The Open EFT of Inflation

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Outline

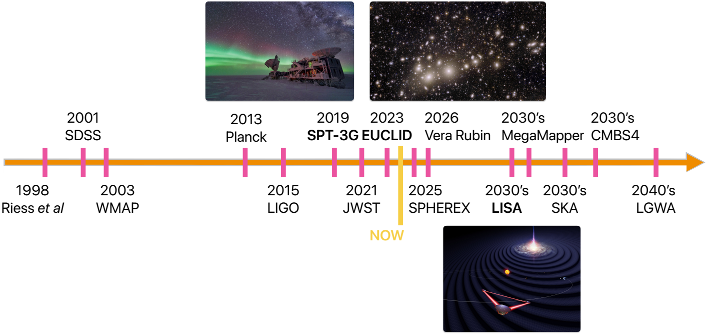
Outline

Aspirations

Learn about **fundamental physics**:

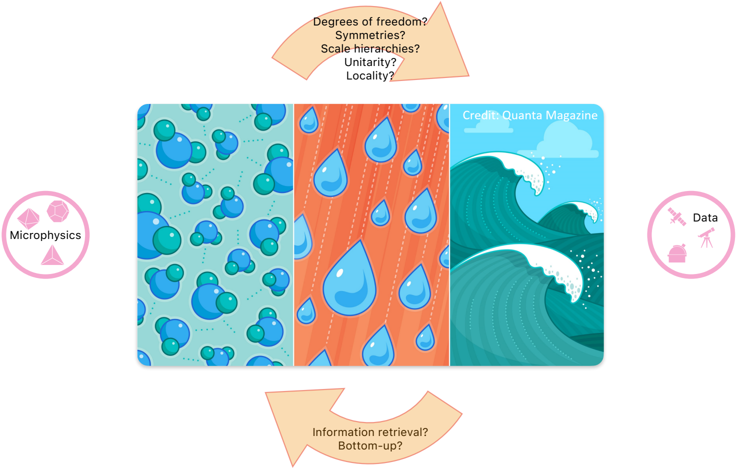
- New degrees of freedom;
- GR at high energy;
- QFT in curved spacetimes;
- Quantum gravity; $\cdots$

A defining moment for **cosmology**:

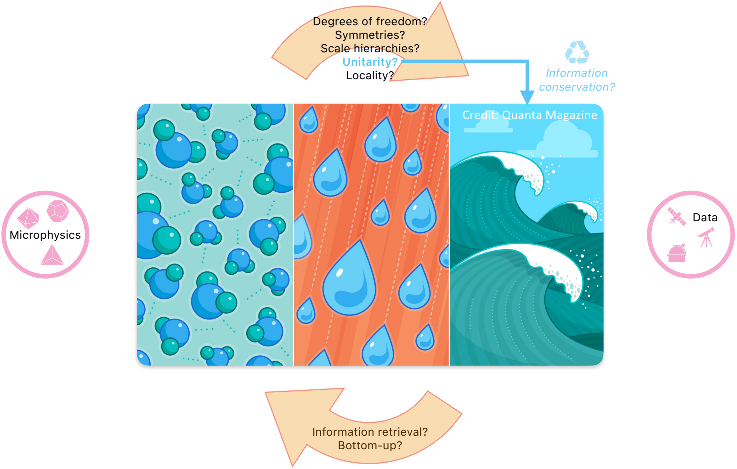


Challenges: i) model degeneracy; ii) enlarged parameter space.

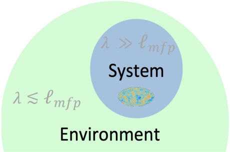
Effective Field Theories



Effective Field Theories



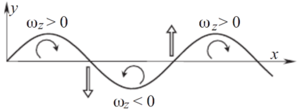
When (non)-unitarity matters



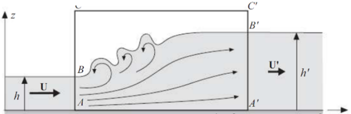
System interacts with environment:
$\Rightarrow$ non-unitary evolution

Dissipation & noise $=$ energy & information losses

Perfect fluid: Wilsonian EFT



Imperfect fluid: non-equilibrium EFT



Credit: [Guyon *et al*, 2012]

What about cosmology?

- **Observable** universe $\neq$ whole; 
- Continuously **evolving**; 
- Always a **medium**; 
- $\exists$ **fluxes** between sectors. 

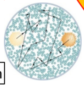
$\Rightarrow$ effective dynamics often non-unitary [Baumann, Nicolis, Senatore & Zaldarriaga, 2010]

Goal: EFTs accounting for dissipation & noise in cosmology

Combining EFTs and Open Quantum Systems

Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$



Martin-Siggia-Rose formalism

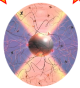
$$P = \int \mathcal{D}x\mathcal{D}\widetilde{x}e^{-S[x,\widetilde{x}]}$$

Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

Stochastic Schrödinger equation

$$\mathrm{d}\widehat{\mathcal{O}} = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right] + \mathrm{d}\widehat{\xi}$$



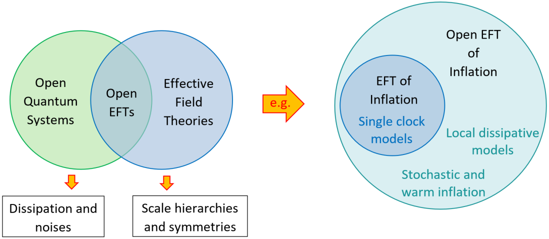
Influence functional

$$\langle\phi_1|\,\widehat{\rho}_{\mathrm{red}}\,|\phi_2\rangle = \int \mathcal{D}\phi_{\pm}e^{iS_{\mathrm{IF}}[\phi_+,\phi_-]}$$

Master equation

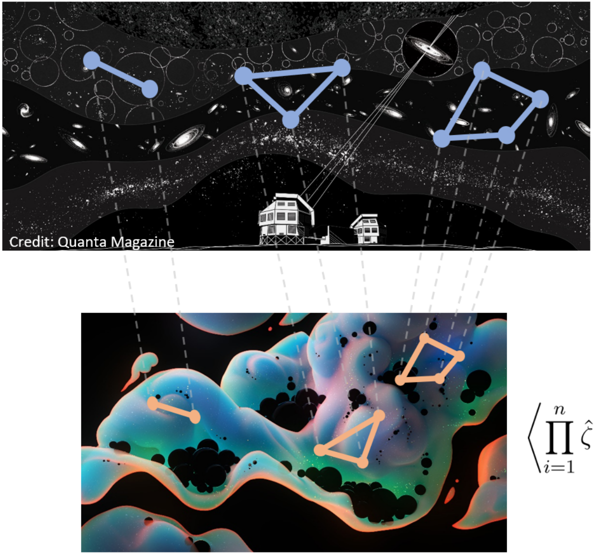
$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{L}[\widehat{\rho}_{\mathrm{red}}]$$

Extend the **embedding power** of EFTs:



Outline

Cosmological correlators



$$\left\langle \prod_{i=1}^{n} \delta(\boldsymbol{k}_i) \right\rangle$$

$$\left\langle \prod_{i=1}^{n} \hat{\zeta}(\boldsymbol{k}_i, \eta_0) \right\rangle$$

Schwinger-Keldysh formalism

Consider some **observable**

$$\widehat{Q} \equiv \widehat{\zeta}(\boldsymbol{x}_1)\widehat{\zeta}(\boldsymbol{x}_2)\cdots\widehat{\zeta}(\boldsymbol{x}_n)$$

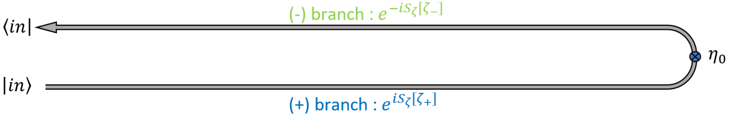
and some unitary **evolution operator** $\widehat{U}(\eta_0, \eta_\mathrm{i})$ so that

$$|\Psi(\eta_0)\rangle = \widehat{U}(\eta_0, \eta_\mathrm{i})\,|\mathrm{BD}\rangle \quad \text{with} \quad \langle\zeta|\,\widehat{U}(\eta_0, \eta_\mathrm{i})\,|\zeta_1\rangle = \int_{\zeta_1}^{\zeta} \mathcal{D}\,[\Phi]\,e^{iS[\Phi]}.$$

