Numerical relativity in the early universe

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Cosmology using numerical relativity
[Living Reviews in Relativity, 2025]
with Katy Clough and Eugene Lim

CHOMBO

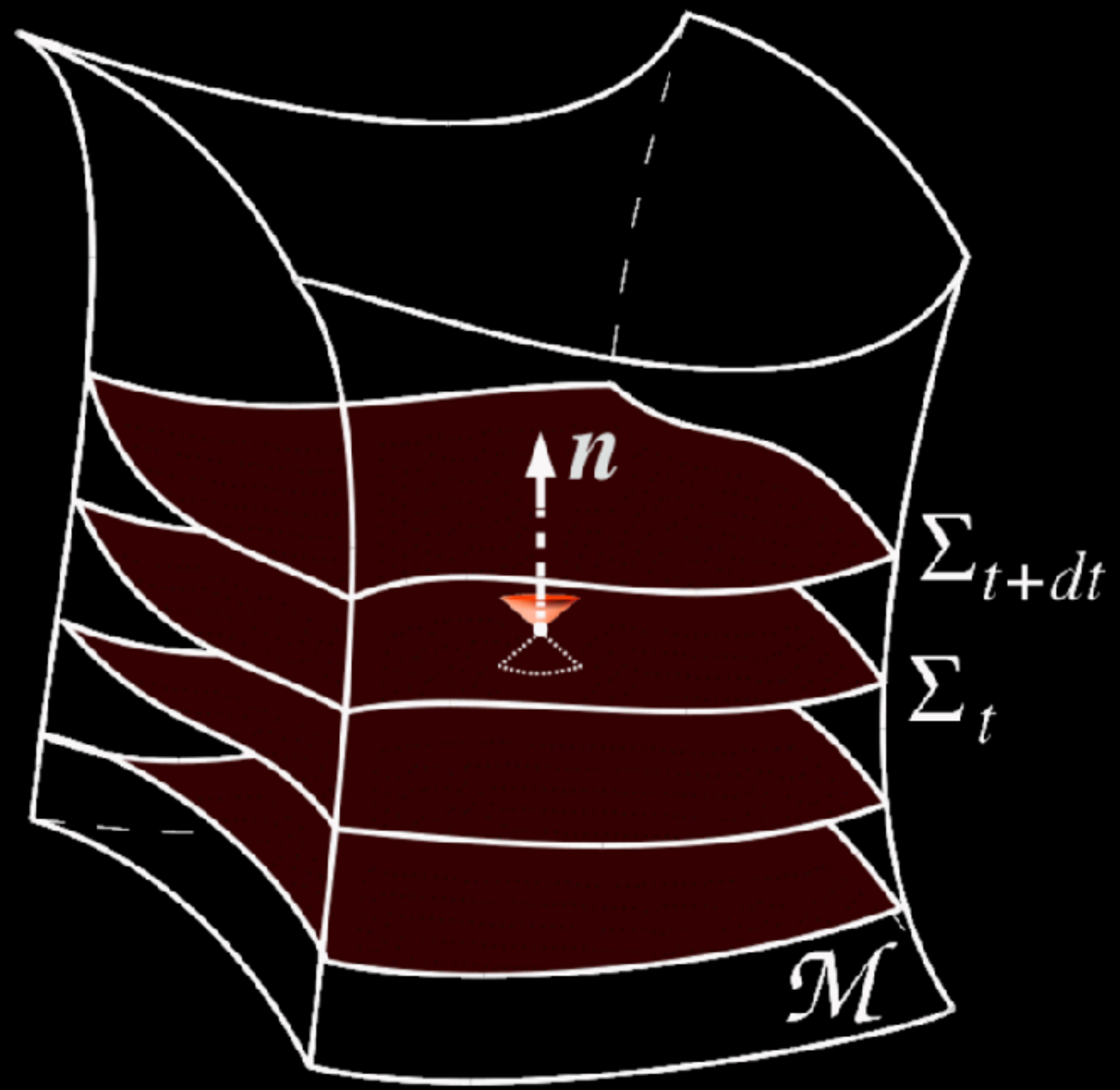
Numerical relativity for cosmologists

Covariant form

$$R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

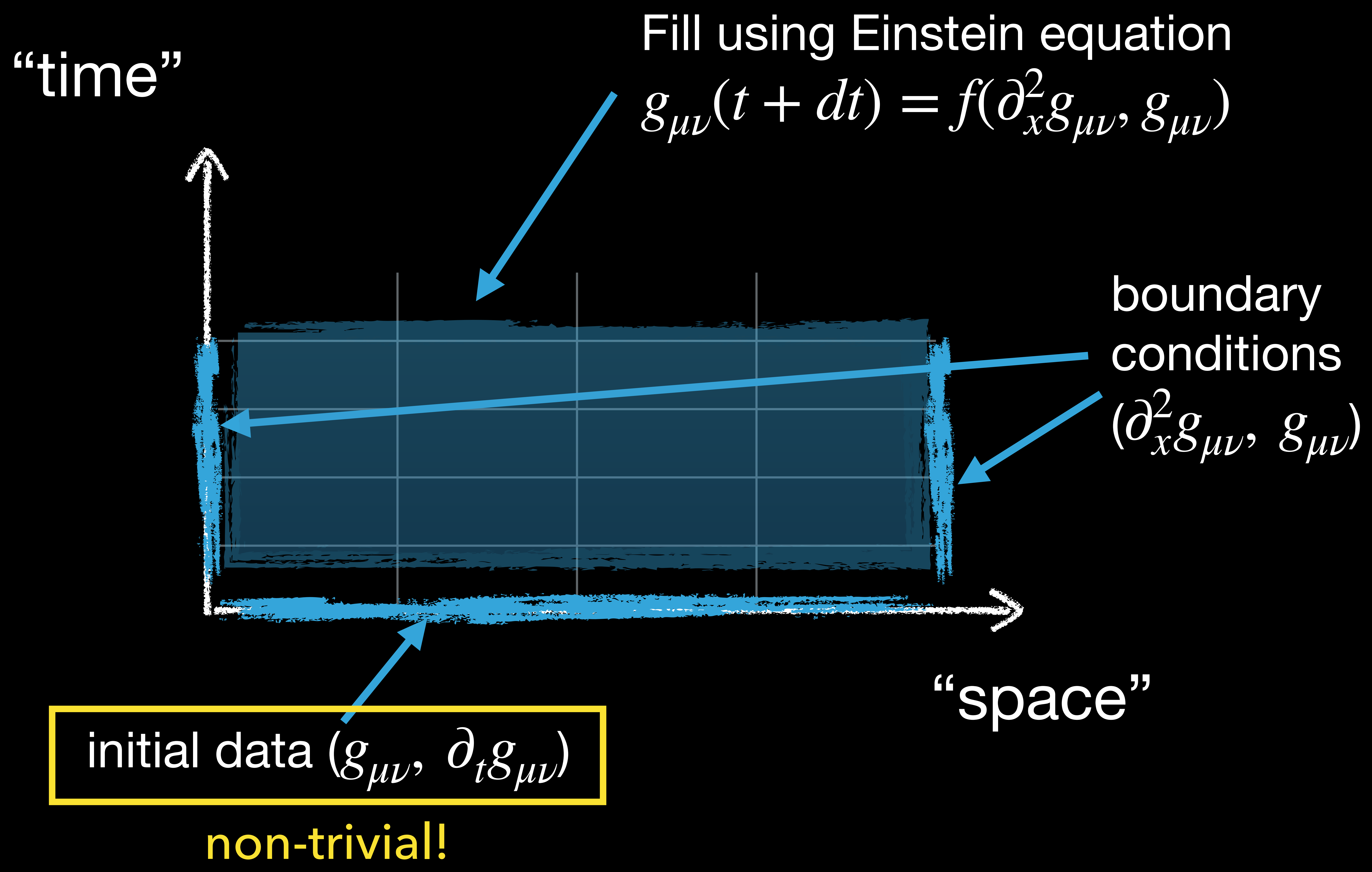
$$\partial_t g_{\mu\nu} = \ldots$$

Initial value form



Arnowitt, Deser, Misner (ADM)
Baumgarte, Shapiro, Shibata, Nakamura (BSSN)

