EPFL

Advancing Cosmological Simulations of Fuzzy Dark Matter with Physics-Informed Neural Networks(PINNs)

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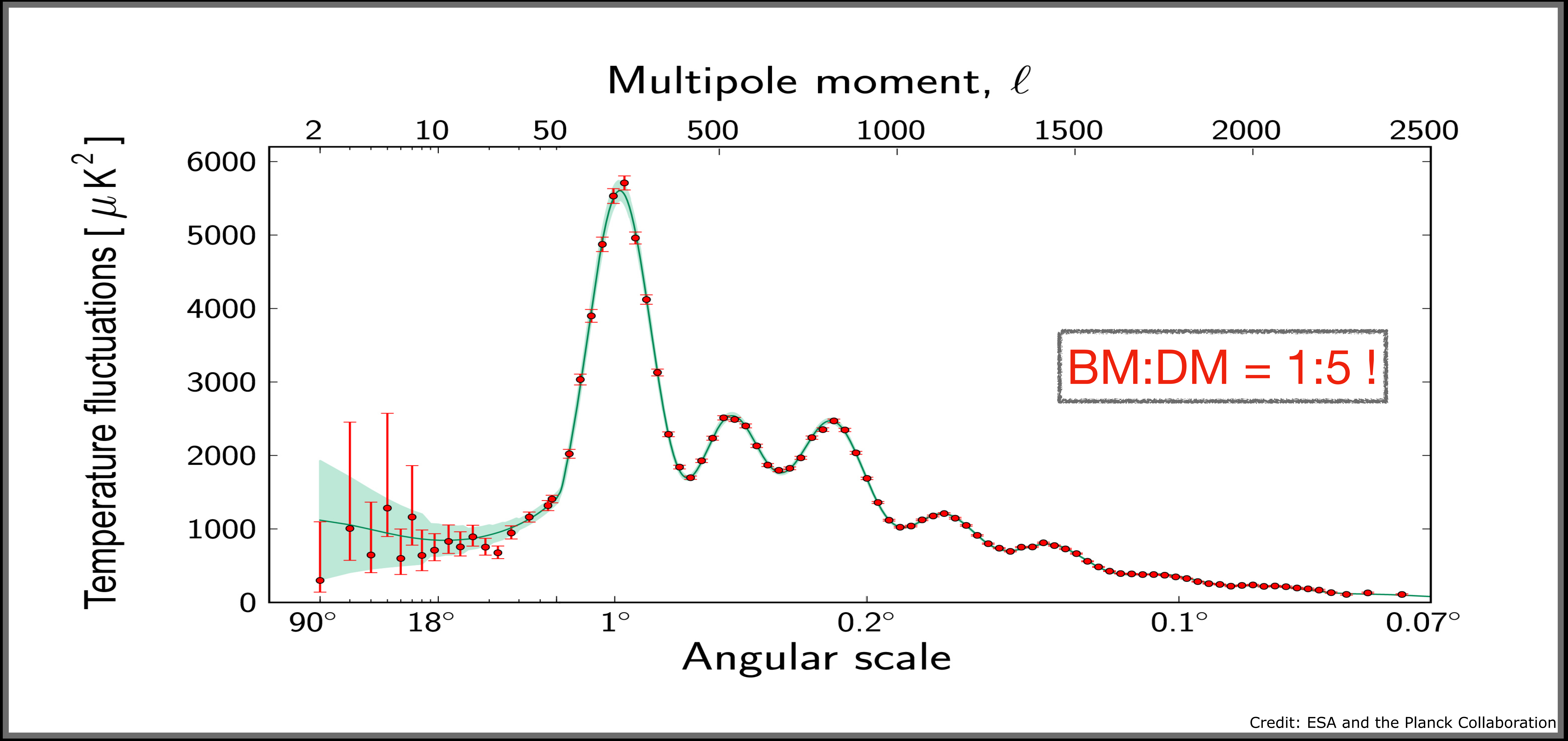
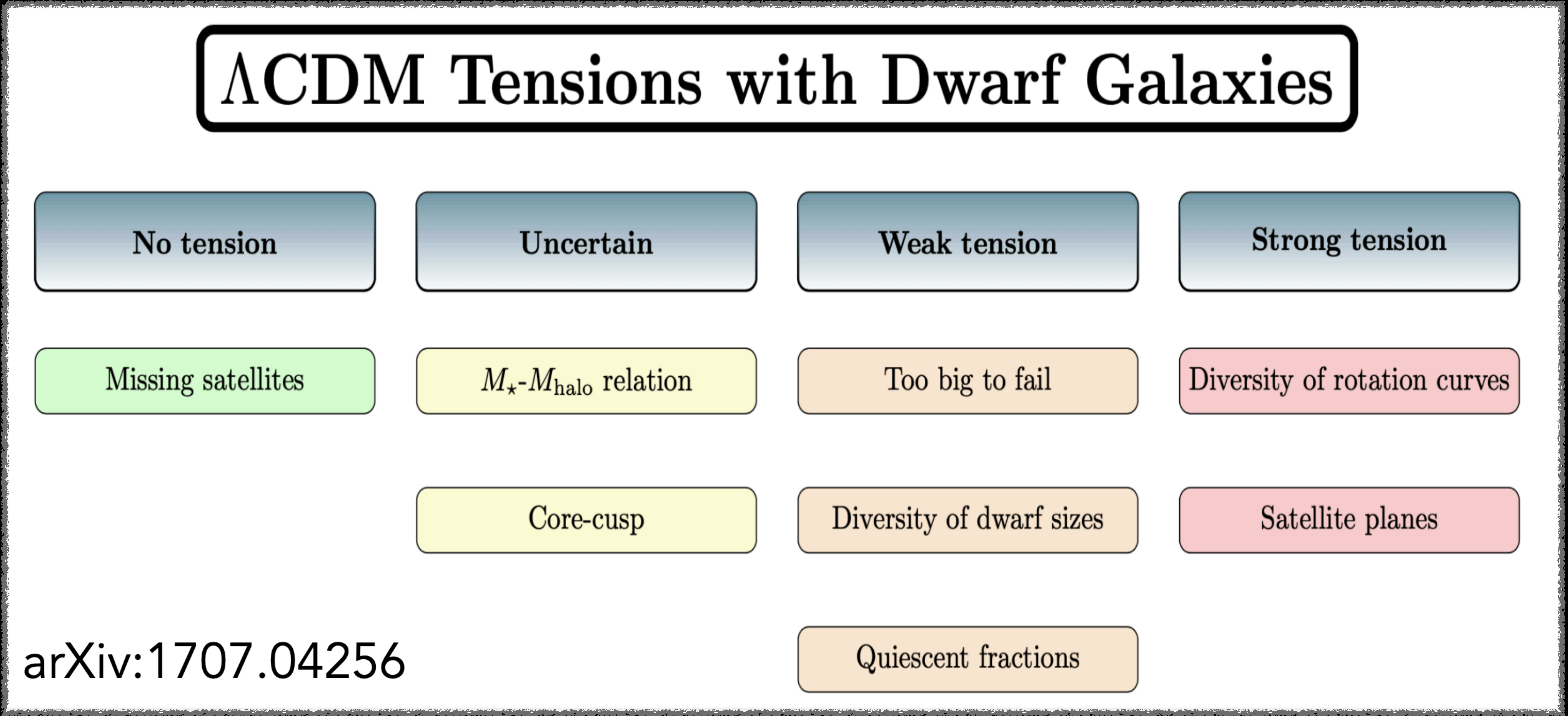
CMB Power Spectrum



ΛCDM Theoretical Fit: $\Omega_b h^2 \approx 0.024, \quad \Omega_m h^2 \approx 0.14$

Small Scale Challenges in CDM Model



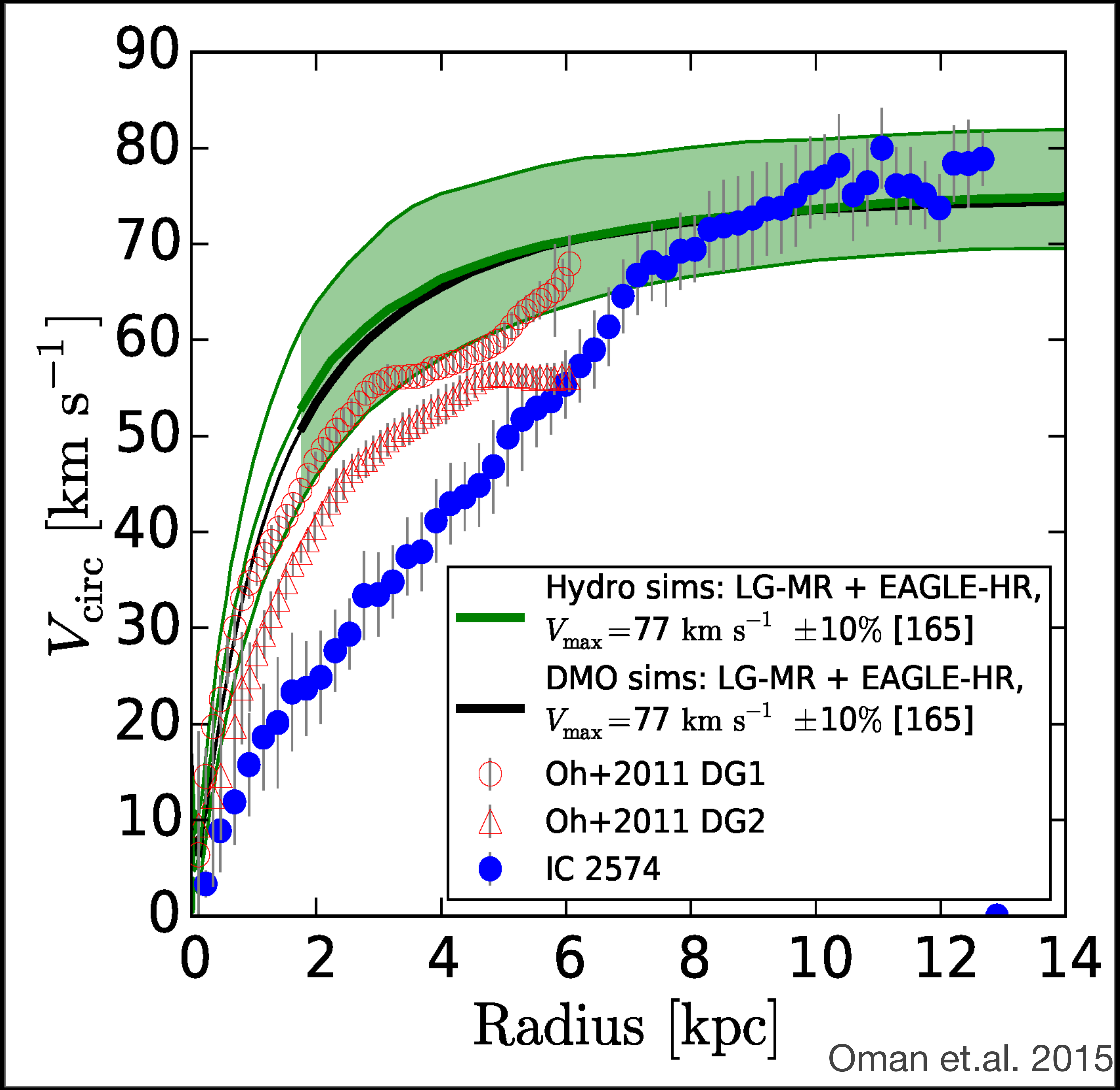
Potential Problem: Absence of **Baryonic Processes** (Feedback, Formation) and/or Nature of **DM**!

Baryonic Processes

Strongly model dependent e.g. feedback sensitivity to the gas threshold for galaxy formation.

Very Difficult to disentangle baryonic effects in the Simulations!

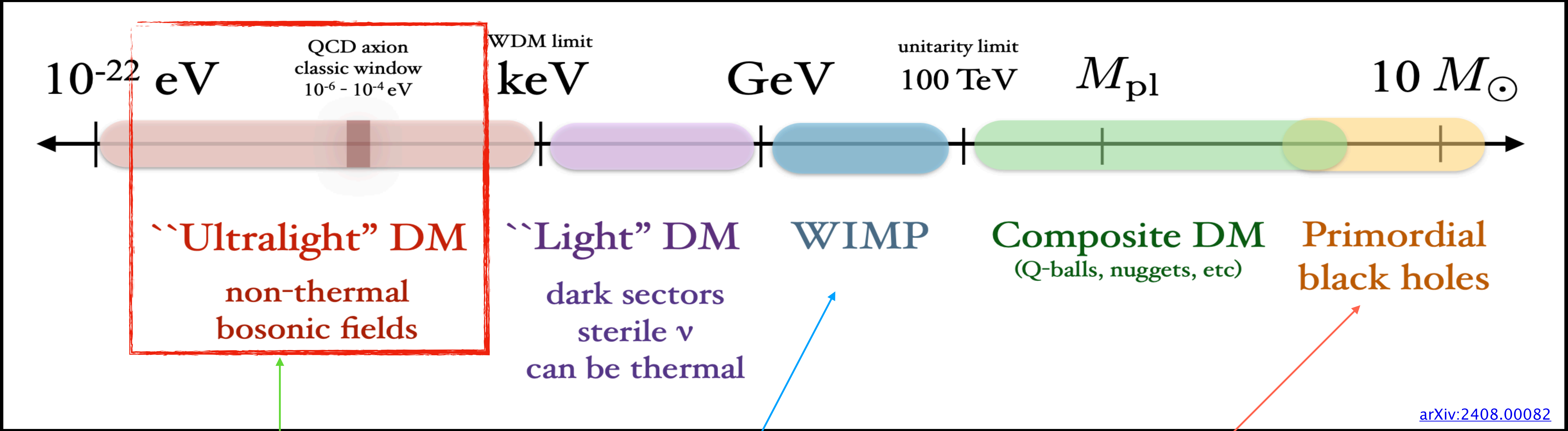
Some outliers like IC 2574 still unexplainable with Feedback!



Alternative Dark Matter Models

Warm Dark Matter (WDM): favored mass range in tension with Lyα observation & abundance of high-z galaxies

Self-interacting Dark Matter (SIDM): Needs fine-tuned cross-sections & struggles to explain full range of observations



Still a viable window
Under observational
costraints!

No Empirical
Detection so far!

