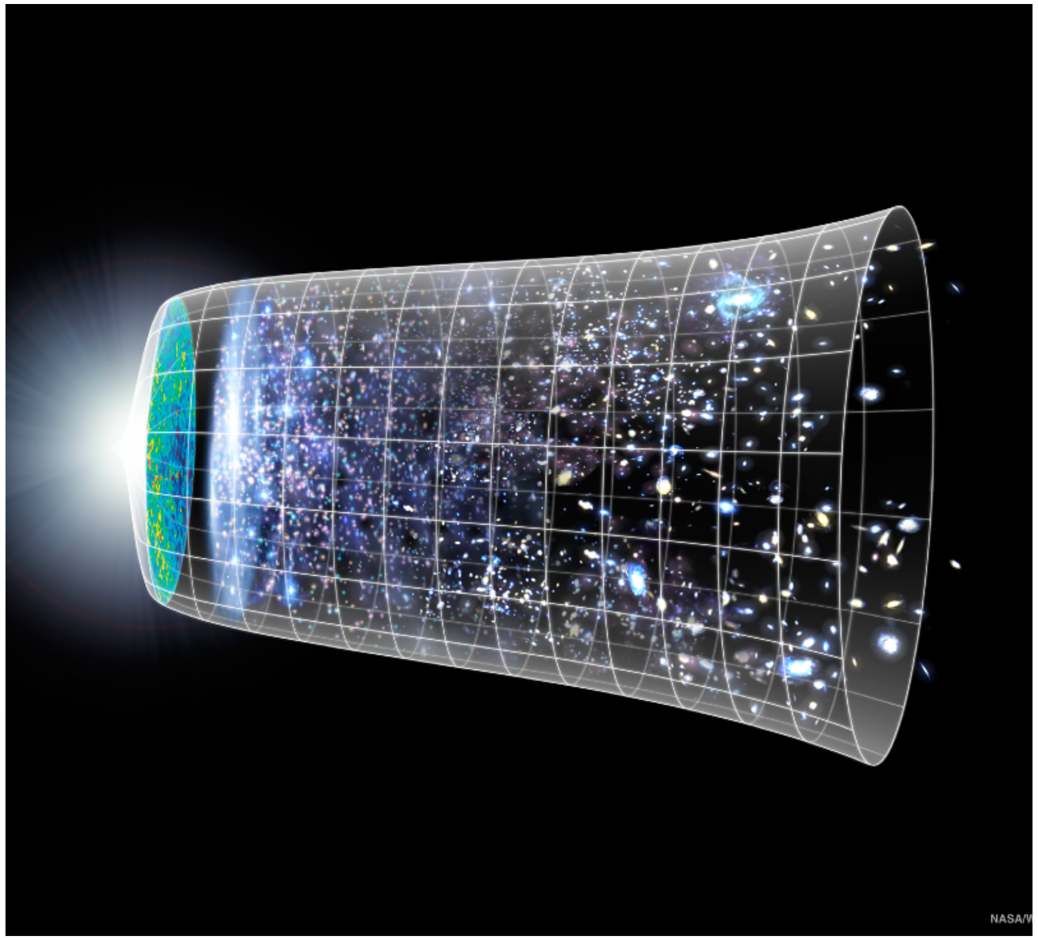
1
Standard sirens: cosmic distances from GWs

Standard sirens: cosmic distances from GWs

How do we extract cosmological information from gravitational wave observations?



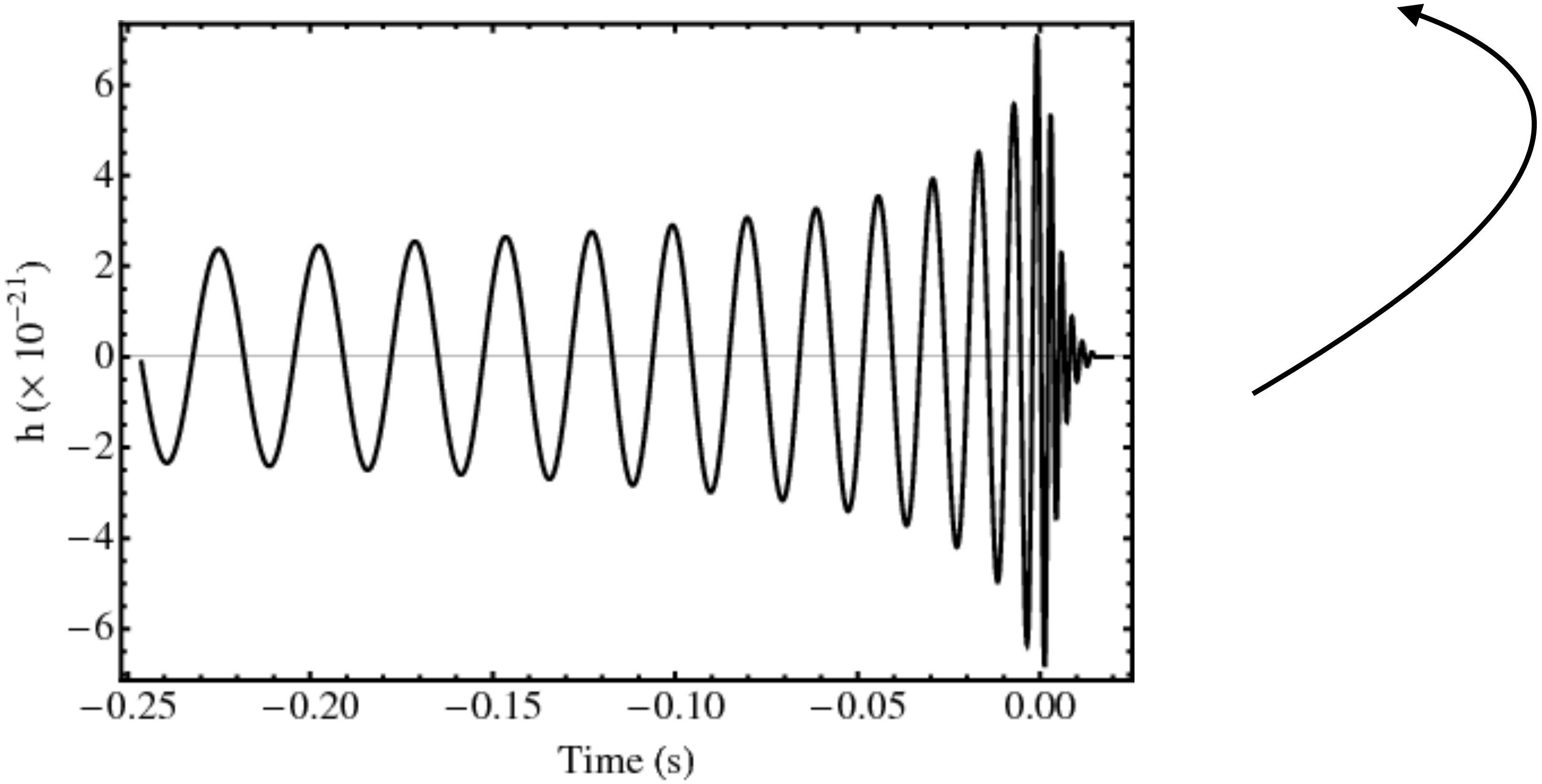
?



Standard sirens: cosmic distances from GWs

Let's now consider the GW emitted by a binary system in circular orbit, which we can rewrite as (only one polarisation considered for simplicity)

$$h_\times(t_s) = \frac{4}{r}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},s}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_c}{c^3}\right)^{-5/8}\tau_s^{5/8}+\Phi_0\right]$$



Standard sirens: cosmic distances from GWs

Let's now consider the GW emitted by a binary system in circular orbit, which we can rewrite as (only one polarisation considered for simplicity)

$$h_\times(t_s) = \frac{4}{r}\left(\frac{G\mathcal{M}_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},s}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_c}{c^3}\right)^{-5/8}\tau_s^{5/8}+\Phi_0\right]$$

If the source is at cosmological distances we need to take into account that the expansion of the universe stretches distances and redshifts time and frequency

$$r \mapsto a(t_o)r \qquad f_s = (1+z)f_o \qquad dt_s = dt_o/(1+z)$$

implying that the waveform at the observer becomes

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

Luminosity distance

$$d_L = (1+z)a(t_o)r$$

Redshifted chirp mass

$$\mathcal{M}_{cz} = (1+z)\mathcal{M}_c$$

[Maggiore, vol.1 (2008)]

Standard sirens: cosmic distances from GWs

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

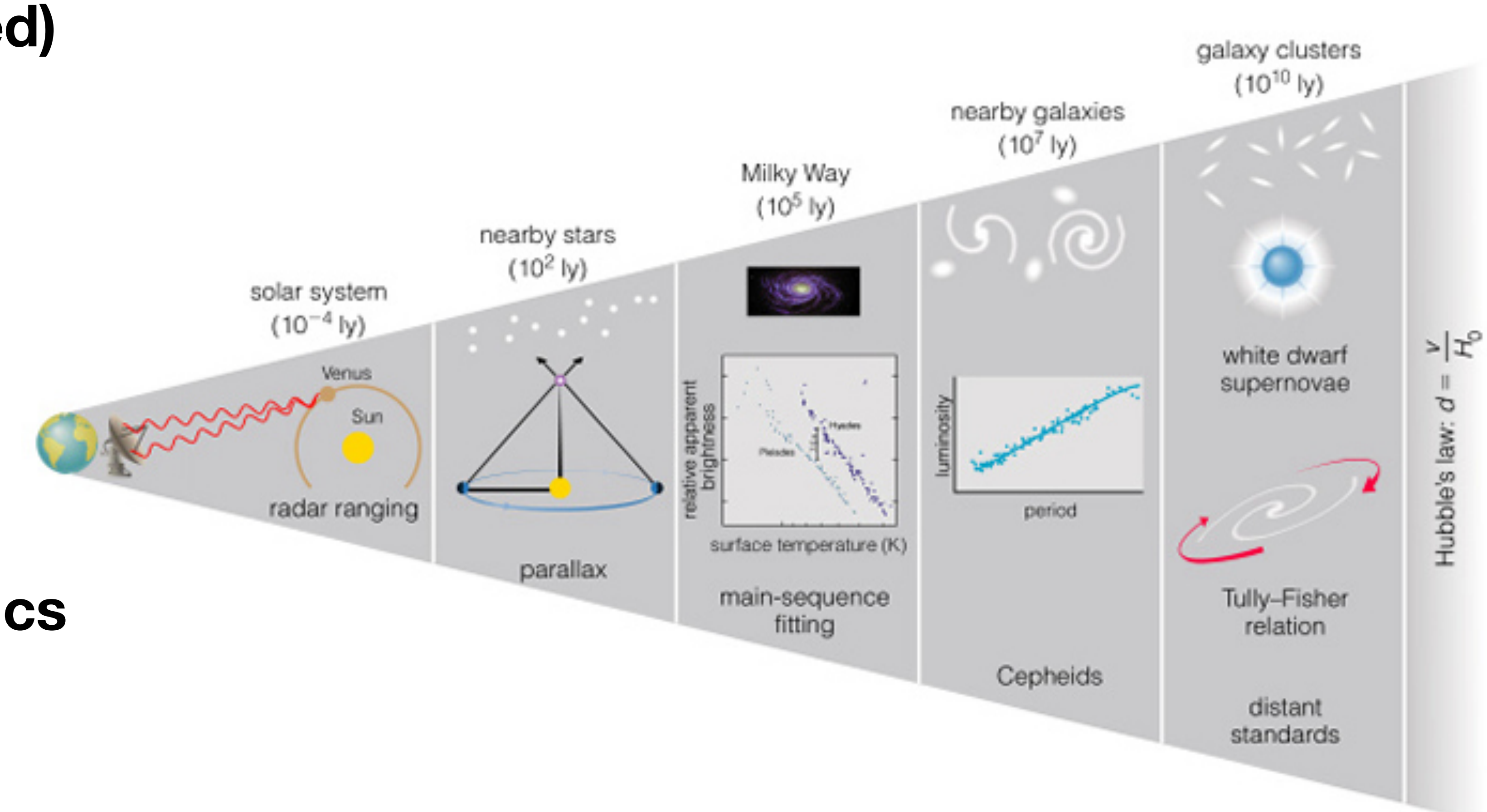
This is the very waveform (in time-domain at the lowest Newtonian order) used to detect GWs and measure the parameters of the system

Most important for cosmology, one can measure the luminosity distance d_L of the source directly from the GW signal without relying on the *cosmic distance ladder* (only GR has been assumed)

This means that GW binaries are absolute cosmological distance indicators!

↓

Free of possible systematics due to distance ladder calibration



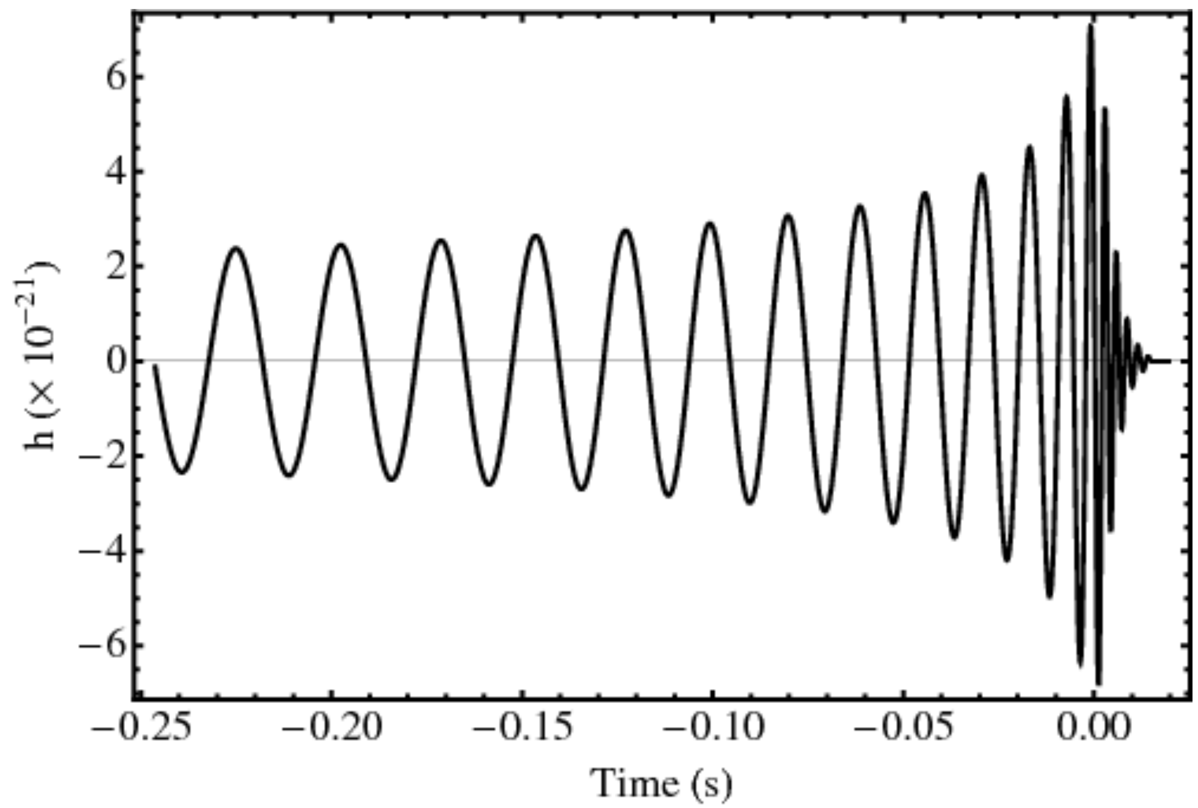
Standard sirens: cosmic distances from GWs

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

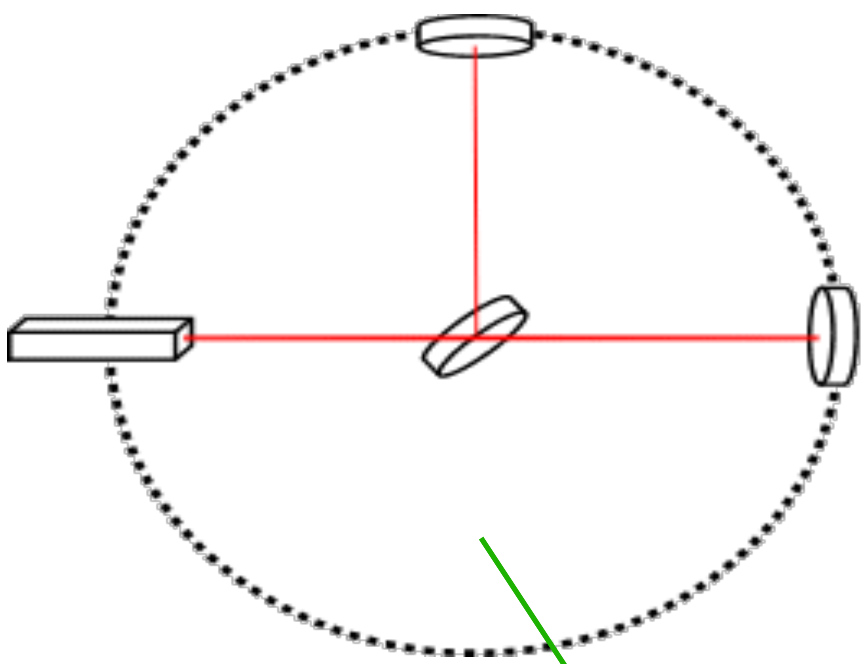
This is the very waveform (in time-domain at the lowest Newtonian order) used to detect GWs and measure the parameters of the system

Most importantly for cosmology, one can measure the luminosity distance d_L of the source directly from the GW signal without relying on the *cosmic distance ladder* (only GR has been assumed)

How do we extract the luminosity distance from GW observations?



Standard sirens: cosmic distances from GWs



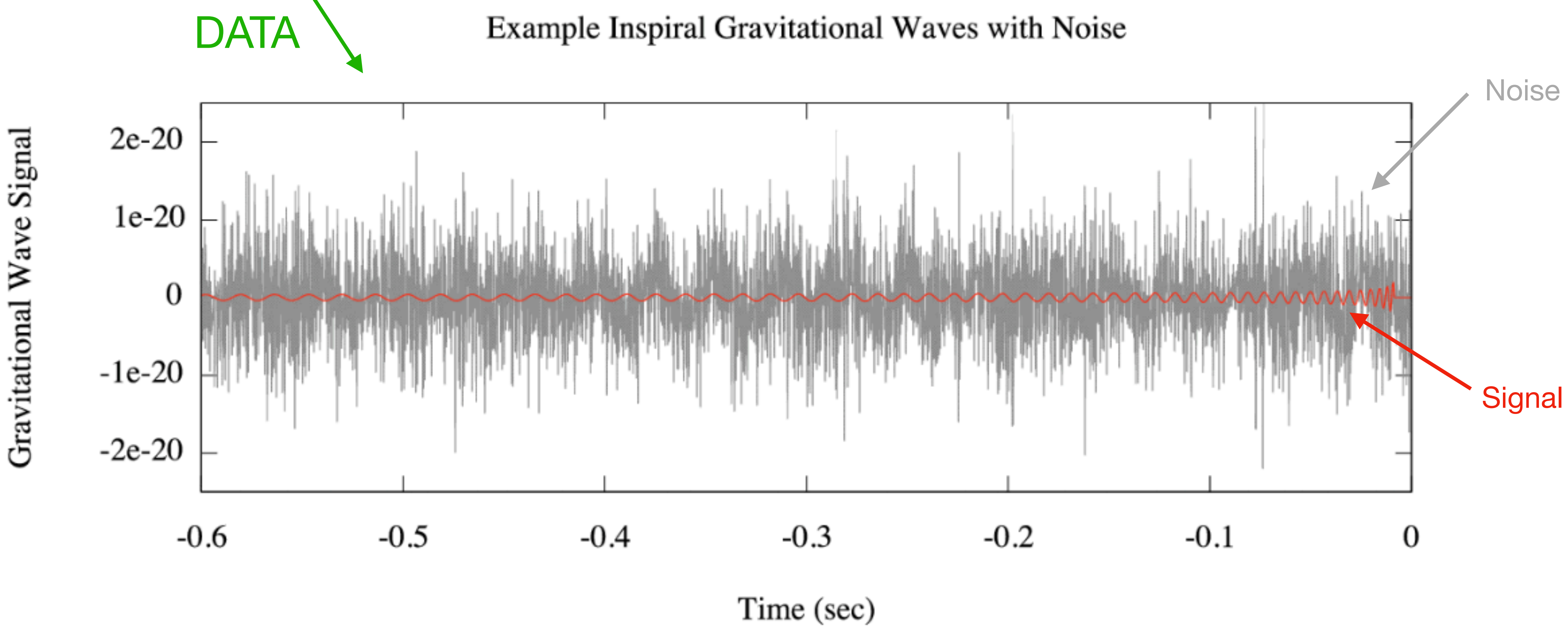
How to extract the signal from observations?

MATCHED FILTERING

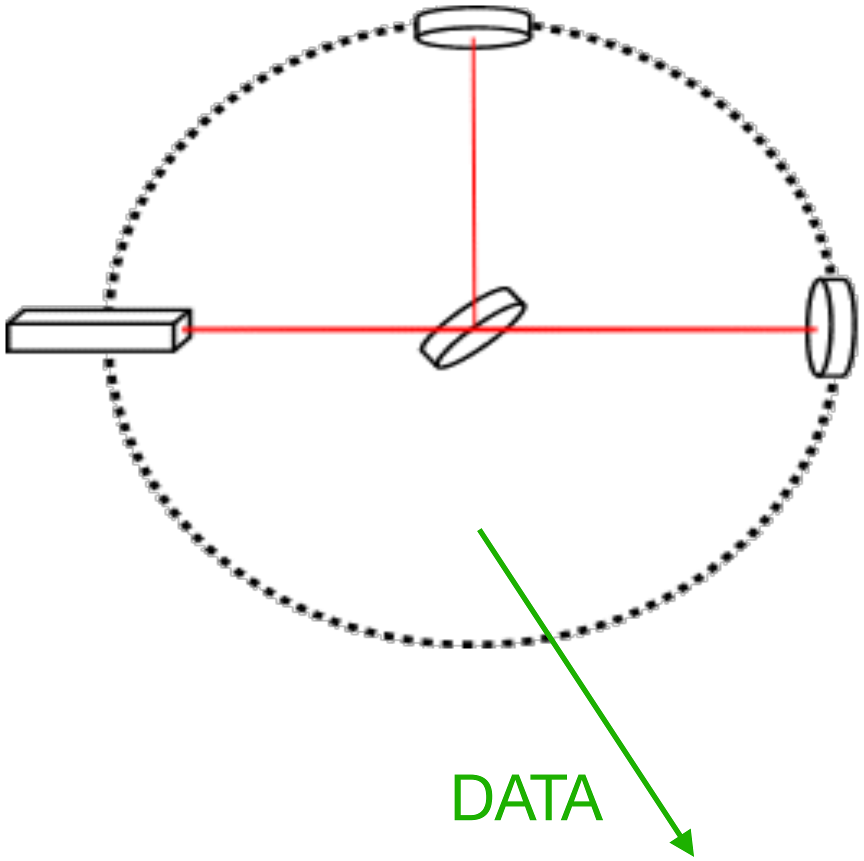
Model the signal theoretically

Find and characterise the signal in the data efficiently

DATA



Standard sirens: cosmic distances from GWs



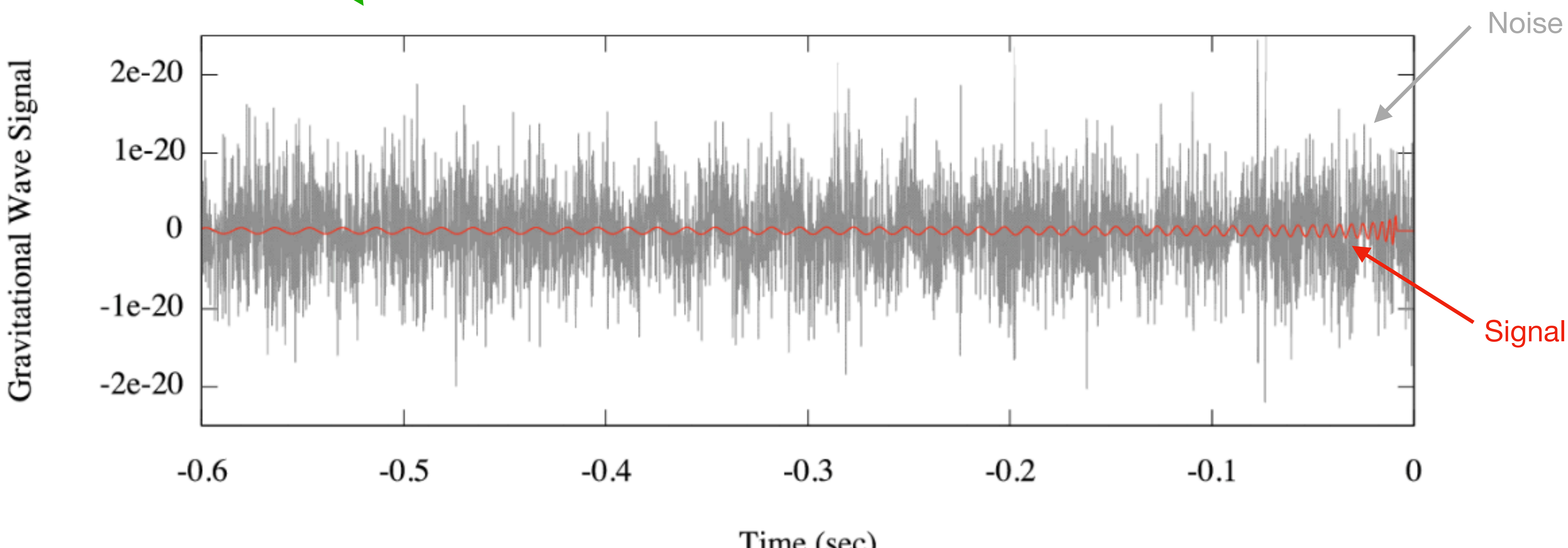
How to extract the signal from observations?

The noise distribution is assumed Gaussian:

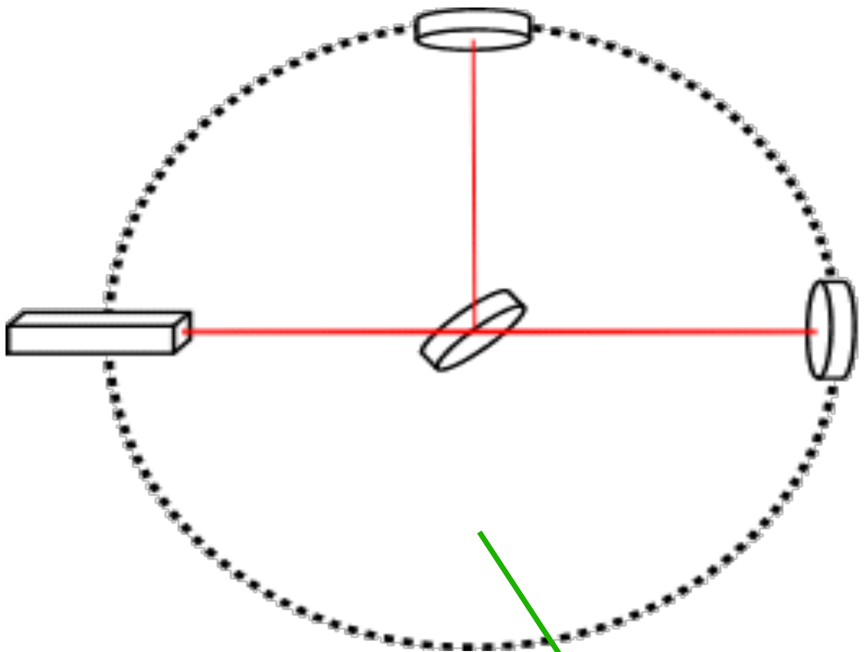
$$p(d|\vec{\theta}) \propto \exp(< n|n >) \propto \exp(< d - h(\vec{\theta})|d - h(\vec{\theta}) >)$$

where $$< a(f)|b(f) > = 2\int df \frac{a(f)b^*(f) + a^*(f)b(f)}{S(f)}$$

Example Inspiral Gravitational Waves with Noise



Standard sirens: cosmic distances from GWs



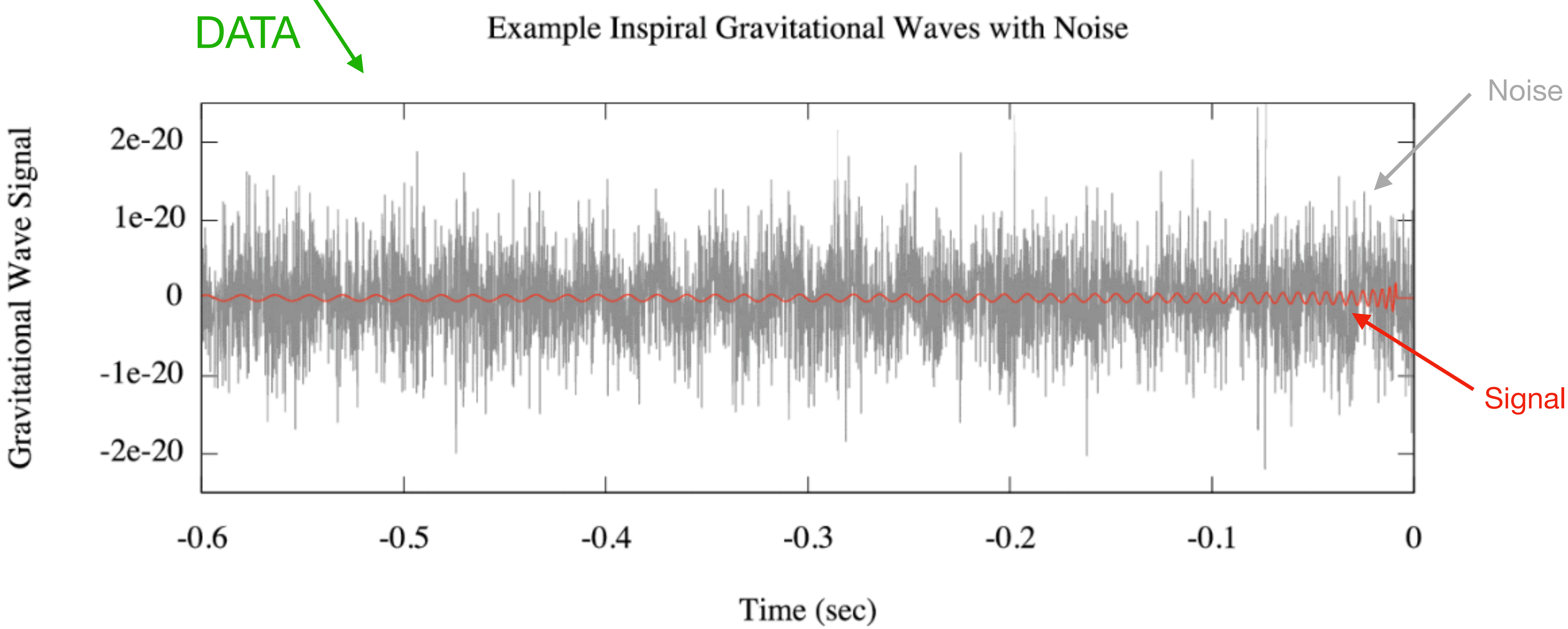
How to extract the signal from observations?

The noise distribution is assumed Gaussian:

$$p(d|\vec{\theta}) \propto \exp(< n|n >) \propto \exp(< d - h(\vec{\theta})|d - h(\vec{\theta}) >)$$

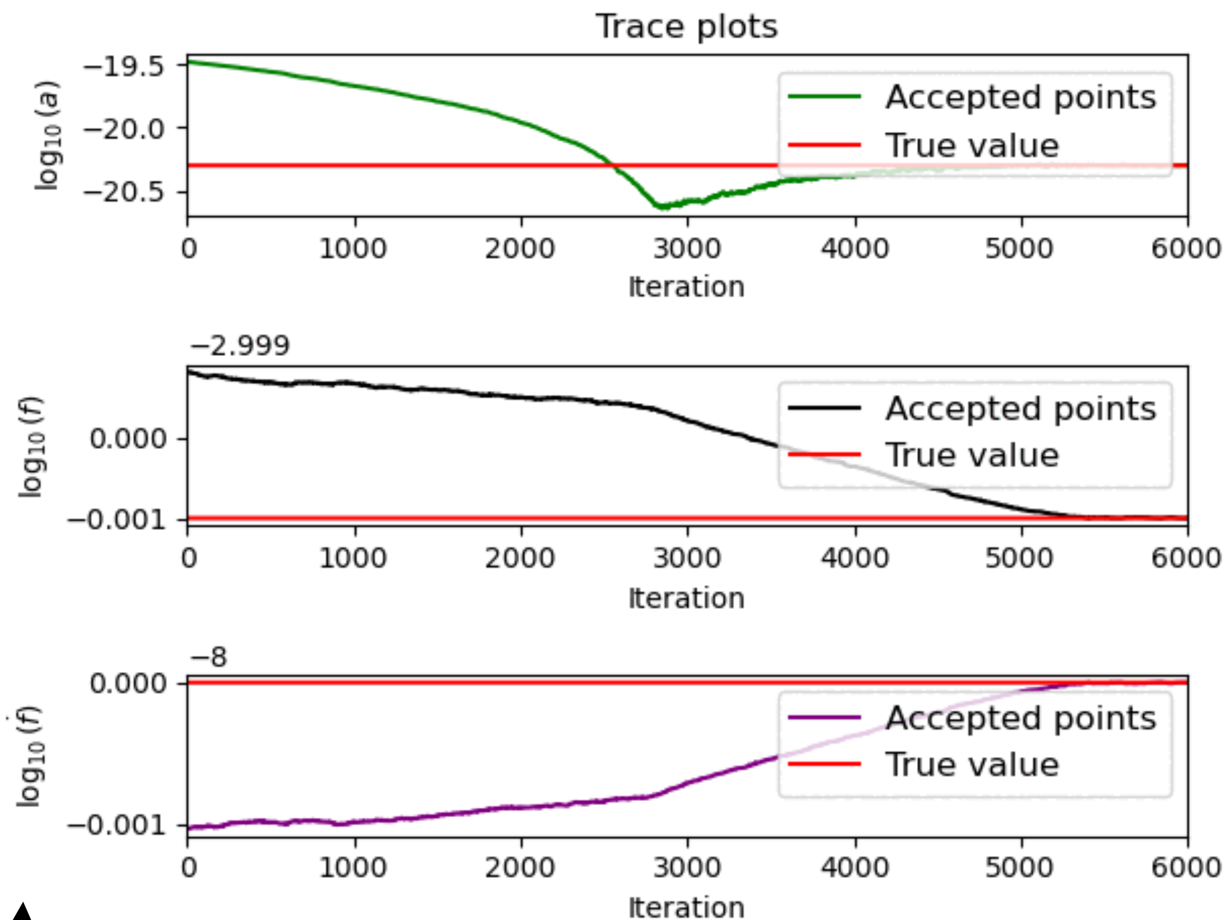
where $$< a(f)|b(f) > = 2\int df \frac{a(f)b^*(f) + a^*(f)b(f)}{S(f)}$$

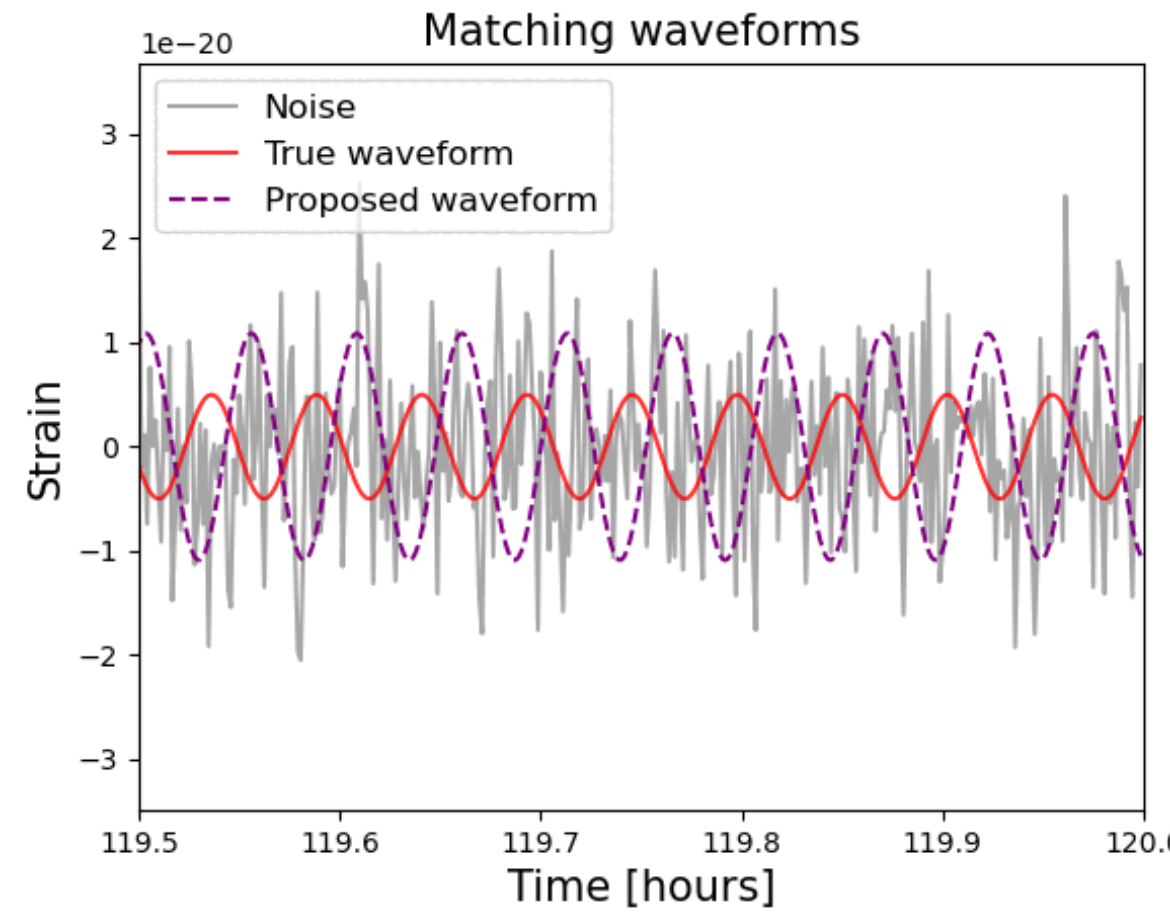
DATA



Standard sirens: cosmic distances from GWs

We must find the signal that better reproduce the observed data:
the <u>parameters</u> of the signal are varied until the best match is found

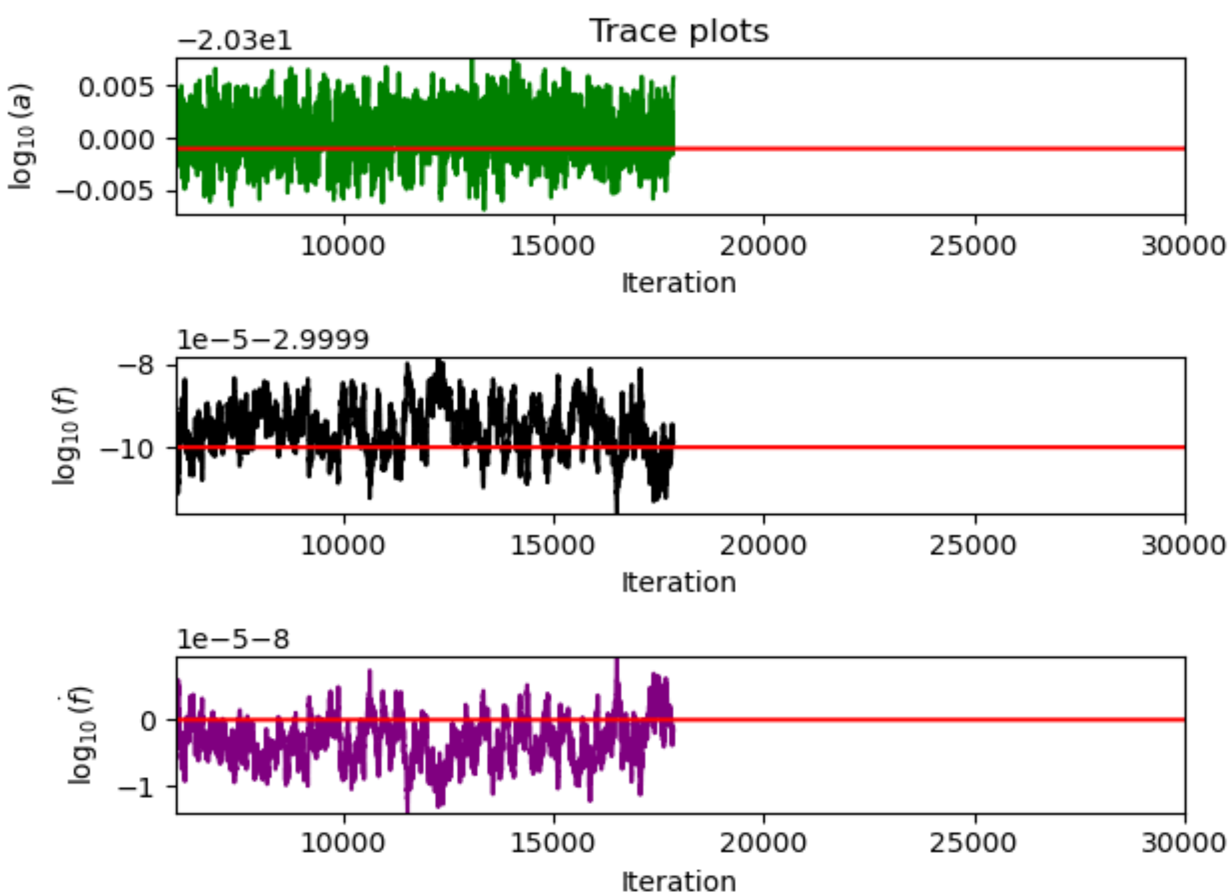


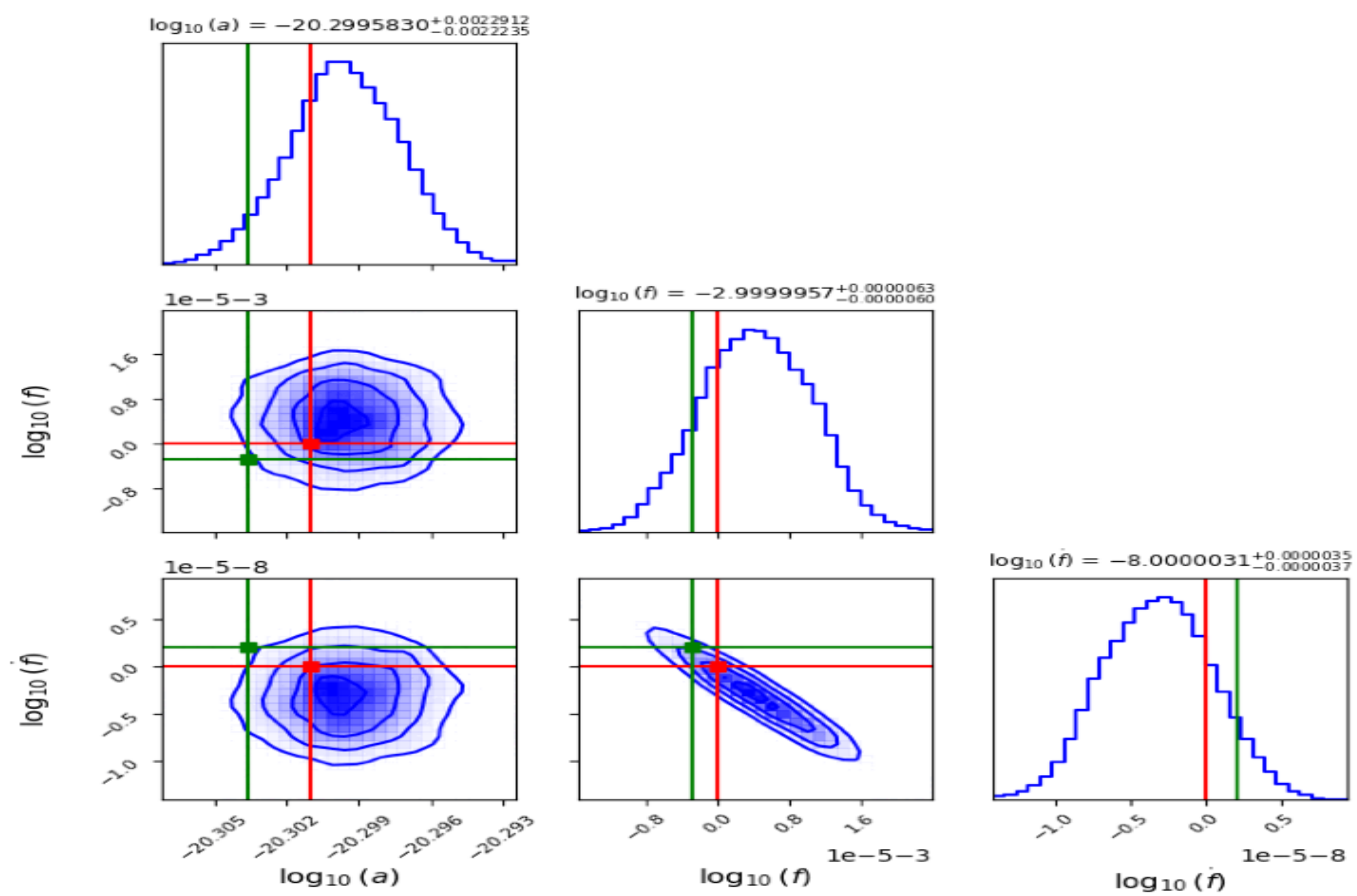


Parameters of the GW signals
(simplified monochromatic signal)

Standard sirens: cosmic distances from GWs

We must find the signal that better reproduce the observed data: the parameters of the signal are varied until the best match is found





Efficient sampling methods (MCMC, Nested Sampling, …) must be applied in order to find the best value of the single parameters and their statistical uncertainties

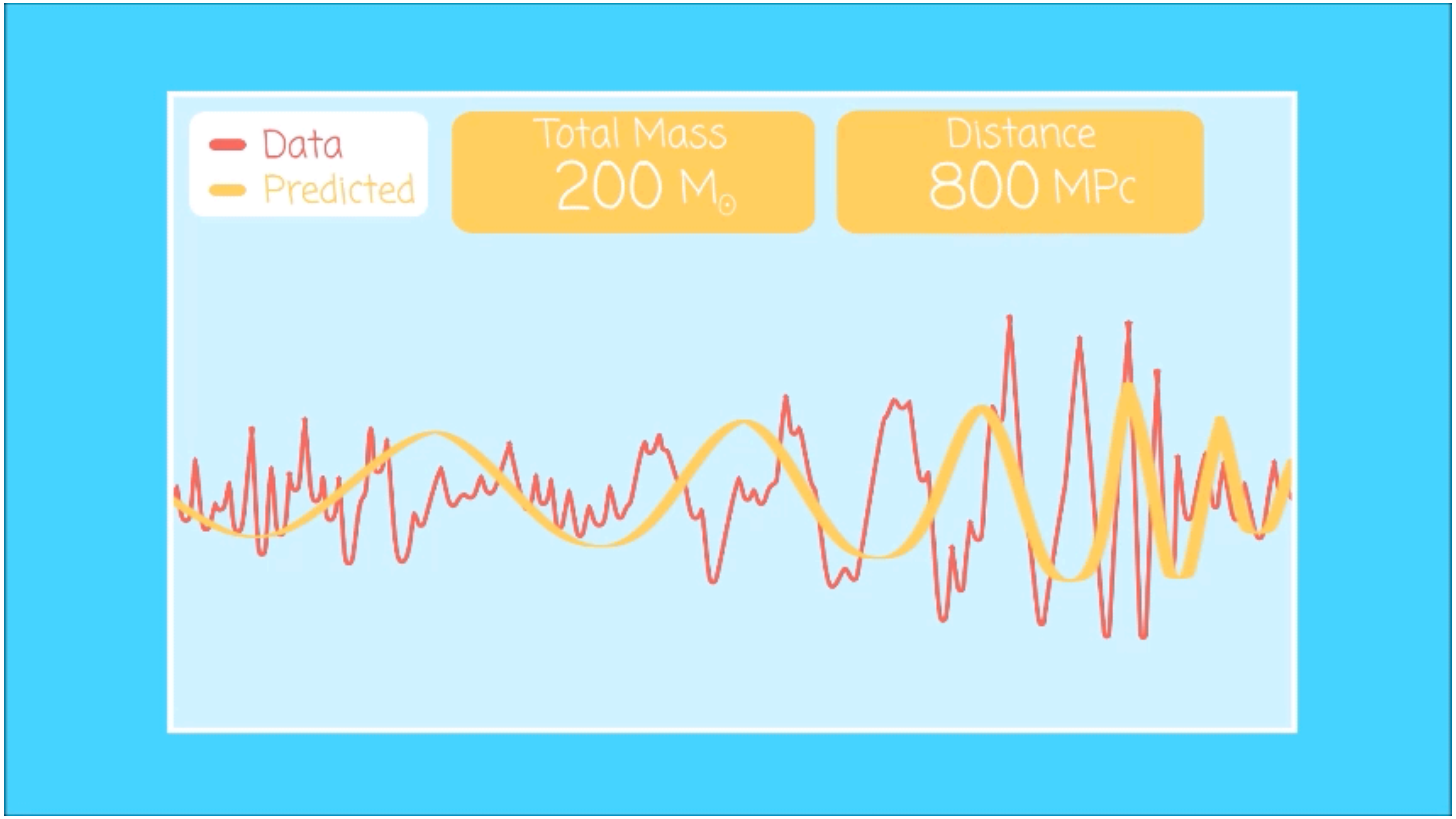
Standard sirens: cosmic distances from GWs

$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

The luminosity distance d_L of the source can thus be directly measured from the GW signal

↓

This means that <u>GW binaries are absolute cosmological distance indicators!</u>



Standard sirens: cosmic distances from GWs

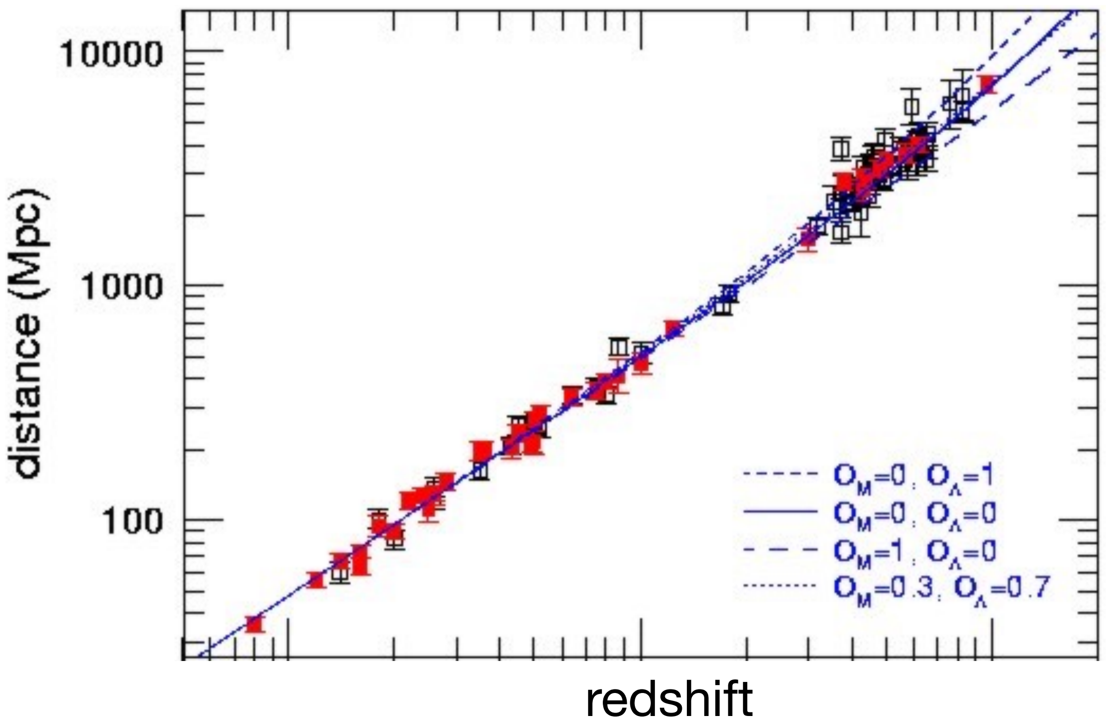
$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta\sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

Note however that the waveform above does not depend explicitly on the redshift z, which cannot thus be measured directly from GWs

One needs independent information on the redshift of the source to do cosmology: for example if both d_L and z are known one can fit the *distance redshift relation*

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{\Omega_k}}\sinh\left[\sqrt{\Omega_k}\int_0^z \frac{H_0}{H(z')}dz'\right]$$

This is very similar to standard candles (supernovae type-Ia), from which the name standard sirens (using the analogy between GWs and sound waves)



[Schutz, *Nature* (1986)]

Standard sirens: cosmic distances from GWs

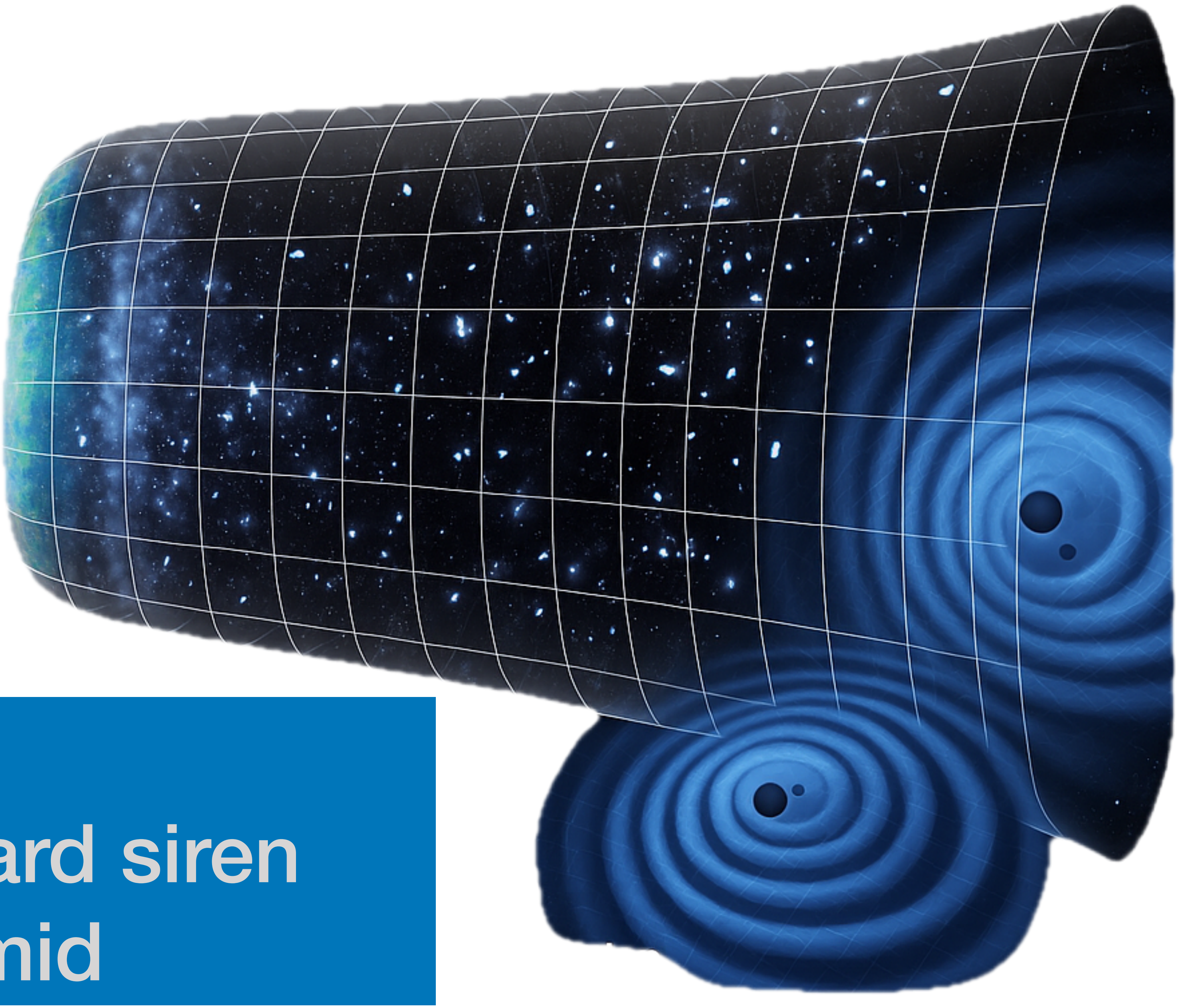
$$h_\times(t_o) = \frac{4}{d_L}\left(\frac{G\mathcal{M}_{cz}}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw},o}}{c}\right)^{2/3}\cos\theta \sin\left[-2\left(\frac{5G\mathcal{M}_{cz}}{c^3}\right)^{-5/8}\tau_o^{5/8}+\Phi_0\right]$$

Note however that the waveform above does not depend explicitly on the redshift z, which cannot thus be measured directly from GWs

One needs independent information on the redshift of the source to do cosmology: for example if both d_L and z are known one can fit the *distance redshift relation*

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{\Omega_k}}\sinh\left[\sqrt{\Omega_k}\int_0^z \frac{H_0}{H(z')}dz'\right]$$

How do we get redhsift information?

