GDR CoPhy - IAP

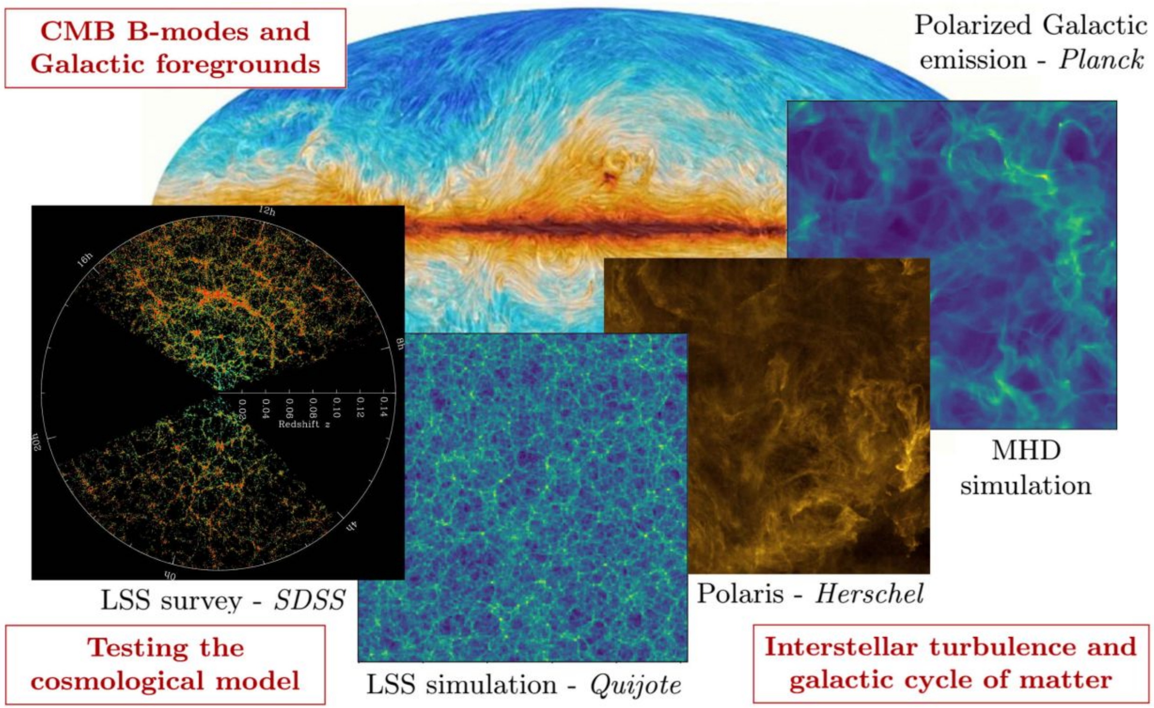
November 20, 2024

s2scat : a software in JAX for generative models on the sphere using Scattering Transforms

Louise Mousset
E. Allys, M. Price, J. Aumont, J.M. Delouis, L. Montier, J. McEwen

LPENS
LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

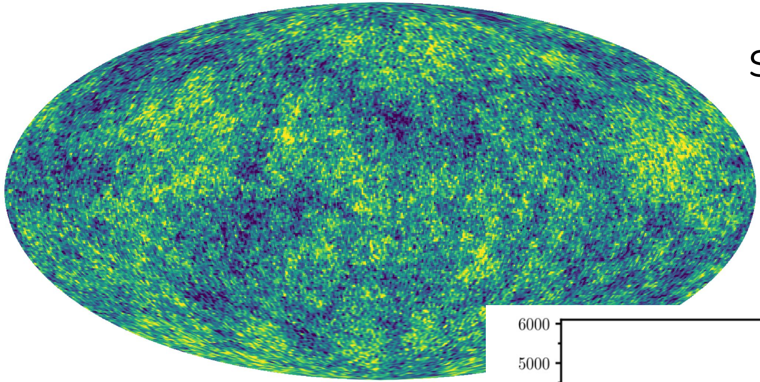
Non Gaussianities in astrophysics



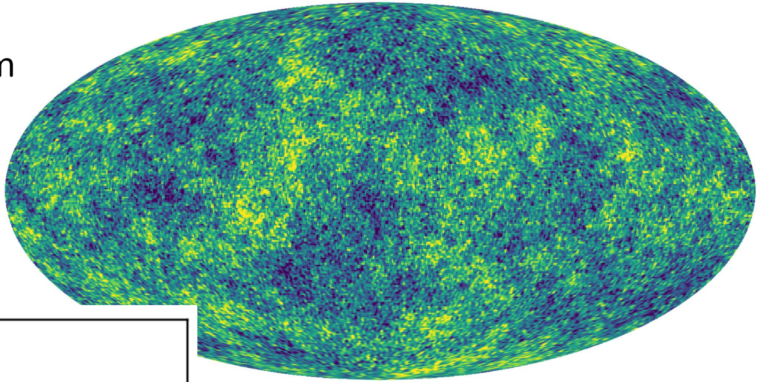
Common difficulty : non-linearity ⇒ non-Gaussian structures
→ Important lever arm for a lot of astrophysical objectives

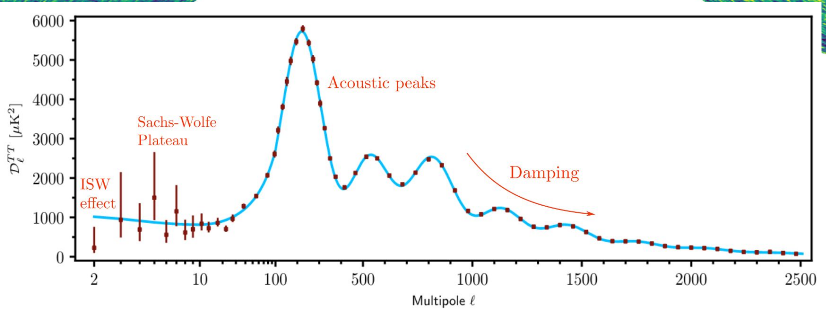
Slide from E. Allys

CMB is a Gaussian field



Same power spectrum





Linear processes → Fourier modes are independent, no coupling between scales → Power spectrum contains all the statistical information

Non Gaussian field

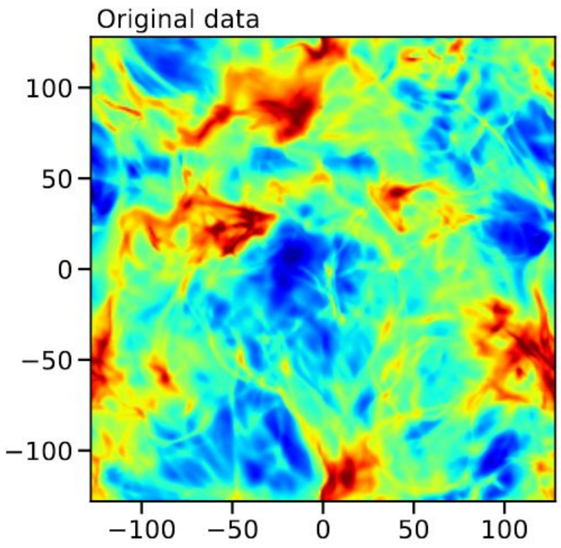
Non linear physical processes: gravity, magneto-hydrodynamics...

→

Coupling between scales, Non Gaussian structures (ex: filaments)

→

Power spectrum is not sufficient



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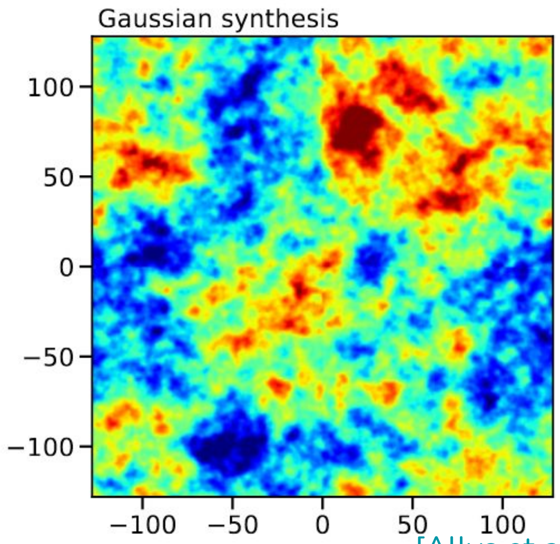

[Allys et al. 2019]

=> Need higher order non-Gaussian statistics for proper characterisation

Scattering Transforms

Scattering Transforms are [Bruna et al. 2013, Allys et al. 2019]

- a set of non-Gaussian summary statistics
- inspired from neural networks, but can be computed without a training stage, even from a single image.
- **directional wavelet filters** to separate the different scales and angles

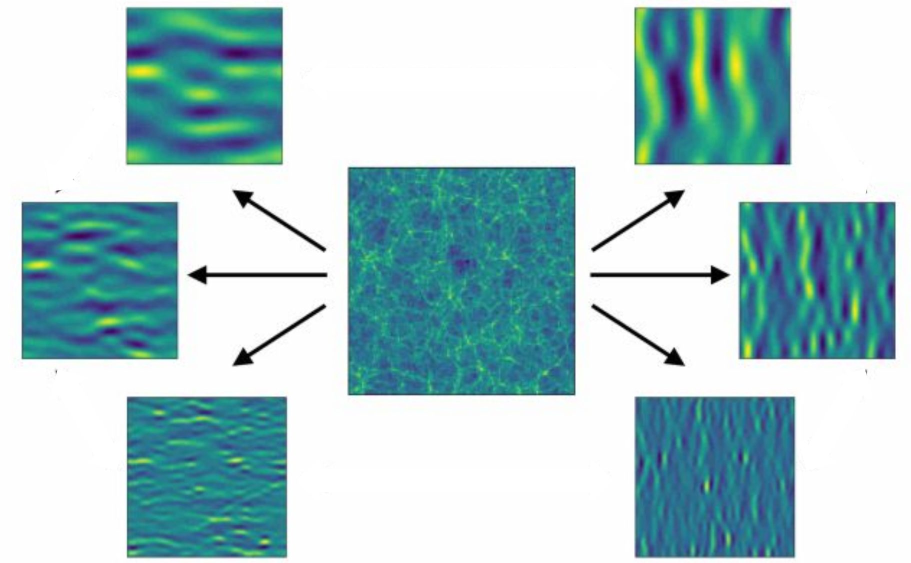

Image from E. Allys

Scattering Transforms

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- a set of non-Gaussian summary statistics
- inspired from neural networks, but can be computed without a training stage, even from a single image.
- **directional wavelet filters** to separate the different scales and angles
- **non linearities** to extract the couplings between scales

