Dynamical Cosmological Constant

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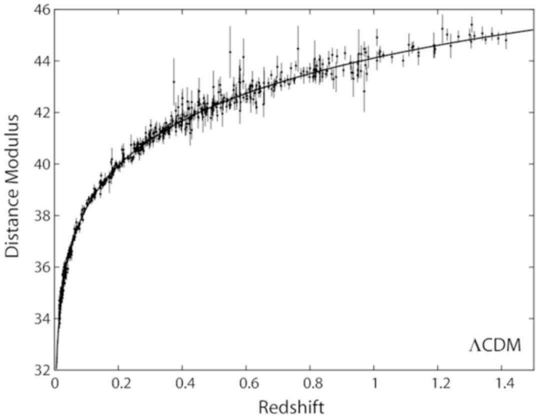
Universe components and expansion

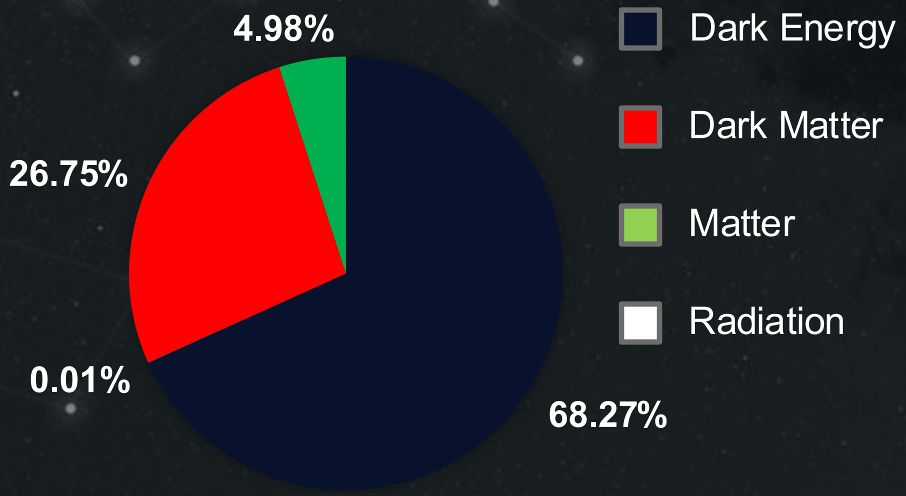
Components of the Universe as perfect fluid with equation of state: $p = w\,\rho$

Energy Momentum Tensor: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$

Supernovae Type IA database.
Luminosity distance vs redshift



$$d_L(z) = \frac{c(1+z)}{H_0}\int_0^z \left[\Omega_M(1+z')^3 + \Omega_\Lambda\right]^{-1/2} dz'$$



*Planck Surveys

Dark Energy and Cosmological constant

Accelerated Expansion of the Universe

In ΛCDM Dark Energy is a Cosmological Constant with equation of state $\boldsymbol{w = -1}$ driving the accelerated expansion. Conservation of the EMT (perfect fluid) implies a constant energy density $\boldsymbol{\rho = \Lambda}$. There are no degrees of freedom and everything is frozen.

Our Proposal

We build a model for a Dynamical Cosmological Constant (DCC) with equation of state $\boldsymbol{w = -1}$ as a dynamical medium with non-trivial fluctuations. We then study the effects of these perturbations on the evolution of the matter density contrast and on the propagation of gravitational waves.

Standard Approaches and Limitations

K-essence scalar Field Theory

Consider a Lagrangian $\mathbf{K}(X,\Phi)$, with a scalar field $\Phi = \phi(t)$, $X = -\frac{1}{2} g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi$ and an EMT of a perfect fluid. The scalar field is perturbed around a background value.

Pathological Dynamics for perturbations

The equation of state $w = -1$ imply a pathological dynamic for the perturbations of the scalar field since the perturbed EMT reads: $\delta\rho = \delta p = 0$.

Our approach

To have a non-trivial and stable dynamics for perturbations, additional degrees of freedom needed to describe the medium.

