UCLouvain

Stochastic trees for stochastic inflation and primordial black hole formation

Chiara Animali

4th December 2025

41st IAP symposium "Inflation 2025"

Inflationary potential and primordial black holes

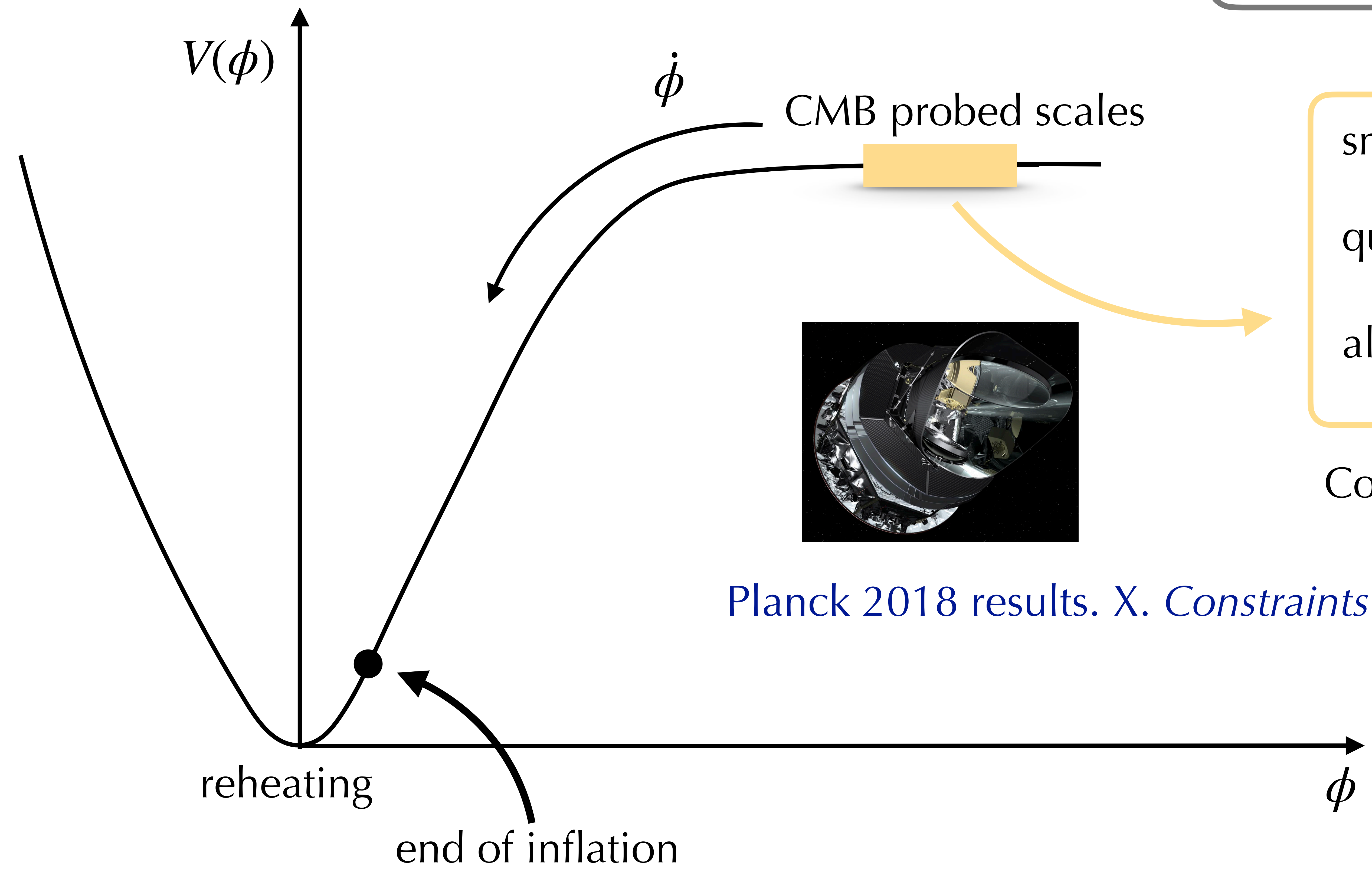
Simplest realisation of inflation: single field, slow roll.

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\pi G} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{8\pi G} \left(\frac{V_{\phi\phi}}{V} \right)$$

$\{\epsilon, |\eta|\} \ll 1$



small perturbations $\zeta \simeq 10^{-5}$

quasi-Gaussian

almost scale invariant

Constrained window ~ 7 e-folds

Planck 2018 results. X. *Constraints on inflation*

Inflationary potential and primordial black holes

Simplest realisation of inflation: single field, slow roll.

$$S_\phi = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right)$$

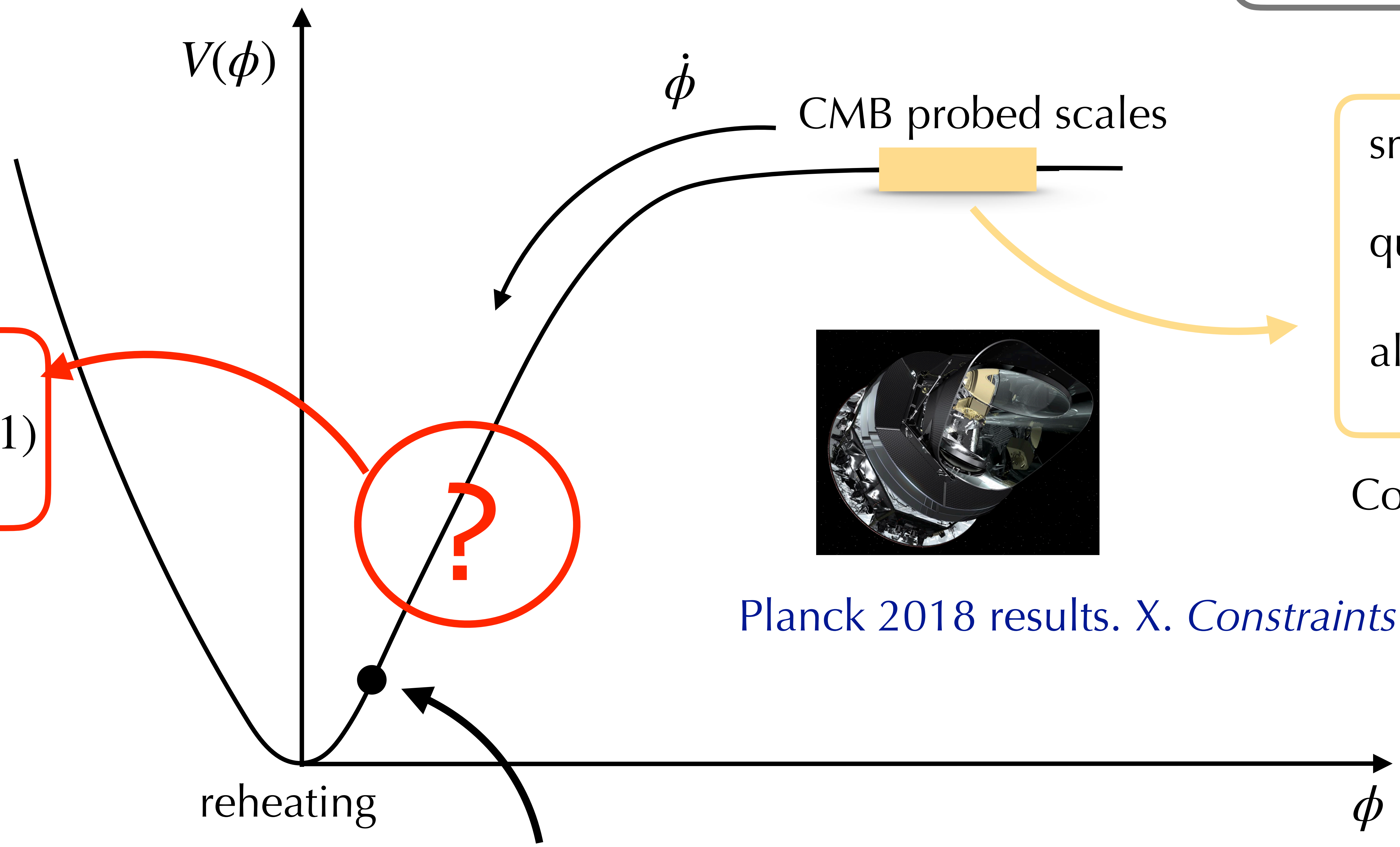
$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\pi G} \left(\frac{V_{,\phi}}{V} \right)^2$$

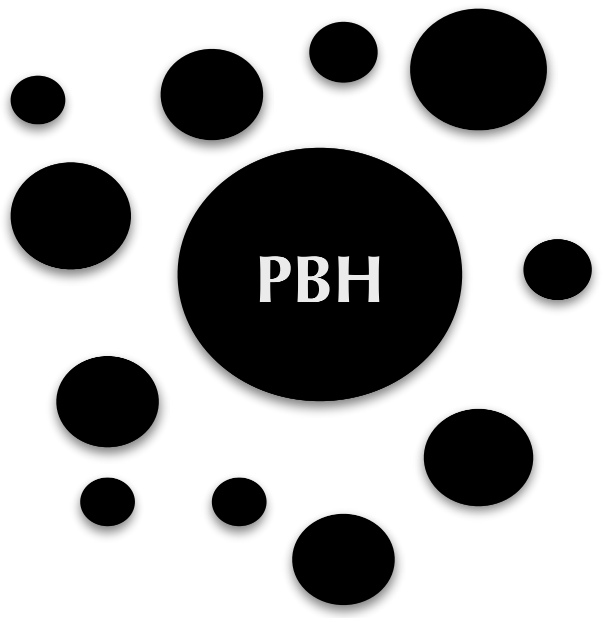
$$\eta = \frac{\dot{\epsilon}}{H\epsilon} = \frac{1}{8\pi G} \left(\frac{V_{\phi\phi}}{V} \right)$$

$\{\epsilon, |\eta|\} \ll 1$

small scales unconstrained

possible enhancement $\zeta \sim \mathcal{O}(1)$





small perturbations $\zeta \simeq 10^{-5}$

quasi-Gaussian

almost scale invariant

Constrained window ~ 7 e-folds

Planck 2018 results. X. *Constraints on inflation*

Primordial black holes

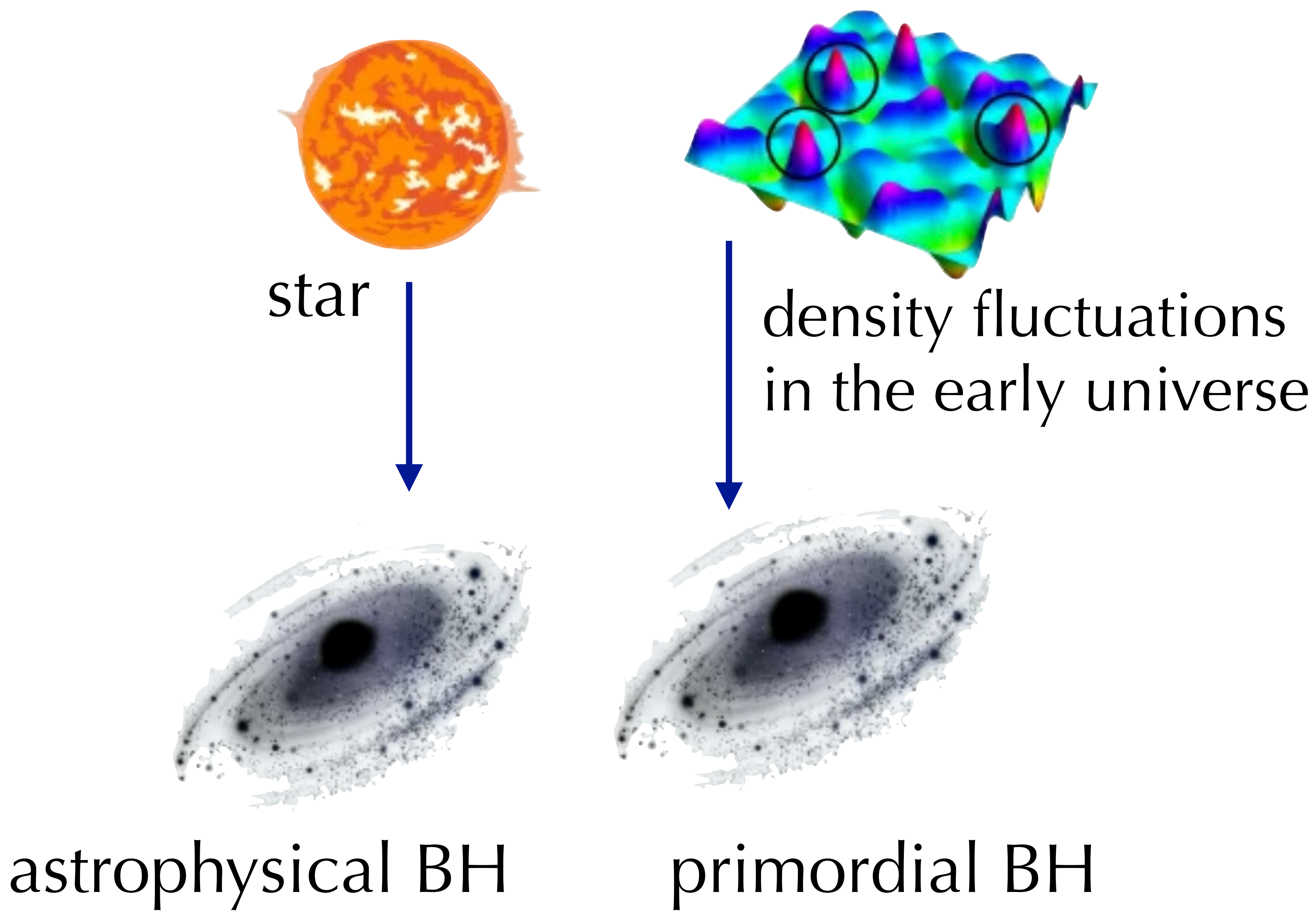
Black holes which could have formed in the early Universe through a non-stellar way.

Zel'dovich & Novikov [1967]
Hawking [1971]
Carr & Hawking [1974]

They may have important astrophysical and cosmological roles :

→ They could be a fraction, or the totality, of the Dark Matter
$(M = 10^{17} - 10^{22}\,\mathrm{g})$.

→ They may explain the existence of progenitors for the merging events observed by LIGO/VIRGO.

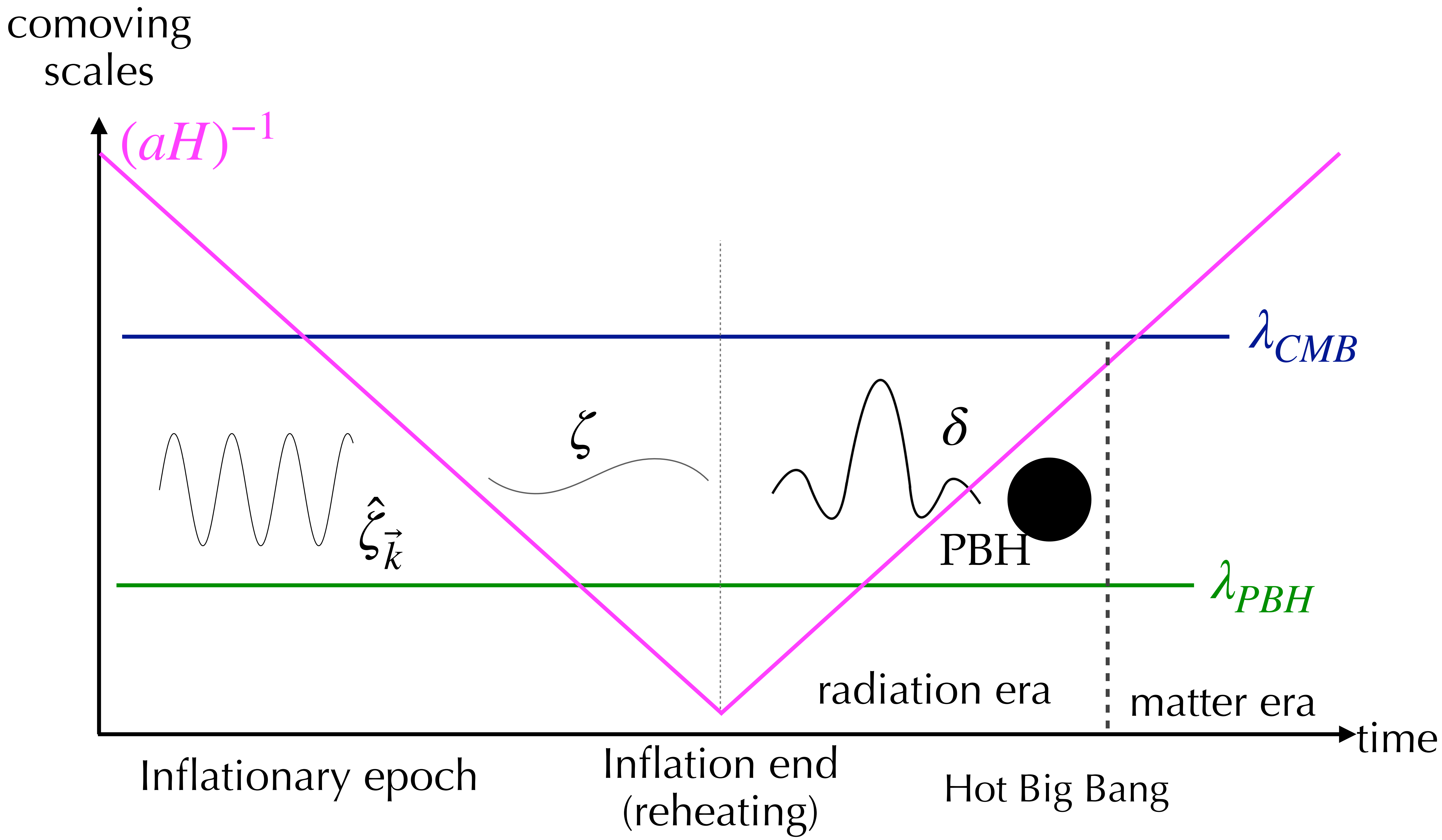


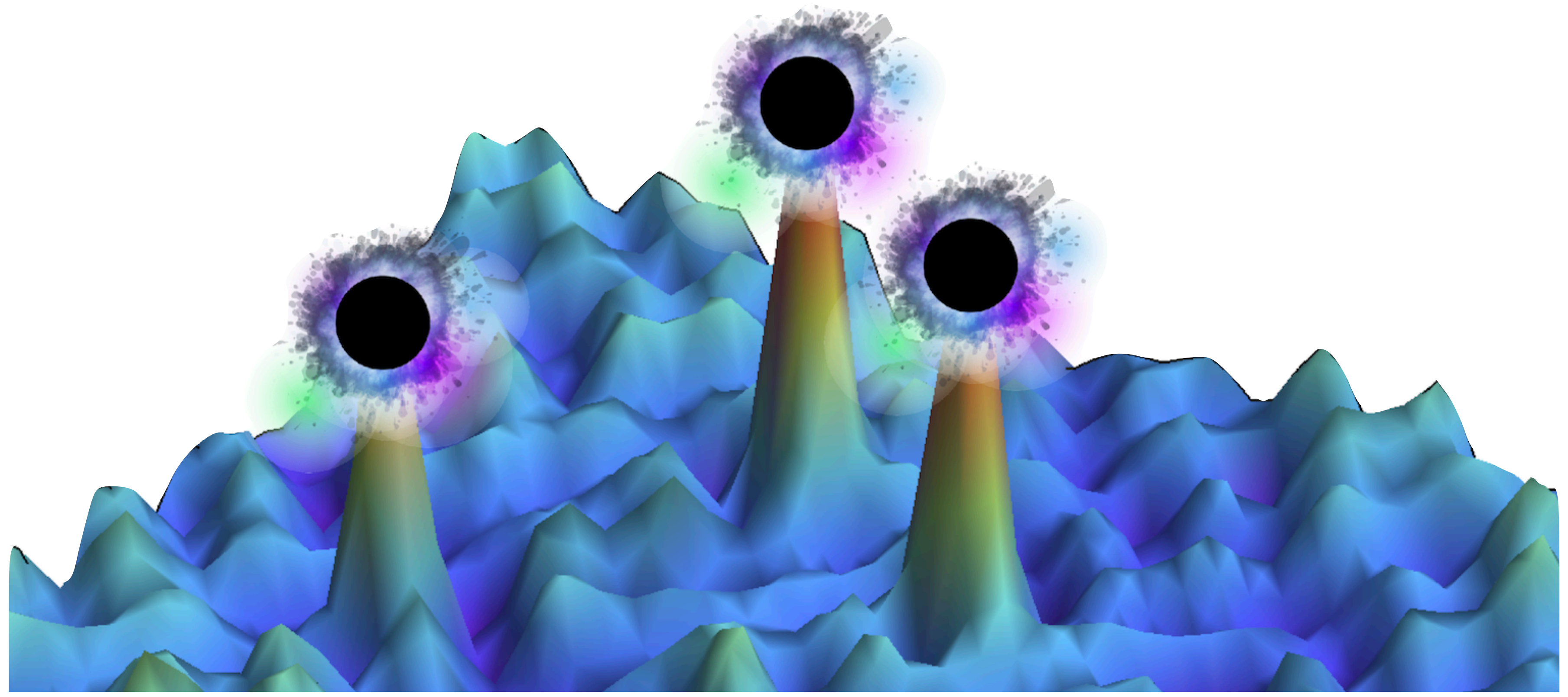
→ They could be the seeds of supermassive black holes in galactic nuclei.

→ They could generate cosmological structures.

Primordial black holes

PBHs may originate from peaks of the density perturbations generated in the early universe.





$$\delta \sim \left.\frac{\delta\rho}{\rho}\right|_{k=aH} \sim \zeta > \zeta_c \sim \mathcal{O}(1)$$

Cosmological perturbation theory

homogeneous background part

$$g_{\mu\nu}(\vec{x},t) = g_{\mu\nu}(t) + \hat{\delta g}_{\mu\nu}(\vec{x},t)$$

$$\phi(\vec{x},t) = \phi(t) + \hat{\delta\phi}(\vec{x},t)$$

small quantised fluctuations

✔ CMB scales: density fluctuations are small.

✘ Small (PBHs) scales: density fluctuations are large.

Quantum field theory in curved spacetime: observational predictions

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2\epsilon_* M_{Pl}^2}\left[1 - 2(C+1)\epsilon_* - 2C(2\epsilon_* - \eta_*) - 2(3\epsilon_* - \eta_*)\log\left(\frac{k}{k_*}\right)\right]$$

$$\mathcal{P}_h(k) = \frac{2H_*^2}{\pi^2 M_{Pl}^2}\left[1 - 2(C+1)\epsilon_* - 2\epsilon_*\log\left(\frac{k}{k_*}\right)\right]$$

$$C = \log 2 + \gamma_E - 2 \simeq -0.7296$$

$$n_T \equiv \frac{d\log\mathcal{P}_h}{d\log k} = -2\epsilon \qquad n_s \equiv 1 + \frac{d\log\mathcal{P}_\zeta}{d\log k} = 1 - 6\epsilon + 2\eta$$

$$r \equiv \frac{\mathcal{P}_h(k_*)}{\mathcal{P}_\zeta(k_*)} \simeq 16\epsilon$$

observational constraints

$$\zeta \propto \left.\frac{\delta T}{T}\right|_{CMB}$$

$$\mathcal{P}_\zeta(k_*) \simeq 2.1 \times 10^{-9}$$

$$n_s = 0.9649 \pm 0.0042$$

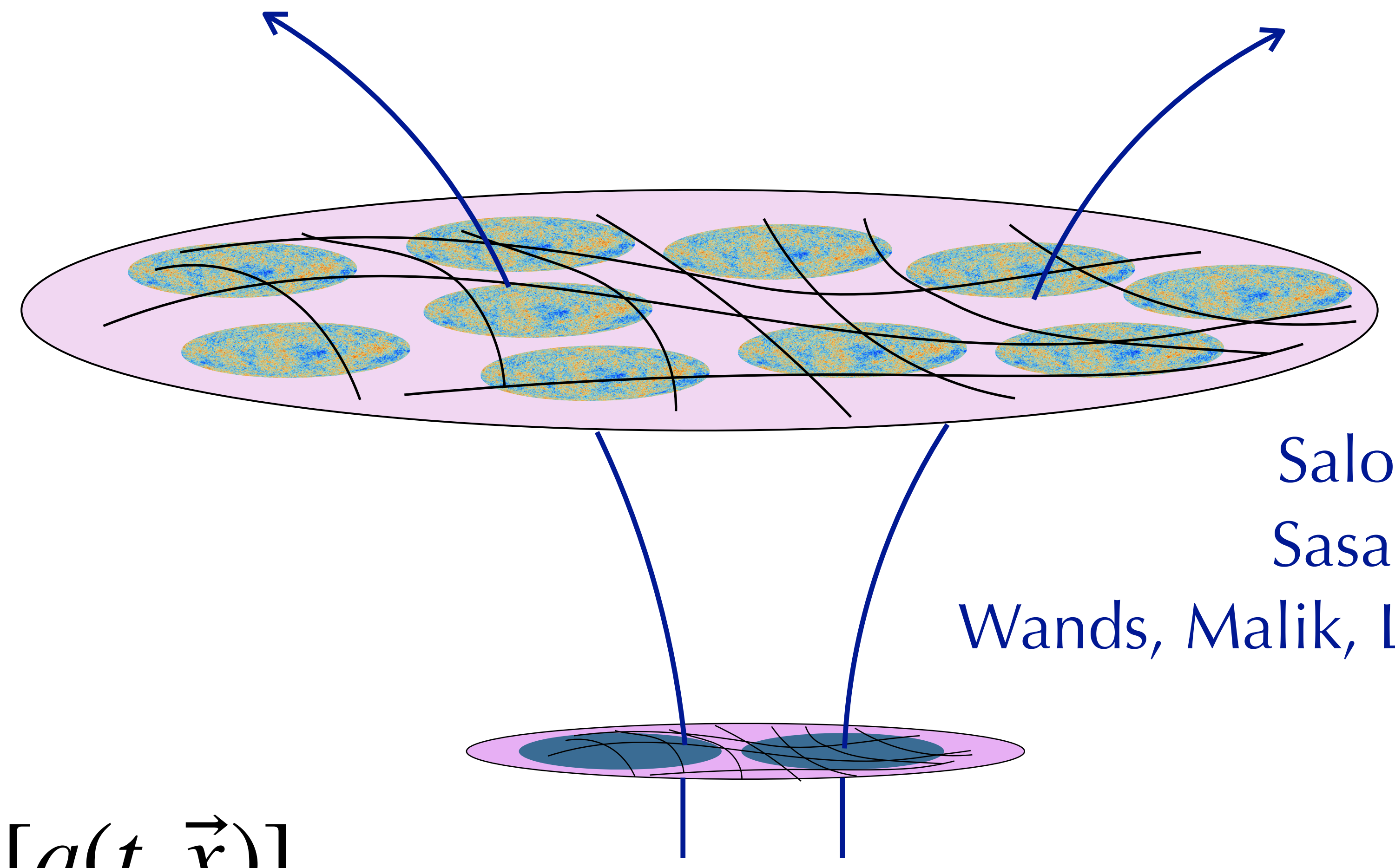
$$r < 0.056$$

Planck 2018 results. X. Constraints on inflation

Large perturbations from inflation: non-perturbative framework

Separate universe approach

At large scales the Universe is an ensemble of independent, locally homogenous and isotropic Hubble-sized patches.



Salopek, Bond [1990]
Sasaki, Stewart [1996]
Wands, Malik, Lyth, Liddle [2000]

Curvature perturbation ζ is the local amount of expansion:

$$\zeta(t,\vec{x}) = N(t,\vec{x}) - \overline{N}(t) \equiv \delta N$$

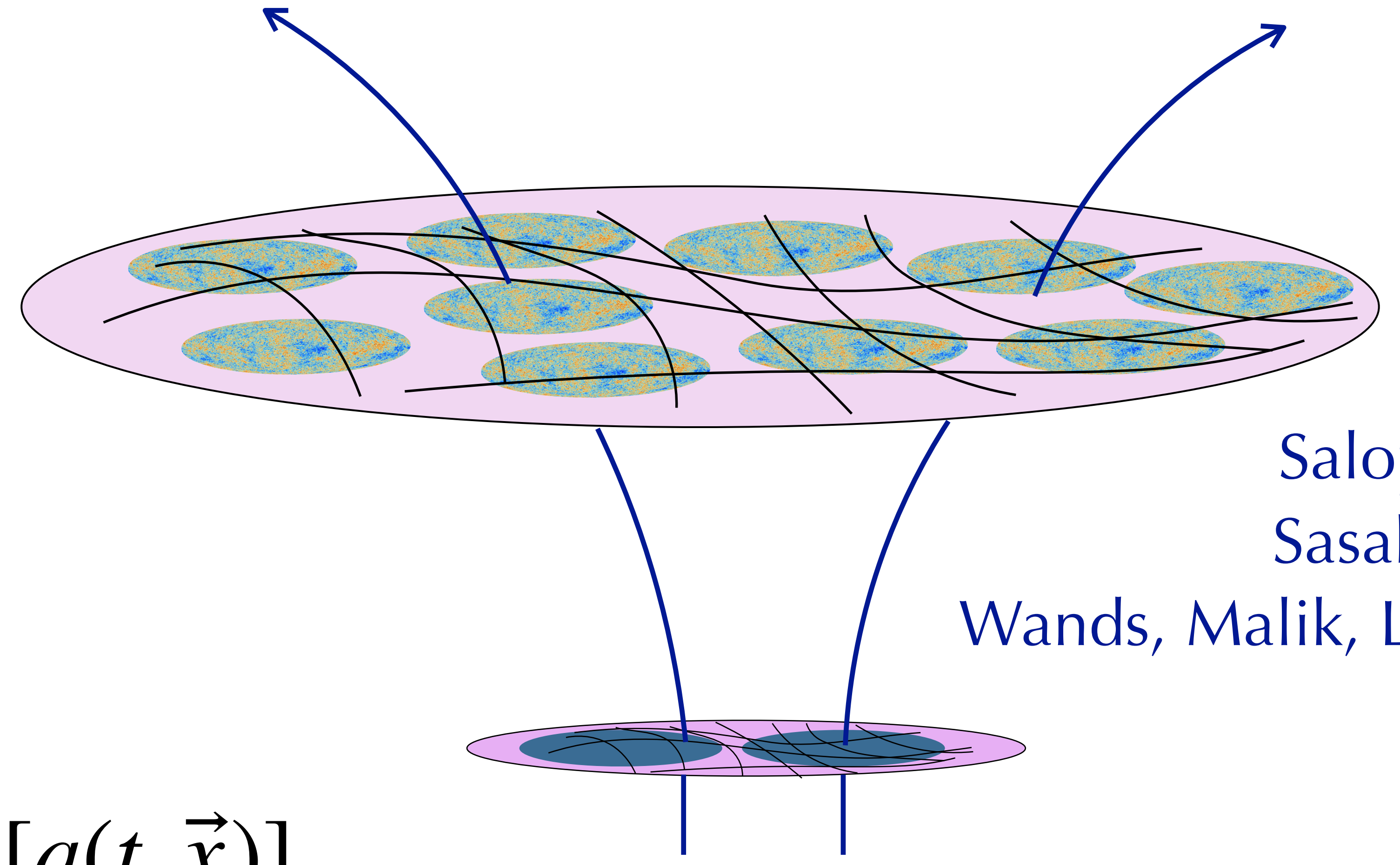
δN formalism

$$N(t,\vec{x}) = \log[a(t,\vec{x})]$$

Large perturbations from inflation: non-perturbative framework

Separate universe approach

At large scales the Universe is an ensemble of independent, locally homogenous and isotropic Hubble-sized patches.