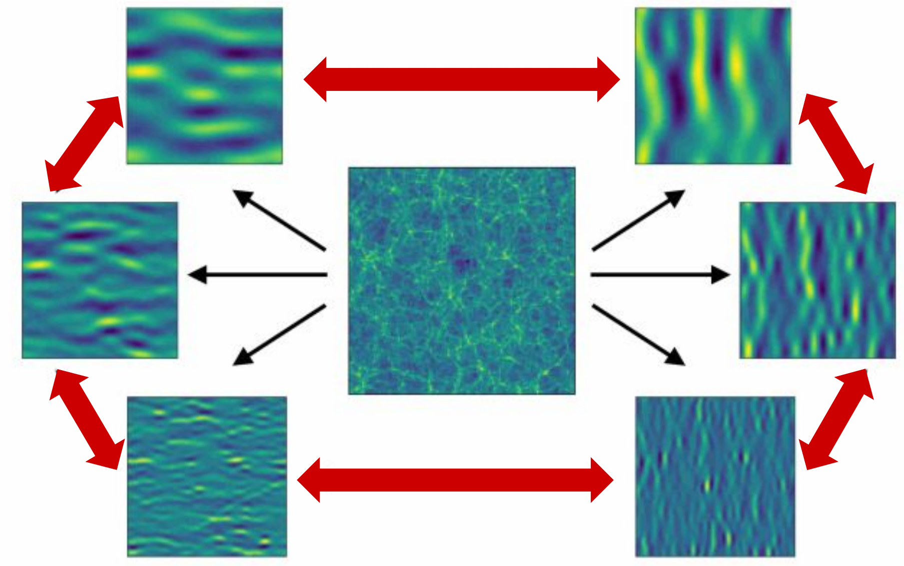
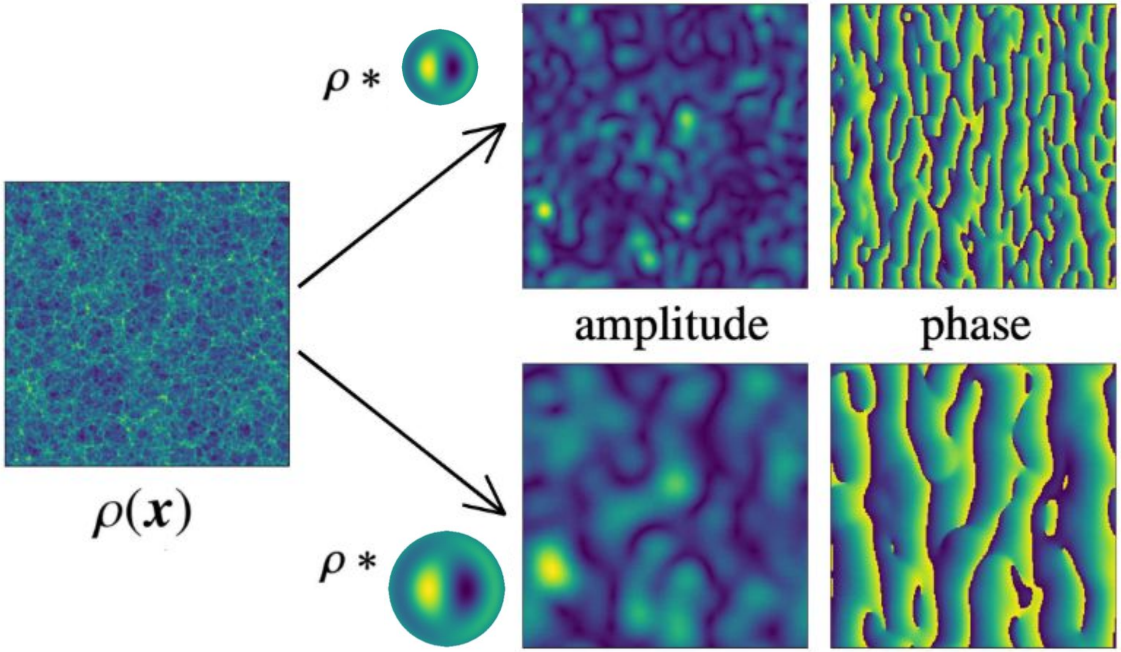
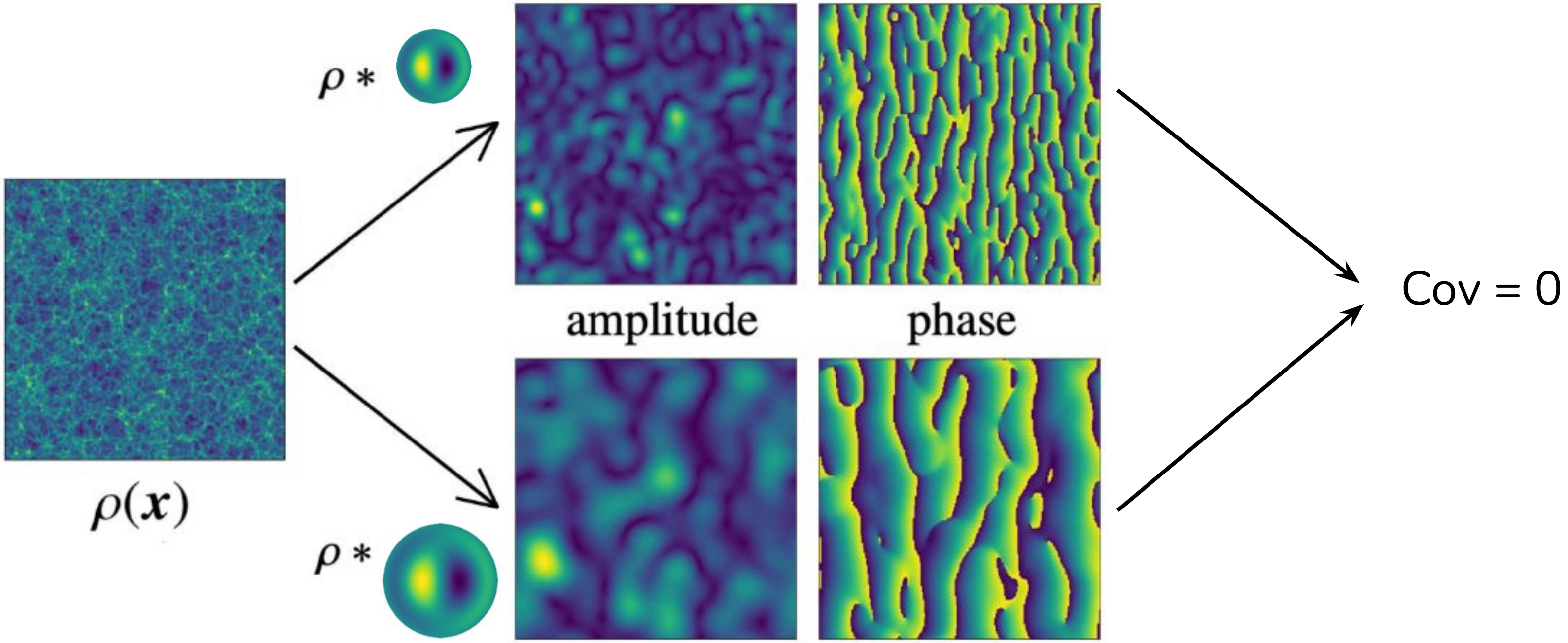


Image from E. Allys

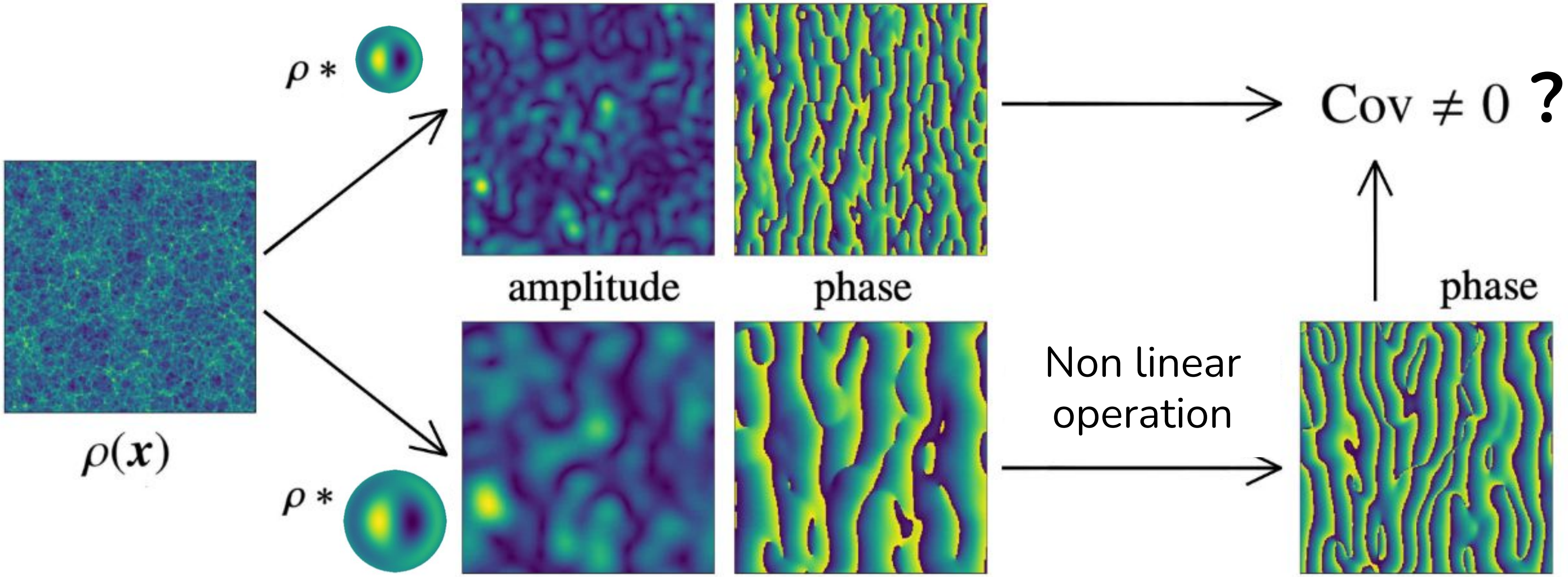
Extract coupling between scales



Extract coupling between scales



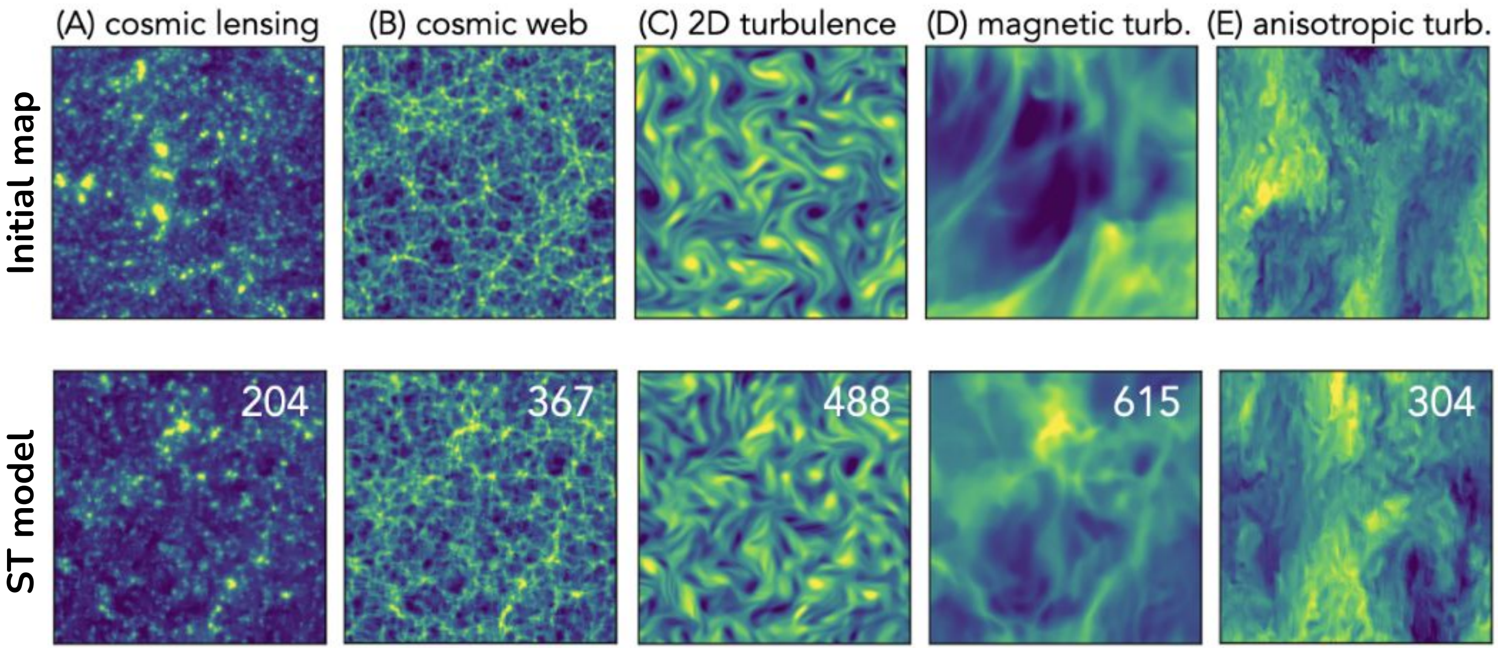
Extract coupling between scales



This is for one pair of scales.
=> Following the same for all scales we get a family of coefficients.

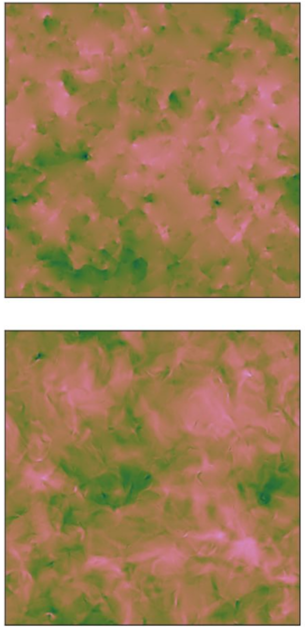
Application 1: Generative models

[Cheng et al. 2023]



[Price et al. 2023]

Cosmic strings

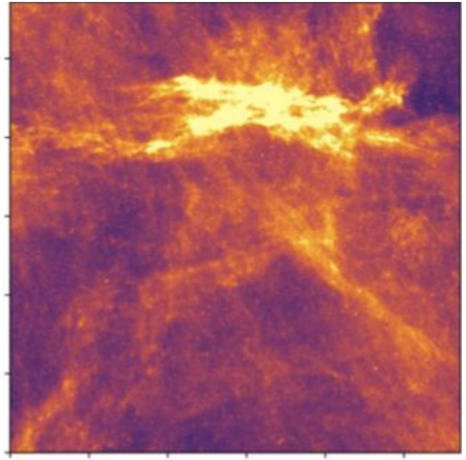


Quantitatively realistic generative models from a few 100 of coefficients only.