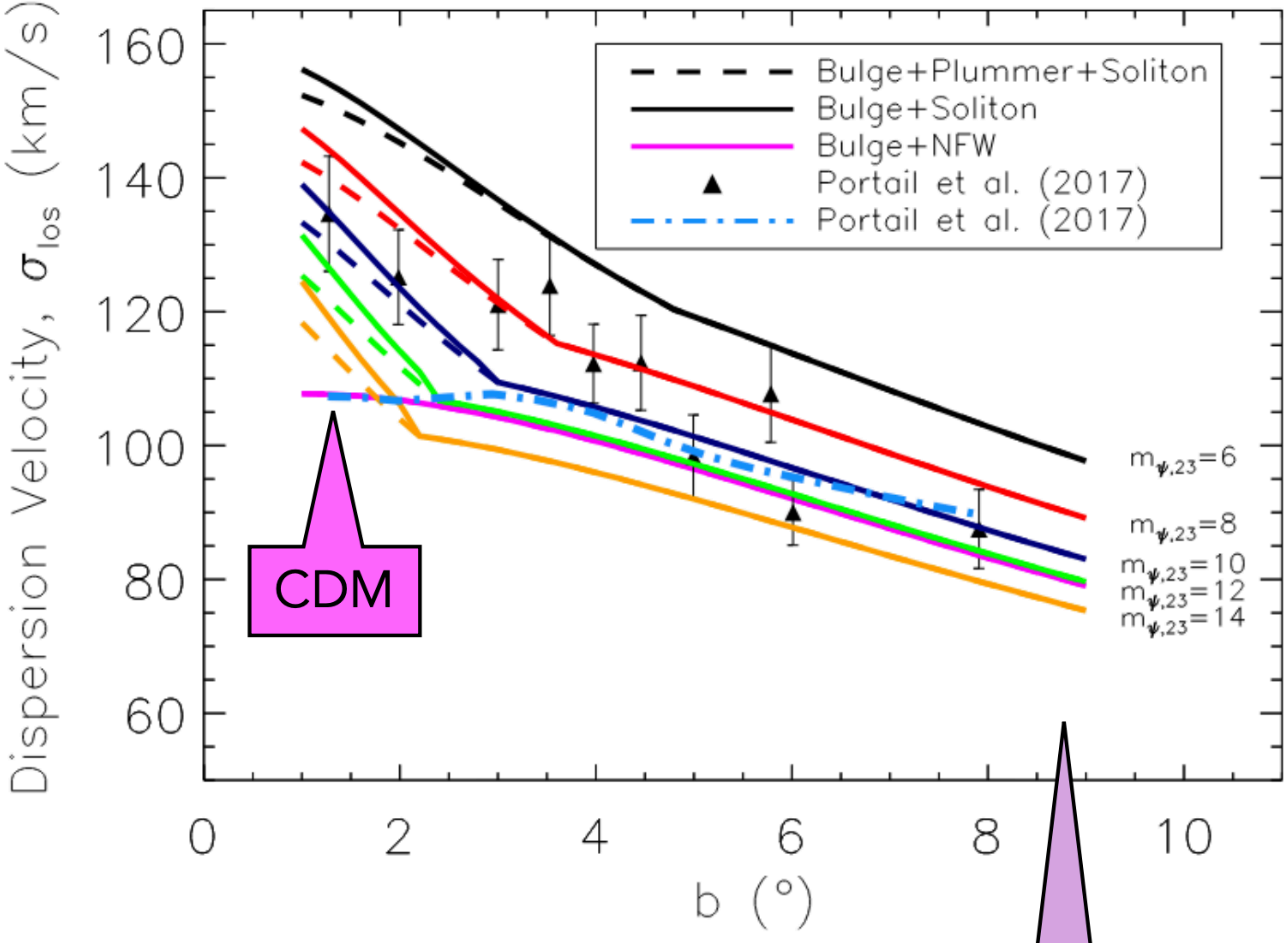
Strong constraints
with microlensing
observations!

Fuzzy Dark Matter

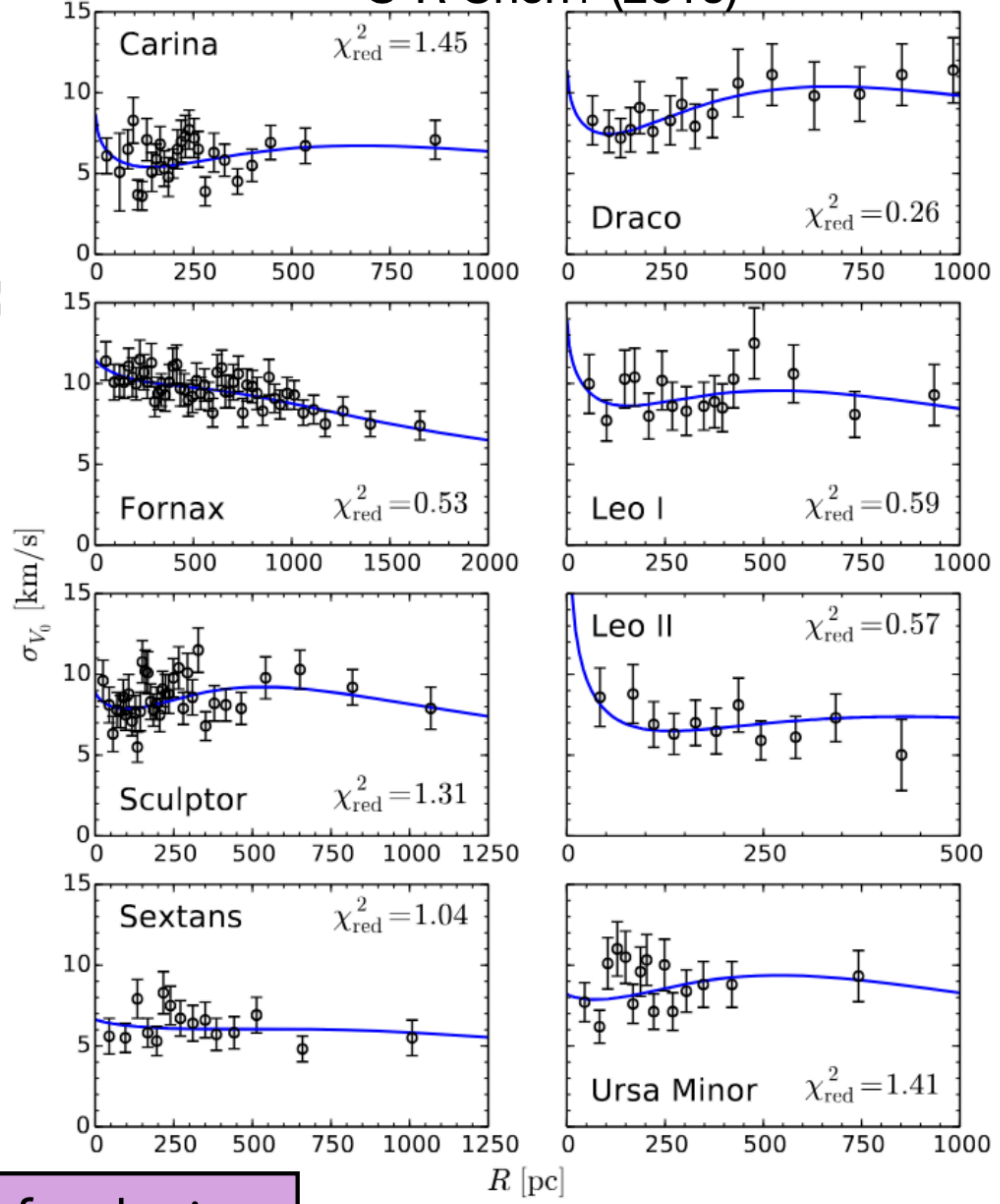
(F(C)DM, BECDM, ULDM, ELBDM, (ultra-light) axion (-like) DM (ULA, ALP))

- ✦ ***Extremely light scalar*** **particle (m $\sim$ 10^{-20} - 10^{-22} eV)**
- ✦ **Non-thermally produced (thus not ultra-hot)**
- ✦ **Clumps to form Bose-Einstein Condensate (BEC)!**
- ✦ **Quantum effects counteract gravity at small scales**
- ✦ **Tiny mass**
  - **— large de-broglie wavelength ($\sim$ 1/m)**
  - **— macroscopic quantum effects at kpc scales**

I. De Martino+ (2020)



S-R Chen+ (2016)



FDM can explain halo profiles of our galaxy & many dwarf galaxies

Fuzzy Dark Matter Equations - I

A. Start with simple scalar field action

$$S = \frac{1}{\hbar c^2}\int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2 - \frac{g}{\hbar^2c^2}\phi^4\right)$$

Note: Corresponds to superfluid dark matter w/o self-interaction ($g \to 0$)

- QCD Axion Case: originates from periodic potential

$$V(\phi) \sim \Lambda^4(1 - \cos(\phi/f_a)) \quad \text{for} \quad \phi \ll f_a$$

B. Rewrite

$$\phi = \sqrt{\frac{\hbar^3 c}{2m}}\frac{1}{2}\mathrm{Re}(\psi e^{-ic^2/\hbar mt}) = \sqrt{\frac{\hbar^3 c}{2m}}(\psi e^{-ic^2/\hbar mt} + \psi^* e^{-ic^2/\hbar mt})$$

Fuzzy Dark Matter Equations - II

C. Take the **non-relativistic limit** with perturbed FLRW metric

$$ds^2 = \left(1 + \frac{2V}{c^2}\right) c^2 dt^2 - a(t)^2 \left(1 - \frac{2V}{c^2}\right) d\vec{x}^2$$

V is the gravitational potential sourcing the perturbations

- Non-relativistic limit: Necessary for non-linear structure formation in universe

D. Result

$$i\hbar \left(\partial_t \psi + \frac{3}{2} H\psi\right) = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

E. With comoving quantities:

$$i\hbar\partial_t\psi_c = -\frac{\hbar^2}{2m}\nabla_c^2\psi_c + mV_c\psi_c$$

$$\psi_c = a^{3/2}\psi$$
$$\nabla_c = a\nabla$$
$$V_c = aV$$

Extensions : self interactions, multiple fields, higher spin fields

1) Multiple Fields

$$\partial_t \psi_j(x,t) = \frac{-i}{\hbar}\left(\frac{-\hbar^2\nabla^2}{2m_j} + m_j\, V(x,t)\right)\psi_j(x,t)\,,$$

$$\nabla^2 V(x,t) = 4\pi G \sum_j |\psi_j(x,t)|^2\,.$$

2) Self-interactions + Multiple fields

$$\partial_t \psi_j(x,t) = \frac{-i}{\hbar}\left(\frac{-\hbar^2\nabla^2}{2m_j} + m_j\, V(x,t) + \frac{\hbar^3}{2m_j^2}\lambda_{jj}|\psi_j(x,t)|^2 + \frac{\hbar^3}{4m_j^2}\sum_k \lambda_{jk}|\psi_k(x,t)|^2\right)\psi_j(x,t)\,,$$

$$\nabla^2 V(x,t) = 4\pi G \sum_j |\psi_j(x,t)|^2\,.$$

3) Self-interactions + Multiple fields + Higher Spin Fields

$$\partial_t \psi_j(x,t) = -i\left(\frac{-\nabla^2}{2m} + mV(x,t) + \frac{\lambda}{2m}|\psi_j|^2 + \frac{\alpha}{m^2}\sum_i \mathbf{S}\cdot\hat{\mathbf{S}}_{ij} + \frac{\xi}{2s+1}\sum_{ik}\hat{A}_{ji}\psi_i^\dagger\psi_k(x,t)\hat{A}_{ki} - i\sum_{ikl} g_{lk}[\hat{S}_l]_{ji}\nabla_k\right)\psi_j(x,t)\,.$$

$$\nabla^2 V = 4\pi G \sum_j |\psi_j|^2\,.$$

Credit: arXiv:2507.00705

Governing Equations

A. Wave Formalism (Schrödinger-Poisson Equations)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

$$\nabla^2 V = 4\pi Gm(|\psi|^2 - |\psi_0|^2)$$

Mean Field Interpretation: Single Macroscopic WF of BEC

B. Madelung Formalism (Fluid Dynamics Representation)

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{m}\vec{\nabla}\left(V - \underbrace{\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}_{= Q}\right)$$

$$\nabla^2 V = 4\pi Gm(\rho - \rho_0)$$

$$\psi = \sqrt{\frac{\rho}{m}}e^{iS}$$

$$\rho = m|\psi|^2$$

$$v = \frac{\hbar}{m}\nabla S$$

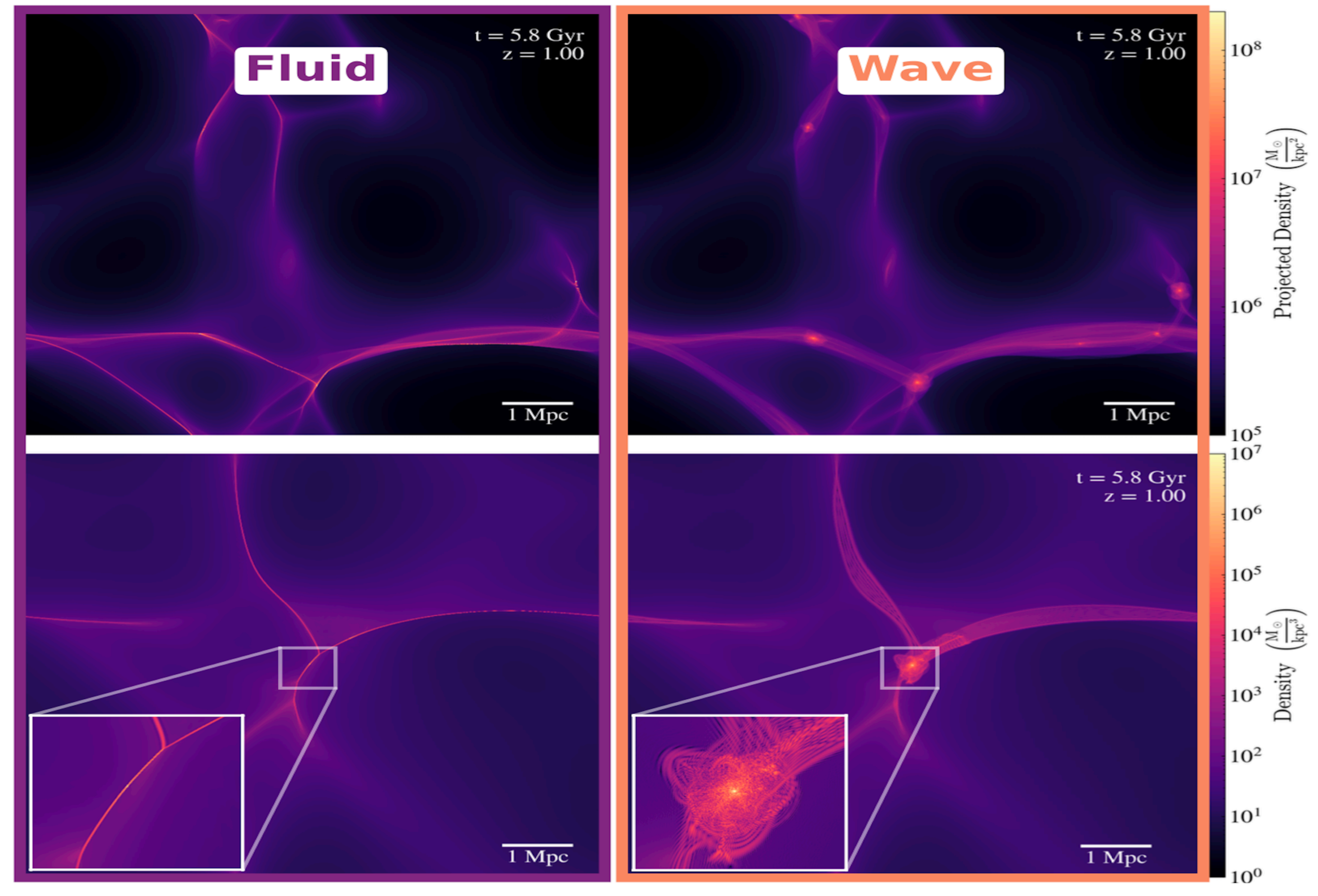
"Quantum Pressure"

Q ill-defined at $\rho = 0$!