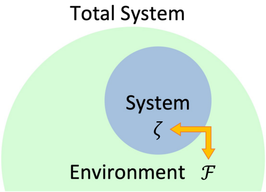
If $S\,[\Phi] = S_\zeta\,[\zeta]$, see [Weinberg, 2005]:

$$\left\langle \widehat{Q}(\eta_0)\right\rangle = \left\langle \Psi(\eta_0)\middle|\widehat{Q}(\eta_0)\middle|\Psi(\eta_0)\right\rangle = \left\langle \mathrm{BD}\middle|\widehat{U}^\dagger(\eta_0,\eta_\mathrm{i})\widehat{Q}(\eta_0)\widehat{U}(\eta_0,\eta_\mathrm{i})\middle|\mathrm{BD}\right\rangle$$

$$= \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\,[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)]\int_{\zeta_1}^{\zeta}\mathcal{D}\,[\zeta_+]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]}\,\langle\zeta_1|\mathrm{BD}\rangle\,\langle\mathrm{BD}|\zeta_2\rangle$$



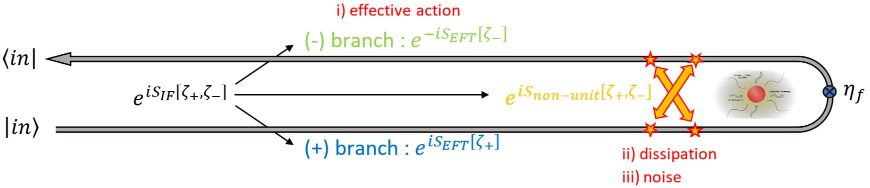
Integrating out an environment



- $S[\Phi] = S_\zeta[\zeta] + S_\mathcal{F}[\mathcal{F}] + S_{\rm int}[\zeta;\mathcal{F}]$ with $\mathcal{F}$ a **hidden sector**.
- *Goal:* tracing out $\mathcal{F}$, the environment being **unobservable**.

Effects of the environment captured by the **Influence Functional** (IF):

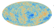
$$\left\langle \widehat{Q}(\eta)\right\rangle = \int \mathrm{d}\zeta\mathrm{d}\zeta_1\mathrm{d}\zeta_2\left[\zeta(\boldsymbol{x}_1)\cdots\zeta(\boldsymbol{x}_n)\right]\int_{\zeta_1}^{\zeta}\mathcal{D}[\zeta_+]\int_{\zeta_2}^{\zeta}\mathcal{D}[\zeta_-]e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]+\boldsymbol{iS_{\rm IF}[\zeta_+;\zeta_-]}}$$



What are the rules obeyed by $S_{\rm IF}[\zeta_+;\zeta_-]$*?*

The Open EFT of Inflation [S.A. Agüí Salcedo, T.C. & E. Pajer, 2404.15416]

Early universe: one scalar degree of freedom $\pi(\boldsymbol{x}, t)$:

Observed ⟨  ⟩ $\Leftrightarrow$ $\langle \hat{\pi}^n \rangle(t) = \int \mathrm{d}\pi \pi^n \mathrm{Prob}_\pi(t)\,.$

- EFT of Inflation [Cheung *et al.*, 2008]: most generic wavefunction

$$\mathrm{Prob}_\pi(t) = |\Psi_\pi(t)|^2 = \left| \int_\Omega^\pi \mathcal{D}\pi \; e^{iS_{\mathrm{eff}}[\pi]} \right|^2$$

- **Dissipation** & **noise**: most generic density matrix

$$\mathrm{Prob}_\pi(t) = \rho_{\pi\pi}(t) = \int_\Omega^\pi \mathcal{D}\pi_+ \int_\Omega^\pi \mathcal{D}\pi_- \; e^{iS_{\mathrm{eff}}[\pi_+, \pi_-]}$$

Physical principles restrict $S_{\mathrm{eff}}[\pi_+, \pi_-]$:

1. Unitarity: {Sys. + Env.} closed $\Rightarrow$ non-equilibrium constraints; [Liu & Glorioso, 2018]
2. Symmetries: in-in coset construction; [Akyuz, Goon & Penco, 2023]
3. Locality: truncatable power counting scheme.

Non-equilibrium constraints [Liu & Glorioso, 2018]

Requiring **Open QFT** originates from a unitary "closed" UV theory:

$$\text{i) } \mathrm{Tr}[\widehat{\rho}] = 1, \qquad \text{ii) } \widehat{\rho}^\dagger = \widehat{\rho} \qquad \text{and} \qquad \text{iii) } \widehat{\rho} \geq 0$$

implies constraints on $S_{\text{eff}}[\pi_+, \pi_-] \equiv S_{\text{unit}}[\pi_+] - S_{\text{unit}}[\pi_-] + S_{\text{non-unit}}[\pi_+, \pi_-]$:

$$\begin{aligned} &\text{i)} && S_{\text{eff}}[\pi_+, \pi_+] = 0, && S_{\text{eff}}[\pi_r, \pi_a = 0] = 0; \\ &\text{ii)} && S_{\text{eff}}[\pi_+, \pi_-] = -S^*_{\text{eff}}[\pi_-, \pi_+], && S_{\text{eff}}[\pi_r, \pi_a] = -S^*_{\text{eff}}[\pi_r, -\pi_a]; \\ &\text{iii)} && \mathfrak{Im} S_{\text{eff}}[\pi_+, \pi_-] \geq 0, && \mathfrak{Im} S_{\text{eff}}[\pi_r, \pi_a] \geq 0, \end{aligned}$$

for $\pi_r = (\pi_+ + \pi_-)/2$ and $\pi_a = \pi_+ - \pi_-$. *Consequences*:

1. $S_{\text{eff}}[\pi_r, \pi_a]$ starts **linear** in π_a;
2. **Odd** powers of π_a are purely real; **even** powers of π_a purely imaginary;
3. Bounds on the **noise coefficients**.

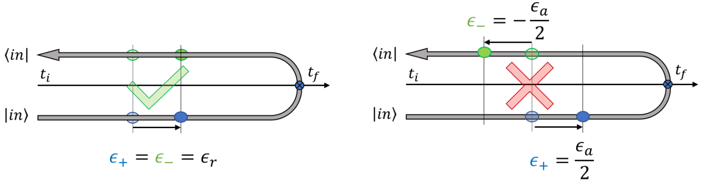
$\Rightarrow$ *Already reduce the scope of available Open EFTs*

Dissipative shift symmetric scalar [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

$S_{\rm unit}\left[\pi_\pm\right]$ invariant under shift$_+$ $\times$ shift$_-$:

$$\pi_\pm(t) \to \pi'_\pm(t) = \pi_\pm(t+\epsilon_\pm) + \epsilon_\pm,$$

but $S_{\rm non-unit}\left[\pi_+;\pi_-\right]$ **is not**: **SSB** shift$_+$ $\times$ shift$_-$ $\to$ shift$_r$:



Retarded $\pi_r = (\pi_+ + \pi_-)/2$ and **advanced** $\pi_a = \pi_+ - \pi_-$ fields:

$$\pi_r(t) \to \pi'_r(t) = \pi_r(t+\epsilon_r) + \epsilon_r, \qquad \pi_a(t) \to \pi'_a(t) = \pi_a(t+\epsilon_r).$$

Building blocks:

$$\pi_a, \quad t+\pi_r, \quad \partial_\mu\pi_a, \quad \partial_\mu(t+\pi_r).$$

Effective functional

- *Quadratic order:* $1 \to 5$ EFT param (1 tadpole constraint):