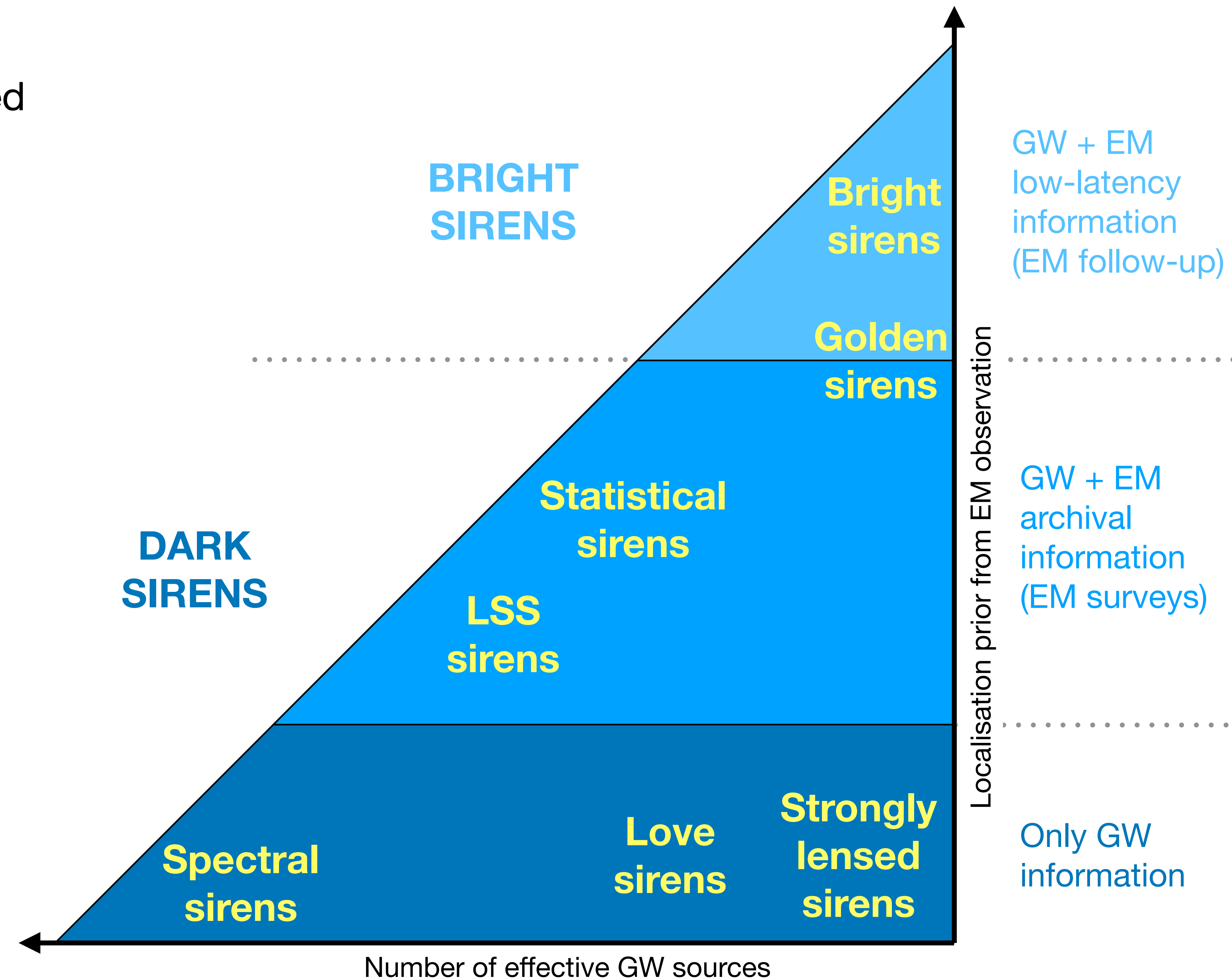


2
The standard siren pyramid

The standard siren Pyramid

Different methods have been developed to obtain <u>redshift infromation</u> for standard sirens

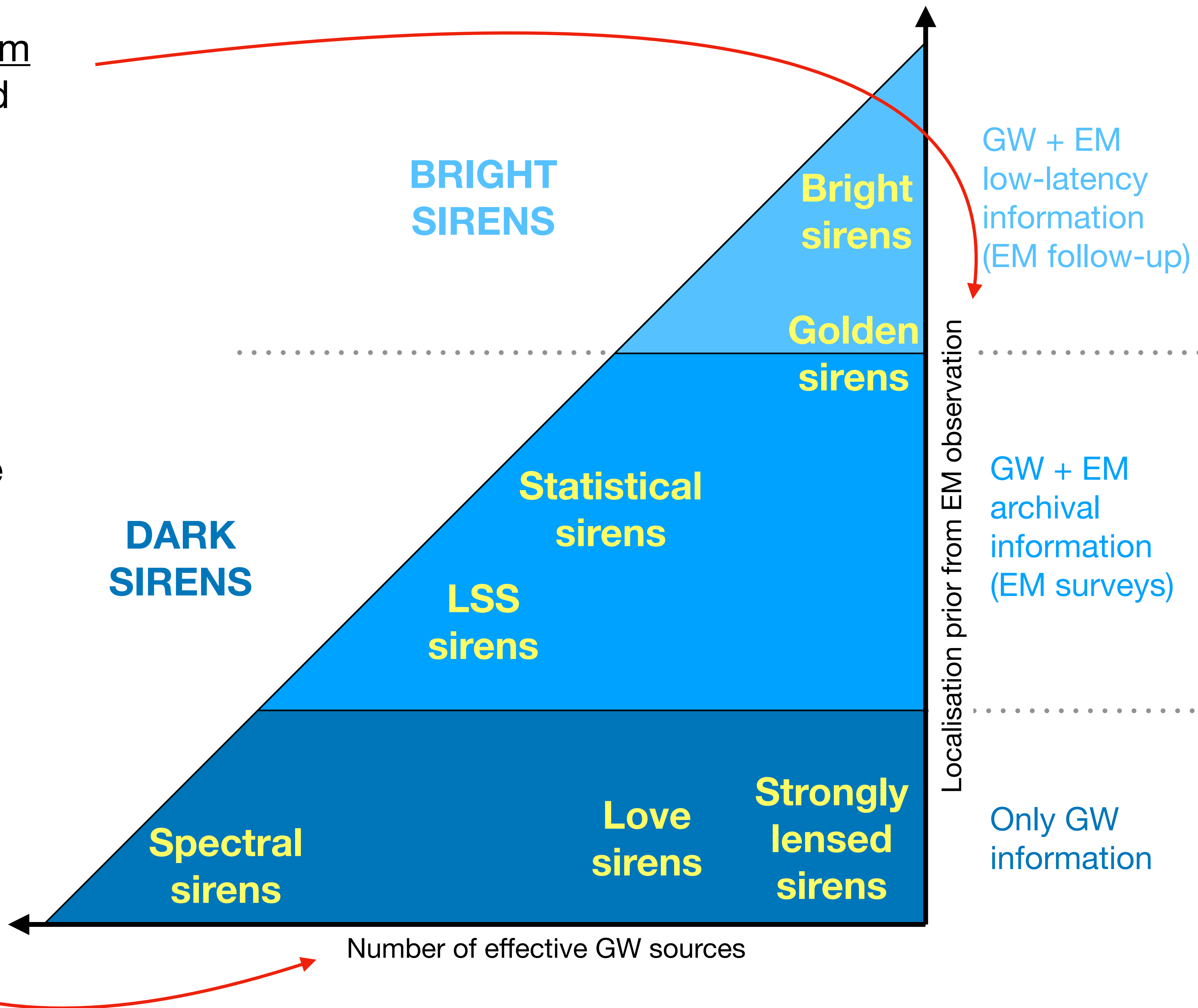
They can be summarised in the **<u>Standard siren pyramid</u>**



The standard siren Pyramid

<u>Localisation prior from EM observation</u> used in the analysis (the more informative the prior the smaller the localisation volume)

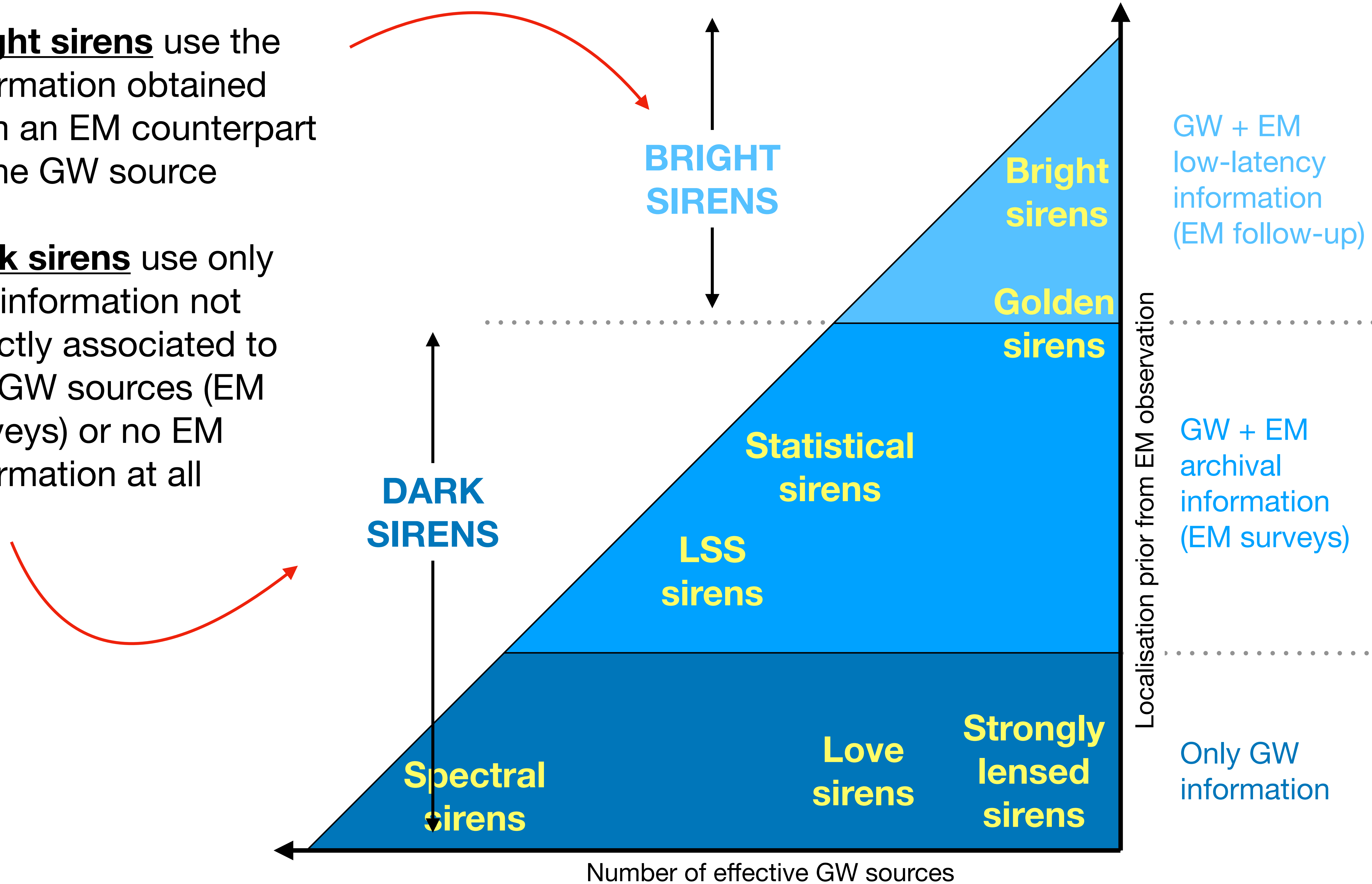
<u>Number of GW sources</u> which effectively contribute to the cosmological inference



The standard siren Pyramid

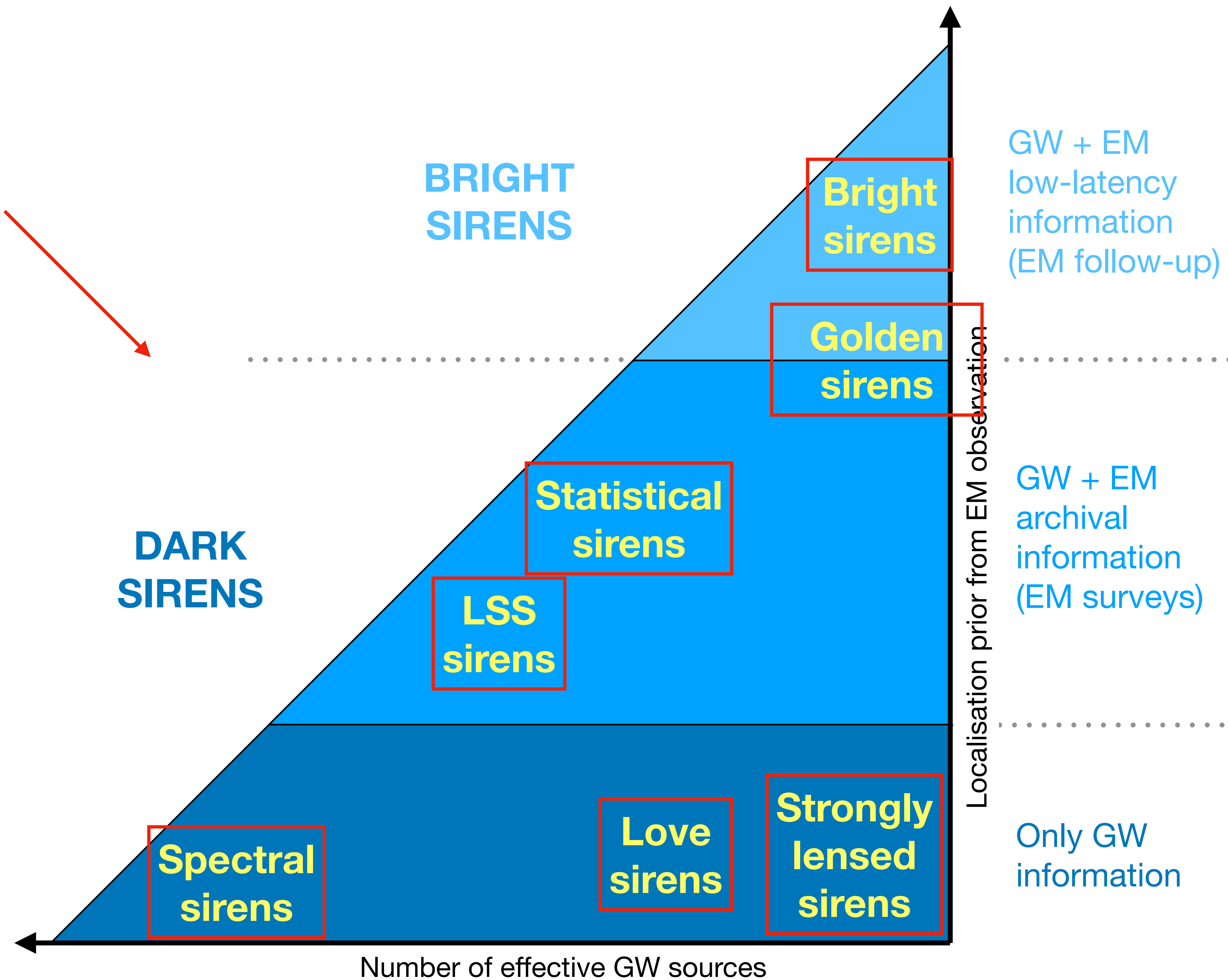
<u>Bright sirens</u> use the information obtained from an EM counterpart to the GW source

<u>Dark sirens</u> use only EM information not directly associated to the GW sources (EM surveys) or no EM information at all



The standard siren Pyramid

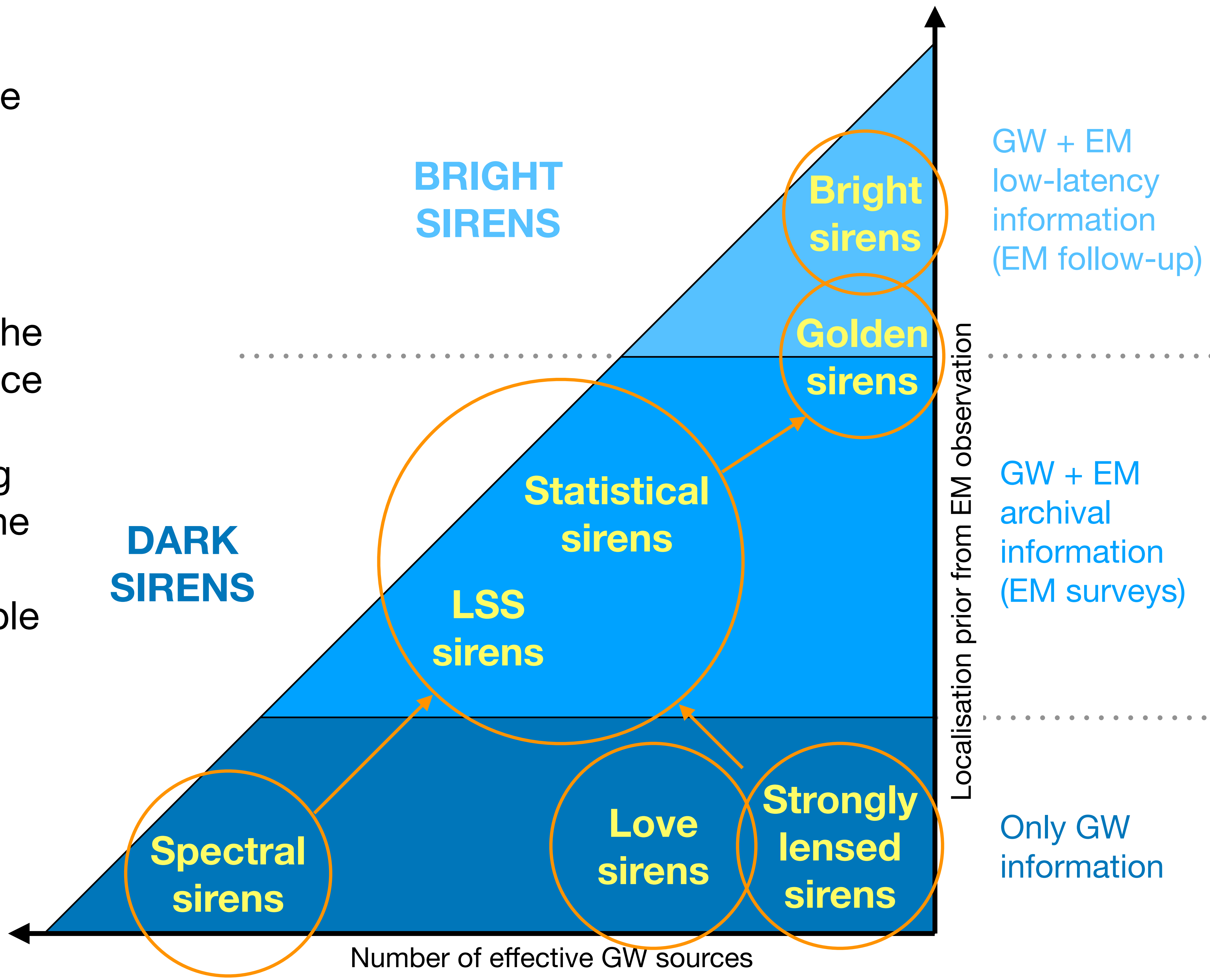
Different standard siren methodologies employing different GW and EM information



The standard siren Pyramid

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

Information helping the cosmological inference can be added progressively starting from the bottom in the cases where EM information is available

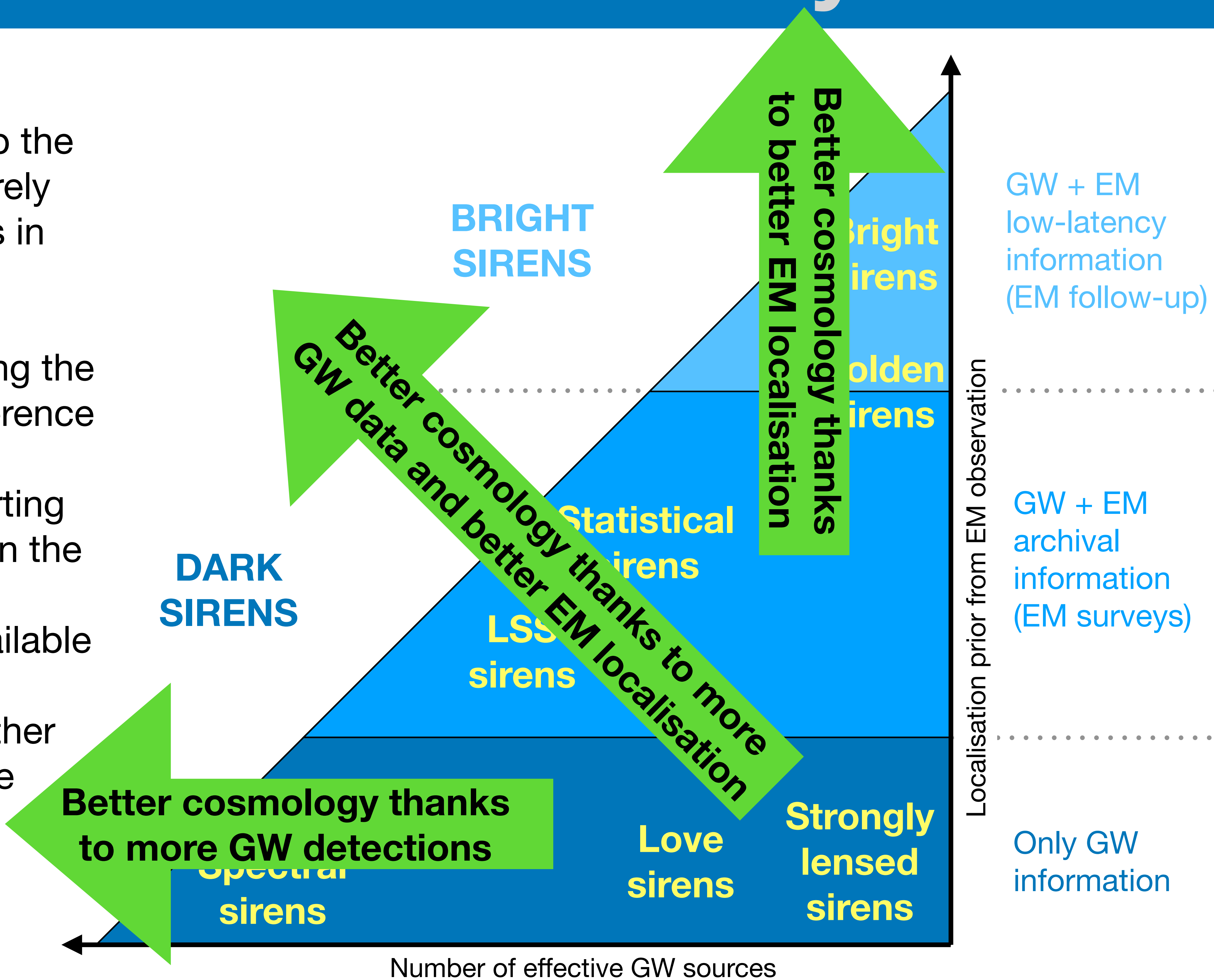


The standard siren Pyramid

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

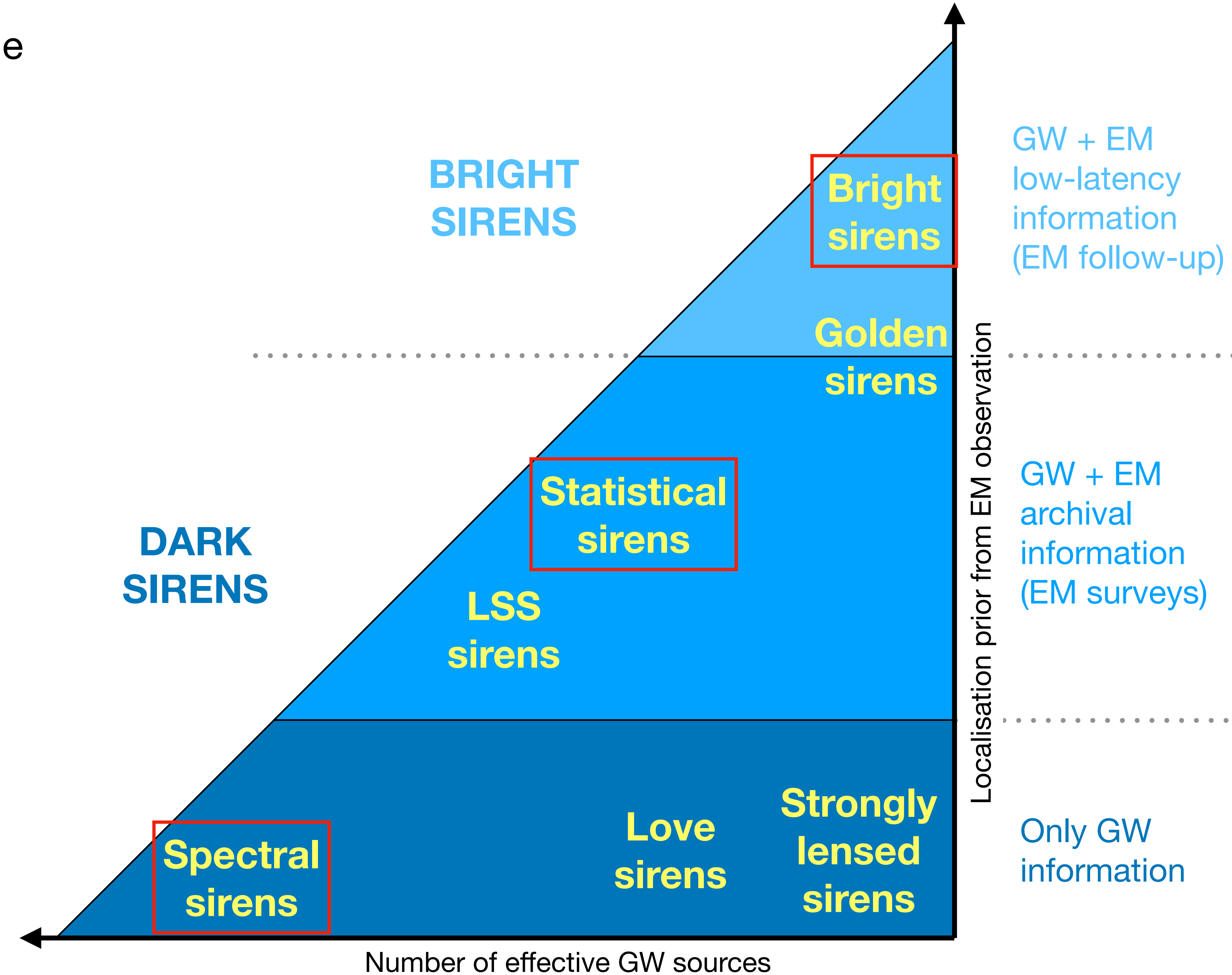
Information helping the cosmological inference can be added progressively starting from the bottom in the cases where EM information is available

In general the farther from the origin the better the cosmological constraints



The standard siren Pyramid

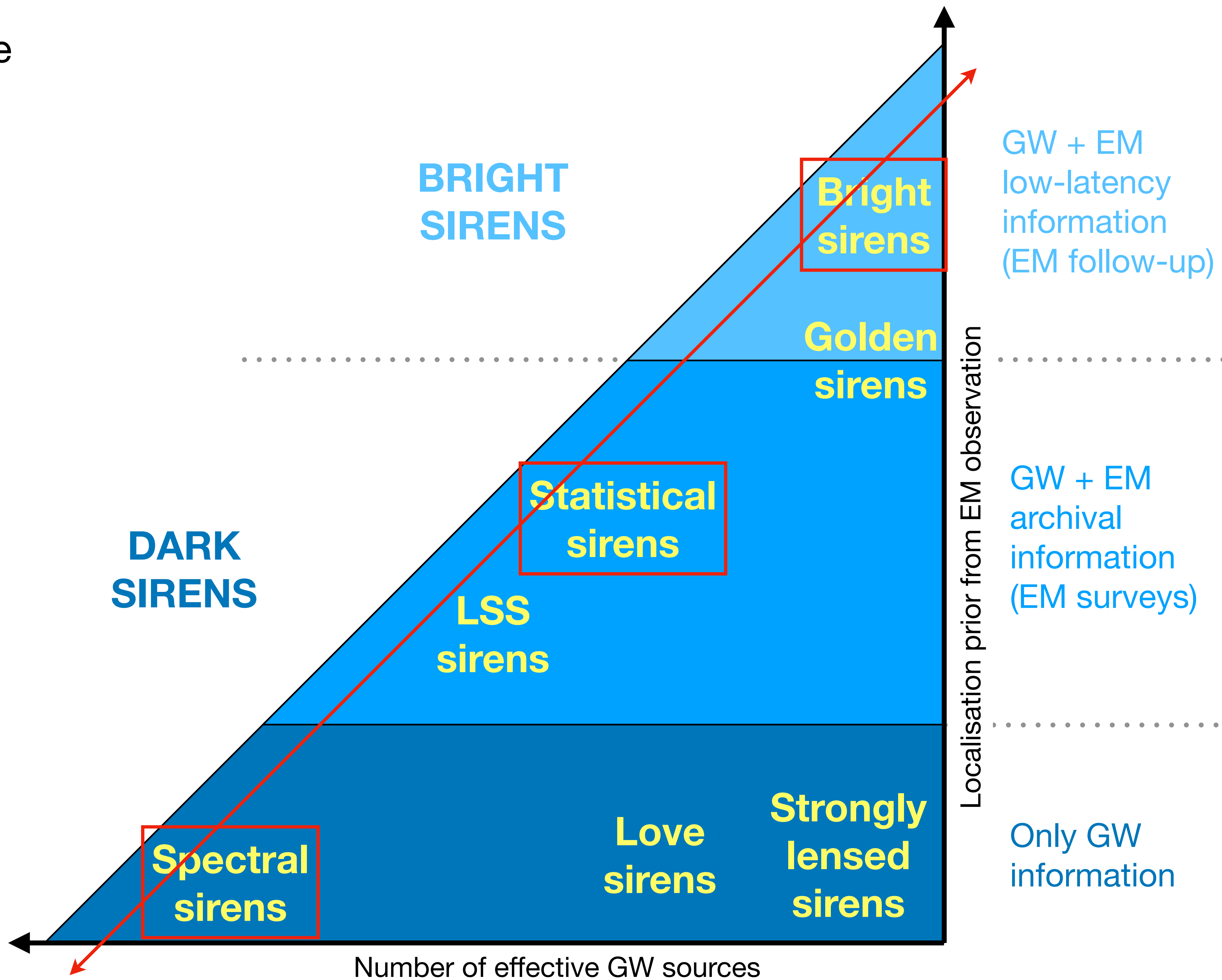
Here we focus on the <u>three main methodologies</u> currently applied to real data



The standard siren Pyramid

Here we focus on the <u>three main methodologies</u> currently applied to real data

They appear on the <u>diagonal</u> of the pyramid which maximises both GW sources and EM information

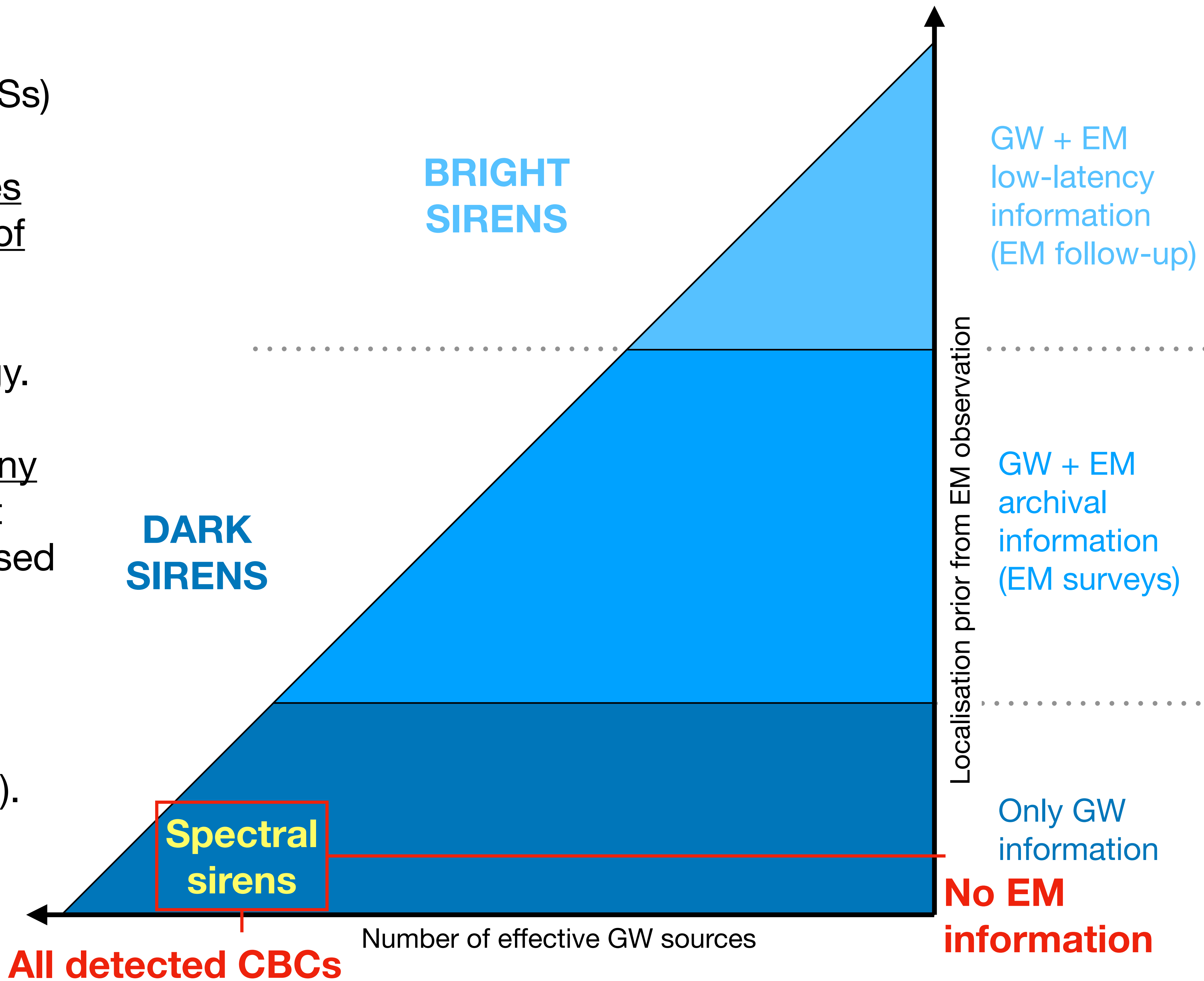


The standard siren Pyramid

<u>Spectral sirens</u>
(BBHs, NSBHs, BNSs)

They exploit <u>features in the distributions of source parameters</u> (mass, rate, …) to constrain cosmology.

They <u>do not need any EM information</u>, but rely on the model used to describe the distribution of parameters of compact binary coalesences (CBCs).



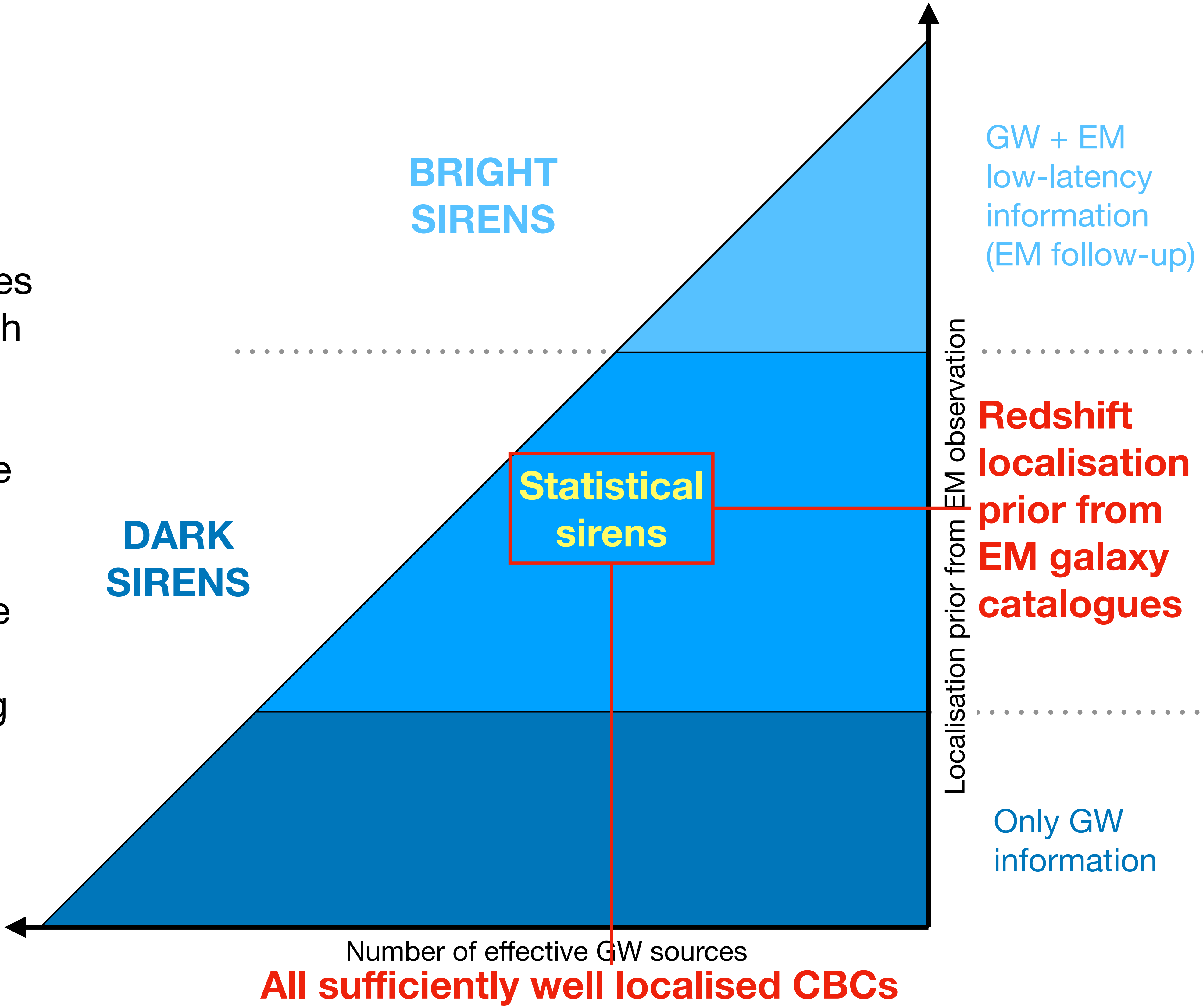
The standard siren Pyramid

<u>Statistical sirens</u>
(well localised BBHs, NSBHs, BNSs)

The sky localisation volume of GW sources is cross-matched with a <u>galaxy catalogue</u>

All galaxies within the localisation volume contribute a <u>redshift</u> value to the inference

Eventually combining enough GW events <u>statistically</u> yields cosmological constraints

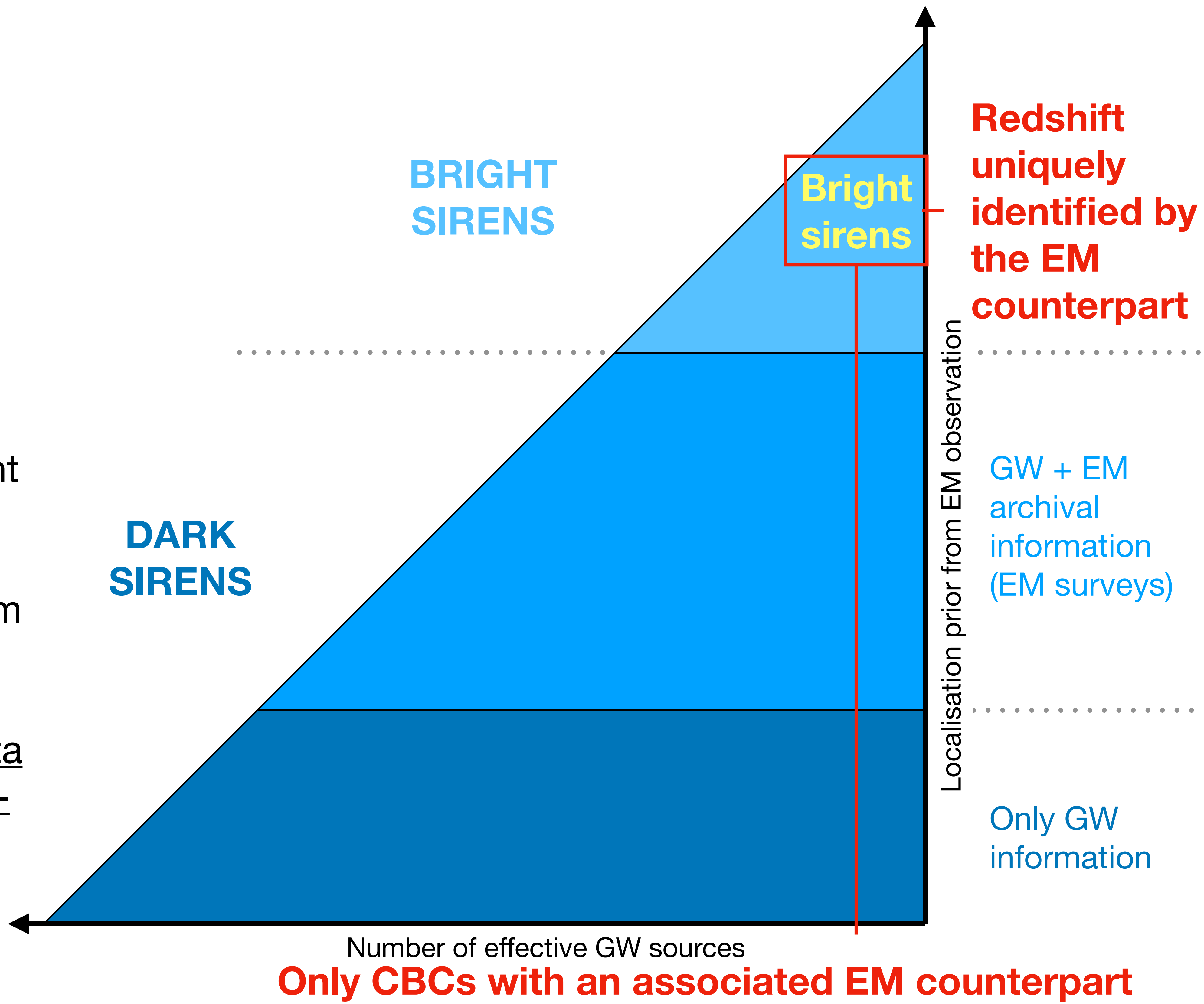


The standard siren Pyramid

<u>Bright sirens</u>
(BNSs, NSBHs with EM counterpart)

The <u>EM counterpart</u> allows for a direct identification of the GW source host galaxy, from which a redshift measurement is obtained

Tight constraints from a low number of events as each provides a <u>single data point in the distance-redshift relation</u>

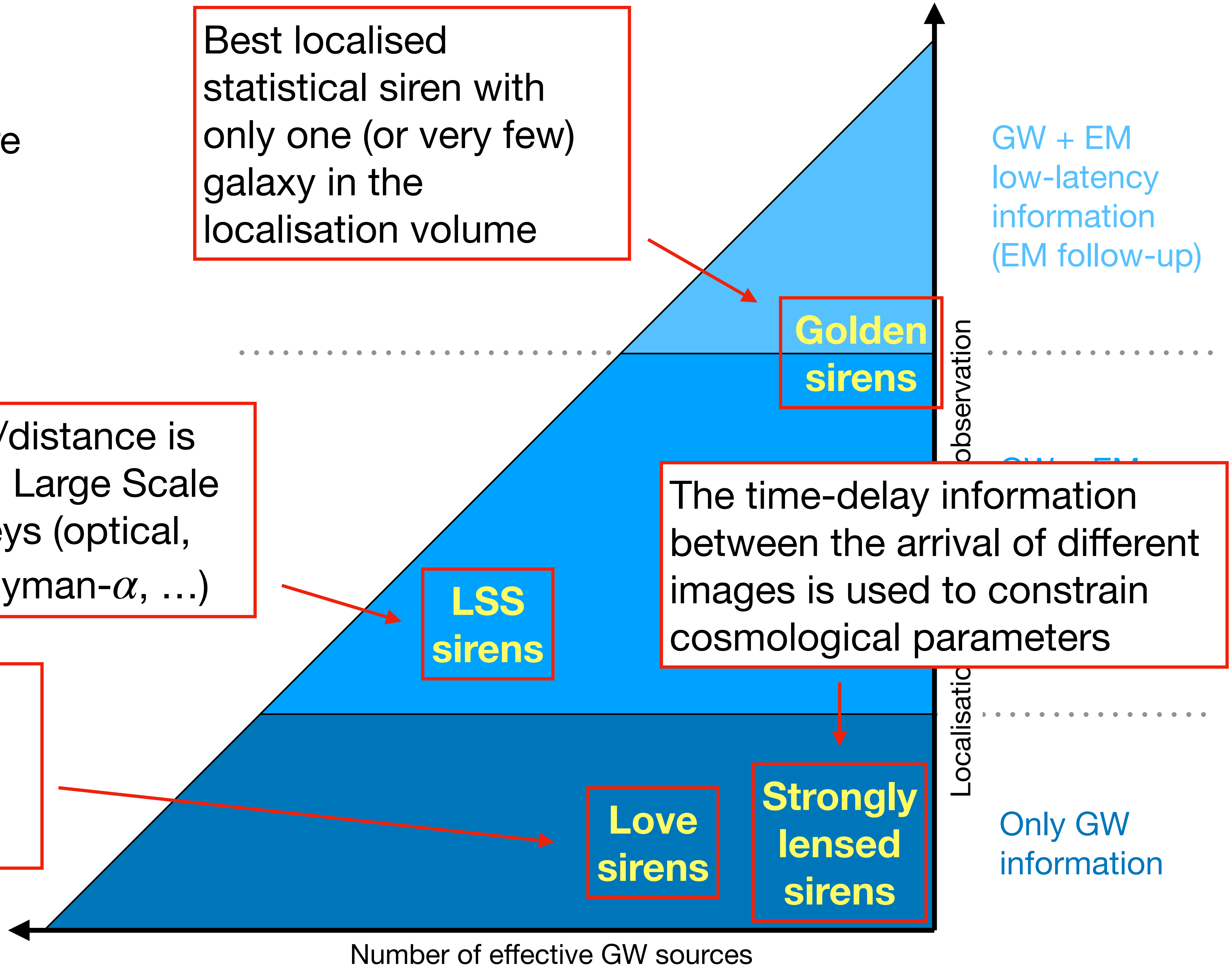


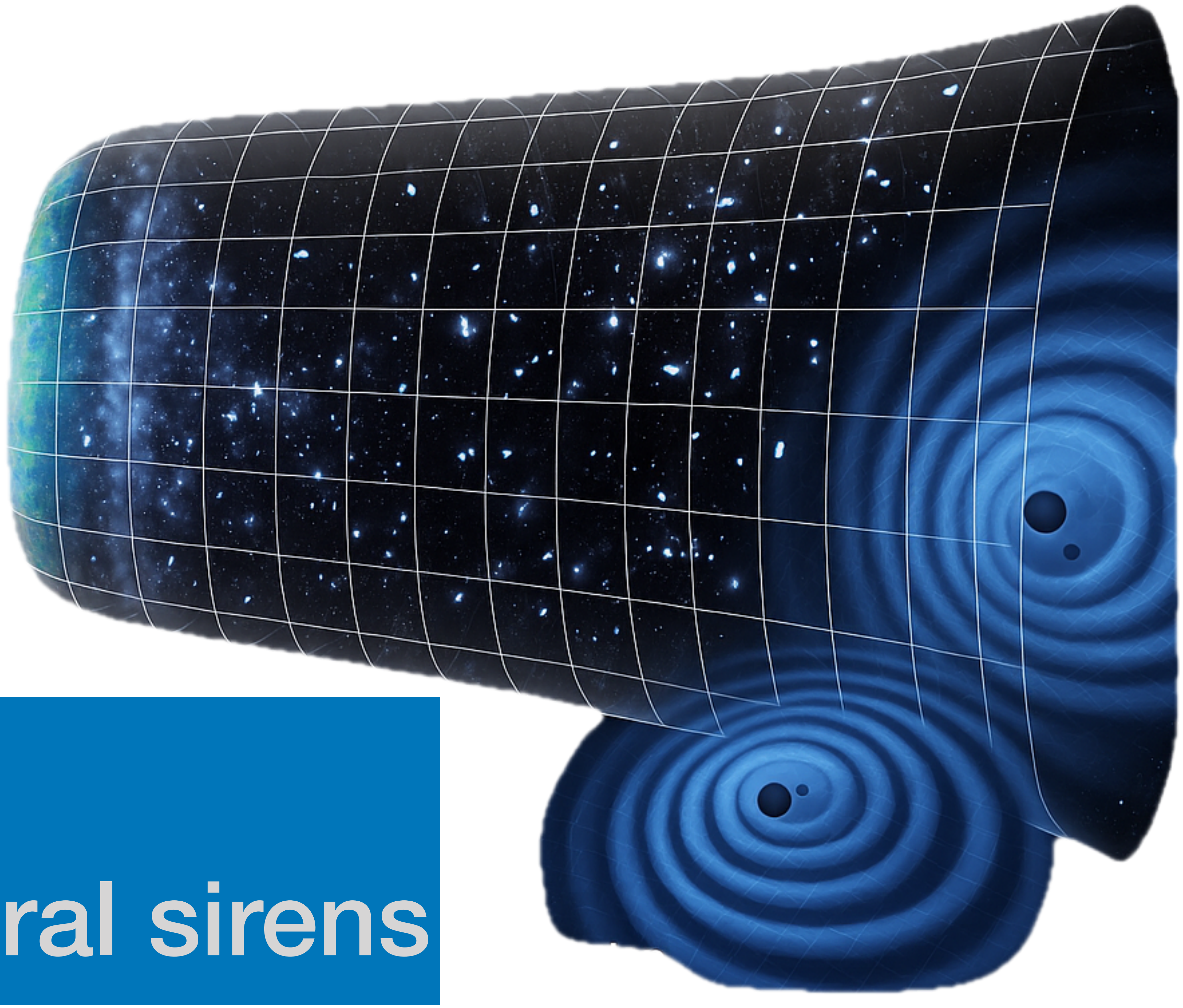
The standard siren Pyramid

The **other siren methods** require further R&D and more observations to be efficiently applied to real data

The GW sky position/distance is cross-correlated with Large Scale Structure (LSS) surveys (optical, infrared, lensing, HI, lyman-α, …)

Information on the EoS of NS is used to break the mass-redshift degeneracy





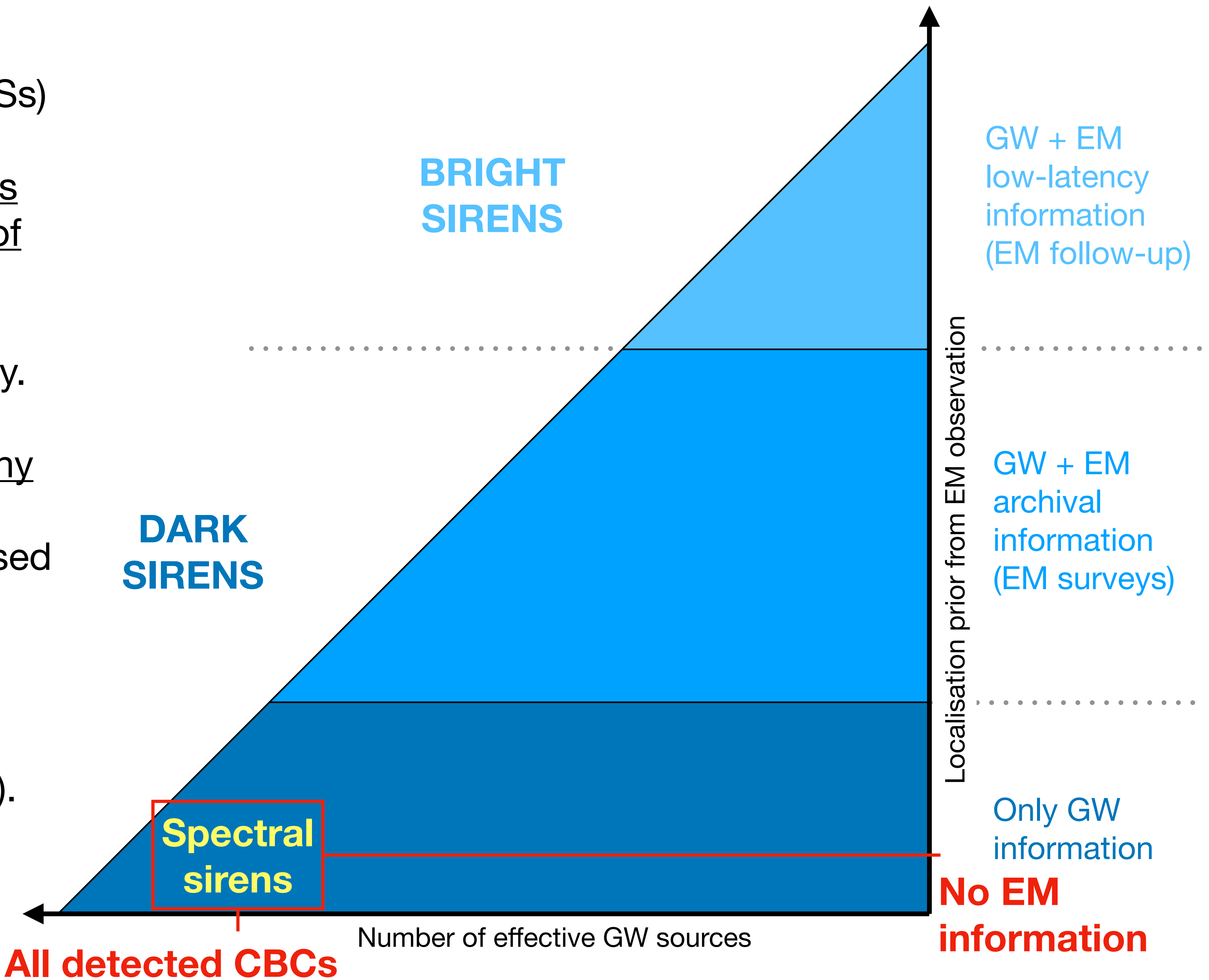
3
Dark spectral sirens

Dark spectral sirens

<u>Spectral sirens</u>
(BBHs, NSBHs, BNSs)

They exploit <u>features in the distributions of source parameters</u> (mass, rate, …) to constrain cosmology.

They <u>do not need any EM information</u>, but rely on the model used to describe the distribution of parameters of compact binary coalesences (CBCs).

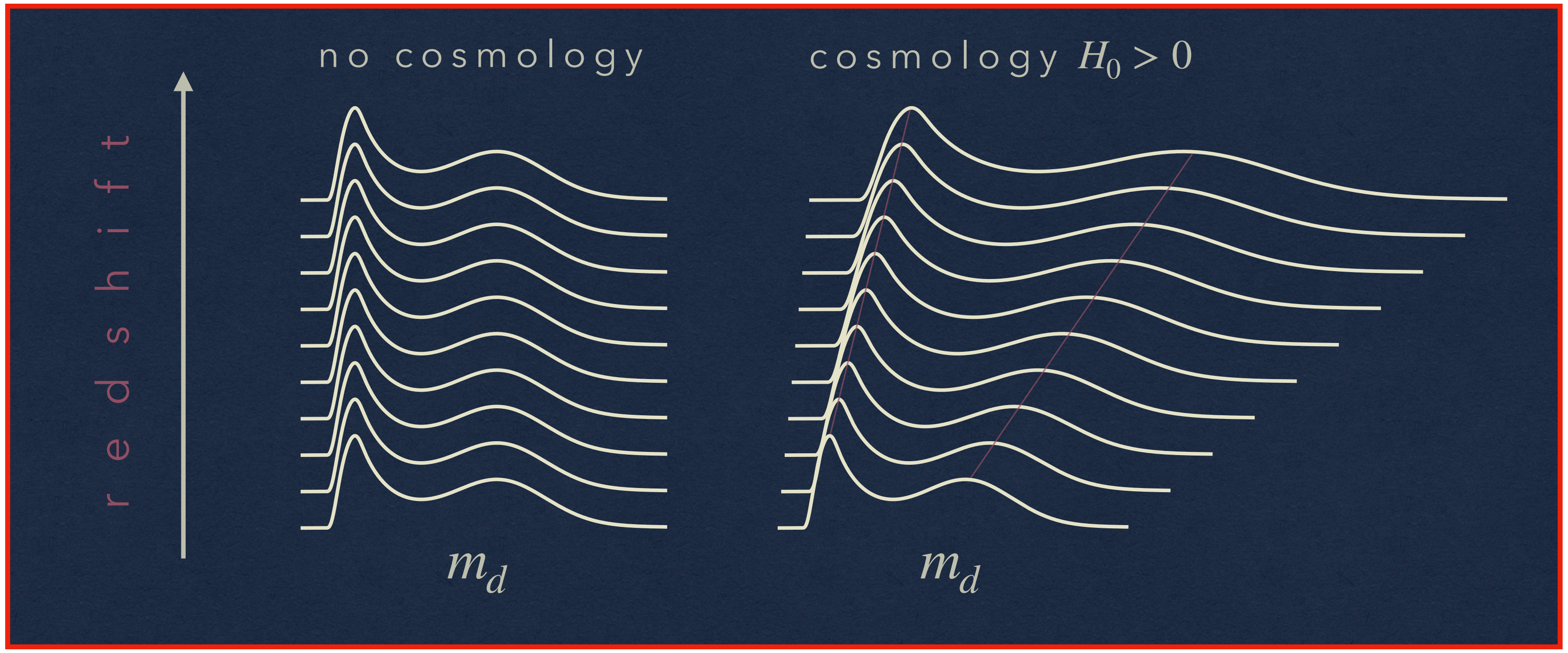


Dark spectral sirens

Spectral sirens
(BBHs, NSBHs, BNSs)

They exploit features in the distributions of source parameters (mass, rate, …) to constrain cosmology.

They do not need any EM information, but rely on the model used to describe the distribution of parameters of compact binary coalesences (CBCs).



Source frame **Observer frame**

$$m_{obs} = (1 + z)m_{src}$$

The observed distribution of certain source parameters (masses, rate, …) depend on cosmology

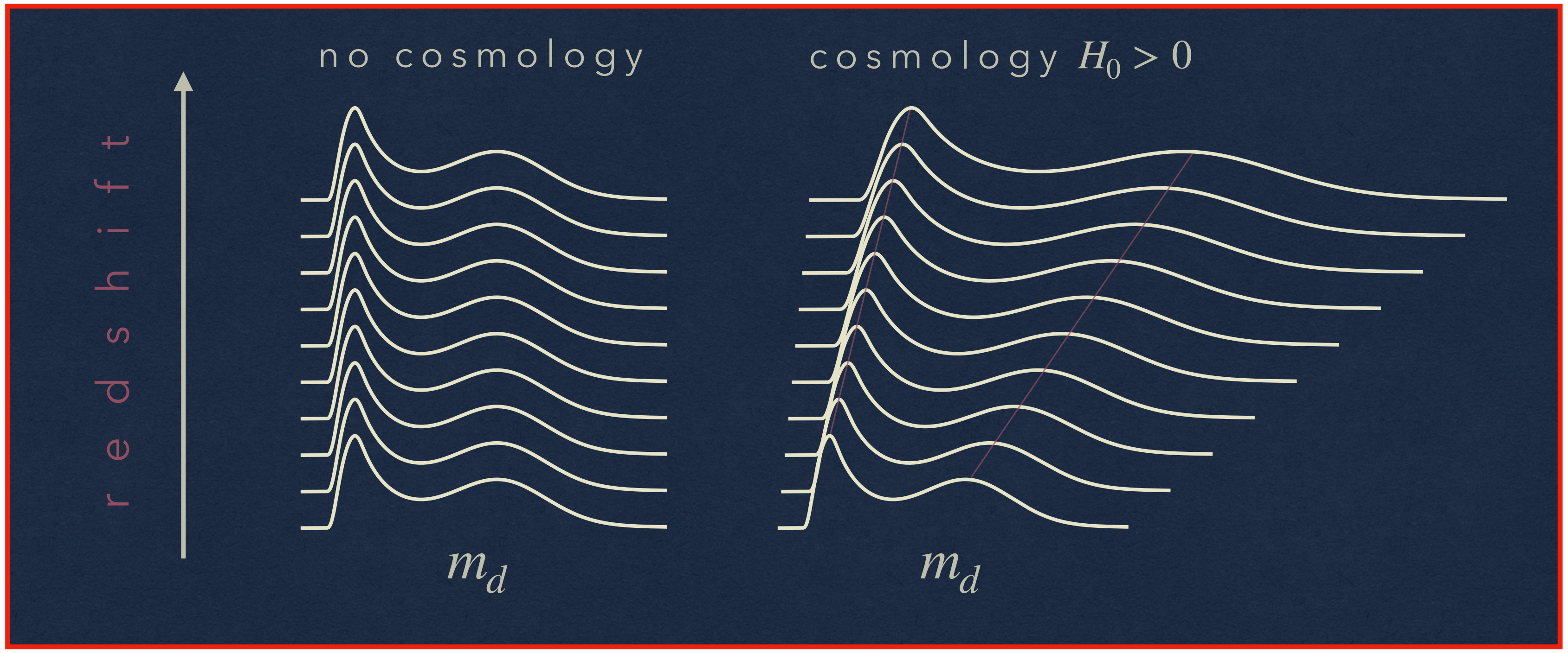
This means that if we knew the distribution at the source we could measure cosmological parameters

Dark spectral sirens

<u>Spectral sirens</u>
(BBHs, NSBHs, BNSs)

They exploit features in the distributions of source parameters (mass, rate, …) to constrain cosmology.

They do not need any EM information, but rely on the model used to describe the distribution of parameters of compact binary coalesences (CBCs).



Source frame **Observer frame**

$$m_{\text{obs}} = (1+z)m_{\text{src}}$$

The observed distribution of certain source parameters (masses, rate, …) depend on cosmology

A parametric model of the source parameter distribution is however all we need as we can infer both source and cosmological parameters simultaneously

Dark spectral sirens

The problem can be well posed using **hierarchical Bayesian inference**

$\vec{\Lambda} = \{\Lambda_1, \Lambda_2, \dots\}$ → Parameters describing the GW population model and the cosmological model

D → Data collected by the GW detectors

Posterior: probability of parameters $\vec{\Lambda}$ given the data D (this is what we want to measure and maximise)

$$p(\vec{\Lambda} \,|\, D) \propto p(D \,|\, \vec{\Lambda}) p(\vec{\Lambda})$$

Bayes Theorem

Likelihood: probability of the data D given the parameters $\vec{\Lambda}$

Prior: initial probability of the parameters $\vec{\Lambda}$ (before making any measurement)

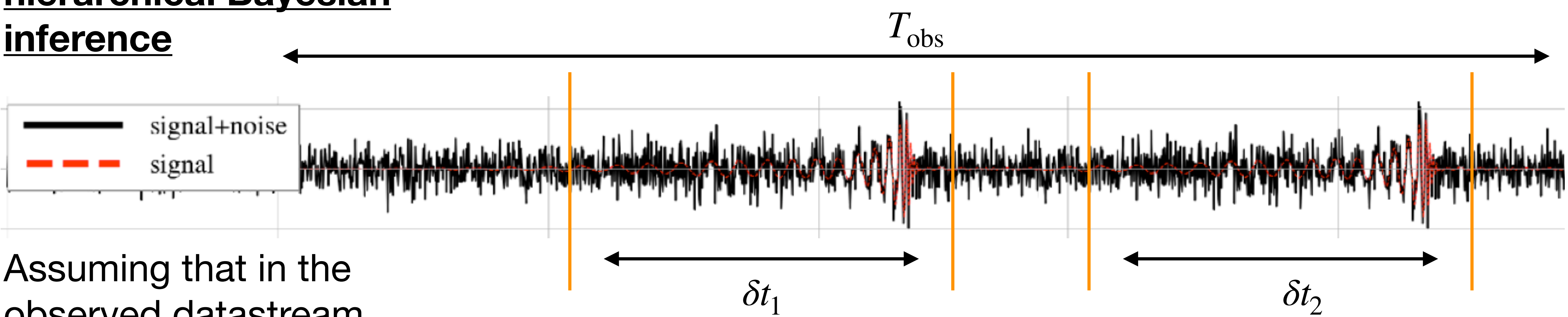
Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **hierarchical Bayesian inference**

$$p(\overrightarrow{\Lambda}\,|\,D) \propto p(D\,|\,\overrightarrow{\Lambda})p(\overrightarrow{\Lambda})$$

Bayes Theorem



Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **Poisson distribution**

$$\text{If} \quad T_{\rm obs} \gg \sum_i \delta t_i \qquad p(D\,|\,\overrightarrow{\Lambda}) \propto e^{N_{\rm exp}(\overrightarrow{\Lambda})} \prod_{i=1}^{N_{\rm obs}} \int d\vec{\theta}\,\mathscr{L}(D_i\,|\,\vec{\theta})\,p_{\rm pop}(\vec{\theta}\,|\,\overrightarrow{\Lambda})$$

$N_{\rm exp} =$ number of expected GW detections

$N_{\rm obs} =$ number of actual GW detections

$p_{\rm pop}(\vec{\theta}\,|\,\overrightarrow{\Lambda}) =$ GW signal parameters distribution (population prior)

$\vec{\theta} =$ GW signal parameters (distance, masses, sky position, …)

$\mathscr{L}(D_i\,|\,\vec{\theta}) =$ likelihood of obtaining the ith data chunck given the GW signal parameters $\vec{\theta}$

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **hierarchical Bayesian inference**

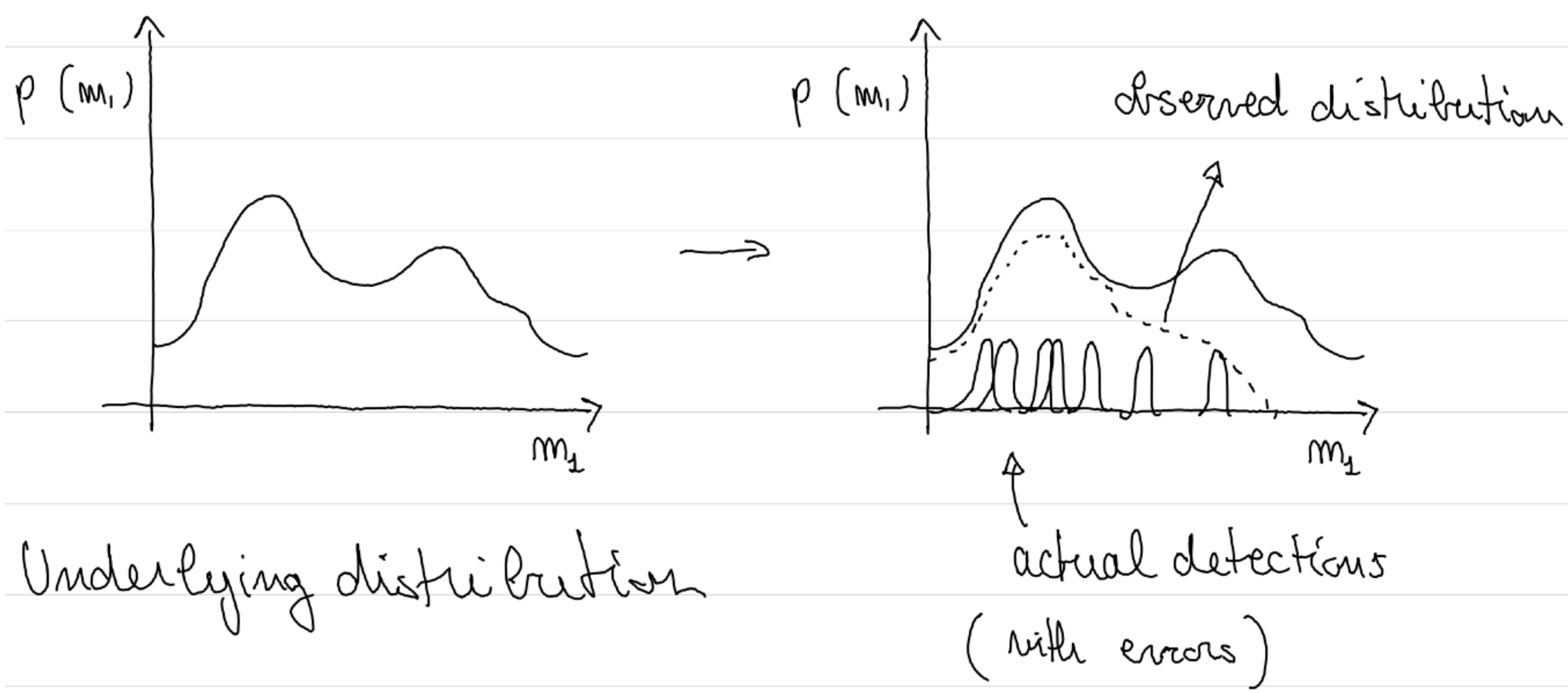
$$p(\vec{\Lambda}\,|\,D) \propto p(D\,|\,\vec{\Lambda})p(\vec{\Lambda})$$

Bayes Theorem

$$p(D\,|\,\vec{\Lambda}) \propto e^{N_{\rm exp}(\vec{\Lambda})} \prod_{i=1}^{N_{\rm obs}} \int d\vec{\theta}\,\mathscr{L}(D_i\,|\,\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})$$

Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **Poisson distribution**

In other words, this is the likelihood of obtaining $N_{\rm obs}$ GW detections in a time $T_{\rm obs}$, from a given population distribution with parameters $\vec{\Lambda}$ (usually called hyperparameters to distinguish them from $\vec{\theta}$)



Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **hierarchical Bayesian inference**

$$p(\overrightarrow{\Lambda} \,|\, D) \propto p(D \,|\, \overrightarrow{\Lambda}) p(\overrightarrow{\Lambda})$$

Bayes Theorem

$$p(D \,|\, \overrightarrow{\Lambda}) \propto e^{N_{\rm exp}(\overrightarrow{\Lambda})} \prod_{i=1}^{N_{\rm obs}} \int d\vec{\theta} \mathscr{L}(D_i \,|\, \vec{\theta}) p_{\rm pop}(\vec{\theta} \,|\, \overrightarrow{\Lambda})$$

Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **Poisson distribution**

WARNING: These assumptions hold for current observations (LVK O3, O4, O5), but they are not satisfied in future observations (LISA, 3G)

The likelihood above must be updated to be applied to future observations by LISA and 3G (open problem)

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **hierarchical Bayesian inference**

$$p(\vec{\Lambda}\,|\,D) \propto p(D\,|\,\vec{\Lambda})p(\vec{\Lambda})$$

Bayes Theorem

$$p(D\,|\,\vec{\Lambda}) \propto e^{N_{\rm exp}(\vec{\Lambda})}\prod_{i=1}^{N_{\rm obs}}\int d\vec{\theta}\mathscr{L}(D_i\,|\,\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})$$

Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **Poisson distribution**

The first term in the likelihood contain the probability of detection, i.e. the information about how the observation has been done. It determines the selection effects and can be written as

$$N_{\rm exp}(\vec{\Lambda}) \propto \int p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})$$

Where

$$p_{\rm det}(\vec{\theta}) = \prod_i \int_{D\in\,\rm detectable} \mathscr{L}(D_i\,|\,\vec{\theta})dD$$

is the probabilitythat the GW signal with parameters $\vec{\theta}$ is detectable (not detected!): it must be marginalised over all possible noise realisations

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **hierarchical Bayesian inference**

$$p(\overrightarrow{\Lambda} \,|\, D) \propto p(D \,|\, \overrightarrow{\Lambda}) p(\overrightarrow{\Lambda})$$

Bayes Theorem

Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **Poisson distribution**

$$p(D \,|\, \overrightarrow{\Lambda}) \propto e^{N_{\rm exp}(\overrightarrow{\Lambda})} \prod_{i=1}^{N_{\rm obs}} \int d\vec{\theta} \mathscr{L}(D_i \,|\, \vec{\theta}) p_{\rm pop}(\vec{\theta} \,|\, \overrightarrow{\Lambda})$$

By taking the following prior choice on the source-frame GW rate $p(N_{\rm src}) \propto 1/N_{\rm src}$ (which is one of the population parameter $\overrightarrow{\Lambda}$ assuming a constant rate) one obtains

$$p(D \,|\, \overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta} \mathscr{L}(D_i \,|\, \vec{\theta}) p_{\rm pop}(\vec{\theta} \,|\, \overrightarrow{\Lambda})}{\int d\vec{\theta} p_{\rm det}(\vec{\theta}) p_{\rm pop}(\vec{\theta} \,|\, \overrightarrow{\Lambda})}$$

which is the likelihood used to evaluate the posterior on the population and cosmological parameters

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

The problem can be well posed using **<u>hierarchical Bayesian inference</u>**

$$p(\vec{\Lambda}\,|\,D) \propto p(D\,|\,\vec{\Lambda})p(\vec{\Lambda})$$

<u>Bayes Theorem</u>

$$p(D\,|\,\vec{\Lambda}) \propto e^{N_{\rm exp}(\vec{\Lambda})} \prod_{i=1}^{N_{\rm obs}} \int d\vec{\theta}\mathscr{L}(D_i\,|\,\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})$$

Assuming that in the observed datastream the detected GW signals do not overlap and that the timelenght of the sum of the data chuncks containing a GW signal is much smaller than the total observational time, the likelihood can be written as (generalised) **<u>Poisson distribution</u>**

Probability of obtaining the GW data D_i given the GW signal with parameters $\vec{\theta}$

$$p(D\,|\,\vec{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathscr{L}(D_i\,|\,\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})}{\int d\vec{\theta}p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})}$$

Probability of detecting the GW signal with parameters $\vec{\theta}$ marginalised over all possible noise realisations

Probability of obtaining the GW parameters $\vec{\theta}$ given the population parameters $\vec{\Lambda}$

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D|\overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta} \mathscr{L}(D_i|\vec{\theta}) p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}{\int d\vec{\theta} p_{\rm det}(\vec{\theta}) p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}$$

Spectral sirens usually rely on two main GW parameters: <u>the luminosity distance and the masses</u>. However other parameters can be taken into account (spins, incl. angle, …). We will only consider these two parameters for the moment, meaning that $\mathscr{L}(D_i|\vec{\theta})$ should be taken as the GW likelihood <u>marginalised</u> over all other parameters.