Fuzzy Dark Matter Simulations

Fluid Solver unable to capture interference effects!

Stick to SP-Equations for evolution!



A. Kunkel et.al. 2024

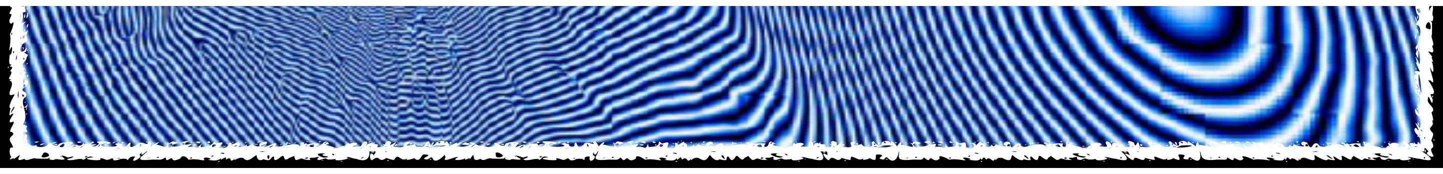
Challenges in Simulating Fuzzy Dark Matter

Schroedinger-Poisson Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi,$$

$$\nabla^2 V = 4\pi G(\rho - \bar{\rho}),$$

Can we solve this with machine learning?



Neural networks are universal function approximators

Theorem (*Cybenko, 1989*)

Let σ be any continuous sigmoidal function. Then, the finite sums of the form

$$g(x) = \sum_{j=1}^{N} w_j^2 \sigma((w_j^1)^T x + b_j^1)$$

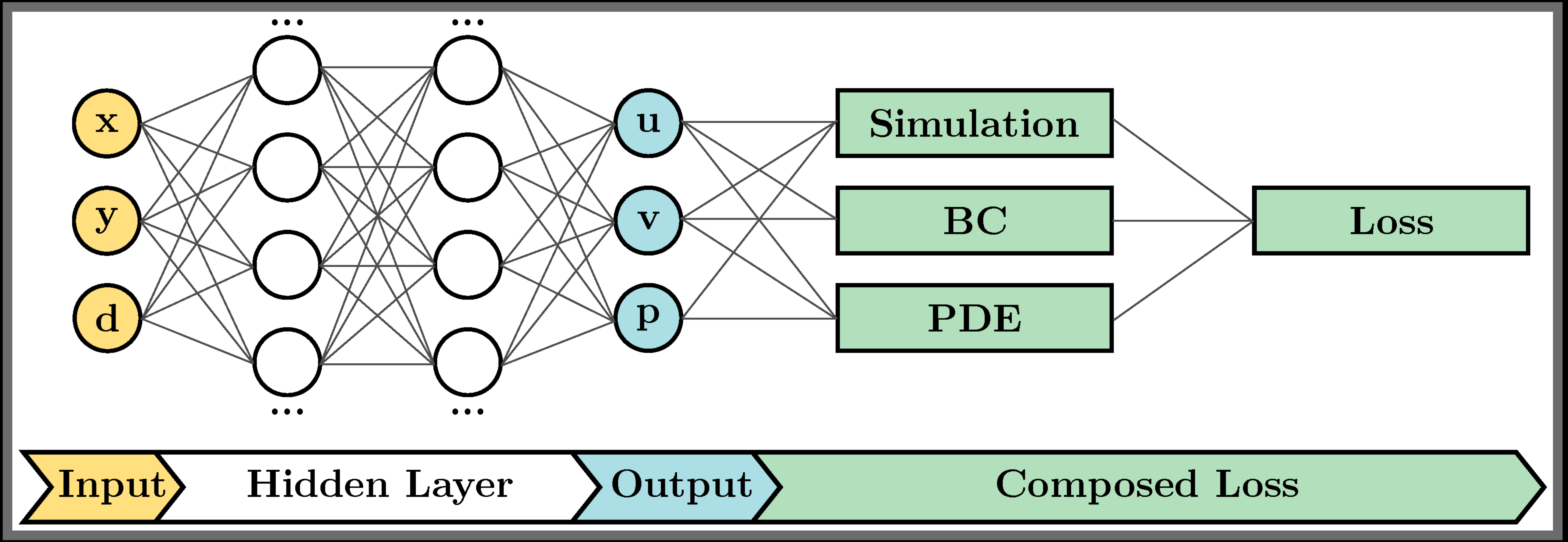
are dense in $C(I_d)$.

Schive, Chieuh, & Broadhurst (2014)

Physics Informed Neural Networks

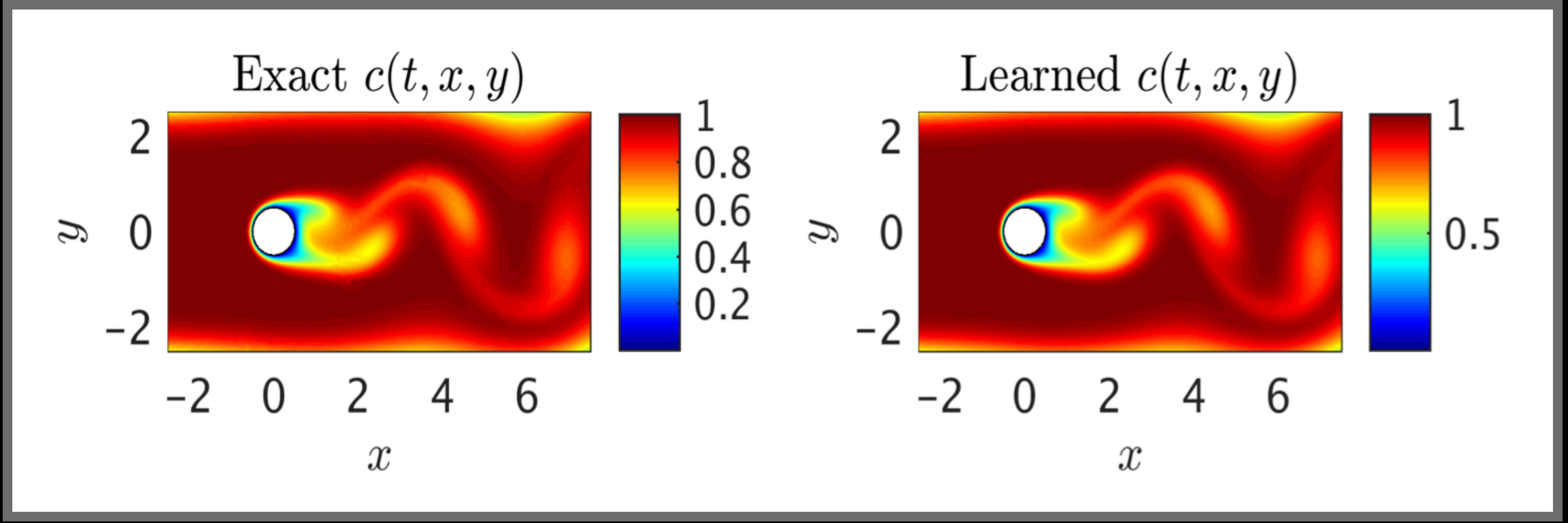
General Framework:

$$\mathscr{D}[NN(X,\theta);\lambda] = f(X), \quad X \in \Omega$$
$$\mathscr{B}[NN(X,\theta);\,] = g(X) \quad X \in \partial\Omega$$



Adapted from F. Pioch et.al.2023

Custom Loss Function: with PDE and boundary conditions as additional constraints



Raissi, Yazdani, Karinadakis 2020

Pretty Successful in Fluid and Climate Simulations!

Schrödinger-Poisson Equations used

$\lambda = \frac{\hbar}{m} \implies$

$$i\frac{\partial}{\partial t}\Psi(\boldsymbol{x},t) = \left(-\frac{\lambda}{2}\nabla^2 + \frac{1}{\lambda}V[\Psi(\boldsymbol{x},t)]\right)\Psi(\boldsymbol{x},t)$$

$$\nabla^2 V[\Psi(\boldsymbol{x},t)] = (|\Psi(\boldsymbol{x},t)|^2 - 1)$$

$\frac{1}{\lambda}$: the strength of potential

$\lambda \to 0$, Gravitational Potential Term is dominant in the SP Equations!

$\lambda \to \infty$, Gravitational Potential Term vanishes, Free Schrodinger Equation representing diffusion!

$\lambda = 1$ throughout this work!

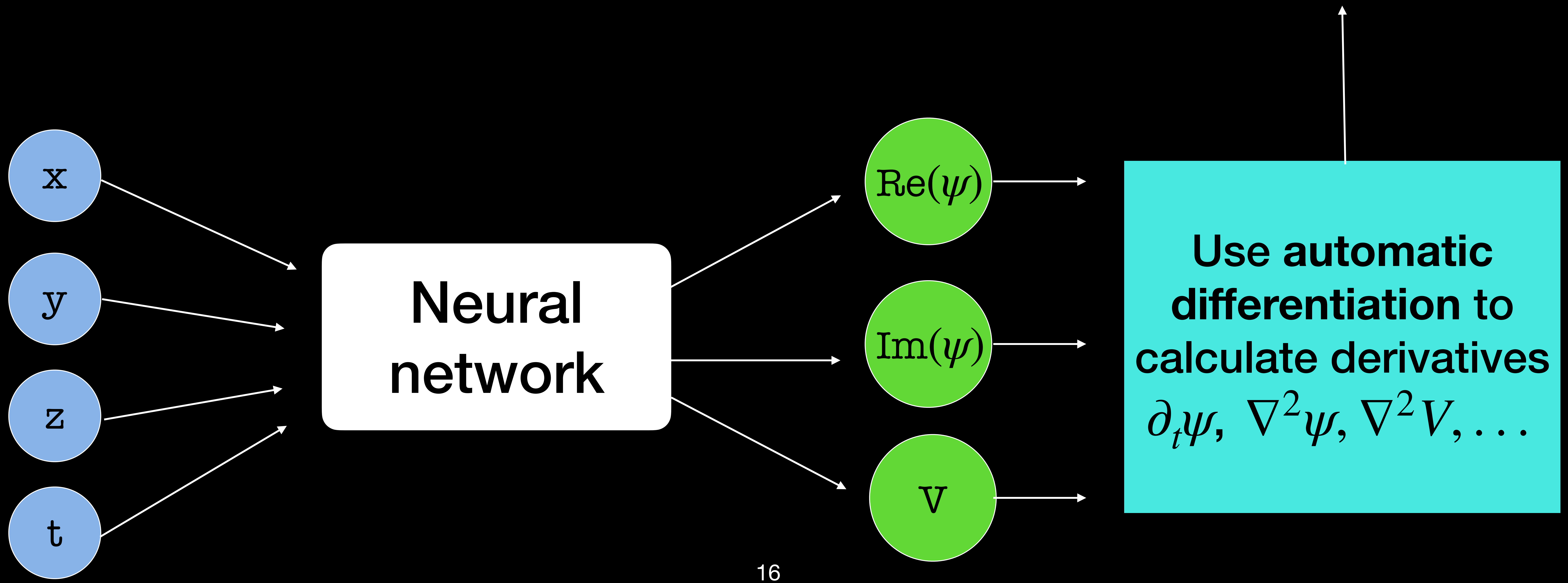
Schrödinger-Poisson Informed Neural Networks (SPINN)