Kinetic term

$$S_{\text{eff}}^{(2)} = \int \mathrm{d}^4x \sqrt{-g} \Big\{ \dot{\pi}_r \dot{\pi}_a - c_s^2 \partial_i \pi_r \partial^i \pi_a$$
$$-\gamma \dot{\pi}_r \pi_a + i \left[\beta_1 \pi_a^2 - (\beta_2 - \beta_4)\, \dot{\pi}_a^2 + \beta_2 \left(\partial_i \pi_a \right)^2 \right] \Big\}$$

Dissipation — Noise

- *Cubic order:* $1 \to 13$ EFT param: EFToI famous for relating operators at different orders because of **non-linearly realised boosts** [López Nacir *et al.*, 2011].

$$\begin{aligned} \text{EFToI}: &\quad \mathcal{L} \supset \left(c_s^2 - 1\right) \left[-2\dot{\pi}_r + (\partial_\mu \pi_r)^2\right] \dot{\pi}_a \\ \text{Dissipation}: &\quad \mathcal{L} \supset \gamma \left[-2\dot{\pi}_r + (\partial_\mu \pi_r)^2\right] \pi_a \\ \text{Noise}: &\quad \mathcal{L} \supset i\beta_4 \left(-\dot{\pi}_a + \partial_\mu \pi_r \partial^\mu \pi_a\right)^2 \end{aligned}$$

Recover and extend EFToI construction.

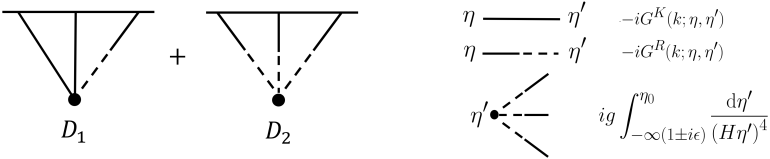
Standard observables

Symmetries ensure existence of nearly scale invariant power spectrum

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = \frac{H^2}{f_\pi^4} \langle \pi^c_{\boldsymbol{k}} \pi^c_{\boldsymbol{k}'} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k} + \boldsymbol{k}) \frac{2\pi^2}{k^3} \Delta^2_\zeta(k).$$

$\Rightarrow \Delta^2_\zeta = 10^{-9}$ obtained by imposing hierarchies of scales.

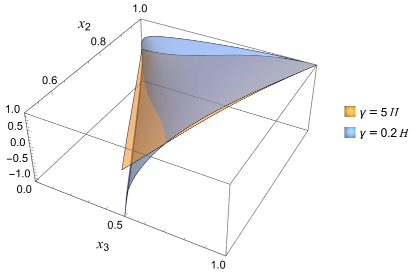
Bispectrum computed in **perturbation theory** using standard in-in rules.



$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = -\frac{H^3}{f_\pi^6} \langle \pi^c_{\boldsymbol{k}_1} \pi^c_{\boldsymbol{k}_2} \pi^c_{\boldsymbol{k}_3} \rangle \equiv (2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B(k_1, k_2, k_3).$$

$$S(x_2, x_3) \equiv (x_2 x_3)^2 \frac{B(k_1, x_2 k_1, x_3 k_1)}{B(k_1, k_1, k_1)}, \qquad f_{\mathrm{NL}}(k_1, k_2, k_3) \equiv \frac{5}{6} \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2 \text{ perms.}}.$$

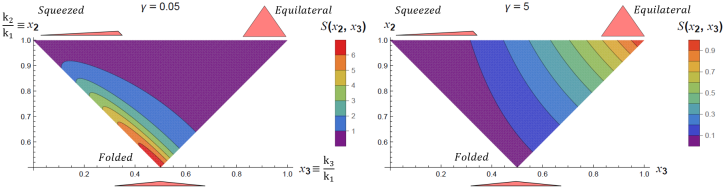
Bispectrum shapes



Main features:

- $\gamma \gg H$: equilateral;
- $\gamma \ll H$: folded;
- Consistency relations;
- Regularized divergence.

Consistent with **flat-space/sub-Hubble** analytic results:



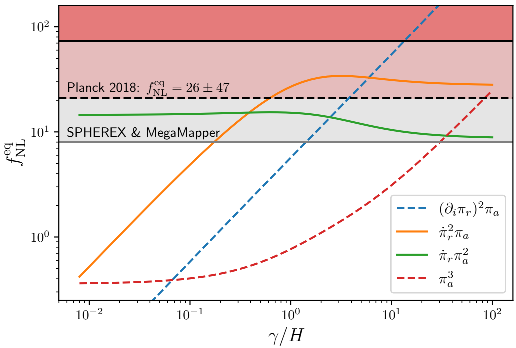
Outline

Matching and $f_{\rm NL}$ with [Creminelli *et al.*, 2305.07695]

UV completion: inflaton ϕ + massive scalar field χ with softly-broken $U(1)$:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - |\partial\chi|^2 + M^2|\chi|^2 - \frac{\partial_\mu\phi}{f}\left(\chi\partial^\mu\chi^* - \chi^*\partial^\mu\chi\right) - \frac{1}{2}m^2\left(\chi^2 + \chi^{*2}\right)\right].$$

$\Rightarrow$ narrow **instability band** in sub-Hubble regime: *local* particle production.



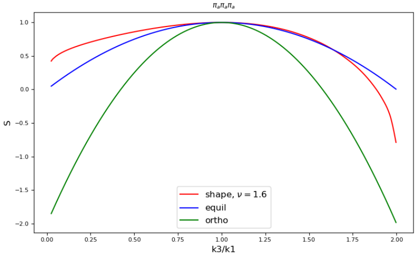
CMB constraints on PNG [with **Bowei Zhang**, **Petar Suman**, in progress]

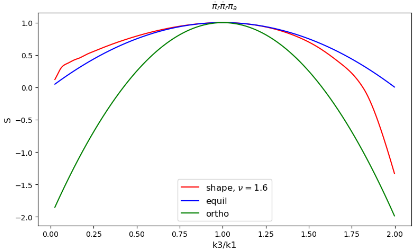
Planck data using **CMB-BEST pipeline** [Fergusson, Liguori & Shellard, 2009], [Fergusson, 2014]

Goal: constrain on EFT coefficients $c_s^2, \gamma, \beta, \cdots$ *to not overproduce PNG*

Operators more or less easy to constrain: at *low dissipation* $\gamma = 0.2H$

- π_a^3: Signal-to-noise $\sim 0.5\sigma$
- $\dot{\pi}_r^2 \pi_a$: Signal-to-noise $\sim 1\sigma$

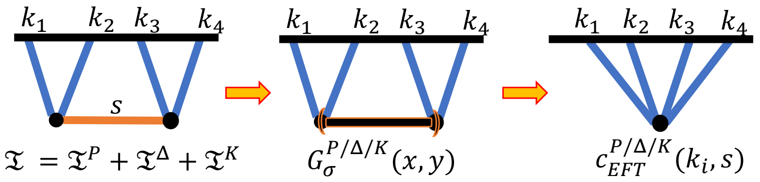




Cosmological Collider Signals [with **Xi Tong**, **Zhehan Qin**, in progress]

$$\mathcal{L}[\varphi, \sigma] = -\frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}M^2\sigma^2 + \frac{1}{2\Lambda}\dot{\varphi}^2\sigma,$$

1. Step 1: Write a non-local Open EFT with **unitary**, **dissipative** and **stochastic** operators.
2. Step 2: Split the signal: $\langle \varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\varphi_{k_4}\rangle_s' = \underbrace{\mathcal{I}^P}_{\text{unitary}} + \underbrace{\mathcal{I}^\Delta}_{\text{dissipation}} + \underbrace{\mathcal{I}^K}_{\text{noise}}.$
3. Step 3: Use **time-convolutionless methods** [Breuer & Petruccione, 2007]: trade non-locality for non-analytic EFT coefficients.



