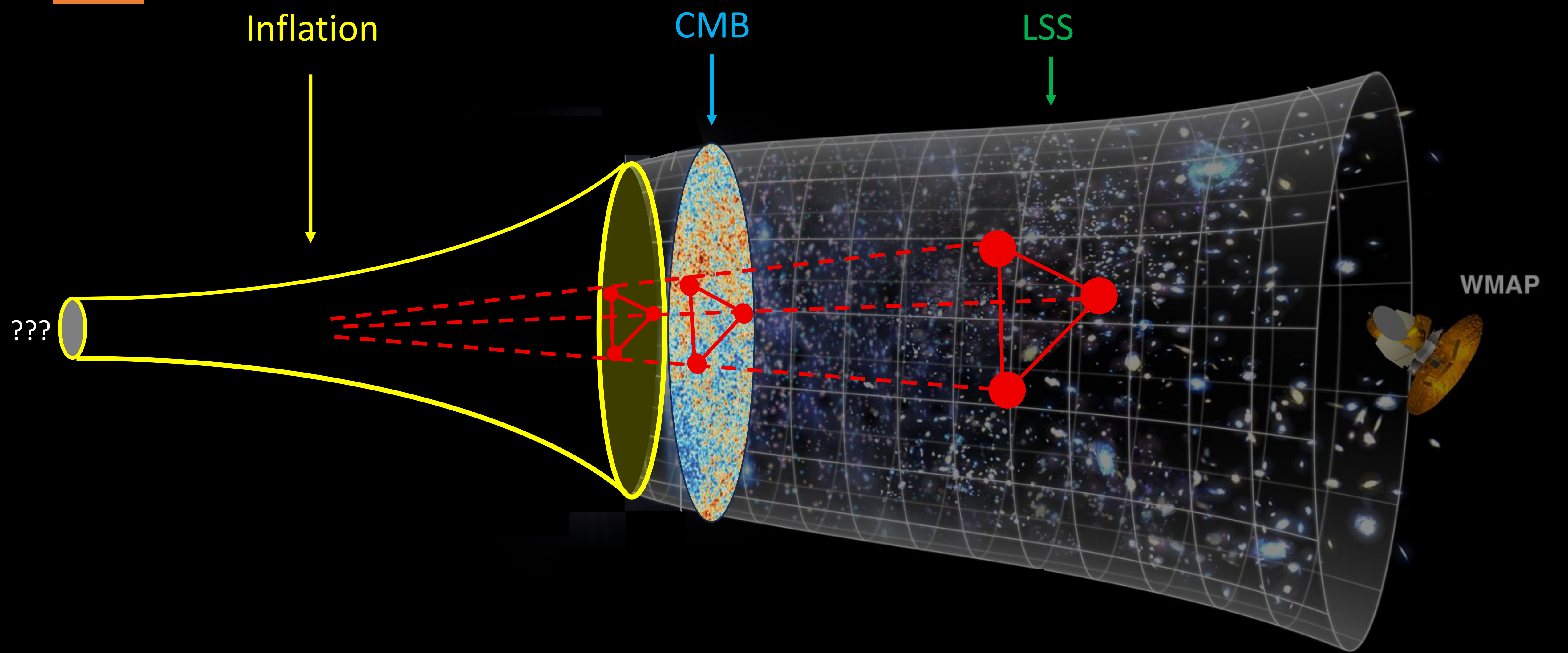
Unitary renormalisation and the breaking of cosmological reality

- Xi Tong
- DAMTP
- With Diksha Jain, Enrico Pajer, David Stefanyszyn, Yuhang Zhu, Tao Liu, Yi Wang and Zhong-Zhi Xianyu