Numerical relativity for cosmologists

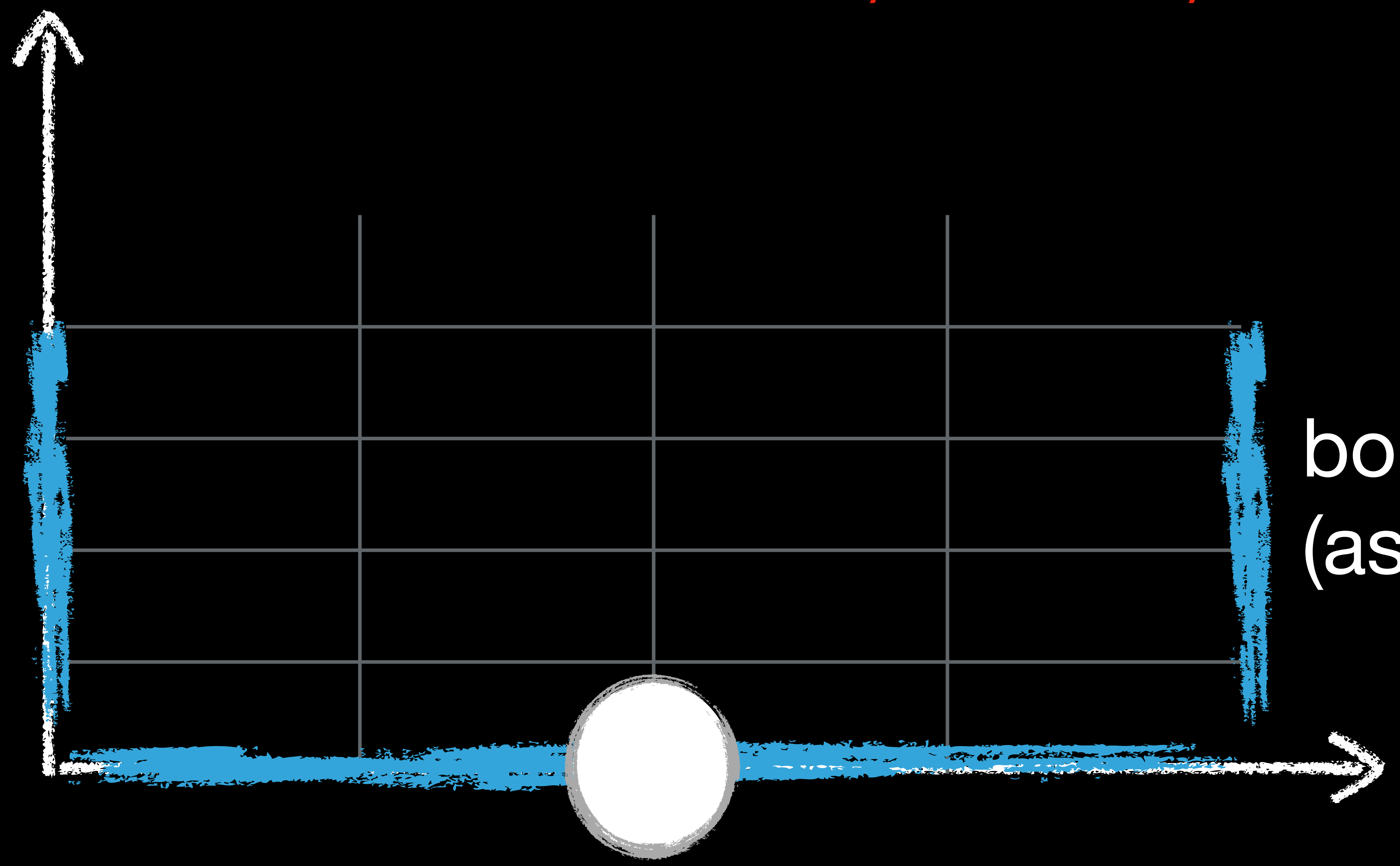


Numerical relativity for cosmologists

Fill using Einstein equation

$\partial_t g_{\mu\nu} = \partial_t^2 g_{\mu\nu} = 0$ (boring!)

“time”



boundary conditions
(asymptotically flat)

“space”

initial data (Schwarzschild metric)

Numerical relativity for cosmologists

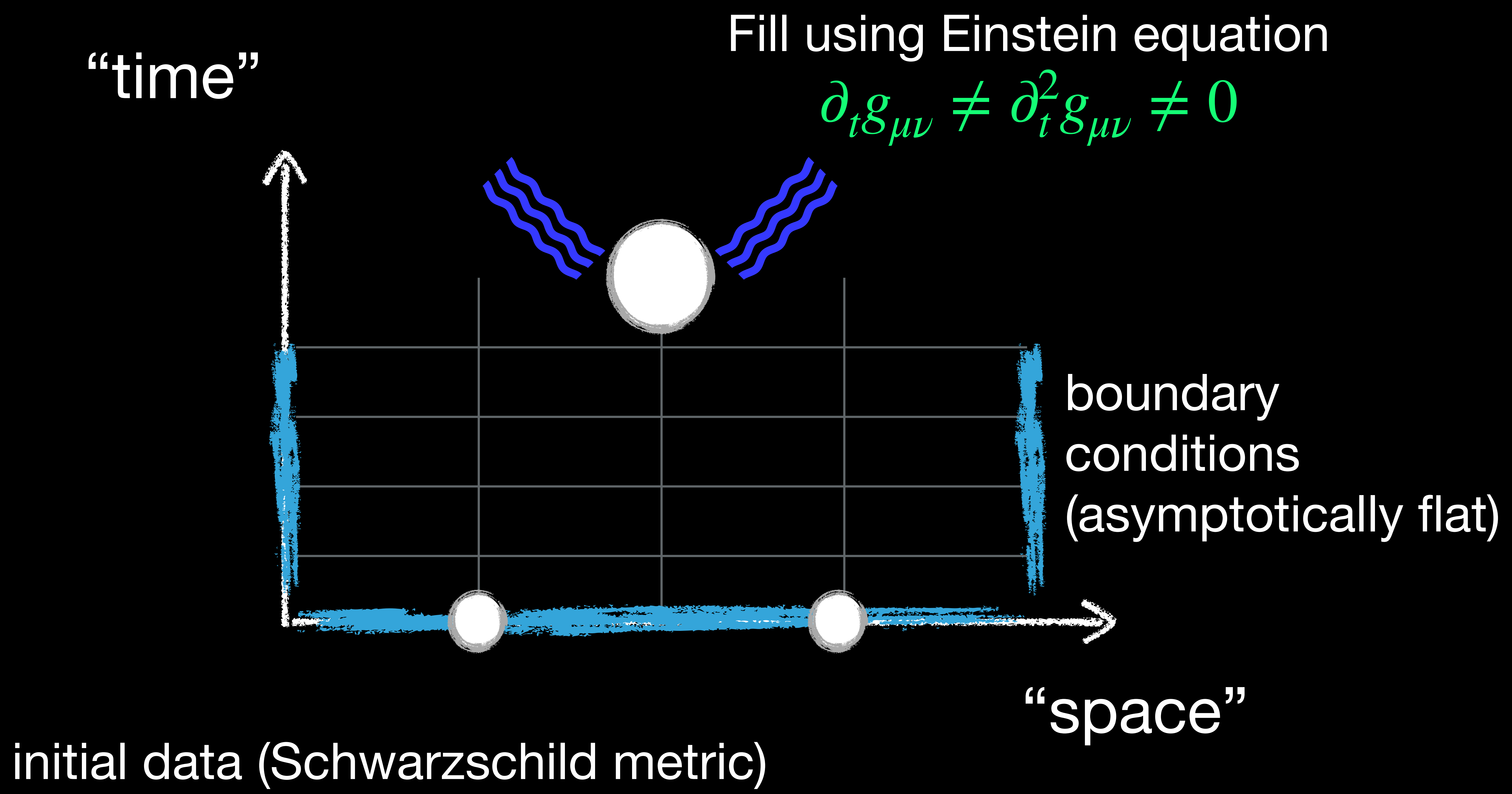
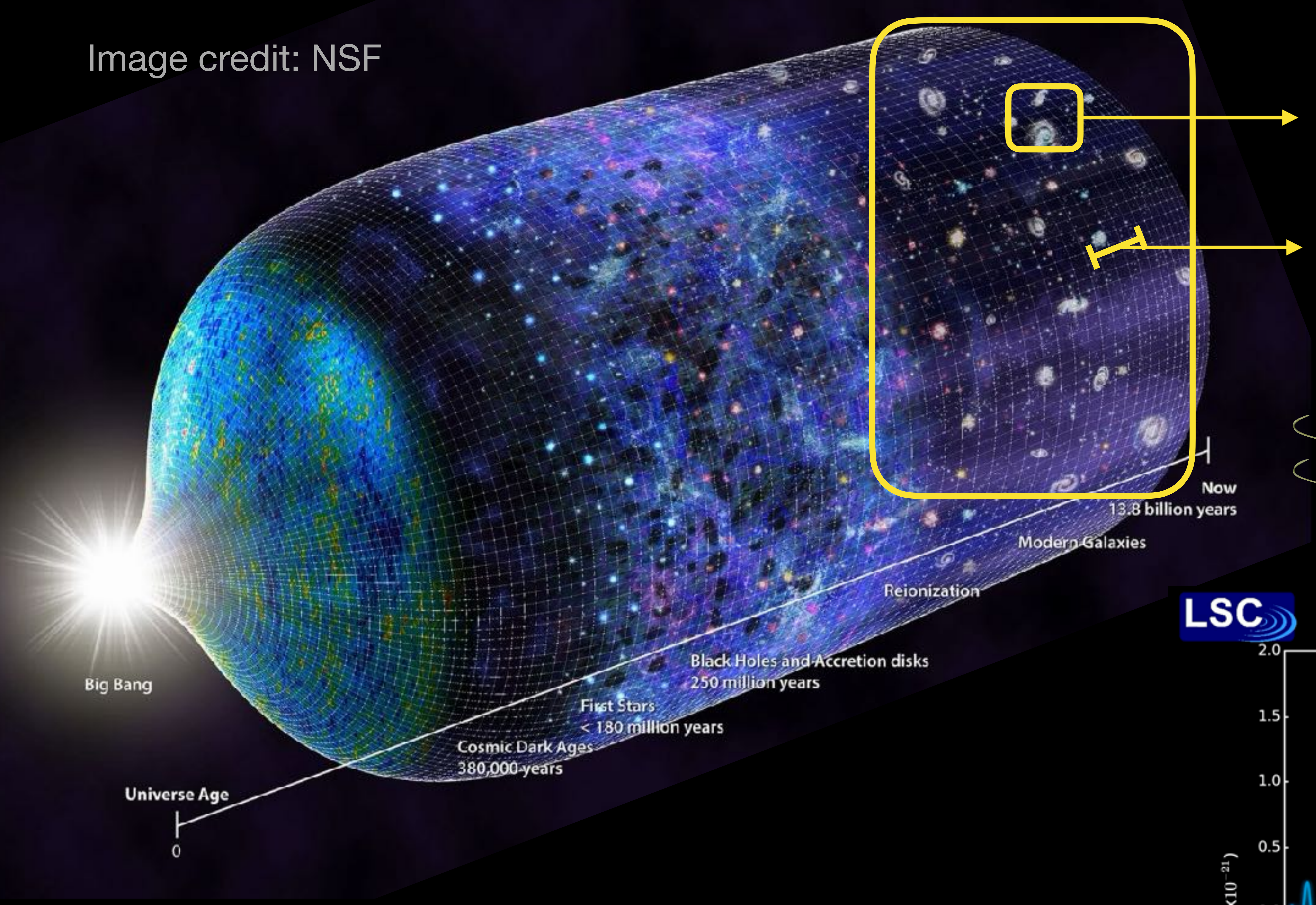