$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi$$

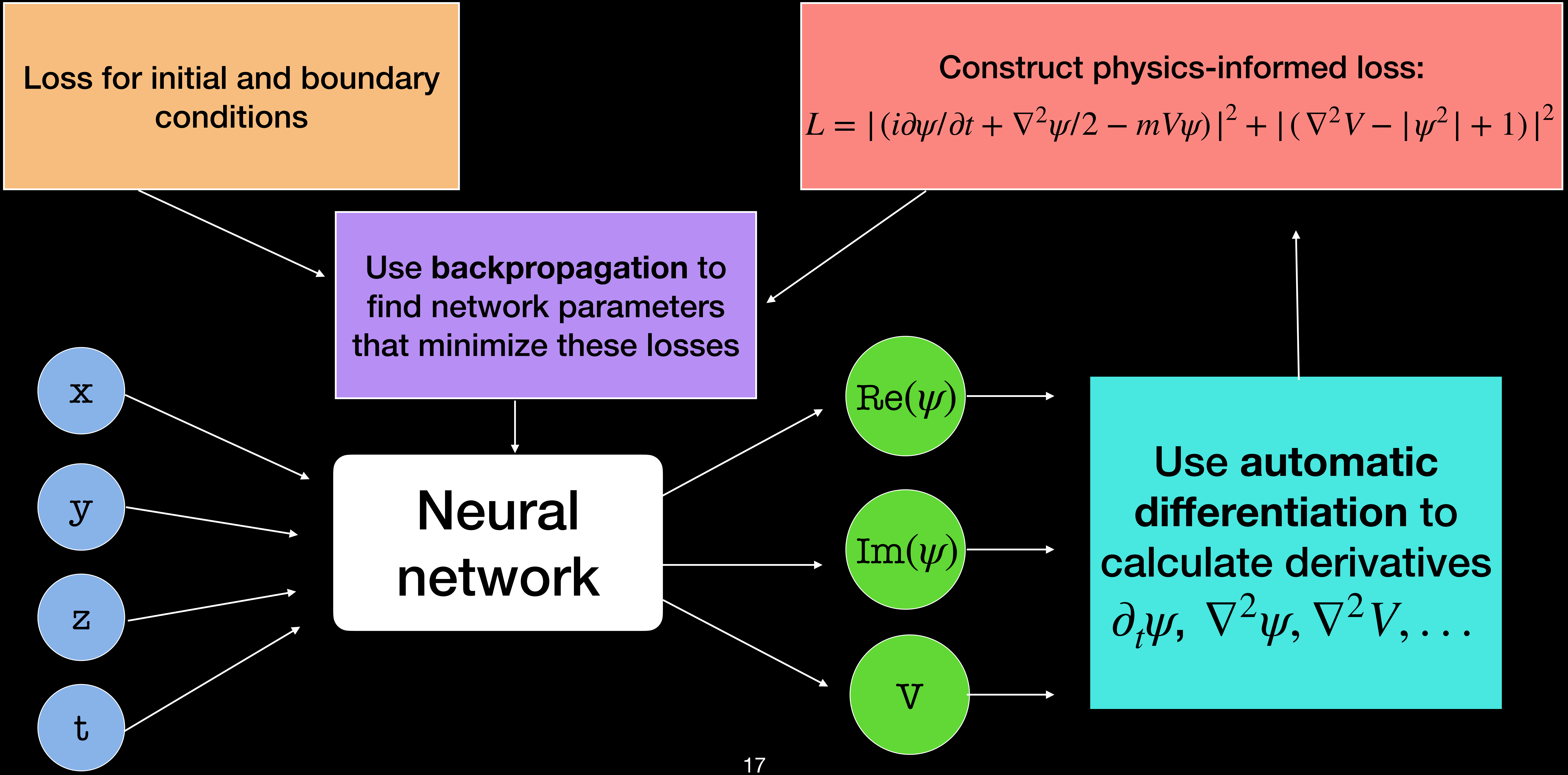
$$\nabla^2 V = 4\pi G m(|\psi|^2 - |\psi_0|^2)$$

Construct physics-informed loss:

$$L = |(i\partial\psi/\partial t + \nabla^2\psi/2 - mV\psi)|^2 + |(\nabla^2 V - |\psi^2| + 1)|^2$$



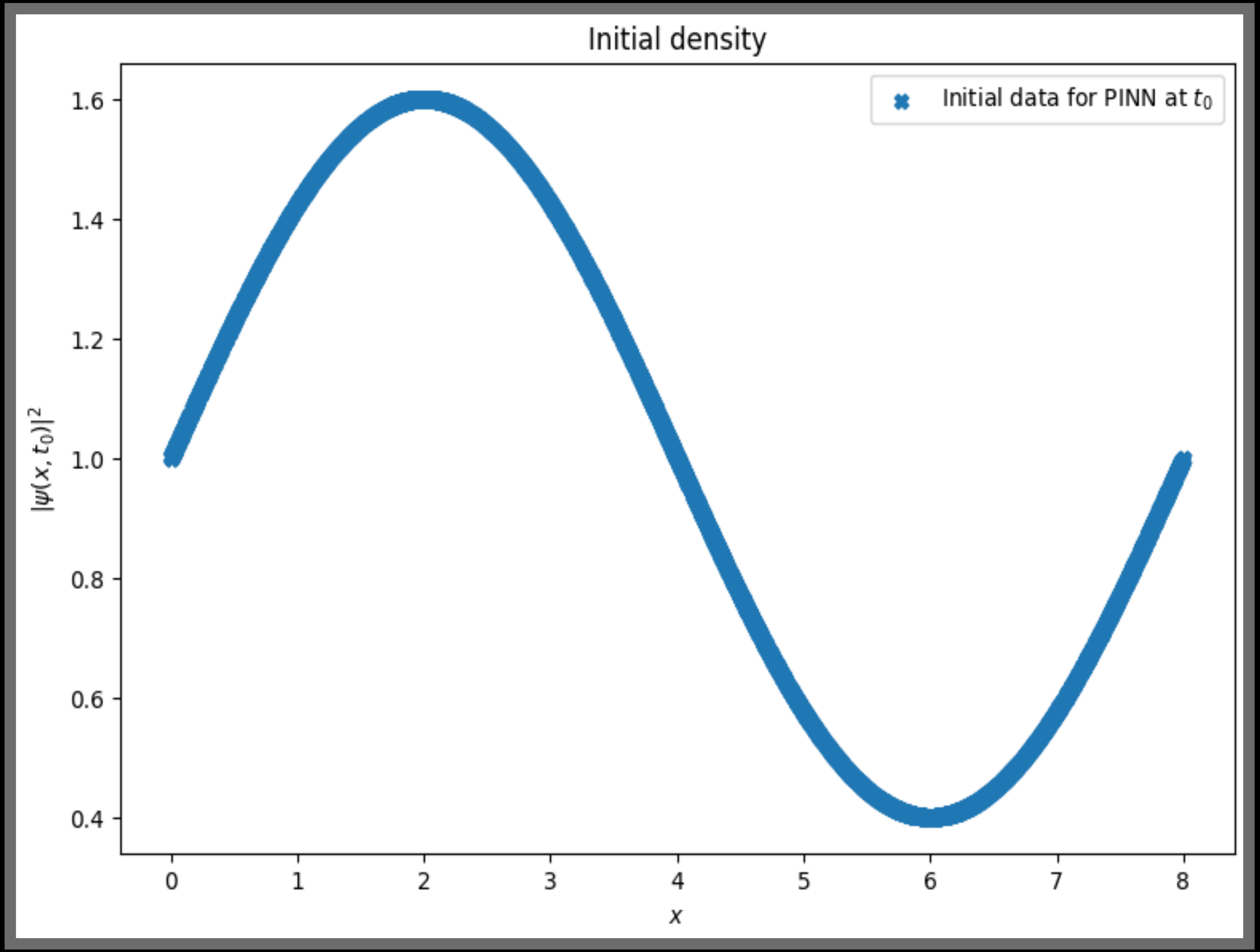
Schrödinger-Poisson Informed Neural Networks (SPINN)



Initial Functions Used

1D Initial Function:

$$\psi(x,0) = \sqrt{1 + 0.6\sin\left(\frac{\pi x}{4}\right)}$$

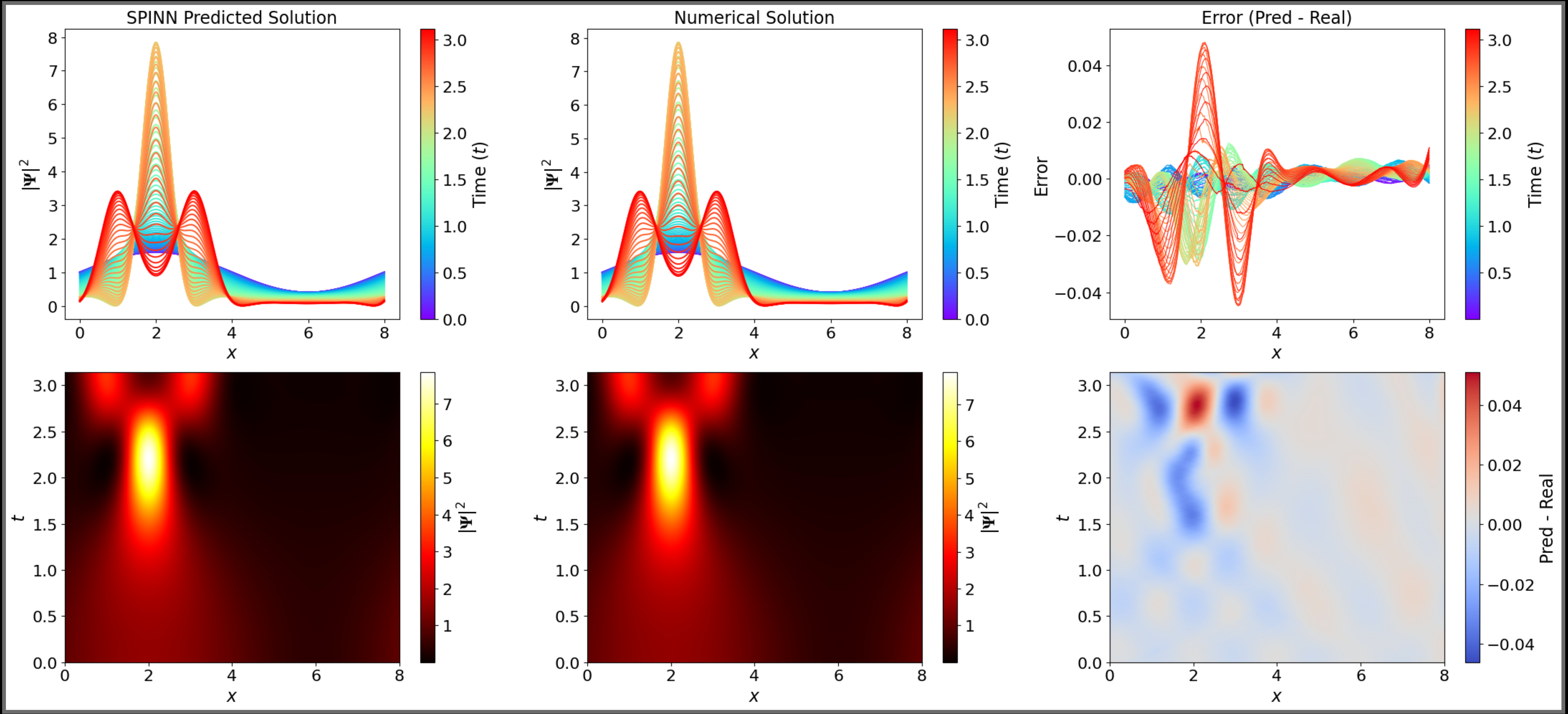


3D Initial Function:

$$\psi(\vec{x},0) = \sqrt{1 + 0.6\sin\left(\frac{\pi x}{4}\right)\sin\left(\frac{\pi y}{4}\right)\sin\left(\frac{\pi z}{4}\right)}$$

Results

Density Predictions in 1D

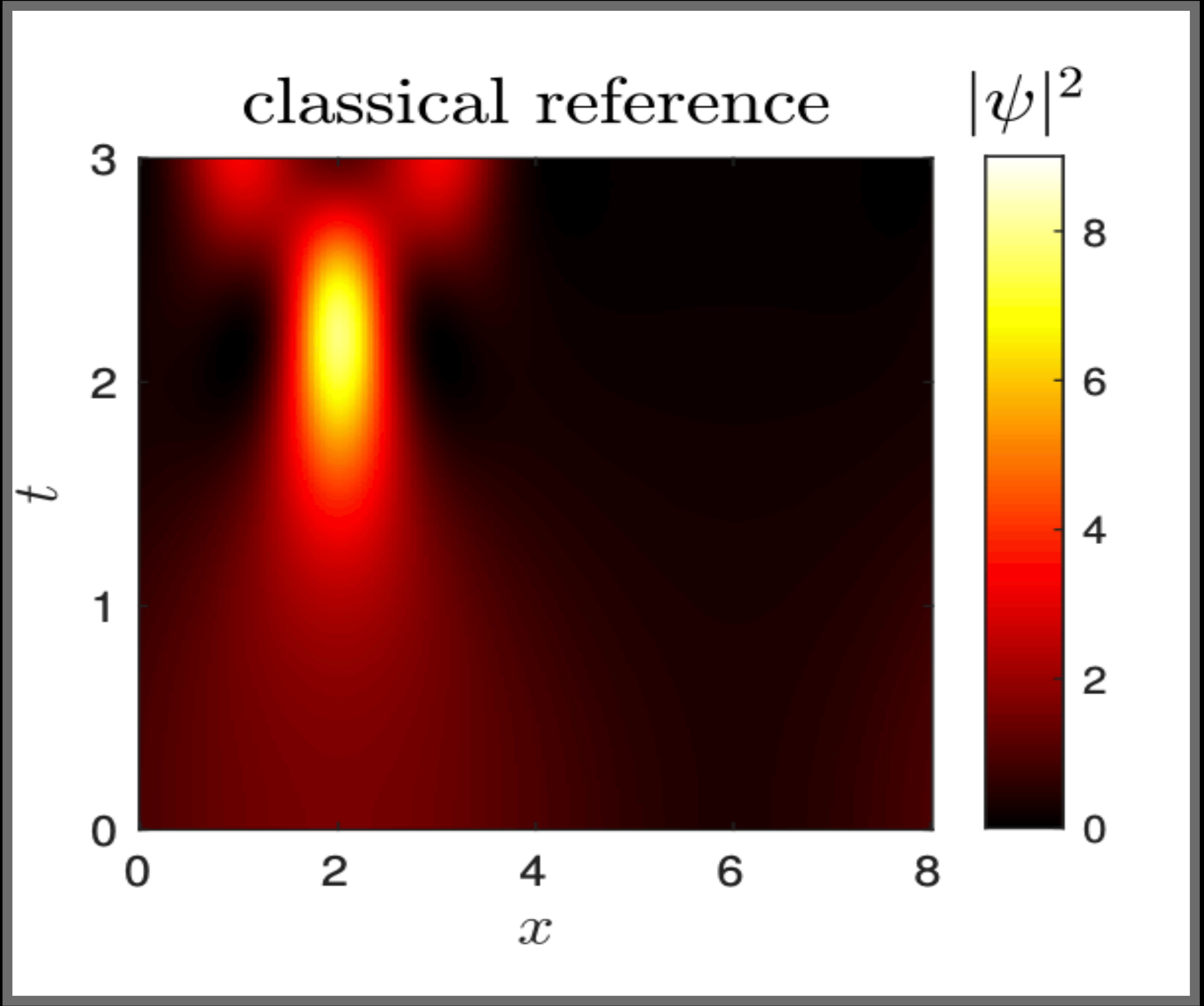


Mishra & Tolley 2025 (Accepted in ApJ)