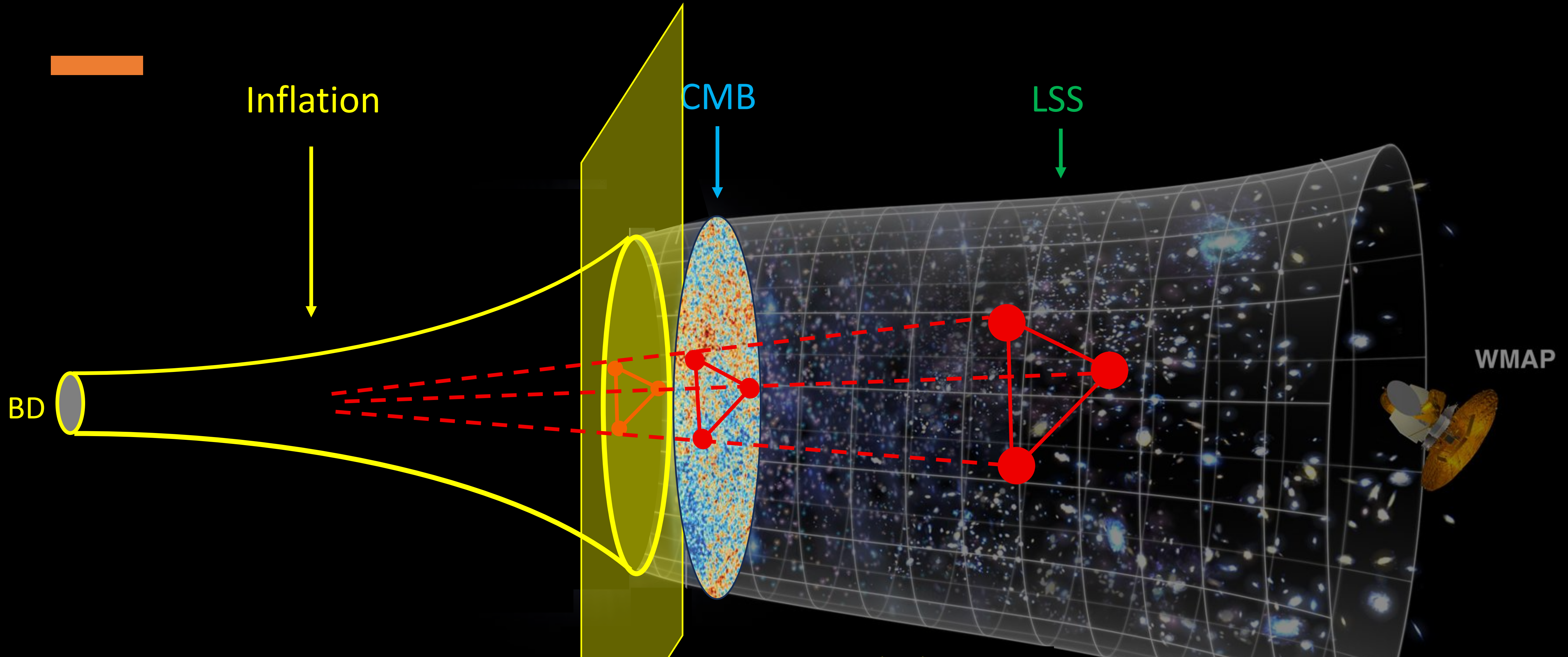
- Based on
  - 2509.02696
  - 2309.07769 (JHEP)
  - 1909.01819 (JHEP)



- Cosmological correlators are a powerful probe of inflationary physics @ $H \leq 10^{13}$ GeV

(DoFs, symmetries, fundamental principles, etc.)

NASA/WMAP Science Team

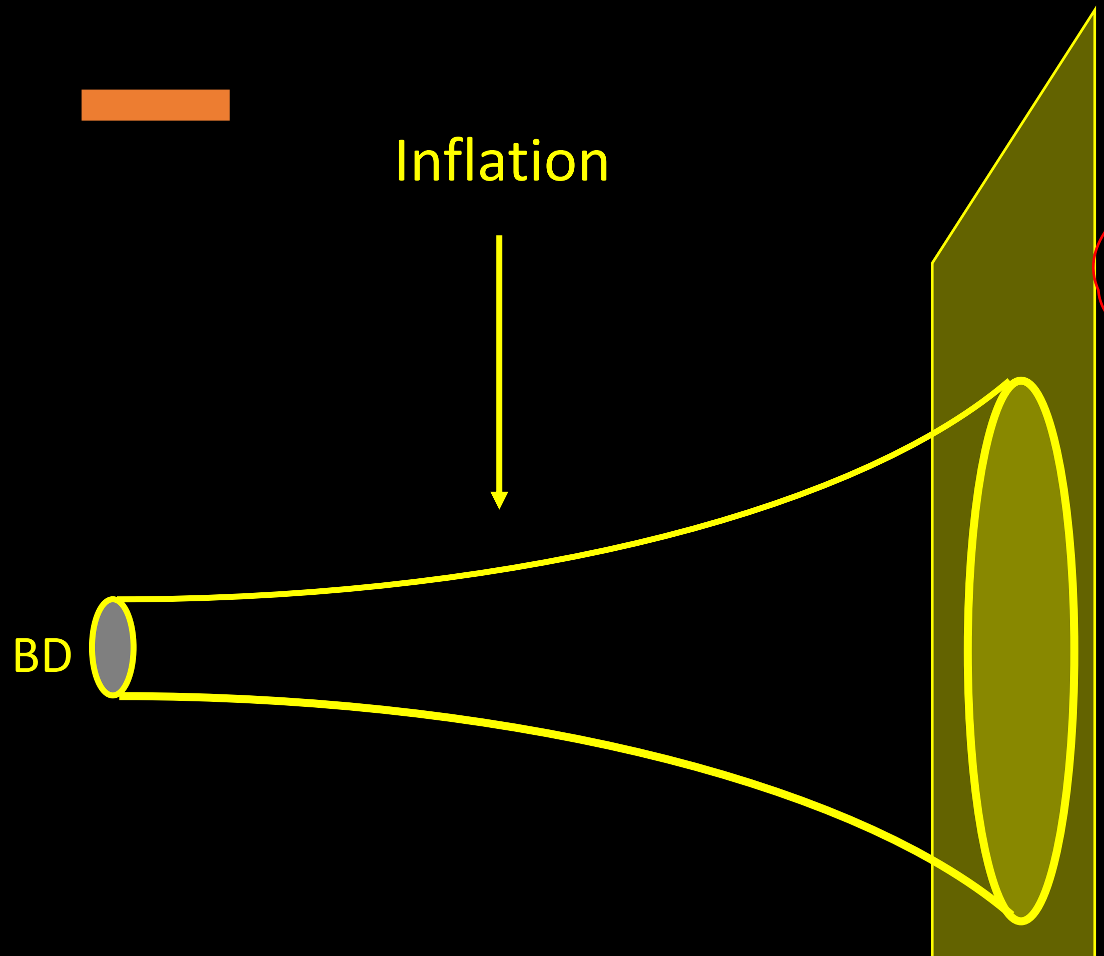


"Boundary"($\eta_0 \to 0$):