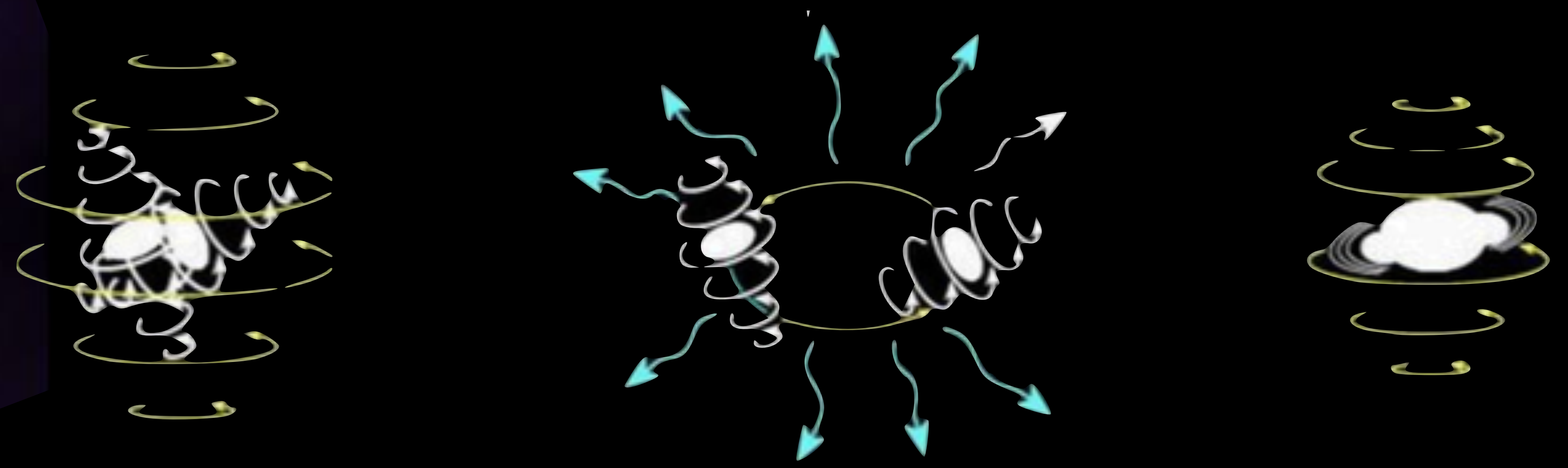
Image credit: NSF

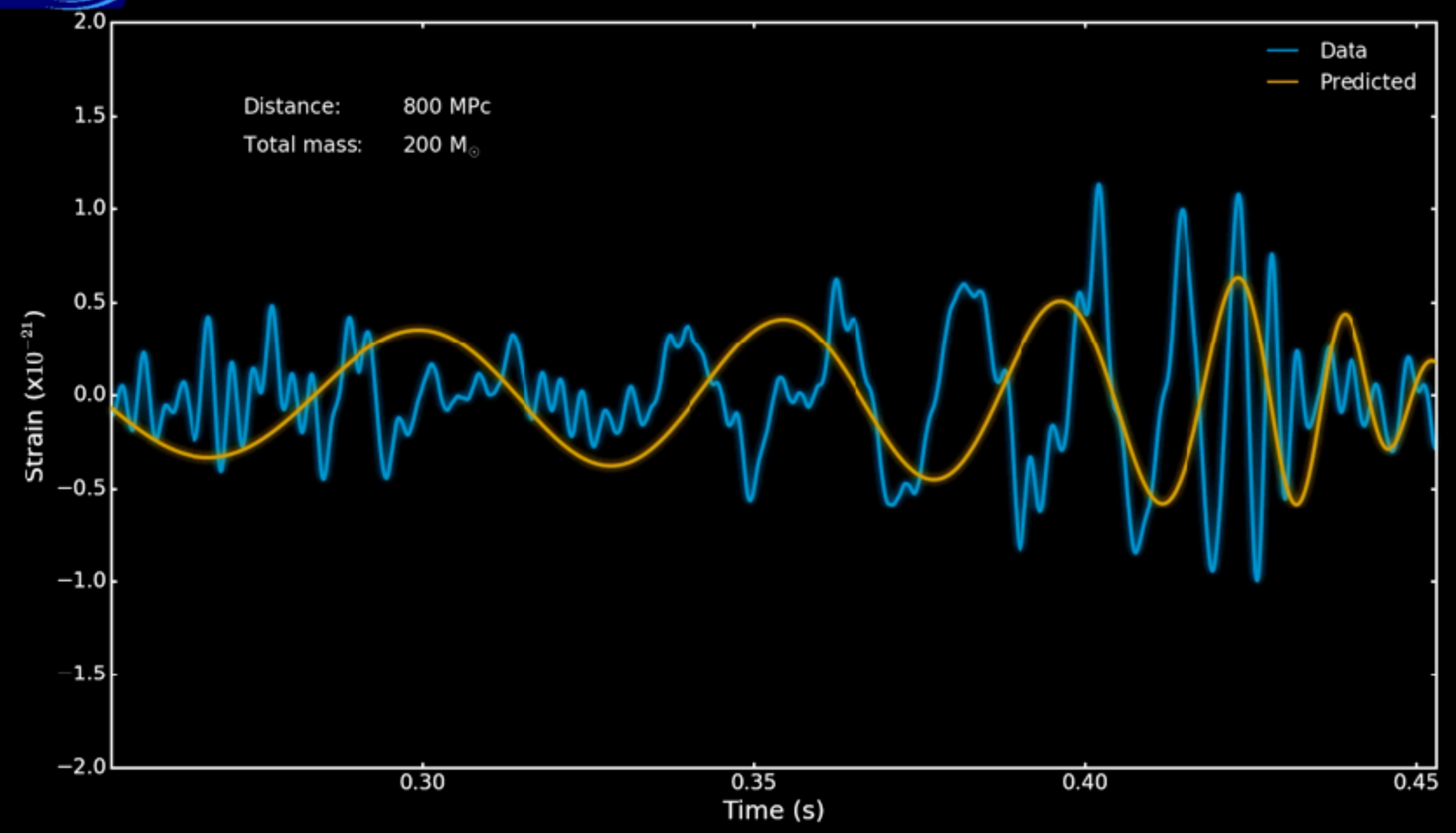


Small (cosmological) scales

Neglect expansion

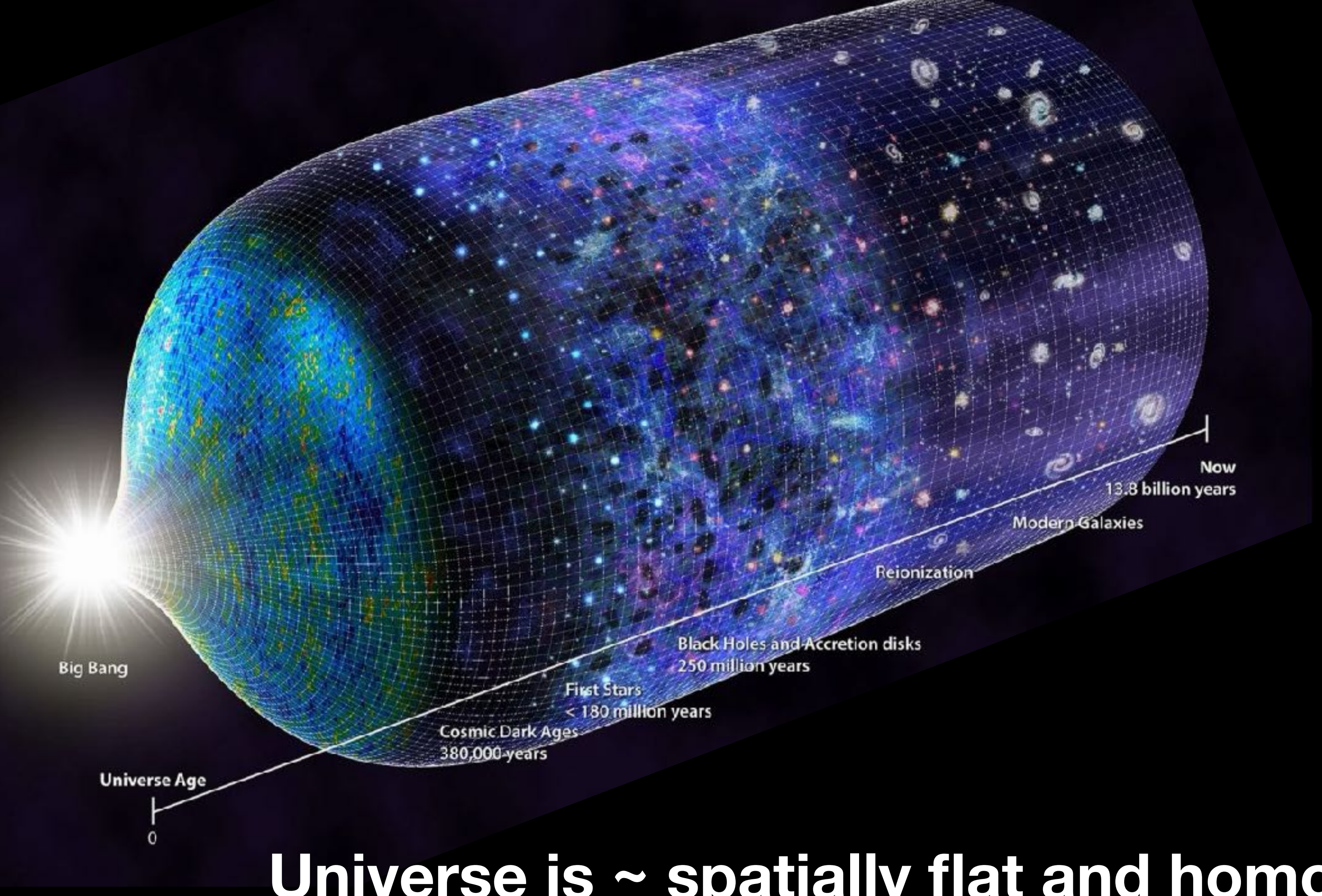
Short (cosmological) timescales





Data & Best-fit Waveform: LIGO Open Science Center (losc.ligo.org); Prediction & Animation: C.North/M.Hannam (Cardiff University)