Application 2: Component separation in Hershel data

[Auclair et al. 2024]

Data = Dust + CIB



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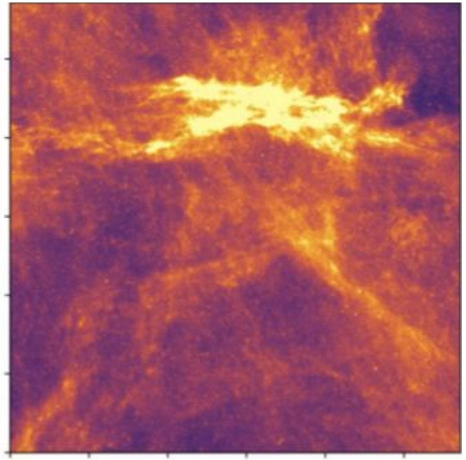
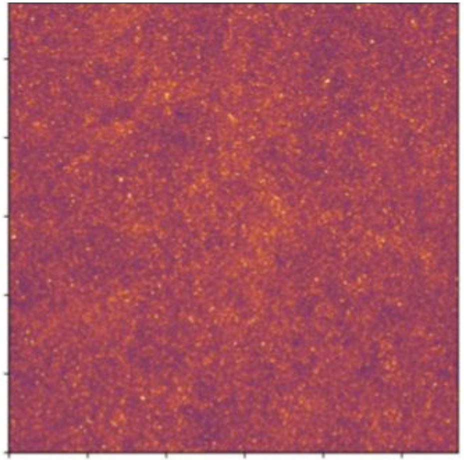


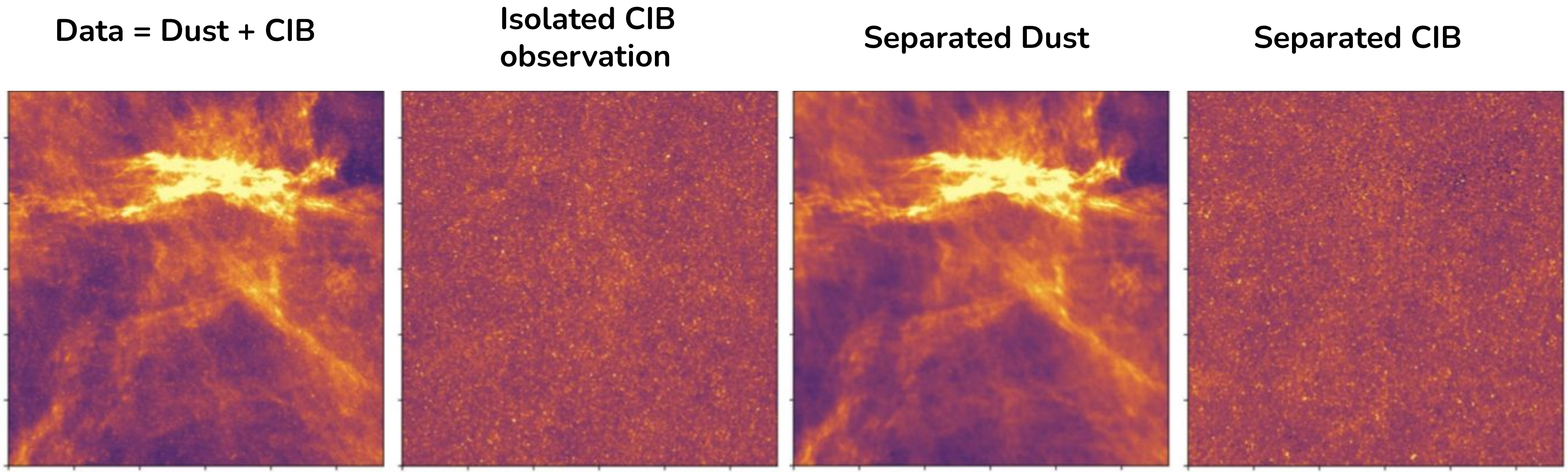
Isolated CIB observation



Compute ST statistics

Application 2: Component separation in Hershel data

[Auclair et al. 2024]



Component separation using only non Gaussian spatial information.
Using only 2 observations.

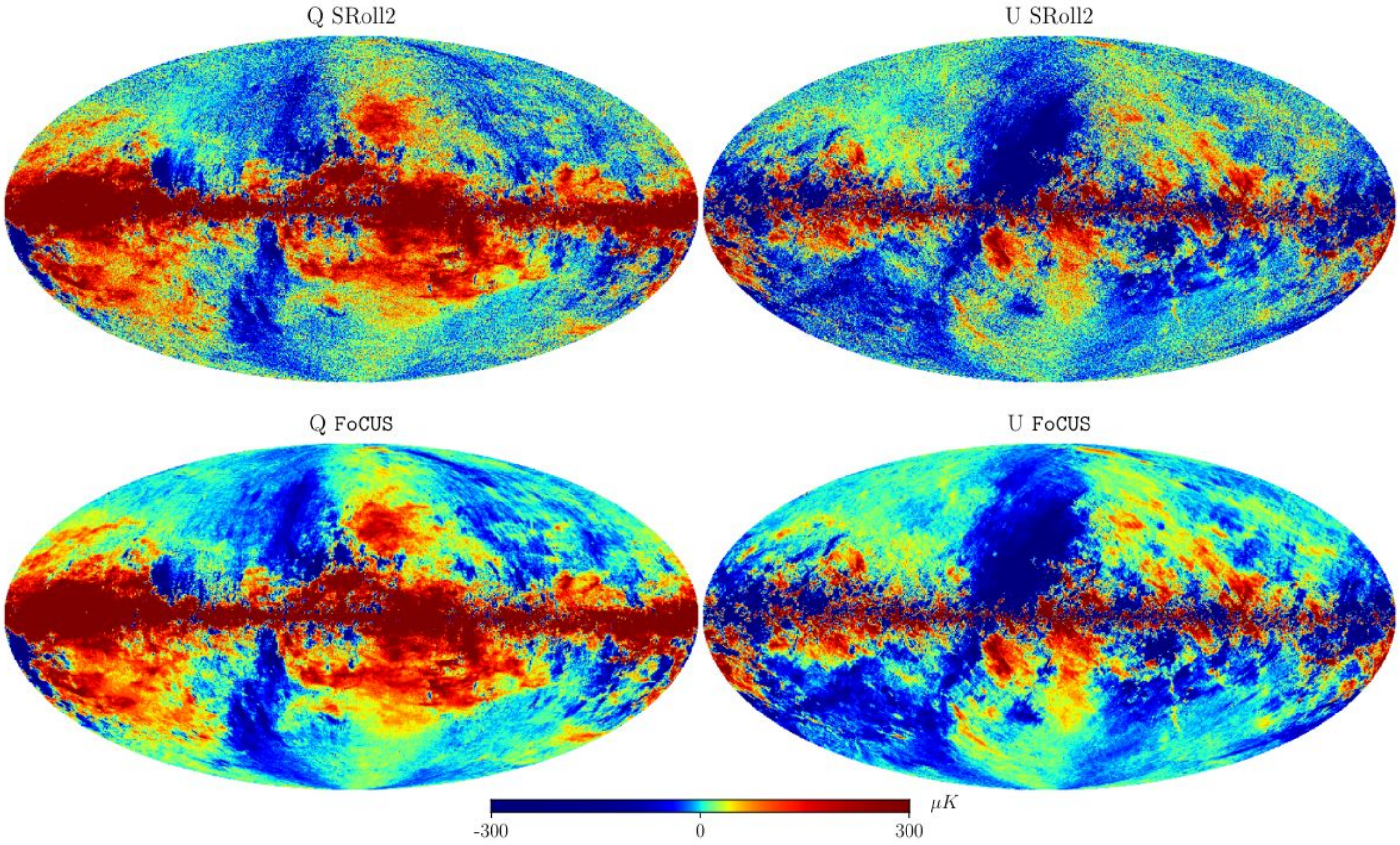
Application 3: Statistical denoising of the Planck maps

[Delouis et al. 2023]

Input: Noise templates for Planck maps.

Deterministic denoising up to SNR≃ 0.1.

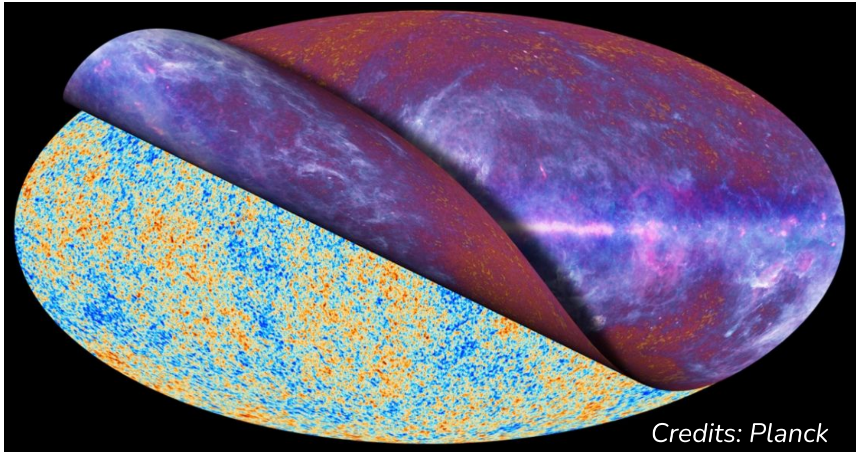
Statistical denoising up to SNR≃0.01.



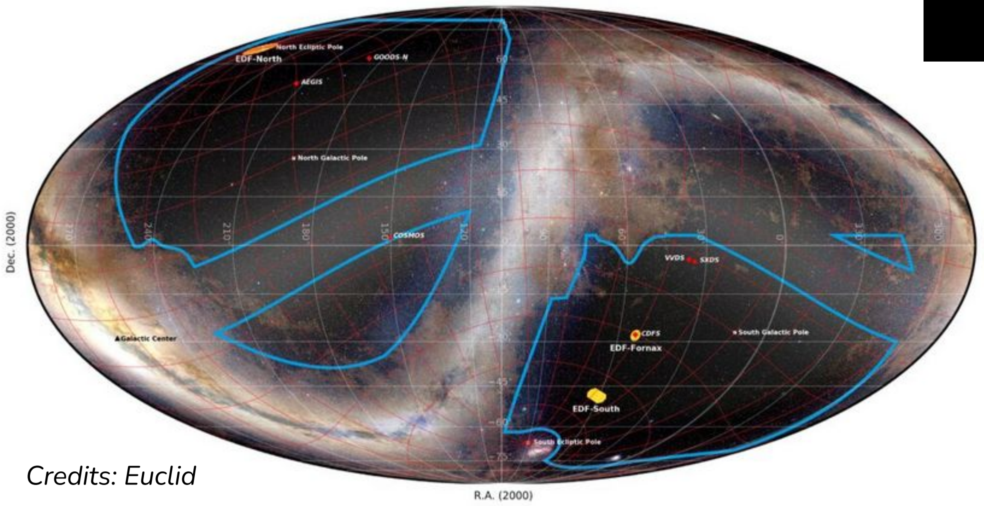
Adapting these tools to spherical maps is needed

Large scale cosmological surveys

- Large Scale Structures : Euclid, Vera Rubin
- CMB : Planck, SPT, ACT, LiteBIRD...



Credits: Planck



Credits: Euclid

Credits: SPT

s2scat

https://github.com/astro-informatics/s2scat

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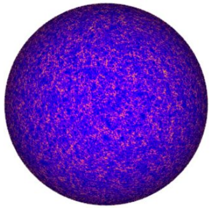
acceleration.

Scattering covariances



We introduce scattering covariances on the sphere in Mousset et al. (2024), which extend to spherical settings similar scattering transforms introduced for 1D signals by Morel et al. (2023) and for planar 2D signals by Cheng et al. (2023). Scattering covariances S are computed by

$$S_1^{\lambda_1} = \langle \, | W^{\lambda_1} I \, | \, \rangle,$$

$$S_2^{\lambda_1} = \langle \, | W^{\lambda_1} I \, |^2 \, \rangle,$$

$$S_3^{\lambda_1,\lambda_2} = \mathrm{Cov}\big[W^{\lambda_1} I, W^{\lambda_1} \, | \, W^{\lambda_2} I \, | \, \big],$$

$$S_4^{\lambda_1,\lambda_2,\lambda_3} = \mathrm{Cov}\big[W^{\lambda_1} \, | \, W^{\lambda_3} I \, | \, , W^{\lambda_1} \, | \, W^{\lambda_2} I \, | \, \big]$$

where $W^{\lambda} I$ denotes the wavelet transform of field I at scale j and direction γ, which we group into a single label $\lambda = (j, \gamma)$.

Paper : https://arxiv.org/abs/2407.07007

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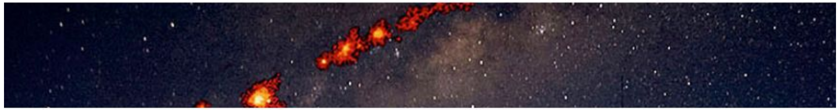

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A&A, 691, A269 (2024)

Generative models of astrophysical fields with scattering transforms on the sphere

L. Mousset[1]★, E. Allys[1], M. A. Price[2], J. Aumont[3], J.-M. Delouis[4], L. Montier[3] and J. D. McEwen[2]

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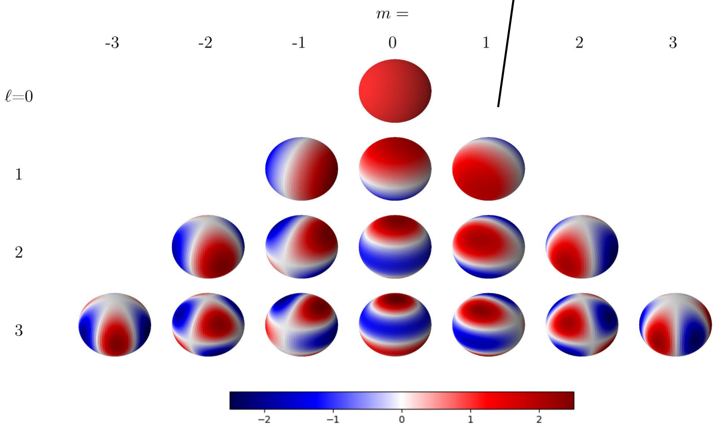
Wavelet filters on the sphere

[Price & McEwen 2023]

Spherical harmonic transform:

$$\Psi(\theta,\varphi) = \sum_{\ell m} \Psi_{\ell m} Y_{\ell m}(\theta,\varphi)$$



$$-\ell \leq m \leq \ell \qquad \ell \sim \frac{\pi}{\theta}$$

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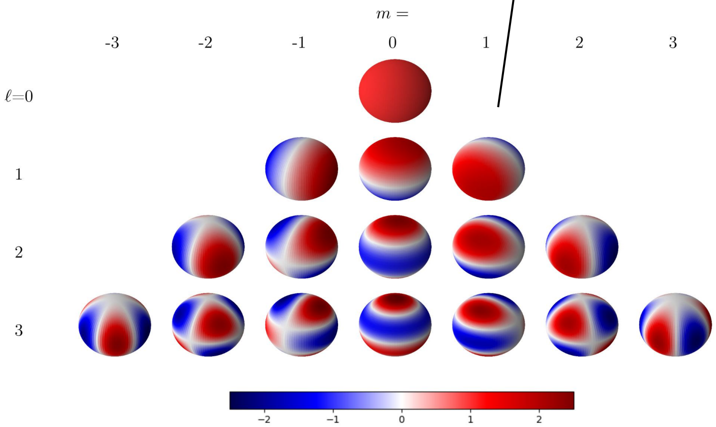