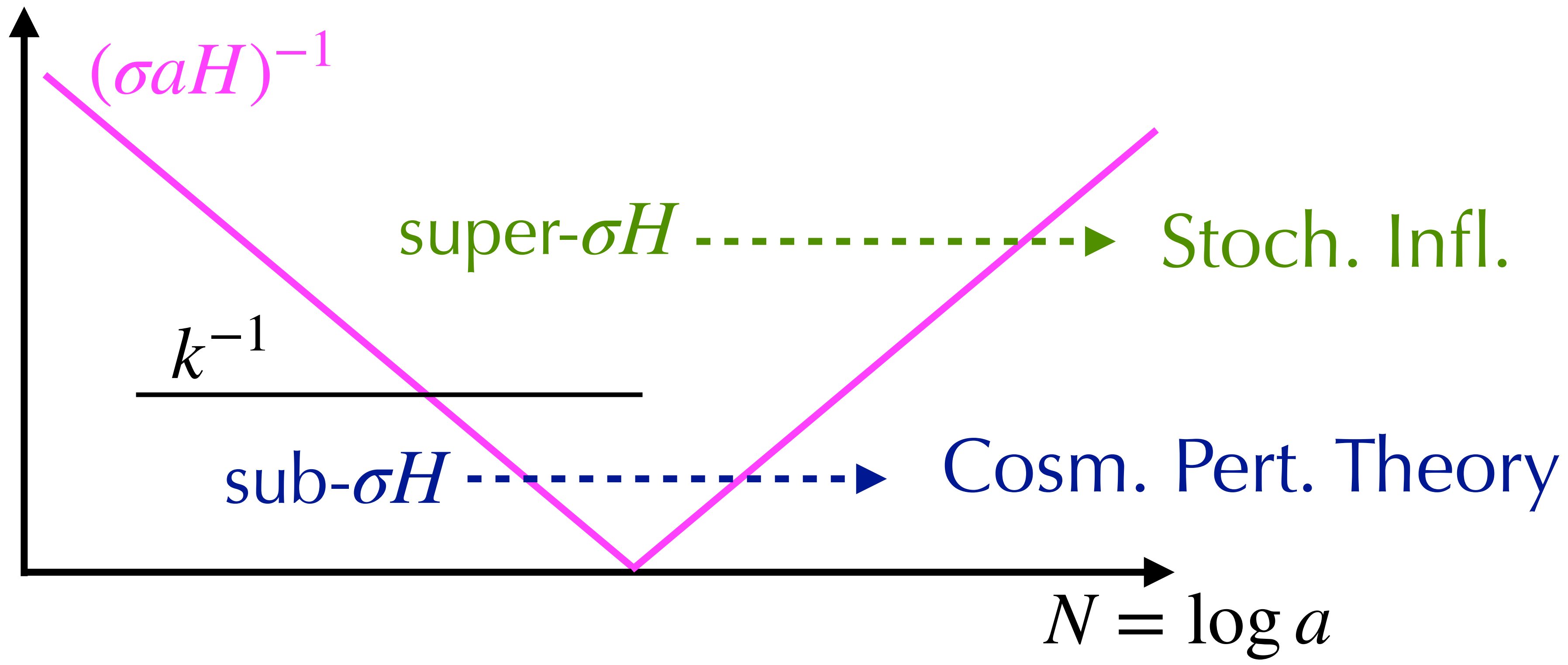
Salopek, Bond [1990]
Sasaki, Stewart [1996]
Wands, Malik, Lyth, Liddle [2000]

Curvature perturbation ζ is the local amount of expansion:

$$\zeta(t,\vec{x}) = N(t,\vec{x}) - \overline{N}(t) \equiv \delta N \qquad \delta N \text{ formalism} \qquad N(t,\vec{x}) = \log[a(t,\vec{x})]$$

Stochastic inflation A. Starobinsky [1986]

$$\hat{\phi}(x)_{\rm cg}(N,\vec{x}) = \int d\vec{k}\, \widetilde{W}\left(\frac{k}{\sigma a(N) H}\right)\left[\phi_{\vec{k}}(N)\, e^{-i\vec{k}\cdot\vec{x}}\, \hat{a}_{\vec{k}} + \text{h.c.}\right]$$



Stochastic classical theory for $\phi_{\rm cg}$:

$$\frac{d\phi_{\rm cg}}{dN} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

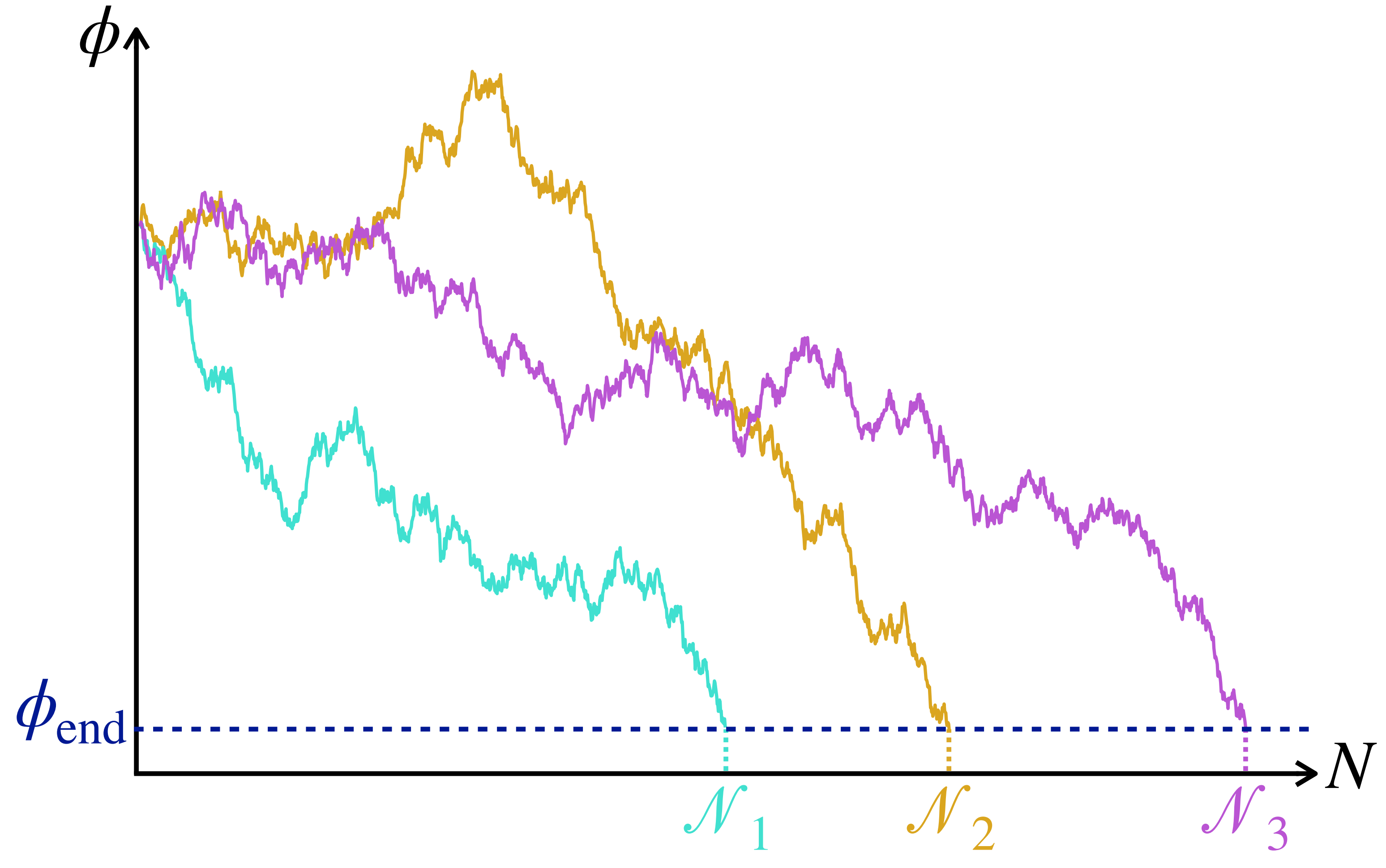
classical drift — quantum diffusion

$V(\phi)$: Inflationary potential

$\xi(N)$: White Gaussian noise

$$\langle \xi(N) \rangle = 0, \ \langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Stochastic-δN formalism



Duration of inflation becomes a stochastic variable: $\mathcal{N}$

Distribution function for the duration of inflation (*first-passage time*):

$$\frac{\partial}{\partial \mathcal{N}} P_{\rm FPT}(\mathcal{N},\phi) = -\frac{V'}{3H^2}\frac{\partial}{\partial \phi} P_{\rm FPT}(\mathcal{N},\phi) + \frac{H^2}{8\pi^2}\frac{\partial^2}{\partial \phi^2} P_{\rm FPT}(\mathcal{N},\phi)$$

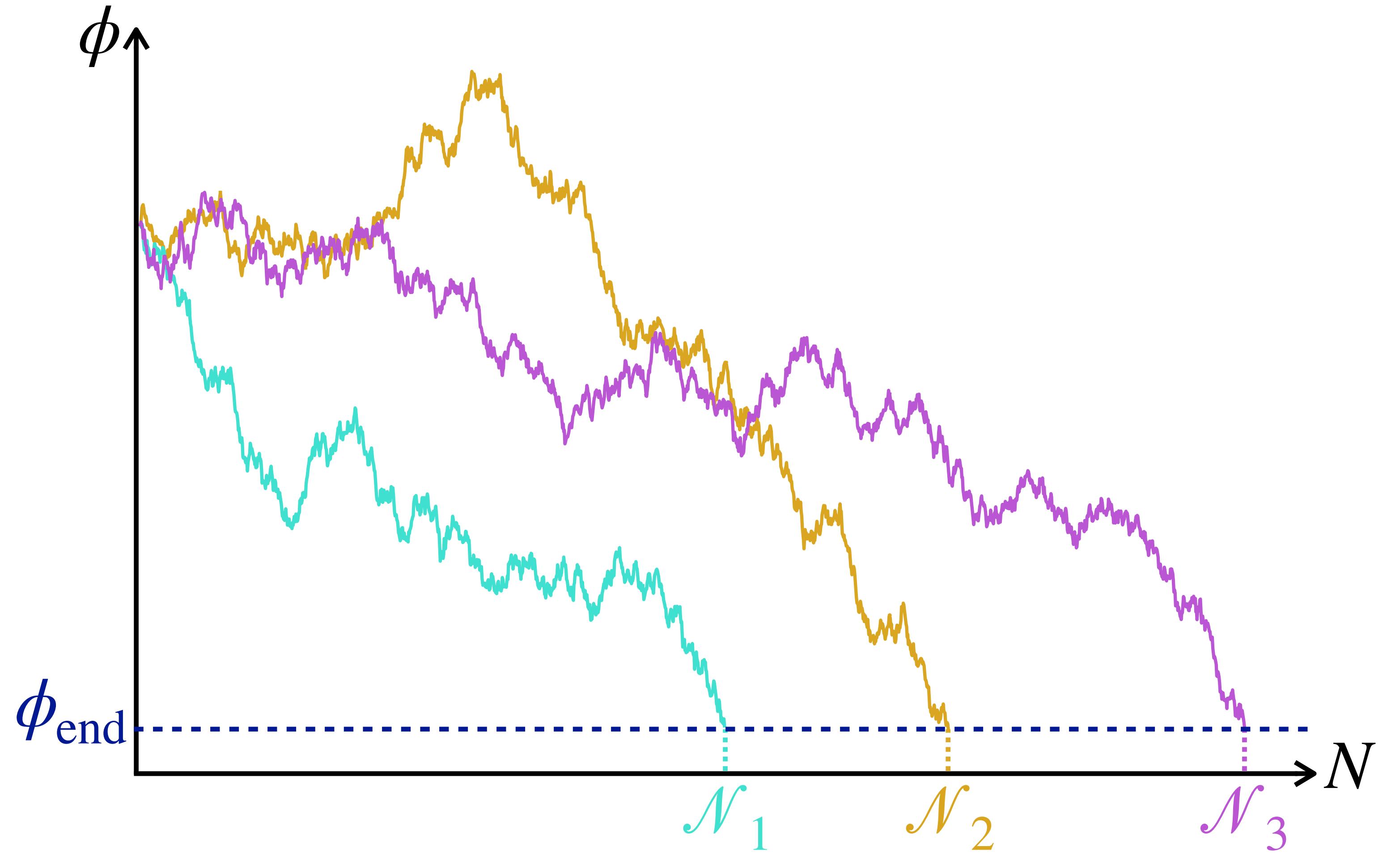
Statistics of ζ from the statistics of $\mathcal{N}$: $\zeta_{cg}(\vec{x}) = \mathcal{N}(\vec{x}) - \langle \mathcal{N} \rangle$

exponential tails

$$P_{\rm FPT}(\mathcal{N},\Phi) = \sum_n a_n(\Phi)\, e^{-\Lambda_n \mathcal{N}}, \quad 0 < \Lambda_0 < \Lambda_1 < \cdots \Lambda_n \xrightarrow{\text{for large values of } \mathcal{N}} P_{\rm FPT}(\mathcal{N},\phi) \simeq a_0(\phi)\, e^{-\Lambda_0 \mathcal{N}}$$

Cannot be captured by perturbative parametrisations ($f_{\rm NL}$, $g_{\rm NL}$,… expansion).

Stochastic-δN formalism



Duration of inflation becomes a stochastic variable: $\mathcal{N}$

Distribution function for the duration of inflation (*first-passage time*):

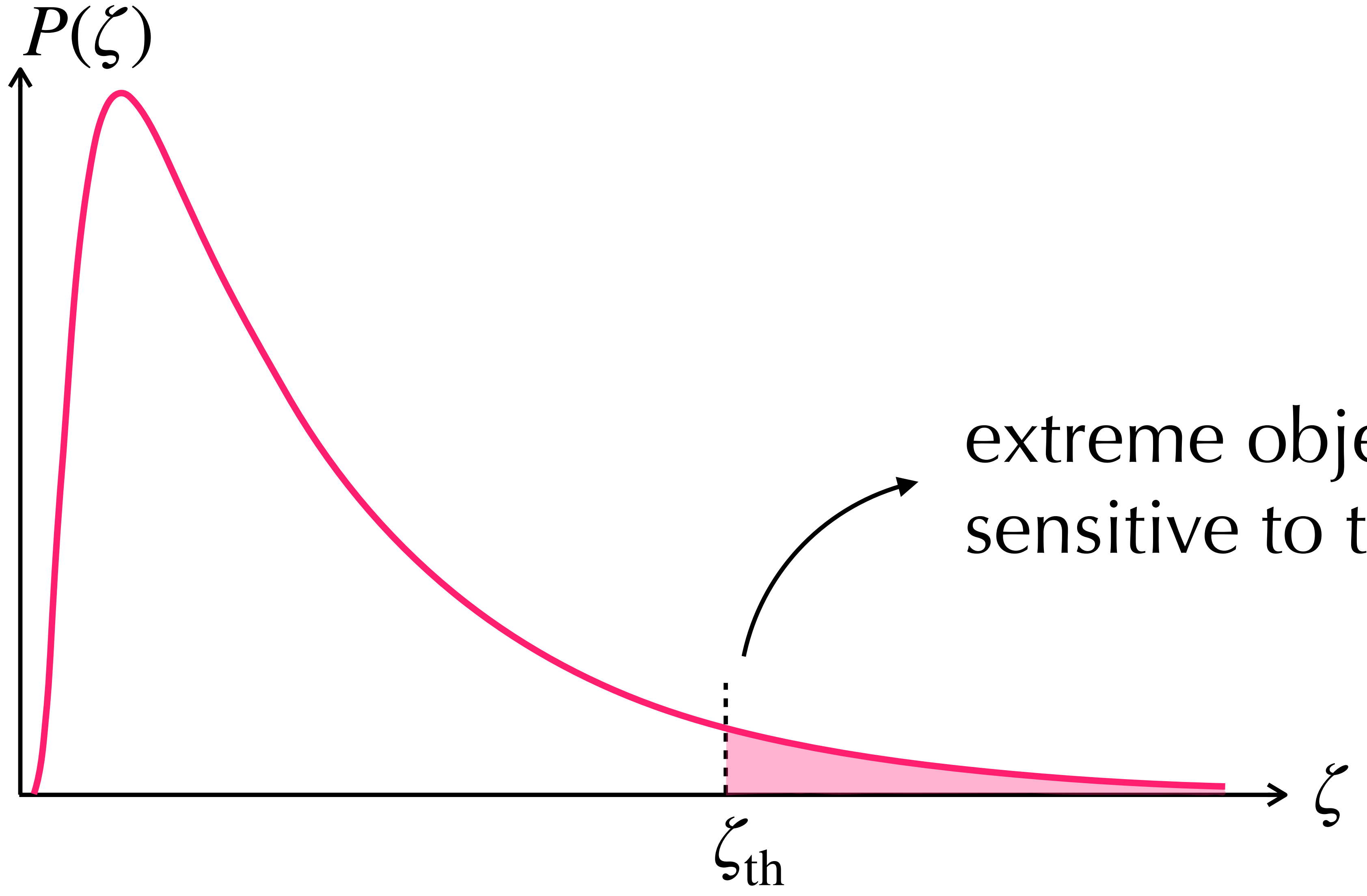
$$\frac{\partial}{\partial \mathcal{N}} P_{\rm FPT}(\mathcal{N},\phi) = -\frac{V'}{3H^2}\frac{\partial}{\partial\phi}P_{\rm FPT}(\mathcal{N},\phi) + \frac{H^2}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P_{\rm FPT}(\mathcal{N},\phi)$$

Statistics of ζ from the statistics of $\mathcal{N}$: $\zeta_{cg}(\vec{x}) = \mathcal{N}(\vec{x}) - \langle \mathcal{N} \rangle$

$$P_{\rm FPT}(\mathcal{N},\Phi) = \sum_n a_n(\Phi)\, e^{-\Lambda_n \mathcal{N}}, \quad 0 < \Lambda_0 < \Lambda_1 < \cdots \Lambda_n \xrightarrow{\text{for large values of } \mathcal{N}} P_{\rm FPT}(\mathcal{N},\phi) \simeq a_0(\phi)\, e^{-\Lambda_0 \mathcal{N}}$$

exponential tails

Cannot be captured by perturbative parametrisations ($f_{\rm NL}$, $g_{\rm NL}$,… expansion).



extreme objects (as primordial black holes) sensitive to tails

PBH abundance: $\beta \simeq \int_{\zeta_c}^{\infty} P(\zeta)\,{\rm d}\zeta$

Going beyond: challenges in the stochastic-δN formalism

When we take a single Langevin realisation, we follow one worldline to its final patch.

Repeating this many times lets us reconstruct the statistics of ζ.

Is the information about the spatial arrangement of patches lost? How to describe spatial correlations?

Going beyond: challenges in the stochastic-δN formalism

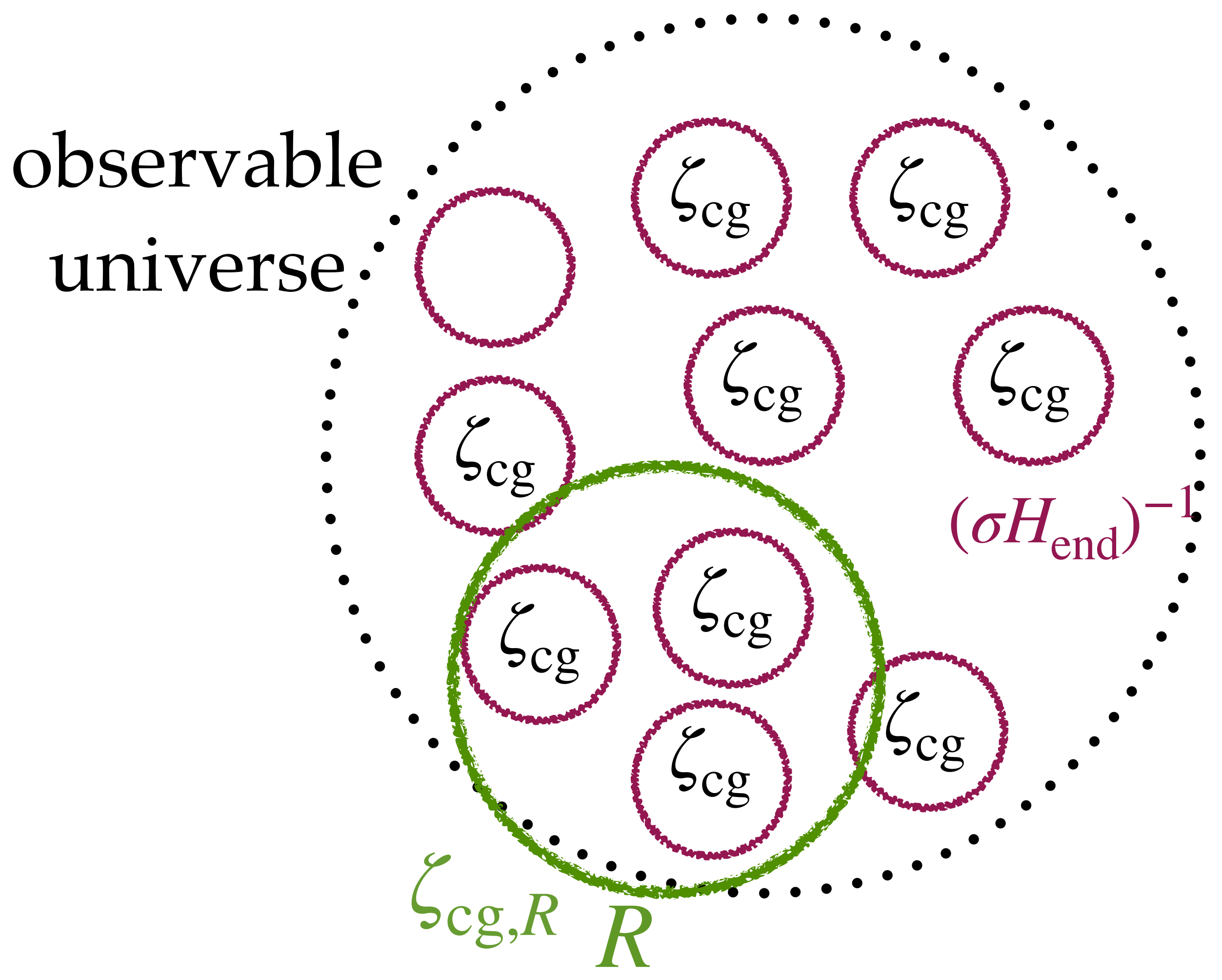
When we take a single Langevin realisation, we follow one worldline to its final patch.

Repeating this many times lets us reconstruct the statistics of ζ.

Is the information about the spatial arrangement of patches lost? How to describe spatial correlations?

→ **Coarse-graining at arbitrary scale R**

(PBHs mass functions, statistics of density contrast, compaction function,…).



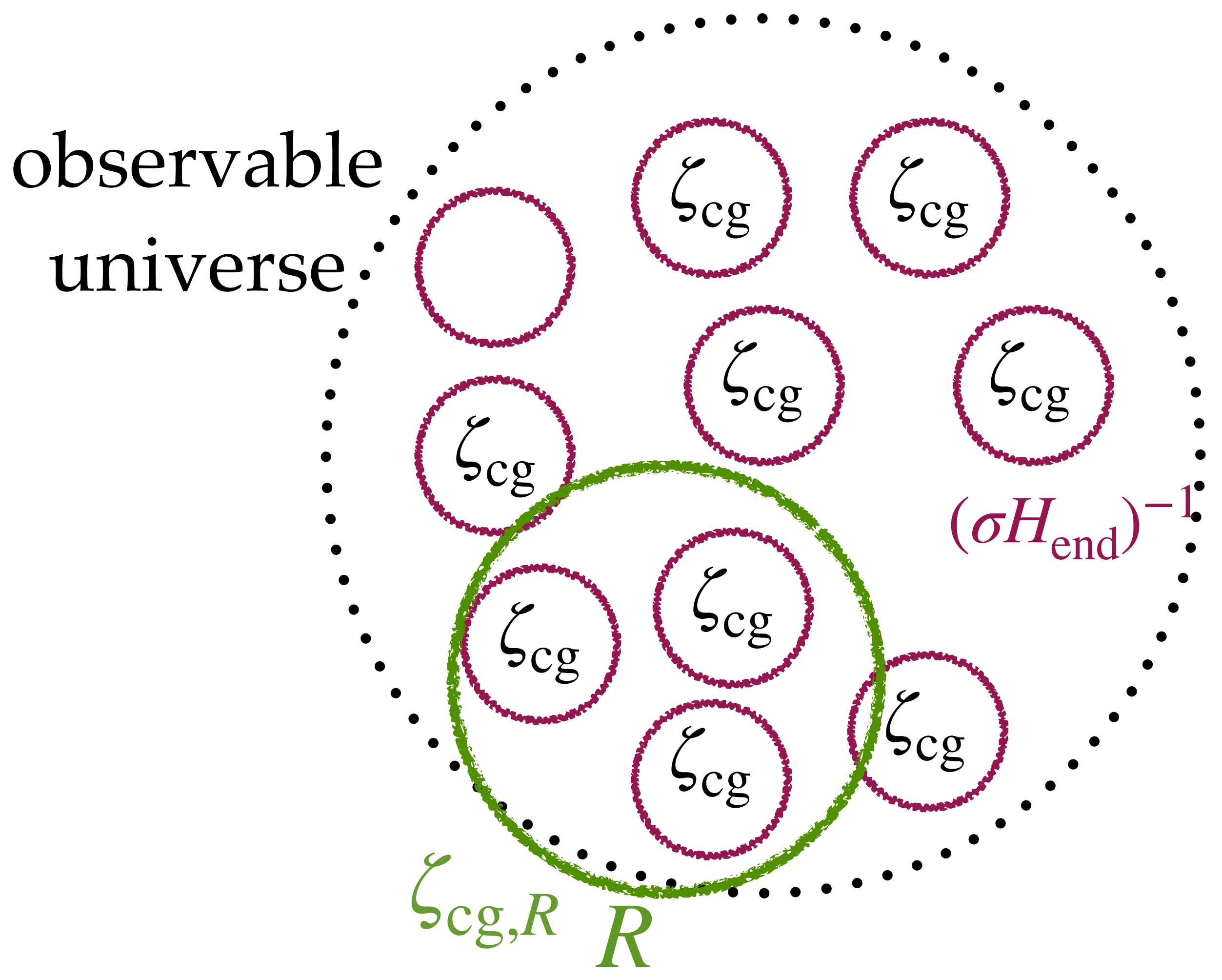
Going beyond: challenges in the stochastic-δN formalism

When we take a single Langevin realisation, we follow one worldline to its final patch.

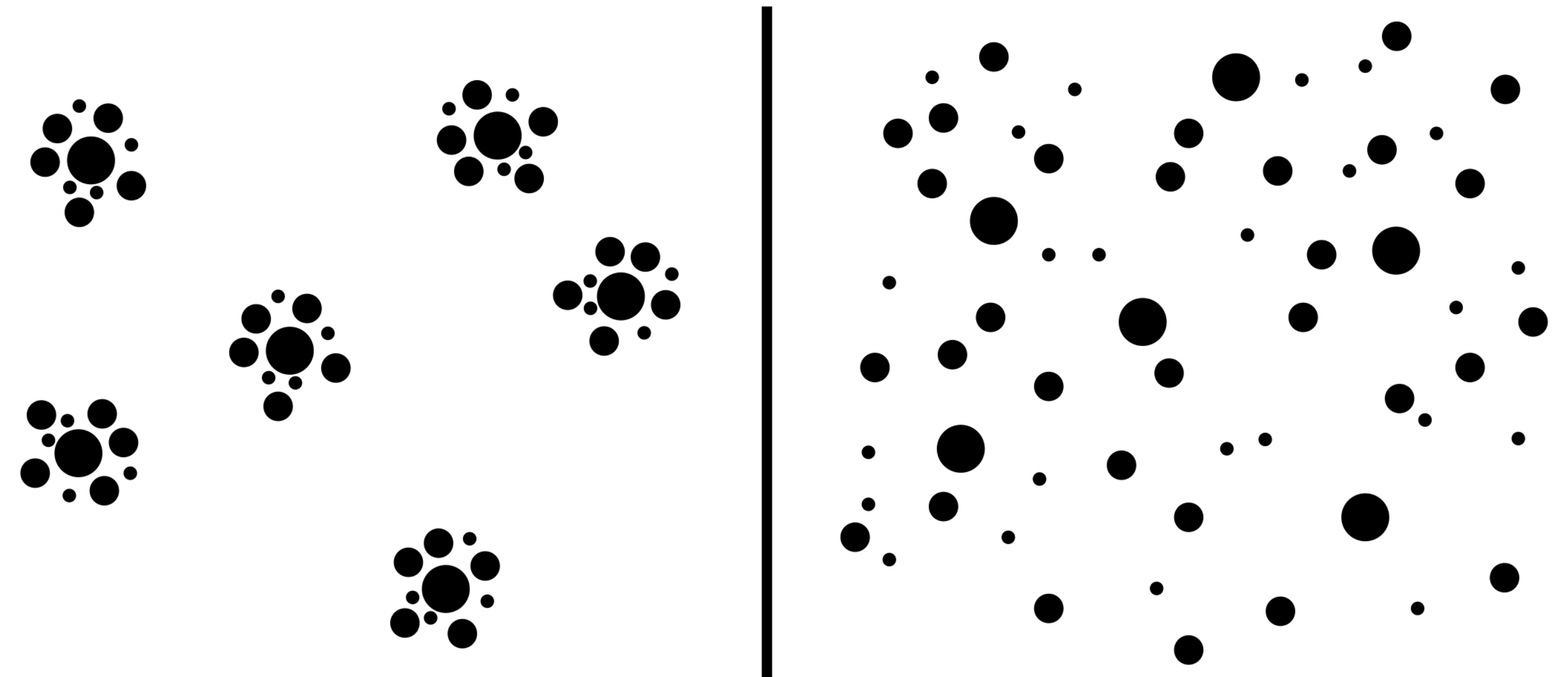
Repeating this many times lets us reconstruct the statistics of ζ.

Is the information about the spatial arrangement of patches lost? How to describe spatial correlations?

→ **Coarse-graining at arbitrary scale R**
(PBHs mass functions, statistics of density contrast, compaction function,…).



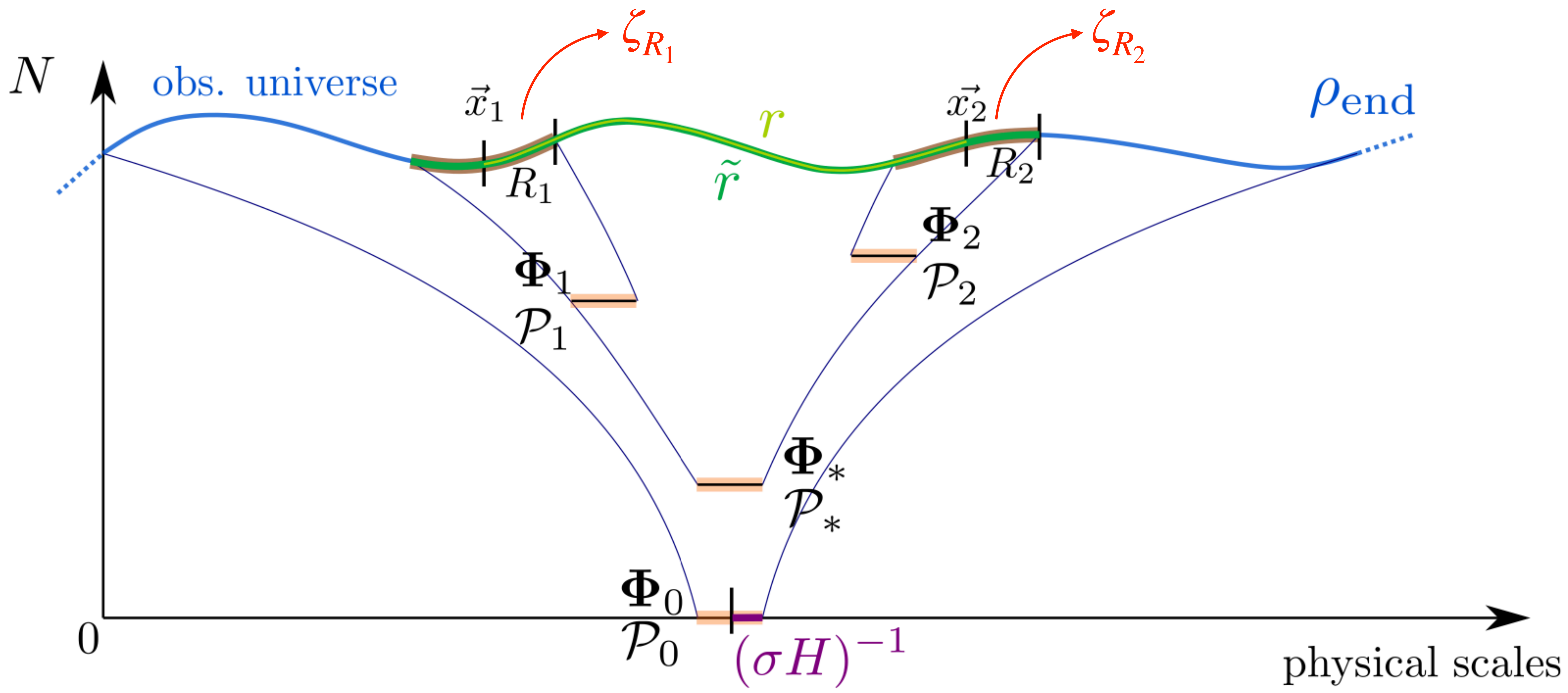
→ **Clustering** properties of PBHs in presence of non-perturbative non Gaussianities (quantum diffusion).



clustered vs non-clustered spatial distribution

Spatial reconstruction: beyond one-point distributions

In the separate-universe framework, distance between two final Hubble patches encoded in the time at which their worldlines became stochastically independent.



Vennin, Ando [2021]
Tada, Vennin [2021]
Animali, Vennin [2024]

$$\zeta_{\mathrm{cg},R_i}(\vec{x}_i) \equiv \zeta_{R_i}(\vec{x}_i) = \mathbb{E}^V_{\mathscr{P}_i}[\mathcal{N}_{\mathscr{P}_0}(\vec{x})] - \mathbb{E}^V_{\mathscr{P}_0}[\mathcal{N}_{\mathscr{P}_0}(\vec{x})]$$

$$\mathcal{N}_{\mathscr{P}_0}(\vec{x}_i) = \mathcal{N}_{\mathscr{P}_0 \to \mathscr{P}_*}(\vec{x}) + \mathcal{N}_{\mathscr{P}_* \to \mathscr{P}_i}(\vec{x}_i) + \mathcal{N}_{\mathscr{P}_i}(\vec{x}_i)$$

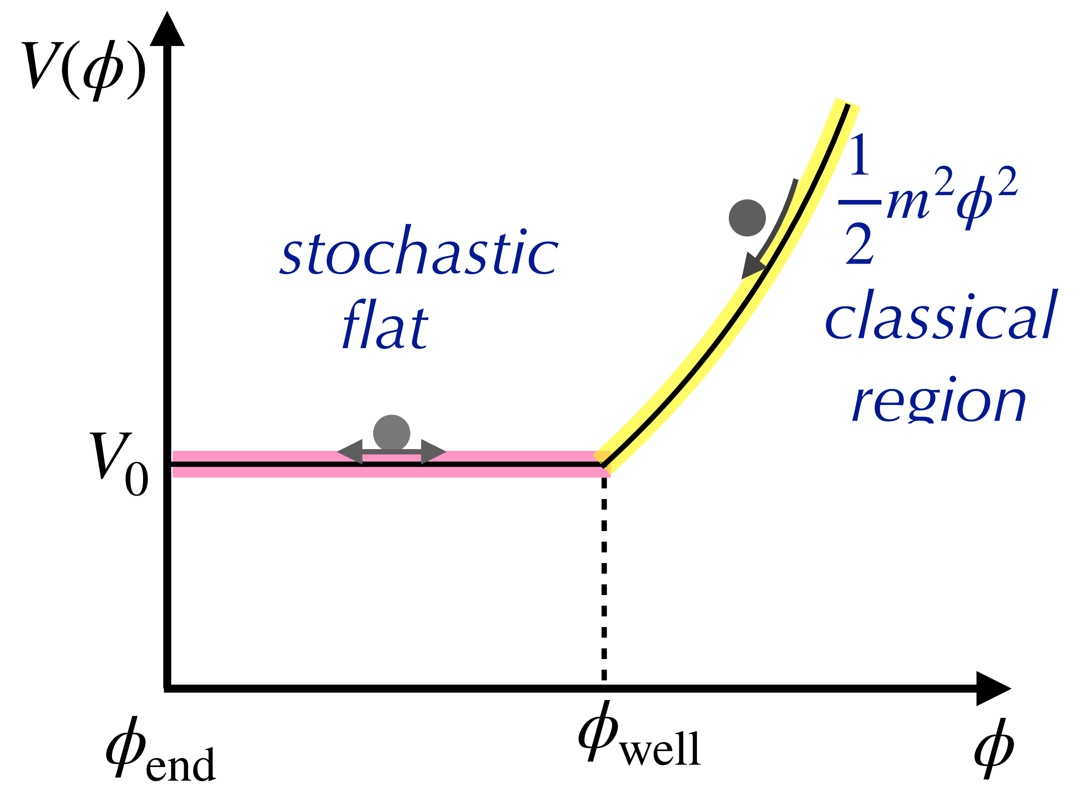
Shared history

Backward-approximation approach

Vennin, Ando [2021]
Vennin, Tada [2022]

Field value at the splitting patch is the field value at a fixed backward number of e-folds $N_{\rm bw}$.