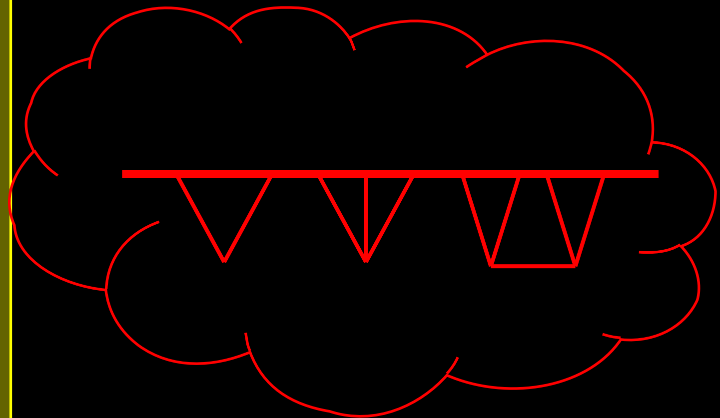
- End of inflation

$$\Psi[\varphi] = \int_{\phi(-\infty)=\mathrm{BD}}^{\phi(\eta_0)=\varphi} \mathcal{D}\phi \, e^{iS[\phi]}$$

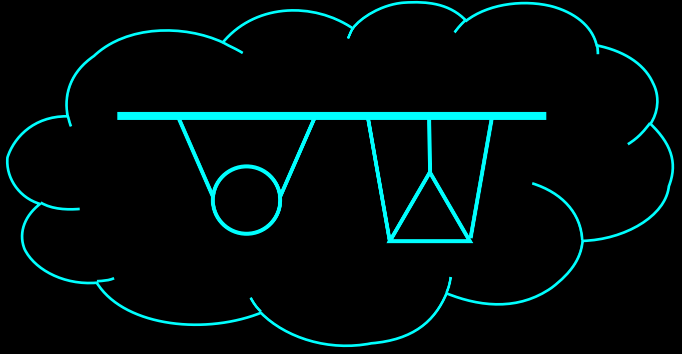
Cosmological wavefunction



- Path integral computed by ***perturbative diagrammatics***



"Classical" ***tree*** diags

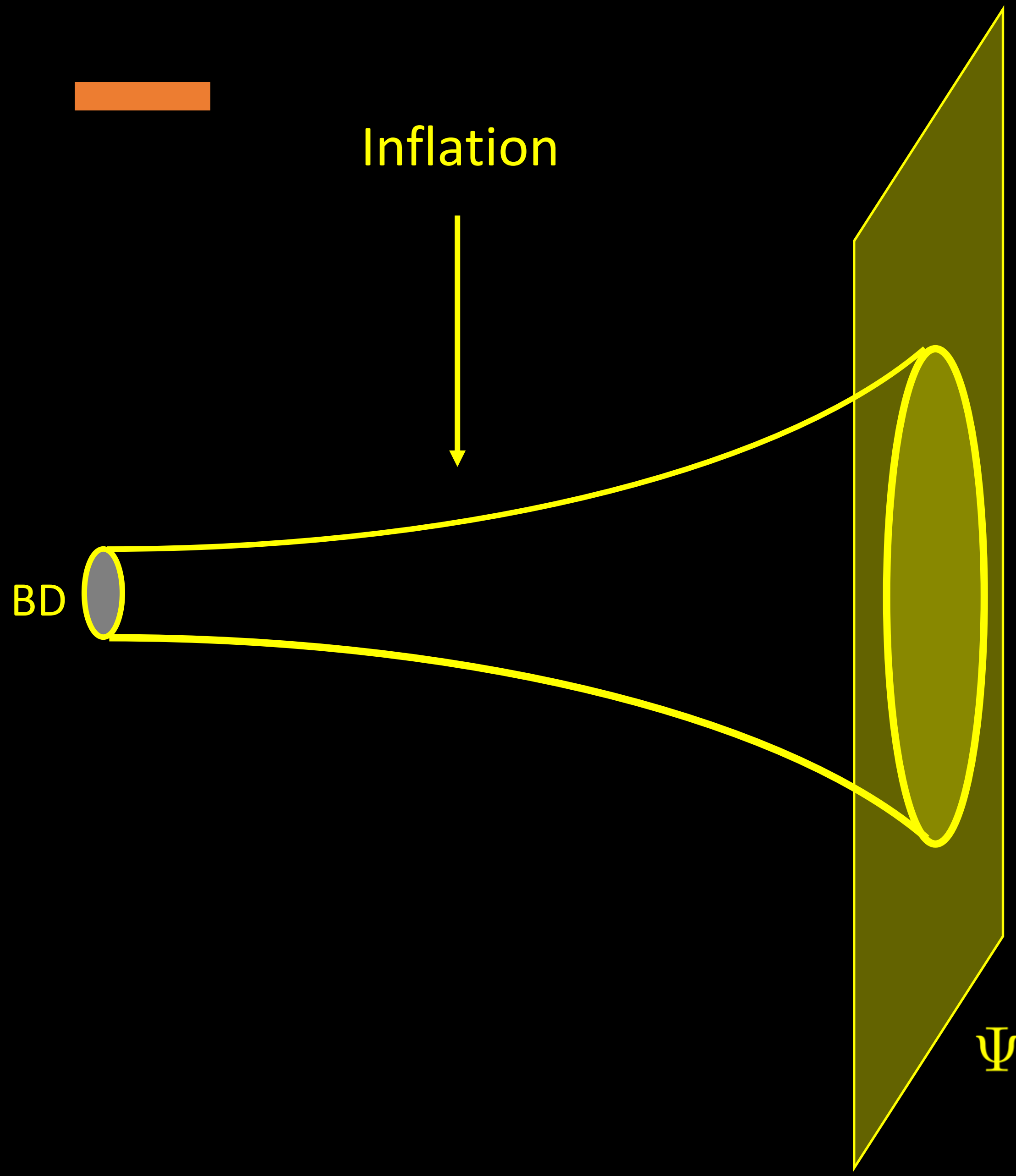


"Quantum" ***loop*** diags

$$= e^{iS[\phi_{\rm cl}]} \times \int \mathcal{D}\delta\phi \, e^{i(S[\phi_{\rm cl}+\delta\phi]-S[\phi_{\rm cl}])}$$

$$\Psi[\varphi] = \int_{\phi(-\infty)=\mathrm{BD}}^{\phi(\eta_0)=\varphi} \mathcal{D}\phi \, e^{iS[\phi]}$$

Cosmological wavefunction