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D|\overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta} \mathscr{L}(D_i|\vec{\theta}) p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}{\int d\vec{\theta} p_{\rm det}(\vec{\theta}) p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}$$

Spectral sirens usually rely on two main GW parameters: the luminosity distance and the masses. However other parameters can be taken into account (spins, incl. angle, …). We will only consider these two parameters for the moment, meaning that $\mathscr{L}(D_i|\vec{\theta})$ should be taken as the GW likelihood marginalised over all other parameters.

The detection probability is usually computed through an injection campaign since for each set of parameters $\vec{\theta}$ one must marginalise over all possible realisation of the noise D.

$$p_{\rm det}(\vec{\theta}) = \prod_i \int_{D \in {\rm detectable}} \mathscr{L}(D_i|\vec{\theta}) dD$$

To compute this probability a clear detection threshold must be defined (for example an SNR threshold).

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D|\overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathscr{L}(D_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}{\int d\vec{\theta}p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}$$

Regarding the population prior we can focus only on three parameters: the two masses (or whatever combination of them) and the luminosity distance:

$$p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda}) = p(m_1^z|\overrightarrow{\Lambda})p(m_2^z|\overrightarrow{\Lambda})p(d_L|\overrightarrow{\Lambda}) = p(m_1|\overrightarrow{\Lambda})p(m_2|\overrightarrow{\Lambda})p(z|\overrightarrow{\Lambda})(1+z)^2\left|\frac{\partial d_L(z)}{\partial z}\right|$$

where

$$m_1^z = (1+z)m_1 \qquad m_2^z = (1+z)m_2 \qquad d_L(z;\overrightarrow{\Lambda}_c) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz'}{E(z;\overrightarrow{\Lambda}_c)}$$

Detector-frame masses **Luminosity distance**

Note that here we assume that this prior is separable into three different factors each depending only on one parameter. If for example either of the mass priors is evolving with redshift, this assumption is no longer valid.

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D|\vec{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathscr{L}(D_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\Lambda})}{\int d\vec{\theta}p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\Lambda})}$$

Regarding the population prior we can focus only on three parameters: the two masses (or whatever combination of them) and the luminosity distance:

$$p_{\rm pop}(\vec{\theta}|\vec{\Lambda}) = p(m_1^z|\vec{\Lambda})p(m_2^z|\vec{\Lambda})p(d_L|\vec{\Lambda}) = p(m_1|\vec{\Lambda})p(m_2|\vec{\Lambda})p(z|\vec{\Lambda})(1+z)^2\left|\frac{\partial d_L(z)}{\partial z}\right|$$

where

$$m_1^z = (1+z)m_1 \qquad m_2^z = (1+z)m_2 \qquad d_L(z;\vec{\Lambda}_c) = \frac{c}{H_0}(1+z)\int_0^z \frac{dz'}{E(z;\vec{\Lambda}_c)}$$

Detector-frame masses **Luminosity distance**

The Jacobian and the luminosity distance (which appears as well within $\mathscr{L}(D_i|\vec{\theta})$) encode the dependency on the cosmological parameters $\vec{\Lambda}_c \in \vec{\Lambda}$ through H_0 and $E(z,\vec{\Lambda}_c)$, which for ΛCDM reads $E(z;\vec{\Lambda}_c) = \sqrt{\Omega_m(1+z)^3 + 1 - \Omega_m}$.

Dark spectral sirens

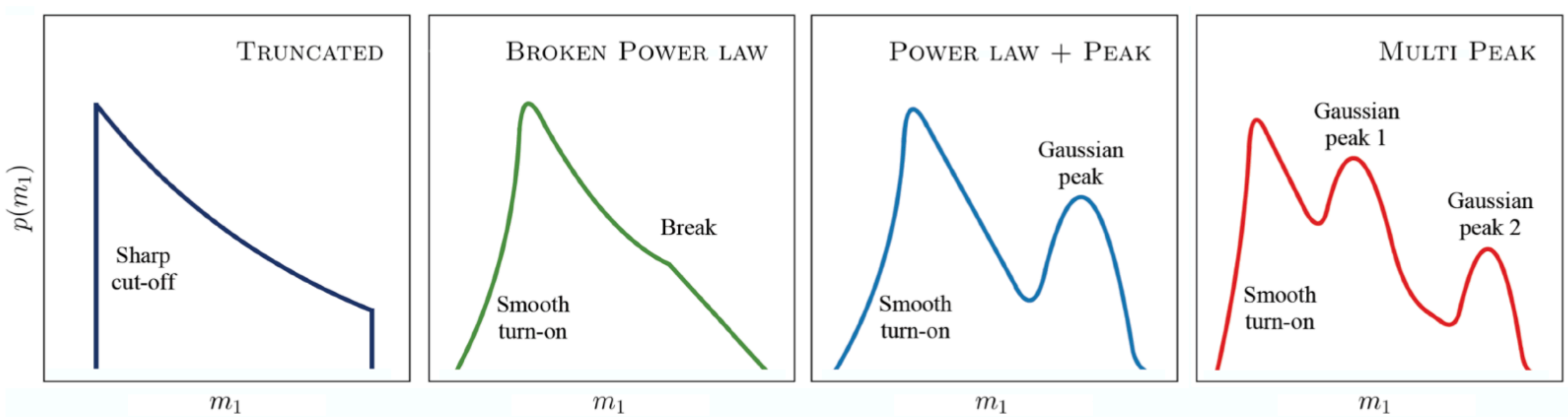
[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D\,|\,\vec{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\,\mathscr{L}(D_i\,|\,\vec{\theta})\,p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})}{\int d\vec{\theta}\,p_{\rm det}(\vec{\theta})\,p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda})}$$

Regarding the population prior we can focus only on three parameters: the two masses (or whatever combination of them) and the luminosity distance:

$$p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda}) = p(m_1^z\,|\,\vec{\Lambda})p(m_2^z\,|\,\vec{\Lambda})p(d_L\,|\,\vec{\Lambda}) = p(m_1\,|\,\vec{\Lambda})p(m_2\,|\,\vec{\Lambda})p(z\,|\,\vec{\Lambda})(1+z)^2\left|\frac{\partial d_L(z)}{\partial z}\right|$$

One must choose a prior model depending on $\vec{\Lambda}$ for the distribution of these parameters.



Example: source-frame primary mass distributions used for the LVK O3 BBH analyses

Dark spectral sirens

[Vitale+, *ArXiv* (2020)] [Mandel+, *MNRAS* (2019)]

$$p(D|\overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathscr{L}(D_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}{\int d\vec{\theta}p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}$$

Regarding the population prior we can focus only on three parameters: the two masses (or whatever combination of them) and the luminosity distance:

$$p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda}) = p(m_1^z|\overrightarrow{\Lambda})p(m_2^z|\overrightarrow{\Lambda})p(d_L|\overrightarrow{\Lambda}) = p(m_1|\overrightarrow{\Lambda})p(m_2|\overrightarrow{\Lambda})p(z|\overrightarrow{\Lambda})(1+z)^2\left|\frac{\partial d_L(z)}{\partial z}\right|$$

One must choose a prior model depending on $\overrightarrow{\Lambda}$ for the distribution of these parameters.

$$p(z|\overrightarrow{\Lambda}) = \frac{a\,(1+z)^b}{1+\left(\frac{1+z}{c}\right)^d}$$

Example: Madau-Dickinson distribution (star formation rate) for the redshift prior

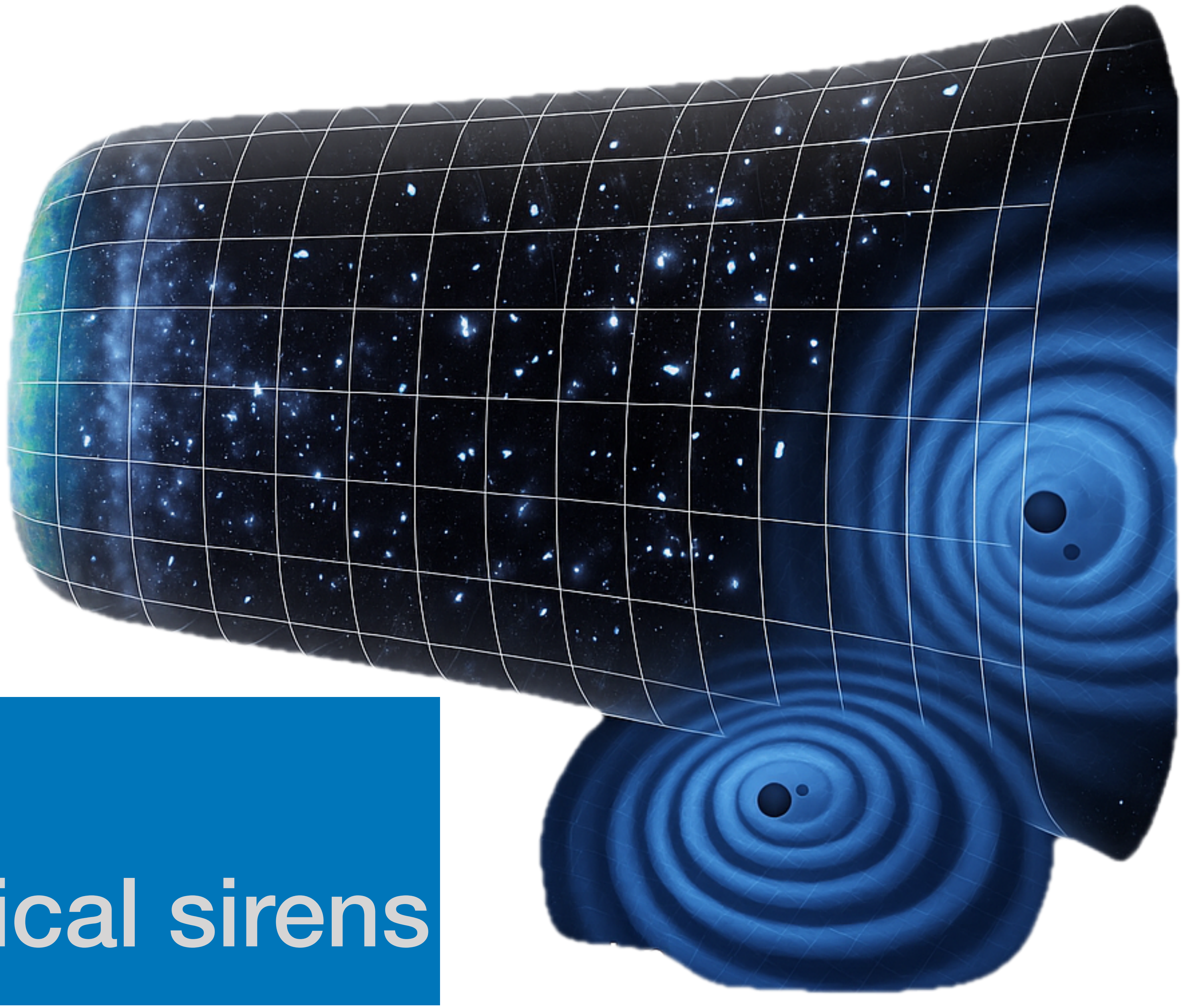
Dark spectral sirens

To summarise here is the lokelihood for spectral sirens:

$$p(D|\overrightarrow{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathscr{L}(D_i|\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}{\int d\vec{\theta}p_{\rm det}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})}$$

where

- $\mathscr{L}(D_i|\vec{\theta})$ is the marginalised GW likelihood, provided by the parameter estimation inference on the GW signal
- $p_{\rm det}(\vec{\theta}|\overrightarrow{\Lambda})$ is the probability of detection, usually computed through an injection campaign
- $p_{\rm pop}(\vec{\theta}|\overrightarrow{\Lambda})$ is the population prior (containing the cosmological information) which must be modelled with parametric functions of $\overrightarrow{\Lambda}$



4
Dark statistical sirens

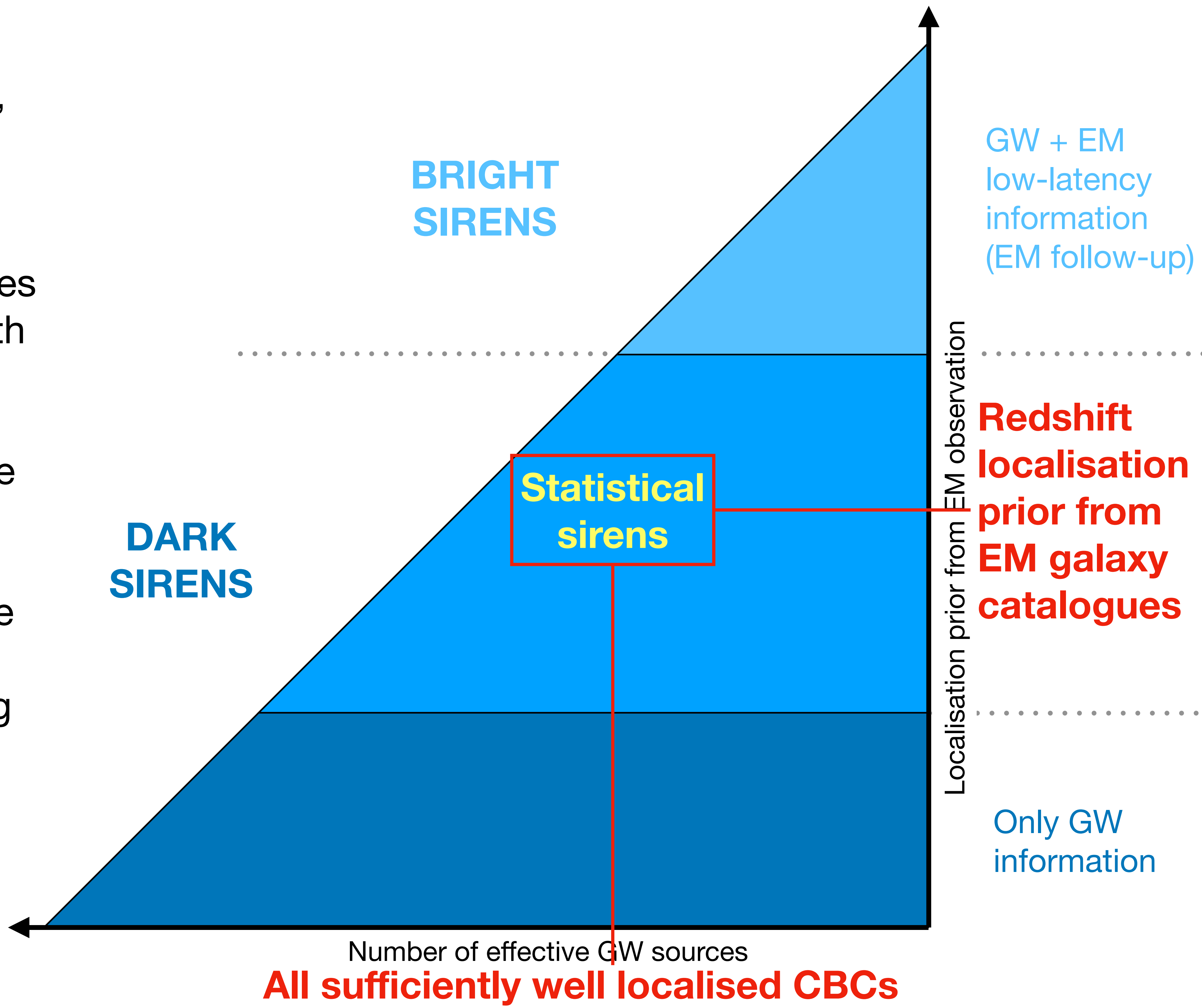
Dark statistical sirens

<u>Statistical sirens</u>
(well localised BBHs, NSBHs, BNSs)

The sky localisation volume of GW sources is cross-matched with a <u>galaxy catalogue</u>

All galaxies within the localisation volume contribute a <u>redshift</u> value to the inference

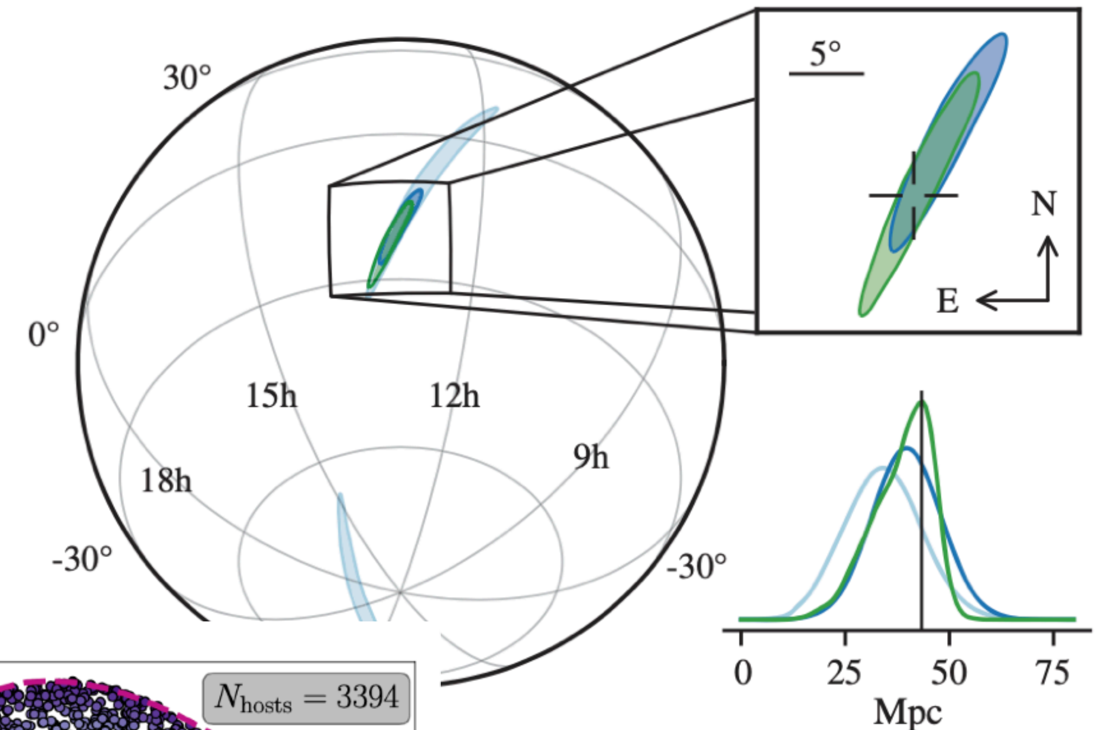
Eventually combining enough GW events <u>statistically</u> yields cosmological constraints

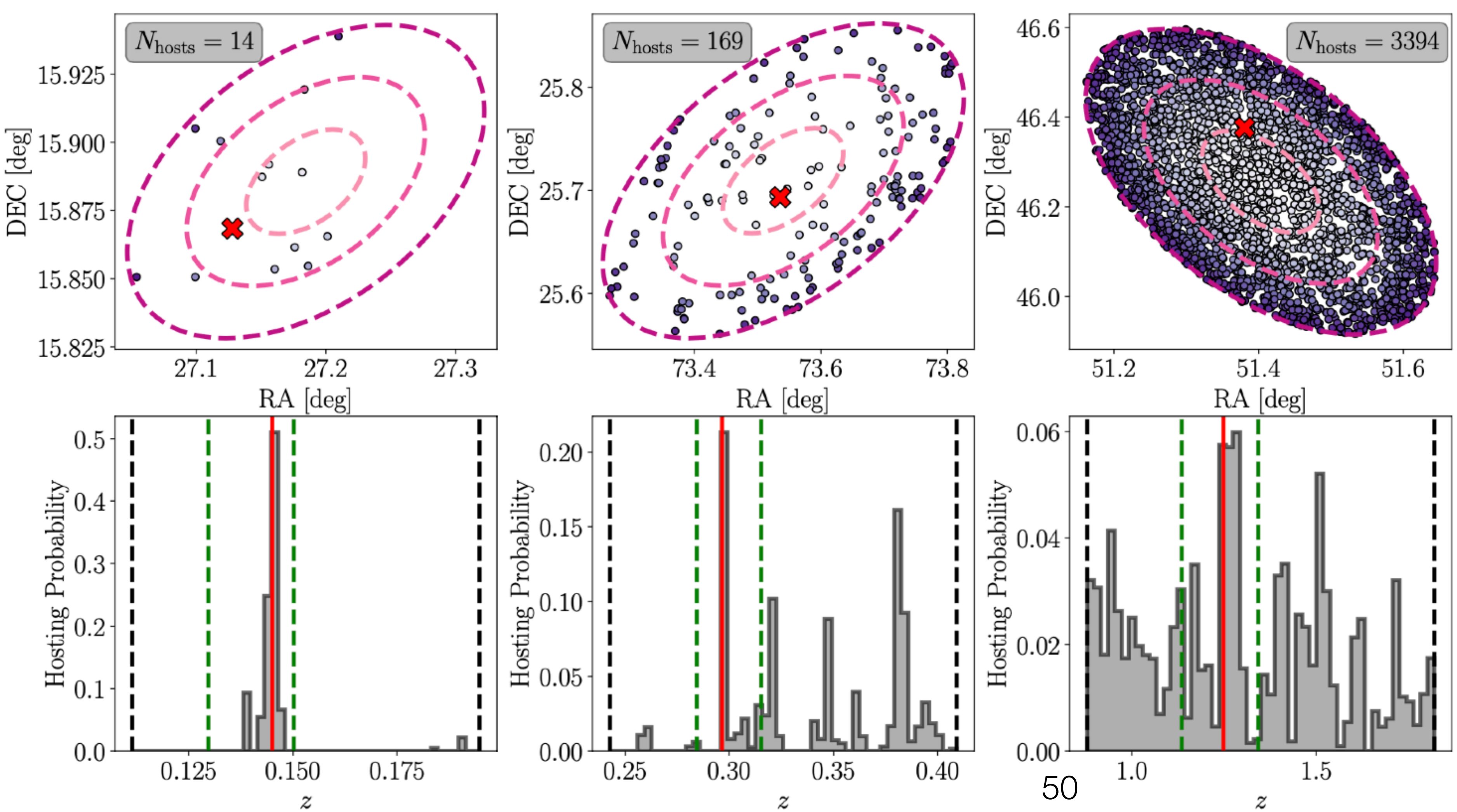


Dark statistical sirens

Statistical sirens exploit EM information coming from galaxy catalogues (or other tracers of cosmic matter) to refine the population prior on th redshift

$$p(z\,|\,\vec{\Lambda})$$





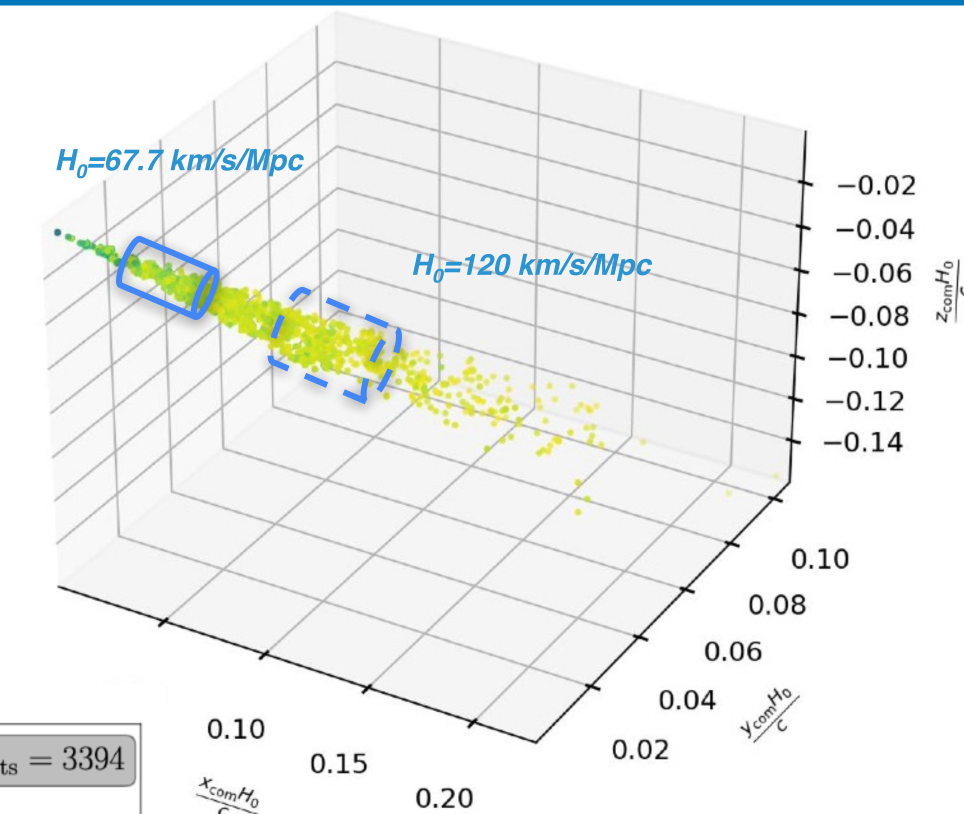
Every galaxy gives a redshift value estimate in which GW events can be localised

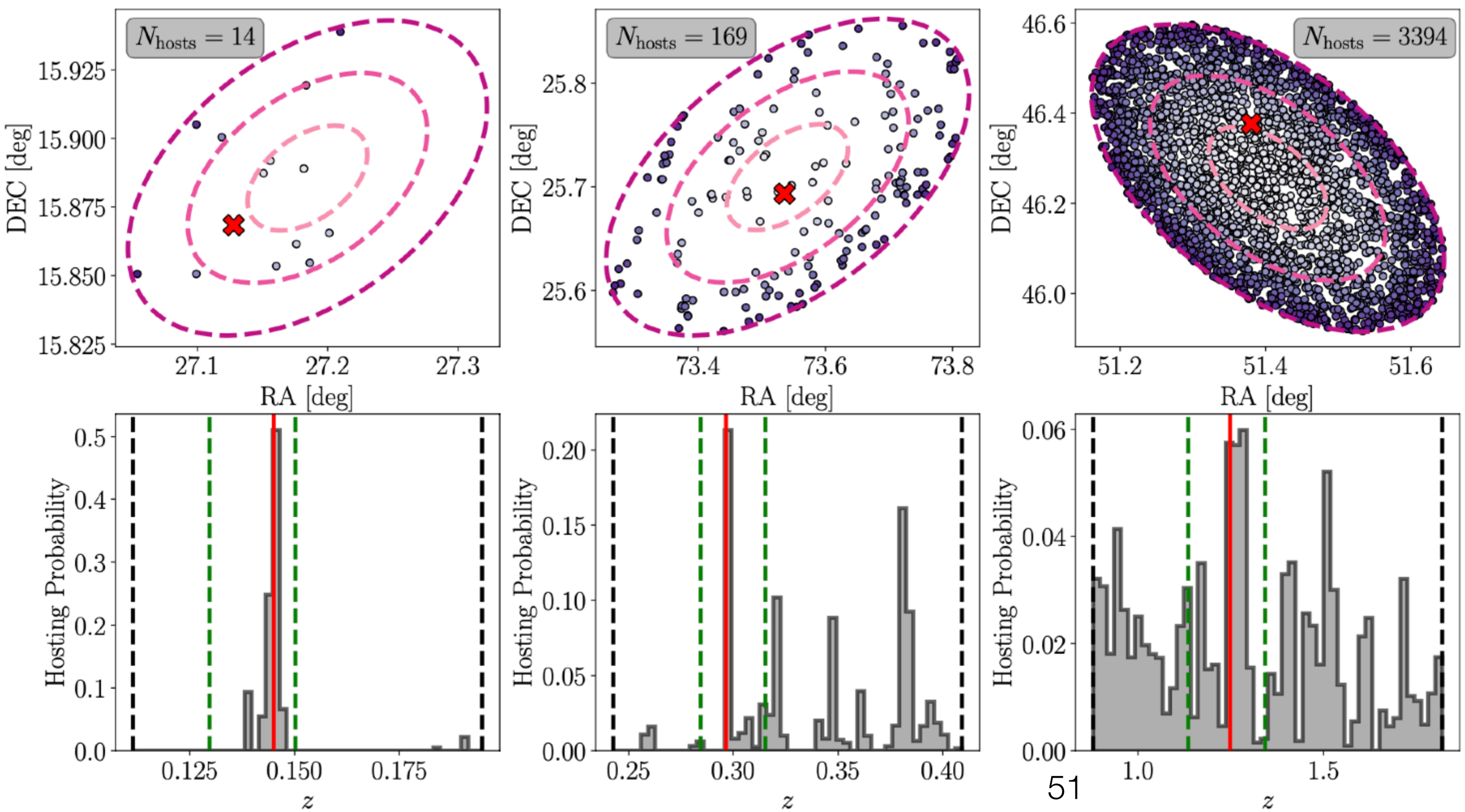
[Schutz, *Nature* (1986)]
[Del Pozzo, *PRD* (2012)]
[Gray+, *PRD* (2020)]
[Gray+, *JCAP* (2023)]

Dark statistical sirens

Statistical sirens exploit EM information coming from galaxy catalogues (or other tracers of cosmic matter) to refine the population prior on th redshift

$$p(z\,|\,\overrightarrow{\Lambda})$$





Every galaxy gives a redshift value estimate in which GW events can be localised

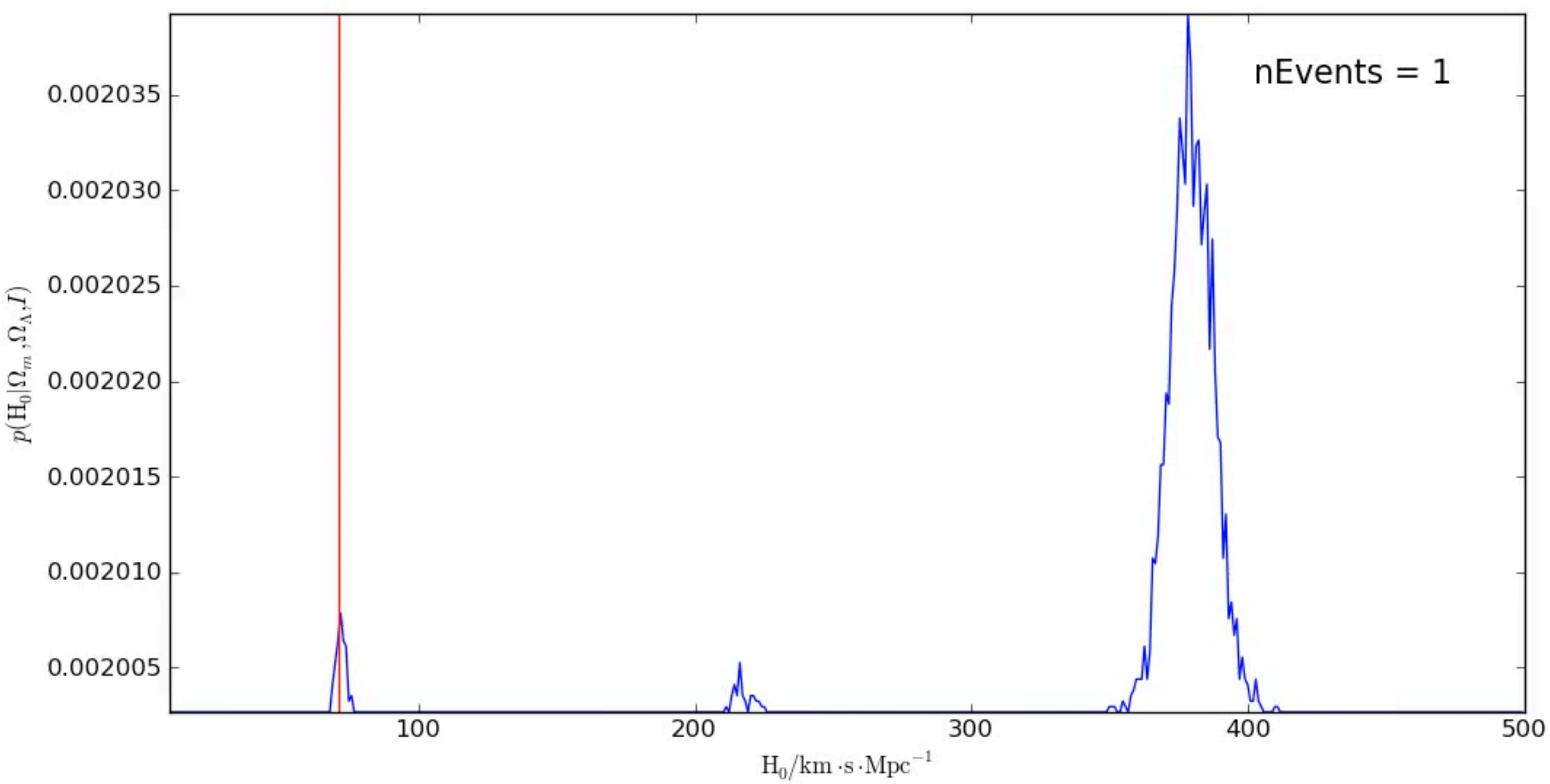
[Schutz, *Nature* (1986)]
[Del Pozzo, *PRD* (2012)]
[Gray+, *PRD* (2020)]
[Gray+, *JCAP* (2023)]

Dark statistical sirens

Statistical sirens exploit EM information coming from galaxy catalogues (or other tracers of cosmic matter) to refine the population prior on th redshift

$$p(z\,|\,\vec{\Lambda})$$

By stacking together the results from many GW events, the redshift estimate given by the host galaxy will add up to the same value of the cosmological paramters, while the contribution given by all other galaxies will cancel out statistically



Credit: W. Del Pozzo

[Schutz, *Nature* (1986)]
[Gray+, *PRD* (2020)]
[Muttoni+, arXiv (2023)]

Dark statistical sirens

For statistical sirens the likelihood is the same as the one used for spectral sirens, except for the population prior which must now contain the localisation information from galaxies:

$$p_{\rm pop}(\vec{\theta}\,|\,\overrightarrow{\Lambda}) = p(m_1^z\,|\,\overrightarrow{\Lambda})p(m_2^z\,|\,\overrightarrow{\Lambda})p(d_L,\hat{\Omega}\,|\,\overrightarrow{\Lambda}) = p(m_1\,|\,\overrightarrow{\Lambda})p(m_2\,|\,\overrightarrow{\Lambda})p(z,\hat{\Omega}\,|\,\overrightarrow{\Lambda})(1+z)^2\left|\frac{\partial d_L}{\partial z}\right|$$

Note that now we kept a dependency on the sky position $\hat{\Omega}$ in the redshift prior as each galaxy is associated to a different value of $\hat{\Omega}$. This is given by

$$p(z,\hat{\Omega}\,|\,\overrightarrow{\Lambda}) \propto p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})\,p_{\rm host}(z,\zeta\,|\,\overrightarrow{\Lambda})$$

where

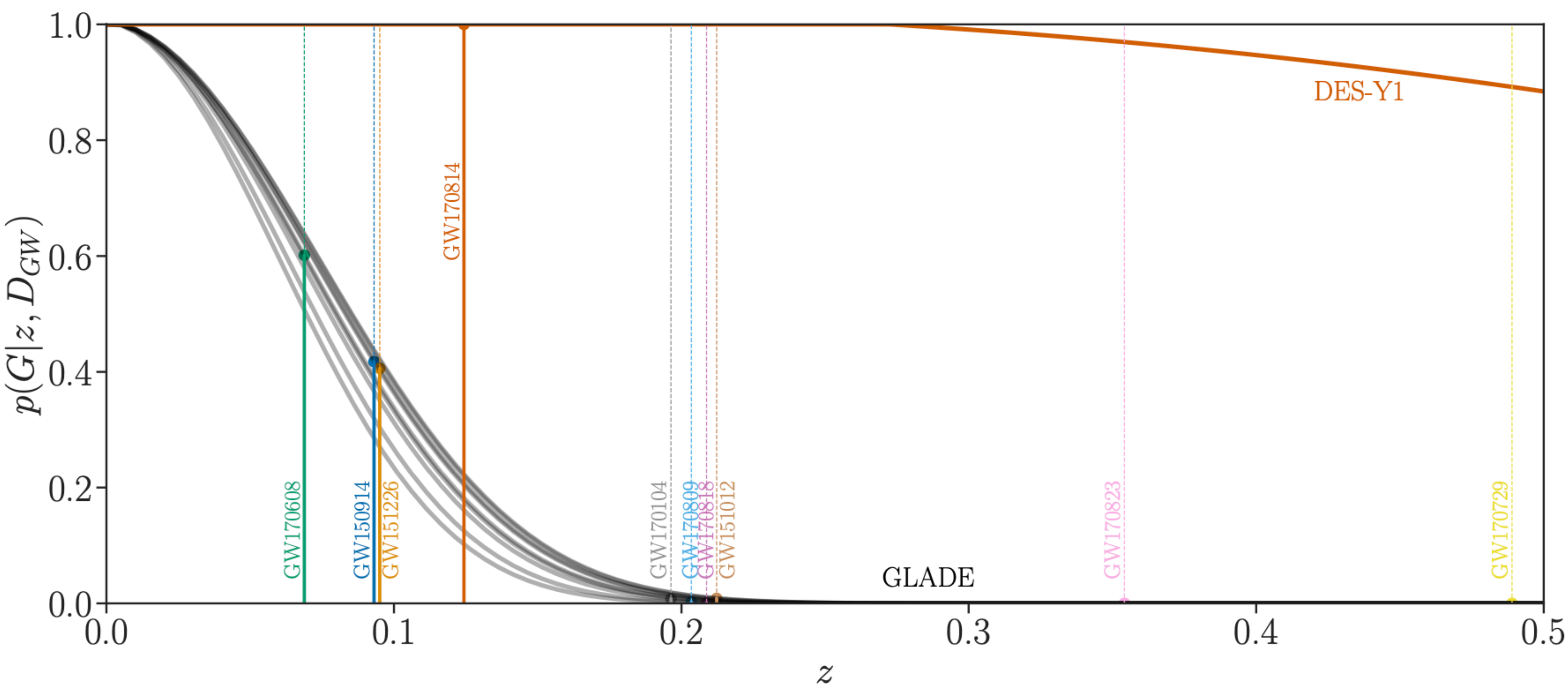
- $p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})$ is the redshift distribution of the galaxy catalogues; usually given by the sum of all redshift measurements of the galaxies, weighted over the GW posterior on $\hat{\Omega}$.
- $p_{\rm host}(z,\zeta\,|\,\overrightarrow{\Lambda})$ is the probability that any given galaxy host a GW event.
- ζ are all observed properties of the galaxies possibly relevant for the inference (luminosity, type, …)

Dark statistical sirens

For statistical sirens the likelihood is the same as the one used for spectral sirens, except for the population prior which must now contain the localisation information from galaxies:

$$p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda}) = p(m_1^z\,|\,\vec{\Lambda})p(m_2^z\,|\,\vec{\Lambda})p(d_L,\hat{\Omega}\,|\,\vec{\Lambda}) = p(m_1\,|\,\vec{\Lambda})p(m_2\,|\,\vec{\Lambda})p(z,\hat{\Omega}\,|\,\vec{\Lambda})(1+z)^2\left|\frac{\partial d_L}{\partial z}\right|$$

However **galaxy catalogues are incomplete**, meaning that several galaxies may not be accounted for.



Dark statistical sirens

For statistical sirens the likelihood is the same as the one used for spectral sirens, except for the population prior which must now contain the localisation information from galaxies:

$$p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda}) = p(m_1^z\,|\,\vec{\Lambda})p(m_2^z\,|\,\vec{\Lambda})p(d_L,\hat{\Omega}\,|\,\vec{\Lambda}) = p(m_1\,|\,\vec{\Lambda})p(m_2\,|\,\vec{\Lambda})p(z,\hat{\Omega}\,|\,\vec{\Lambda})(1+z)^2\left|\frac{\partial d_L}{\partial z}\right|$$

However **galaxy catalogues are incomplete**, meaning that several galaxies may not be accounted for. For this reason the statistical siren analysis must be generalised with the following redshift prior

$$p(z,\hat{\Omega}\,|\,\vec{\Lambda}) \propto f(z,\hat{\Omega})p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})\,p_{\rm host}(z,\zeta\,|\,\vec{\Lambda}) + \left[1 - f(z,\hat{\Omega})\right]p_{\rm out}(z\,|\,\vec{\Lambda})$$

Fraction of galaxies we expect in the catalogue at $z, \hat{\Omega}$.

Same as for spectral siren (e.g. Madau-Dickinson)

Dark statistical sirens

For statistical sirens the likelihood is the same as the one used for spectral sirens, except for the population prior which must now contain the localisation information from galaxies:

$$p_{\rm pop}(\vec{\theta}\,|\,\vec{\Lambda}) = p(m_1^z\,|\,\vec{\Lambda})p(m_2^z\,|\,\vec{\Lambda})p(d_L,\hat{\Omega}\,|\,\vec{\Lambda}) = p(m_1\,|\,\vec{\Lambda})p(m_2\,|\,\vec{\Lambda})p(z,\hat{\Omega}\,|\,\vec{\Lambda})(1+z)^2\left|\frac{\partial d_L}{\partial z}\right|$$

However **galaxy catalogues are incomplete**, meaning that several galaxies may not be accounted for. For this reason the statistical siren analysis must be generalised with the following redshift prior

$$p(z,\hat{\Omega}\,|\,\vec{\Lambda}) \propto f(z,\hat{\Omega})\,p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})\,p_{\rm host}(z,\zeta\,|\,\vec{\Lambda}) + \left[1 - f(z,\hat{\Omega})\right] p_{\rm out}(z\,|\,\vec{\Lambda})$$

$f(z,\hat{\Omega})$ is usually computed via the **Schechter function** $f_{\rm Sch}(M,z)$ (which estimates the distribution of galaxies in absolute magnitude):

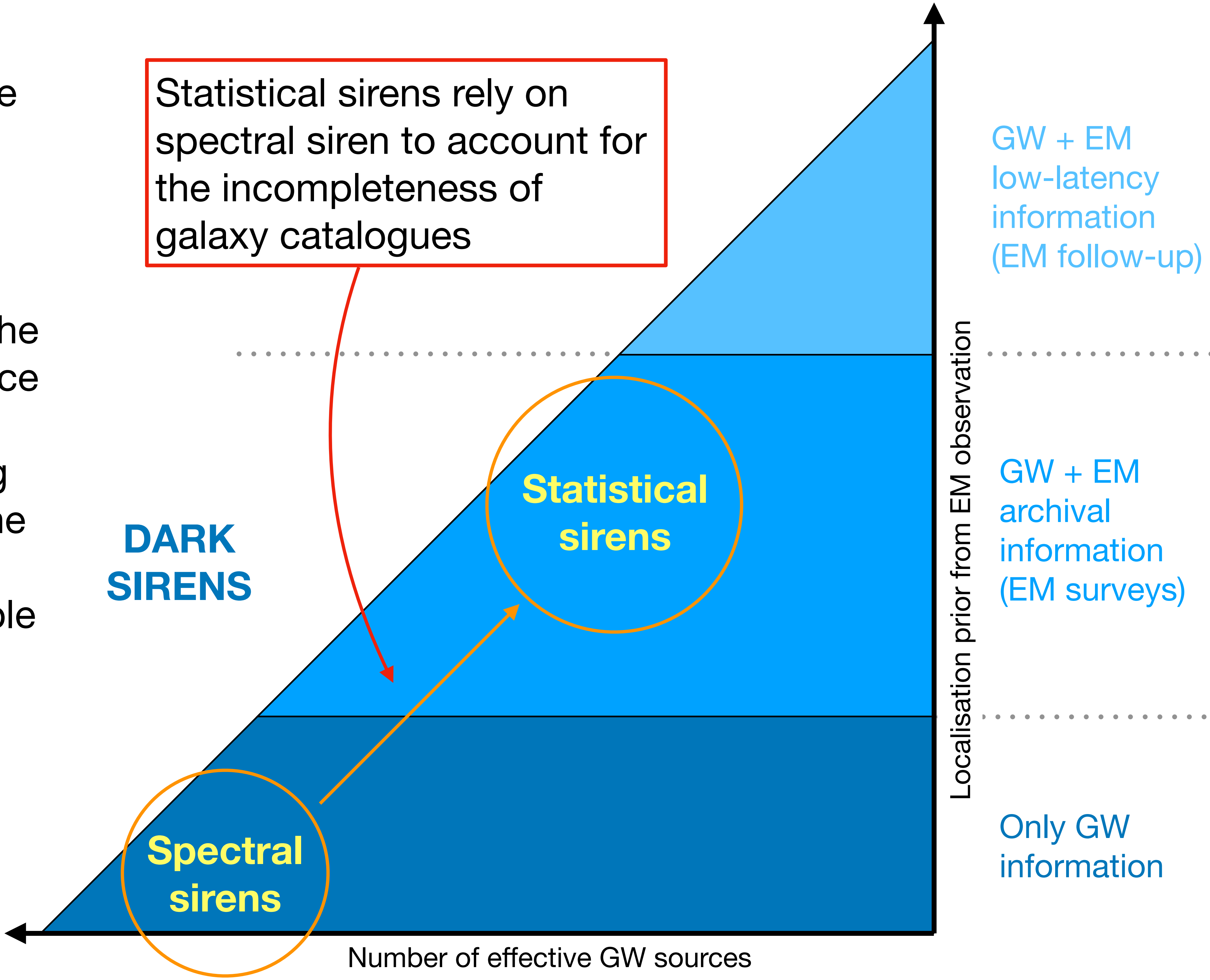
$$f(z,\hat{\Omega}) = \frac{\int_{M_{\rm thr}}^{\infty} f_{\rm Sch}(M,z)p_{\rm host}(z,M\,|\,\vec{\Lambda})dM}{\int_{M_{\rm faint}}^{\infty} f_{\rm Sch}(M,z)p_{\rm host}(z,M\,|\,\vec{\Lambda})dM}$$

where $M_{\rm thr}$ and $M_{\rm faint}$ are the apparent magnitude and faint end boundaries of the galaxy catalogue survey considered.

The standard siren Pyramid

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

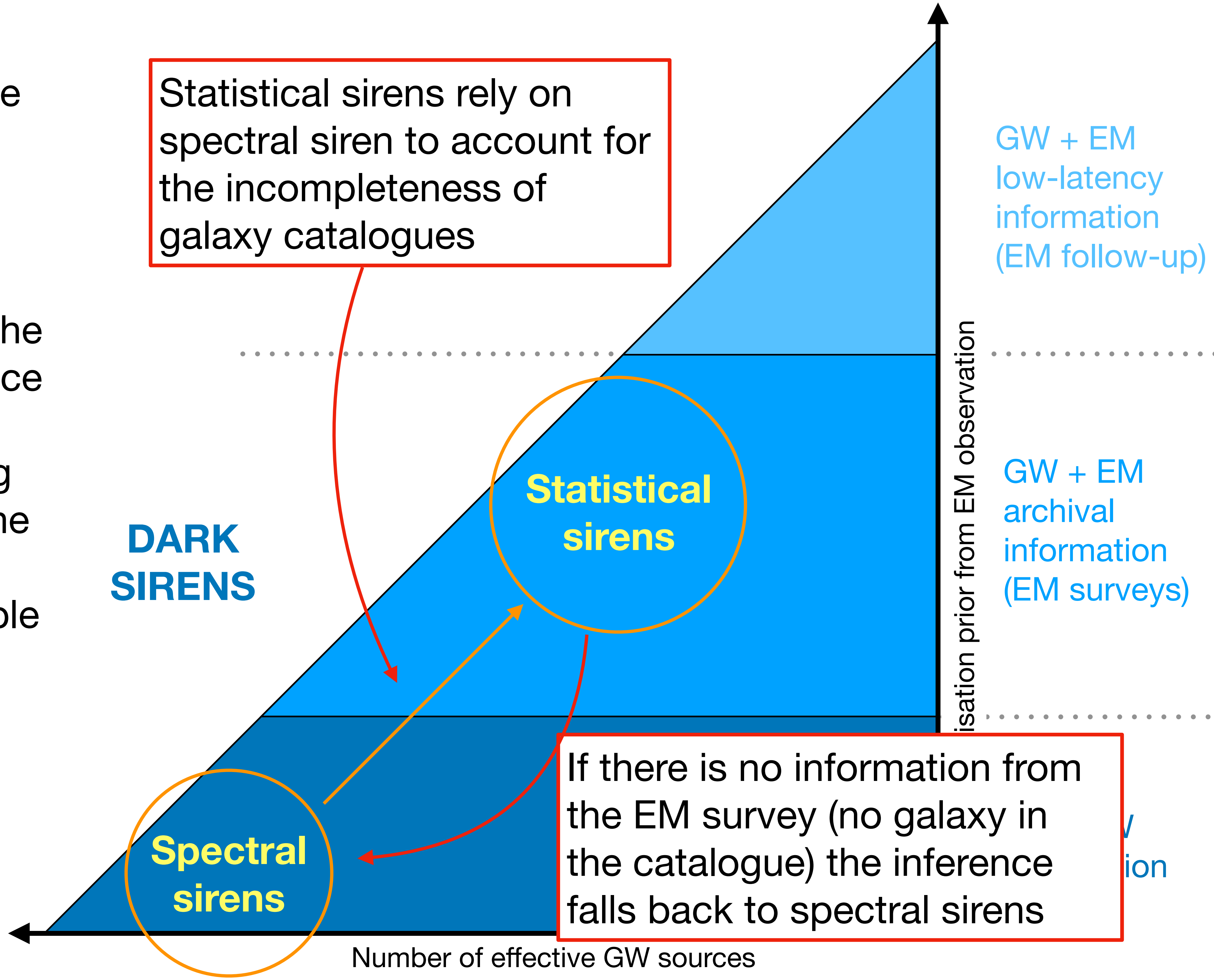
Information helping the cosmological inference can be added progressively starting from the bottom in the cases where EM information is available

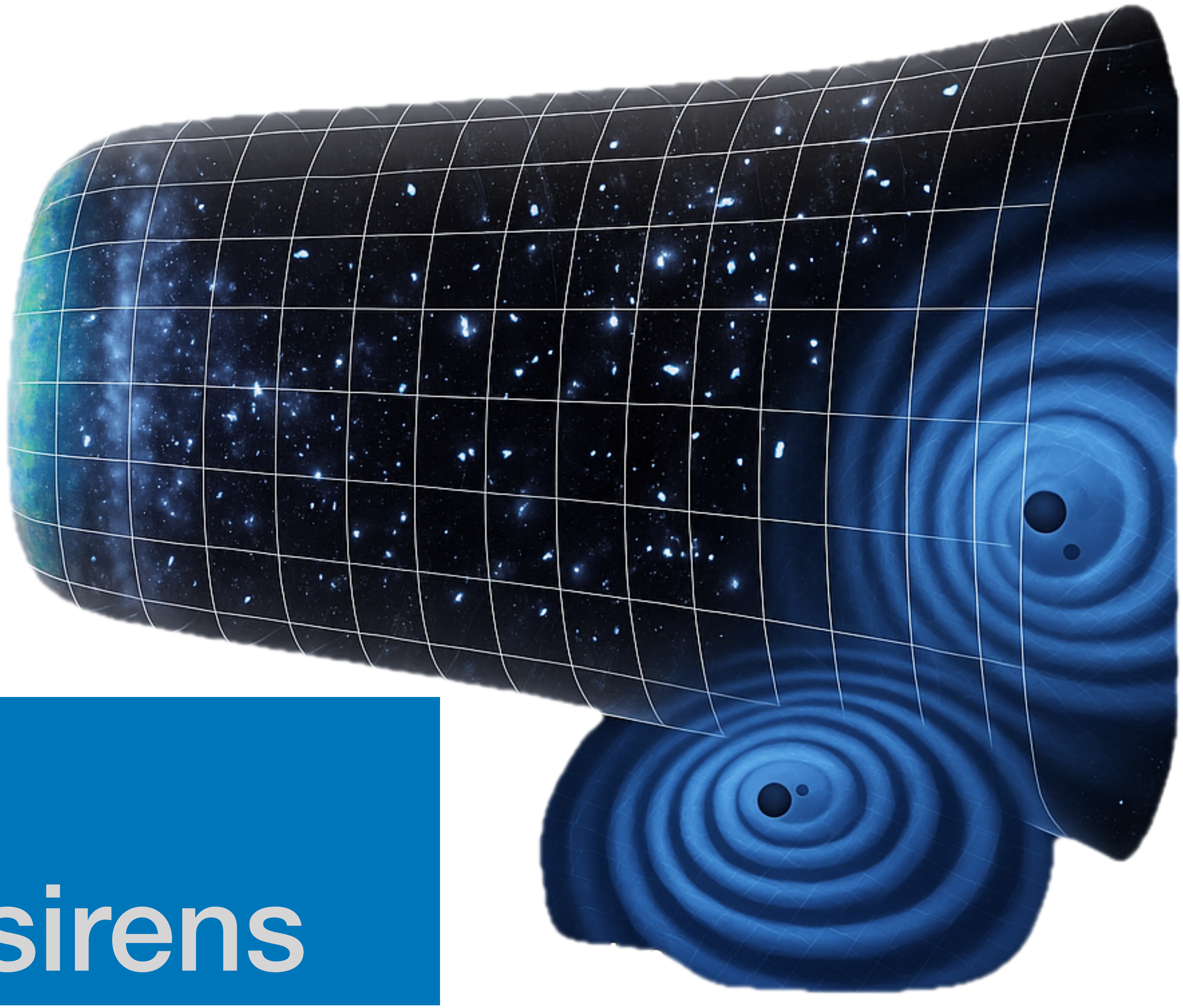


The standard siren Pyramid

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

<u>Information</u> helping the cosmological inference <u>can be added progressively</u> starting from the bottom in the cases where EM information is available





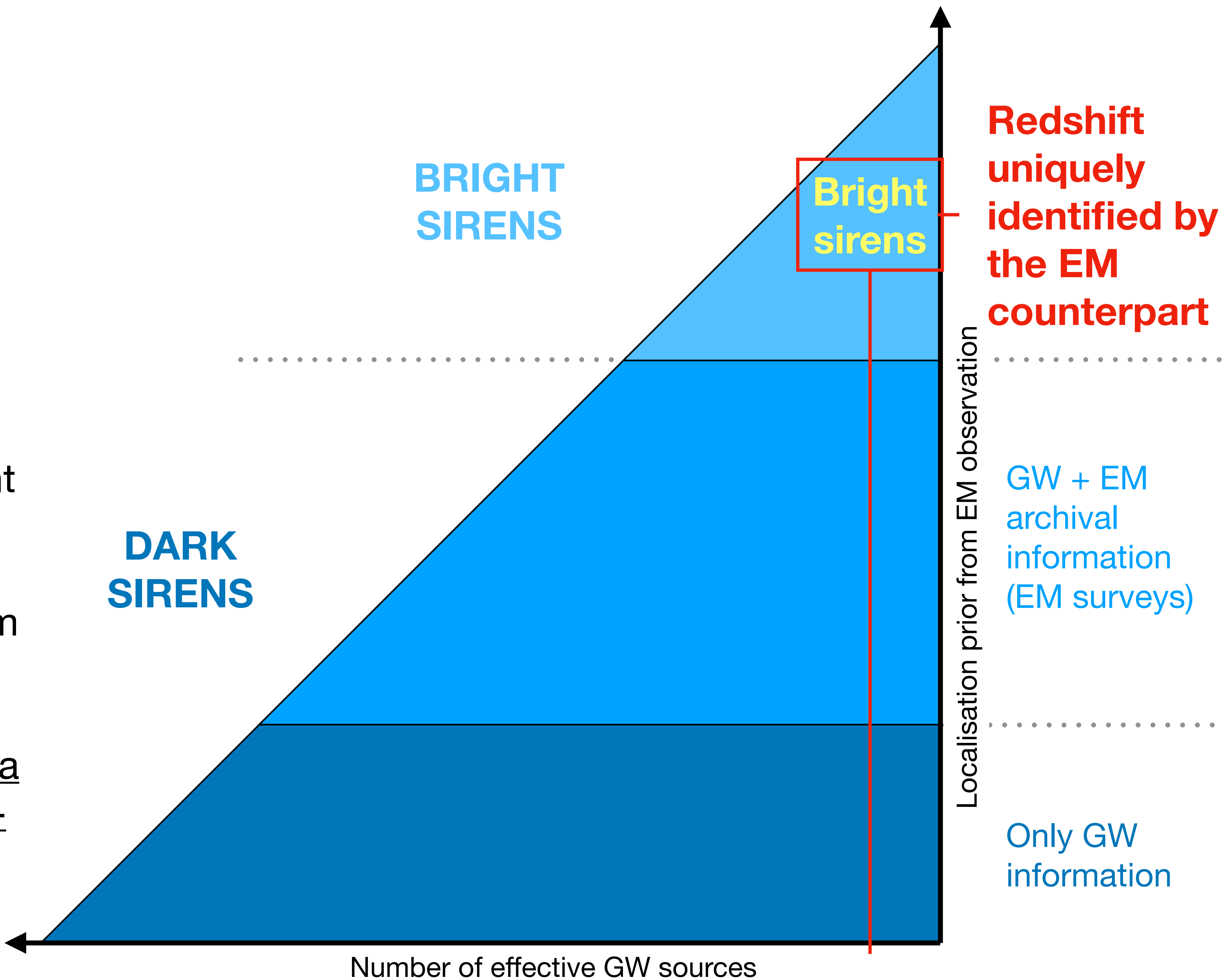
5
Bright sirens

Bright sirens

<u>Bright sirens</u>
(BNSs, NSBHs with EM counterpart)

The <u>EM counterpart</u> allows for a direct identification of the GW source host galaxy, from which a redshift measurement is obtained

Tight constraints from a low number of events as each provides a <u>single data point in the distance-redshift relation</u>

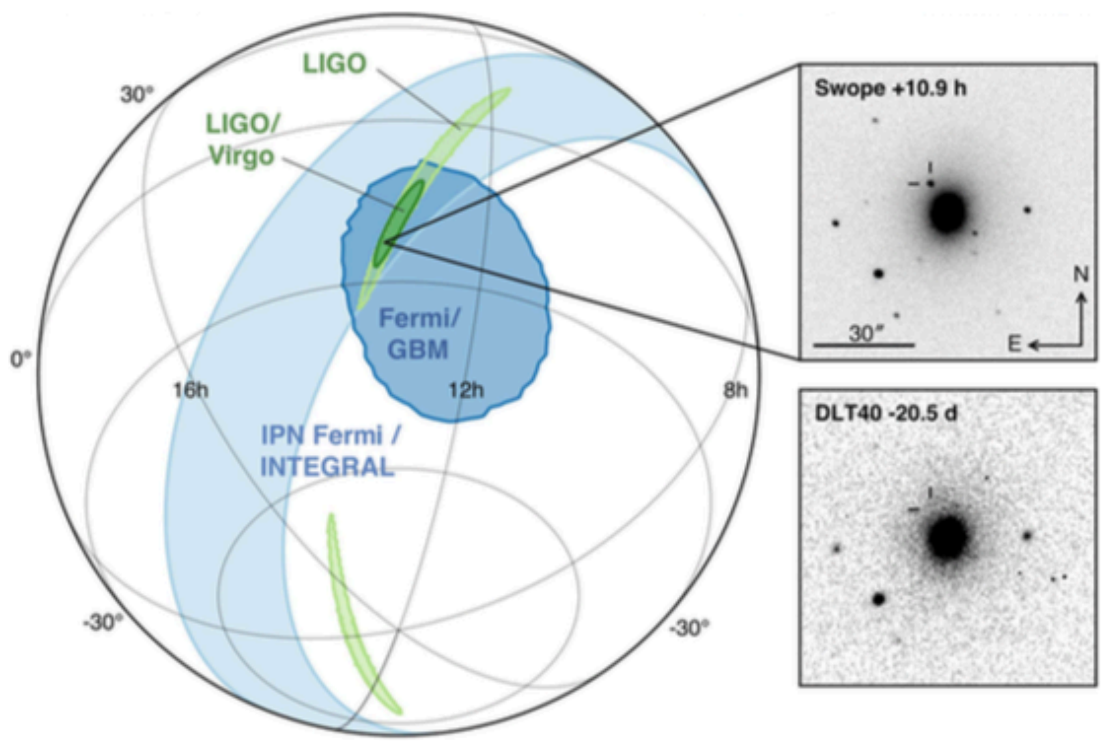


Only CBCs with an associated EM counterpart

Bright sirens

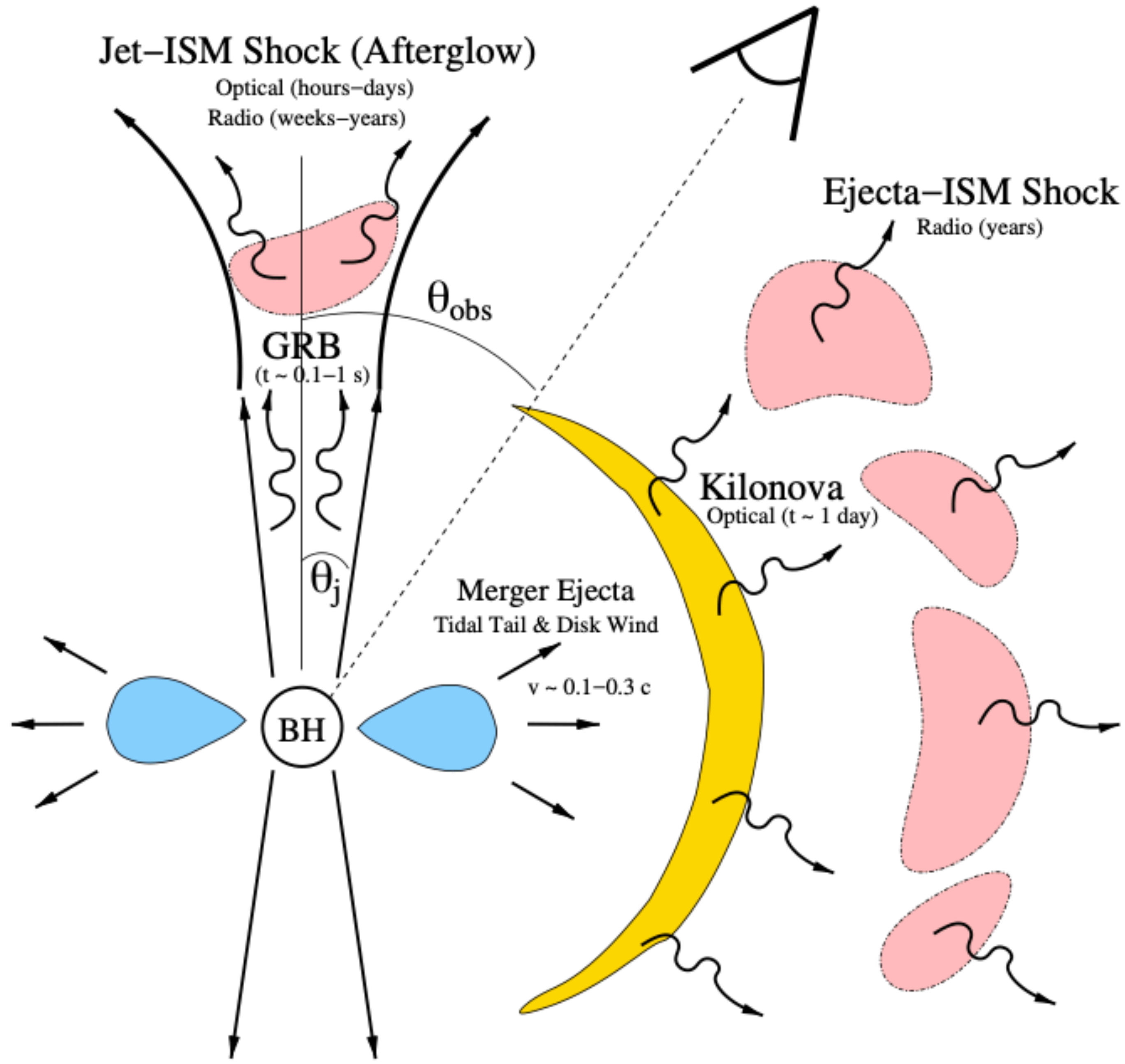
Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information