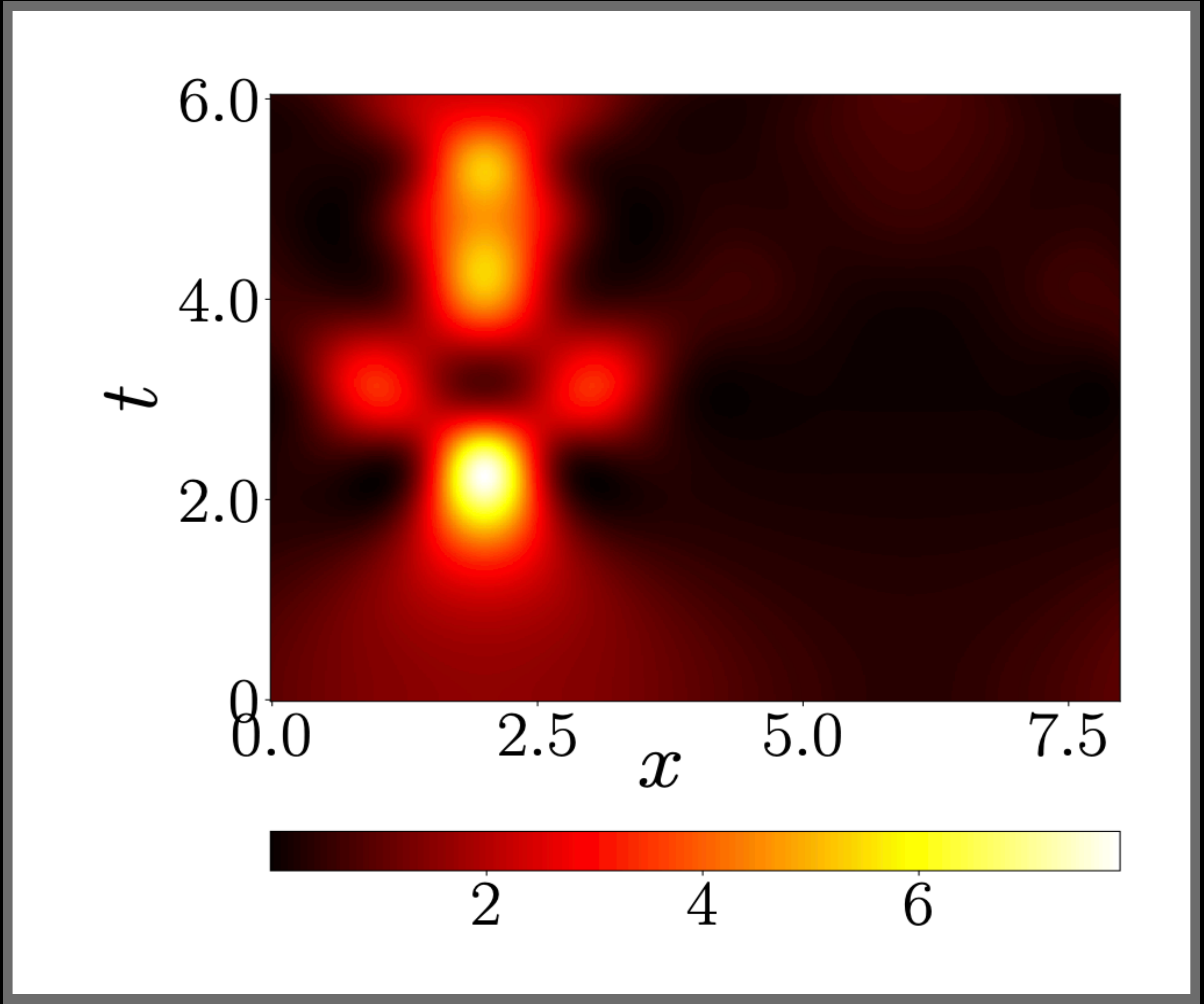
Overdensities collapse as expected!

Decent Match with Spectral Method ✓

Comparison with existing references



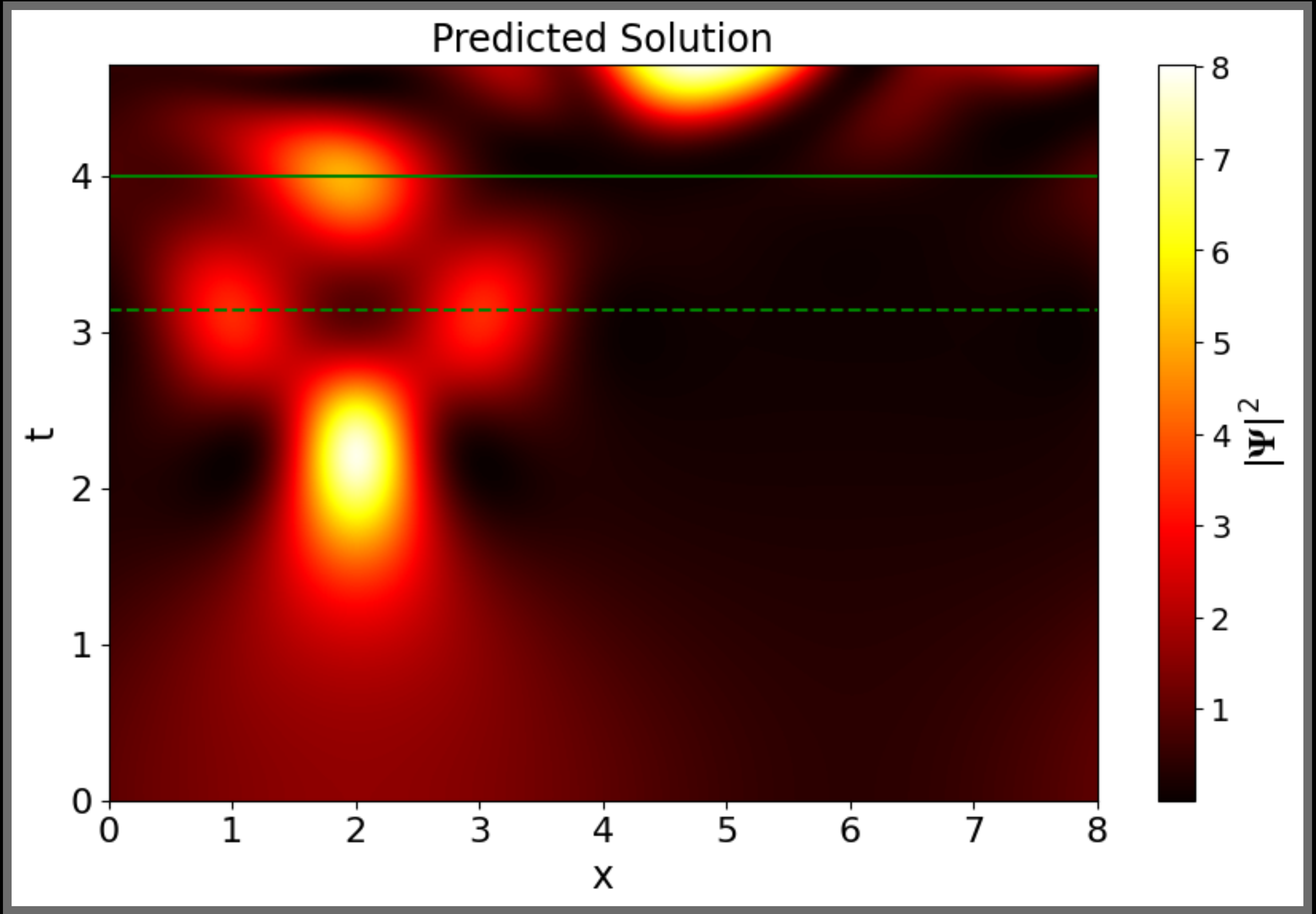
arXiv:2101.05821

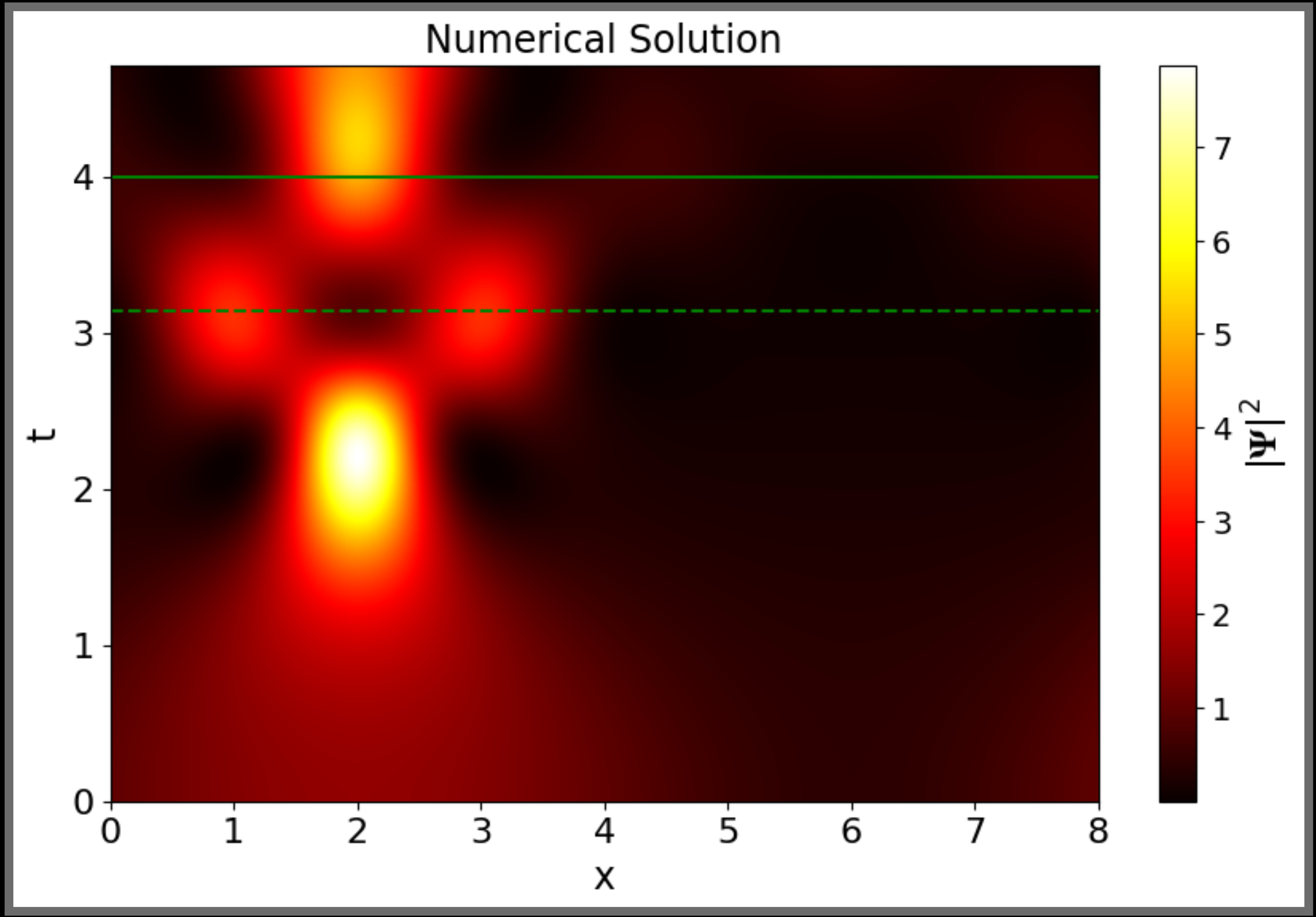


arXiv:2307.06032

Well agrees with existing works!

