MATTEO BRAGLIA

Inflation 2025, IAP,
December 3, 2025

INFLATIONARY POWER SPECTRA AT ONE-LOOP

MB, Pinol 2504.07926,
MB, Pinol 2504.13136,
MB, Cespedes, Pinol in progress

Funded by
the European Union

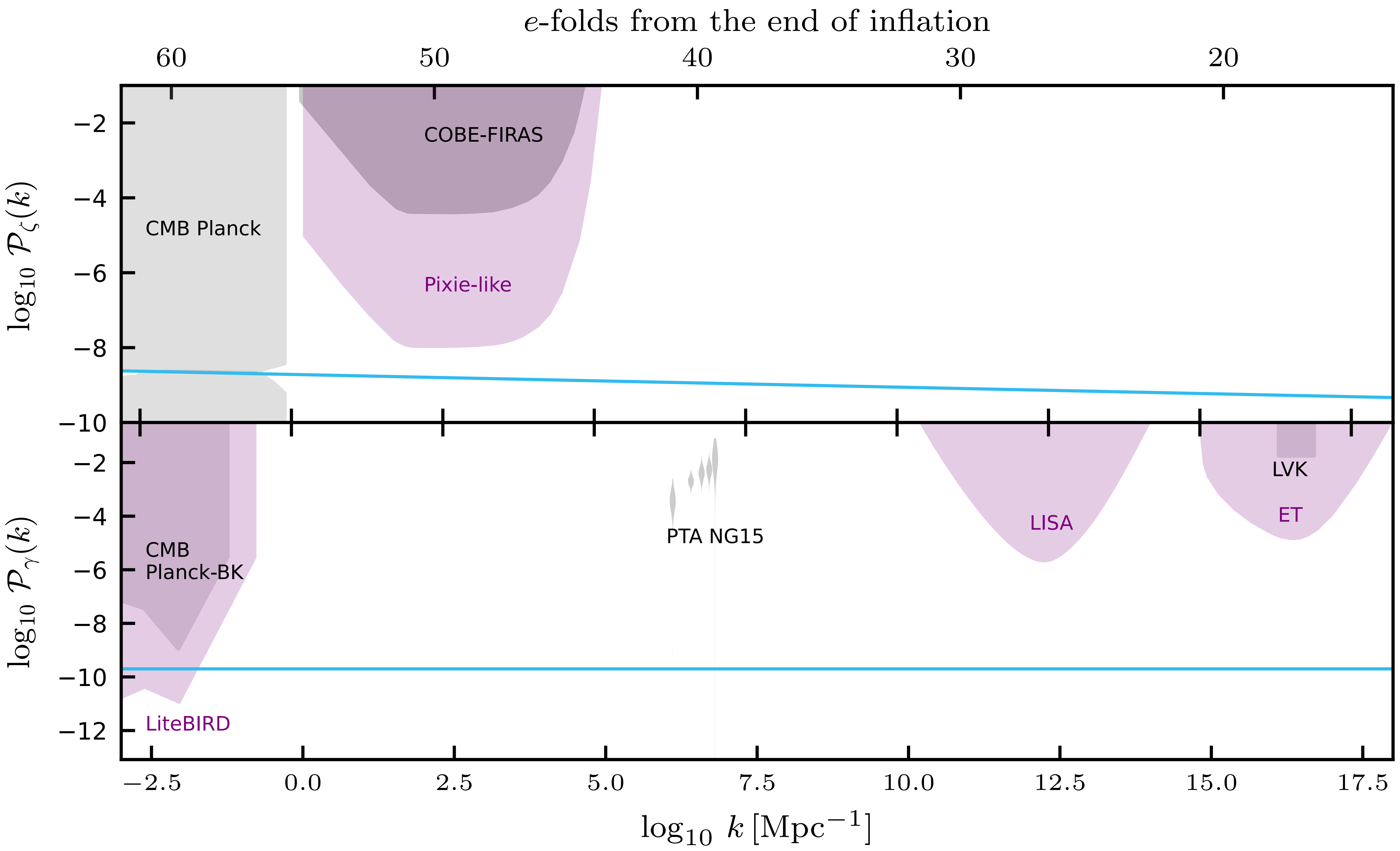
Outline of this talk

- Motivations.
- The Effective Field Theory (EFT) of inflationary fluctuations
- Calculation of 1-loop power spectra in de Sitter
- Preliminary results on scale-dependent scenarios

(MY) MOTIVATIONS

LARGE SCALE CONSTRAINTS

(See Fabio's talk)



$$\epsilon \ll 1, \quad |\eta| \equiv \left|\frac{\dot{\epsilon}}{H\epsilon}\right| \ll 1, \quad |\eta_2| \equiv \left|\frac{\dot{\eta}}{H\eta}\right| \ll 1,$$

$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}, \quad A_s = \frac{H_*^2}{8\pi^2 \epsilon_*}, \quad n_s - 1 = -2\epsilon_* - \eta_*$$

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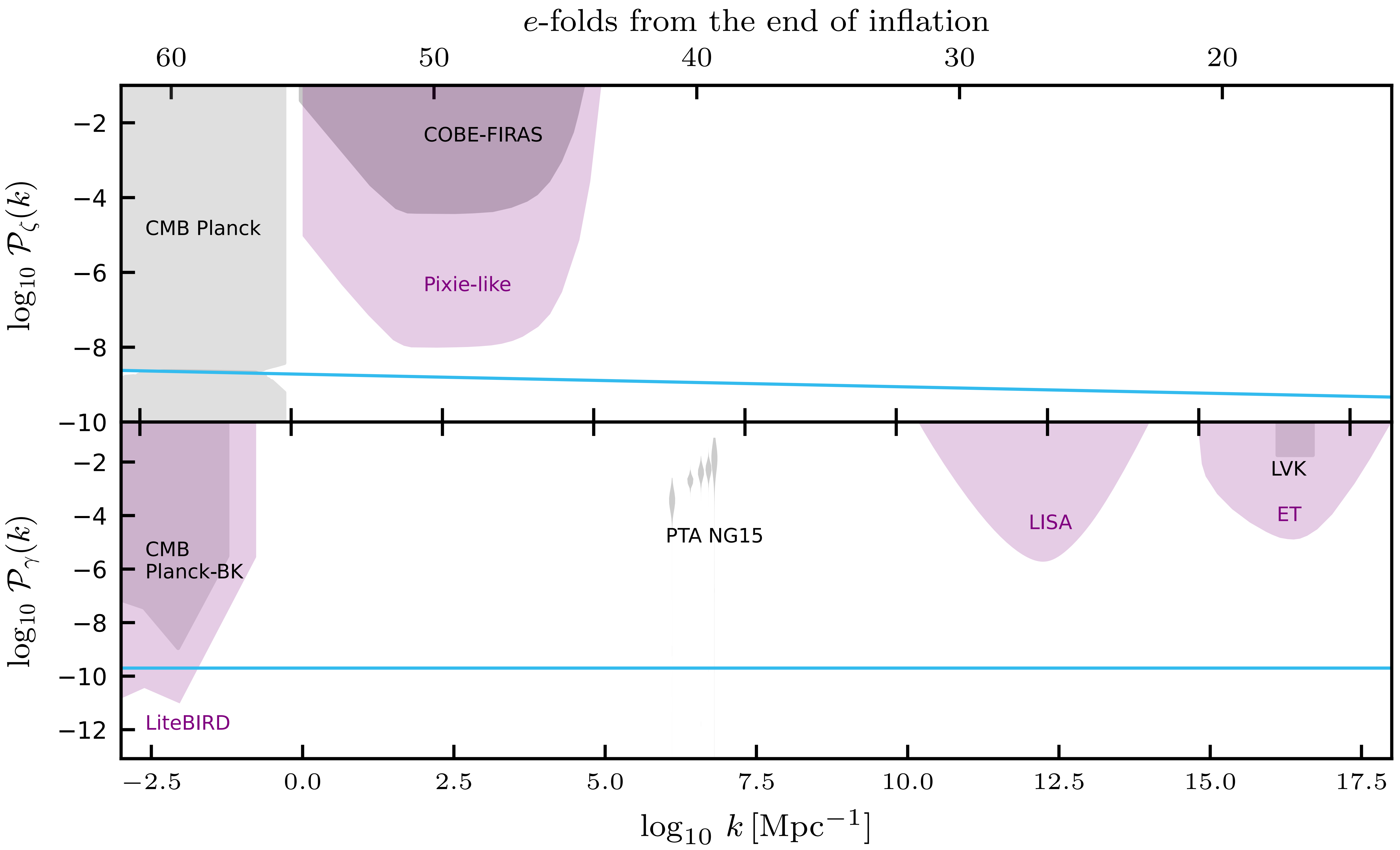

Tree-level predictions (free theory)

$$\bar{\mathcal{H}}^{(2)} = \frac{1}{M_{\rm Pl}^2}\frac{p_\zeta^2}{4a^3\epsilon} + a\epsilon M_{\rm Pl}^2\left(\partial_i\zeta\right)^2$$

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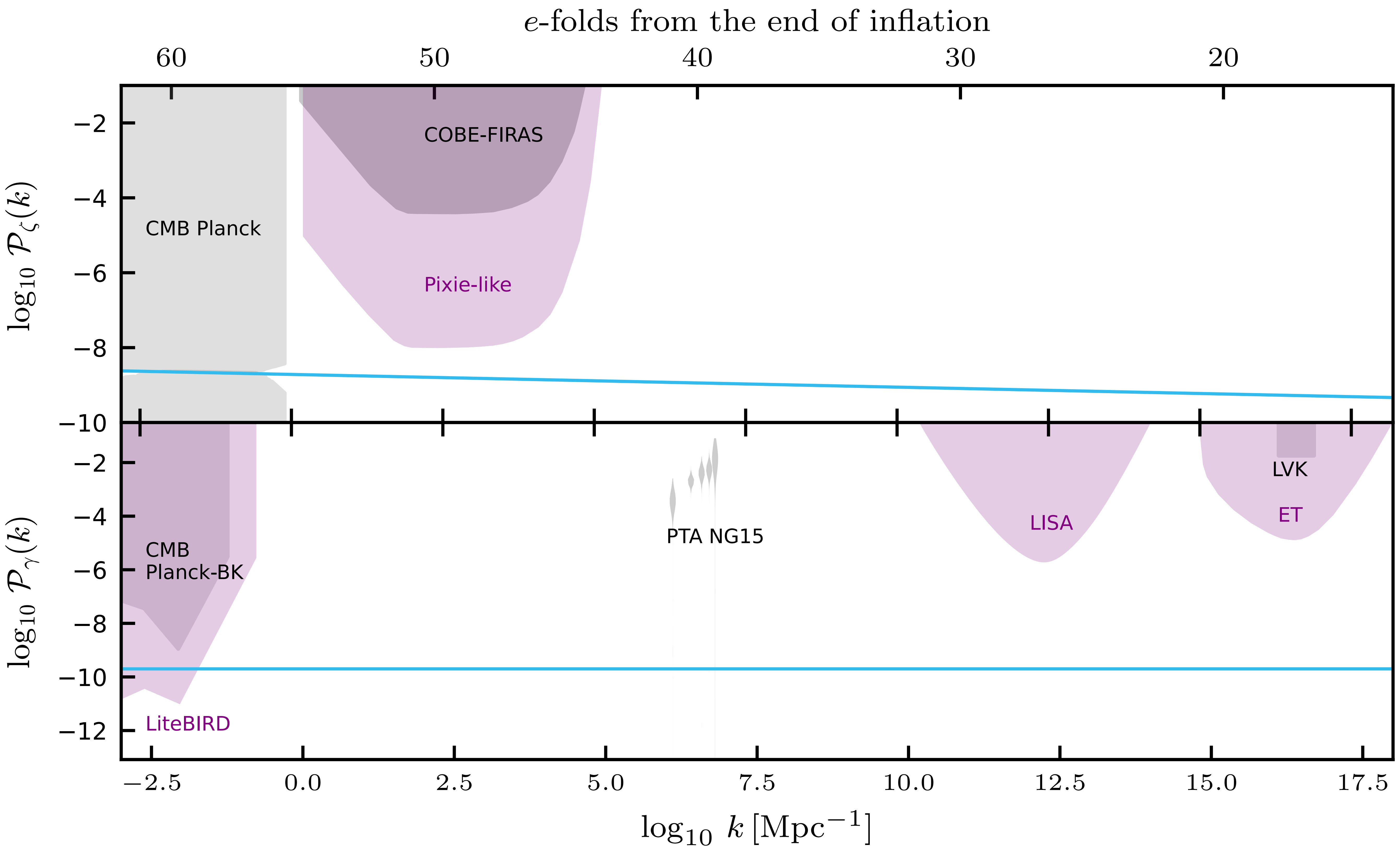

