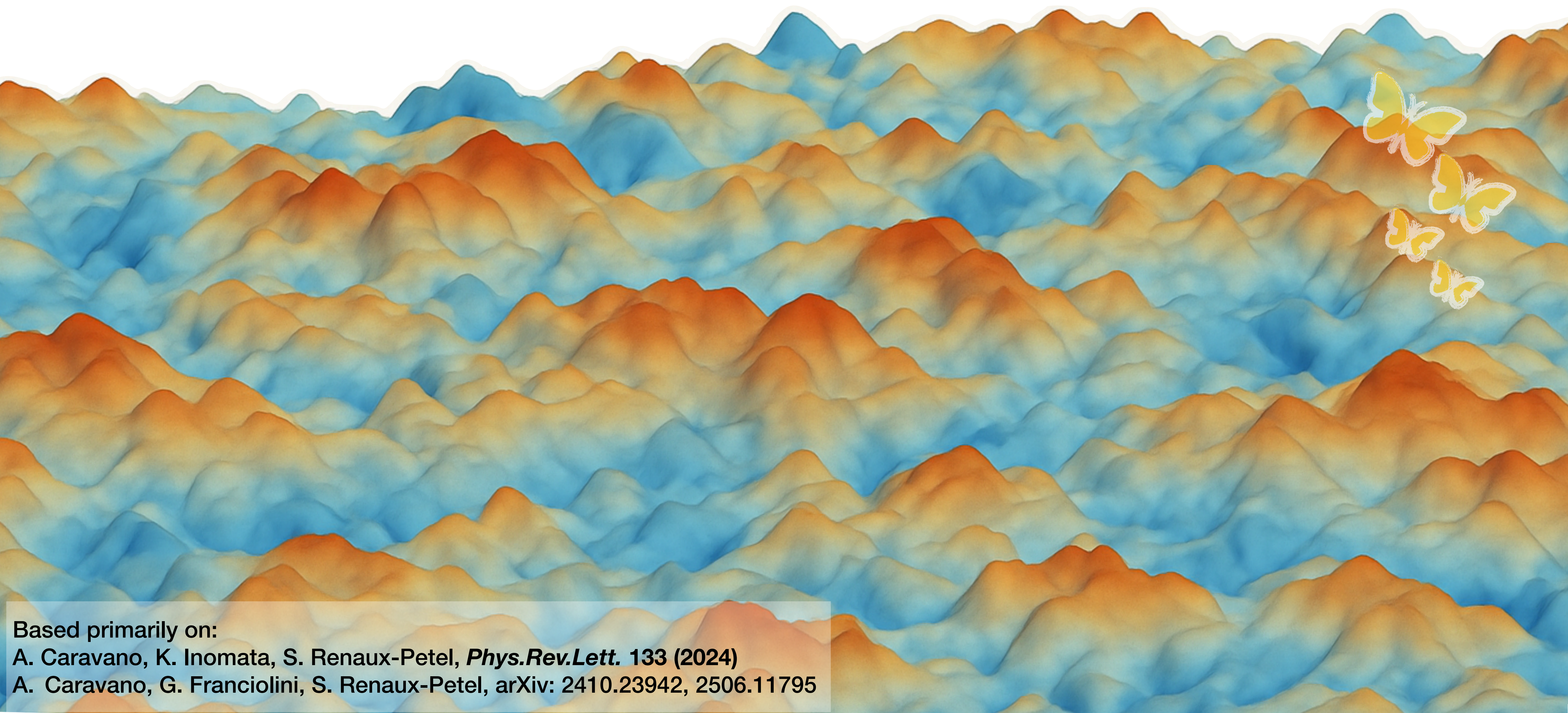
A Tale of Cosmic Butterflies

non-perturbative journey into the physics of inflation

Angelo Caravano (He/Him) - University of Amsterdam



Based primarily on:

A. Caravano, K. Inomata, S. Renaux-Petel, ***Phys.Rev.Lett.* 133 (2024)**

A. Caravano, G. Franciolini, S. Renaux-Petel, arXiv: 2410.23942, 2506.11795

Roadmap

0) Motivation: why simulating inflation?

AC, Komatsu, Lozanov, Weller — 2102.06378, 2110.10695, 2204.12874

AC — 2209.13616, 2506.11797

Jamieson, **AC**, Komatsu — 2507.22285

1) Lattice simulations of inflation

2) Example: **inflationary butterfly effect**

AC, K. Inomata, S. Renaux-Petel — 2403.12811

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3) Some ongoing work

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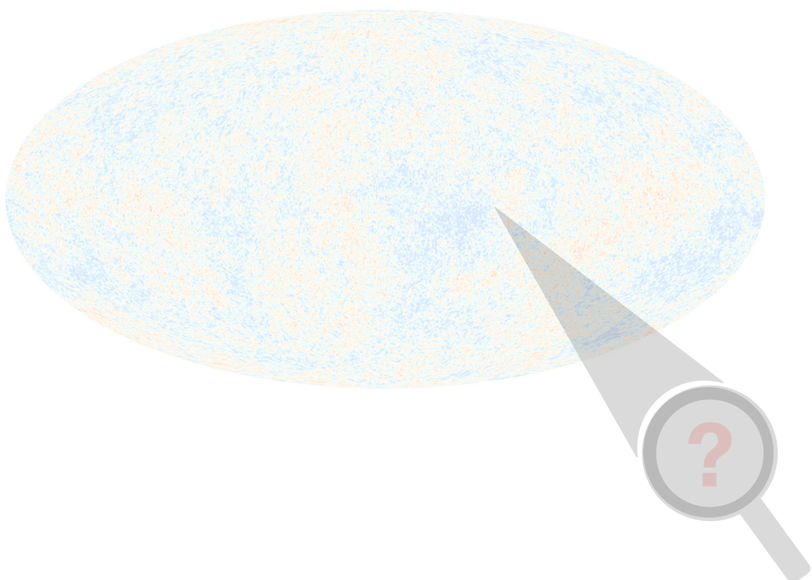
Simulations are useful when dealing with **nonlinear** and/or **non-perturbative** physics.

Examples: non-Gaussianity, amplified perturbations, particle production.

Today, focus on the following question:

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?
Can we observe it?



Yes! But we need a large amplification:

$$P_{\zeta,\mathrm{CMB}} \sim 10^{-9} \longrightarrow \mathcal{P}_\zeta \sim 10^{-2} - 10^{-4}$$

$$\zeta \sim 10^{-5} \qquad \zeta \sim 10^{-1} - 10^{-2}$$

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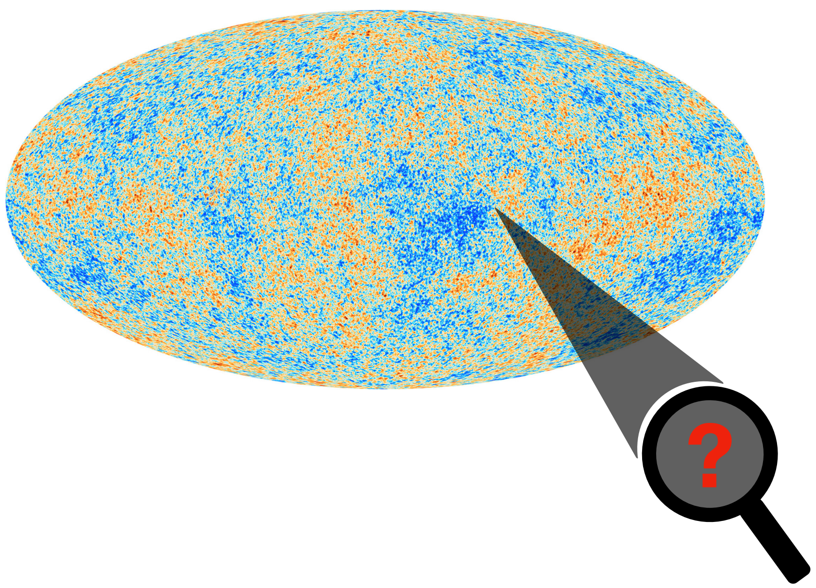

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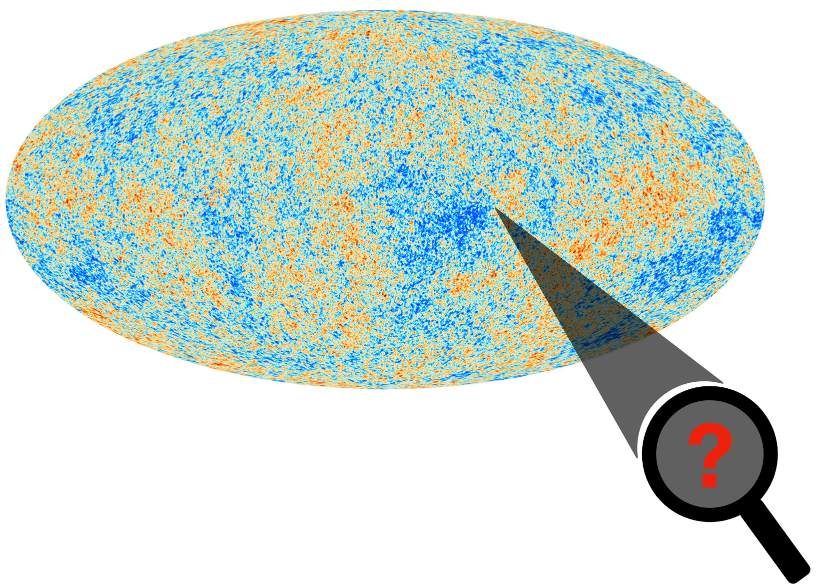

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Lattice simulations of inflation

[**AC** 2506.11797]

- Lattice simulations: known tool to study non-perturbative cosmological phenomena.

  Examples: reheating, cosmological phase transitions

My goal: Develop lattice techniques for inflation

This journey started **here at IAP in 2018!**

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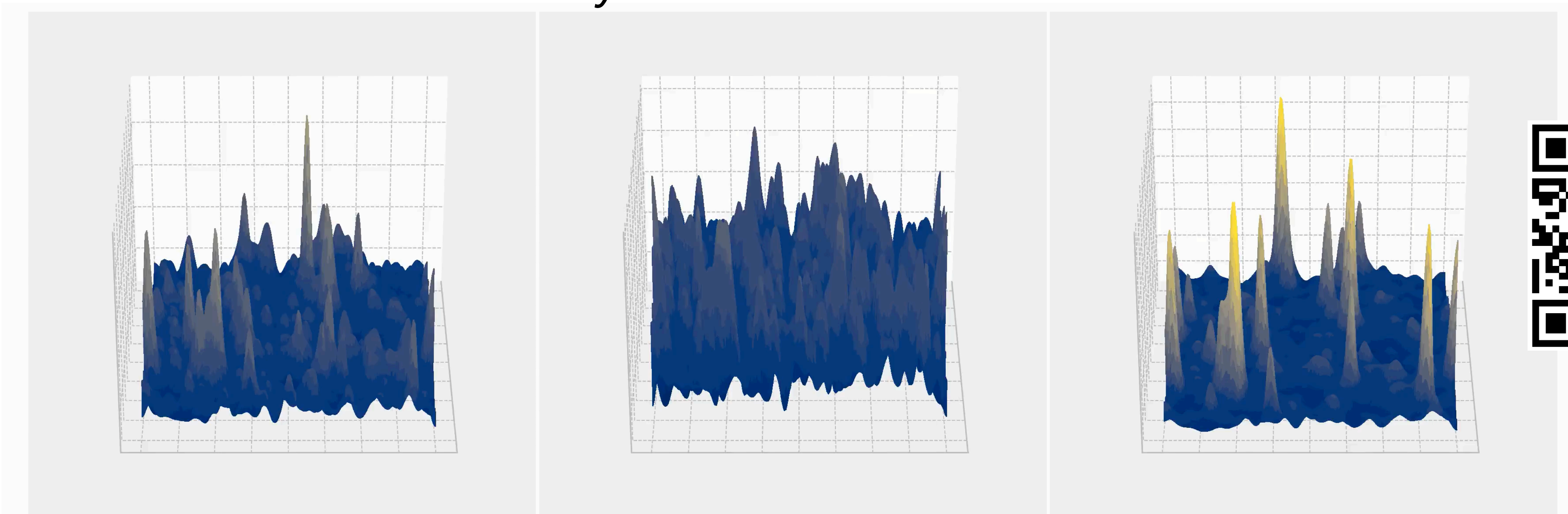
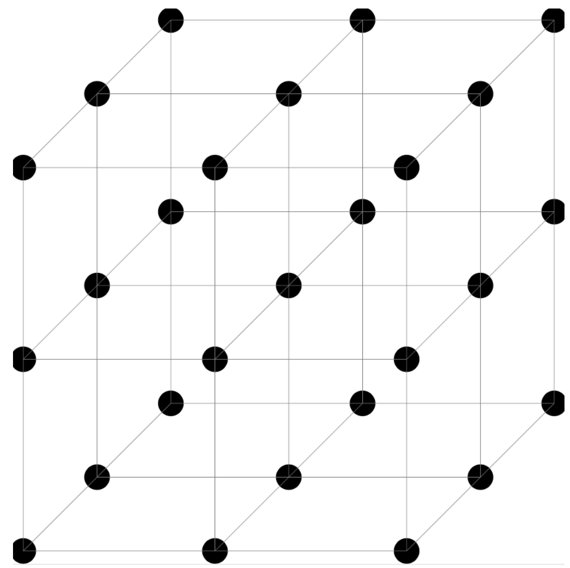
Public code: *InflationEasy:* A C++ Lattice Code for Inflation



Lattice simulations

Put the continuous inflationary universe on a discrete cubic lattice:

$\phi(\vec{x}, t)$ →



$\phi_i(t)$

$$\phi(\vec{x}, t) = \bar{\phi}(t) + \delta\phi(\vec{x}, t)$$

& perturbation theory on $\delta\phi$

→

Nonlinear, non-perturbative evolution of ϕ_i

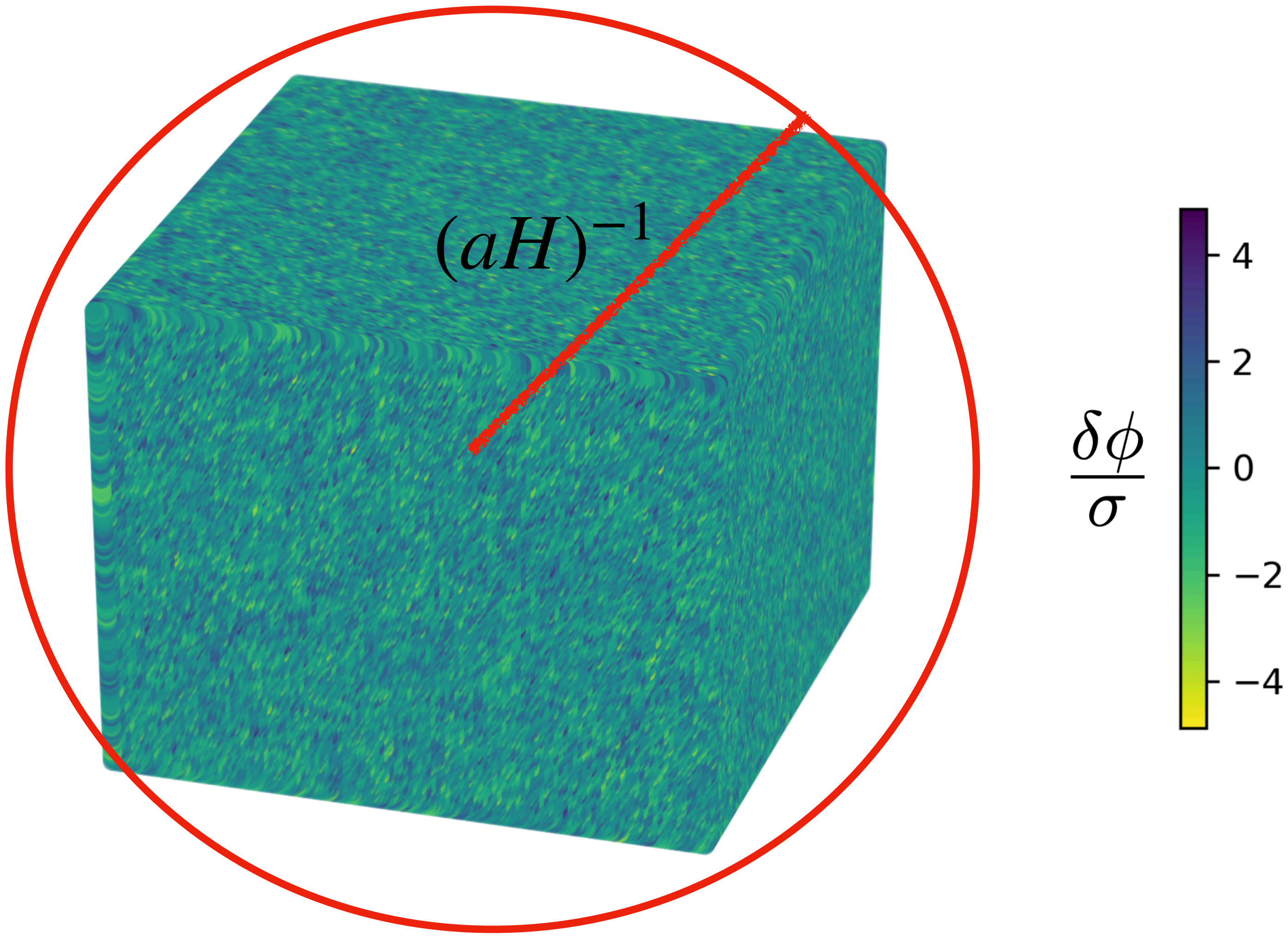
Numerically solve the classical eqs:

$$\frac{\partial \mathscr{L}}{\partial \phi_i} = \frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\phi}_i}\right)$$

Lattice simulations of inflation

[AC 2506.11797]

Start with quantum fluctuations on sub-horizon box:



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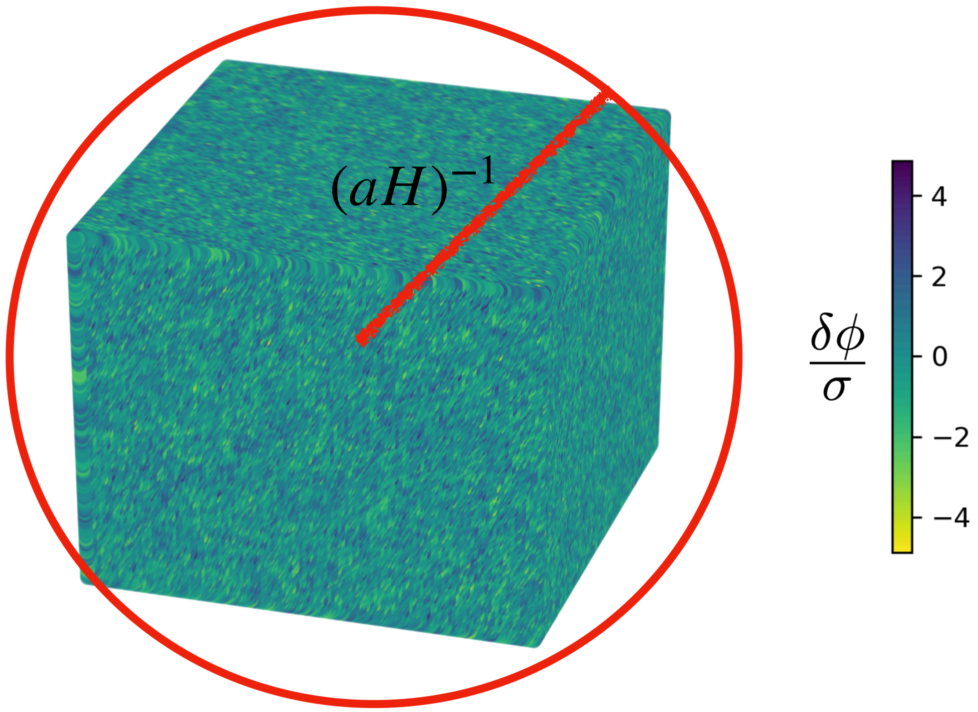


$$\hat{\phi}(\vec{n}) = \sum_{\vec{m}} \left[\hat{a}_{\vec{m}}\, u(\vec{\kappa}_{\vec{m}})\, e^{i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} + \hat{a}^{\dagger}_{\vec{m}}\, u^{\dagger}(\vec{\kappa}_{\vec{m}})\, e^{-i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} \right]$$

$$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau}$$

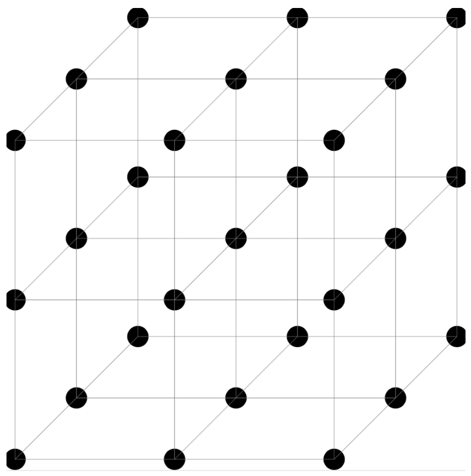
"Discrete Bunch Davies" [AC+ 2102.06378]

$$\hat{a}_{\vec{m}} = e^{i2\pi \hat{Y}_{\vec{m}}} \sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

$\hat{X}_{\vec{m}}, \hat{Y}_{\vec{m}}$ uniform randoms between 0 and 1: **"stochastic" approximation of quantum noise**

Lattice approach: evolution

[AC 2506.11797]



Solve numerically for all lattice points:

$$\phi''(\vec{n}) + 2H\phi'(\vec{n}) - \nabla^2\phi(\vec{n}) + a^2\frac{\partial V}{\partial \phi}(\vec{n}) = 0$$

+ Friedmann equation for scale factor $\frac{d^2a}{d\tau^2} = \frac{1}{6}\left(\langle\rho\rangle - 3\langle p\rangle\right)a^3$

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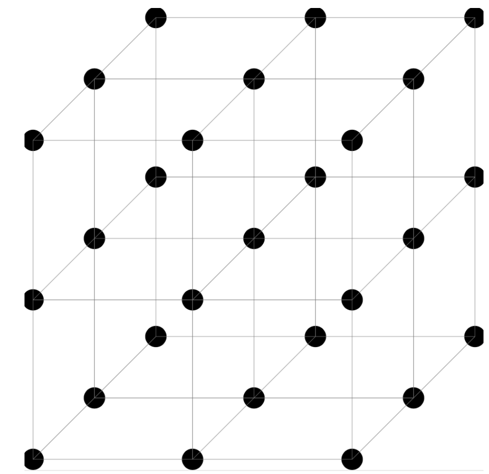

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Assuming **unperturbed metric** $ds^2 = a^2(-d\tau^2 + d\vec{x}^2)$ because:

- $\delta g_{ij} \equiv 0$ (spatially flat gauge)
- $\delta g_{0\mu} \propto \epsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{M_{\rm Pl}^2 H^2} \to 0$, known as "decoupling limit" of gravity $M_{\rm Pl} \to \infty$

Cheung++ [0709.0293]
Behbahani++[1111.3373]
Creminelli++[2401.10212]
…

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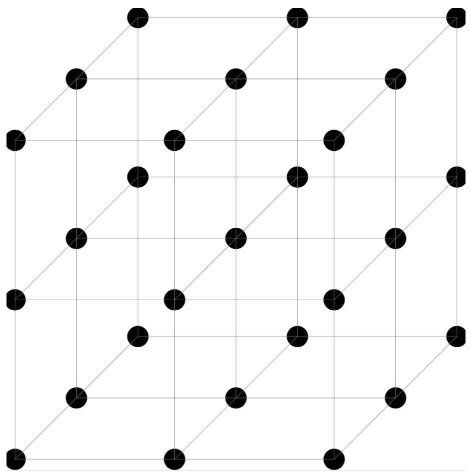

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Cheung++ [0709.0293]
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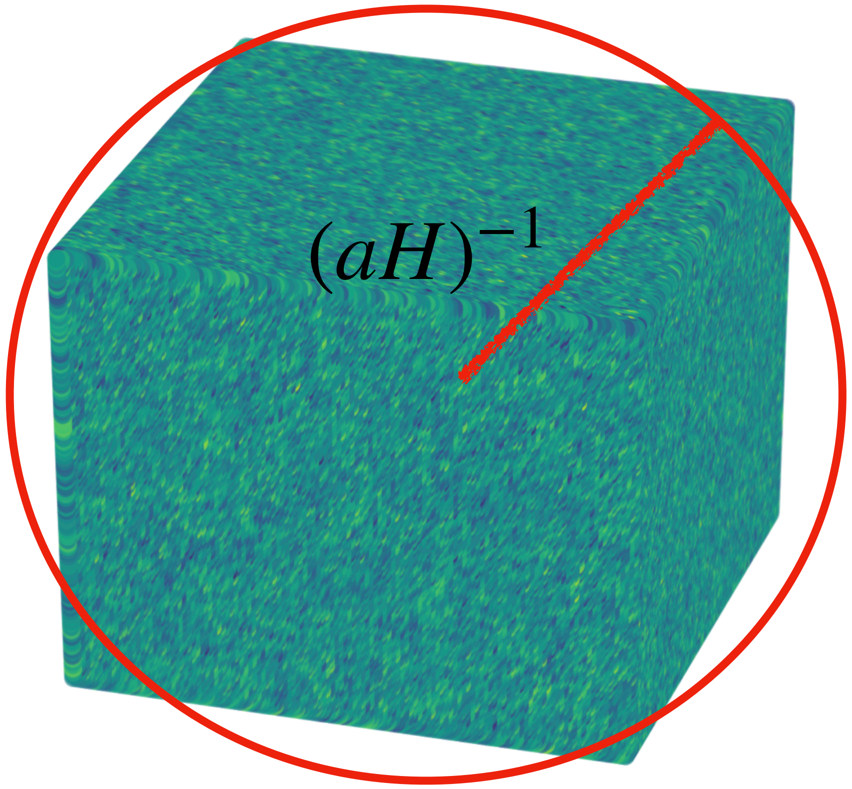
Creminelli++[2401.10212]
…

<u>Not a fundamental limitation:</u>

Gravity can be perturbatively included **AC**, Peloso [2407.13405]
Jamieson, **AC**, Komatsu [2507.22285]

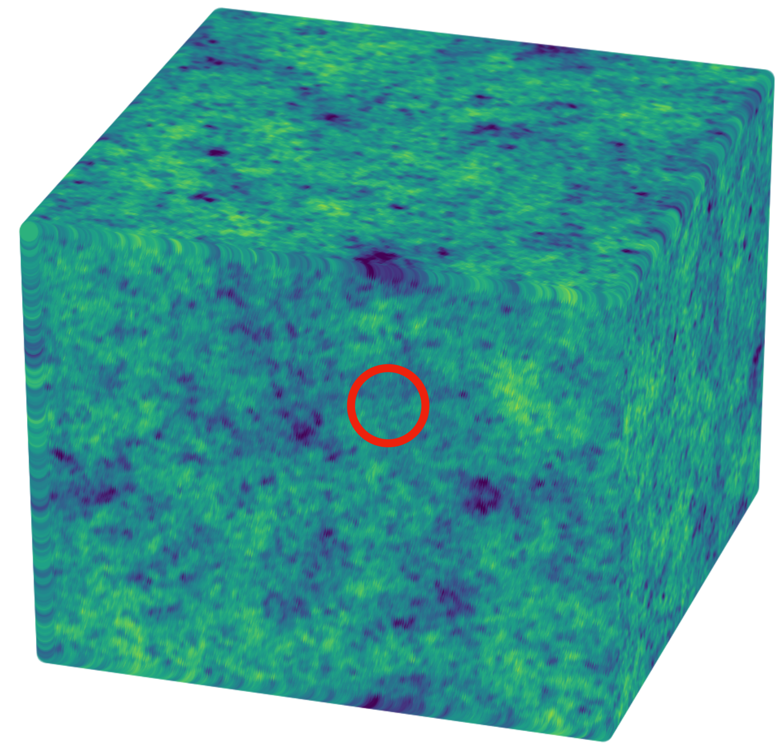
Lattice simulations of Inflation

[AC 2506.11797]