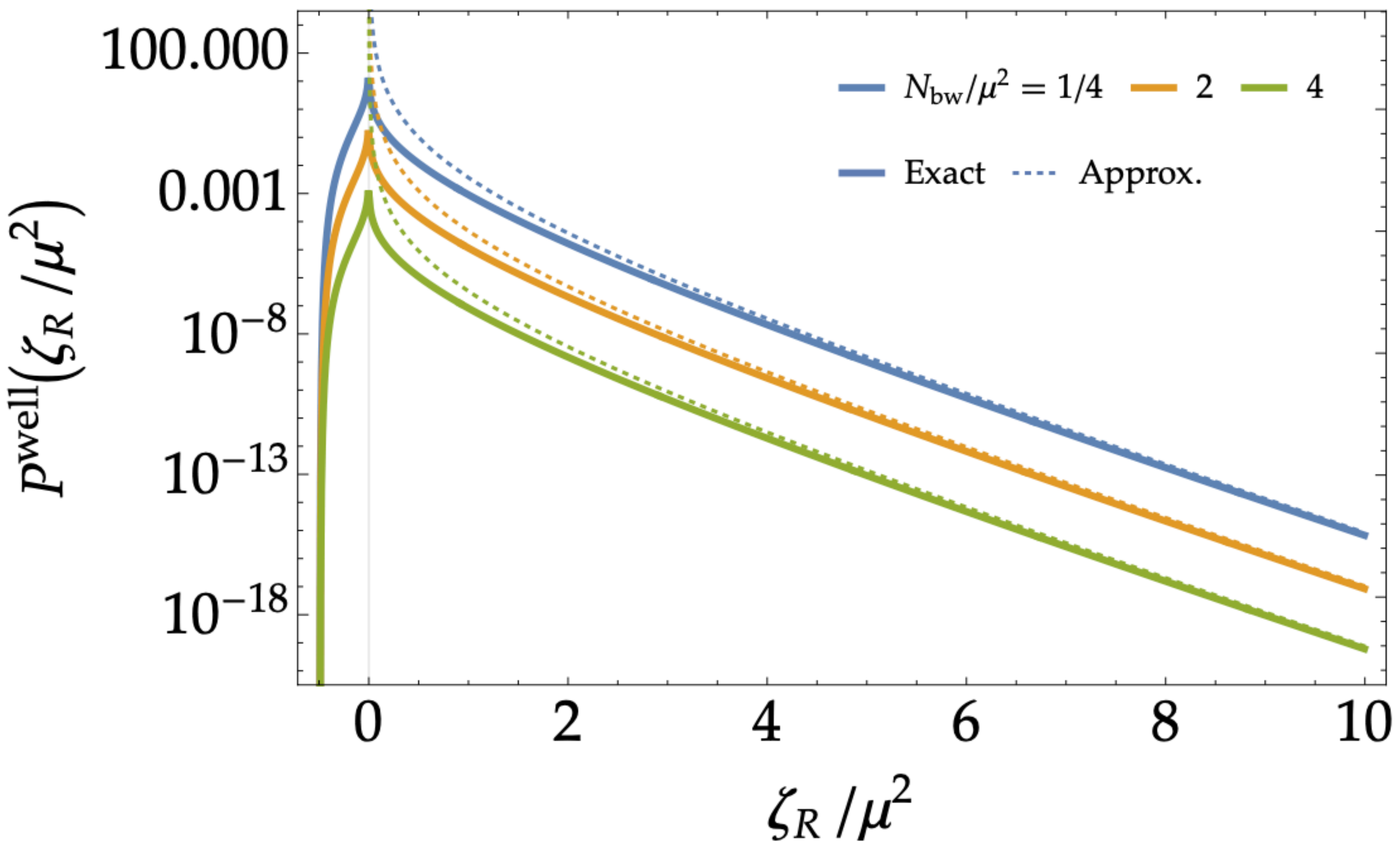
$$P_{\rm bw}(\Phi_*, N_{\rm bw}) = P_{\rm FPT}(N_{\rm bw}, \Phi_*)\frac{\int_0^\infty {\rm d}N P(\Phi_*, N \,|\, \Phi_0, 0)}{\int_{N_{\rm bw}}^\infty {\rm d}N_{\rm tot}\, P_{\rm FPT}(N_{\rm tot}, \Phi_0)}$$

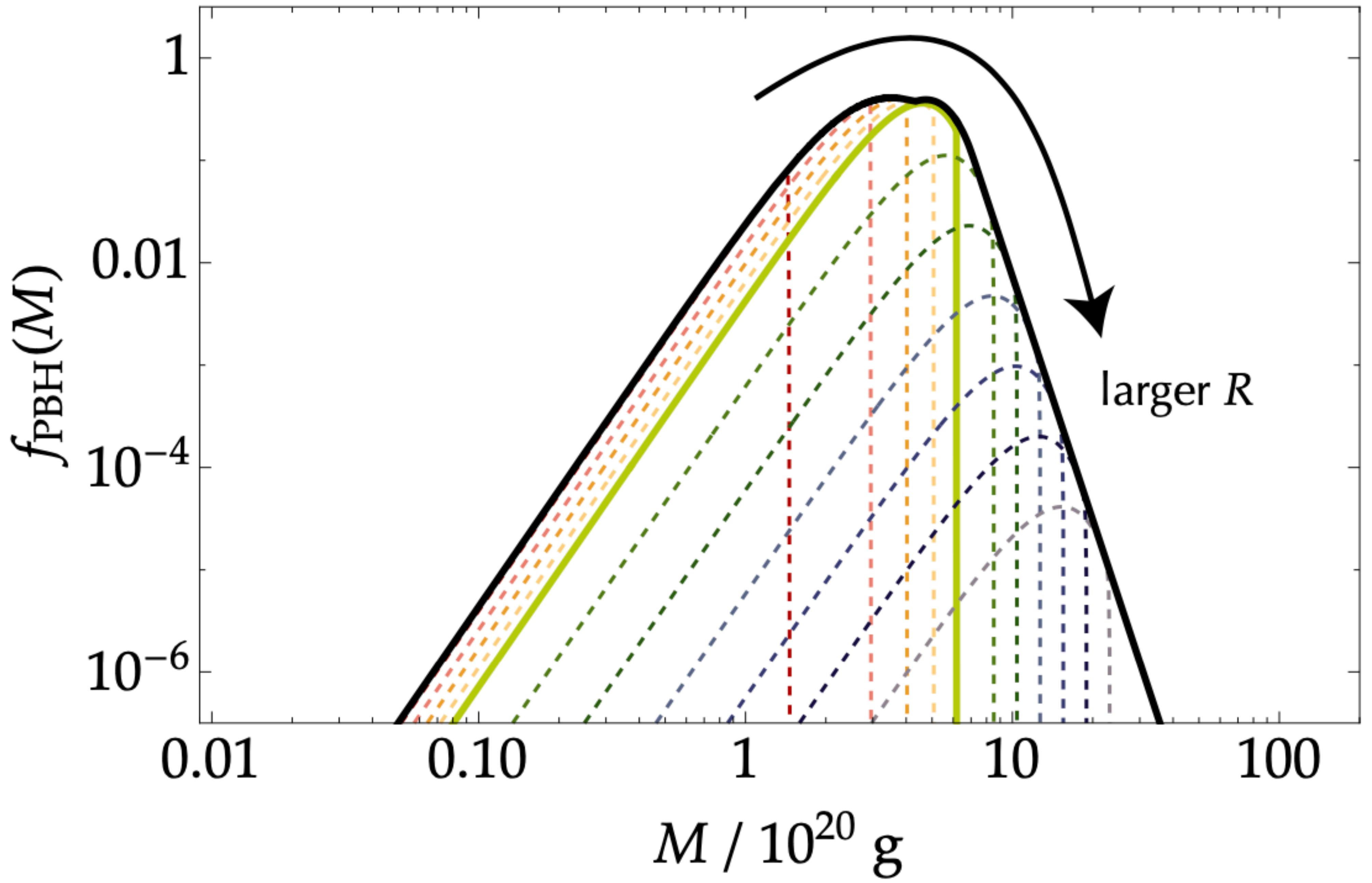


Statistics of coarse-grained fields:

$$P(\zeta_R) = \int_\Omega {\rm d}\Phi_*\, P_{\rm bw}[\Phi_* \,|\, N_{\rm bw}(R)]\, P_{{\rm FPT},\Phi_0\to\Phi_*}[\zeta_R - \langle \mathcal{N}(\Phi_*)\rangle + \langle \mathcal{N}(\Phi_0)\rangle]$$

$$\mu = \frac{1}{\sqrt{6}}$$

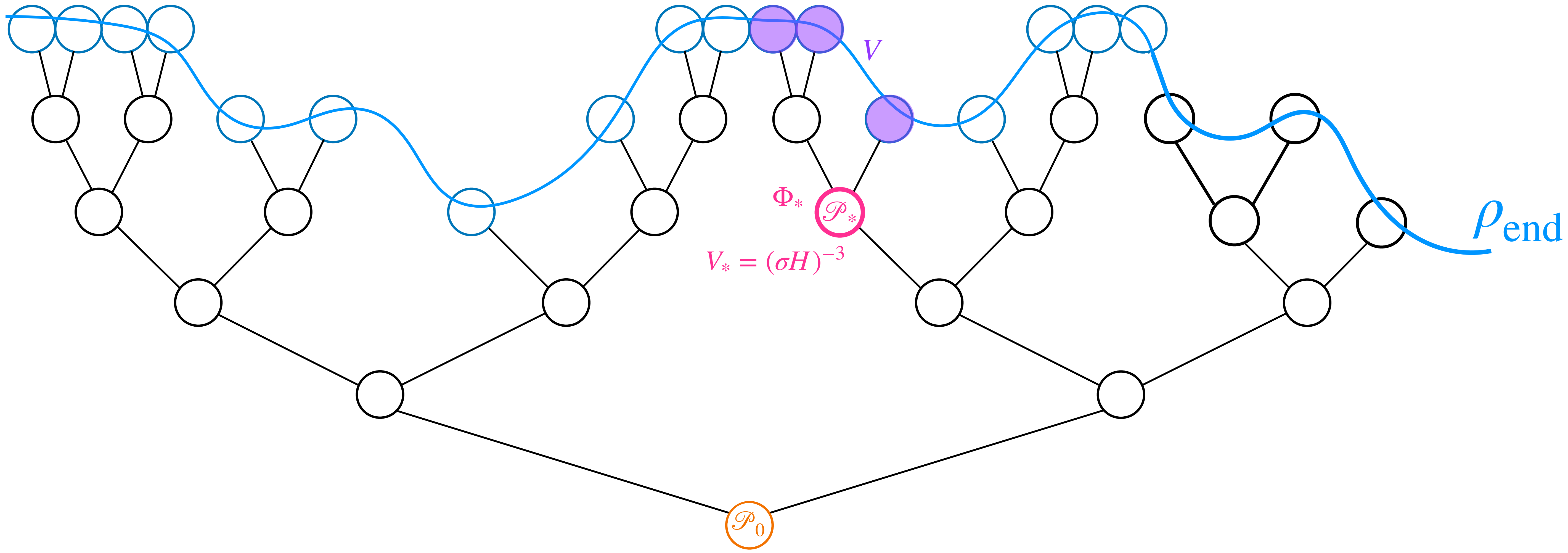




Forward and backward statistics

Animali, Vennin [2024]

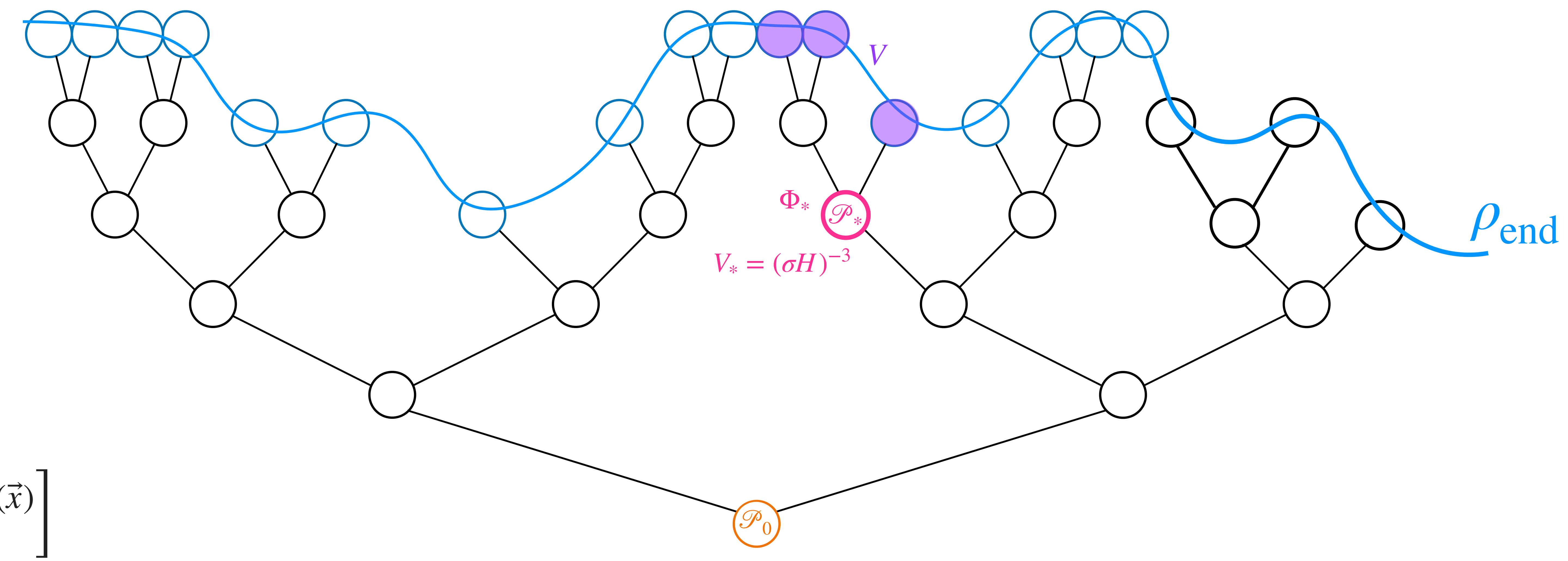
Relation between field values and physical distances encoded in the separate-universe structure of a universe which inflates stochastically.



Forward and backward statistics

Animali, Vennin [2024]

Relation between field values and physical distances encoded in the separate-universe structure of a universe which inflates stochastically.



$$V = \int_{\mathscr{P}_*} \mathrm{d}\vec{x}\, e^{3\mathcal{N}_{\mathscr{P}_*}(\vec{x})} = \mathbb{E}_{\mathscr{P}_*}\left[e^{3\mathcal{N}_{\mathscr{P}_*}(\vec{x})}\right]$$

$$P(\Phi_* \,|\, V, \Phi_0) = \frac{P(V\,|\,\Phi_*)P(\Phi_*\,|\,\Phi_0)}{P(V)} = \frac{P(V\,|\,\Phi_*)P(\Phi_*\,|\,\Phi_0)}{\int \mathrm{d}\Phi_*\, P(V\,|\,\Phi_*)P(\Phi_*\,|\,\Phi_0)}$$

$$W \equiv \mathbb{E}^V_{\mathscr{P}_*}\left[\mathcal{N}_{\mathscr{P}_*}(\vec{x})\right] = V^{-1}\int_{\mathscr{P}_*} e^{3\mathcal{N}_{\mathscr{P}_*}(\vec{x})}\mathcal{N}_{\mathscr{P}_*}(\vec{x})\mathrm{d}\vec{x} = V^{-1}\mathbb{E}_{\mathscr{P}_*}\left[e^{3\mathcal{N}_{\mathscr{P}_*}(\vec{x})}\mathcal{N}_{\mathscr{P}_*}(\vec{x})\right]$$

$$\zeta_{\rm cg} = \mathbb{E}^V_{\mathscr{P}_*}(\mathcal{N}_{\mathscr{P}_0}) - \mathbb{E}^V_{\mathscr{P}_0}(\mathcal{N}_{\mathscr{P}_0}) = \mathcal{N}_{\mathscr{P}_0\to\mathscr{P}_*} + W - \mathbb{E}^V_{\mathscr{P}_0}(\mathcal{N}_{\mathscr{P}_0})$$

Solutions of Fokker-Planck, adjoint Fokker Planck equations

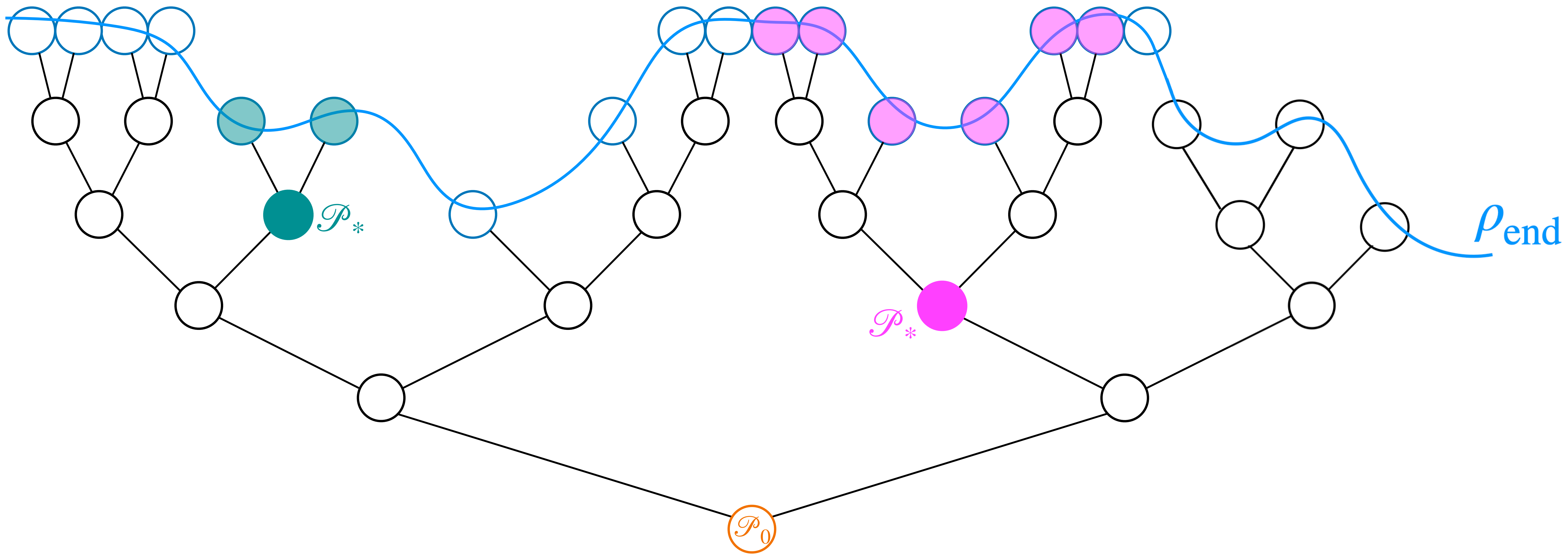
$$P(\zeta_R) \propto P(\mathcal{N}_{\mathscr{P}_0\to\mathscr{P}_*}, W\,|\,V, \Phi_0) = \int \mathrm{d}\Phi_* P^V(\mathcal{N}_{\mathscr{P}_0\to\mathscr{P}_*})P_{\rm FP}(\Phi_*, \mathcal{N}_{\mathscr{P}_0\to\mathscr{P}_*}\,|\,\Phi_0)\,\frac{P(V, W\,|\,\Phi_*)}{P(V)}$$

→ *Not straightforward to compute analytically*

Volume weighting

Animali, Vennin [2024]

Different regions of the universe inflate by different amounts $\mathcal{N}$:
they contribute differently to ensemble averages computed by local observers on the end-of-inflation hypersurface.



Distributions with respect to which observable quantities are defined should be **volume weighted.**

$$P^V_{\mathrm{FPT},\Phi_0}(\mathcal{N}) = \frac{P_{\mathrm{FPT},\Phi_0}(\mathcal{N})\, e^{3\mathcal{N}}}{\int_0^\infty \mathrm{d}\mathcal{N} P_{\mathrm{FPT},\Phi_0}(\mathcal{N})\, e^{3\mathcal{N}}}$$

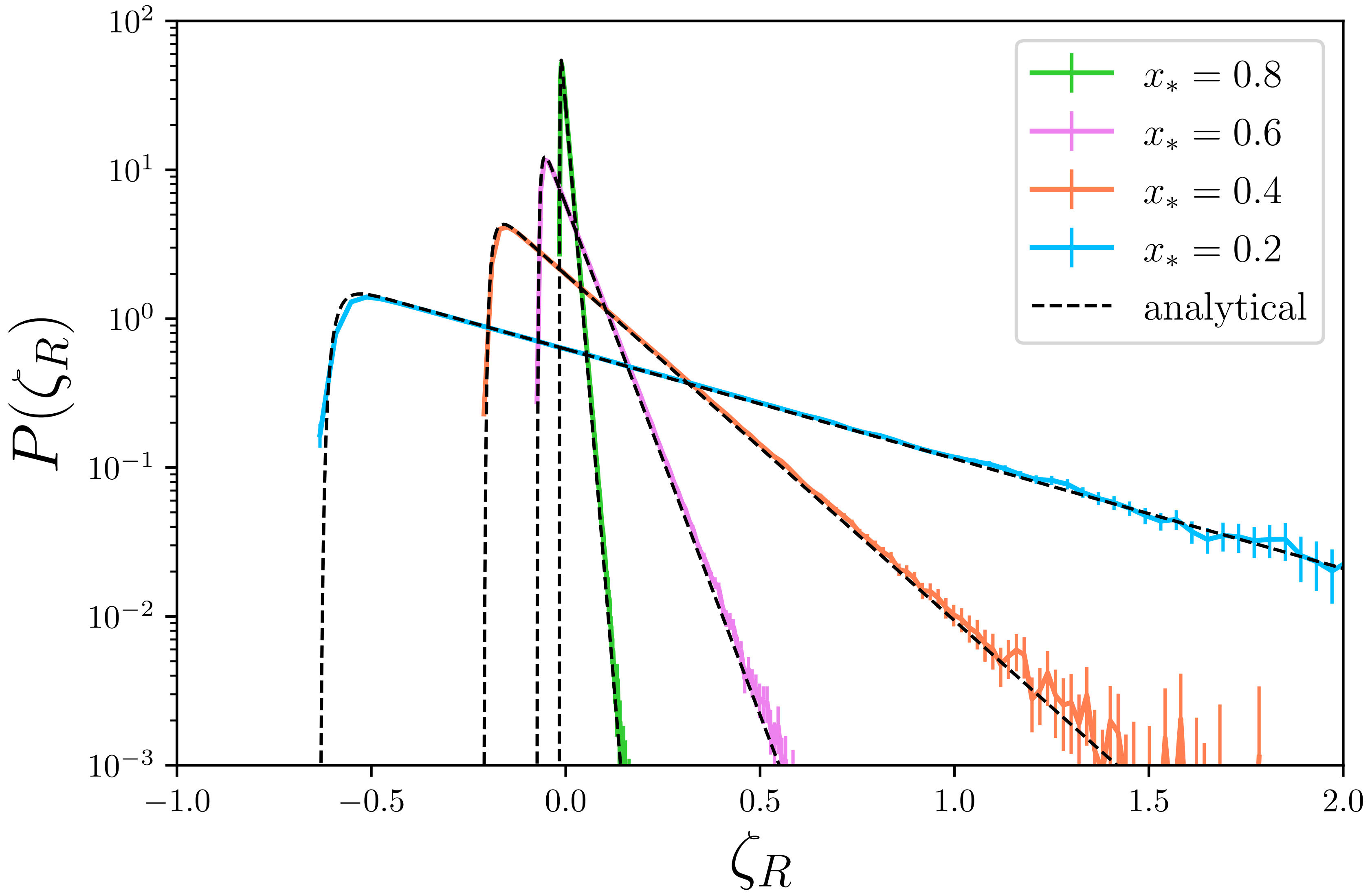
$$\zeta_{\rm cg}(\vec{x}) = \mathcal{N}_{\mathscr{P}_0}(\vec{x}) - \mathbb{E}^V_{\mathscr{P}_0}(\mathcal{N}_{\mathscr{P}_0}) \qquad P(\zeta_{\rm cg}\,|\,\Phi_0) = P^V_{\mathrm{FPT},\Phi_0}(\zeta_{\rm cg} + \mathbb{E}^V_{\mathscr{P}_0}(\mathcal{N}_{\mathscr{P}_0}))$$

For $P_{\mathrm{FPT},\Phi_0}(\mathcal{N}) \propto e^{-\Lambda\mathcal{N}}$ and $\Lambda \leq 3$ the volume-weighted distribution is not well-defined. "eternal inflation"

Large-volume approximation: one-point distributions

Animali, Vennin [2024]

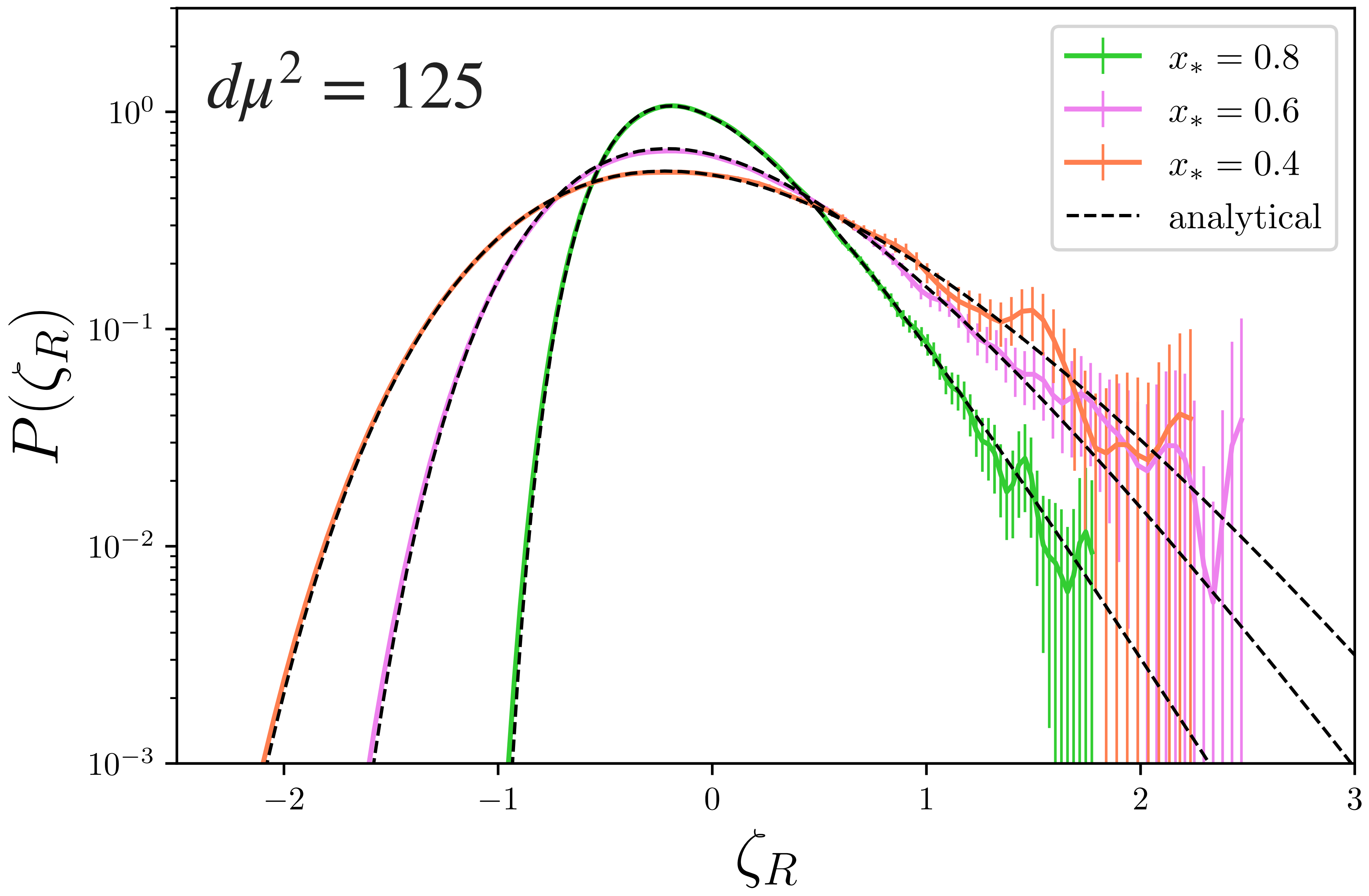
$v = v_0$



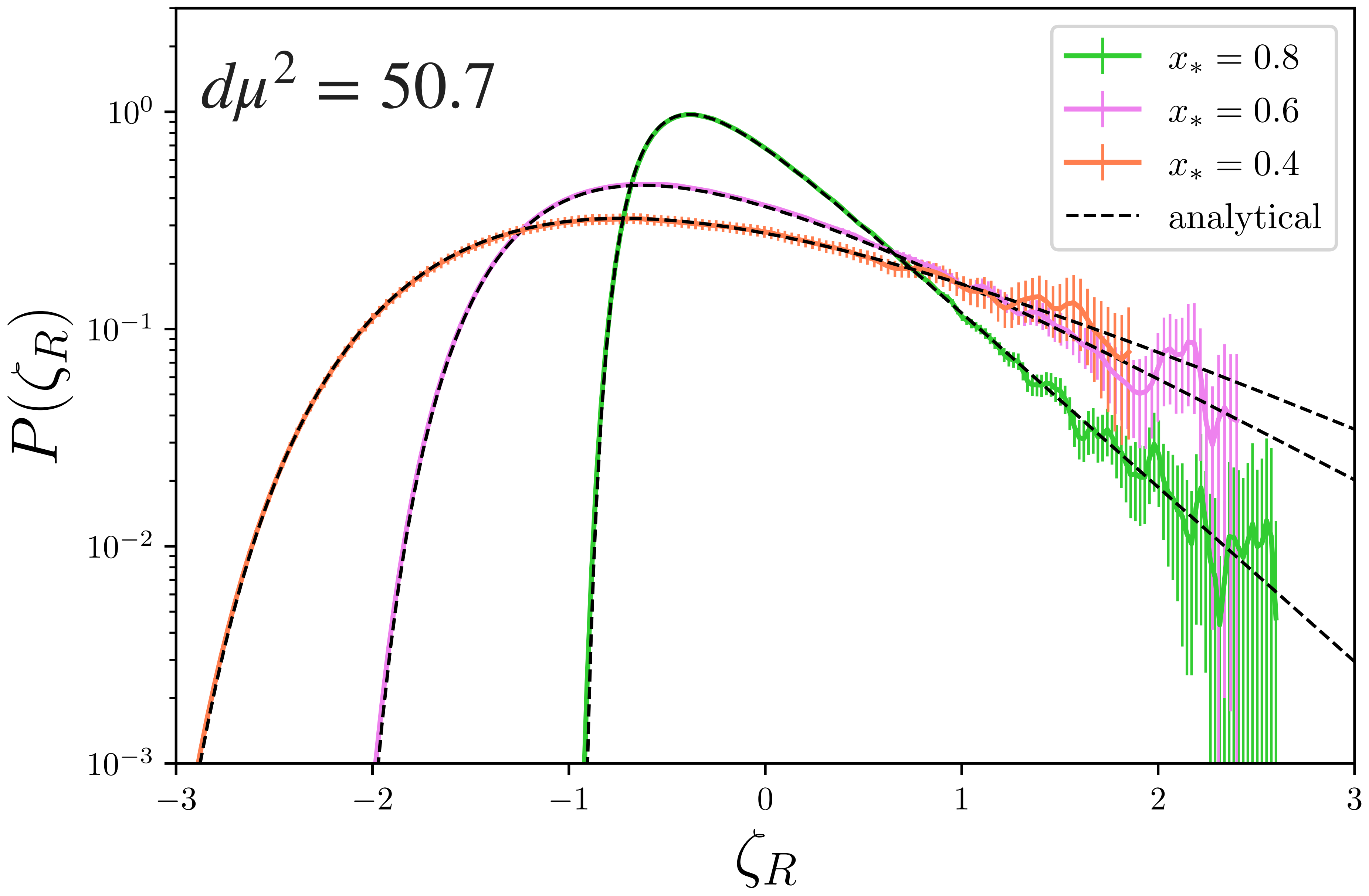
Tail behaviour: $$P(\zeta_R) \simeq \frac{\pi \cos\left[\sqrt{3}(1-x_*)\,\mu\right]}{(1-x_*)^2\,\mu^2} e^{\left[3-\frac{\pi^2}{4(1-x_*)^2\mu^2}\right]\left\{\zeta_R + \frac{\mu}{2\sqrt{3}}(1-x_*)\tan\left[\sqrt{3}\mu\,(1-x_*)\right]\right\}}$$

exponential-tail profile

$v = v_0(1 + \alpha\phi/M_{\mathrm{Pl}})$



$v = v_0(1 + \alpha\phi/M_{\mathrm{Pl}})$

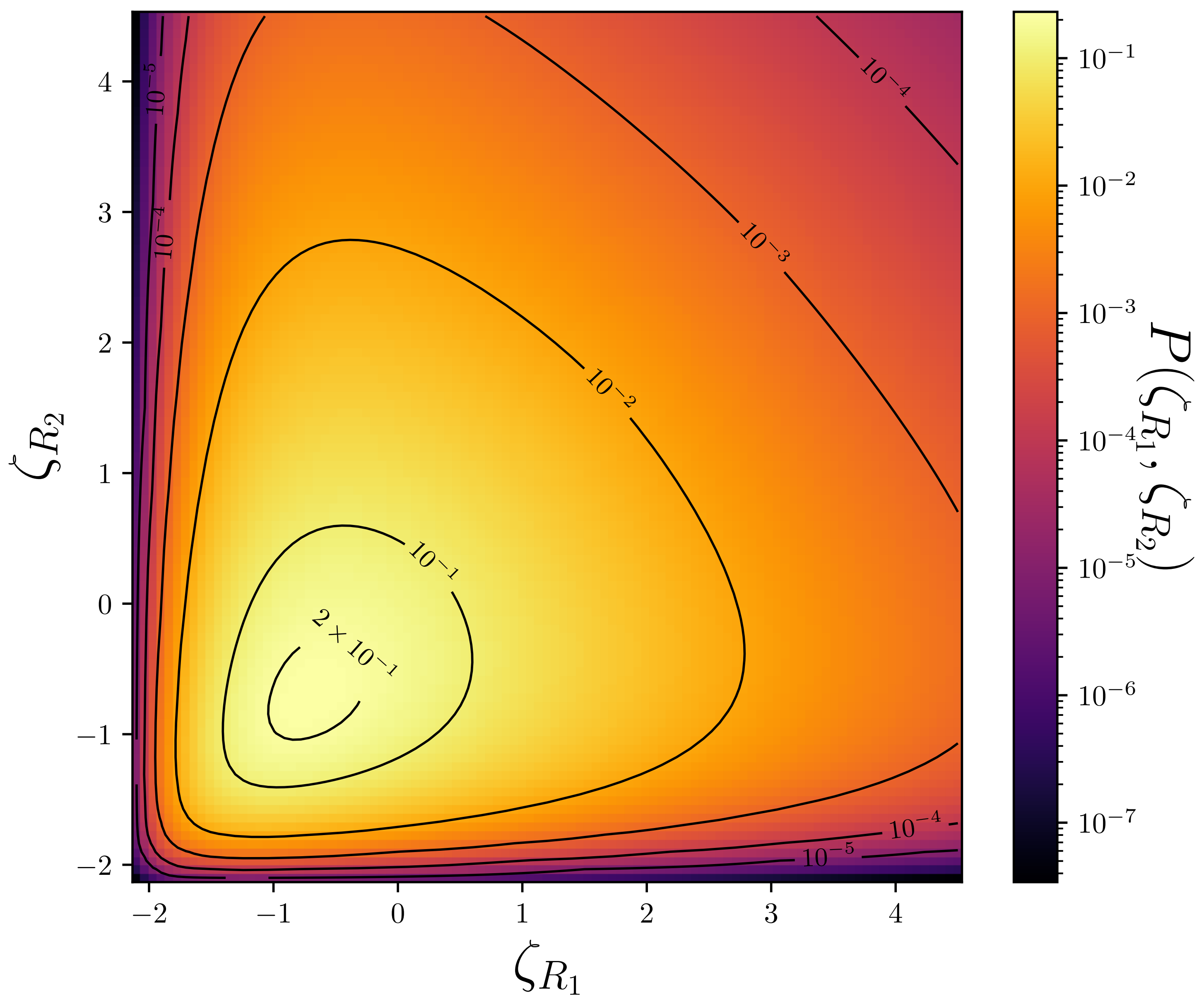


$\alpha\Delta\phi_{\mathrm{well}}/(v_0 M_{\mathrm{Pl}}) \equiv d\mu^2 \to \infty$:

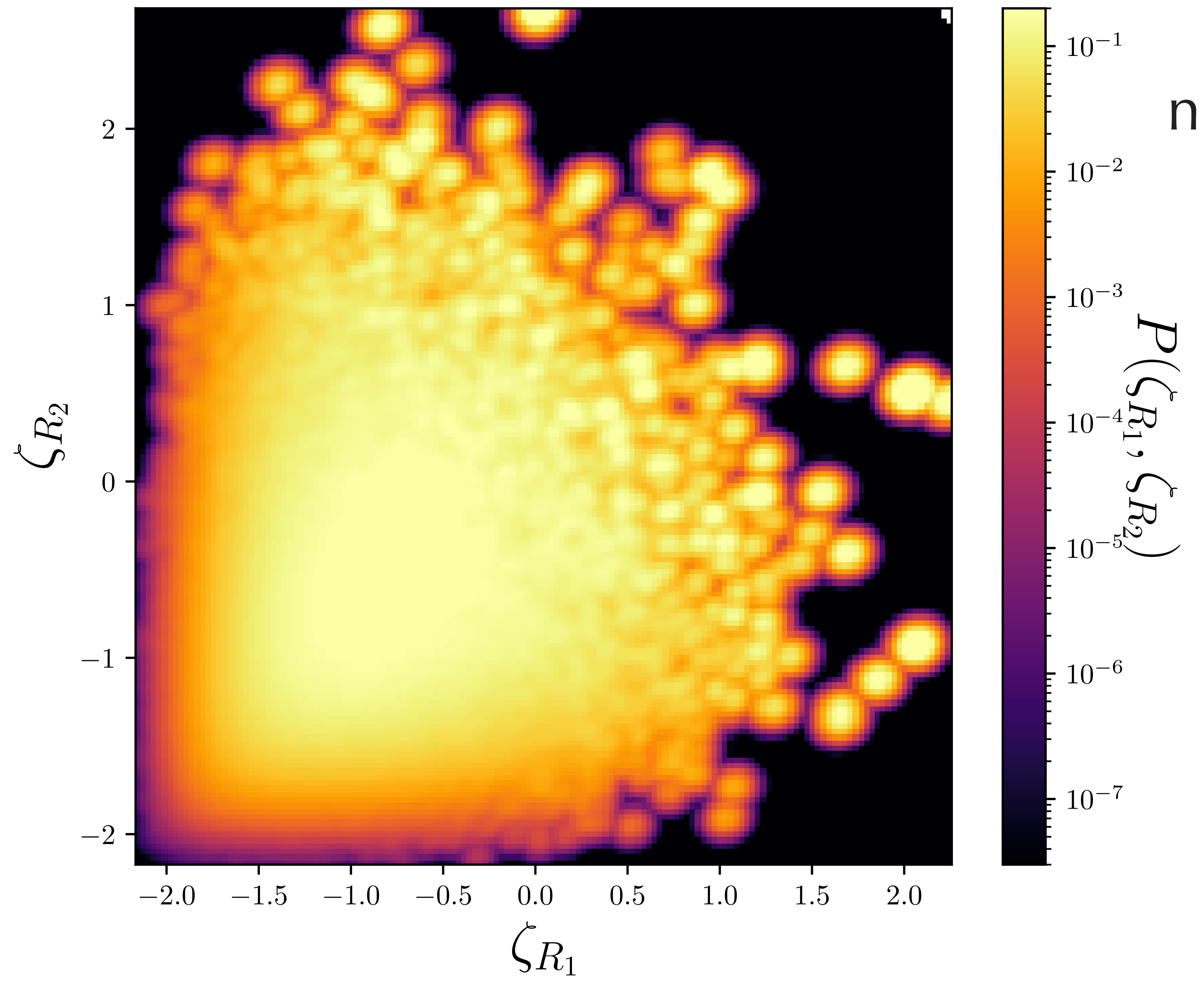
classical limit

Animali, Vennin [2024]

Large-volume approximation: two-point distributions

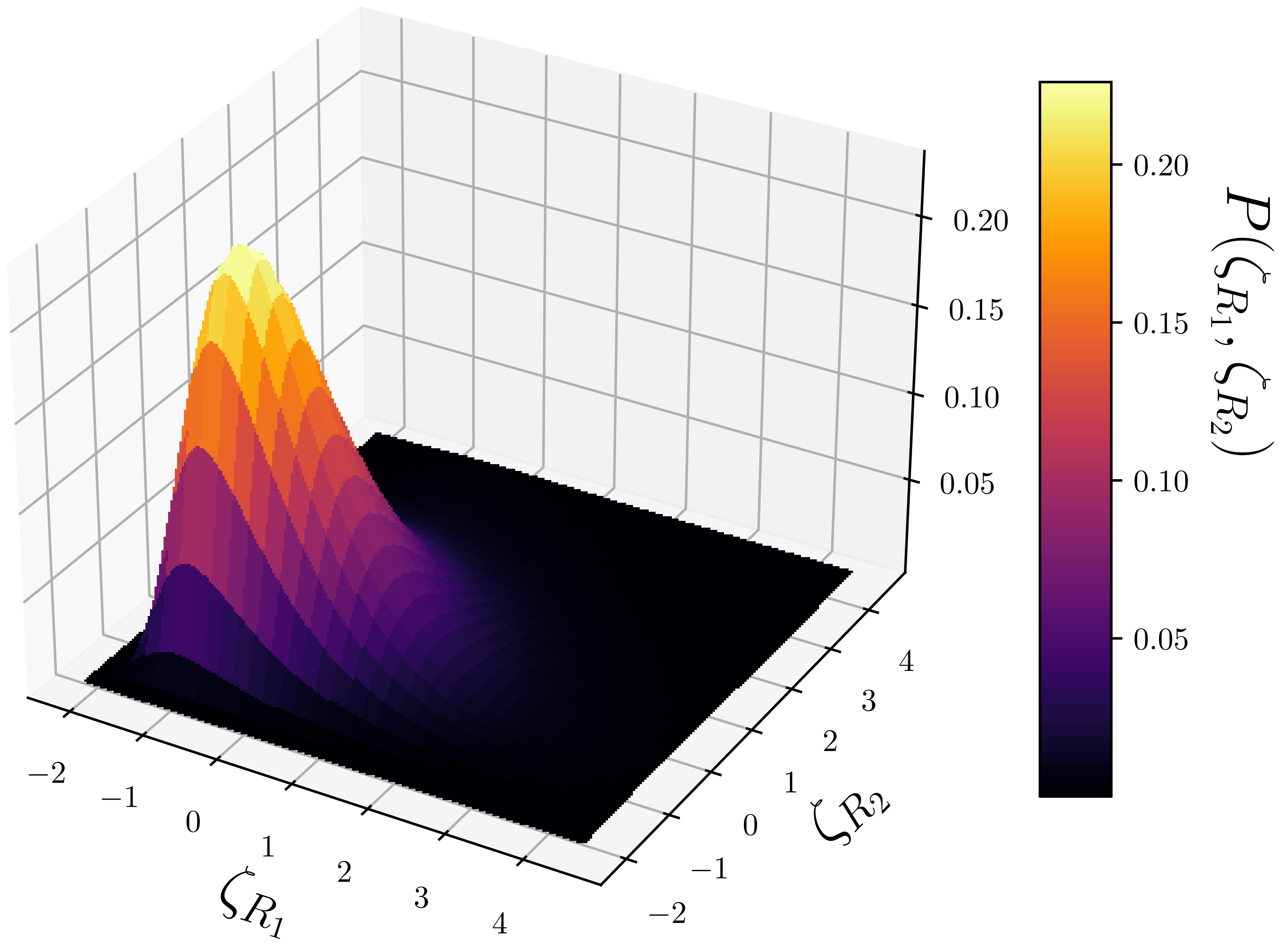


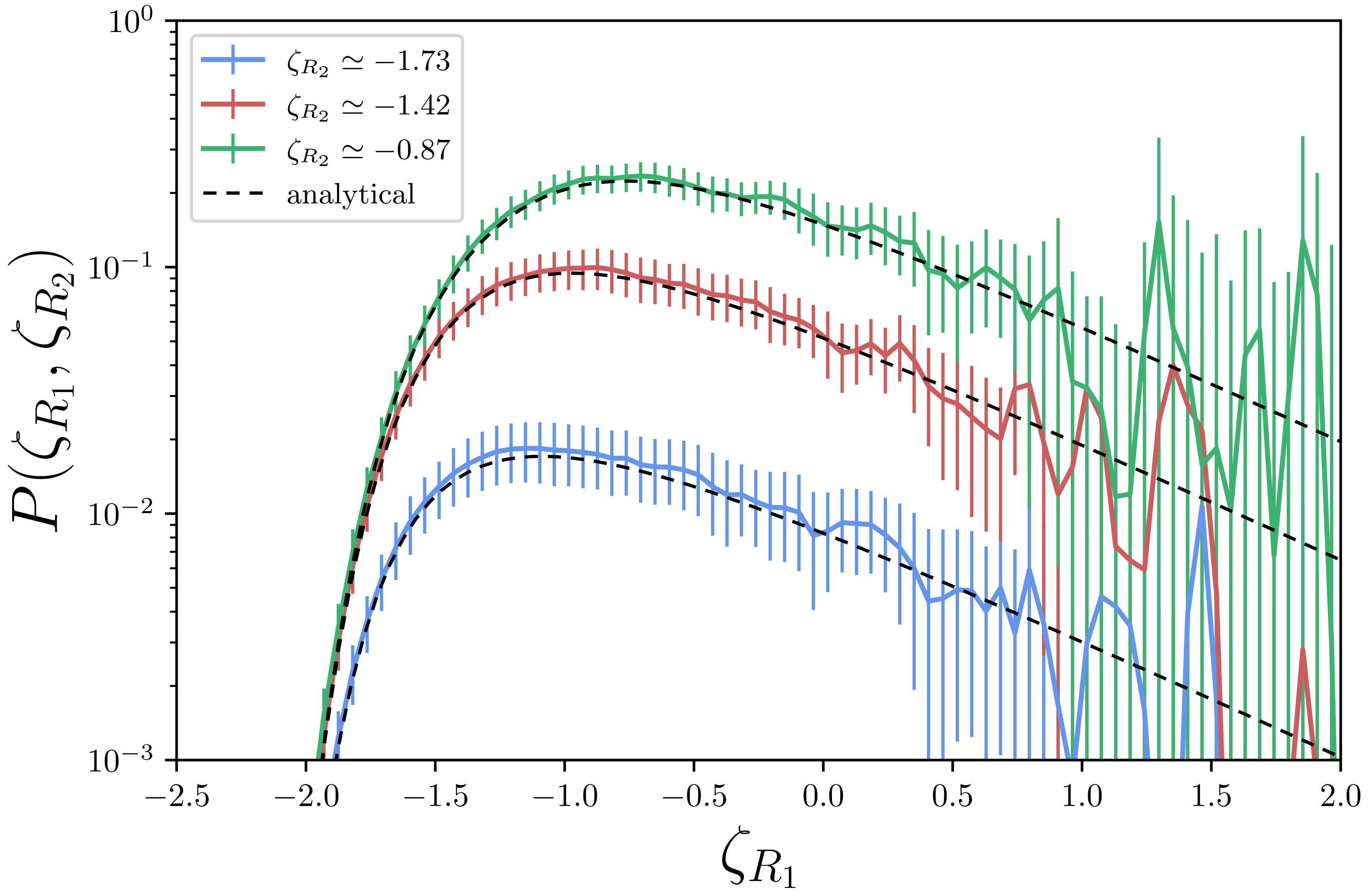
analytical approx. results



numerical simulations

$v = v_0(1 + \alpha\phi/M_{\rm Pl})$
$d\mu^2 = 50.7$





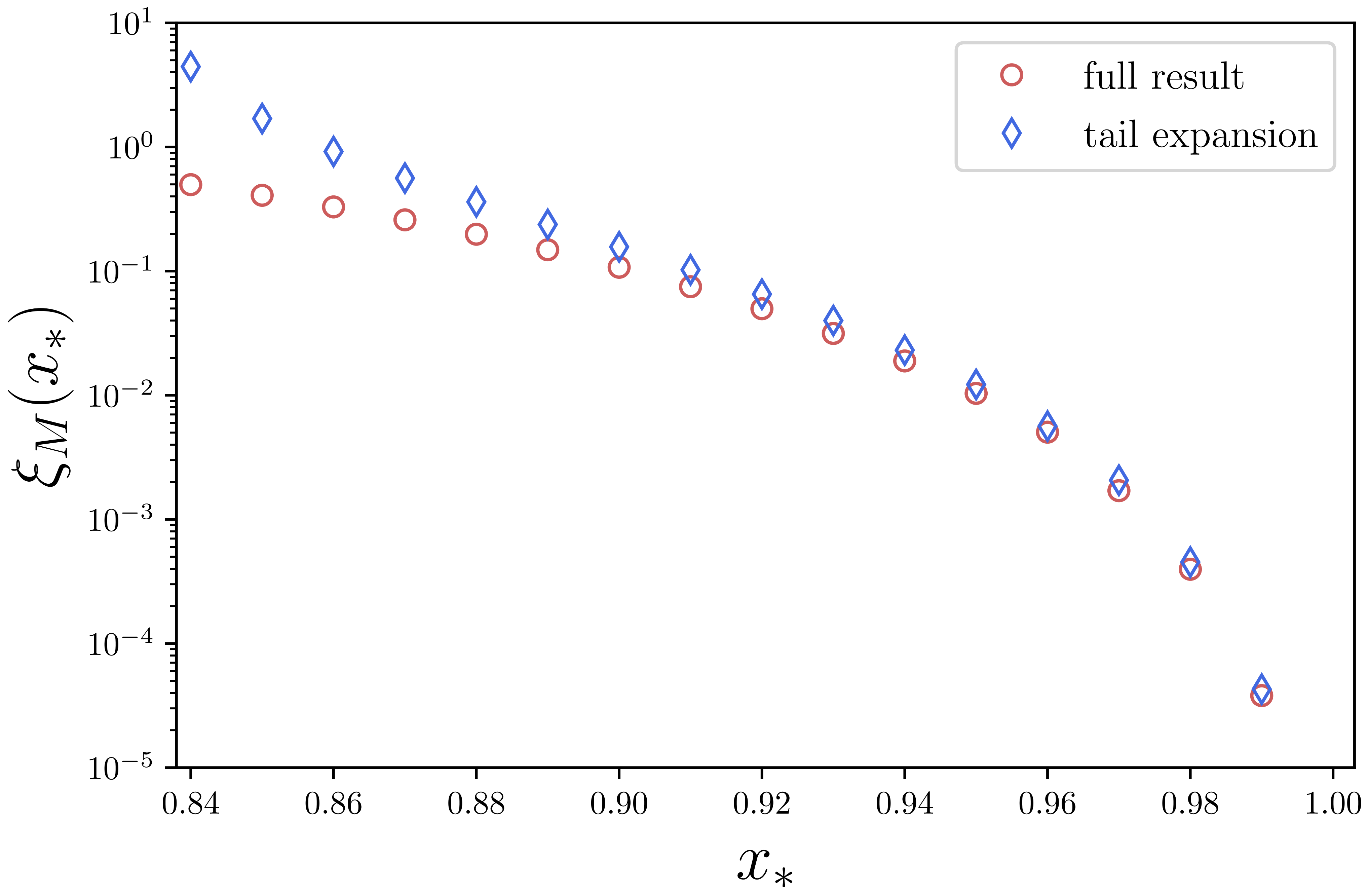
Clustering from quantum diffusion

Animali, Vennin [2024]

Reduced correlation: $\xi_{M_1,M_2}(r) = \dfrac{p(M_1,\vec{x}\,;\,M_2\,,\vec{x}+\vec{r})}{p_{M_1}\,p_{M_2}} - 1$

$$P(\zeta_{R_1},\zeta_{R_2}) = P(\zeta_{R_1})\,P(\zeta_{R_1})\,\frac{a_V(x_*,x_1)}{a_V(x_0,x_1)}\frac{a_V(x_*,x_2)}{a_V(x_0,x_2)}\int \mathrm{d}\mathcal{N}\,P^V_{\mathrm{FPT},x_0\to x_*}(\mathcal{N}_{x_0\to x_*})\,e^{\left[\frac{\mu^2 d^2}{2}+\frac{\pi^2}{\mu^2(1-x_1)^2}+\frac{\pi^2}{\mu^2(1-x_2)^2}-6\right]\mathcal{N}_{x_0\to x_*}}$$

Reduced correlation: large-distance behaviour



Peculiar structure of the two-point distribution on the tail:

$$P(\zeta_{R_1},\zeta_{R_2}) \simeq F(R_1,R_2,r)P(\zeta_{R_1})P(\zeta_{R_2}) \longrightarrow \xi = F(R_1,R_2,r) - 1$$

Reduced correlation does not depend on the formation threshold.

→ Universal clustering behaviour for all tail-born structures.

→ In the large-threshold limit, Gaussian clustering is suppressed by the ratio between the squared threshold and the field variance: clustering is always larger when quantum diffusion is included.

Alternative way:

implement stochastic inflation on stochastic trees,

modelling inflationary expansion as a branching process

Animali, Auclair, Blachier, Vennin [2025]

Stochastic trees for inflation

Stochastic trees for inflation

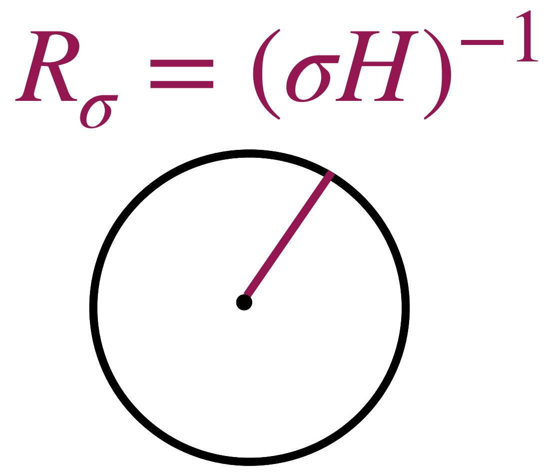
Reference: single-field slow-roll model

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\langle \xi(N) \rangle = 0$$
$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

White Gaussian noise



Hubble patch

Stochastic trees for inflation

Reference: single-field slow-roll model

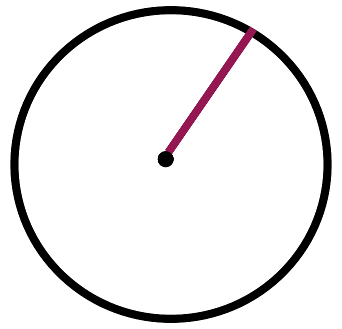
$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\langle \xi(N) \rangle = 0$$
$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

White Gaussian noise

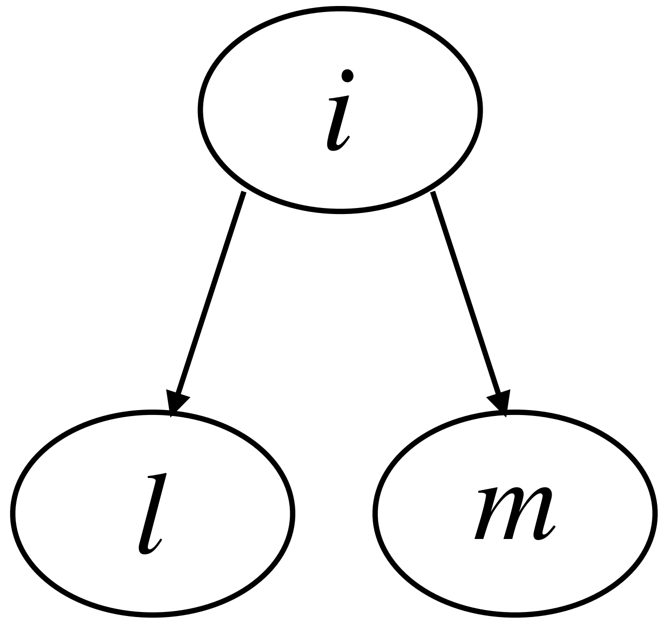
$R_\sigma = (\sigma H)^{-1}$



Hubble patch

$N = 0$

$N = \log(2)/3$



elementary vertex

Children patches have no future causal contact:
separate universe implemented.

Stochastic trees for inflation

Reference: single-field slow-roll model

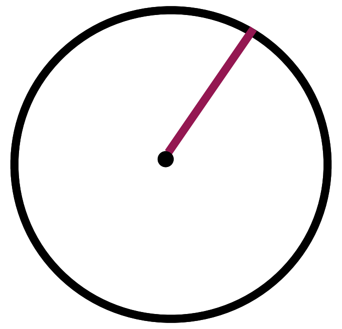
$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\langle \xi(N) \rangle = 0$$
$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

White Gaussian noise

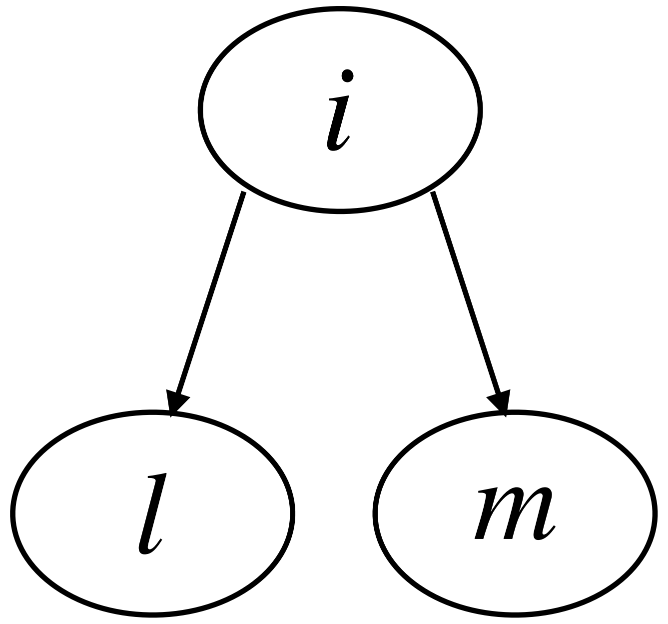
$R_\sigma = (\sigma H)^{-1}$



Hubble patch

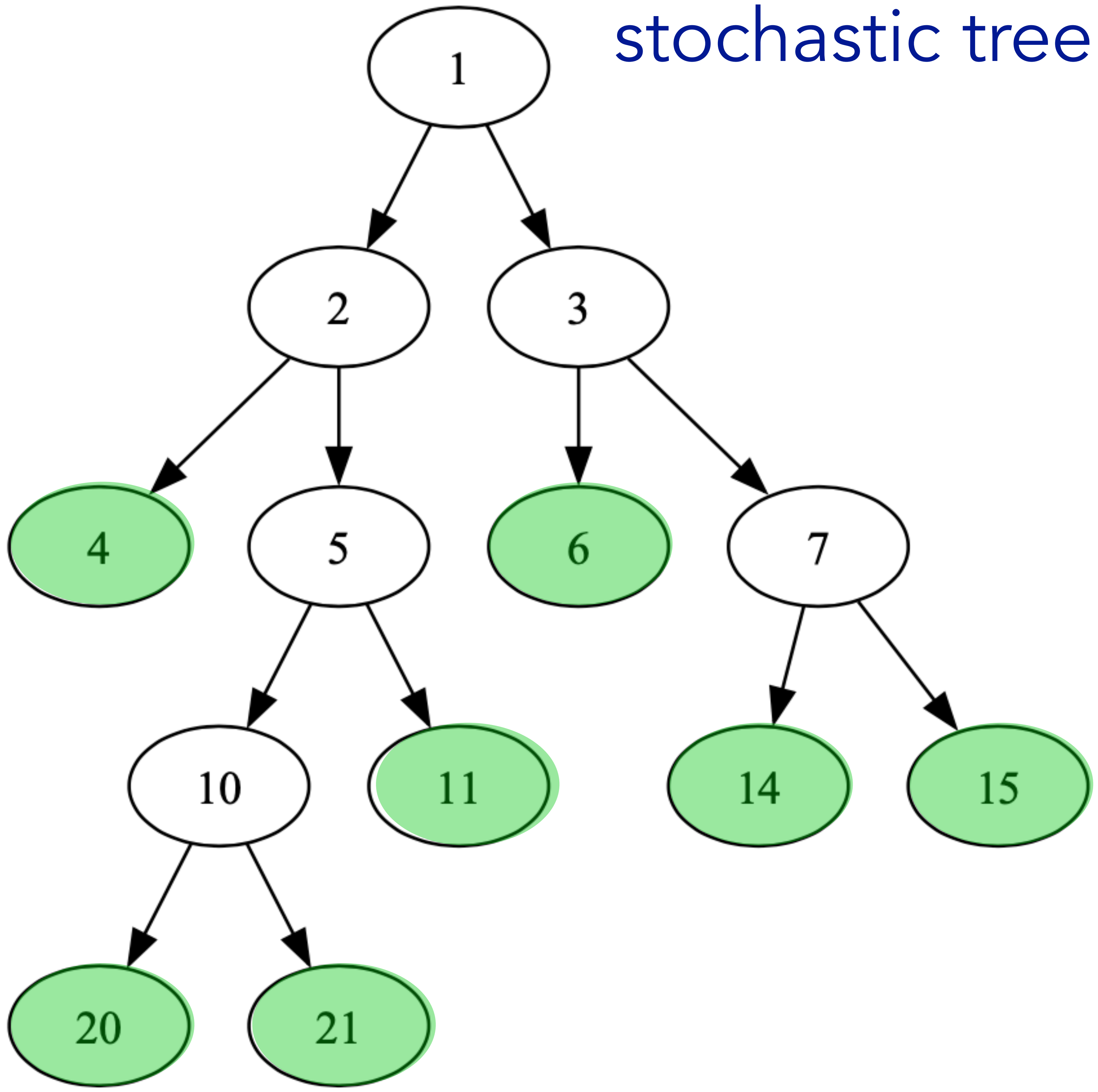
$N = 0$

$N = \log(2)/3$



elementary vertex

recursive iteration

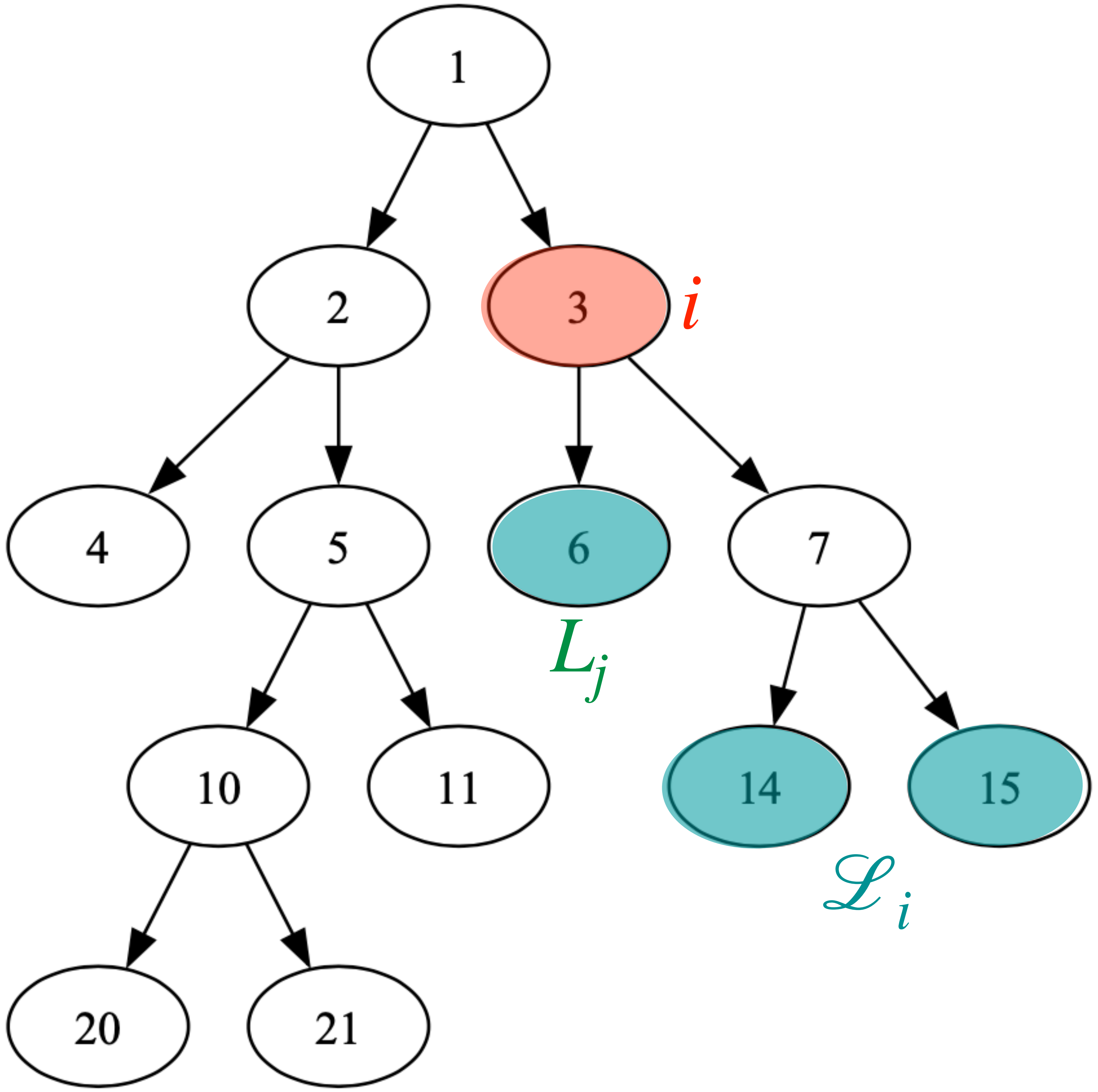


stochastic tree

Children patches have no future causal contact: separate universe implemented.