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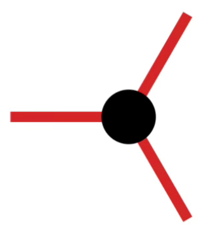
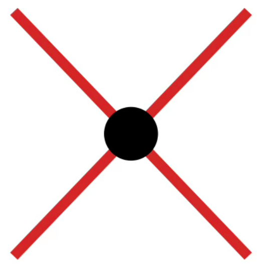
Loop corrections (interactions)

$$\mathcal{H}^{(3)} \propto (\epsilon, \eta)^2 \zeta^3$$



$$\mathcal{H}^{(4)} \propto (\epsilon, \eta, \eta_2)^3 \zeta^4$$



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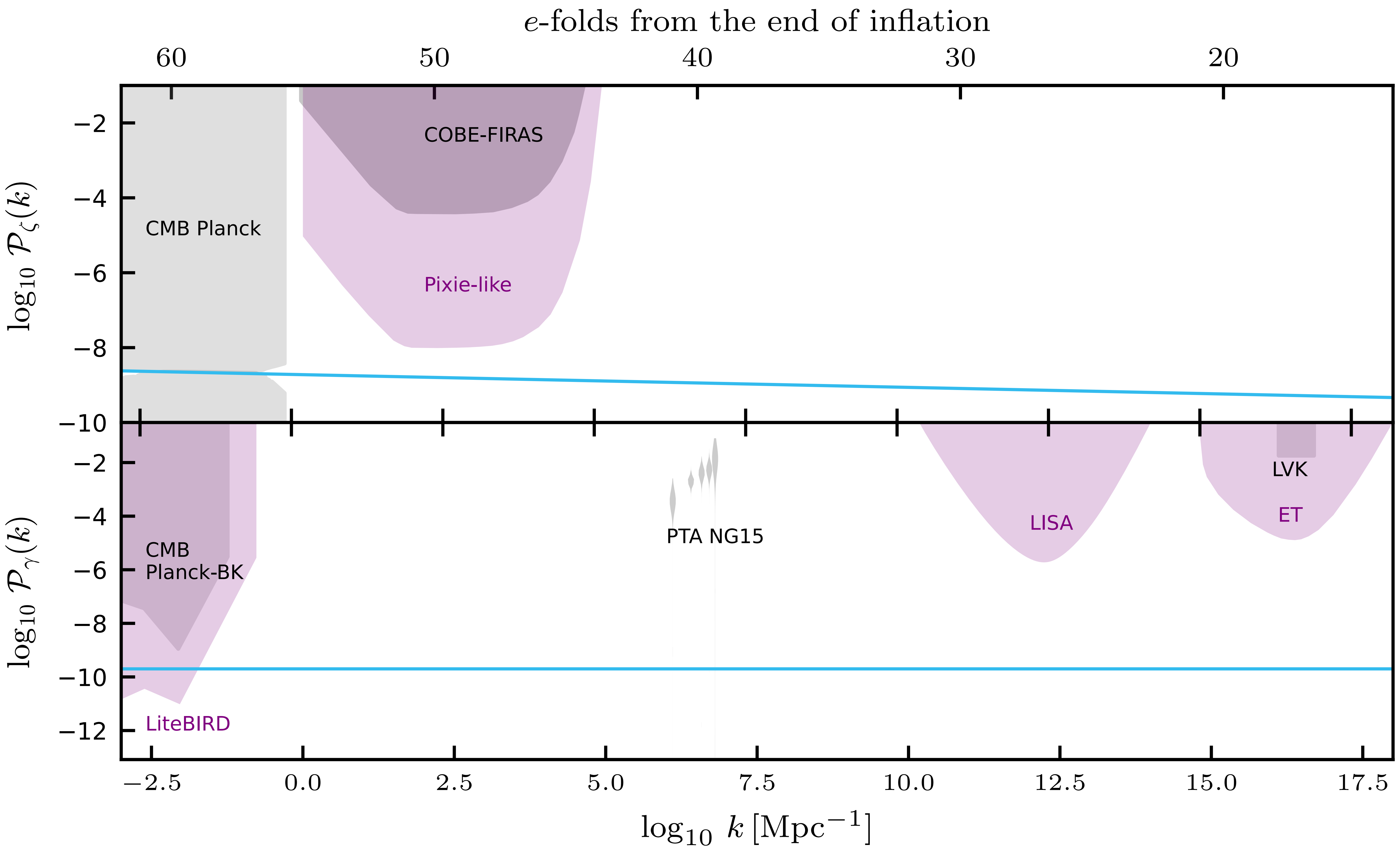

Loop corrections are suppressed by the SR parameters at large scales.

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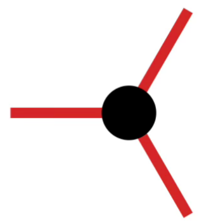

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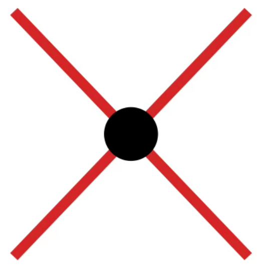
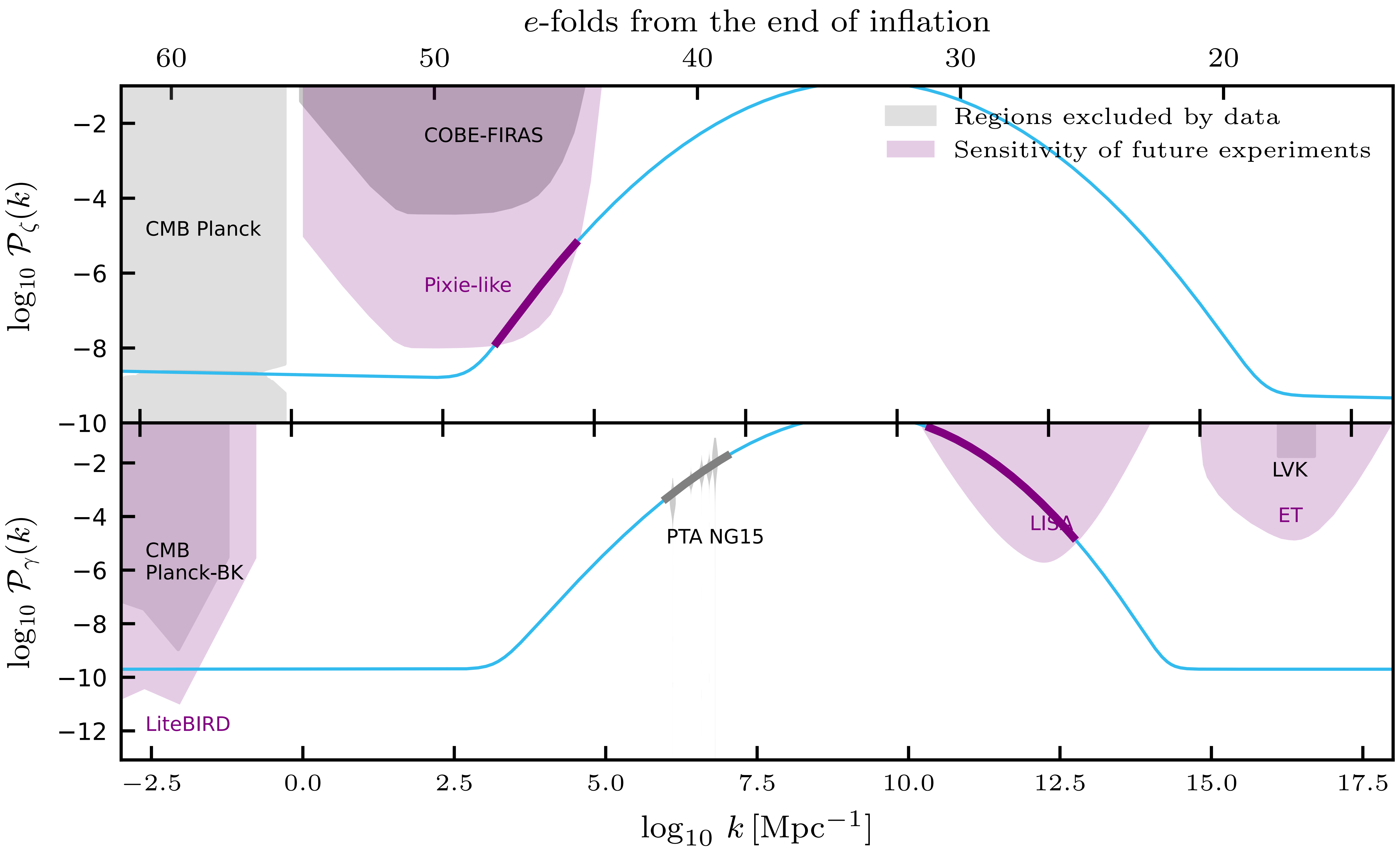


SMALLER SCALES

(See Juan's and Jacopo's talks)



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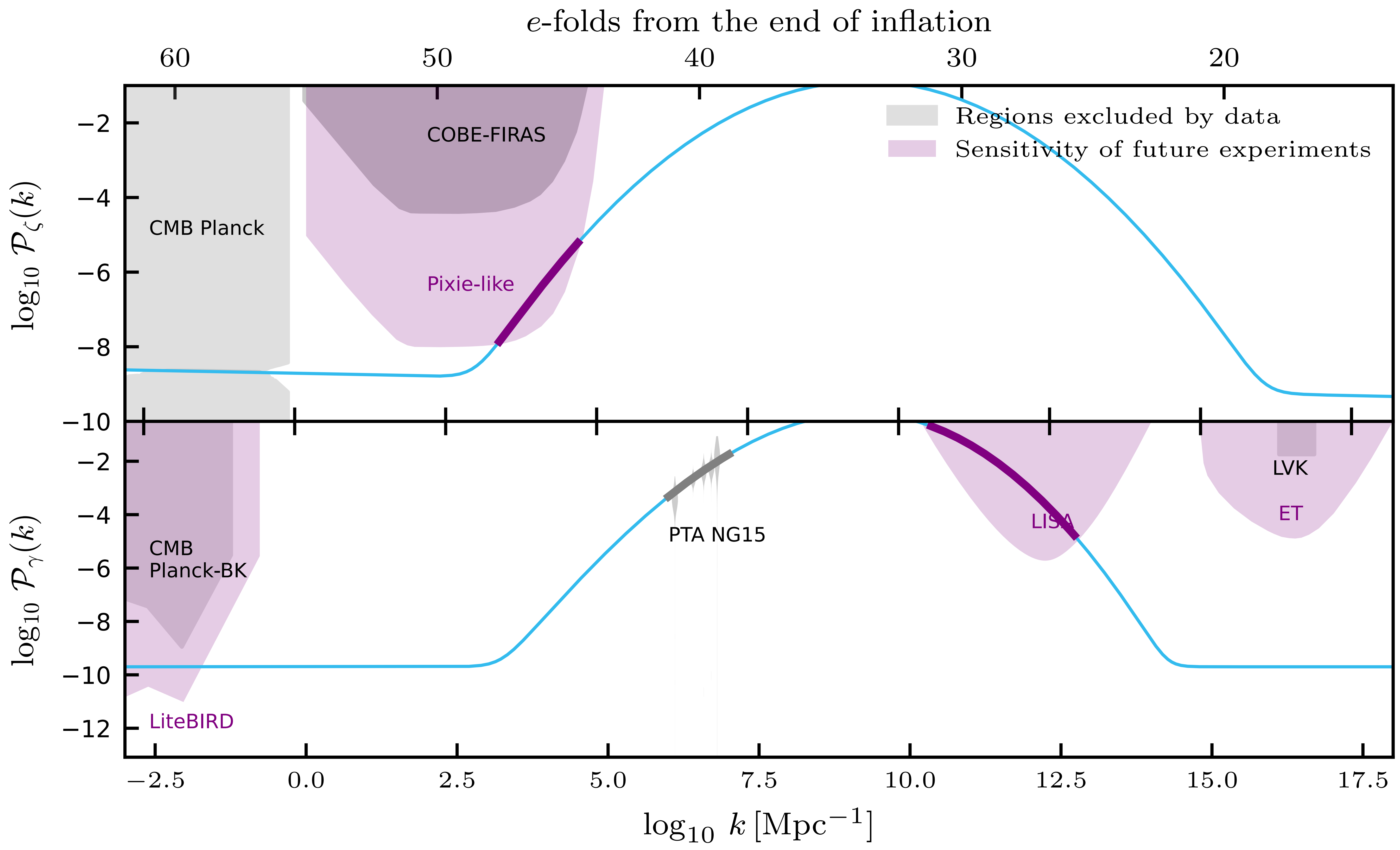

Loop corrections can be enhanced by the SR parameters at small scales.

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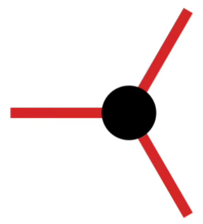