Dynamical Model: Basics

A generic perfect fluid has a Lagrangian of the form $\boldsymbol{U(b)}$ described by three scalar fields $\boldsymbol{\Phi^a}$. With $\boldsymbol{w=-1}$, no dynamics.

$$b = (\mathrm{Det}[B^{ab}])^{1/2}, \quad B^{ab} = g^{\mu\nu}\partial_\mu\Phi^a\partial_\nu\Phi^b, \quad a,b = 1,2,3.$$

Add a scalar field $\boldsymbol{\Phi^0}$ to have more operators that represent a coupled fluid/superfluid $\boldsymbol{U(b,y,\chi)}$ but still with $\boldsymbol{w=-1}$ no dynamics.

$$\chi = (-g^{\mu\nu}\partial_\mu\Phi^0\partial_\nu\Phi^0)^{1/2}. \quad y = u^\mu\partial_\mu\Phi^0 \quad v_\mu = \chi^{-1}\partial_\mu\Phi^0.$$

Healty dynamics only by adding a solid component through the operators $\boldsymbol{\tau_Y, \tau_Z}$. $\boldsymbol{B}$ is the matrix $\boldsymbol{B^{ab}}$

$$\tau_1 = \mathrm{Tr}(\boldsymbol{B}), \qquad \tau_Y = \frac{\mathrm{Tr}(\boldsymbol{B}^2)}{\tau_1^2}, \qquad \tau_Z = \frac{\mathrm{Tr}(\boldsymbol{B}^3)}{\tau_1^3};$$

Lagrangian	Medium type	d.o.f.	Properties
$U(\chi)$	Superfluid	1	Zero kinetic term
$U(b)$	Perfect barotropic fluid	$2+1$	Zero kinetic term
$U(b,y)$	Perfect fluid	$2+1$	Zero kinetic term
$U(b,y,\chi)$	fluid/superfluid	$2+2$	Zero kinetic term
$U(b,\tau_Y,\tau_Z)$	Solid	$2+1$	Zero kinetic term
$U(b,y,\chi,\tau_Y,\tau_Z)$	Supersolid	$2+2$	Healthy dynamics

Summary of all type of medium one can build starting from four scalar fields.

Dynamical Model: Basics

- Fluctuations introduced as 4 scalar fields

$$\Phi_0 = t + \pi_0 \, , \Phi_a = x_a + \pi_a \quad , a = 1, 2, 3$$

- Perturbations of Φ^a decomposed in **two transverse vector modes** $\pi_T^{1,2}$ and a **longitudinal scalar mode:** π_l

- Fields minimally coupled with gravity. U Lagrangian

$$S = \int d^4x\sqrt{-g}U(\partial\Phi_0 \, , \partial\Phi_a)$$

- Modified Energy Momentum tensor with anisotropic terms and additional velocity.

$$T_{\mu\nu} = (U - bU_b)g_{\mu\nu} + \left(yU_y - bU_b\right)u_\mu u_\nu + \chi U_\chi v_\mu v_\nu + Q_{\mu\nu}^{(y)} U_{\tau y} + Q_{\mu\nu}^{(z)} U_{\tau z}$$

- Mass parameters defined from derivatives of U.

$$M_0, M_1, M_2, M_3, M_4$$

Dynamical Model: Stability Conditions

Two regions of stability: standard and anomalous. Stability associated with anisotropic term. In the standard region is impossible to have $\boldsymbol{w = -1}$

$$H = \frac{\omega_1}{2}\left(\Pi_{c1}^2 + \varphi_{c1}^2\right) - \frac{\omega_2}{2}\left(\Pi_{c2}^2 + \varphi_{c2}^2\right)$$

Anomalous stability condition with $\mathrm{w} = -1$.
Energy not positive definite. Hamiltonian composed of two harmonic oscillators but with opposite sign.

$$\lim_{M_2 \to 0} \omega_{1,2}^2 = -k^2$$

Catastrophic effect? By studying the evolution of DCC fluctuations and their effects on the propagation of Gravitational waves and on structure formation this doesn't seem to be true.

Cosmological Perturbations

- We studied the dynamic of perturbations of this DCC, analyzing also the effects on the evolution of the matter density contrast and on propagation of gravitational waves.