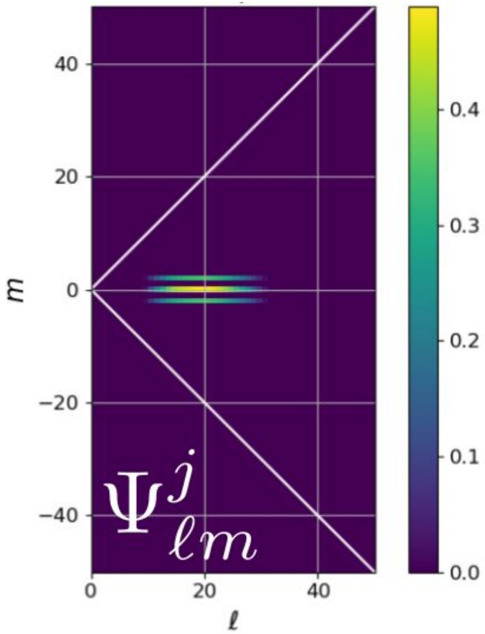
$$-\ell \leq m \leq \ell \qquad \ell \sim \frac{\pi}{\theta}$$

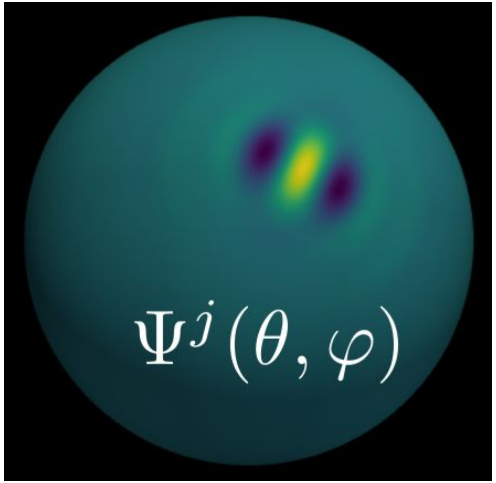
Wavelet filters:

$$\Psi^j_{\ell m} = \sqrt{\frac{2\ell+1}{8\pi^2}}\,\kappa^j_\ell\,\zeta_{\ell m}$$

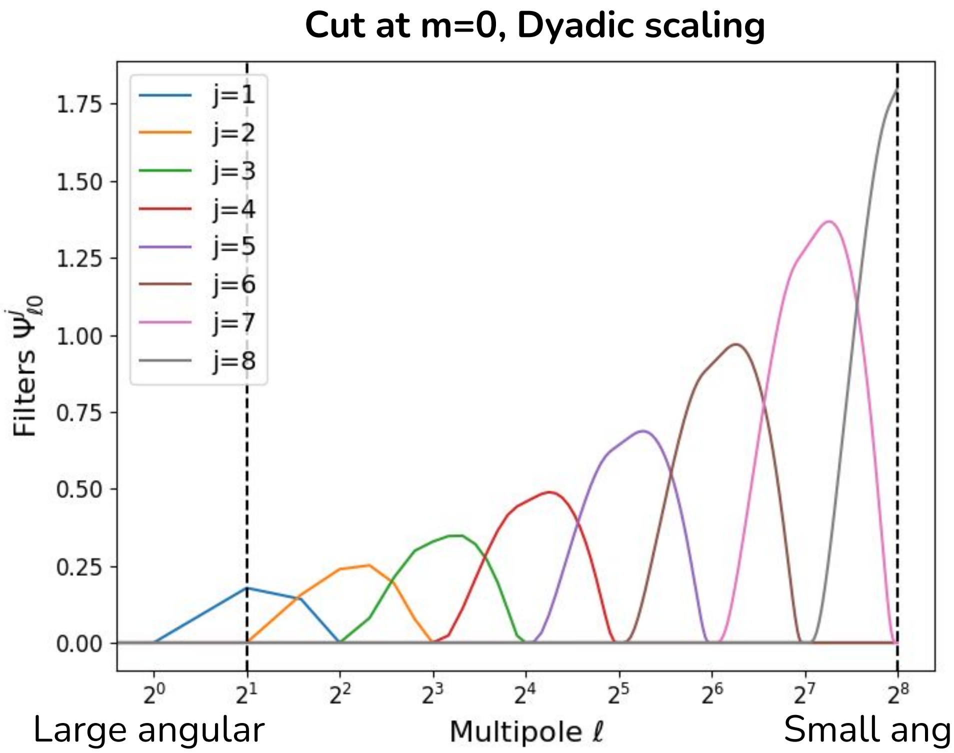
Filter scale

Directionality



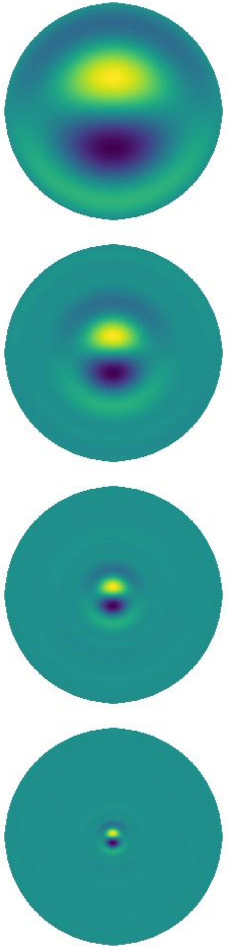


Filter set scaling



Large angular scales

Small angular scales



$j = 1$

$j = 2$

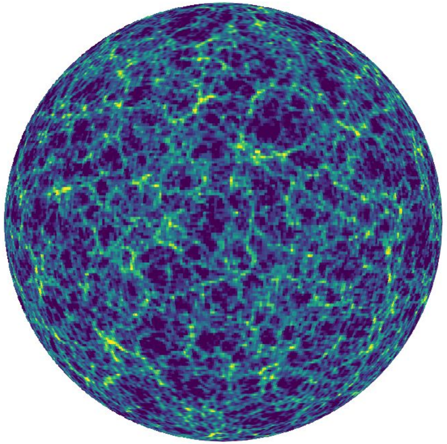
$j = 3$

$j = 4$

Wavelet transform

[McEwen et al. 2015, Price & McEwen 2023]
S2FFT Python package

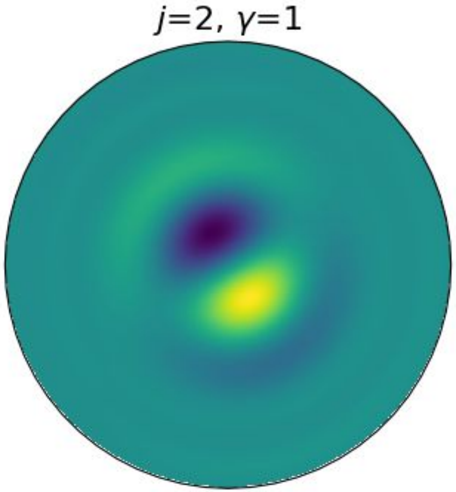
Field I



Directional convolution

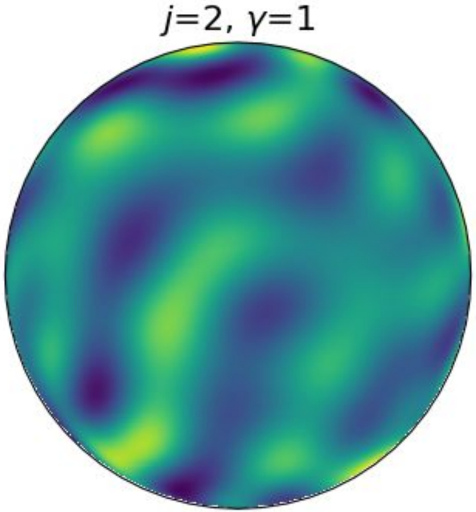
$\star$

Filter Ψ



Wavelet transform

$I \star \Psi^{j,\gamma}$



Directional convolution performed in harmonic space.

Scattering covariance coefficients S_1, S_2, S_3, S_4

[Morel et al. 2023, Cheng et al. 2023]

Coefficients associated to a single scale and a single angle:

$$S_1^{\lambda_1} = \langle |I \star \Psi^{\lambda_1}| \rangle$$

$$S_2^{\lambda_1} = \langle |I \star \Psi^{\lambda_1}|^2 \rangle$$

Averages over pixels

$$\lambda = (j, \gamma)$$

Scale (j), Angle (γ)

Scattering covariance coefficients S_1, S_2, S_3, S_4

[Morel et al. 2023, Cheng et al. 2023]

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Averages over pixels

$$\lambda = (j, \gamma)$$

Scale — Angle

Coefficients to probe the coupling between scales:

$$S_3^{\lambda_1,\lambda_2} = \mathrm{Cov}\left[I \star \Psi^{\lambda_1}, |I \star \Psi^{\lambda_2}| \star \Psi^{\lambda_1}\right]$$

$$S_4^{\lambda_1,\lambda_2,\lambda_3} = \mathrm{Cov}\left[|I \star \Psi^{\lambda_3}| \star \Psi^{\lambda_1}, |I \star \Psi^{\lambda_2}| \star \Psi^{\lambda_1}\right]$$

For instance, with $j = [1, 8]$ and $\gamma = [1, 5]$

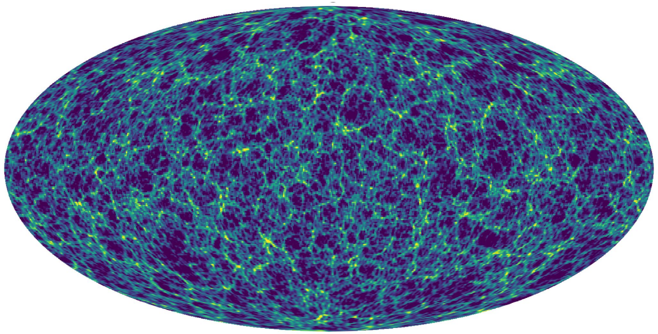
$\Rightarrow \sim 10^3$ coefficients.

Maximum entropy generative model

Summary statistics:

$$\phi(x) = \{\langle x \rangle, Var(x), S_1, S_2, S_3, S_4\}$$

Target ϕ_t



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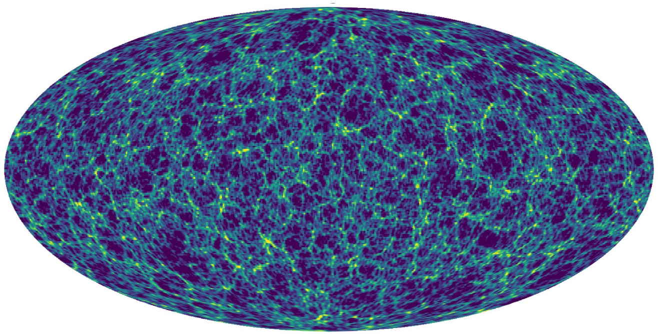

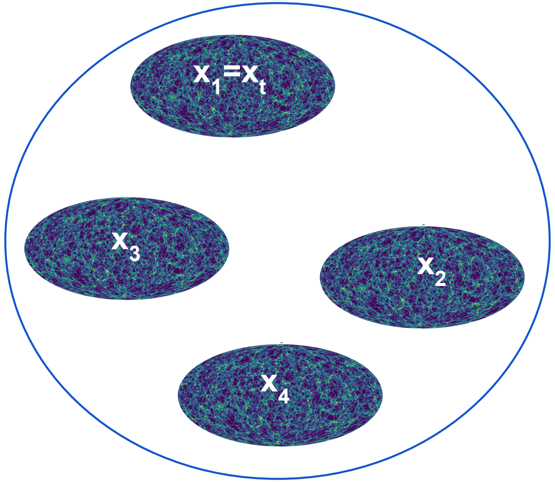

Microcanonique ensemble:

$$\Omega_\varepsilon = \left\{x : |\phi(x) - \phi(x_t)|^2 < \varepsilon\right\}$$
$$\mathcal{P}(x_1) = \mathcal{P}(x_2) = \mathcal{P}(x_i)$$

Maximum entropy generative model

Summary statistics:

$$\phi(x) = \{\langle x \rangle, Var(x), S_1, S_2, S_3, S_4\}$$

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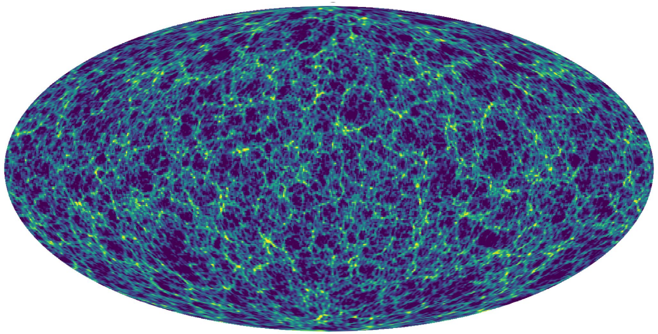