Origin of the Cosmological Collider

$$\lim_{s\to 0}\langle\varphi_{k_1}\varphi_{k_2}\varphi_{k_3}\varphi_{k_4}\rangle' \supset \underbrace{\mathcal{A}^{\mathrm{L}}\sin\left(\mu\ln\frac{k_{12}}{k_{34}}\right)}_{\text{local signal}} + \underbrace{\mathcal{A}^{\mathrm{NL}}\cos\left(\mu\ln\frac{4k_{12}k_{34}}{s^2}+\delta\right)}_{\text{non-local signal}}$$

	Parity-even sector	Parity-odd sector
Local signal	Unitary	Stochastic
Non-local signal	Stochastic	Stochastic

EFT coeff **non-analytic in** $\mu = \sqrt{M^2/H^2 - 9/4}$ (analytic part = background)

$$\lim_{s\to 0} c_{\text{unit.}}(k_1,k_2,s;\eta) = \left[-\frac{ik_1k_2\eta^2}{32\Lambda^2\mu}e^{ik_{12}\eta}E_{i\mu-3/2}(ik_{12}\eta) + (\mu\to-\mu)\right]$$
$$+ (k_1\to -k_1) + (k_2\to -k_2) + (k_{1,2}\to -k_{1,2})$$

$$\lim_{s\to 0} c_{\text{stoch.}}(k_1,k_2,s;\eta) = \left[-\frac{k_1k_2\eta^2}{16\pi\Lambda^2}\left(-\frac{s\eta}{2}\right)^{-2i\mu}\Gamma^2(i\mu)e^{ik_{12}\eta}E_{i\mu-3/2}(ik_{12}\eta) + (\mu\to-\mu)\right]$$
$$+ (k_1\to -k_1) + (k_2\to -k_2) + (k_{1,2}\to -k_{1,2})$$

Stochastic inflation [with **Sebastian Cespedes**, in progress]

Q?: Can a field produce its own bath?

Answer is **yes**! Stochastic inflation from Open EFTs [Burgess, Holman, Tasinato, 1512.00169]

Smooth filter function: $\varphi^L(\boldsymbol{k}, t) = \Omega_\Lambda(k)\varphi(\boldsymbol{k}, t)$, $\varphi^S(\boldsymbol{k}, t) = [1 - \Omega_\Lambda(k)]\varphi(\boldsymbol{k}, t)$

System-environment interactions [Pinol, Renaux-Petel & Tada, 2008.07497], [Li, 2507.02070]

$$S_{\mathrm{cross}} = \int_{\boldsymbol{k}} \int \mathrm{d}t a^3(t)\dot{\Omega}_\Lambda(k) \left[\dot{\varphi}_r^L \varphi_a^S - \varphi_r^L \dot{\varphi}_a^S - \dot{\varphi}_a^L \varphi_r^S + \varphi_a^L \dot{\varphi}_r^S\right].$$

1. Integrate out $\varphi^S_{r,a}$: *influence-functional* for $\varphi^L_{r,a}$ [Lombardo, 0412069];
2. Super-Hubble *gradient expansion*: $k \ll aH$ [Gorbenko & Senatore, 1911.00022].

Consequences:

- Dissipation gradient-suppressed: $G^R_S \dot{\varphi}^L_r \dot{\varphi}^L_a \sim [k/(aH)]^3$
- Leading noise: $G^K_S \dot{\varphi}^L_a \dot{\varphi}^L_a \sim H^2/(2k^3) \Rightarrow$ [Starobinsky, 1986]

Non-equilibrium Renormalization Group

Schwinger-Keldysh path integral prepares a density matrix:

$$P_\Lambda[\phi^\ell] = \int_\Omega^{\phi^\ell} \mathcal{D}\varphi_r \int_\Omega^0 \mathcal{D}\varphi_a e^{iS_{\mathrm{eff}}^\Lambda[\varphi_r,\varphi_a]} \underbrace{=}_{\text{nPI}} e^{i\Gamma_\Lambda[\boldsymbol{\Phi}(\phi^\ell),\boldsymbol{G}(\phi^\ell),\boldsymbol{B}(\phi^\ell),\cdots]}$$

Coarse-graining and Polchinski equation [Polchinski, 1984] $\frac{\mathrm{d}}{\mathrm{d}\Lambda} Z_\Lambda[J_r, J_a] = 0$

$\Rightarrow$ generates **Fokker-Planck equation** for $P_\Lambda[\phi^\ell]$.

At $0PI$, resum classical non-linearities: in terms of $\Phi_\alpha[\phi^\ell]$ *classical field solutions*:

$$\frac{\partial P_\Lambda}{\partial \log \Lambda} = 2\int \mathrm{d}^4x \frac{\delta}{\delta \Phi_\alpha(x)}\left[\frac{\partial \Phi_\alpha^\Lambda(x)}{\partial \log\Lambda} P_\Lambda\right] - 2i \int \mathrm{d}^4x \int \mathrm{d}^4y \frac{\partial G_\Lambda^{\alpha\beta}(x,y)}{\partial \log \Lambda}\frac{\delta^2 P_\Lambda}{\delta\Phi_\beta(x)\delta\Phi_\alpha(y)}$$

Identify $\Lambda = \epsilon a H$ with $\epsilon \ll 1$: **recover stochastic inflation and its corrections**.

Fixed points, β-functions, universality classes?

Conclusion

Realistic environments are noisy and dissipative:
*We need **systematic modelling** for **upcoming data**.*

Open EFT of Inflation:

1. Unitarity, locality, scale invariance: dissipative shift symmetric scalar in dS;
2. Smoking gun near folded triangles in **primordial non-Gaussianities**.

Power counting, renormalization, boostrap, entropy bounds

Matching:

1. Gauge-inflation models with a **scale hierarchy**;
2. Non-local Cosmological Collider Signal as an **environmental noise**;
3. Stochastic inflation as a **non-equilibrium renormalization group**.

Axion inflation, warm inflation, Open Cosmological Collider

Beyond scalar perturbations: tensor modes $\Rightarrow$ Lennard Dufner's talk

Backup

Thomas Colas

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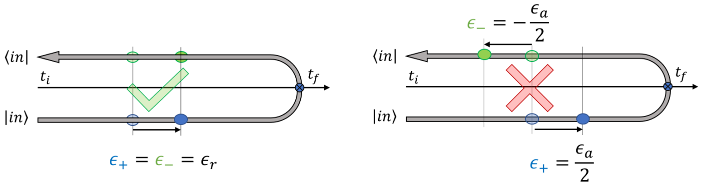
In-in coset construction [Hongo *et al.*, 2018], [Akyuz, Goon & Penco, 2023]

Two **simplifications** from [Cheung *et al.*, 2008]: [regime of validity? ⇒ later]

1. *Decoupling limit*: Mixing $\pi/\delta g$ small as long as $E \sim H \gg E_{\rm mix} \sim \epsilon^{1/2} H$

⇒ **enough** to construct theory of **dissipative shift symmetric scalar**:



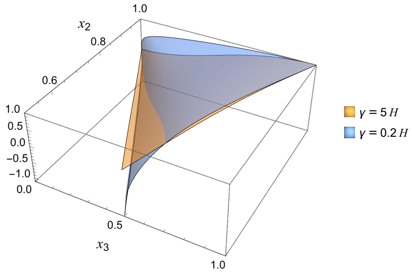
$S_{\rm eff}\left[\pi_r, \pi_a\right]$ invariant under *retarded time diffeomorphism*:

$$t \to t + \epsilon_r : \qquad \pi_r \to \pi_r - \epsilon_r, \qquad \pi_a \to \pi_a.$$

Building blocks: π_a, $t + \pi_r$, $\partial_\mu \pi_a$, $\partial_\mu (t + \pi_r)$.

2. *Derivative expansion*: locality and truncatable power counting scheme.

Primordial non-Gaussianities

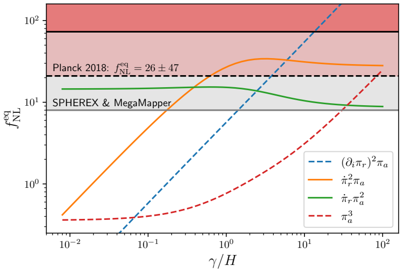


Main features:

- $\gamma \gg H$: equilateral shape;
- $\gamma \ll H$: **new** folded signal;
- Consistency relations hold.