Cosmology for numerical relativists



$$ds^2 = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right)$$

Universe is ~ spatially flat and homogeneous

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

Hubble parameter = rate of expansion $H(t) \equiv \frac{\dot{a}}{a}$

In ADM language: $K = -3H$

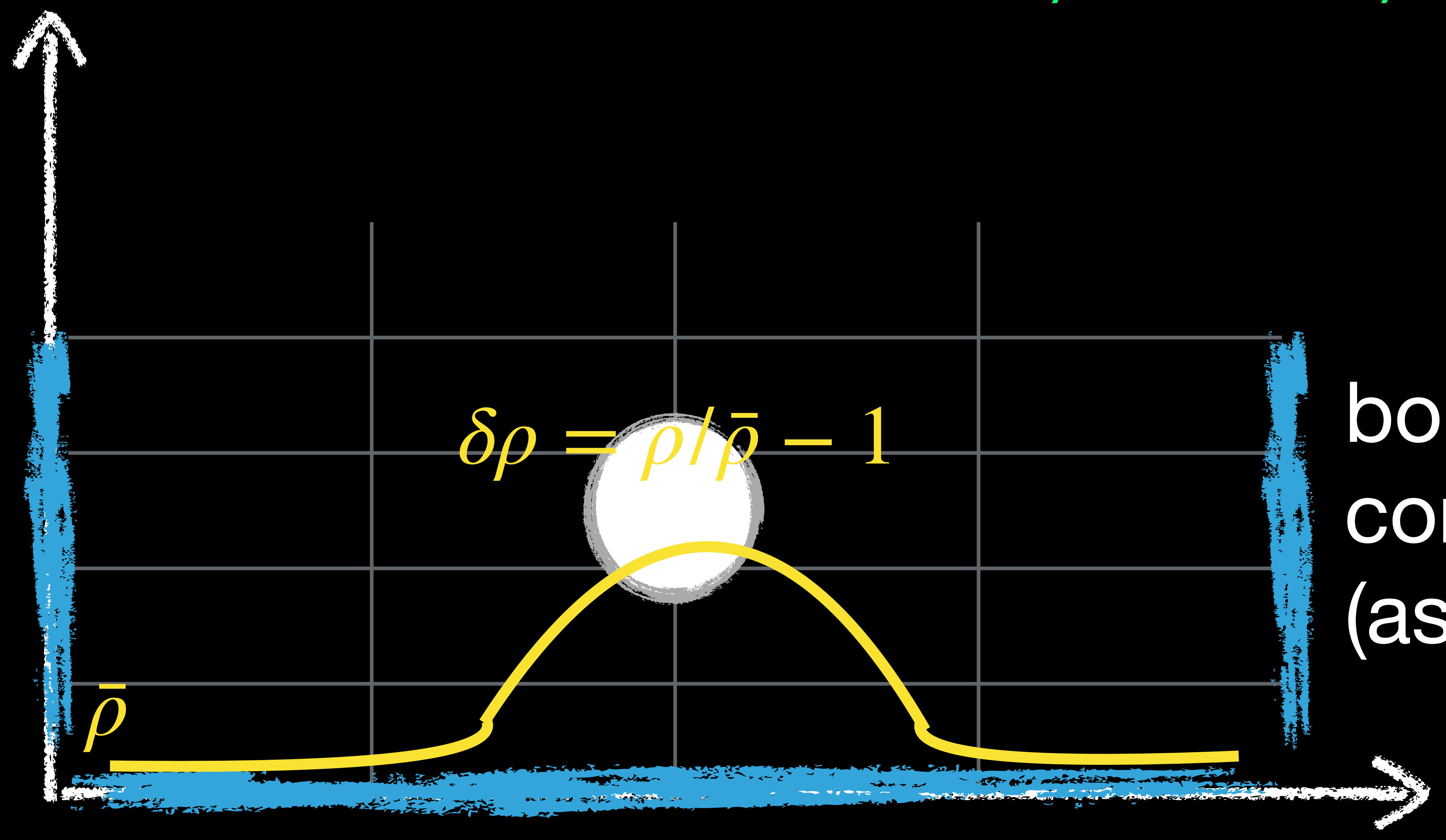
FLRW: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$ $\qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

Primordial black hole formation

Fill using Einstein equation

$$\partial_t g_{\mu\nu} \neq \partial_t^2 g_{\mu\nu} \neq 0$$

“time”

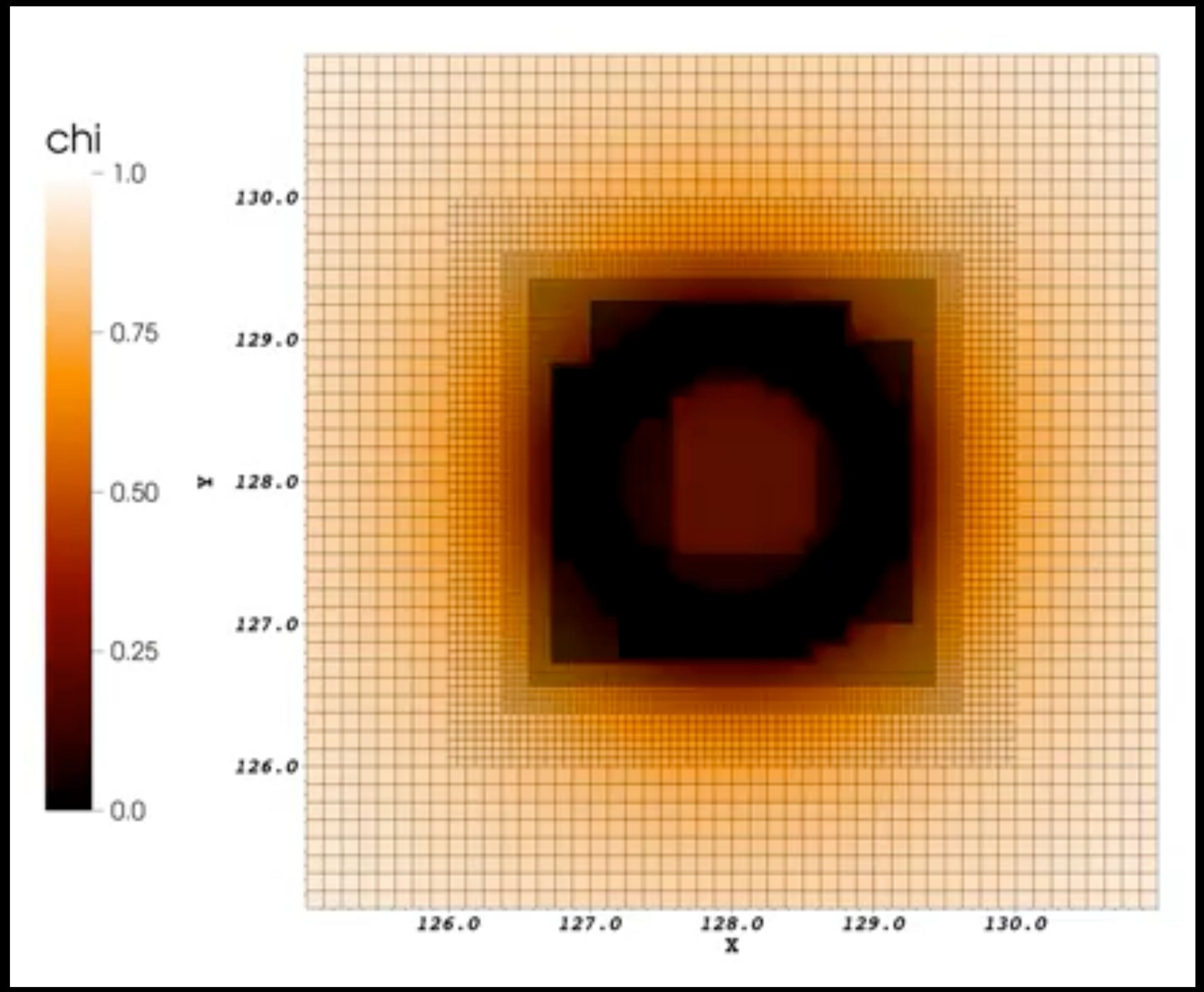


boundary conditions (asymptotically flrw)