- Correlators are extracted from the WF coeffs via the ***Born rule***

$$\langle \varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\rangle = \frac{\int \mathcal{D}\varphi \left|\Psi[\varphi]\right|^2 \varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi \left|\Psi[\varphi]\right|^2}$$

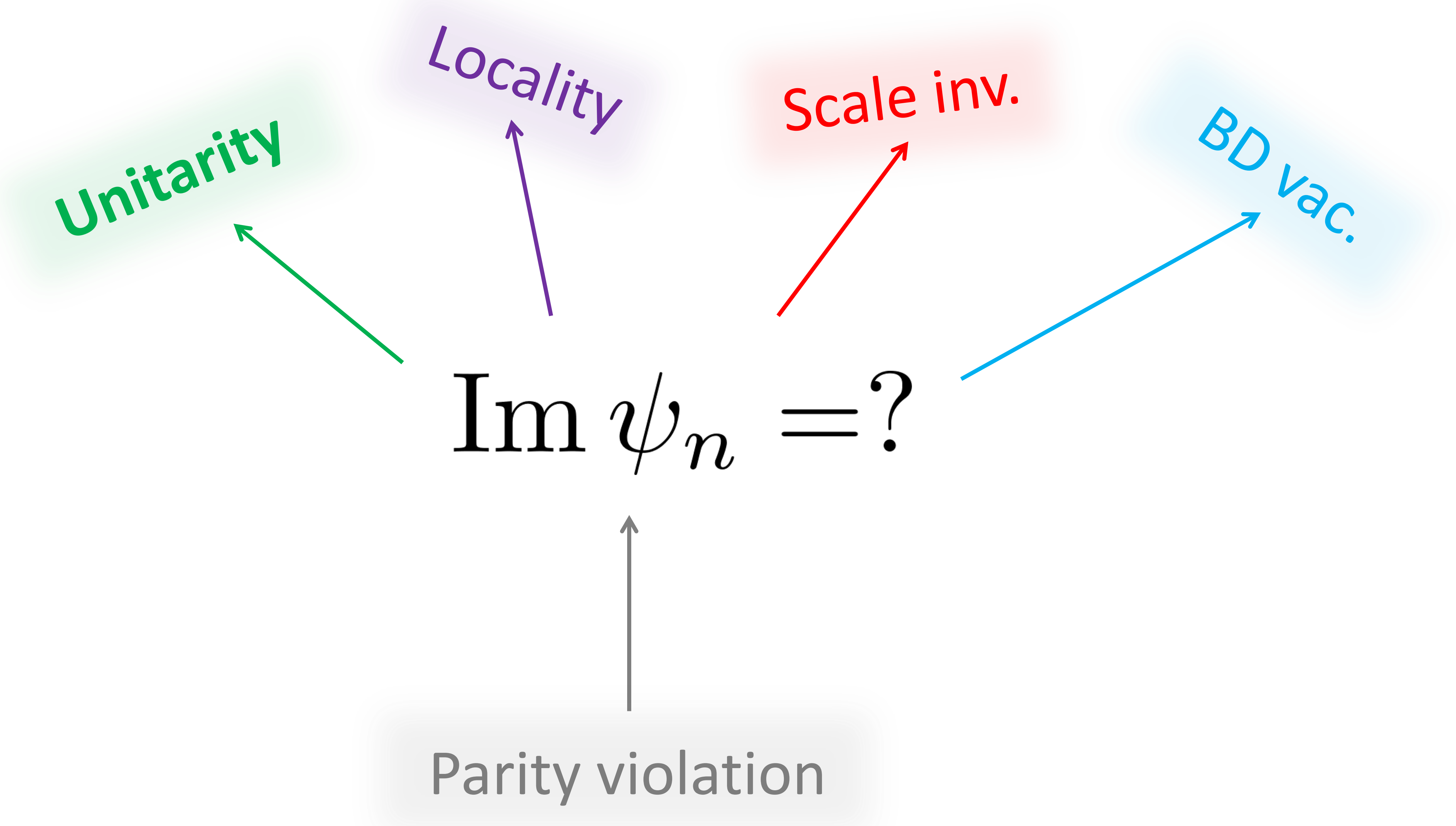
$$\equiv \exp\left[\sum_{n=2}^{\infty}\frac{1}{n!}\underbrace{\left(\psi_n^{0\mathrm{L}}+\psi_n^{1\mathrm{L}}+\psi_n^{2\mathrm{L}}+\cdots\right)}_{\text{N-pt wavefunction coeffs}}\varphi^n\right]$$

$$\Psi[\varphi] = \int_{\phi(-\infty)=\mathrm{BD}}^{\phi(\eta_0)=\varphi}\mathcal{D}\phi\, e^{iS[\phi]}$$