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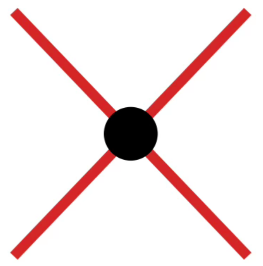
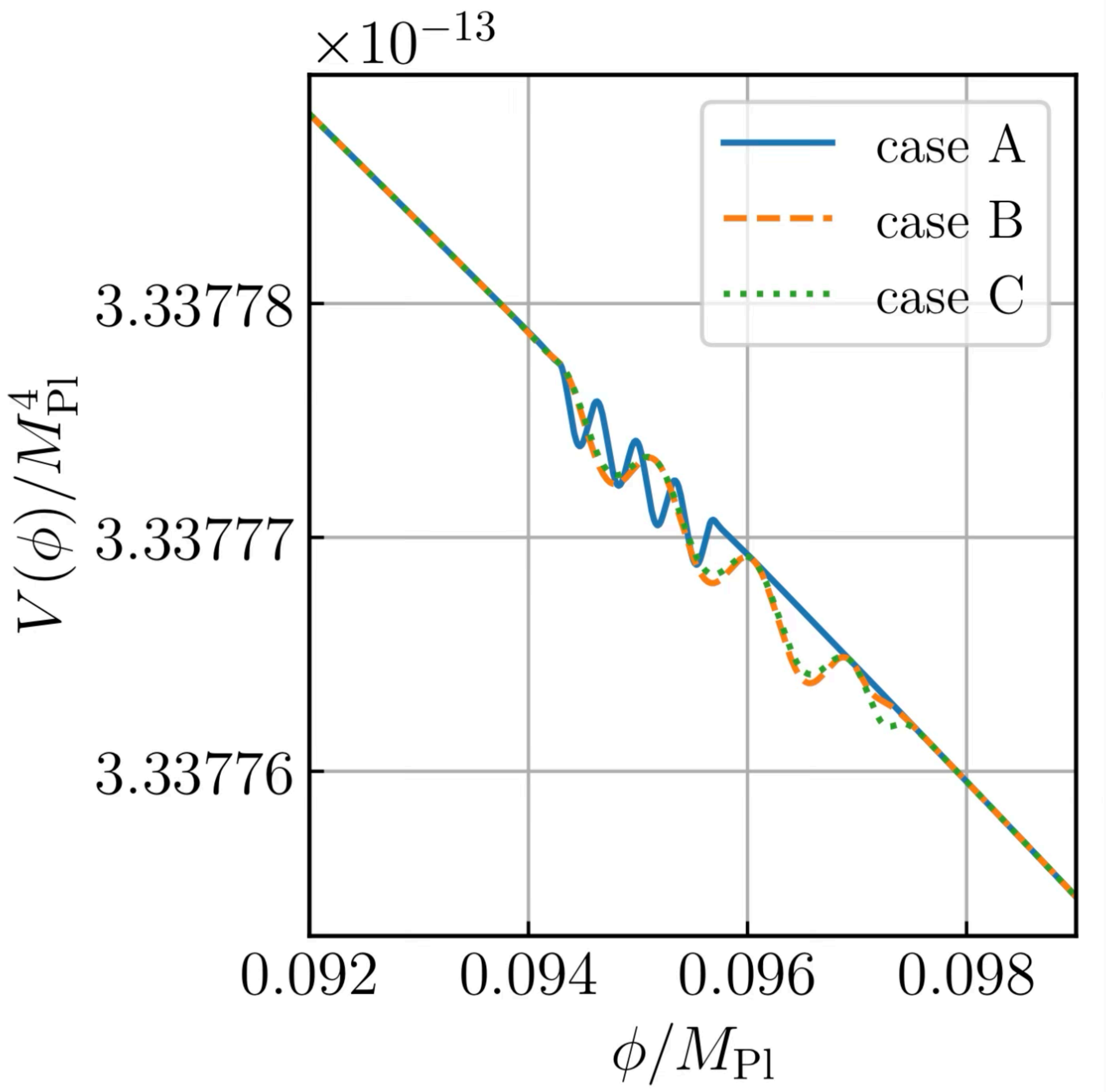
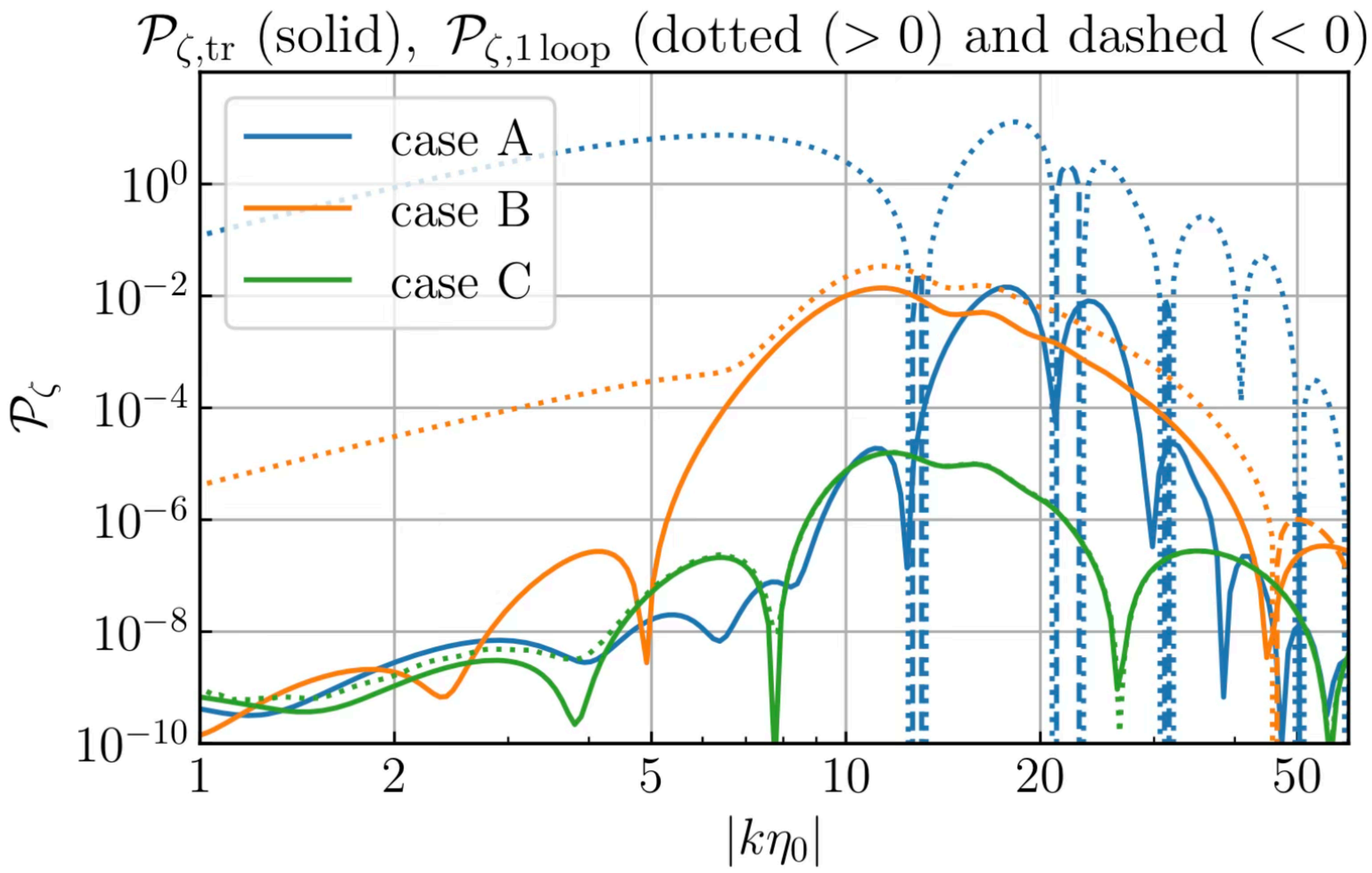


LOOP CORRECTIONS AT SMALL SCALES

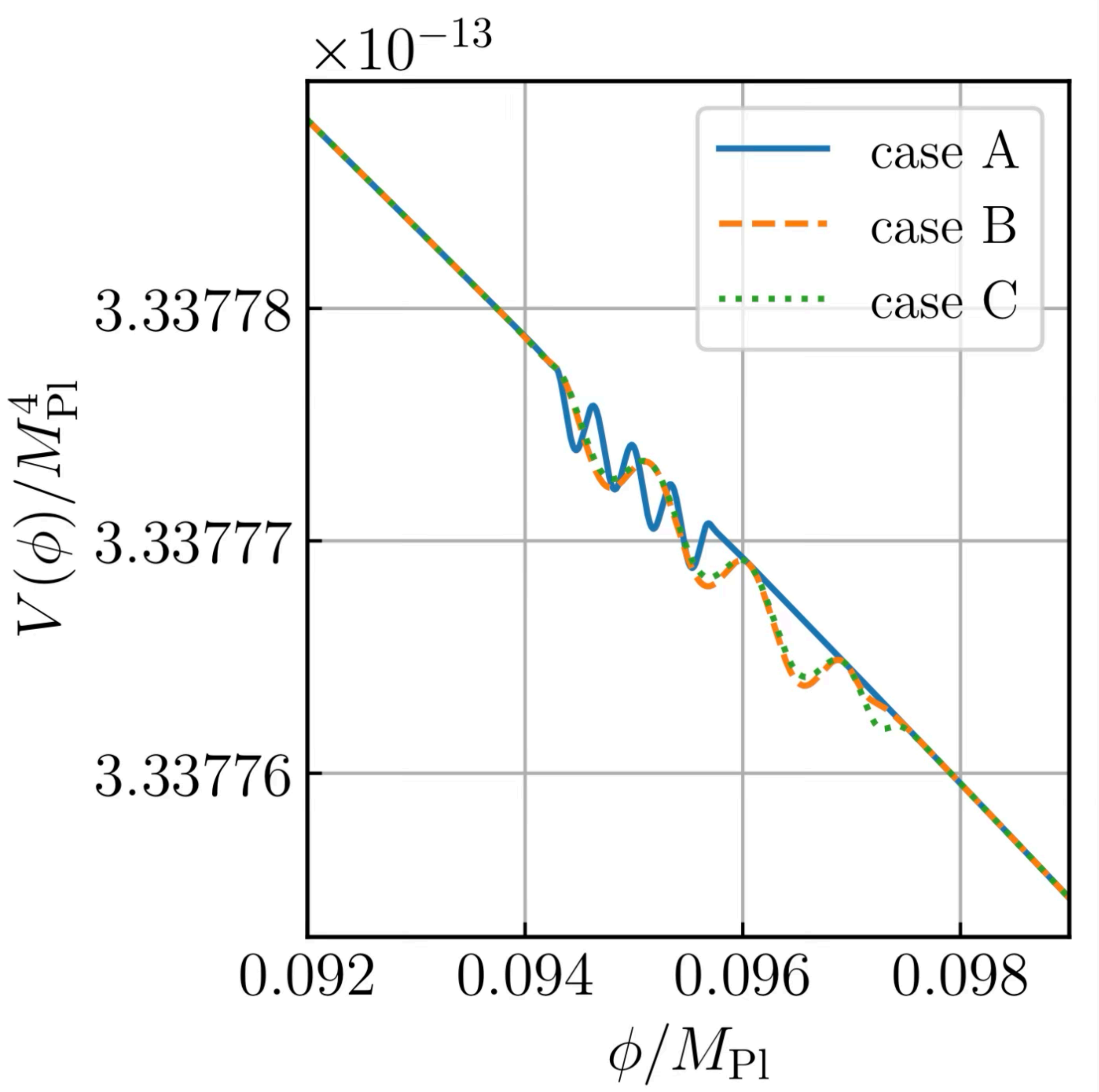


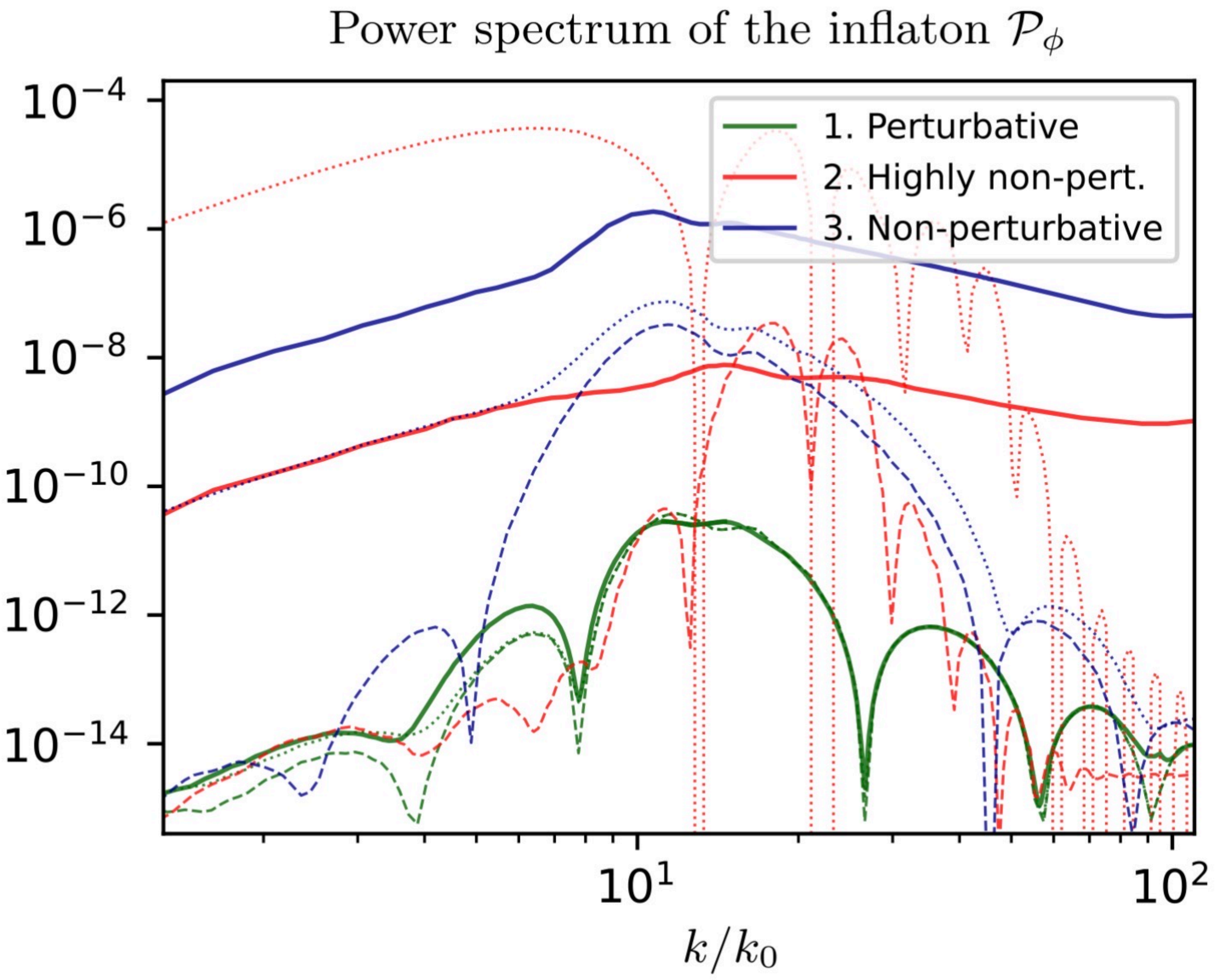


Inomata, **MB**, Chen, Renaux-Petel 2211.02586

We estimated the loop correction to be large *near* the peak in the power spectrum.