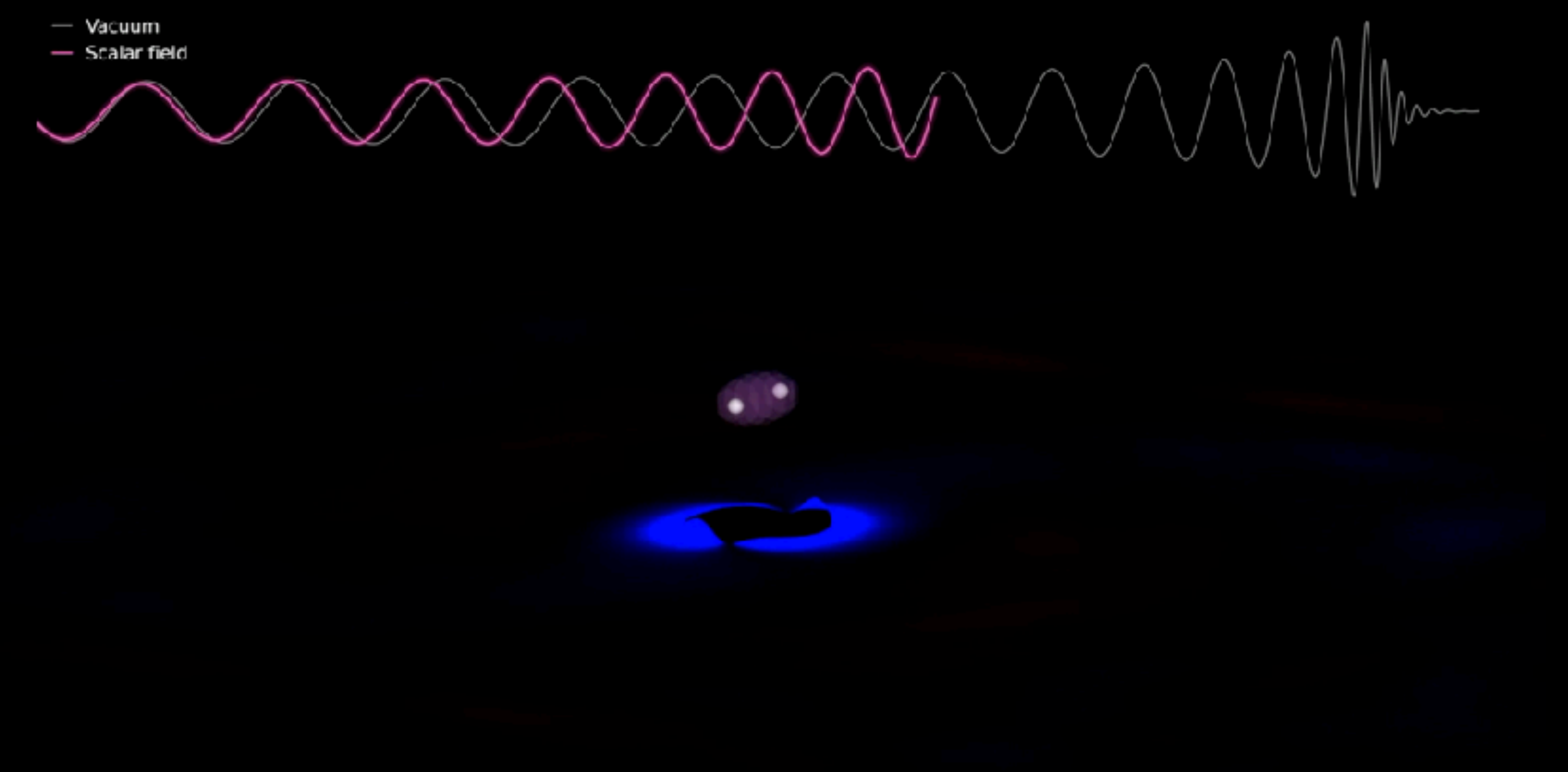
“space”

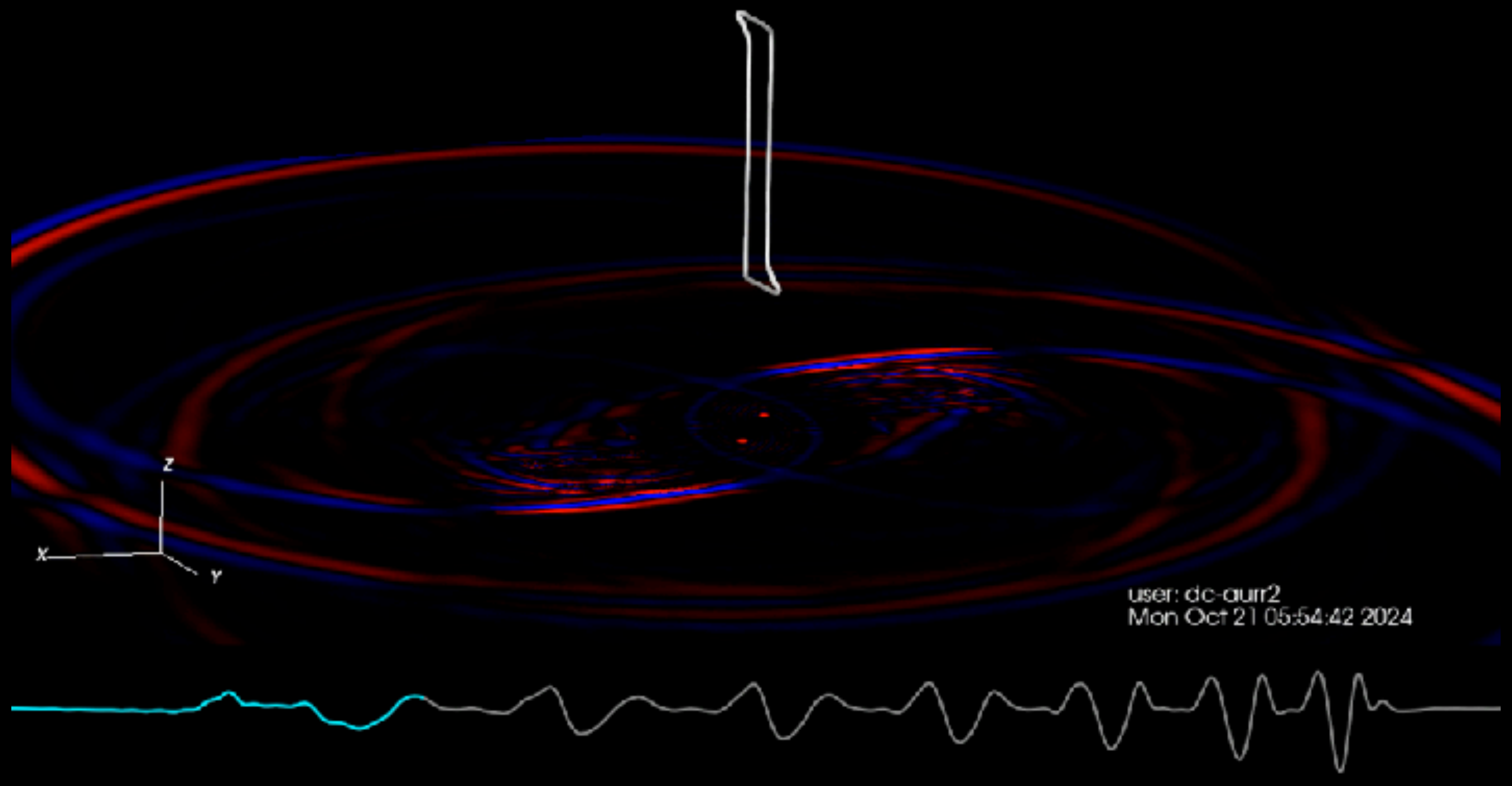
initial data (overdensity)

Numerical Relativity with AMR



Figueras et al.





Dark matter around BBHs   Roy et al., 2025

Cosmic strings with field theory + GR   (In preparation)

