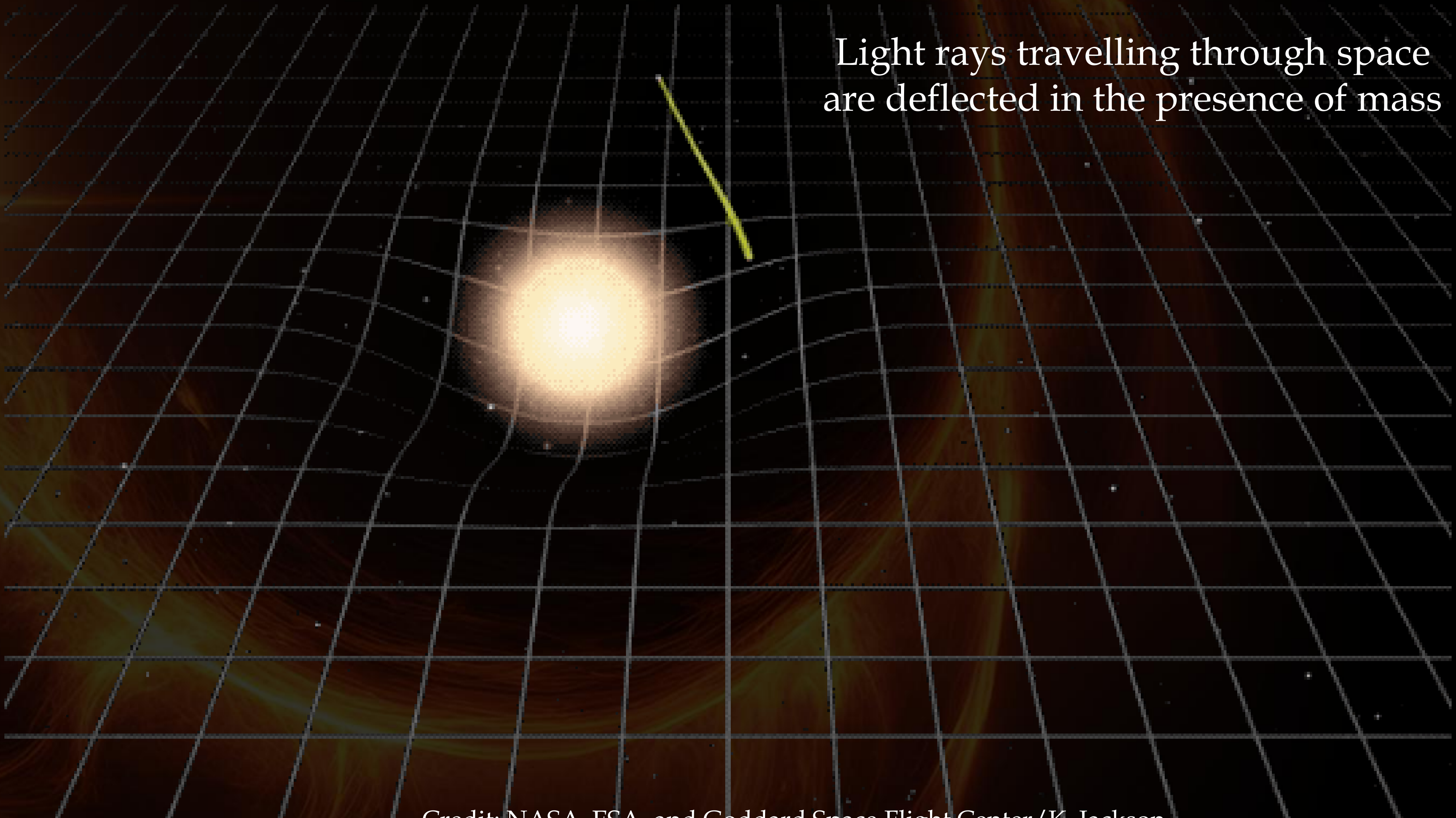
Strong lensing cosmology and the line of sight

Daniel Johnson

Pierre Fleury, Julien Larena

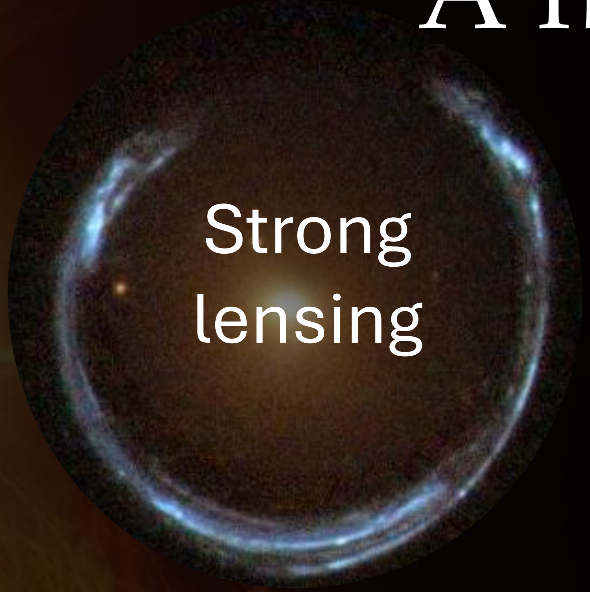
Gravitational lensing



Light rays travelling through space are deflected in the presence of mass

Credit: NASA, ESA, and Goddard Space Flight Center/K. Jackson

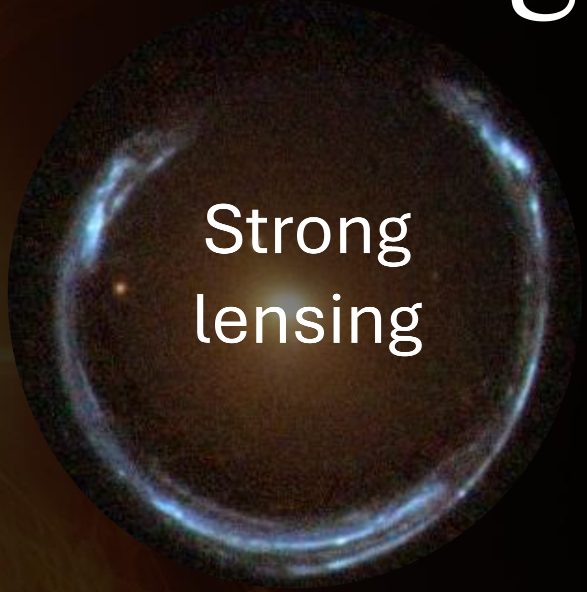
A historical division



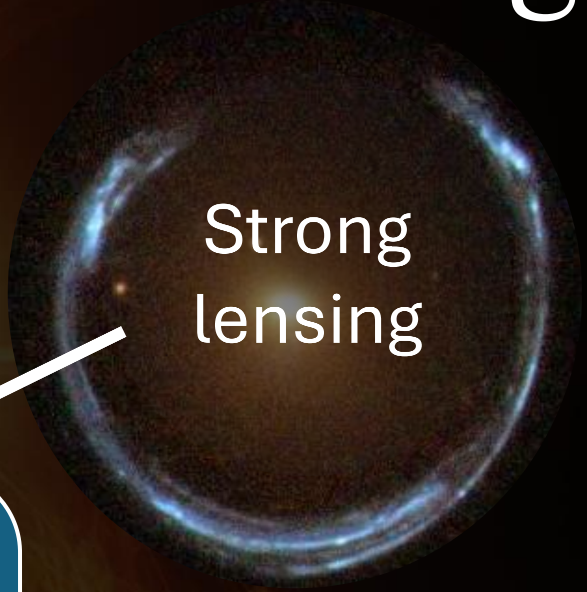
Strong, non-linear distortions by a galaxy or galaxy cluster, often resulting in multiple images

Small distortions to magnifications and shapes – an integrated effect along the line of sight

Use in cosmology

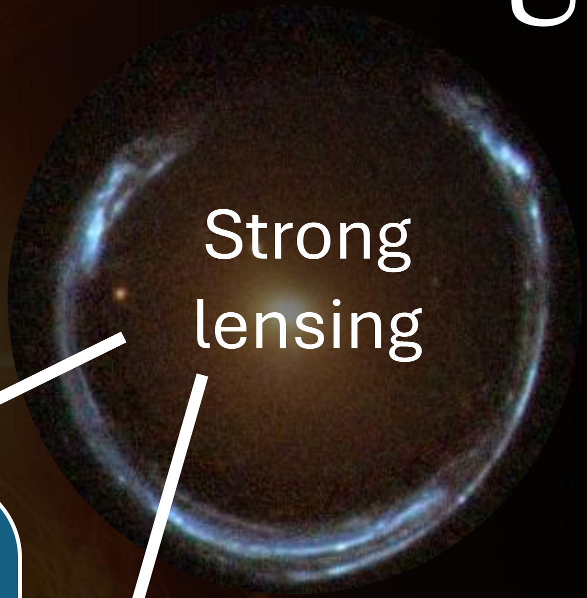


Use in cosmology



H_0 via time delay cosmography

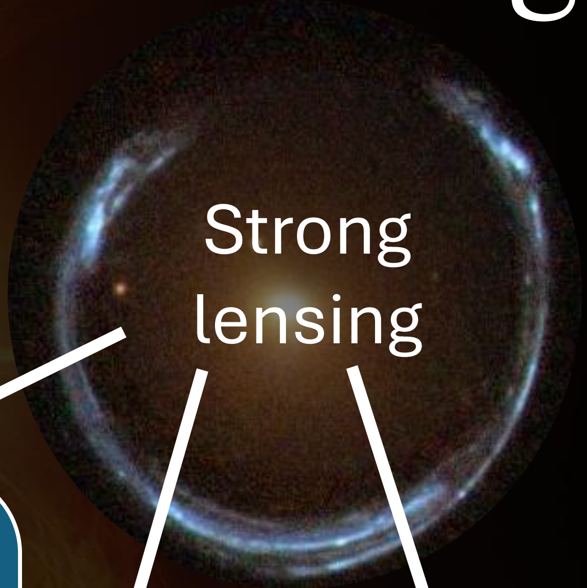
Use in cosmology



H_0 via time delay cosmography

$\Omega_m / w(z)$ from distance ratios in compound lensing

Use in cosmology



Strong lensing

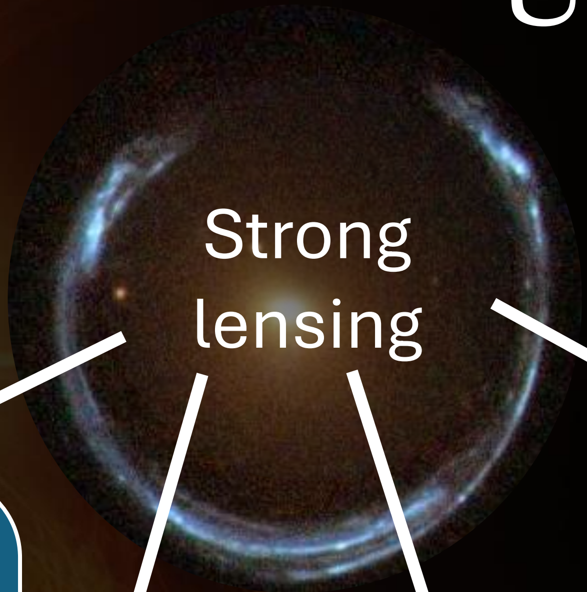
H_0 via time delay cosmography

$\Omega_m/w(z)$ from distance ratios in compound lensing

Galaxy dark matter content by comparing light and mass profiles

Weak lensing

Use in cosmology

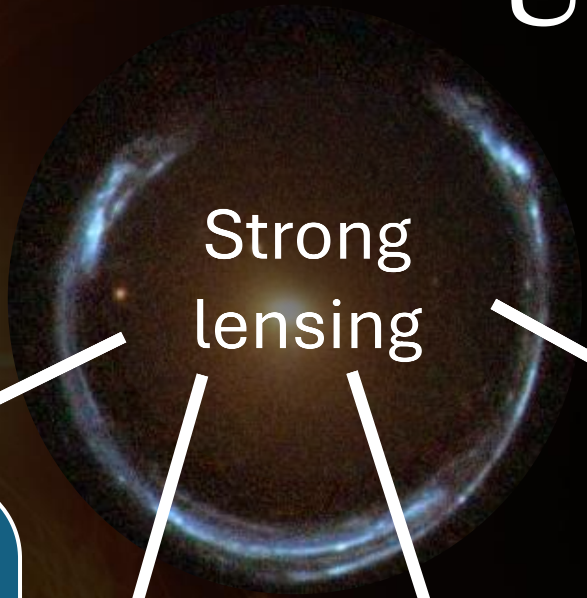


Strong lensing

- H_0 via time delay cosmography
- $\Omega_m/w(z)$ from distance ratios in compound lensing
- Galaxy dark matter content by comparing light and mass profiles
- Dark matter substructures from perturbations to Einstein rings

Weak lensing

Use in cosmology



Strong lensing

- H_0 via time delay cosmography
- $\Omega_m/w(z)$ from distance ratios in compound lensing
- Galaxy dark matter content by comparing light and mass profiles
- Dark matter substructures from perturbations to Einstein rings

Weak lensing

- The matter power spectrum

Use in cosmology

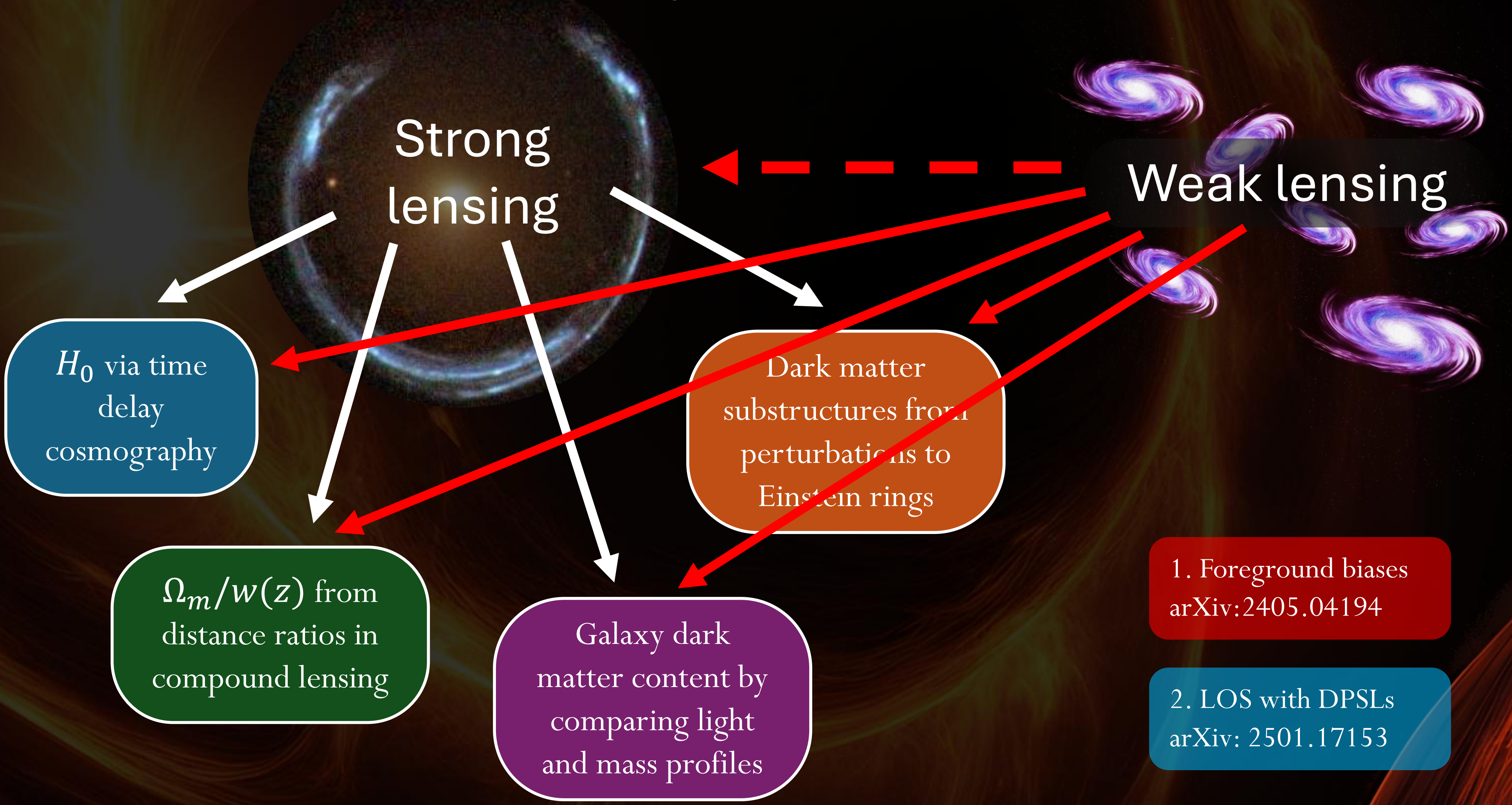
Strong lensing

- H_0 via time delay cosmography
- $\Omega_m / w(z)$ from distance ratios in compound lensing
- Galaxy dark matter content by comparing light and mass profiles
- Dark matter substructures from perturbations to Einstein rings

Weak lensing

- Ω_m, σ_8
- The matter power spectrum

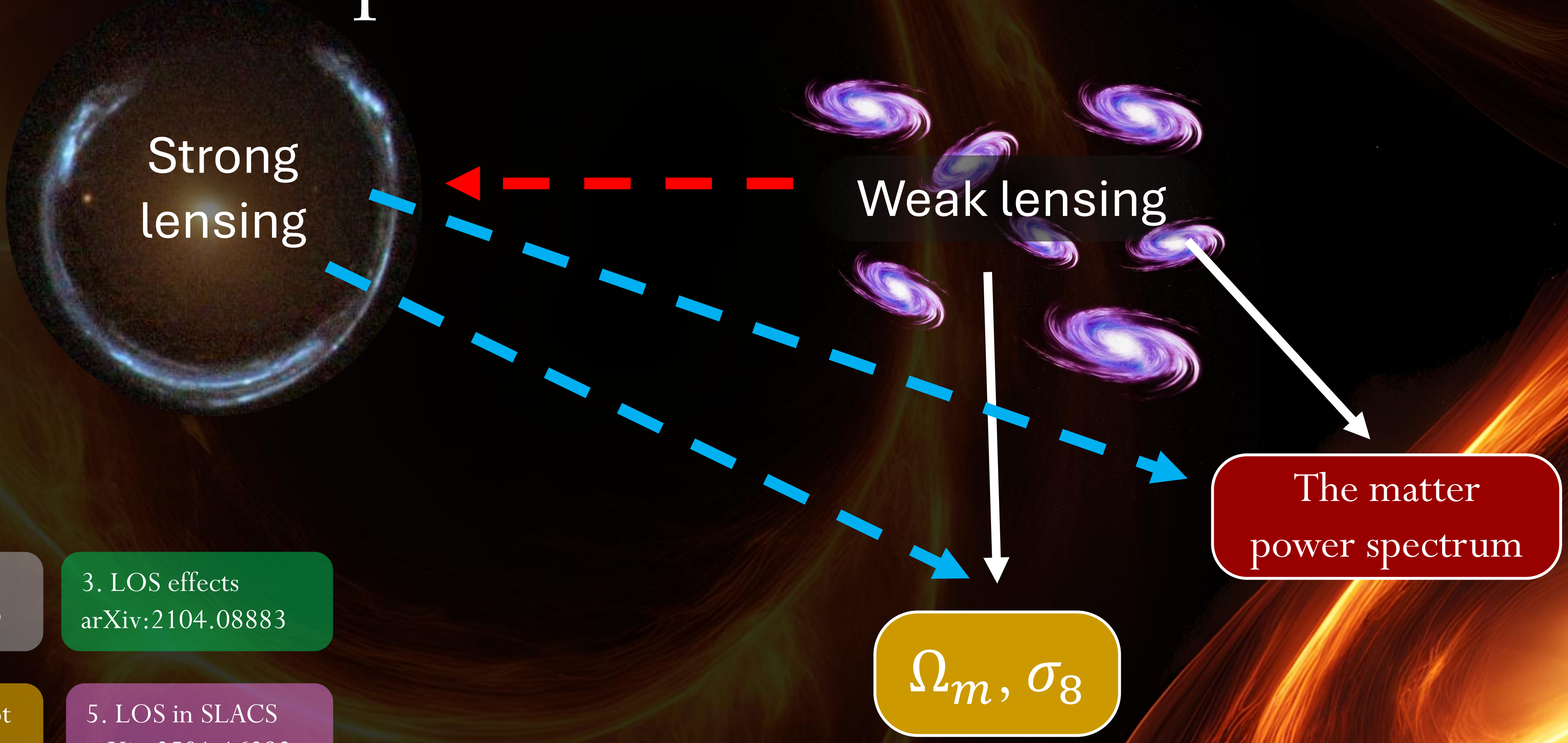
But they're not independent!



1. Foreground biases arXiv:2405.04194

2. LOS with DPSLs arXiv: 2501.17153

Perhaps this could be useful?