“sub-horizon” box

Nonlinear evolution →



“super-horizon” box
(frozen)

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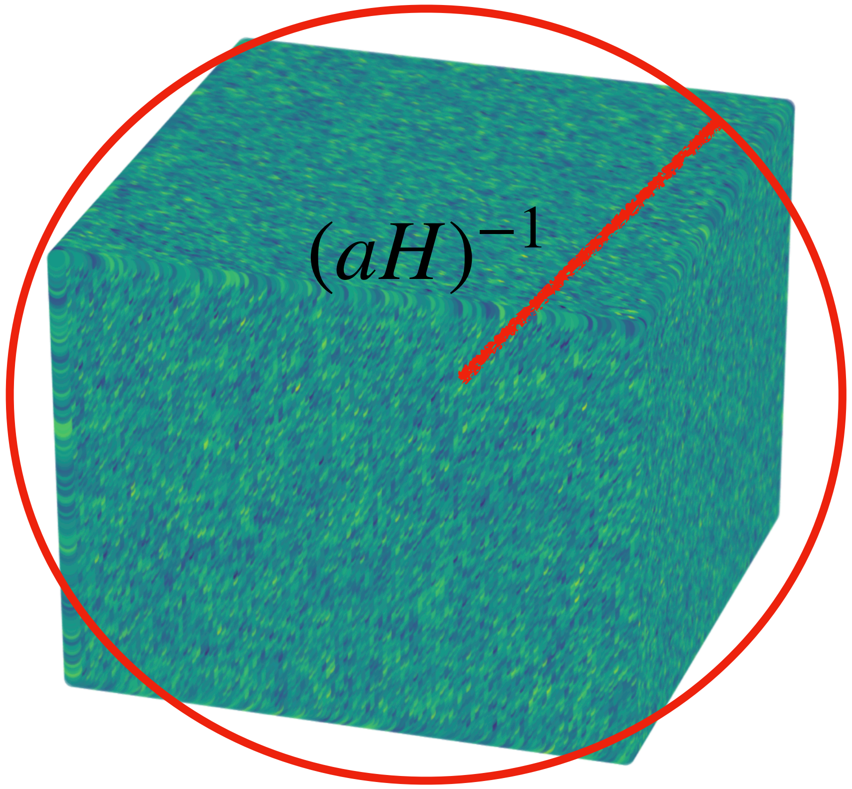

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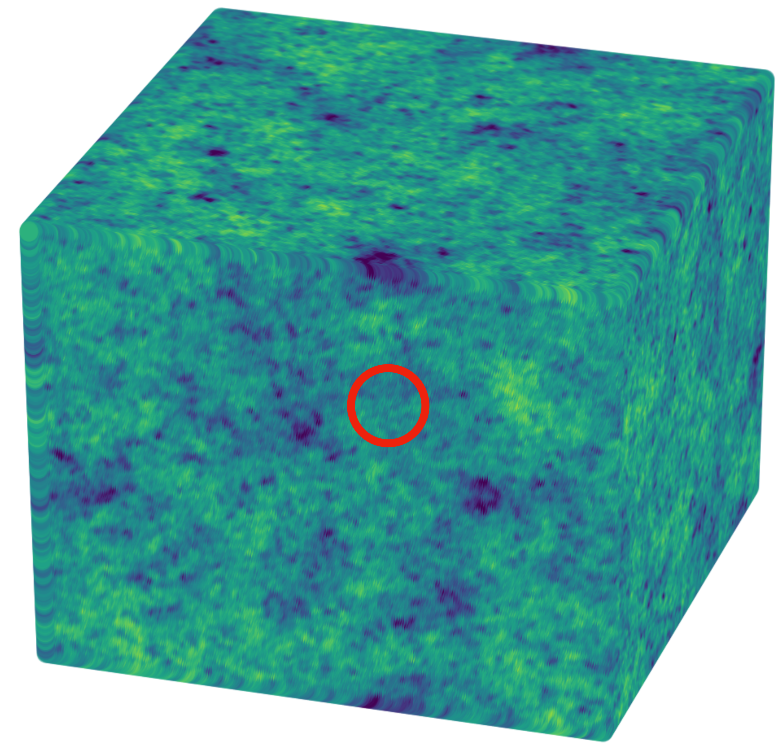

"super-horizon" box
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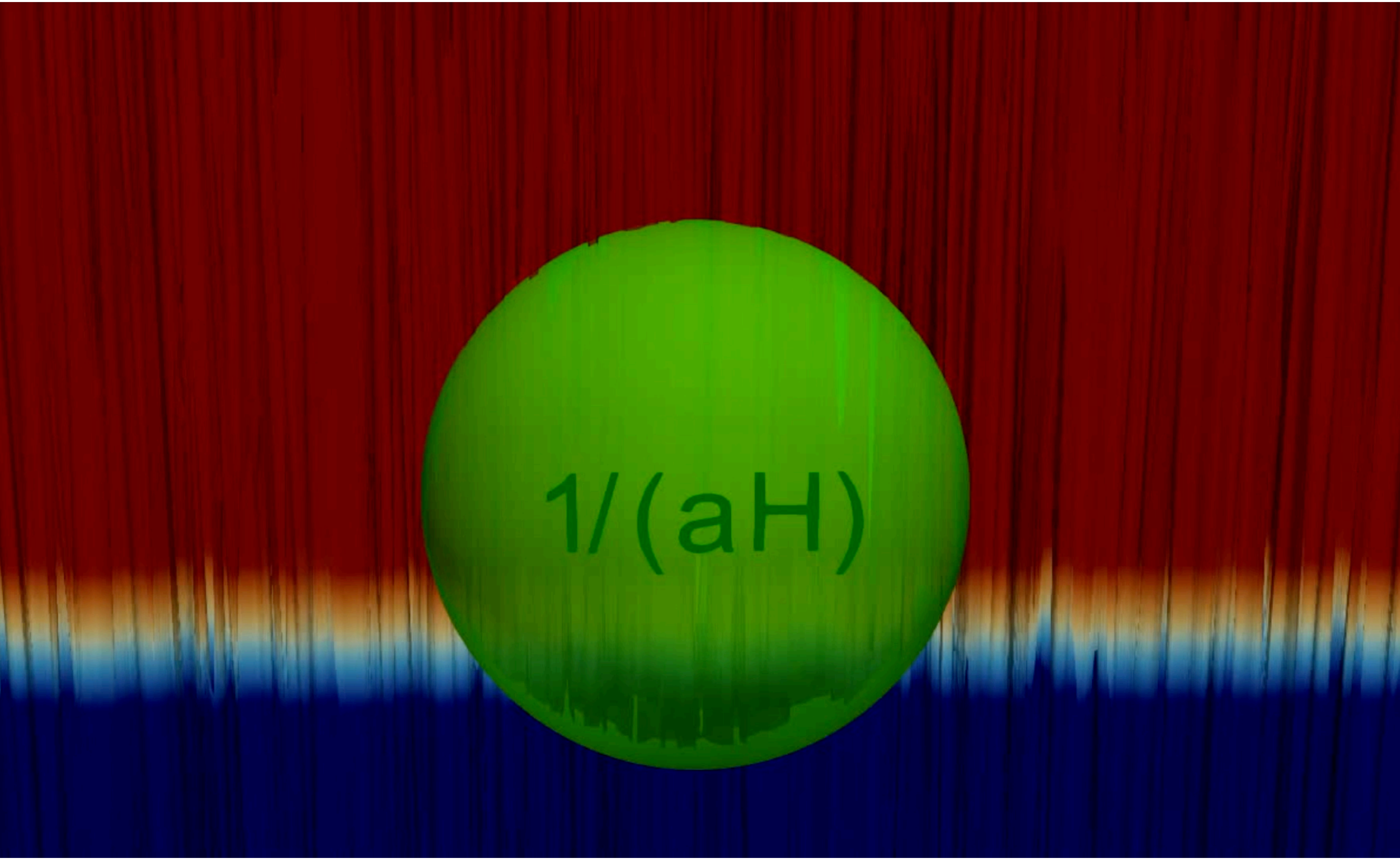
- Key point: non-perturbative $\phi(\vec{x},t) \neq \bar{\phi}(t) + \delta\phi(\vec{x},t)$
- Assumptions: 1) Neglect gravitational interaction fixed metric $ds^2 = a(\tau)\left(-d\tau^2 + d\vec{x}^2\right)$

  2) Semi-classical approach (neglect quantum tunneling, interference, etc…)


1/(aH)

Quick detour: non-Gaussianity

Interactions → $\zeta = \zeta_G + \zeta_{NL}$ Unknown

To deal with this, we typically write: $\zeta_{NL} \simeq f_{NL} K[\zeta_G, \zeta_G]$

example, local NGs: $\zeta_{NL} = f_{\rm NL} \zeta_G^2$

Quick detour: non-Gaussianity

AC, Komatsu, Lozanov, Weller [2204.12874]
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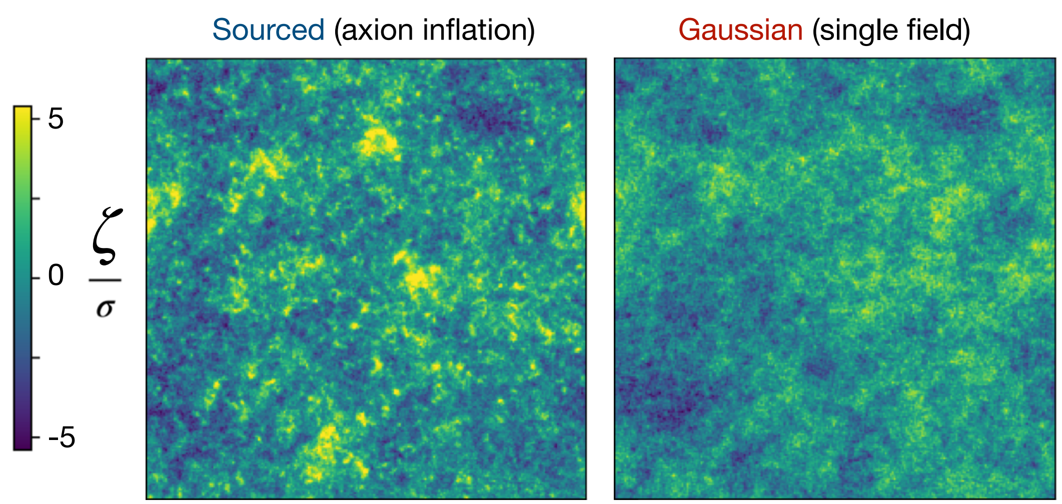
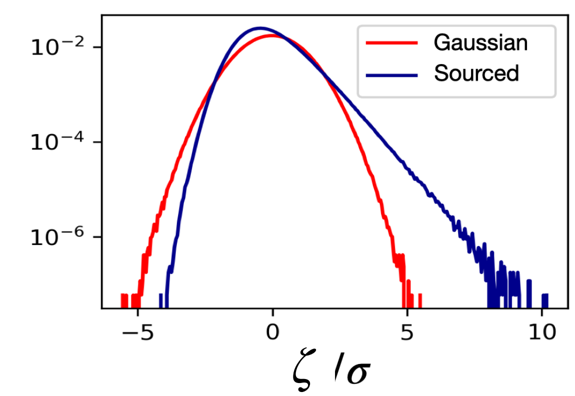
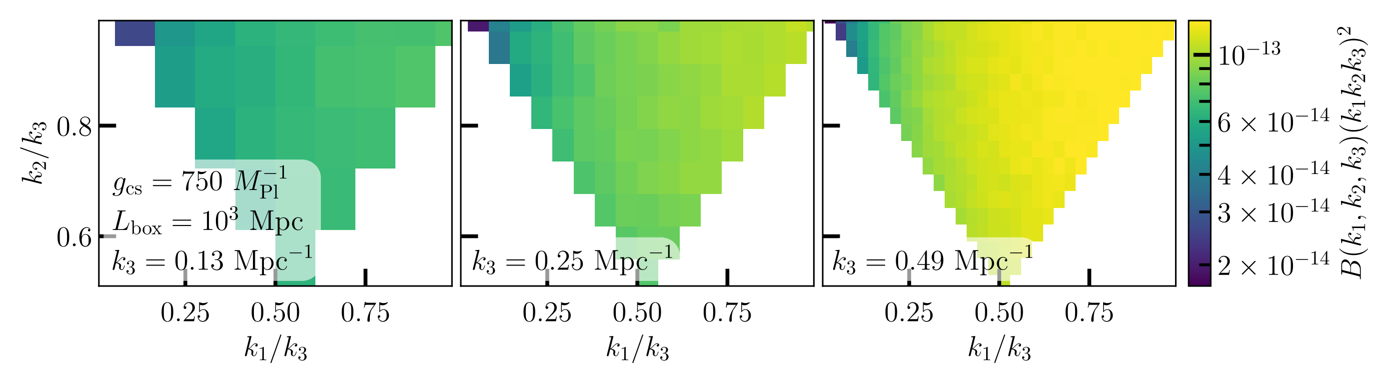
Thanks to the simulation, we finally know the nonlinear ζ





$\sigma = \sqrt{\langle \zeta^2 \rangle}$



We learned that
$\zeta \neq \zeta_G + f_{NL} K[\zeta_G, \zeta_G]$

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- 2.1) Oscillatory potential
- 2.2) Ultra-slow-roll inflation

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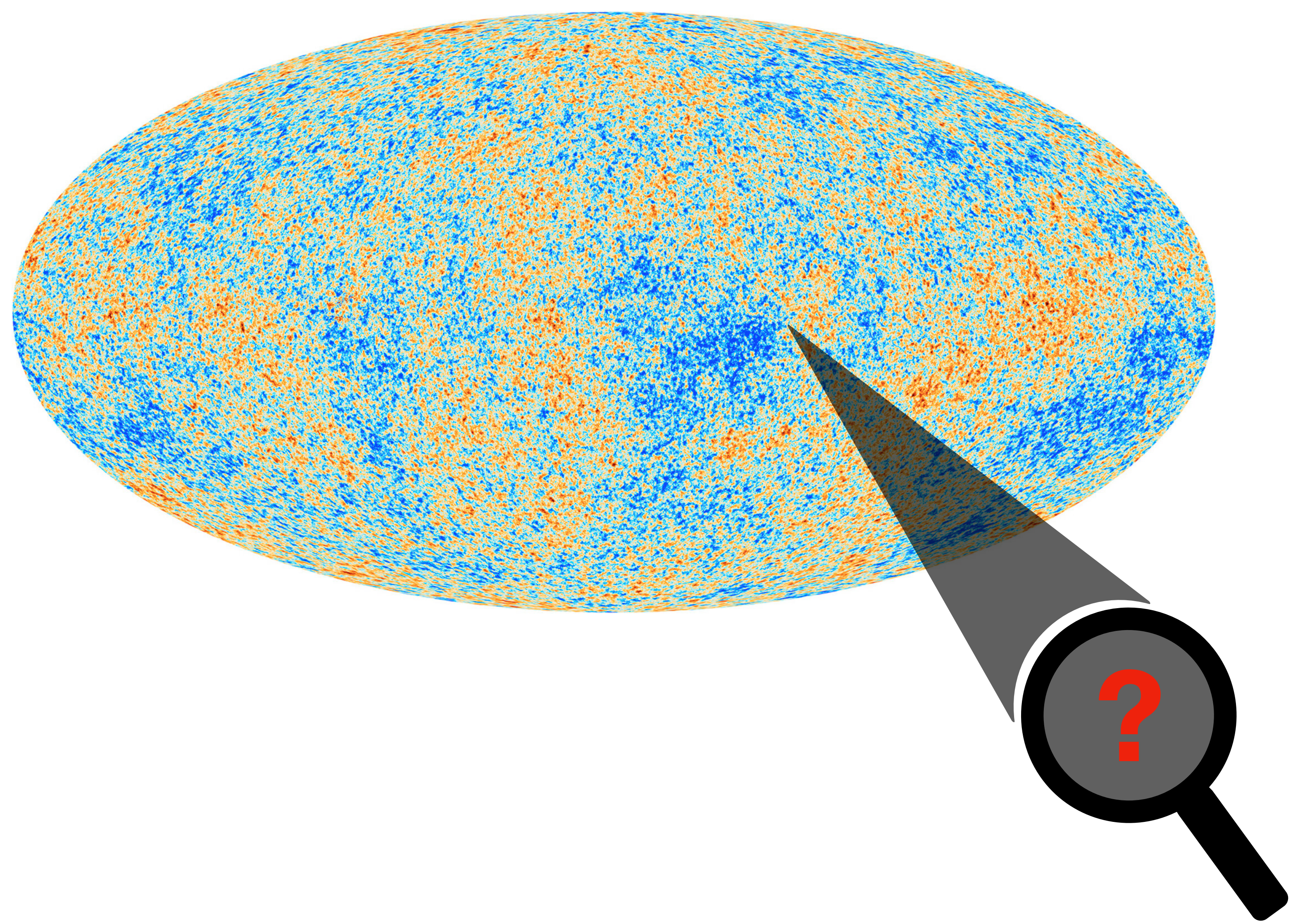
Inflation at small scales

What is the physics of inflation at scales $\lambda \ll \lambda_{CMB}$?



Inflation generates fluctuations at scales $\sim e^{40}$ smaller than CMB scales

Inflation on small scales

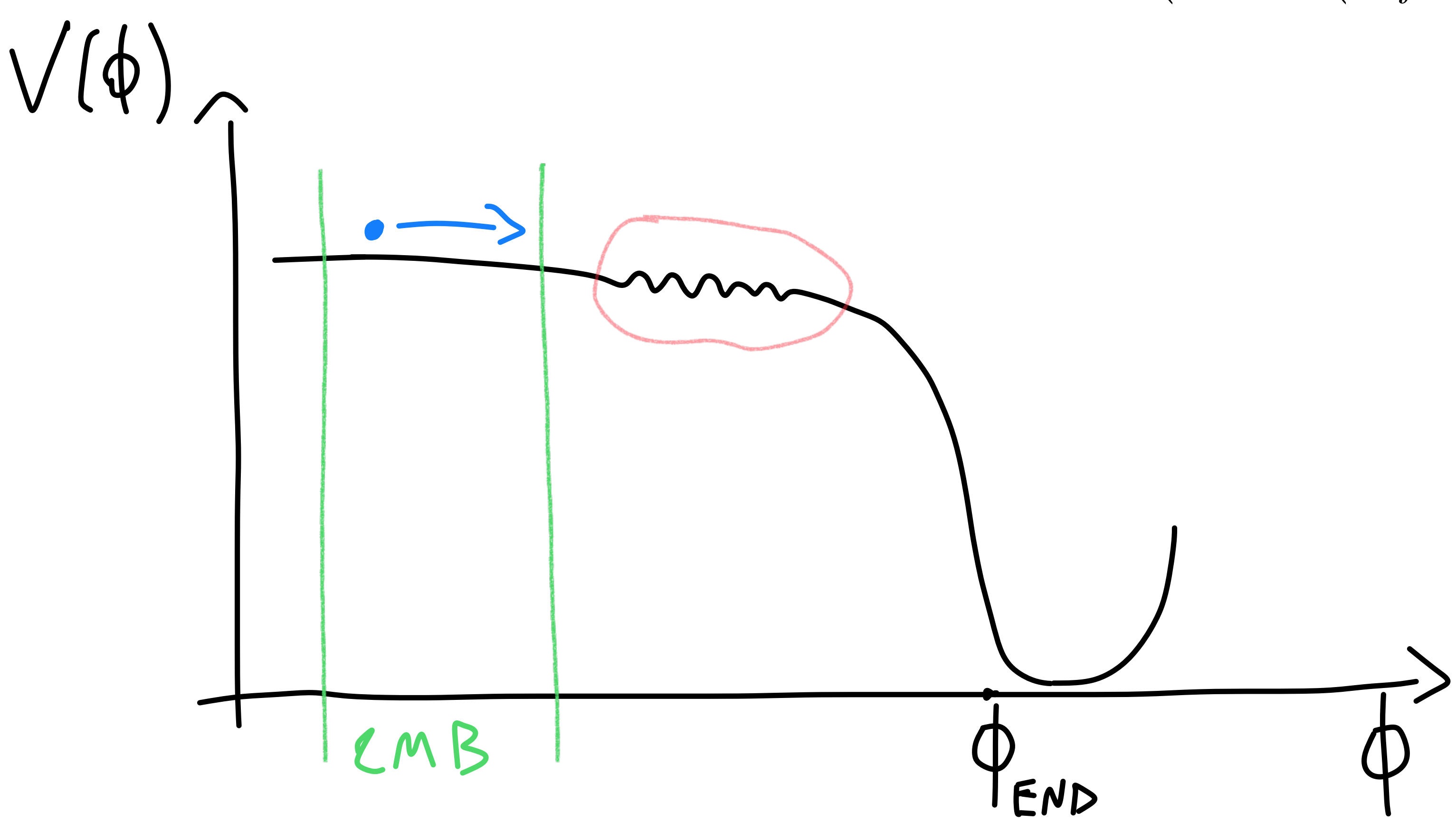
Toy model: a **small-scale modification** of the inflaton potential

$$V(\phi) = V_{sr}(\phi) + \Lambda^4 W(\phi)\left[\cos\left(\frac{\phi - \phi_0}{f}\right) - 1\right]$$

Slow-roll potential

Localised oscillation

$$W(\phi) = \frac{1}{4}\left(1 + \tanh\left(\frac{\phi - \phi_0}{f}\right)\right)\left(1 + \tanh\left(\frac{\phi_0 - \phi + \Delta\phi}{f}\right)\right)$$



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oscillation → parametric resonance → exponential growth of perturbations → observable!

Oscillatory potential

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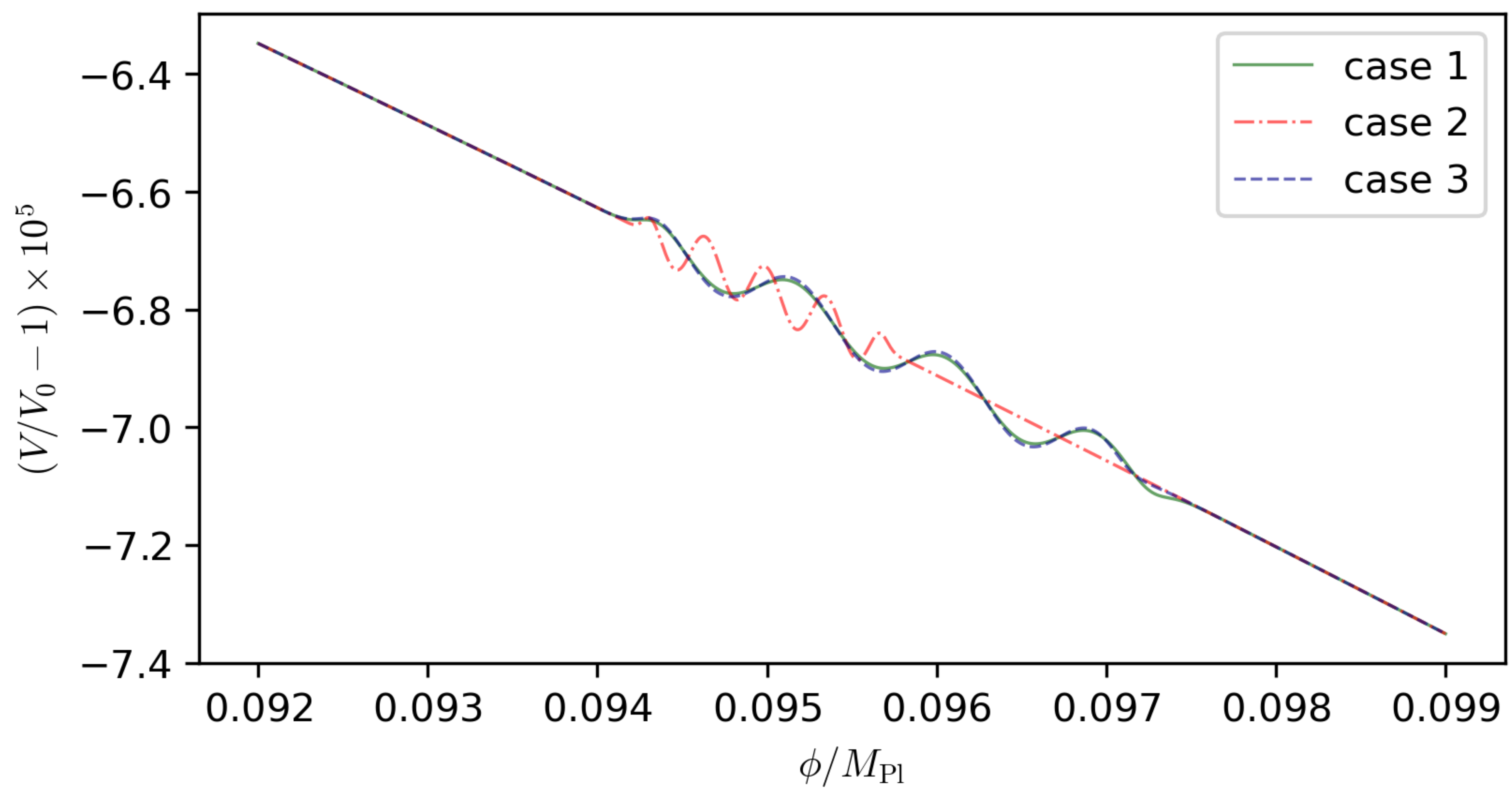
Let's consider the following three cases:



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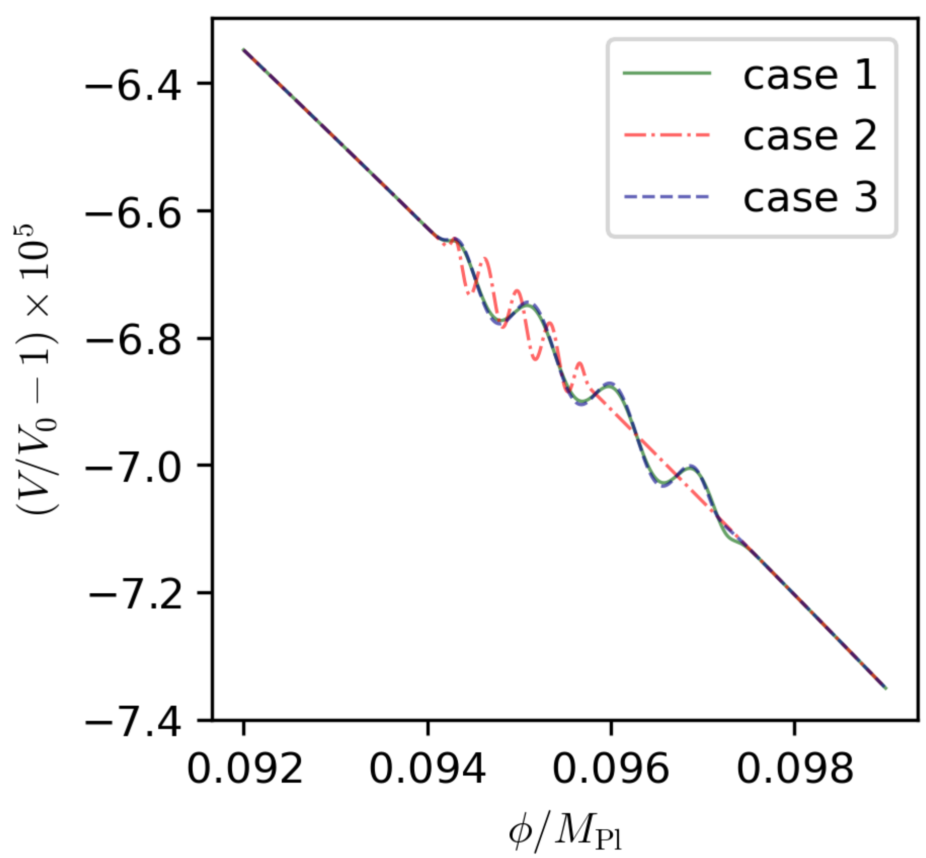