FOREST: FOtran Recursive Exploration of Stochastic Trees https://doi.org/10.5281/zenodo.15235932

Stochastic trees: curvature perturbation at the end of inflation



Physical volume emerging from node i : $V_i = \sum_{i \in \mathscr{L}_i} V(L_j)$

$V(L_j) \propto e^{3(\mathscr{N}_{i\to j} - \Delta N)}$

Expansion from node i,
volume-averaged over the child leaves $\mathscr{L}_i$: $W_i = \frac{1}{V_i} \sum_{j \in \mathscr{L}_i} V(L_j) \mathscr{N}_{i\to j}$

Curvature perturbation coarse-grained over a single leaf:

$$\zeta_{V_j}(\vec{x}_j) = \mathscr{N}_{1\to j} - W_1$$

Curvature perturbation coarse-grained over set of leaves descending from a branching node:

$$\zeta_i \equiv \zeta_{V_i}(\vec{x}_i) = \frac{1}{V_i} \sum_{j \in \mathscr{L}_i} V_j \left(\mathscr{N}_{1\to i} + \mathscr{N}_{i\to j} - W_1\right) = \mathscr{N}_{1\to i} + W_i - W_1$$

Harvesting primordial black holes

PBH formation takes place in region of high curvature.

Curvature perturbation ζ is not a local quantity: $\zeta_{V_j}(\vec{x}_i) = N_{1\to j} - W_1$

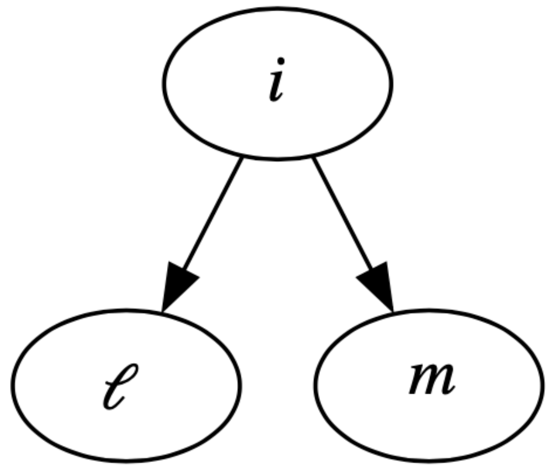
Other cosmological fields are more suitable:

$\delta(\vec{x}) \simeq -\frac{2(1+w)}{5+3w}\frac{1}{a^2H^2}\nabla^2\zeta(\vec{x})$ (linear) density contrast

$\mathscr{C}(r) = \frac{3(1+w)}{5+3w}\left\{1-[1+r\zeta'(r)]^2\right\}$ compaction function

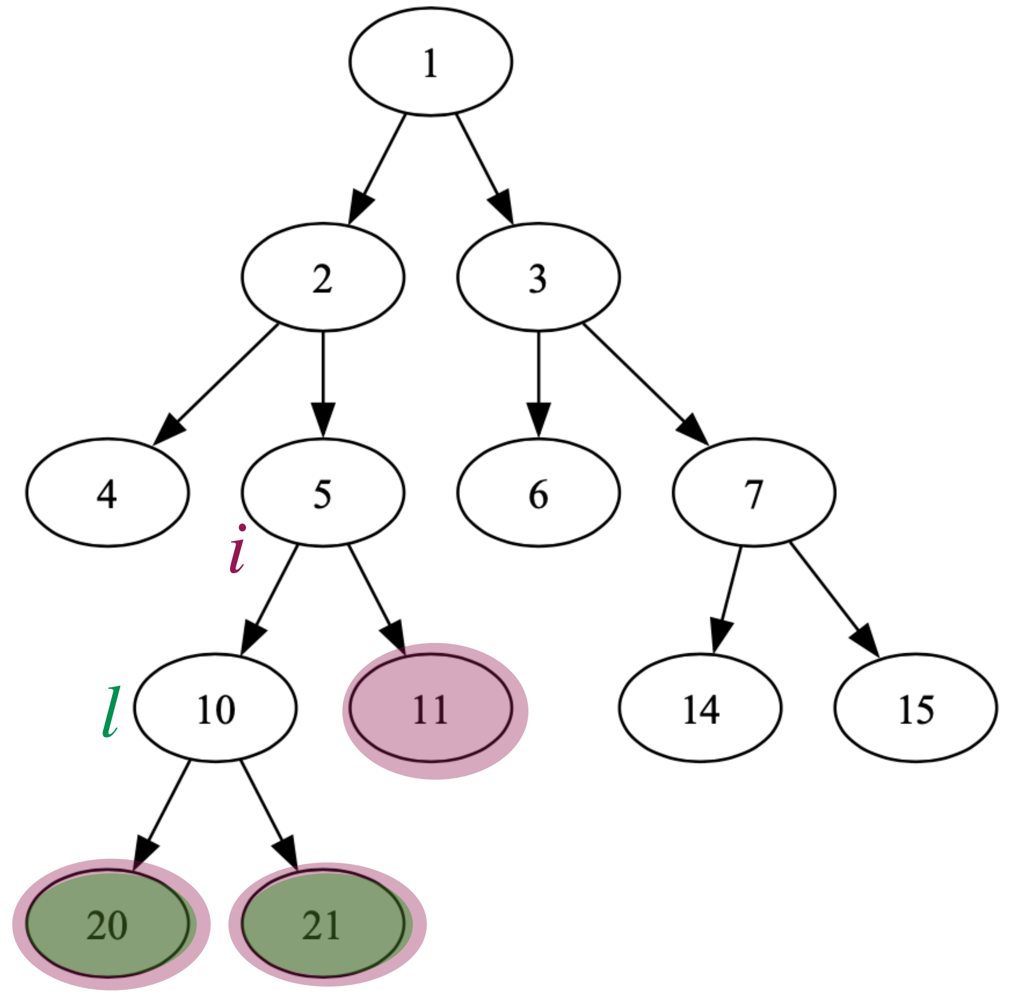
"Coarse-shelled" curvature perturbation proxy: $\Delta\zeta(\vec{x}) = \zeta_{R_1}(\vec{x}) - \zeta_{R_2}(\vec{x})$ Tada, Vennin [2021]

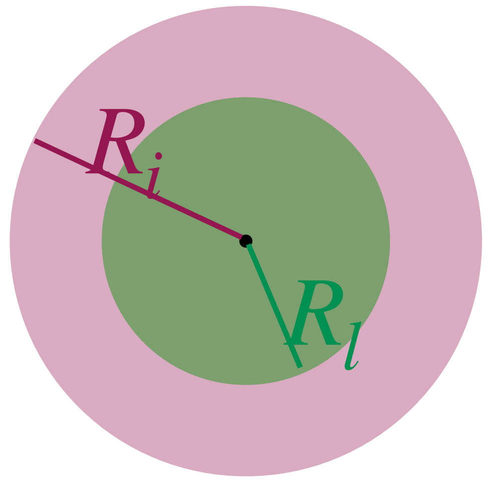
Coarse-shelled curvature perturbation



$\zeta_{li} = \zeta_l - \zeta_i$ curvature perturbation in node l relative to its local background i.

Concentric spheres approximation:





$V_l = 4/3\pi R_l^3$
$V_i = 4/3\pi R_i^3$

Nodes for which $\zeta_{li} > \zeta_{li,\mathrm{c}}$ collapse into PBHs:

$$\zeta_{li,\mathrm{c}} = 3\log\left(\frac{R_i}{R_l}\right)\left[1 - \sqrt{1 - \left(\frac{5+3w}{3+3w}\right)\mathscr{C}_\mathrm{c}}\right] = \frac{1}{2}\log\left(\frac{V_i}{V_l}\right)$$

for $w = 1/3$ and $\mathscr{C}_\mathrm{c} = 0.5$.

$$\zeta_{li}/\zeta_{li,\mathrm{c}} = \frac{2}{\log(V_i/V_l)}\frac{V_m}{V_i}(W_l - W_m)$$

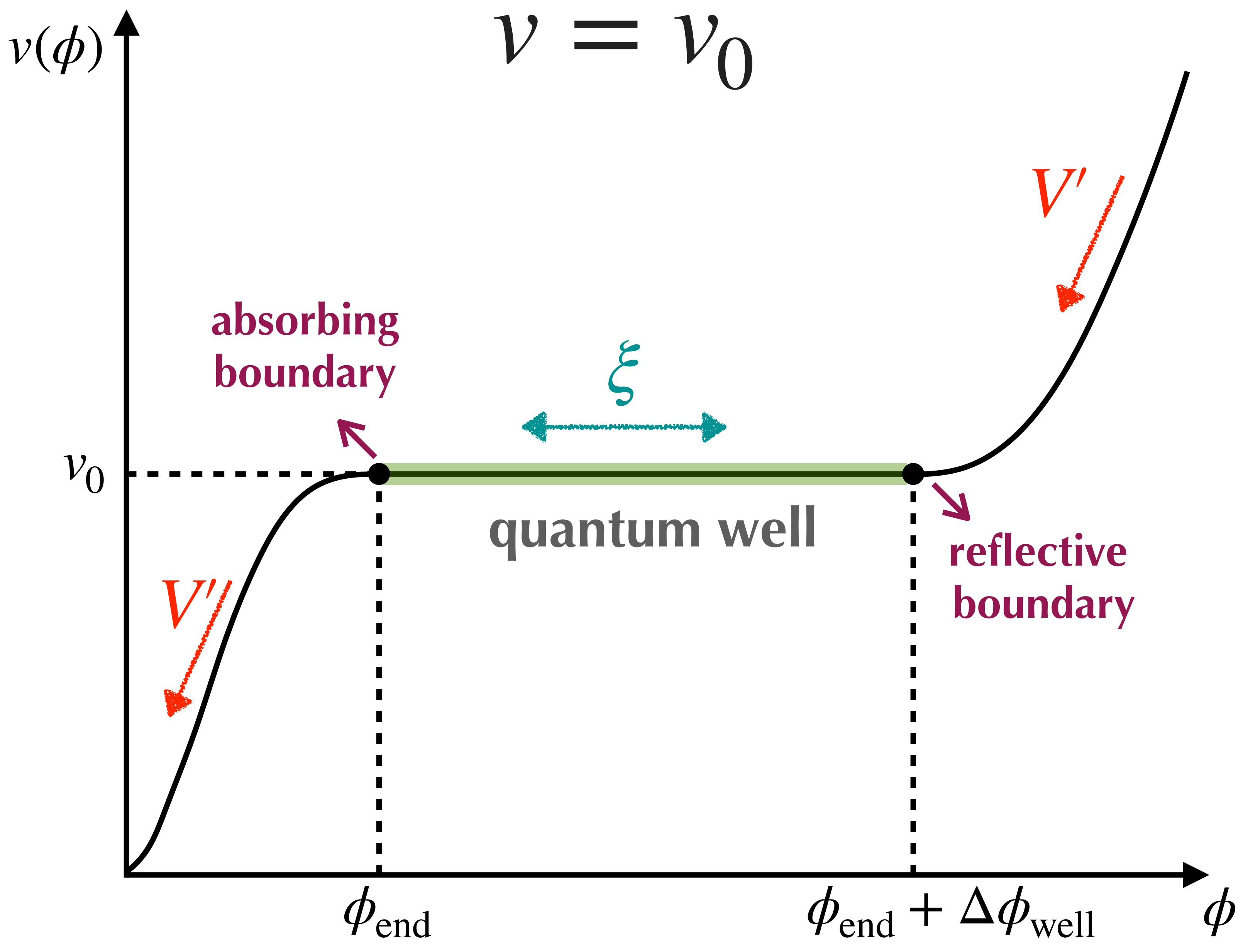
→ $= 0$ if $W_l = W_m$ balanced tree

→ $\simeq 2(W_l - W_m)$ if $V_l \gg V_m$

Collapse happens at asymmetric nodes.

Application: flat-well toy model

Pattison, Vennin, Assadullahi, Wands [2017]
Ezquiaga, Garcia-Bellido, Vennin [2020]
Animali, Vennin [2024]



$$x = (\phi - \phi_{\rm end})/\Delta\phi_{\rm well} \in [0,1]$$

$$\mu^2 = \frac{\Delta\phi_{\rm well}^2}{v_0 M_{\rm Pl}^2}$$

$$\chi_{\mathcal{N}}(t,\phi) = \frac{\cos[\sqrt{it}\,\mu\,(x-1)]}{\cos[\sqrt{it}\,\mu]}$$

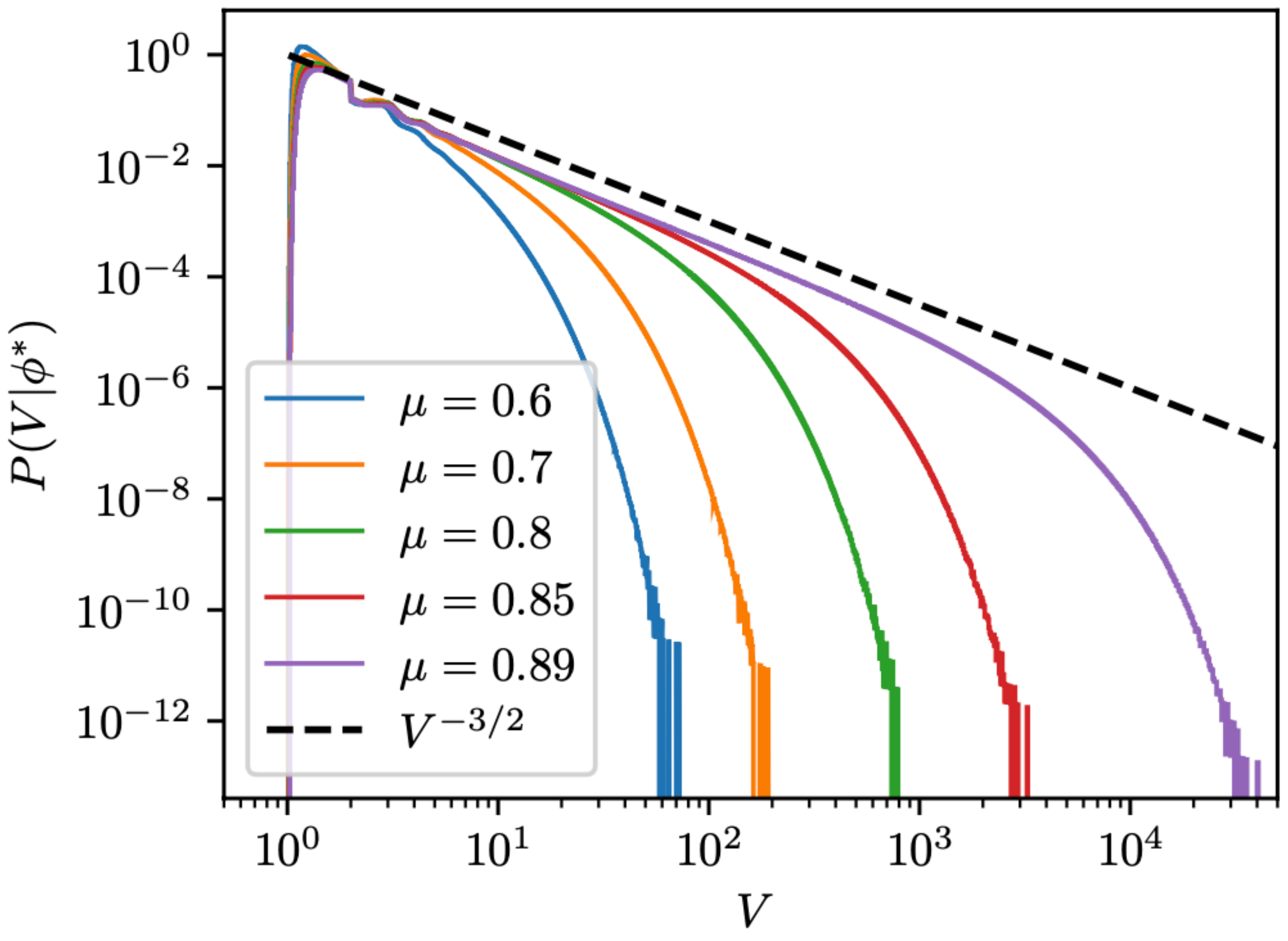
$$P_{\rm FPT,\phi}(\mathcal{N}) = -\frac{\pi}{2\mu^2}\vartheta_2'\left(\frac{\pi}{2}x, e^{-\frac{\pi^2}{\mu^2}\mathcal{N}}\right)$$

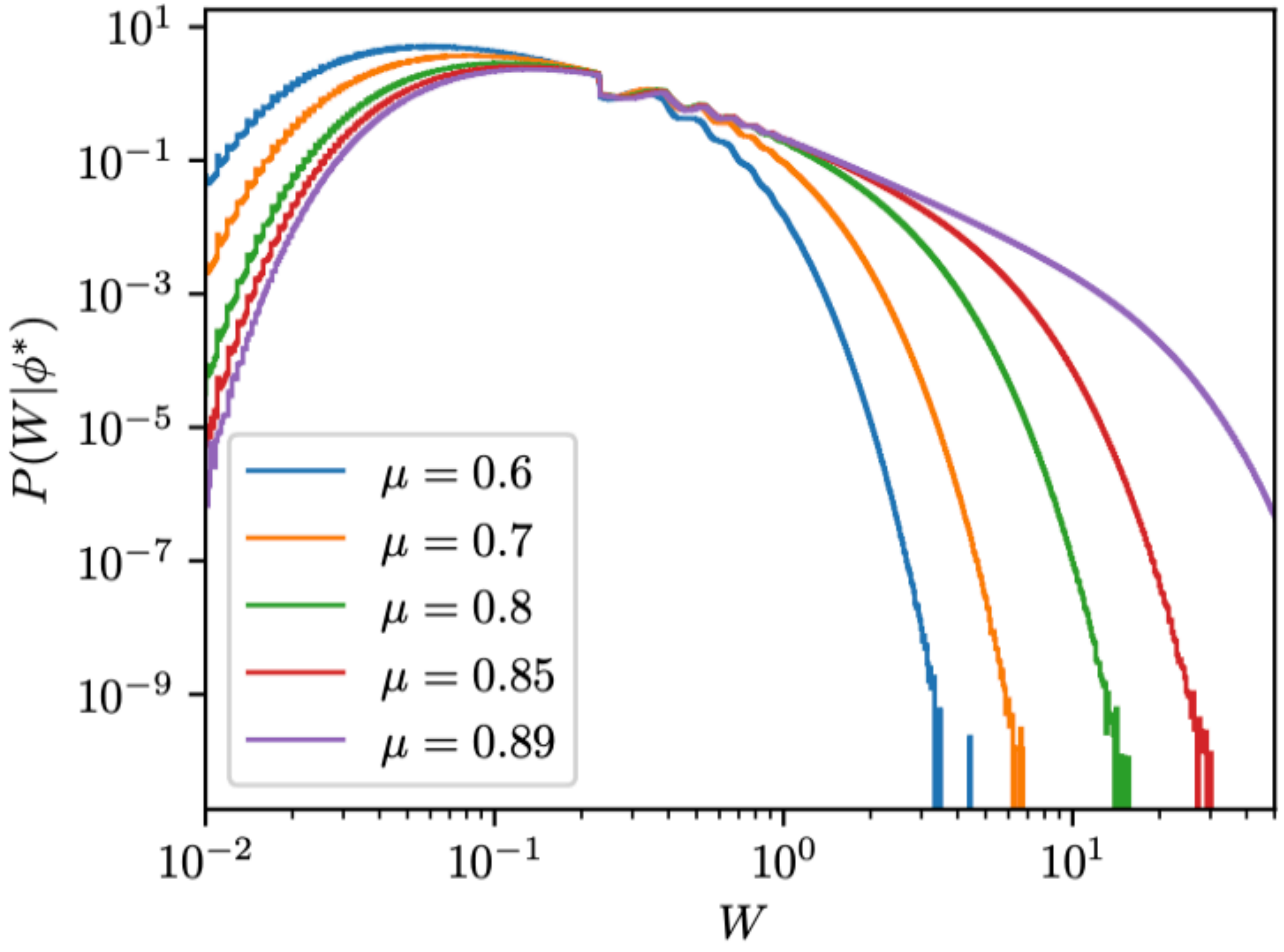
$$\langle V \rangle = \langle e^{3\mathcal{N}} \rangle = \frac{\cos[\sqrt{3}\mu(1-x)]}{\cos(\sqrt{3}\mu)}$$

$$\mu \geq \mu_{\rm c} = \frac{\pi}{2\sqrt{3}}$$ eternal inflation

Probability distributions over the trees

Forward statistics of the volume V and of the volume-averaged expansion W :





$$P(V|\phi_*) \propto e^{z_* V} V^{-3/2} .$$

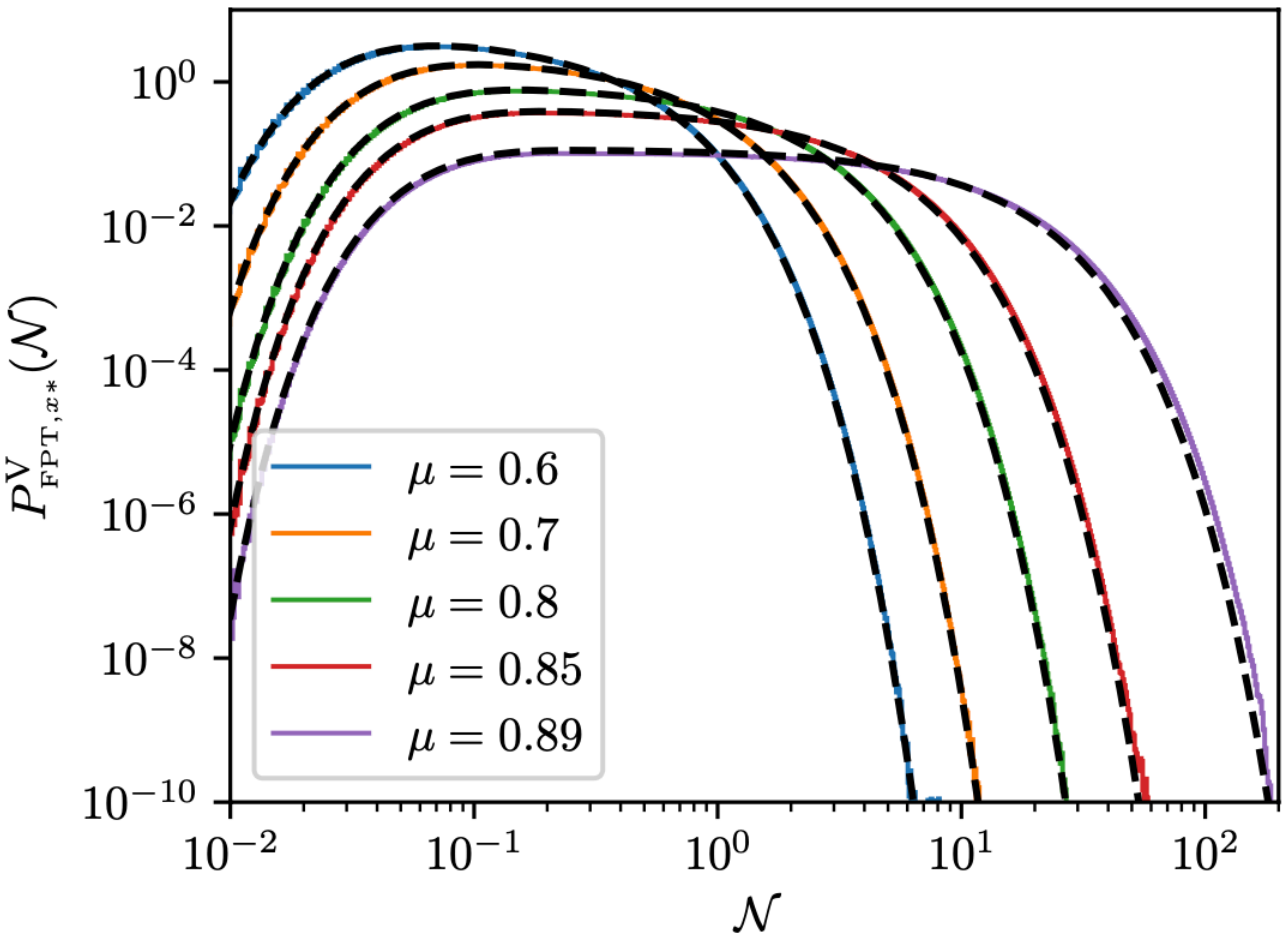
For $\mu \to \mu_c$ $P(V) \propto V^{-3/2}$.

Expected from "bacteria models" (Galton-Watson processes):

Bacteria non extinction → eternal inflation

Probability distributions over the leaves

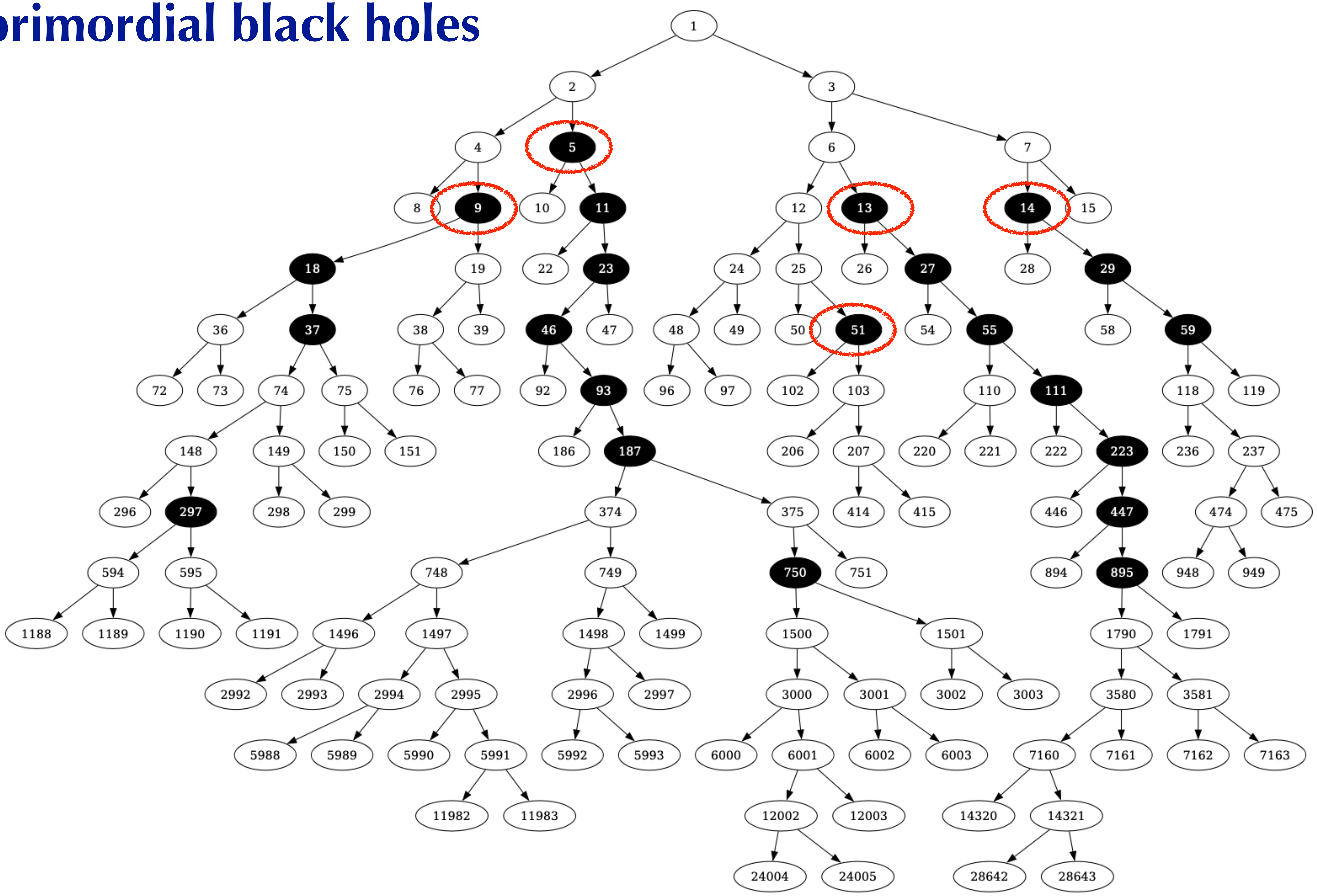
Volume-weighted first-passage-time distribution through the end-of-inflation hypersurface.



$$P^V_{\mathrm{FPT},x_*}(\mathcal{N}) = \frac{P_{\mathrm{FPT,x_*}}(\mathcal{N})\, e^{3\mathcal{N}}}{\int_0^\infty \mathrm{d}\mathcal{N} P_{\mathrm{FPT,x_*}}(\mathcal{N})\, e^{3\mathcal{N}}}$$

$$P_{\mathrm{FPT},x_*}(\mathcal{N}) = -\,\pi/(2\mu^2)\vartheta_2'(\pi/2\, x_*, e^{-\pi^2/\mu^2\mathcal{N}})$$

Harvesting primordial black holes



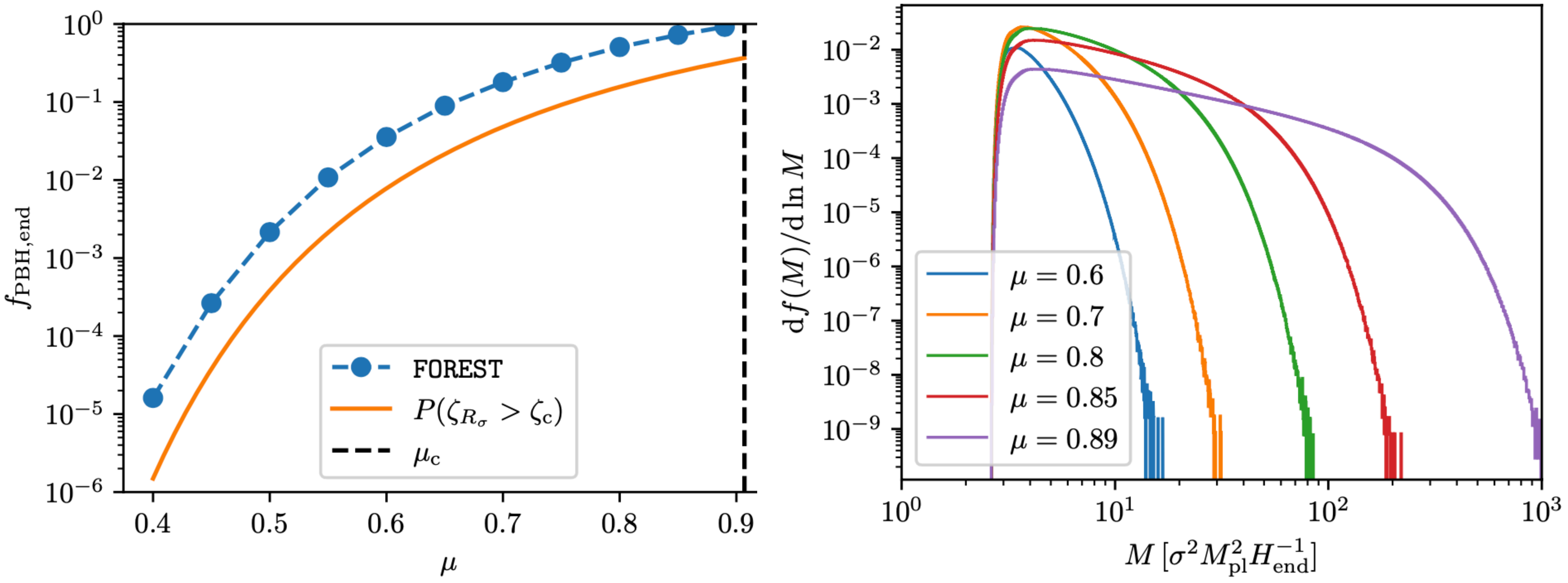
Nested PBH formation along a branch analogous to the cloud-in cloud problem.

Only the highest (“oldest”) nodes are kept in the PBH inventory. Cloud-in-cloud effects naturally accounted for.

Distribution of primordial black holes

Fraction of the universe at the end of inflation that will eventually collapse into PBHs.