LOS inference
arXiv:1610.01599

3. LOS effects
arXiv:2104.08883

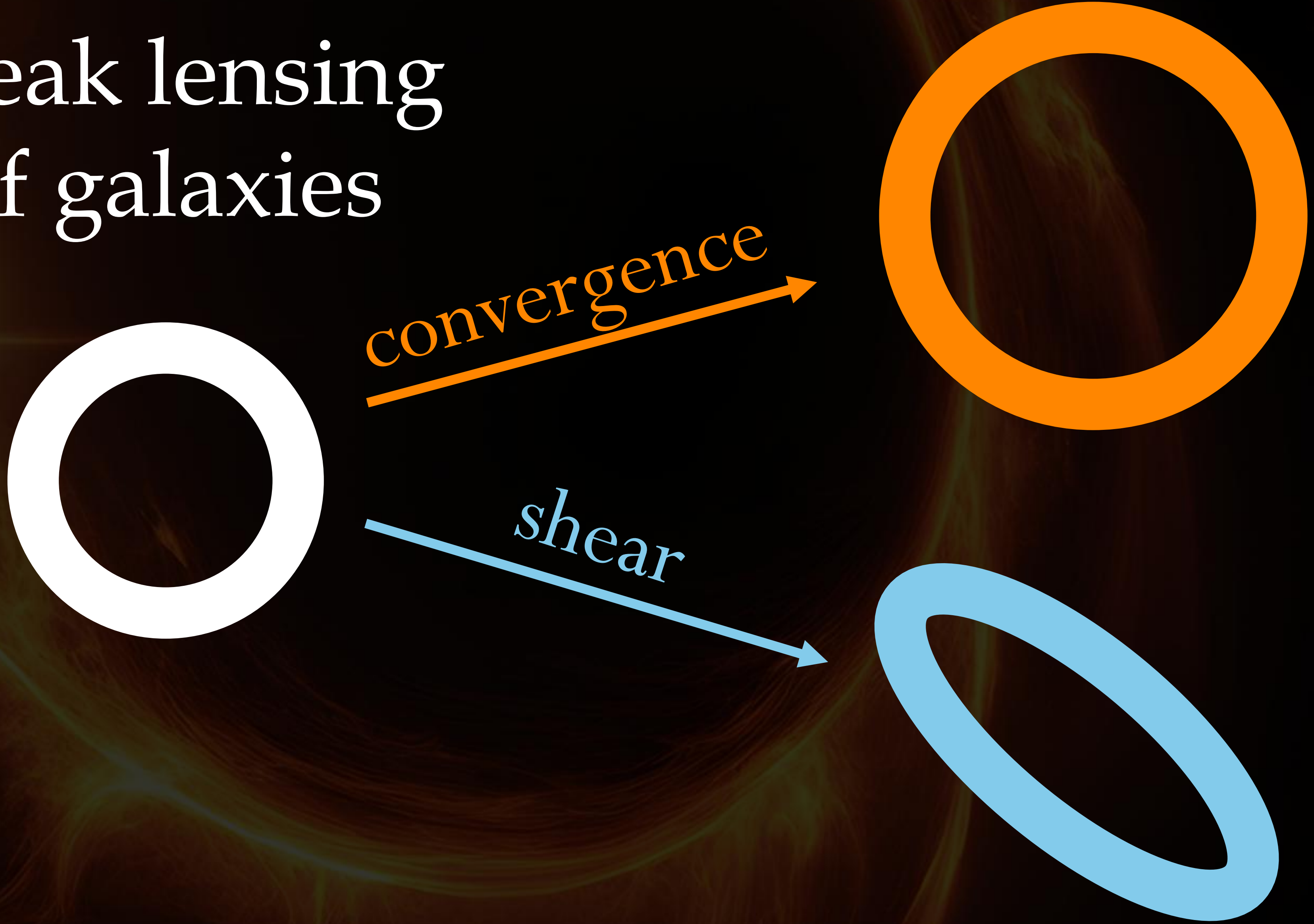
4. Proof of concept
arXiv:2210.07210

5. LOS in SLACS
arXiv:2501.16292

Weak lensing of galaxies

Weak lensing of galaxies

Weak lensing of galaxies



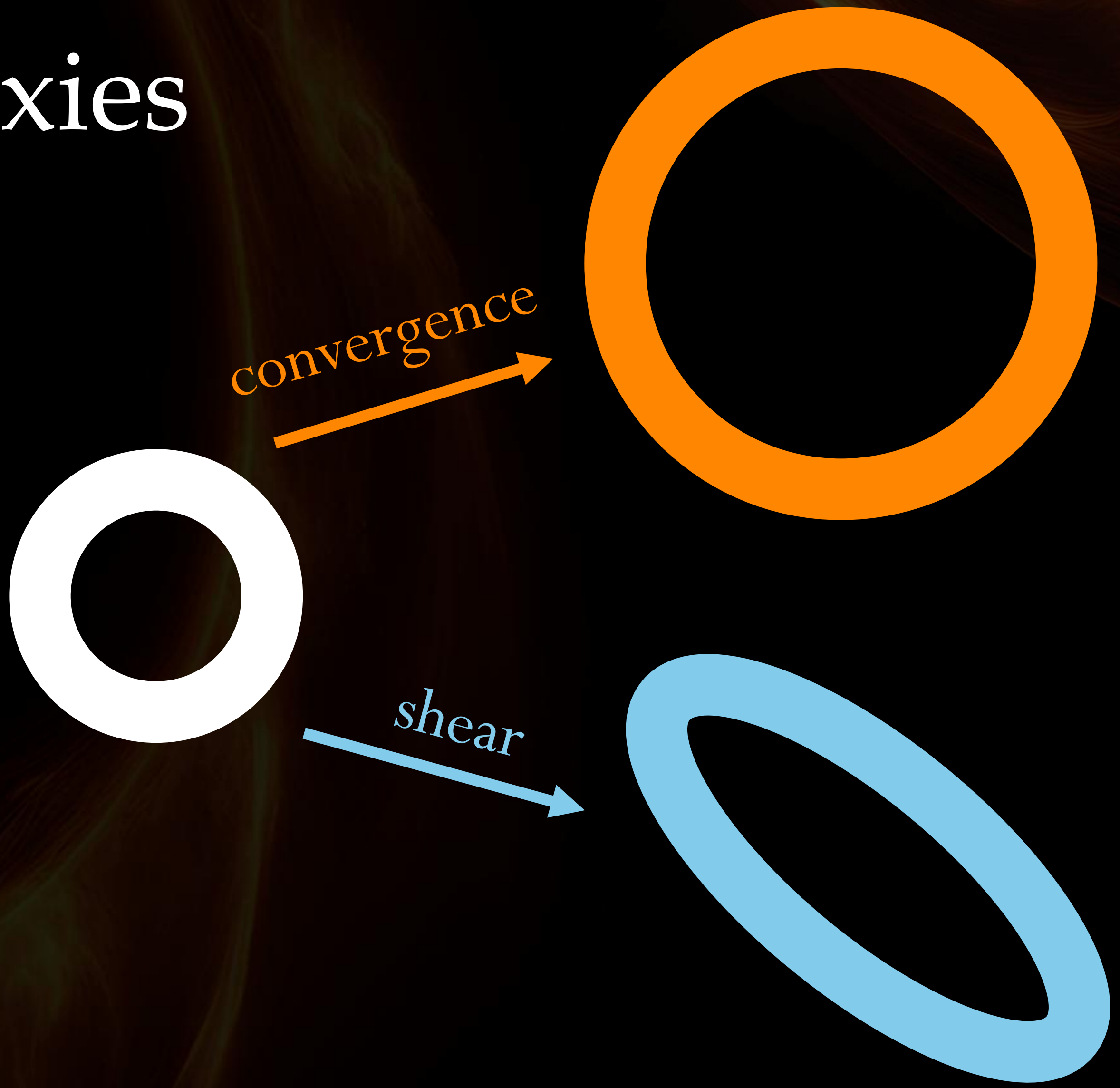
Weak lensing of galaxies

Convergence and shear

$$\beta = A\theta$$

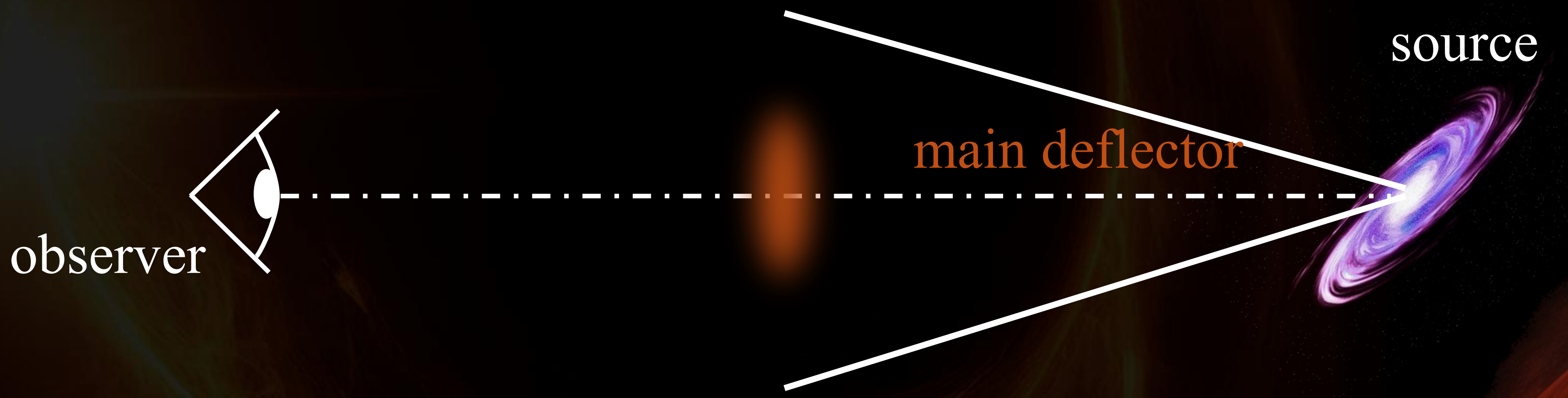
source position

image position

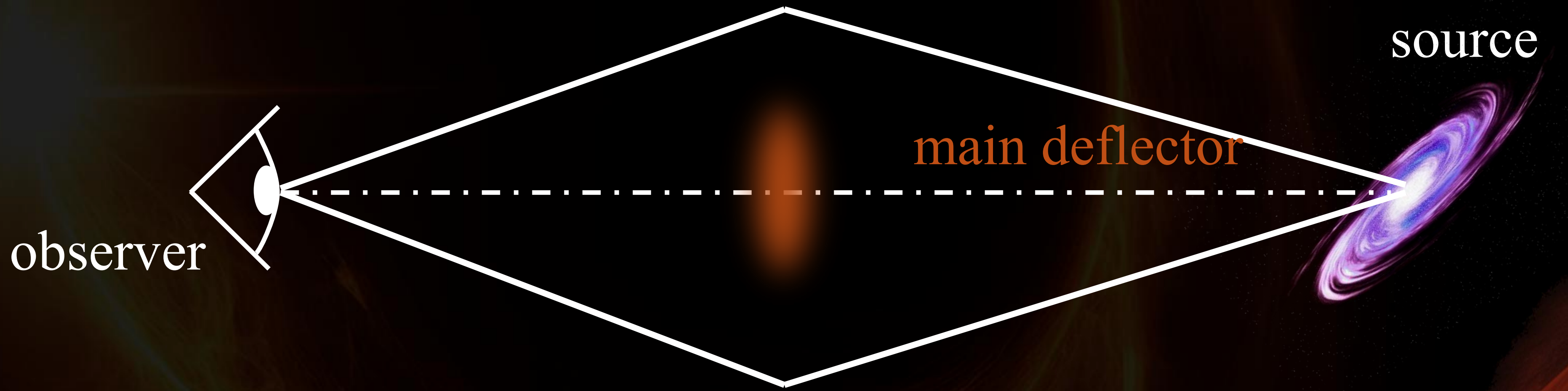


Strong lensing

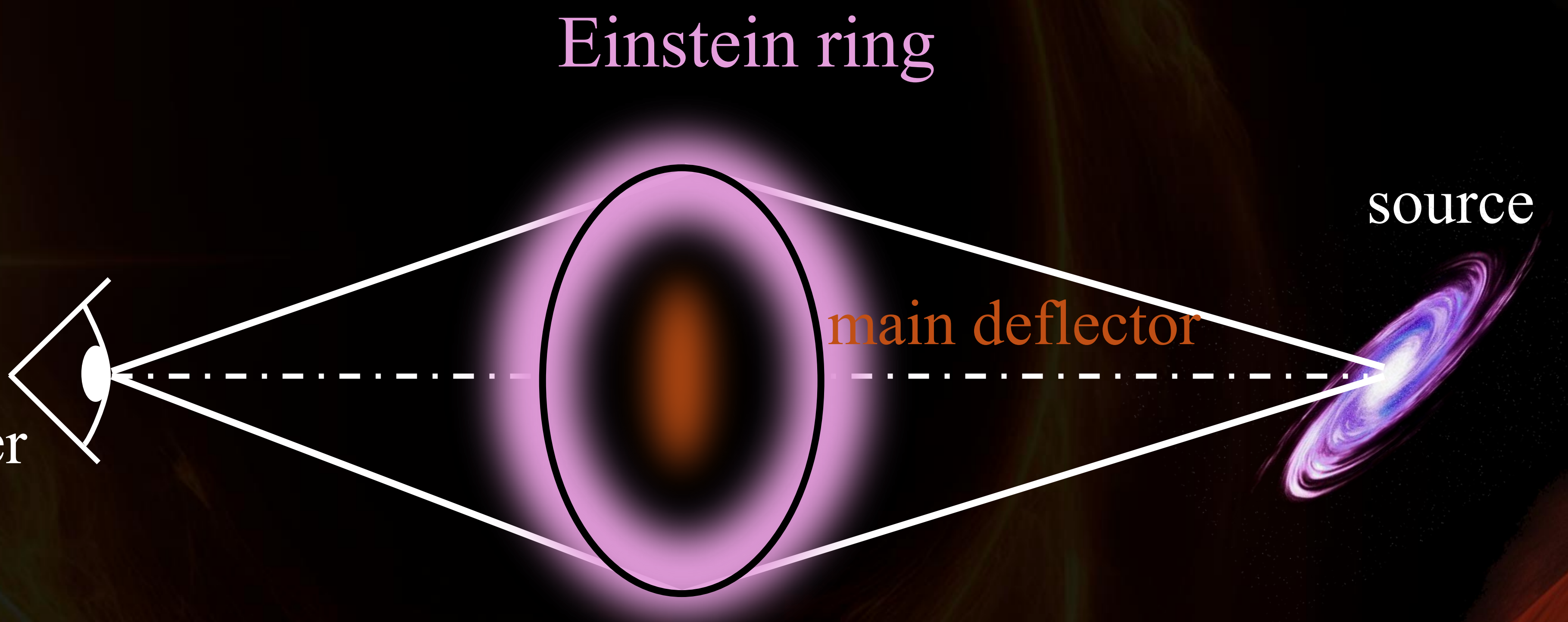
Strong lensing



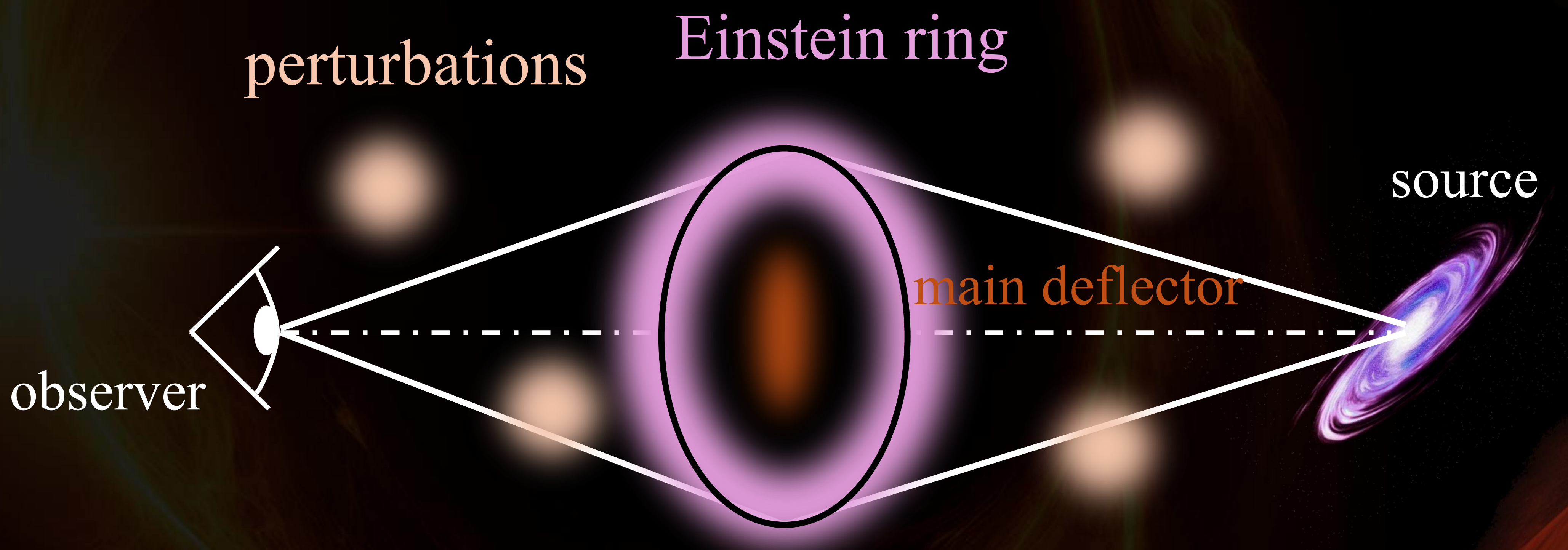
Strong lensing



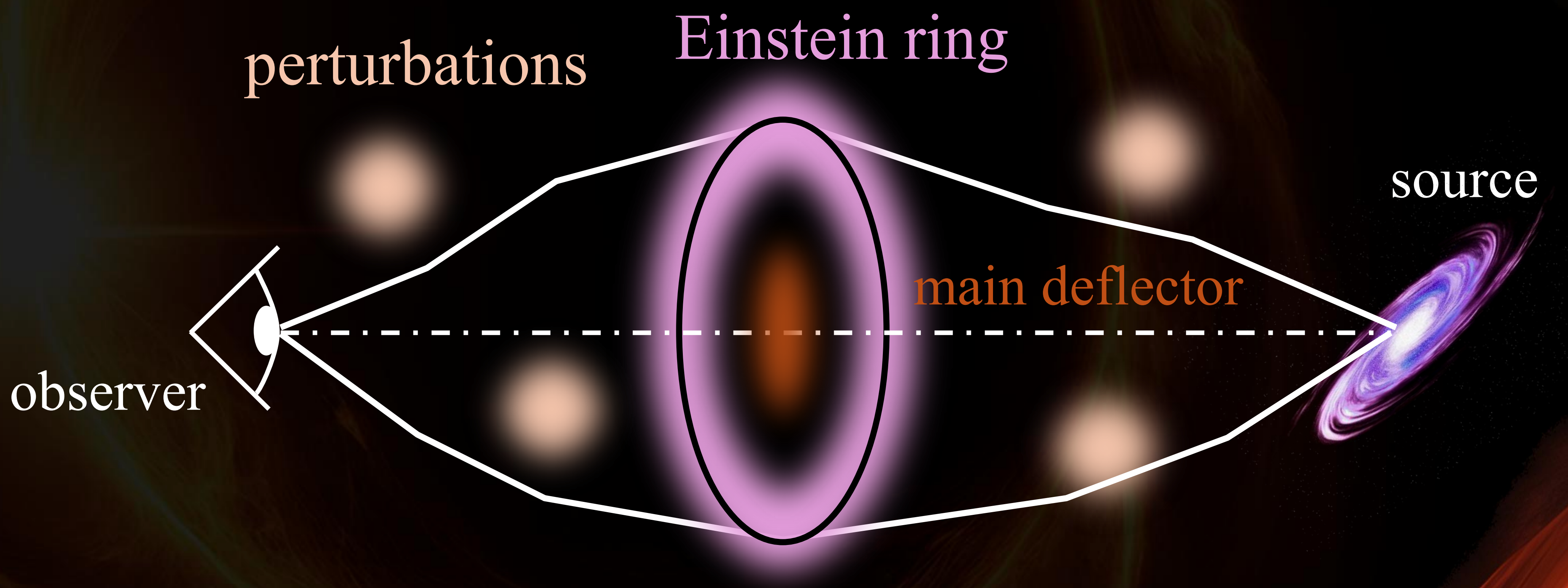
Strong lensing



Weak lensing of strong lensing

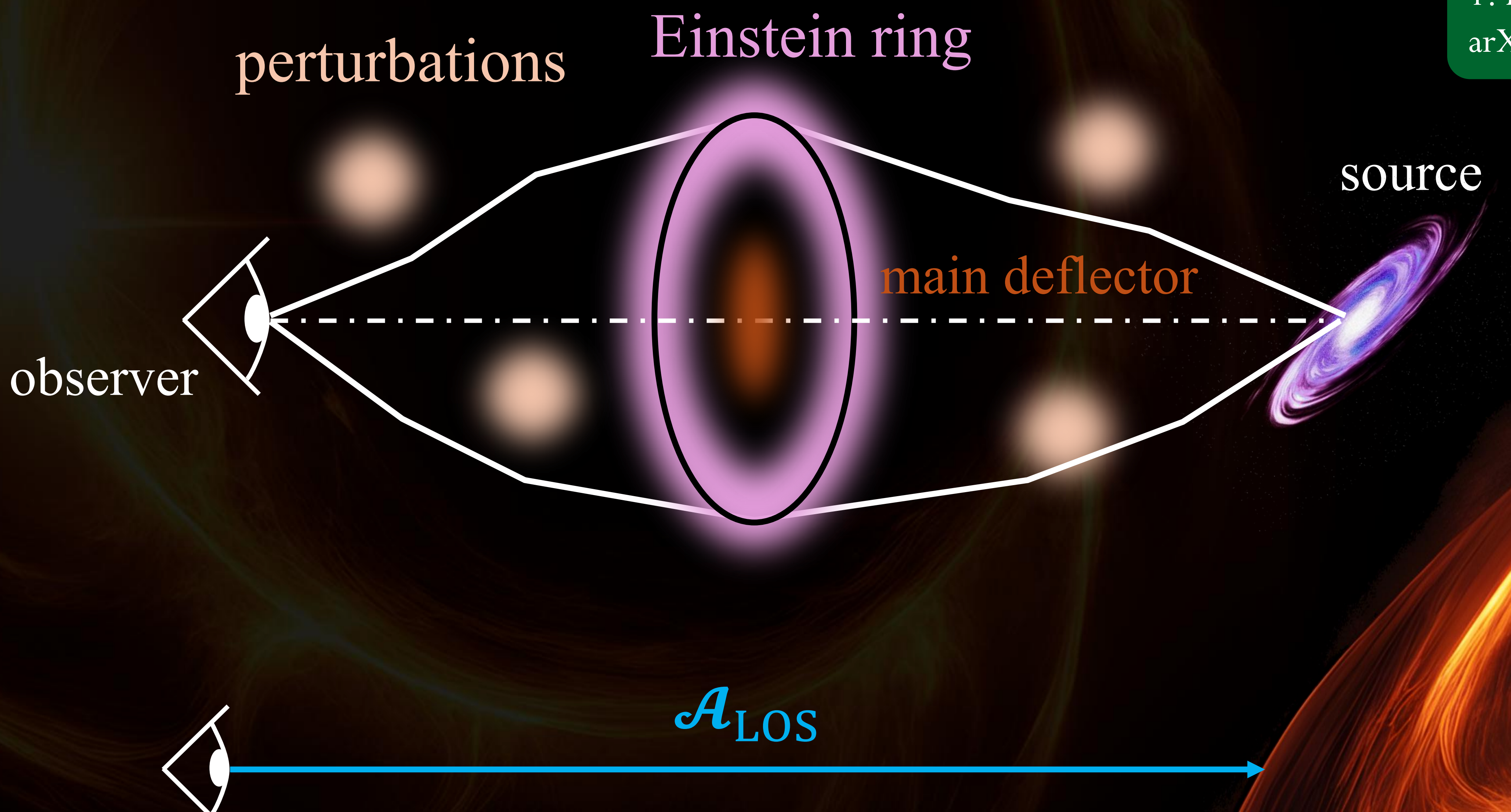


Weak lensing of strong lensing



Weak lensing of strong lensing

1. LOS effects
arXiv:2104.08883

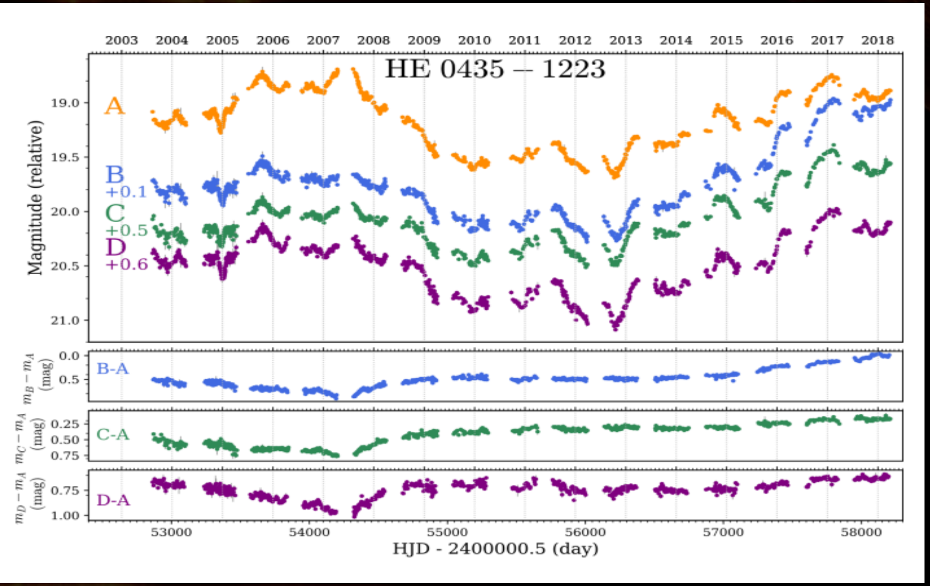


The work I'll talk about:

1. How the line of sight biases measurements

2. The line of sight as a cosmological probe → σ_8, Ω_m

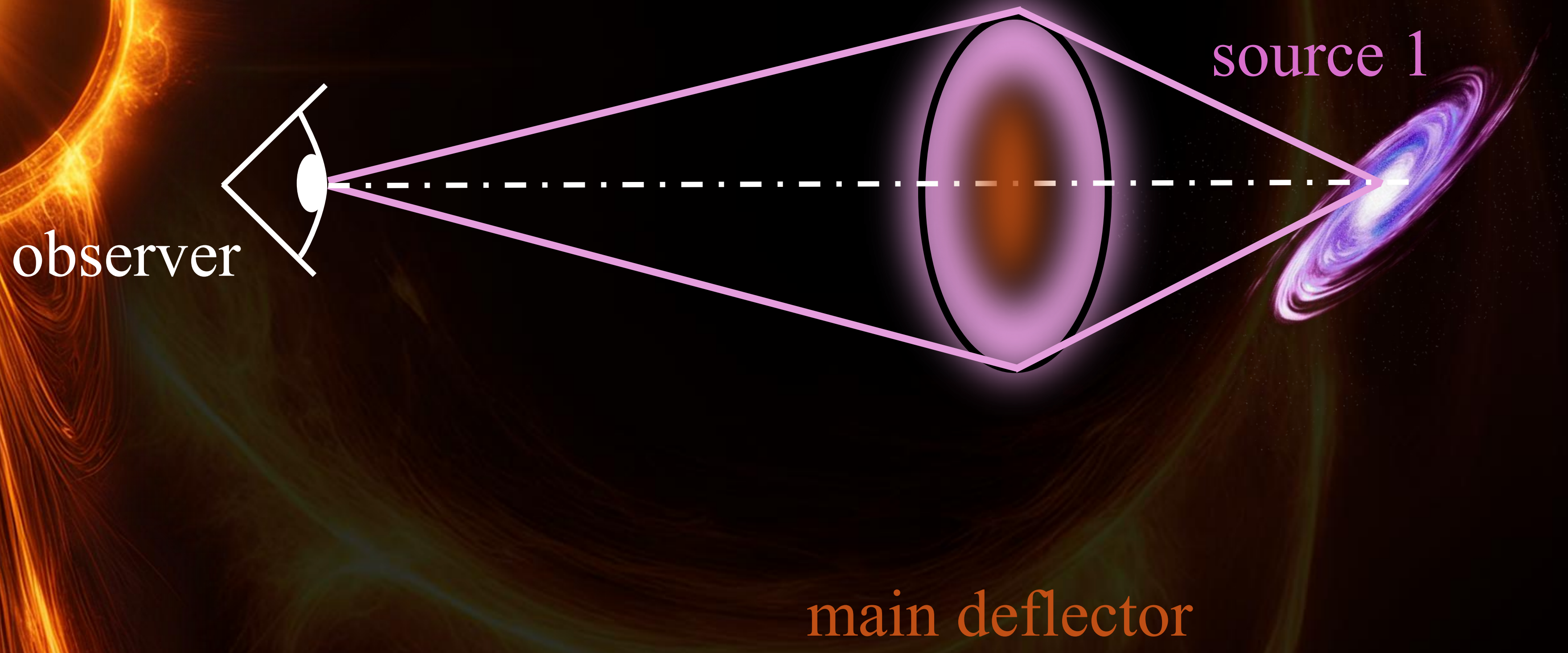
Time delay cosmography



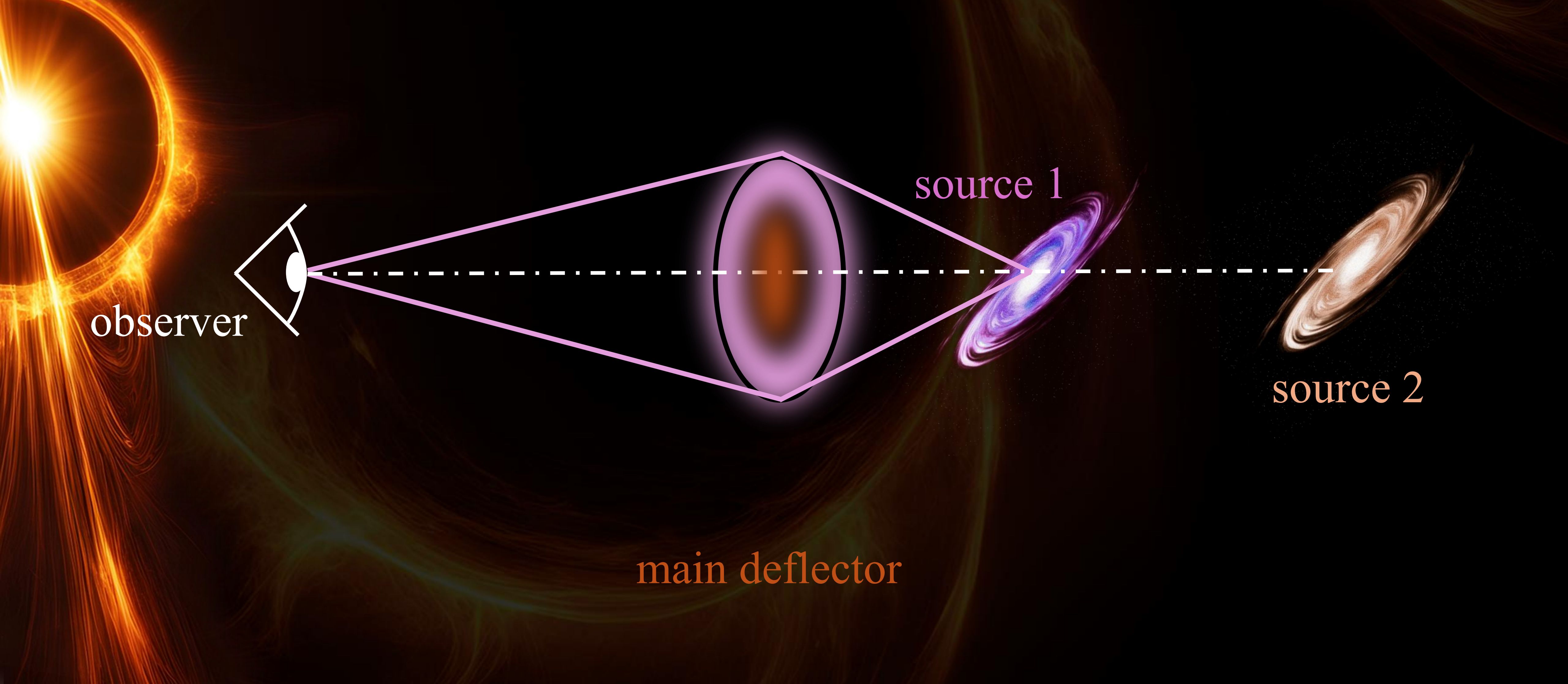
Credit: S. Birrer et al.