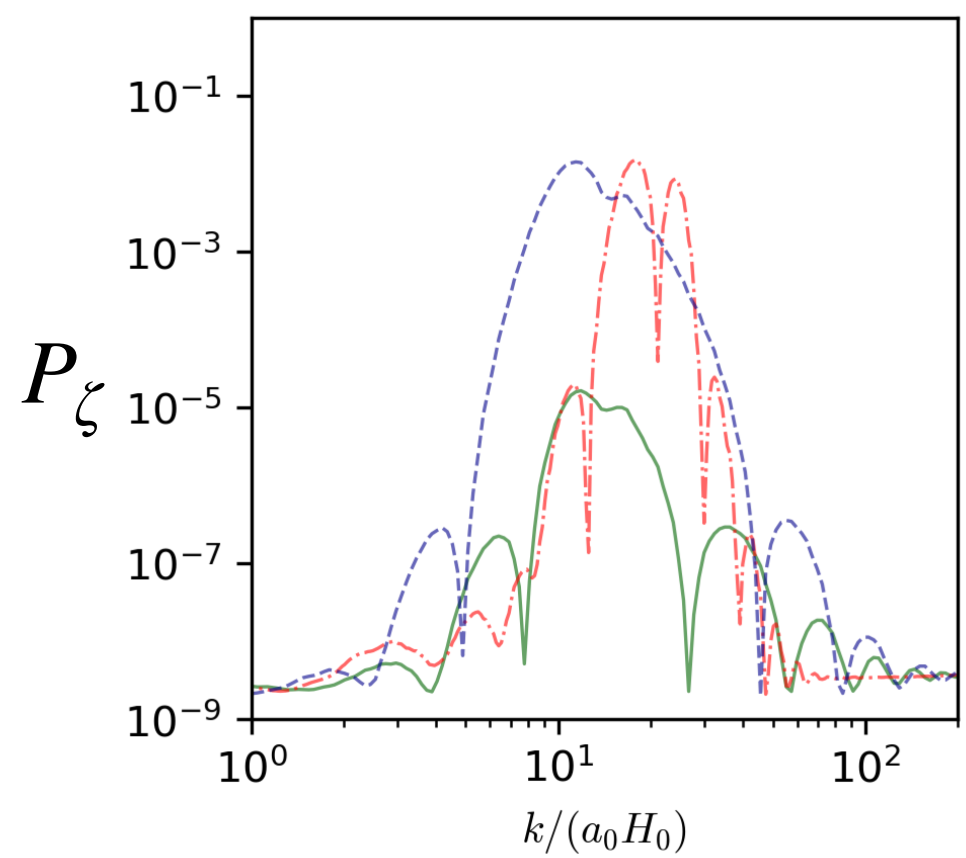

Case 1: $P_\zeta \simeq 10^{-5}$

Case 2: $P_\zeta \simeq 10^{-2}$

Case 3: $P_\zeta \simeq 10^{-2}$

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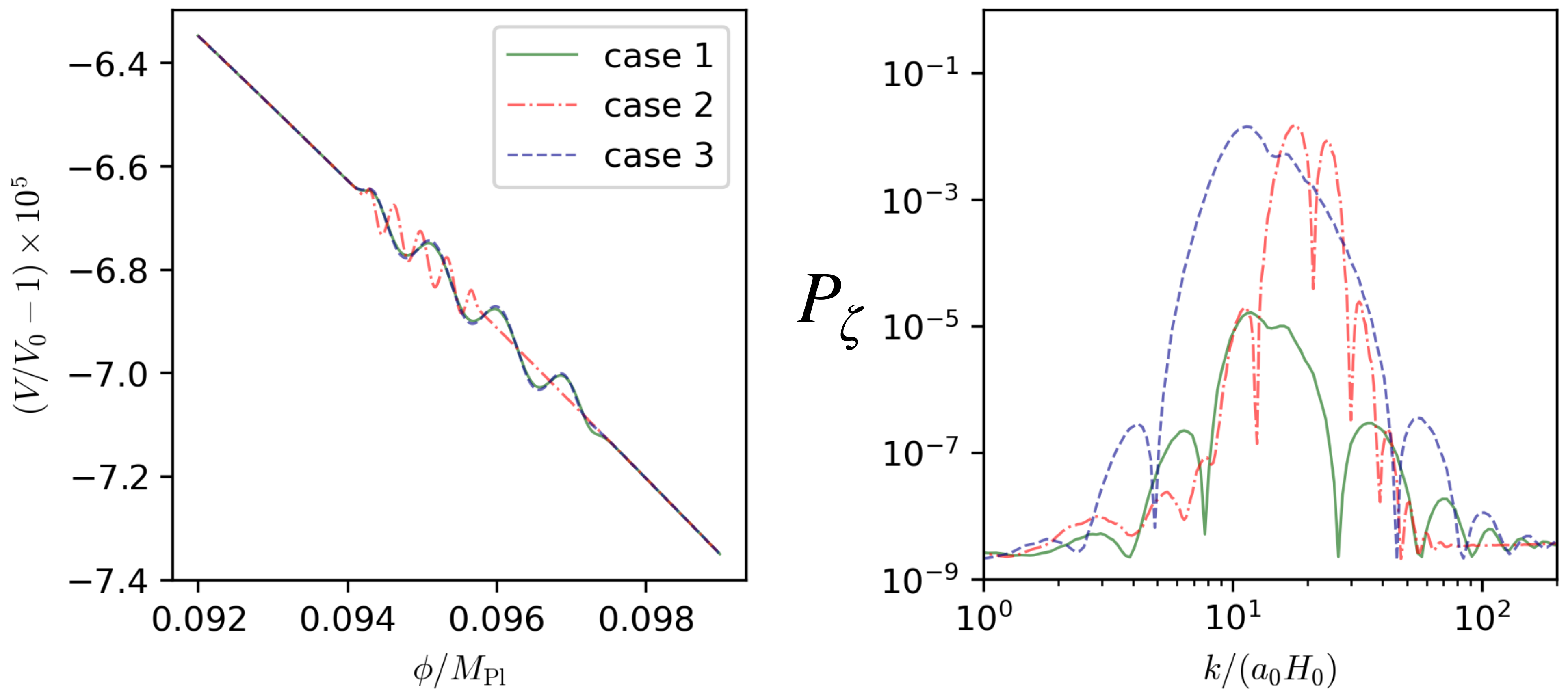

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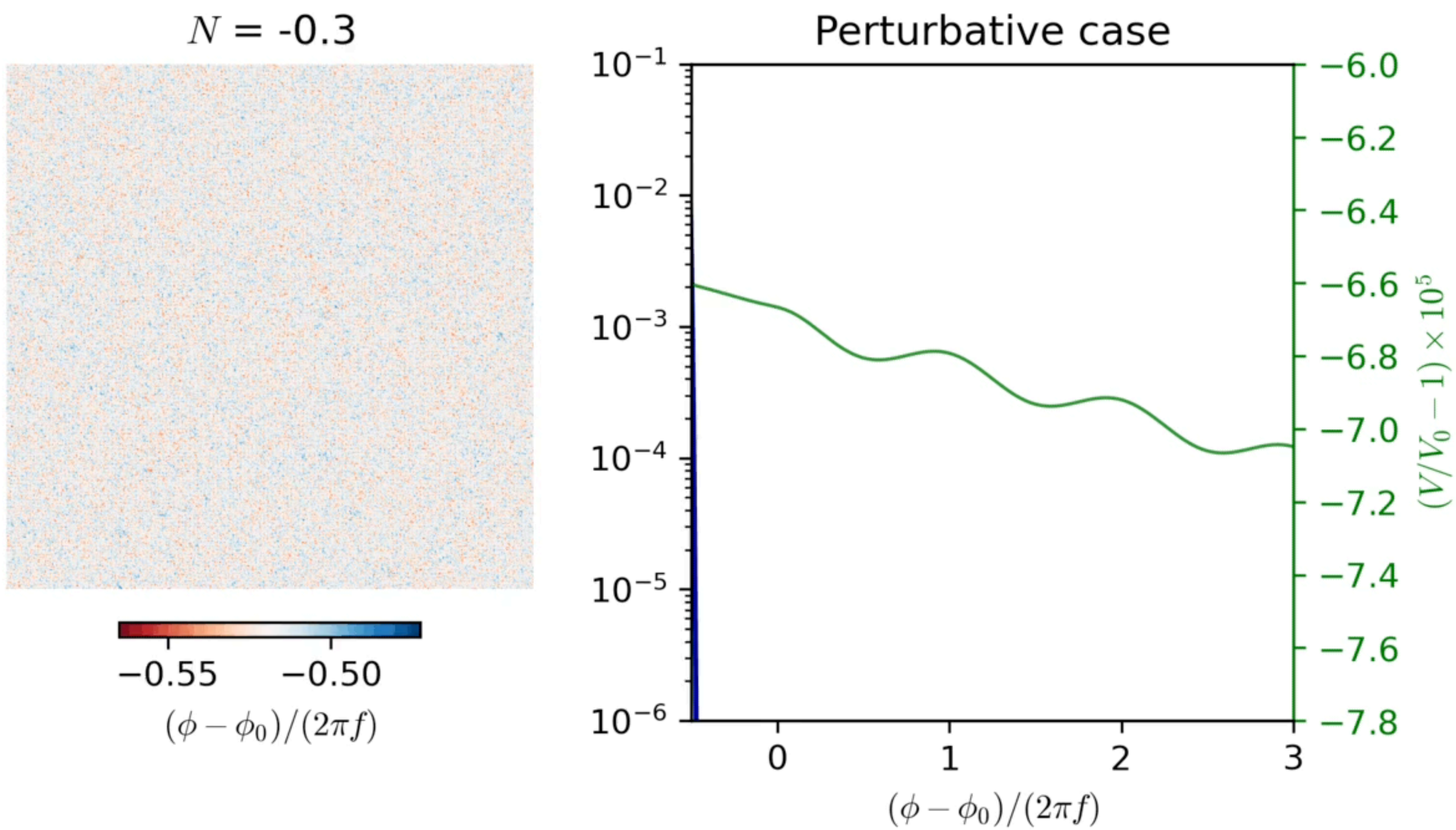
[K. Inomata, M. Braglia, X. Chen, S. Renaux-Petel 2211.02586]

$$P_{\zeta,1-\rm loop} \gtrsim P_{\zeta,\rm tree}$$

In case 3 and 2, but not 1

Case 1. $(P_\zeta \sim 10^{-5})$

Case 1 is perturbative



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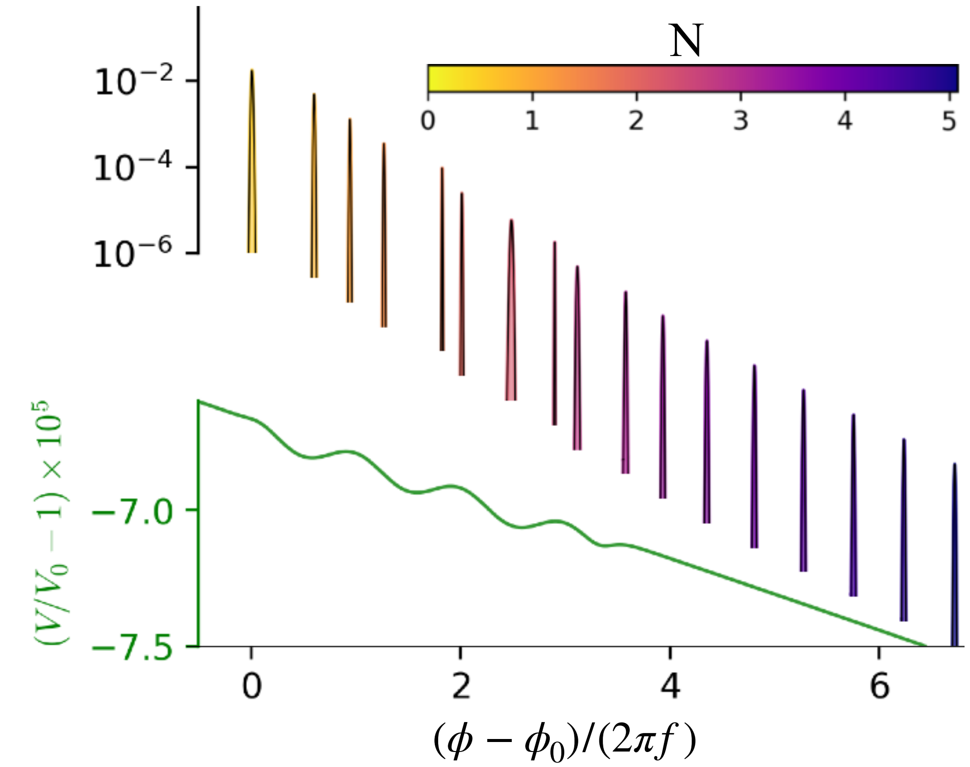
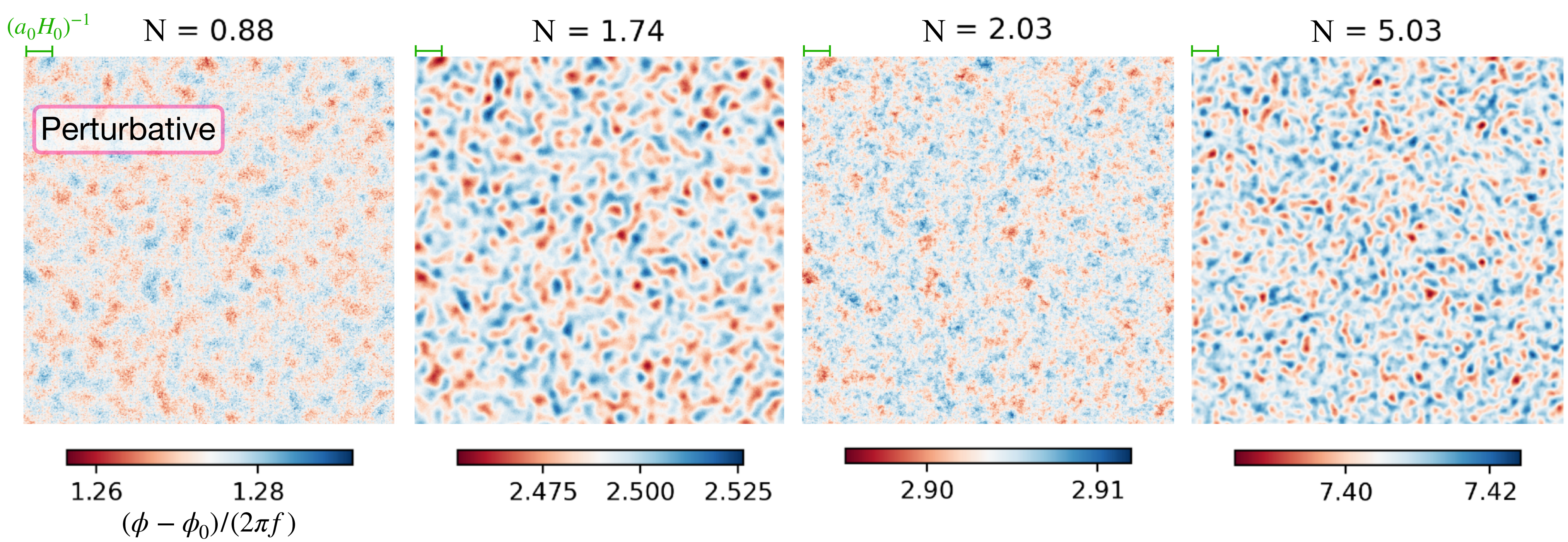
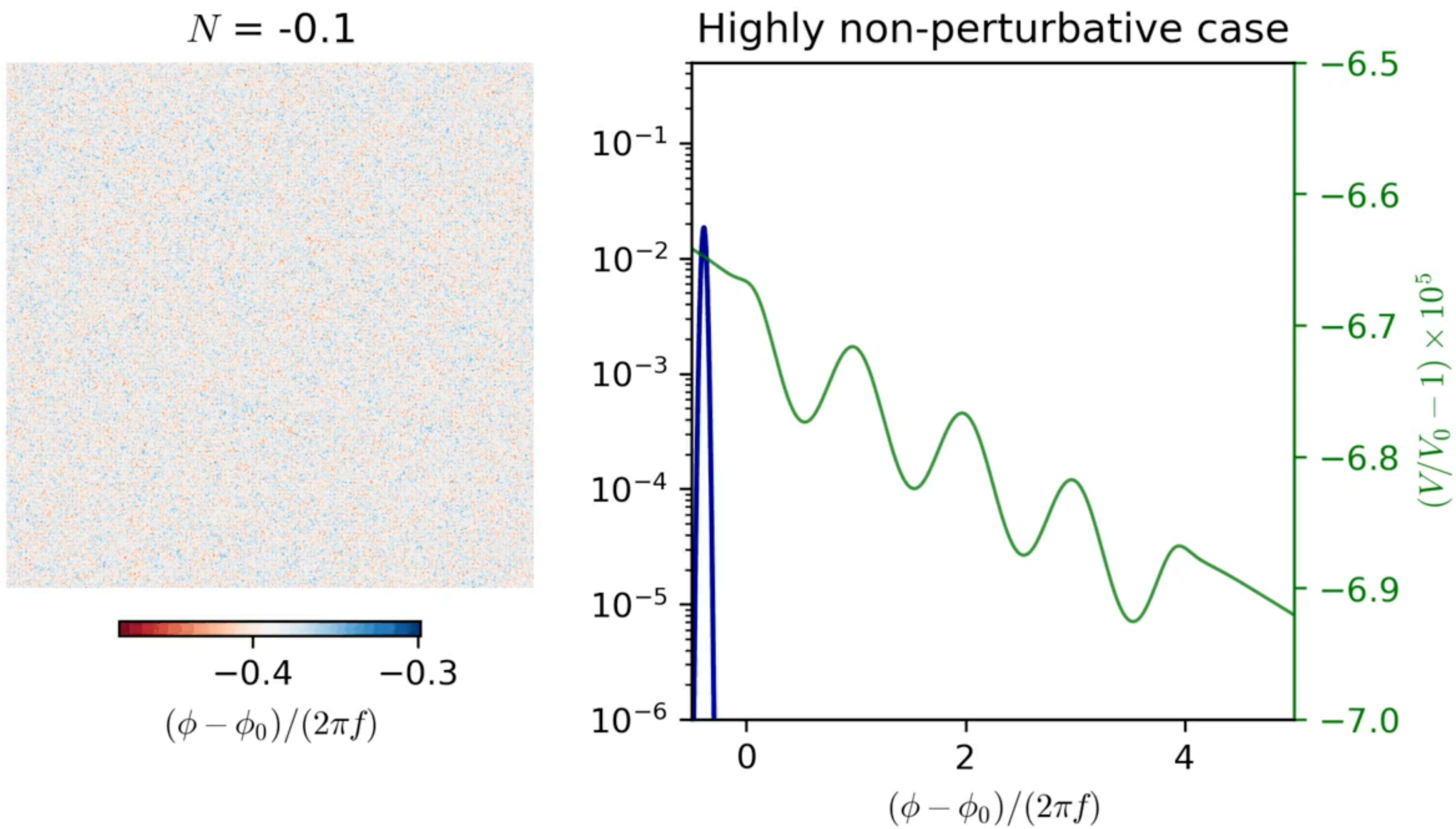




Case 2. $(P_\zeta \sim 10^{-2})$

Case 2 is highly non-perturbative:
Inflaton is stuck inside the oscillatory potential



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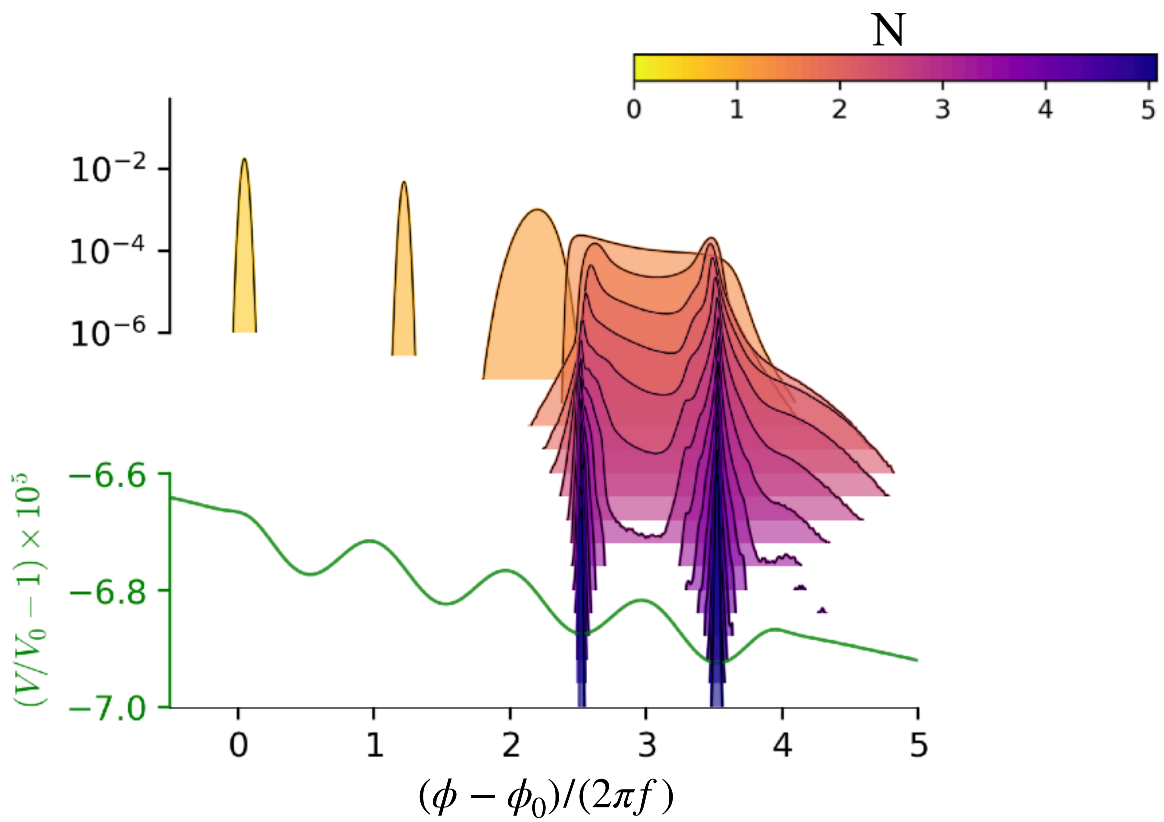
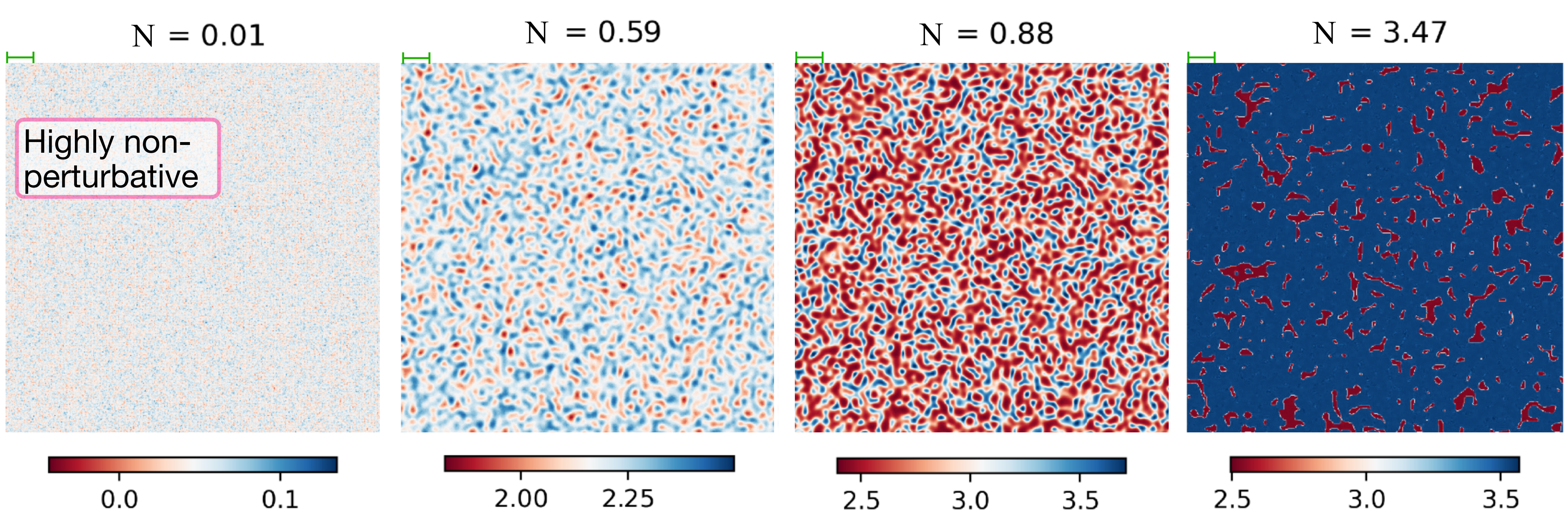
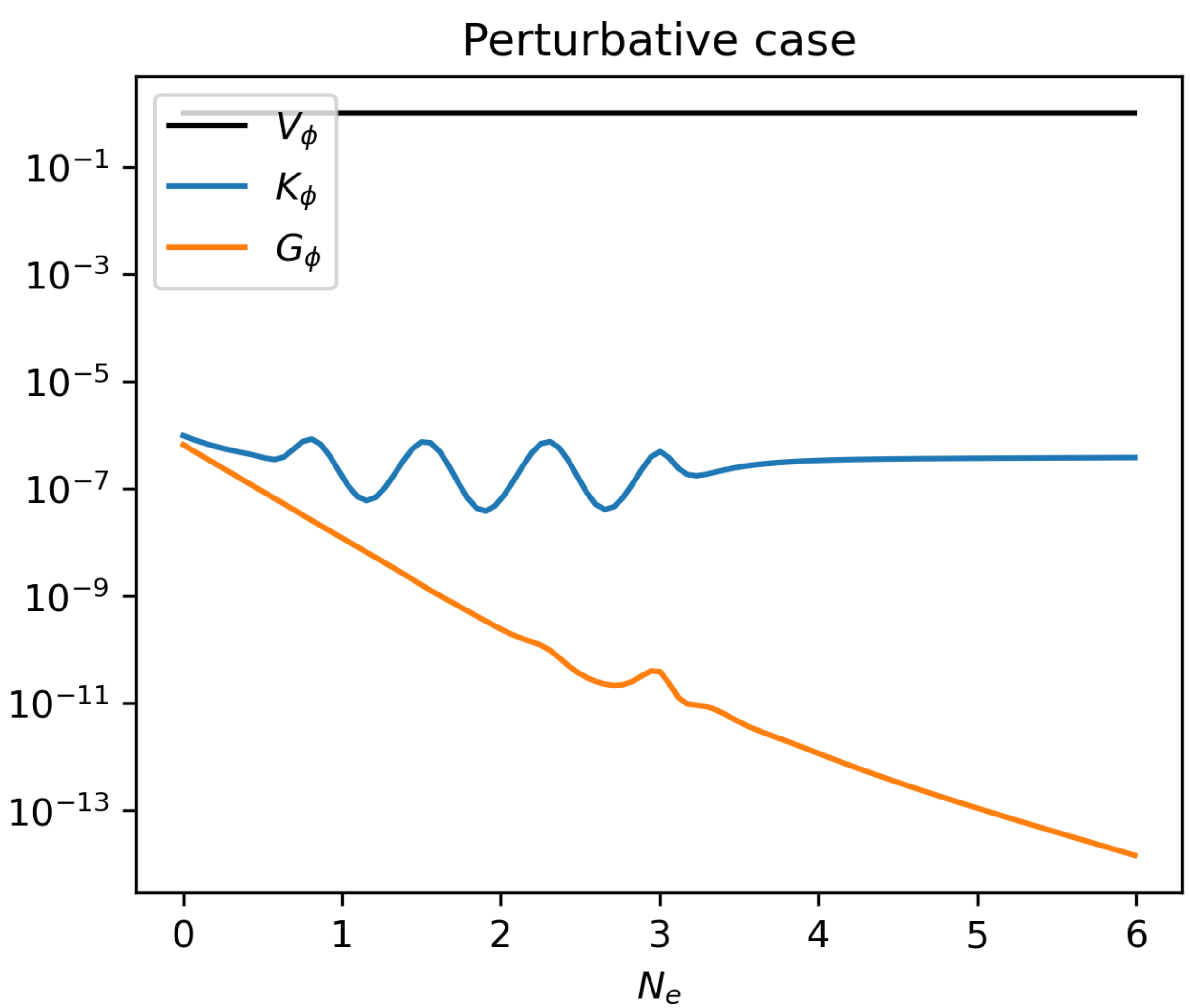
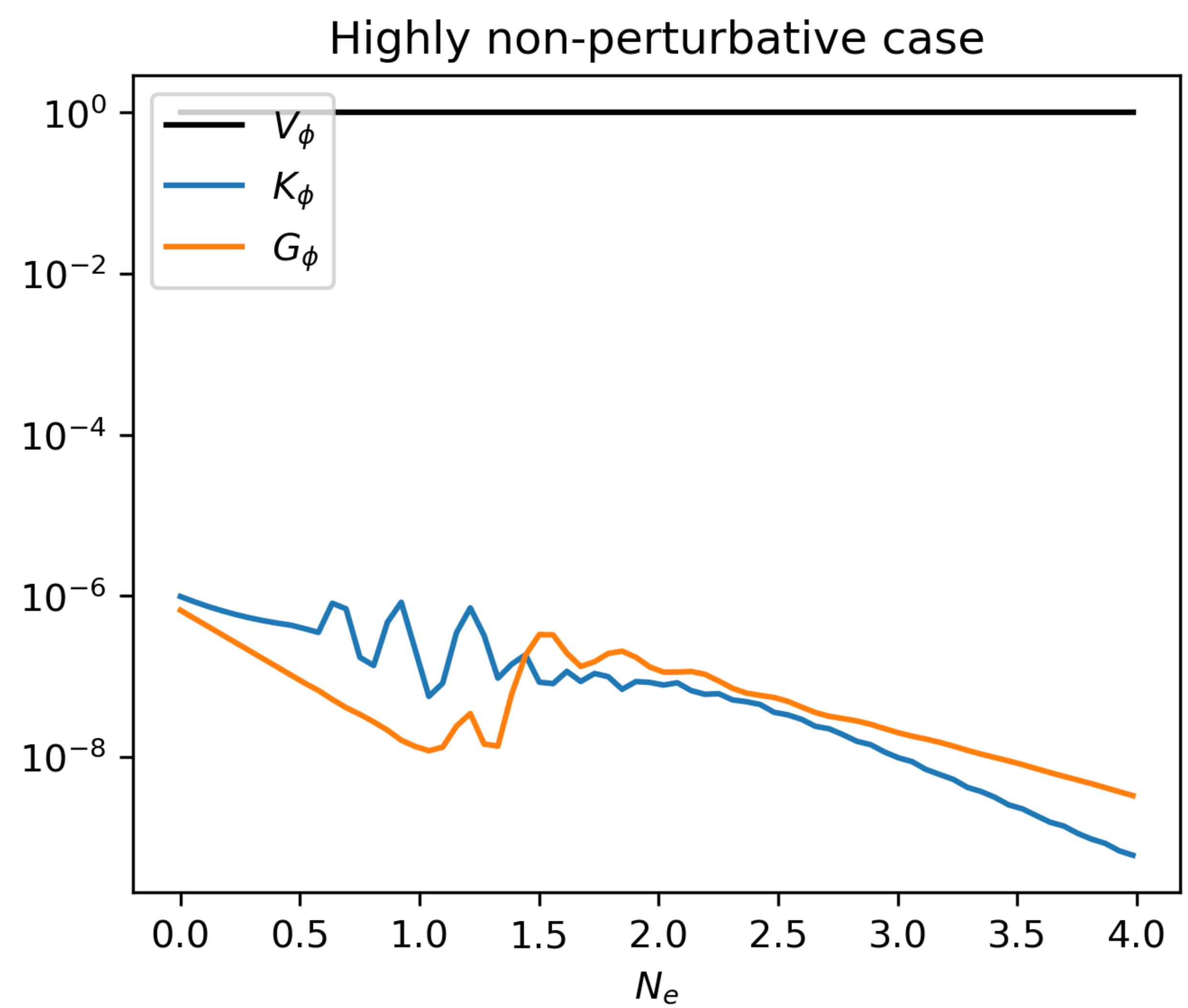