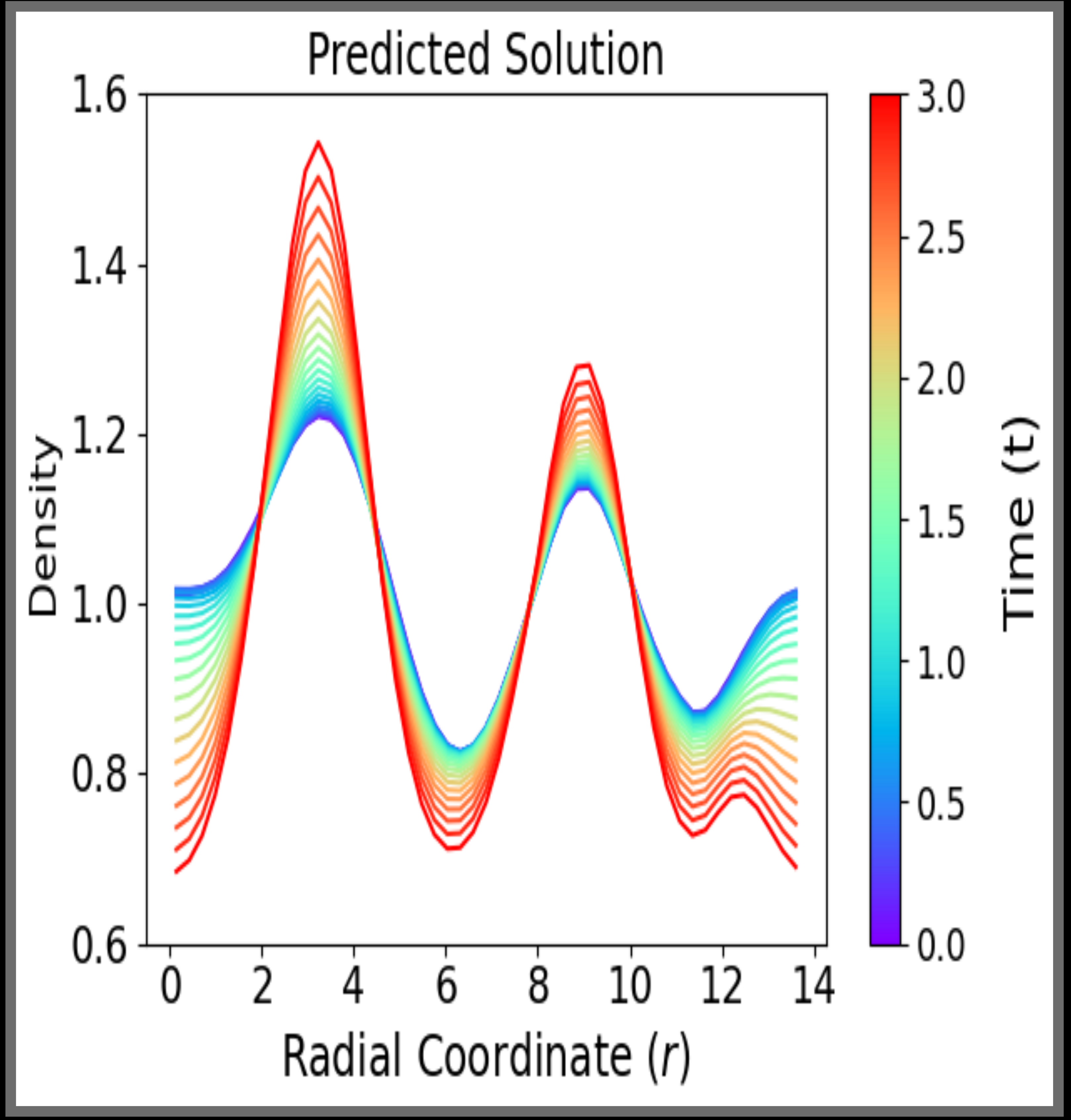
Extrapolation

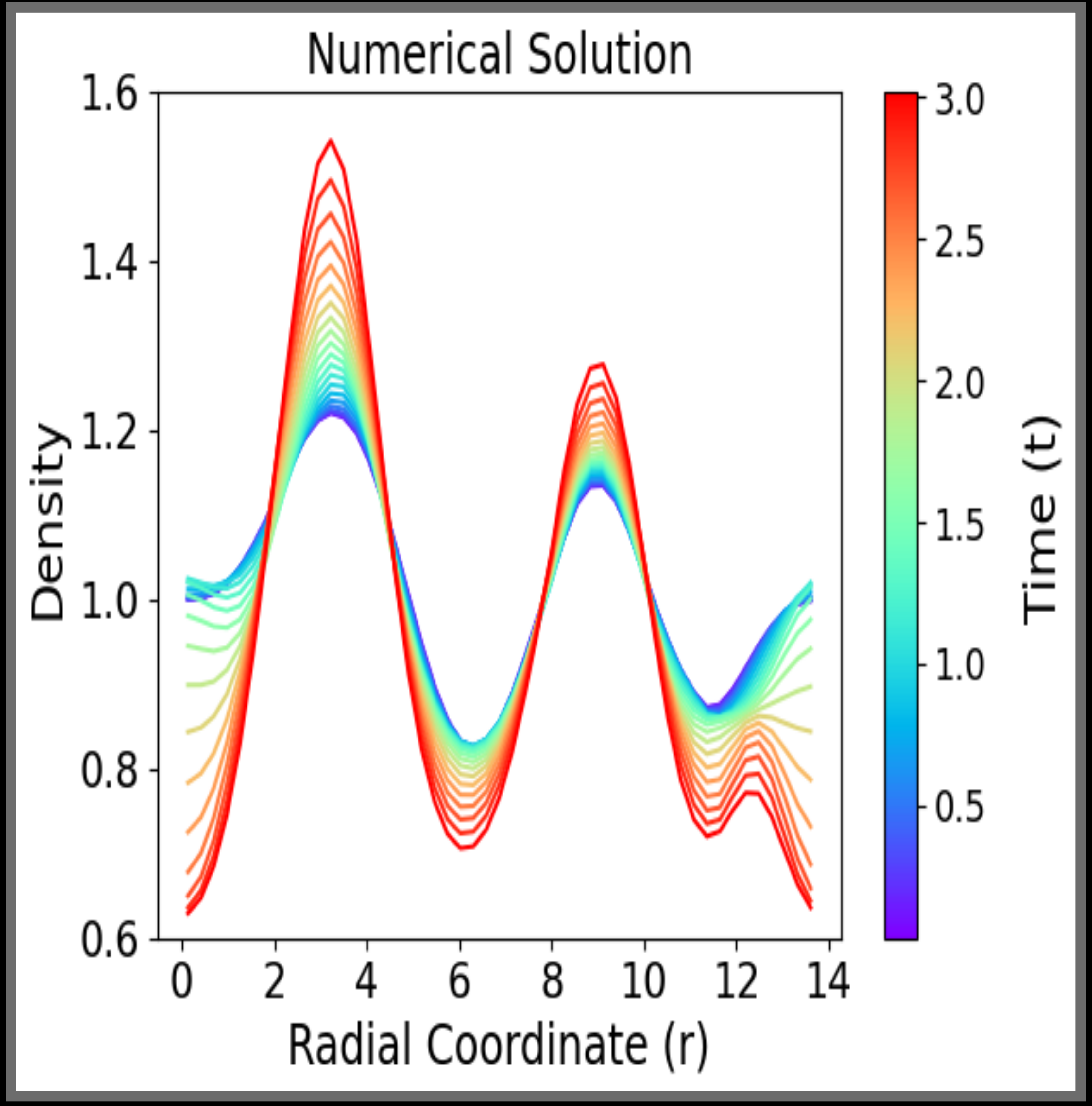


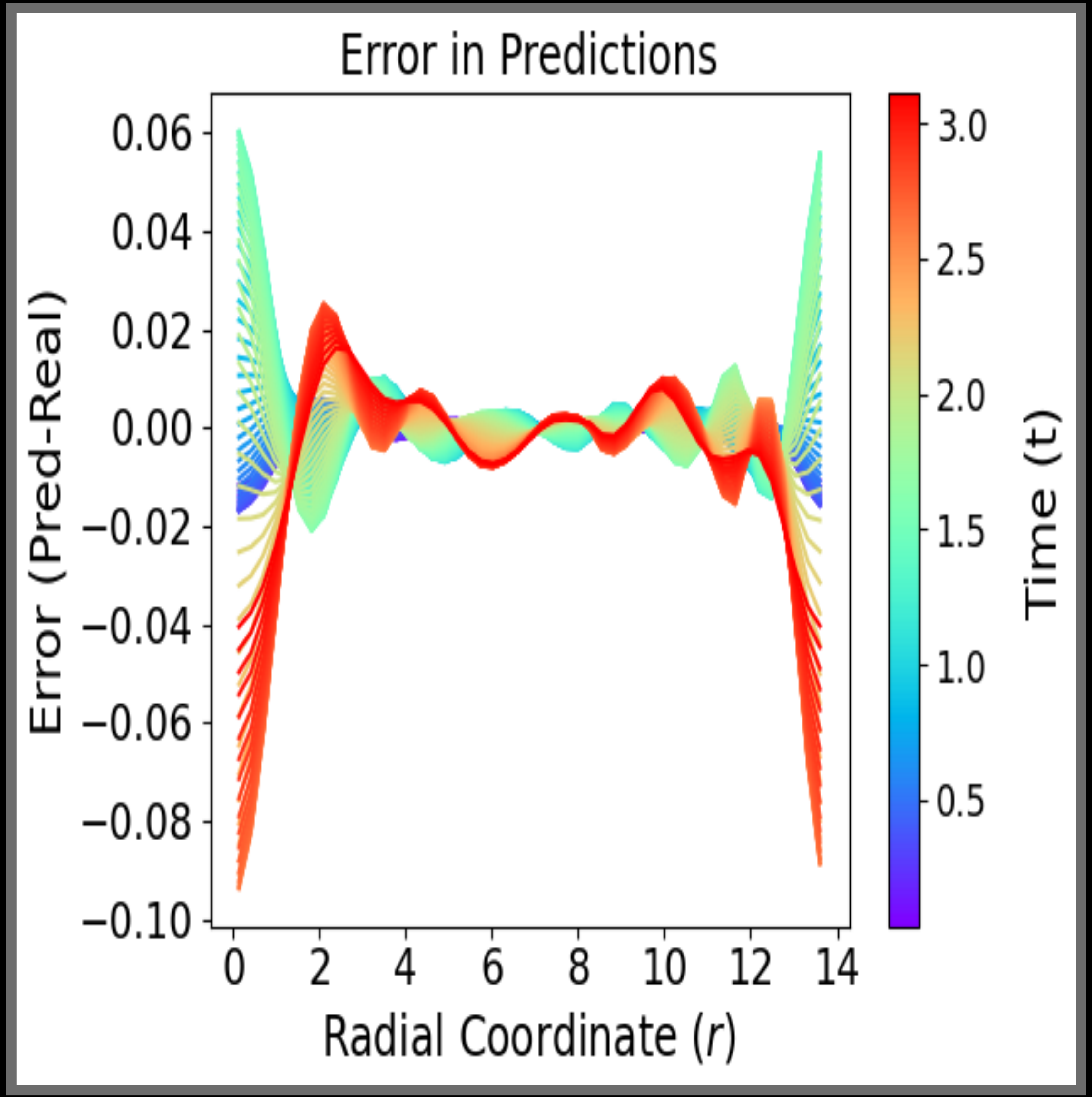


Evidence of SPINNs' ability to generalize and predict beyond trained time intervals !

Density Predictions in 3D

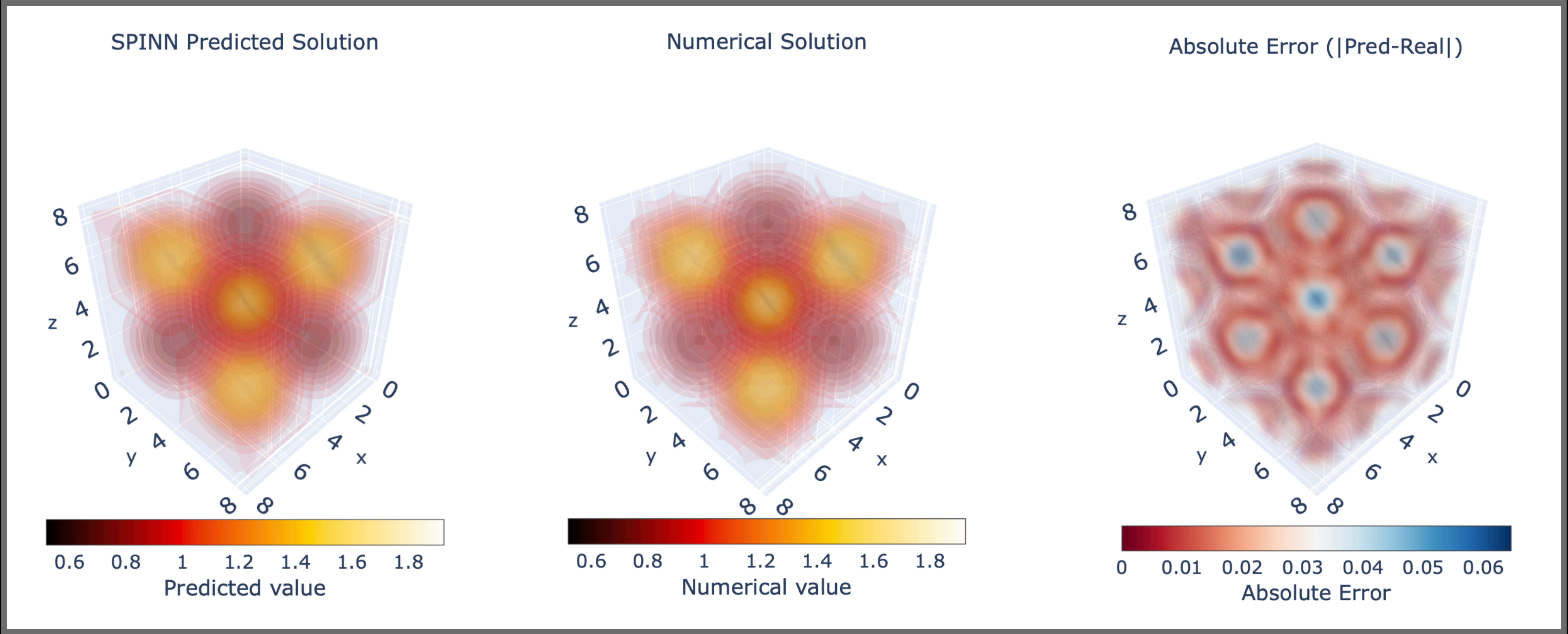






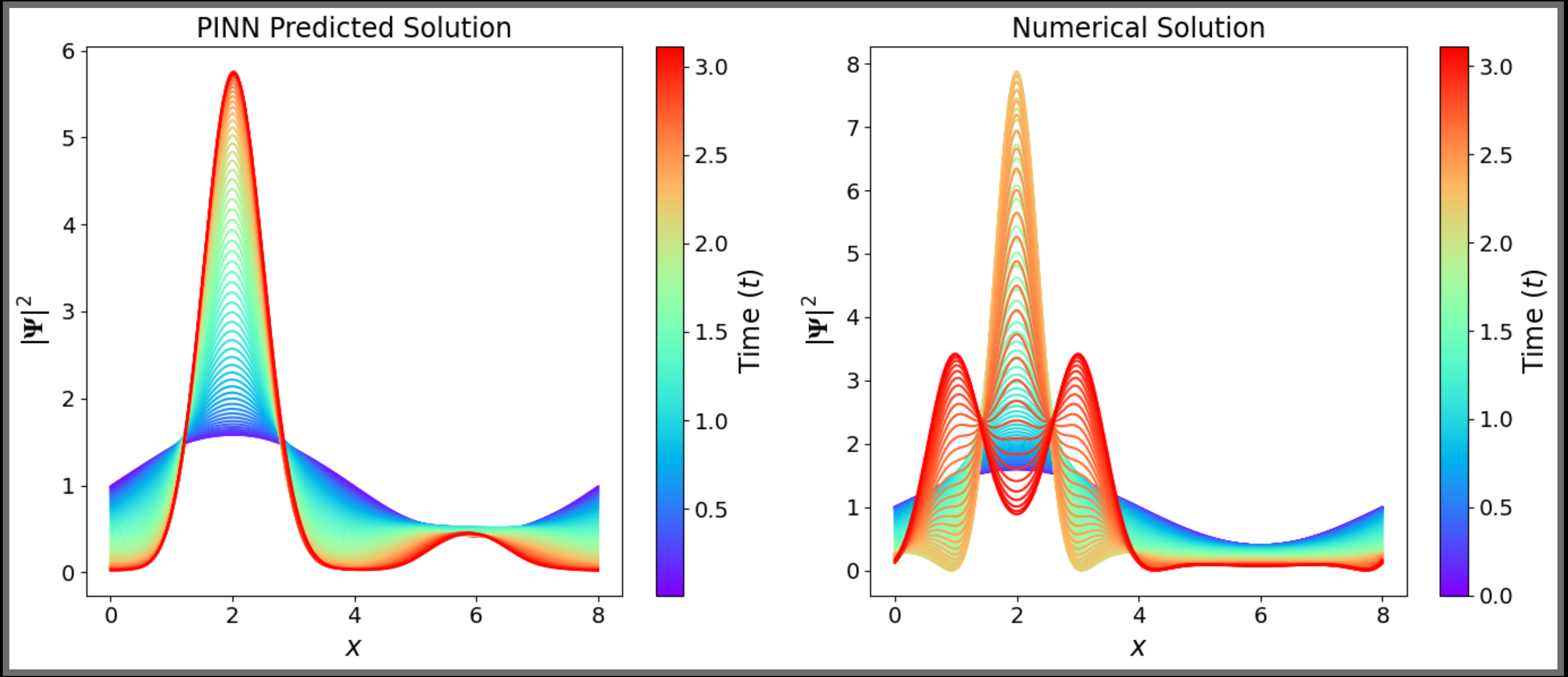
Overdensity collapse, well extends to 3D!