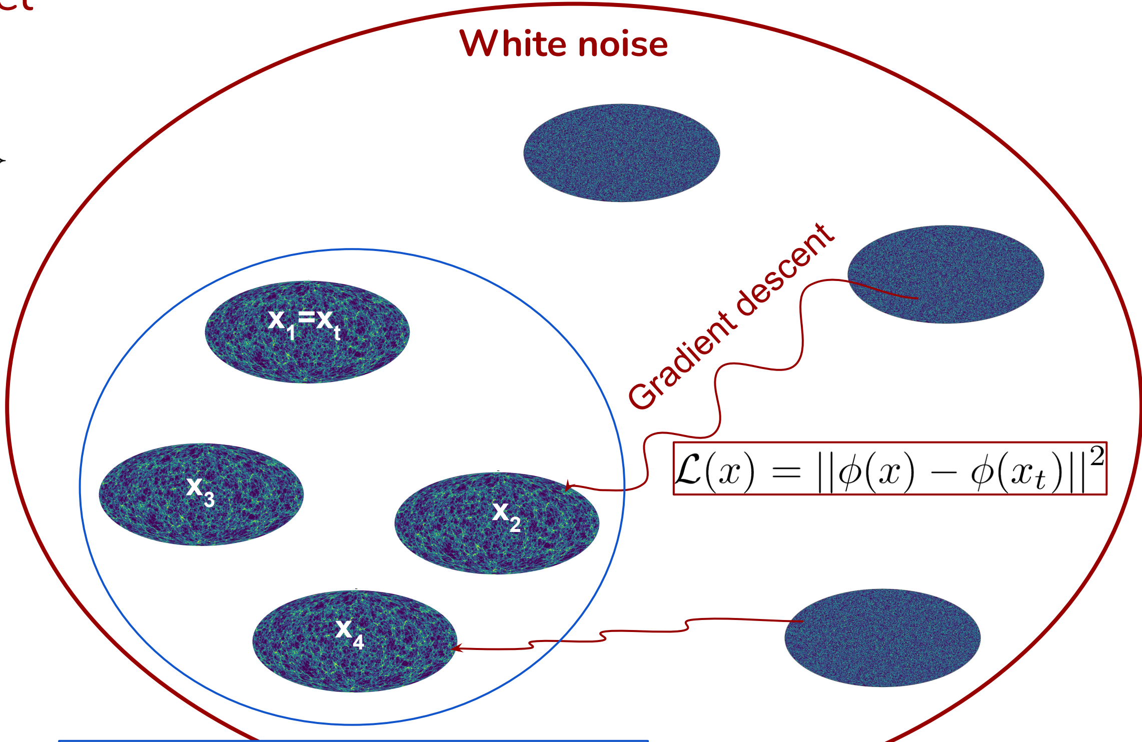

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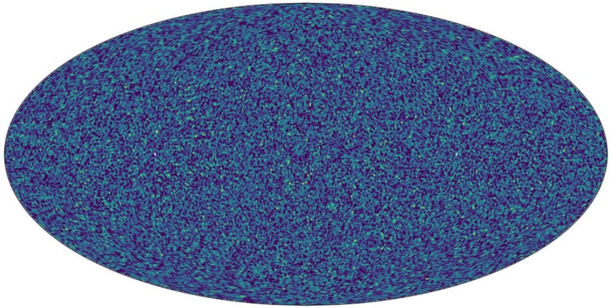
$$\mathcal{P}(x_1) = \mathcal{P}(x_2) = \mathcal{P}(x_i)$$

The gradient descent in practice

Start (white gaussian noise)

$\phi_{\rm start}$

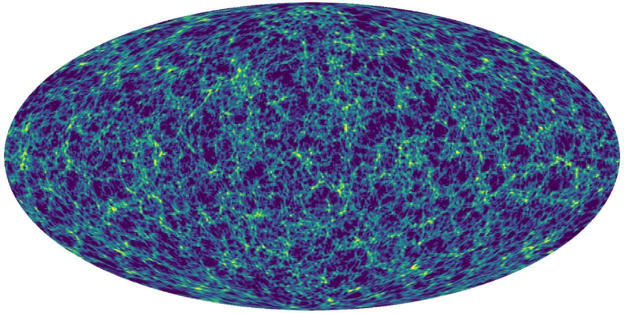


GRADIENT DESCENT
Iterate on the pixels or on the harmonic coefficients

JAX => Auto-differentiable
Using GPU
Minimizer from Optax or jaxopt

Generated

$x_{\rm end} \in \Omega_\varepsilon \qquad \phi_{\rm end} \simeq \phi_t$



Difficulty :

- Directional convolution performed in harmonic space
- Non-linear operation (modulus) performed in map space

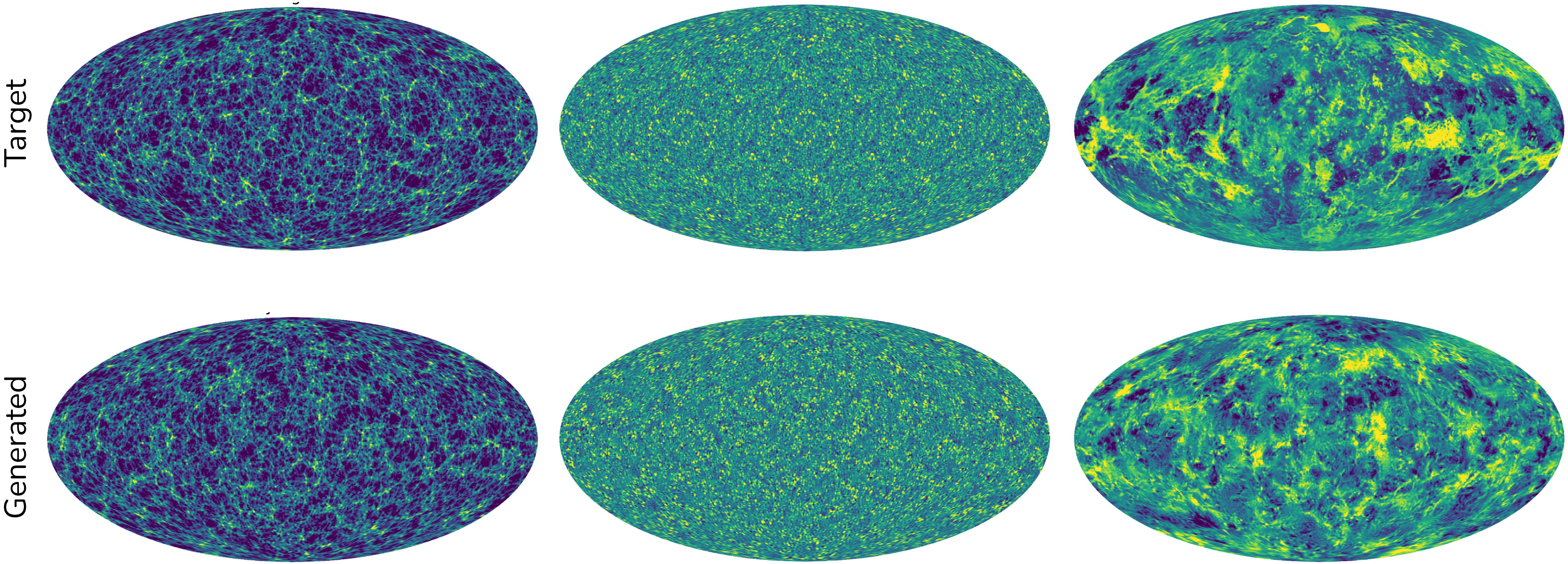
=> Change space at each iteration in the gradient descent

Visual validation

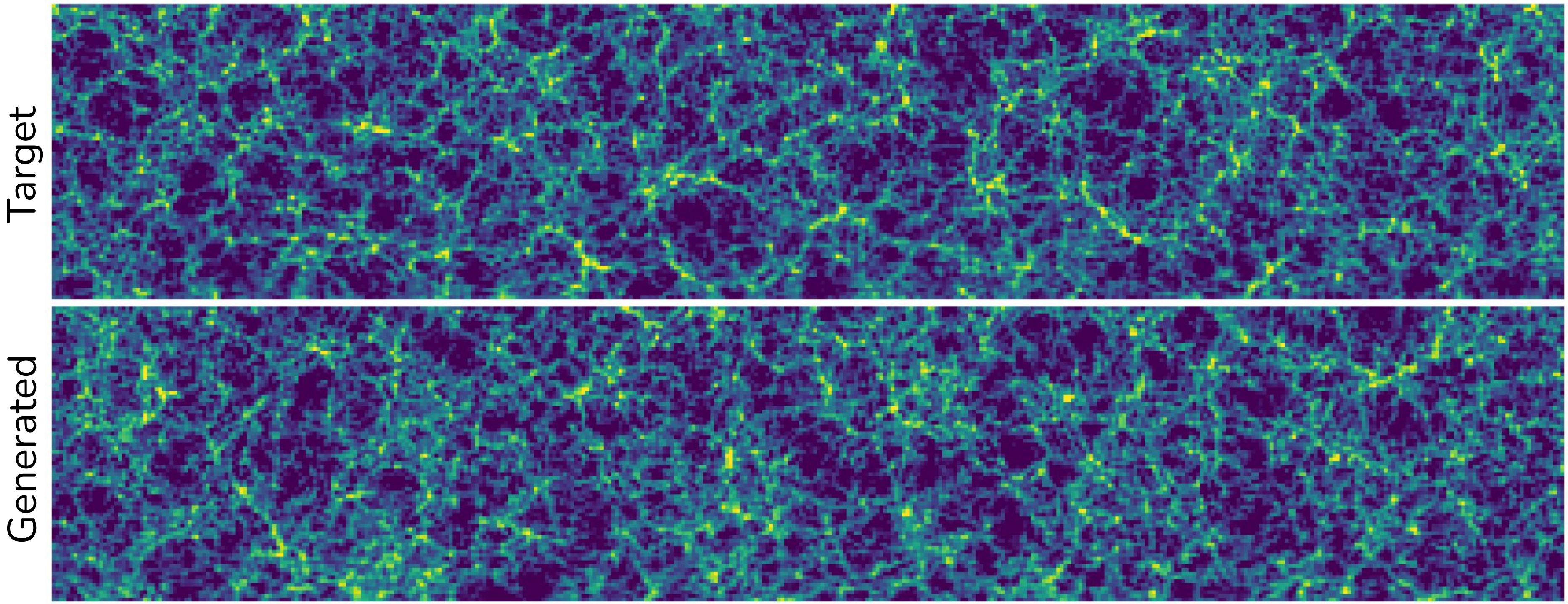
Large Scale Structures
N-body simulation [Kacprzak et al. 2022]

Thermal Sunyaev-Zeldovitch effect (tSZ) [Ade et al. 2019]