Case 2. $(P_\zeta \sim 10^{-2})$

Why is this happening? Let's look at the **energy**:

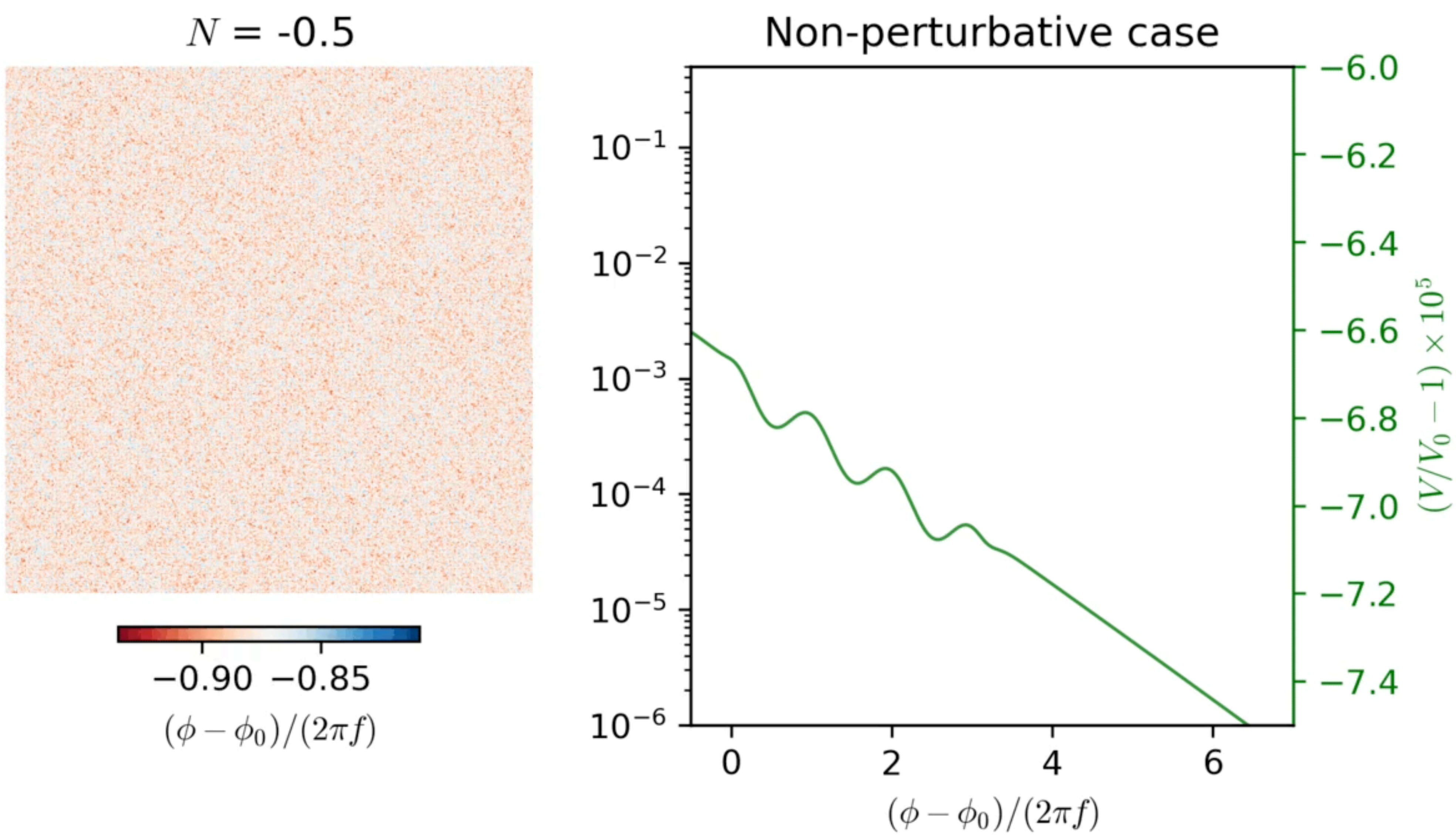




Case 3. $(P_\zeta \sim 10^{-2})$

Case 3: Only some patches are stuck in the resonant potential!

The rest continues slow-rolling



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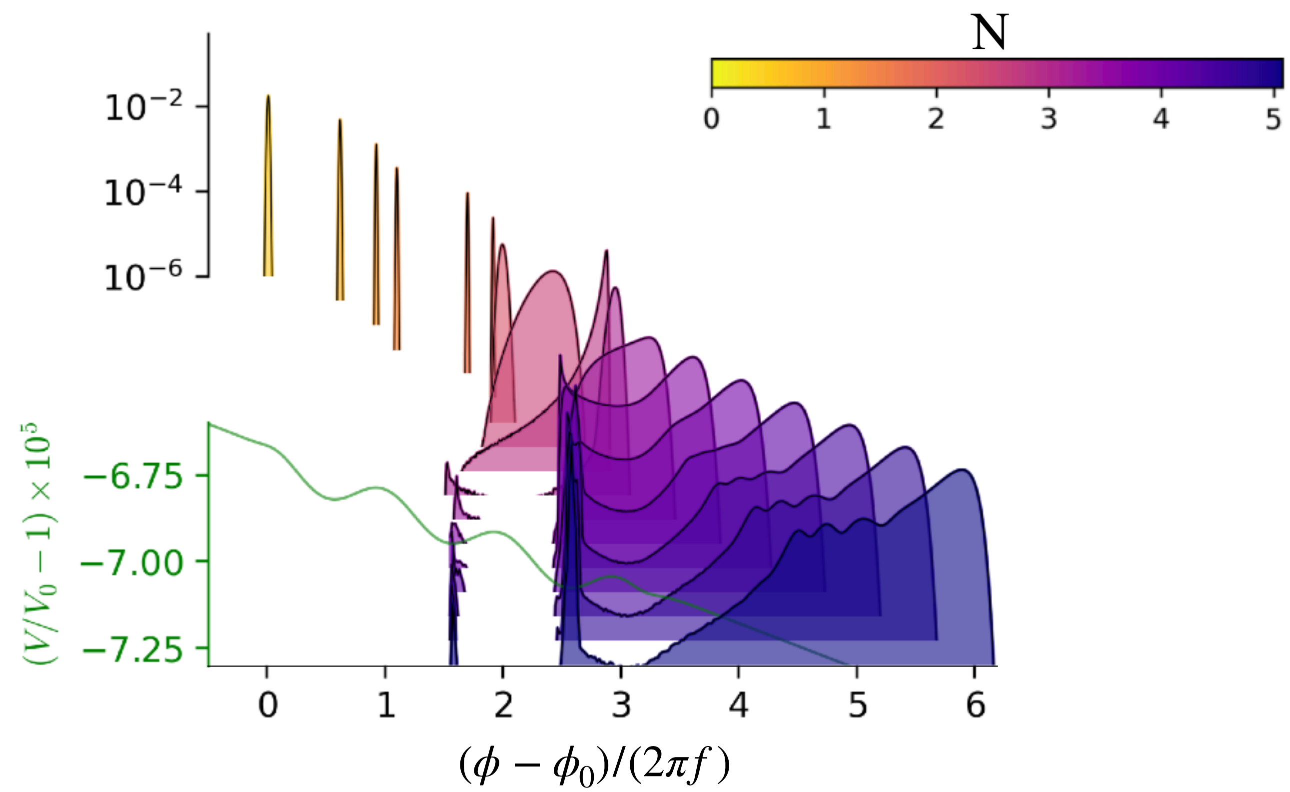
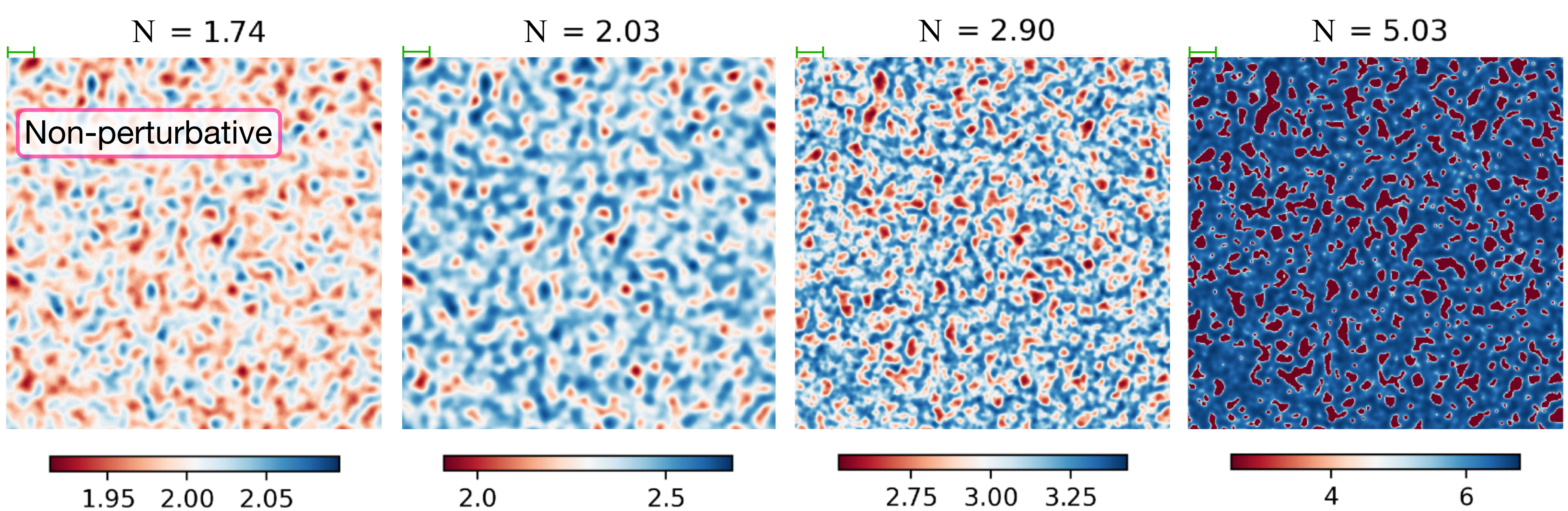


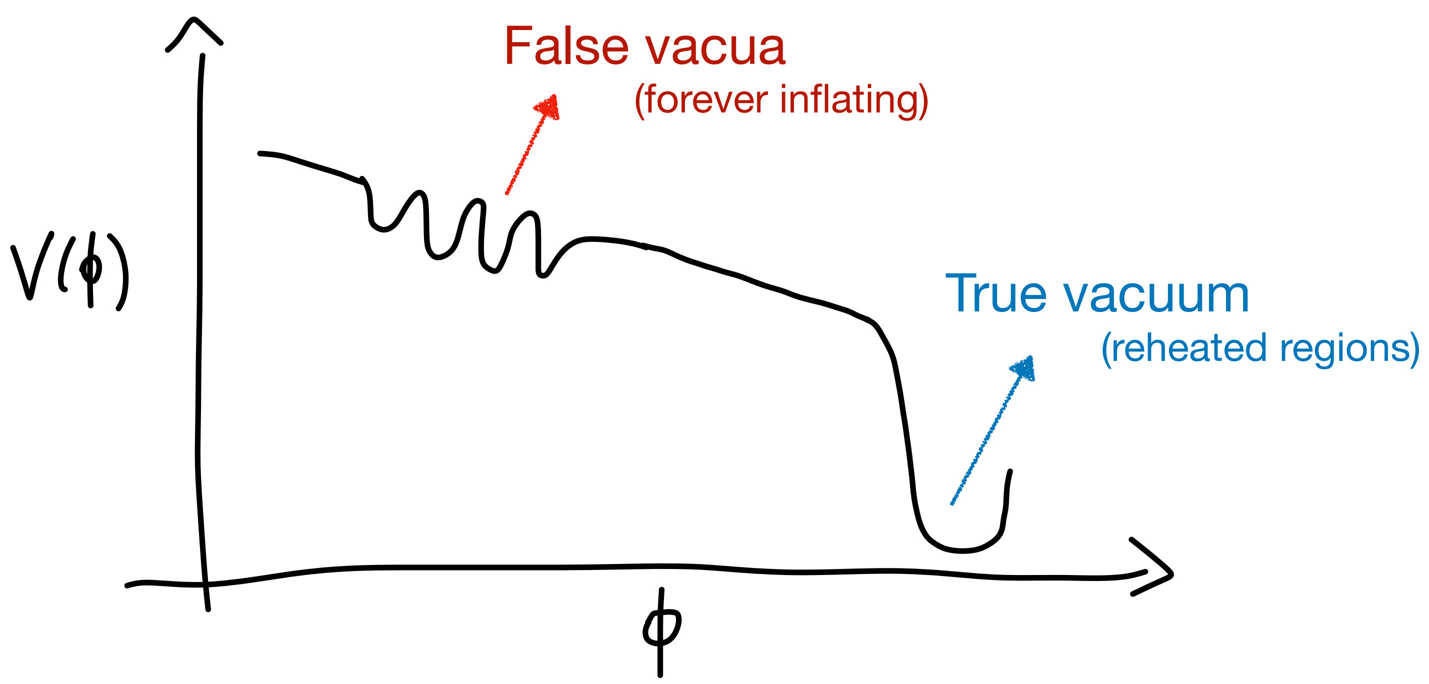


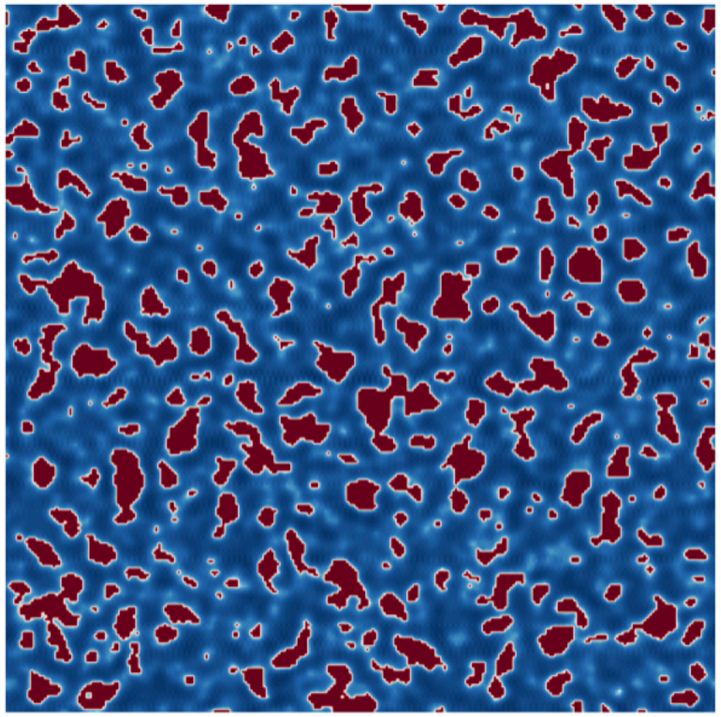
Case 3: inflaton trapping

Case 3:

What happens to the trapped regions at the end of inflation?

Their fate is <u>analogous to false vacuum trapping</u>.





Inflaton trapping and PBHs

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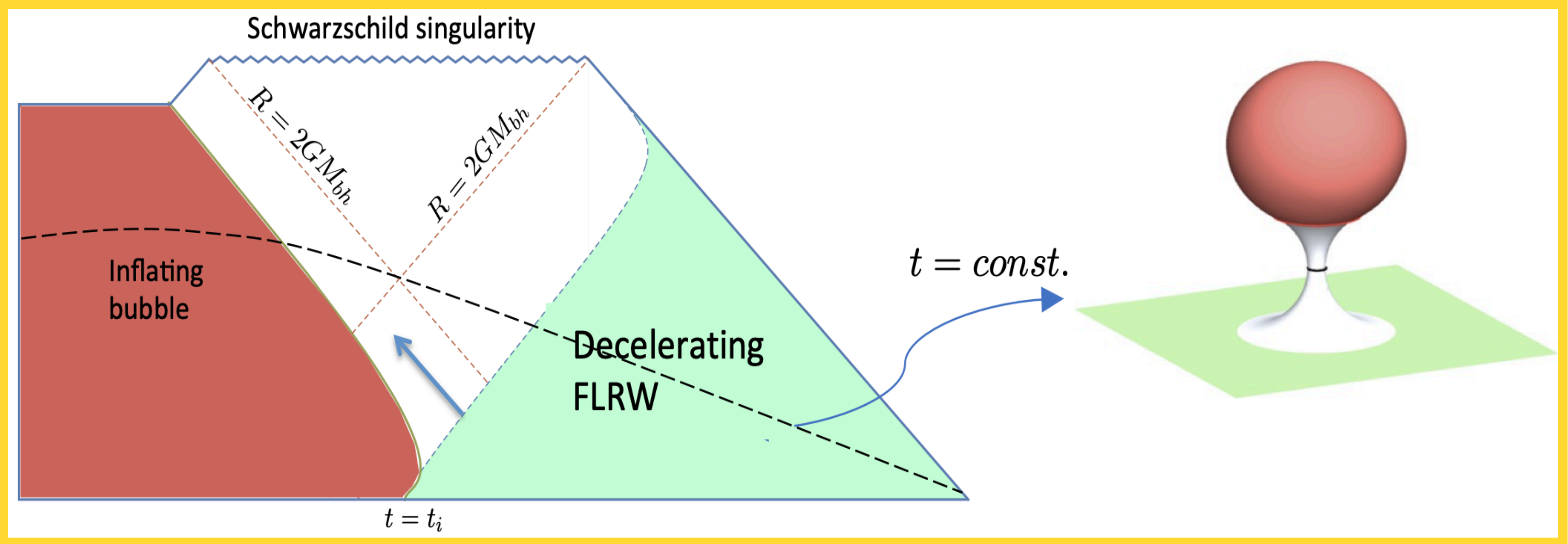
Their fate is analogous to false vacuum trapping.



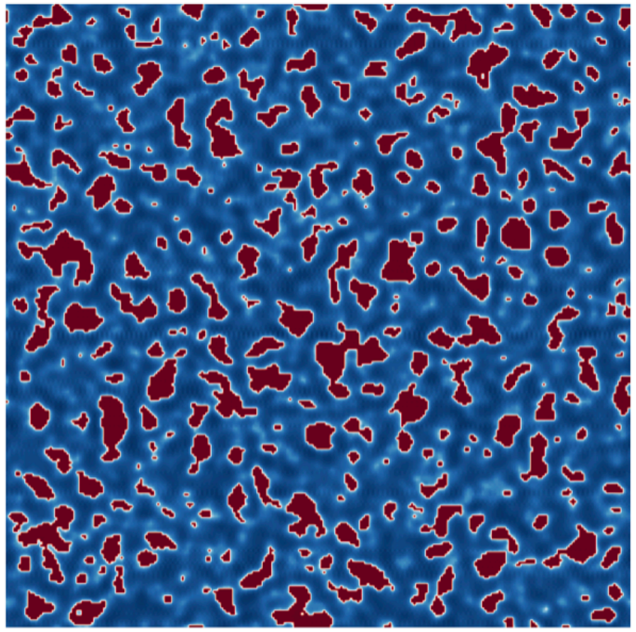


Figure credit:
[J. Garriga, A. Vilenkin, J. Zhang arXiv:1512.01819]

The trapped regions become PBHs at the end of inflation! (in the form of baby universes)

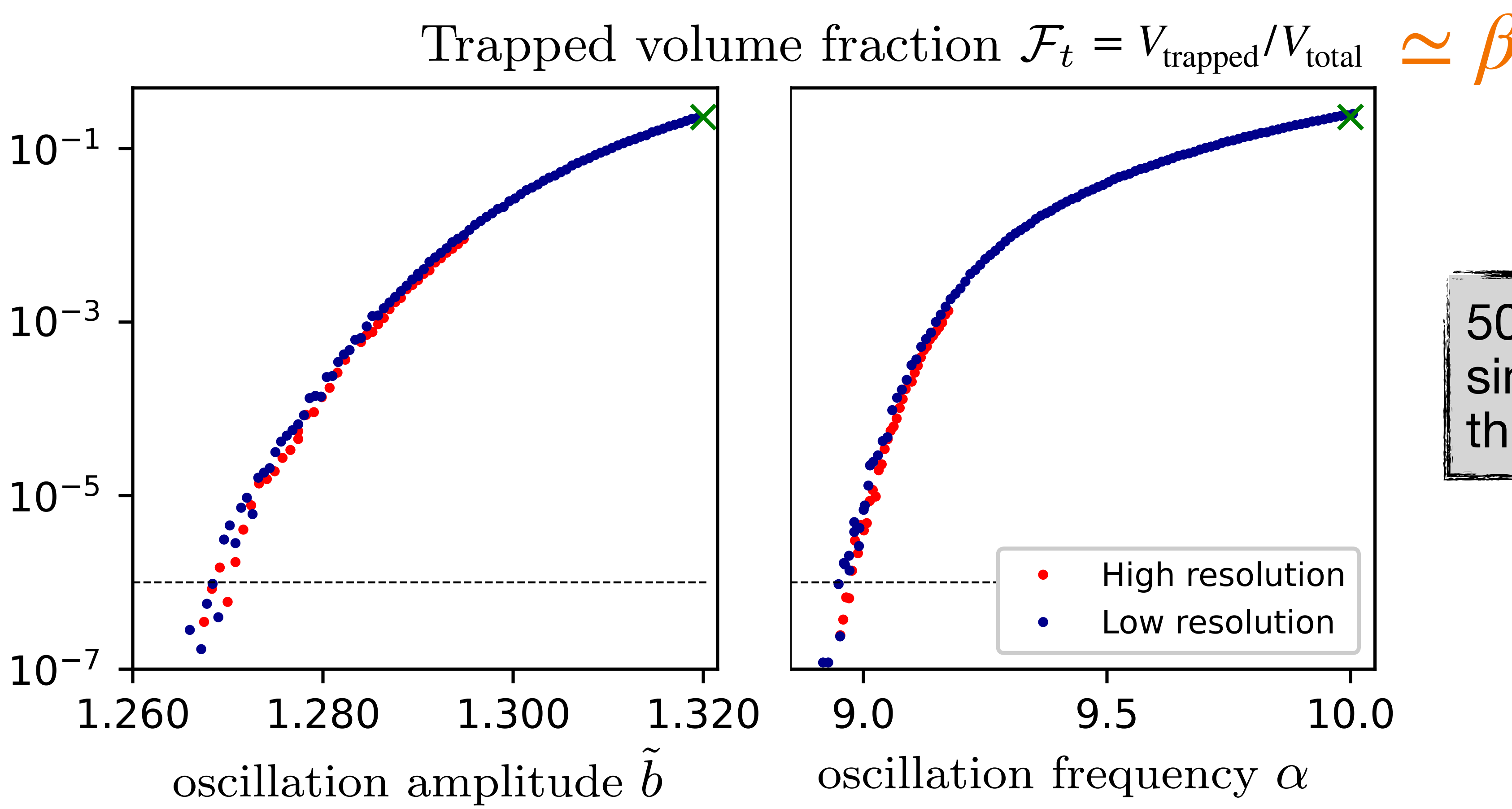
PBH abundance

Case 3:

The trapped regions become PBHs at the end of inflation!