Cosmological wavefunction are useful because…

… they make ***fundamental principles*** manifest

$$\operatorname{Im}\psi_n = ?$$


Unitarity
Locality
Scale inv.
BD vac.
$\mathrm{Im}\,\psi_n = ?$
Parity violation

The tree-level reality: $\mathrm{Im}\,\psi_n^{\mathrm{0L}} = 0$

[Liu, **Tong**, Wang & Xianyu, 2019]
[Stefanyszyn, **Tong** & Zhu, 2023]

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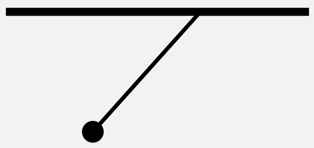
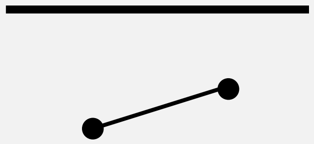
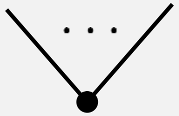
- Feynman rules:



$$= K_k(\eta) = (1 - ik\eta)e^{ik\eta}$$



$$\begin{aligned} &= G_k(\eta_1, \eta_2) \\ &= \frac{iH^2}{k^3}(k\eta_1 \cos k\eta_1 - \sin k\eta_1) \\ &\quad \times (1 - ik\eta_2)e^{ik\eta_2}\theta(\eta_1 - \eta_2) \\ &\quad + (\eta_1 \leftrightarrow \eta_2) \end{aligned}$$



$$= i\lambda \int_{-\infty(1-i\epsilon)}^{0} \frac{\mathrm{d}\eta}{(-H\eta)^4} \mathcal{L}_{\mathrm{int}}(\eta\partial_\eta, -i\mathbf{k}\eta)$$

The tree-level reality: $\mathrm{Im}\,\psi_n^{0\mathrm{L}} = 0$

[Liu, **Tong**, Wang & Xianyu, 2019]
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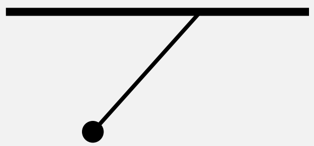

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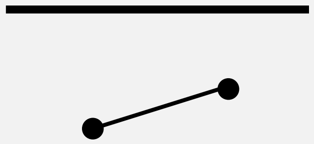

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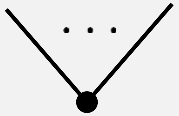

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- i & η always appear together (BD vac.)
- ∂_η & $\mathbf{k}$ always appear with η (scale inv.)
- λ 's are real (unitarity)

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[Liu, **Tong**, Wang & Xianyu, 2019]
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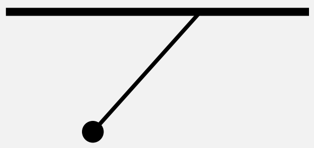
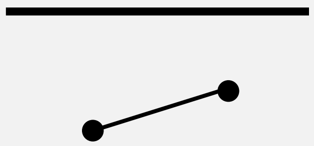


$\Rightarrow \in \mathbb{R}$



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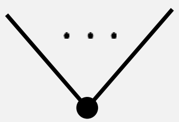

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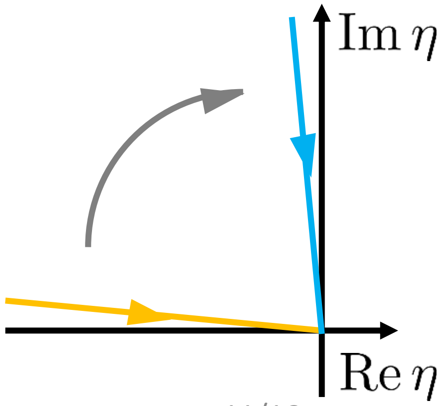
➢ Wick rotation $\eta = i\chi$ (locality)



Now, what does this cosmological reality imply?

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$$\varphi^*(\mathbf{k}) = \varphi(-\mathbf{k})$$

$\Rightarrow$ Real $=$ Parity-even!