$$\Delta t \propto \frac{D_{\rm od} D_{\rm os}}{D_{\rm ds}} \propto \frac{1}{H_0}$$

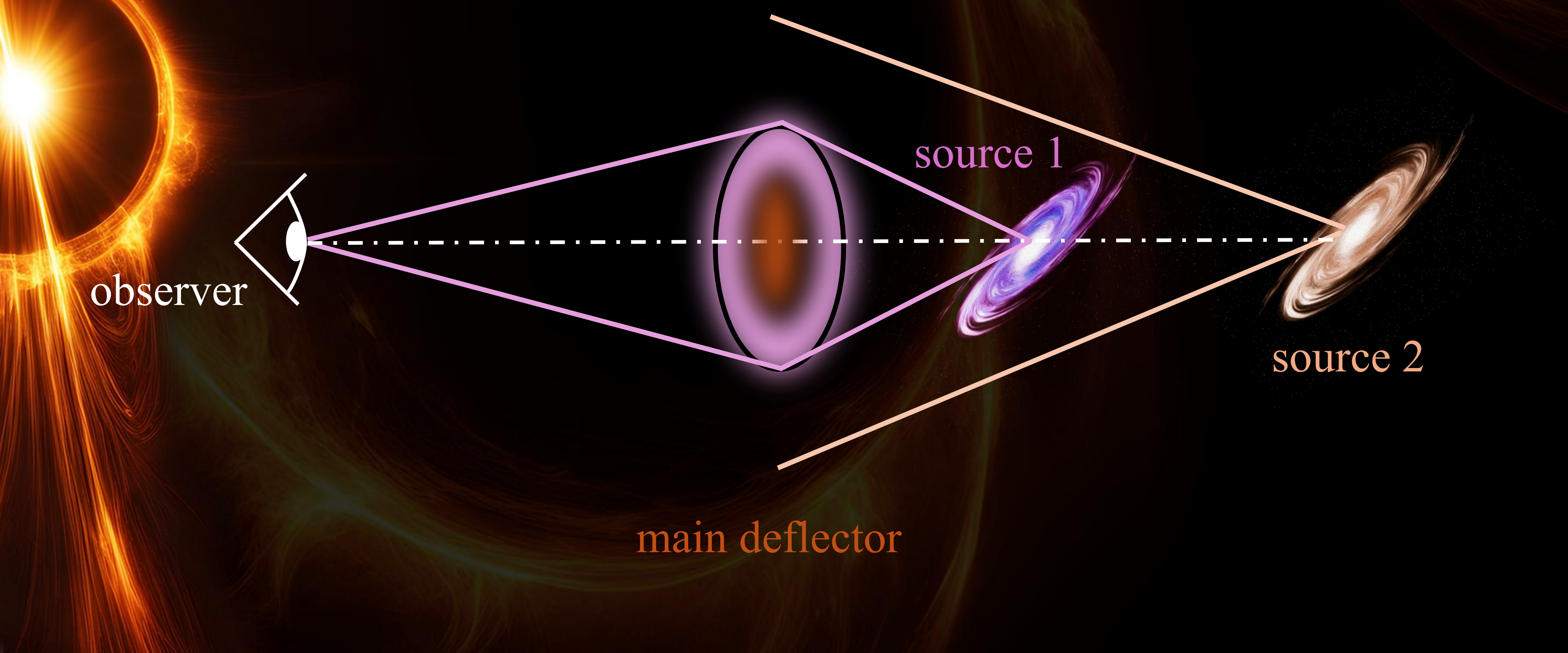
Double-source-plane lenses



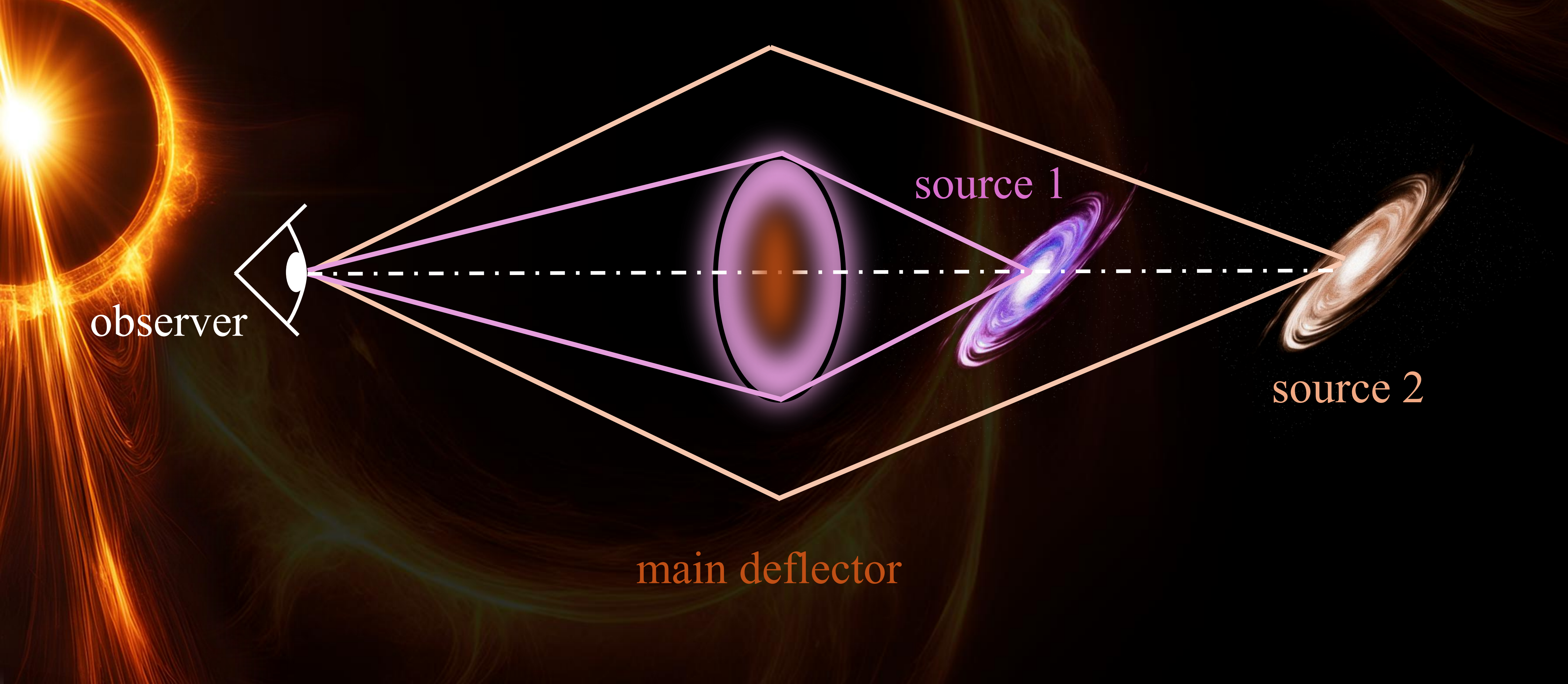
Double-source-plane lenses



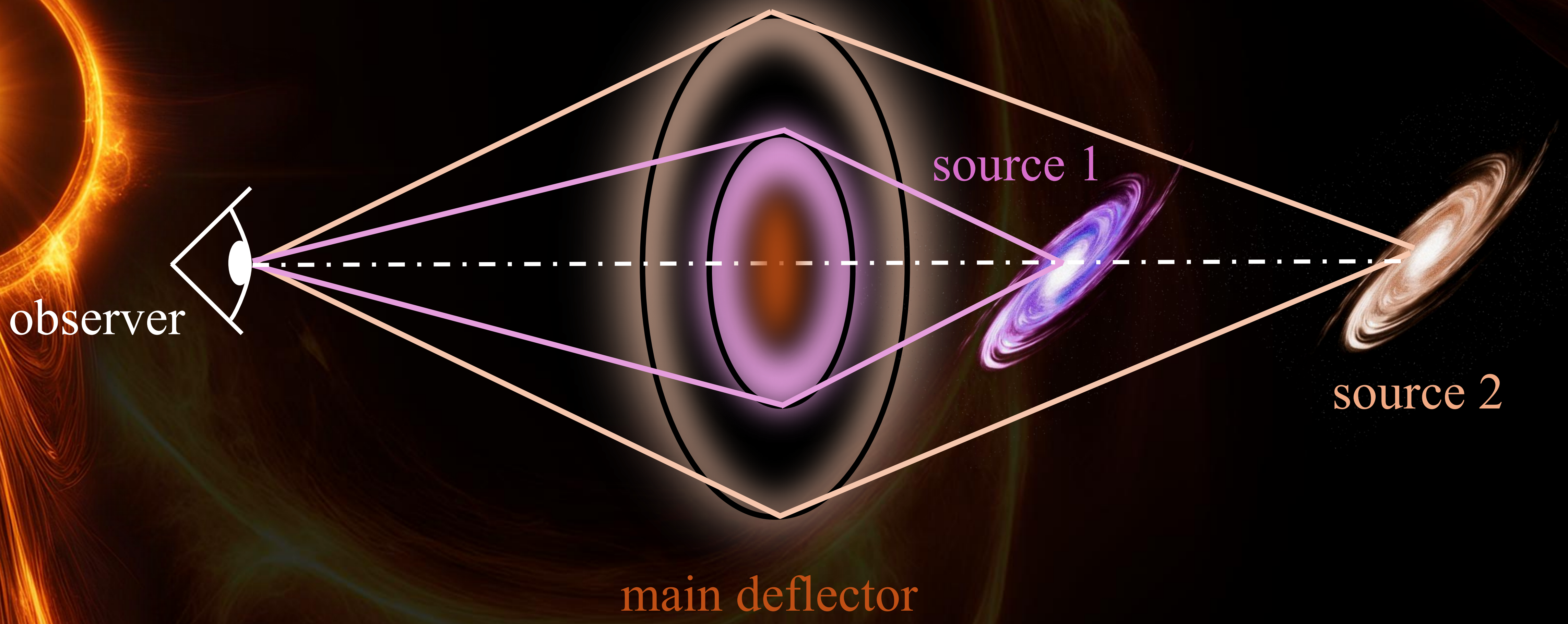
Double-source-plane lenses



Double-source-plane lenses

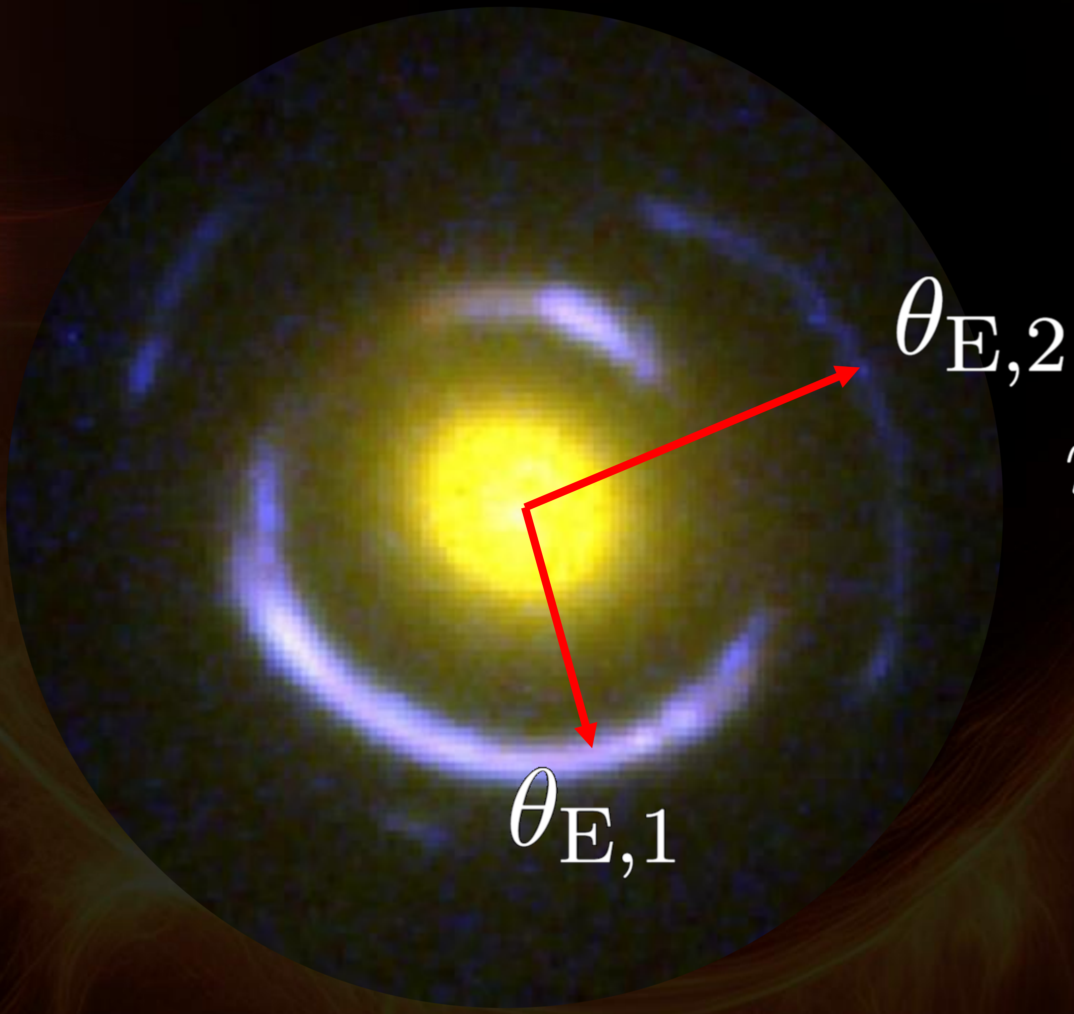


Double-source-plane lenses



Double-source-plane lenses

Double-source-plane lenses



$$\eta = \frac{D_{\mathrm{s1}} D_{\mathrm{ds2}}}{D_{\mathrm{ds1}} D_{\mathrm{s2}}} \approx \frac{\theta_{\mathrm{E},2}}{\theta_{\mathrm{E},1}}$$

Convergence and distances

Projected surface mass density

$$\tilde{D}_{ij} = (1 - \kappa_{ij}) D_{ij}$$

Real spacetime

FLRW background

Convergence and distances

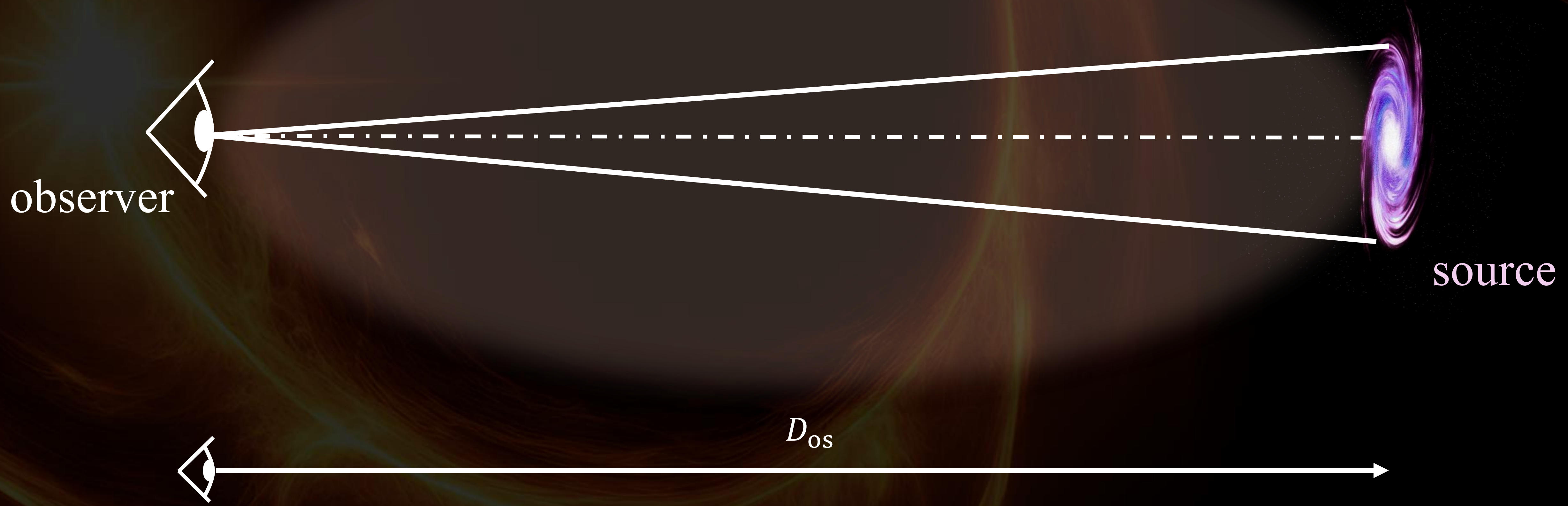
Projected surface mass density

$$\tilde{D}_{ij} = (1 - \kappa_{ij})D_{ij}$$

unmeasurable!!

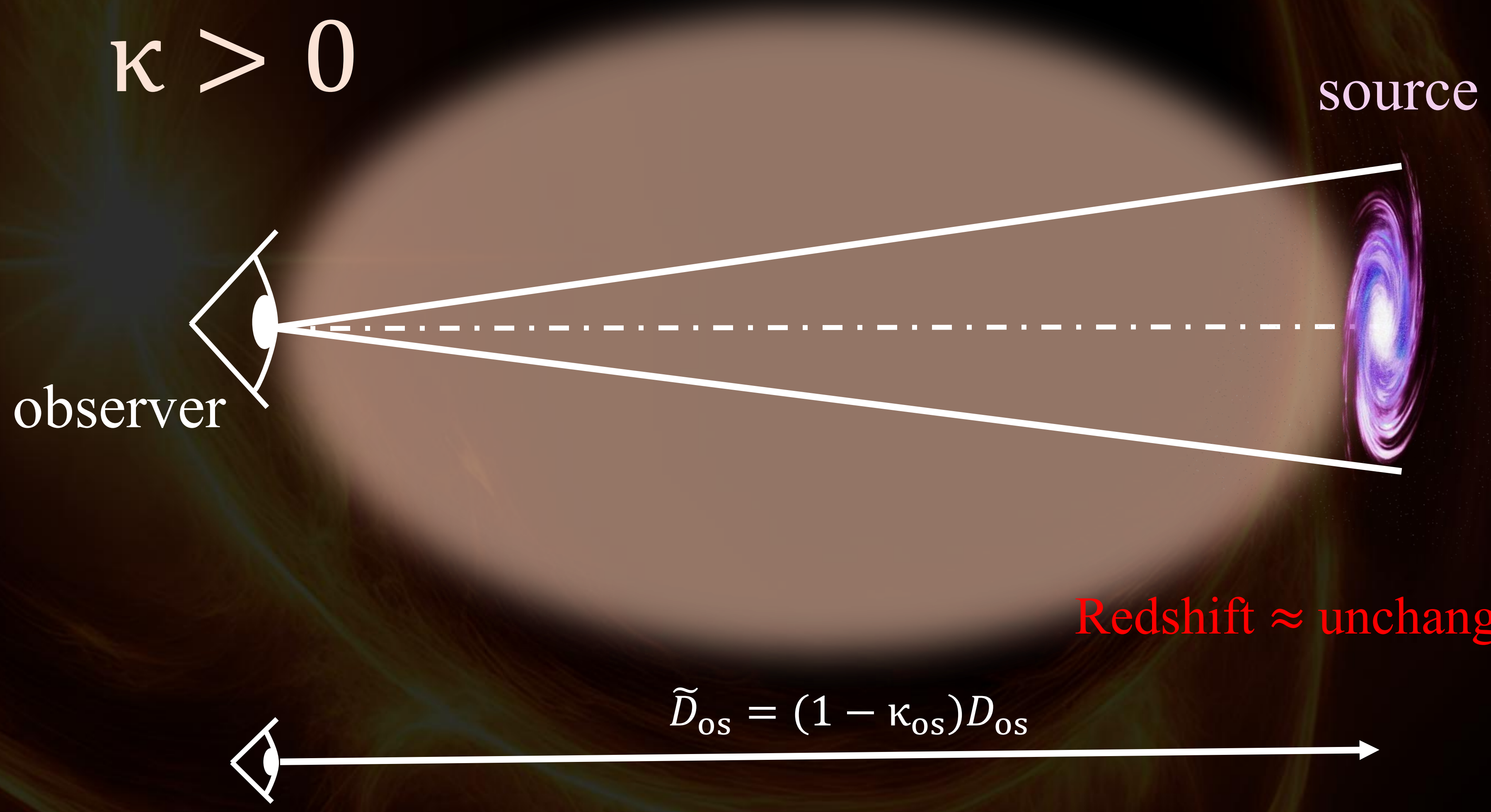
The MSD and angular diameter distances

$\kappa = 0$



The MSD and angular diameter distances

$\kappa > 0$

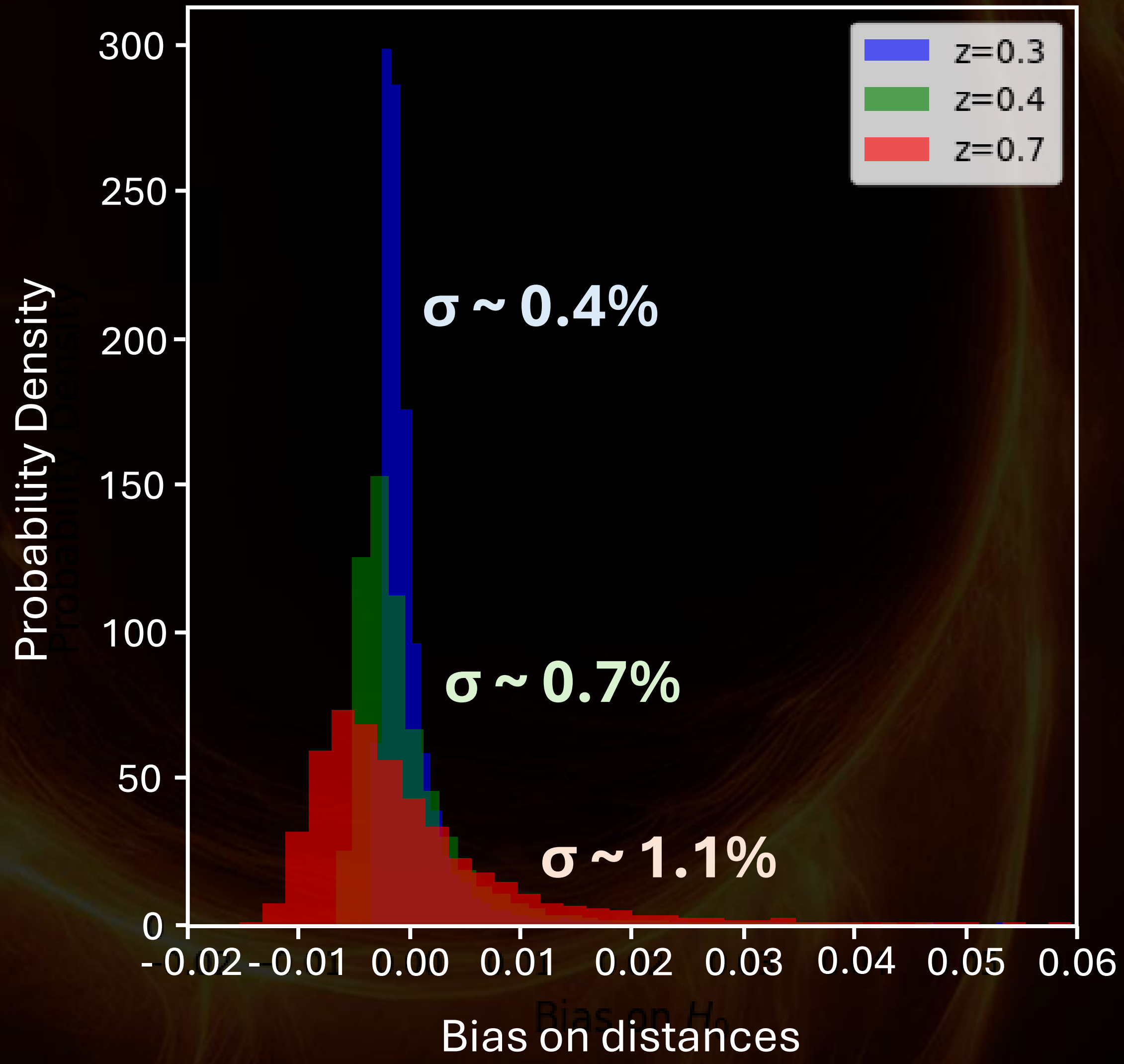


Overdense line of sight
→ shorter distance
compared to expectation
from cosmology

Redshift ≈ unchanged

$$\tilde{D}_{\mathrm{os}} = (1 - \kappa_{\mathrm{os}}) D_{\mathrm{os}}$$

How big is the effect?