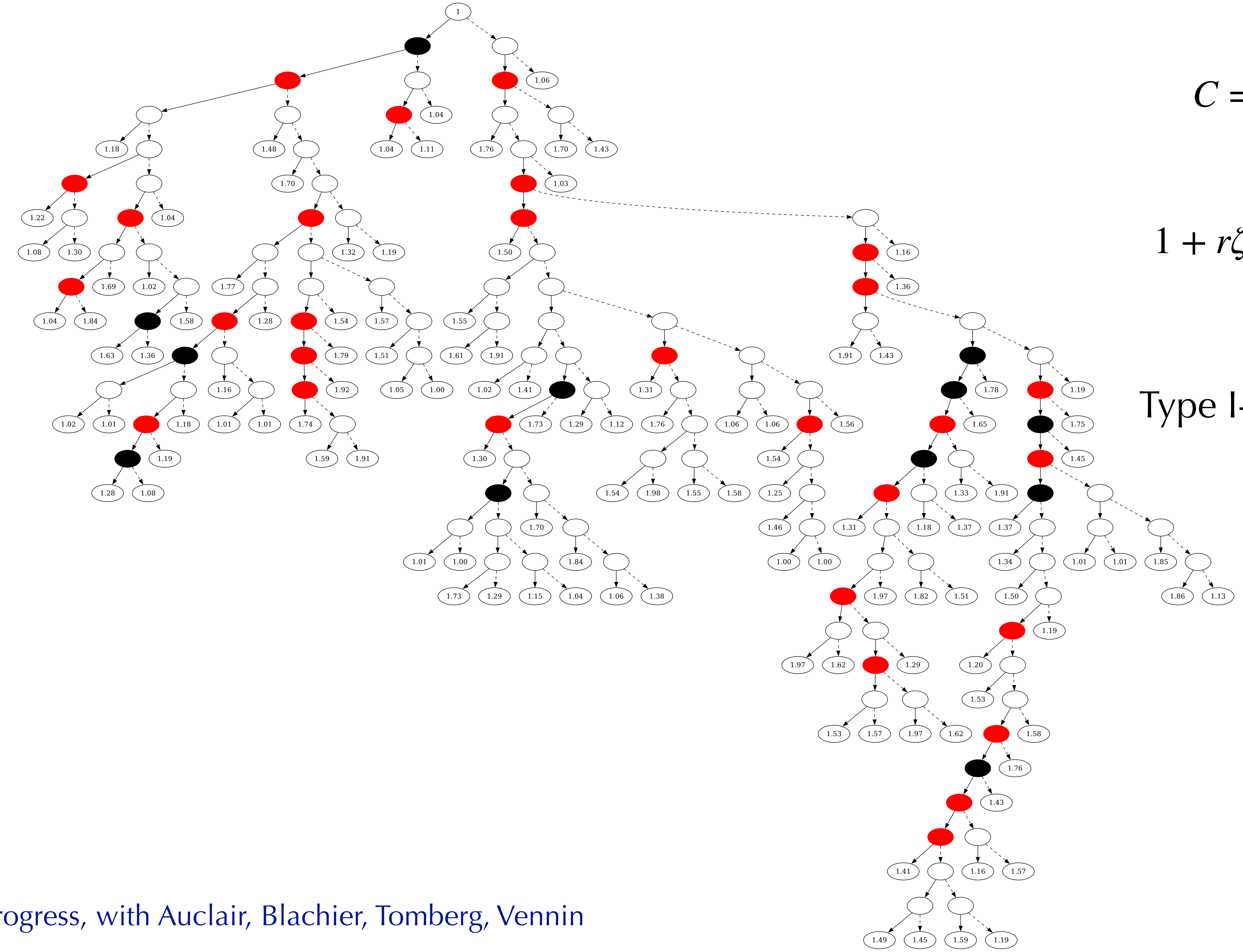
Mass distribution: $M_H(R_i) \simeq M_{\rm Pl}^2 R_i^2 H_{\rm end}$



$\mathrm{d}f/\mathrm{d}\log M \propto M^{-\alpha},\ \alpha \approx 2/3$

Compaction function and type I/type II perturbations from stochastic trees

The compaction function can be reconstructed by considering three-node hierarchies (three generations):



$$C = \frac{3(1+w)}{5+3w}\left\{1 - \left[1 + r\zeta'(r)\right]\right\}$$

$$1 + r\zeta'(r) = \frac{1}{3}\left(\frac{\mathrm{d}V}{\mathrm{d}\log r}\right)^{-1}\frac{\mathrm{d}^2 V}{\mathrm{d}(\log r)^2}$$

Type I-II perturbations can be distinguished.

In progress, with Auclair, Blachier, Tomberg, Vennin

Conclusions

- Large perturbations from inflation should be described with non-perturbative methods, as the stochastic-δN formalism.
- A characteristic signature is the presence of non-Gaussian, exponential-type tails. Relevant for rare event (PBHs).
- The stochastic δN formalism can be extended beyond one-point statistics.
- In the stochastic framework, approximations are required to relate physical distances at the end of inflation to the field-space configuration when those scales emerged from a Hubble patch during inflation.
- Stochastic inflation can be efficiently implemented on stochastic trees, modeling the inflationary expansion as a branching process.
- Statistical properties of curvature perturbations and other cosmological fields embedded in the tree structure.
- Stochastic trees are ideal tools to “harvest” primordial black holes, directly addressing the cloud-in-cloud problem.
- Power-law behaviour followed by exponential tails characterises forward statistics over the trees and over the leaves, and also the mass function of primordial black holes, in simple toy models.

Open challenges

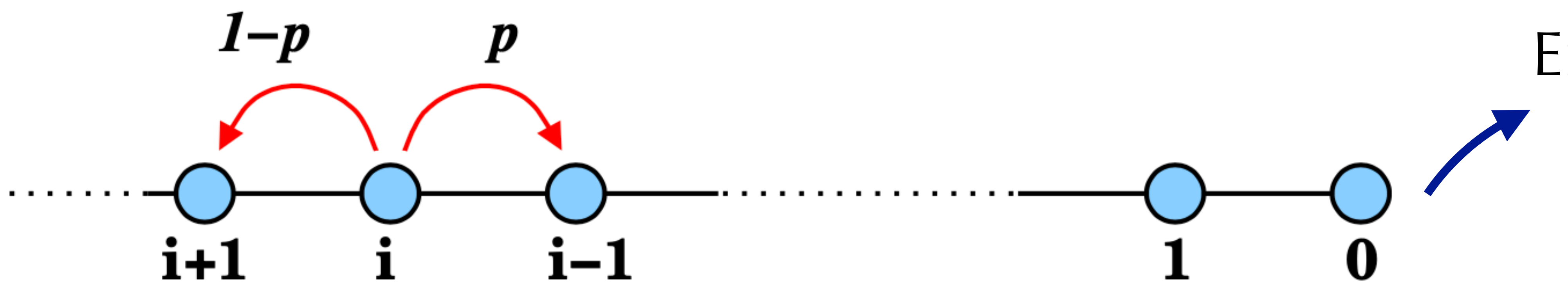
- Volume weighting leads to eternal inflation:

  local observers only have access to a finite region around them, in which inflation has ended.
  Can a formalism expressed solely in terms of backward quantities avoid eternal inflation?

Time-reversed stochastic inflation [Blachier, Ringeval 2025]

- How to go beyond analytically ($P(V), P(W)$)?

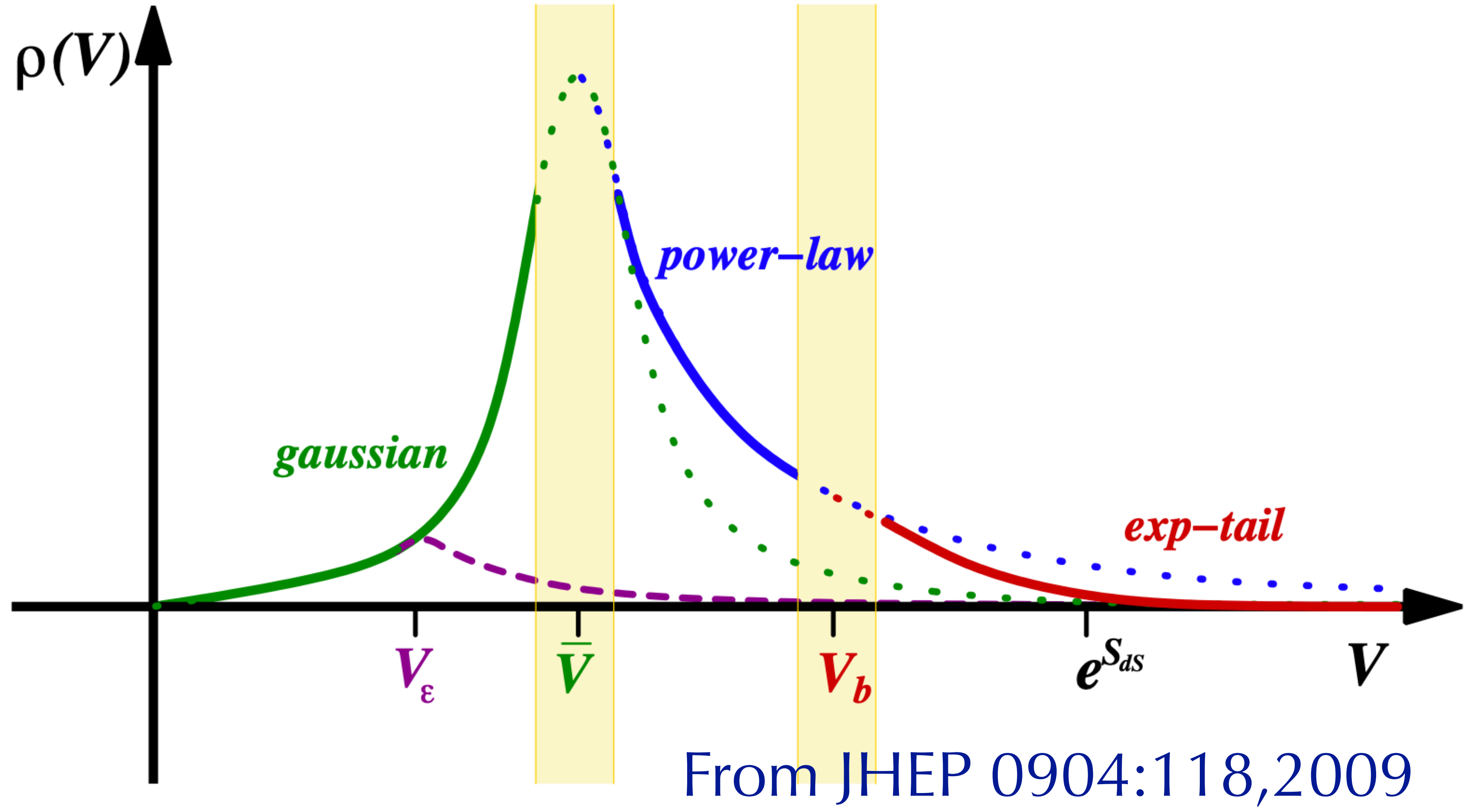
Creminelli, Dubovsky, Nicholas, Senatore, Zaldarriaga [2008]
Dubovsky, Senatore, Villadoro [2009]



End site : bacterium dies

Multi-type Galton-Watson process

Bacteria model of inflation



From JHEP 0904:118,2009

- Ultra-slow roll, clustering, power spectrum… from stochastic trees.

In progress with Auclair, Blachier, Tomberg, Vennin

Open challenges

Thanks!

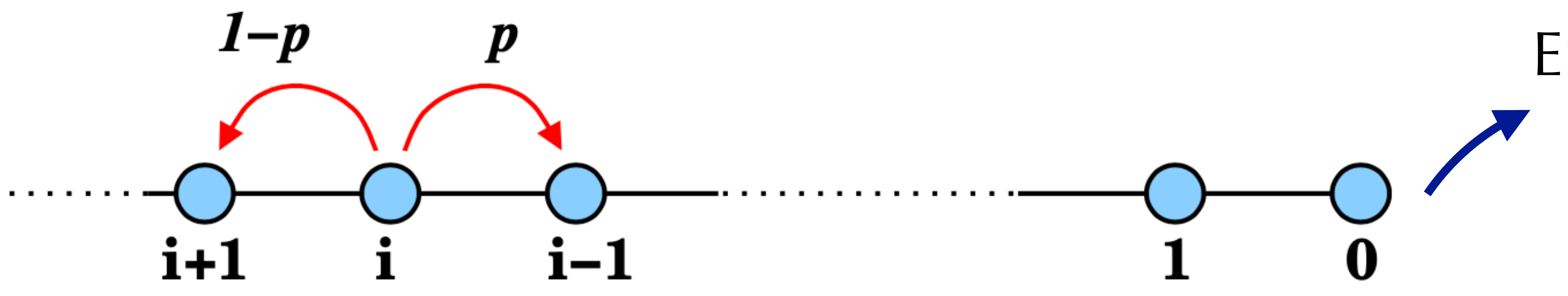
- Volume weighting leads to eternal inflation:

local observers only have access to a finite region around them, in which inflation has ended.
Can a formalism expressed solely in terms of backward quantities avoid eternal inflation?

Time-reversed stochastic inflation [Blachier, Ringeval 2025]

- How to go beyond analytically ($P(V), P(W)$)?

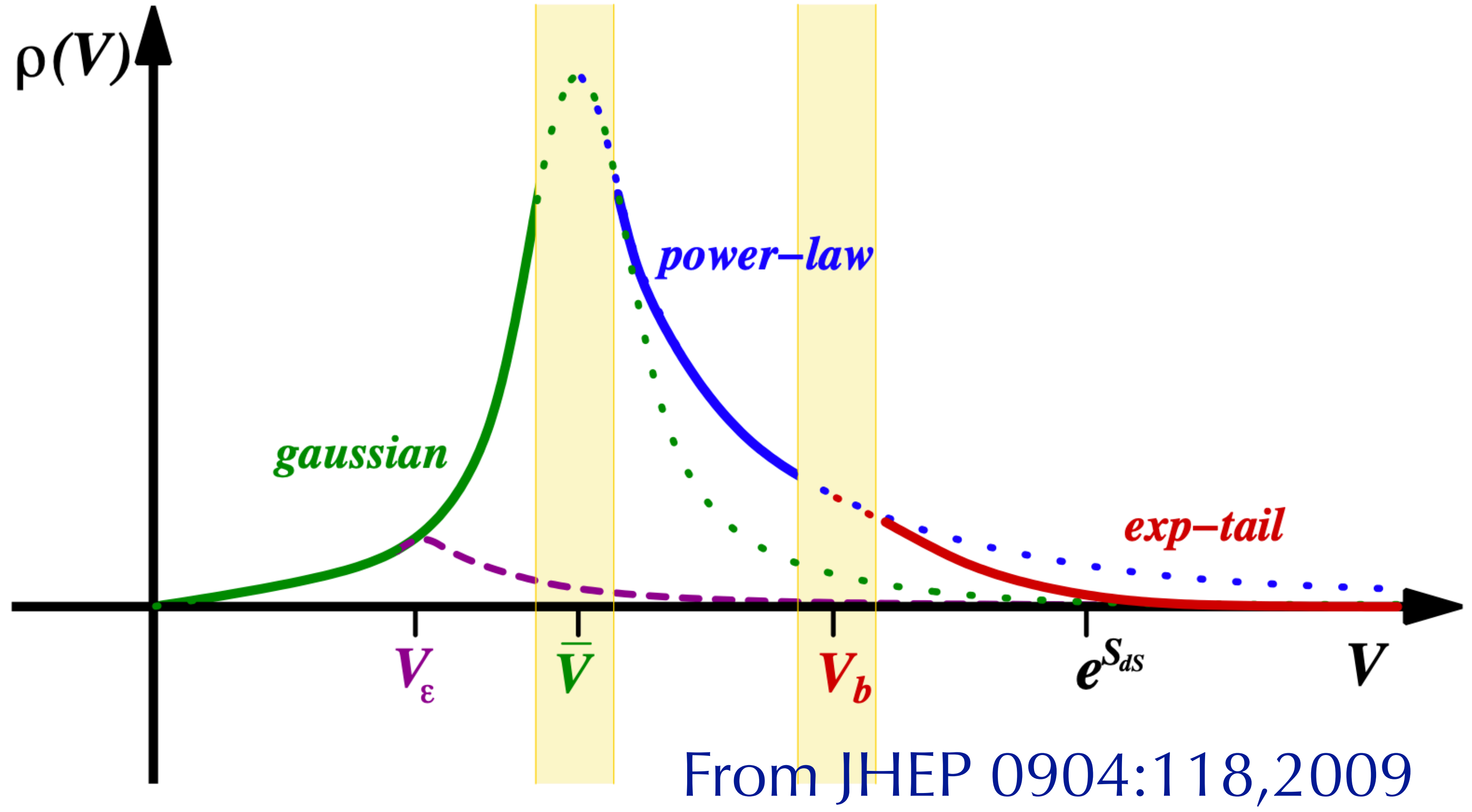
Creminelli, Dubovsky, Nicholas, Senatore, Zaldarriaga [2008]
Dubovsky, Senatore, Villadoro [2009]



End site : bacterium dies

Multi-type Galton-Watson process

Bacteria model of inflation



From JHEP 0904:118,2009

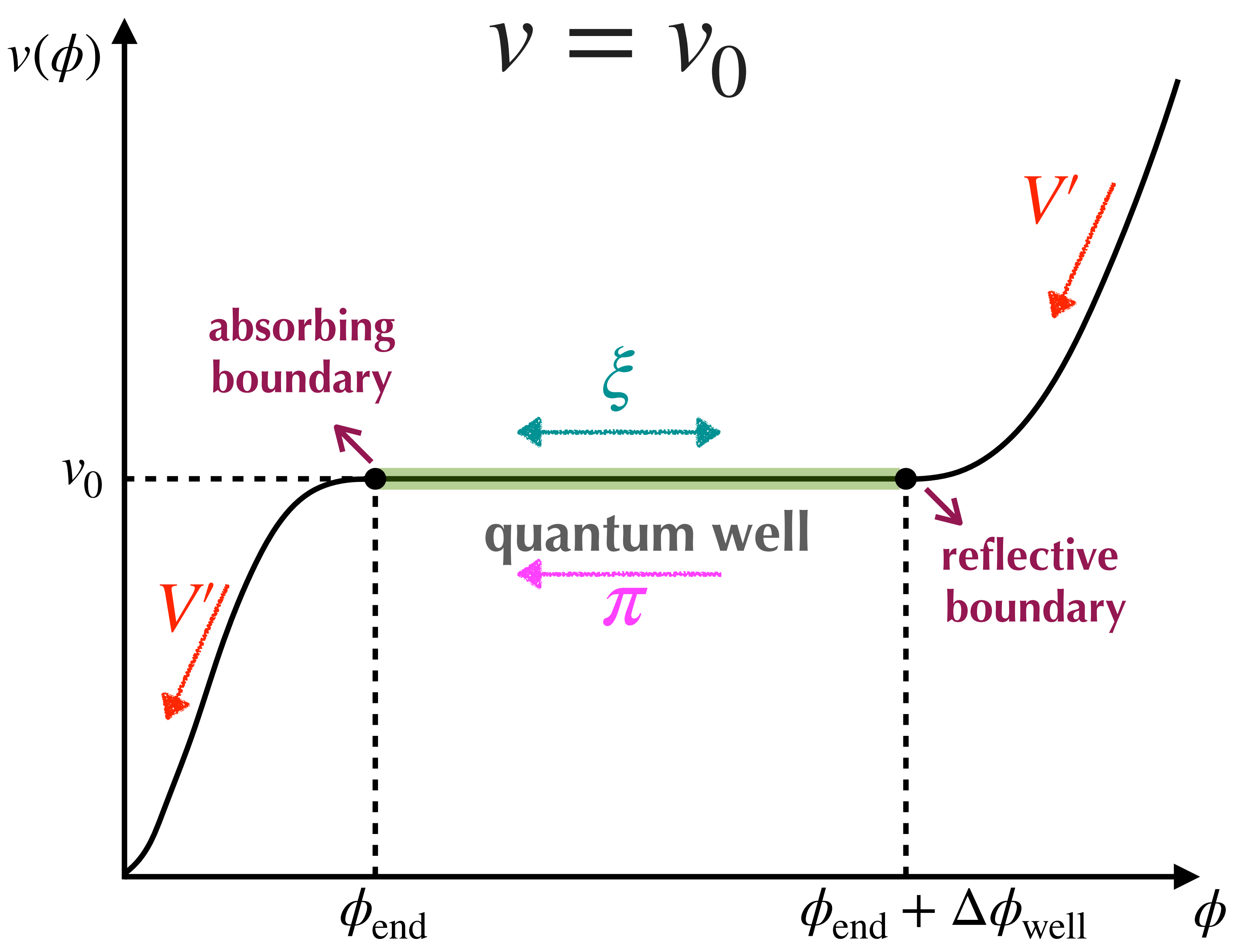
- Ultra-slow roll, clustering, power spectrum… from stochastic trees.

In progress with Auclair, Blachier, Tomberg, Vennin

Backup slides

Ultra slow-roll model

Pattison, Vennin, Wands, Assadullahi [2021]



$$x = (\phi - \phi_{\rm end})/\Delta\phi_{\rm well} \in [0,1] \qquad \mu^2 = \frac{\Delta\phi_{\rm well}^2}{v_0 M_{\rm Pl}^2}$$

Initial field velocity

$$y = \frac{\pi}{\pi_{\rm crit}}, \quad \pi_{\rm crit} = -3\Delta\phi_{\rm well} \qquad \pi = \frac{\mathrm{d}\phi}{\mathrm{d}N}$$

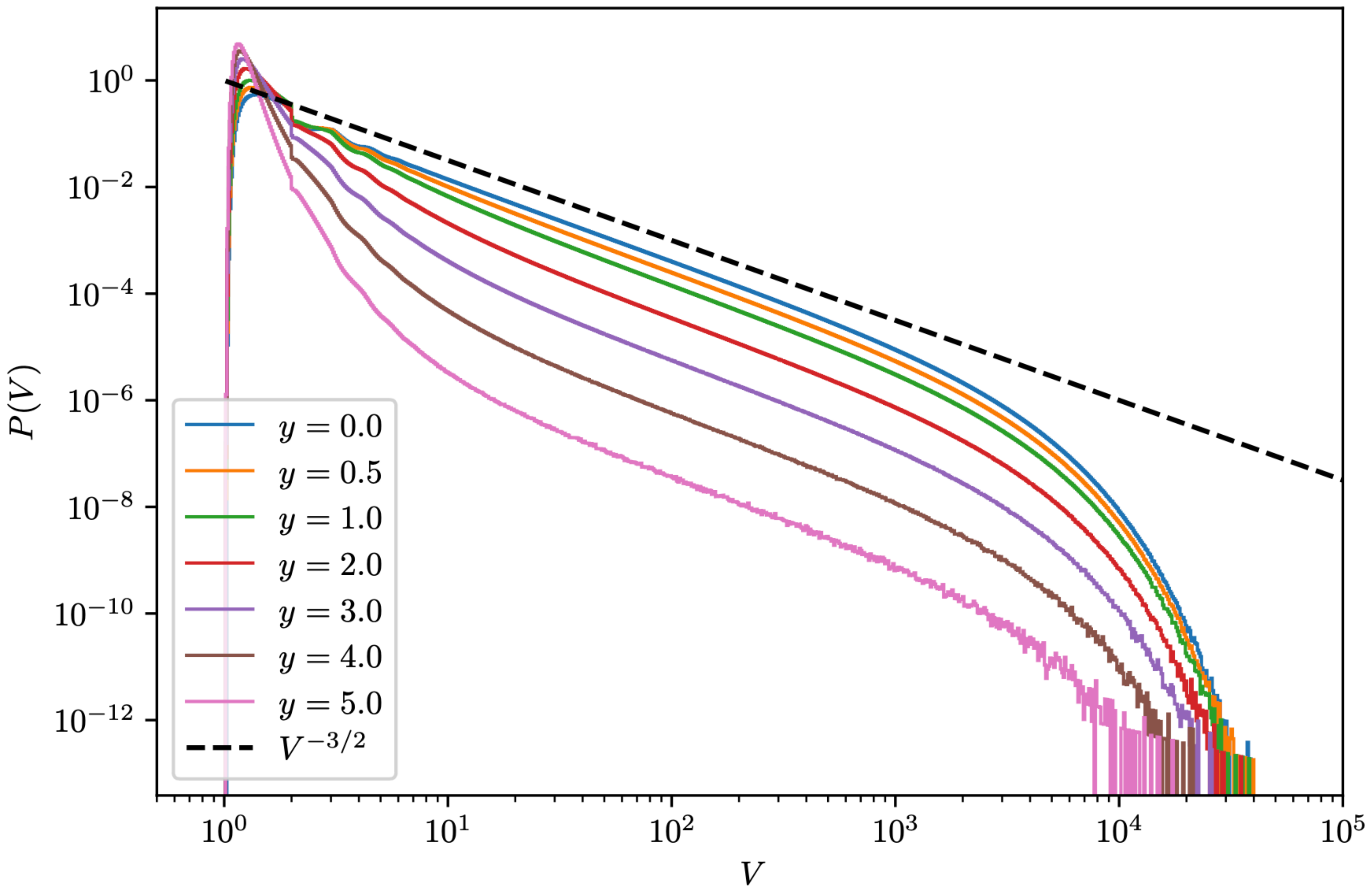
$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}N} = -3y + \dfrac{\sqrt{2}}{\mu}\xi(N) \\ \dfrac{\mathrm{d}y}{\mathrm{d}N} = -3y \end{cases}$$

$y \ll 1$ stochastic limit

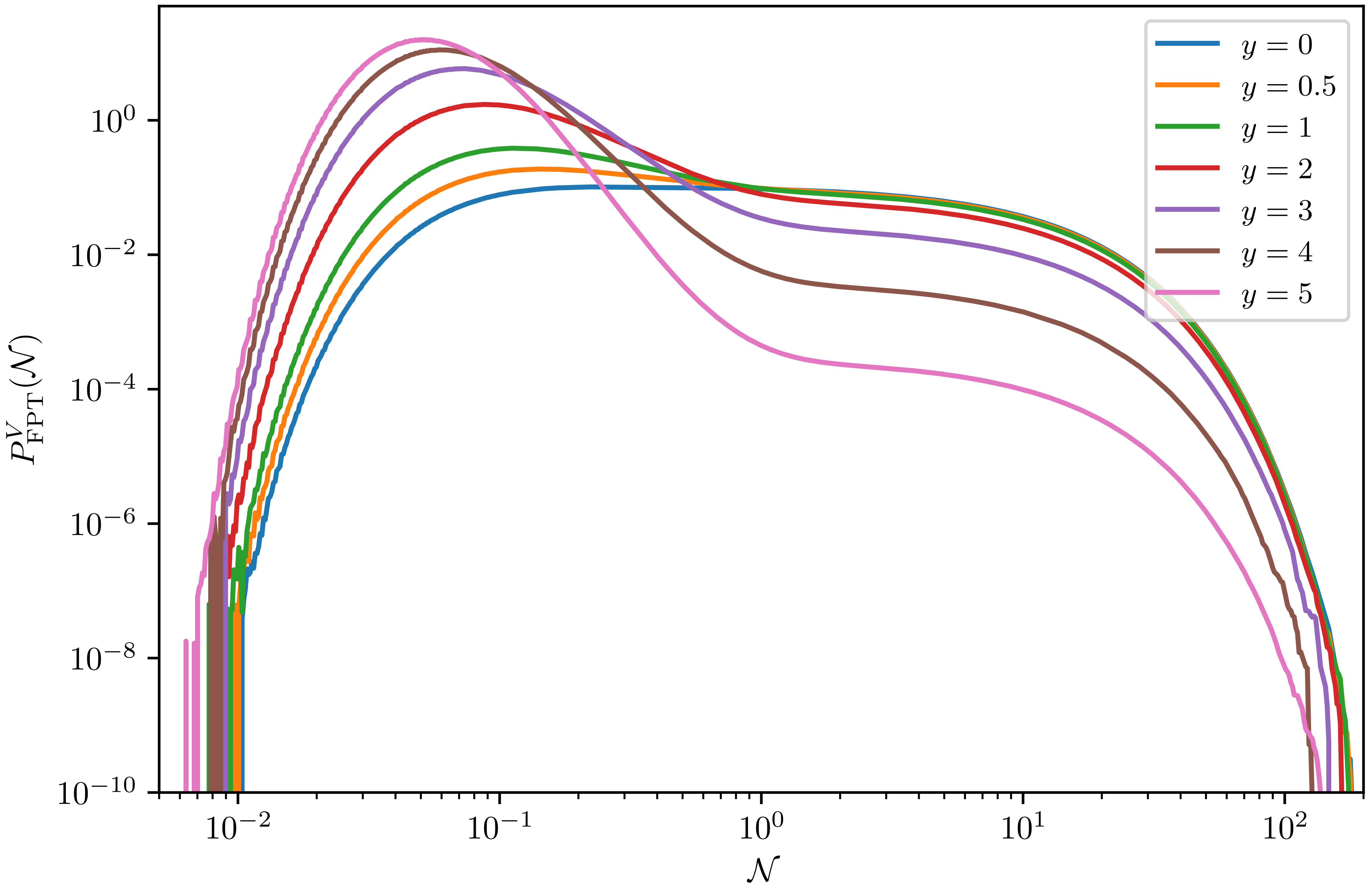
$y \gg 1$ classical limit

Ultra slow-roll model

Volume distribution



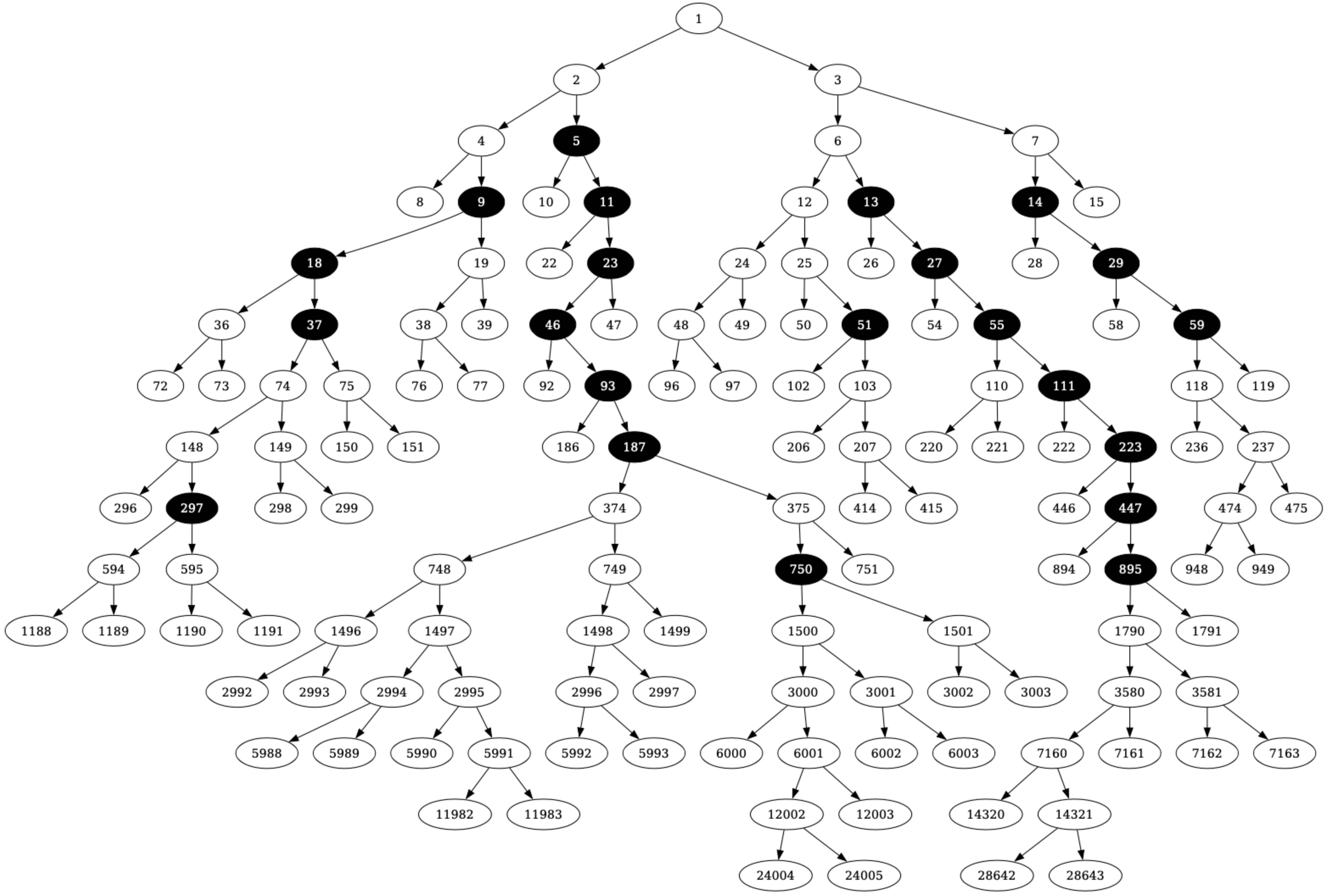
Volume-weighted FPT distribution

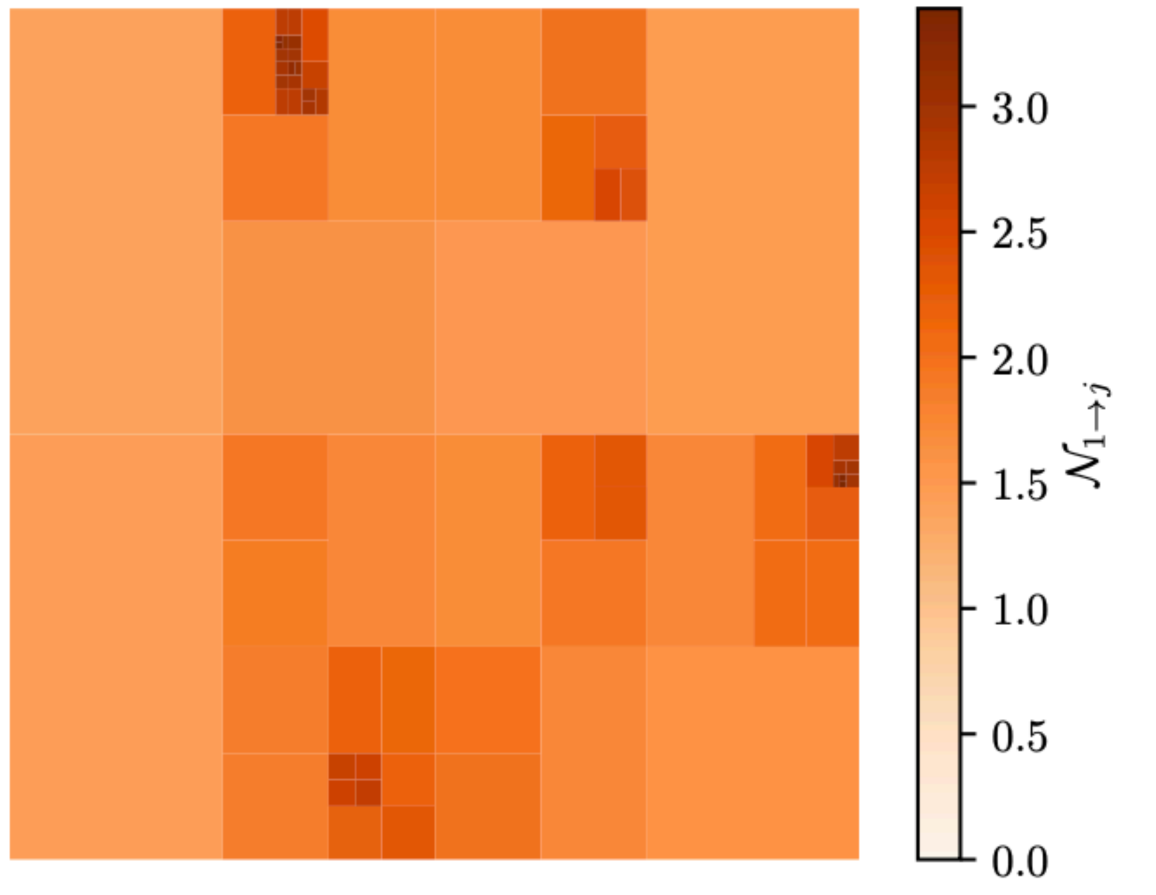


Power-law behaviour $P(V) \propto V^{-3/2}$ followed by exponential tails even for velocity $y > 1$.