A no-go theorem on parity violation

- Unitarity
- locality
- Scale inv.
- BD vac.
- Tree

} No PV correlators

$$\langle\varphi(\mathbf{k}_1)\cdots\varphi(\mathbf{k}_n)\rangle^{\mathrm{PO}} = 0$$

in massless scalar EFTs

[Liu, **Tong**, Wang & Xianyu, 2019]
[Cabass, Jazajeri, Pajer & Stefanyszyn, 2022]

Generalisable to

- Higher spins
- Other dimensions

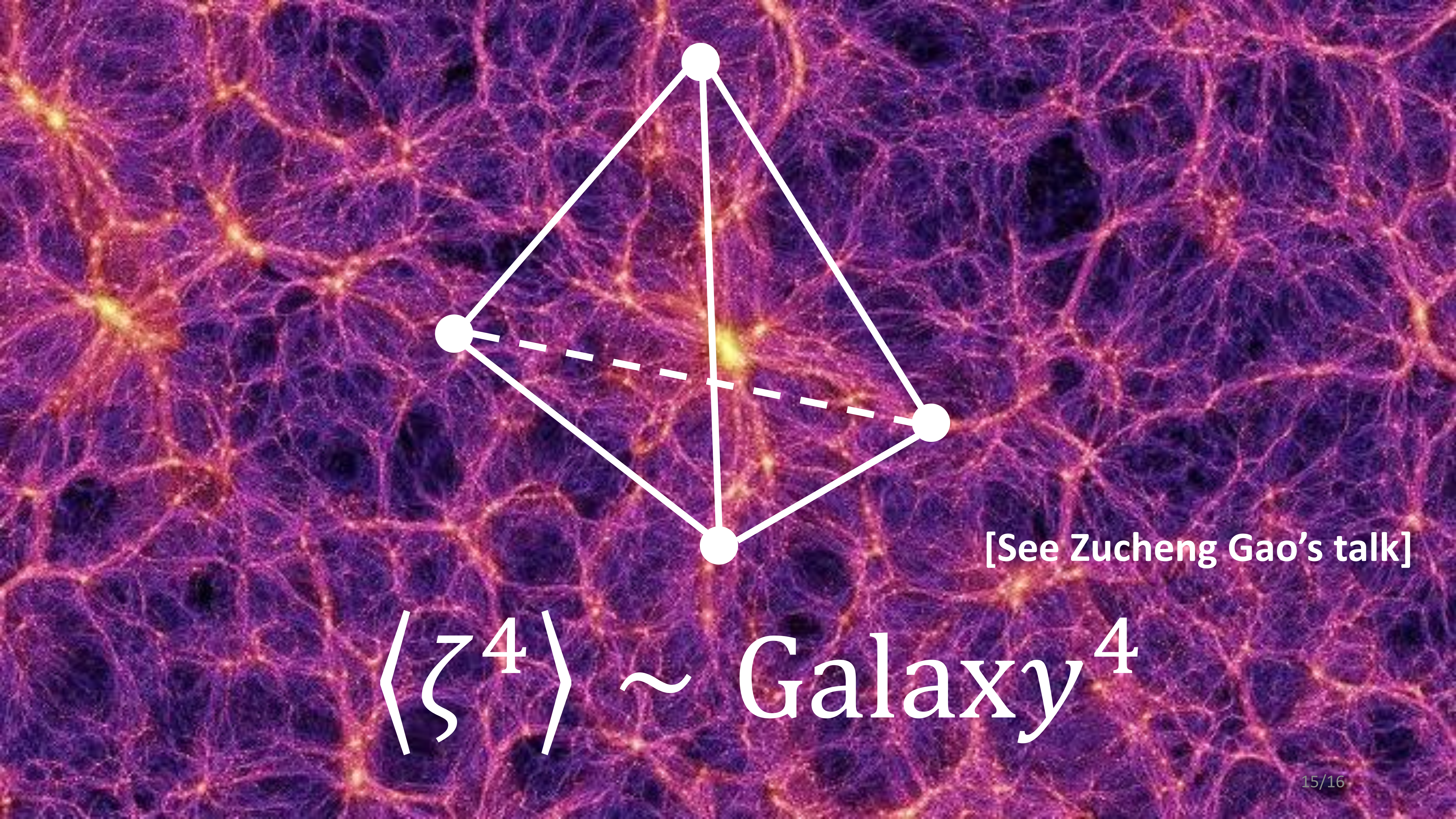
[Cabass, Jazajeri, Pajer & Stefanyszyn, 2022]
[Goodhew, Thavanesan & Wall, 2024]
[Thavanesan, 2025]

➢ An infinite & invisible tower of PV operators:

$$\mathcal{L}_{\mathrm{int}} = \frac{g}{2}\ddot{\phi}^2 + \cdots + \frac{\lambda}{3!}\dot{\phi}^3 + \cdots + c_{10}\epsilon_{ijk}\phi\partial_i\phi\partial_j\dot{\phi}\partial_k\partial^2\phi + \cdots$$

dynamical but ***invisible***

➢ A ***null test*** on the ***fundamental principles***



[See Zucheng Gao's talk]

$$\langle \zeta^4 \rangle \sim \text{Galax}y^4$$

✓ Cosmo PV is growingly active.

◆ But how about loops??