How many PBHs?

Mass fraction in PBHs at the time of formation



500 lattice simulations in this plot

Inflationary Butterfly Effect

Lorenz (1972):

"Can the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?" [1,2]

[1]: E. N. Lorenz, American Association for the Advancement of Science (1972).
[2]: E. N. Lorenz, Deterministic Nonperiodic flow (1972).

Finite systems of deterministic ordinary nonlinear differential equations may be designed to represent forced dissipative hydrodynamic flow. Solutions of these equations can be identified with trajectories in phase space. For those systems with bounded solutions, it is found that nonperiodic solutions are ordinarily unstable with respect to small modifications, so that slightly differing initial states can evolve into considerably different states. Systems with bounded solutions are shown to possess bounded numerical solutions.

A simple system representing cellular convection is solved numerically. All of the solutions are found to be unstable, and almost all of them are nonperiodic.

The feasibility of very-long-range weather prediction is examined in the light of these results.

Special thanks are due to Miss Ellen Fetter for handling the many numerical computations and preparing the graphical presentations of the numerical material.

Inflationary Butterfly Effect

Lorenz (1972):

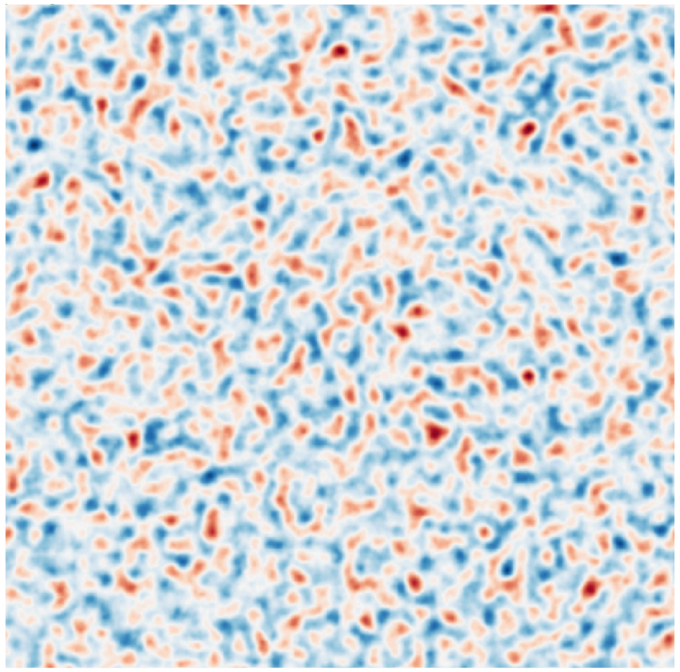
"Can the Flap of a Butterfly's Wings in Brazil Set Off a Tornado in Texas?" [1,2]

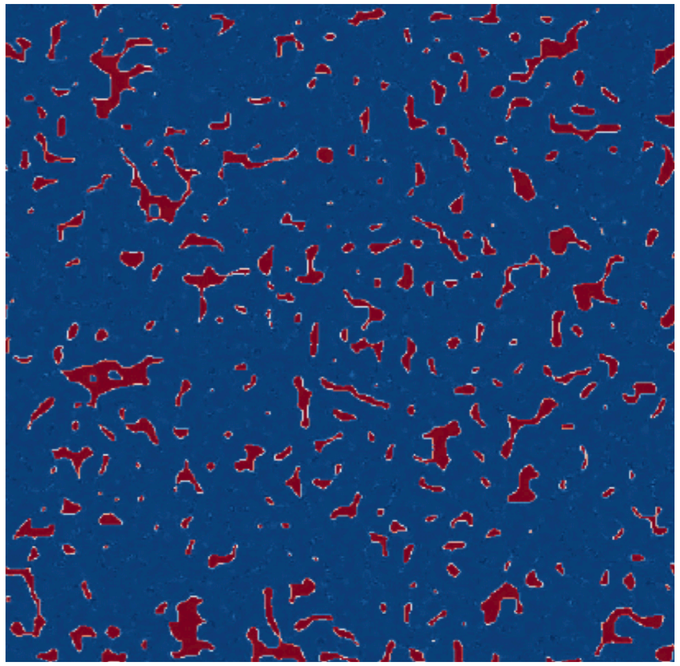
Can tiny, small-scale quantum fluctuations affect the dynamics of the entire Universe?

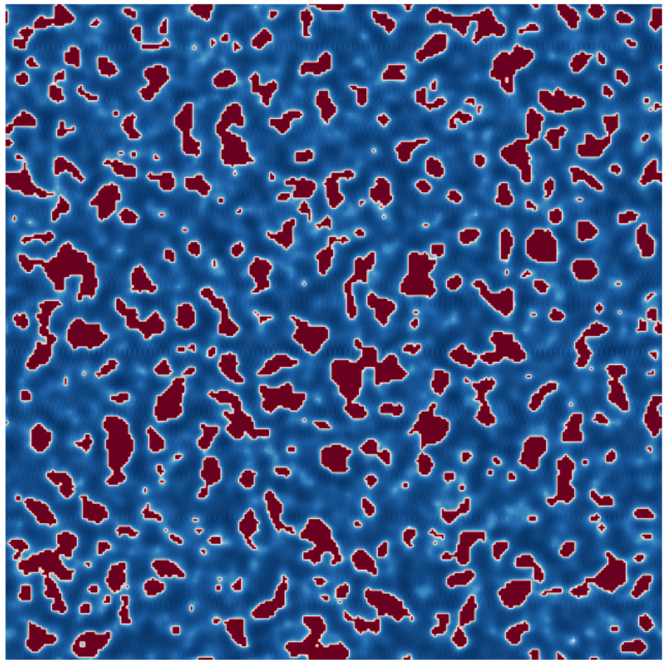
Inflationary Butterfly Effect

Main lesson:

<u>Non-perturbative physics</u> at small scales can have drastic effects on the inflationary dynamics when $\mathscr{P}_\zeta \sim 10^{-2}$



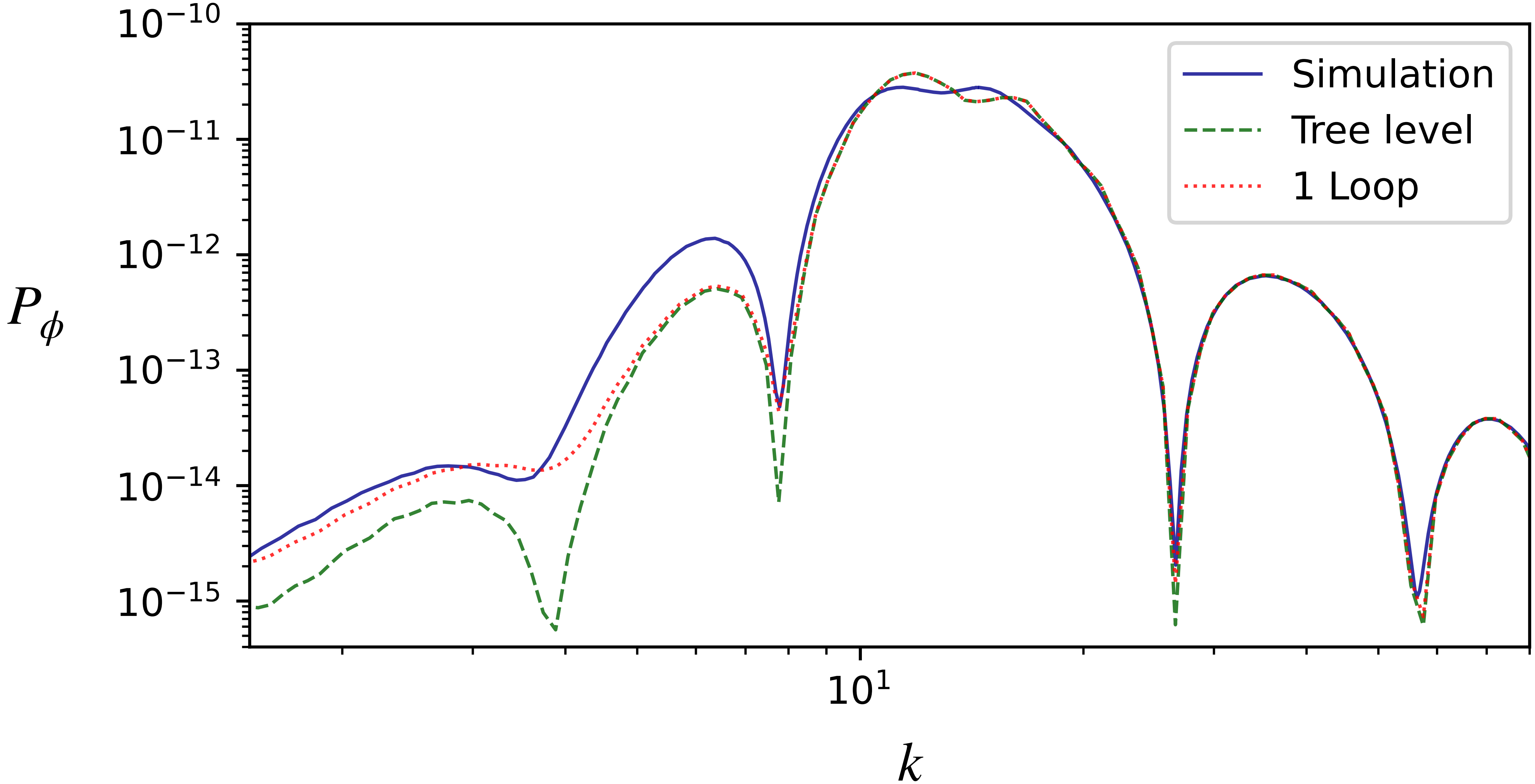




Loop effects

In the perturbative setup (case 1),

first **quantitative comparison** between full nonlinear, tree-level and 1-loop



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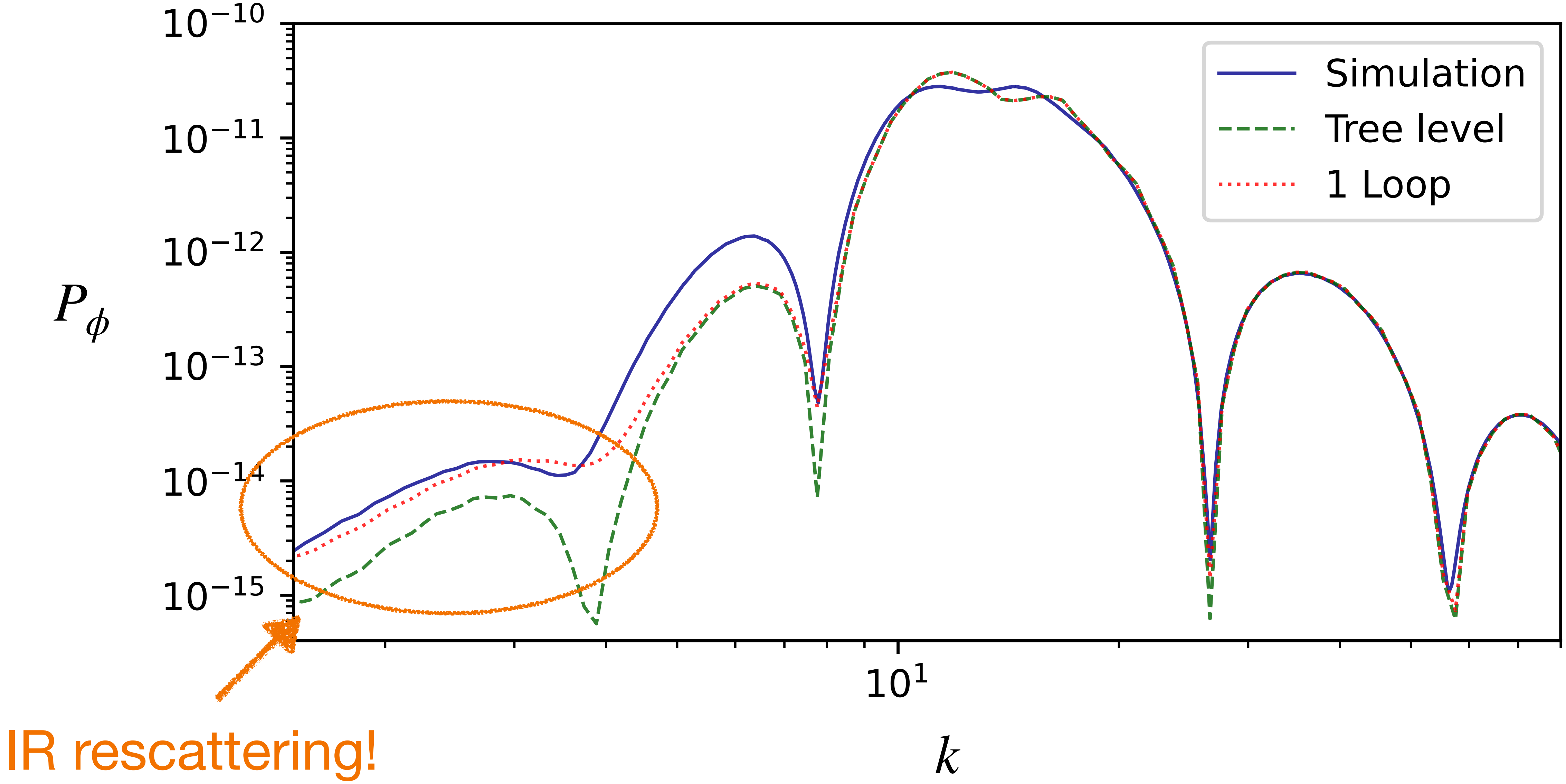

Fumagalli, Bhattacharya, Peloso, Renaux-Petel, Witkowski [2307.08358]

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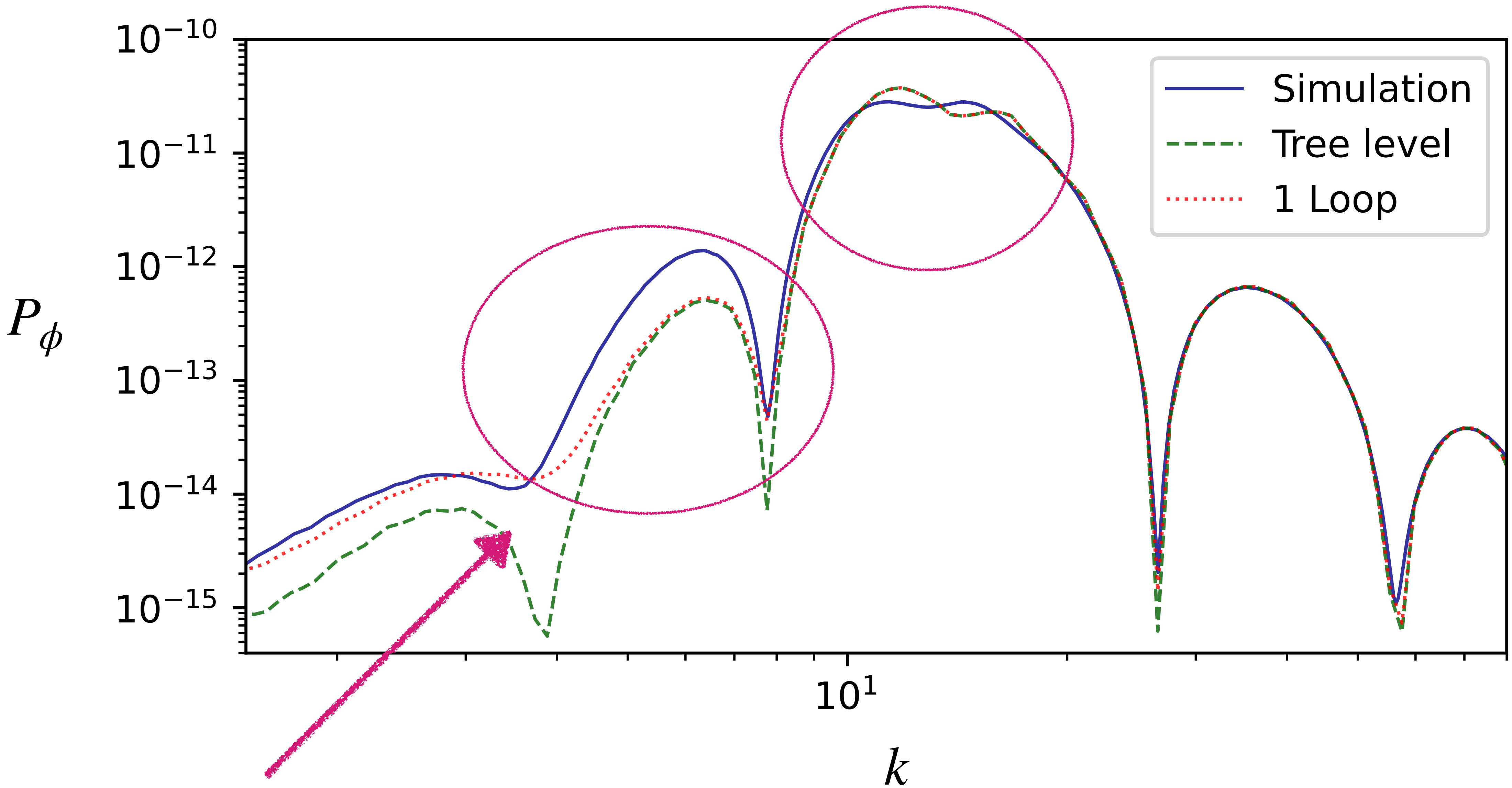


Beyond 1-loop?? Other corrections?

Roadmap

0) Motivation: why simulating inflation?

AC, Komatsu, Lozanov, Weller — 2102.06378, 2110.10695, 2204.12874

AC — 2209.13616, 2506.11797

1) Lattice simulations of inflation

2) Example: **inflationary butterfly effect**

2.1) Oscillatory potential

2.2) Ultra-slow-roll inflation

AC, K. Inomata, S. Renaux-Petel — 2403.12811

AC, G. Franciolini, S. Renaux-Petel — 2410.23942, 2506.11795

3) Some ongoing work

AC, G. Franciolini, S. Renaux-Petel, [2410.23942]
[2506.11795]

Ultra-Slow-Roll inflation

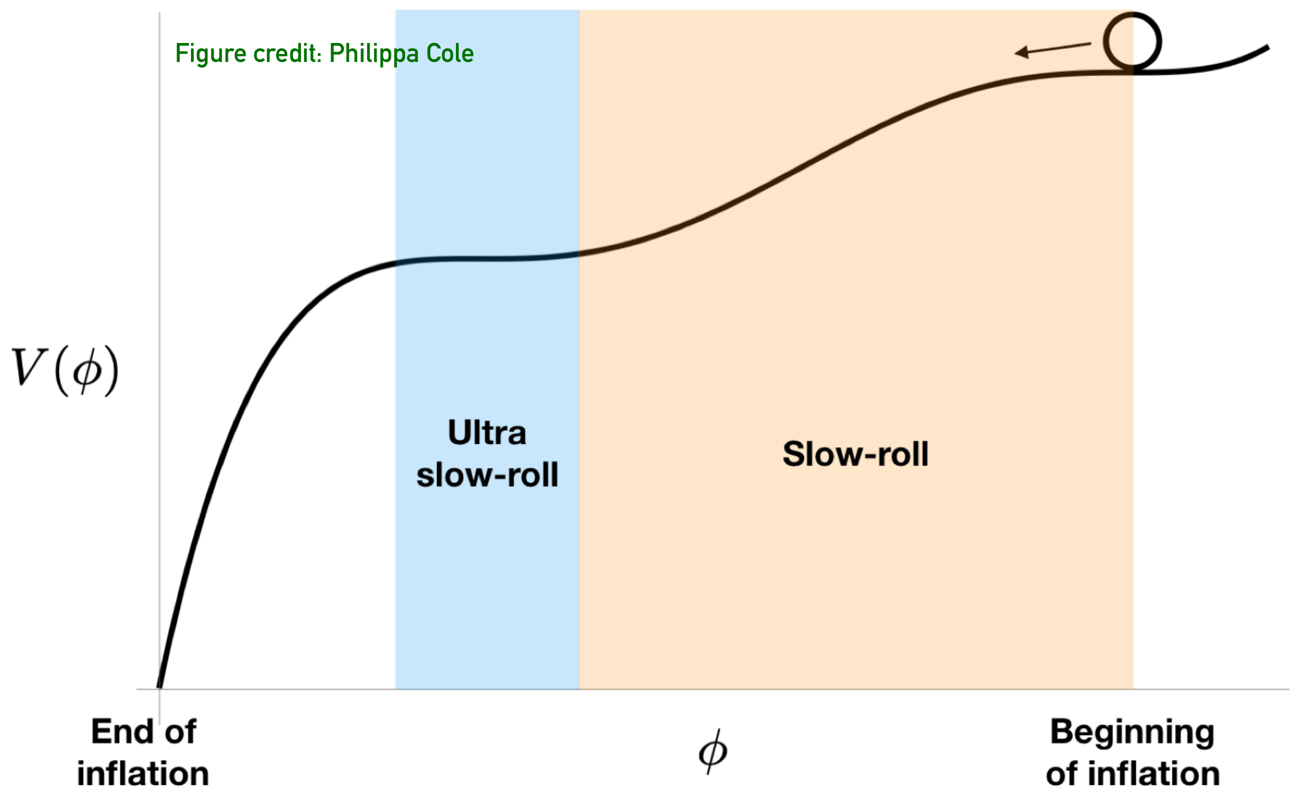
A well-known mechanism to enhance density fluctuations is an inflection point

Fluctuations amplified via a <u>deceleration</u> of the inflaton

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1$$

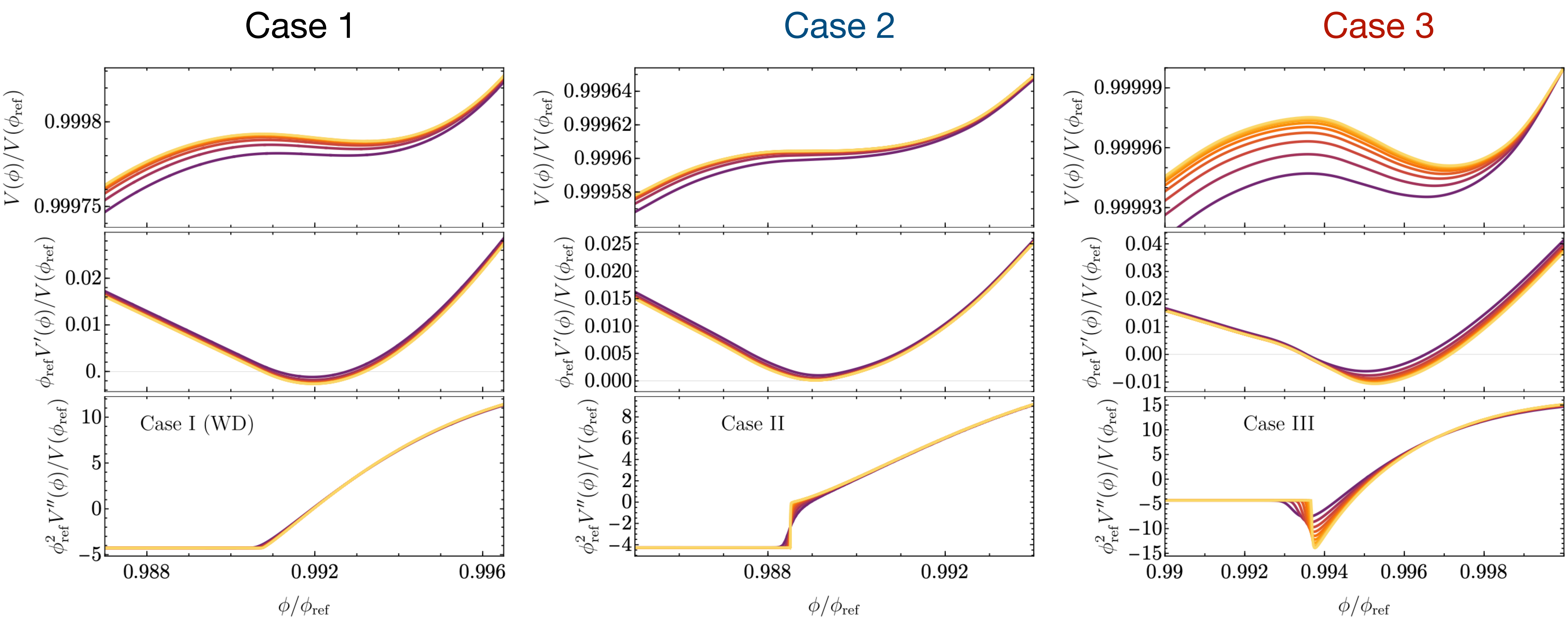
$$|\eta_H| = |\frac{\dot{\epsilon}_H}{H\epsilon_H}| \sim 1$$