Example: BNS mergers and GRBs



[Schutz, *Nature* (1986)]
[LVC+, *ApJL* (2017)]

Example: BNS mergers

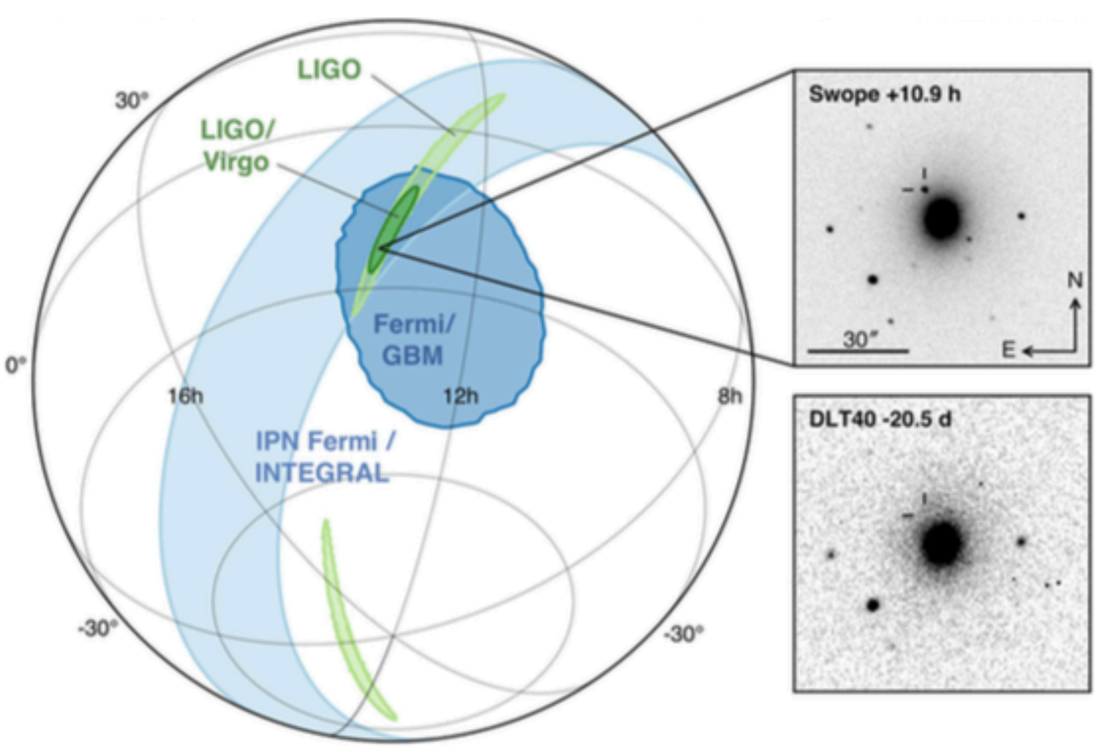


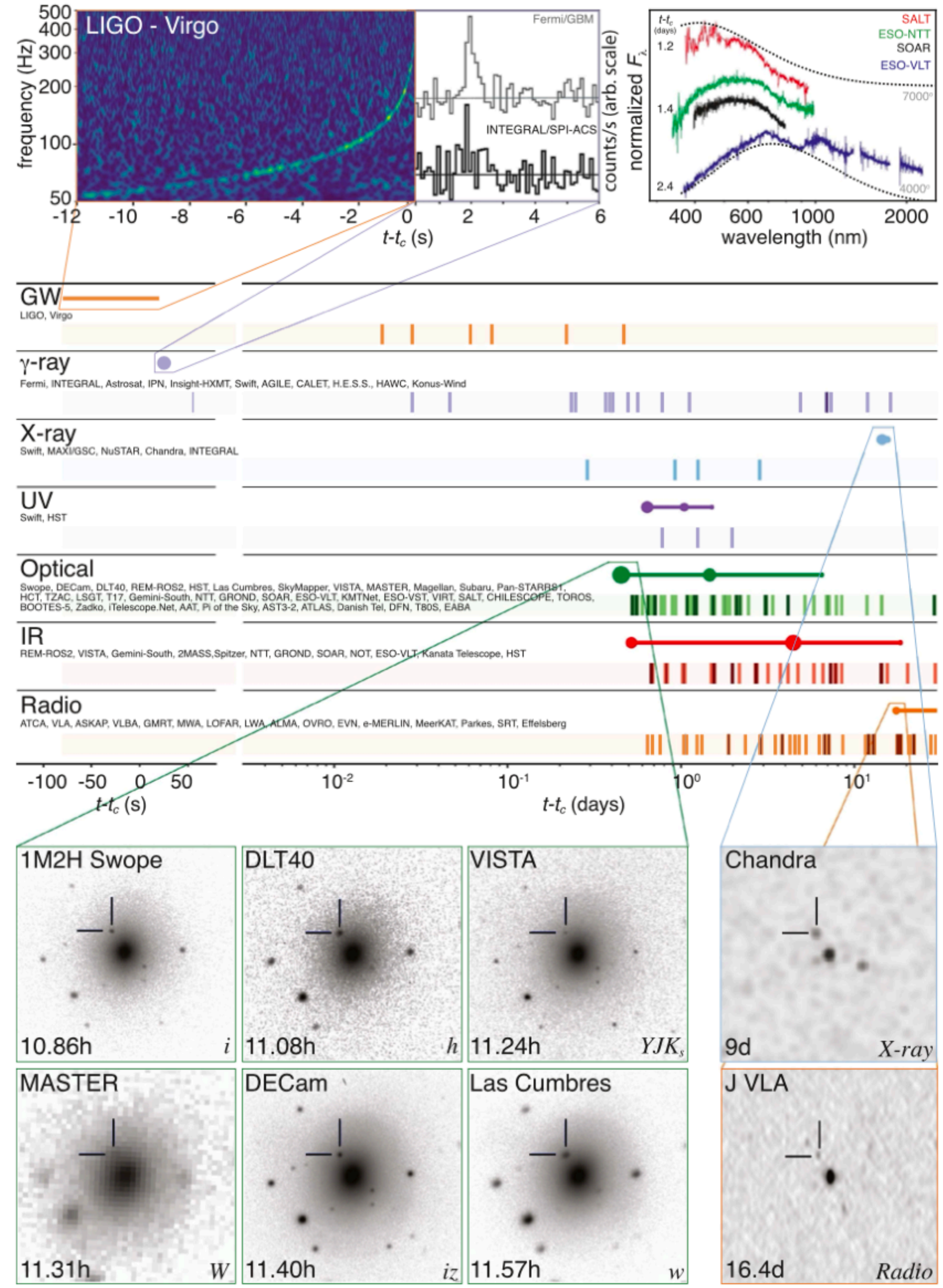
[Metzger&Berger, *ApJ* (2012)]

Bright sirens

Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information

Example: BNS mergers and GRBs





[Schutz, *Nature* (1986)]
[LVC+, *ApJL* (2017)]

Example: GW170817

[LVC+, *ApJL* (2017)]

Bright sirens

Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information

The EM counterpart usually allows for the unique identification of the host galaxy (or more generally to a few possible host galaxies)

The redshift prior can thus be simplified to

$$p(z,\hat{\Omega}\,|\,\vec{\Lambda}) \propto f(z,\hat{\Omega})p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})\,p_{\rm host}(z,\zeta\,|\,\vec{\Lambda}) + \left[1 - f(z,\hat{\Omega})\right]p_{\rm out}(z\,|\,\vec{\Lambda})$$

$$\propto p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})$$

since $f(z,\hat{\Omega}) = 1$ and $p_{\rm host}(z,\zeta\,|\,\vec{\Lambda}) = 1$ (host galaxy identified).

In this case $p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal})$ is simply the redshift measurement (posterior) of the host galaxy, which for a Gaussian estimate and accurate sky localisation reads

$$p_{\rm gal}(z,\hat{\Omega},\zeta\,|\,D_{\rm gal}) \propto e^{\left(\frac{z-z_{\rm gal}}{\sigma_{z,\rm gal}}\right)^2}\delta(\hat{\Omega}-\hat{\Omega}_{\rm gal})$$

Bright sirens

Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information

The EM counterpart usually allows for the unique identification of the host galaxy (or more generally to a few possible host galaxies)

The redshift prior can thus be simplified to

$$p(z, \hat{\Omega} \,|\, \vec{\Lambda}) \propto f(z, \hat{\Omega}) p_{\rm gal}(z, \hat{\Omega}, \zeta \,|\, D_{\rm gal})\, p_{\rm host}(z, \zeta \,|\, \vec{\Lambda}) + \left[1 - f(z, \hat{\Omega})\right] p_{\rm out}(z \,|\, \vec{\Lambda})$$

$$\propto p_{\rm gal}(z, \hat{\Omega}, \zeta \,|\, D_{\rm gal})$$

since $f(z, \hat{\Omega}) = 1$ and $p_{\rm host}(z, \zeta \,|\, \vec{\Lambda}) = 1$ (host galaxy identified).

If $p_{\rm gal}(z, \hat{\Omega}, \zeta \,|\, D_{\rm gal})$ is perfectly localised $p_{\rm gal}(z, \hat{\Omega}, \zeta \,|\, D_{\rm gal}) \propto \delta(z - z_{\rm gal})\delta(\hat{\Omega} - \hat{\Omega}_{\rm gal})$ then at low redshift the full hierarchical likelihood can be approximated as

$$p(D \,|\, \vec{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{p(D_i \,|\, d_L(z; \vec{\Lambda}_c), \hat{\Omega}_{\rm gal})}{H_0^3}$$

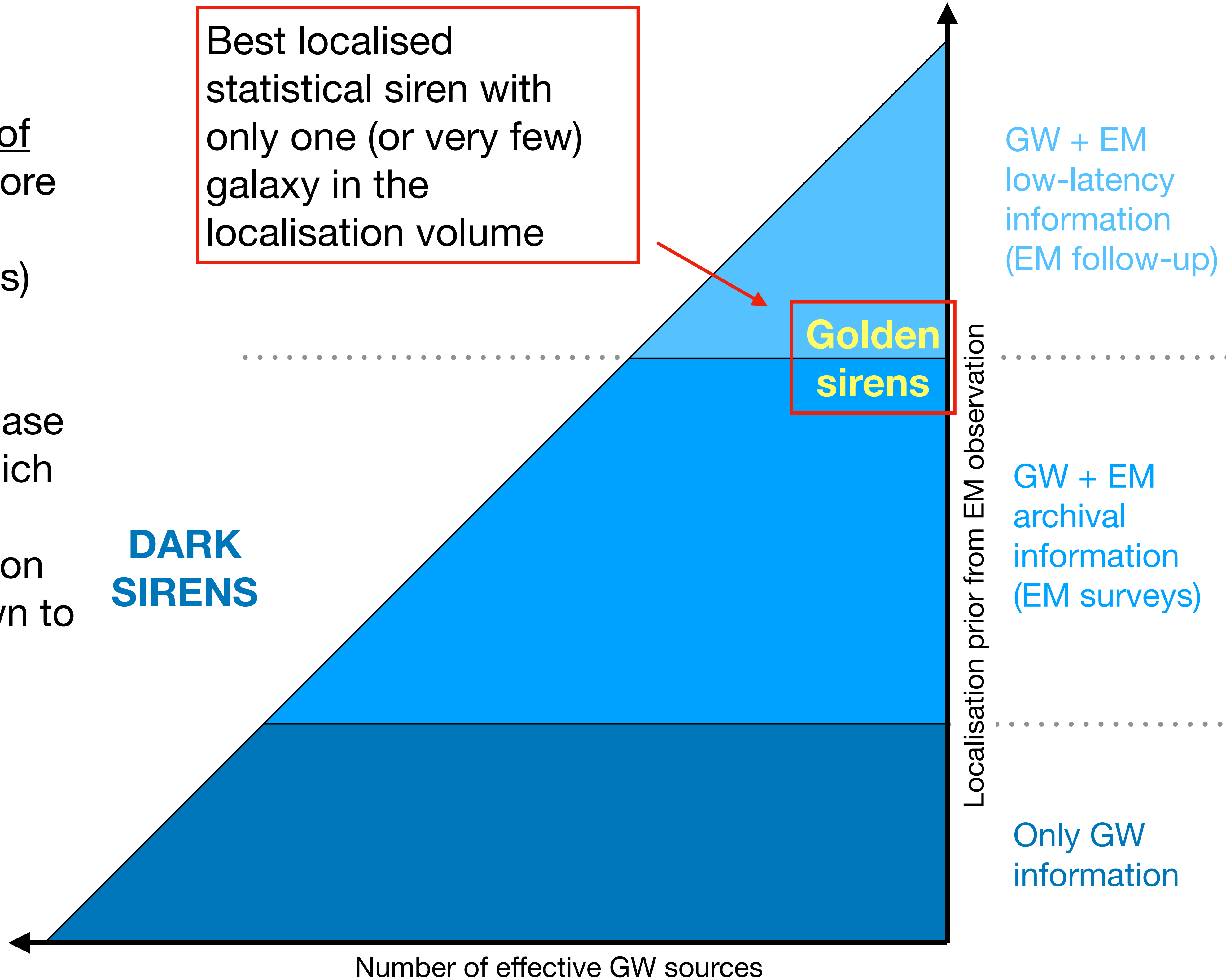
→ Direct product of the GW measurement salong the galaxy line of sight

→ Probability of detection at low-z proportional to the comoving volume

Bright sirens

The EM counterpart usually allows for the unique identification of the host galaxy (or more generally to a few possible host galaxies)

This is however the case of **golden sirens**, which do not assume EM counterpart information but are localised down to a single host galaxy

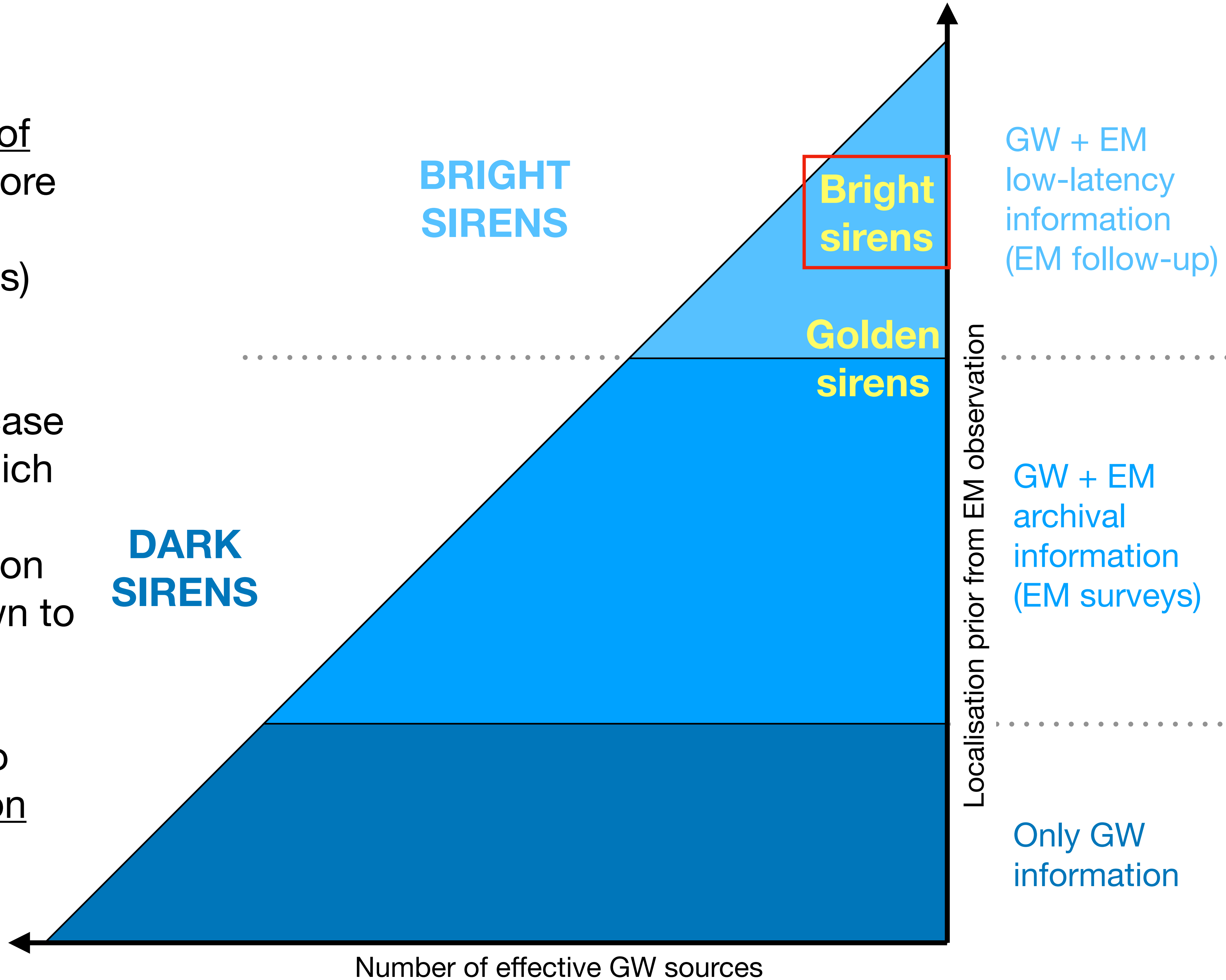


Bright sirens

The EM counterpart usually allows for the unique identification of the host galaxy (or more generally to a few possible host galaxies)

This is however the case of **golden sirens**, which do not assume EM counterpart information but are localised down to a single host galaxy

Bright sirens can also exploit EM information from the counterpart itself



Bright sirens

Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information.

Using this additional EM information implies a modifcation of the spectral siren likelihood as follows:

$$p(D|\vec{\Lambda}) \propto \prod_{i=1}^{N_{\rm obs}} \frac{\int d\vec{\theta}\mathcal{L}_{\rm GW}(D_i^{\rm GW}|\vec{\theta})\mathcal{L}_{\rm EM}(D_i^{\rm EM}|\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\Lambda})}{\int d\vec{\theta}p_{\rm det}^{\rm GW+EM}(\vec{\theta})p_{\rm pop}(\vec{\theta}|\vec{\Lambda})}$$

where now we must take into account the possible measurement of source parameters directly by the EM observations of the counterpart, $\mathcal{L}_{\rm EM}(D_i^{\rm EM}|\vec{\theta})$, and that the detection probability now must be computed for the EM observations as well, $p_{\rm det}^{\rm GW+EM}(\vec{\theta})$.

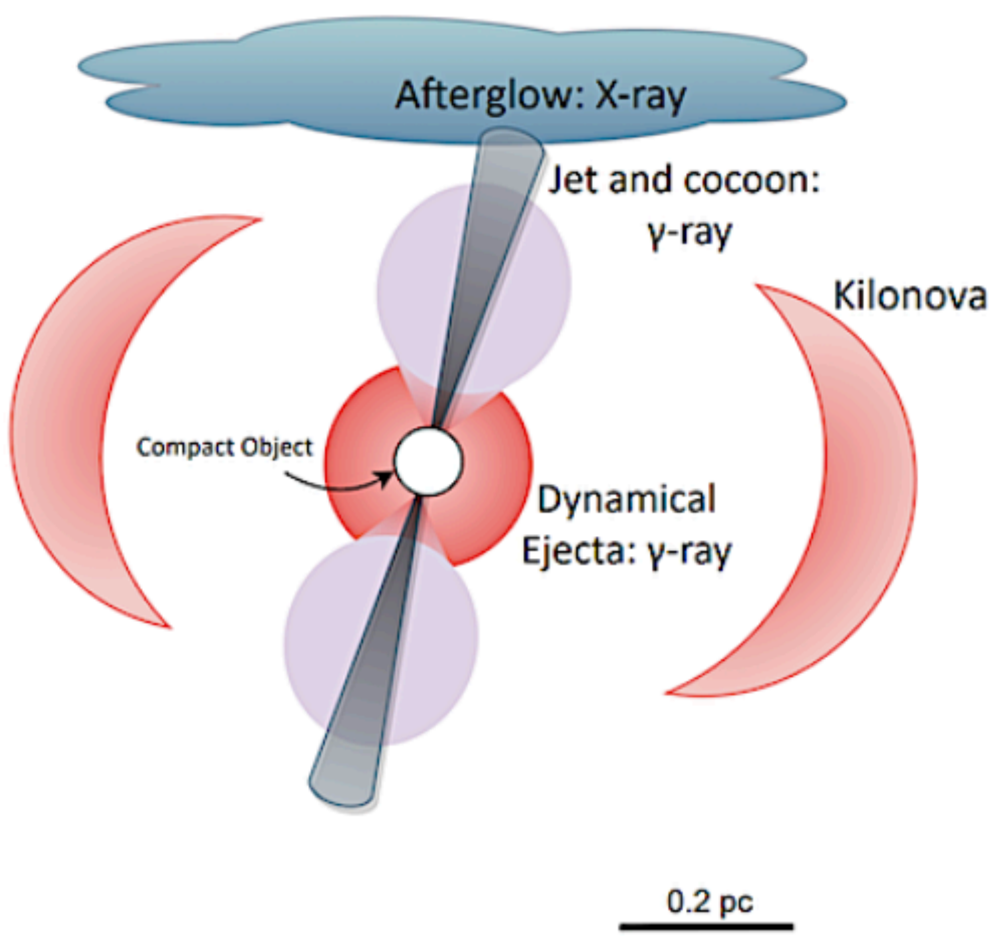
Besides d_L (or better z) and $\hat{\Omega}$, the EM counterpart may provide information on the binary's orbital inclination angle $\cos\iota$

$$\mathcal{L}_{\rm EM}(D_i^{\rm EM}|\vec{\theta}) = \mathcal{L}_{\rm EM}(D_i^{\rm EM}|z, \hat{\Omega}, \cos\iota)$$

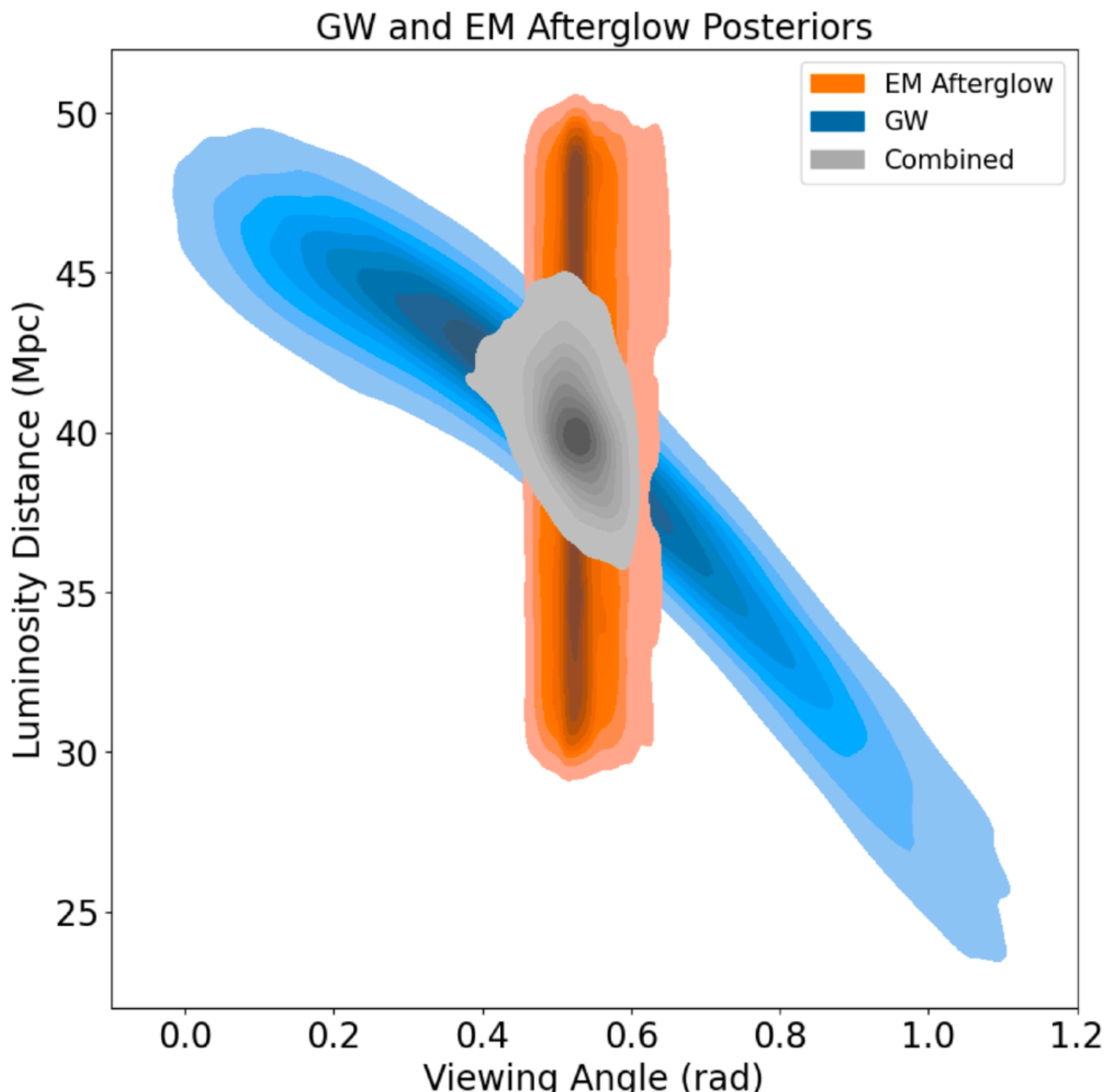
Bright sirens

Bright sirens are associated to an EM counterpart which brings additional information on top of the GW (and galaxy catalogue) information.

For example, the afterglow of BNS merger can be modelled to yield information on the inclination angle



(a) First days to years post-merger

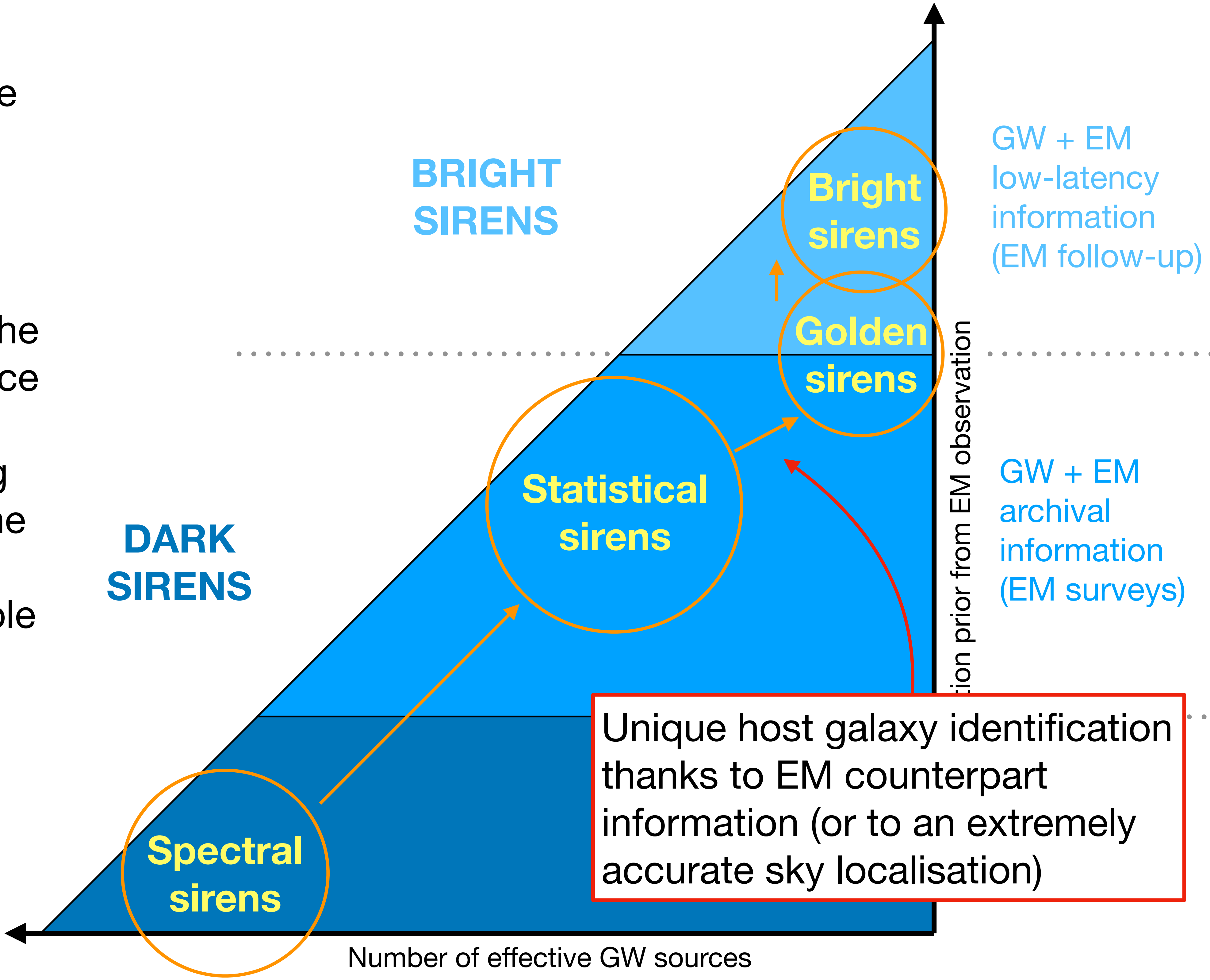


[Perkins+, *arXiv* (2023)]
[Palmese+, *PRD* (2024)]

Bright sirens

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

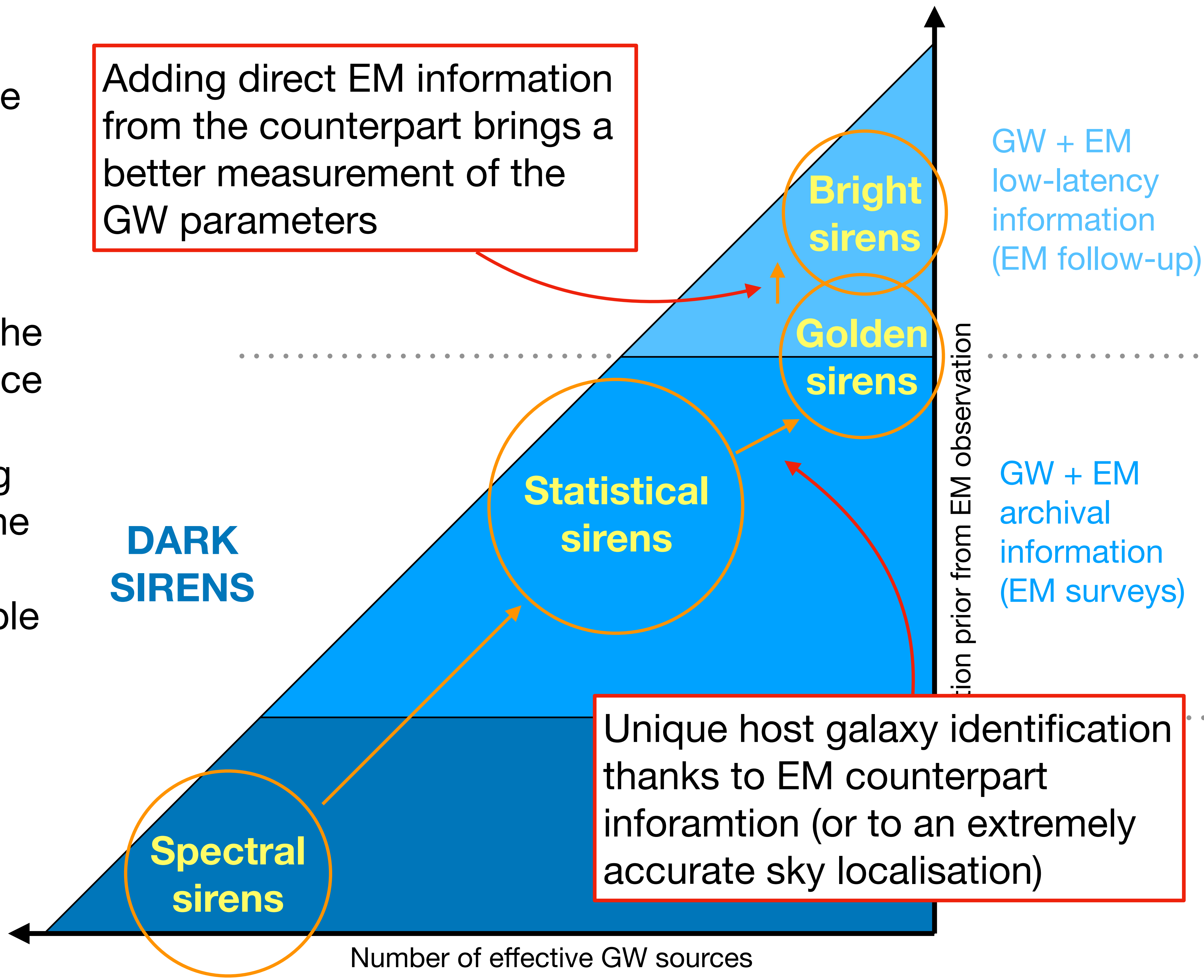
Information helping the cosmological inference can be added progressively starting from the bottom in the cases where EM information is available

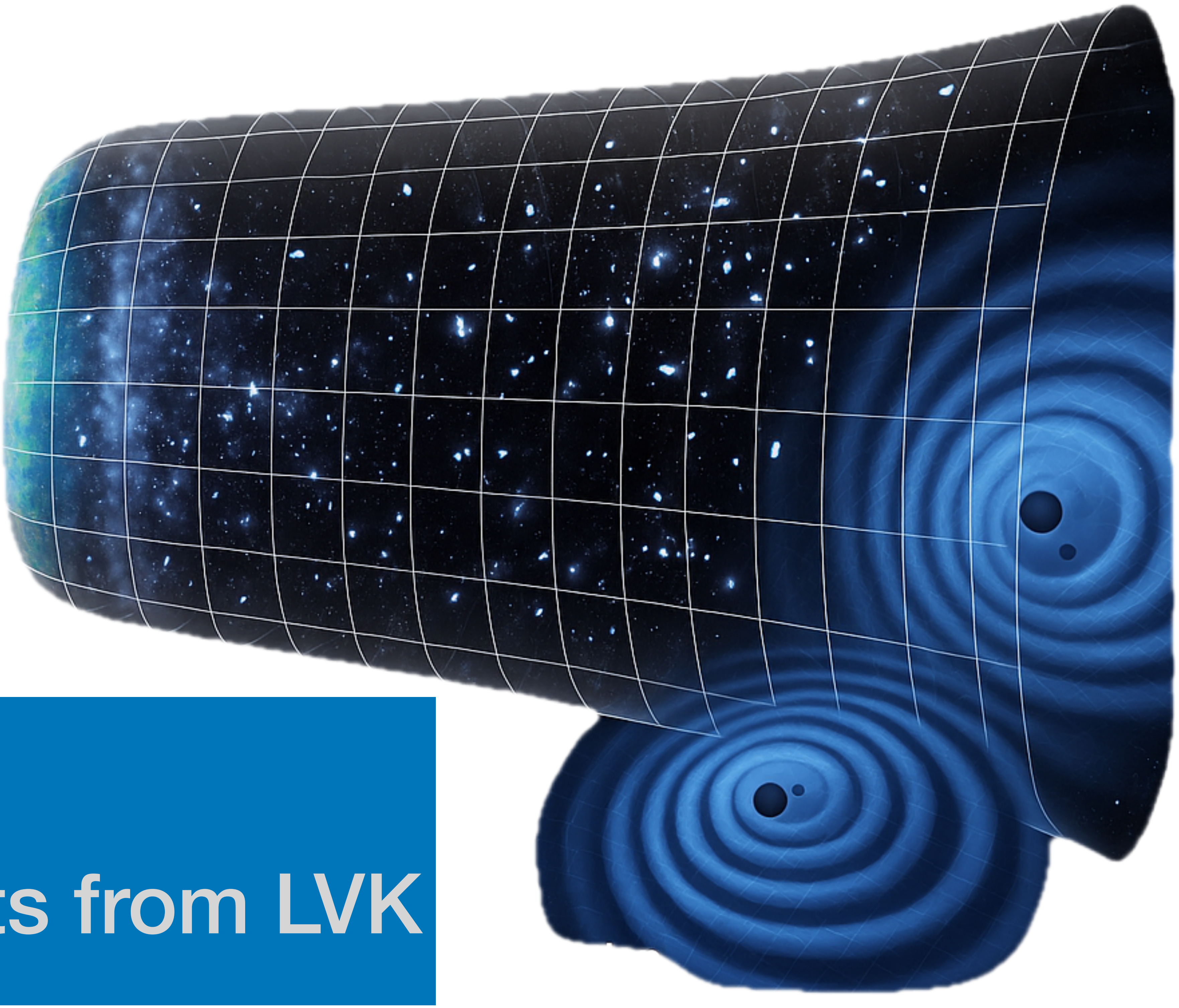


Bright sirens

Standard siren methodologies up the pyramyd usually rely on other methods in the lower layers

Information helping the cosmological inference can be added progressively starting from the bottom in the cases where EM information is available



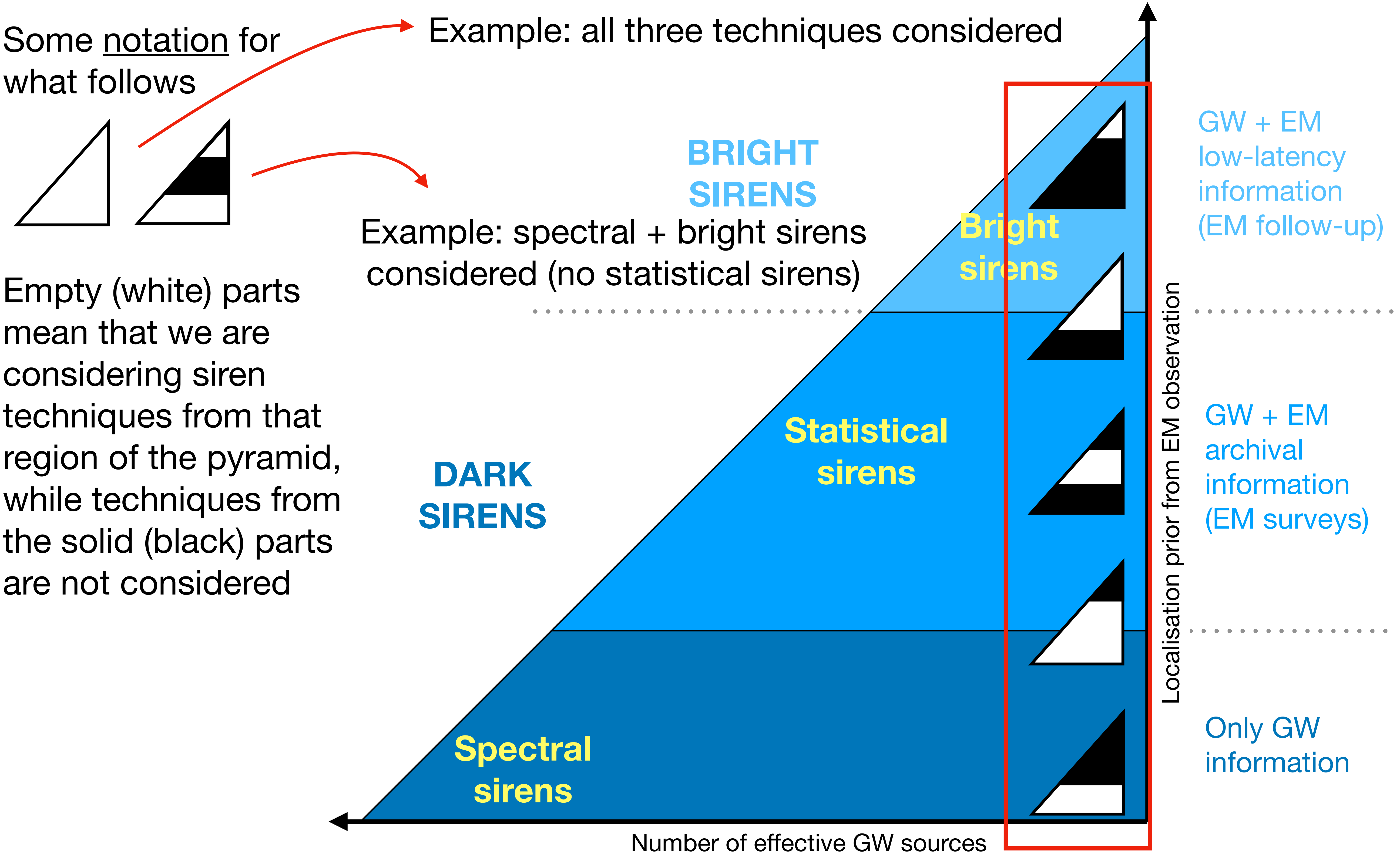


6
Current results from LVK

The standard siren Pyramid

Some notation for what follows

Empty (white) parts mean that we are considering siren techniques from that region of the pyramid, while techniques from the solid (black) parts are not considered



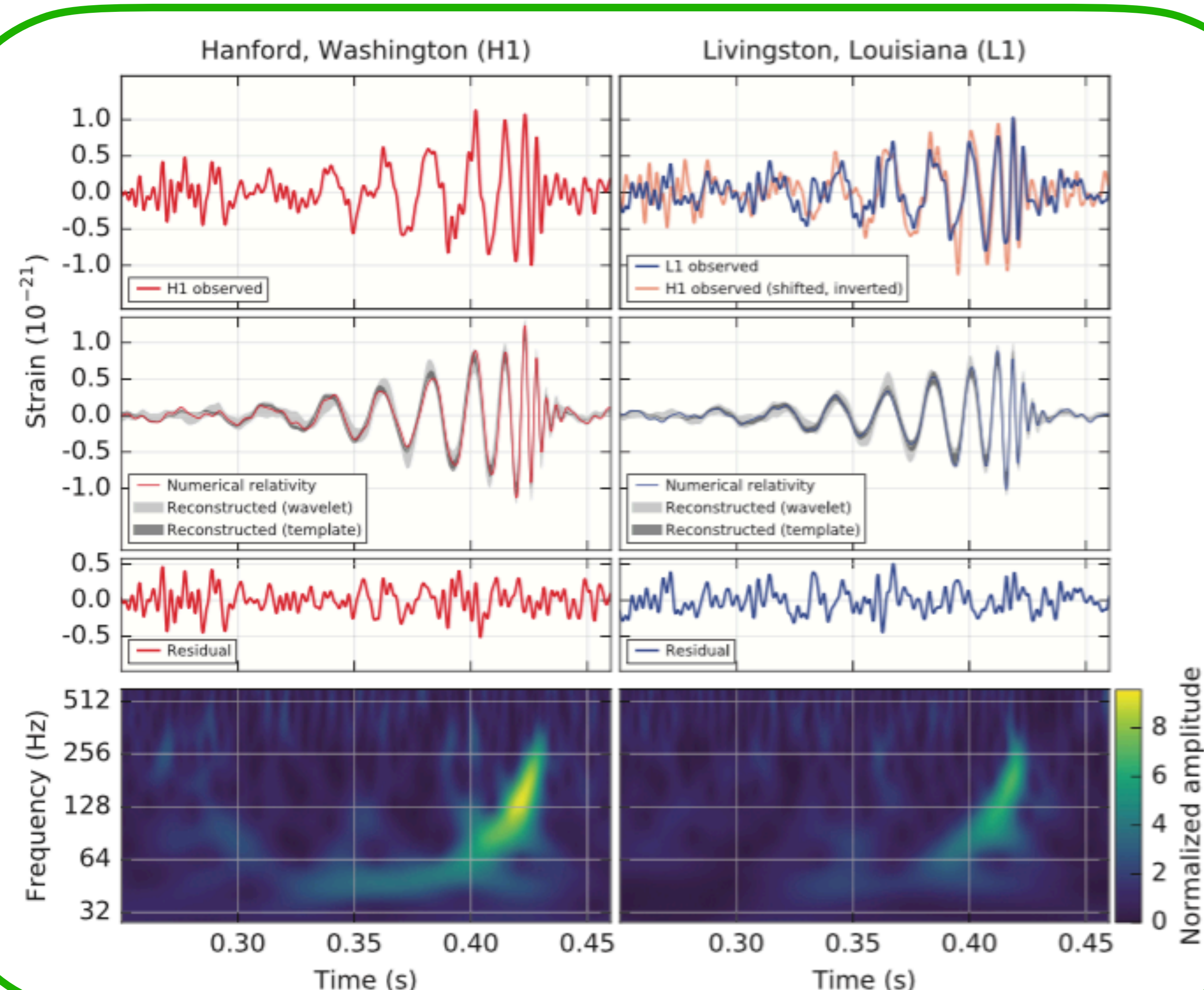
Current results from LVK

<u>Status of Earth-based GW observations:</u>

Current results from LVK

Status of Earth-based GW observations:

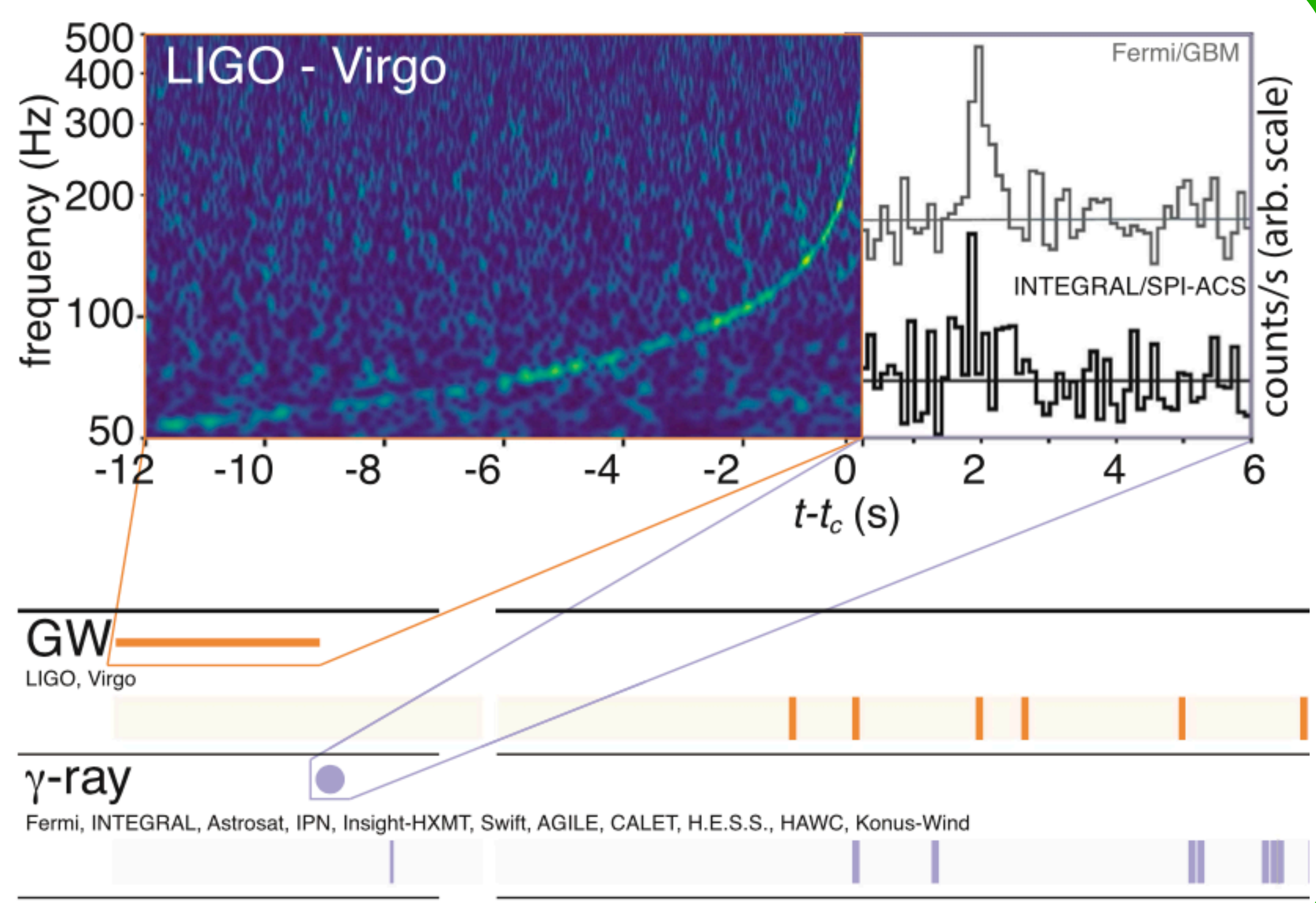
- O1: 2015 (completed), LIGO only, 4 months of data, 3 BBHs detected
- O2: 2017 (completed), LIGO(+VIRGO for GW1708xx only), 6 months of data, 7 BBHs + 1 BNS with EM counterpart (GW170817) [LVC, *PRX* (2019)]
- O3: 2019 (completed), LIGO+VIRGO, ~1 year of data, 79 events,
  73 BBH +2 BNS +2 NSBH +2 NS-NS/BH [LVK, *PRX* (2020)] [LVK, *ApJL* (2021)] [LVK, *arXiv* (2021)]
- O4: started May 2023 ends October 2025 (?), LIGO+VIRGO(+KAGRA)
- O5: ~2028
  LIGO India may join



Current results from LVK

Status of Earth-based GW observations:

- O1: 2015 (completed), LIGO only, 4 months of data, 3 BBHs detected
- O2: 2017 (completed), LIGO(+VIRGO for GW1708xx only), 6 months of data, 7 BBHs + 1 BNS with EM counterpart (GW170817) [LVC, *PRX* (2019)]
- O3: 2019 (completed), LIGO+VIRGO, ~1 year of data, 79 events, 73 BBH +2 BNS +2 NSBH +2 NS-NS/BH [LVK, *PRX* (2020)] [LVK, *ApJL* (2021)] [LVK, *arXiv* (2021)]
- O4: started May 2023 ends October 2025 (?), LIGO+VIRGO(+KAGRA)
- O5: ~2028 LIGO India may join



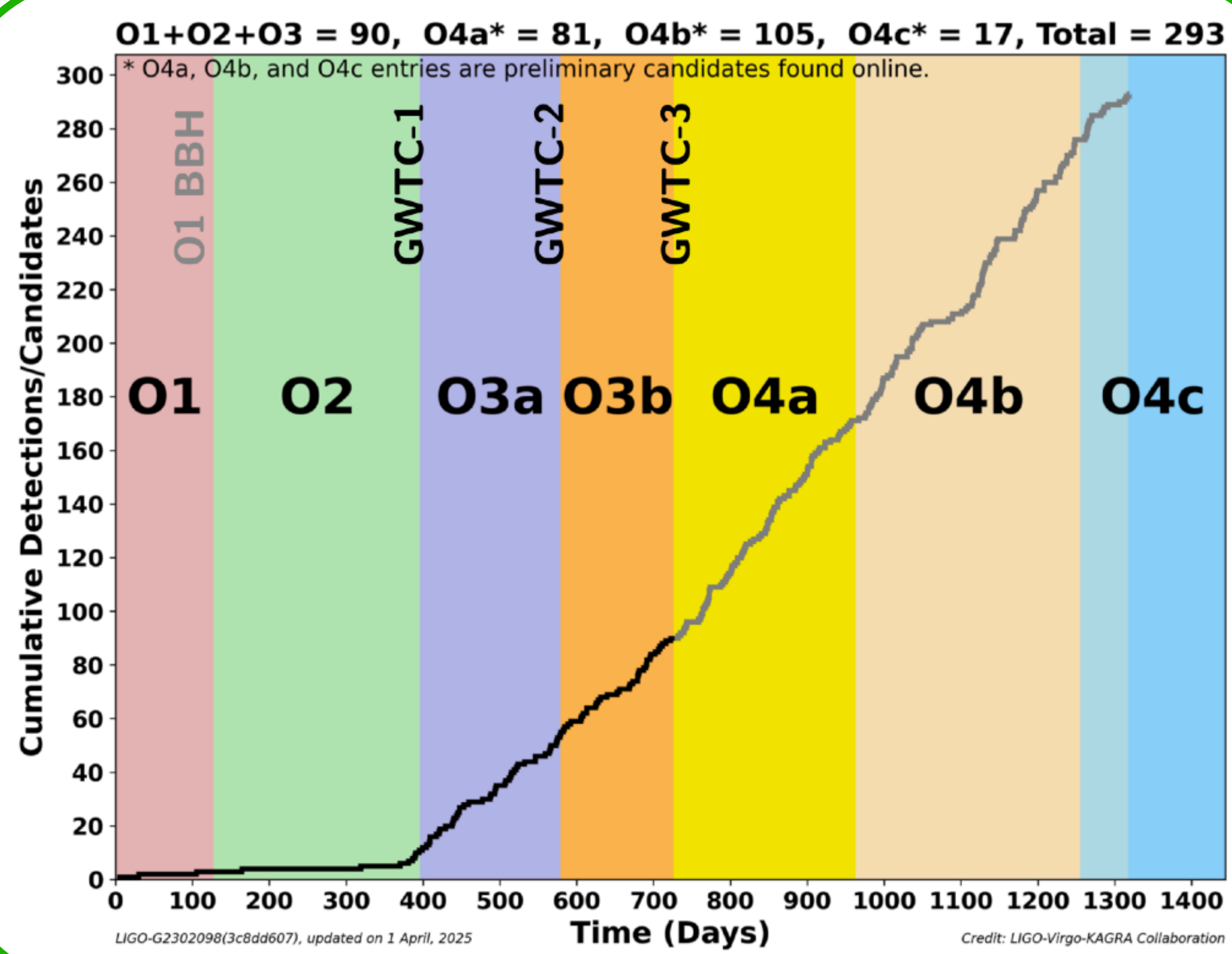
Current results from LVK

Status of Earth-based GW observations:

- O1: 2015 (completed), LIGO only, 4 months of data, 3 BBHs detected
- O2: 2017 (completed), LIGO(+VIRGO for GW1708xx only), 6 months of data, 7 BBHs + 1 BNS with EM counterpart (GW170817) [LVC, *PRX* (2019)]
- O3: 2019 (completed), LIGO+VIRGO, ~1 year of data, 79 events, 73 BBH +2 BNS +2 NSBH +2 NS-NS/BH [LVK, *PRX* (2020)] [LVK, *ApJL* (2021)] [LVK, *arXiv* (2021)]
- O4: started May 2023 ends October 2025 (?), LIGO+VIRGO(+KAGRA)
- O5: ~2028 LIGO India may join

90 high-significance GW events in total up to O3

More than 200 preliminary GW candidates from O4

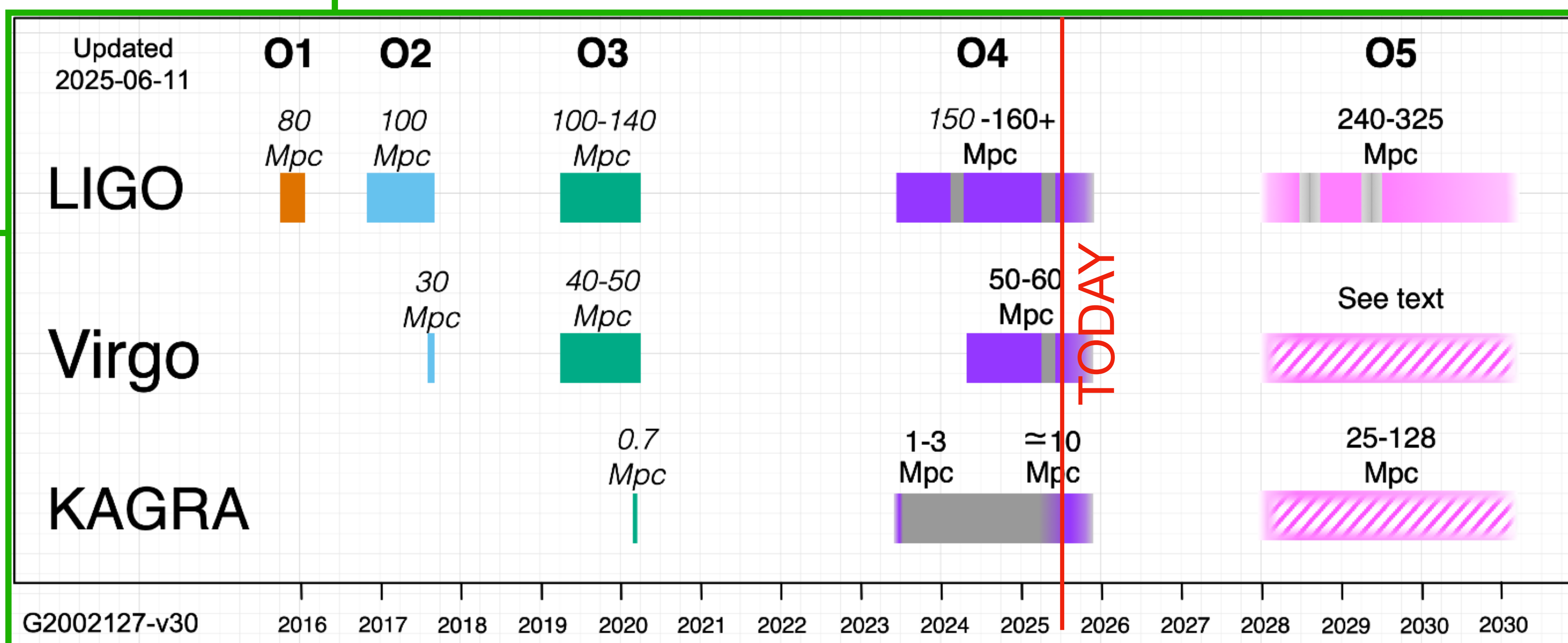


Current results from LVK

<u>Status of Earth-based GW observations:</u>

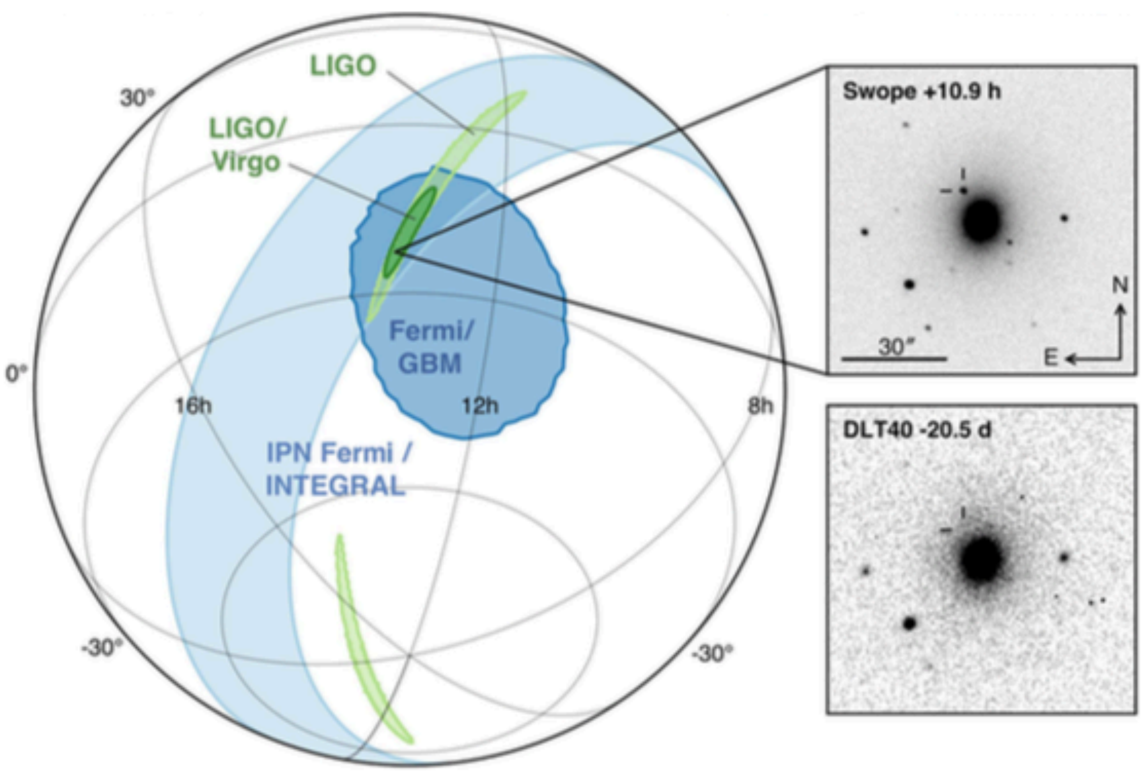
- O1: 2015 (completed), LIGO only, 4 months of data, <u>3 BBHs</u> detected
- O2: 2017 (completed), LIGO(+VIRGO for GW1708xx only), 6 months of data, 7 BBHs + 1 BNS with EM counterpart (GW170817) [LVC, *PRX* (2019)]
- O3: 2019 (completed), LIGO+VIRGO, ~1 year of data, 79 events, 73 BBH +2 BNS +2 NSBH +2 NS-NS/BH [LVK, *PRX* (2020)] [LVK, *ApJL* (2021)] [LVK, *arXiv* (2021)]
- O4: started May 2023 ends October 2025 (?), LIGO+VIRGO(+KAGRA)
- O5: ~2028 LIGO India may join