The tree-level story: $\mathrm{Im}\,\psi_n^{0\mathrm{L}} = 0$

[Liu, **Tong**, Wang & Xianyu, 2019]
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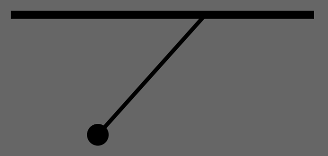
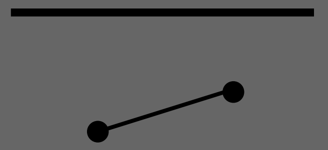
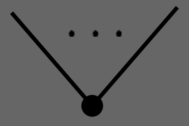
- Feynman rules:



$\Rightarrow \in \mathbb{R}$



$\Rightarrow \in \mathbb{R}$



$\Rightarrow \in \mathbb{R}$

- i & η always appear together (BD vac. + locality)
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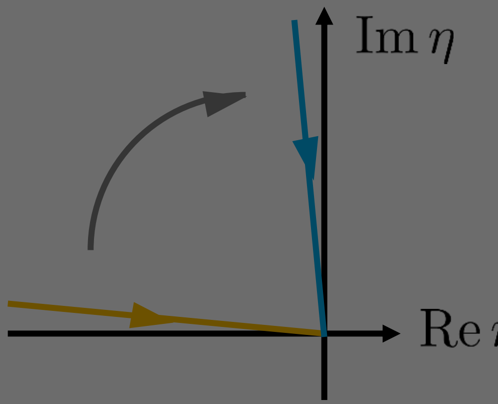
➢ Wick rotation $\eta = i\chi$



Where's the "tree" assumption?

How about loops?

- At 1 loop:

Manifestly real after Wick rot. $\eta = i\chi$

$$\psi_n^{1L} = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \int_{-\infty}^{0} \underbrace{\left[\prod_{v=1}^{V} i\, d\eta_v f_v(\eta\partial_\eta, -i\mathbf{k}\eta)\right] \left[\prod_{e=1}^{n} K_e(\eta_e, k_e)\right] \left[\prod_{i=1}^{I} G_e(\eta_i, \eta_i', q_i)\right]} = \text{Real?}$$

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Careful: UV div. !

How about loops?

- At 1 loop:

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Careful: UV div. !

- A divergent integral of real numbers may not be real anymore…

$$(+\infty)^* - (+\infty) = 0?$$

- Classic example:

$$1 + 2 + 3 + \cdots \to -\frac{1}{12}$$

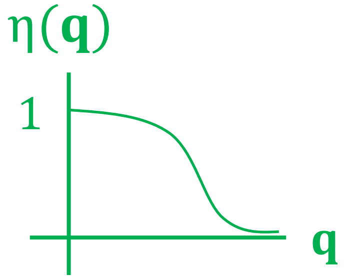
∞ needs *interpretation*…

The breaking of cosmological reality

- (η-) regularisation:

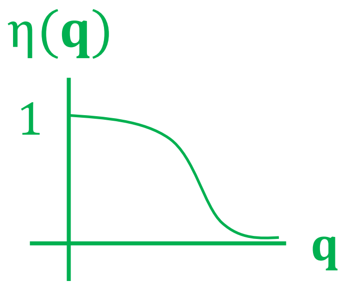
[Padilla & Smith, 2024,2024]
(Minkowski)

$$\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \to \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \times \eta(\mathbf{q})$$



Not necessarily real!

The breaking of cosmological reality



- (η-) regularisation:

[Padilla & Smith, 2024,2024]
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$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \to \int \frac{d^3\mathbf{q}}{(2\pi)^3} \times \eta(\mathbf{q})$$

Not necessarily real!

- Not any η goes in dS/inflation…

Unitarity

Scale inv.

Causality

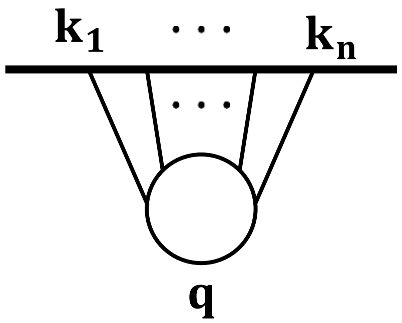
Unitary & analytic η-regs

$$\eta = \rho\left(\frac{q}{ik} \Big/ \frac{\Lambda}{H}\right)$$

$$\rho(0) = 1\ ,$$

$$\rho(\infty e^{i\alpha}) = 0\ ,\ -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$$

A 1-loop universality



Real by tree-level reality

$$= \psi_n^{1\mathrm{L}} = \int_{\min}^{\infty} \frac{\mathrm{d}q_+}{q_+} f(\{k\}, \{q\}, \{\mathbf{k}\}) \times \rho\left(\frac{q_+/k_T}{i\Lambda/H}\right)$$

+ counterterms

$$= \cancel{\sum_{m=\text{even}} f_m C_m[\rho] \left(\frac{\Lambda}{H}\right)^m} + f_0\left(\log \frac{\cancel{\Lambda}}{H} + \frac{i\pi}{2} + \cancel{\gamma[\rho]} + g_0\right) + \cdots$$

- A ***universal*** imaginary part for all 1-loop renormalised WFs:

$$\left(\mu \frac{\partial}{\partial \mu} - \frac{2}{\pi} \mathrm{Im}\right) \widehat{\psi}_n^{1\mathrm{L}} = 0$$

- Any 1-loop topologies
- Any bulk fields with ***integer spin*** and ***light mass***
- Any ***IR-conv.*** interactions
- Any U & A η-regs (***uncountably*** many)

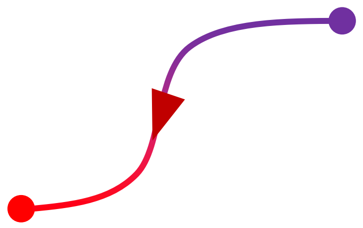
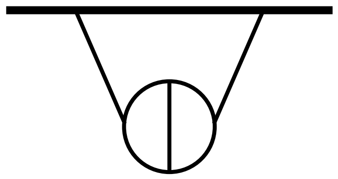
Summary

- ✓ Cosmological WF and correlators are useful
- ✓ WFs satisfy ***reality*** at tree level
- ✓ Translates to a no-go thrm. for PV with obs. consequences
- ✓ Yet spontaneously ***broken*** by UV divs loops
- ✓ By U & A, breaking is ***universal*** and hints at a connnection to RG in dS

$$\left(\mu\frac{\partial}{\partial\mu}-\frac{2}{\pi}\mathrm{Im}\right)\widehat{\psi}_n^{1\mathrm{L}}=0$$

Thanks for your attention!