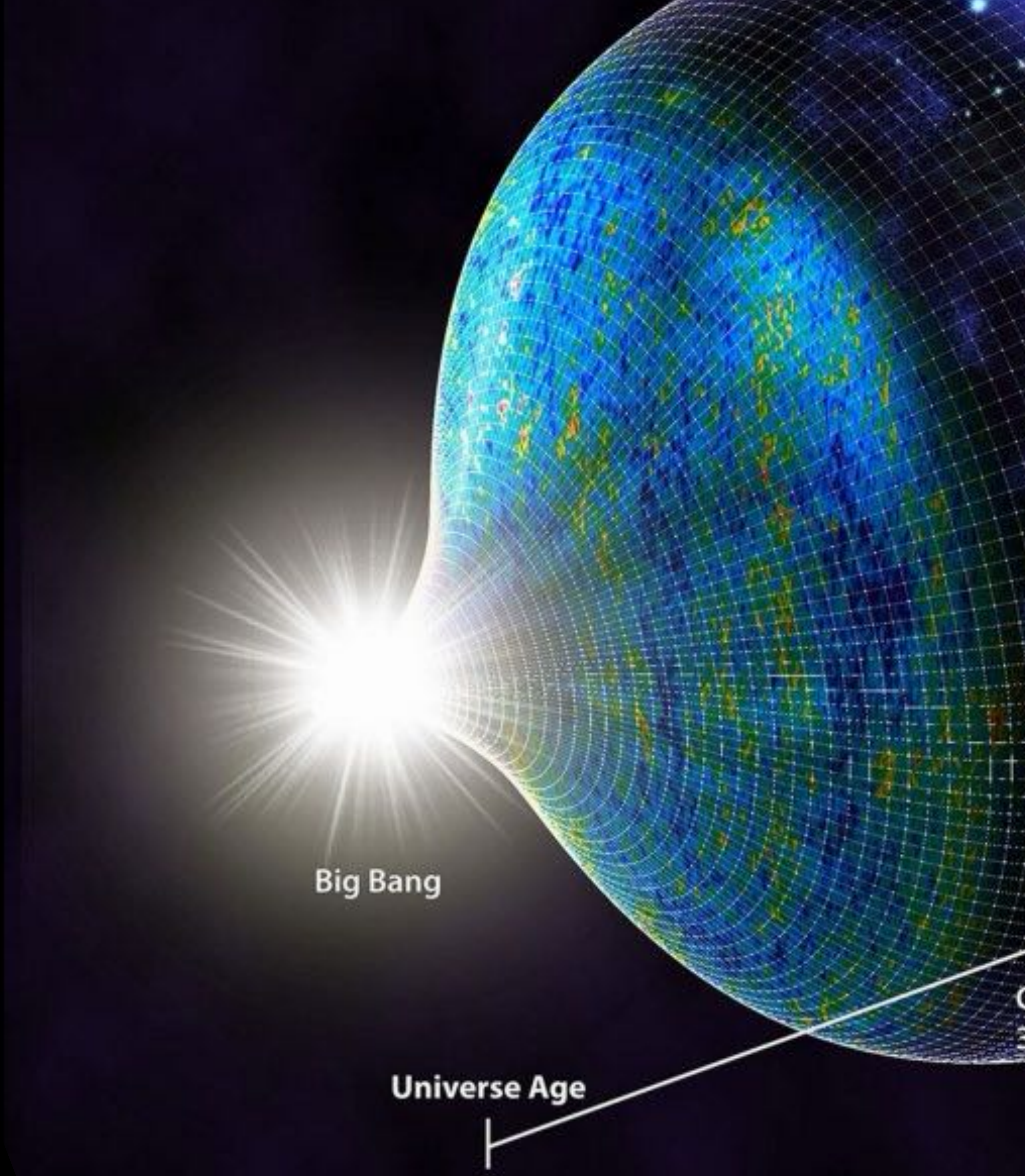
Inflation

Reheating

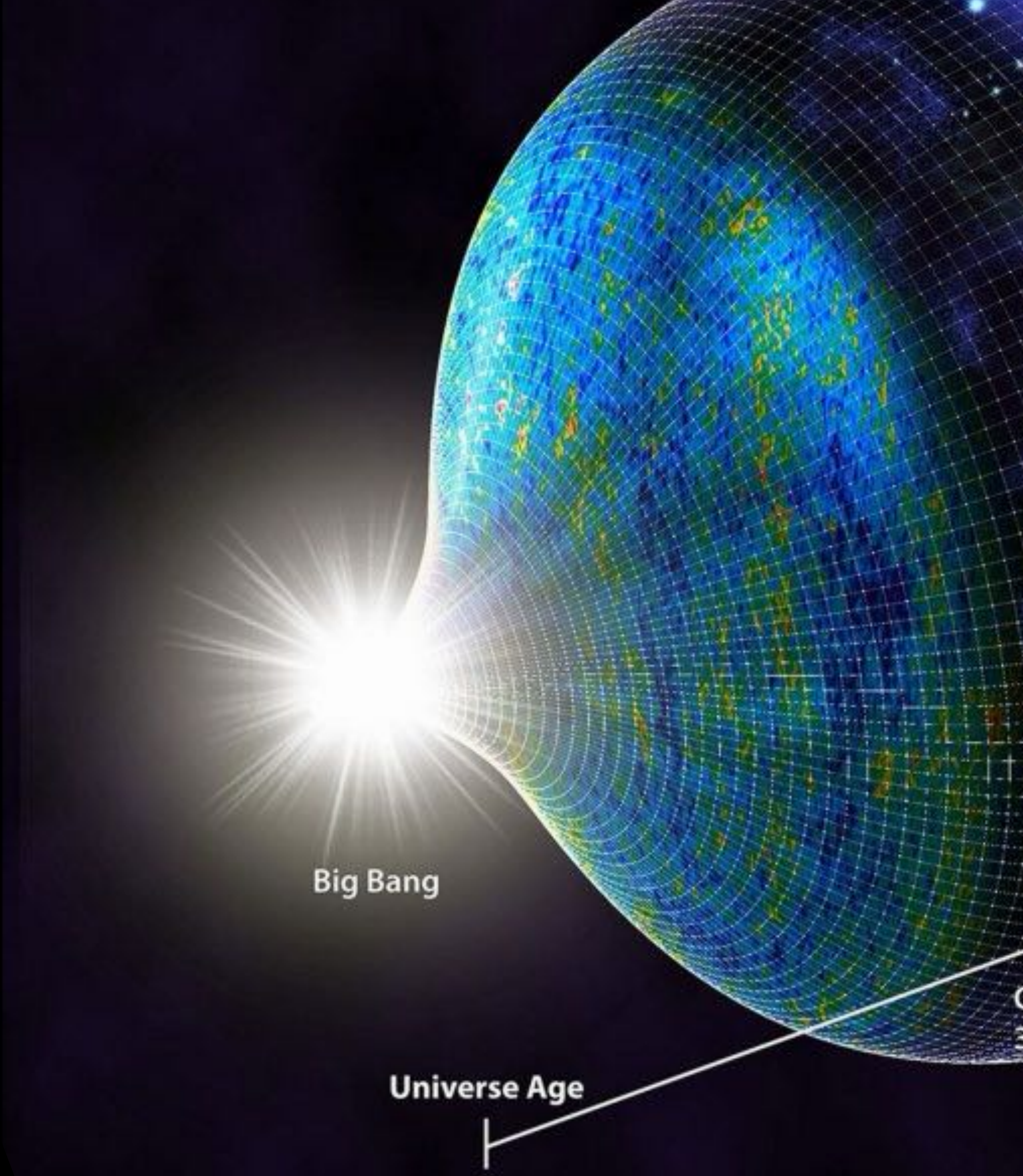
Primordial black holes

For more see:
Cosmology using numerical relativity [2409.01939]

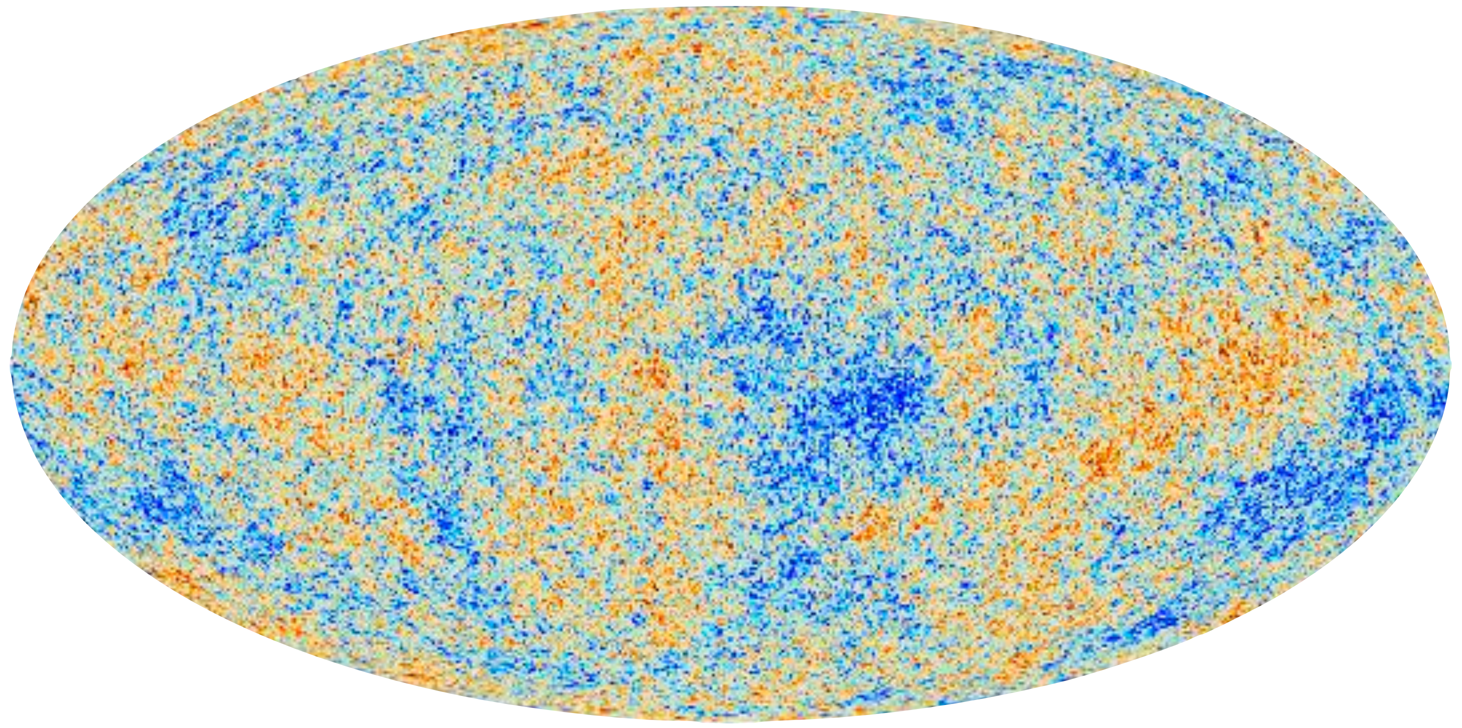
Cosmological singularities,
bouncing models,
bubble collisions,
phase transitions,
topological defects…