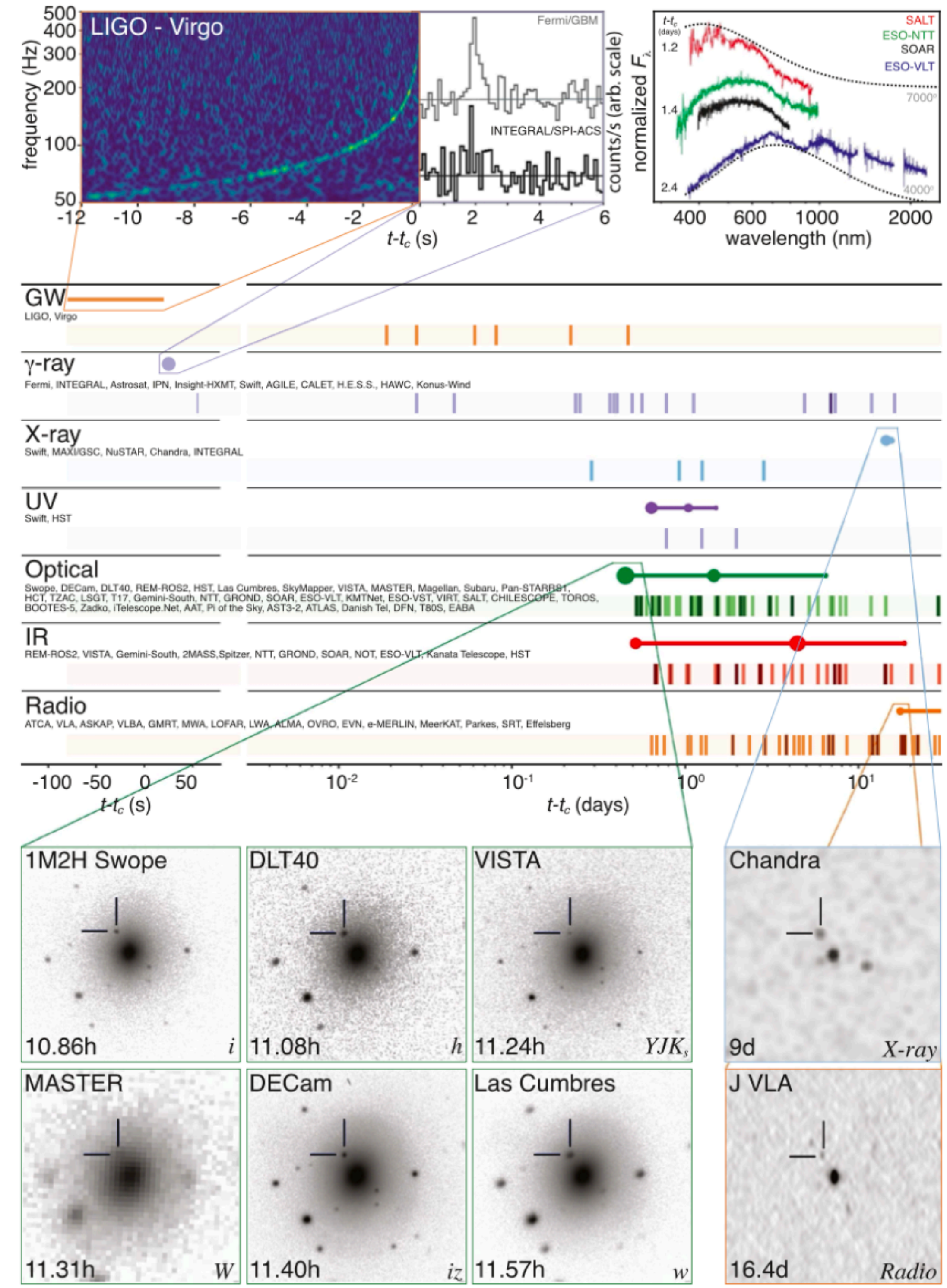
https://observing.docs.ligo.org/plan



Current results from LVK

GW170817: the first ever (bright) standard siren





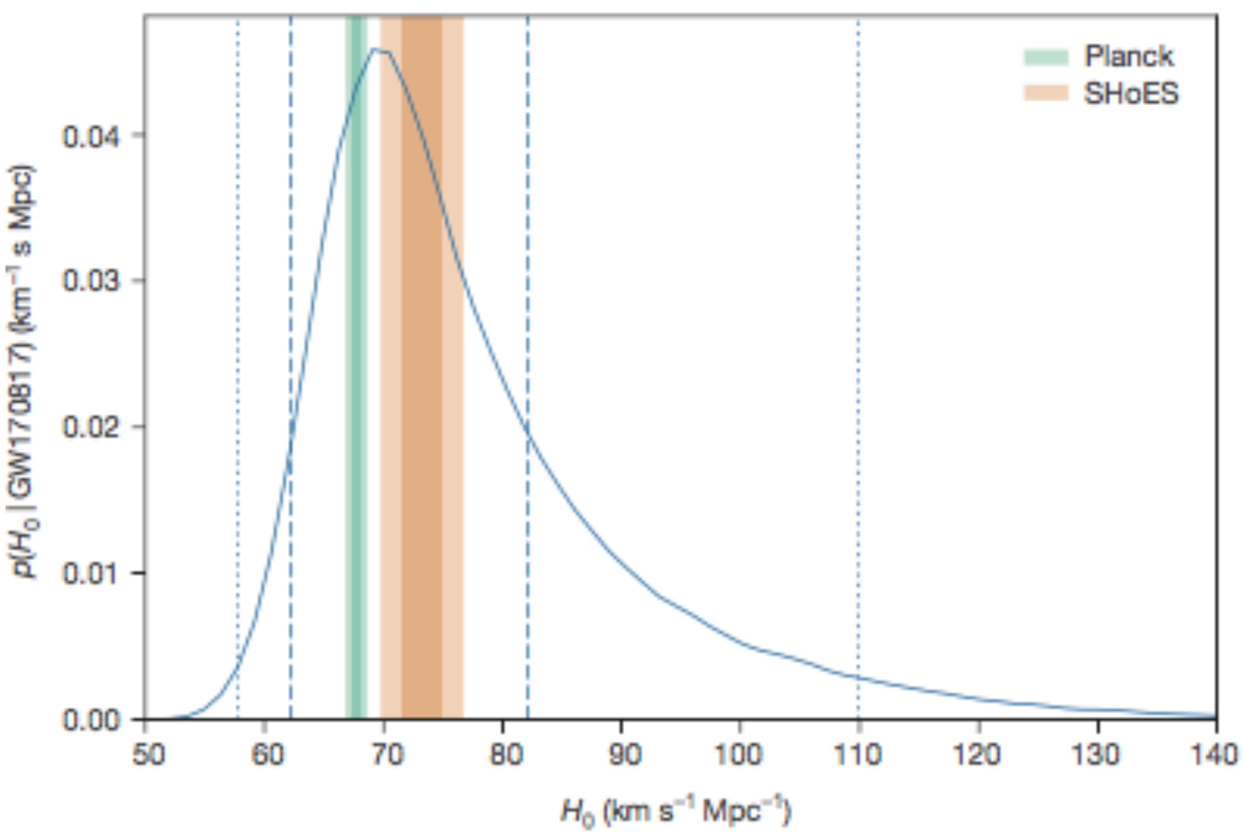
The identification of an EM counterpart yielded the first cosmological measurements with GW standard sirens

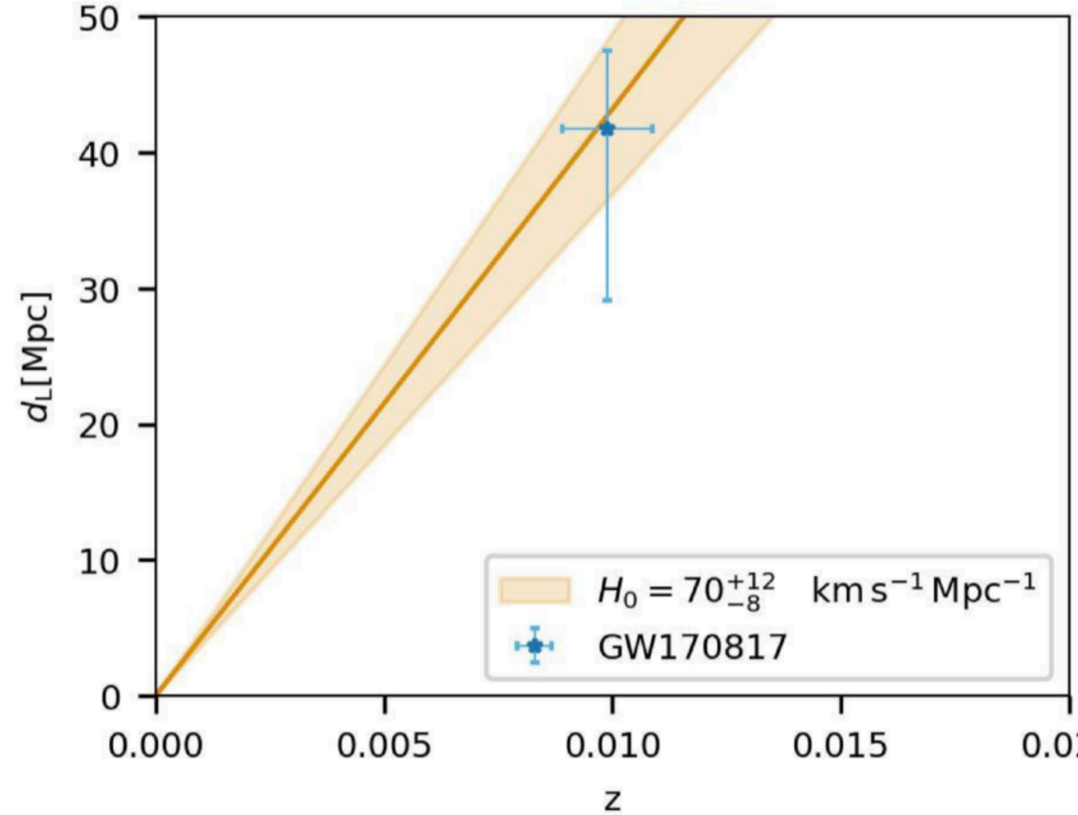
$$H_0 = 70^{+12}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68.3% C.I.)

[LVC+, *Nature* (2017)]

Current results from LVK

GW170817: the first ever (bright) standard siren





The identification of an EM counterpart yielded the first cosmological measurements with GW standard sirens

$$H_0 = 70^{+12}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68.3% C.I.)

[LVC+, *Nature* (2017)]

Low-redshift event ($z = 0.01$): only H_0 can be measured (**Hubble law**)

$$d_L(z) \simeq \frac{c}{H_0} z \quad for \;\; z \ll 1$$

Results largely in agreement with EM constraints (SNIa/CMB), but not yet competitive with them

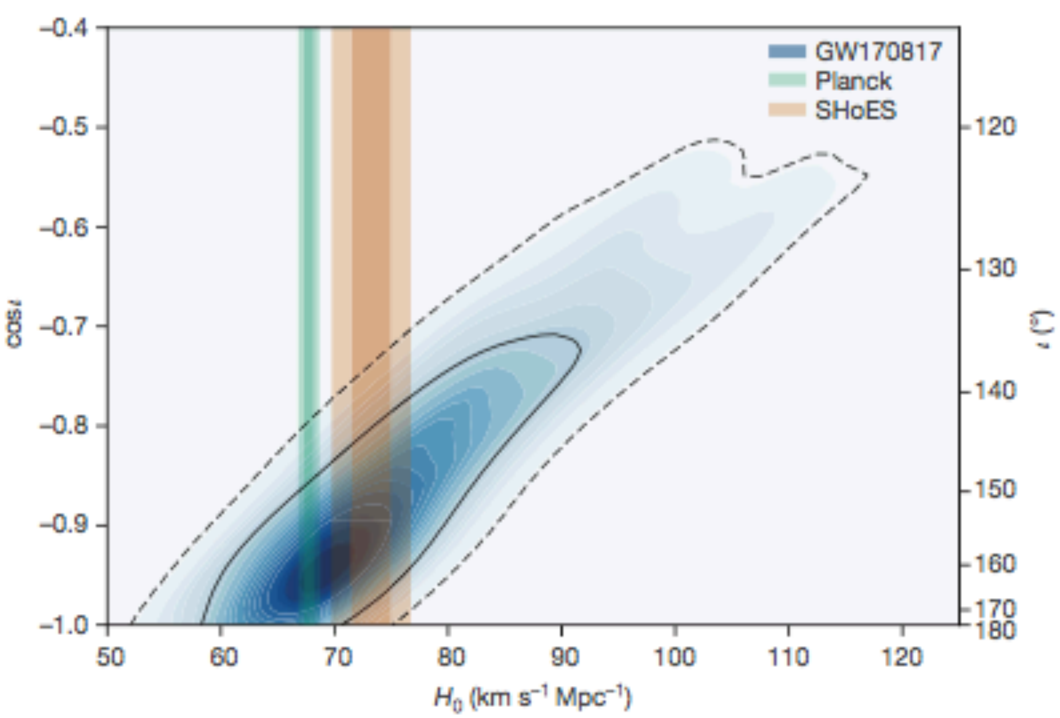
Current results from LVK

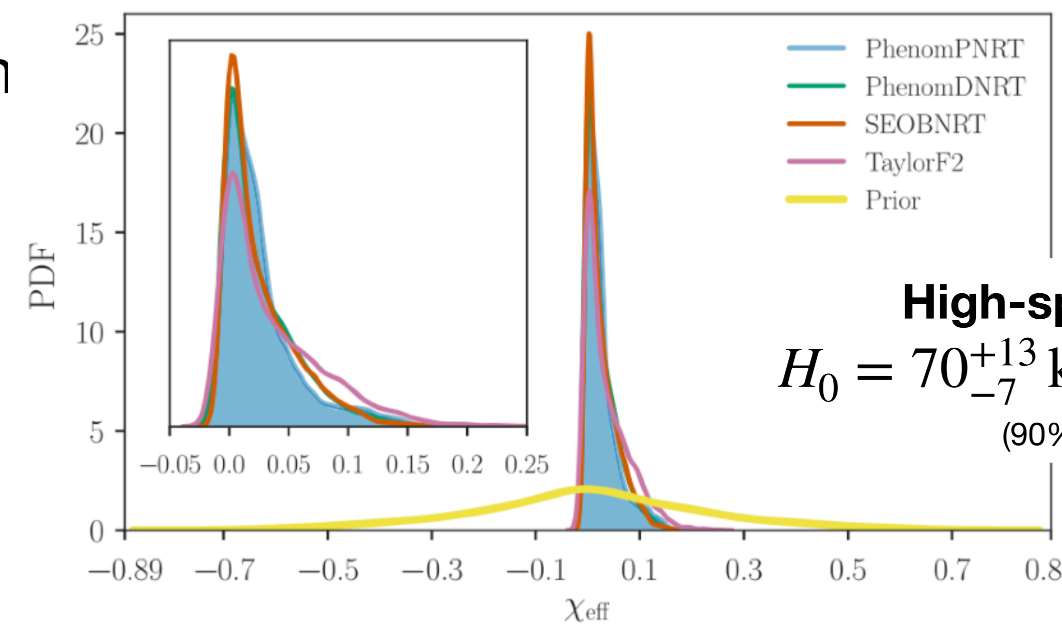
GW170817: the first ever (bright) standard siren

The distance-inclination degeneracy is one of the main source of uncertainty in GW distance measurements.

Correlation with other waveform parameters can thus impact cosmological measurements.

The **spin prior choice** in particular has been found to affect H0 measurements with GW170817

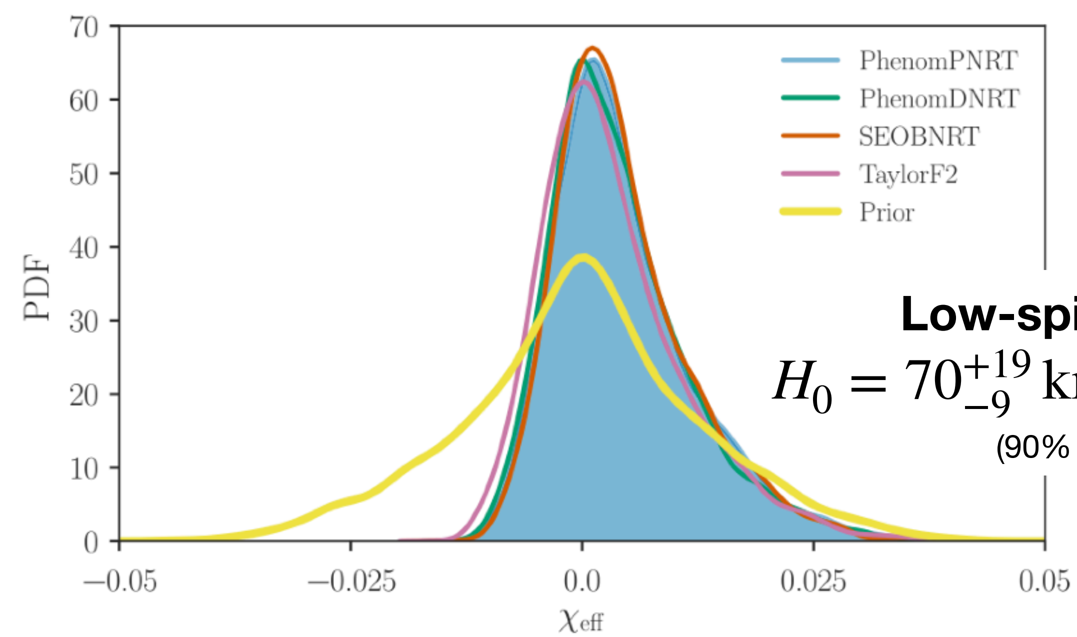




High-spin prior

$$H_0 = 70^{+13}_{-7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

(90% C.I.)



Low-spin prior

$$H_0 = 70^{+19}_{-9}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

(90% C.I.)

[LVC, *PRX* (2019)]

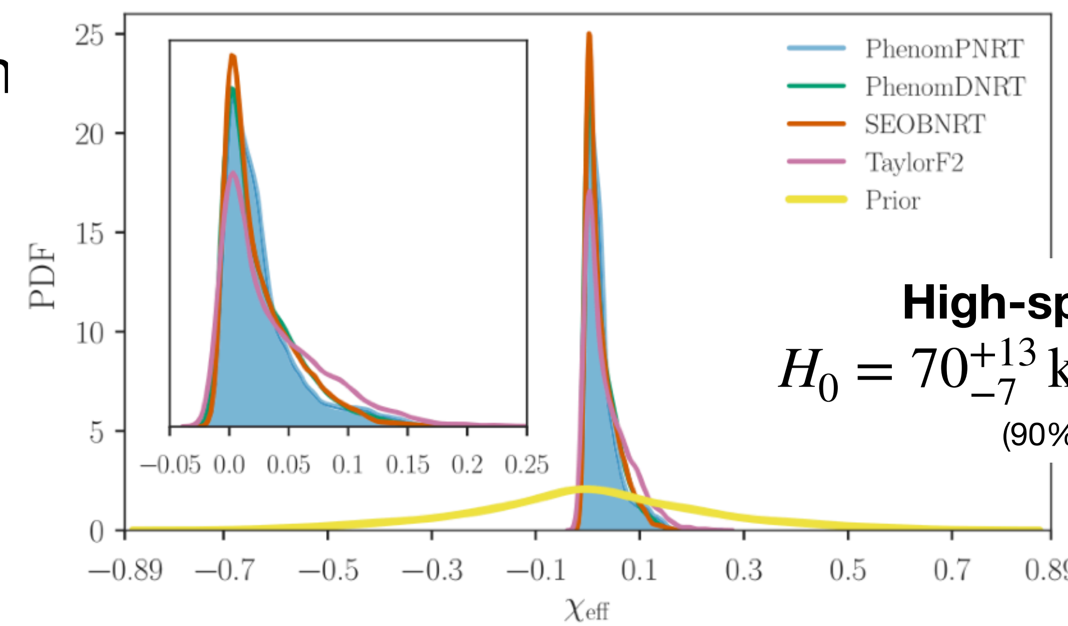
Current results from LVK

GW170817: the first ever (bright) standard siren

The distance-inclination degeneracy is one of the main source of uncertainty in GW distance measurements.

Correlation with other waveform parameters can thus impact cosmological measurements.

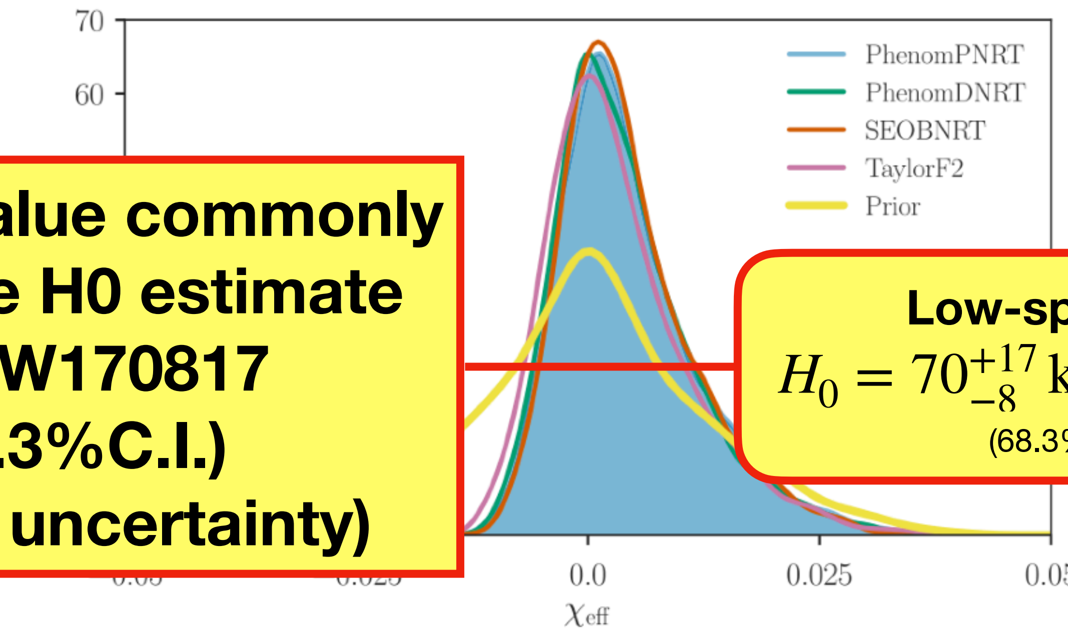
The **spin prior choice** in particular has been found to affect H0 measurements with GW170817



High-spin prior

$$H_0 = 70^{+13}_{-7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

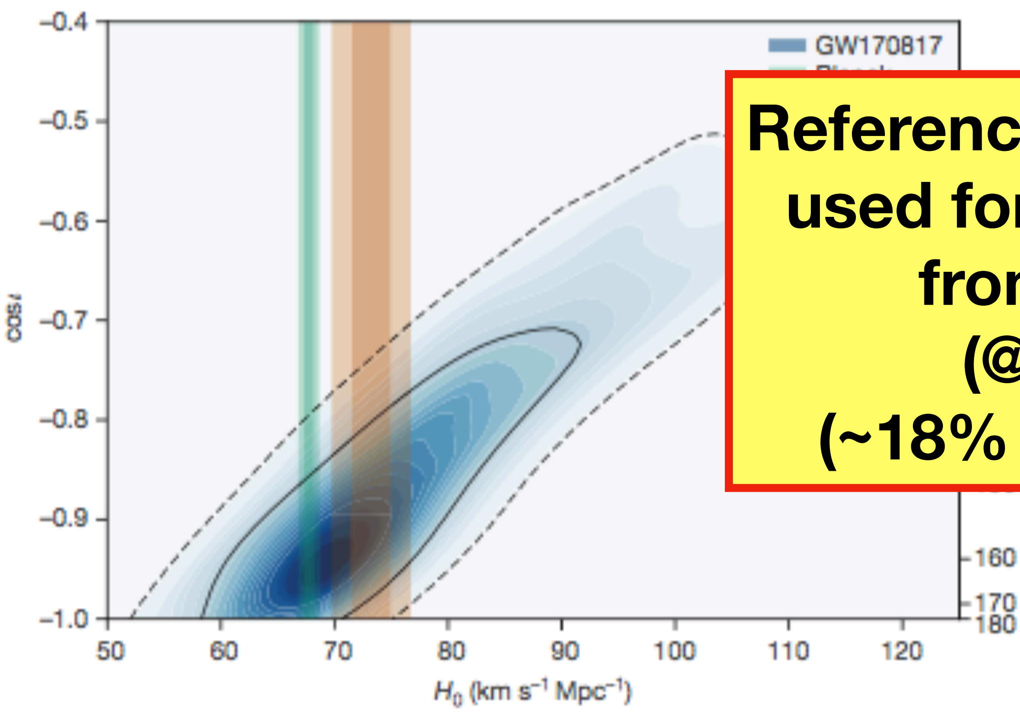
(90% C.I.)



Low-spin prior

$$H_0 = 70^{+17}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

(68.3% C.I.)



Reference value commonly used for the H0 estimate from GW170817 (@ 68.3%C.I.) (~18% rel. uncertainty)

[LVC, *PRX* (2019)]

Current results from LVK

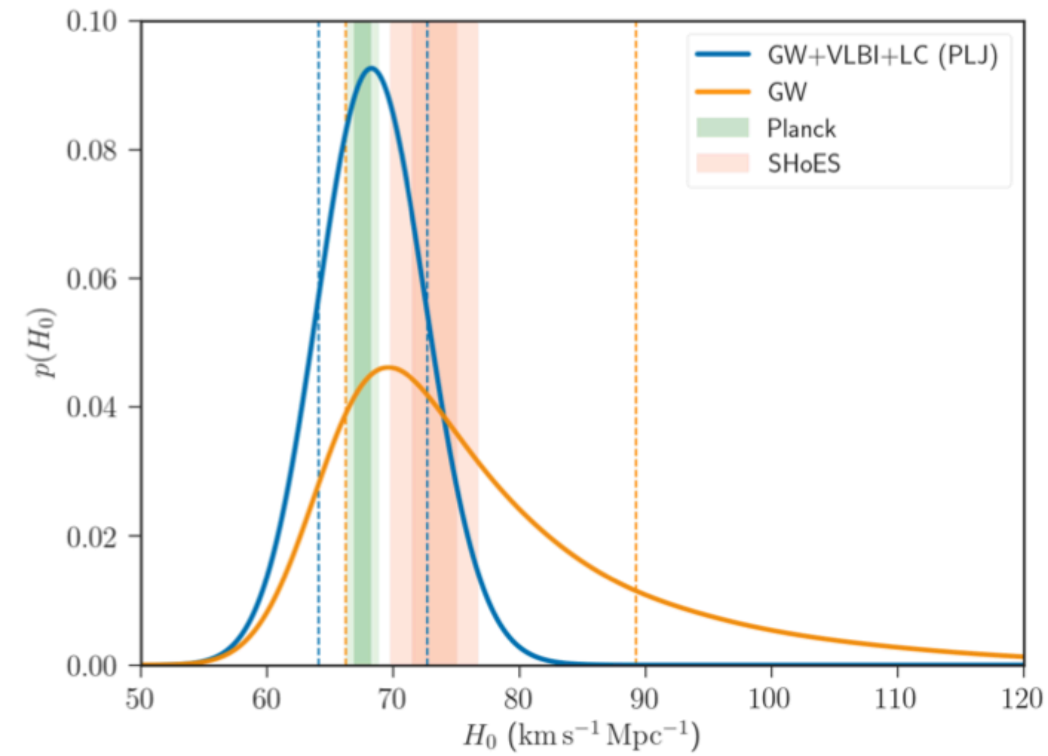
GW170817: the first ever (bright) standard siren

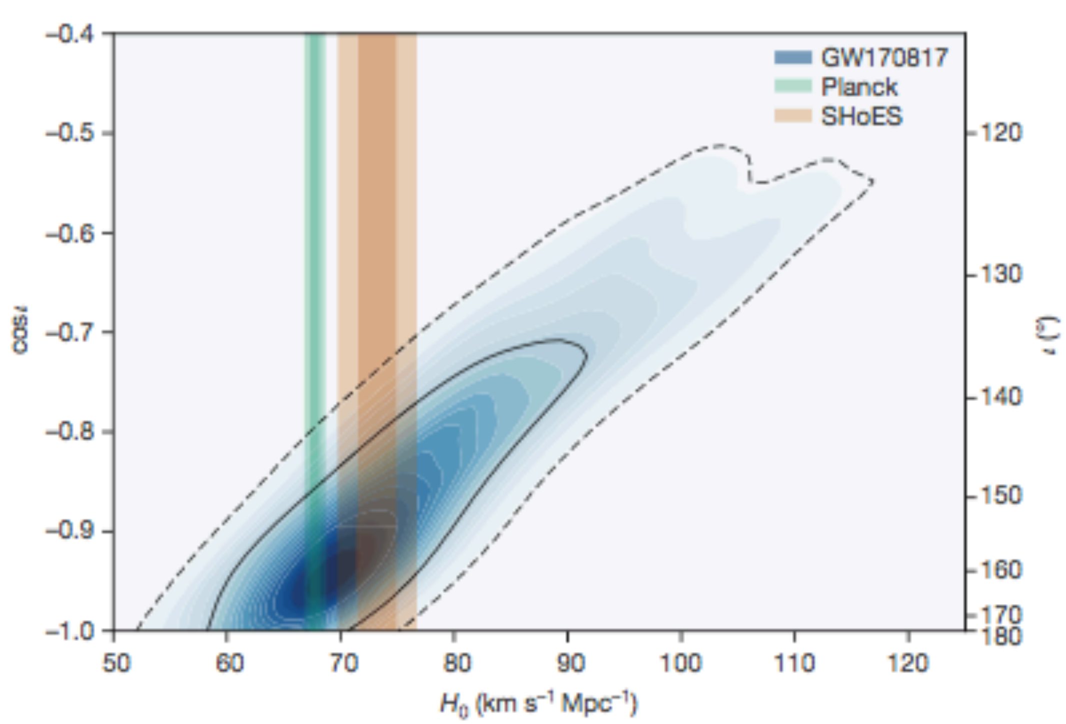
The distance-inclination degeneracy can be broken/alleviated with EM measurements of the viewing angle of the emitted radio jet via the afterglow:

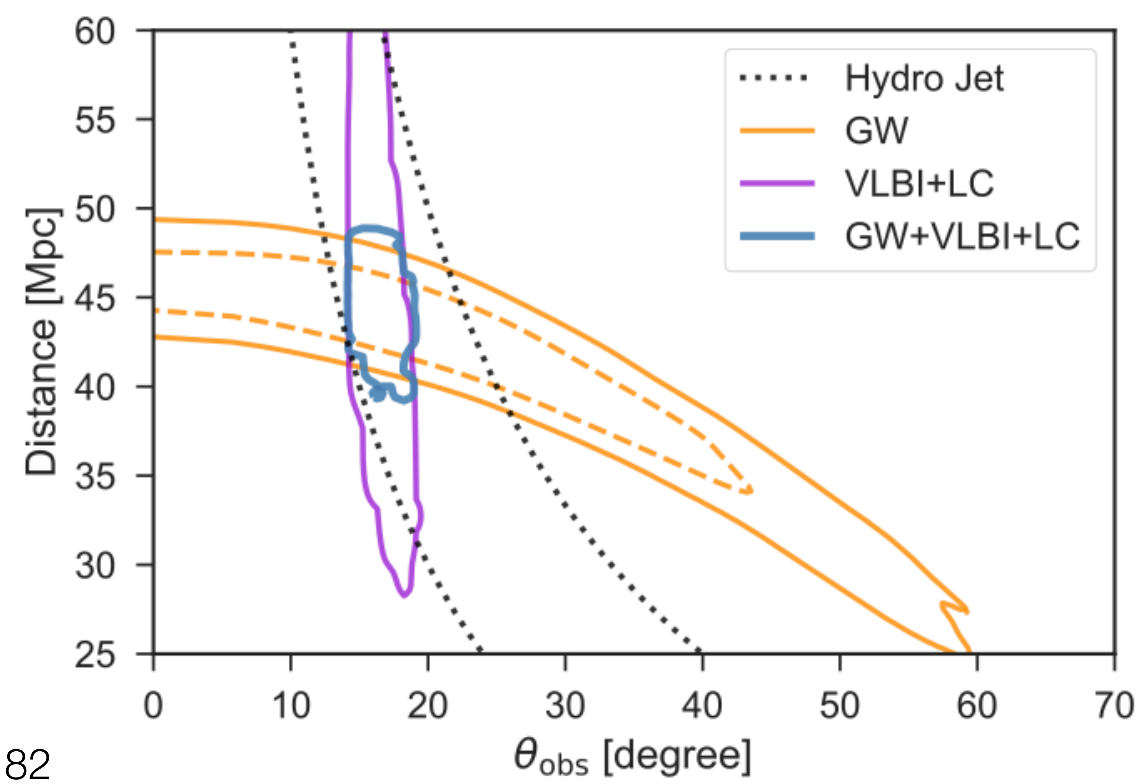
$$H_0 = 70.3^{+5.3}_{-5.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

H_0 measured at 7% level, but dependent on jet/afterglow modelling.

[Hotokezaka+, *Nature* (2019)]
[Palmese+, *PRD* (2024)]

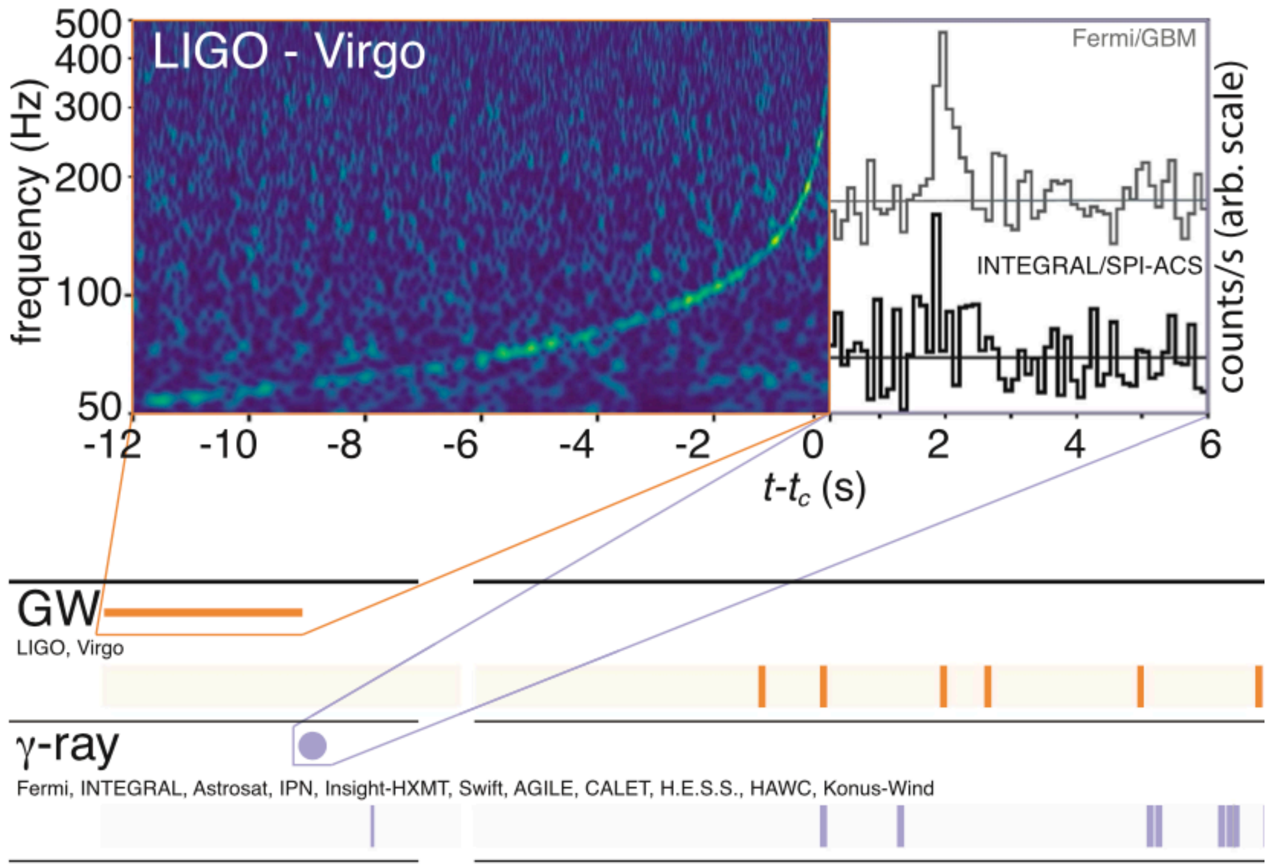






Current results from LVK

GW170817: the first ever (bright) standard siren



[LVC+, *ApJL* (2017)]

The coincident GW-EM detection of GW170817 puts stringent constraints on the speed of GW:

$$c_T = c^{+7\times10^{-16}}_{-3\times10^{-15}}$$

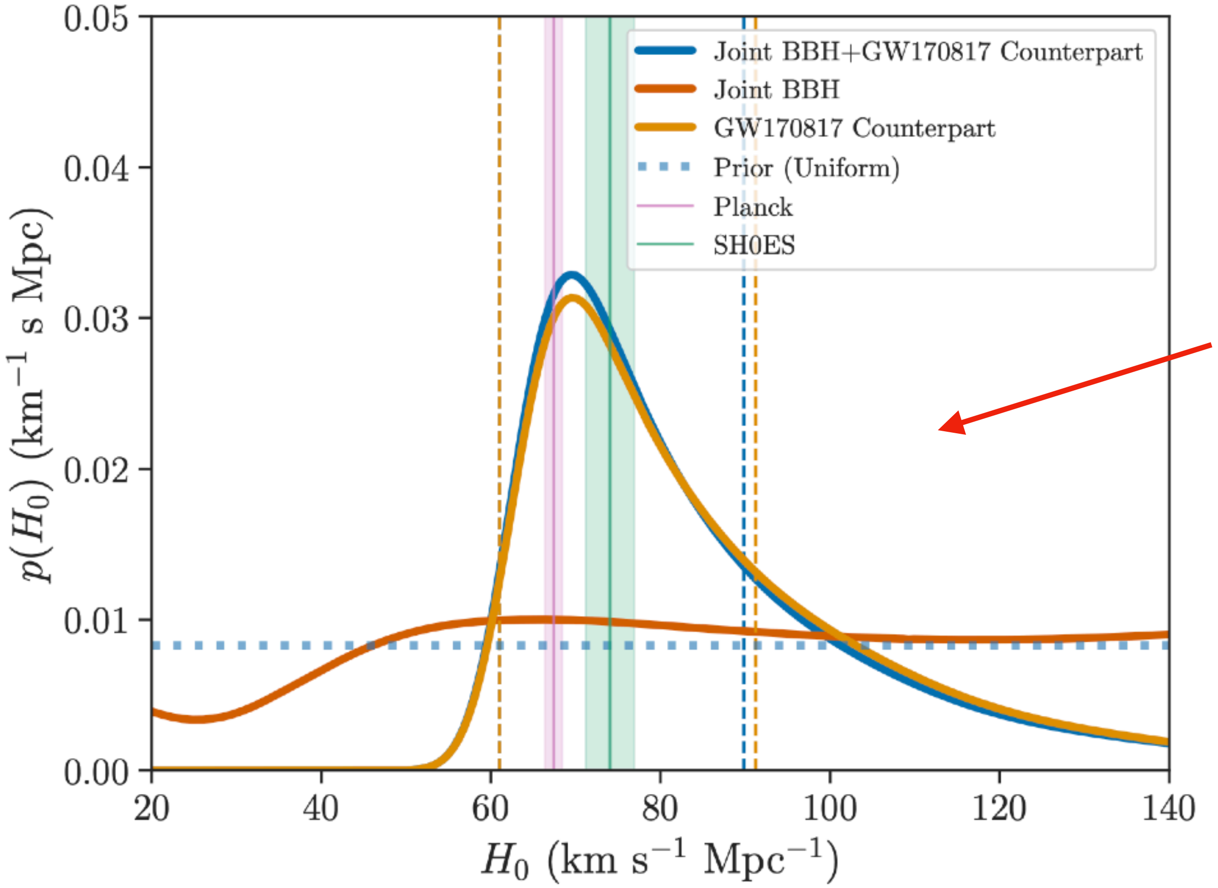
This observation rules out several modified gravity models predicting $c_T \neq c$ [see e.g. 1807.09241 and refs therein]

The low redshift of GW170817 however do not allow for any relevant constraints on the GW friction ν

[Belgacem+, *PRD* (2018)]

Current results from LVK

The <u>statistical dark siren method</u> has then been applied to combine BBHs events:

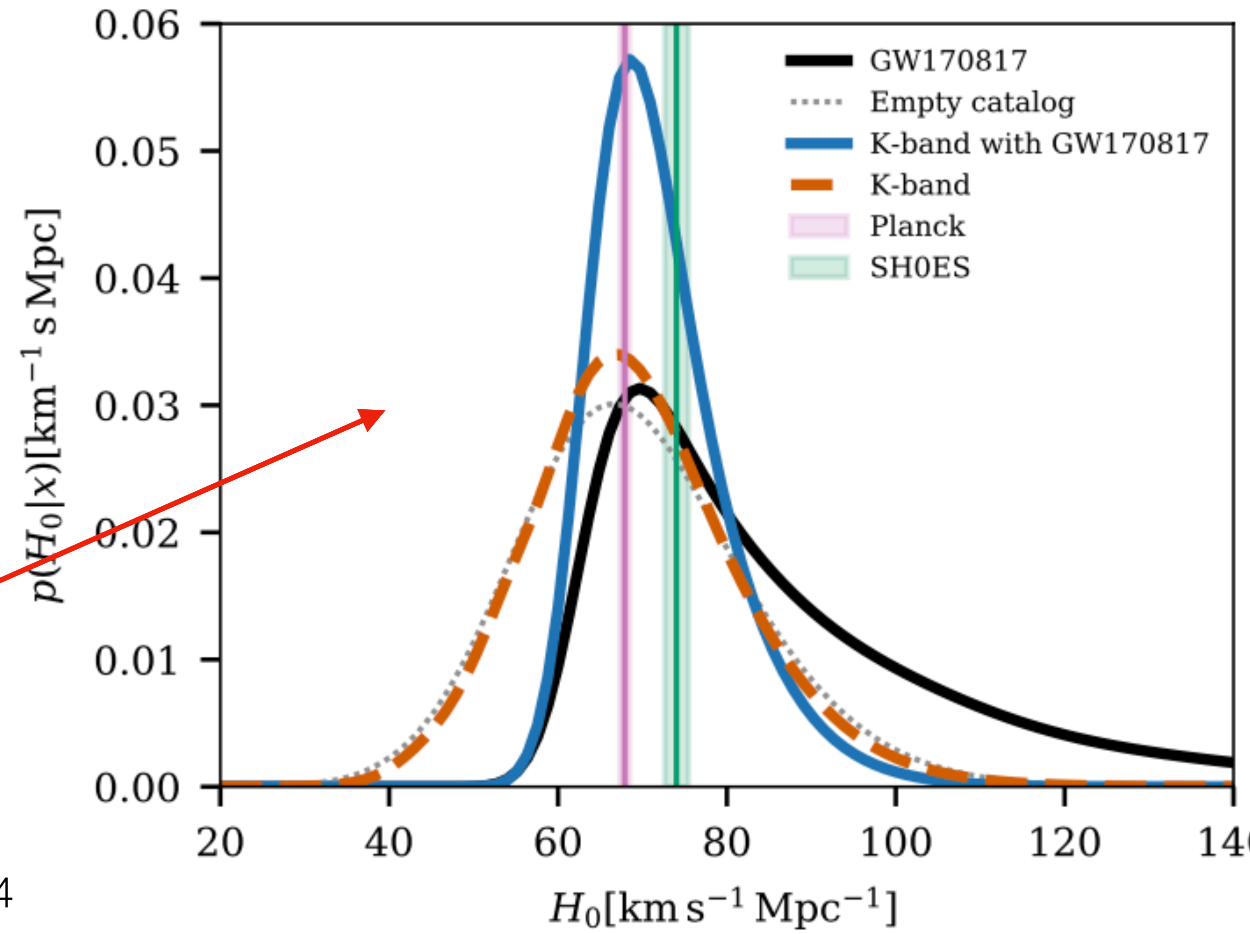


LVC results with all O1 and O2 events combined (w/o GW170817): [LVC, *ApJ* (2020)]

$$H_0 = 75^{+39}_{-14}\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \quad \text{(90\% C.I.)}$$

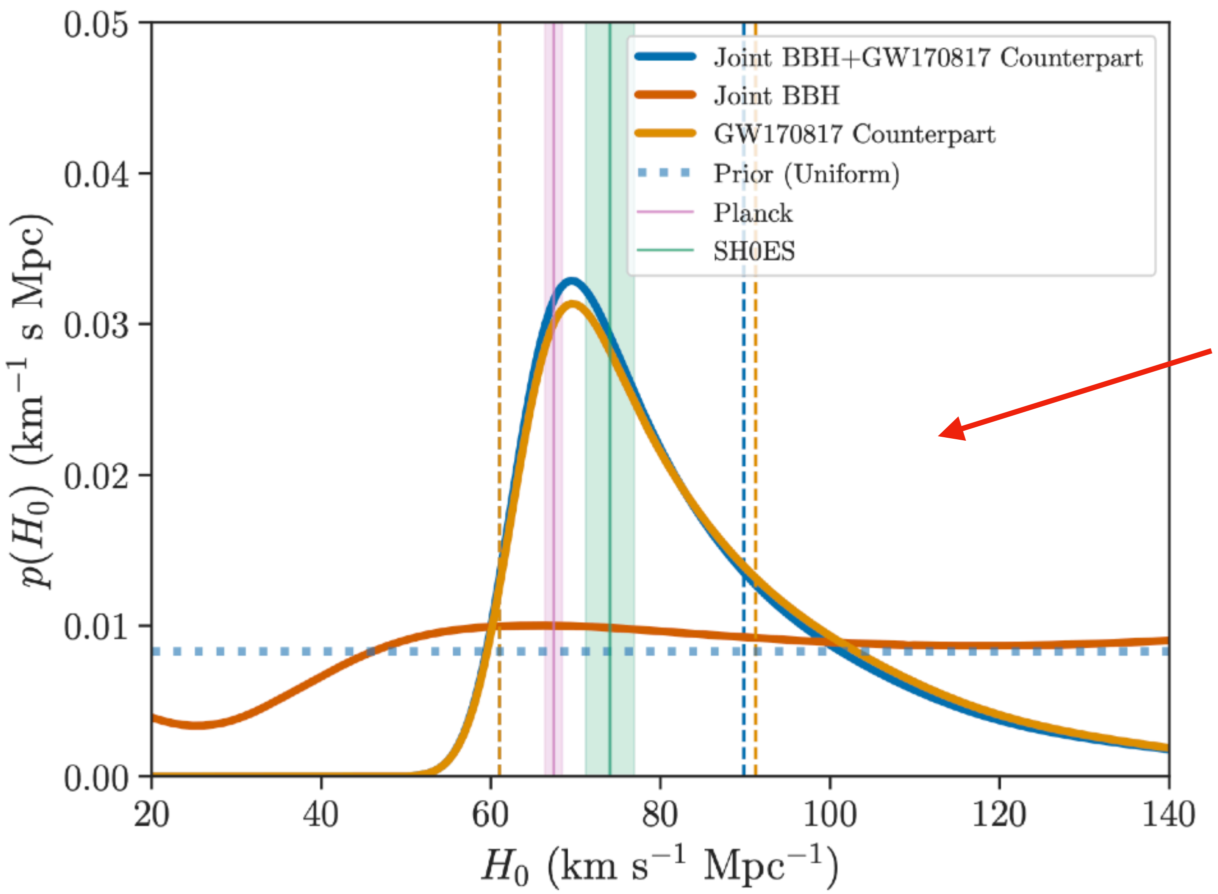
LVC results with all events from O1, O2, O3 (w/o GW170817): [LVK, *ApJ* (2023)]

$$H_0 = 67^{+13}_{-12}\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \quad \text{(68\% C.I.)}$$



Current results from LVK

The <u>statistical dark siren method</u> has then been applied to combine BBHs events:



LVC results with all O1 and O2 events combined (w. GW170817): [LVC, *ApJ* (2020)]

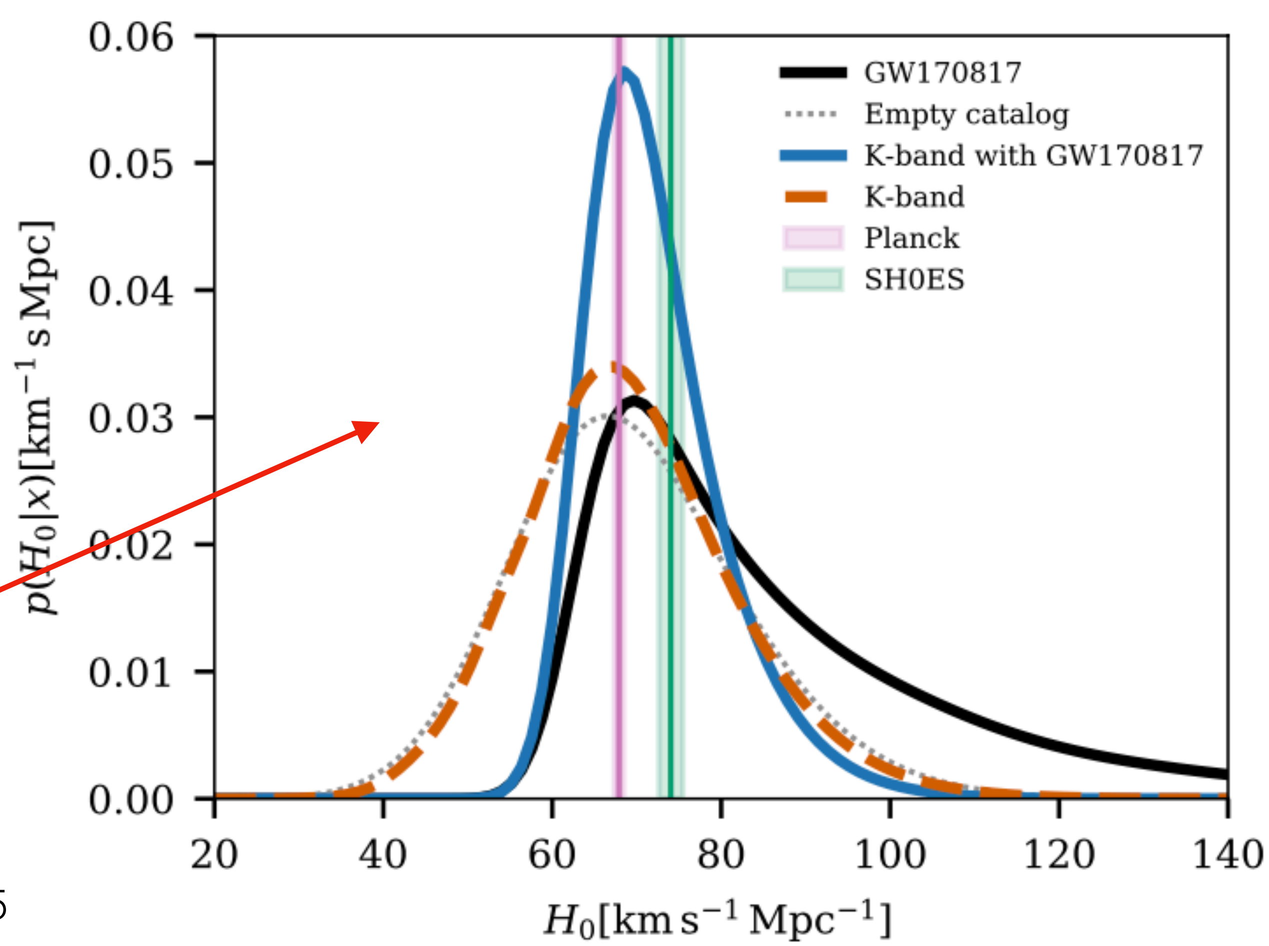
$H_0 = 69^{+16}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68.3% C.I.)

(4% improvement over GW170817 only)

LVC results with all events from O1, O2, O3 (w. GW170817): [LVK, *ApJ* (2023)]

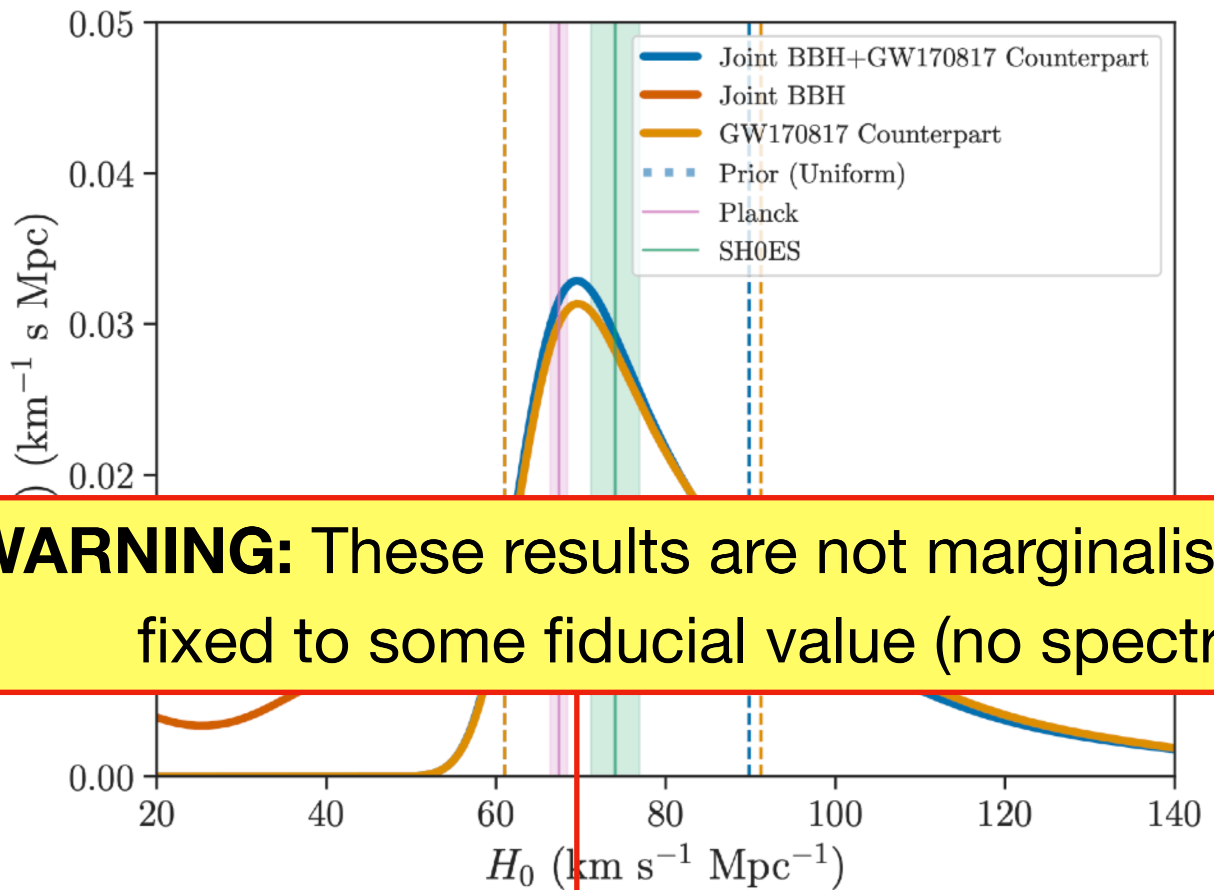
$H_0 = 68^{+8}_{-6}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

(40% improvement over O2 results)



Current results from LVK

The <u>statistical dark siren method</u> has then been applied to combine BBHs events:



LVC results with all O1 and O2 events combined: [LVC, *ApJ* (2020)]

$$H_0 = 69^{+16}_{-8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68.3% C.I.)

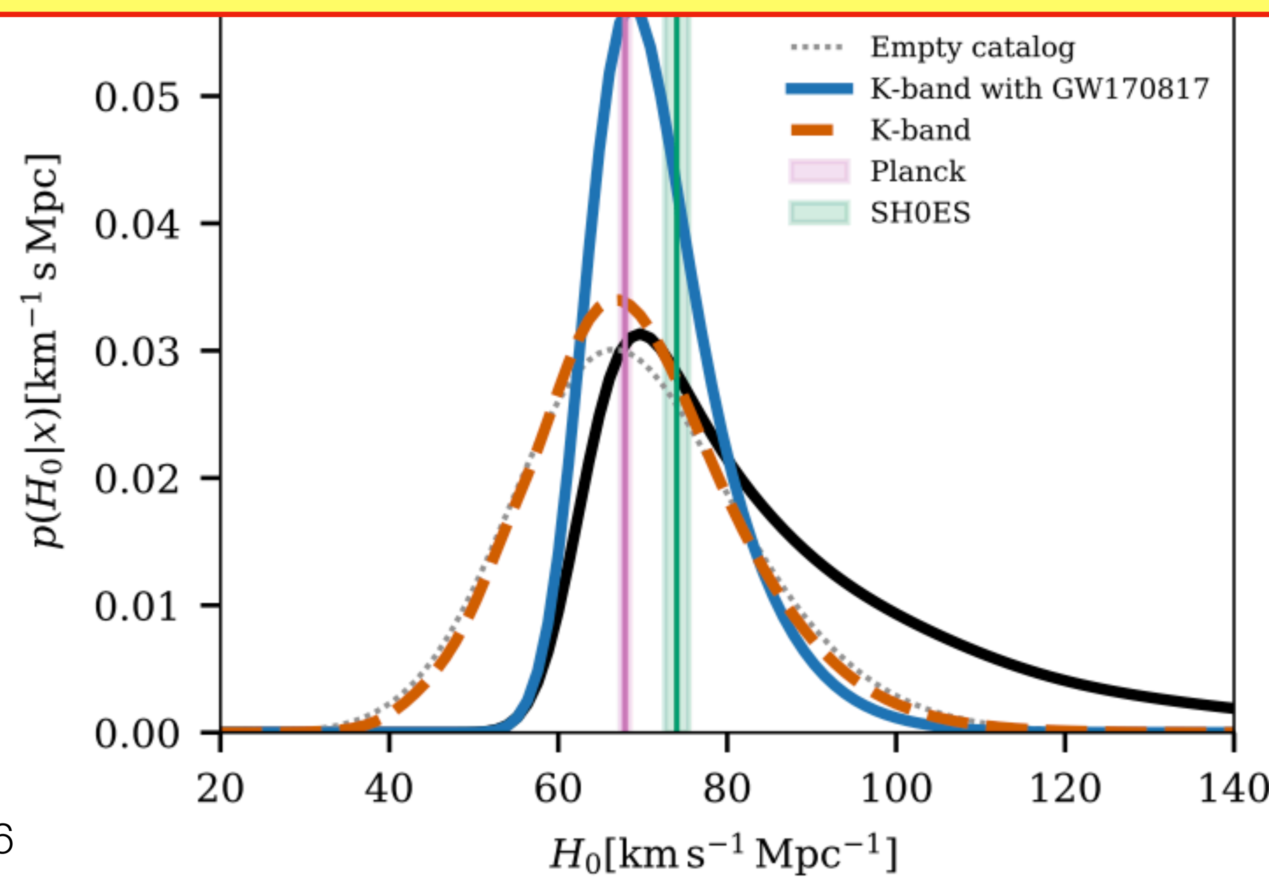
(4% improvement over GW170817 only)

WARNING: These results are not marginalised over population parameters which have been fixed to some fiducial value (no spectral siren analyses) ⇒ overoptimistic results

LVC results with all events so far combined (O1+O2+O3): [LVK, *ApJ* (2023)]

$$H_0 = 68^{+8}_{-6}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68% C.I.)