So-called "ultra slow-roll" phase



Ultra-Slow-Roll inflation

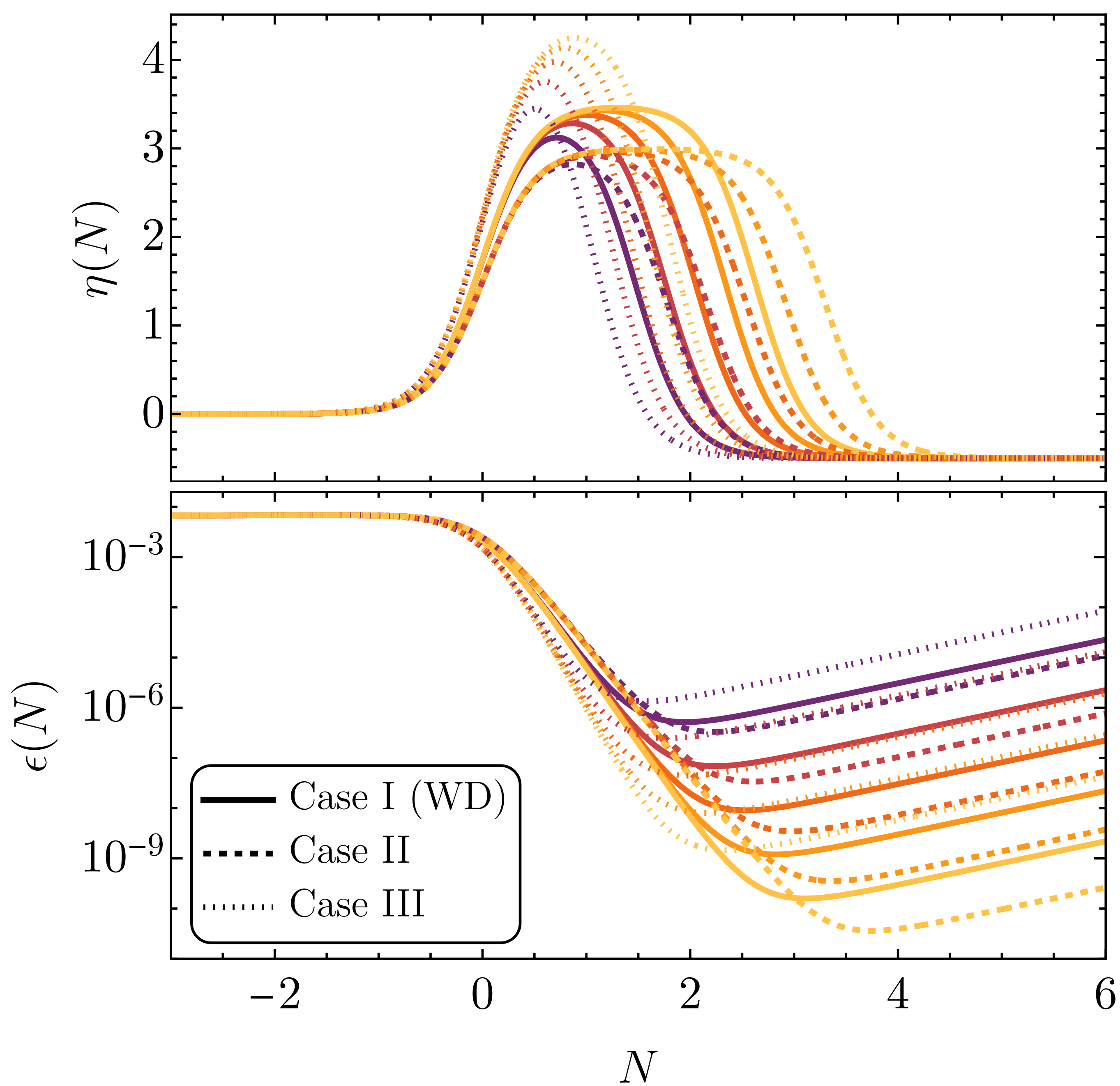
A systematic study of USR potentials:



$$\frac{\partial^3 V(\phi)}{\partial \phi^3} \sim 0 \qquad \frac{\partial^3 V(\phi)}{\partial \phi^3} > 0 \qquad \frac{\partial^3 V(\phi)}{\partial \phi^3} < 0$$

Ultra-Slow-Roll inflation

A systematic study of USR potentials:



AC, G. Franciolini, S. Renaux-Petel, [2410.23942]
AC, G. Franciolini, S. Renaux-Petel, [2506.11795]

Ultra-Slow-Roll inflation

$\frac{\partial^3 V(\phi)}{\partial \phi^3}$ is the leading self-interaction of the inflaton: $V(\bar{\phi} + \delta\phi) = \sum_n \frac{\delta\phi^n}{n!} \frac{\partial^n V(\phi)}{\partial \phi^n}\Big|_{\bar{\phi}}$

Case 1

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} \sim 0$$

↓

Free theory
Aka “Wands duality”

Case 2

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} > 0$$

↓

Repulsive
self-interaction

Case 3

$$\frac{\partial^3 V(\phi)}{\partial \phi^3} < 0$$

↓

Attractive
self-interaction

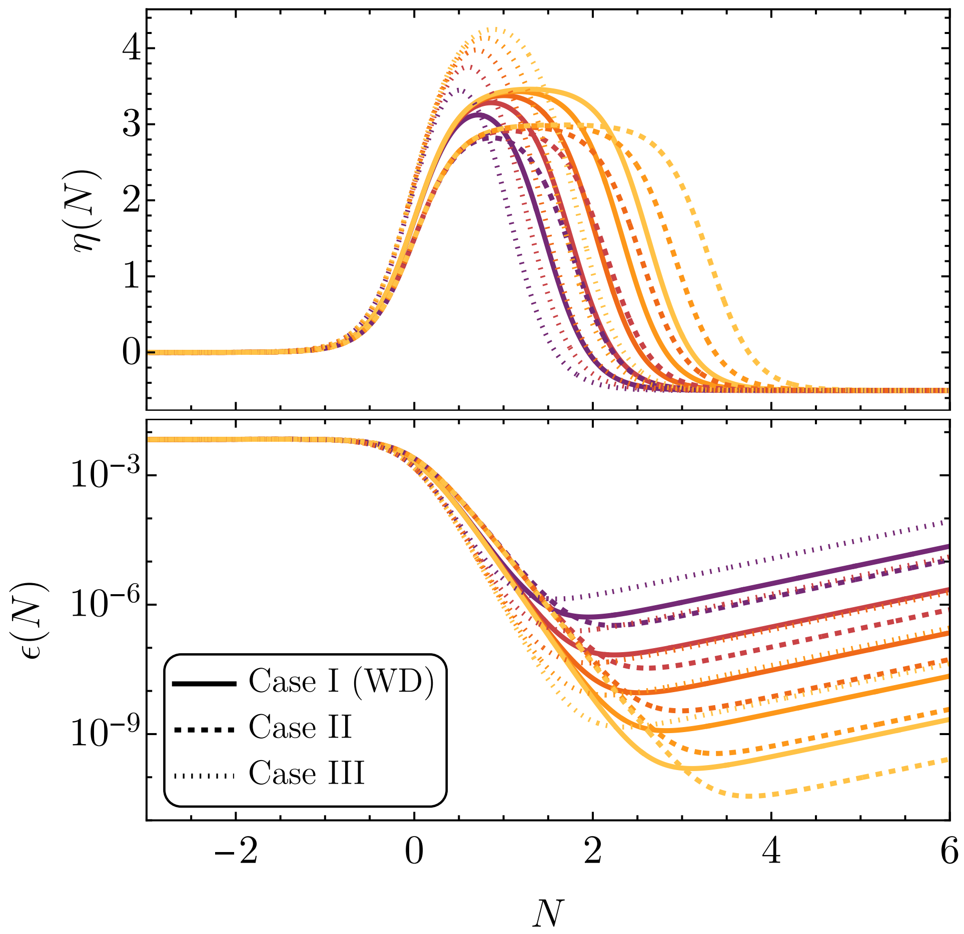
Ultra-Slow-Roll inflation

Wands duality: [D. Wands (1998)]

Evolution of scalar field perturbation is invariant (dual) under the transformation of the background:

$$\eta \to 3 - \eta$$

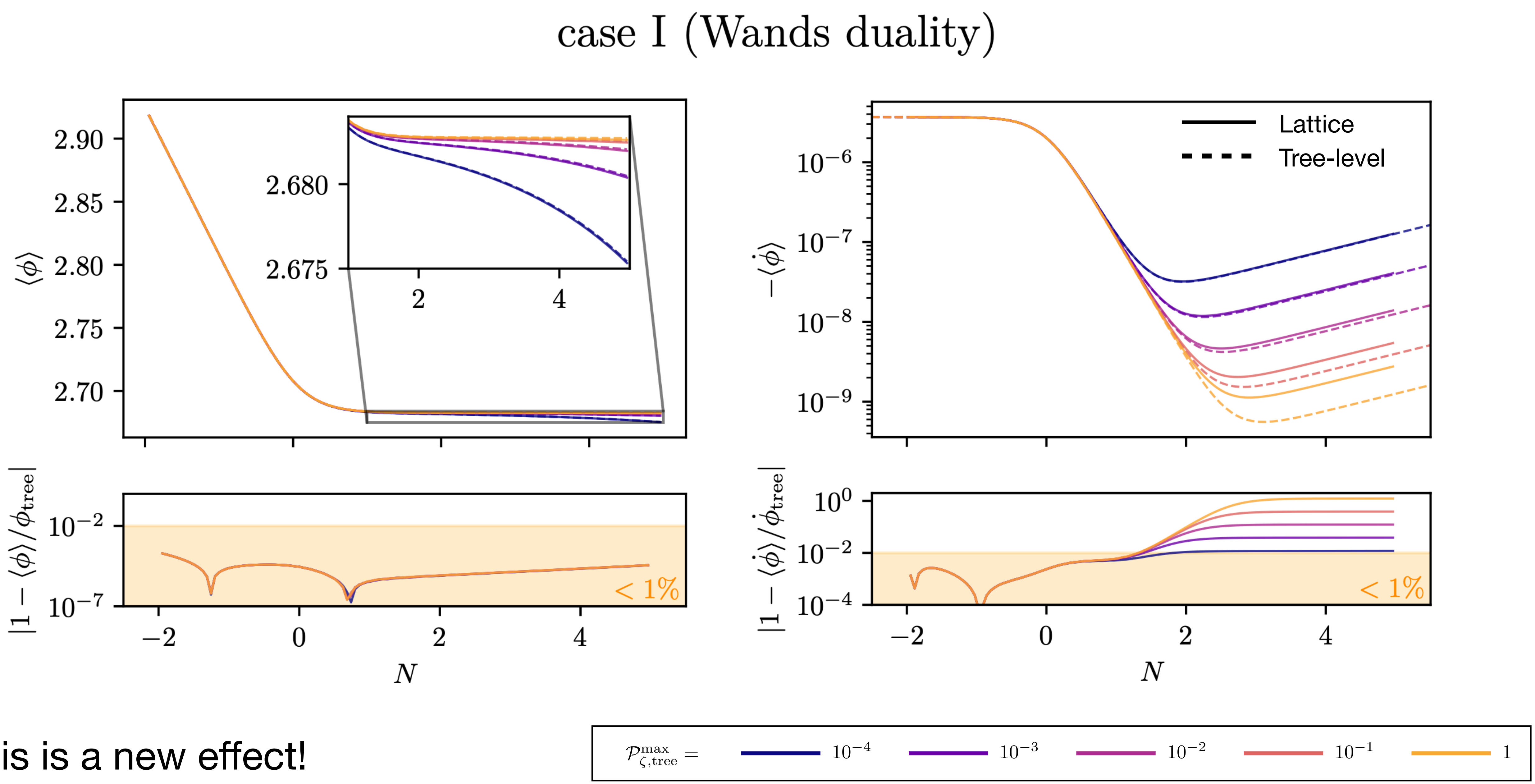
Our potential in case 1 is constructed so that $\eta_{USR} = 3 - \eta_{SR,2}$, so the theory is approximately free



Ultra-Slow-Roll inflation

Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution



This is a new effect!

AC, G. Franciolini, S. Renaux-Petel, [2410.23942]
AC, G. Franciolini, S. Renaux-Petel, [2506.11795]

Ultra-Slow-Roll inflation

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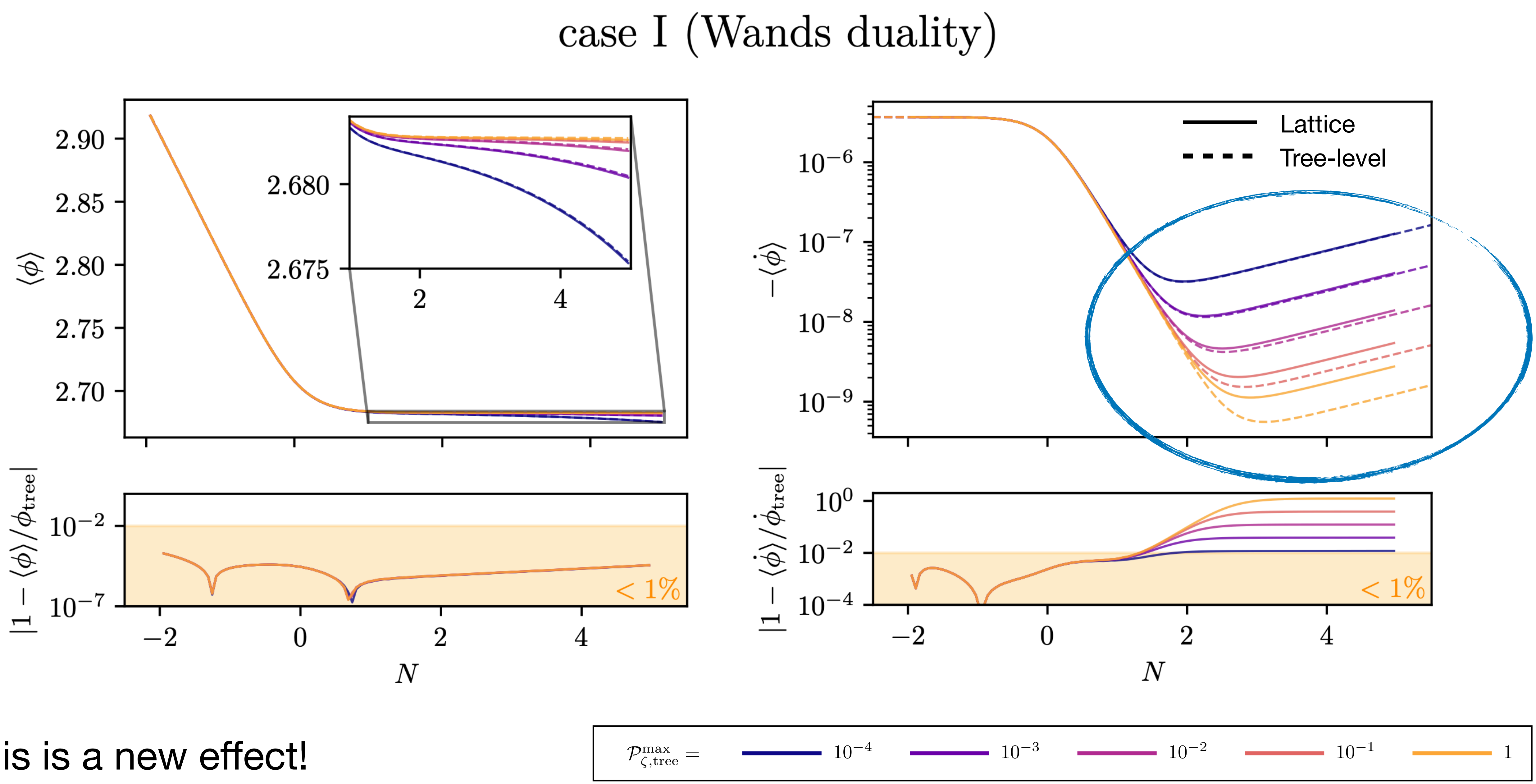

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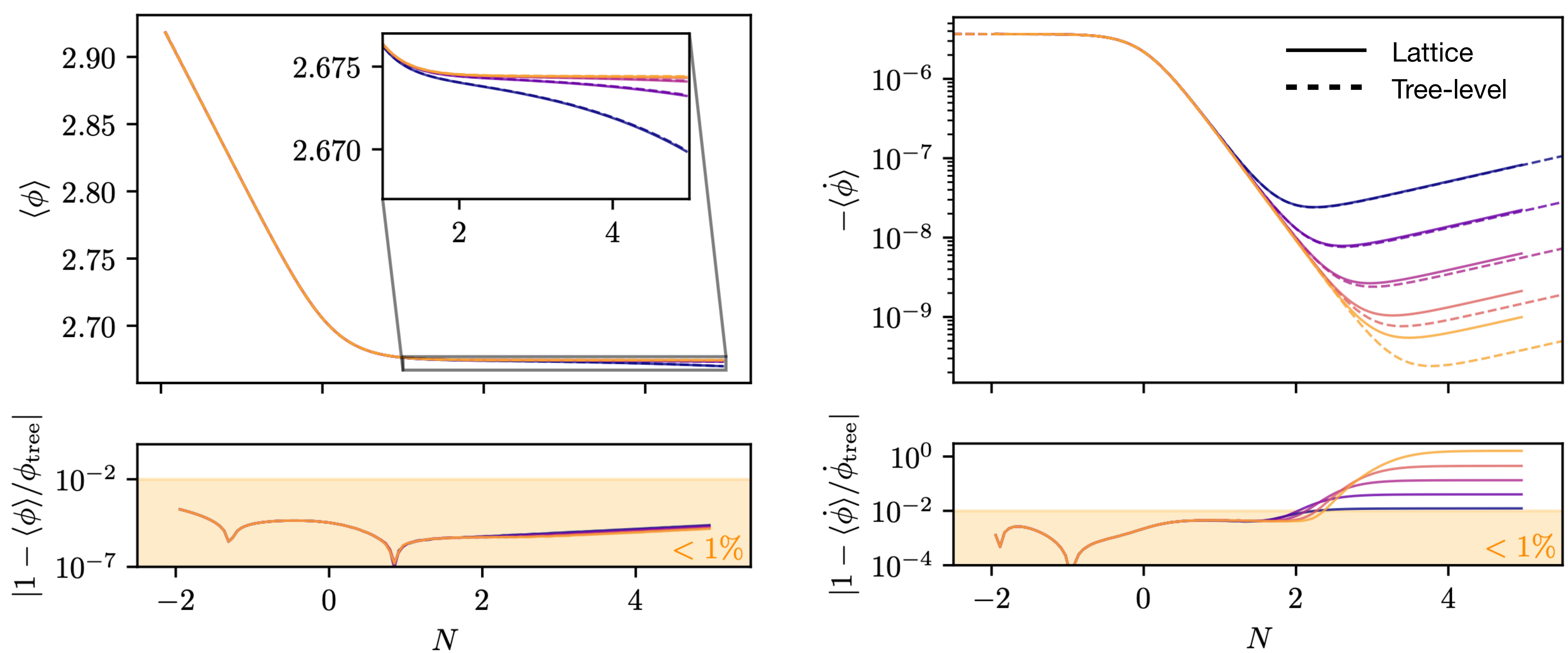
Ultra-Slow-Roll inflation

Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution

case II (repulsive)



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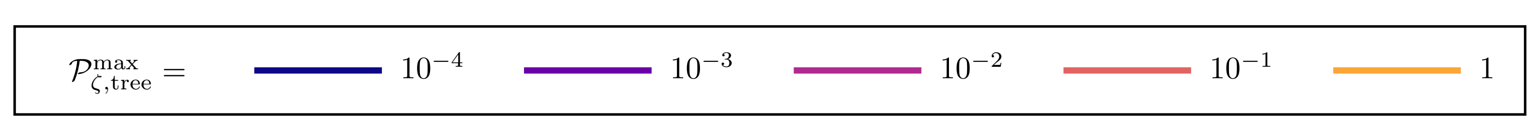

Ultra-Slow-Roll inflation

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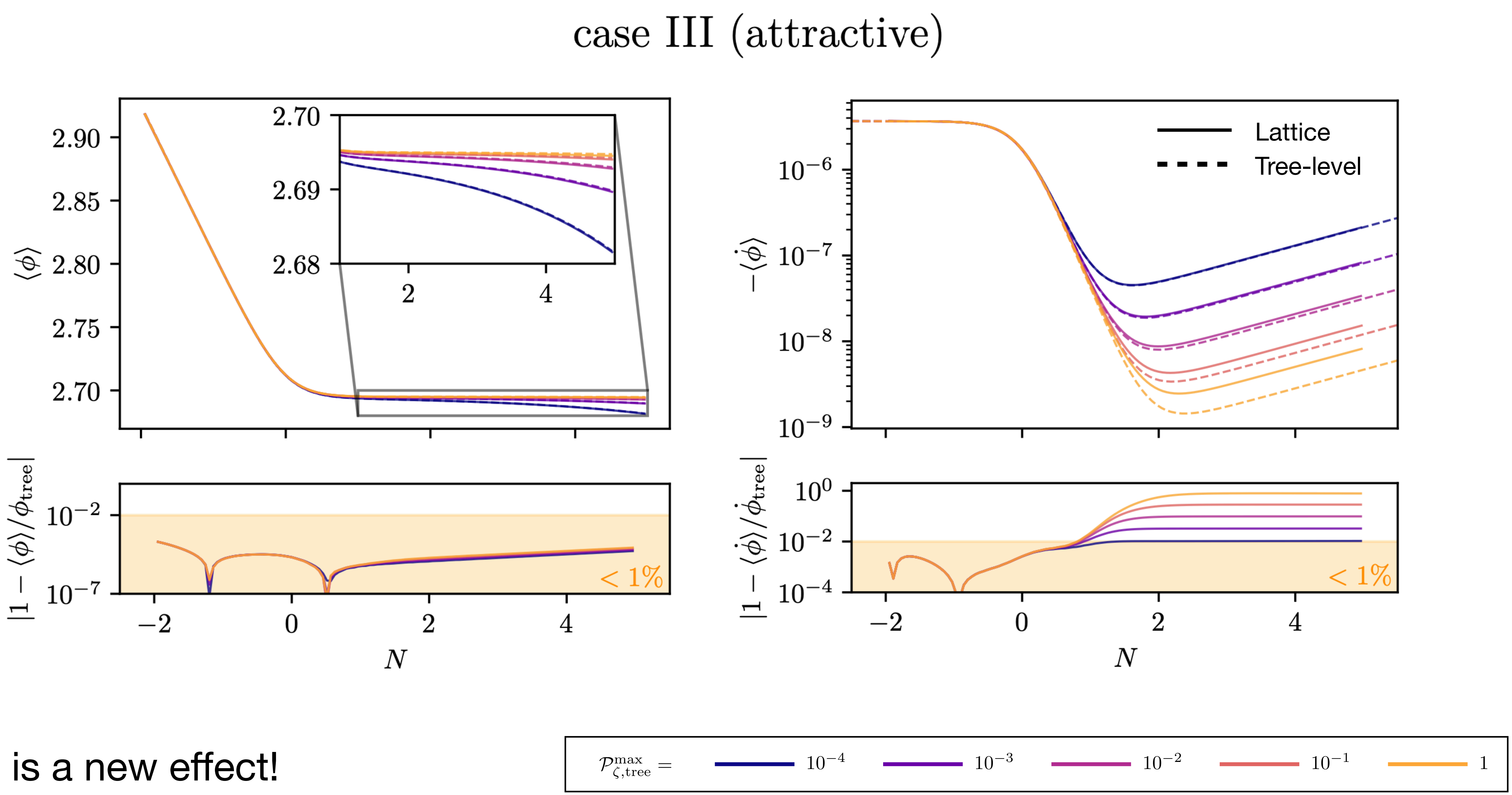


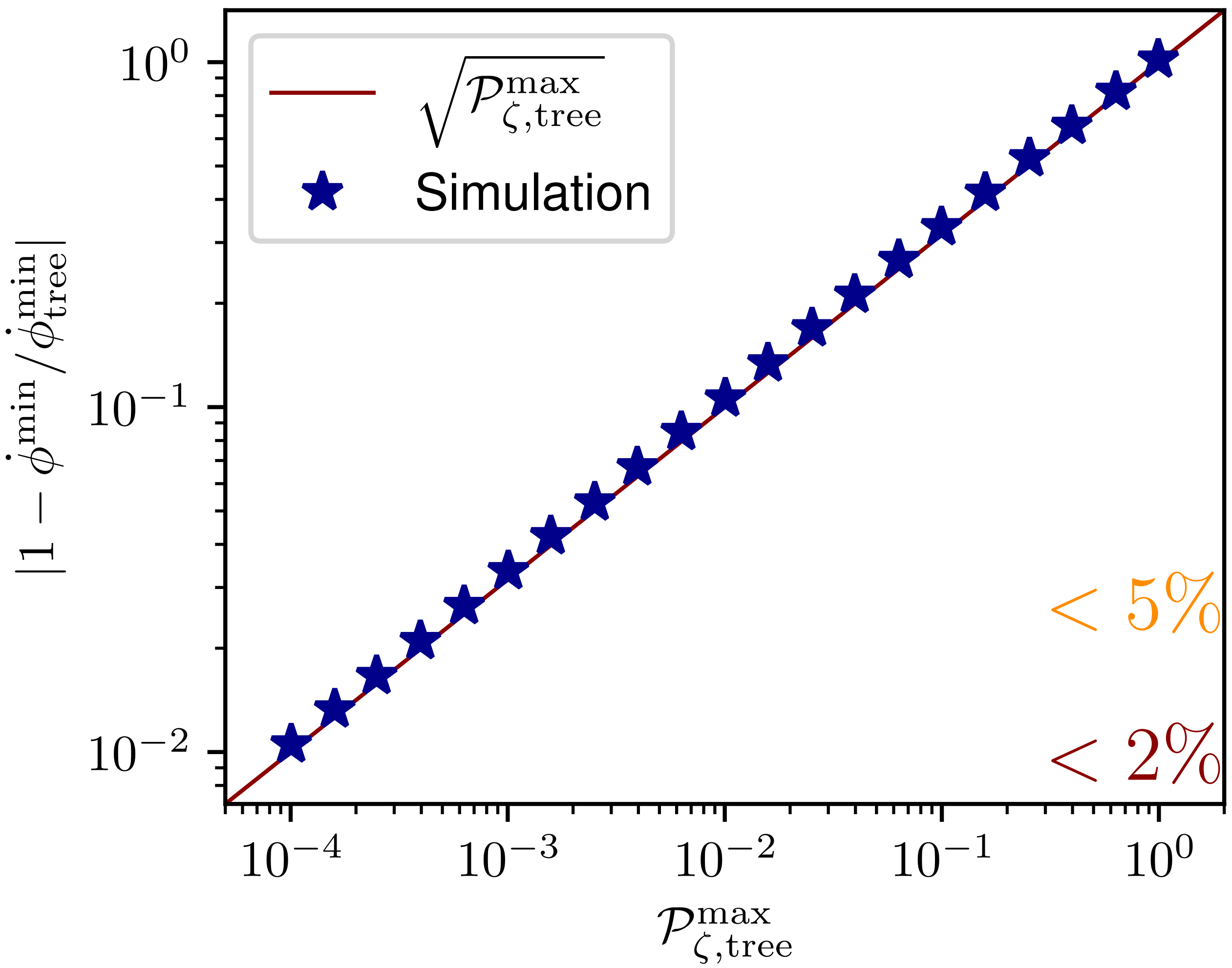
This is a new effect!