LOOP CORRECTIONS AT SMALL SCALES

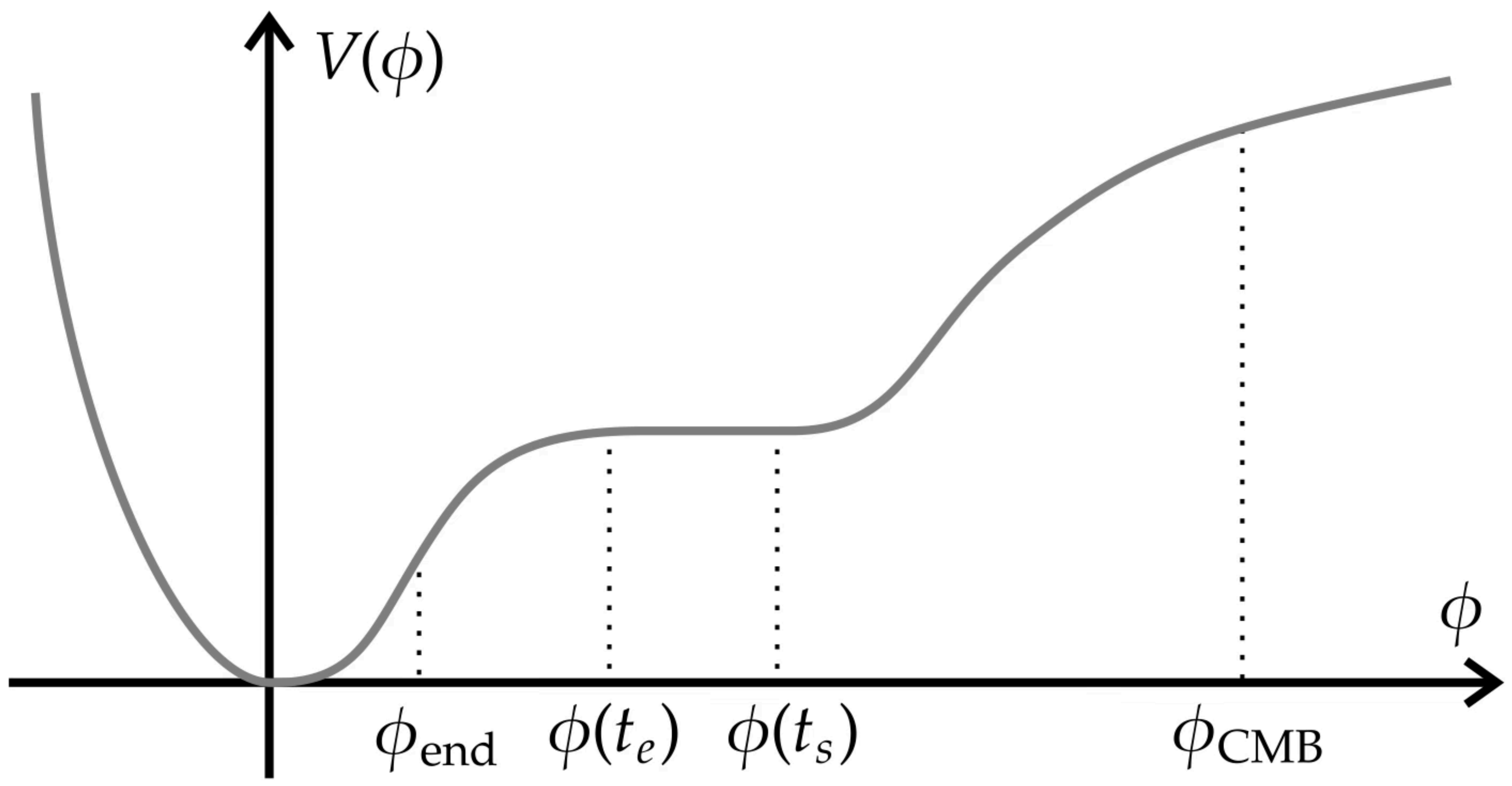


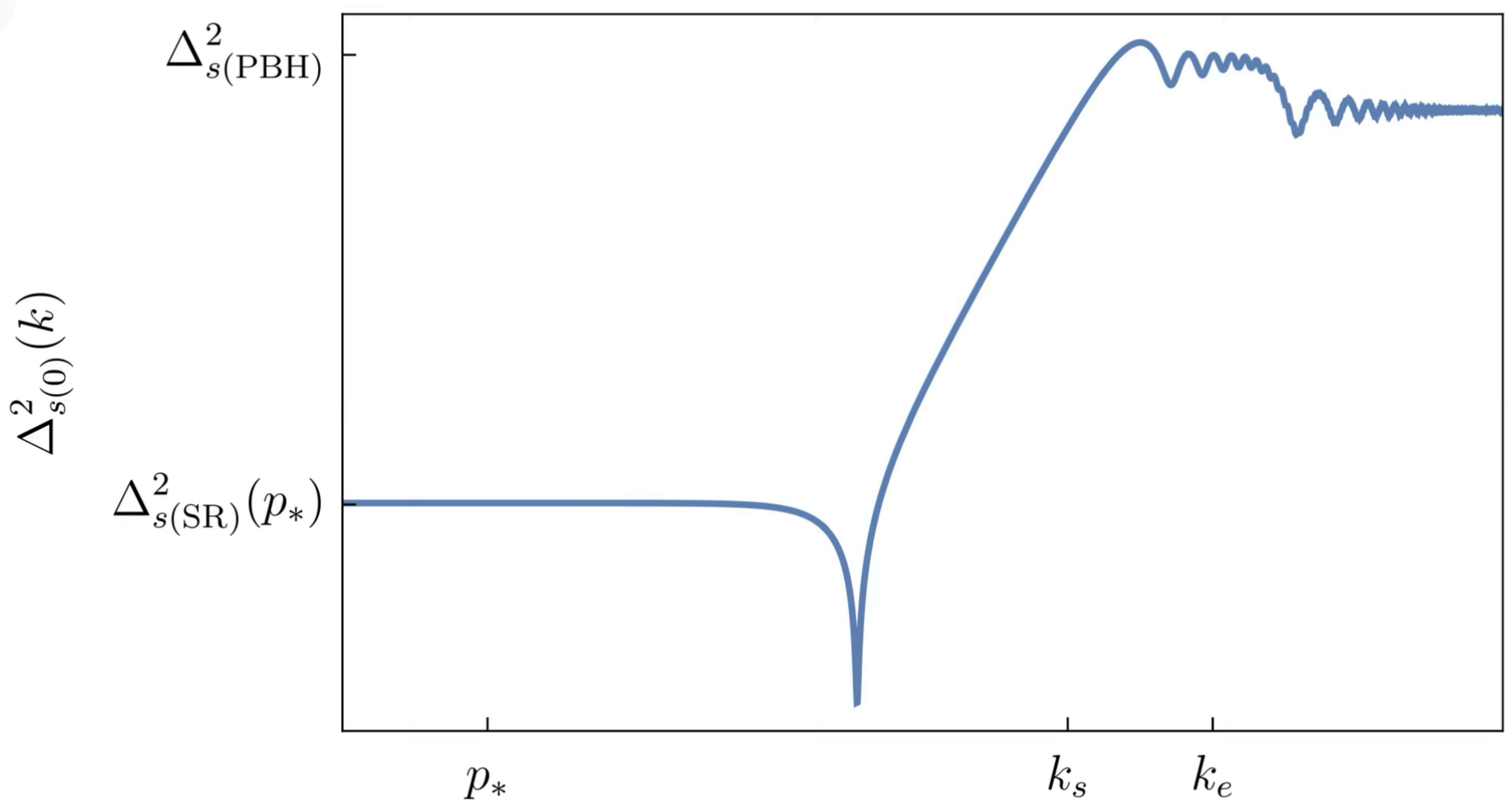


Caravano, Inomata, Renaux-Petel 2403.12811

Lattice simulations confirmed that the fully-nonlinear power spectrum differs from the tree-level on. (See Angelo's talk)

LOOP CORRECTIONS AT LARGE SCALES FROM SMALL SCALES (?)



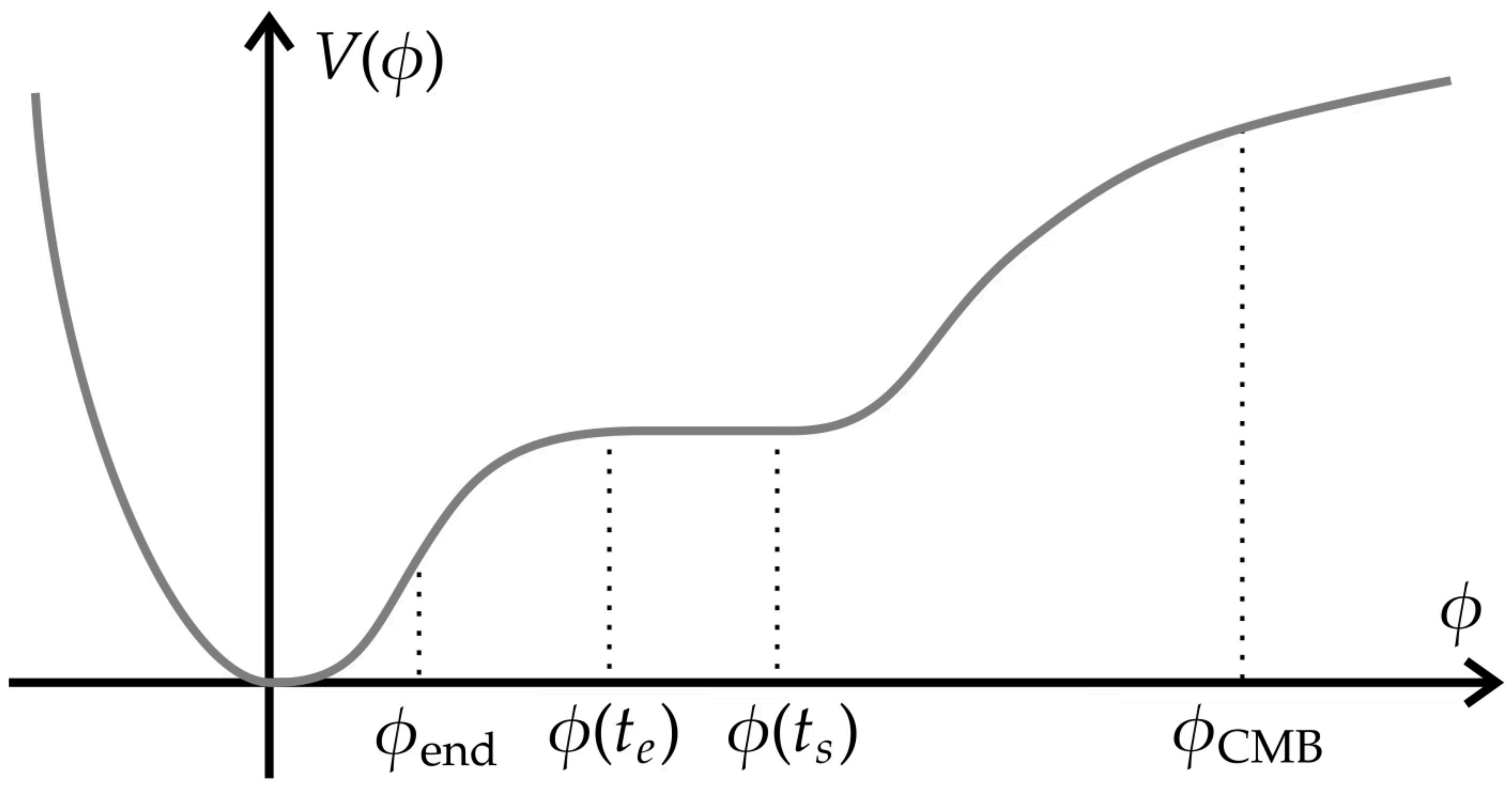


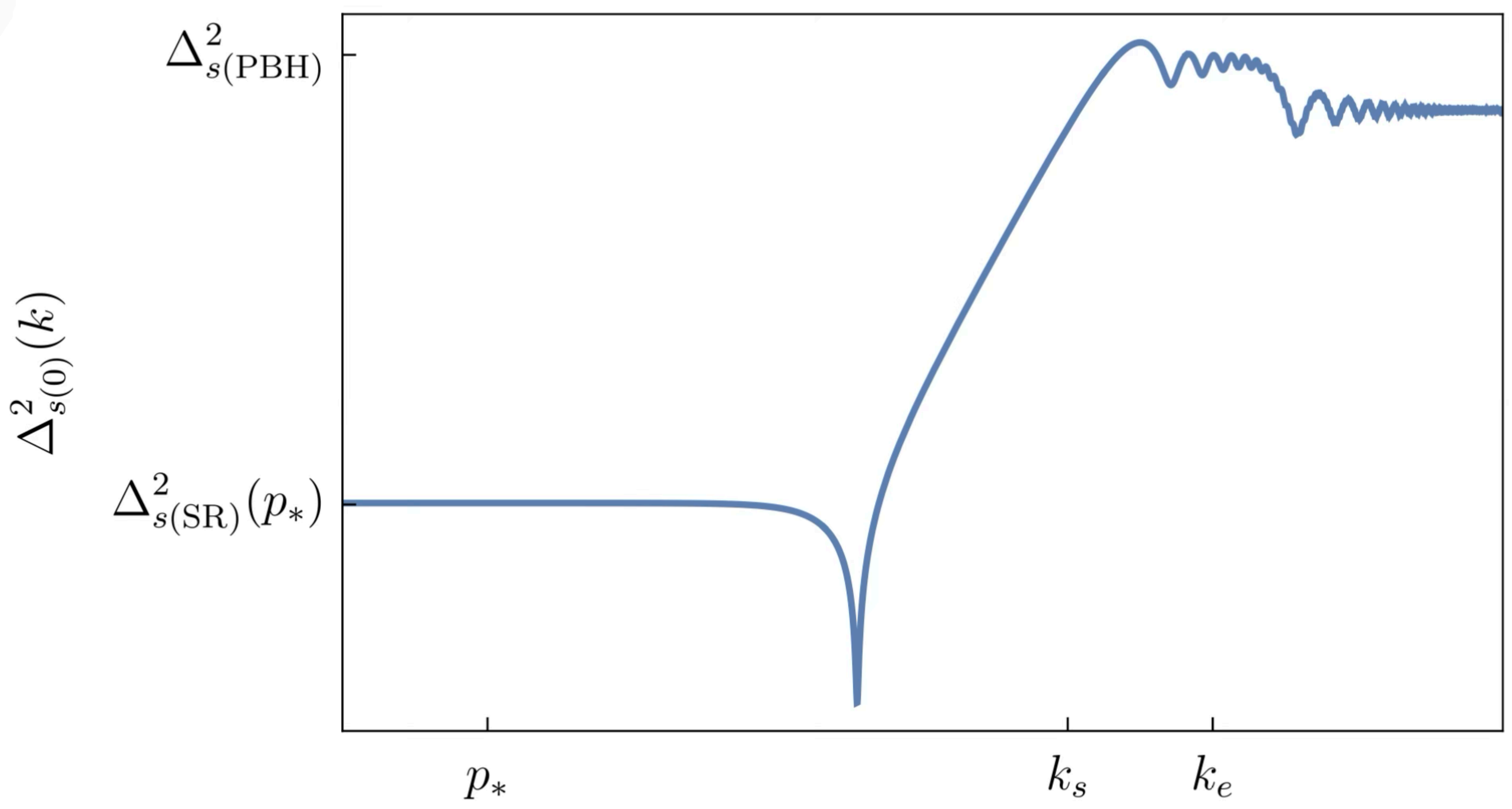
Kristiano & Yokoyama 2211.03395

By requiring that the loop-correction at large scales be smaller that the tree-level, Kristiano and Yokoyama set the tight bound:

$$\Delta^2_{s(\rm PBH)} \ll \frac{1}{(\Delta\eta)^2} \simeq 0.03$$

LOOP CORRECTIONS AT LARGE SCALES FROM SMALL SCALES (?)





Kristiano & Yokoyama 2211.03395

"We find models producing appreciable amount of PBHs generically induce too large one-loop correction on large scale probed by CMB radiation. We therefore conclude that PBH formation from single-field inflation is ruled out."