(40% improvement over O2 results)

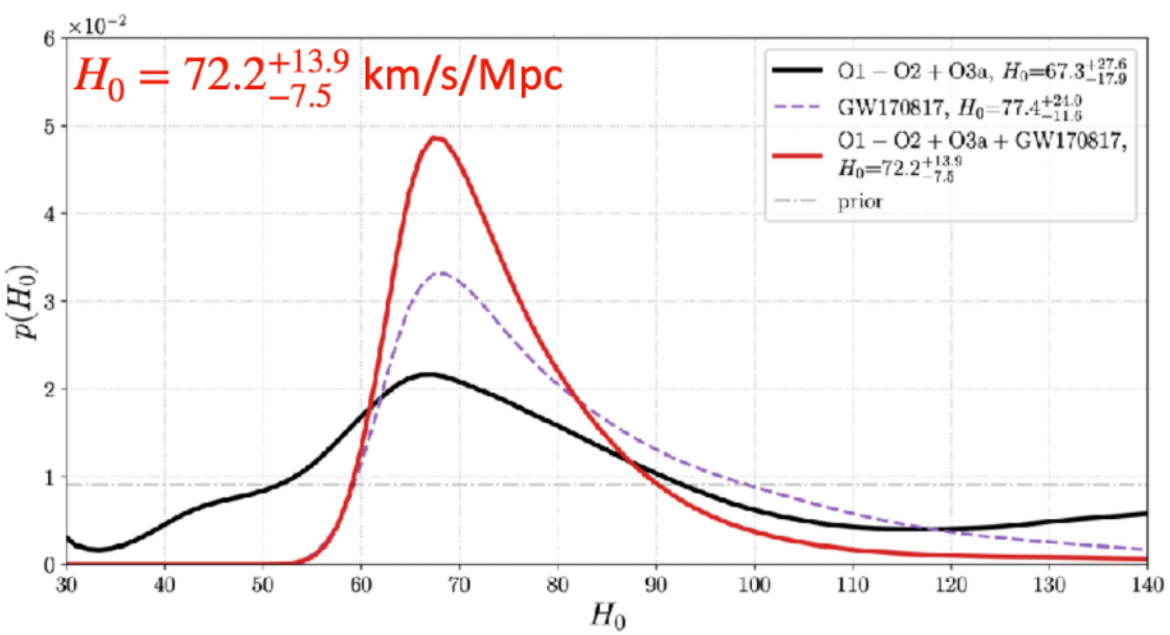


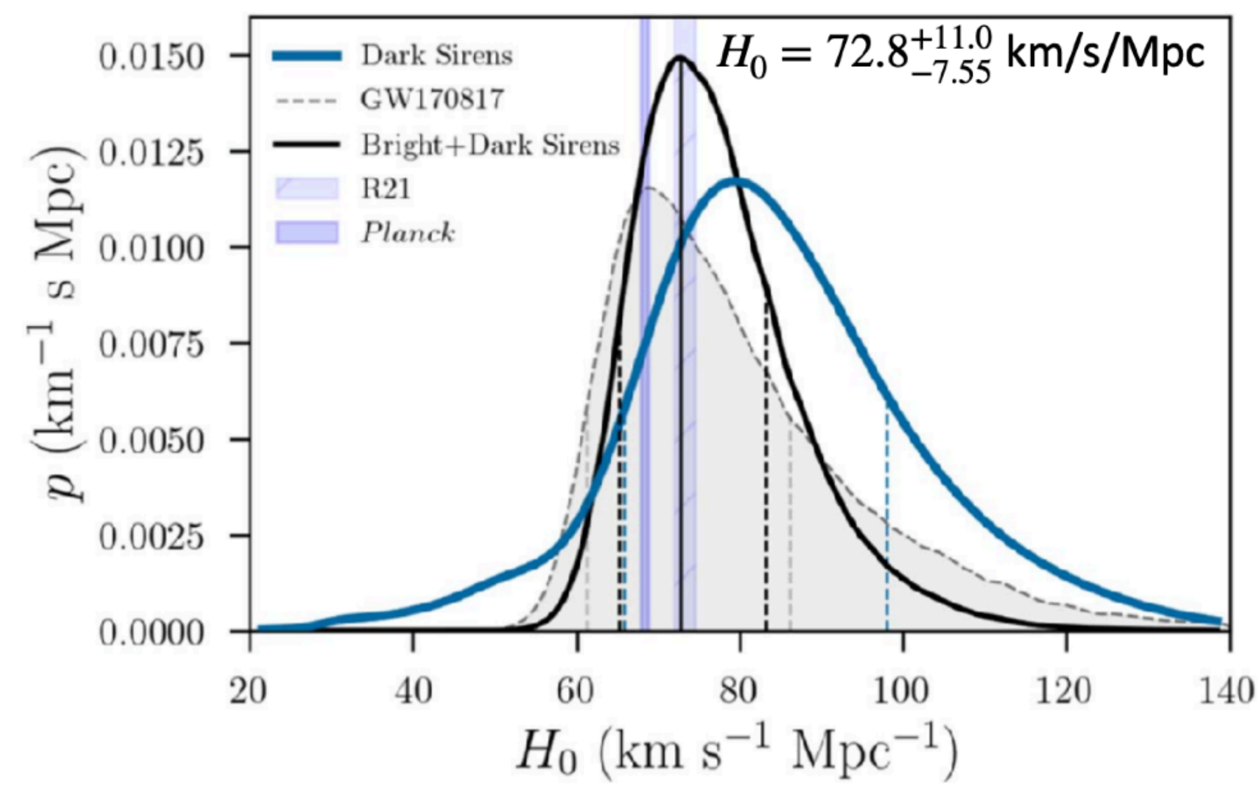
Current results from LVK

Other recent dark siren studies using LVK observatories

Finke et al., JCAP (2021)
[with GWTC-2 catalog and GLADE galaxy catalog]

Palmese et al., ApJ (2023)
[with GWTC-3 catalog and DESI Legacy Survey]





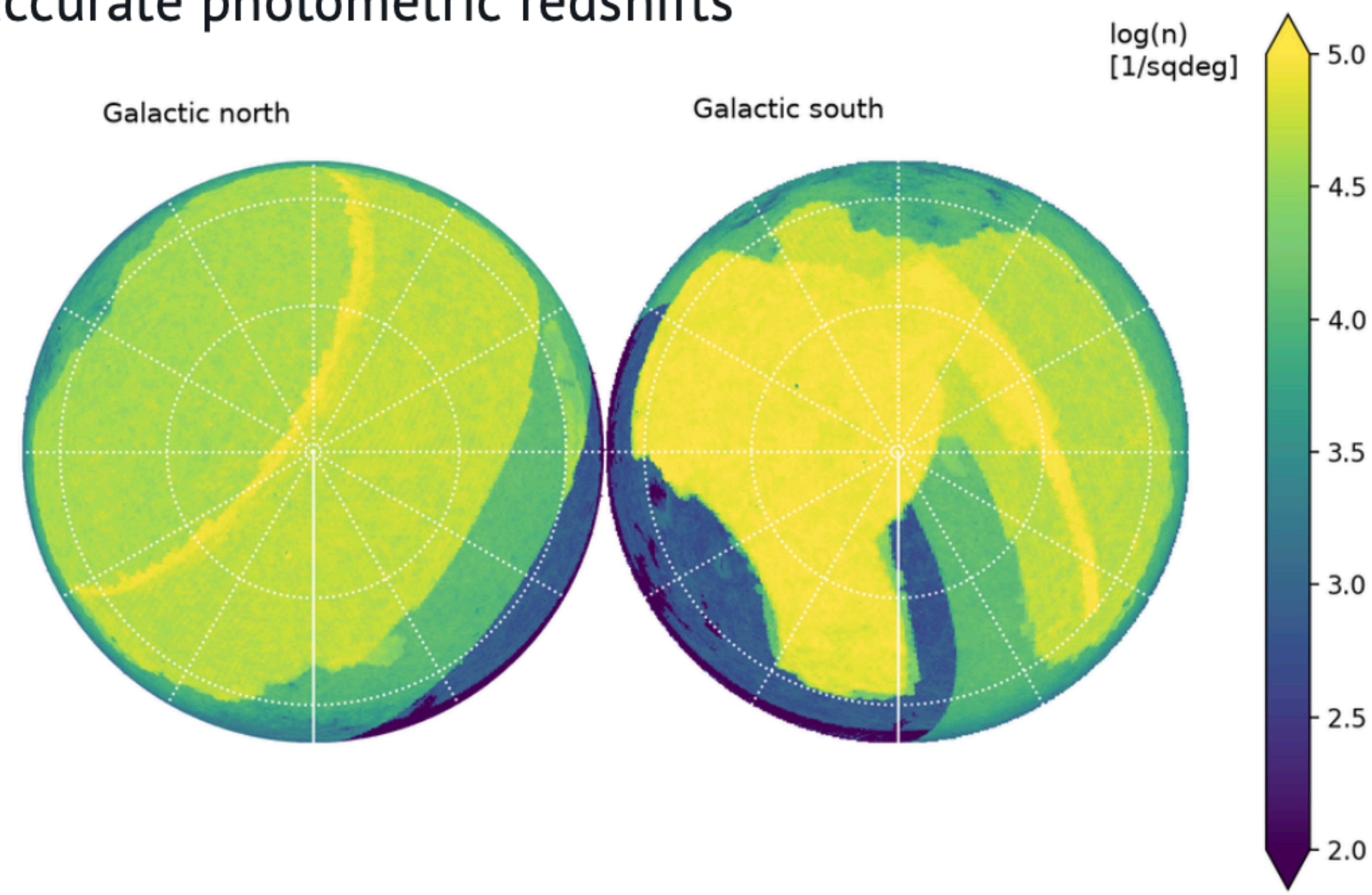
Results well in agreement with LVK ones

Slide curtesy of D. Laghi

Current results from LVK

What to expect from O4 with the statistical dark siren method

- 2 orders of magnitude more galaxies (1.3 billion)
- Legacy Survey + SGA, Pan-STARRS, CatWISE, SDSS, SkyMapper
- More photometric bands
- More accurate photometric redshifts

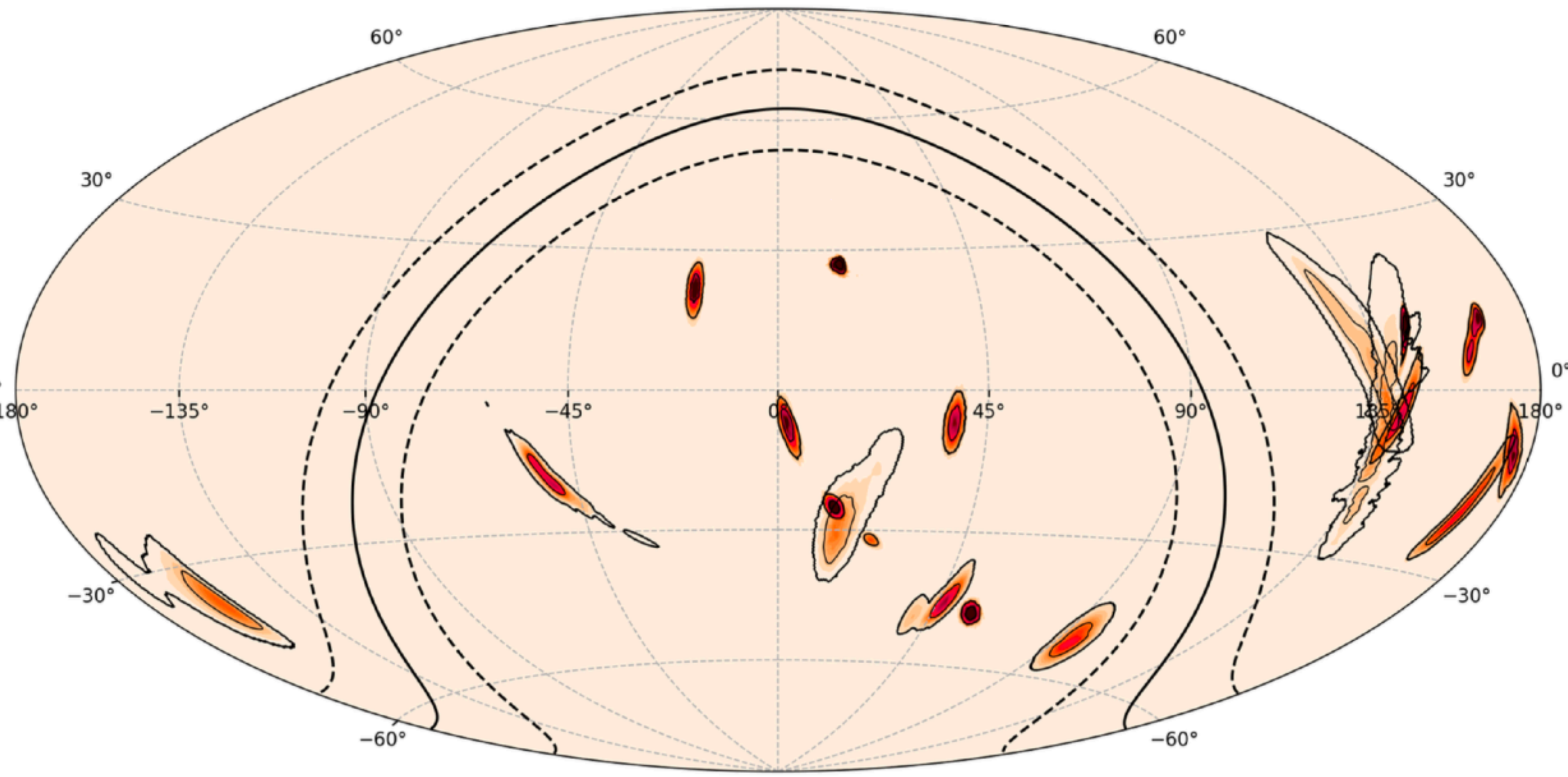


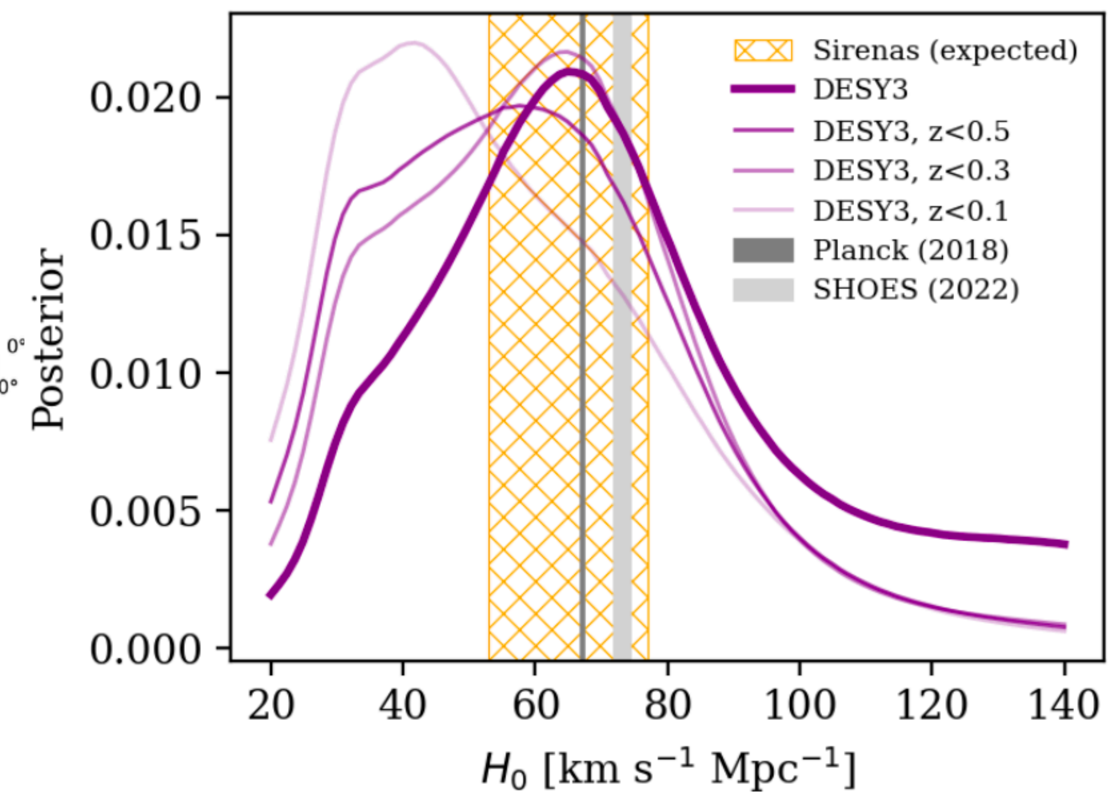
LVK O4b/c cosmological results will use **UpGLADE**, an updated version of GLADE+ (used in O3 and O4a)

Dark sirens results alone should become comparable with or better than GW170817

Current results from LVK

The <u>statistical dark siren method</u> can benefit from targeted EM observations



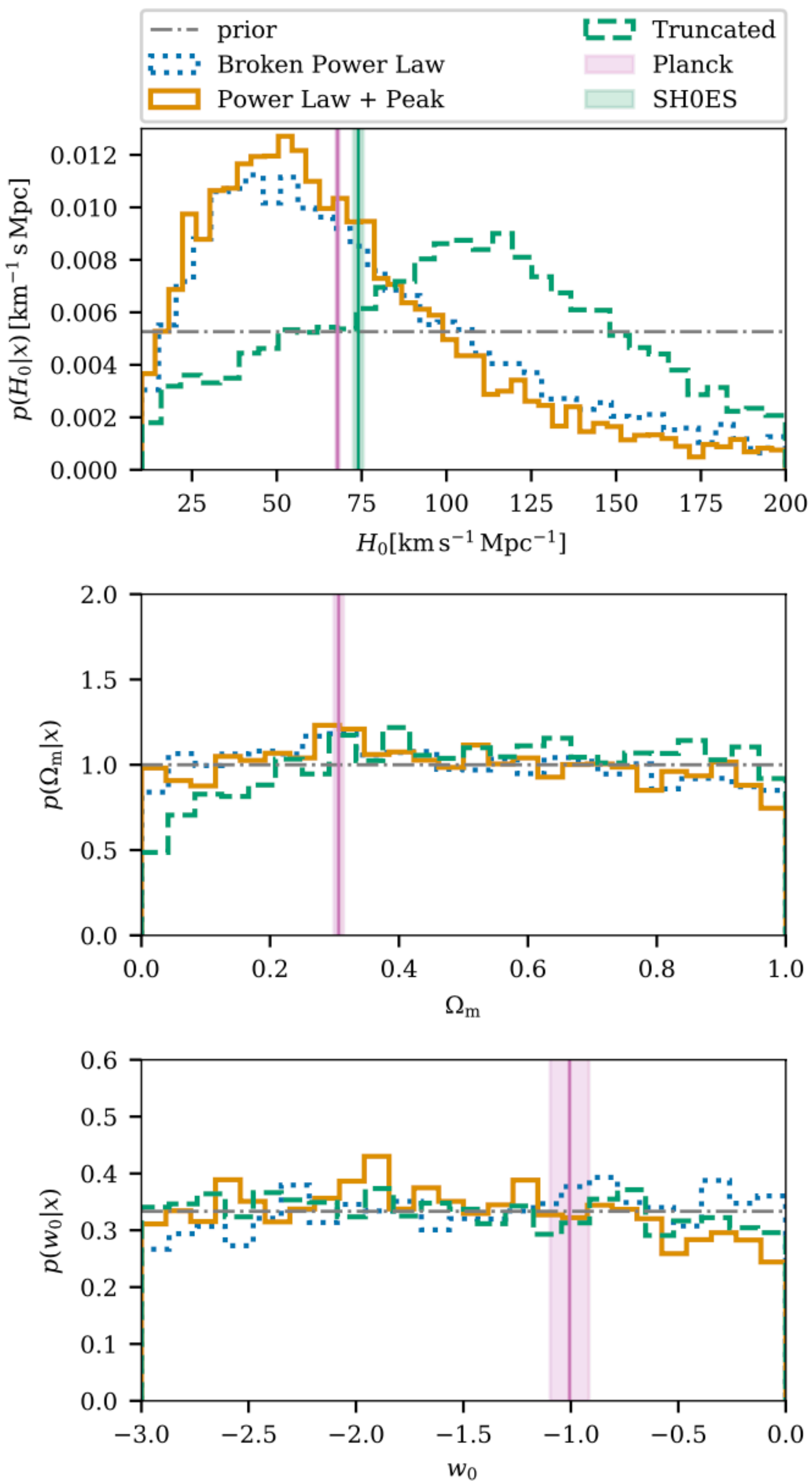


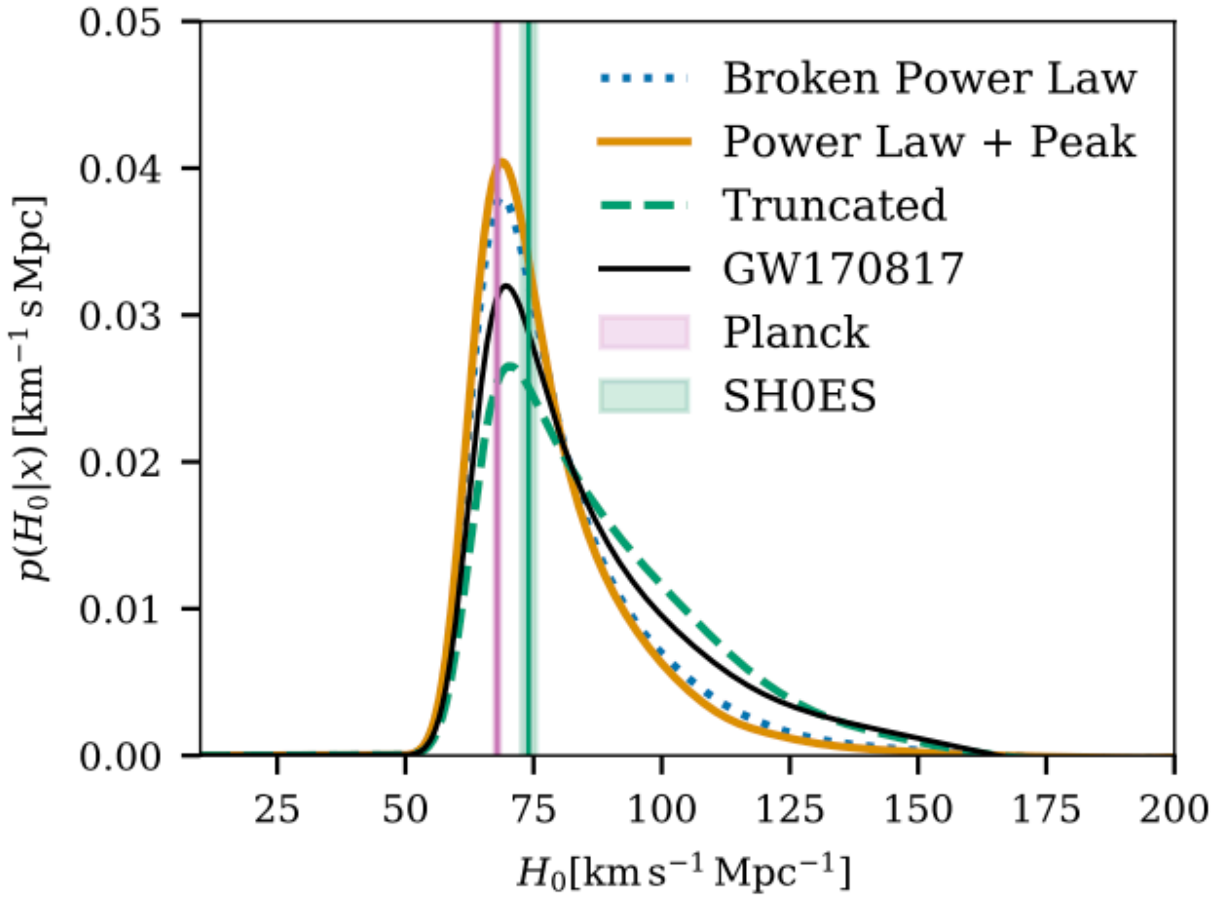
Completeness of galaxy catalogs is the main limitation for statistical dark sirens
Dedicated observations can "fill the gaps" in galaxy catalogs where GW events are localised

[LVC, *ApJ* (2020)]
[Soares-Santos+, *DECam Survey proposal* (2025)]

Current results from LVK

Finally the <u>spectral dark siren method</u> has been applied to current events (O1+O2+O3):





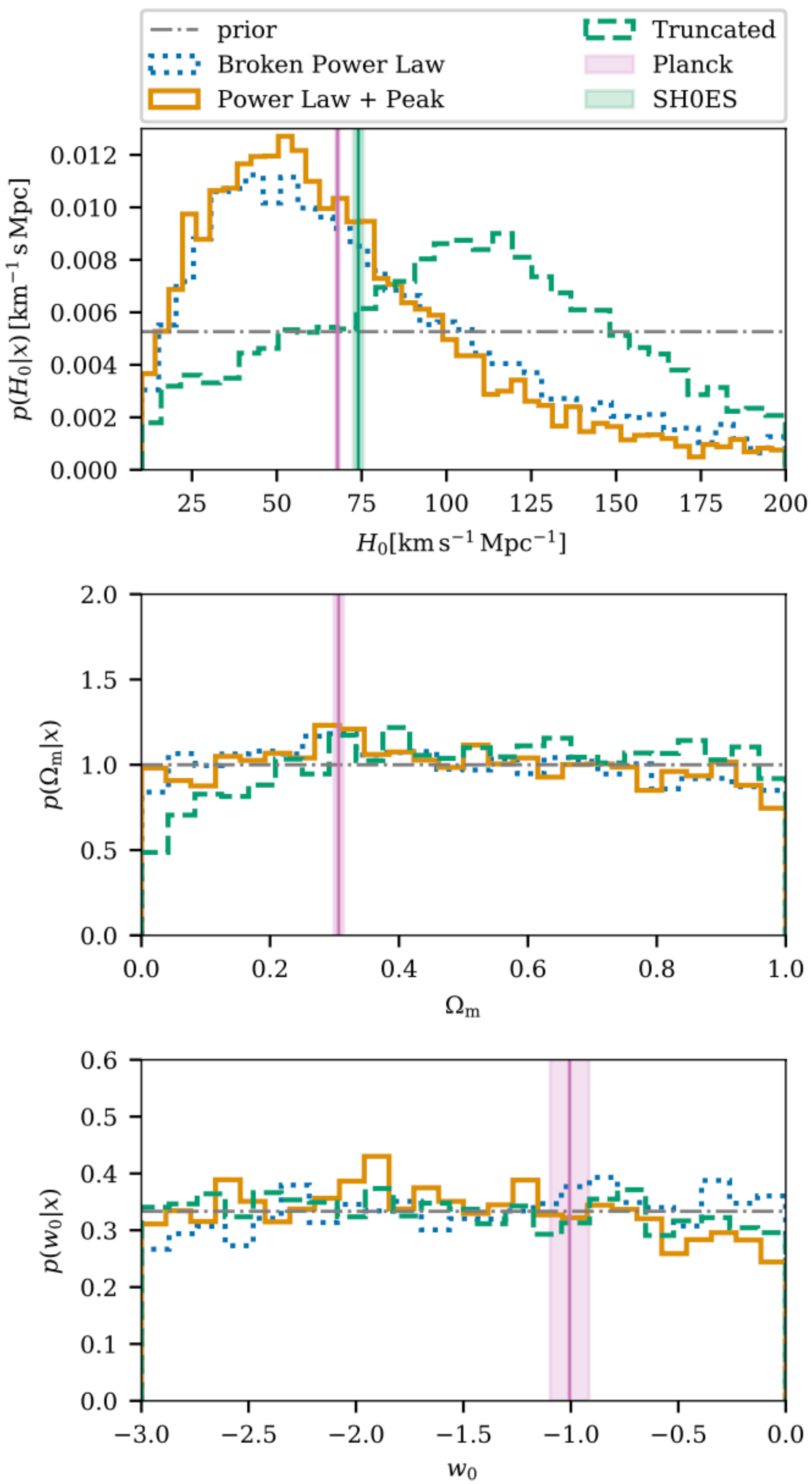
Posteriors are informative on H_0 (not on other cosmo parameters) but strongly depend on population model (w/o GW170817)

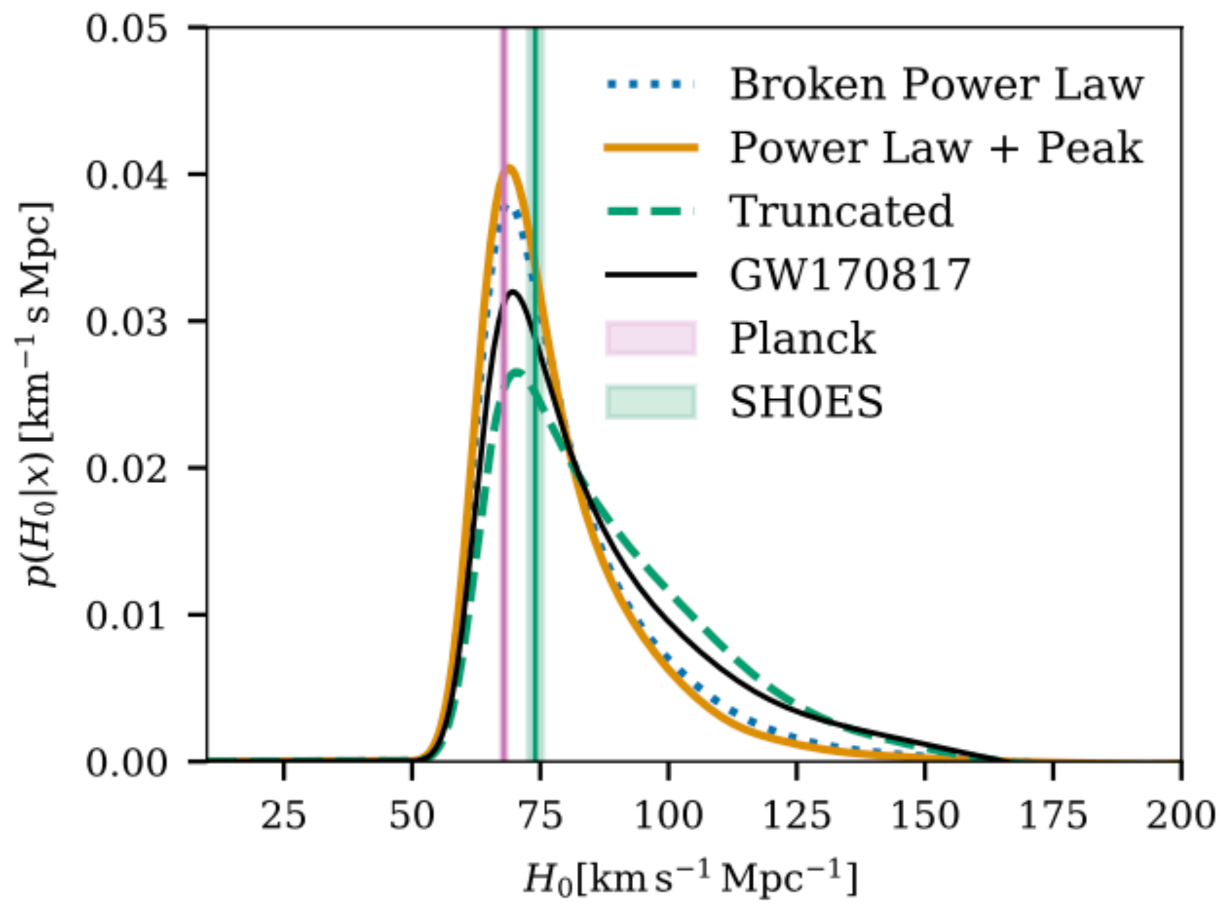
$$H_0 = 50^{+37}_{-30}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68% C.I.)

[LVK, *ApJ* (2023)]

Current results from LVK

Finally the spectral dark siren method has been applied to current events (O1+O2+O3):





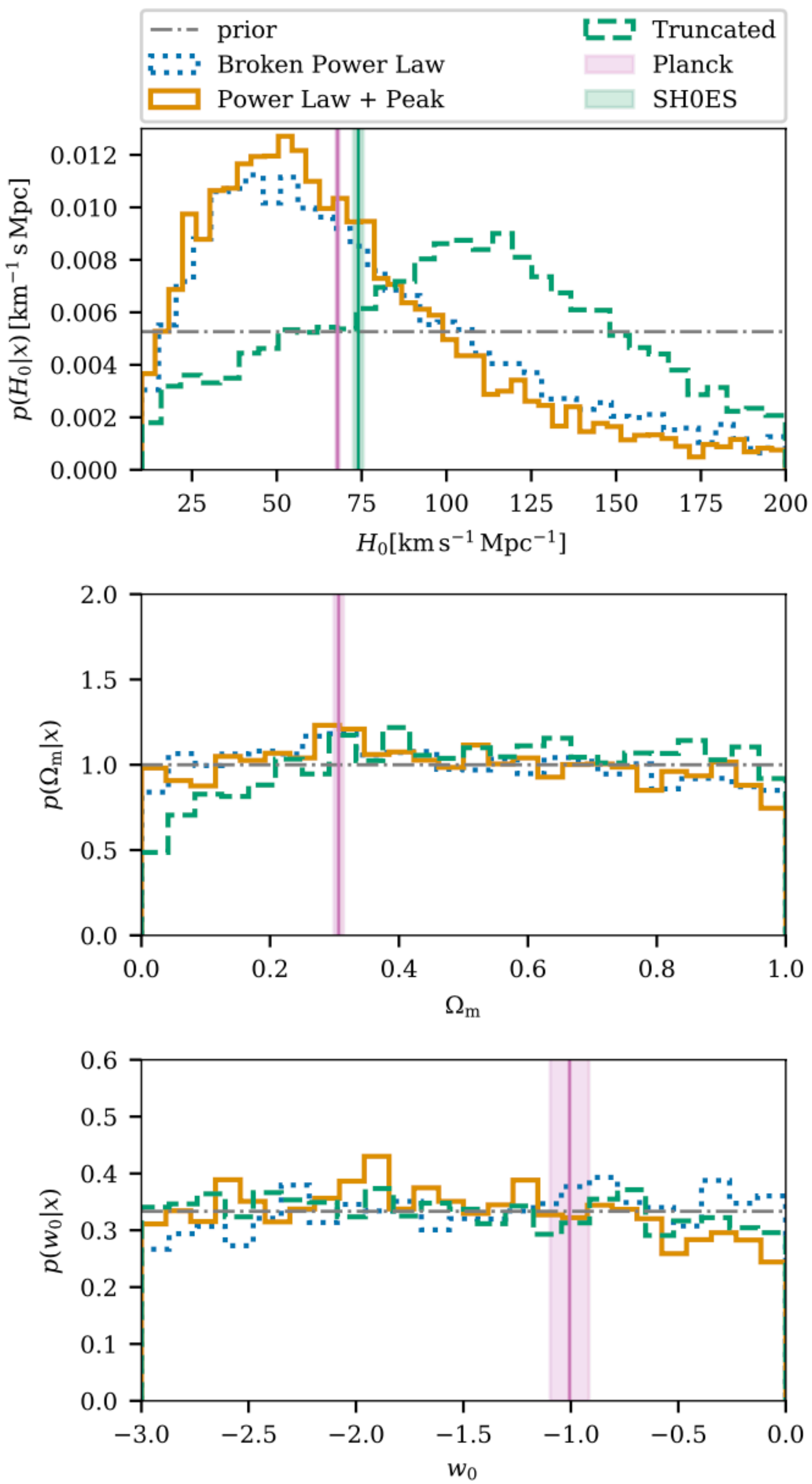
Posteriors are informative on H_0 (not on other cosmo parameters) but strongly depend on population model (w. GW170817)

$H_0 = 68^{+12}_{-6}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

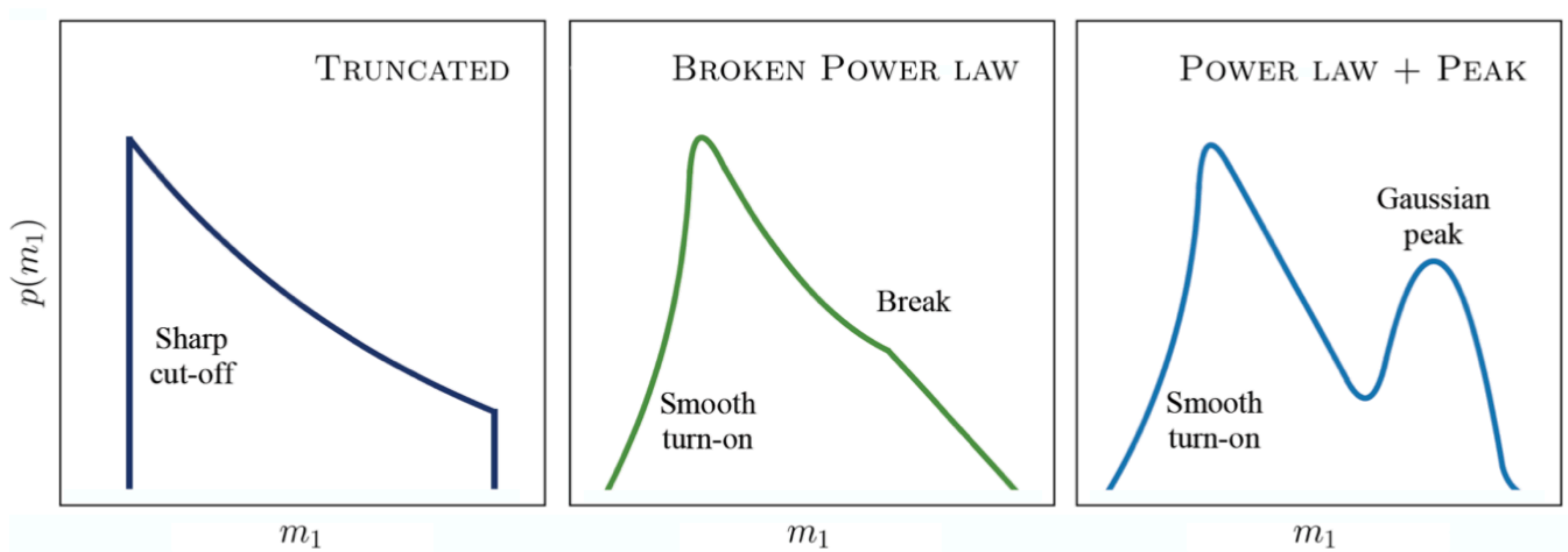
(17% improvement over O2 results) [LVK, *ApJ* (2023)]

Current results from LVK

Finally the <u>spectral dark siren method</u> has been applied to current events (O1+O2+O3):



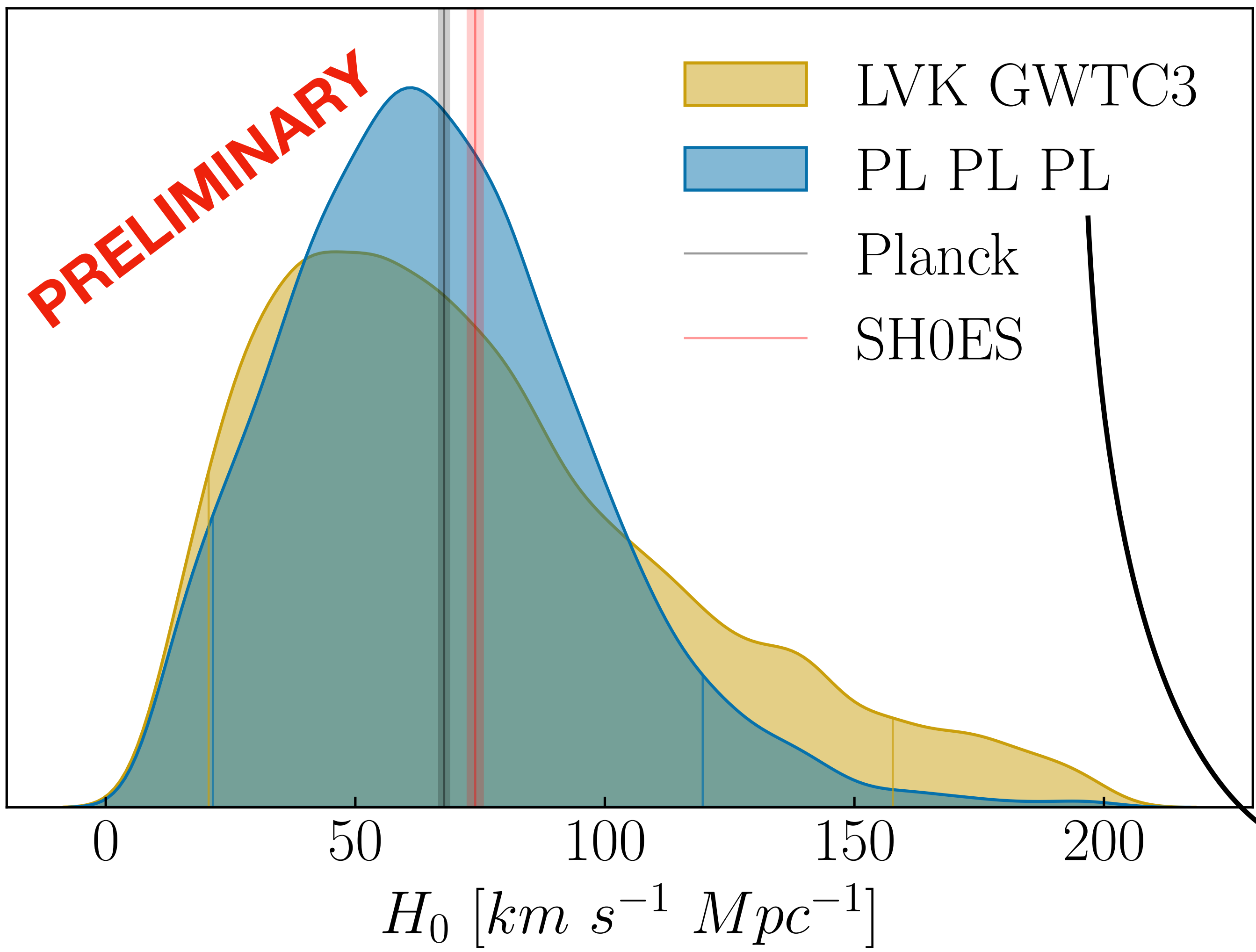
Mismodelling of the source-frame mass function is the main limitation for spectral dark sirens



[LVK, *ApJ* (2023)]

Current results from LVK

Finally the <u>spectral dark siren method</u> has been applied to current events (O1+O2+O3):



Mismodelling of the source-frame mass function is the main limitation for spectral dark sirens

The more features are present in the mass function **the better** the cosmological constraints

Model with 3 features
(one more than LVK model but comparable/better fit to O3 data)

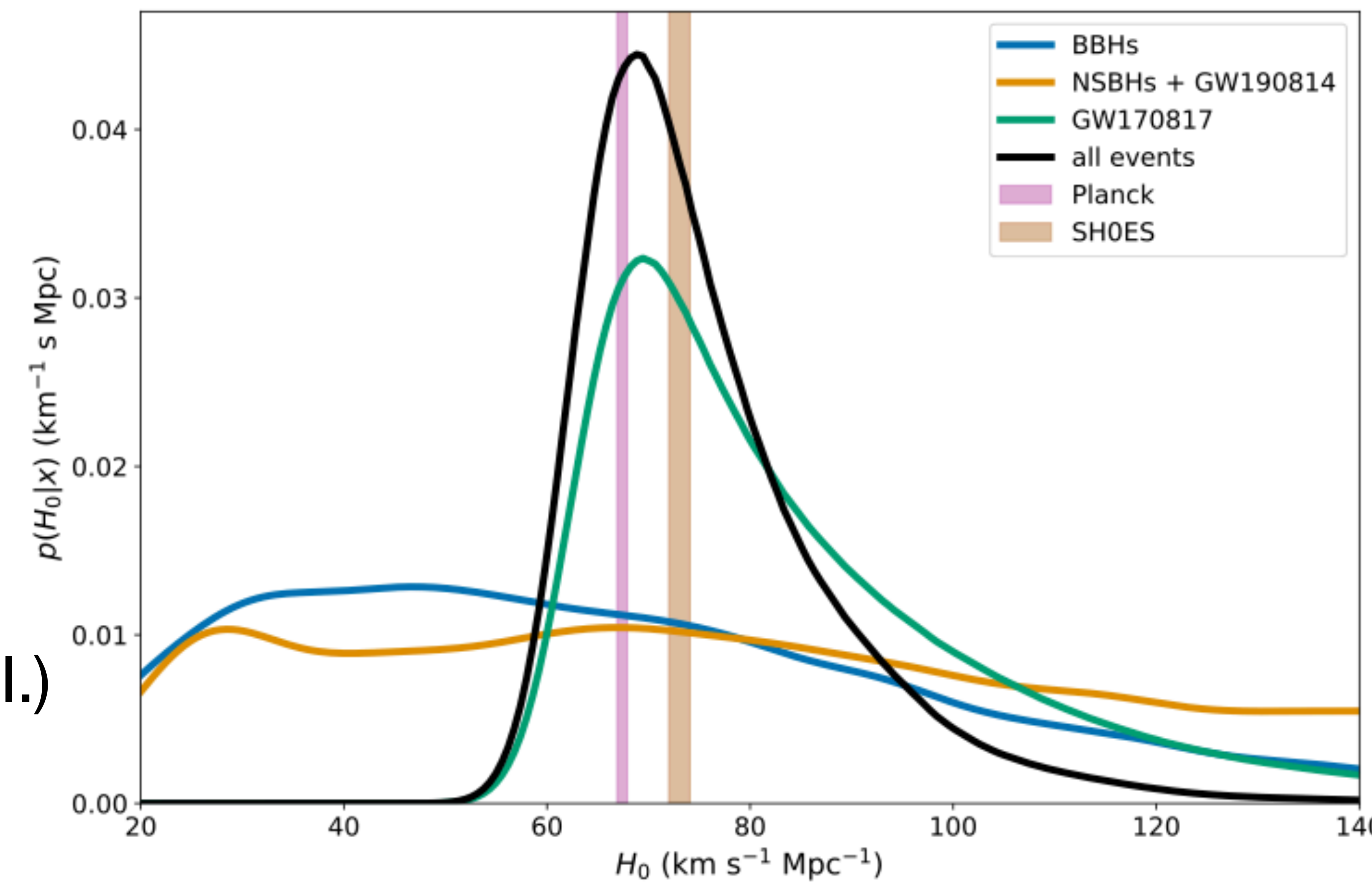
[Gennari+, *in preparation*]

Current results from LVK

Latest observational results obtained by combining all sirens methods

A joint analysis combining all dark sirens methods (statistical+spectral) + GW170817 provides the best constraint so far from O3 (including marginalisation over population parameters)

$$H_0 = 69^{+12}_{-7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$ (68% C.I.)



This represents a ~20% improvement over GW170817 only results

LVK O4 cosmological results will combine all standard siren methods (bright+spectral+statistical) for the first time

[Gray+, *JCAP* (2023)]
[Mastrogiovanni+, *PRD* (2023)]

The standard siren Pyramid

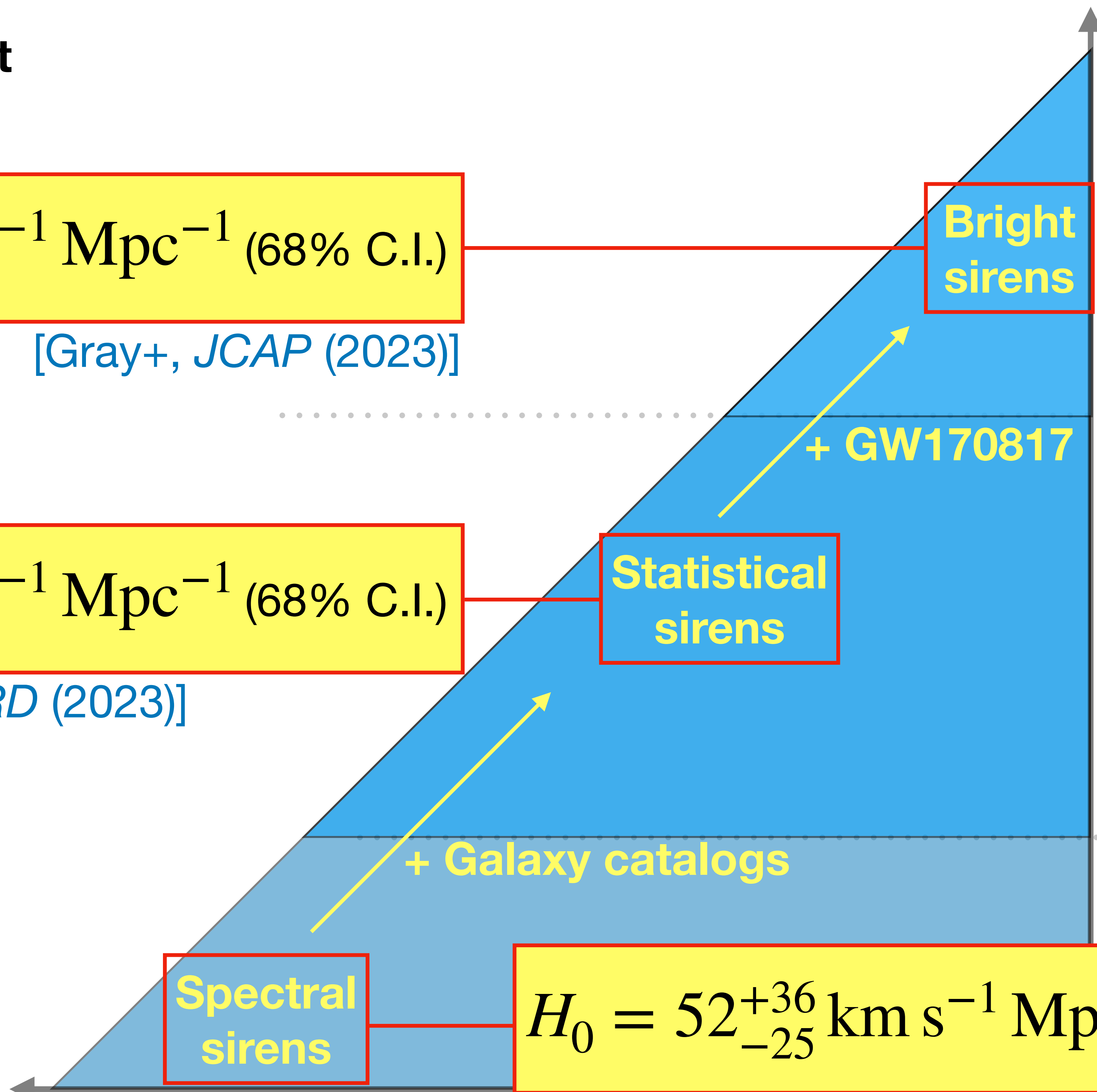
Summary of current constraints

$H_0 = 69^{+12}_{-7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

[Gray+, *JCAP* (2023)]

$H_0 = 71^{+35}_{-30}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

[Mastrogiovanni+, *PRD* (2023)]



$H_0 = 52^{+36}_{-25}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

[Mastrogiovanni+, *PRD* (2023)]

The standard siren Pyramid

Summary of current constraints

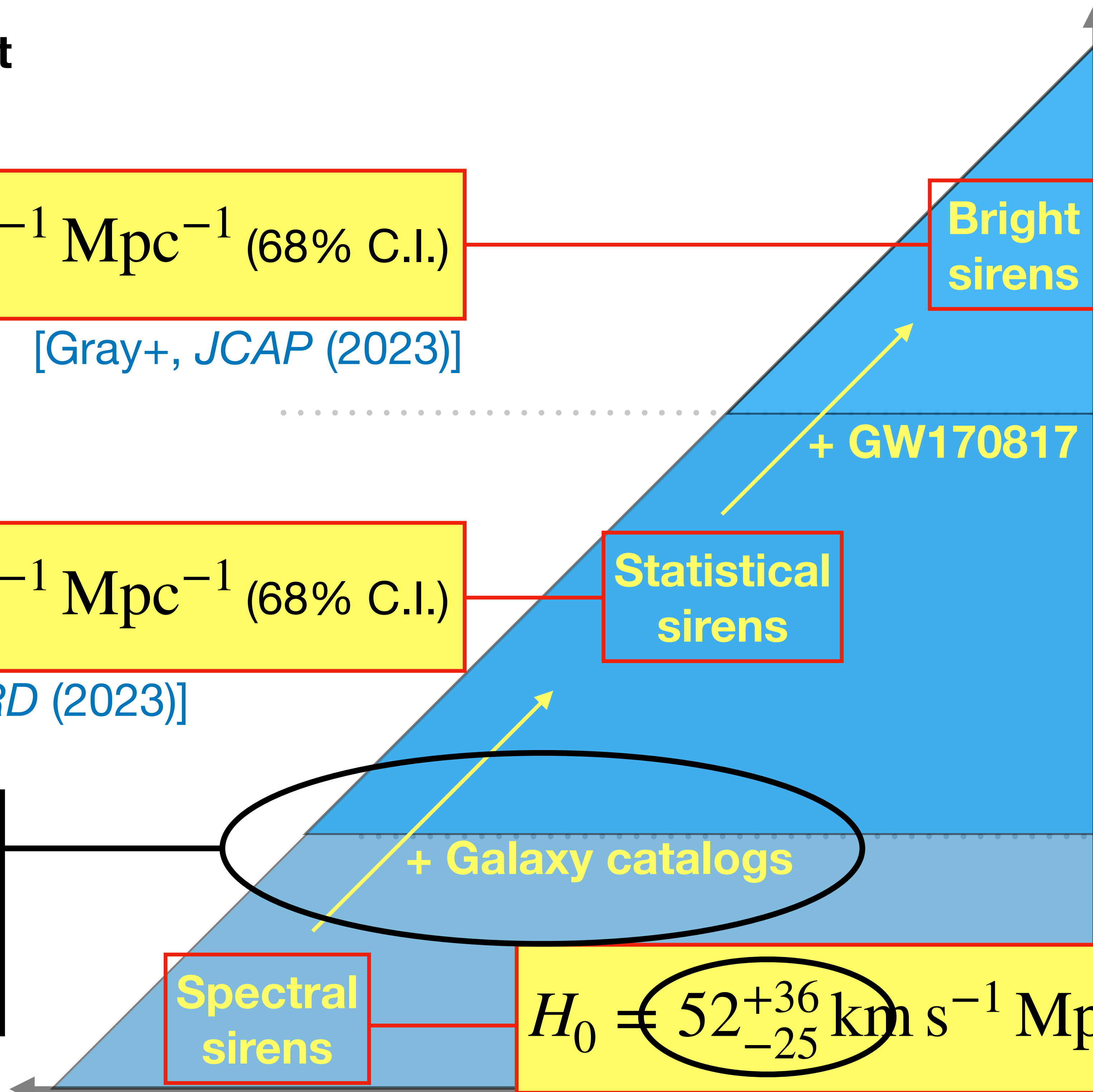
$H_0 = 69^{+12}_{-7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

[Gray+, *JCAP* (2023)]

$H_0 = 71^{+35}_{-30}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

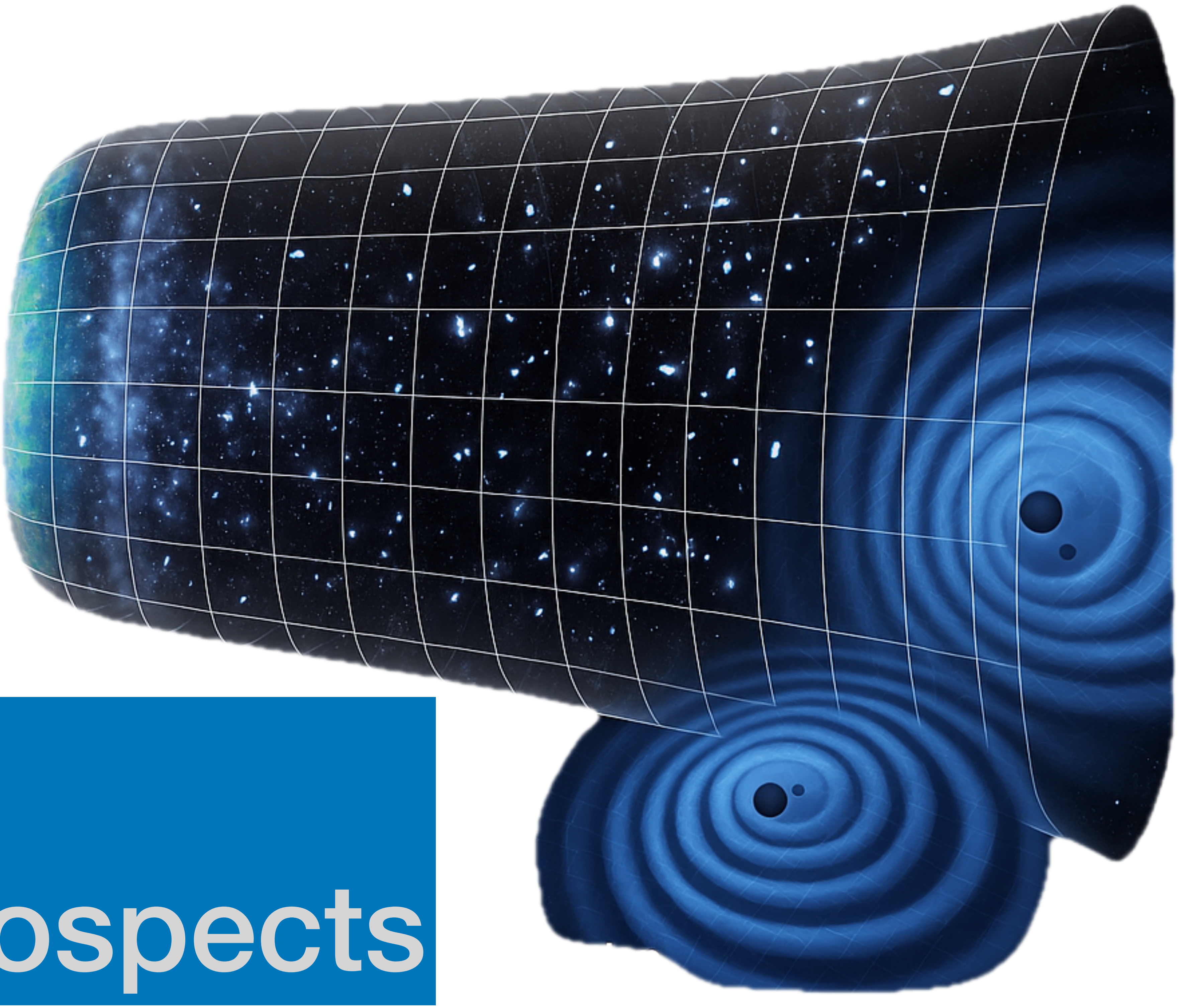
[Mastrogiovanni+, *PRD* (2023)]

At the moment galaxy catalogs are not providing much useful information!



$H_0 = 52^{+36}_{-25}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (68% C.I.)

[Mastrogiovanni+, *PRD* (2023)]

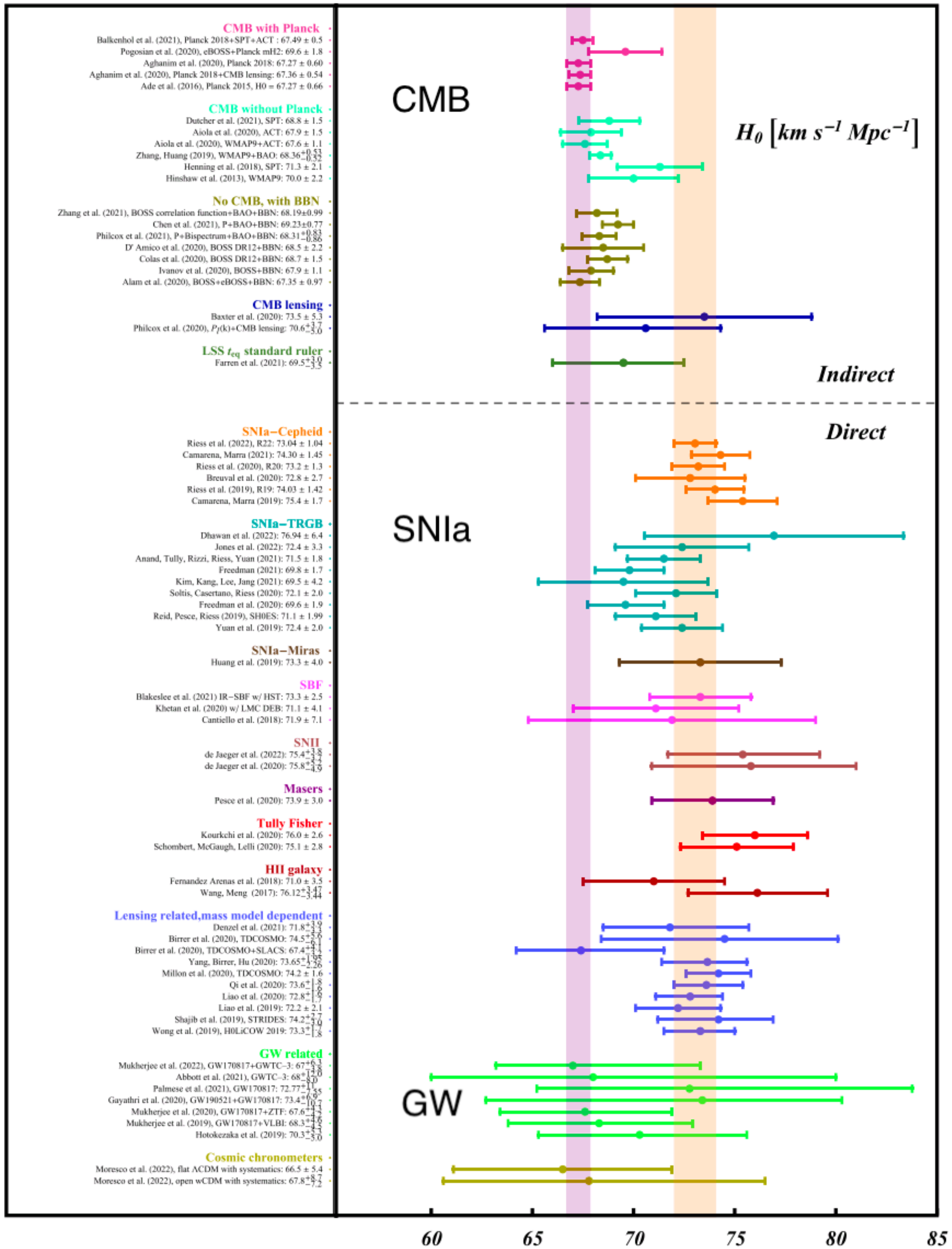


7
Future prospects

Future prospects

The Hubble tension

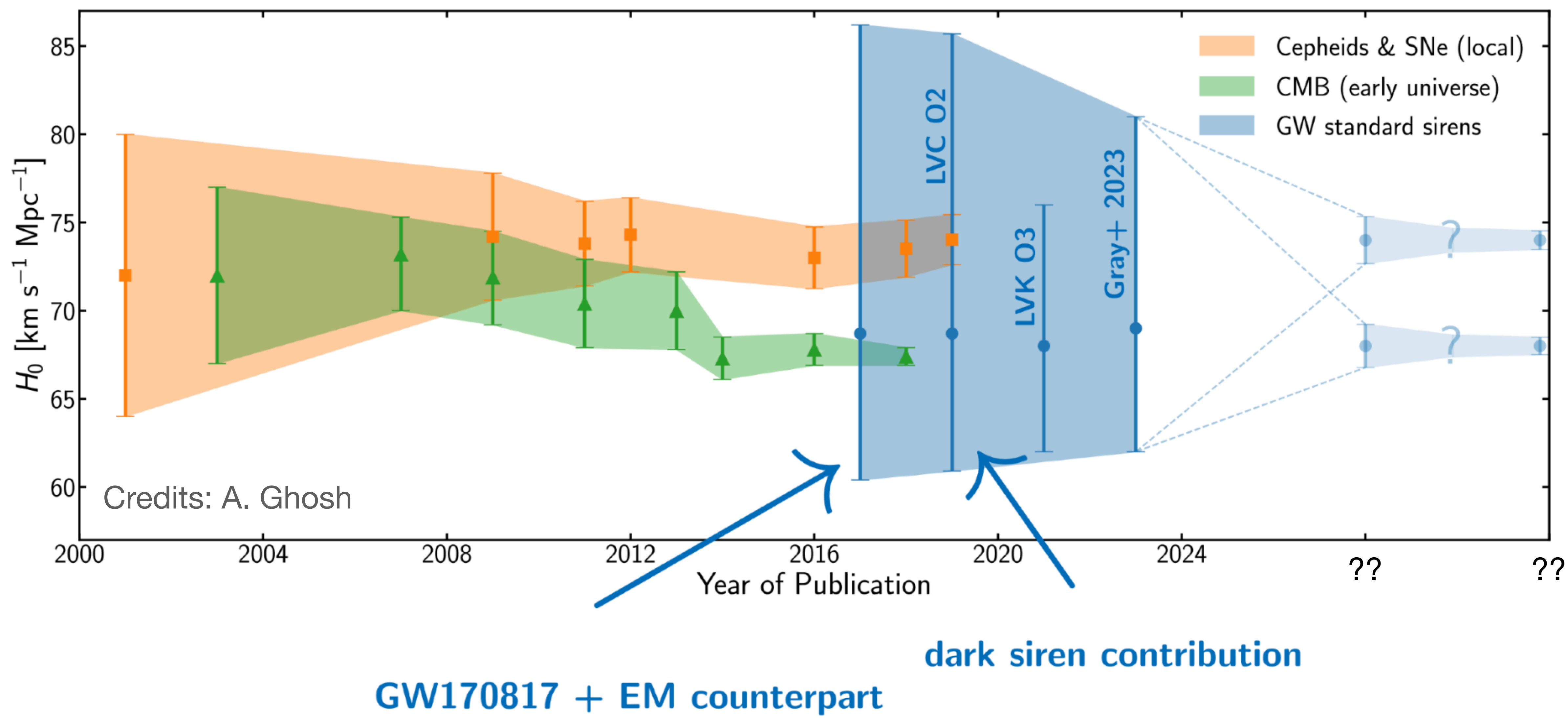
A few % constraints on H0 with GWs could solve the current tension between local and CMB measurements



[Abdalla+, *JHEAp* (2022)]

Future prospects

<u>FUTURE PROSPECTS WITH LVK:</u> When will we obtain a few % measurement of the Hubble constant with GWs?