Ultra-Slow-Roll inflation

Result #1:

We find **backreaction**, i.e. an effect of fluctuations on the background evolution

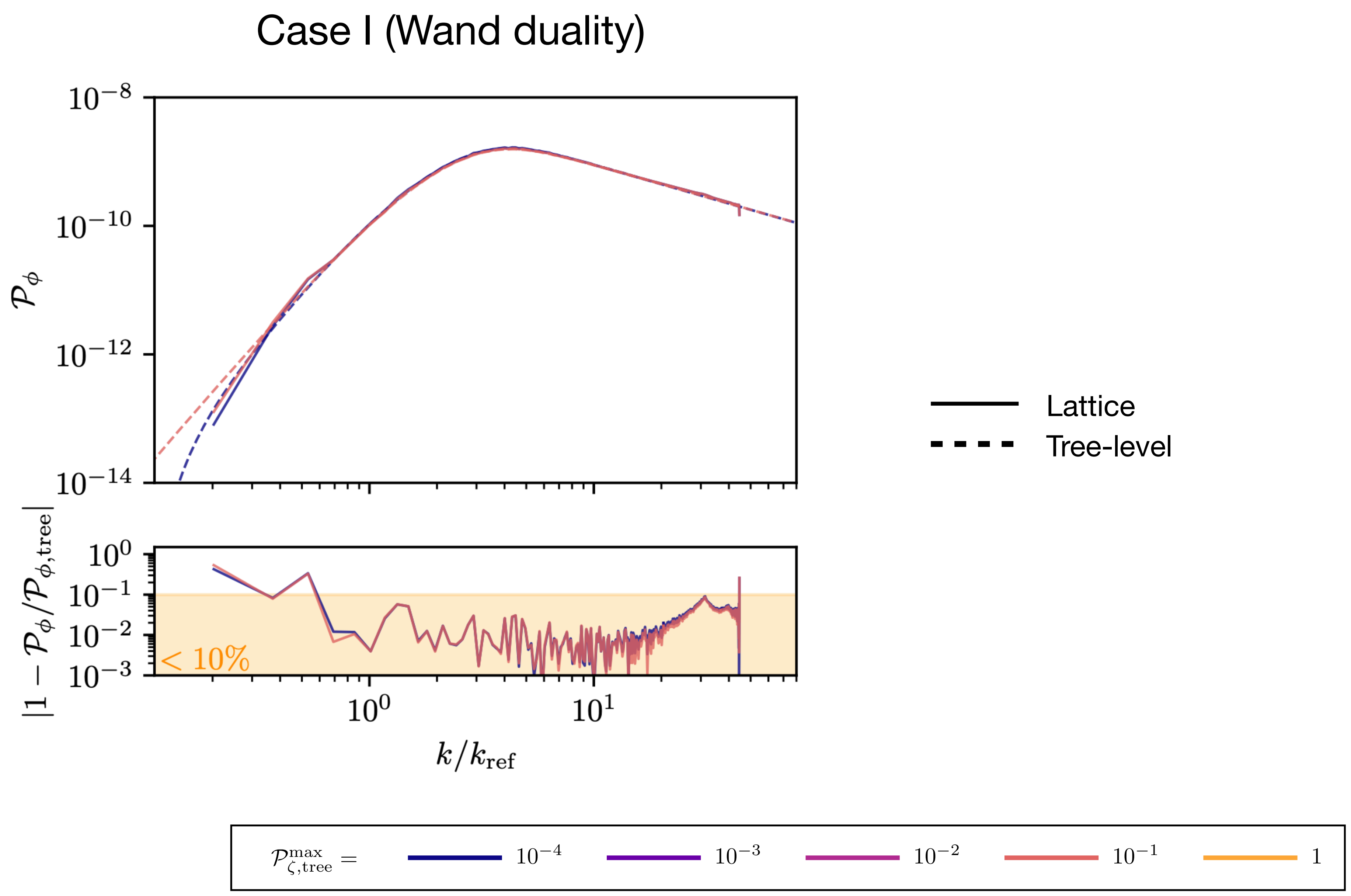
Backreaction follows a simple fitting formula: $\dot{\phi} = \dot{\phi}_{\rm tree}\left(1 + \sqrt{\mathscr{P}^{\rm max}_{\zeta,\rm tree}}\right)$



Ultra-Slow-Roll inflation

Result #2:

How **nonlinearity** affects inflaton fluctuations



Ultra-Slow-Roll inflation

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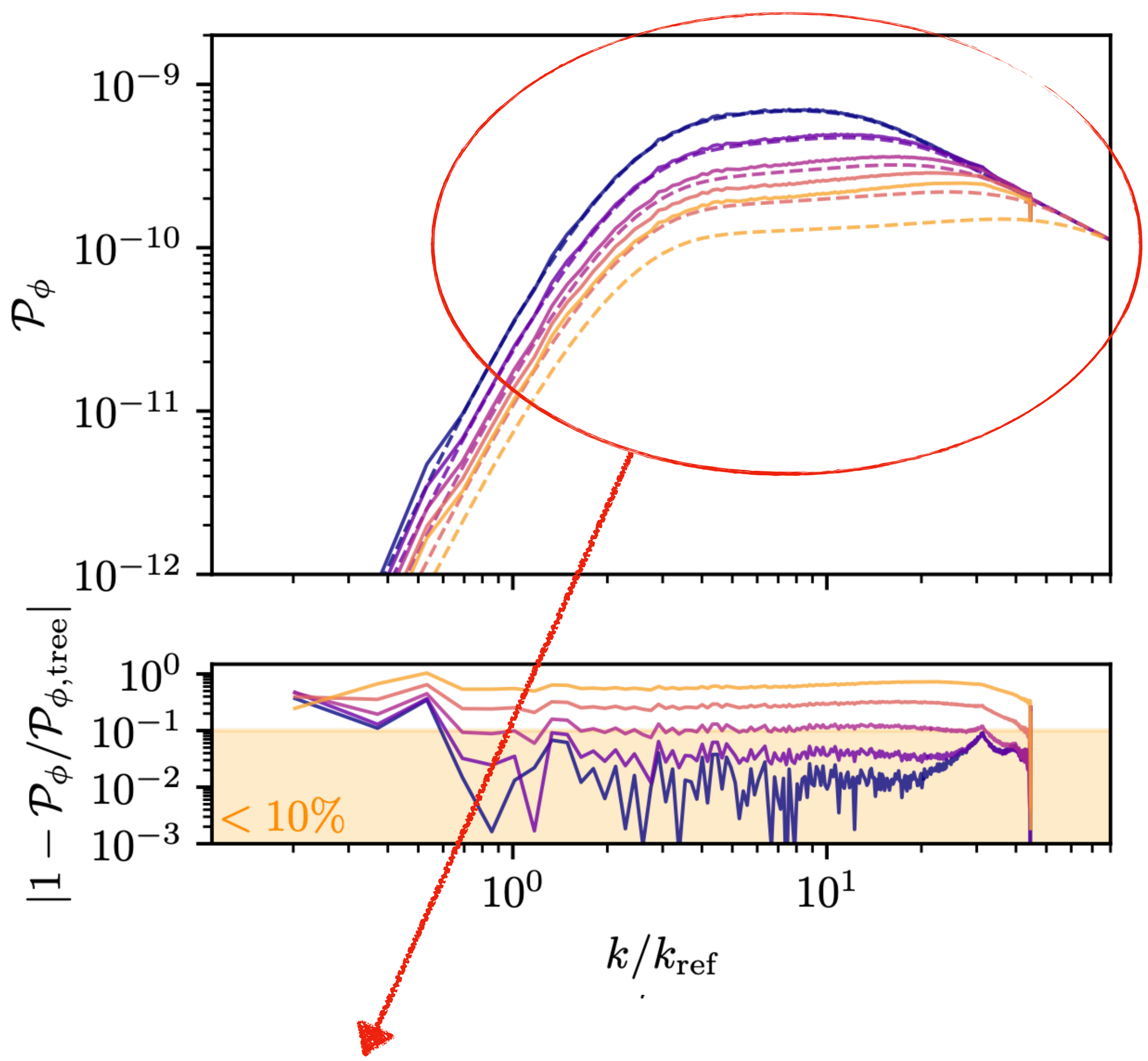
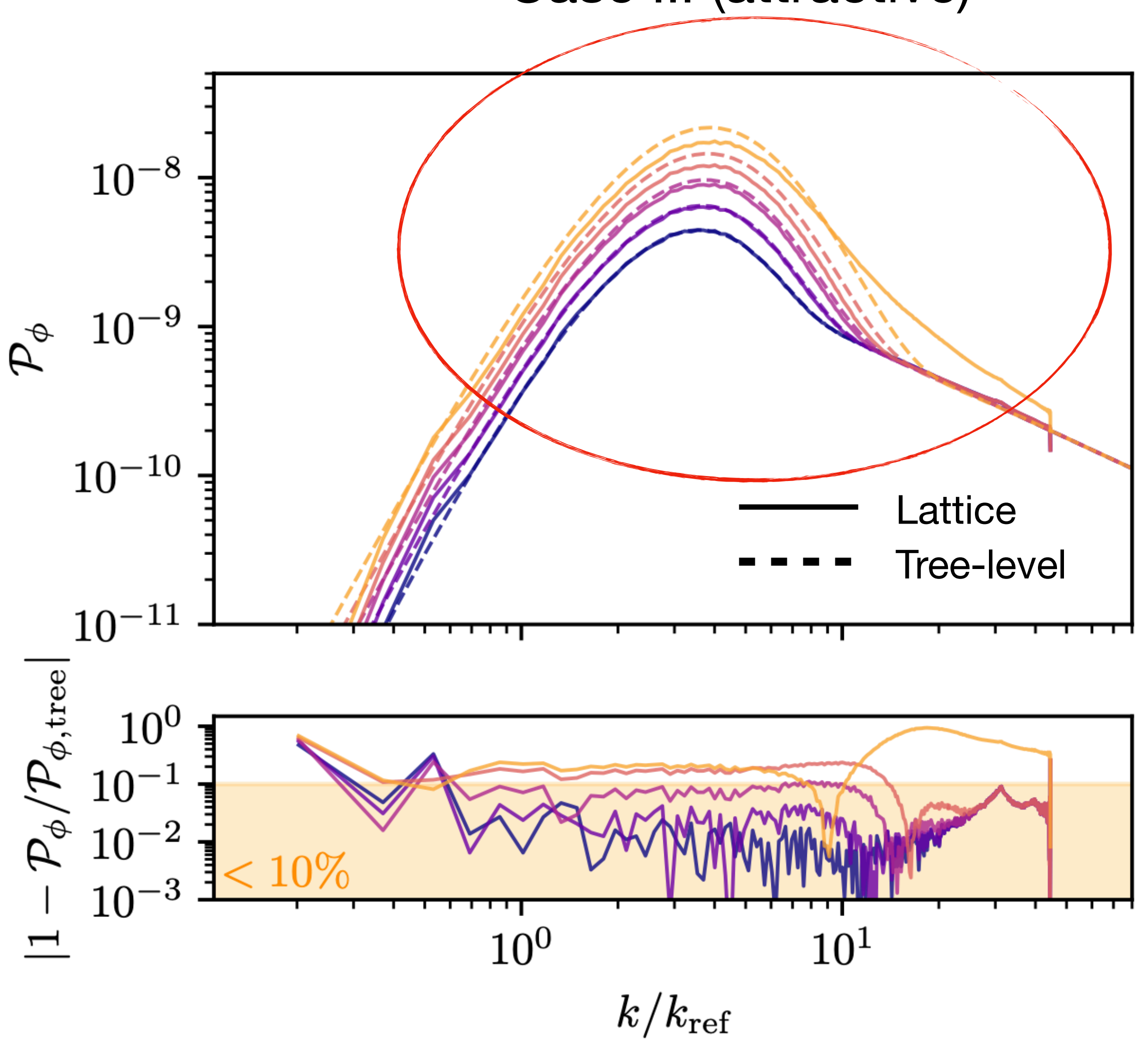
Case II (repulsive)



Case III (attractive)

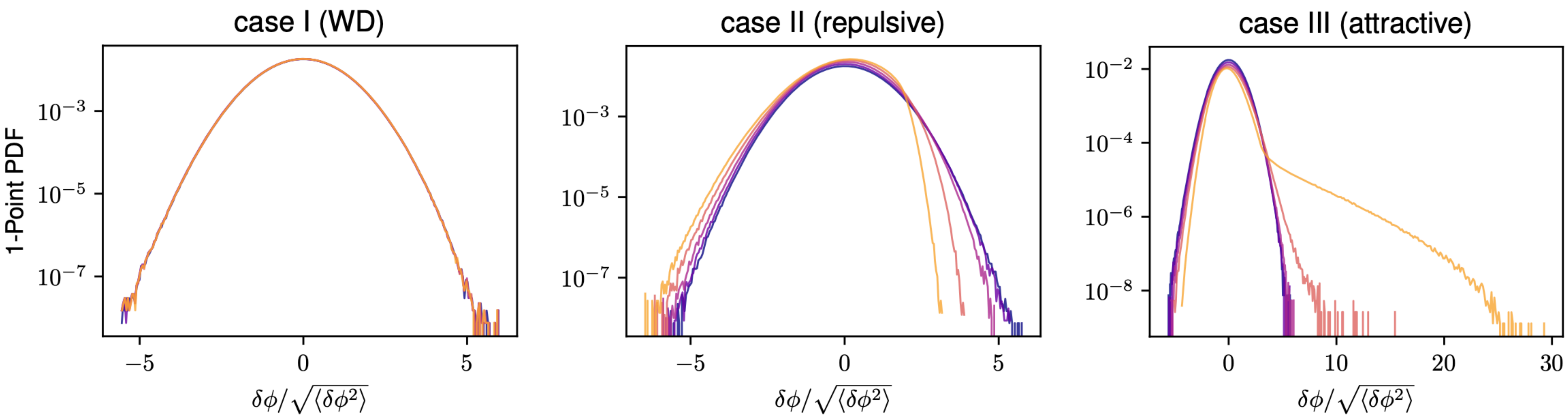


Loop effects

Ultra-Slow-Roll inflation

Result #2:

How **nonlinearity** affects inflaton fluctuations



Ultra-Slow-Roll inflation

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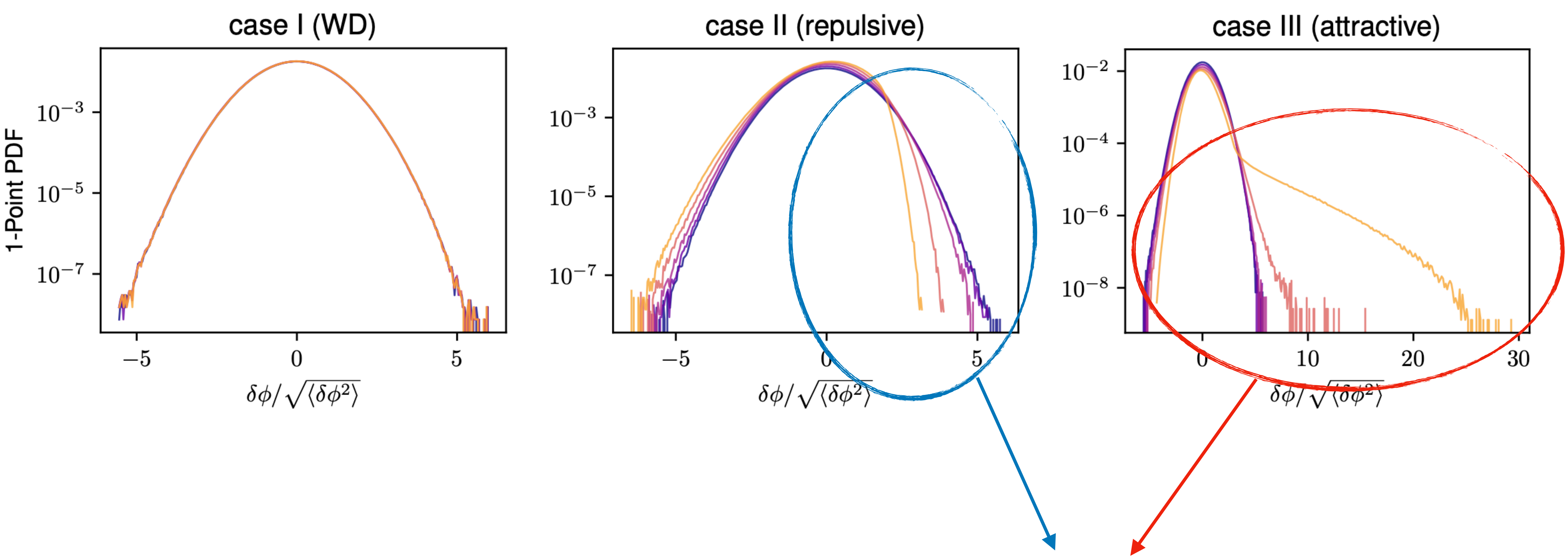
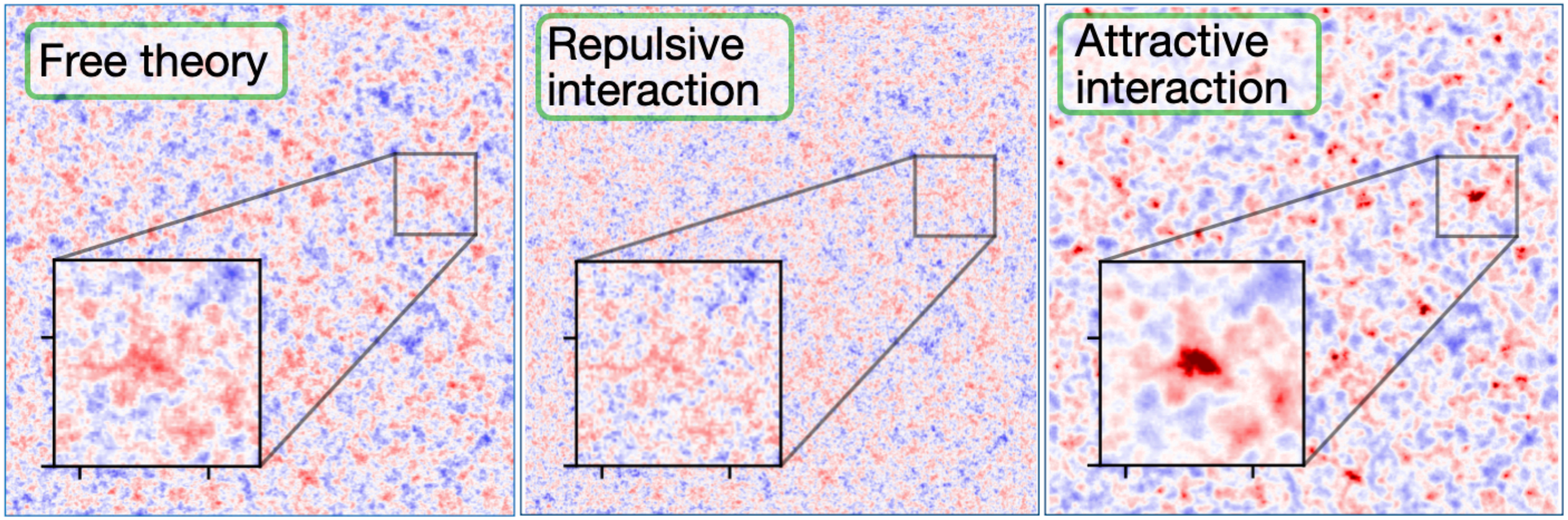


repulsive/**attractive** nature

Ultra-Slow-Roll inflation

Self-interactions matter.

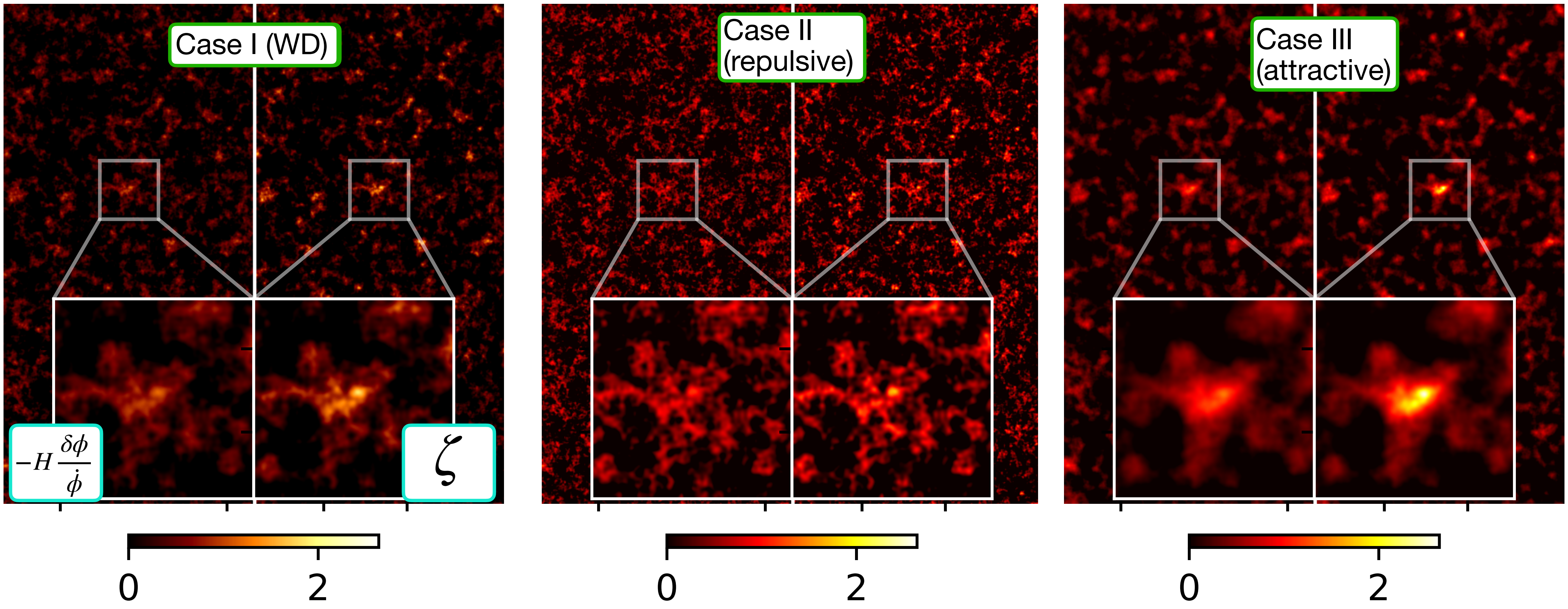
These interactions typically happen on (slightly) sub-horizon scales, and are **neglected by coarse-graining** in stochastic approaches



Ultra-Slow-Roll inflation

So far, we only looked at the inflaton field ϕ

We calculate ζ in a fully nonlinear way using a δN technique applied to simulation data



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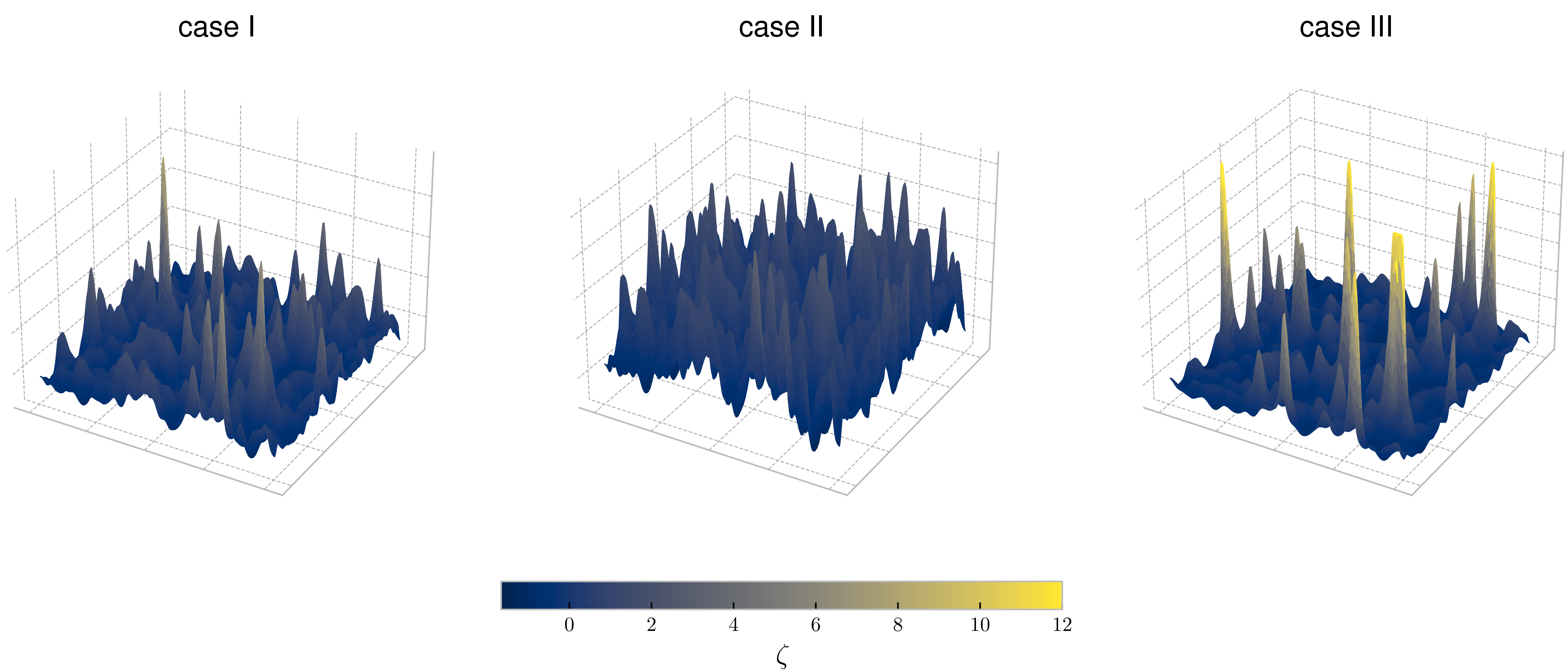

Ultra-Slow-Roll inflation

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We calculate ζ in a fully nonlinear way using a δN technique applied to simulation data

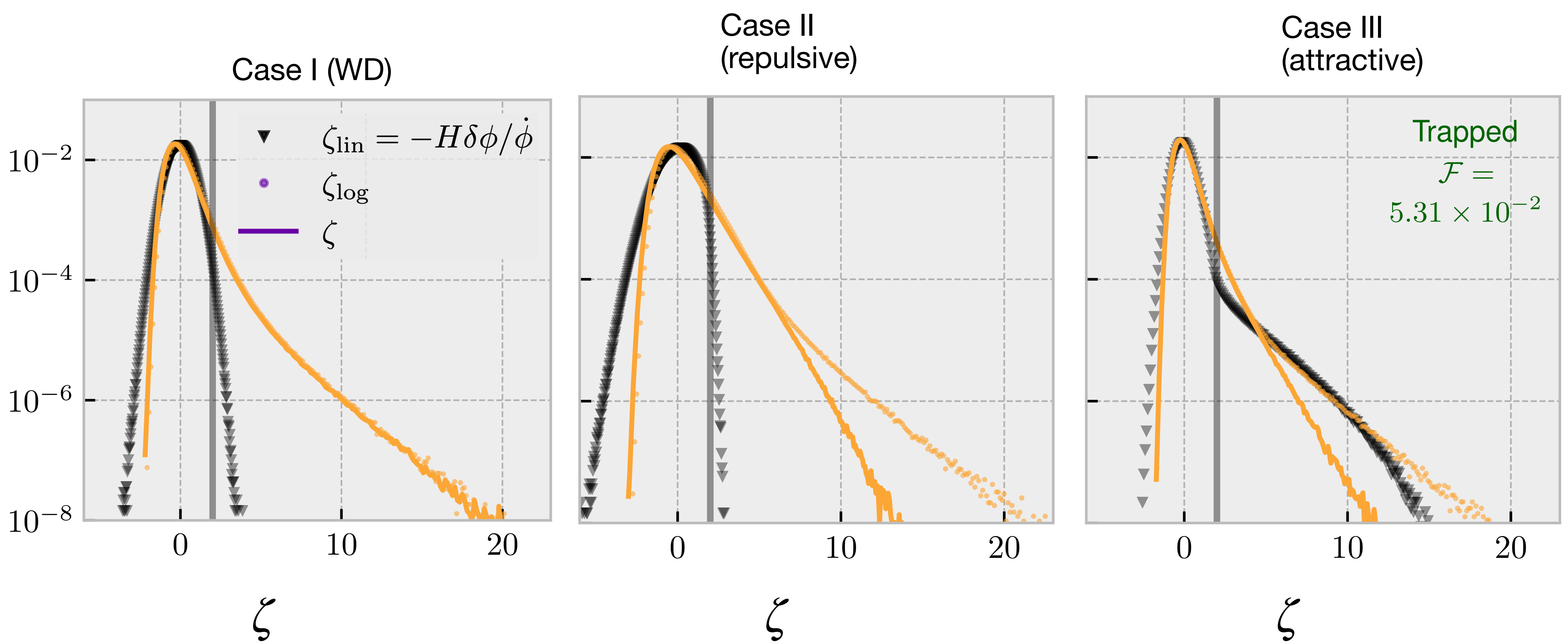
In all our models,
$\eta_{III} =$ constant. →

$$\zeta(\vec{x}) = \frac{1}{\eta} \log\left(1 - \eta\, H \frac{\delta\phi(\vec{x})}{\dot{\phi}}\right)$$

Ultra-Slow-Roll inflation

So far, we only looked at the inflaton field ϕ

It is interesting to see how the logarithmic relation breaks for very large fluctuations:



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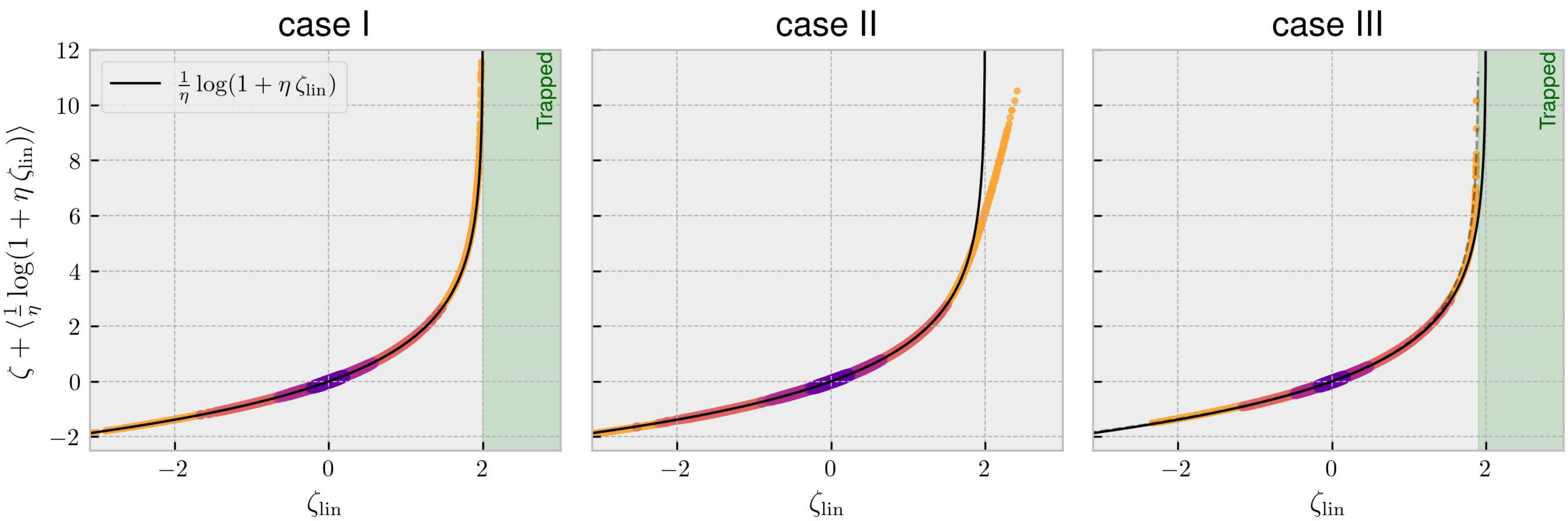

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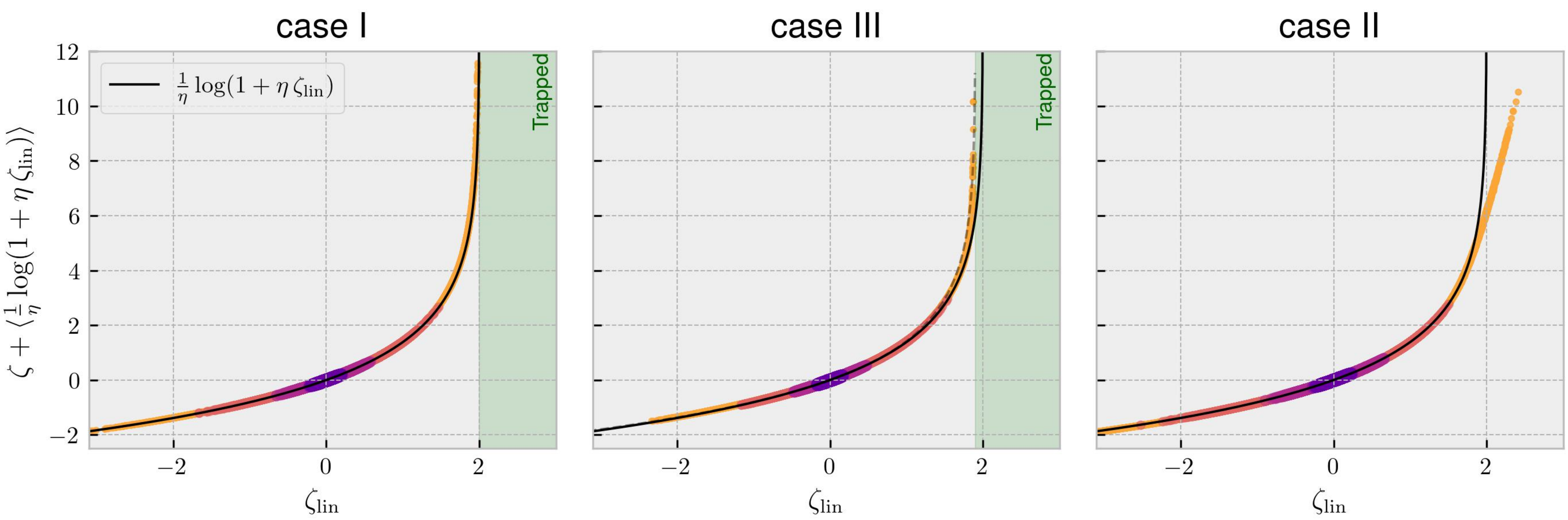


What goes wrong with the log relation: **nonlinear $\neq$ nonperturbative**

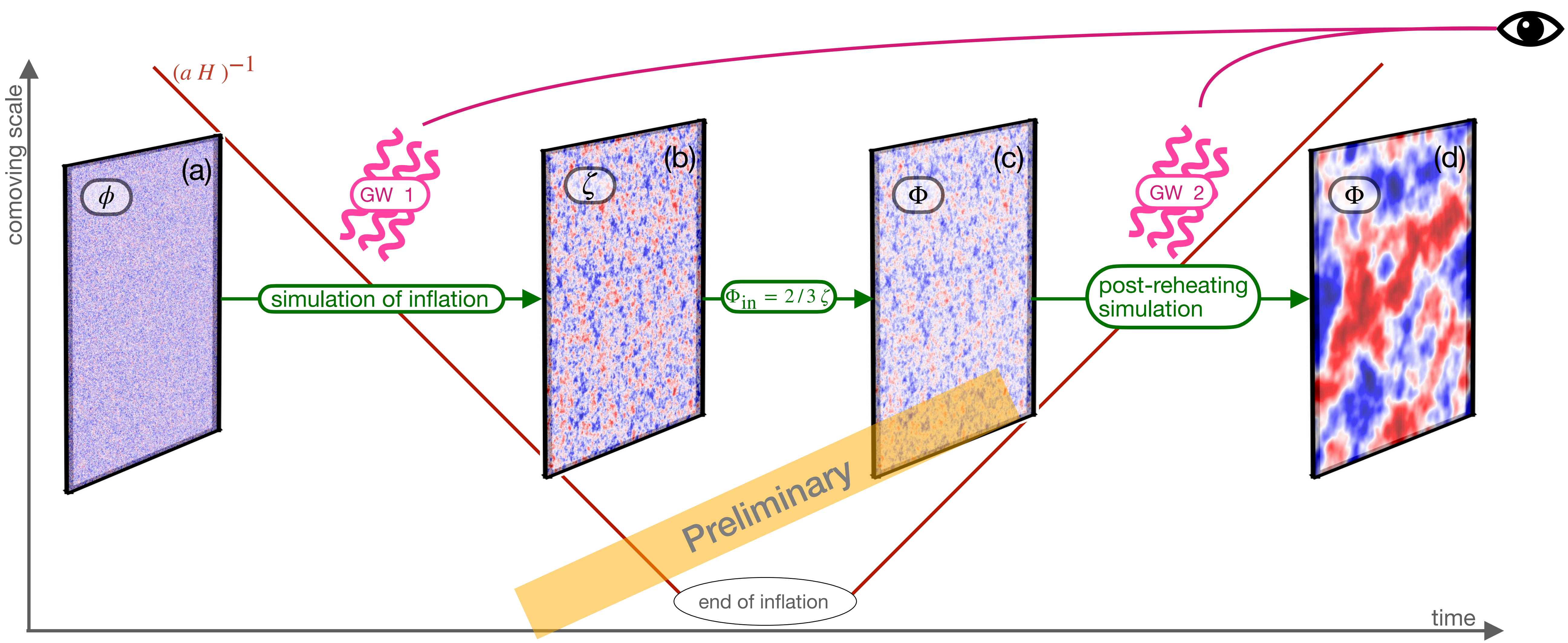
The notion of a unique background is lost

AC, G. Franciolini, S. Renaux-Petel, A. Riotto **[in prep.]**

Ultra-Slow-Roll inflation

Some ongoing work:

Fully nonlinear calculations of GWs from inflation:



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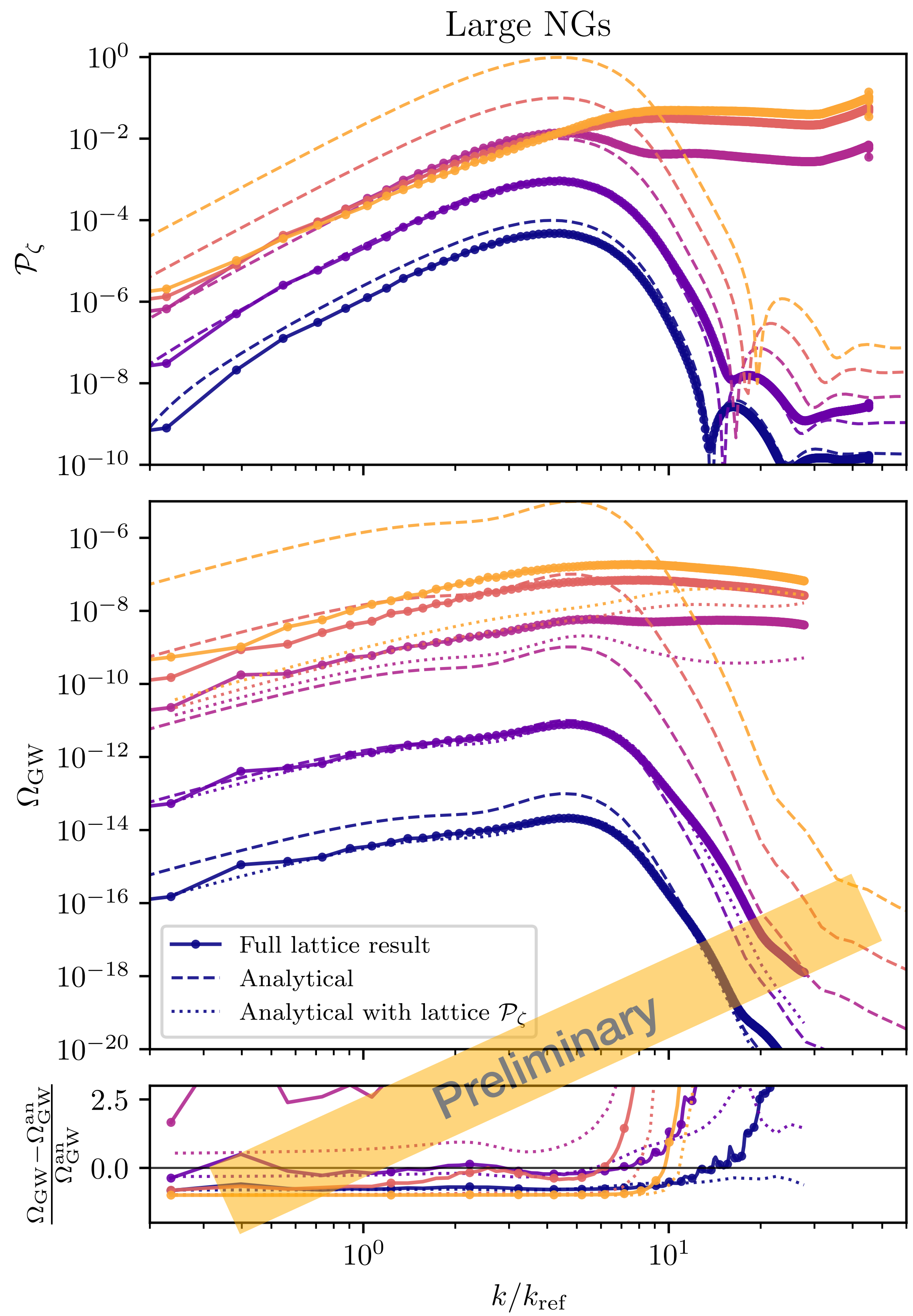
Stay tuned!



Summary

- Lattice simulations of inflation are a **new technique**, made publicly available

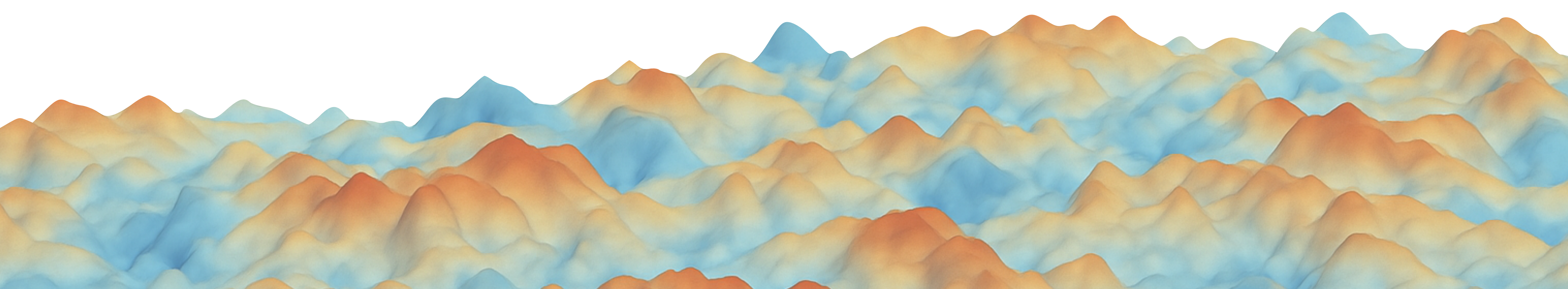
- We finally know what happens **when perturbation theory breaks** down in inflation.

  Applied to the small scale physics of inflation, but there are a lot of other applications!

- What's next?

  - Develop techniques to calculate **measurable quantities** directly from the simulation (e.g GW spectrum).

  - Explore more models, e.g. **multi-field** inflation

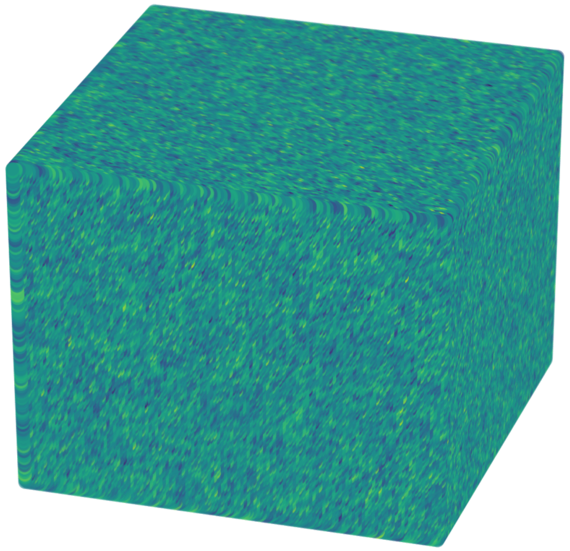



Thank you for the attention!

Backup slides

Lattice simulation: initial conditions

[AC 2209.13616]



- $$\hat{\phi}(\vec{n}) = \sum_{\vec{m}} \left[\hat{a}_{\vec{m}}\, u(\vec{\kappa}_{\vec{m}})\, e^{i\frac{2\pi}{N}\vec{n}\cdot\vec{m}} + \hat{a}^{\dagger}_{\vec{m}}\, u^{\dagger}(\vec{\kappa}_{\vec{m}})\, e^{-i\frac{2\pi}{N}\vec{n}\cdot\vec{m}}\right]$$

  $\vec{n}$ = lattice site, $\quad n_i, m_i \in 1,\ldots,N$. $\quad \vec{\kappa}_{\vec{m}} = \frac{2\pi}{L}\vec{m}$

- Discrete Bunch-Davies spectrum: [AC+ 2102.06378]

  $$u(\vec{\kappa}) = \frac{L^{3/2}}{a\sqrt{2\omega_{\vec{\kappa}}}} e^{-i\omega_{\vec{\kappa}}\tau}, \qquad \omega^2_{\vec{\kappa}} = k^2_{\text{eff}}(\vec{\kappa}) + m^2$$

  (discrete dispersion relation)

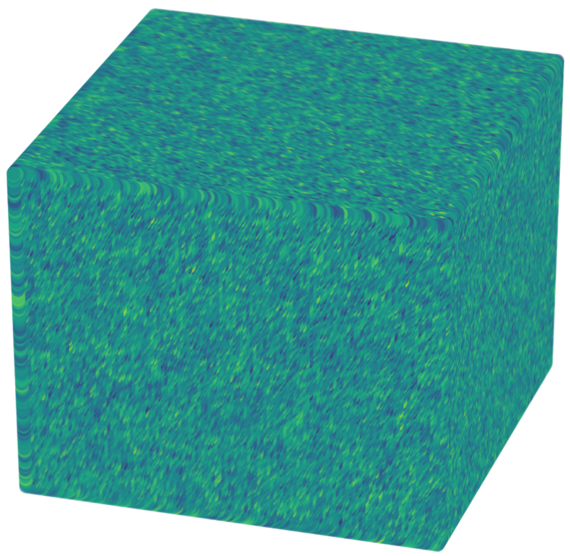
- Stochastic approximation:

  $$\hat{a}_{\vec{m}} = e^{i2\pi\hat{Y}_{\vec{m}}}\sqrt{-\ln(\hat{X}_{\vec{m}})/2},$$

  $\hat{X}_{\vec{m}}, \hat{Y}_{\vec{m}}$ uniform randoms between 0 and 1

Lattice approach: initial conditions

[AC 2209.13616]



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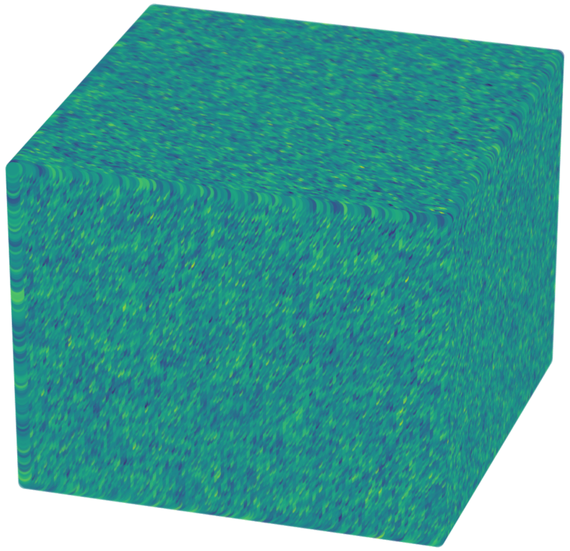

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(discrete dispersion relation)

$$k^2_{\rm eff}(\vec{\kappa}_{\vec{m}}) = \frac{4}{(dx)^2}\left[\sin^2\left(\frac{\pi m_1}{N}\right) + \sin^2\left(\frac{\pi m_2}{N}\right) + \sin^2\left(\frac{\pi m_3}{N}\right)\right].$$

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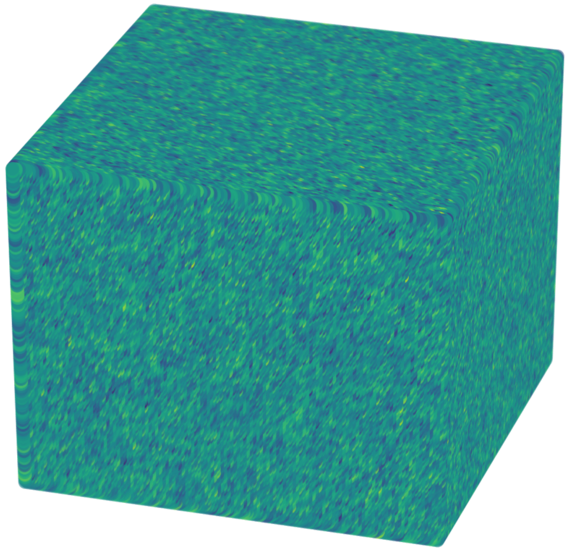
[AC 2209.13616]



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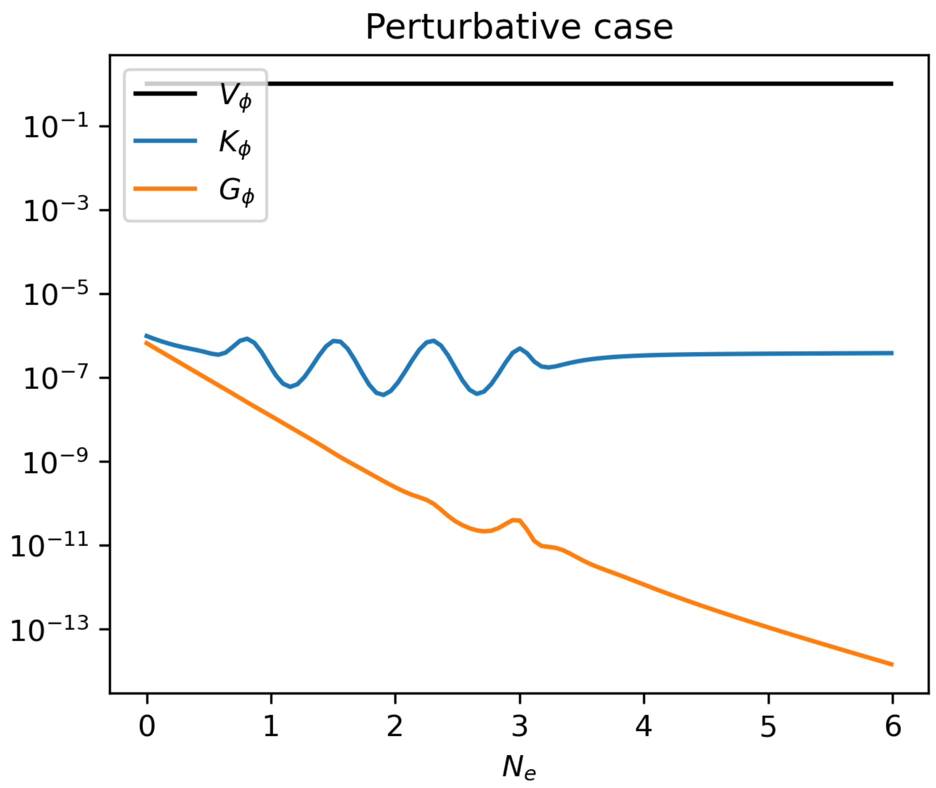
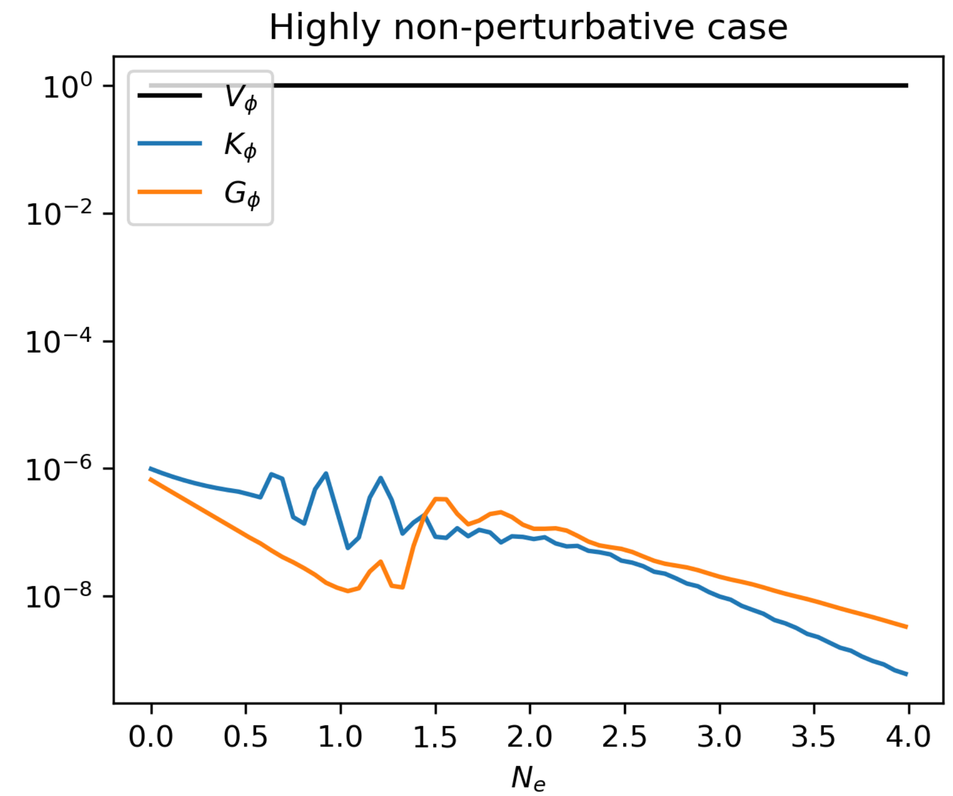
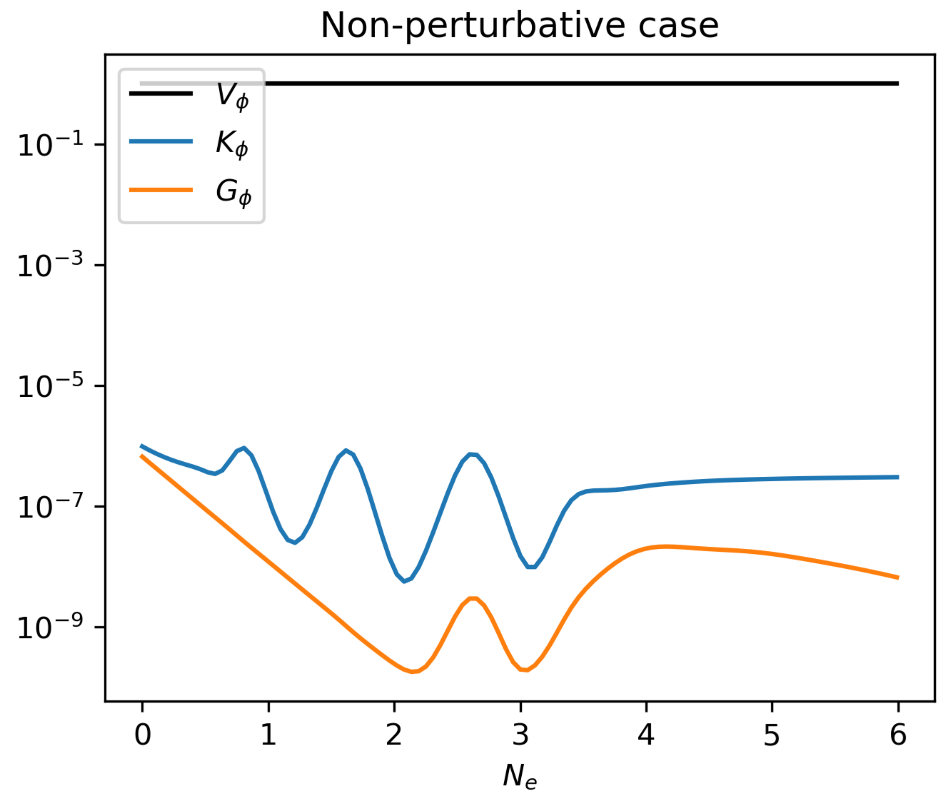
AC, S. Renaux-Petel, K. Inomata [2403.12811]

Energy contributions in oscillatory potentials







Energy contributions in USR

AC, G. Franciolini, S. Renaux-Petel, [2410.23942]