1. Foreground biases arXiv:2405.04194

2. LOS with DPSLs arXiv: 2501.17153

- Small additional uncertainty
- Asymmetric
- Increases with redshift

When is this important?

- Whenever you observe distances, luminosities or shapes to high precision

When is this important?

- Whenever you observe distances, luminosities or shapes to high precision
  - H0 constraints from strong lensing time delays

When is this important?

- Whenever you observe distances, luminosities or shapes to high precision
  - H0 constraints from strong lensing time delays
  - w(z) constraints from double source plane lenses

When is this important?

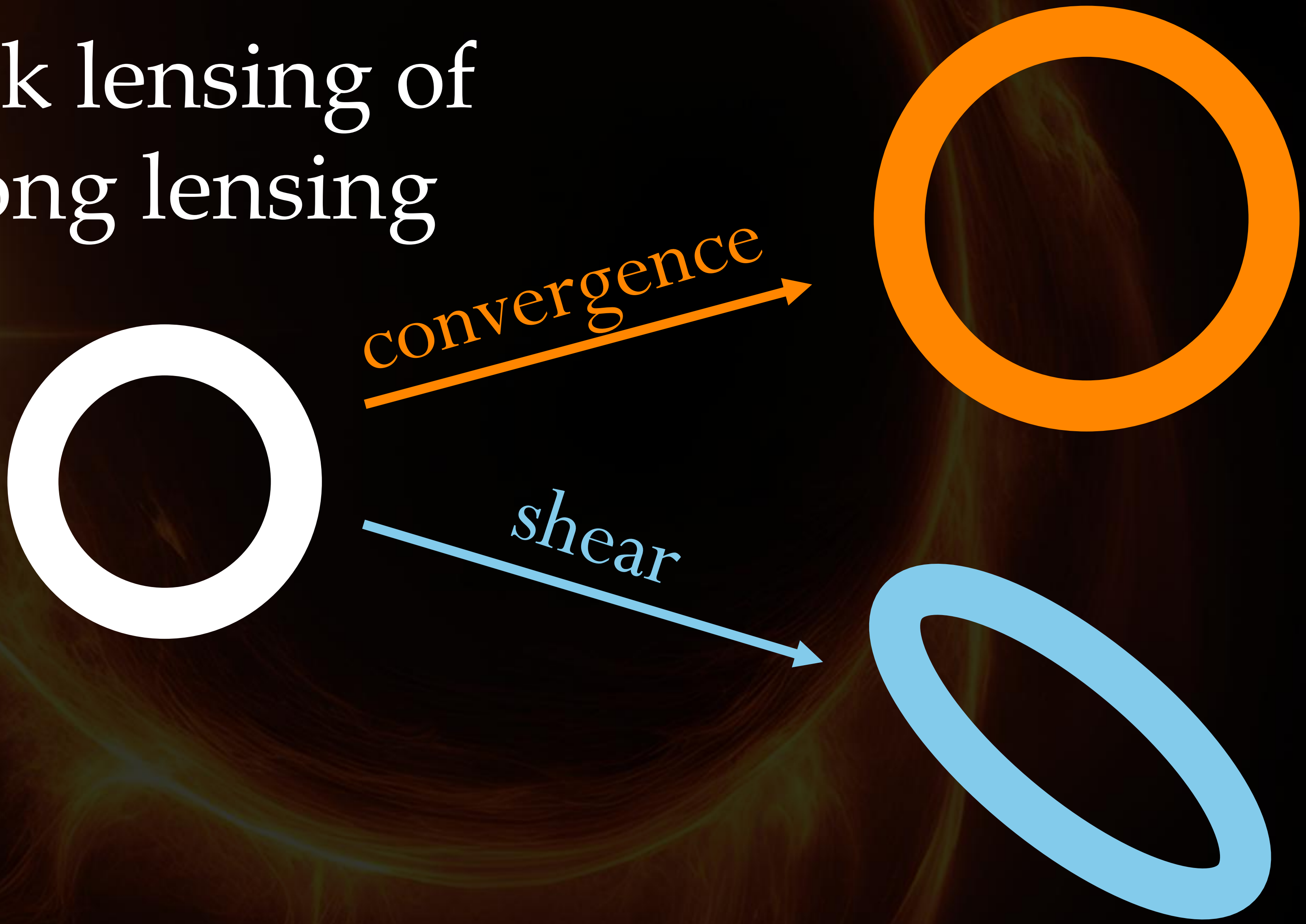
- Whenever you observe distances, luminosities or shapes to high precision
  - H0 constraints from strong lensing time delays
  - w(z) constraints from double source plane lenses
  - distance measurements from supernovae

When is this important?

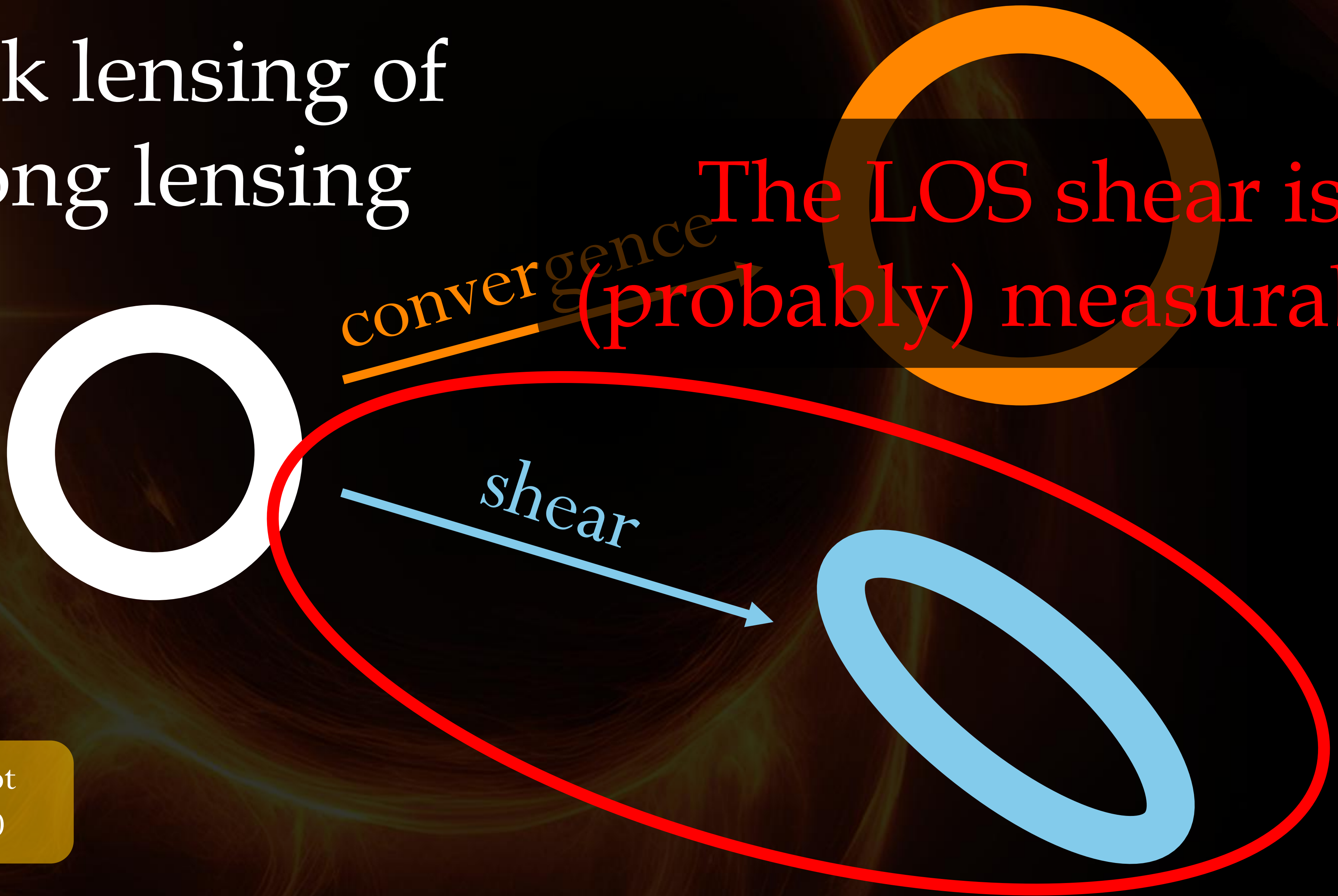
- Whenever you observe distances, luminosities or shapes to high precision
  - H0 constraints from strong lensing time delays
  - w(z) constraints from double source plane lenses
  - distance measurements from supernovae

- Generally mitigated by large sample sizes, but selection effects and other subtleties may cause problems

Weak lensing of strong lensing



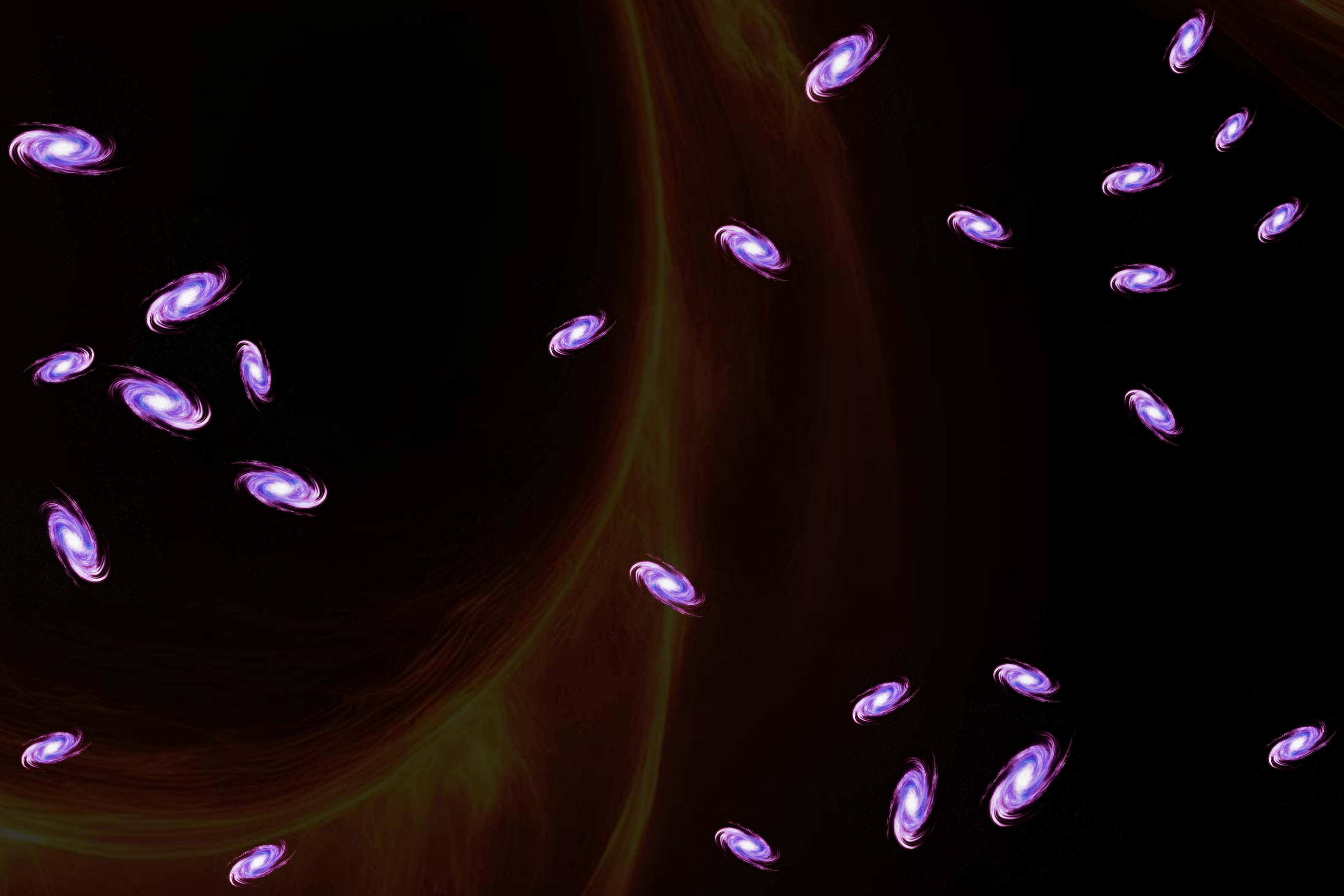
Weak lensing of strong lensing



The LOS shear is (probably) measurable!