Few % accuracy on H_0 possible only in the most optimistic O5 scenario

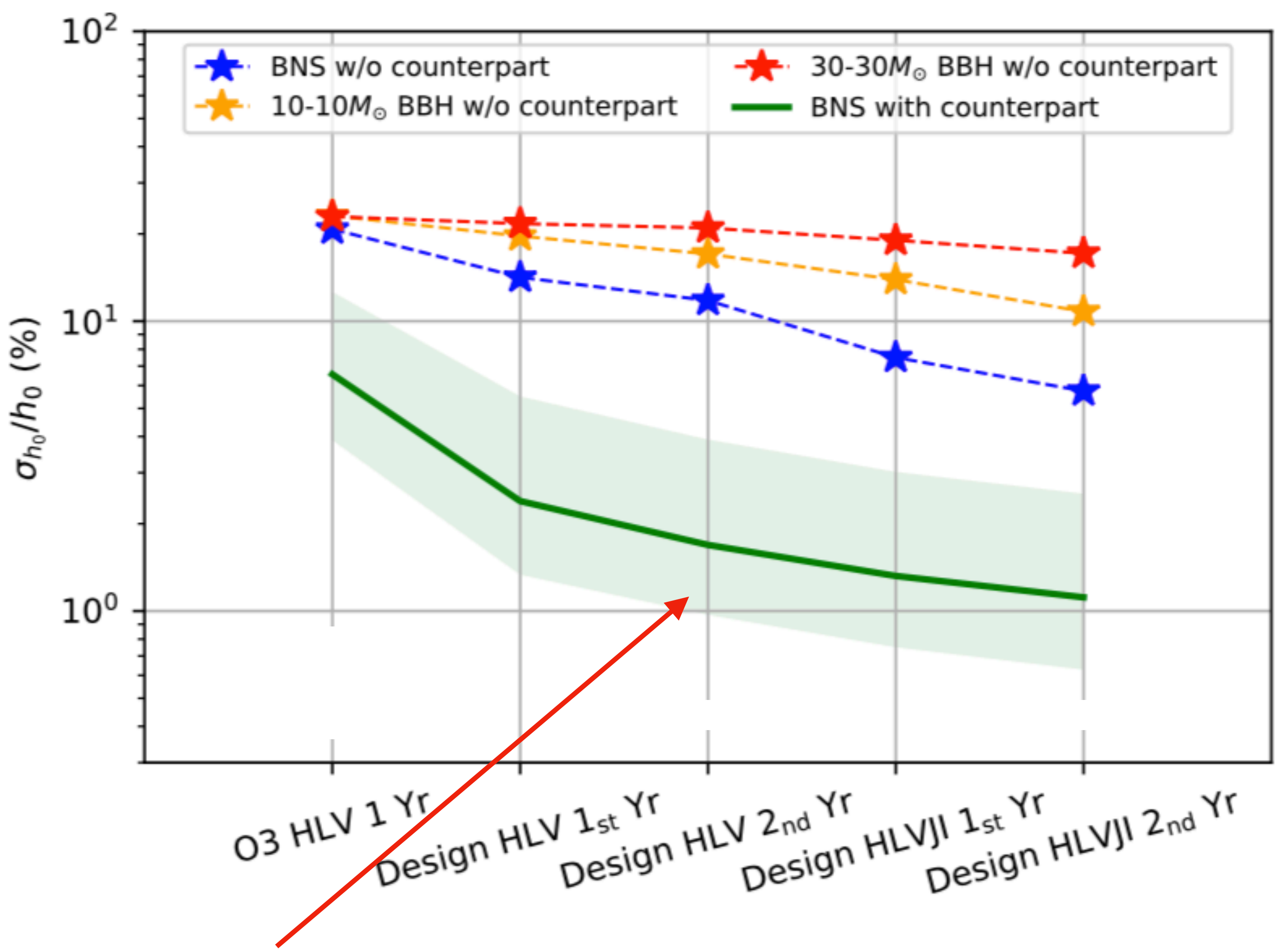
[Kiendrebeogo+, *ApJ* (2023)]

Post-O5 (>2030) observations needed to solve the <u>Hubble tension</u>

Future prospects

FUTURE PROSPECTS WITH LVK:

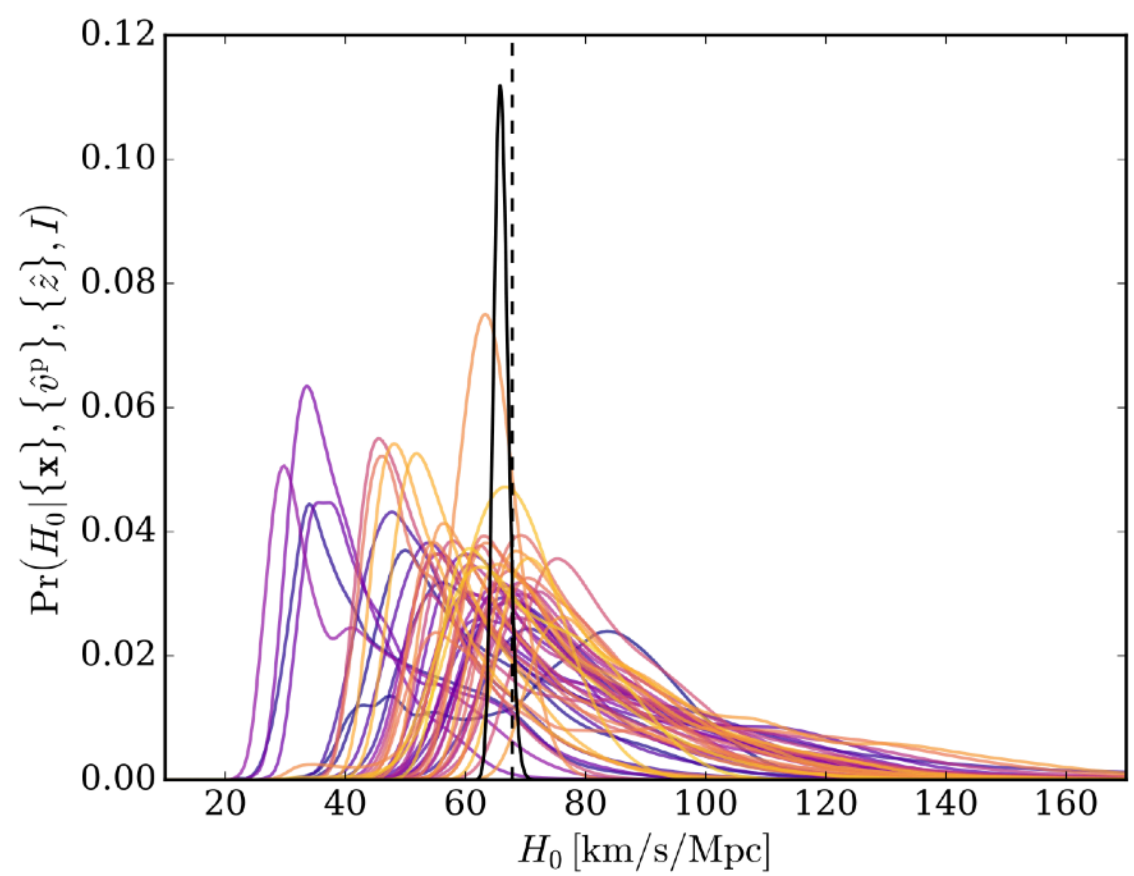
In order to get to few % we need to detect around 50 bright sirens (no EM information)



Very optimistic (based on pre-O3 BNS rates)

[Chen+, *Nature* (2018)]
[Chen+, *ApJL* (2020)]

BNSs with EM counterpart:
~2% constraint on H_0 with ~50 events (but systematics!)

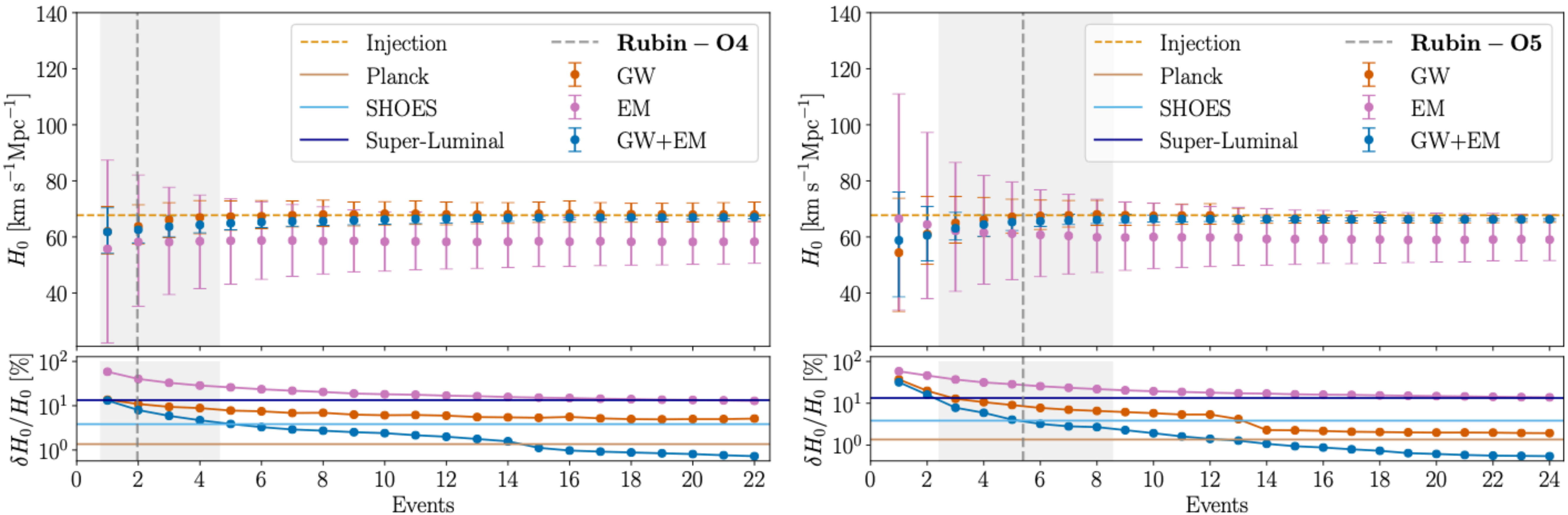


[Feeney+, *PRL* (2019)]

Future prospects

FUTURE PROSPECTS WITH LVK:

In order to get to few % one could also use the loudest events with EM information



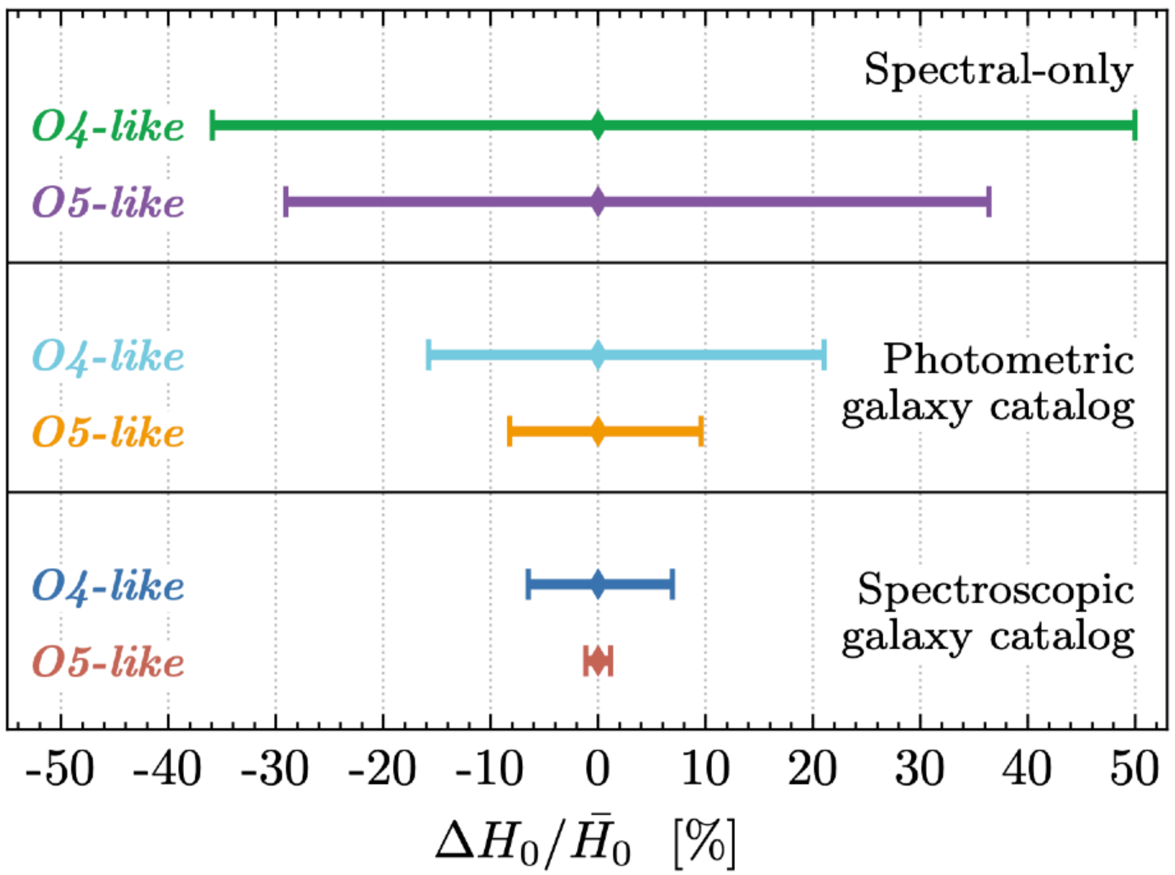
Few % accuracy on H_0 possible only in the most optimistic O5 scenario

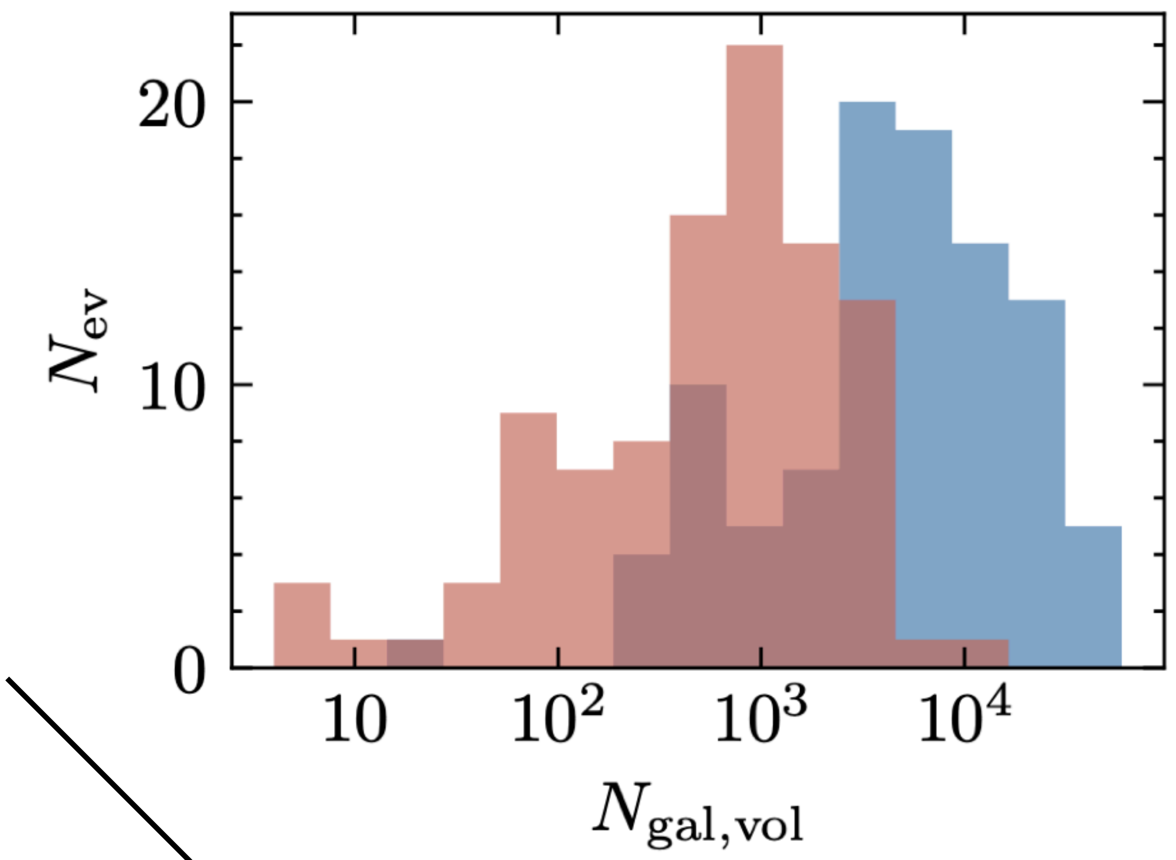
[Kiendrebeogo+, *ApJ* (2023)]

Run	Telescope	BNS	NSBH
EM annual number of detections			
O4	ZTF	$0.43^{+0.58}_{-0.26}$	$0.13^{+0.24}_{-0.11}$
	Rubin	$1.97^{+2.68}_{-1.2}$	$0.03^{+0.06}_{-0.03}$
O5	ZTF	$0.43^{+0.44}_{-0.2}$	$0.09^{+0.12}_{-0.06}$
	Rubin	$5.39^{+6.59}_{-2.99}$	$0.43^{+0.59}_{-0.28}$

Future prospects

<u>FUTURE PROSPECTS WITH LVK:</u> Dark sirens (spectral+statistical) could deliver a few % constraints on H0 only in the most optimistic O5 scenario





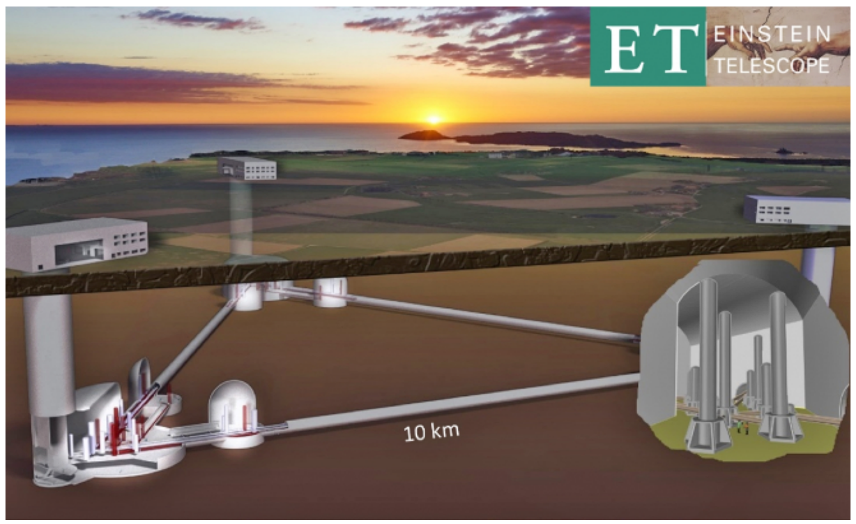
Few % accuracy on H_0 possible only in the most optimistic O5 scenario

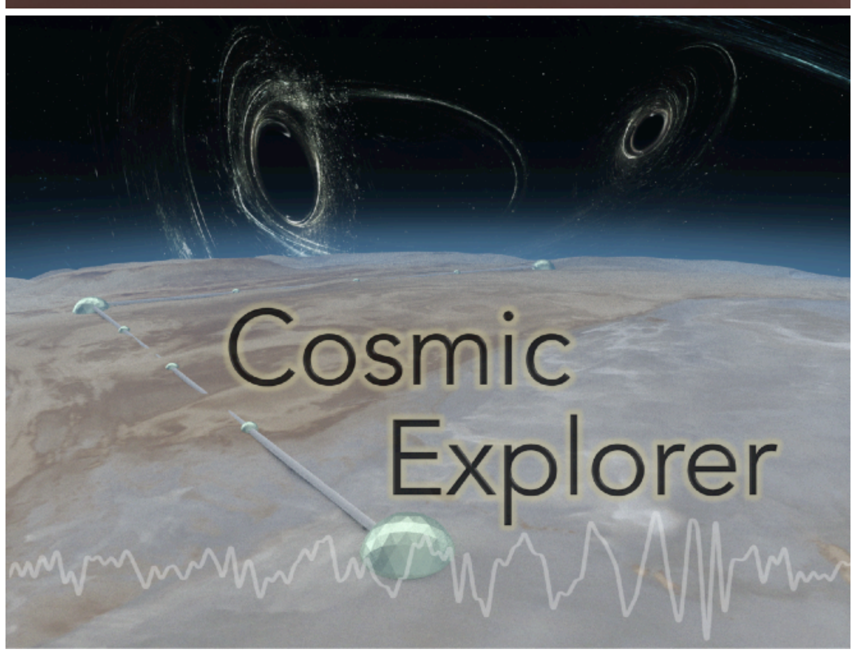
Optimistic! Galaxy catalogs fully complete, 5 detectors (HLVKI), simplified GW PE, …

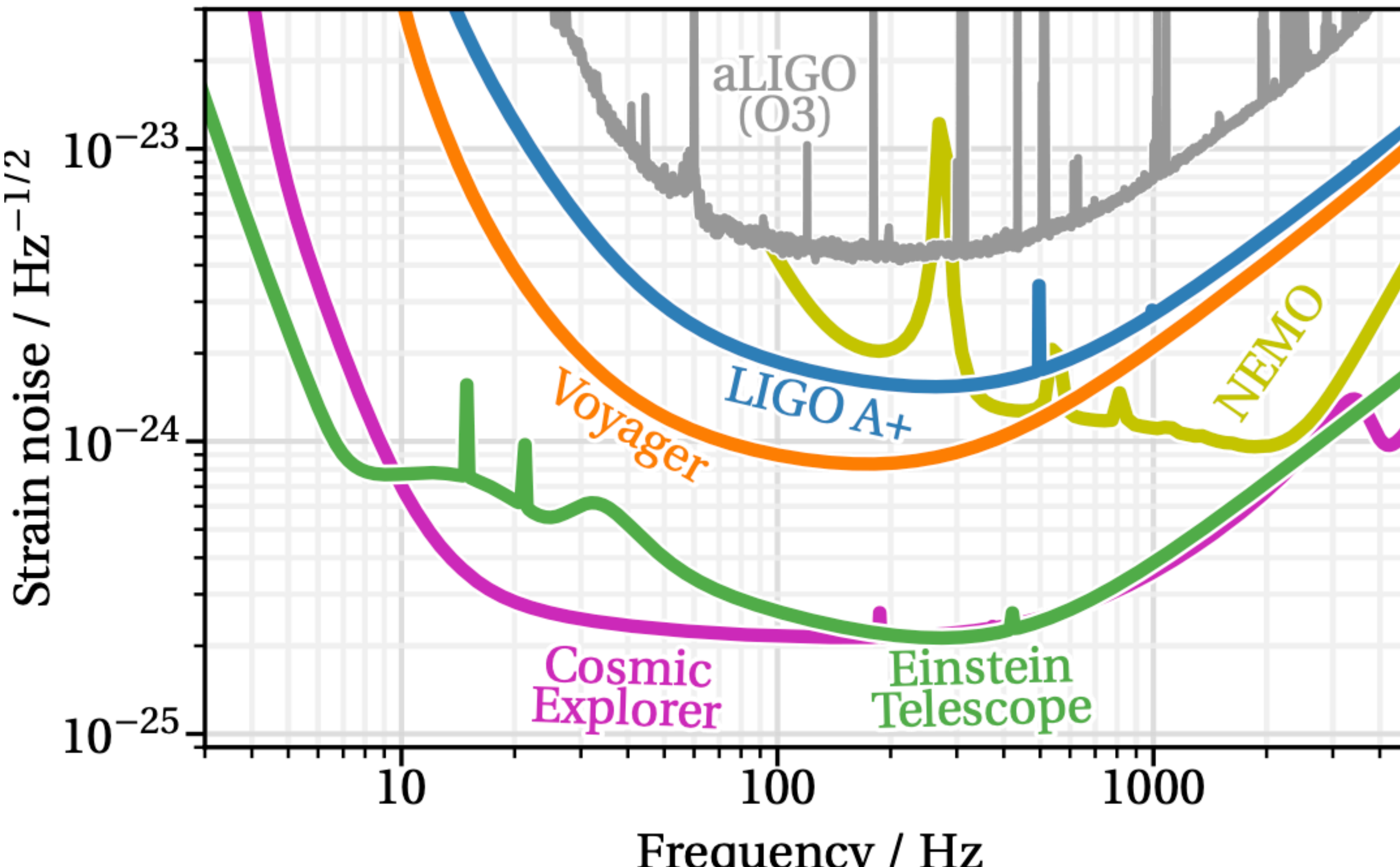
[Borghi+, *ApJ* (2024)]

Future prospects

Around 2040 <u>3G detectors</u> will turn GW observations into precise cosmological probes:



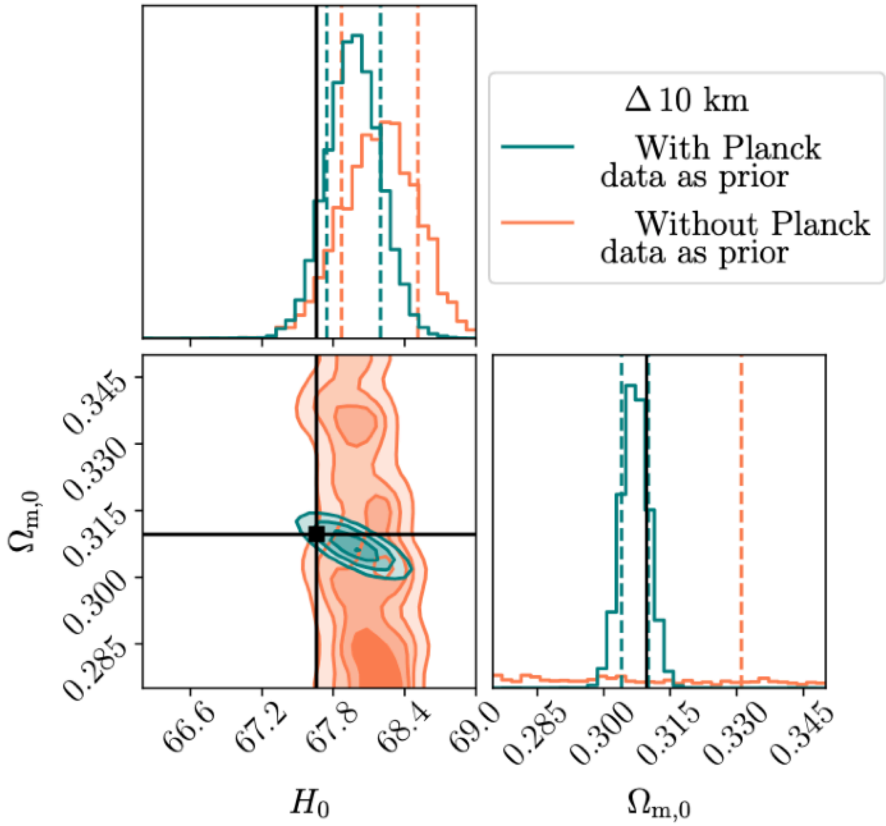


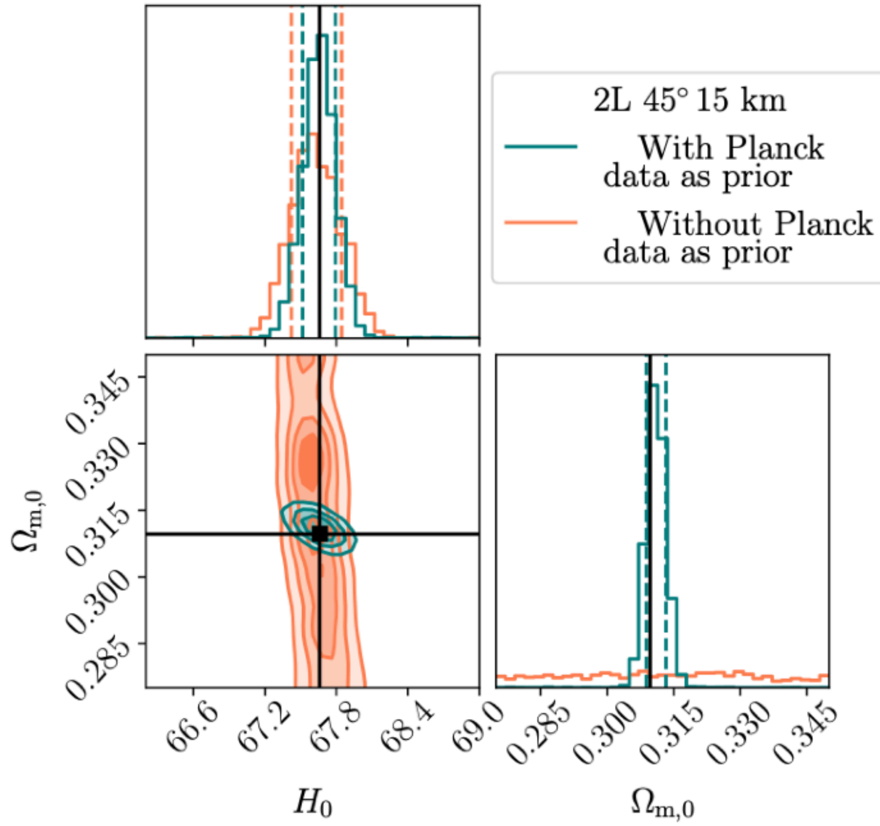


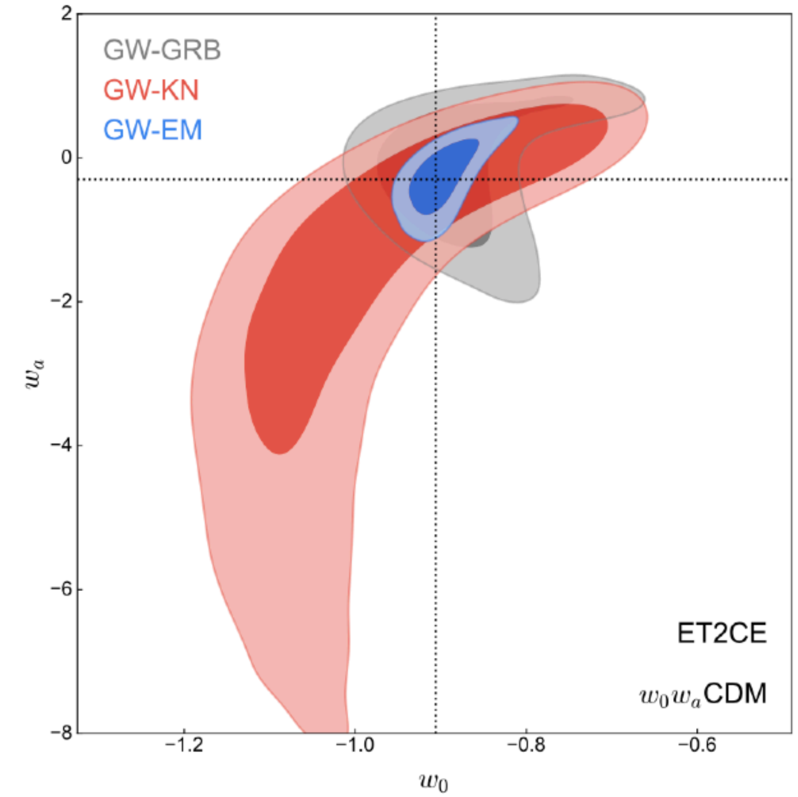
One order of magnitude sensitivity improvement w.r.t. LIGO, similar frequency range

Future prospects

Around 2040 <u>3G detectors</u> will turn GW observations into precise cosmological probes:







<u>Einstein Telescope</u> and *<u>Cosmic Explorer</u>* will guarantee:

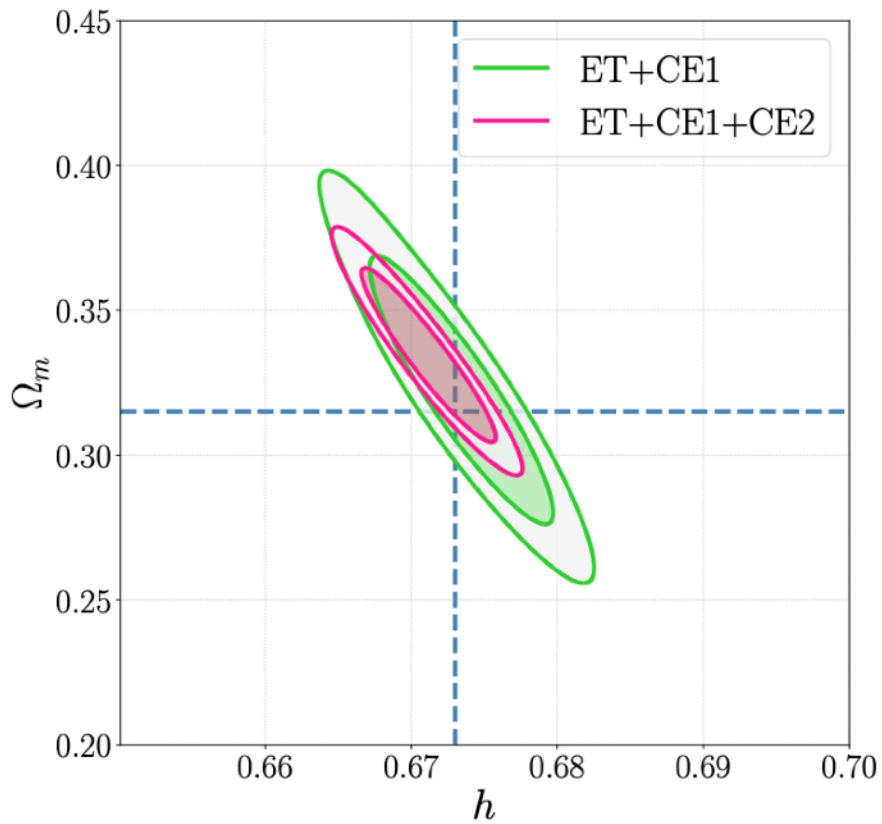
- % constraints on H_0 or better
- Improved constraints on dark energy
- Strong GW-only tests of GR at cosmic distances

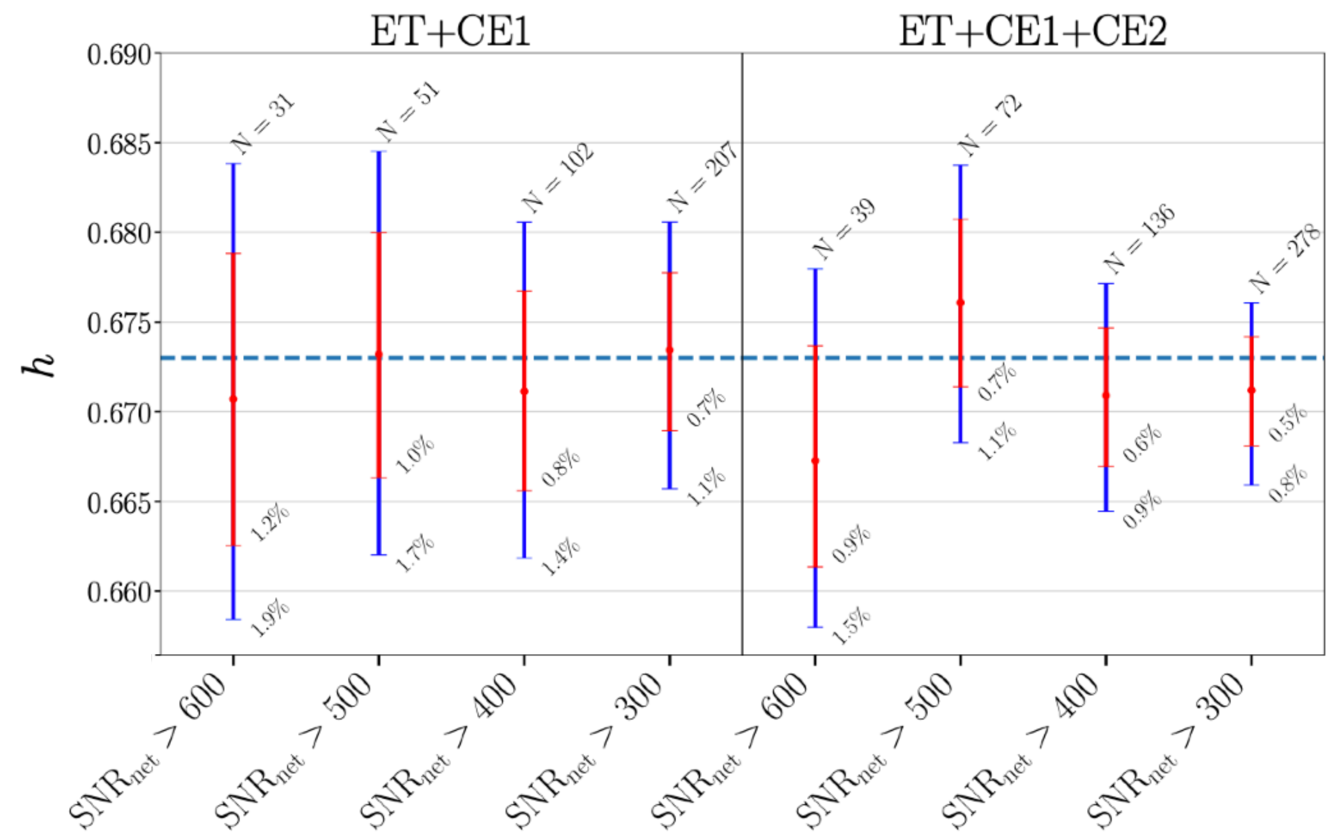
Forecasts with <u>bright</u> sirens

[ET Blue Book, *arXiv* (2025)]
[Han+, *arXiv* (2025)]

Future prospects

Around 2040 3G detectors will turn GW observations into precise cosmological probes:





Einstein Telescope and *Cosmic Explorer* will guarantee:

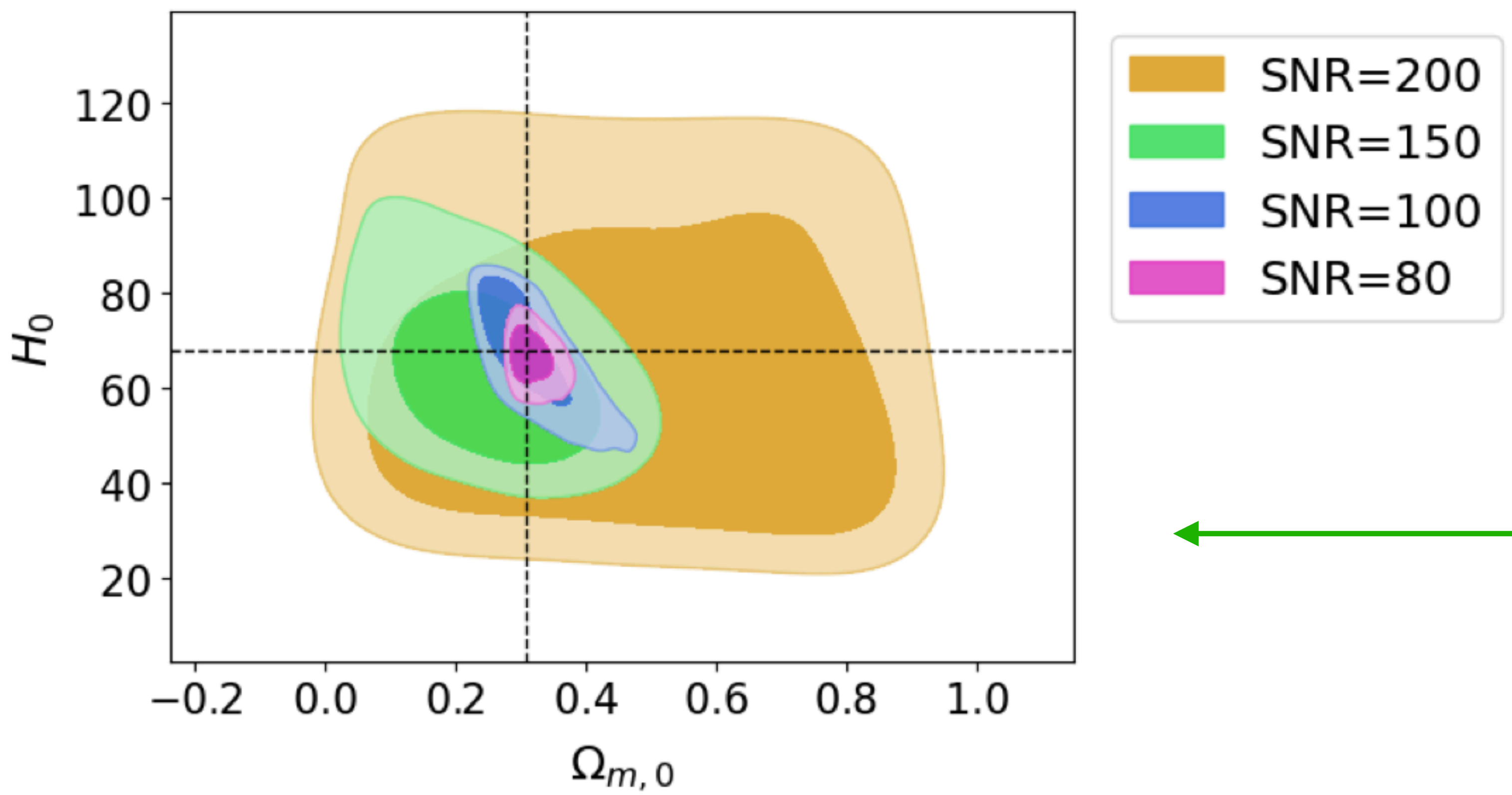
- % constraints on H_0 or better
- Improved constraints on dark energy
- Strong GW-only tests of GR at cosmic distances

Forecasts with statistical dark sirens

[Muttoni+, *PRD* (2023)]
[ET Blue Book, *arXiv* (2025)]

Future prospects

Around 2040 <u>3G detectors</u> will turn GW observations into precise cosmological probes:



Forecasts with <u>spectral</u> siren method (limited to high SNR events)

<u>Einstein Telescope</u> and *<u>Cosmic Explorer</u>* will guarantee:

- % constraints on H_0 or better
- Improved constraints on dark energy
- Strong GW-only tests of GR at cosmic distances

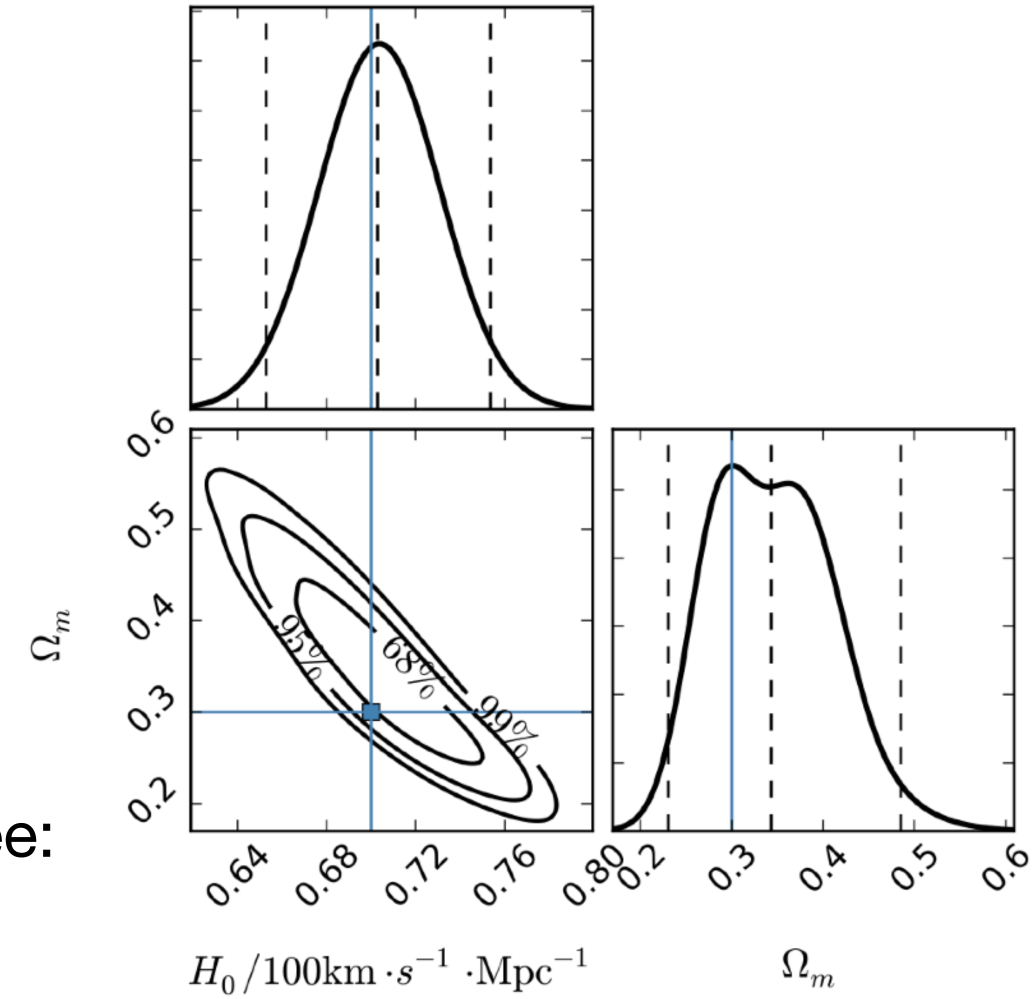
Moreover 3G detectors may not need EM information to get the redshift of BNSs, but use their mass function and/or the EoS to do cosmology and test GR/LCDM

[Califano+, *ArXiv* (2025)]

Future prospects

Around 2040 <u>3G detectors</u> will turn GW observations into precise cosmological probes:

Forecasts with <u>Love siren</u> method



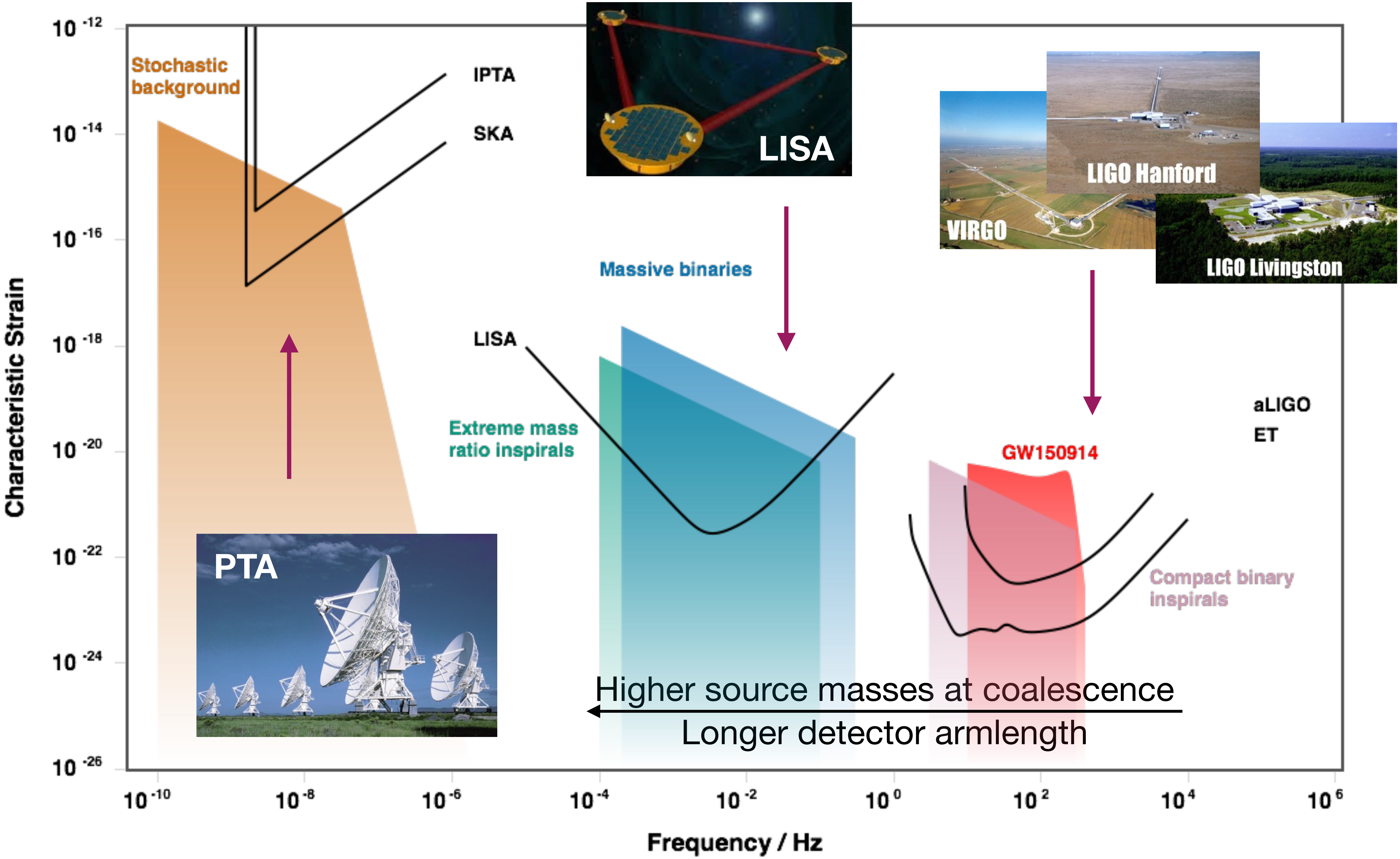
<u>Einstein Telescope</u> and *<u>Cosmic Explorer</u>* will guarantee:

- % constraints on H_0 or better
- Improved constraints on dark energy
- Strong GW-only tests of GR at cosmic distances

Moreover 3G detectors may not need EM information to get the redshift of BNSs, but use their mass function and/or the EoS to do cosmology and test GR/LCDM

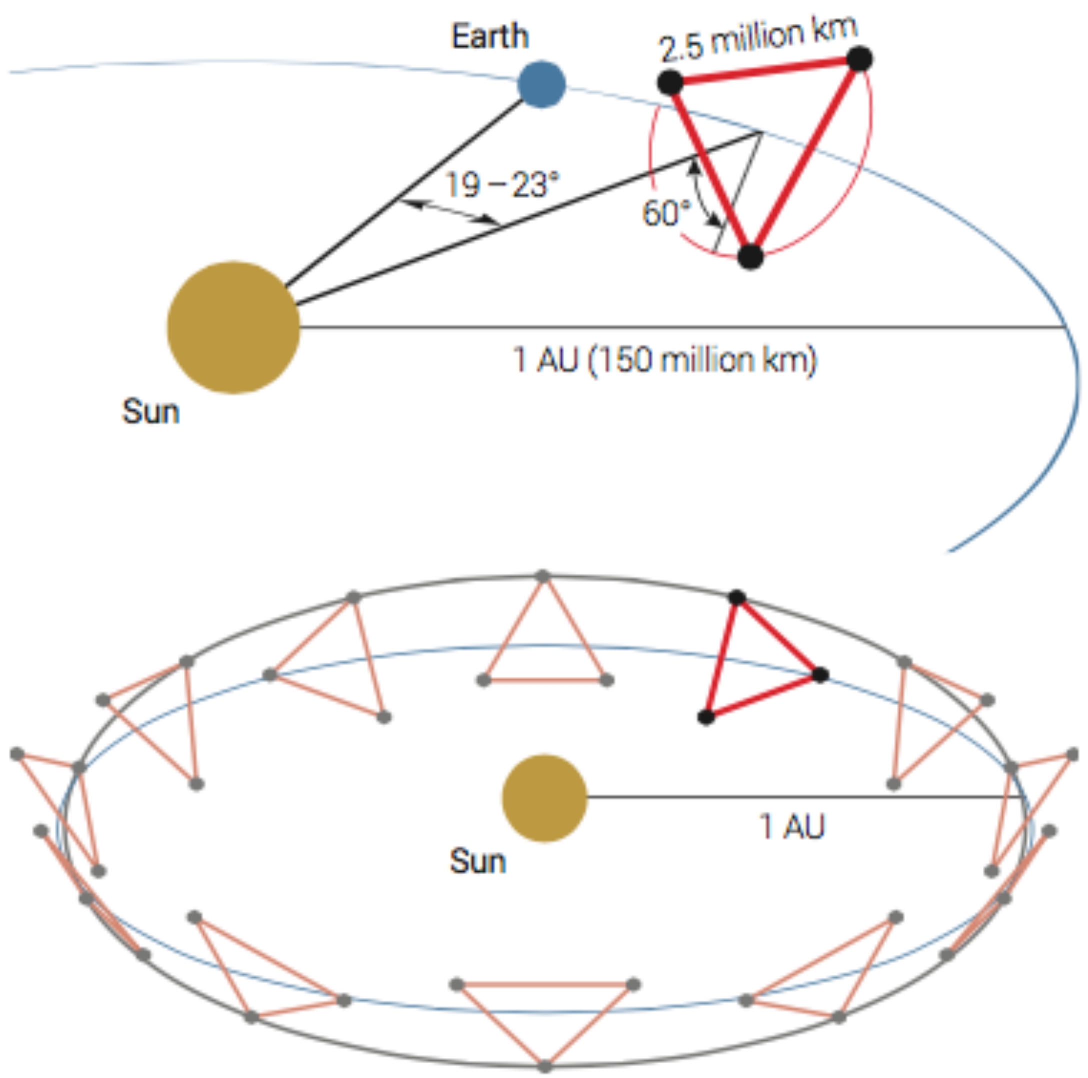
[Taylor&Gair, *PRD* (2012)]
[Del Pozzo+, *PRD* (2017)]
[Finke+, *Phys. Dark Univ.* (2021)]

Future prospects



Future prospects

Laser Interferometer Space Antenna



[LISA, *ArXiv* (2017)]
[LISA, *ArXiv* (2024)]

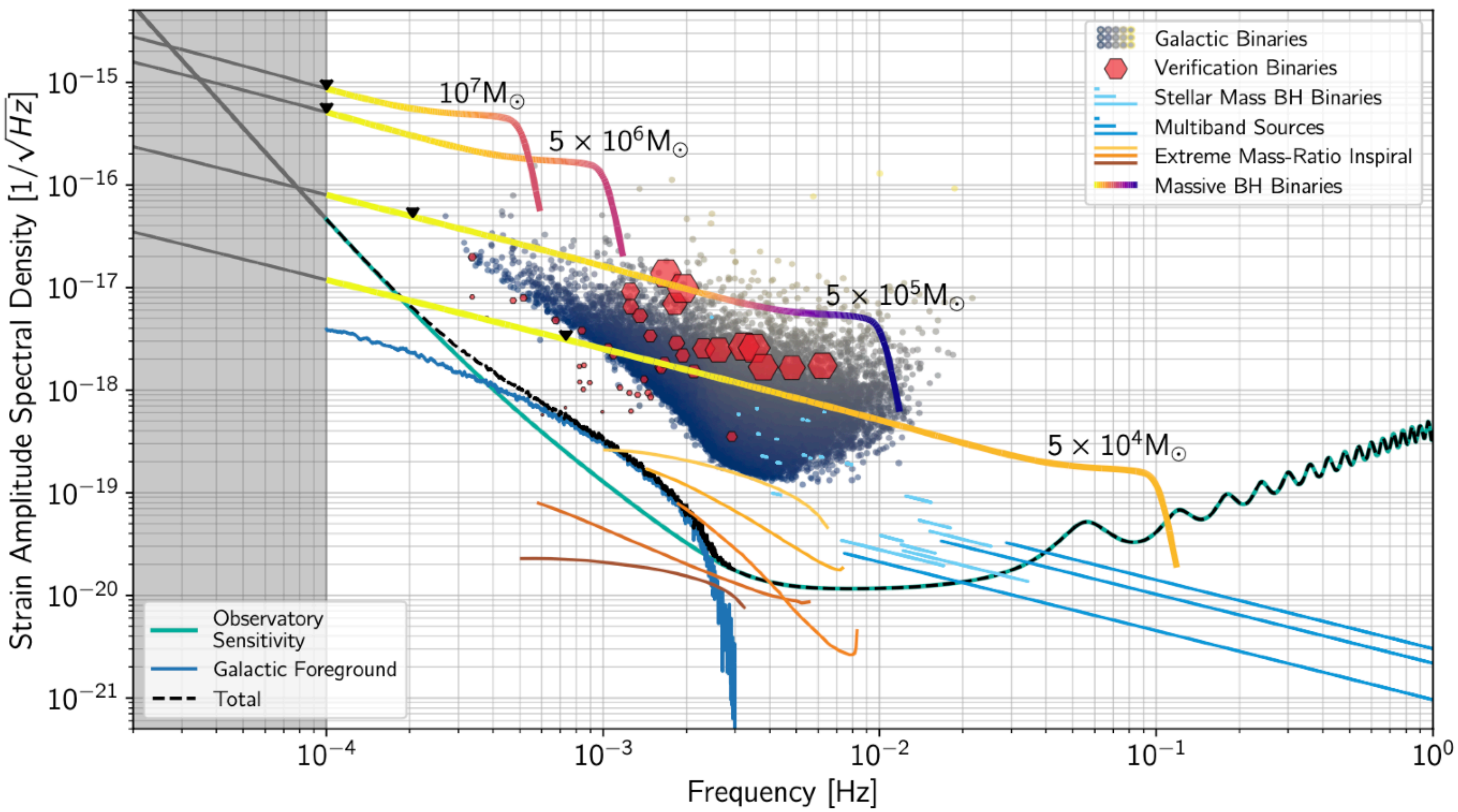
Design:

- Near equilateral triangular formation in heliocentric orbit
- 6 laser links (3 active arms)
- Arm-length: 2.5 million km
- Scince observations: 4 to 10 yrs
- **Adopted by ESA in 2024 !**
- Launch: mid-2030s

Review on
Cosmology with LISA
LISA CosWG, Liv. Rev. Rel. (2023)
arXiv:2204.05434

Future prospects

Laser Interferometer Space Antenna



Standard siren sources:

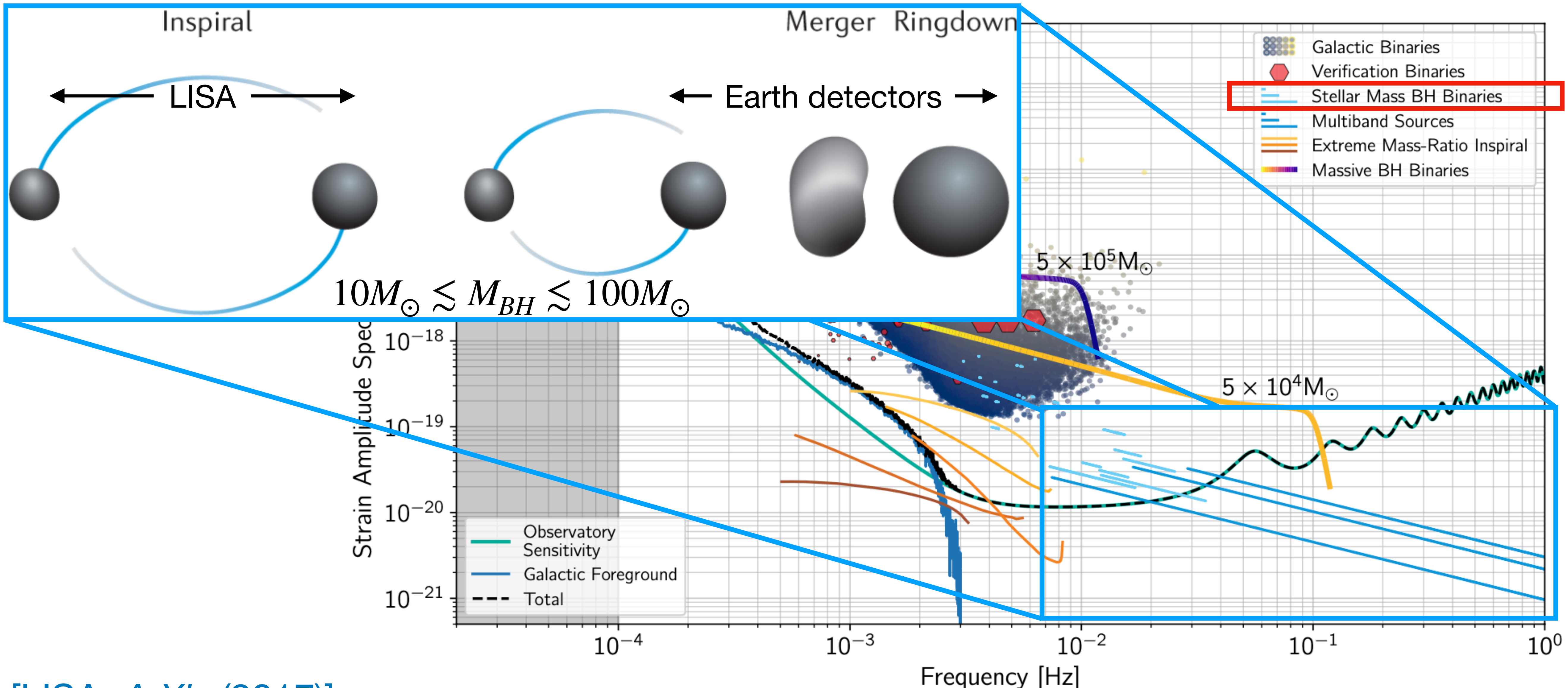
- ~~Stellar-mass BBHs ($10-100\,M_\odot$)~~
- Intermediate-mass BBHs? ($\gtrsim 100\,M_\odot$)
- Extreme mass ratio inspirals (EMRIs)
- Massive Black Hole Binaries* ($10^4-10^7\,M_\odot$)

[LISA, *ArXiv* (2017)]
[LISA, *ArXiv* (2024)]

*EM counterparts expected

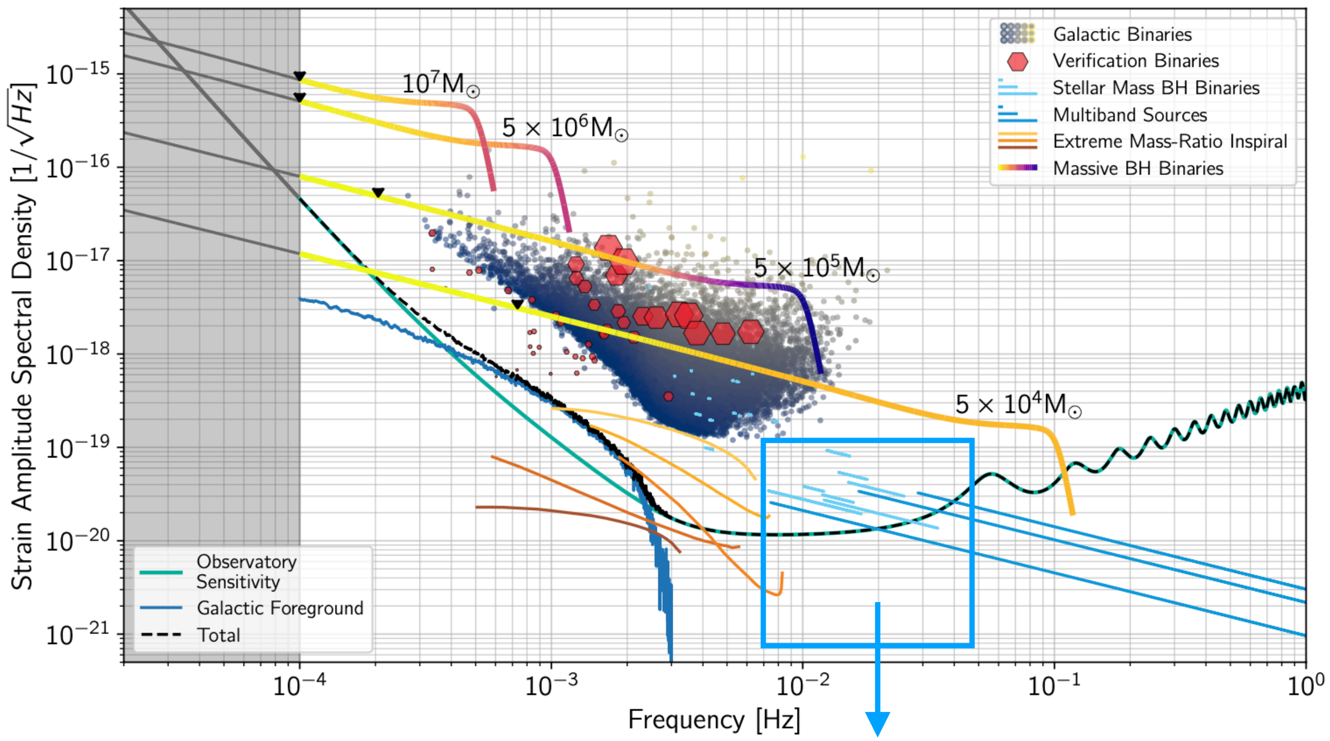
Future prospects

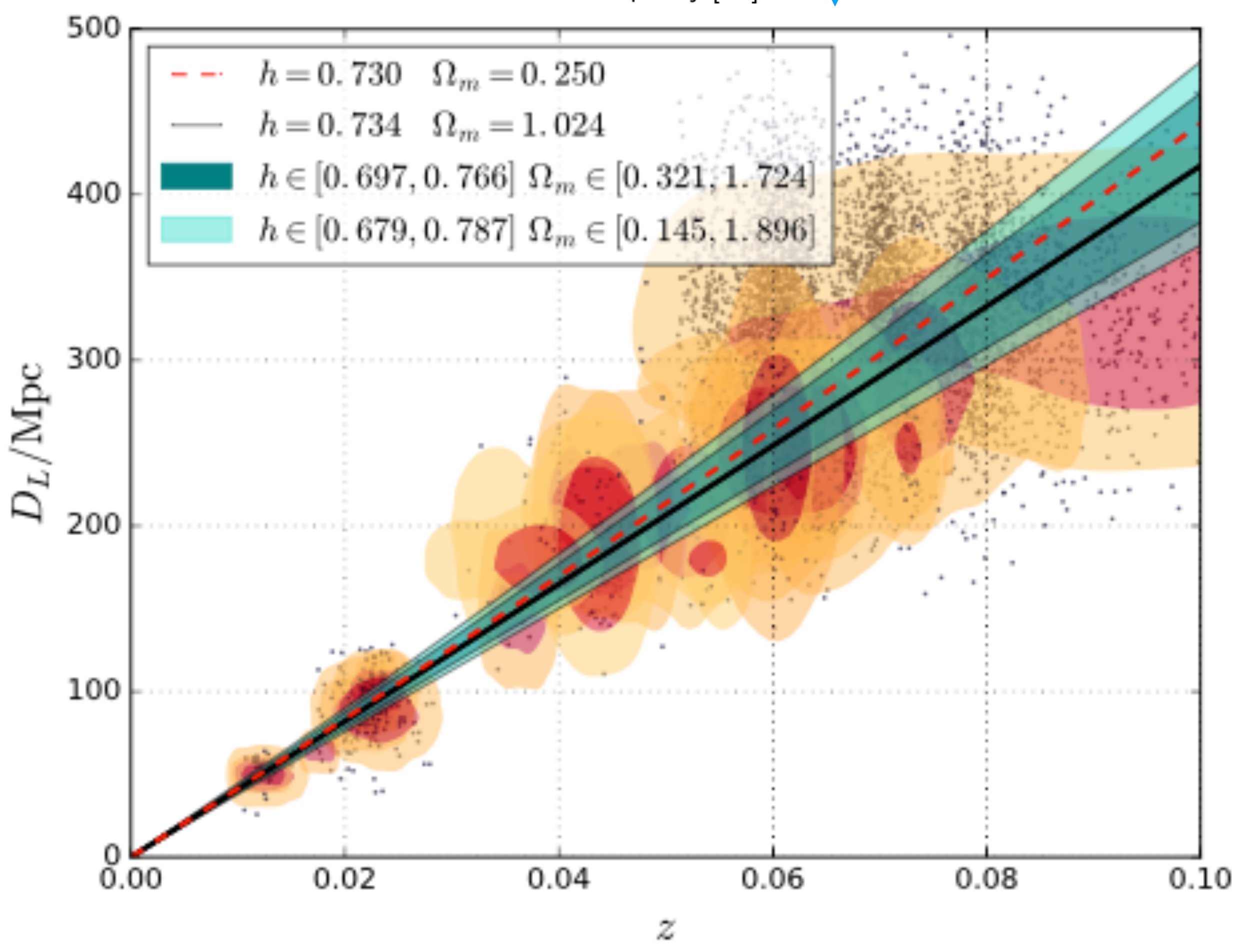
LISA can detect the inspiral of a few stellar mass BBHs up to $z \gtrsim 0.1$



[LISA, *ArXiv* (2017)]
[LISA, *ArXiv* (2024)]

Future prospects



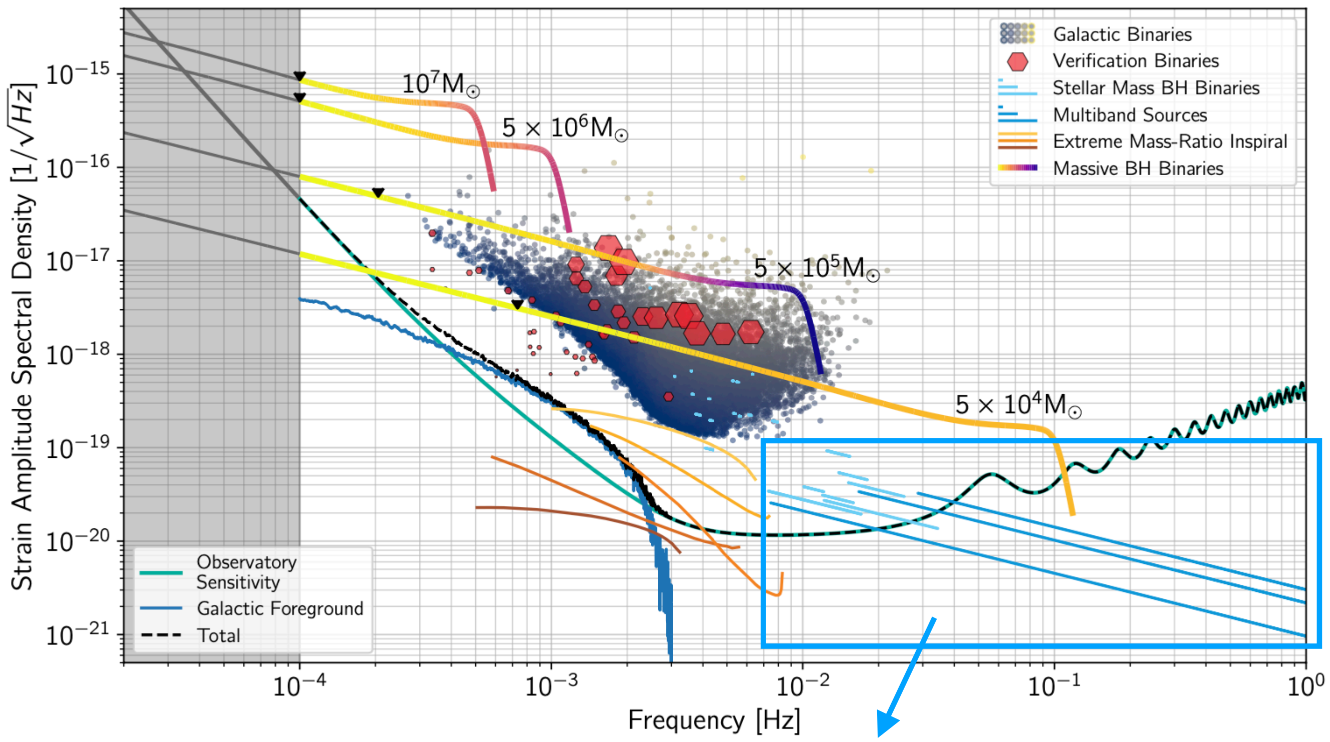


Stellar-mass BBHs

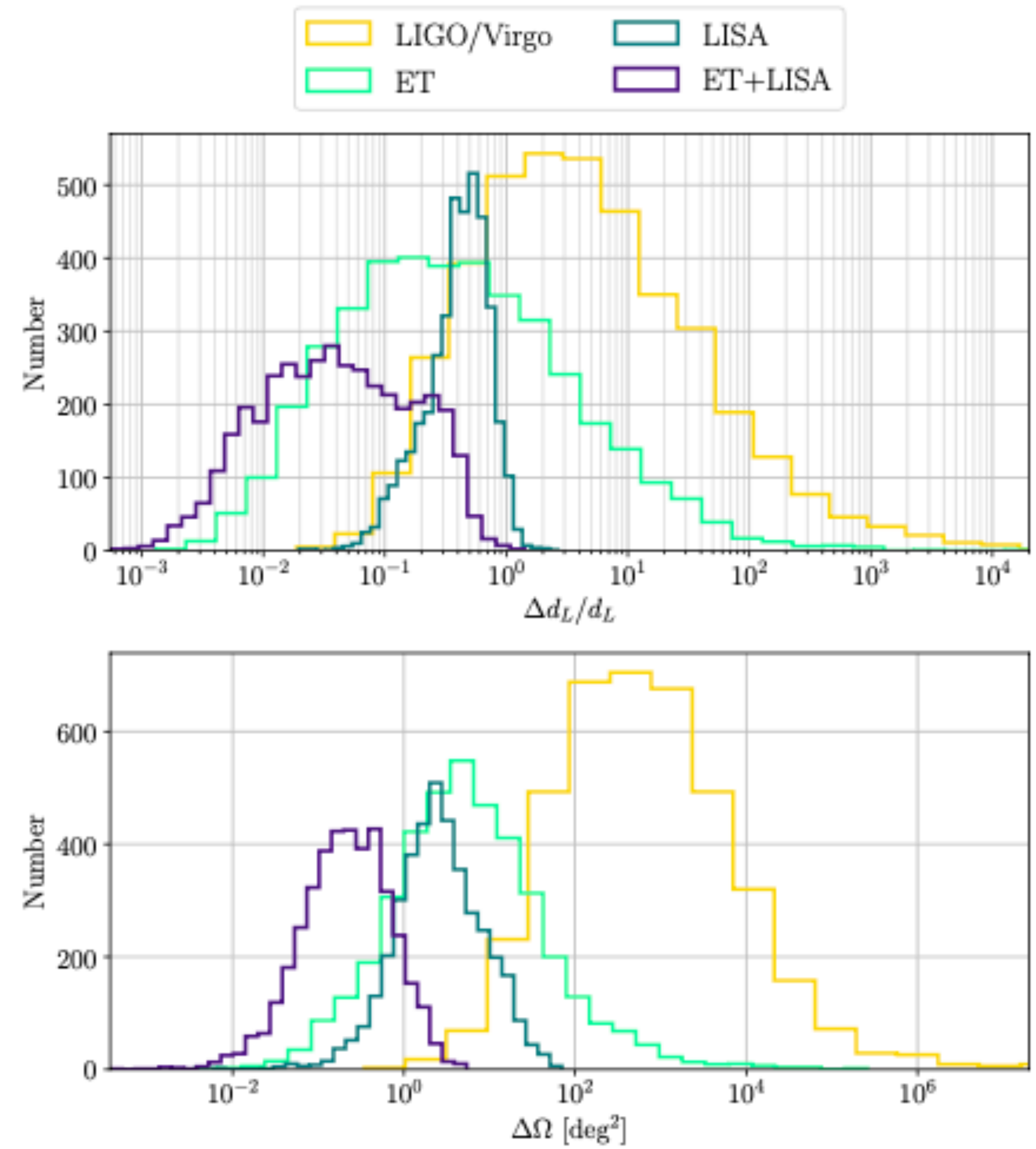
- Redshift range: $z \lesssim 0.1$
- No EM counterparts expected
- LISA detections: ~~~50/yr (optimistic)~~
  ~few/yr
- Useful as standard sirens:
  - If $\Delta d_L / d_L < 0.2$
  - If $\Delta\Omega \sim 1\ \mathrm{deg}^2$
  - $\Rightarrow$ ~~~ 5 standard sirens / yr~~
    ~0.1 standard sirens / yr
- **<u>Expected results:</u>**
  - ~~H_0 to few %~~
    H_0 not measured

[Kyutoku & Seto, *PRD* (2017)]
[Del Pozzo+, *MNRAS* (2018)]

Future prospects



Multi-band IMBHs ?