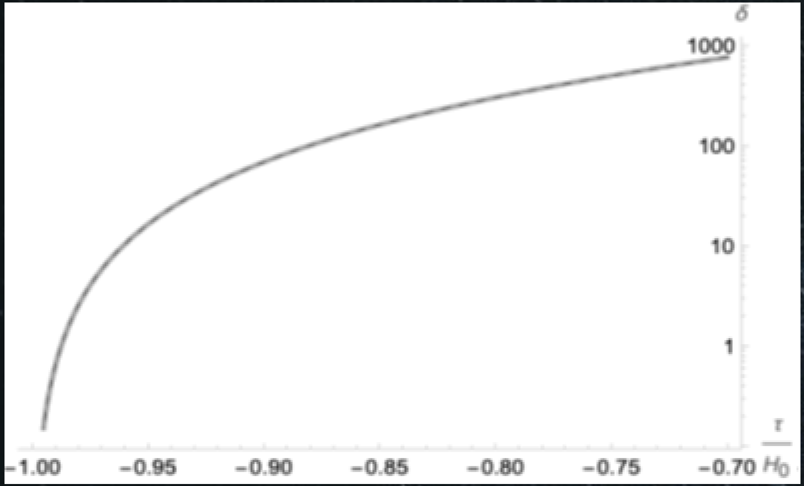
- We got the evolution equations for DCC perturbations π_0, π_l and for the matter density contrast δ_m from Einstein equations starting from a perturbed FRW metric.

- We worked in conformal time τ, using a DeSitter scale factor $a(\tau) = -\frac{1}{\tau H_0}$ where H_0 is the today Hubble constant . We also used Newtonian Gauge.

- Due to the coupling of the scalar fields, it was possible to solve analytically the equations only in certain limits such as the **Large Scale** $\frac{k}{H_o} \ll 1$ or **Small scale** $\frac{k}{H_0} \gg 1$

Result: Matter Density Contrast

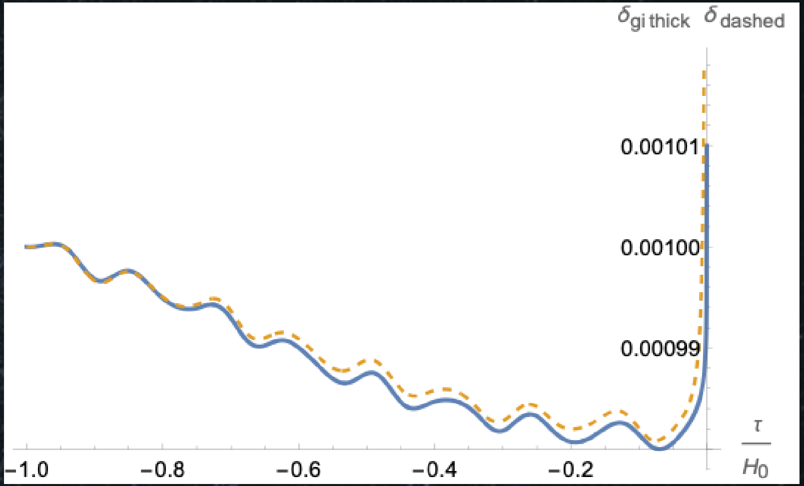
In ΛCDM $\boldsymbol{\delta}$ remain constant at all scales



Matter density contrast at $\frac{k}{H_0} = \mathbf{10^{-3}}$

$\boldsymbol{\delta}$ grows as a power law at large scales.

$\delta_{gi} = \delta - 3\,\mathcal{H}\,v_m$

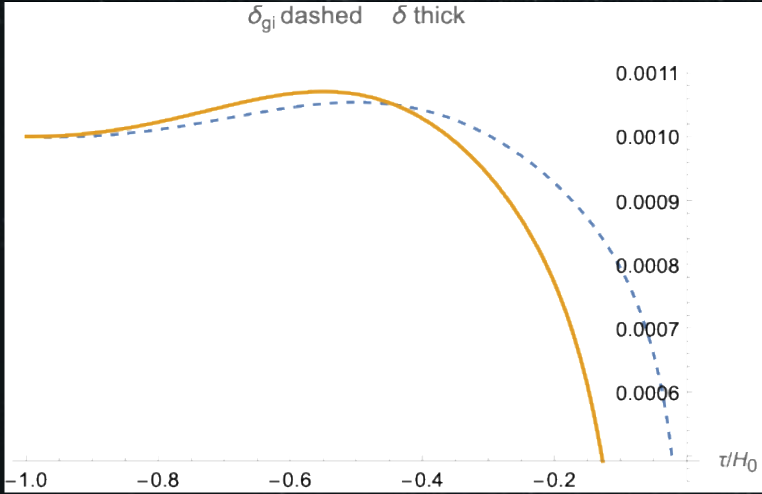


Matter density contrast at $\frac{k}{H_0} = 100$

Oscillations induced by DCC at small scales.