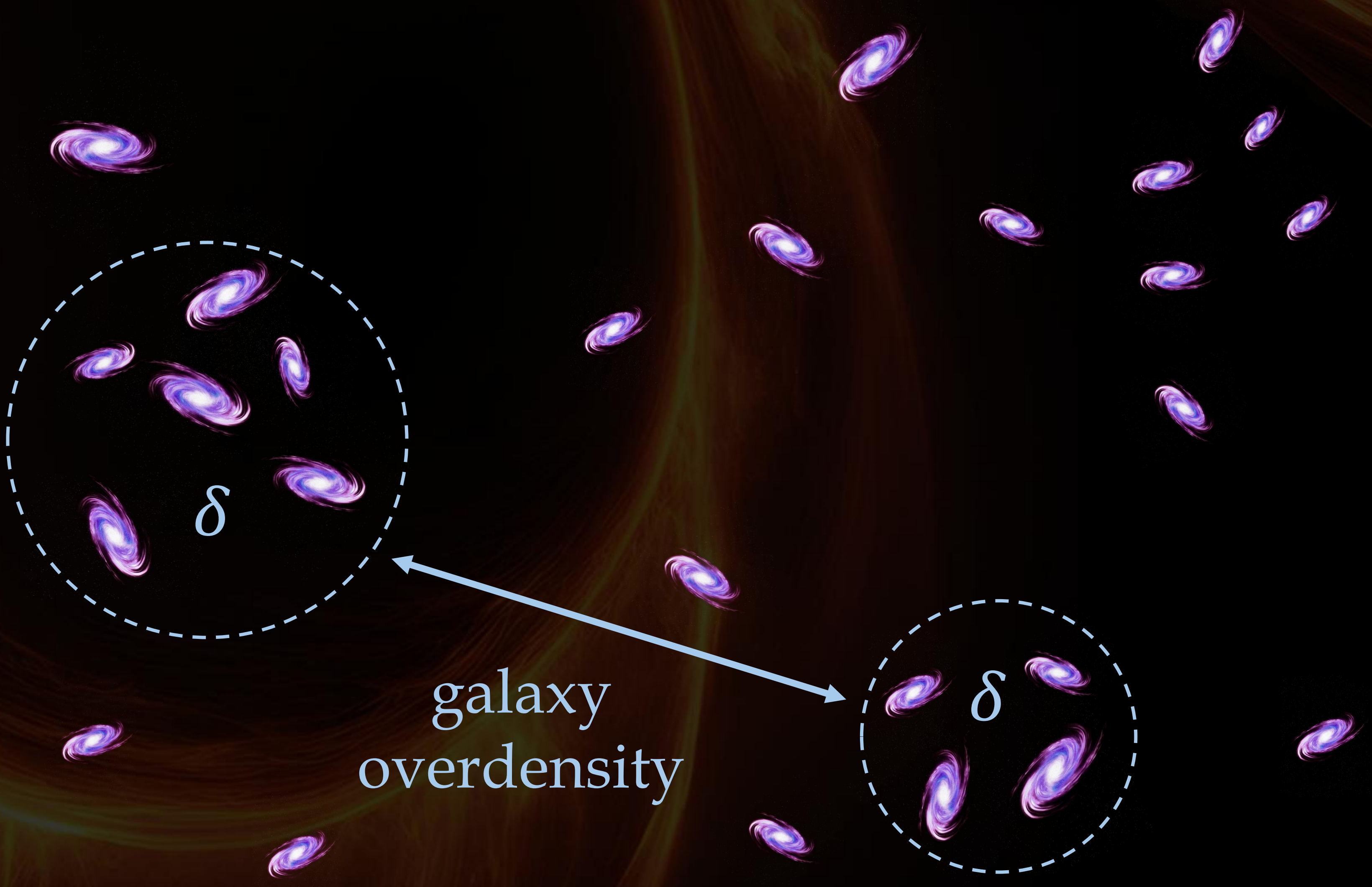
4. Proof of concept
arXiv:2210.07210

The 3x2pt correlation scheme



The 3x2pt correlation scheme

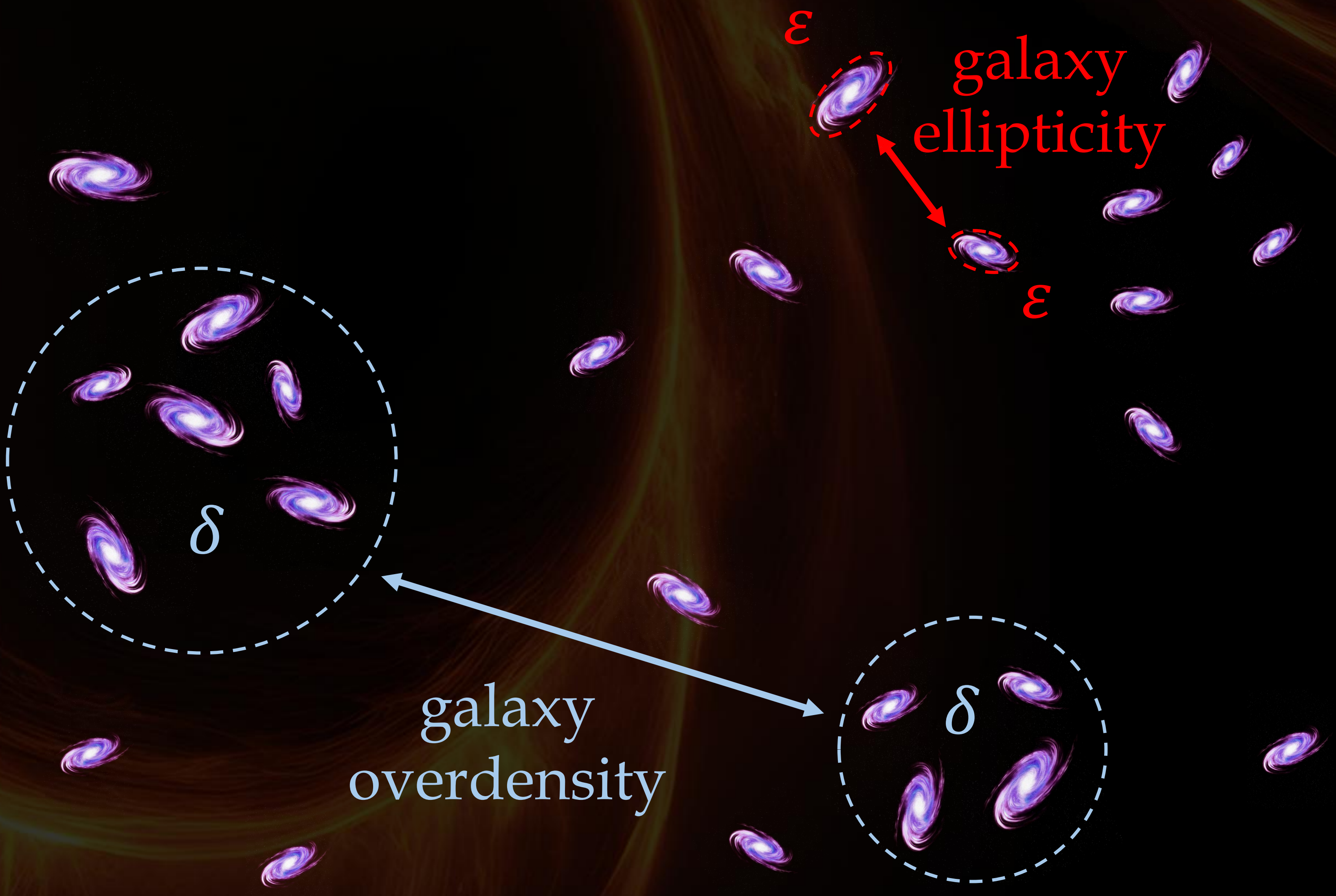
$\langle \delta \times \delta \rangle$



The 3x2pt correlation scheme

$\langle \delta \times \delta \rangle$

$\langle \varepsilon \times \varepsilon \rangle$

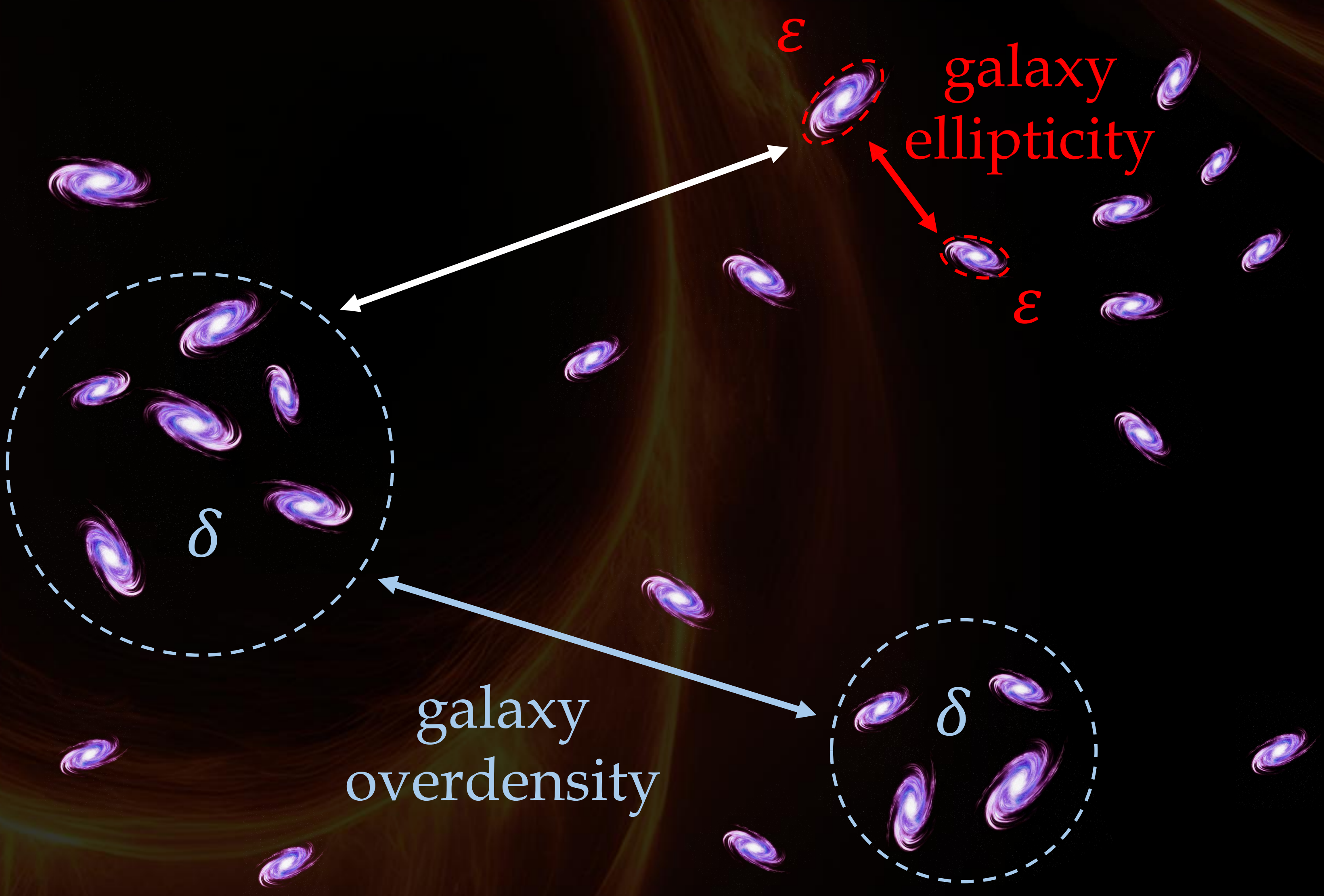


The 3x2pt correlation scheme

$\langle \delta \times \delta \rangle$

$\langle \varepsilon \times \varepsilon \rangle$

$\langle \delta \times \varepsilon \rangle$



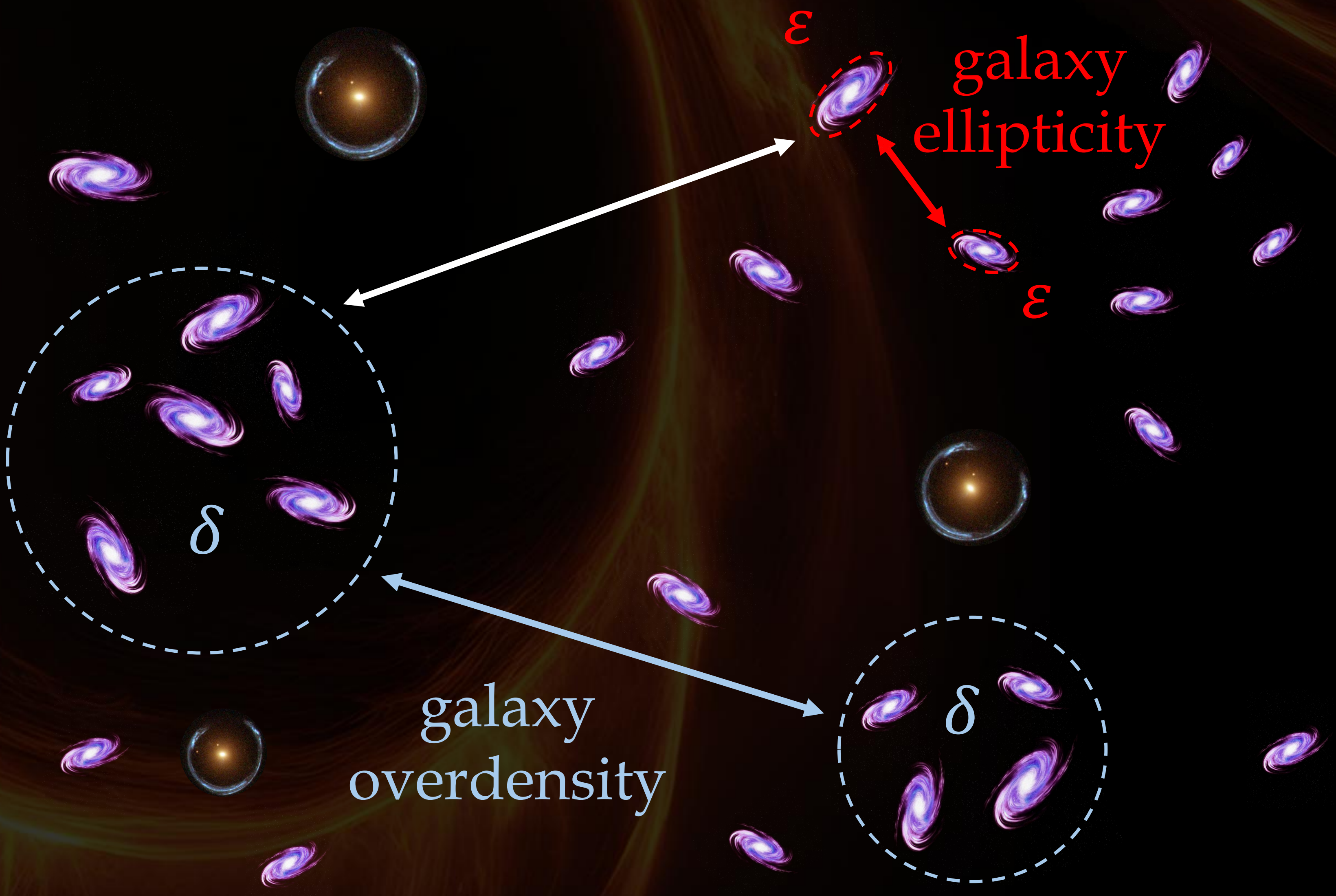
The 3x2pt correlation scheme

$\sim 10^5$ lenses with *Euclid*

$\langle \delta \times \delta \rangle$

$\langle \varepsilon \times \varepsilon \rangle$

$\langle \delta \times \varepsilon \rangle$



The 6x2pt correlation scheme

$\langle \delta \times \delta \rangle$

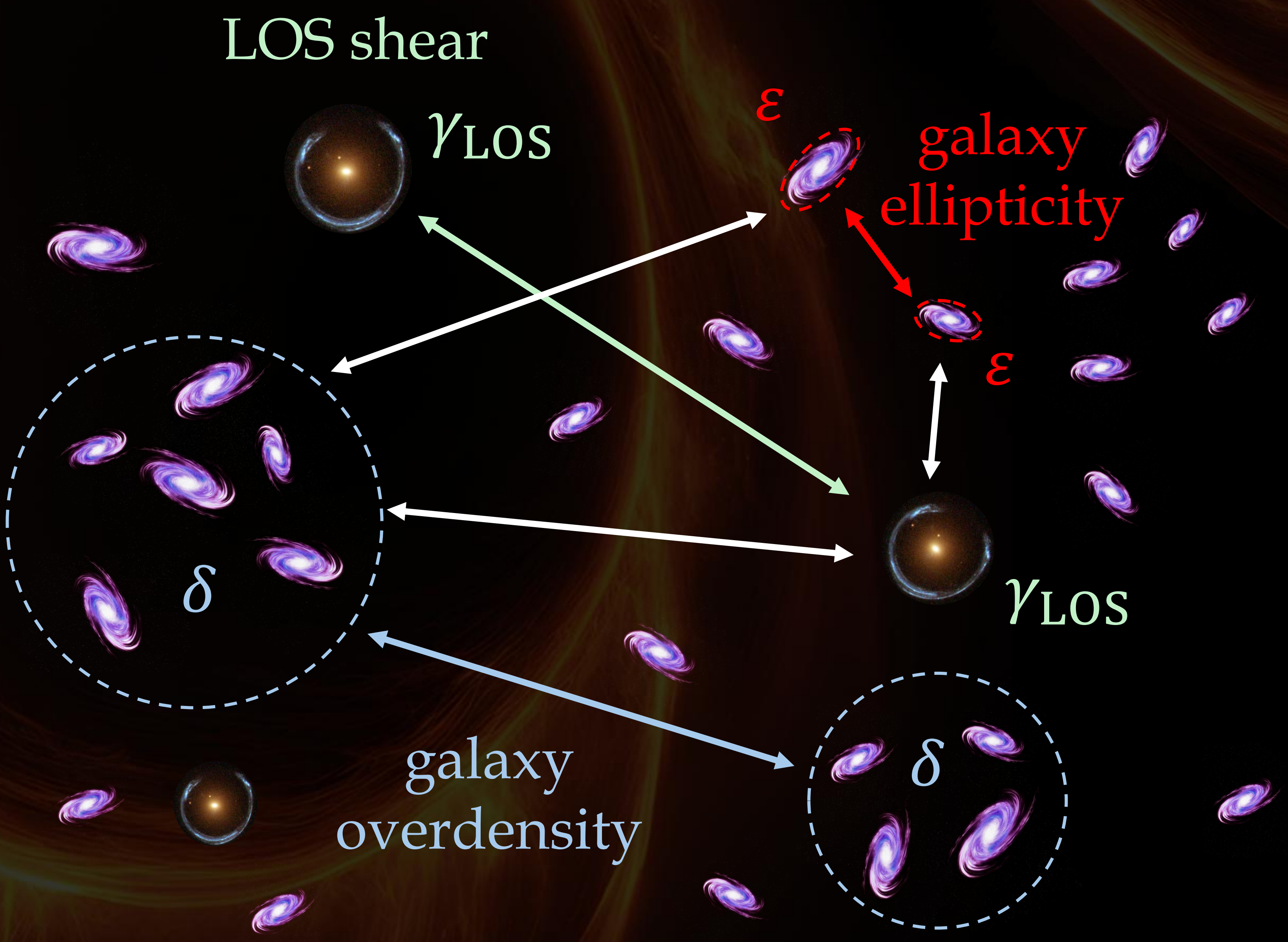
$\langle \varepsilon \times \varepsilon \rangle$

$\langle \delta \times \varepsilon \rangle$

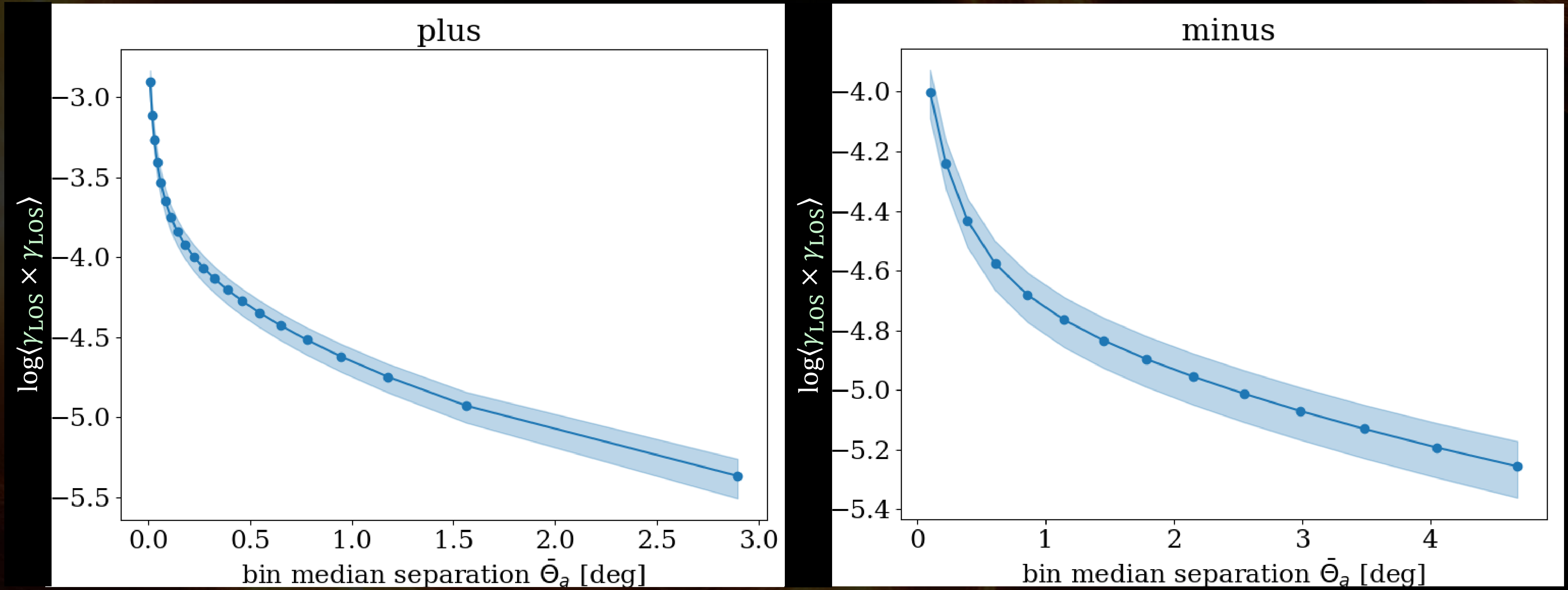
$\langle \gamma_{\mathrm{LOS}} \times \gamma_{\mathrm{LOS}} \rangle$

$\langle \gamma_{\mathrm{LOS}} \times \delta \rangle$

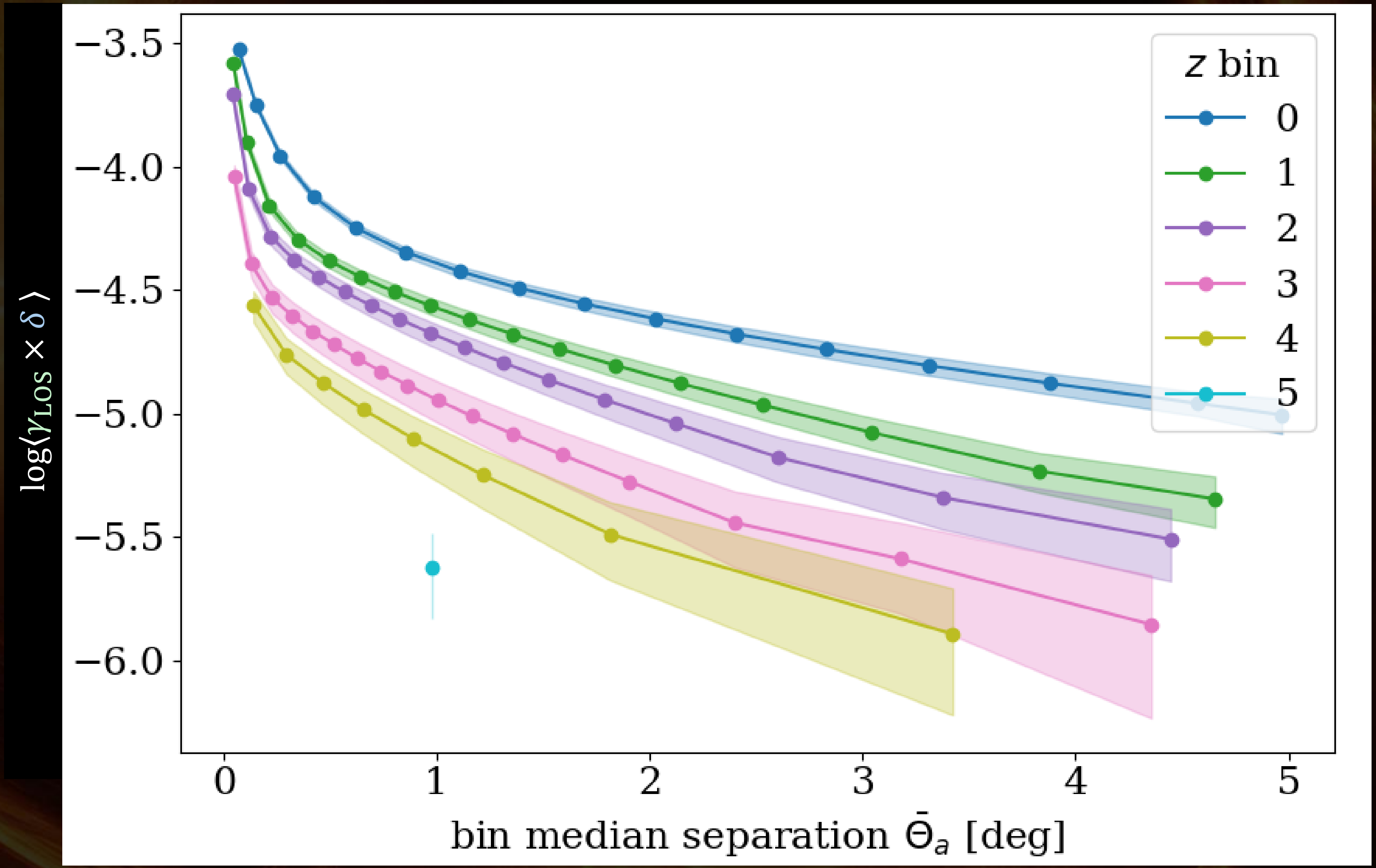
$\langle \gamma_{\mathrm{LOS}} \times \varepsilon \rangle$



LOS shear × LOS shear correlation function



$\sigma_{\rm LOS} = 1\%$ $N_{\rm lens} = 10^5$

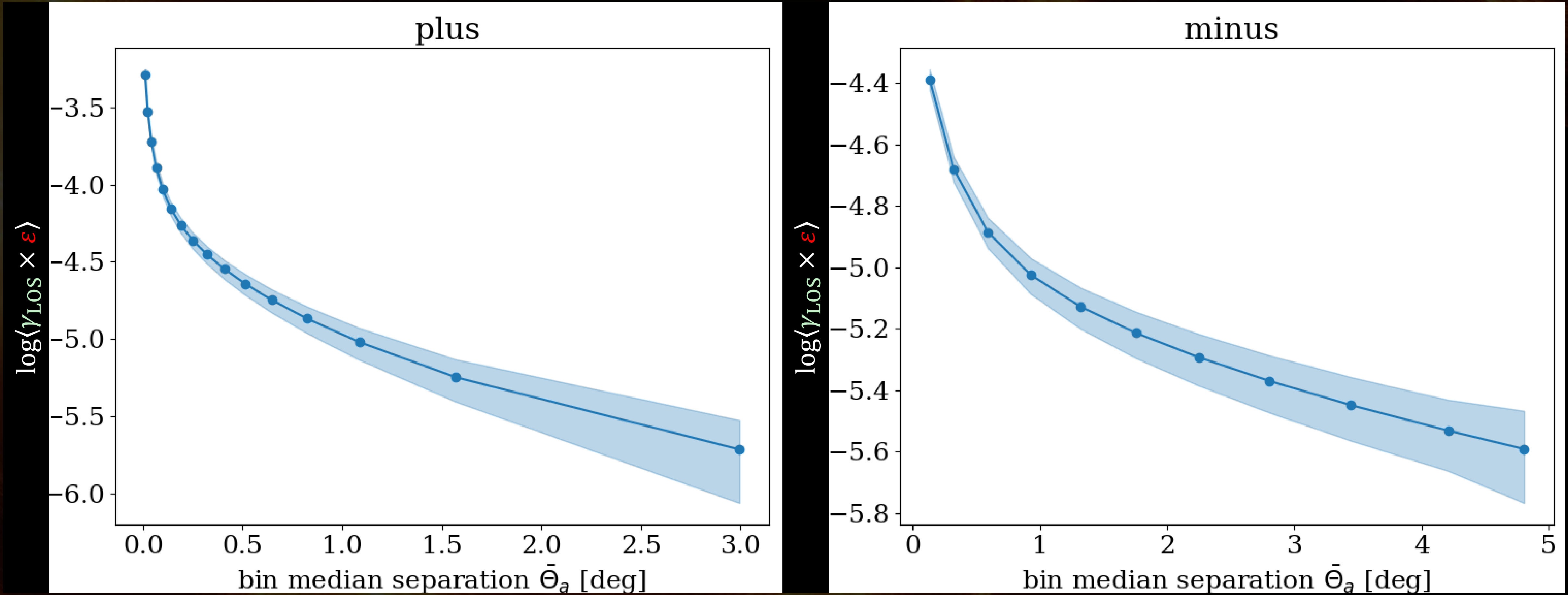


LOS shear × galaxy density correlation function

$\sigma_{\rm LOS} = 5\%$

$N_{\rm lens} = 10^5$

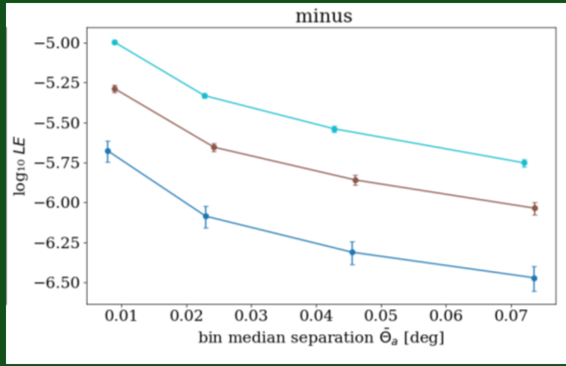
LOS shear × galaxy ellipticity correlation function



$\sigma_{\rm LOS} = 10\%$ $\quad N_{\rm lens} = 10^4$

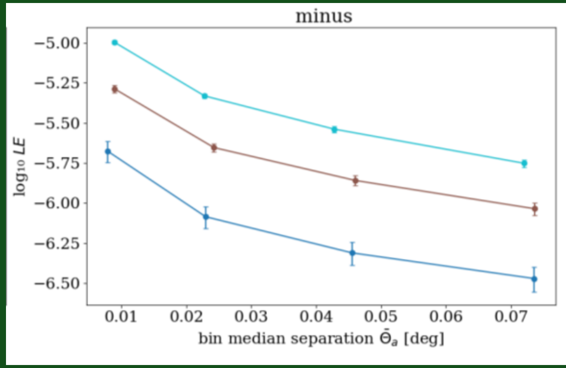
Observed Correlation Functions

How the relationship between observables changes as their separation increases



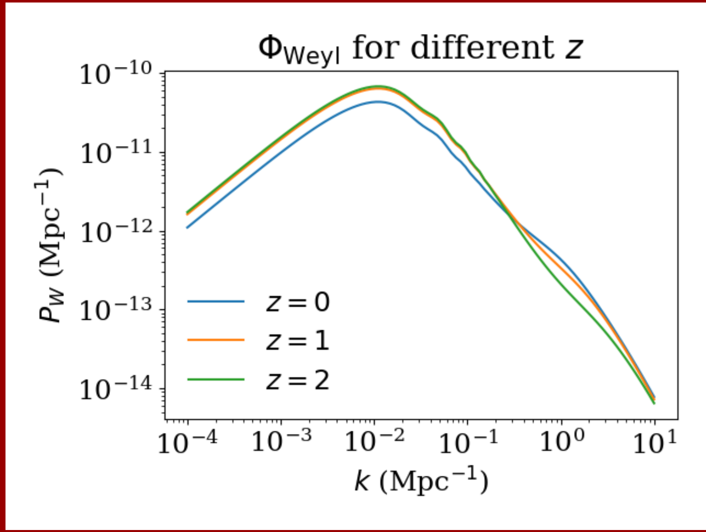
Observed Correlation Functions

How the relationship between observables changes as their separation increases



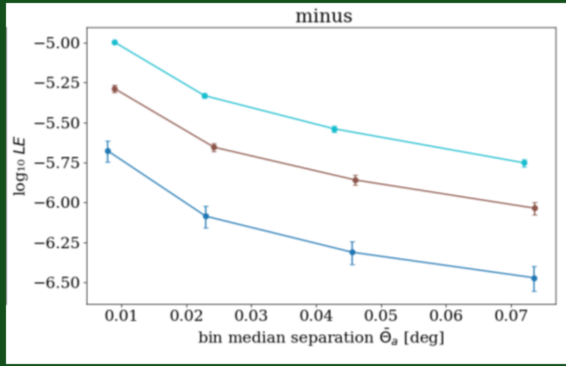
Matter power spectrum

How matter is distributed through the universe



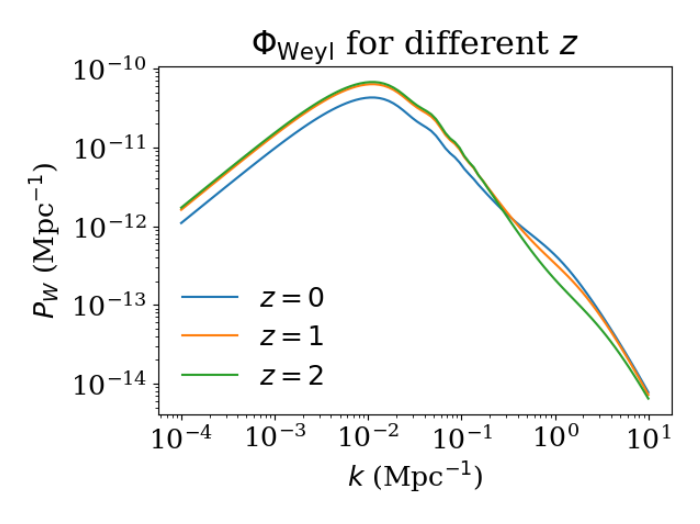
Observed Correlation Functions

How the relationship between observables changes as their separation increases



Matter power spectrum

How matter is distributed through the universe



Cosmological parameters

Covariance matrices

Cosmic covariance

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2}\Big[\int_{\Omega}\frac{d^2 l''}{\Omega}\,\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|l''+l'-l|)\\
&+\int_{\Omega}\frac{d^2 l''}{\Omega}\,\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|l''+l'-l|)\cos 4(\psi''-\psi_{l''+l'-l})\\
&+\int_{\Omega}\frac{d^2 l''}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l''-l|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l''+l'|)\\
&+\int_{\Omega}\frac{d^2 l''}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l''-l|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l''+l'|)\cos 4(\psi_{l''+l'}-\psi_{l''-l})\Big]\,, \qquad (1.1)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|l''+l-l|)\cos 4(\psi'-\psi_{l''+l'-l})\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|l''+l'-l|)\cos 4(\psi'-\psi'')\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi'-\psi_{l-l''})\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi'-\psi_{l'+l''})\Big]\,, \qquad (1.2)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|l''+l'-l)\cos 4(\psi_{l''+l'-l}-\psi)\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\cos 4(\psi''-\psi)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|l''+l'-l)\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\cos 4(\psi_{l-l''}-\psi)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\\
&+\int_{\Omega_{a''}}\frac{d^2 l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l'')\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi_{l'+l''}-\psi)\Big]\,, \qquad (1.3)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2}\Big[\int_{\Omega''_a}\frac{d^2 l''}{\Omega''_a}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|l''+l'-l|)\cos 4(\psi-\psi')\\
&+\int_{\Omega''_a}\frac{d^2 l''}{\Omega''_a}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|l''+l'-l|)\cos 4(\psi''+\psi_{l''+l'-l}-\psi'-\psi)\\
&+\int_{\Omega''_a}\frac{d^2 l''}{\Omega''_a}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi-\psi')\\
&+\int_{\Omega''_a}\frac{d^2 l''}{\Omega''_a}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi_{l'+l''}+\psi_{l-l''}-\psi-\psi')\Big]. \qquad (1.4)
\end{aligned}$$