LOOP CORRECTIONS AT LARGE SCALES FROM SMALL SCALES (?)

Their paper sparked an intense debate.

Some agree with the calculation.
Firouzjahi 2303.12025, Choudhury Gangopadhyay, Sami 2301.10000…

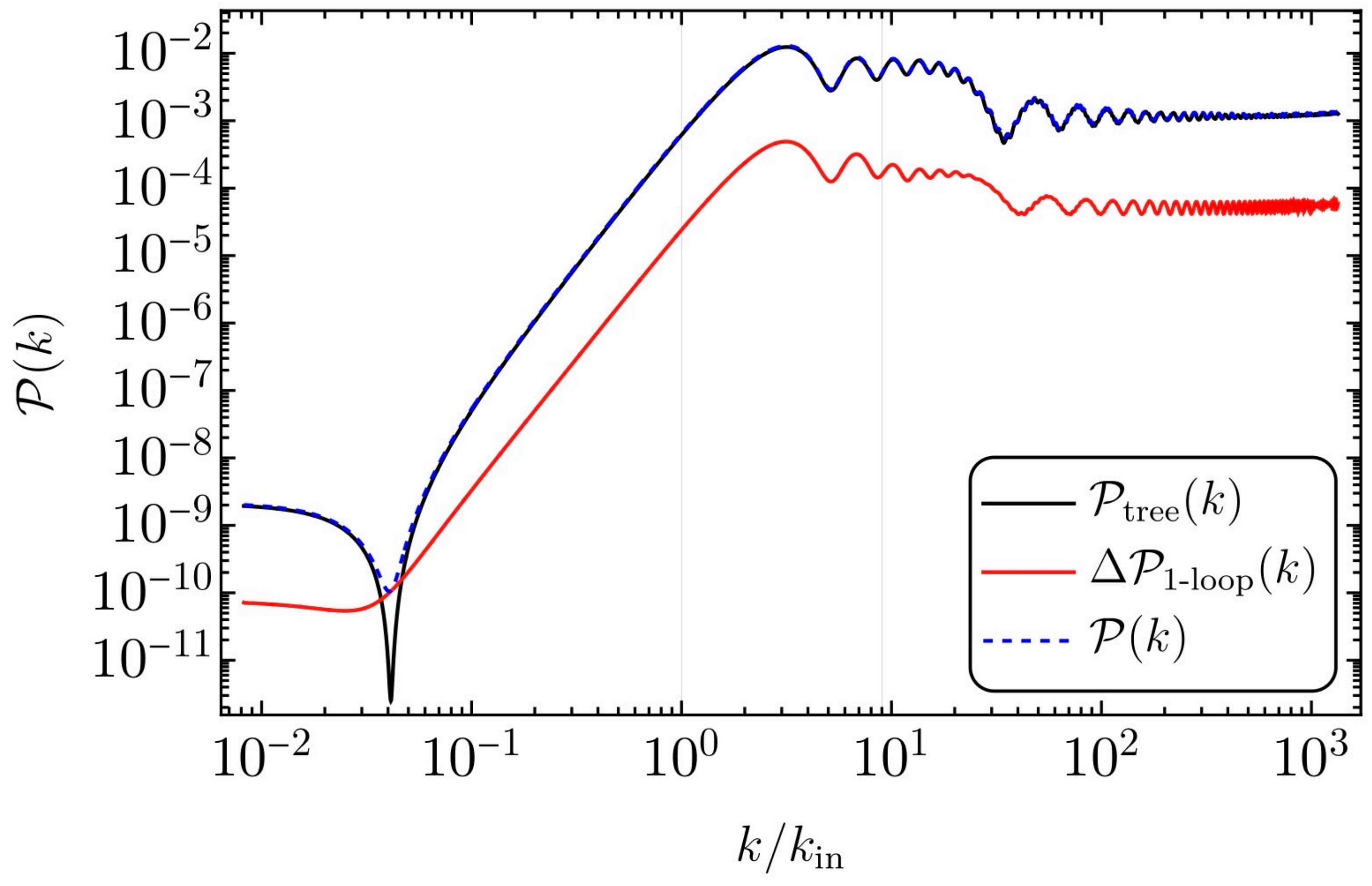
Some claim it does impact the spectrum, with the exact size depending on the duration of the non-SR stage.

Franciolini,Iovino, Taoso, Urbano 2305.03491, Davies, Iacconi, Mulryne 2312.05694, Iacconi, Mulryne, Seery 2312.12424, Ballesteros & Gambina 2404.07916

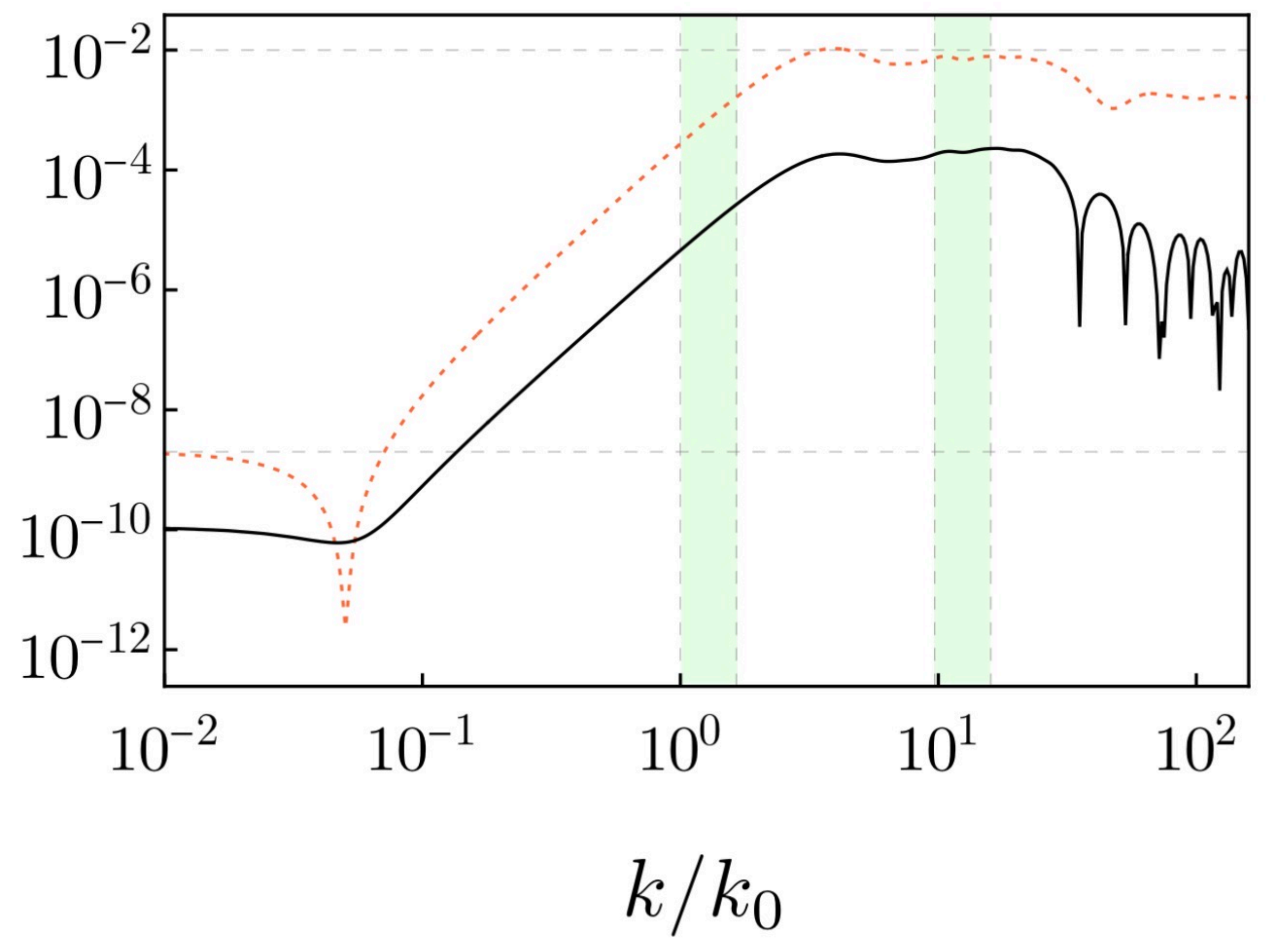
Some claim the loop-correction at large scales is *exactly zero*

Fumagalli 2305.19263 2408.08926, Tada, Terada, Tokuda 2308.04732, Inomata 2403.04682, Kawaguchi, Tsujikawa, Yamada 2407.19742, Ema, Hong, Jinno, Mukaida 2506.15780…

A complete calculation (in a sense to be defined shortly) is still missing.



Franciolini et al 2305.03491



Ballesteros & Gambina 2404.07916

WHAT'S SO COMPLICATED

14:00	**Zhong-Zhi Xianyu, Analytical routes to massive inflationary correlators**	*IAP*	14:00 - 14:45
	Hayden Lee, A Hidden Pattern in Cosmological Correlators	*IAP*	14:45 - 15:30
15:00	**Coffee break**	*IAP*	15:30 - 16:00
16:00	**Arthur Poisson, de Sitter momentum space**	*IAP*	16:00 - 16:15
	Nathan Belrhali, Dual space for cosmological correlators	*IAP*	16:15 - 16:30
	Xi Tong, Unitary renormalisation and the quantum breaking of cosmological reality	*IAP*	16:30 - 16:45
	Xiangwei Wang, Interact or Twist, Cosmological Correlators from Field Redefinitions Revisited	*IAP*	16:45 - 17:00
17:00			

WHAT'S SO COMPLICATED

- Gauge ambiguities make it hard to identify dominant interactions.
- The correlators often contain UV $(k \to \infty)$, IR $(k \to 0)$, and secular $(\tau \to 0)$ divergences.
- The interactions backreact on the background. This must be taken into account.

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EFT OF INFLATION TO THE RESCUE

- Dominant interactions are easily extracted
- This makes it easy to identify the counterterms to remove divergences
- The non-linearly realized symmetries make it straightforward to take into account backreaction effects

THE EFT OF INFLATIONARY FLUCTUATIONS

GENERALITIES

During inflation time-diffeomporphisms are spontaneously broken by the time-dependent background $H(t)$.

It is convenient to work in the unitary gauge, where the NG boson is 'eaten' by the metric. In this gauge, the building blocks are geometric quantities that transform properly under spatial diffeomorphisms:

$$\delta g^{00}, \delta K^{ij}, \ {}^{(3)}R_{ij}; \ \nabla_\mu; f(t)$$

The most general EFT Lagrangian consistent with these symmetries is:

$$\mathcal{L} = \underbrace{\frac{M_{\rm Pl}^2}{2}\left(R - 2(\dot{H} + 3H^2)\right) + M_{\rm Pl}^2 \dot{H}\, g^{00}}_{\text{model–independent}} \ + \ \underbrace{F^{(2)}\left(\delta g^{00},\ \delta K_{ij},\ {}^{(3)}R_{ij};\ \nabla_\mu;\ t\right)}_{\text{model–dependent}}$$

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 0709.0293

MODEL DEPENDENCE

We rely on a derivative expansion. Each derivative will bring a factor of H/Λ where $\Lambda \gg H$ is the cutoff scale of the EFT.

# of derivatives

1

$$F^{(2)} = \frac{M_2^4}{2!}\left(\delta g^{00}\right)^2 + \frac{M_3^4}{3!}\left(\delta g^{00}\right)^3 + \dots$$

2

$$F^{(2)} \supset -\bar{M}_1^3 \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2}\delta K_{ij}\delta K^{ij} - \frac{\bar{M}_3^2}{2}\delta K^2 + \dots$$

$$\frac{m_1^2}{2}\left(\nabla^0 \delta g^{00}\right)^2 + m_2^2 \nabla^0 \delta g^{00} \delta K + \frac{m_3^2}{2} h^{ij}\left(\nabla_i \delta g^{00}\right)\left(\nabla_j \delta g^{00}\right) + \dots$$

3

$$F^{(2)} \supset \bar{m}_1^2 \delta^{(3)} R + \bar{m}_2^2 \delta g^{00} \delta^{(3)} R + \frac{\alpha_1}{2}\delta^{(3)} R_{ij} \delta^{(3)} R^{ij} + \frac{\alpha_2}{2} \nabla^0 \delta K_{ij} \nabla^0 \delta K^{ij} + \dots$$

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# of derivatives

Hierarchy of operators

$$\underbrace{M_2^4\left(\delta g^{00}\right)^2}_{\sim\Lambda^4_{\text{new}}\delta g^2} \gg \underbrace{\bar{M}_1^3\delta g^{00}\delta K}_{\sim\Lambda^3_{\text{new}}H\delta g^2} \gg \underbrace{\bar{M}_2^2\delta K_{ij}\delta K^{ij} \sim \bar{m}_1^2\delta^{(3)}R}_{\sim\Lambda^2_{\text{new}}H^2\delta g^2} \gg \dots$$

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Our minimal EFT Lagrangian is:

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THE STÜCKLEBERG TRICK

We make explicit the effects of the symmetry breaking by introducing the NG boson $\pi(t,\vec{x})$, such that $t+\pi(t,\vec{x})$ invariant under full diffs.

We use the remaining gauge freedom to work in the spatially flat gauge:

$$\zeta(t,\vec{x}) = -H\pi + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}H\pi^2\right) - \frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{1}{6}H\pi^3\right)$$

We are interested in the tansformation of the building block δg^{00}:

$$\delta g^{00} \to \widetilde{\delta g}^{00} = -2\dot{\pi} - \dot{\pi}^2 + \frac{(\partial_i\pi)^2}{a^2} + \delta g^{00}_{\text{flat}}(1+\dot{\pi})^2 + 2\delta g^{0i}_{\text{flat}}(1+\dot{\pi})\partial_i\pi + \delta h^{ij}_{\text{flat}}\partial_i\pi\partial_j\pi$$

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Non-linearly realized symmetries

Keeping the lapse and the shift is non trivial

DECOUPLING LIMIT

The NG boson mixes with the gravitational degrees of freedom.