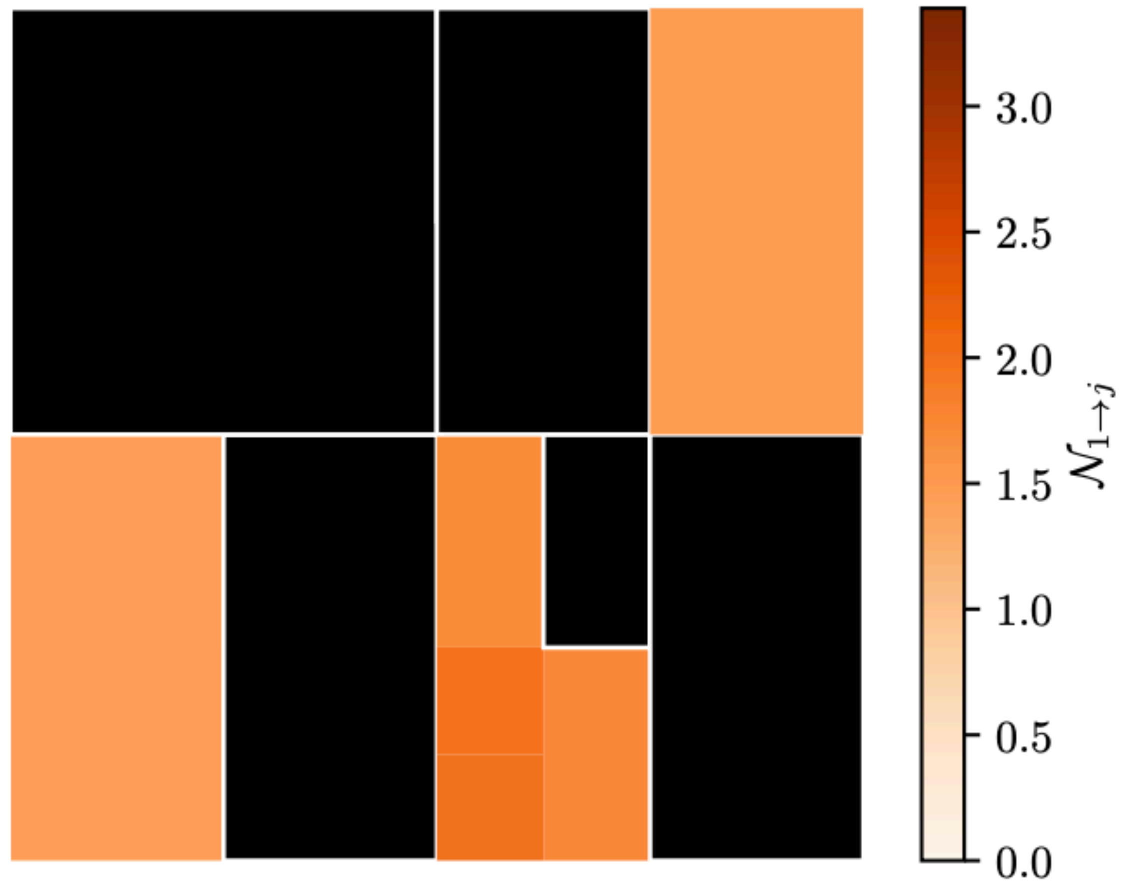
Classical regime characterised by non-Gaussian tails where PBHs form.

Harvesting primordial black holes





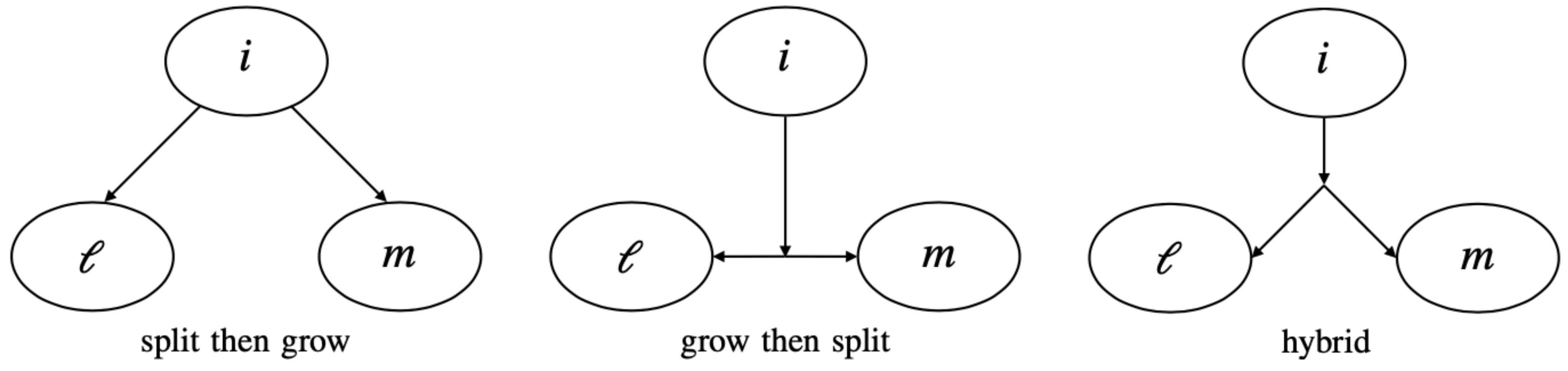
(a) Without PBHs.



(b) With PBHs.

Discretisation artefacts: branching times

Different branching prescriptions:



Node i grows up to a time $\alpha \Delta N$, then splits, and the child branches are evolved independently for $(1-\alpha)\Delta N$ with $0 \leq \alpha \leq 1$.

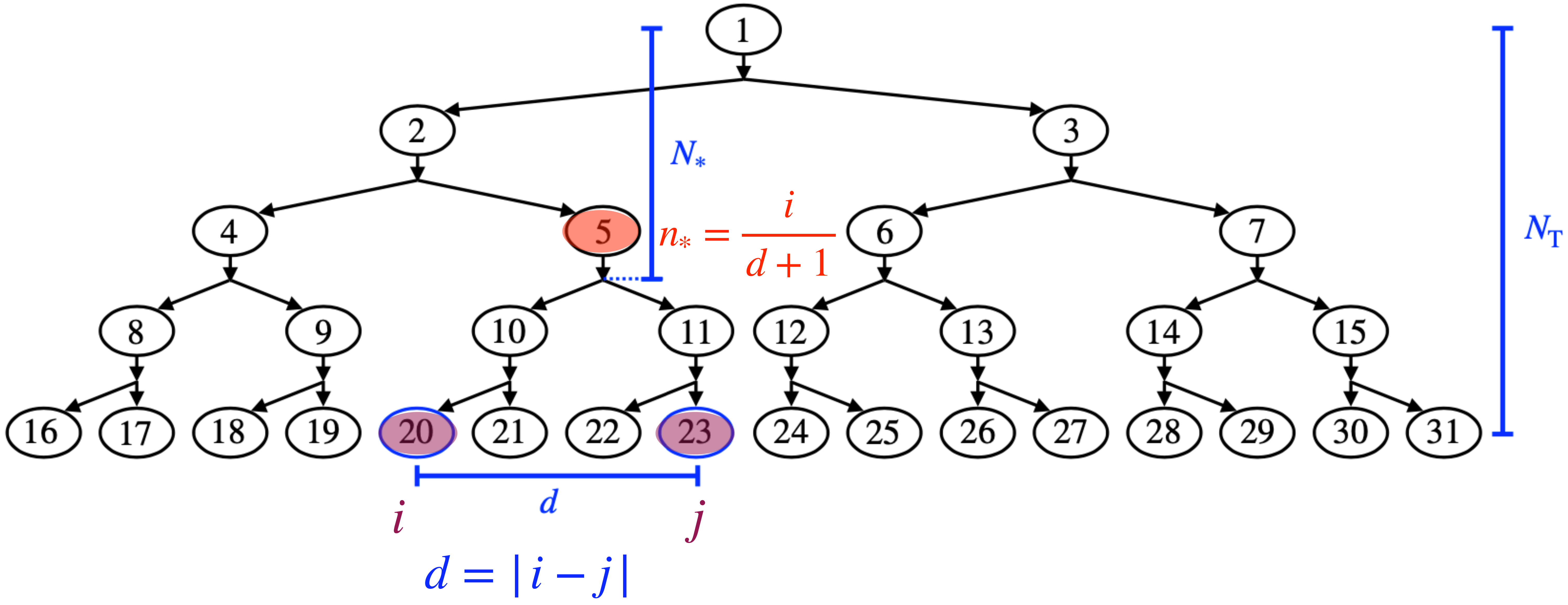
Large scale properties do not depend on the choice of α

$$N_* = N_T - \Delta N_* = N_T - \left[\frac{\log(d+1)}{\log(2)} - \alpha \right] \Delta N$$

<u>Light test scalar field in a fixed binary tree</u>

$$(d+1)V_\sigma = \frac{4}{3}\pi (d_\mathrm{P}/2)^3 \Rightarrow d+1 = \left(\frac{d_\mathrm{P} \sigma H}{2} \right)^3$$

$$\Delta N_* = \log(H d_\mathrm{P}) + \log(2^{-1-\alpha/3}\sigma)$$



$$P(\phi_i, \phi_j) = \int \mathrm{d}\phi_* P(\phi_* \,|\, \phi_1, N_*) P(\phi_i \,|\, \phi_*, \Delta N_*) P(\phi_j \,|\, \phi_*, \Delta N_*)$$

depends on α only through ΔN_*

(α dependence reabsorbed in σ)

Discretisation artefacts: branching times

Explicit example: light test field with $V(\phi) = \frac{1}{2}m^2\phi^2$ in de-Sitter universe

Gaussian solution for the stochastic problem:

Starobinsky & Yokoyama [1994]

$$P(\phi \,|\, \phi_{\rm in}, N) = \frac{e^{-\frac{[\phi - \bar{\phi}(N,\phi_{\rm in})]^2}{2s^2(N)}}}{\sqrt{2\pi s^2(N)}}$$

$$\bar{\phi}(N, \phi_{\rm in}) = \phi_{\rm in} e^{-\frac{m^2}{3H^2}N}$$

$$s^2(N) = \frac{3H^4}{8\pi^2 m^2}\left(1 - e^{-\frac{2m^2}{3H^2}N}\right)$$

$$P(\phi_i, \phi_j) = \frac{1}{\sqrt{(2\pi)^2 \det \Sigma}} e^{-\frac{1}{2}(\Delta\phi_i, \Delta\phi_j)\cdot\Sigma^{-1}\cdot\begin{pmatrix}\Delta\phi_i \\ \Delta\phi_j\end{pmatrix}}$$

$$\Delta\phi_i = \phi_i - \bar{\phi}(N_{\rm T}, \phi_1)$$

$$\Sigma_{ii} = \langle \Delta\phi_i^2 \rangle = \Sigma_{jj} = \langle \Delta\phi_j^2 \rangle = s^2(N_T)$$

$$\Sigma_{ij} = \langle \Delta\phi_i \Delta\phi_j \rangle = s^2(N_*) e^{-\frac{2m^2}{3H^2}\Delta N_*}$$

From computation in QFT + renormalisation, late-time limit:

N. A. Chernikov and E. A. Tagirov [1968]

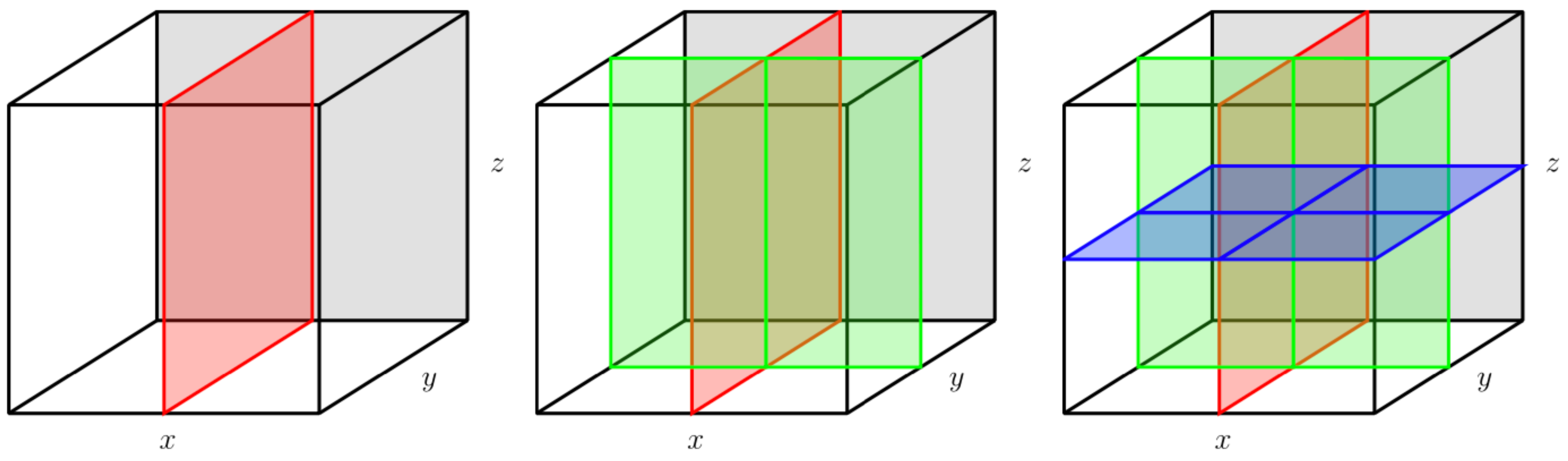
E. A. Tagirov [1973]

T. S. Bunch and P. C. W. Davies [1978]

$$\Sigma_{ii} = \frac{3H^4}{8\pi^2 m^2} \qquad \Sigma_{ij} \propto (Hd_{\rm p})^{-\frac{2m^2}{3H^2}}$$

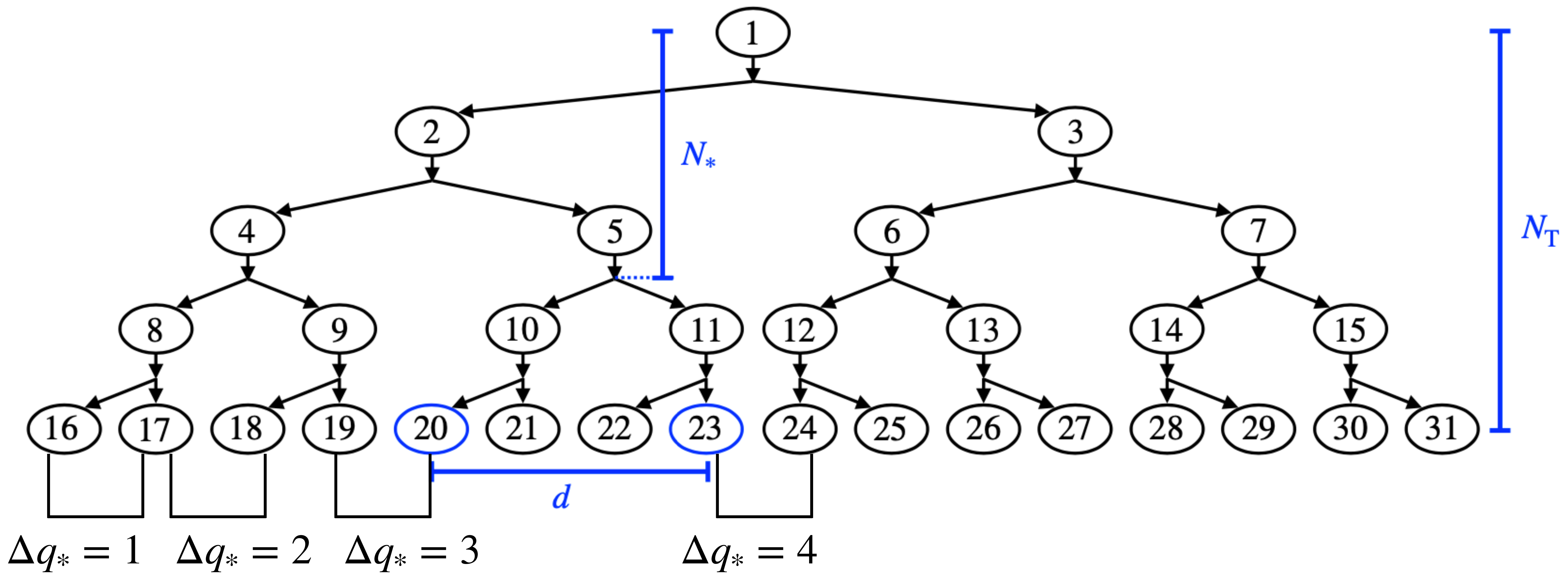
in agreement with the above result

Discretisation artefacts: branching surfaces



(a) ΔN: we divide the x-axis. (b) $2\Delta N$: we divide the y-axis. (c) $3\Delta N$: we divide the z-axis.

Branching surfaces breaks the homogeneity of FLRW spacetime.



Topological distance Δq_* not directly mapped to the geometrical distance d at the end of inflation.

$\Delta N_*(i, j)$ hence $P(\phi_i, \phi_j)$ not just a function of $|i - j|$: breaking of space-translation invariance.

Discretisation artefacts: branching surfaces

Two-point correlation at physical distance d_P should be defined by averaging over all pairs of two leaves distant by d on the end-of-inflation hyper surface $\Sigma(d_\mathrm{P}) = \dfrac{1}{2^{q_\mathrm{T}} - d} \displaystyle\sum_{i=2^{q_\mathrm{T}}}^{2^{q_\mathrm{T}+1}-d-1} \Sigma_{i,i+d}$.

Counting function for the number of pairs: $\beta(d,q) = \begin{cases} 2^q d \quad \text{if} \quad d \leq 2^{q_\mathrm{T}-q-1} \\ 2^{q_\mathrm{T}} - 2^q d \quad \text{if} \quad 2^{q_\mathrm{T}-q-1} \leq d \leq 2^{q_\mathrm{T}-q} \\ 0 \quad \text{if} \quad d \geq 2^{q_\mathrm{T}-q} \end{cases}$.

$$\Sigma\left(d_\mathrm{P}\right) = \frac{1}{2^{q_\mathrm{T}} - d} \sum_{q=0}^{q_\mathrm{T}-1} \beta(d,q) \Sigma\left[\left(q_\mathrm{T} - q\right)\Delta N\right] = \frac{3H^4}{8\pi^2 m^2} \frac{e^{-a q_\mathrm{T}}}{2^{q_\mathrm{T}} - d} \left[2^{q_\mathrm{T}}\left(e^{a q_*} - 1\right) - \frac{2\left(e^a - 1\right) d \left(2^{q_*} e^{a q_*} - 1\right)}{2 e^a - 1}\right]$$

$q_* = \lfloor q_\mathrm{T} - \ln(d)/\ln(2) \rfloor$

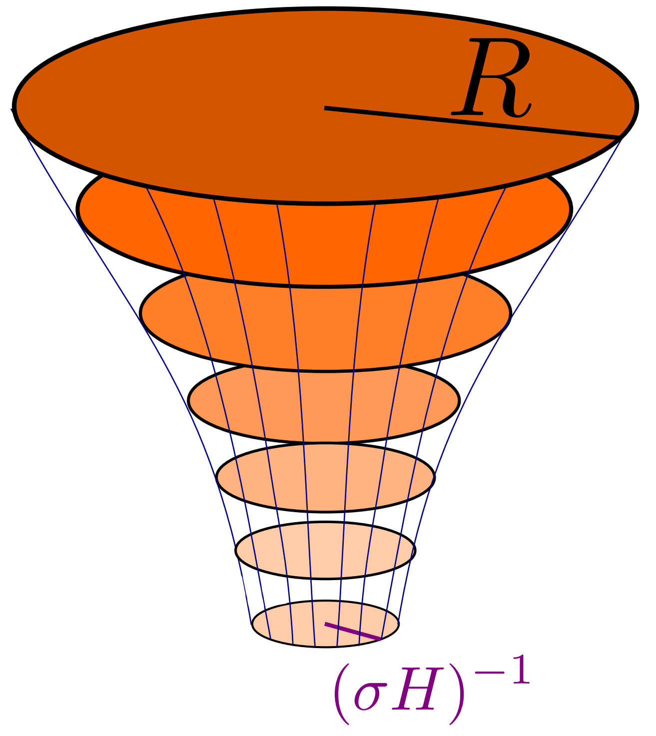
$a = 2m^2 \Delta N / (3H^2)$

At large distances ($1 \ll d \ll D \Rightarrow q_* \gg 1$): $\Sigma\left(d_\mathrm{P}\right) \simeq \dfrac{3H^4}{8\pi^2 m^2} \left[\dfrac{e^{3a}}{2e^a - 1} - e^{3a - a q_*}\right] \left(\sigma H d_\mathrm{P}\right)^{-\frac{2m^2}{3H^2}} \underset{a \ll 1}{\simeq} \dfrac{3H^4}{8\pi^2 m^2} \left[1 - e^{-\frac{2m^2}{3H^2} N_*}\right] \left(\sigma H d_\mathrm{P}\right)^{-\frac{2m^2}{3H^2}}$

(slow roll)

consistent with previous result and with QFT computation

Large-volume approximation



$R^3 \gg (\sigma H)^{-3}$

Ensemble average over the set of final leaves ⟶ Stochastic average of a single element within the ensemble

$V \to \langle V \rangle \qquad P(V \mid \Phi_*) \simeq \delta_\mathrm{D}(V - V_* \langle e^{3\mathcal{N}_{\Phi_*}} \rangle) \qquad \langle e^{3\mathcal{N}_{\Phi_*}} \rangle = \int_0^\infty P_{\mathrm{FPT},\Phi_*}(\mathcal{N}) e^{3\mathcal{N}} \mathrm{d}\mathcal{N}$

$W \to \langle W \rangle \qquad W \simeq \langle \mathcal{N}_{\Phi_*} \rangle_V = \dfrac{\langle \mathcal{N}_{\Phi_*} e^{3\mathcal{N}_{\Phi_*}} \rangle}{\langle e^{3\mathcal{N}_{\Phi_*}} \rangle}$

$$\zeta_R(\vec{x}_0) = \mathcal{N}_{\mathcal{P}_0 \to \mathcal{P}_*}(\vec{x}_0) + W(\mathcal{P}_*) - \mathbb{E}^V_{\mathcal{P}_0}[\mathcal{N}_{\mathcal{P}_0}(\vec{x})] \longrightarrow \zeta_R \simeq \mathcal{N}_{\mathcal{P}_0 \to \mathcal{S}_*} + \langle \mathcal{N}_{\Phi_*} \rangle_V - \langle \mathcal{N}_{\Phi_0} \rangle_V$$

$$P(\zeta_R \mid \Phi_0) = \int_{\mathcal{S}_*} \mathrm{d}\Phi_* P^V_{\mathrm{FPTL},\Phi_0 \to \mathcal{S}_*}(\mathcal{N}_{\mathcal{P}_0 \to \mathcal{S}_*} = \zeta_R - \langle \mathcal{N}_{\Phi_*} \rangle_V + \langle \mathcal{N}_{\Phi_0} \rangle_V, \Phi_* \mid \Phi_0)$$

$\mathcal{S}_*$: *hypersurface of constant mean forward volume*
$\langle e^{3\mathcal{N}_{\Phi_*}} \rangle = R^3$

first-passage time and location distribution

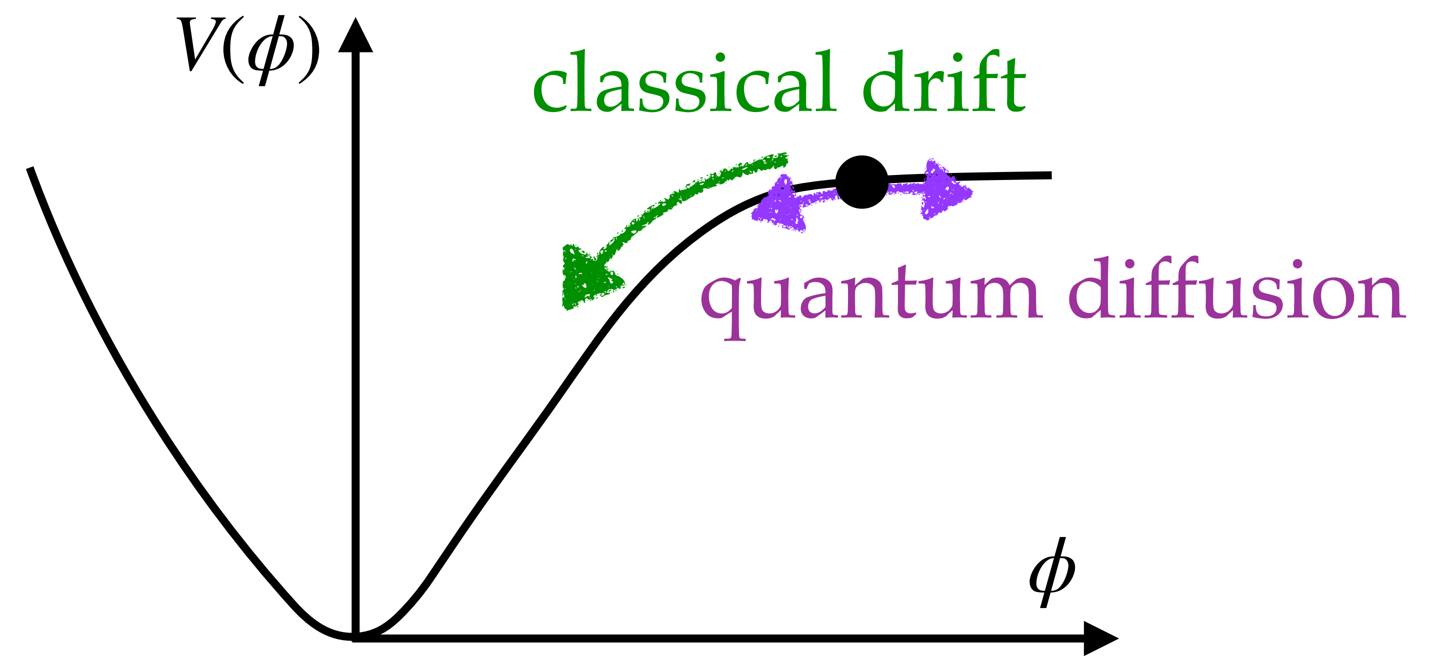
$$P^V_{\mathrm{FPTL},\Phi_0 \to \mathcal{S}_*}(\mathcal{N}_{\Phi_0 \to \mathcal{S}_*}, \Phi_* \mid \Phi_0) = P^V_{\mathrm{FPT},\Phi_0 \to \mathcal{S}_*}(\mathcal{N}_{\Phi_0 \to \mathcal{S}_*}) P(\Phi_* \mid \mathcal{N}_{\Phi_0 \to \mathcal{S}_*})$$

Single-clock models

$\Phi \to \phi$: single-field models of inflation along a dynamical attractor (slow roll).

Hypersurfaces $\mathcal{S}_*$ of fixed mean final volume reduce to **single points.**

Backward fields become **deterministic** quantities.



$$P(\zeta_R) = P^V_{\mathrm{FPT},\phi_0\to\phi_*}\left(\zeta_R - \langle \mathcal{N}_{\phi_*}\rangle_V + \langle \mathcal{N}_{\phi_0}\rangle_V\right)$$

$$P(\zeta_{R_1},\zeta_{R_2}) = \int \mathrm{d}\mathcal{N}_{\phi_0\to\phi_*}(\mathcal{N}_{\phi_0\to\phi_*}) P^V_{\mathrm{FPT},\phi_*\to\phi_1}\left(\zeta_{R_1} - \mathcal{N}_{\phi_0\to\phi_*} + \langle \mathcal{N}_{\phi_0}\rangle_V - \langle \mathcal{N}_{\phi_1}\rangle_V\right) P^V_{\mathrm{FPT},\phi_*\to\phi_2}\left(\zeta_{R_2} - \mathcal{N}_{\phi_0\to\phi_*} + \langle \mathcal{N}_{\phi_0}\rangle_V - \langle \mathcal{N}_{\phi_2}\rangle_V\right)$$

Power spectrum from the two-point statistics

Two-point correlation function of coarse-grained fields:

$$\langle \zeta_{R_1}\zeta_{R_2}\rangle = \int \mathrm{d}\zeta_{R_1}\int \mathrm{d}\zeta_{R_2} P(\zeta_{R_1},\zeta_{R_2})\zeta_{R_1}\zeta_{R_2} = \langle \mathcal{N}^2_{\phi_0\to\phi_*}\rangle_V - \langle \mathcal{N}_{\phi_0\to\phi_*}\rangle_V^2 \equiv \langle \delta\mathcal{N}^2_{\phi_0\to\phi_*}\rangle_V = \langle \delta\mathcal{N}^2_{\phi_0}\rangle_V - \langle \delta\mathcal{N}^2_{\phi_*}\rangle_V$$

no dependence on the coarse-graining scales R_1, R_2 .