2

Sparsity covariance

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2L}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(|l-l'|)\right]\\
&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\frac{1}{2}\xi^{+}_{\gamma_{\rm LOS}}(|l-l'|)\xi^{+}_{\epsilon}(0)\right]\\
&\quad+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(0)\\
&\quad+\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)+\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\right]\Big\}\,, \qquad (1.5)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2L}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi'-\psi_{l-l'})\right]\\
&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{-}_{\gamma_{\rm LOS}}(|l-l'|)\xi^{+}_{\epsilon}(0)\cos 4(\psi'-\psi_{l-l'})\right]\\
&\quad+\frac{2\delta_{aa'}\delta_{bb'}\Omega}{LG_b\Omega_a}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l).\,, \qquad (1.6)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2L}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]\\
&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{-}_{\gamma_{\rm LOS}}(|l-l'|)\xi^{+}_{\epsilon}(0)\cos 4(\psi_{l-l'}-\psi)\right]\\
&\quad+\frac{2\delta_{aa'}\delta_{bb'}\Omega}{LG_b\Omega_a}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\,, \qquad (1.7)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]\\
&=\frac{1}{2L}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(|l-l'|)\cos 4(\psi-\psi')\right]\\
&\quad+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\epsilon}(0)\xi^{+}_{\gamma_{\rm LOS}}(|l-l'|)\cos 4(\psi-\psi')\right]\\
&\quad+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(0)\\
&\quad+\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2 l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)+\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\right]\Big\}. \qquad (1.8)
\end{aligned}$$

Noise covariance

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{+}_{\rm LOS}(|l-l'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\rm LOS}(0)\right]\sigma_\epsilon^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{+}_{\epsilon}(|l-l'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2\,, \qquad (1.9)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{-}_{\rm LOS}(|l-l'|)\cos 4(\psi_{l-l'}-\psi')\right]\sigma_\epsilon^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi')\right]\sigma_n^2\,, \qquad (1.10)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{-}_{\rm LOS}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]\sigma_\epsilon^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]\sigma_n^2\,, \qquad (1.11)
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{+}_{\rm LOS}(|l-l'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\rm LOS}(0)\right]\sigma_\epsilon^2\\
&\quad+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2 l}{\Omega_a}\int_{\Omega_{a'}}d^2 l'\,\xi^{+}_{\epsilon}(|l-l'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2 \qquad (1.12)
\end{aligned}$$

Covariance matrices

Cosmic covariance

$$\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2}\Big[\int_{\Omega}\frac{d^2l''}{\Omega}\,\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|l''+l'-l|)$$
$$+\int_{\Omega}\frac{d^2l''}{\Omega}\,\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|l''+l'-l|)\cos 4(\psi''-\psi_{l''+l'-l})$$
$$+\int_{\Omega}\frac{d^2l''}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l''-l|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l''+l'|)$$
$$+\int_{\Omega}\frac{d^2l''}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l''-l|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l''+l'|)\cos 4(\psi_{l''+l'}-\psi_{l''-l})\Big]\,,\quad(1.1)$$

$$\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|l''+l-l|)\cos 4(\psi'-\psi_{l''+l'-l})$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|l''+l'-l|)\cos 4(\psi'-\psi'')$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi'-\psi_{l-l''})$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi'-\psi_{l'+l''})\Big]\,,\quad(1.2)$$

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2}\Big[\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{-}_{\epsilon}(|l''+l'-l)\cos 4(\psi_{l''+l'-l}-\psi)$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\cos 4(\psi''-\psi)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{+}_{\epsilon}(|l''+l'-l)$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\cos 4(\psi_{l-l''}-\psi)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)$$
$$+\int_{\Omega_{a''}}\frac{d^2l''}{\Omega_{a''}}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l'')\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi_{l'+l''}-\psi)\Big]\,,\quad(1.3)$$

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2}\Big[\int_{\Omega''_a}\frac{d^2l''}{\Omega''_a}\xi^{+}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{+}_{\epsilon}(|l''+l'-l|)\cos 4(\psi-\psi')$$
$$+\int_{\Omega''_a}\frac{d^2l''}{\Omega''_a}\xi^{-}_{\gamma_{\rm LOS}}(l'')\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{-}_{\epsilon}(|l''+l'-l|)\cos 4(\psi''+\psi_{l''+l'-l}-\psi'-\psi)$$
$$+\int_{\Omega''_a}\frac{d^2l''}{\Omega''_a}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi-\psi')$$
$$+\int_{\Omega''_a}\frac{d^2l''}{\Omega''_a}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l-l''|)\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\,\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(|l'+l''|)\cos 4(\psi_{l'+l''}+\psi_{l-l''}-\psi-\psi')\Big].\quad(1.4)$$

2

Sparsity covariance

$$\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2L}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(|l-l'|)\right]$$
$$+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\frac{1}{2}\xi^{+}_{\gamma_{\rm LOS}}(|l-l'|)\xi^{+}_{\epsilon}(0)\right]$$
$$+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(0)$$
$$+\int_{\Omega_a}\frac{d^2l}{\Omega_a}\left[3\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)+\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\right]\Big\}\,,\quad(1.5)$$

(21 of these)

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2L}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]$$
$$+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{-}_{\gamma_{\rm LOS}}(|l-l'|)\xi^{+}_{\epsilon}(0)\cos 4(\psi_{l-l'}-\psi)\right]$$
$$+\frac{2\delta_{aa'}\delta_{bb'}\Omega}{LG_b\Omega_a}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\,,\quad(1.7)$$

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]$$
$$=\frac{1}{2L}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(|l-l'|)\cos 4(\psi-\psi')\right]$$
$$+\frac{\delta_{bb'}}{2G_b}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l')+\xi^{+}_{\epsilon}(0)\xi^{+}_{\gamma_{\rm LOS}}(|l-l'|)\cos 4(\psi-\psi')\right]$$
$$+\frac{\delta_{aa'}\delta_{bb'}\Omega}{2LG_b\Omega_a}\Big\{\xi^{+}_{\gamma_{\rm LOS}}(0)\xi^{+}_{\epsilon}(0)$$
$$+\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}\frac{d^2l'}{\Omega_{a'}}\left[3\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{-}_{\gamma_{\rm LOS}\times\epsilon}(l)+\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\xi^{+}_{\gamma_{\rm LOS}\times\epsilon}(l)\right]\Big\}.\quad(1.8)$$

Noise covariance

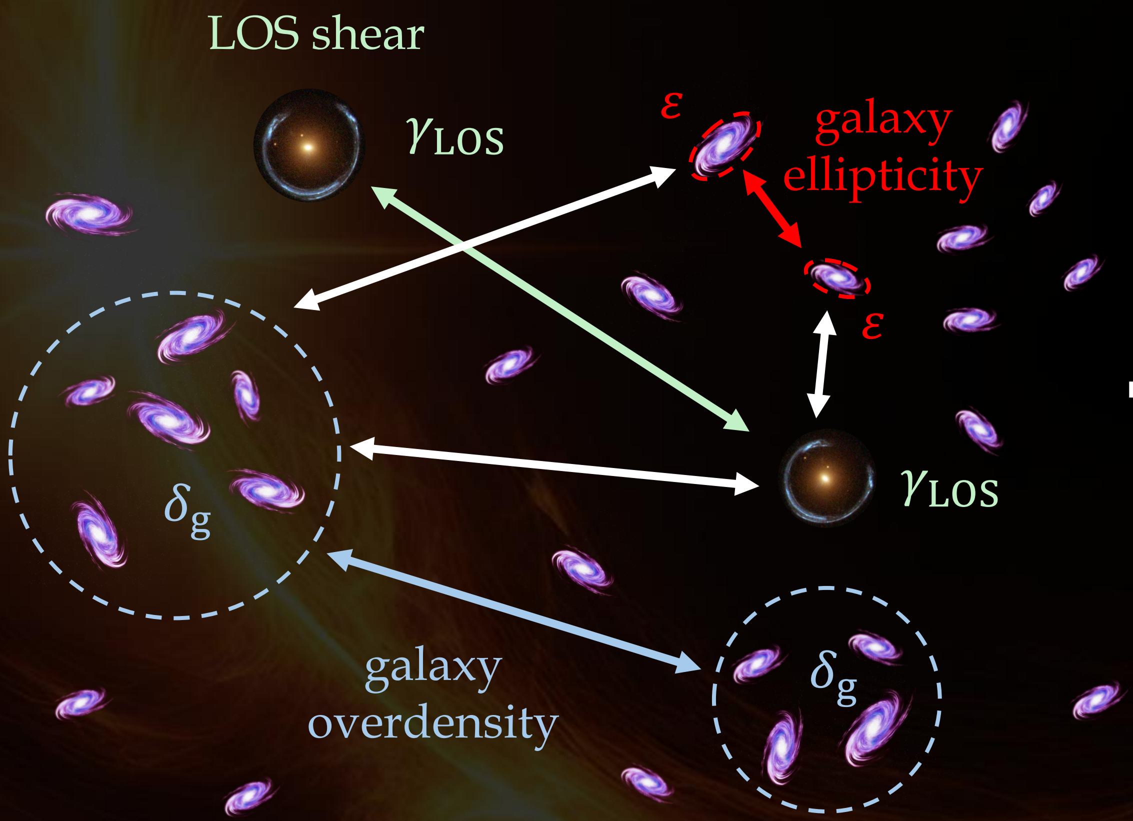
$$\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{+}_{\rm LOS}(|l-l'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\rm LOS}(0)\right]\sigma_\epsilon^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{+}_{\epsilon}(|l-l'|)+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2\,,\quad(1.9)$$

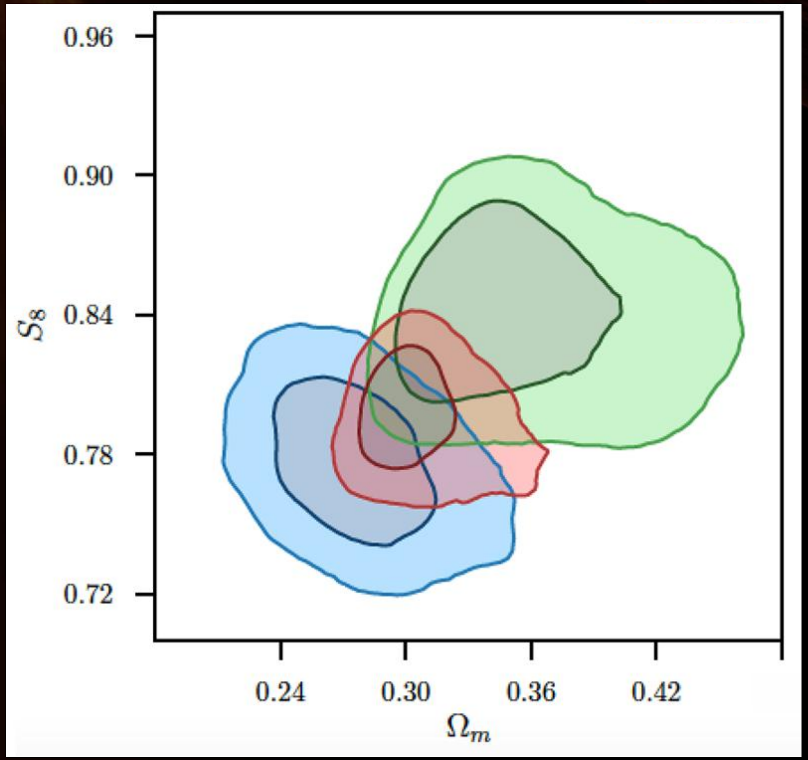
$$\left[\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{-}_{\rm LOS}(|l-l'|)\cos 4(\psi_{l-l'}-\psi')\right]\sigma_\epsilon^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi')\right]\sigma_n^2\,,\quad(1.10)$$

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{+}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=-\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{-}_{\rm LOS}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]\sigma_\epsilon^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{-}_{\epsilon}(|l-l'|)\cos 4(\psi_{l-l'}-\psi)\right]\sigma_n^2\,,\quad(1.11)$$

$$\left[\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a,b),\hat{\xi}^{-}_{\gamma_{\rm LOS}\times\epsilon}(a',b')\right]=\frac{\Omega}{4LG_b\Omega_a}\delta_{aa'}\delta_{bb'}\sigma_n^2\sigma_{\epsilon_0}^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{L}{\Omega}\delta_{bb'}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{+}_{\rm LOS}(|l-l'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\rm LOS}(0)\right]\sigma_\epsilon^2$$
$$+\frac{\Omega}{2LG_b\Omega_a}\left[\frac{G_{b'}}{\Omega}\int_{\Omega_a}\frac{d^2l}{\Omega_a}\int_{\Omega_{a'}}d^2l'\,\xi^{+}_{\epsilon}(|l-l'|)\cos 4(\psi-\psi')+\delta_{aa'}\delta_{bb'}\,\xi^{+}_{\epsilon}(0)\right]\sigma_n^2\quad(1.12)$$

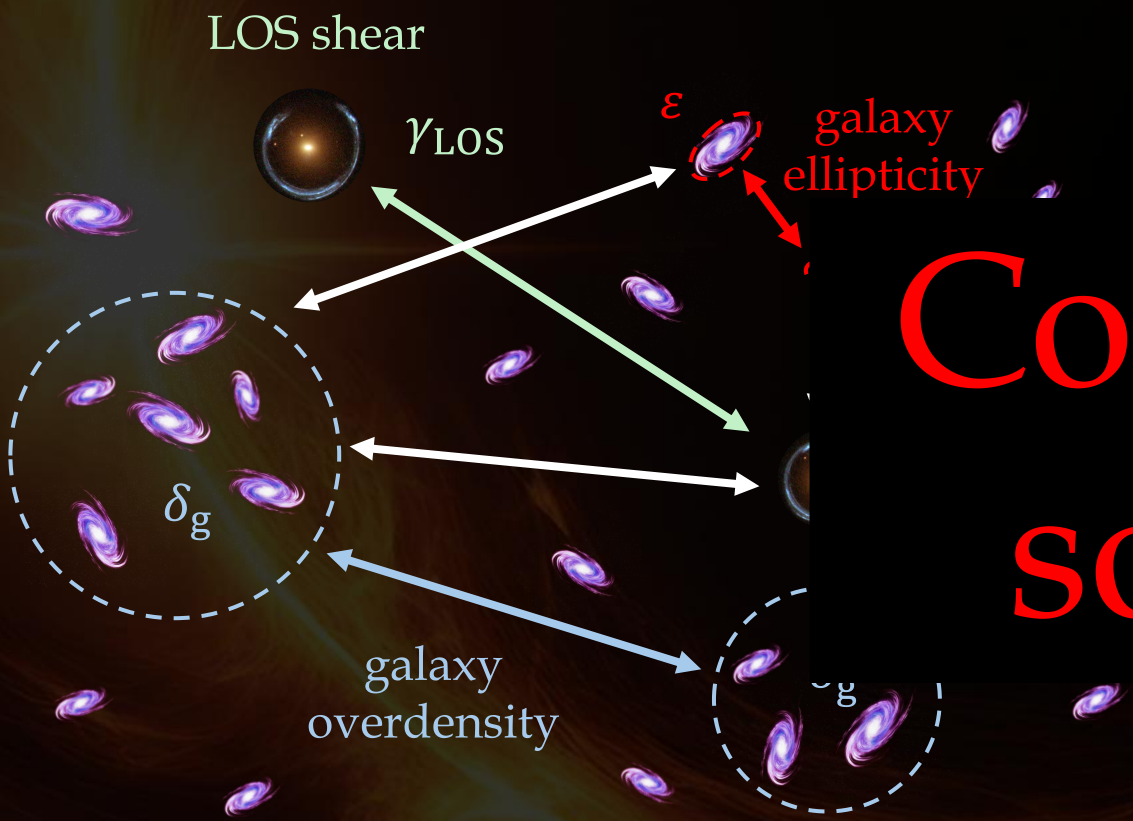
The goal



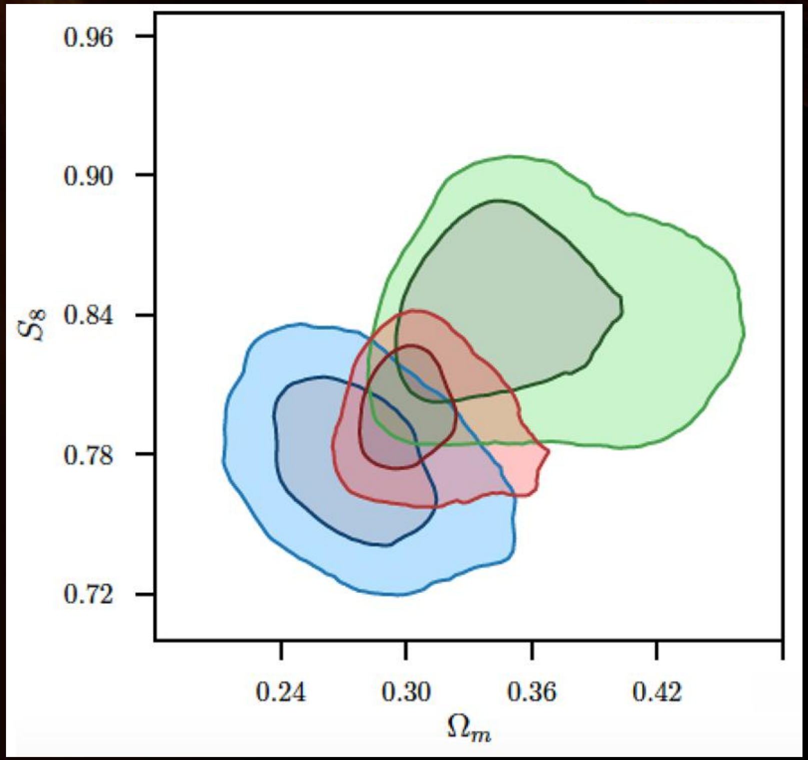


Credit: T. M. C. Abbott *et al.*, Phys. Rev. D (2018)

The goal



Coming soon!



Credit: T. M. C. Abbott *et al.*, Phys. Rev. D (2018)

Conclusions

- Gravitational lensing is a powerful observational tool in cosmology

Conclusions

- Gravitational lensing is a powerful observational tool in cosmology
- Matter along the line of sight has a non-negligible effect on strong lensing observables, as with any measure of distance, shape or luminosity

Conclusions

- Gravitational lensing is a powerful observational tool in cosmology
- Matter along the line of sight has a non-negligible effect on strong lensing observables, as with any measure of distance, shape or luminosity
- Ignoring or under-parameterising these effects can lead to precise but inaccurate cosmological constraints

Conclusions

- Gravitational lensing is a powerful observational tool in cosmology
- Matter along the line of sight has a non-negligible effect on strong lensing observables, as with any measure of distance, shape or luminosity
- Ignoring or under-parameterising these effects can lead to precise but inaccurate cosmological constraints
- Measurements of these effects offers a new cosmological probe

Thanks for listening!

daniel.johnson@umontpellier.fr

Further reading:

Daniel Johnson, Thomas Collett, Tian Li, Pierre Fleury,
Line-of-sight effects on double source plane lenses, arXiv:2501.17153

Daniel Johnson, Pierre Fleury, Julien Larena, Lucia Marchetti,
Foreground biases in strong gravitational lensing, JCAP 10 (2024) 055, arXiv:2405.04194

Natalie B. Hogg, Anowar J. Shajib, Daniel Johnson, Julien Larena,
Line-of-sight shear in SLACS strong lenses, arXiv:2501.16292

Natalie B. Hogg, Pierre Fleury, Julien Larena, Matteo Martinelli,
Measuring line-of-sight shear with Einstein rings: a proof of concept, MNRAS 520 (2023) 04, arXiv:2210.07210

Pierre Fleury, Julien Larena, Jean-Philippe Uzan,
Line-of-sight effects in strong gravitational lensing, JCAP 08 (2021) 024, arXiv:2104.08883

The minimal lens model

Fleury et al. 2021, 2104.08883

$$\tilde{\beta} = \boldsymbol{\mathcal{A}}_{\mathrm{LOS}}\boldsymbol{\theta} - \frac{\mathrm{d}\psi_{\mathrm{eff}}}{\mathrm{d}\boldsymbol{\theta}}$$

(A single main lens + tidal line-of-sight effects)

The minimal lens model

Fleury et al. 2021, 2104.08883

$$\tilde{\boldsymbol{\beta}} = \mathcal{A}_{\rm LOS}\boldsymbol{\theta} - \frac{\mathrm{d}\psi_{\rm eff}}{\mathrm{d}\boldsymbol{\theta}}$$

"External" convergence and shear

$$\psi_{\rm eff}(\boldsymbol{\theta}) \equiv \psi[\mathcal{A}_{\rm od}\boldsymbol{\theta}]$$

Foreground convergence and shear

The effective potential

$$\psi_{\mathrm{eff}}(\boldsymbol{\theta}) \equiv \psi[\mathcal{A}_{\mathrm{od}}\boldsymbol{\theta}]$$

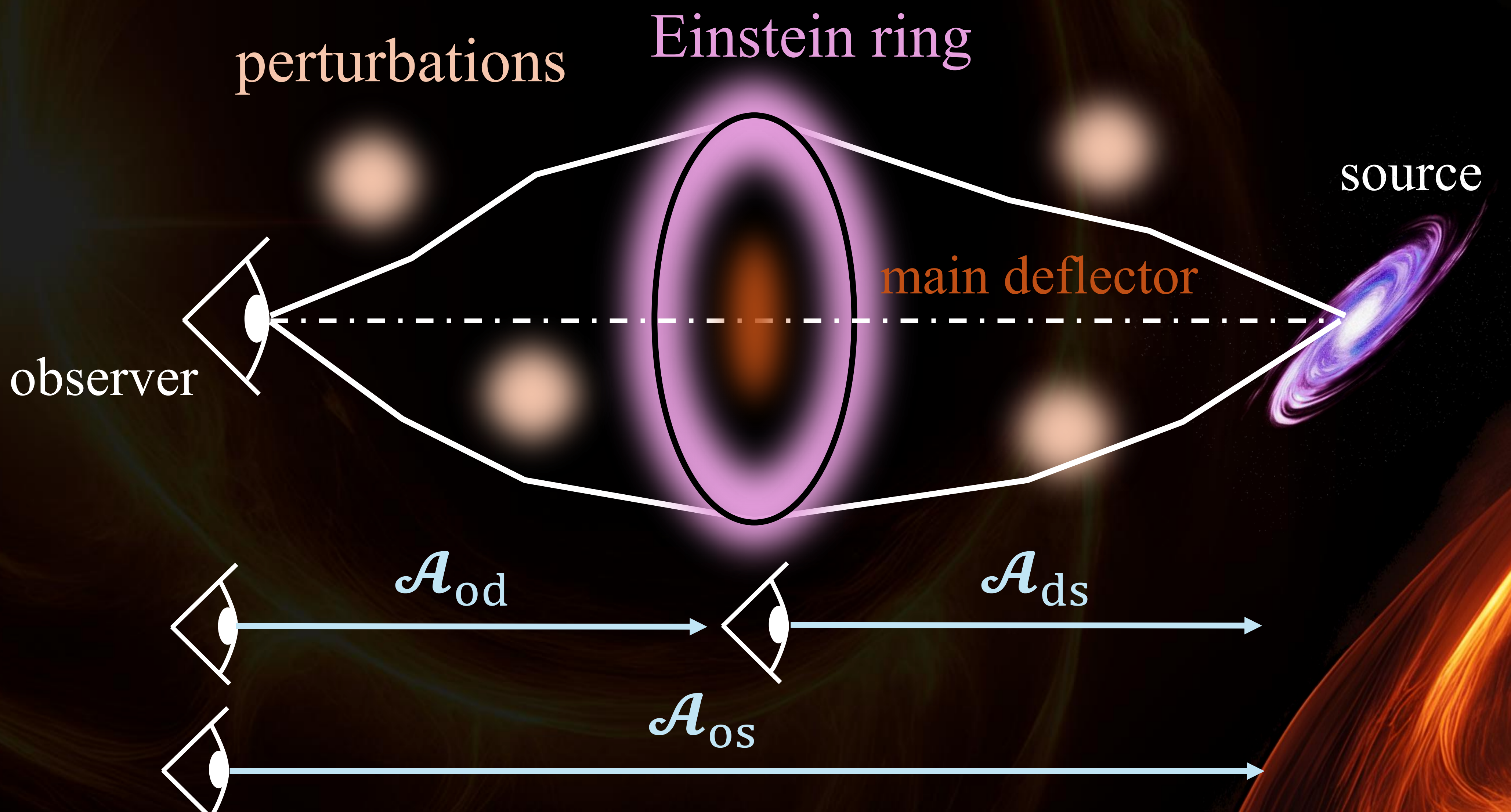
Sufficiently complicated main lens model should absorb foreground effects, but caution is needed when interpreting parameters

The effective potential

$$\psi_{\mathrm{eff}}(\boldsymbol{\theta}) \equiv \psi[\mathcal{A}_{\mathrm{od}}\boldsymbol{\theta}]$$

Sufficiently complicated main lens model should absorb foreground effects, but caution is needed when interpreting parameters

Weak lensing of strong lensing



The MSD and angular diameter distances

The time delay distance

$$\Delta t \propto \frac{\tilde{D}_{\mathrm{od}} \tilde{D}_{\mathrm{os}}}{\tilde{D}_{\mathrm{ds}}} \propto \frac{1-\kappa_{\mathrm{od}}}{H_0} \frac{1-\kappa_{\mathrm{os}}}{1-\kappa_{\mathrm{ds}}}$$

Velocity dispersion measurements

$$\sigma \propto \frac{\tilde{D}_{\mathrm{os}}}{\tilde{D}_{\mathrm{ds}}} \propto \frac{1-\kappa_{\mathrm{os}}}{1-\kappa_{\mathrm{ds}}}$$

Velocity dispersion constraints

Teodori et al. 2022, 2201.05111

$$\sigma^2(\boldsymbol{\theta}) = \underbrace{\frac{1-\kappa_{\rm LOS}}{1-\kappa_{\rm od}}}_{\text{The MSD factor}} \underbrace{\frac{D_{\rm os}}{D_{\rm ds}}}_{\text{independent of } H_0} \underbrace{J(\theta, \theta_{\rm E}, \gamma)}_{\text{Depends only on lensing observables}}$$

Foreground shear effects (ellipticity is biased)

$$\varepsilon_{\text{eff}} \equiv \varepsilon + 2(5-\gamma)g_{\text{od}}$$

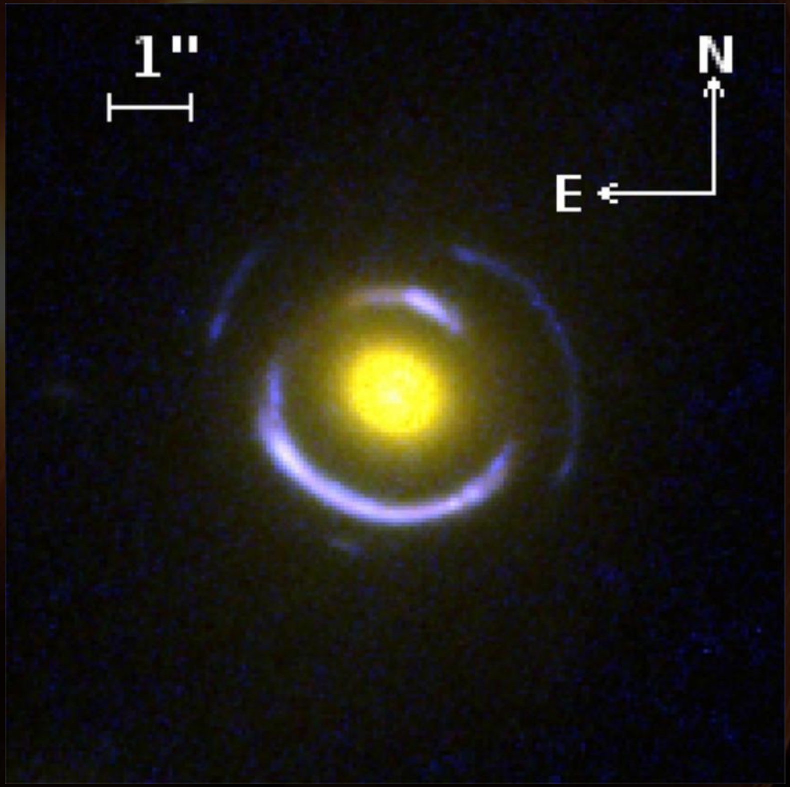
If the foreground shear is left out of a model, you will measure an effective ellipticity. However, other parameters should be unaffected

Double source plane lensing

Two Einstein radii!