Matching with [Creminelli *et al.*, 2305.07695]

- inflaton ϕ
- massive scalar field χ
- softly-broken $U(1)$



New challenge: dynamical gravity

Details of matching with [Creminelli *et al.*, 2305.07695]

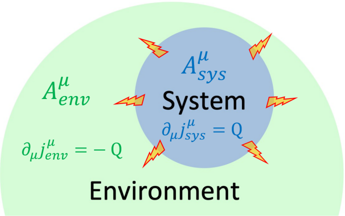
	Parameters:						
UV completion	M	m	f	ρ	ξ	γ	$\nu_{\mathcal{O}}/\nu_{\mathcal{O}^3}$
Open EFT	f_π^2	c_s	γ	γ_2	β_1	β_5	δ_1

Matching:	$f_\pi^2 = \rho f$
$c_s = 1$	$\gamma = \frac{\xi m^4}{\pi M f^2} e^{2\pi\xi}$
$\Gamma = \pi\xi\gamma$	$\beta_1 = \frac{\nu_{\mathcal{O}}}{2\rho f}\frac{m^4}{f^2}$
$\beta_5 = 2\pi\xi\beta_1$	$\delta_1 = \frac{\nu_{\mathcal{O}^3}}{6\sqrt{\rho f}}\frac{m^6}{f^3}$

Outline

Open gauge theories

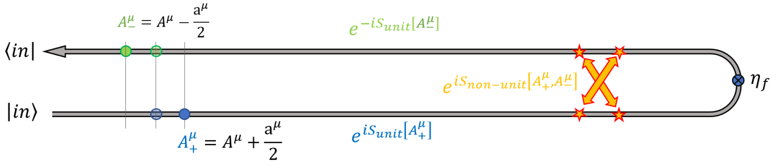
[S.A. Agüí Salcedo, T.C. & E. Pajer, 2412.12299]: *Dissipative theory* for of light in a medium.



- Gauge symmetry: $U(1)_r \times U(1)_a$
- Global symmetry:

$$U(1)_r \times U(1)_a \to U(1)_r$$

Keldysh basis: retarded $A^\mu = \left(A^\mu_+ + A^\mu_-\right)/2$; advanced $a^\mu = A^\mu_+ - A^\mu_-$.



Retarded & advanced gauge transformation

Retarded gauge transformation $\epsilon_+ = \epsilon_- = \epsilon_r$:

$$A^\mu \to A^\mu + \partial^\mu \epsilon_r \,, \qquad a^\mu \to a^\mu \,.$$

Advanced gauge transformation $\epsilon_+ = -\epsilon_- = \epsilon_a$:

$$A^\mu \to A^\mu \,, \qquad a^\mu \to a^\mu + \partial^\mu \epsilon_a \,.$$

- Global $U(1)_a$ broken: $B_a^\mu = a^\mu + \partial^\mu \varphi_a$ with $\varphi_a \to \varphi_a - \epsilon_a$.
- Unitary gauge: $\varphi_a = 0$ and **retarded gauge invariance** only.

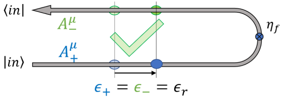
Effective functional constructed out of $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and a^μ:

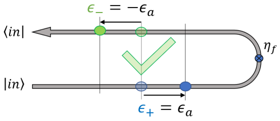
$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl} \right) + a^\mu j_\mu \right]$$

$$S_2 = \int_{\omega,\boldsymbol{k}} i a^\mu N_{\mu\nu} a^\nu \,, \qquad S_{n\geq 3} = \cdots$$

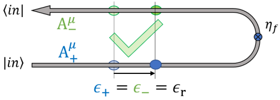
Summary

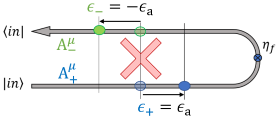
1. **Unitary**: $\Delta S_{\text{eff}}^{\text{adv}} = 0 \quad \Rightarrow \quad \partial^\mu j_\mu = 0$: Maxwell in medium;



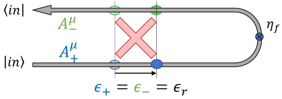


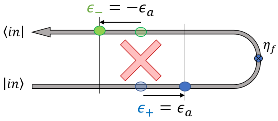
2. **Non-unitary**: $\Delta S_{\text{eff}}^{\text{adv}} \neq 0 \quad \Rightarrow \quad \partial^\mu j_\mu \neq 0$: modified charge conservation;





3. **Conductor**: $\Delta S_{\text{eff}}^{\text{ret}} \neq 0 \quad \Rightarrow \quad$ new d.o.f.: dissipative Proca theory.





Recovering electromagnetism in a medium

From S_{eff}, obtain modified Gauss and Ampère laws:

$$\frac{\delta S_{\text{eff}}}{\delta a^0} = 0 \qquad \Rightarrow \qquad \nabla.\mathbf{E} = j_0 + \xi_0,$$

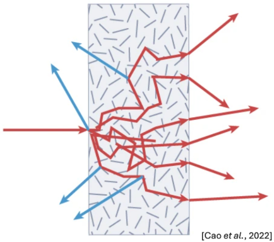
$$\frac{\delta S_{\text{eff}}}{\delta a^i} = 0 \qquad \Rightarrow \qquad \gamma_2 \mathbf{E} + \gamma_3 \nabla \times \mathbf{B} - 2\gamma_4 \mathbf{B} = \mathbf{j} + \xi,$$

and a noise constraint: *modified charge conservation in the system*

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

Properties:

- Dispersive medium: $n = 1/\sqrt{-\gamma_3}$;
- Dissipative medium: $\gamma_2 = -i\omega + \Gamma$;
- Anisotropic medium: γ_4;
- Random medium: ξ^μ.

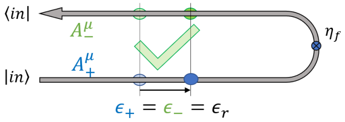


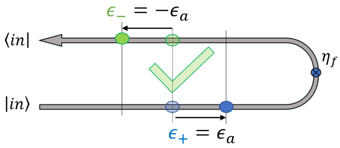
[Cao *et al.*, 2022]

Unitary limit

$$\Delta S_1^{\rm ret} = 0 \qquad \text{but} \qquad \Delta S_1^{\rm adv} = \int_{\omega,\boldsymbol{k}} (i\omega + \gamma_2)\epsilon_a i k_i F^{0i}.$$

- For general γ_2, $\Delta S_1^{\rm adv} \neq 0$;
- When $\gamma_2 = -i\omega$, $\Delta S_1^{\rm adv} = 0$.





Unitary theory with $S_1[A_+, A_-] = S[A_+] - S[A_-]$ and

$$S[A] = \frac{1}{4}\int \mathrm{d}^4 x \left[F^{\mu\nu}F_{\mu\nu} + (c_s^2 - 1)F^{ij}F_{ij} + \theta F^{\mu\nu}\tilde{F}^{\mu\nu}\right]$$

Recover Maxwell in a medium.

Deformed advanced gauge

When $\gamma_2 \neq -i\omega$, does it exist ***physically equivalent*** *advanced configurations?*

1. Retarded gauge invariance: $M_{\mu\nu}k^\nu = 0$ where $k^\mu = (\omega, \mathbf{k})$.
2. $\exists$ "right kernel" $\Rightarrow$ $\exists$ "left kernel" such that $v^\mu M_{\mu\nu} = 0$.
3. M is non-Hermitian $\Rightarrow$ different left and right kernels: $v^\mu = (i\gamma_2, \boldsymbol{k})$.

Conclusion: retarded gauge invariance generates advanced gauge invariance.

S_1 remains unchanged under

$$A^\mu \to A^\mu + \epsilon_r k^\mu, \qquad a^\mu \to a^\mu + \epsilon_a v^\mu.$$

Not broken but rather **deformed**! [Liu & Glorioso, 2018], [Akyuz, Goon & Penco, 2023].

Advanced gauge redundancy allows us to reduce number of advanced components.