ADM constraints set: $\delta g^{00}_{\rm flat} = 2\epsilon H \pi$. Neglecting metric perturbations in e.g. $\left(\delta g^{00}\right)^2$ implies:

$$\left\{\delta g^{00}_{\rm flat}\dot{\pi},\, \left(\delta g^{00}_{\rm flat}\right)^2\right\} \ll \dot{\pi}^2 \qquad \dot{\pi} \sim \omega \times \pi$$

Around horizon crossing

$$\epsilon \ll 1$$

No conditions on η, η_2

FREE THEORY AND LEADING ITERACTIONS (in the decoupling and $c_s \to 1$ limit)

$$\mathcal{H}_{\text{free}} = \frac{a^3 \epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 + \frac{c_s^2}{a^2}(\partial_i \pi)^2\right]$$

with

$$\frac{1}{c_s^2} - 1 = \frac{2M_2^4}{\epsilon H^2 M_{\text{Pl}}^2}$$

$$\pi_k^I(\tau) = \frac{1}{\sqrt{4\epsilon c_s k^3} M_{\text{Pl}}}(1 + i c_s k\tau)\, e^{-i c_s k \tau}$$

Mode functions

Propagators

$$\mathcal{P}_\pi^{\text{tree}}(x) = \frac{(1 + c_s^2 x^2)}{8\pi^2 \epsilon\, c_s M_{\text{Pl}}^2}$$

Tree-level power spectrum

FREE THEORY AND LEADING INTERACTIONS (in the decoupling and $c_s \to 1$ limit)

$$\mathcal{H}_{\text{free}} = \frac{a^3 \epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 + \frac{c_s^2}{a^2}(\partial_i \pi)^2\right] \quad \text{with} \quad \frac{1}{c_s^2} - 1 = \frac{2M_2^4}{\epsilon H^2 M_{\text{Pl}}^2}$$

$$a\mathcal{H}_{\text{int}}^{(3)} = -a^2 \epsilon \eta H^3 M_{\text{Pl}}^2 \left[\frac{1}{c_s^2}\pi \pi'^2 - \pi(\partial_i \pi)^2\right]$$

$$a\mathcal{H}_{\text{int}}^{(4)} = \frac{a^2}{2}\epsilon \eta H^4 M_{\text{Pl}}^2 \left[\frac{\eta - \eta_2}{c_s^2}\pi^2 \pi'^2 + (\eta + \eta_2)\pi^2(\partial_i \pi)^2\right]$$

One must be careful of derivative interactions. In particular quartic Hamiltonian and Lagrangians differ.

FREE THEORY AND LEADING INTERACTIONS (in the decoupling and $c_s \to 1$ limit)

$$\mathcal{H}_{\text{free}} = \frac{a^3 \epsilon H^2 M_{\text{Pl}}^2}{c_s^2}\left[\dot{\pi}^2 + \frac{c_s^2}{a^2}(\partial_i \pi)^2\right] \qquad \text{with} \qquad \frac{1}{c_s^2} - 1 = \frac{2M_2^4}{\epsilon H^2 M_{\text{Pl}}^2}$$

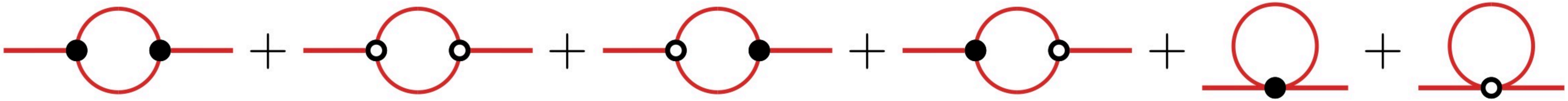
$$a\mathcal{H}_{\text{int}}^{(3)} = -a^2 \epsilon \eta H^3 M_{\text{Pl}}^2 \left[\frac{1}{c_s^2}\pi\pi'^2 - \pi(\partial_i \pi)^2\right]$$

$$a\mathcal{H}_{\text{int}}^{(4)} = \frac{a^2}{2}\epsilon \eta H^4 M_{\text{Pl}}^2 \left[\frac{\eta - \eta_2}{c_s^2}\pi^2\pi'^2 + (\eta + \eta_2)\pi^2(\partial_i \pi)^2\right]$$

One must be careful of derivative interactions. In particular quartic Hamiltonian and Lagrangians differ.

ONE LOOP POWER SPECTRUM OF SCALARS

BARE LOOP CORRECTIONS



All these diagrams are divergent. We use dimensional regularization to identify them.
Dim-reg modifies both the integral measure and the mode functions.

$$\pi_k(\tau) = \frac{\sqrt{\pi} e^{i\pi\delta/4} c_s^{-(1+\delta)/2}}{2\sqrt{2\epsilon}} \frac{1}{M_{\rm Pl}} \left(\frac{H}{\mu}\right)^{\delta/2} \frac{(-c_s k\tau)^{(3+\delta)/2}}{k^{(3+\delta)/2}} H^{(1)}_{(3+\delta)/2}(-c_s k\tau)$$

$$\int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} f(\vec{k}) \mapsto \int \frac{\mathrm{d}^d k}{(2\pi)^d} f(\vec{k}) = \Omega_{d-3} \int_0^\infty \mathrm{d}k\, k^{d-1} \int_0^\pi \mathrm{d}\theta \sin^{d-2}\theta \int_0^\pi \mathrm{d}\varphi \sin^{d-3}\varphi\, f(k,\theta,\varphi)$$

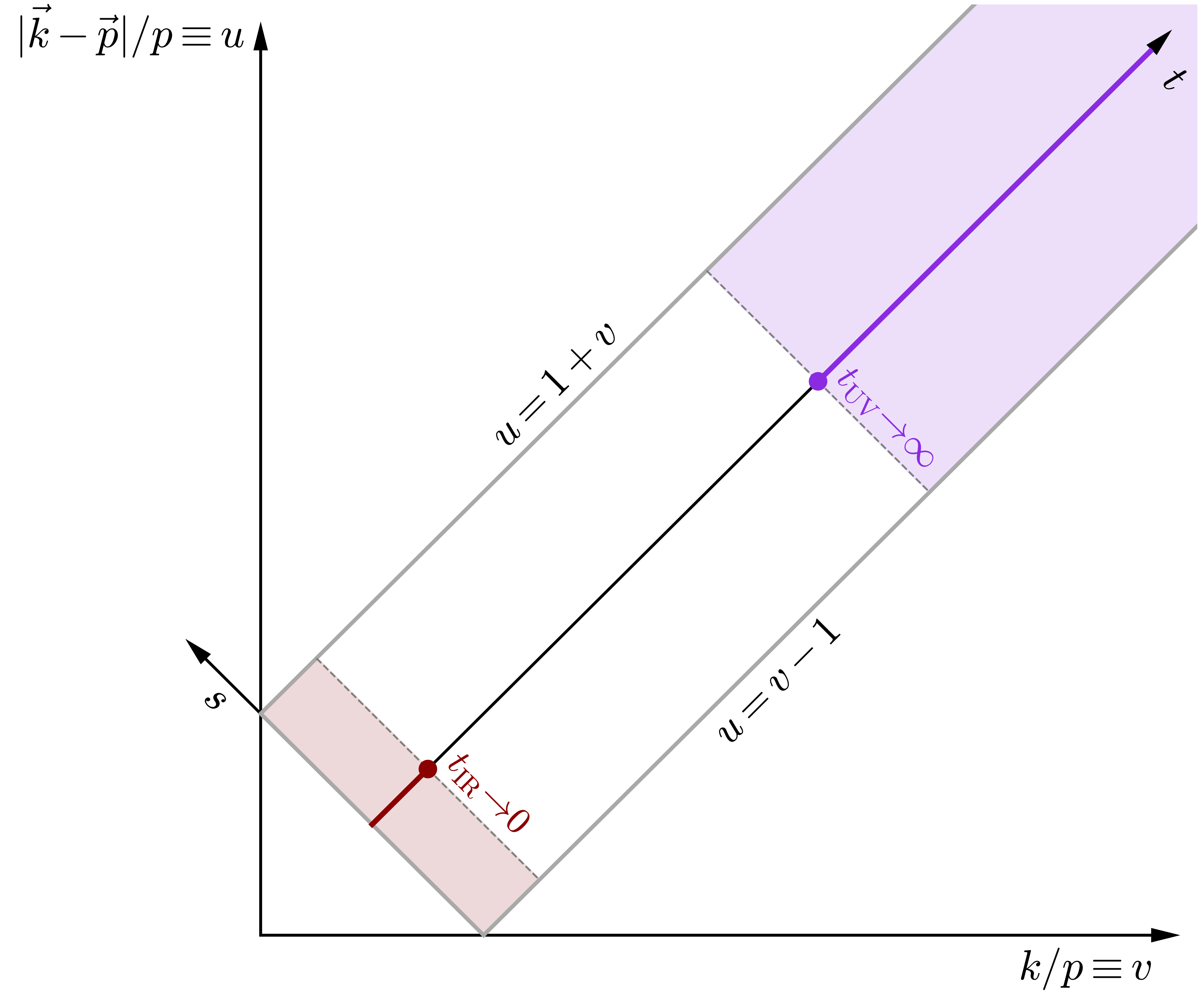
BARE LOOP CORRECTIONS: CALCULATION

We expand the integrand (except the k^{δ} regulator) at linear order in δ, and perform time and angular integrals.

Senatore & Zaldarriaga 1203.6354

We introduce an 'fake' cutoff $t_{\rm UV}$, and expand the integrand for $t \to \infty$ to identify the pole in δ. The result is independent of $t_{\rm UV}$

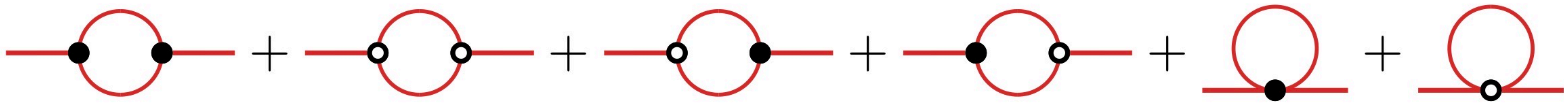
Ballesteros, Gambina, Riccardi 2411.19674



We regulate IR divs with a comoving cutoff $t_{\rm IR}$ as we are not concerned with them in our work.

BARE LOOP CORRECTIONS: RESULT

$x \equiv -k\tau$



$$= \mathcal{P}^{\text{b, UV}}_{\pi,\,1\text{L}}(x) + \mathcal{P}^{\text{b, IR}}_{\pi,\,1\text{L}}(x) + \mathcal{P}^{\text{b, fin}}_{\pi,\,1\text{L}}(x)$$

$$\mathcal{P}^{\text{b, UV}}_{\pi,\,1\text{L}}(x) = -\,(1 + c_s^2 x^2)\mathcal{P}^{\text{tree}^2}_{\pi,\,0} H^2 \frac{\eta(\eta - 2\eta_2)}{4}\left[\frac{1}{\delta} + 2\log\left(\frac{H}{\mu}\sqrt{\pi e^{-\gamma_{\text{E}}}}\right)\right]$$

To remove by hand with counterterms.

$$\lim_{x\to 0} \mathcal{P}^{\text{bare}}_{\pi,\,1\text{L}}(x) \sim -\mathcal{P}^{\text{tree}^2}_{\pi,\,0} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x$$

This cannot be removed by hand!

RENORMALIZATION AND COUNTERTERMS

We look for a minimal set of quadratic operators to remove UV divergences at all times.

Lowest order in derivatives: renormalization of the speed of sound

$$\mathcal{L} \supset \frac{M_2}{2}\left(\delta g^{00}\right)^2 = \frac{M^4_{2,\,\mathrm{ren}}}{2}\left(\delta g^{00}\right)^2 + \frac{\delta M_2^4}{2}\left(\delta g^{00}\right)^2$$
$$\to \epsilon H^2 M_{\mathrm{Pl}}^2 \left(\frac{1}{c_s^2} - 1\right)\dot{\pi}^2 - \delta_{c_s^2}\epsilon H^2 M_{\mathrm{Pl}}^2 \dot{\pi}^2 + \ldots$$

Next order in derivatives, minimal set of non-degenerate operators

$$\mathcal{L} \supset -\frac{\bar{M}_3^2}{2}\delta K^2 + \frac{m_3^2}{2} h^{ij}\left(\nabla_i \delta g^{00}\right)\left(\nabla_j \delta g^{00}\right) \to -\delta_1 \frac{(\partial^2\pi)^2}{a^4} - \delta_2 \frac{(\partial_i \dot{\pi})^2}{a^2} + \ldots$$

Bordin, Cabass, Creminelli, Vernizzi 1706.03758

RENORMALIZATION AND COUNTERTERMS

$$\sum_{i=c_s^2,\,1,\,2}$$

$$
\begin{aligned}
&= \delta_{c_s^2}\, \mathcal{P}_{\pi,\,0}^{\mathrm{tree}^2}\, \pi^2 c_s^3 M_{\mathrm{Pl}}^2 \epsilon \left(-1 + c_s^2 x^2\right) \left[1 + \delta \log\left(\frac{H}{\mu}\right)\right] + \delta_{c_s^2} \mathcal{P}_{\pi,\,1\mathrm{L}}^{\delta_{c_s^2},\,\mathrm{fin}}(x) \\
&- \delta_1\, \mathcal{P}_{\pi,\,0}^{\mathrm{tree}^2}\, \frac{\pi^2 M_{\mathrm{Pl}}^2}{2 c_s} \left(5 + 5 c_s^2 x^2 + 2 c_s^4 x^4\right) \left[1 + \delta \log\left(\frac{H}{\mu}\right)\right] + \delta_1 \mathcal{P}_{\pi,\,1\mathrm{L}}^{\delta_1,\,\mathrm{fin}}(x) \\
&+ \delta_2\, \mathcal{P}_{\pi,\,0}^{\mathrm{tree}^2}\, \frac{\pi^2 c_s M_{\mathrm{Pl}}^2}{2} \left(1 + c_s^2 x^2 + 2 c_s^4 x^4\right) \left[1 + \delta \log\left(\frac{H}{\mu}\right)\right] + \delta_2 \mathcal{P}_{\pi,\,1\mathrm{L}}^{\delta_2,\,\mathrm{fin}}(x)
\end{aligned}
$$

The combination $\delta_{c_s^2} = 0 \quad \delta_1 = c_s^2 \delta_2, \qquad \delta_2 = -\frac{1}{\delta} \frac{H^2}{M_{\mathrm{Pl}}^2} \frac{\eta(\eta - 2\eta_2)}{8\pi^2 c_s}$ removes the UV pole, but

$$\lim_{x \to 0} \mathcal{P}_{\pi,\,1\mathrm{L}}^{\mathrm{bare}}(x) \sim -\mathcal{P}_{\pi,\,0}^{\mathrm{tree}^2} H^2 \frac{1}{2} \eta(2\eta - \eta_2) \log x$$

…not the secular divergence!