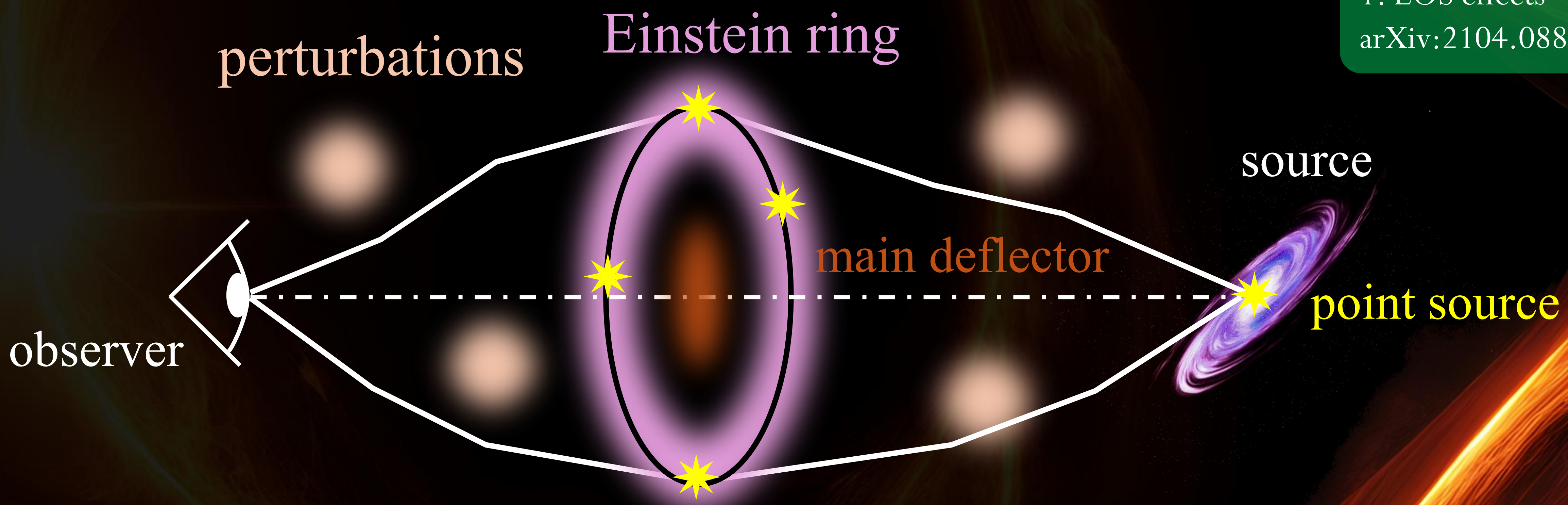
Main observable – the cosmological scaling factor:

$$\eta = \frac{D_{\mathrm{s1}} D_{\mathrm{ds2}}}{D_{\mathrm{ds1}} D_{\mathrm{s2}}} \approx \frac{\theta_{\mathrm{E,2}}}{\theta_{\mathrm{E,1}}}$$



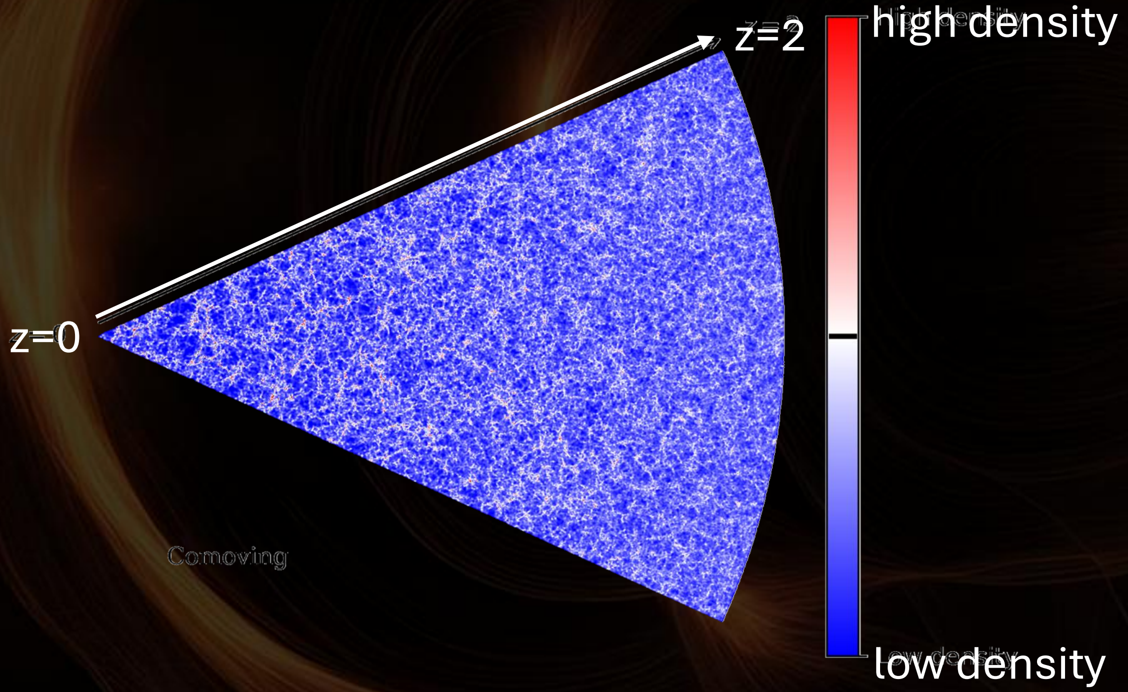
Weak lensing of strong lensing

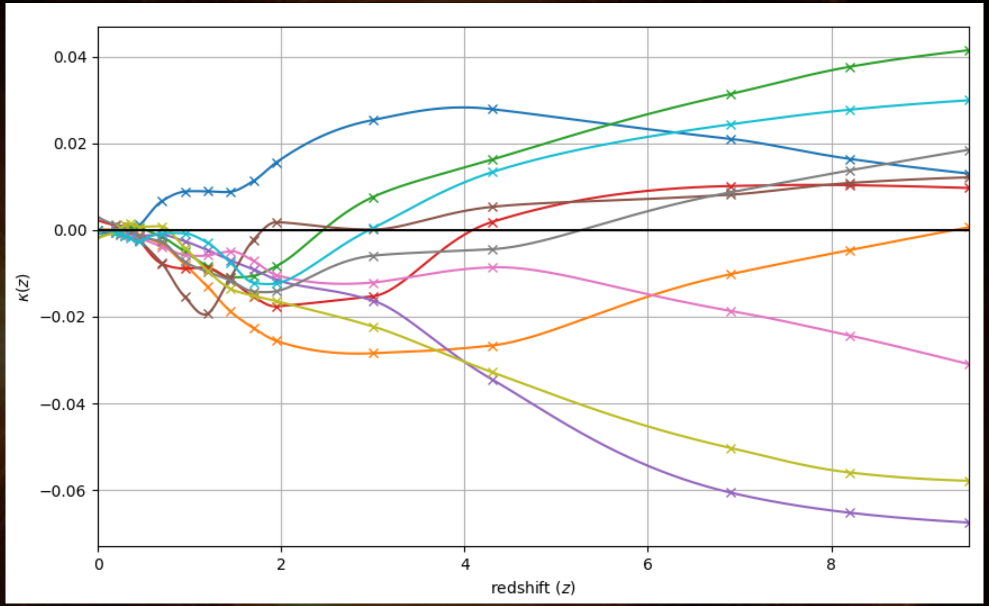
1. LOS effects
arXiv:2104.08883



$$\mathcal{A}_{\mathrm{LOS}} \approx \mathcal{A}_{\mathrm{od}} + \mathcal{A}_{\mathrm{os}} - \mathcal{A}_{\mathrm{ds}}$$

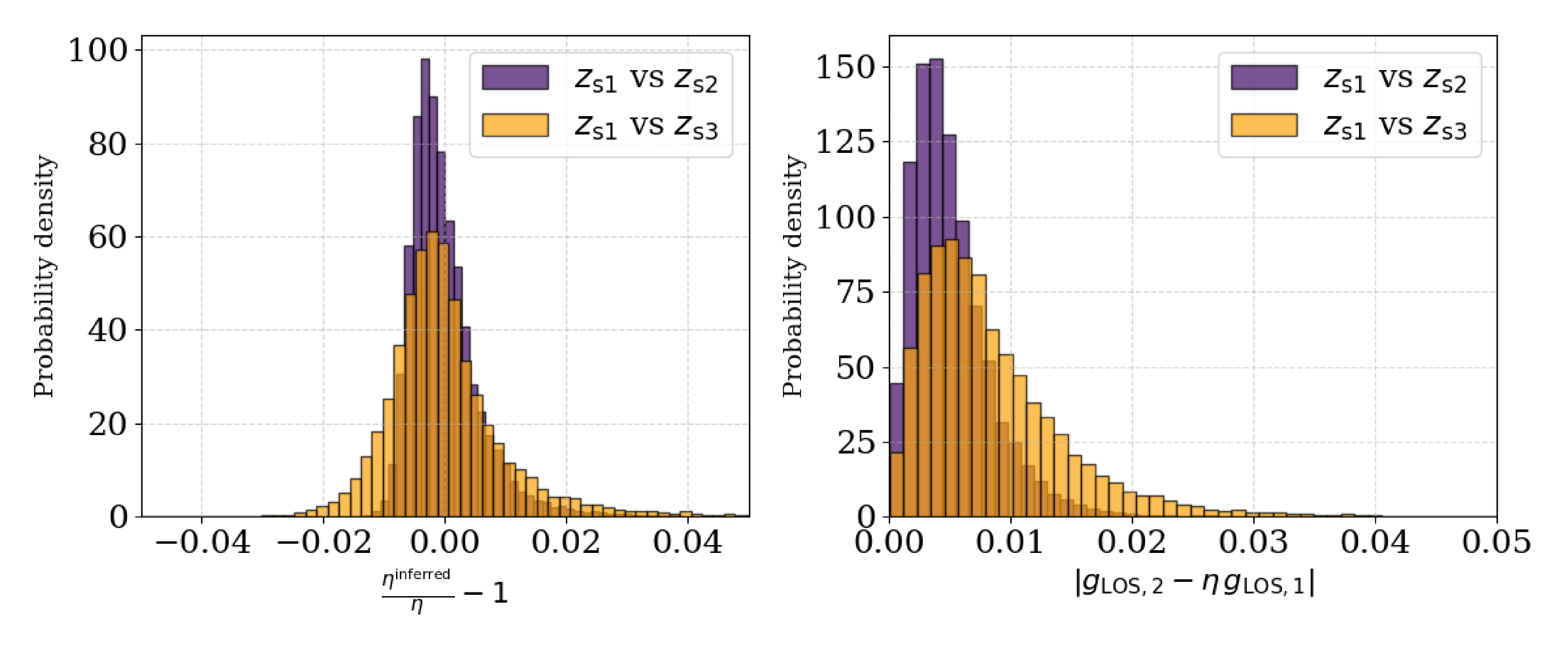
Generating lines of sight



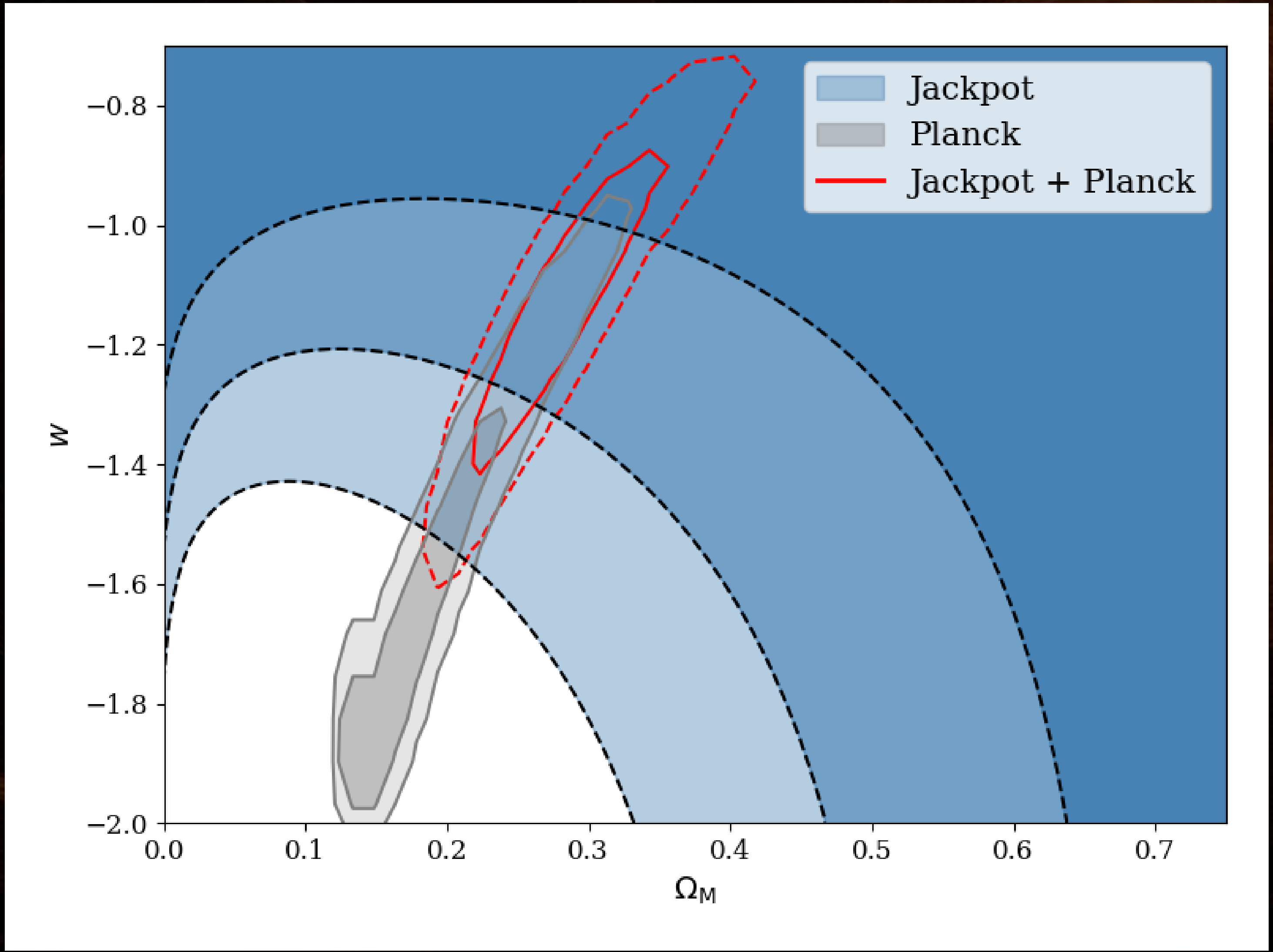


- Interpolate between datapoints
- Integrate to determine background terms

The Jackpot lens



The Jackpot lens



Velocity dispersion constraints

$$\sigma^2(\boldsymbol{\theta}) = \underbrace{\frac{1-\kappa_{\mathrm{LOS}}}{1-\kappa_{\mathrm{od}}}}_{\text{The MSD factor}} \underbrace{\frac{D_{\mathrm{os}}}{D_{\mathrm{ds}}}}_{\text{independent of } H_0} \underbrace{J(\theta, \theta_{\mathrm{E}}, \gamma)}_{\text{Depends only on lensing observables}}$$

Comments on MSD
arXiv: 2201.0511

3/4 – the line of sight as a source of uncertainty

Comments on MSD
arXiv: 2201.0511

A problem!

The factor we need

$$1 - \kappa_{\mathrm{LOS}}$$

What velocity dispersion measurements give us

$$\frac{1 - \kappa_{\mathrm{LOS}}}{1 - \kappa_{\mathrm{od}}}$$

Comments on MSD
arXiv: 2201.0511

A problem!

The factor we need $1 - \kappa_{\mathrm{LOS}}$

What velocity dispersion measurements give us $\dfrac{1 - \kappa_{\mathrm{LOS}}}{1 - \kappa_{\mathrm{od}}}$

$$1 - \kappa_{\mathrm{ext}}$$

Historically, both terms were simply identified as the external convergence, but these are not the same!

Comments on MSD
arXiv: 2201.0511

A problem!

The factor we need $1 - \kappa_{\mathrm{LOS}}$

What velocity dispersion measurements give us $\dfrac{1 - \kappa_{\mathrm{LOS}}}{1 - \kappa_{\mathrm{od}}}$

$$\left.\begin{array}{l} 1 - \kappa_{\mathrm{LOS}} \\ \dfrac{1 - \kappa_{\mathrm{LOS}}}{1 - \kappa_{\mathrm{od}}} \end{array}\right\} \quad 1 - \kappa_{\mathrm{ext}}$$

Historically, both terms were simply identified as the external convergence, but these are not the same!

Leads to a bias on the inferred H_0 value

$$\frac{H_0^{\mathrm{inferred}}}{H_0} \approx 1 + \kappa_{\mathrm{od}}$$

The MSD and H_0

$$H_0 = (1 - \kappa_{\rm LOS}) H_0^{\rm model}$$

true value

line-of-sight convergence

modelled value

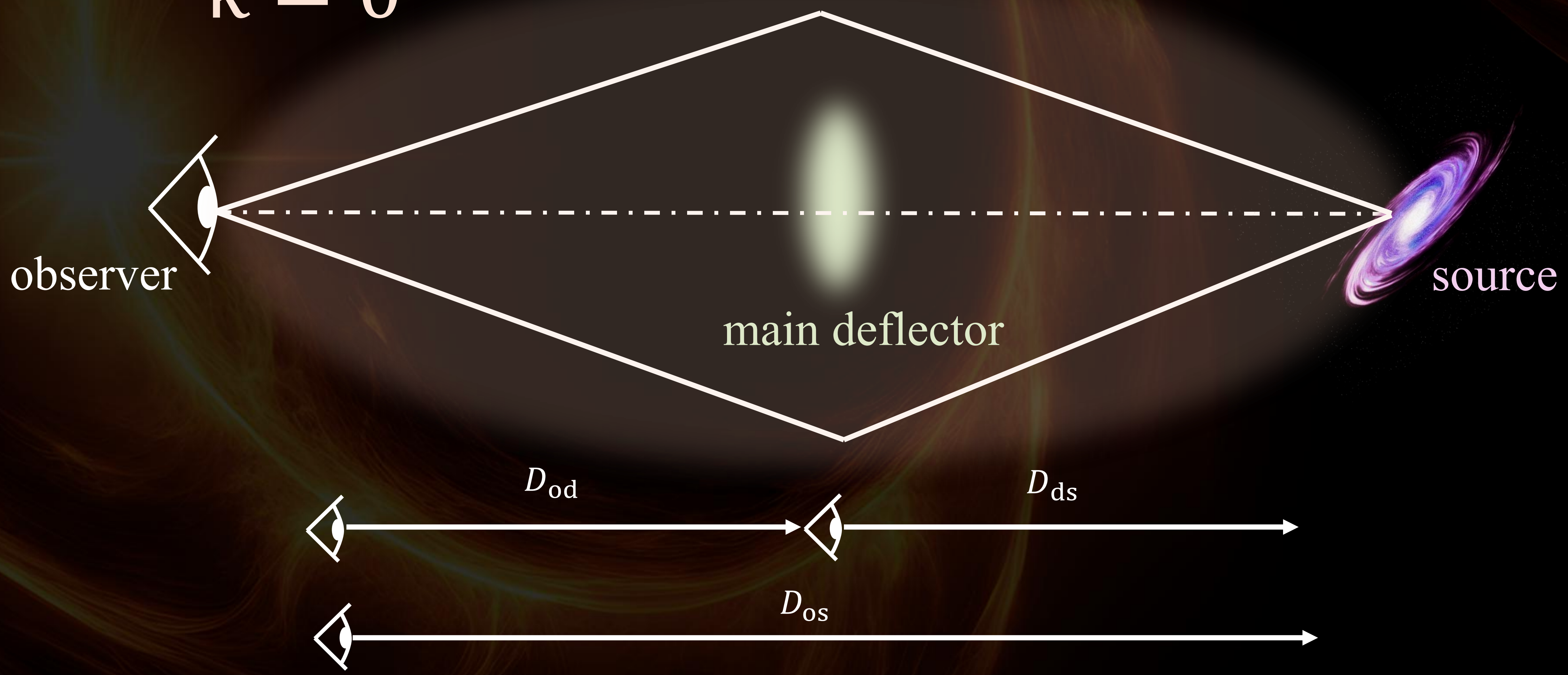
Double-source-plane lenses

$$\eta = \frac{D_{\mathrm{s1}} D_{\mathrm{ds2}}}{D_{\mathrm{ds1}} D_{\mathrm{s2}}}$$