Noise constraint

Add some noise: $N_{\mu\nu}$ positive semi-definite [Kamenev, 2011], [Breuer & Petruccione, 2007]

$$S = \int d^4x \left[a^\mu M_{\mu\nu} A^\nu + i a^\mu N_{\mu\nu} a^\nu \right],$$

Hubbard-Stratonovich trick:

$$\mathcal{Z} = \int [\mathcal{D}A^\mu] \int [\mathcal{D}a^\mu] \int [\mathcal{D}\xi_\mu] \exp\left[\int d^4x \; i\boldsymbol{a^\mu} \left(\boldsymbol{M_{\mu\nu} A^\nu} - j_\nu - \xi_\nu \right) - \frac{1}{4} \xi_\mu (N^{-1})^{\mu\nu} \xi_\nu \right]$$

Advanced gauge symmetry $\boldsymbol{a^\mu \to a^\mu + \epsilon_a v^\mu}$ induces noise constraint:

$$v^\mu (j_\mu + \xi_\mu) = 0.$$

Meaning: $\gamma_2 = \Gamma - i\omega$ leads to

$$\partial^\mu (j_\mu + \xi_\mu) = \Gamma (j_0 + \xi_0).$$

Conservation of the total current $\Leftrightarrow$ ***Non-conservation of the system's current***

Advanced Stueckelberg [Lau, Nishii & Noumi, 2412.21136]

Stueckelberg trick: $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}_a^\mu = a^\mu + \partial^\mu X_a \to \mathcal{A}_a^\mu$ invariant

$$S_{\rm eff} = \int_{\omega,\boldsymbol{k}} \left[\mathcal{A}_{a0} ik_i F^{0i} + \mathcal{A}_{ai}\left(\gamma_2 F^{0i} + \gamma_3 ik_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl}\right) - \mathcal{A}_{a\mu}\left(j^\mu + \xi^\mu\right)\right],$$

$$= S_{\rm eff}^{\rm old} + \int_{\omega,\boldsymbol{k}} X_a \left[(i\omega + \gamma_2) ik_i F^{0i} + \partial_\mu\left(j^\mu + \xi^\mu\right)\right].$$

X_a e.o.m. recovers noise constraint: for $\gamma_2 = \Gamma - i\omega$,

$$\frac{\delta S_{\rm eff}}{\delta X_a} = 0 \quad \Rightarrow \quad \partial^\mu (j_\mu + \xi_\mu) = \Gamma(j_0 + \xi_0).$$

using on-shell relation $\delta S_{\rm eff}/\delta a_0 = 0$ that is $ik_i F^{0i} = -\left(j^0 + \xi^0\right)$.

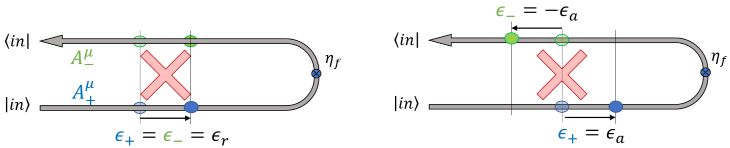
Lessons:

- Manifest advanced gauge invariance at the price of X_a;
- If $\partial^\mu(j_\mu + \xi_\mu) = 0$, only solution is $\gamma_2 = -i\omega$, *i.e.* no dissipation $\Gamma = 0$.

Proca mass term

Adding a mass term breaks **both** retarded and advanced gauge symmetries:

$$S_{\text{eff}} = \int_{\omega,\boldsymbol{k}} ik_i a_0 F^{0i} - i\omega a_i F^{0i} - m^2 a_\mu A^\mu$$



Stueckelberg trick:

- $X_a \to X_a - \epsilon_a$ s.t. $\mathcal{A}^\mu_a = a^\mu + \partial^\mu X_a \to \mathcal{A}^\mu_a$ invariant;
- $X_r \to X_r - \epsilon_r$ s.t. $\mathcal{A}^\mu_r = A^\mu + \partial^\mu X_r \to \mathcal{A}^\mu_r$ invariant

$$S_{\text{eff}} = S^{\text{old}}_{\text{eff}} - m^2 \int \mathrm{d}^4 x \left[\partial_\mu X_a \partial^\mu X_r - a_\mu \partial^\mu X_r - \partial_\mu X_a A^\mu\right]$$

Decoupling: when $E \gg m$, mixing between X and A negligeable.

Breaking retarded gauge invariance triggers new d.o.f.

Dispersion relations

Gauge fixing

- retarded Coulomb gauge: $\partial_i A^i = 0$

$$\exists\ \epsilon_r \quad \text{s.t.} \quad k_i A'^i = 0, \quad \text{where} \quad A'^\mu = A^\mu + \epsilon_r k^\mu .$$

- advanced Coulomb gauge: $\partial_i a^i = 0$

$$\exists\ \epsilon_a \quad \text{s.t.} \quad k_i a'^i = 0, \quad \text{where} \quad a'^\mu = a^\mu + \epsilon_a v^\mu .$$

Eigenvalues of the kinetic matrix: 1 constrained dof, 2 propagating dof

$$\left(k^2\,, i\gamma_2\omega + \gamma_3 k^2 + 2\gamma_4 k, i\gamma_2\omega + \gamma_3 k^2 - 2\gamma_4 k\right).$$

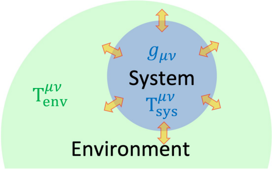
Introduce $\gamma_2 = \Gamma - i\omega$, $\gamma_3 = -c_s^2$:

$$\omega^2 + i\Gamma\omega - c_s^2 k^2 \pm 2\gamma_4 k = 0 \quad \Rightarrow \quad \omega = -i\frac{\Gamma}{2} \pm \sqrt{c_s^2 k^2 - (\Gamma/2)^2 \mp 2\gamma_4 k}\,.$$

Outline

Open gravity

Dissipative theory for a **massless spin 2 graviton**: theory of gravity in a medium.



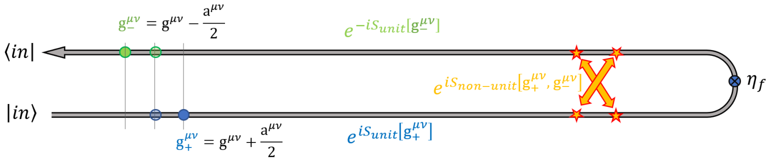
Diffeomorphisms invariance:

$$g^{\mu\nu}_{\pm}(x) \to \frac{\partial(x^\mu + \xi^\mu_\pm)}{\partial x^\alpha}\frac{\partial(x^\nu + \xi^\nu_\pm)}{\partial x^\beta} g^{\alpha\beta}_{\pm}(x)$$

for each branch of SK path integral contour.

Keldysh basis: **retarded** $g_{\mu\nu}$ and **advanced** $a^{\mu\nu}$ metric:

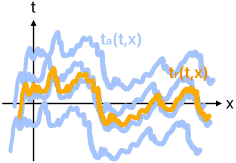
$$g = \frac{g_+ + g_-}{2} = \bar{g} + \delta g, \qquad \text{and} \qquad a = g_+ - g_- = \delta a$$



A tale of two clocks

Fluctuating clocks: $\phi_\pm(t, \mathbf{x})$, i.e.

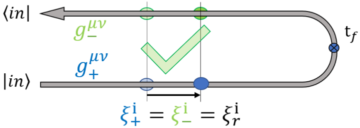
- $\phi_r(t, \mathbf{x}) \Leftrightarrow t_r(t, \mathbf{x})$: average clock;
- $\phi_a(t, \mathbf{x}) \Leftrightarrow t_a(t, \mathbf{x})$: stochasticity.