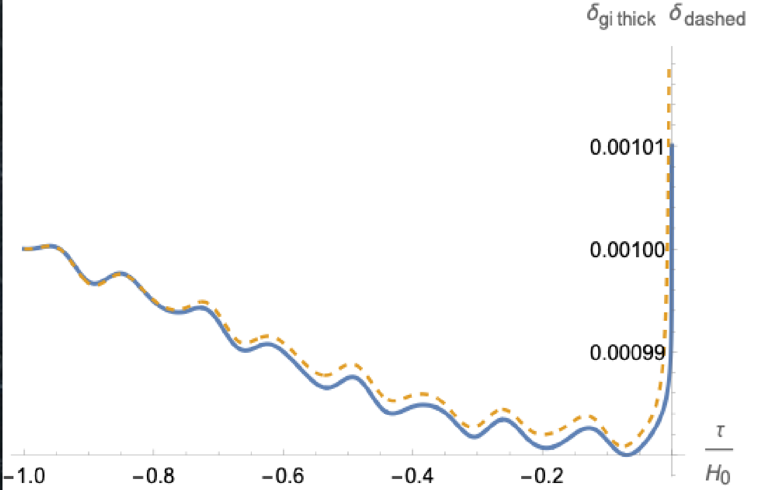
Result: Matter Density Contrast

δ intermediate case. Incomplete oscillations



Matter density contrast at $\frac{k}{H_0} = 10$

δ Small scale case. Growth when $\tau \to 0$ due to the scale factor: $a(\tau) = -\frac{1}{\tau H_0}$



Matter density contrast at $\frac{k}{H_0} = 100$

Result: Gravitational Waves

- The equation of motion for Gravitational Waves in this DCC dominated Universe is:

$$\chi''_{ij} + 2\mathcal{H}\chi'_{ij} + \left(k^2 + a^2 M_2\right)\chi_{ij} = 0$$

- The solution in De Sitter is combination of Bessel functions J_{ν_T} and Y_{ν_T} :

$$\chi_{ij}(\tau) = a^{-\frac{3}{2}}\epsilon_{ij}\left[\chi_0 J_{\nu_T}(-k\tau) + \chi_1 Y_{\nu_T}(-k\tau)\right]$$

$$\nu_T = (9 - 4\frac{M_2}{\mathcal{H}_0})^{\frac{1}{2}}$$

- At **small scales** oscillatory behavior with amplitude $\propto\ a^{-1}$ as in the cosmological constant case.

- At **large scales**, with $M_2 > 0$, the amplitude grows logarithmically

$$\chi_{ij} \sim \epsilon_{ij} a^{-\frac{3}{2}+\frac{\nu_T}{2}}$$

Thanks!

Do you have any questions?

Giuseppe Di Donato
Les Houches 10/07/2025

Extra Slides

The Dynamical Cosmological Constant model has the following characteristic:

- The medium has an **equation of state** with $w = -1$.
- **Four scalar fields** to have **non-trivial dynamics for its perturbations**.
- The presence of **anisotropic stress** is **crucial for stability** of the system.
- **Stability** of the model with $w = -1$ is reached in an **anomalous region** where the **Hamiltonian is non positive definite**

Implications

1. The **matter density contrast grows at large scales.** Instead shows **induced oscillations at small scales.**

2. **Amplitude** of gravitational waves grows logarithmically in the **large scales** limit. **Standard evolution** in the **small scales** limit.