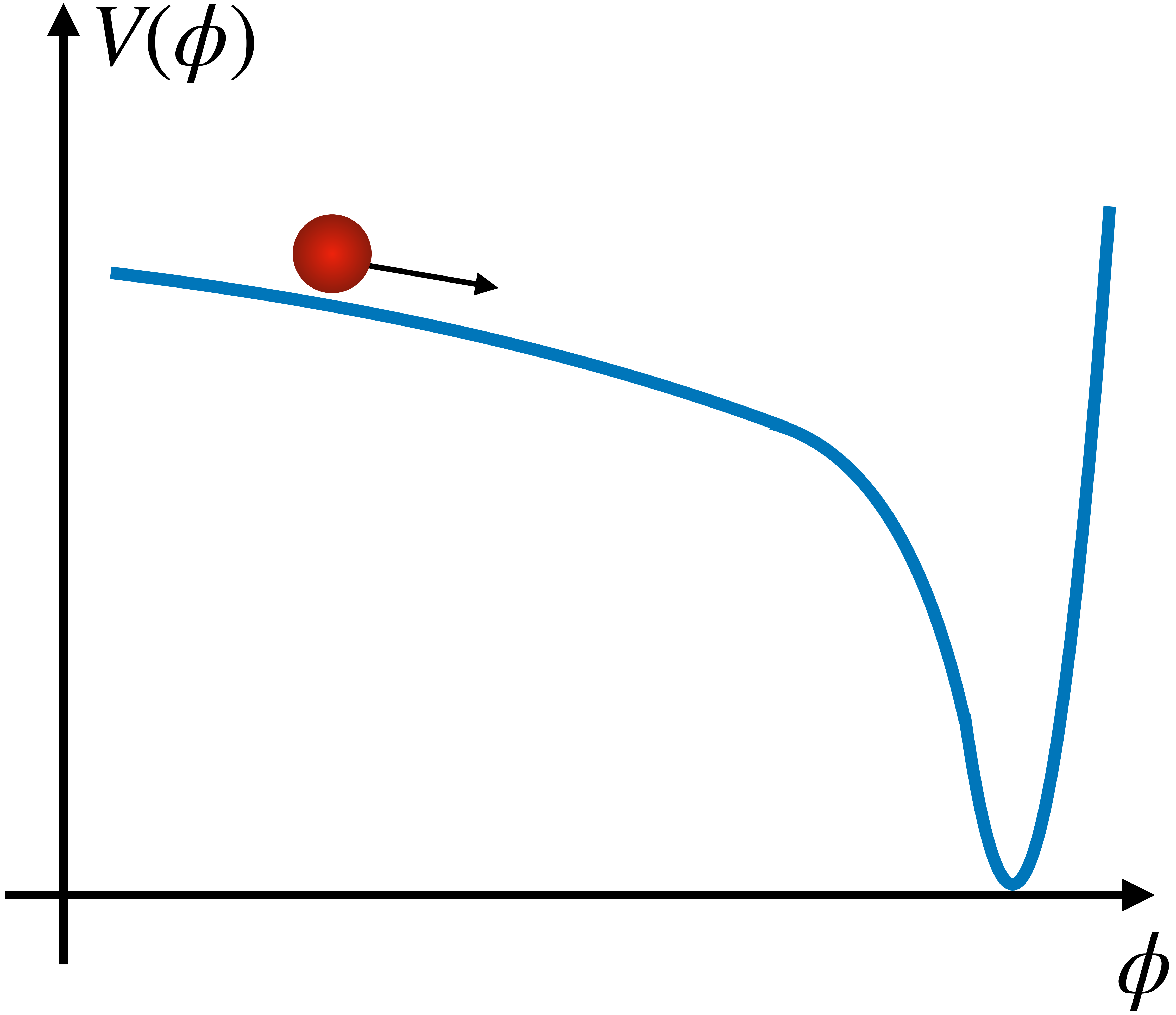


Inflation



Inflation



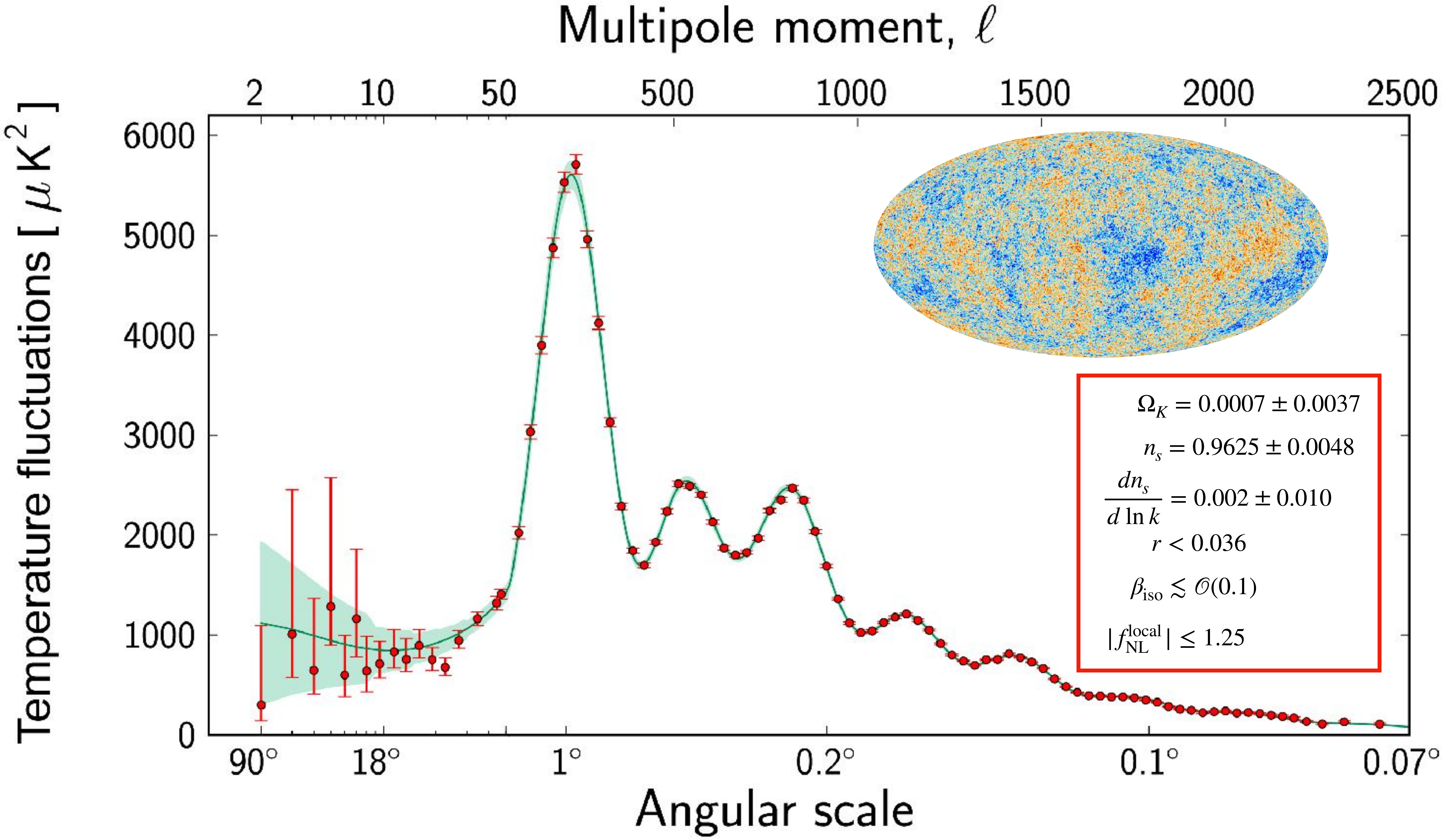


$$a(t) \sim e^{Ht}$$

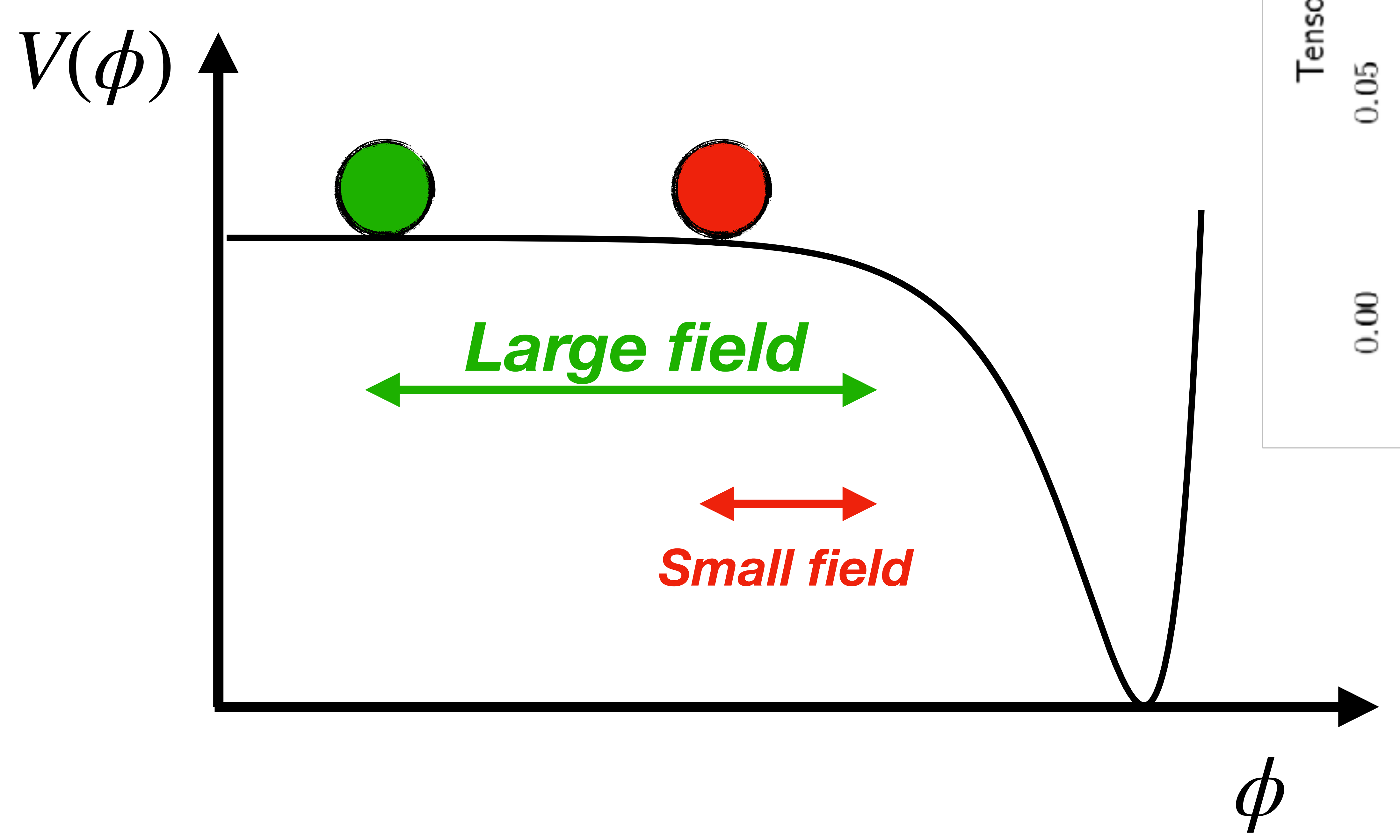
Explains why the universe is so homogeneous and flat