TADPOLES

To keep our in-in theory defined around the correct background we impose tadpole cancellation. The only operators generating linear interactions are:

$$\mathcal{L} \supset -g^{00}\delta c(t) - M_{\rm Pl}^2 \delta\Lambda(t)$$

Pimentel, Senatore, Zaldarriaga 1203.6651

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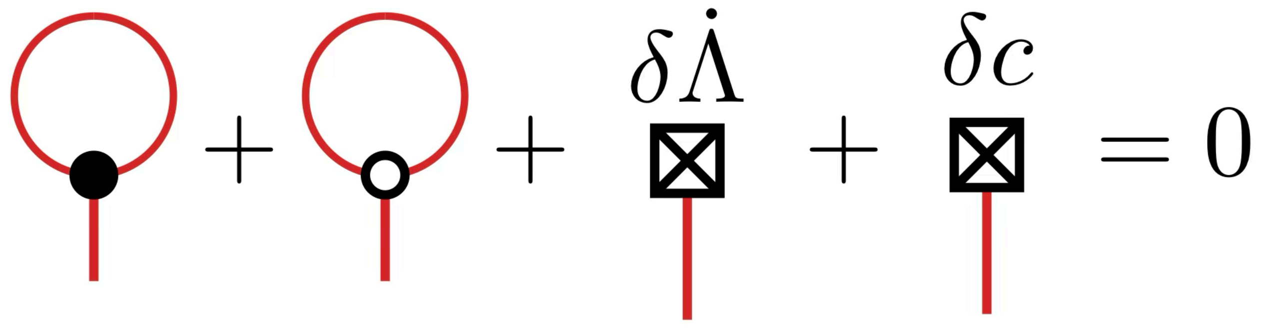
Pimentel, Senatore, Zaldarriaga 1203.6651

$$a\mathcal{H}^{(1)}_{\rm c.t.} = a^4 M_{\rm Pl}^2\, \delta\dot{\Lambda}\, \pi - 2a^4\, \delta c\, \dot{\pi}$$



$$\delta\dot{\Lambda}(\tau_1) = \frac{3}{16\pi^2 c_s^3}\frac{H^5}{M_{\rm Pl}^2}\eta \qquad \delta c(\tau_1) = -\frac{1}{16\pi^2 c_s^3} H^4 \eta$$

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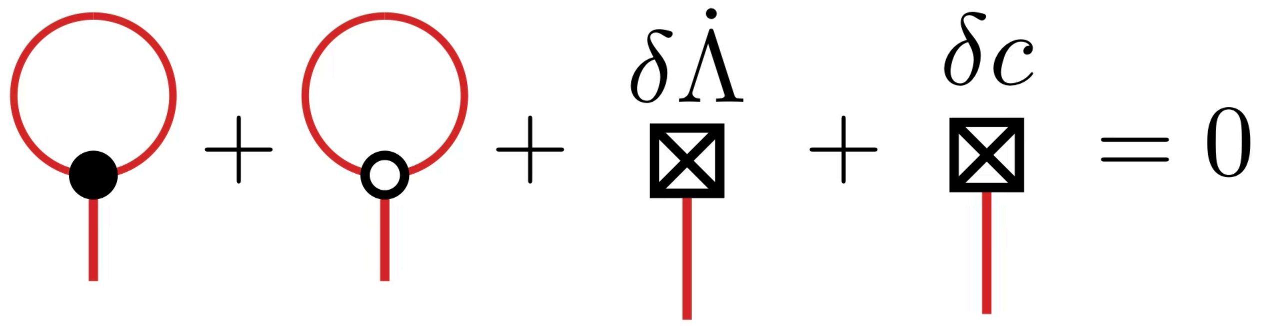
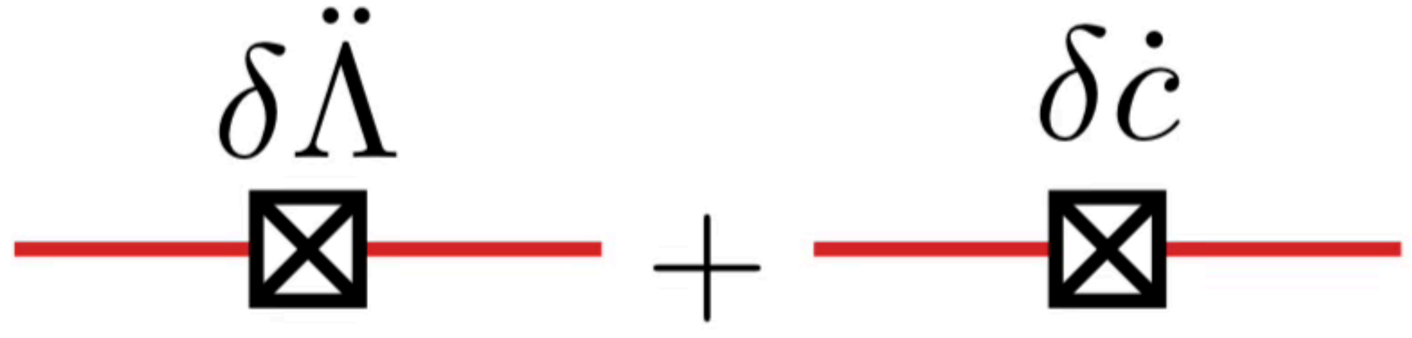
$$a\mathcal{H}^{(1)}_{\rm c.t.} = a^4 M_{\rm Pl}^2\,\delta\dot\Lambda\,\pi - 2a^4\,\delta c\,\dot\pi$$

$$a\mathcal{H}^{(2)}_{\rm c.t.} = \frac{a^4 M_{\rm Pl}^2}{2}\,\delta\ddot\Lambda\,\pi^2 - 2a^4\,(\delta\dot c - \eta H\,\delta c)\,\pi\dot\pi$$





$$\delta\dot\Lambda(\tau_1) = \frac{3}{16\pi^2 c_s^3}\frac{H^5}{M_{\rm Pl}^2}\eta \qquad \delta c(\tau_1) = -\frac{1}{16\pi^2 c_s^3}H^4\eta$$

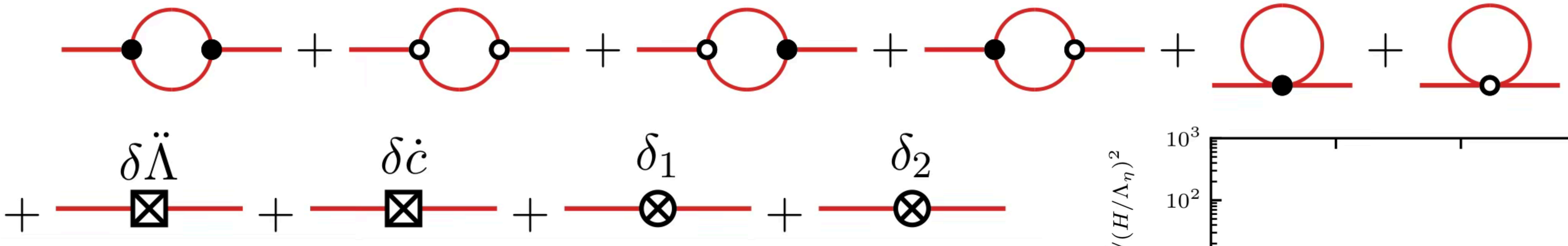
$$\lim_{x\to 0}\mathcal{P}^{\rm c.t.,(\Lambda,\,c)}_{\pi,\,1L}(x) \sim \mathcal{P}^{\rm tree^2}_{\pi,\,0} H^2\frac{1}{2}\eta(2\eta - \eta_2)\log x$$

TADPOLES

$$\mathcal{P}_{\pi,0}^{\text{tree}^2} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x - \mathcal{P}_{\pi,0}^{\text{tree}^2} H^2 \frac{1}{2}\eta(2\eta - \eta_2)\log x = 0$$

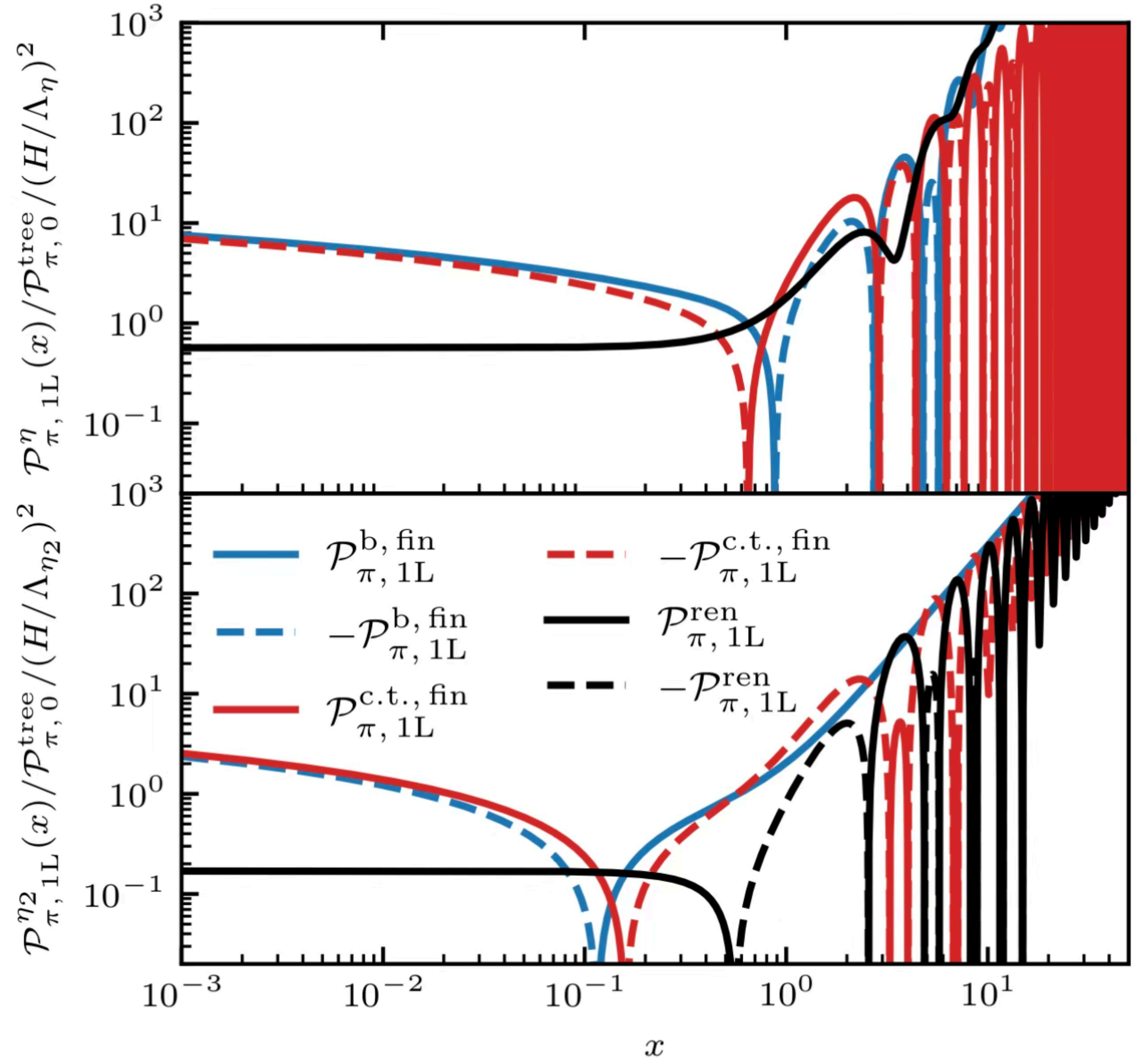
Thanks to the non-linearly realized symmetries, imposing tadpole cancellation cancels the secular divergence!

TOTAL RENORMALIZED SPECTRUM



The result at the end of inflation can be written in terms of strong coupling scales $\Lambda_\eta^2 = M_{\rm Pl}^2 \frac{\epsilon c_s}{\eta^2}\,, \quad \Lambda_{\eta_2}^2 = M_{\rm Pl}^2 \frac{\epsilon c_s}{\eta\eta_2}$

$$\frac{\mathcal{P}^{\rm ren}_{\pi,1{\rm L},0}}{\mathcal{P}^{\rm tree}_{\pi,0}} = \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_\eta}\right)^2\left[\frac{269-290\log(2)}{160} - \frac{1}{4}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}t_{\rm IR}^2\right)\right] + \frac{1}{8\pi^2}\left(\frac{H}{\Lambda_{\eta_2}}\right)^2\left[\frac{11}{24} + \frac{1}{2}\log\left(\frac{H}{\mu}\sqrt{\frac{\pi}{4c_s^2 e^{\gamma_E}}}t_{\rm IR}\right)\right]$$