Extra Slides

- Lagrangian of a gyroscopic system

$$\mathcal{L}^{(2)} = \frac{M_{pl}^2}{2}[\dot{\varphi}^t \mathcal{K} \dot{\varphi} + 2\varphi^t \mathcal{D} \dot{\varphi} - \varphi^t \mathcal{M} \varphi], \quad \varphi^t = (k^2 \pi_l, k\pi_0);$$

- In canonical form Kinetic matrix diagonal, the other two matrices linked to three parameters

$$\mathcal{D} \to D = \begin{pmatrix} 0 & d \\ -d & 0 \end{pmatrix}, \quad \mathcal{M} \to M = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$

- We found solution of this kind: $\varphi_{1,2} \propto e^{-i\,\omega_{1,2}\,t}\,\theta_{1,2}$
Linear stability with $\omega_{1,2}$ **real** and $\omega_{1,2} > 0$ and leads to a condition on the parameters.

$$m_{1,2}^2 < 0, \qquad d^2 \geq \frac{(\sqrt{-m_1^2} + \sqrt{-m_2^2})^2}{4}$$

$$M_0 > \frac{2}{3}M_2, \quad M_1 > 0, \quad M_1 + \sqrt{M_0^2 - \frac{2M_0M_2}{3}} < M_0.$$

Extra Slides

Mass Parameters

$$M_0 = \frac{\phi'^2 (U_{\chi\chi} + 2U_{y\chi} + U_{yy})}{2a^2}, \quad M_1 = -\frac{U_\chi \phi'}{a},$$
$$M_2 = -\frac{4(U_{\tau Y} + U_{\tau Z})}{9}, \quad M_3 = \frac{27a^{-6}U_{bb} - 8(U_{\tau Y} + U_{\tau Z})}{54},$$
$$M_4 = \frac{\phi'\{-[a^3(U_\chi + U_y)] + U_{b\chi} + U_{by}\}}{2a^4}. \tag{B1}$$

With $w = -1$ masses are time independent and related.

$$\frac{M_4}{M_0} = 1, \qquad M_2 - 3(M_3 - M_4) = 0.$$

Extra Slides

- Perturbation corresponding to gravitational waves (traceless and transverse):

$$ds^2 = a^2(\eta_{\mu\nu}dx^\mu dx^\nu + \chi_{ij}dx^i dx^j)$$

$$\chi_{ij}\delta^{ij} = \partial^j \chi_{ij} = 0$$

- Gravitational waves from the action

$$S = \int dx^4 \sqrt{-g}\left[\frac{1}{2}R + U\big(b, y, \chi, \tau_y, \tau_z\big)\right]$$

- Relation between Bardeen Potentials

$$2\,\mathrm{a}^2\mathrm{M}_2\pi_1 - \Phi + \Psi = 0$$

- Matter density contrast equation

$$\delta_m'' + \mathcal{H}\delta_m' + k^2\Psi - 3\Phi'' - 3\mathcal{H}\Phi' = 0.$$

Extra Slides

Vector Modes equation of propagation

$$\pi_{gi}^{i\,\prime\prime} + \frac{4\mathcal{H}(a^2 c_1 H_0^2 + k^2)}{2a^2 c_1 H_0^2 + k^2}\pi_{gi}^{i\,\prime} + \frac{c_2(2a^2 c_1 H_0^2 + k^2)}{c_1}\pi_{gi}^i = 0,$$

Vector Modes: Small Scales

$$\pi_{\rm gi}^i = \sin\left(\frac{\sqrt{c_2}k\tau}{\sqrt{c_1}}\right)\left[\frac{\sqrt{\frac{2}{\pi}}\sqrt[4]{c_1}\tilde{q}_2^i(c_2k^2\tau^2 - 3c_1)}{c_2^{5/4}} + 3\sqrt{\frac{2}{\pi}}\left(\frac{c_1}{c_2}\right)^{3/4} k\,\tau\tilde{q}_1^i\right] + \cos\left(\frac{\sqrt{c_2}k\tau}{\sqrt{c_1}}\right)\left[\frac{\sqrt{\frac{2}{\pi}}\sqrt[4]{c_1}\tilde{q}_1^i(c_2k^2\tau^2 - 3c_1)}{c_2^{5/4}} - 3\sqrt{\frac{2}{\pi}}\left(\frac{c_1}{c_2}\right)^{3/4} k\,\tau\tilde{q}_2^i\right],$$

Vector Modes: Large Scales

$$\pi_{\rm gi}^i = q_1^i(-k\tau)^{(3-\sqrt{9-8c_2})/2} + q_2^i(-k\tau)^{(3+\sqrt{9-8c_2})/2},$$

No growing modes for vector degrees of freedom, only oscillations at small scales and decreasing modes at large scales