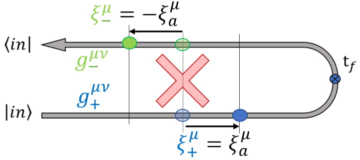


Gauge fixing. Work in *unitary gauges*: $\phi_+(t, \mathbf{x}) = \phi_-(t, \mathbf{x}) = \bar{\phi}(t)$:

$$\phi_r(t, \mathbf{x}) = \bar{\phi}(t) \quad \text{and} \quad \phi_a(t, \mathbf{x}) = 0 \qquad \text{i.e.} \qquad t_r = t \quad \text{and} \quad t_a = 0.$$

Following EFT of Inflation [Cheung *et al.*, 2008] and Dark Energy [Gubitosi, Piazza & Vernizzi, 2013]:





$$\text{4d-diff}_+ \times \text{4d-diff}_- \xrightarrow{\text{open}} \text{4d-diff}_r \xrightarrow{\text{clock}} \text{3d-diff}_r \,.$$

Unitary gauges

Most generic functional invariant under retarded spatial diffeomorphisms.

$$S_{\text{eff}} = \int \mathrm{d}^4x\sqrt{-g}\sum_{\ell=0}(g^{00}+1)^\ell\Big[M_{\mu\nu,\ell}a^{\mu\nu} + iN_{\mu\nu\rho\sigma,\ell}a^{\mu\nu}a^{\rho\sigma} + \cdots\Big]$$

with $M_{\mu\nu,\ell}$ and $N_{\mu\nu\rho\sigma,\ell}$ rank-2 and 4 cotensors under *retarded spatial diffs*.

$$M_{00,\ell} = \gamma^{tt}_{1,\ell} + \gamma^{tt}_{2,\ell}K + \gamma^{tt}_{3,\ell}K^2 + \gamma^{tt}_{4,\ell}K_{\alpha\beta}K^{\alpha\beta} + \gamma^{tt}_{5,\ell}\nabla^0K + \gamma^{tt}_{6,\ell}R + \gamma^{tt}_{7,\ell}R^{00};$$

$$M_{0\mu,\ell} = \gamma^{ts}_{1,\ell}R^0{}_\mu + \gamma^{ts}_{2,\ell}\nabla_\mu K + \gamma^{ts}_{3,\ell}\nabla_\beta K^\beta{}_\mu;$$

$$\begin{aligned} M_{\mu\nu,\ell} = {} & g_{\mu\nu}\left(\gamma^{ss}_{1,\ell} + \gamma^{ss}_{2,\ell}K + \gamma^{ss}_{3,\ell}K^2 + \gamma^{ss}_{4,\ell}K_{\alpha\beta}K^{\alpha\beta} + \gamma^{ss}_{5,\ell}\nabla^0K + \gamma^{ss}_{6,\ell}R + \gamma^{ss}_{7,\ell}R^{00}\right) \\ & + \gamma^{ss}_{8,\ell}K_{\mu\nu} + \gamma^{ss}_{9,\ell}\nabla^0K_{\mu\nu} + \gamma^{ss}_{10,\ell}K_{\mu\alpha}K^\alpha{}_\nu + \gamma^{ss}_{11,\ell}KK_{\mu\nu} + \gamma^{ss}_{12,\ell}R_{\mu\nu} + \gamma^{ss}_{13,\ell}R_\mu{}^0{}_\nu{}^0 \\ & + \gamma^{\mathrm{P.O.}}_{1,\ell}\epsilon_\mu{}^{\alpha\beta0}\nabla_\alpha K_{\beta\nu} + \gamma^{\mathrm{P.O.}}_{2,\ell}\epsilon_\mu{}^{\alpha\beta0}R_{\alpha\beta}{}^0{}_\nu. \end{aligned}$$

and similarly for $N_{\mu\nu\rho\sigma,\ell} \Rightarrow$ can be studied **systematically**.

Background evolution

Modified Friedmann equations with α_i functions of EFT coefficients:

$$\text{Open: } \begin{cases} 3M_{\rm Pl}^2 H^2 = \alpha_1 + \alpha_2 H \,, \\ 2M_{\rm Pl}^2 \dot{H} = \alpha_3 + \alpha_4 H \,, \end{cases} \quad \text{vs} \quad \text{Closed: } \begin{cases} 3M_{\rm Pl}^2 H^2 = \rho_\phi \,, \\ 2M_{\rm Pl}^2 \dot{H} = -(\rho_\phi + P_\phi) \,, \end{cases}$$

1. *Bulk viscosity* [Weinberg, 1971]: $\alpha_4 = 3\zeta$

$$2M_{\rm Pl}^2 \dot{H} = -\left[\rho_\phi + (P_\phi - 3H\zeta)\right] ,$$

2. *Brane-world gravity* [Dvali, Gabadadze & Porrati, 2000]: $\alpha_2 = \pm 3M_{\rm Pl}^2/r_c$

$$3M_{\rm Pl}^2(H^2 \pm H/r_c) = \rho_\phi ,$$

$\Rightarrow$ Modified continuity equation: $\dot{\alpha}_1 + \left(\dot{\alpha}_2 H + \alpha_2 \dot{H}\right) - 3H\left(\alpha_3 + \alpha_4 H\right) = 0$

3. *Interacting dark sector*: $Q = -(\dot{\alpha}_2 H + \alpha_2 \dot{H}) + 3\alpha_4 H^2$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = Q.$$

Reintroducing the scalar

Restore covariance by *retarded* and *advanced* time diffeomorphism:

$$S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}] \to S_{\text{eff}}[g_{\mu\nu}, a_{\mu\nu}, \pi_r, \pi_a]\,,$$

- **retarded Stückelberg trick**: $t_r \to t_r + \pi_r(x)$:

$$g^{00} \to g^{00} + 2g^{0\mu}\partial_\mu \pi_r + g^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r,$$
$$a^{00} \to a^{00} + 2a^{0\mu}\partial_\mu \pi_r + a^{\mu\nu}\partial_\mu\pi_r\partial_\nu\pi_r.$$

- **advanced Stückelberg trick**: $t_a \to t_a - \pi_a(x)$:

$$g_{\mu\nu} \to g_{\mu\nu},$$
$$a^{\mu\nu} \to a^{\mu\nu} + 2\nabla^{(\mu}\epsilon_a^{\nu)}.$$

Decoupling limit: slow-roll $\epsilon \ll 1$ (**even with dissipation & noise**):

$$\begin{aligned} -a^{00} + (g^{00}/2)g_{\mu\nu}a^{\mu\nu} &\to 2\dot{\pi}_a - 2\partial^\mu\pi_r\partial_\mu\pi_a\,, \\ g_{\mu\nu}a^{\mu\nu} &\to 2\dot{\pi}_a + 6H\pi_a\,, \\ 1 + g^{00} &\to -2\dot{\pi}_r + (\partial_\mu\pi_r)^2, \end{aligned}$$

$\Rightarrow$ *recover Open EFToI & more: primordial GWs*

Primordial gravitational waves

Transverse and traceless (TT) sector: $g_{ij} = a^2(t)\left(\delta_{ij} + h_{ij}\right)$, $a^{ij} = a^{-2} h^a_{ij}$:

$$S^{(2)} = \int \mathrm{d}^4x\sqrt{-g}\,\frac{M_{\mathrm{Pl}}^2}{4c_T^2}\,h^a_{ij}\Big\{\ddot{h}_{ij} - c_T^2\frac{\nabla^2}{a^2}h_{ij} + \left(\Gamma_T + 3H\right)\dot{h}_{ij} + \frac{\chi_T}{a}\epsilon_{imn}\left(\partial_m\dot{h}_{nj} + 2H\partial_m h_{nj}\right) + i\,\frac{\beta_T}{M_{\mathrm{Pl}}^2}h^a_{ij}\Big\}.$$

1. Speed of propagation c_T^2;
2. Dissipation Γ_T;
3. Birefringence χ_T;
4. Noise β_T.

Effects correlated with **background** and **scalar sector** [Lau, Nishii & Noumi, 2024]:

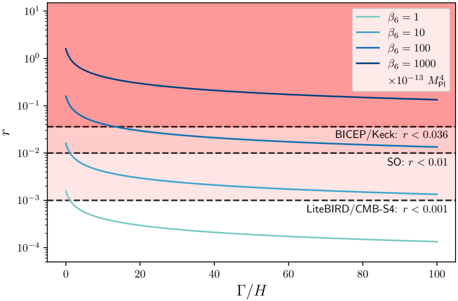
$$S_{\mathrm{eff}} \supset \int \mathrm{d}^4x\sqrt{-g}\,\gamma^{ss}_{8,\ell}K_{\mu\nu}a^{\mu\nu}.$$

- Background with bulk viscosity: $\zeta = 2\gamma^{ss}_{8,\ell}/3$.
- Dissipation in tensor sector: $\Gamma_T = 2\gamma^{ss}_{8,\ell}/M_{\mathrm{Pl}}^2$.