SCALE DEPENDENT SCENARIOS

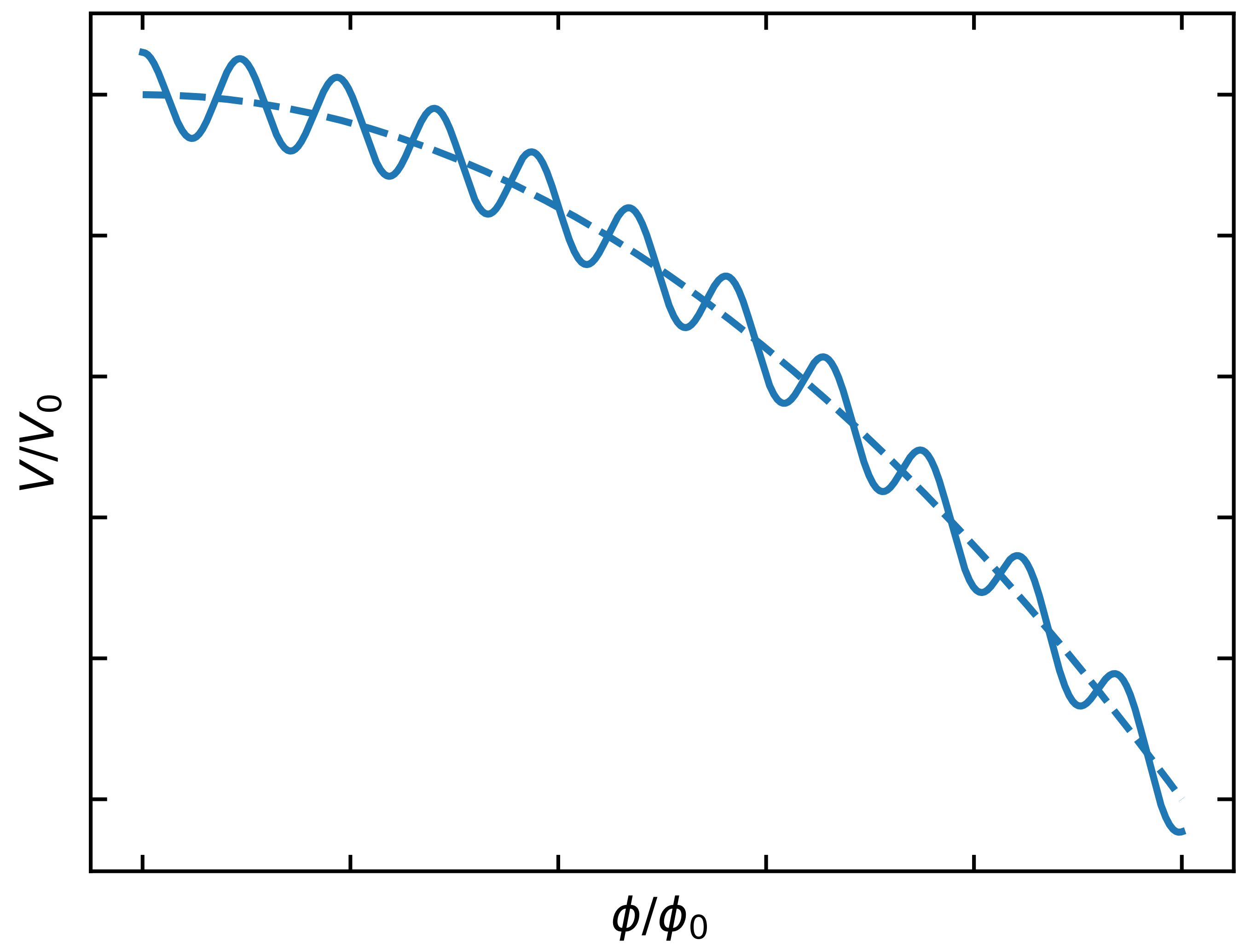
MB, Cespedes, Pinol in progress

SCALE DEPENDENT SCENARIOS (PRELIMINARY)

We consider two simple examples of primordial features chosen to illustrate typical scale dependent pattern and allow for analytical calculations.

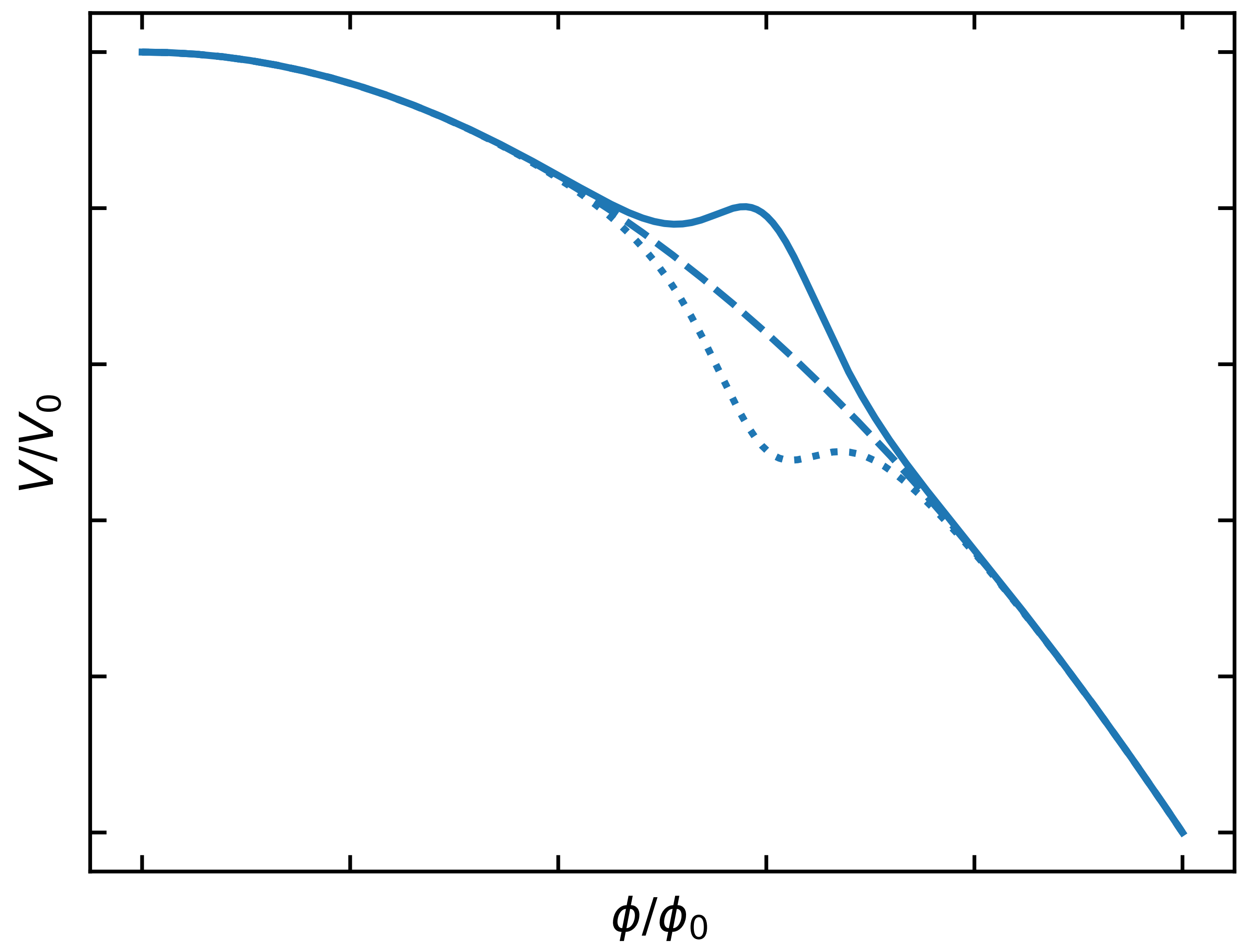
Resonant features: periodic oscillation in the background.

$$\Delta\mathcal{P}^{\rm res}(k) \sim \sin\left[\frac{\omega}{H}\log\left(\frac{k}{k_{\rm res}}\right)\right]$$



Sharp feature: momentary departure of background quantity from the attractor.

$$\Delta\mathcal{P}^{\rm sharp}(k) \sim \sin\left(2\frac{k}{k_0}\right)$$



SCALE DEPENDENT SCENARIOS (PRELIMINARY)

We introduce a time dependence in the background as

$$\epsilon(\tau) = \epsilon_0 + \Delta\epsilon^{\rm i}(\tau) \qquad {\rm i = res,\ sharp}$$

At tree level this induces a scale dependent correction at tree-level

$$\Delta\mathcal{P}^{\rm i}_{\rm tree}(k,\,\tau) = A^{\rm i}_{\rm tree} f^{\rm i}_{\rm tree}(k,\,\tau)$$

What about loops? If the scale dependence of loops differs from the tree level we may hope to measure them

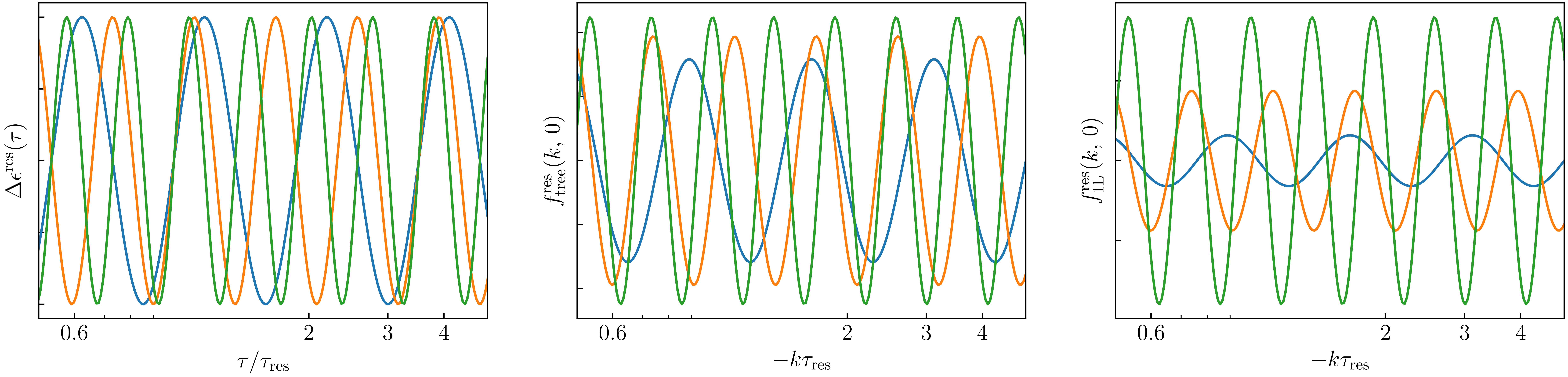
UV DIVERGENCE (PRELIMINARY)

So far we have computed the UV pole, which is highly non-trivial to remove at all scales and times

$$\Delta\mathcal{P}^{\mathrm{i}}_{1\mathrm{L}}(k,\,\tau) = A^{\mathrm{i}}_{1\mathrm{L}} f^{\mathrm{i}}_{1\mathrm{L}}(k,\,\tau)\left[\frac{1}{\delta} + 2\log\left(\frac{H}{\mu}\sqrt{\pi e^{-\gamma_{\mathrm{E}}}}\right)\right]$$

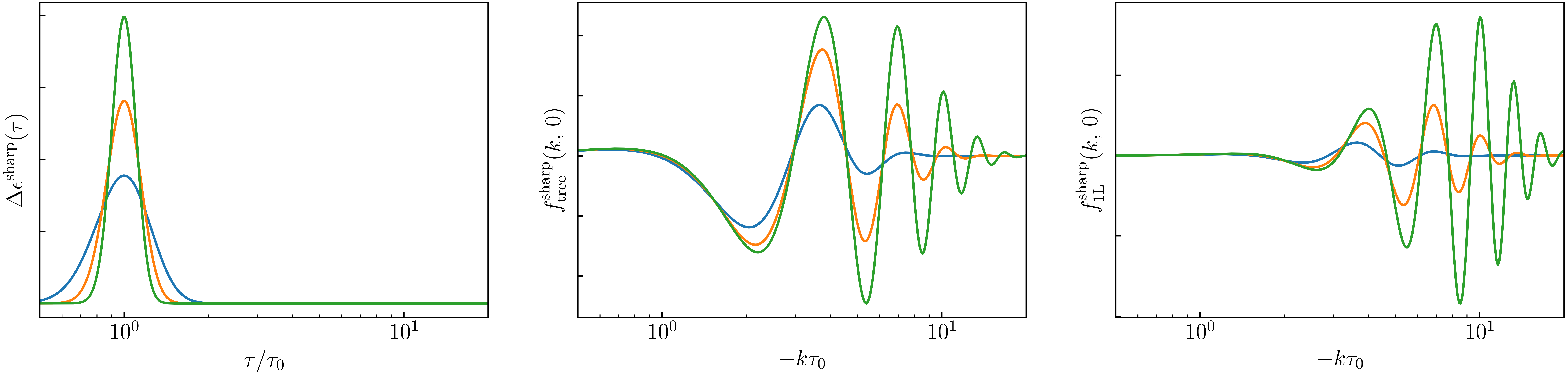
To draw conclusions we need to compute all the finite terms, but this already give us some insights.

UV DIVERGENCE: RESONANT FEATURES (PRELIMINARY)



Both the tree level and the loop oscillate with the same frequency: they cannot be distinguished by observations.

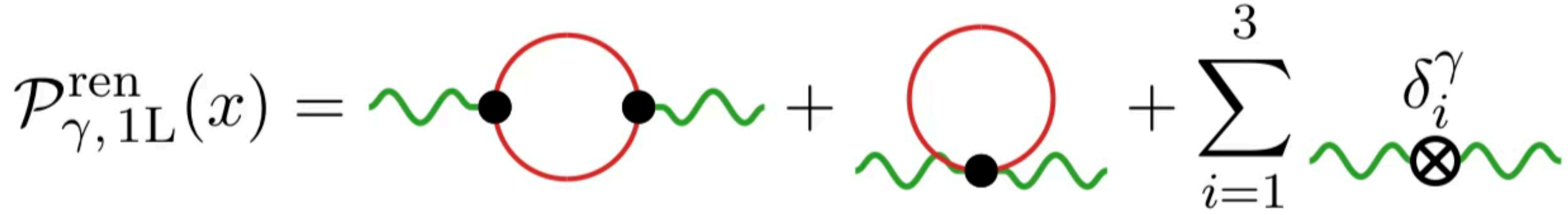
UV DIVERGENCE: SHARP FEATURES (PRELIMINARY)



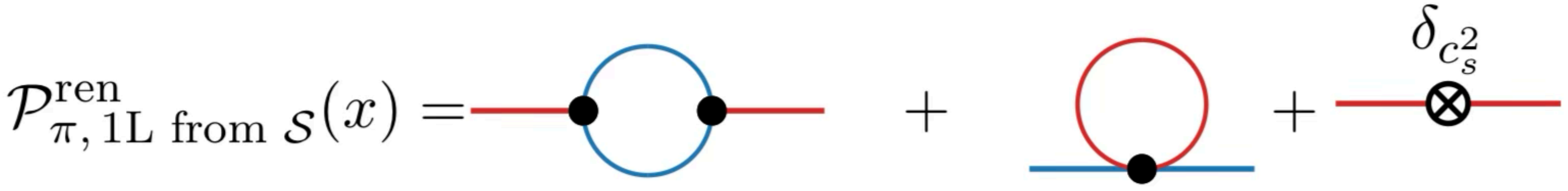
Both the tree level and the loop oscillate with the same frequency, but phases and envelope differ: they can be distinguished by observations.

CONCLUSIONS

- We successfully renormalized the EFT of inflation at order H^2/Λ^2.
- Scalar-induced GWs



- Correction to the power spectrum of curvature perturbations from conformal fields



- Ongoing work suggest scale dependent scenarios may be observable