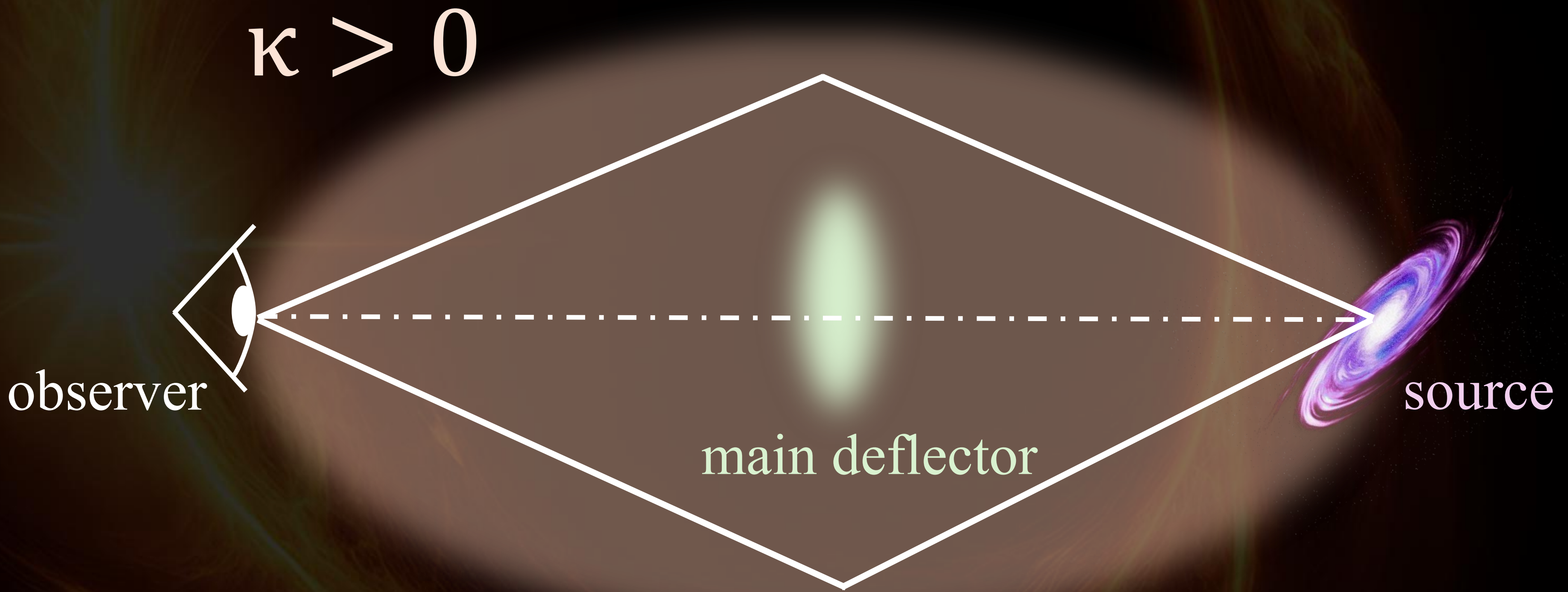
$$\frac{\eta^{\mathrm{inferred}}}{\eta} = \frac{(1-\kappa_{\mathrm{ds1}})(1-\kappa_{\mathrm{s2}})}{(1-\kappa_{\mathrm{s1}})(1-\kappa_{\mathrm{ds2}})}$$

The MSD and angular diameter distances

$\kappa = 0$



The MSD and angular diameter distances



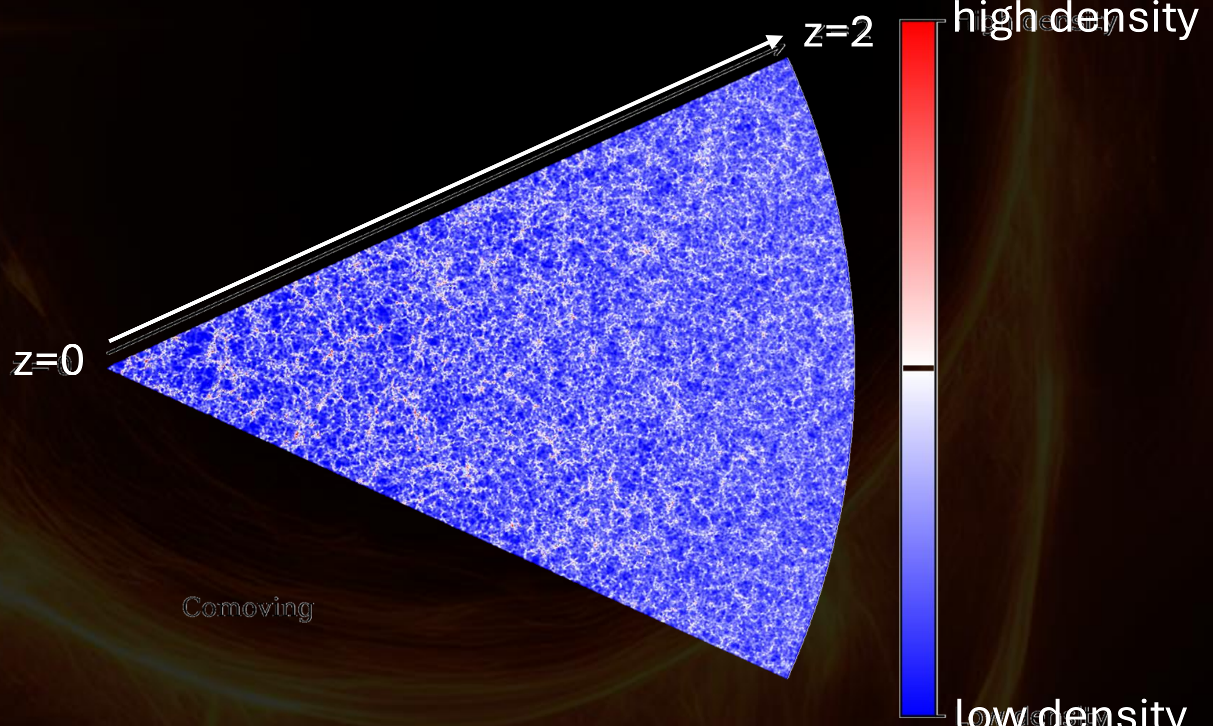
$\widetilde{D}_{\mathrm{od}} = (1 - \kappa_{\mathrm{od}}) D_{\mathrm{od}}$

$\widetilde{D}_{\mathrm{ds}} = (1 - \kappa_{\mathrm{ds}}) D_{\mathrm{ds}}$

$\widetilde{D}_{\mathrm{os}} = (1 - \kappa_{\mathrm{os}}) D_{\mathrm{os}}$

How big is the effect?

We make use of the high-resolution dark matter only ray tracing results of Breton et al. 2018 (1803.04294), which gives convergence and shear values for lines-of-sight for WMAP-7 best fit cosmology

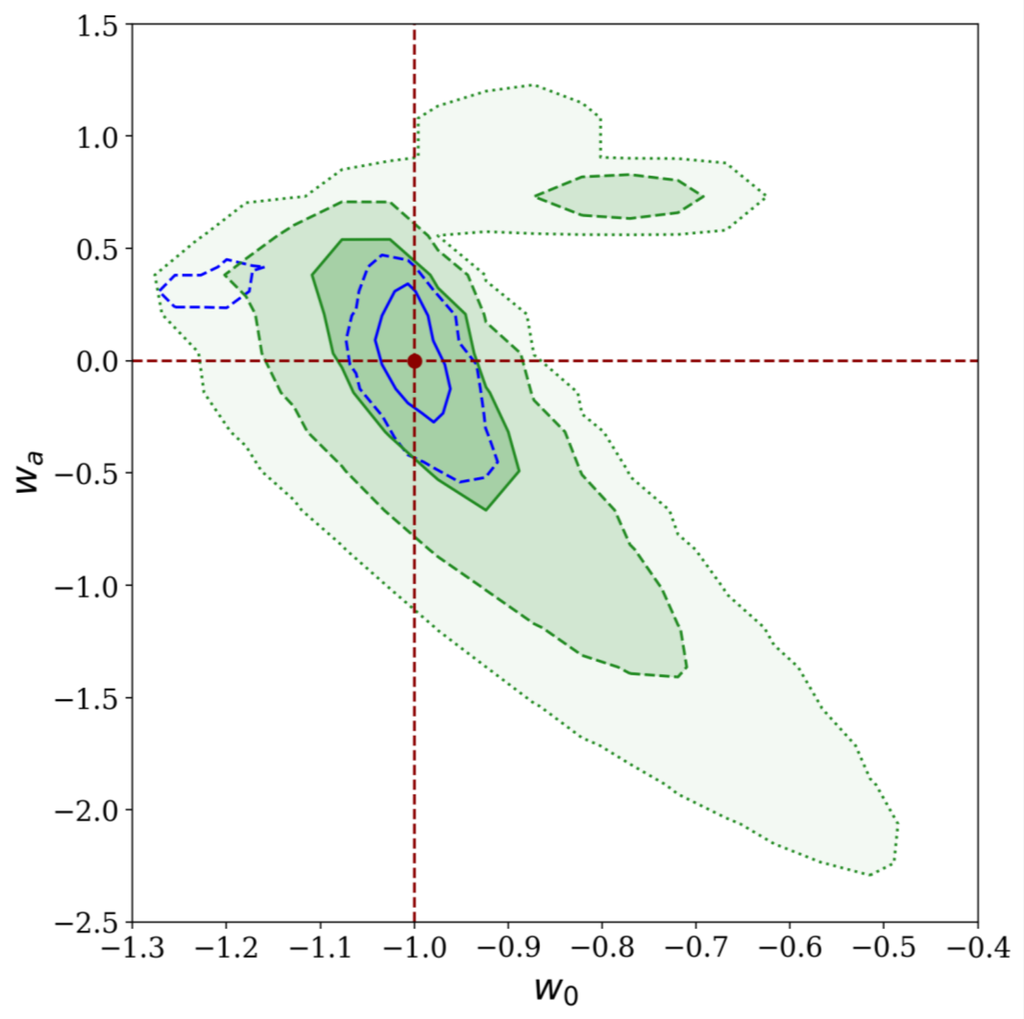


2. Foreground biases
arXiv:2405.04194

3. LOS with DPSLs
arXiv: 2501.17153

How big is the effect for $w(z)$?

3. LOS with DPSLs
arXiv: 2501.17153

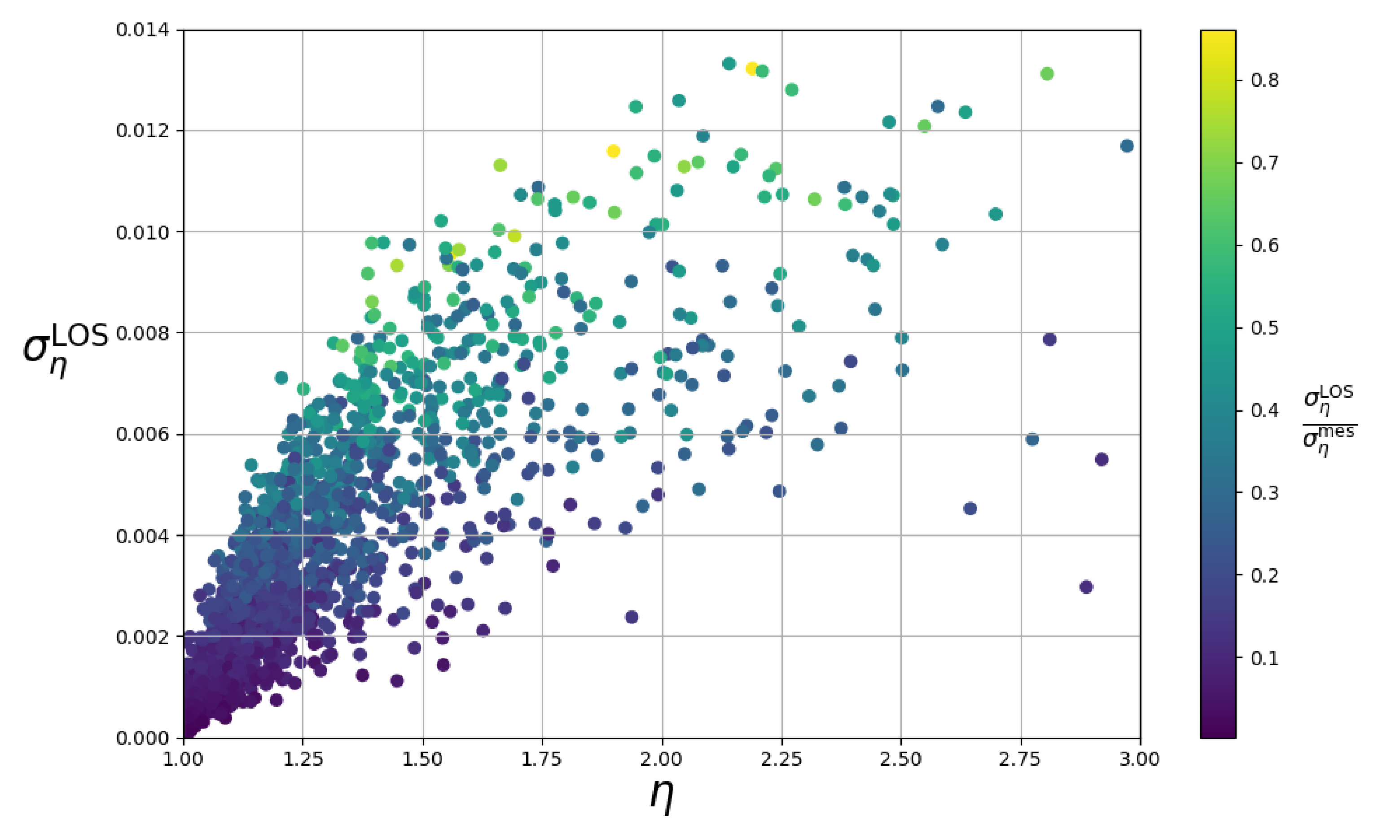


$$\frac{\eta^{\text{inferred}}}{\eta} = \frac{(1-\kappa_{\text{ds1}})(1-\kappa_{\text{s2}})}{(1-\kappa_{\text{s1}})(1-\kappa_{\text{ds2}})}$$

2-sigma outliers in ~35% of cases

How big is the effect for η?

3. LOS with DPSLs
arXiv: 2501.17153



$$\frac{\eta^{\mathrm{inferred}}}{\eta} = \frac{(1-\kappa_{\mathrm{ds1}})(1-\kappa_{\mathrm{s2}})}{(1-\kappa_{\mathrm{s1}})(1-\kappa_{\mathrm{ds2}})}$$

Is it systematic?

No systematic error unless there are selection effects

If overdense foregrounds are typical, H_0 would be overestimated

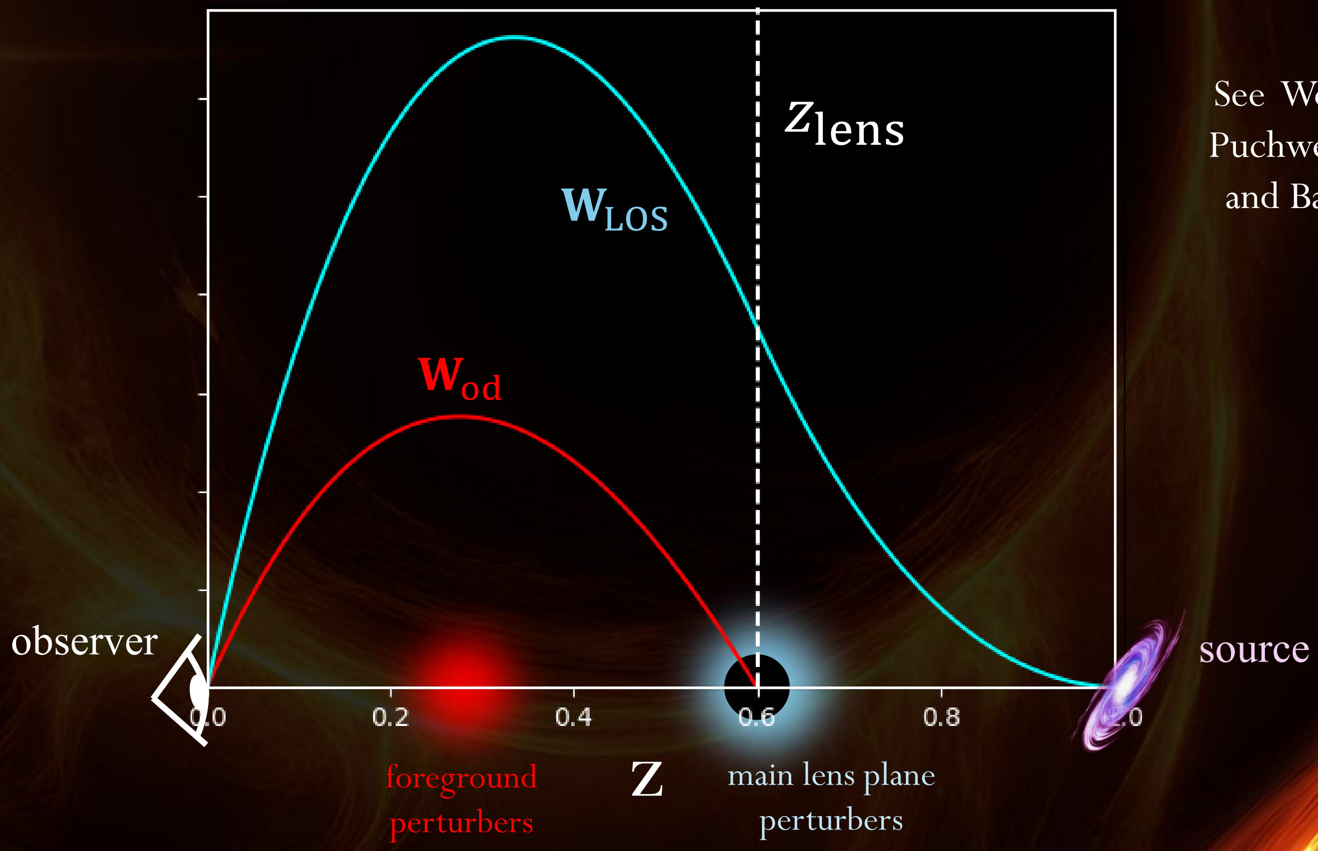
Evidence from the literature to suggest overdense lines of sight are typical

(Keeton et al. 2008, astro-ph/9610163, Bayliss et al. 2014, 1312.3637, Bartelmann et al. 1997, astro-ph/9707167; Meneghetti et al. 2013, 1303.3363; Puchwein et al. 2009, 0904.0253, Wells et al. 2024, 2403.10666)

$$\langle \kappa_{\mathrm{od}} \rangle > 0 \implies \langle H_0^{\mathrm{inferred}} \rangle > H_0$$

Is it systematic?

Debate in the literature - matter associated with main lens (affects $\mathcal{A}_{\rm LOS}$), or independent matter along the line-of-sight (affects $\mathcal{A}_{\rm od}$)?



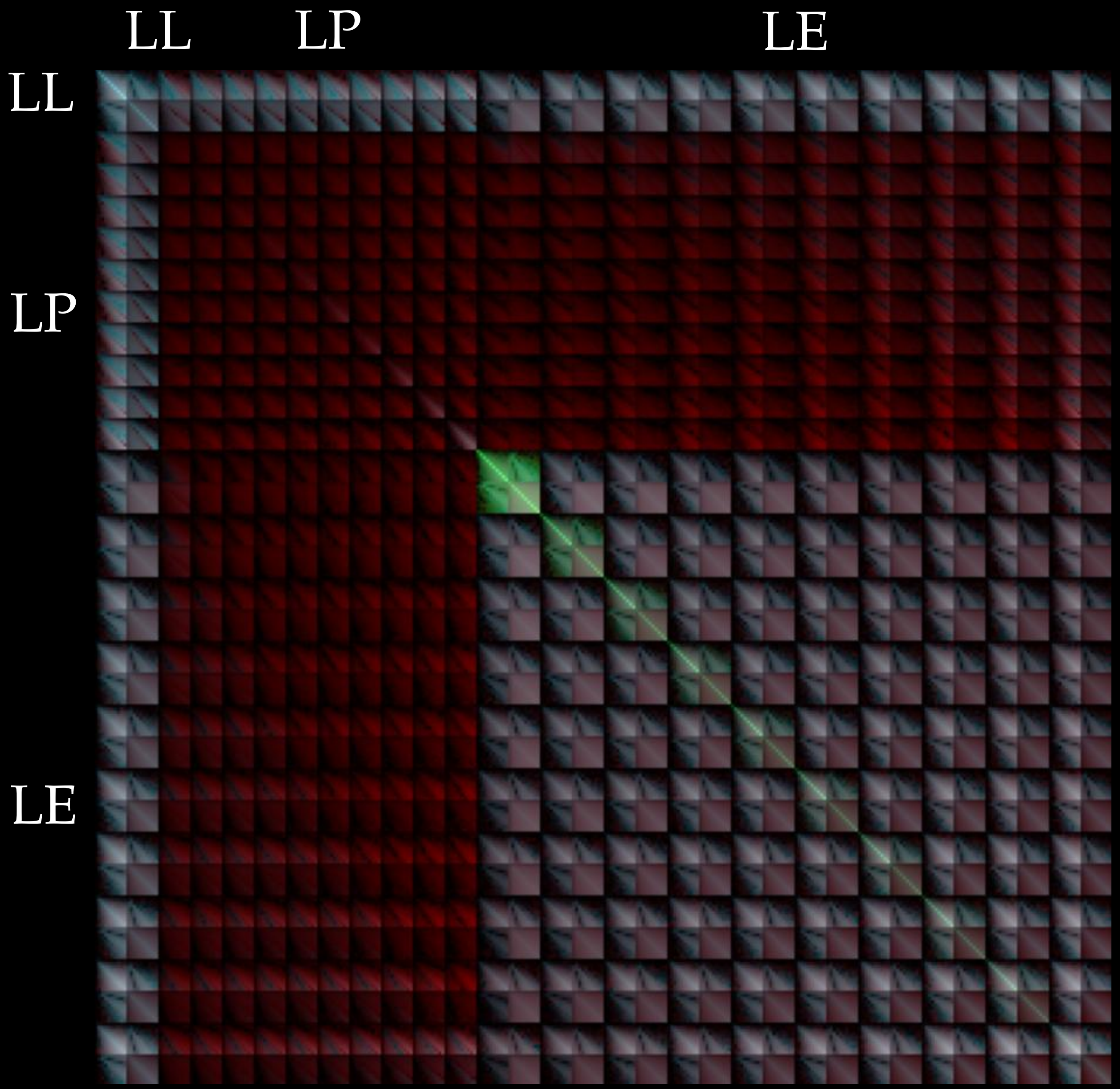
See Wong et al. 2019, 1907.04869, Puchwein, Hilbert 2009, 0904.0253 and Bayliss et al. 2014, 1312.3637

Covariance matrices

noise

sparsity

cosmic

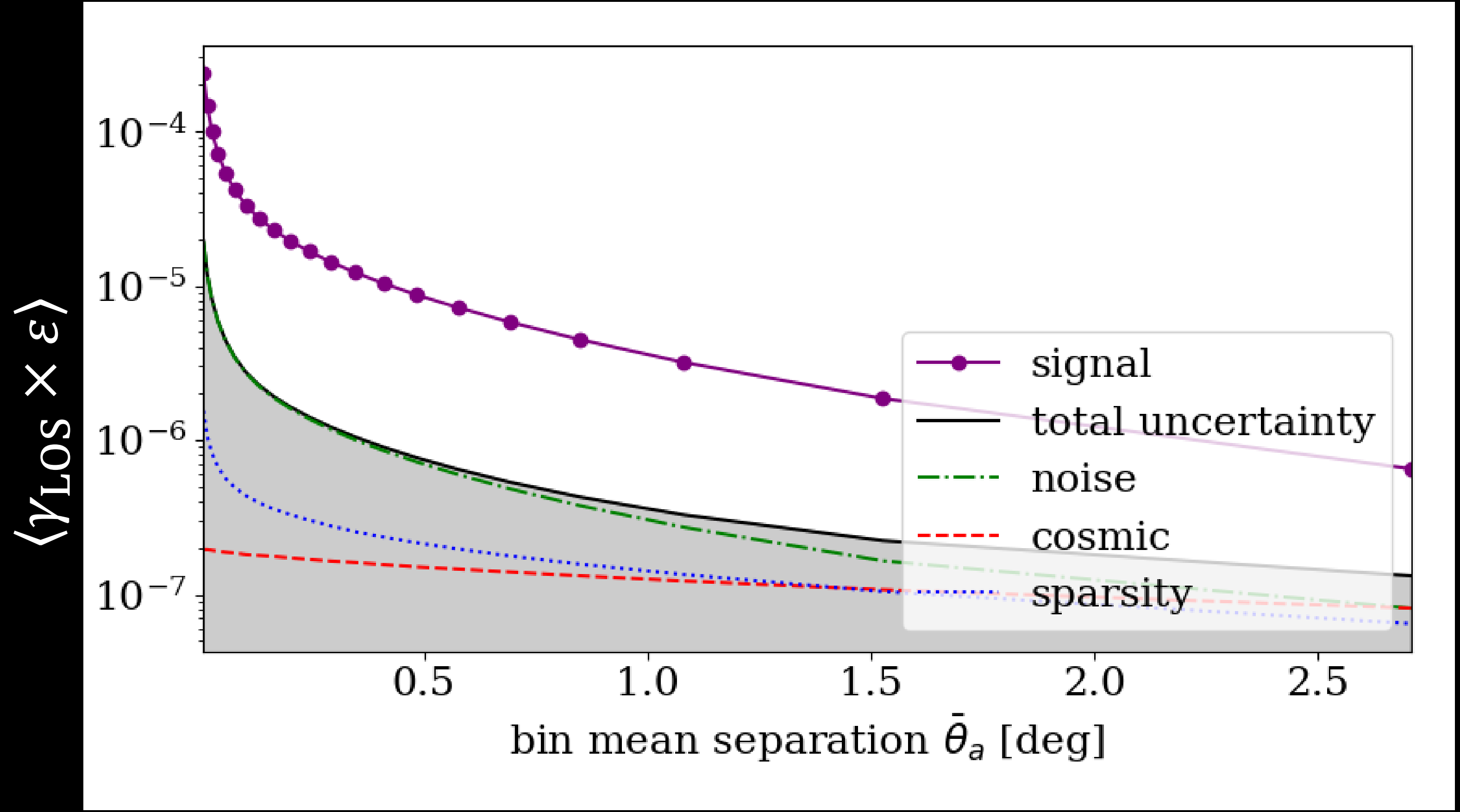


Sources of uncertainty

noise

cosmic

sparsity



Sparsity covariance

Beyond $\sigma_{\mathrm{LOS}} < \sim 5\%$,
no real improvements
without more lenses