3D Cubes Comparisons



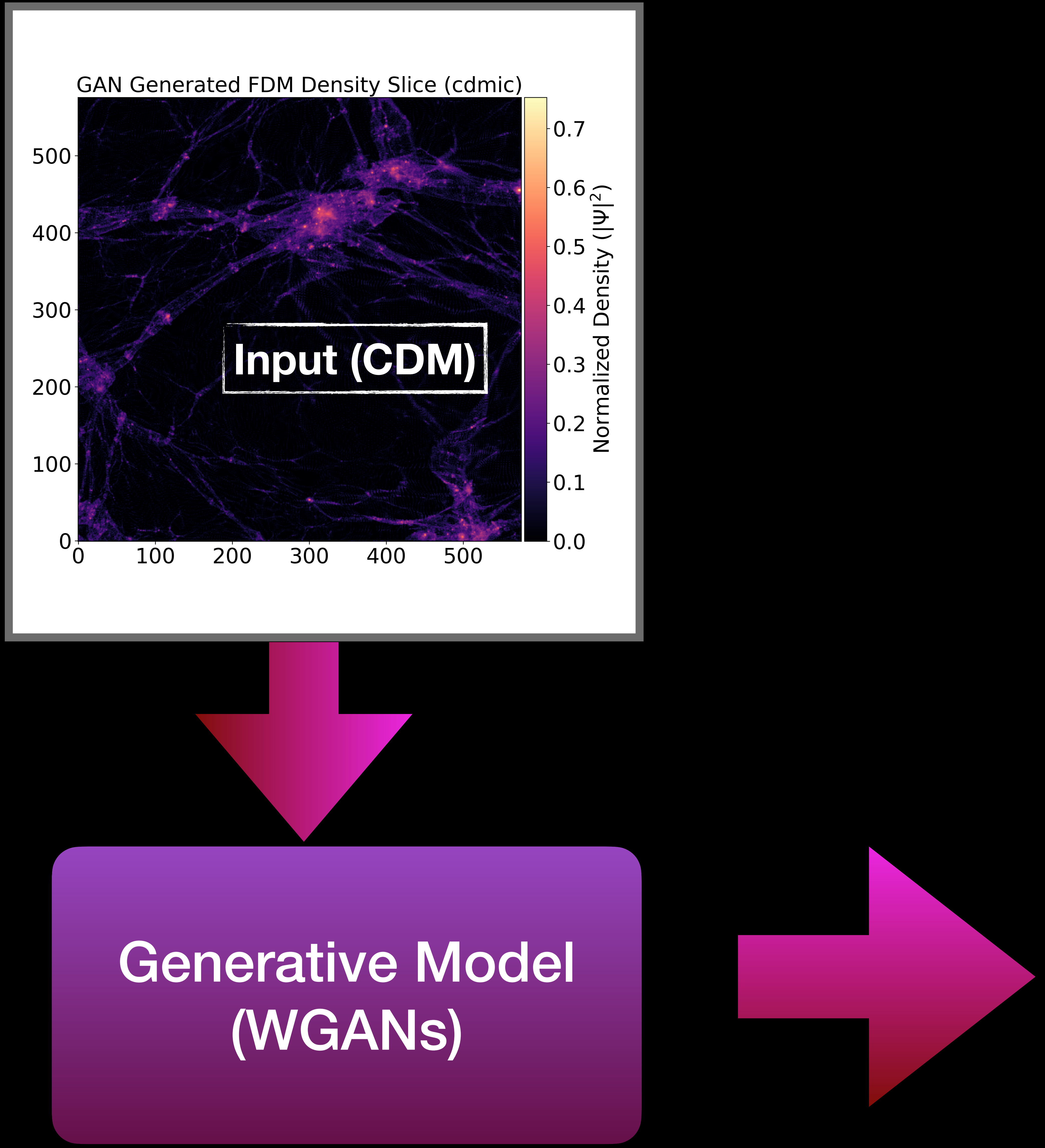
Maximum (abs) error of 6% for 3D cubes!

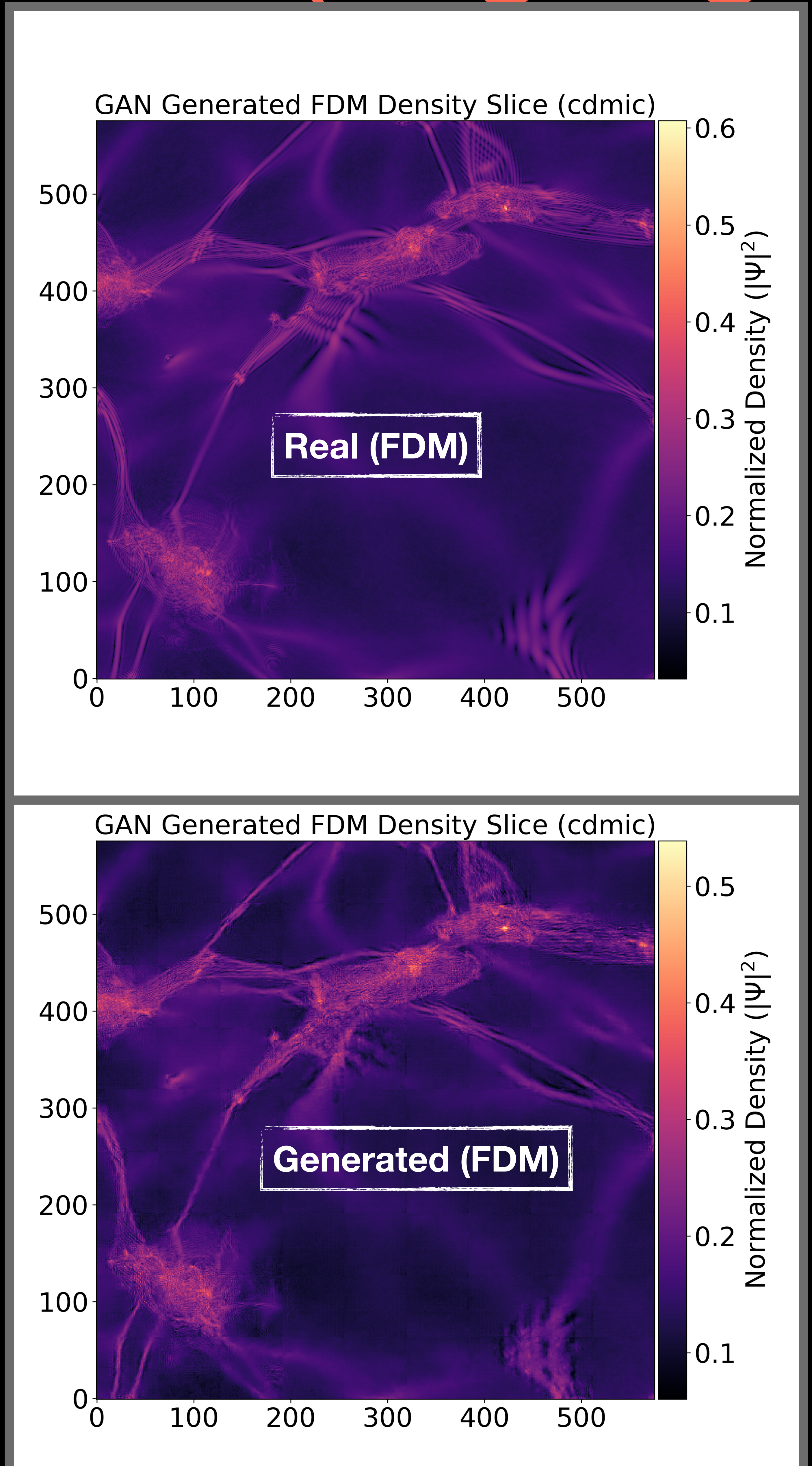
Results with Madelung Formalism



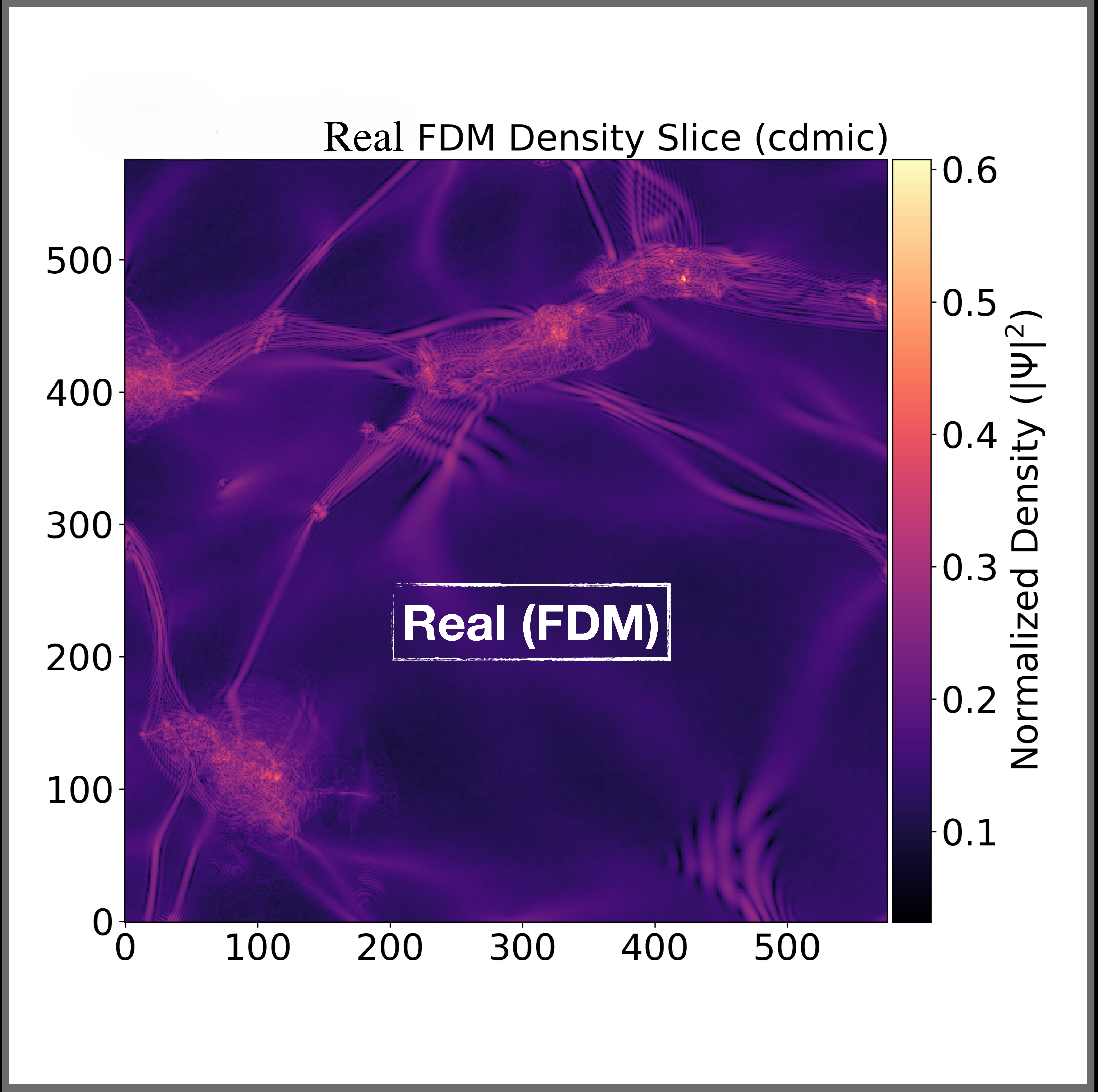
Doesn’t learn the interference features !