In Fourier space: $\zeta_{R_i}(\vec{x}_i) = \int \frac{\mathrm{d}\vec{k}}{(2\pi)^{3/2}}\,\zeta_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}_i}\,\widetilde{W}\left(\frac{kR_i}{a}\right)$

$$\langle \zeta_{R_1}\zeta_{R_2}\rangle = \int_0^\infty \mathrm{d}\ln k\,\mathcal{P}_\zeta(k)\widetilde{W}\left(\frac{kR_1}{a}\right)\widetilde{W}\left(\frac{kR_2}{a}\right)\widetilde{W}\left(\frac{kr}{a}\right)$$

$r > R_1, R_2$ ⟶ $\langle \zeta_{R_1}\zeta_{R_2}\rangle = \int_0^\infty \mathrm{d}\ln k\,\mathcal{P}_\zeta(k)\widetilde{W}\left(\frac{kr}{a}\right)$

Differentiation w.r.t. r:

$$\mathcal{P}_\zeta(k) = -\frac{\partial}{\partial \ln r}\langle \zeta_{R_1}\zeta_{R_2}\rangle\Big|_{r=a_{\mathrm{end}}/k} = \frac{\partial}{\partial \ln r}\langle \delta\mathcal{N}_{\phi_*}\rangle^2\Big|_{r=a_{\mathrm{end}}/k}$$

$$\mathcal{P}_\zeta(k) = \frac{r}{\tilde{r}}\left[\frac{1}{3}\frac{\partial}{\partial\phi_*}\ln\langle e^{3\mathcal{N}_{\phi_*}}\rangle - \frac{\partial}{\partial\phi_*}\ln H(\phi_*)\right]^{-1}\frac{\partial}{\partial\phi_*}\langle \delta\mathcal{N}^2_{\phi_*}\rangle_V\Big|_{\langle e^{3\mathcal{N}_{\phi_*}}\rangle^{1/3}=\frac{1}{2}\frac{r}{\tilde{r}}\frac{a_{\mathrm{end}}\sigma H(\phi_*)}{k}}$$

$\tilde{r} = r + R_1 + R_2$

$r \gg R_1, R_2 \to \frac{r}{\tilde{r}} \simeq 1$

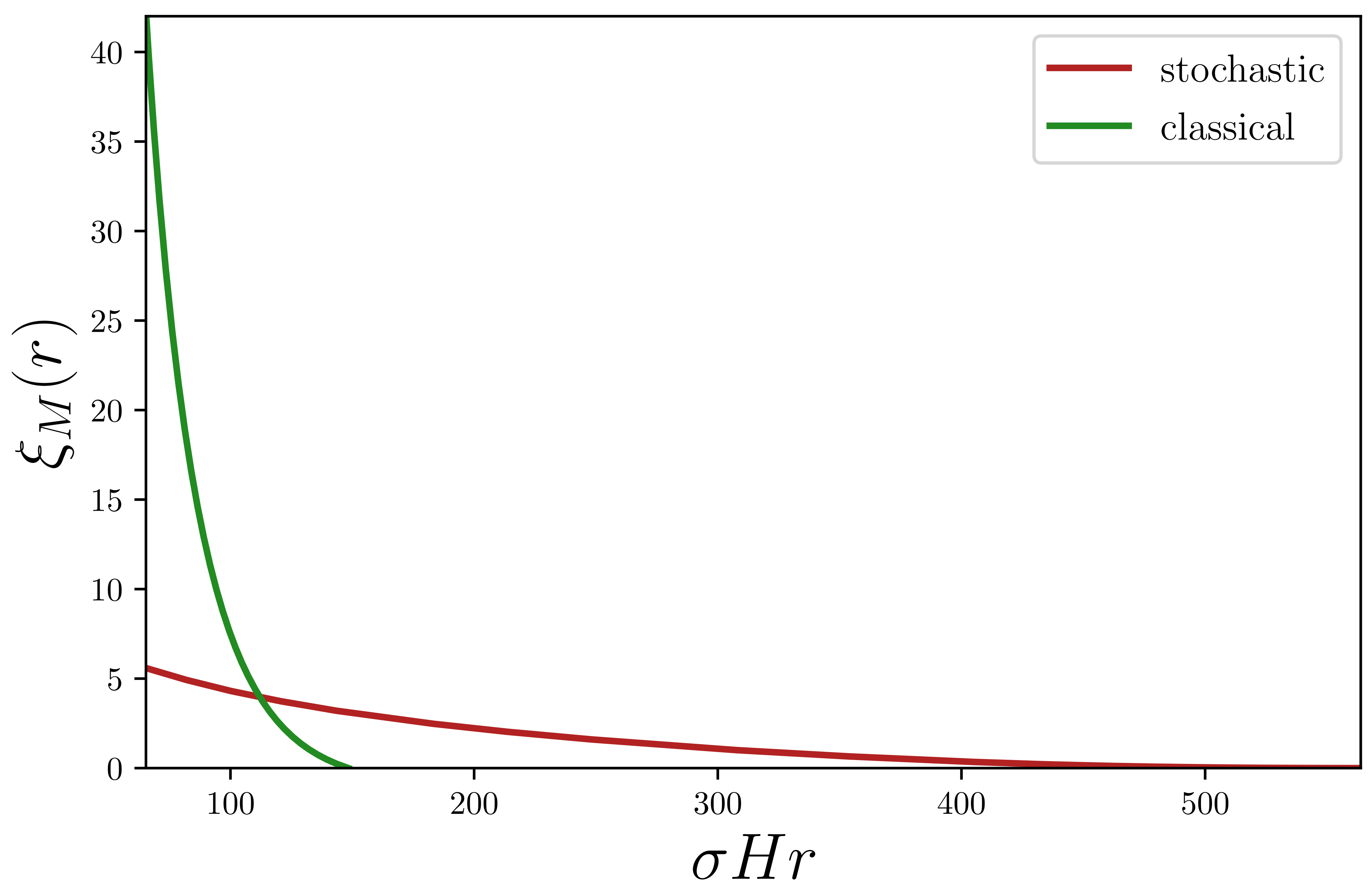
$\partial \ln N/\partial\phi \simeq \sqrt{\epsilon_1/2}/M_{\mathrm{Pl}}$

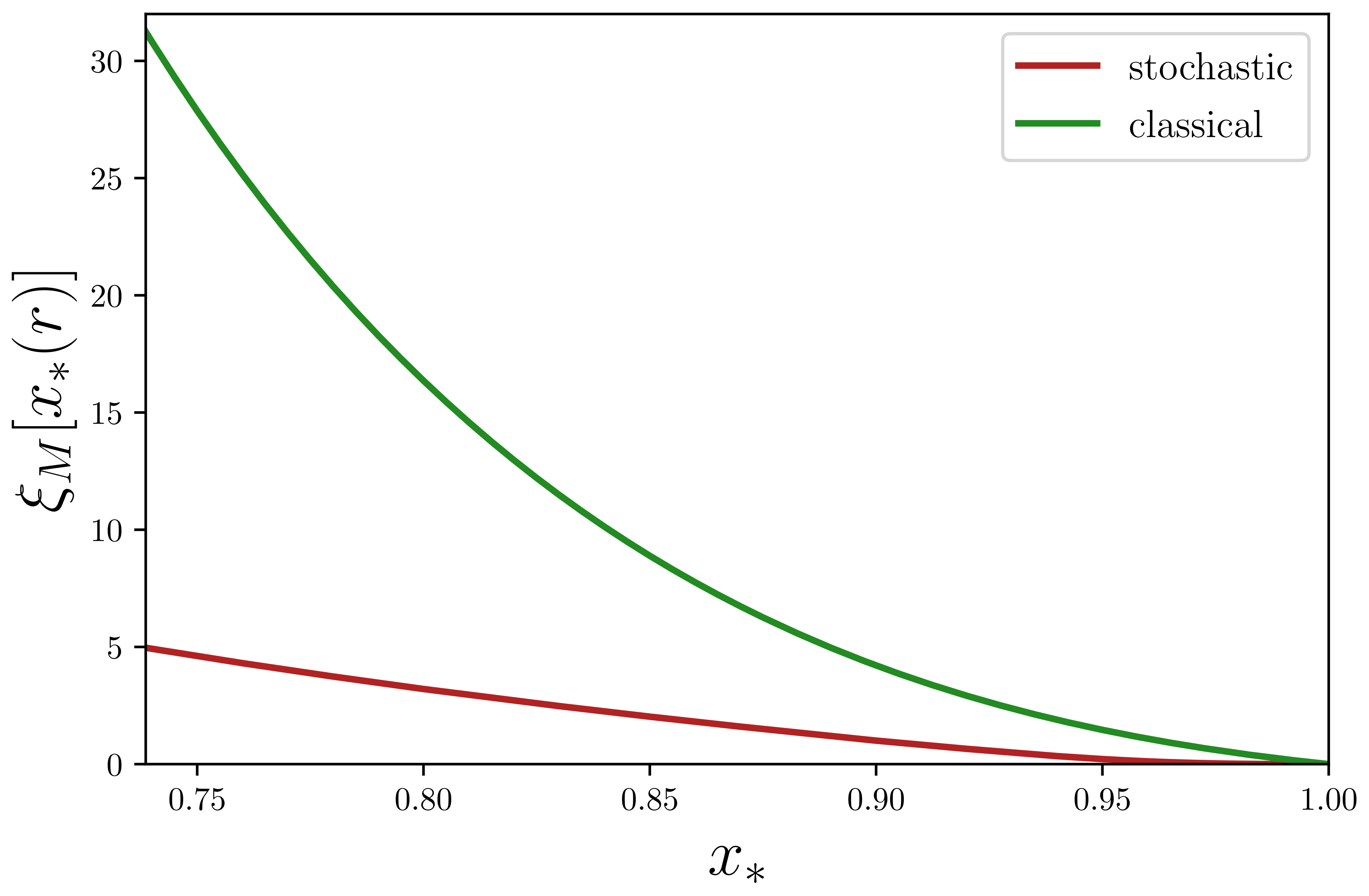
c.f.r. V. Vennin and A. A. Starobinsky [2015]
T. Fujita, M. Kawasaki, Y. Tada and T. Takesako [2013]

Same expression at l.o. in slow roll neglecting volume weighting and defining ϕ_* via $\langle\mathcal{N}\rangle$ and not via $\langle e^{3\mathcal{N}}\rangle$.

Comparison with the classical limit

Reduced correlation





larger distances r are covered in the stochastic calculation than in its classical counterpart

different relation between scales and field values: $r_{\max}^{\text{class}} = e^{1/d}$ versus $\tilde{r}_{\max}^{\text{stoch}} = 2\langle e^{3\mathcal{N}} \rangle_{x=1}^{1/3}$

PBHs are correlated over longer distances once quantum diffusion is taken into account.

If $\xi(x_*, x_1, x_2)$ functions are compared rather than $\xi(r, R_1, R_2)$ the clustering profiles are similar: field-scale distortion main reason for the large difference.

Stochastic-δN formalism: exponential tails

Full PDF of the first passage time

Characteristic function (includes all moments):

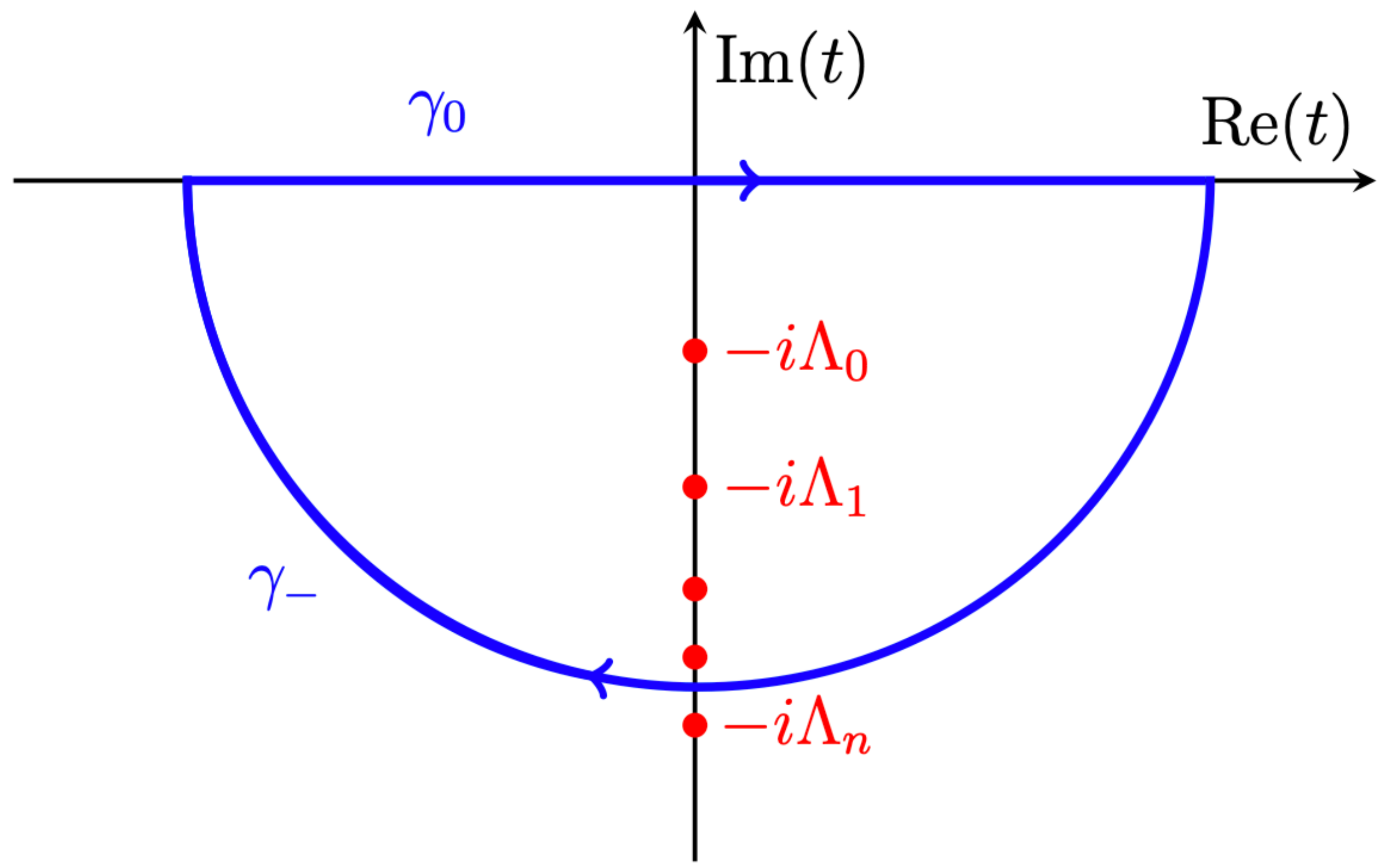
$$\chi(t,\Phi) \equiv \langle e^{it\mathcal{N}} \rangle = \int_{-\infty}^{\infty} e^{it\mathcal{N}} P(\mathcal{N},\Phi)\, d\mathcal{N} \longrightarrow \mathcal{L}^{\dagger}_{FP} \cdot \chi(t,\Phi) = -\, i\, t\, \chi(t,\Phi) \longrightarrow P(\mathcal{N},\Phi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-it\mathcal{N}} \chi(t,\Phi)\, dt$$

Useful trick: pole expansion

Ezquiaga, Garcia-Bellido, Vennin (2020)

$$\chi(t,\Phi) = \sum_n \frac{a_n(\Phi)}{\Lambda_n - i\, t} + g(t,\Phi)$$

$$P(\mathcal{N},\Phi) = \sum_n a_n(\Phi)\, e^{-\Lambda_n \mathcal{N}} \qquad 0 < \Lambda_0 < \Lambda_1 < \cdots \Lambda_n$$



Tail of the PDF of $\mathcal{N}$ (hence ζ) has an exponential fall-off behaviour.

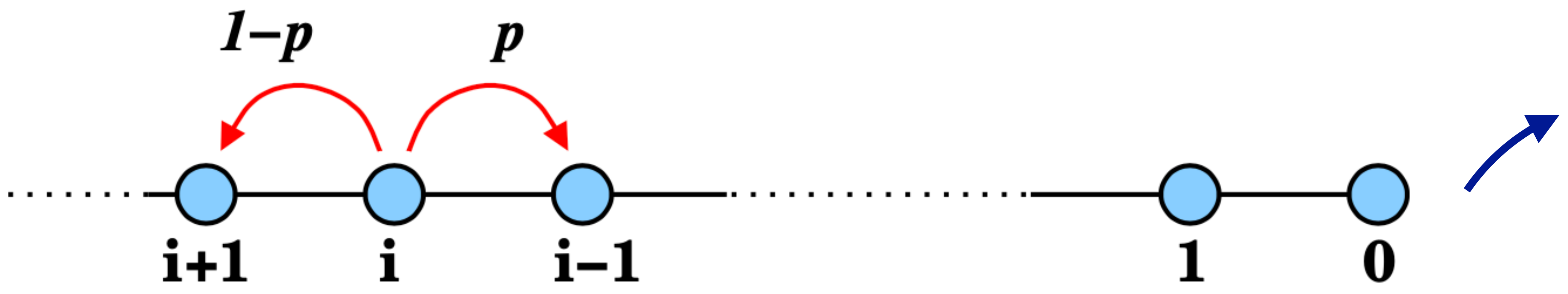
This type of non-Gaussianities cannot be captured by perturbative parametrisations (such as the $f_{\rm NL}$, $g_{\rm NL}$ expansion).

Going beyond

Is it possible to go beyond the large volume approximation?

Creminelli, Dubovsky, Nicholas, Senatore, Zaldarriaga [2008]
Dubovsky, Senatore, Villadoro [2009]

Bacteria model of inflation



End site : bacterium dies

Multi-type Galton-Watson process

Bacteria live on discrete set of positions along a line, replicating into N copies at each time step.

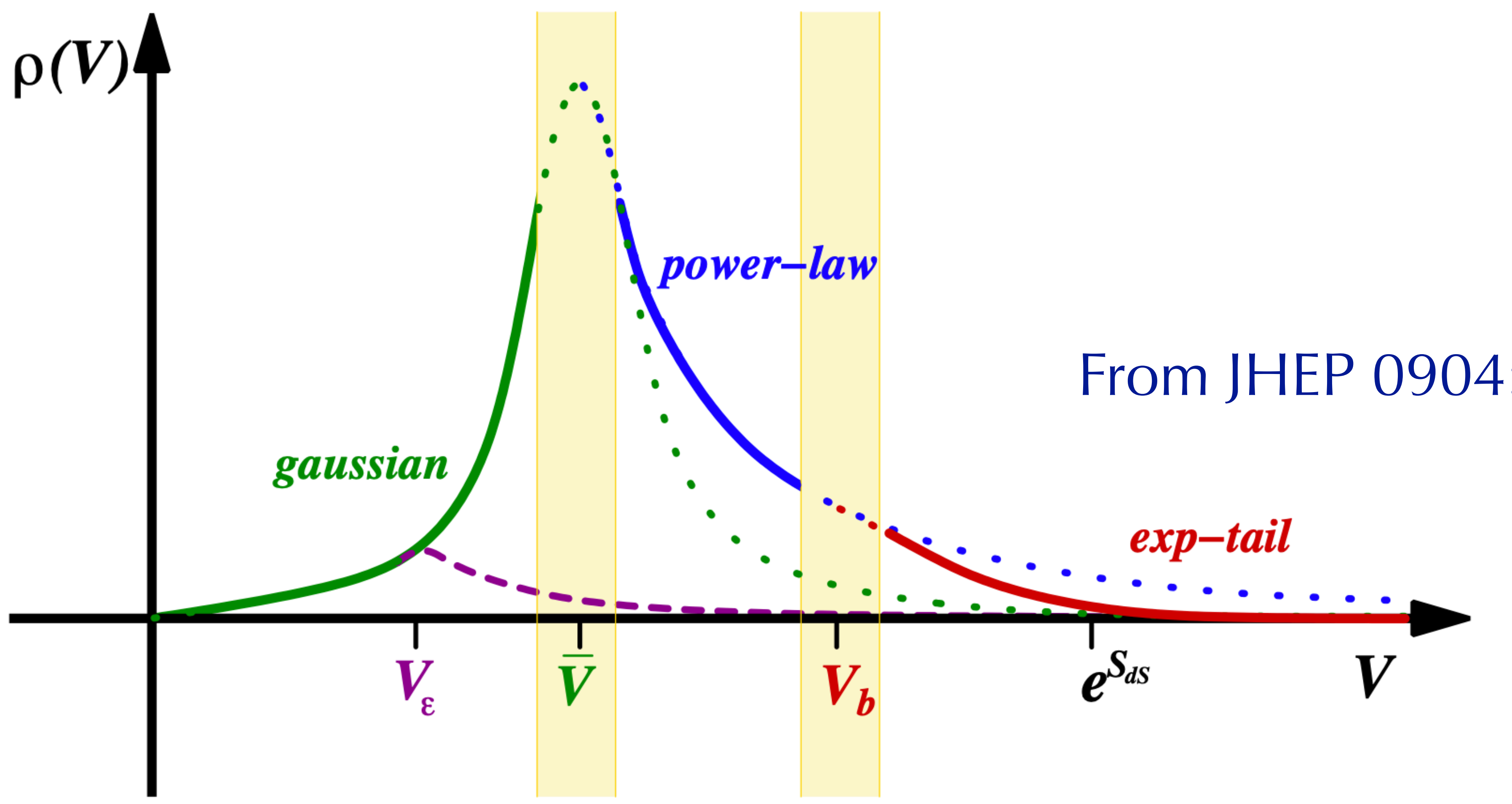
Bacteria → Hubble patches

Sites → Inflaton values

Random hopping → Quantum diffusion

Difference in $(1-p)$ and p → Drift

Number of dead bacteria → Final volume

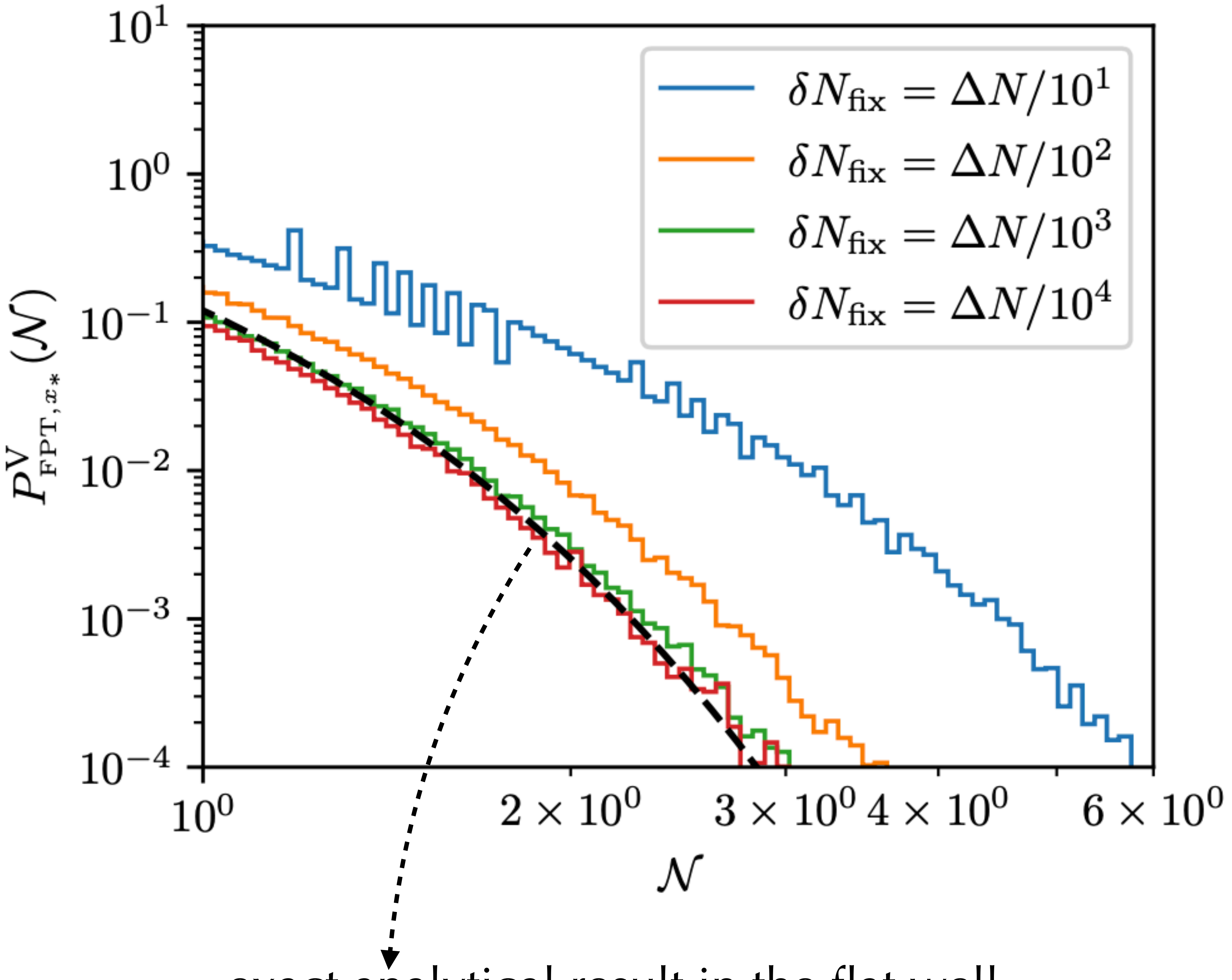


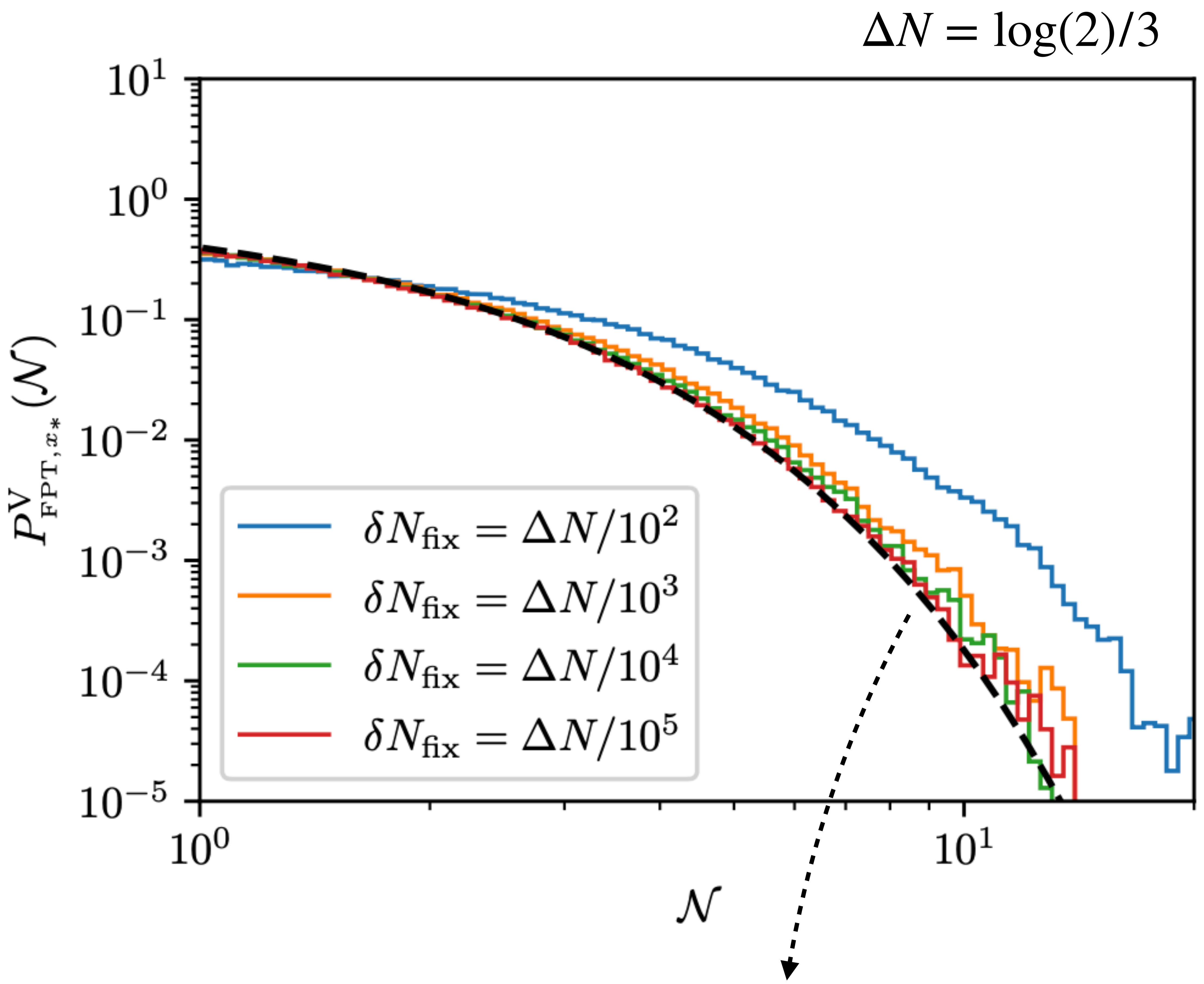
From JHEP 0904:118,2009

Forest: convergence test

Euler-Maruyama method with varying step δN used to solve Langevin equations.

Using a too large $\delta N_{\rm fix}$ overestimates the FPT.





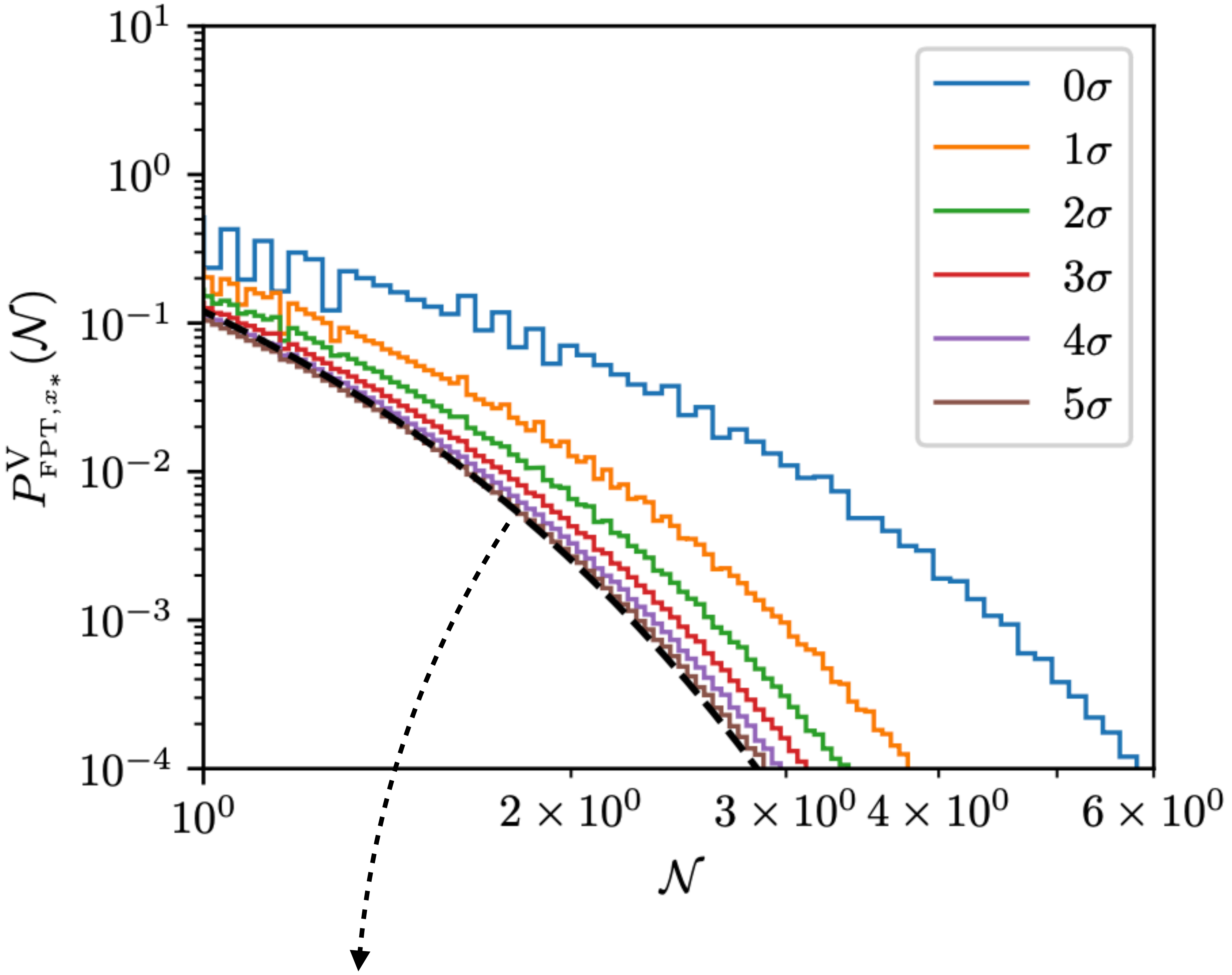
exact analytical result in the flat well

exact analytical result in the flat well

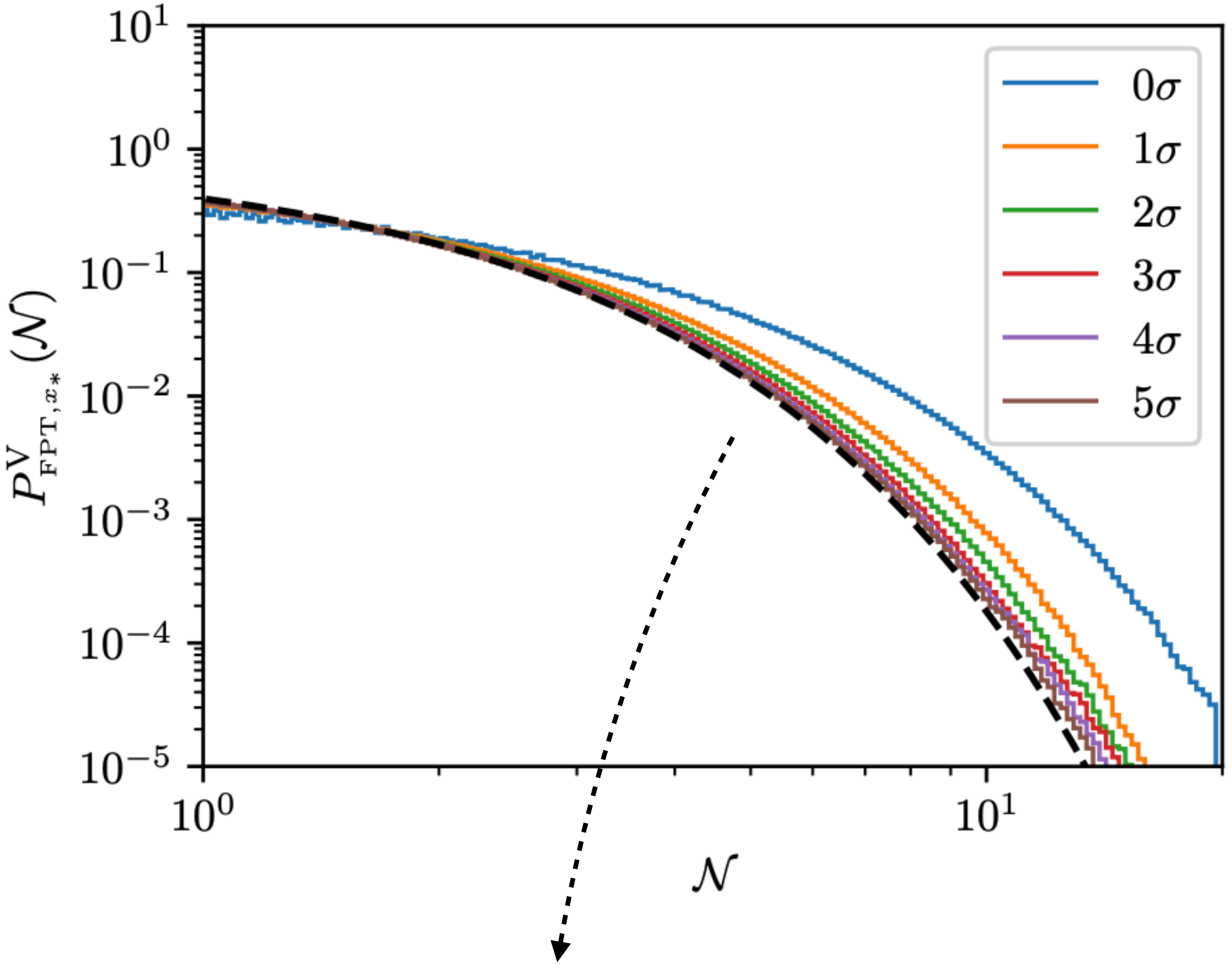
Forest: convergence test

Varying step δN to limit probability of barrier crossing to 5σ and to avoid double crossing that spoil FPT estimation:

$$\delta N = \min \left\{ \delta N_{\rm fix}, \frac{3[2\pi M_{\rm Pl}(\phi - \phi_{\rm end})]^2}{\kappa V(\phi)} \right\} \qquad \kappa = 5$$



exact analytical result in the flat well



exact analytical result in the flat well