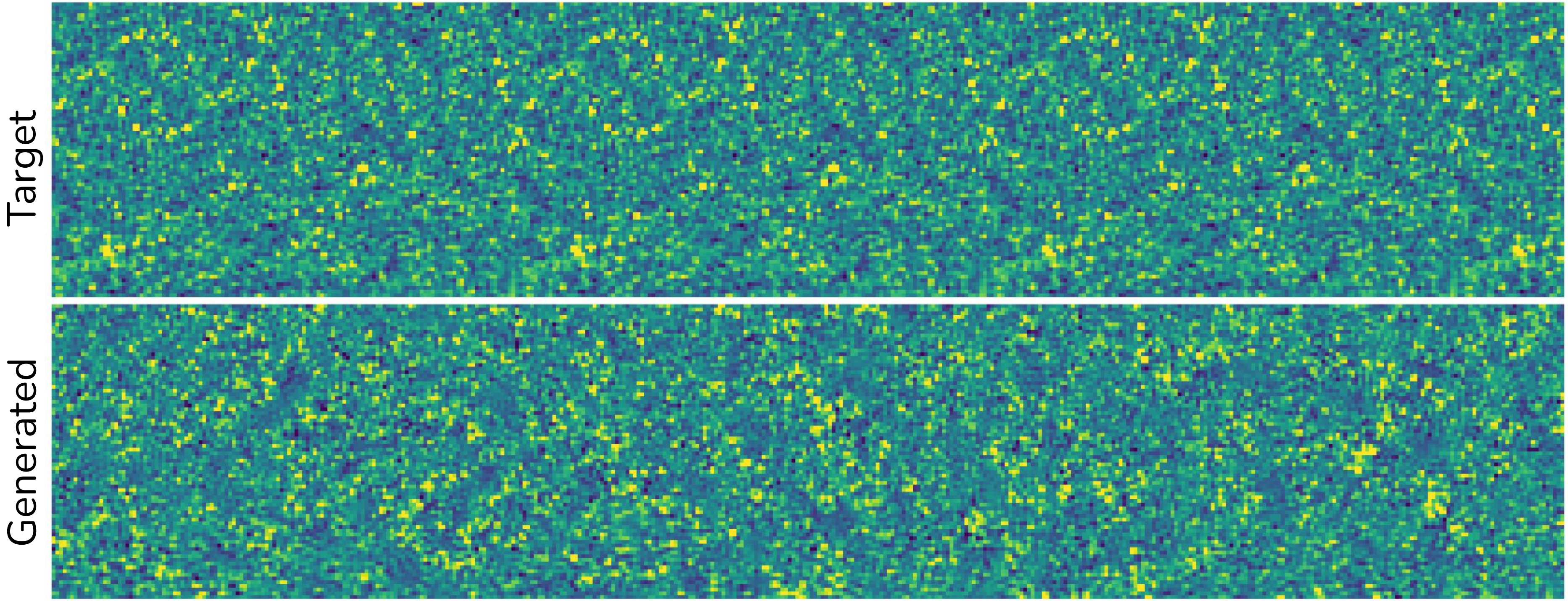
Venus surface



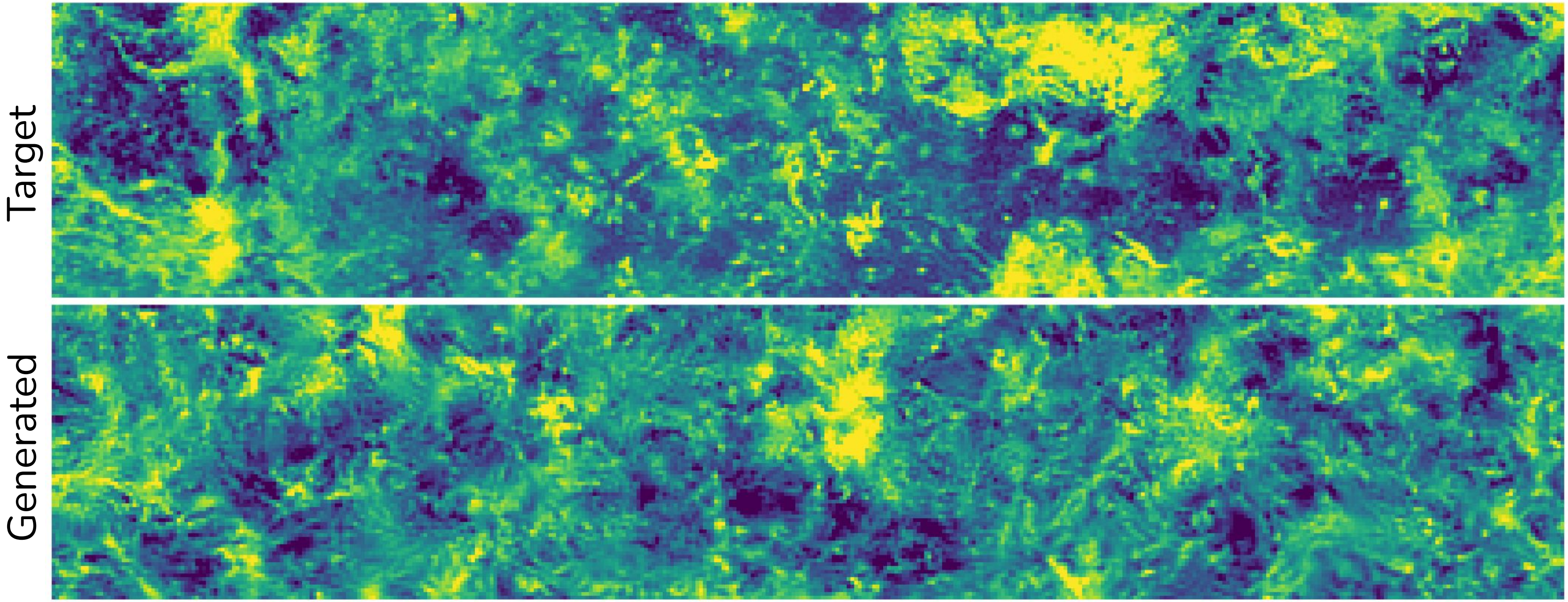
Zoom on a patch - Weak lensing



Zoom on a patch - tSZ



Zoom on a patch - Venus



Comparison with a Gaussian generative model

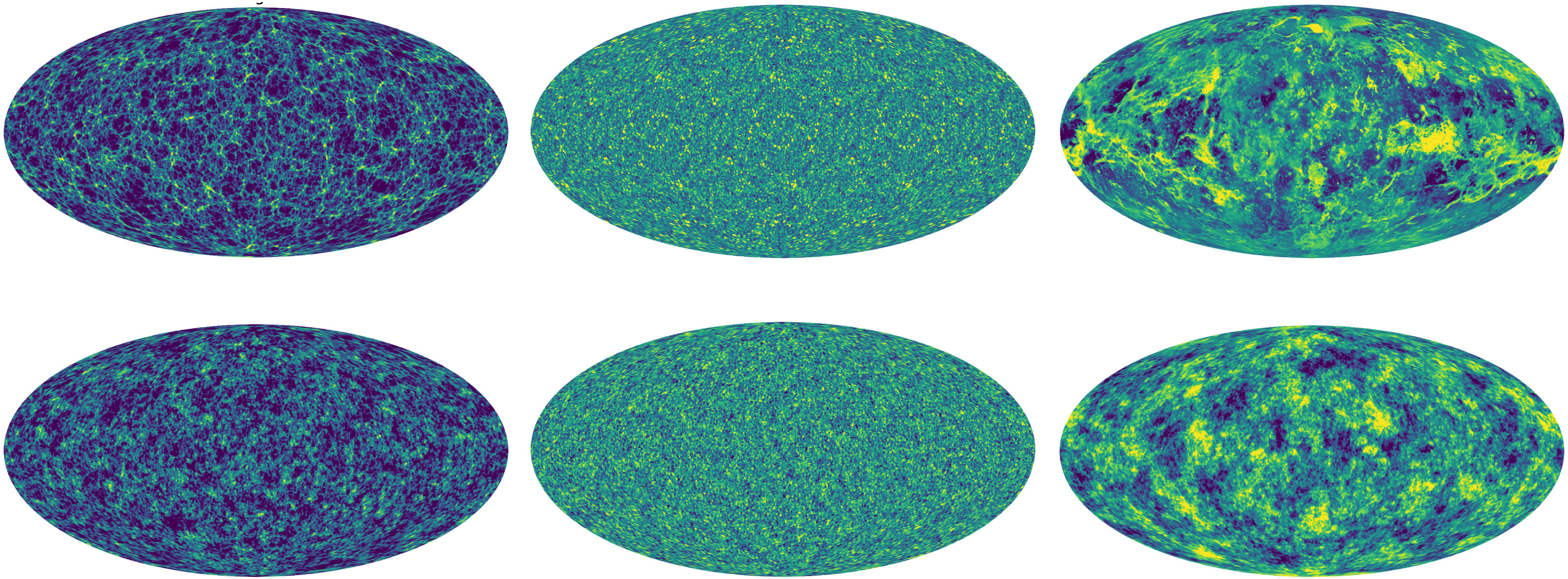
Large Scale Structures
N-body simulation [Kacprzak et al. 2022]

Thermal Sunyaev-Zeldovitch effect (tSZ) [Ade et al. 2019]

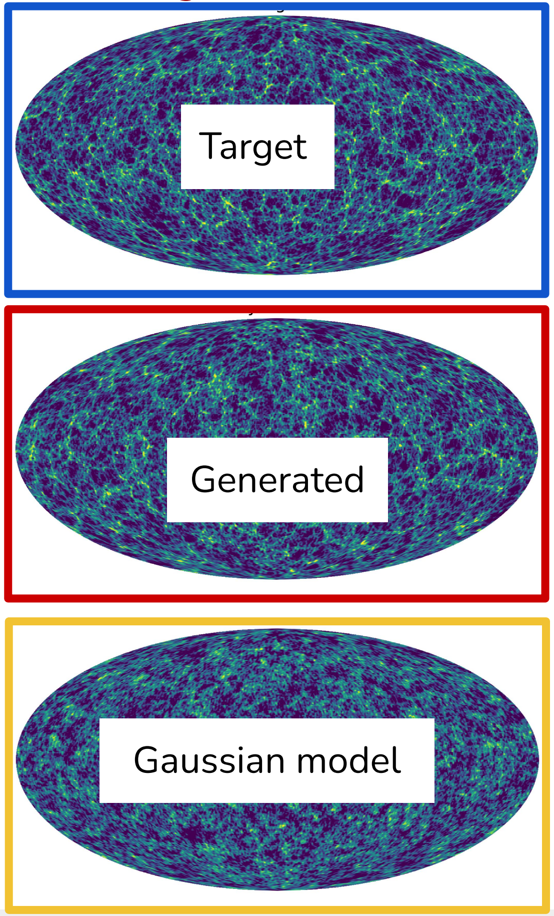
Venus surface

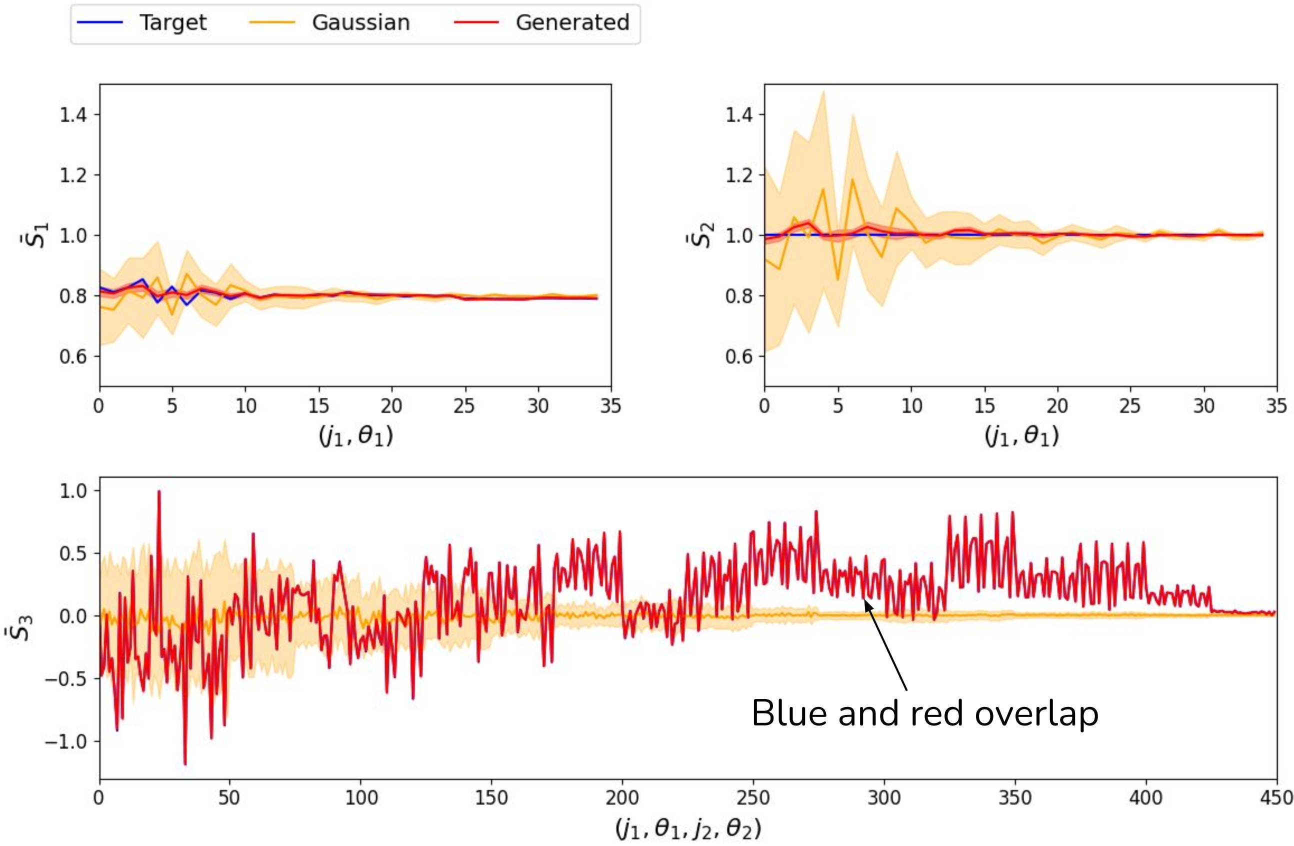
Target

Generated Gaussian



Scattering covariances for the LSS

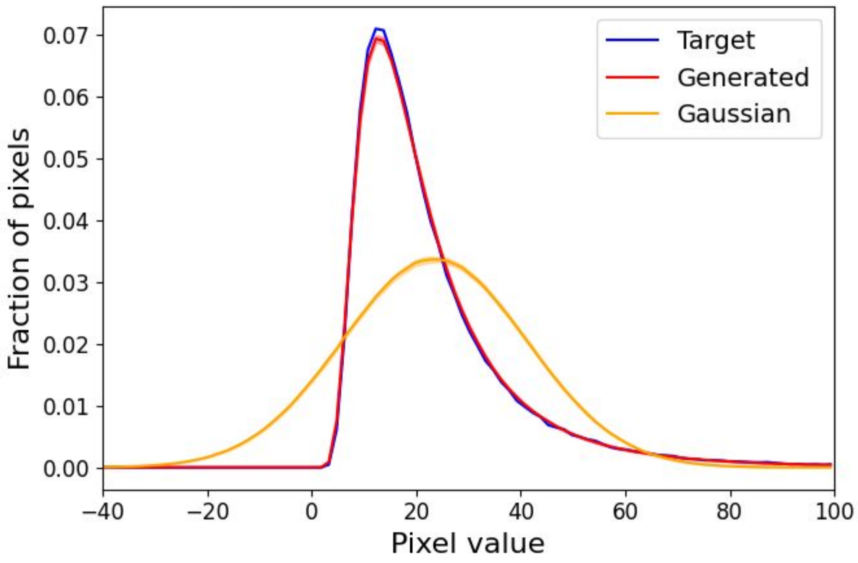




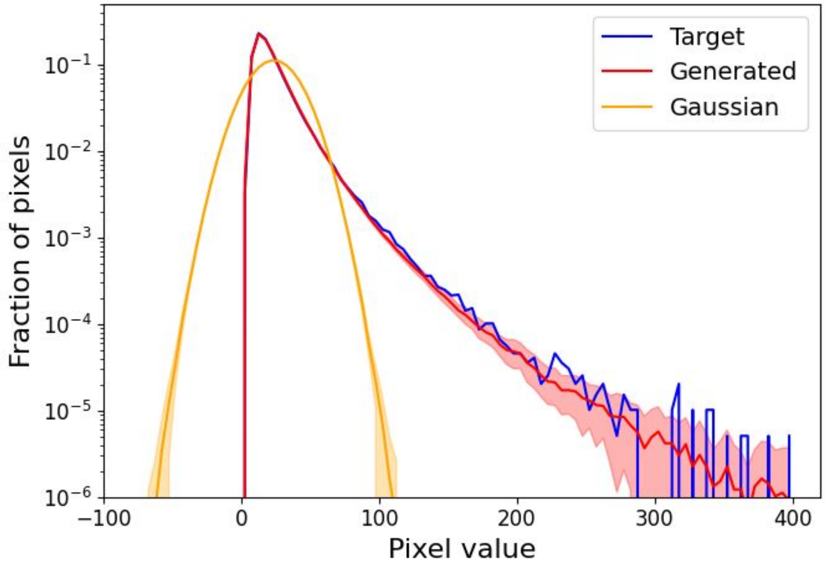
Statistical validation - Probability Density Function (PDF)

