

hands on

$$\begin{aligned} \langle \delta^3(\underline{x}) \rangle &= \langle (\delta^{(1)} + \delta^{(2)} + \cancel{\delta^{(3)}})^3(\underline{x}) \rangle \\ &= \underbrace{\langle \delta^{(1)}(\underline{x})^3 \rangle}_{=0} + 3 \langle \delta^{(1)}(\underline{x})^2 \delta^{(2)}(\underline{x}) \rangle \end{aligned}$$

for Gaussian initial field

$$\begin{aligned} &= 3 \int \frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^9} \exp[i \underline{x} \cdot (\underline{k}_1 + \underline{k}_2 + \underline{k}_3)] \langle \delta^{(1)}(\underline{k}_1) \delta^{(1)}(\underline{k}_2) \delta^{(2)}(\underline{k}_3) \rangle \\ &\quad \xrightarrow{\text{redefine}} \uparrow \quad \text{plug in } F_2 \\ &= 3 \int \frac{d^3k_1 d^3k_2 d^3\tilde{k}_3 d^3k_4}{(2\pi)^{12}} \exp[i \underline{x} \cdot (\underline{k}_1 + \underline{k}_2 + \tilde{\underline{k}}_3 + \underline{k}_4)] F_2(\tilde{\underline{k}}_3, \underline{k}_4) \\ &\quad \xrightarrow{\langle \delta^{(1)}(\underline{k}_1) \delta^{(1)}(\underline{k}_2) \delta^{(1)}(\tilde{\underline{k}}_3) \delta^{(1)}(\underline{k}_4) \rangle} \\ &\quad (2\pi)^6 \left[ \delta_D(\underline{k}_1 + \underline{k}_2) \delta_D(\tilde{\underline{k}}_3 + \underline{k}_4) P_L(k_1) P_L(\tilde{k}_3) \right. \\ &\quad + \delta_D(\underline{k}_1 + \tilde{\underline{k}}_3) \delta_D(\underline{k}_2 + \underline{k}_4) P_L(k_1) P_L(k_2) \\ &\quad \left. + \delta_D(\underline{k}_1 + \underline{k}_4) \delta_D(\underline{k}_2 + \tilde{\underline{k}}_3) P_L(k_1) P_L(\tilde{k}_2) \right] \end{aligned}$$

$$= 6 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} F_2(-\underline{k}_1, -\underline{k}_2) P_L(k_1) P_L(k_2)$$

$$\uparrow \quad d^3k = dk k^2 dm d\theta$$

$$= 6 \cdot \left( \frac{5}{7} \cdot 1 + \frac{2}{7} \cdot \frac{1}{3} \right) \langle \delta_L^2 \rangle^2 = 6 \cdot \frac{17}{21} \langle \delta_L^2 \rangle^2 = \frac{34}{7} \sigma^4$$

$$\frac{\langle \delta^3 \rangle}{\langle \delta^2 \rangle^2} = \frac{34}{7}$$

$$\alpha(\underline{k}_1, \underline{k}_2) \quad \text{angular average} \quad \bar{\alpha} = 1$$

$$F_2 = \frac{5}{7} \alpha + \frac{2}{7} \beta$$

$$\beta(\underline{k}_1, \underline{k}_2) \quad \bar{\beta} = 1/3$$

or bispectrum

$$\langle \delta^{(1)}(\underline{k}_1) \delta^{(1)}(\underline{k}_2) \delta^{(2)}(\underline{k}_3) \rangle \approx \dots$$

+ cyclic

$$\Rightarrow B_{112}(k_1, k_2, k_3) = 2 \left( F_2(\underline{k}_1, \underline{k}_2) P_L(k_1) P_L(k_2) \right)$$