Intermediate mass black holes (IMBHs) can be used in **multi-band analyses** since their merger can be observed by ground-based detectors and their inspiral by LISA

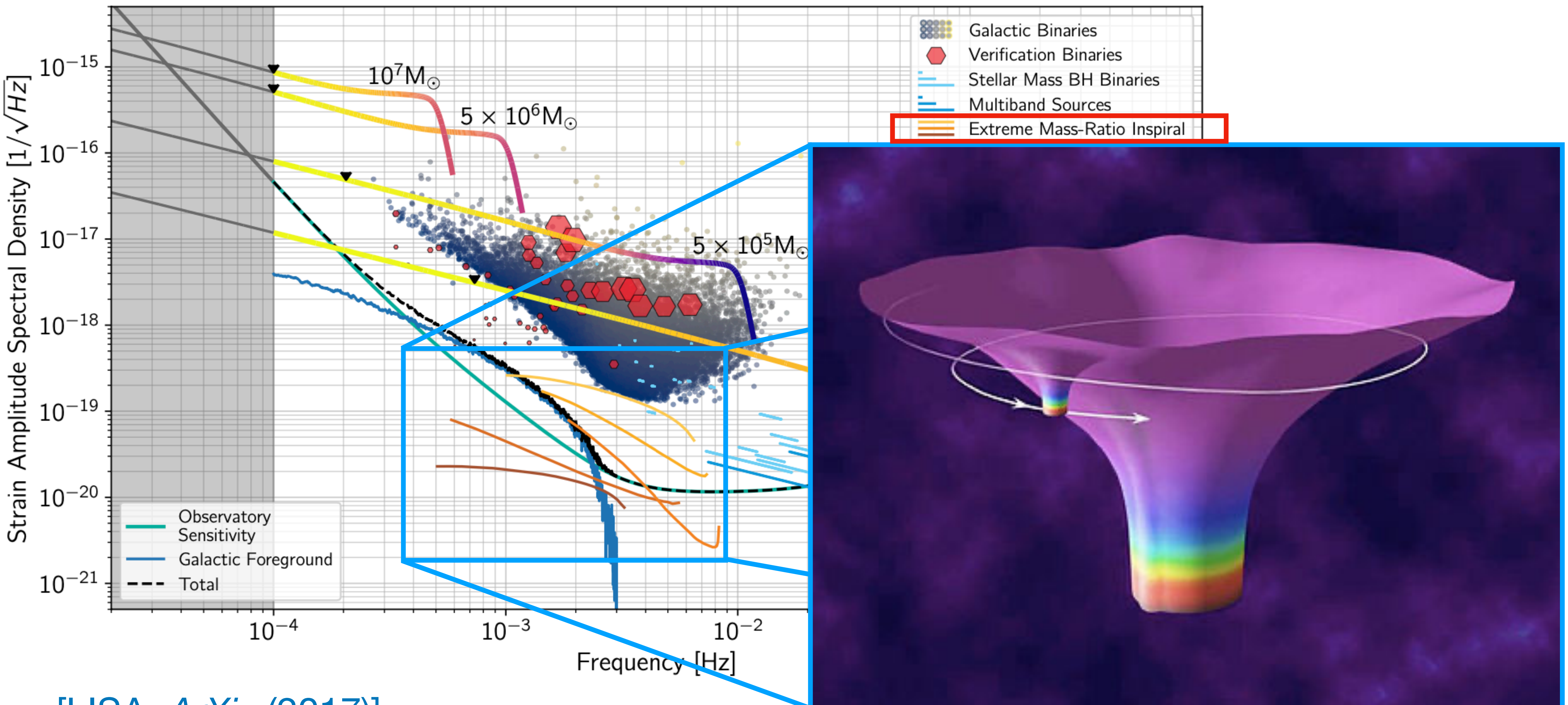
Expected results:

- H_0 to few % with $\mathcal{O}(10)$ IMBHs (rates yet unknown)

[Muttoni+, *PRD* (2022)]

Future prospects

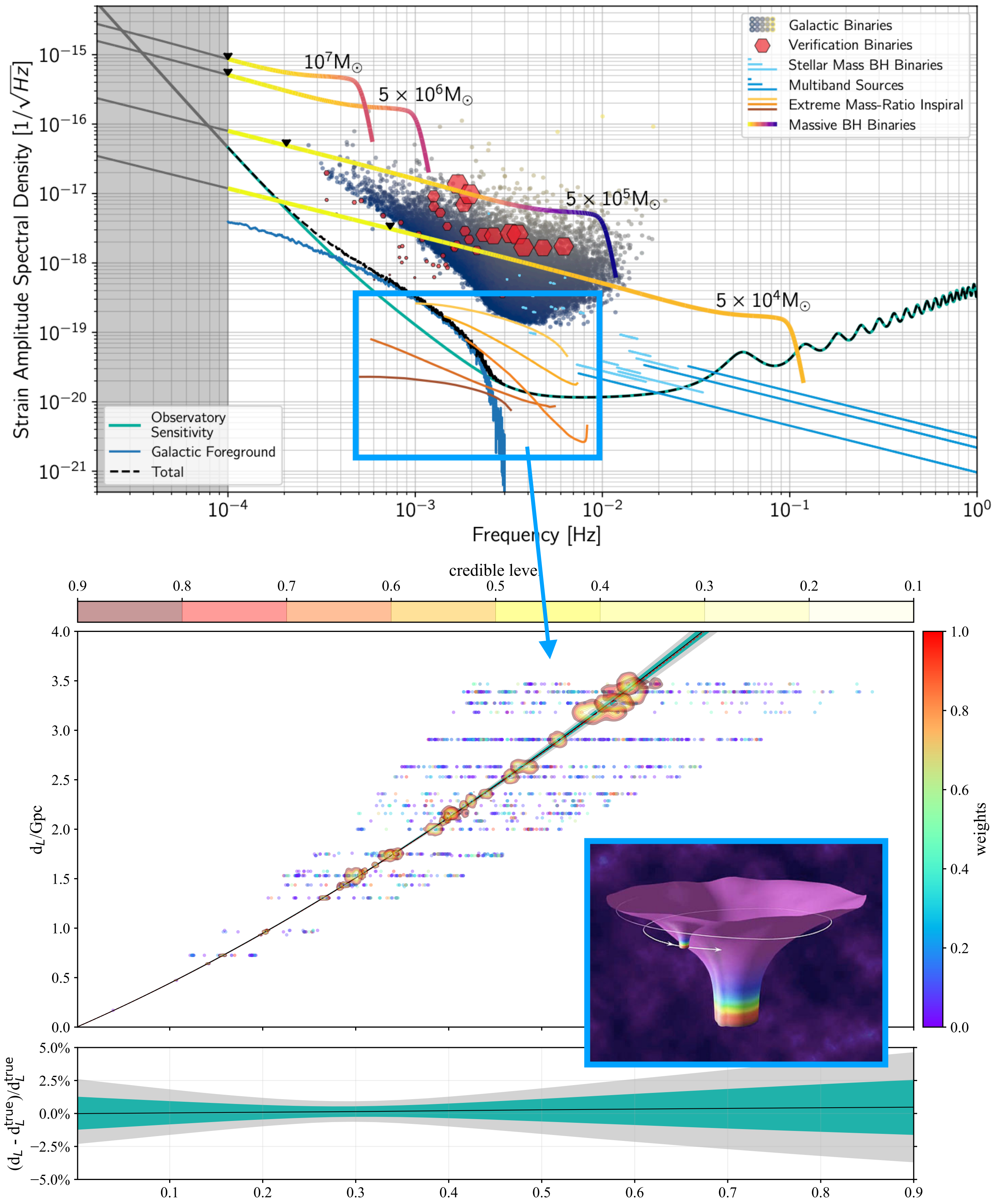
LISA can detect up to thousands of extreme mass ratio inspiral (EMRI) events up to $z \gtrsim 4$



[LISA, *ArXiv* (2017)]
[LISA, *ArXiv* (2024)]

[Babak+, PRD (2017), arXiv:1703.09722]

Future prospects



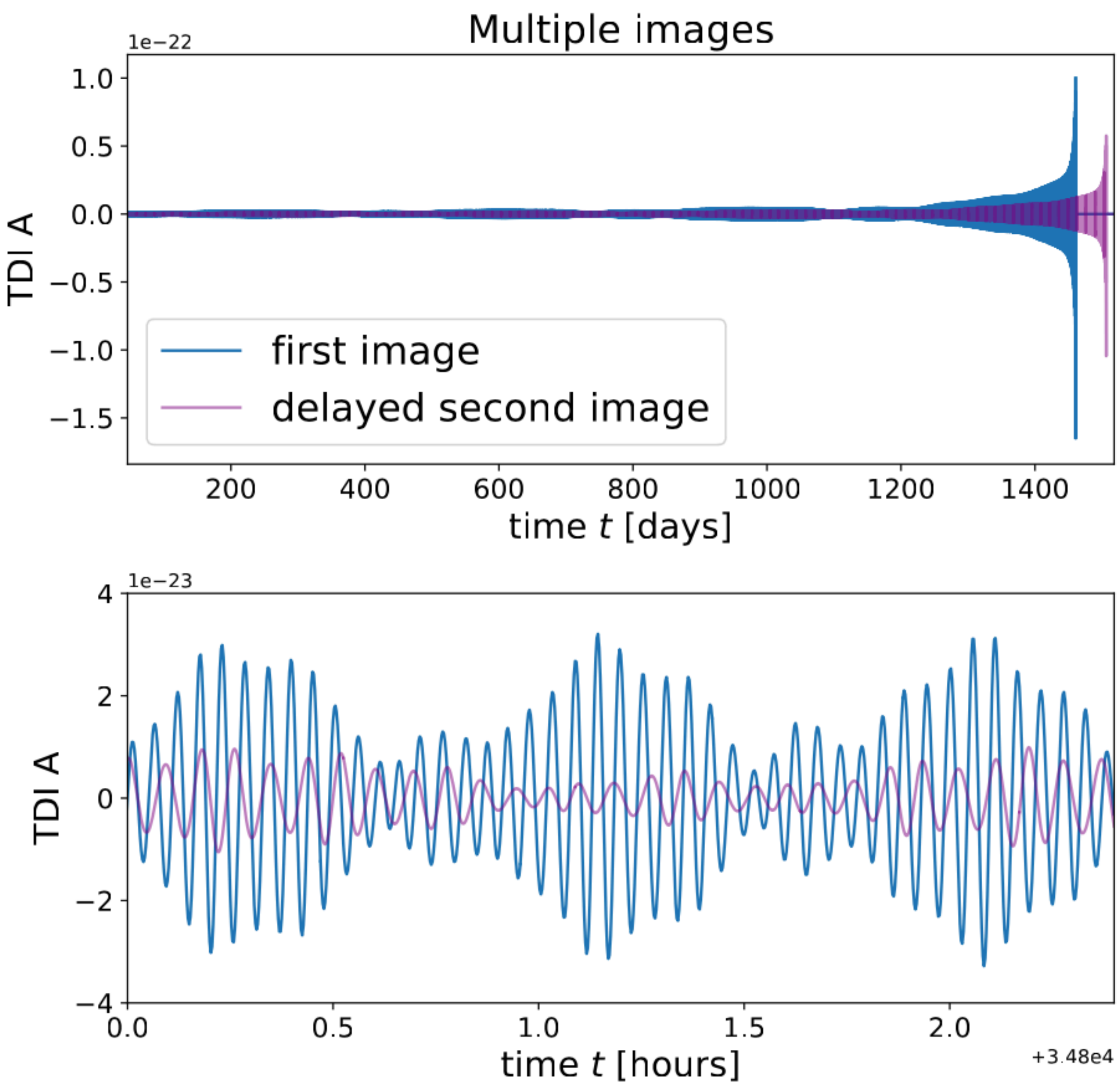
EMRIs

- Redshift range: $0.1 \lesssim z \lesssim 4$
- No EM counterparts expected
- LISA detections: from 1 to 1000/yr
- Useful as standard sirens:
  - $0.1 \lesssim z \lesssim 1$
  - If $\Delta d_L / d_L < 0.1$
  - If $\Delta\Omega < 2\ \mathrm{deg}^2$
  - $\Rightarrow$ ~ 1 to 100 standard sirens / yr
- **Expected results:**
  - H_0 between 1 and 10 %
  - w_0 between 5 and 10 %

[MacLeod & Hogan, *PRD* (2008)]
[Laghi+, *MNRAS* (2021)]

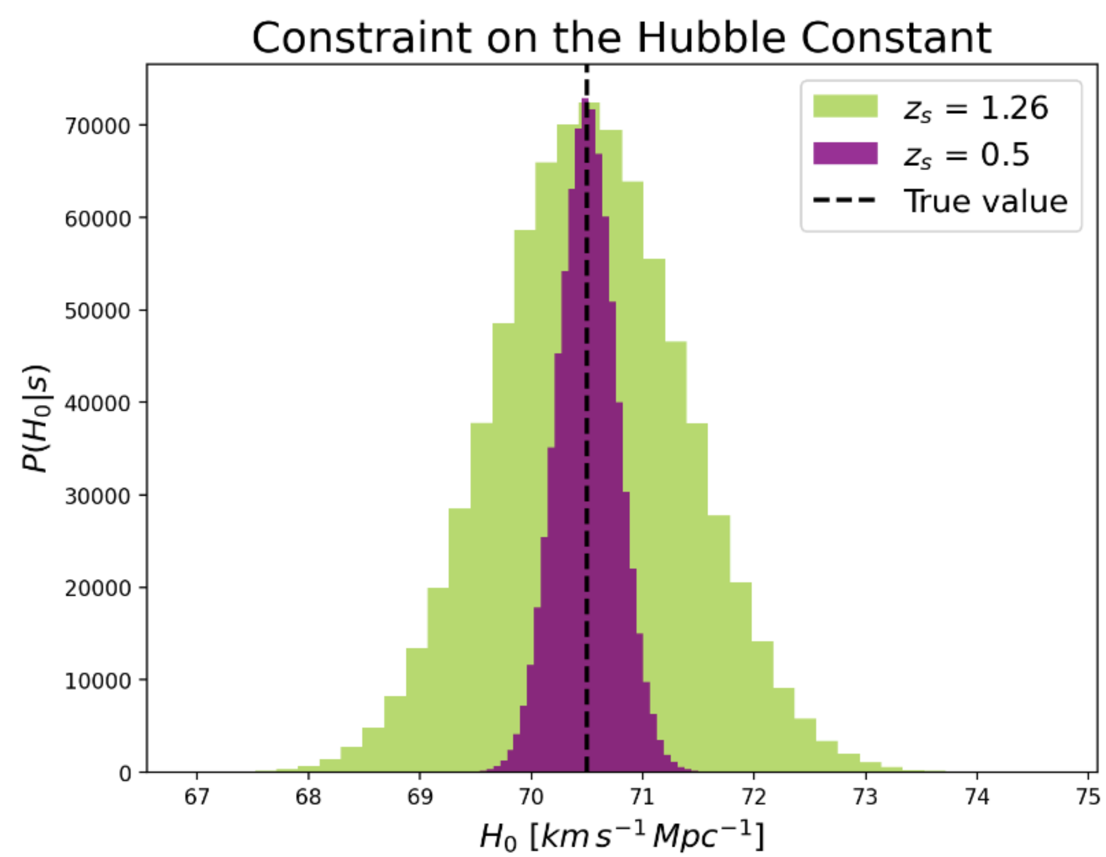
Future prospects

Strongly lensed EMRIs



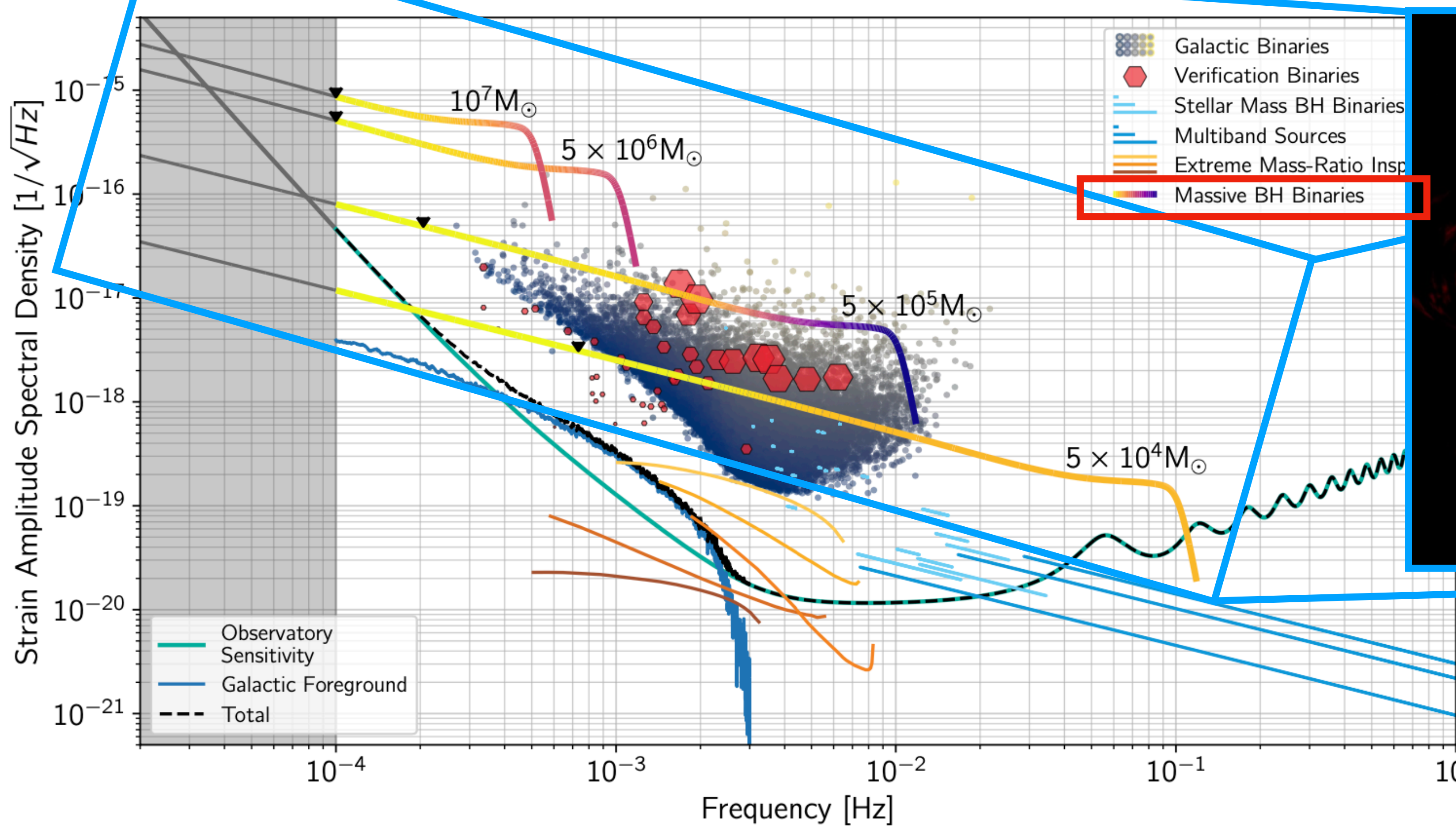
[Toscani+, *PRD* (2024)]

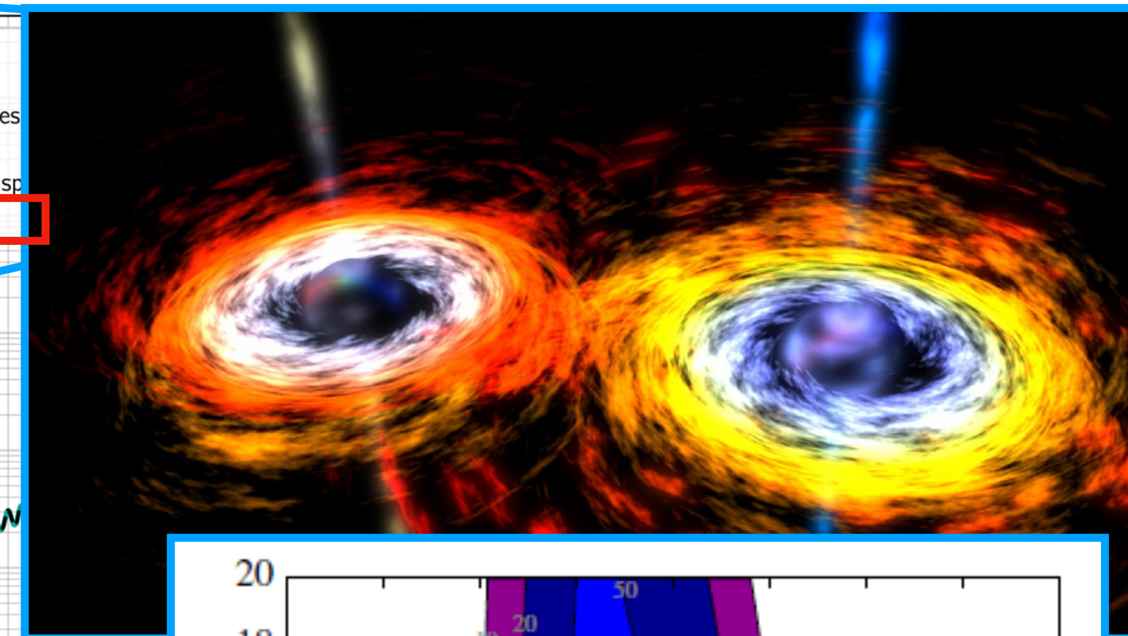
- Redshift range source: $0.5 \lesssim z \lesssim 2$
- <u>Lensed host galaxy may be identified</u>
- LISA detections: 0 to 10/yr
- **<u>Expected results with one LEMRI (with host galaxy identified):</u>**
  - H_0 at 1% or better (assuming Ω_m)

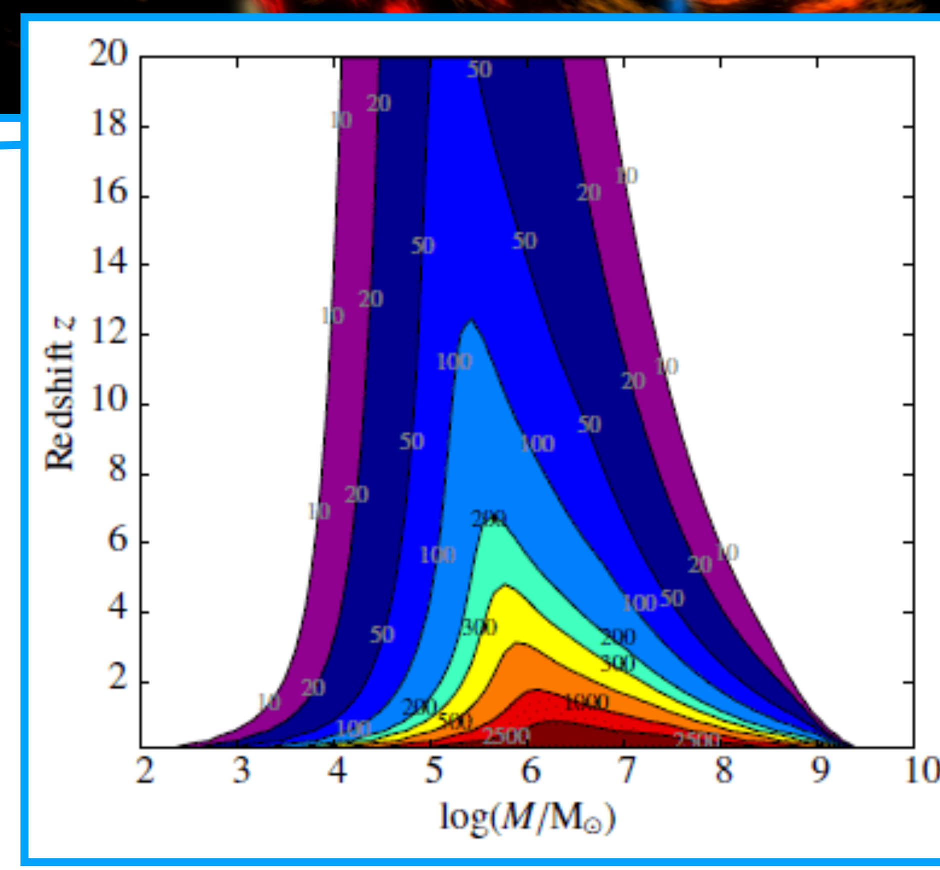


Future prospects

LISA can detect up to hundreds of massive black hole binary mergers up to $z \sim 20$



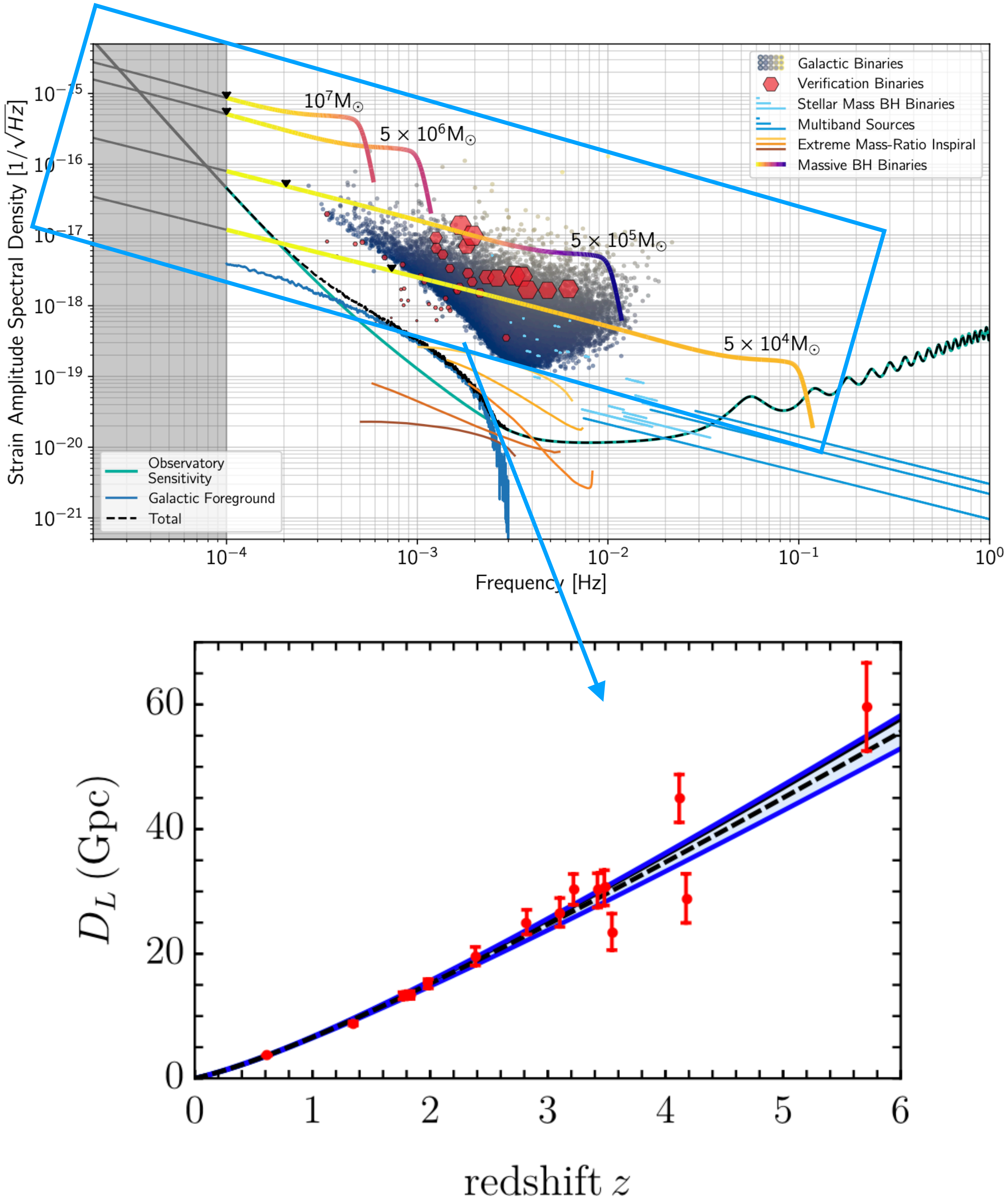




[LISA, *ArXiv* (2017)]
[LISA, *ArXiv* (2024)]

[Klein+, PRD (2016)]

Future prospects



[Speri+, *PRD* (2021)]

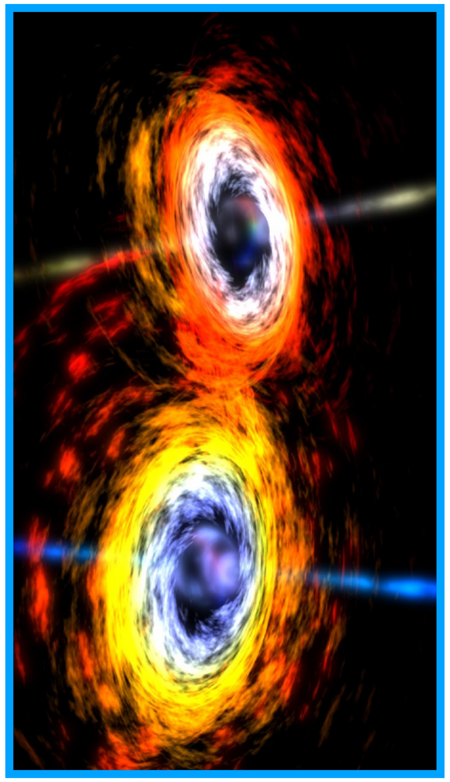
MBHBs

- Redshift range: $z \lesssim 20$
- <u>EM counterparts expected</u>
- LISA detections: 1 to 100/yr
- Useful as standard sirens:
  - $z \lesssim 7$
  - If $\Delta d_L / d_L \lesssim 0.1$ (include lensing)
  - If $\Delta\Omega < 10\ \mathrm{deg}^2$
  - $\Rightarrow$ 1 to 5 standard sirens / yr (with EM counterpart)
- <u>**Expected results:**</u>
  - H_0 to few %
  - “Precise” high-z cosmography

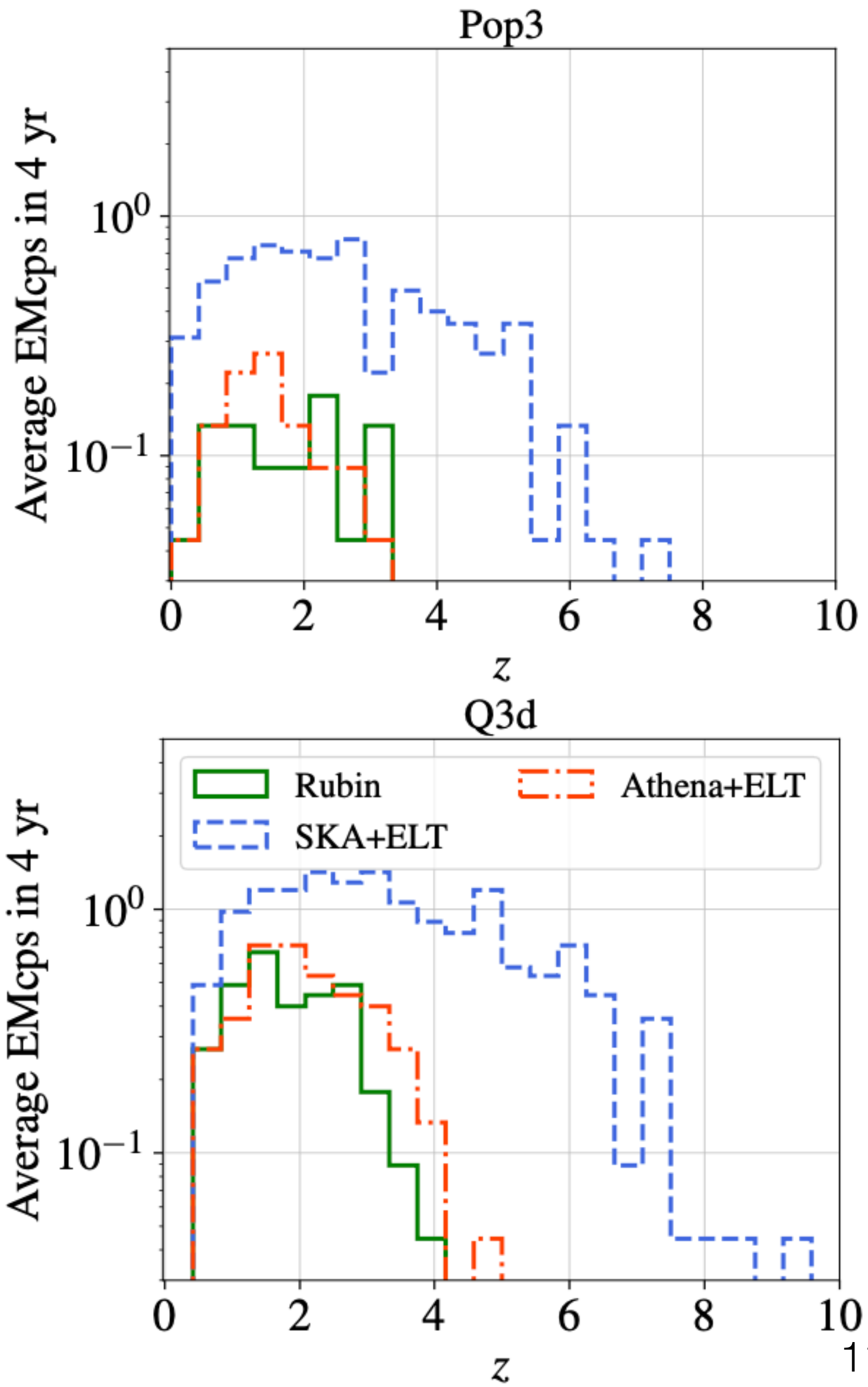
[Tamanini+, *JCAP* (2016)]
[Mangiagli+, *PRD* (2025)]

Future prospects

(In 4 yr)	Standard	w Obsc./Colli. radio
Light	6.4	1.6
Heavy	14.8	3.3
Heavy-no-delays	20.7	3.5



[Mangiagli+, *PRD* (2022)]



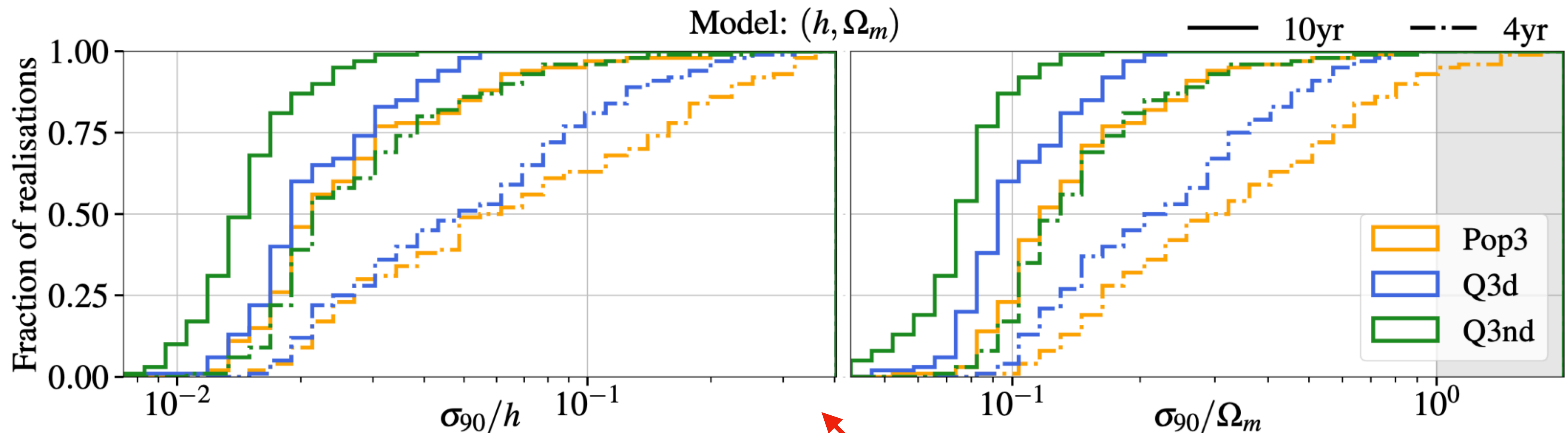
MBHBs

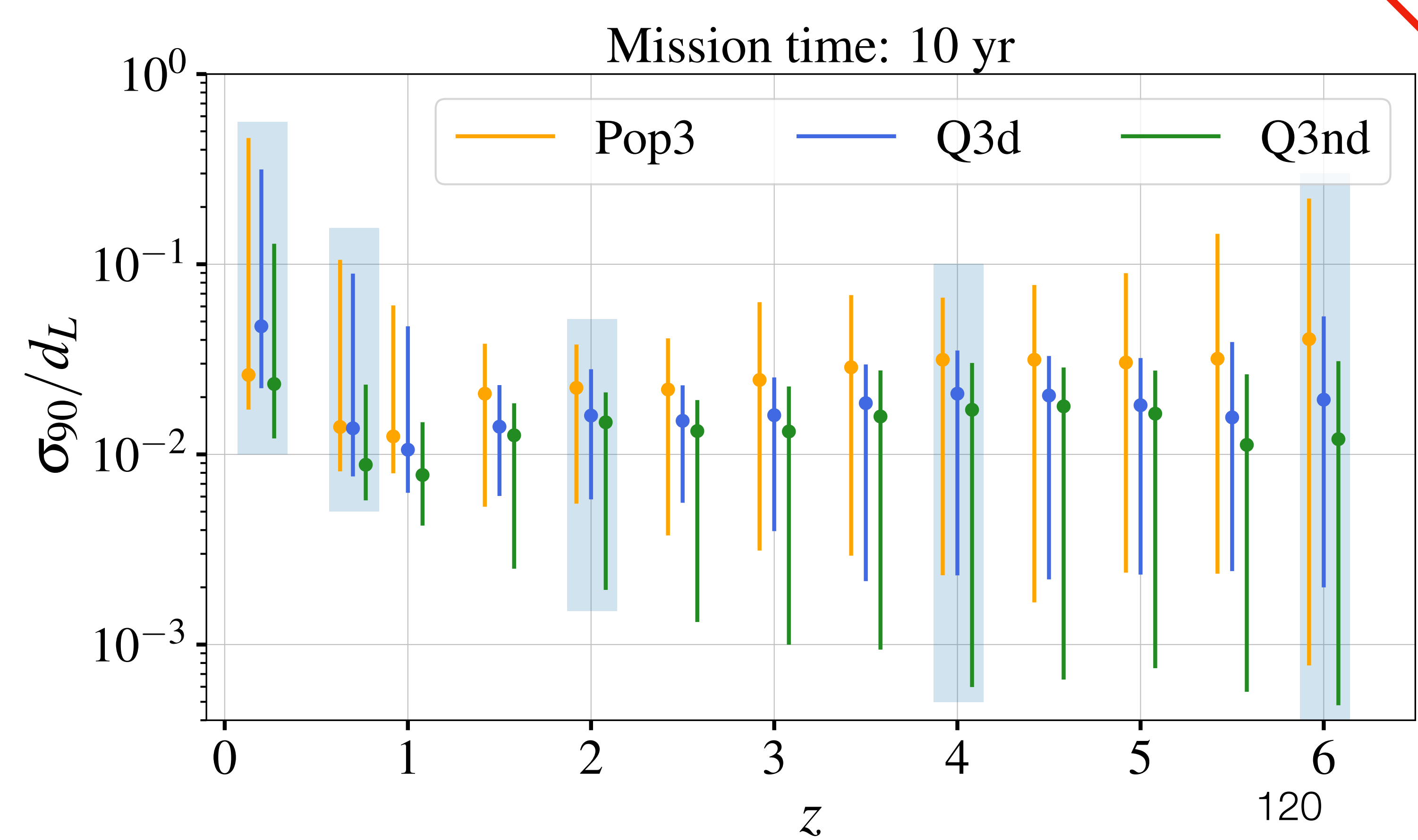
- Redshift range: $z \lesssim 20$
- EM counterparts expected
- LISA detections: 1 to 100/yr
- Useful as standard sirens:
  - $z \lesssim 7$
  - If $\Delta d_L / d_L \lesssim 0.1$ (include lensing)
  - If $\Delta\Omega < 10\ \mathrm{deg}^2$
  - $\Rightarrow$ 1 to 5 standard sirens / yr (with EM counterpart)
- **Expected results:**
  - H_0 to few %
  - "Precise" high-z cosmography

[Tamanini+, *JCAP* (2016)]
[Mangiagli+, *PRD* (2025)]

Future prospects

MBHBs





Expected results:

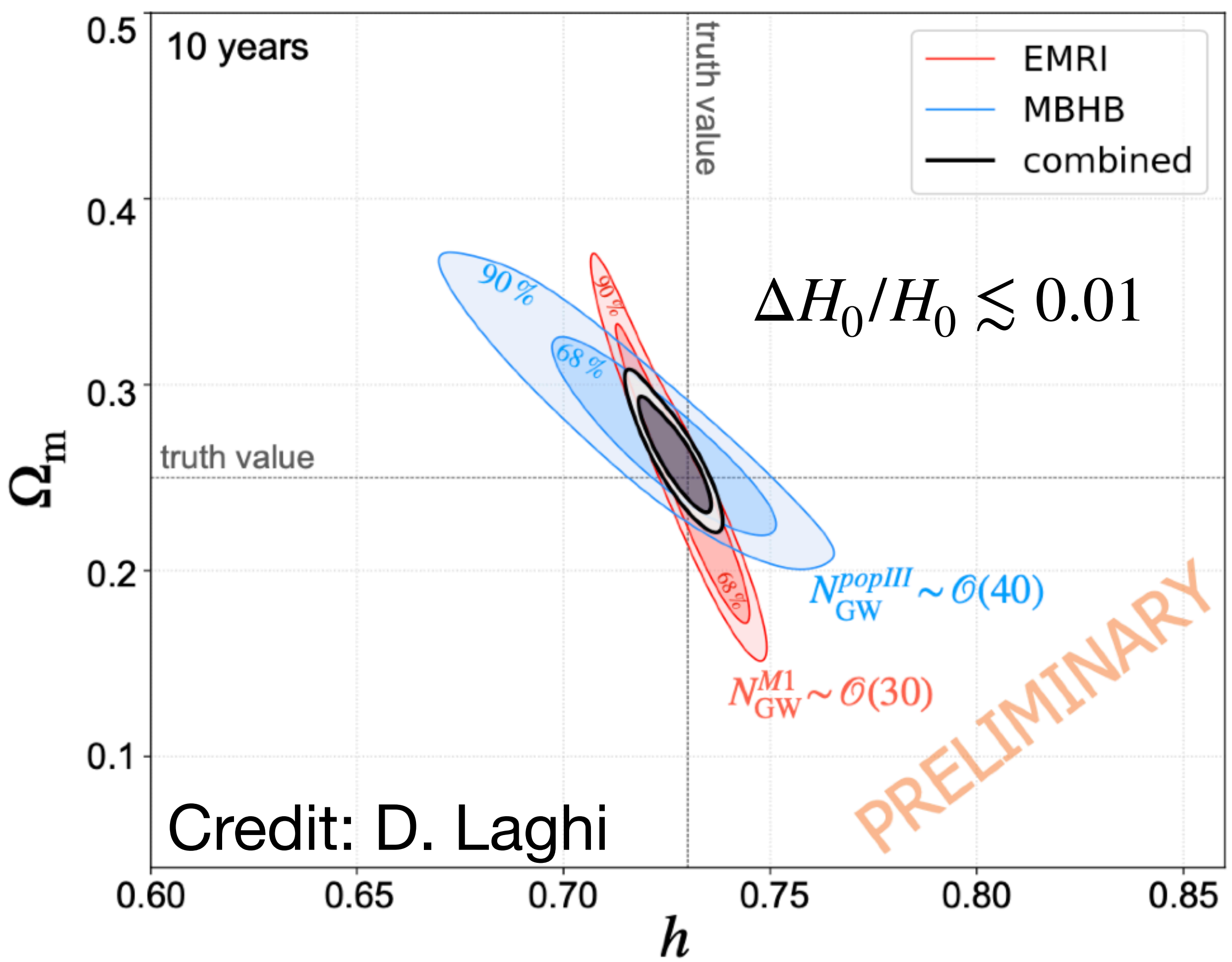
- H_0 to few %
- "Precise" high-z cosmography
    - $d_L(z)$ to few % @ $z \gtrsim 2$

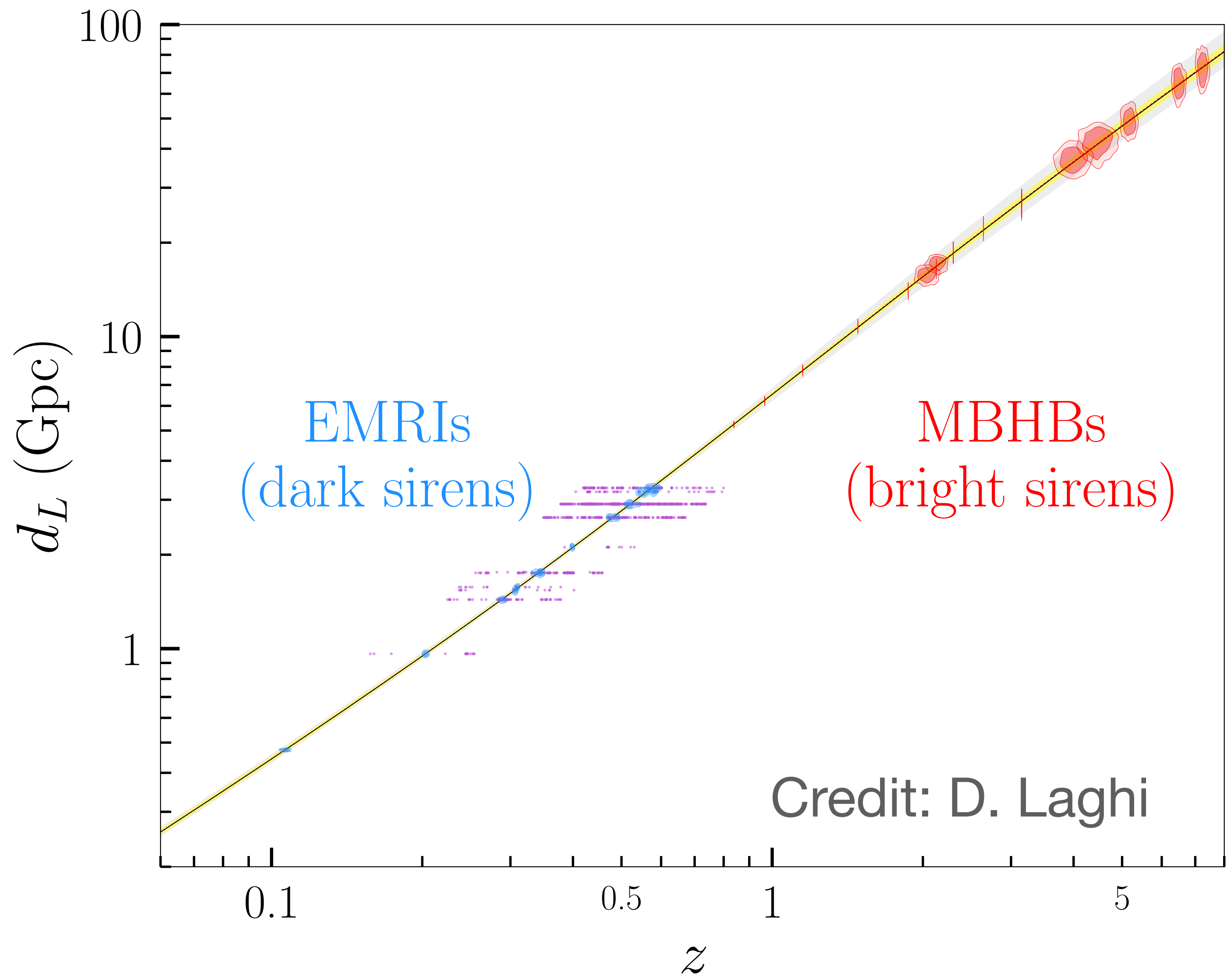
[Mangiagli+, *PRD* (2025)]

Future prospects

The combination of different standard sirens will allow LISA to measure the expansion of the universe from $z \sim 0.01$ to $z \sim 10$

Expected results for ΛCDM:

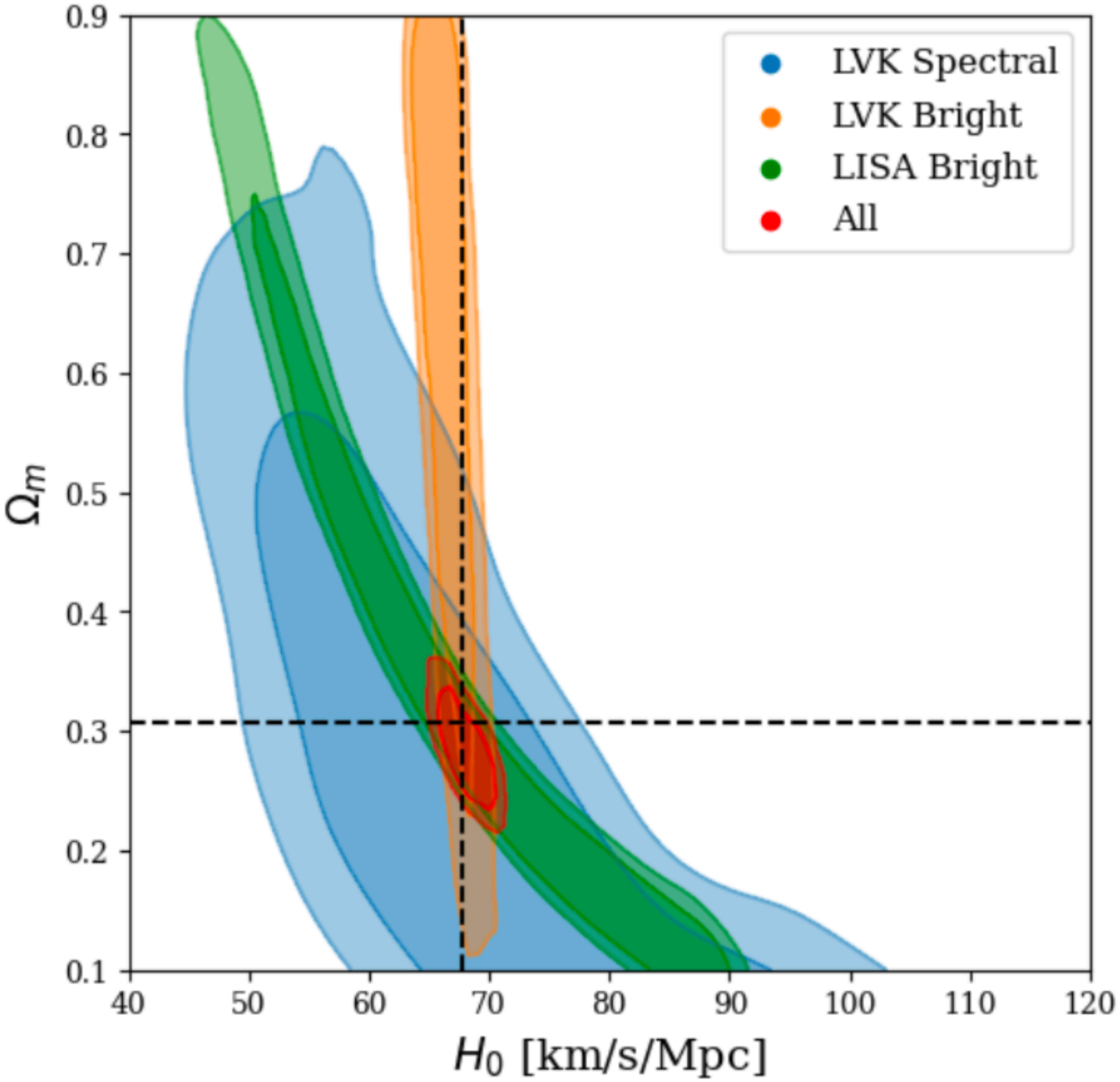




[Tamanini, *J. Phys. Conf. Ser.* (2017)]
[Laghi, Tamanini+, *in prep.*]

Future prospects

The combination of different standard sirens will allow LISA to measure the expansion of the universe from $z \sim 0.01$ to $z \sim 10$



Similarly, combining LISA high-z MBHBs with LVK low-z sirend will break degeneracies in ΛCDM

[Salvarese+, *arXiv* (2025)]

Future prospects

LISA Source	Redshift Range	Detection Rates	Redshift Measured (Bright Sirens)	Well Localised (Dark Sirens)	$\frac{\Delta H_0}{H_0}$	More
SOBHBs	$\lesssim 0.1$	$\lesssim 1/\mathrm{yr}$	None	$\lesssim 0.1/\mathrm{yr}$	None	
IMBHs?	$\lesssim 0.1$	$\lesssim 10/\mathrm{yr}\,(?)$	None	$\lesssim 2/\mathrm{yr}\,(?)$	~2%	Multiband
EMRIs	$\lesssim 4$	$\lesssim 1000/\mathrm{yr}$	None	$\lesssim 100/\mathrm{yr}$ @ $z \lesssim 1$	1-10%	$\Delta w_0 \lesssim 0.1$
LEMRIs	$\lesssim 4$	$\lesssim 10/\mathrm{yr}$	$\lesssim 1/\mathrm{yr}$ @ $z \lesssim 2$	$\lesssim 10/\mathrm{yr}\,(?)$ @ $z \lesssim 1$	~1%	
MBHBs	$\lesssim 20$	$\lesssim 100/\mathrm{yr}$	$\lesssim 3/\mathrm{yr}$ @ $z \lesssim 7$	$\lesssim 10/\mathrm{yr}\,(?)$ @ $z \lesssim 2$	2-10%	High-z Analyses
LMBHBs	$\lesssim 20$	$\lesssim 1/\mathrm{yr}$	$\lesssim 0.1/\mathrm{yr}\,(?)$ @ $z \lesssim 2$	$\lesssim 0.1/\mathrm{yr}\,(?)$ @ $z \lesssim 2$	~10%	High-z Analyses
Combined			$\lesssim 3/\mathrm{yr}$	$\lesssim 100/\mathrm{yr}$	$\lesssim 1\,\%$	High-z and dark energy Analyses

Conclusions

- Standard sirens are excellent distance indicators:
  - They do not require calibration and are not affected by possible systematics in the cosmic distance ladder
  - They can be used with or without an EM counterpart
    - Bright and dark sirens
  - New cosmological tests complementary to EM observations

- Current observations with ground-based detectors:
  - First standard (bright) siren discovered: GW170817
    - First GW measurement of H_0
  - Dark sirens results provides significant improvement on top of GW170817

- Future prospects:
  - Future observations useful to solve tension on H_0
  - 3G detectors and LISA will bring precise GW cosmology and will test LCDM at high-redshift