LSS

y-axis linear scaling



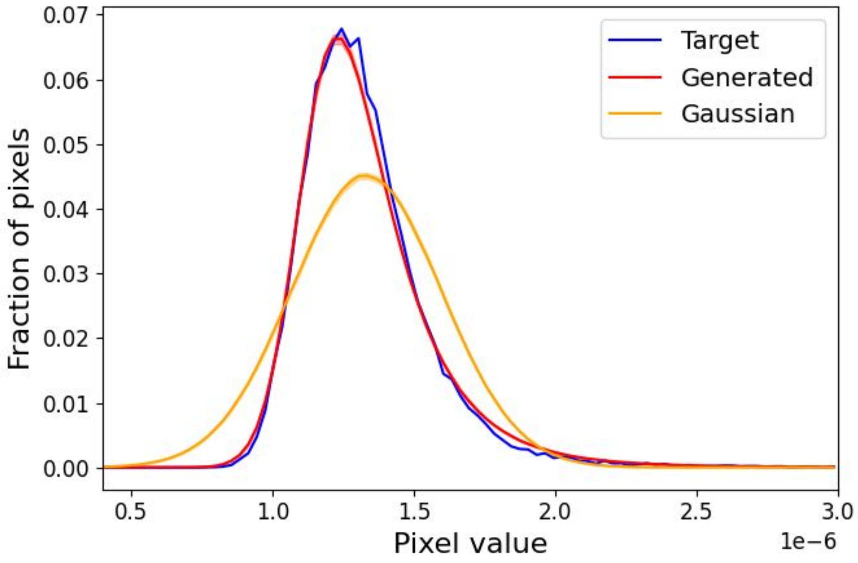
y-axis log scaling



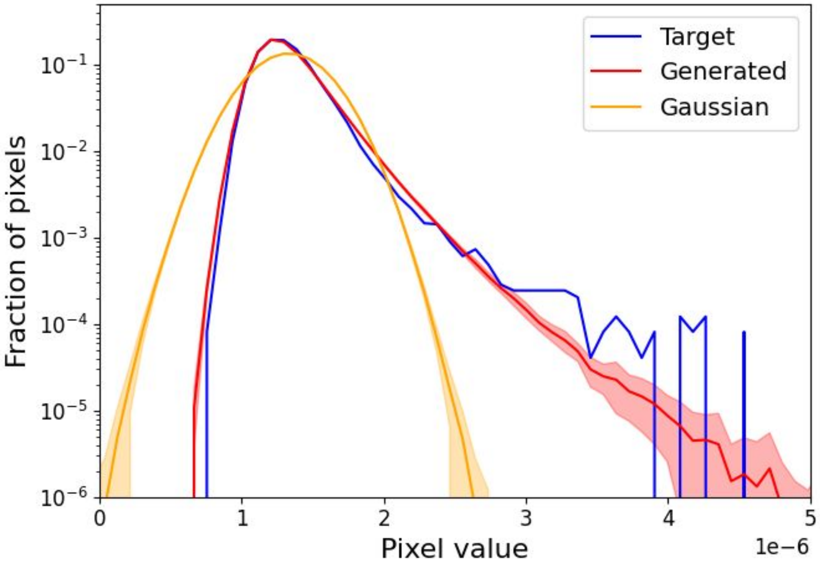
Statistical validation - Probability Density Function (PDF)

tSZ

y-axis linear scaling



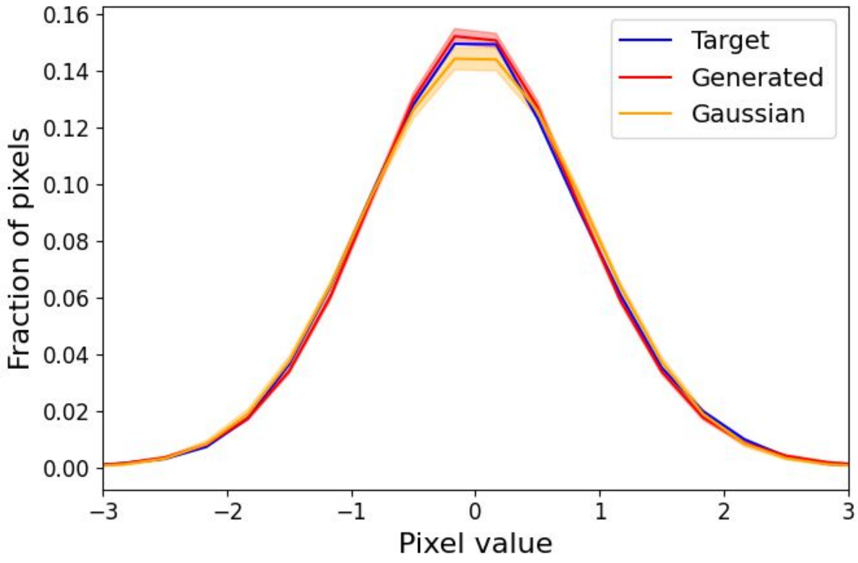
y-axis log scaling



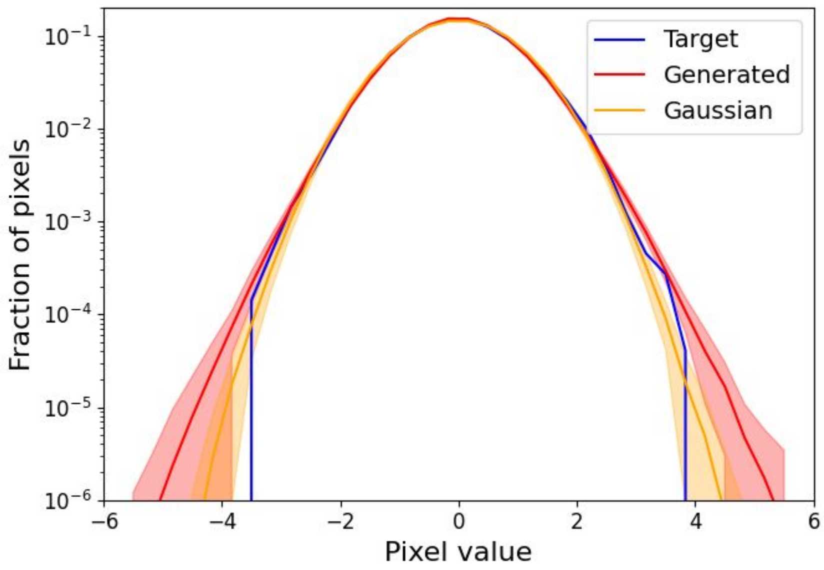
Statistical validation - Probability Density Function (PDF)

Venus

y-axis linear scaling

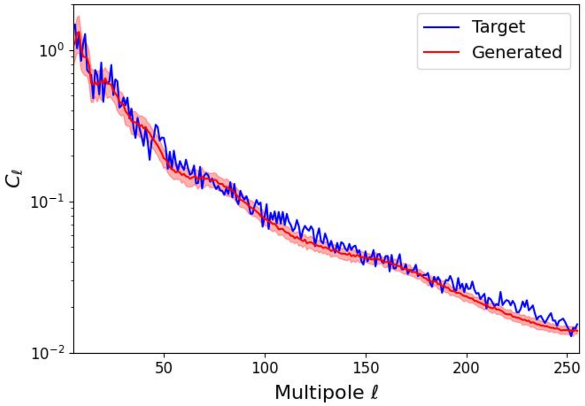


y-axis log scaling

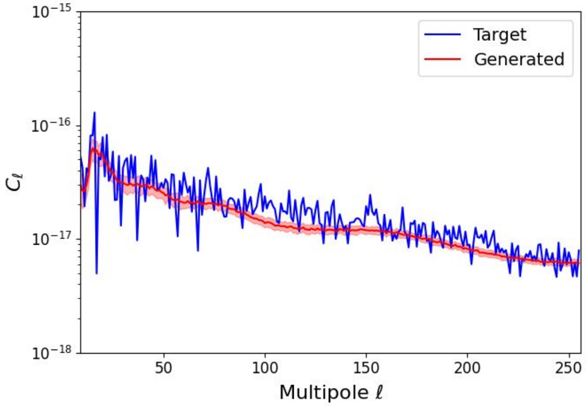


Statistical validation - Angular power spectrum

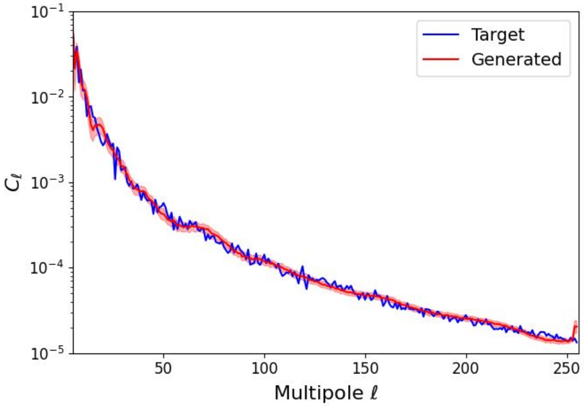
LSS



tSZ

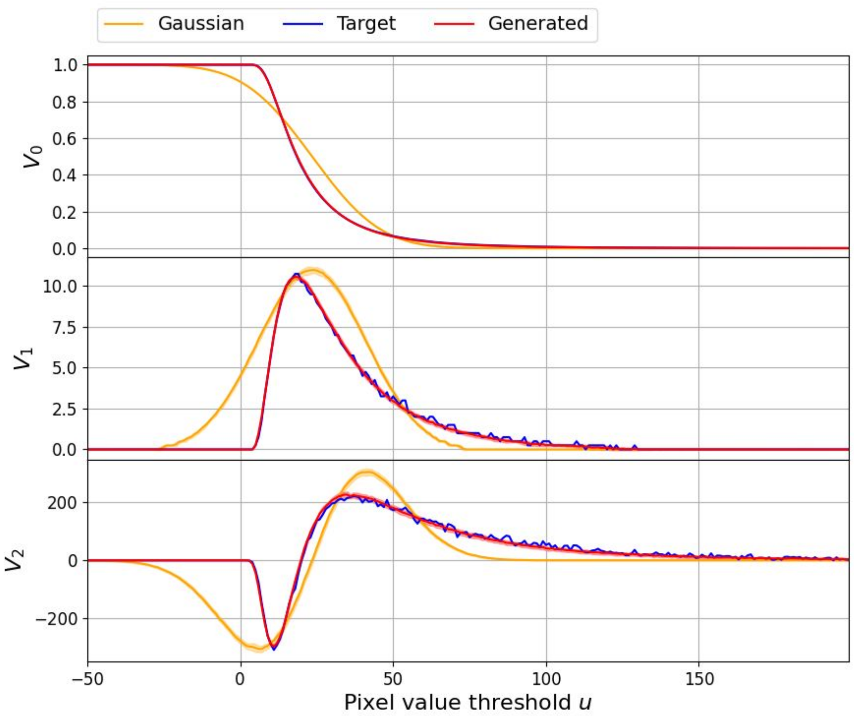


Venus



Statistical validation - Minkowski functionals

LSS



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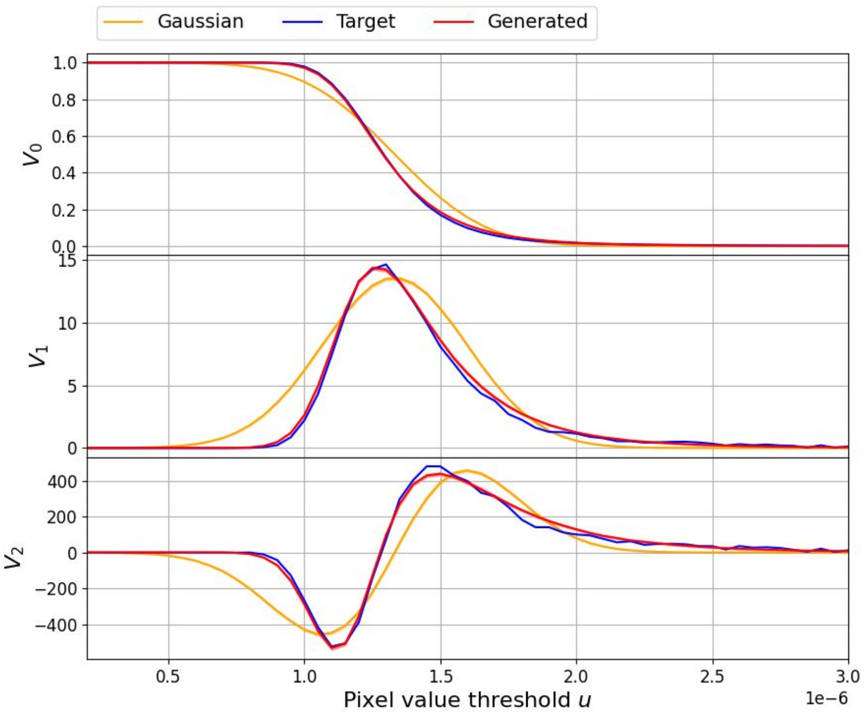
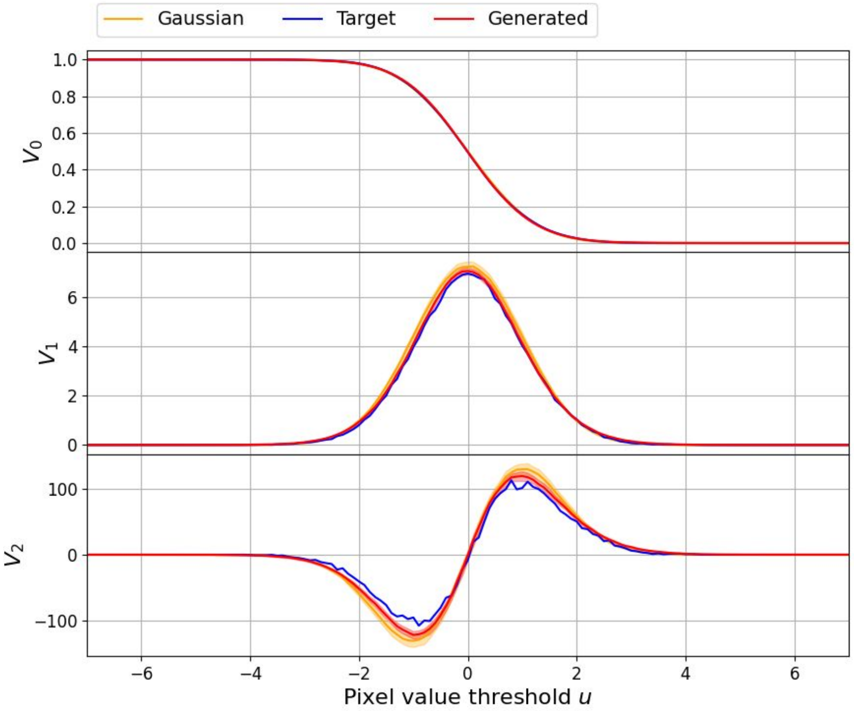
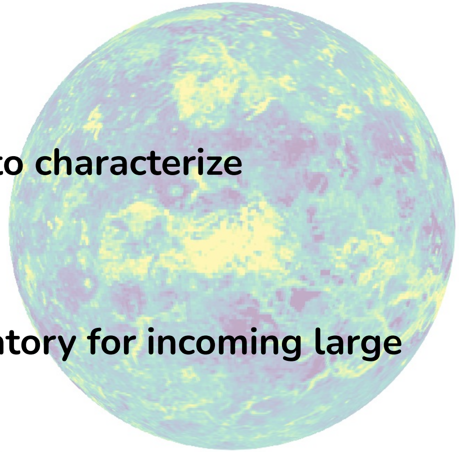