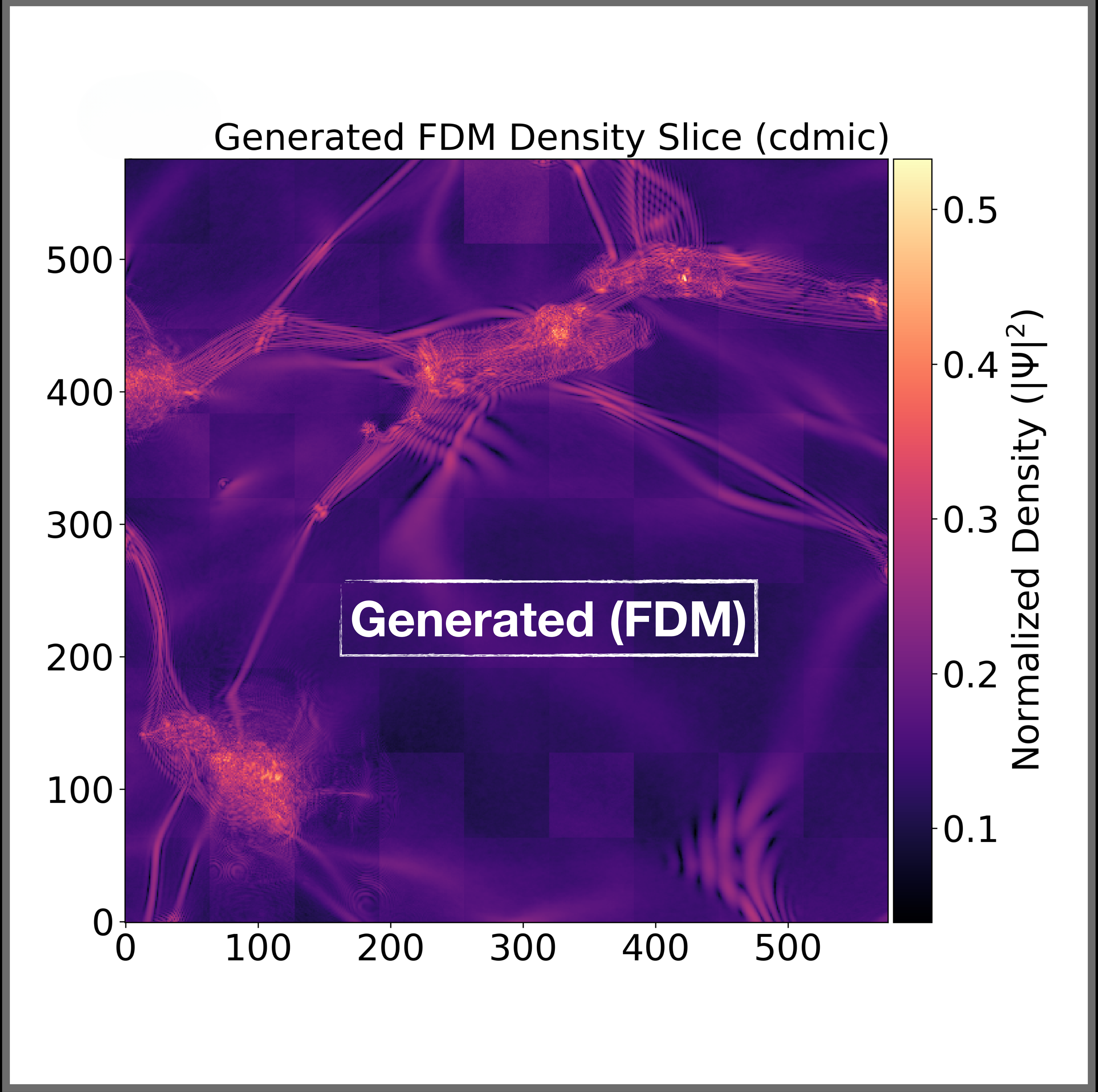
Current Hybrid Approach-WGANs (Ongoing work)





Current Hybrid Approach-DDPMs(Ongoing work)





Work in Progress!
(Still to scale to larger times)

01 **Unsupervised FDM PINNs with Intial conditions same as CDM case**

02 **Supervised PINNs using large-scale CDM simulations as additional data constraint**

03 **Generative Models for painting-in small-scale features**

04 **Reproducing Core-Halo Relations for FDM with PINNs**

THANK YOU!

SCAN ME

Arxiv Link

Question?

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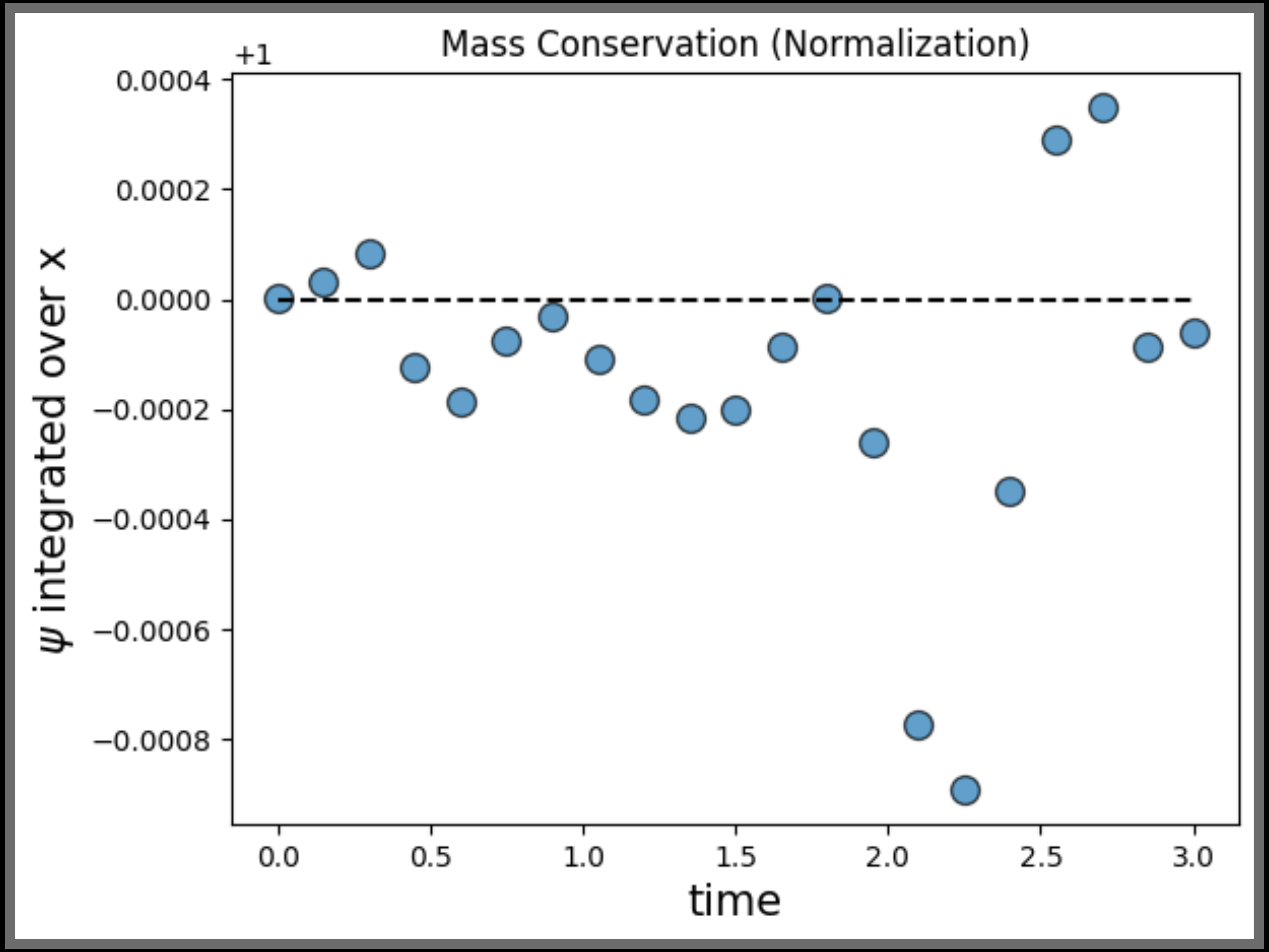
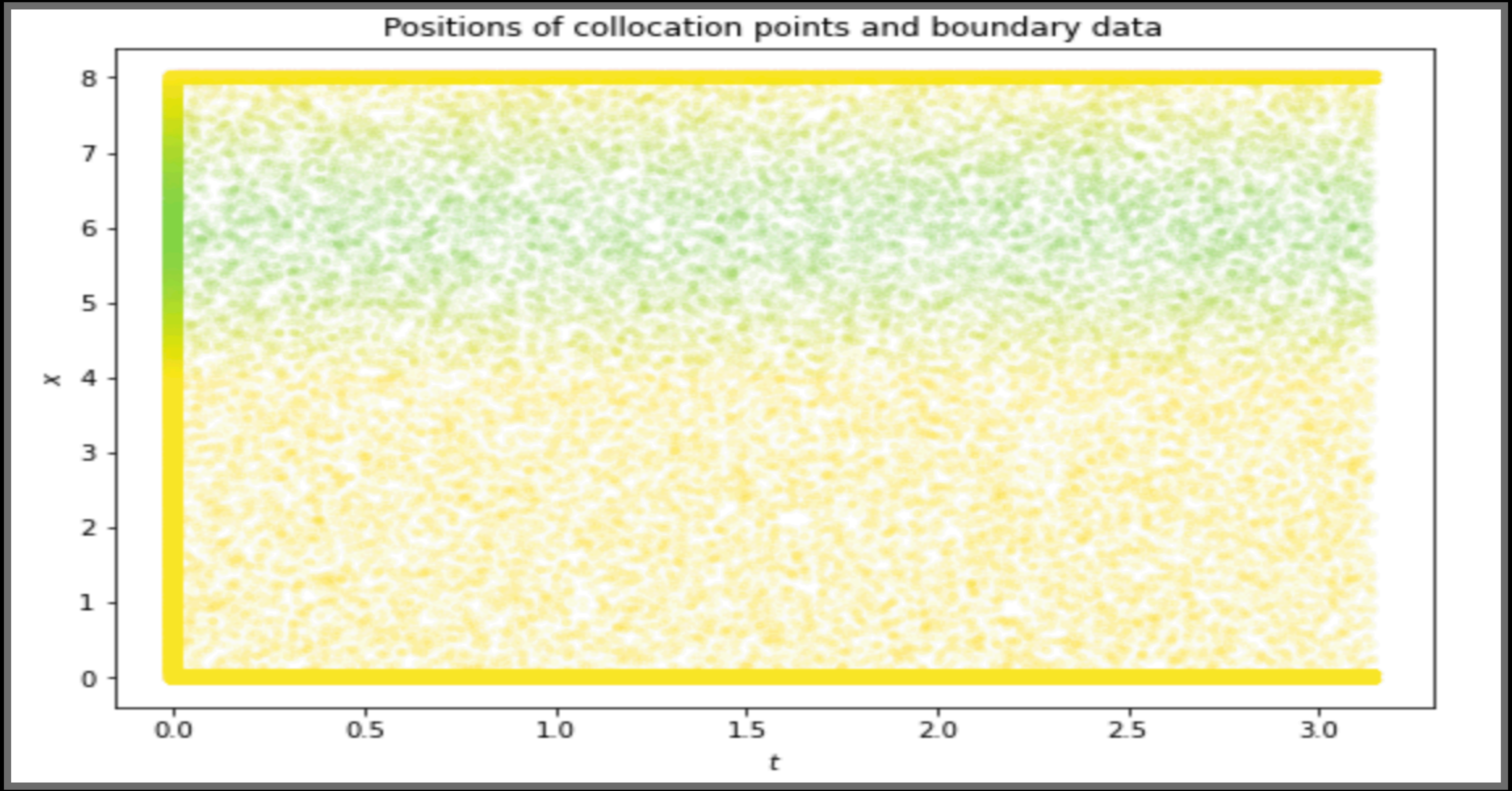
Checks on Density Predictions



Mass is largely conserved ✔

Collocation Points



$x \in [0,8] \quad t \in [0,\pi]$

Dense enough to learn the solution!

Periodic Boundary Conditions

Example in x-direction for Real part of Wavefunction and Potential:

Periodicity for Real Part:

$$\Re(\psi)(x=0,y,z,t) = \Re(\psi)(x=L,y,z,t)$$
$$\partial_x \Re(\psi)(x=0,y,z,t) = \partial_x \Re(\psi)(x=L,y,z,t)$$

Periodicity for Potential:

$$V(x=0,y,z,t) = V(x=L,y,z,t)$$
$$\partial_x V(x=0,y,z,t) = \partial_x V(x=L,y,z,t)$$

Loss Term for Boundary

$$MSE_b(\theta) = \frac{1}{N_b}\sum_{n=1}^{N_b}\left[\left|\Re_\theta(\Psi)(X_n^b) - \Re_b(\Psi)(X_n^b)\right|^2 + \left|\Im_\theta(\Psi)(X_n^b) - \Im_b(\Psi)(X_n^b)\right|^2 + \left|V_\theta(X_n^b) - V_b(X_n^b)\right|\right]$$

Residual Functions

Residual Contributions for Schrodinger + Poisson equations

$$\mathscr{R}_{\mathfrak{R}(\Psi)}(X) = \partial_t \mathfrak{R}_\theta(\Psi) + \frac{1}{2}\left(\sum_{i=1}^{d} \partial_{x_i}^2 \mathfrak{I}_\theta(\Psi)\right) - V_\theta \cdot \mathfrak{I}_\theta(\Psi)$$

$$\mathscr{R}_{\mathfrak{I}(\Psi)}(X) = \partial_t \mathfrak{I}_\theta(\Psi) - \frac{1}{2}\left(\sum_{i=1}^{d} \partial_{x_i}^2 \mathfrak{R}_\theta(\Psi)\right) + V_\theta \cdot \mathfrak{R}_\theta(\Psi)$$

$$\mathscr{R}_V(X) = \sum_{i=1}^{d} \partial_{x_i}^2 V_\theta - \left((\mathfrak{R}_\theta(\Psi)^2 + \mathfrak{I}_\theta(\Psi)^2) - 1.0\right)$$

Loss Term for Enforcing PDEs

$$MSE_{PDE}(\theta) = \frac{1}{N_r}\sum_{n=1}^{N_r}\left[\left|\mathscr{R}_{\mathfrak{R}(\Psi)}(X_n^r)\right|^2 + \left|\mathscr{R}_{\mathfrak{I}(\Psi)}(X_n^r)\right|^2 + \left|\mathscr{R}_V(X_n^r)\right|^2\right]$$

Numerical Method (Mocz et. al. 2017)

2nd Order Unitary Spectral Method

- ✦ Calculate potential:

$$V = \text{IFFT}\left(-\frac{1}{k^2}\text{FFT}\left(4\pi Gm(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$

Kick

- ✦ Full-Step 'Drift' in Fourier Space:

$$\psi \leftarrow \textbf{IFFT}\left(exp[-i\Delta t(\hbar/m)k^2/2]\textbf{FFT}(\psi)\right)$$

Drift

- ✦ Update the potential:

$$V \leftarrow \textbf{IFFT}\left(-\frac{1}{k^2}\textbf{FFT}\left(4\pi Gm(|\psi|^2 - |\psi_0|^2)\right)\right)$$

- ✦ Another Half-Step 'Kick':

$$\psi \leftarrow exp[-i(m/\hbar)(\Delta t/2)V]\psi$$

Kick