Tensor-to-scalar ratio

Without birefringence ($\chi_T = 0$): define $\nu_\Gamma \equiv 3/2 + \Gamma_T/(2H)$

$$\Delta_h^2 = \frac{\beta_T}{M_{\rm Pl}^4} 2^{2\nu_\Gamma} \frac{\Gamma(\nu_\Gamma - 1)\Gamma(\nu_\Gamma)^2}{\Gamma(\nu_\Gamma - \frac{1}{2})\Gamma(2\nu_\Gamma - \frac{1}{2})} \propto \begin{cases} \frac{\beta_T}{M_{\rm Pl}^4}, & \text{for} \quad \Gamma_T \ll H, \\ \frac{\beta_T}{M_{\rm Pl}^4}\sqrt{\frac{H}{\Gamma_T}}\left[1 + \mathcal{O}\left(\frac{H}{\Gamma_T}\right)\right], & \text{for} \quad \Gamma_T \gg H. \end{cases}$$



⚠ In general, $\beta_T \propto f(\Gamma_T)$:

- Thermal equilibrium: $\beta_T \propto \Gamma_T T$ (FDR);
- What about out-of-equilibrium?

BICEP/Keck bound: $r < 0.036 \quad \Rightarrow \quad \beta_T \lesssim (0.002 M_{\rm Pl})^4$

A glimpse on what to expect

- Dissipative and stochastic Einstein Equations: $G_{\mu\nu} + \Gamma \mathcal{D}_{\mu\nu} = T_{\mu\nu} + \xi_{\mu\nu}$
- Non-conserved stress-energy tensor: $\nabla_\mu T^{\mu\nu} \neq 0$

Phenomenology:

- **Background**: Viscous cosmology & Interacting dark sectors

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = \Gamma \quad \text{and} \quad \dot{\rho}_m + 3H(\rho_m + p_m) = -\Gamma$$

- **Clustering** $\Rightarrow$ redshift space distortion (RSD) and weak lensing (WL)

$$k^2\langle\psi\rangle = -4\pi G\mu(a,k)a^2\rho_m\langle\delta\rangle, \qquad k^2\frac{\langle\psi+\phi\rangle}{2} = -4\pi G\Sigma(a,k)a^2\rho_m\langle\delta\rangle$$

- **Gravitational waves** $\Rightarrow$ GW production, propagation and dissipation

$$\ddot{h}_{ij} + \Gamma\dot{h}_{ij} + c_T^2 h_{ij} + \chi\epsilon_{ilm}k_m h_{jl} = T_{ij} + \xi_{ij}$$

Rich phenomenology to explore,
eventually ***already constrained from data****.*

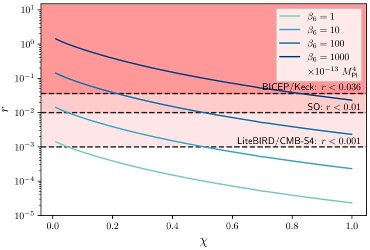
Including birefringence

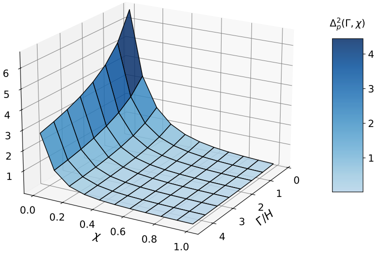
$$\left[\partial_\eta^2 - \left(\frac{2+\frac{\Gamma_T}{H}}{\eta} \mp \chi_T k\right)\partial_\eta + k^2\right] G^R_{\times,+}(k;\eta,\eta') = H^2\eta^2\delta(\eta-\eta').$$

Analytic expression in terms of Appell F_2 function: $\kappa_\pm \equiv \mp \frac{i\chi_T}{\sqrt{1-\chi_T^2}}(\nu_T - 1/2)$

$$\Delta^2_{\times,+} = \frac{\beta_T/M_{\mathrm{Pl}}^4}{2\pi^2\nu_\Gamma^2\chi_T^3} F_2\left(3; \tfrac{1}{2}+\nu_\Gamma-\kappa_\pm, \tfrac{1}{2}+\nu_\Gamma+\kappa_\pm; 2\nu_\Gamma+1, 2\nu_\Gamma+1; \mp\sqrt{1-\frac{1}{\chi_T^2}}, \pm\sqrt{1-\frac{1}{\chi_T^2}}\right)$$

Recover **instability band** at low dissipation:





Late-universe: clockless example

Subset that preserves retarded time-diff: **no dynamical scalar**

$$S_{\rm eff} = \int {\rm d}^4 x \sqrt{-g}\, (f_1 R_{\mu\nu} + f_2 g_{\mu\nu} + f_3 T_{\mu\nu} + {\rm h.d.})\, a^{\mu\nu}.$$

$\Rightarrow$ *Very few ingredients!* To lowest order, e.o.m. takes the form:

$$G_{\mu\nu} + \tilde{f}_1 R g_{\mu\nu} = \tilde{f}_2 T_{\mu\nu} + \tilde{f}_3 T g_{\mu\nu}.$$

w.l.o.g., separating **Standard Model**, **dark matter** and **cosmological constant**:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\rm N}\left[T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu} + (\lambda^{\rm SM} T^{\rm SM} + \lambda^{\rm DM} T^{\rm DM}) g_{\mu\nu}\right].$$

If dark matter $\simeq$ perfect fluid, absorb $\lambda^{\rm DM} T^{\rm DM} g_{\mu\nu}$ into equation of state $w^{\rm DM}$:

$\simeq$ *Gravitational Aether* [Afshordi, 2008] | *Massive Gravity* [Isham, Salam & Strathdee, 1971]

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = G_{\rm N}\left[T^{\rm SM}_{\mu\nu} + T^{\rm DM}_{\mu\nu} + \lambda^{\rm SM} T^{\rm SM} g_{\mu\nu}\right].$$

Blueprint for cosmological analysis [with F. McCarthy, ACT & SO]

- Does not come from an action ⇒ **evade [Lovelock, 1971] theorem**;
- Dissipative dark matter:

$$\nabla^\mu T^{\rm SM}_{\mu\nu} = 0\,, \qquad \nabla^\mu T^{\rm DM}_{\mu\nu} + \lambda^{\rm SM}\partial_\nu T^{\rm SM} = 0\,.$$

- To **UV complete**, need to find sector s.t. $\langle T^{\rm env}_{\mu\nu}\rangle = \lambda^{\mathsf{SM}} T^{\mathsf{SM}} g_{\mu\nu}$;
- One-parameter extension of Λ-CDM ⇒ data-analysis CMB + BAO.

Background ⇒ Hubble diagram function of redshift z:

$$\frac{H(z)}{H_0} = \sqrt{\Omega_\Lambda + \Omega_{\rm DM}(1+z)^{3(1+w_{\rm DM})} + \Omega_{\rm SM}(1+z)^{3(1+w_{\rm SM})}\left[1+\lambda(1-3w_{\rm SM})\right]}\,.$$

Clustering ⇒ redshift space distortion (RSD) and weak lensing (WL):

$$k^2\langle\psi\rangle = -4\pi G\mu(a,k)a^2\rho_m\langle\delta\rangle, \qquad k^2\frac{\langle\psi+\phi\rangle}{2} = -4\pi G\Sigma(a,k)a^2\rho_m\langle\delta\rangle$$