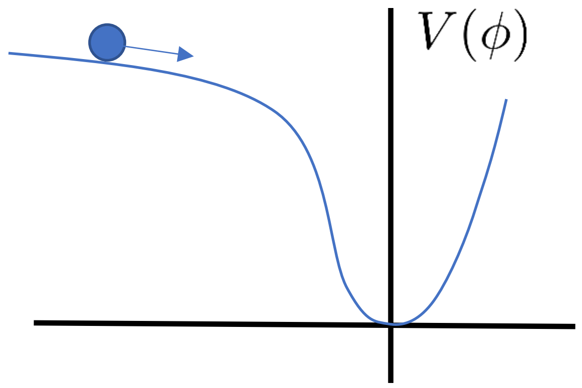
Outlooks

- ☐ Connection to RG? → 
- ☐ **Parity violation as a "scale anomaly"?** → $\langle \phi^n \rangle_{\mathrm{PO}} = \frac{i\pi}{2} \mu \frac{\partial}{\partial \mu} \langle \phi^{n-1} \pi \rangle_{\mathrm{PO}}\,?$
- ☐ More on η reg in cosmology → $\eta(k_{\mathrm{IR}})\,?$
- ☐ Fermions and higher loops? → 
- ☐ Etc.

Back ups

A minimal set up

- Single-field slow-roll inflation



$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_p^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_{\mathrm{int}}\right]$$

- Quasi-de Sitter (dS) background

$$\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right) \quad \text{with} \quad a(t) = e^{Ht} = -\frac{1}{H\eta}$$

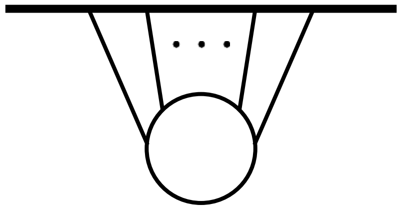
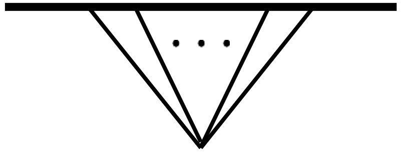
- A single massless scalar DoF with (***IR convergent***) self-interactions

$$\mathcal{L}_{\mathrm{int}} = \mathcal{L}_{\mathrm{int}}(\partial\phi, \partial^2\phi, \cdots) = \frac{\lambda}{3!}\dot{\phi}^3 + \cdots$$

(Isolate the ***sub-horizon*** physics)

Shift symmetric

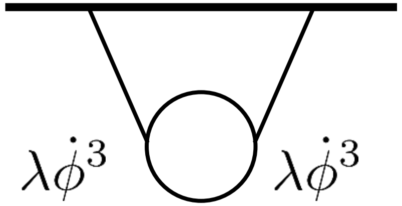
Checking with dim regs

Imaginary part	$\psi_n^{\rm 1L}$ 		$\psi_n^{\rm ct}$ 		✓
dim reg ($d = 3 - \epsilon,\ m_\phi^2 = 0$)	0	+	$\frac{i\pi}{2}$	=	$\mathrm{Im}\, \widehat{\psi}_n^{\rm 1L} = \frac{\pi}{2}$
mass-dim reg ($d = 3 - \epsilon,\ m_\phi^2 = -\frac{3}{2}\epsilon H^2$)	$\frac{i\pi n}{4}$	+	$-\frac{i\pi(n-2)}{4}$	=	$\mathrm{Im}\, \widehat{\psi}_n^{\rm 1L} = \frac{\pi}{2}$

Counterterms can’t solve the issue

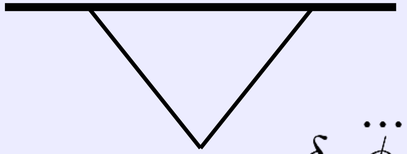
- Renormalisation

$$\widehat{\psi}_2^{\rm 1L} = \psi_2^{\rm 1L} + \psi_2^{\rm ct}$$



$$\frac{1}{15}\frac{\lambda^2 H^2}{(4\pi)^2}k^3\left(\frac{1}{\epsilon}+\frac{i\pi}{2}+\text{real \& fin.}\right)$$

$$\frac{1}{15}\frac{\lambda^2 H^2}{(4\pi)^2}k^3\left(\log\frac{\Lambda}{H}+i\theta+\text{real \& fin.}\right)$$



$$\frac{1}{15}\frac{\lambda^2 H^2}{(4\pi)^2}k^3\left(-\frac{1}{\epsilon}\right)$$

$$\frac{1}{15}\frac{\lambda^2 H^2}{(4\pi)^2}k^3\left(-\log\frac{\Lambda}{H}\right)$$

Real by the tree-level reality

Hermitian counterterms ***cannot*** alter the imaginary part