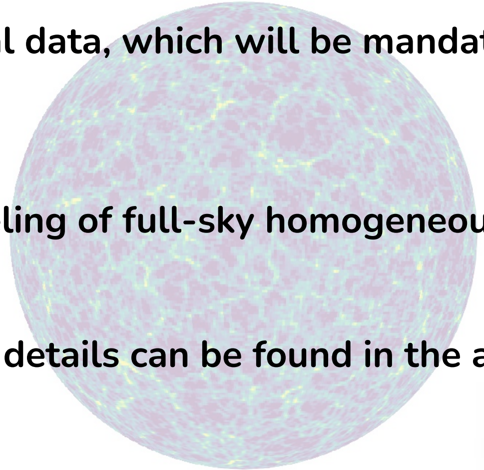
Statistical validation - Minkowski functionals

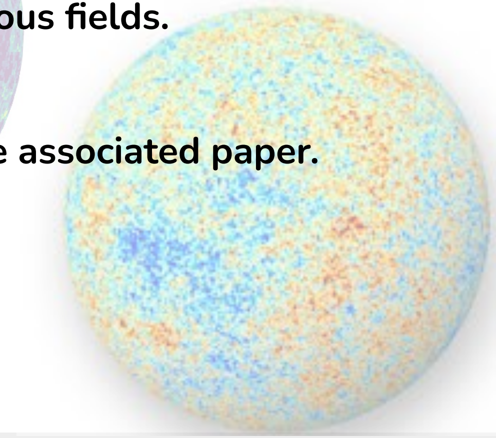
Venus



Summary







- Scattering transforms are efficient low variance summary statistics to characterize non-Gaussian fields.

- We have adapted these tools to spherical data, which will be mandatory for incoming large scale surveys.

- We validated s2scat on generative modeling of full-sky homogeneous fields.

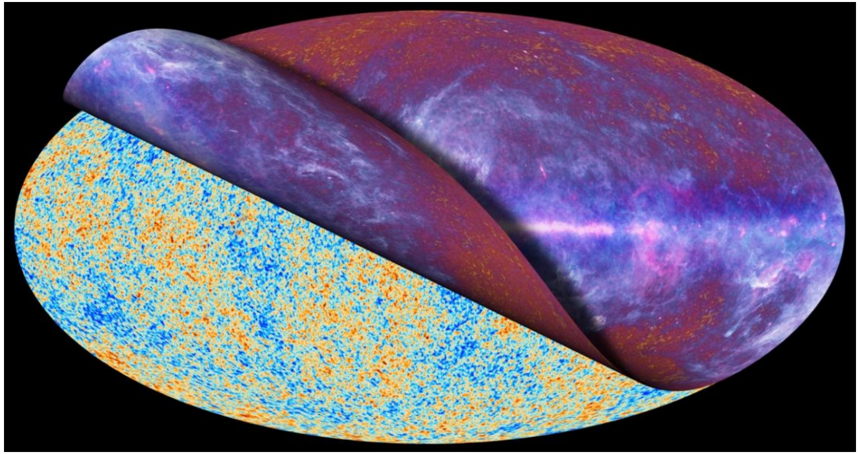
- The software is available on GitHub and details can be found in the associated paper.

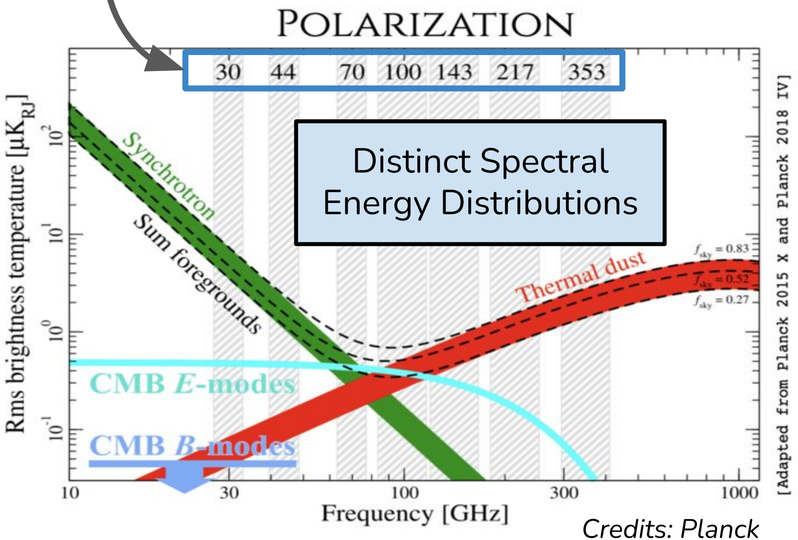
Future applications

We have a low dimensional generative model applicable to a broad range of physical fields.
=> Can be plug into different algorithms.

One example : Traditional component separations for CMB.

Observation in several frequency bands



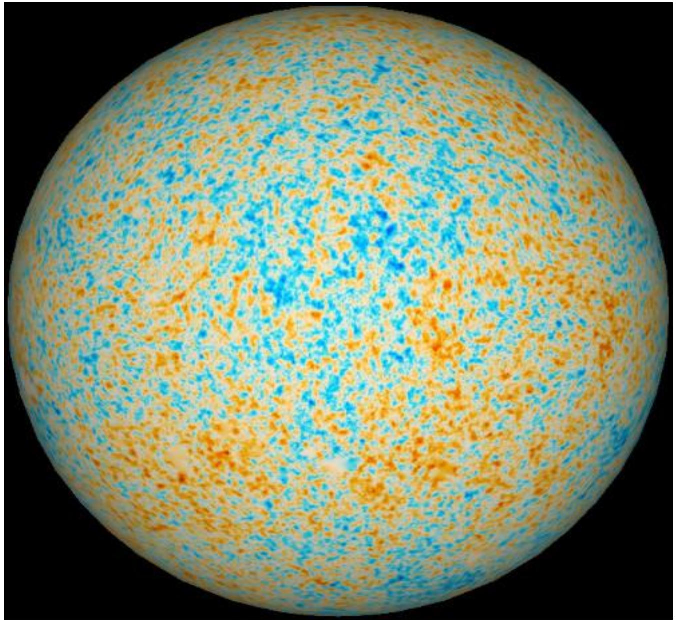


Distinct Spectral Energy Distributions

Credits: Planck

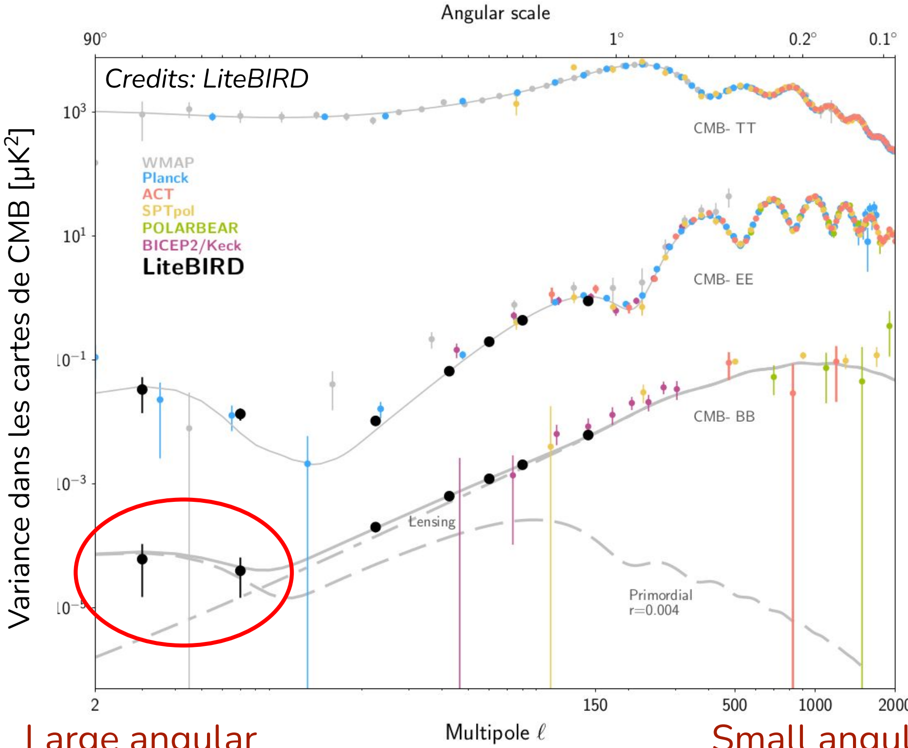
Thank you for your attention!

A critical example for large scale measurement



Spherical maps
Planar approximation not valid

Angular power spectrum



Large angular scales

Small angular scales

Directional convolution on the sphere

[McEwen et al. 2015, Price & McEwen 2023]
S2FFT Python package

$$W^j_{\ell m n} = \frac{8\pi^2}{2\ell+1} I_{\ell m} \Psi^{j*}_{\ell n}$$

Inverse Wigner Transform

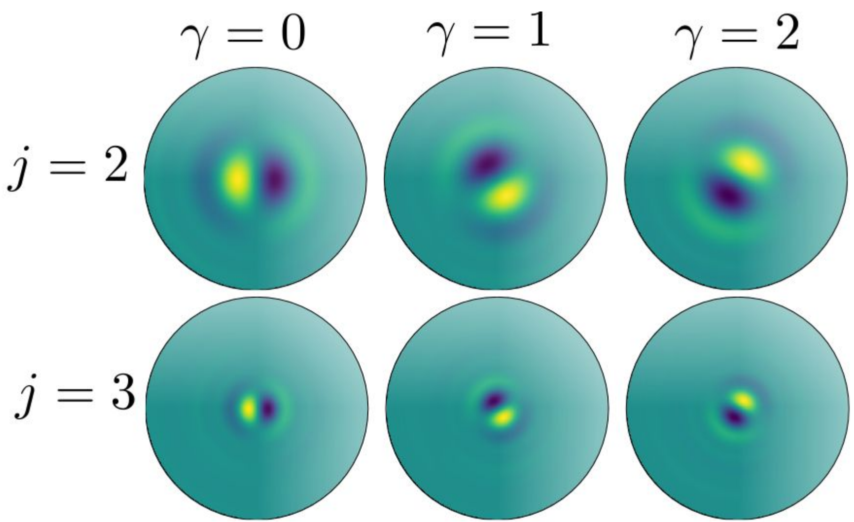
$$W^j(\alpha, \beta, \gamma) \rightarrow W^{j\gamma}(\theta, \varphi)$$

Euler angles

Scale

Angle

Convolution of a Dirac map:



Computational benchmarking

Table 1. Computational benchmarking.

Pre-compute Mode			
Bandlimit	Forward	Gradient	JIT Compilation
256	15 ms	30 ms	20 s
512	100 ms	200 ms	25 s
Recursive Mode			
Bandlimit	Forward	Gradient	JIT Compilation
256	120 ms	300 ms	90 s
1024	5 s	10 s	3 m
2048	20 s	50 s	6 m

Notes. Results of the SC transform provided by `s2scat`. These results were recovered on a single NVIDIA A100 40GB GPU, although it is possible to run across multiple GPUs. In our analysis we generate spherical images through 400 iterations to be conservative. In practice, however, we find that $\sim$ 100 iterations is typically sufficient, in which case an image at $L = 256$ can be generated in $\sim$ 4s. Furthermore, batched generation can dramatically decrease per sample compute time. For example, 20 images at $L = 256$ can be generated in $\sim$ 12s, corresponding to $\sim$ 0.5s per sample.