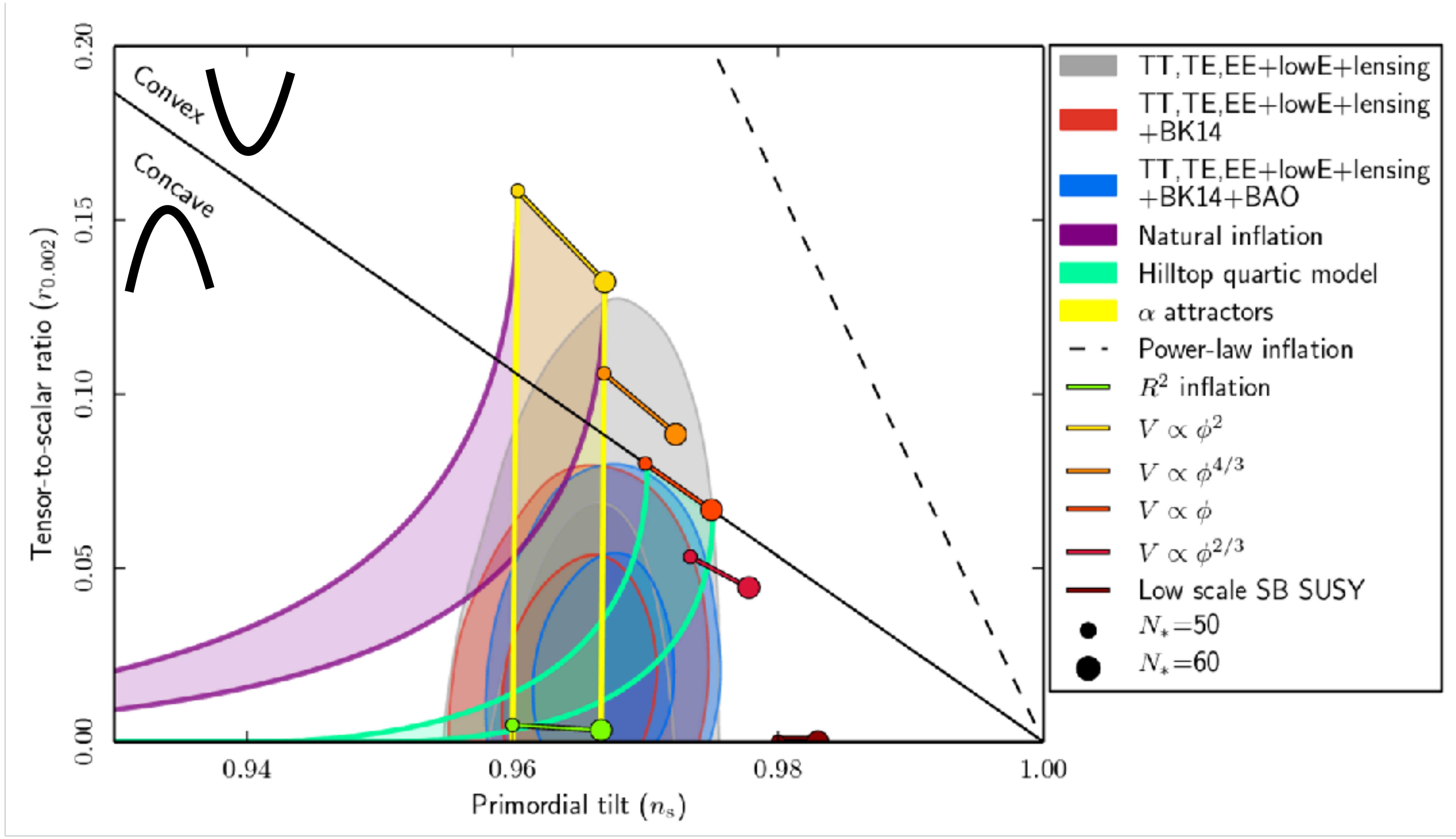
Mechanism for spectrum of scale invariant fluctuations

Inflation

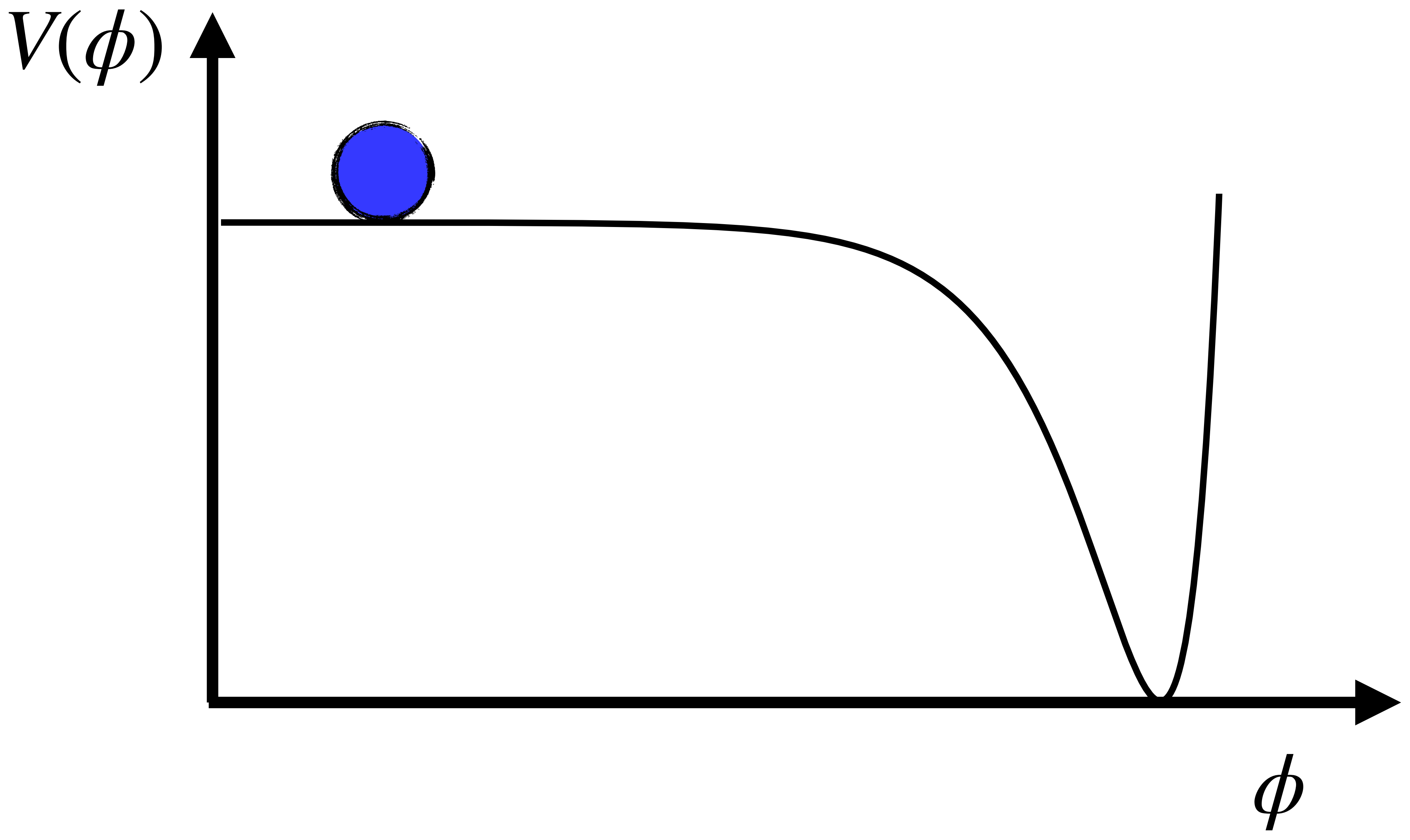

Multipole moment, ℓ
2
10
50
500
1000
1500
2000
2500
Temperature fluctuations [μK^2]
6000
5000
4000
3000
2000
1000
0
90°
18°
1°
0.2°
0.1°
0.07°
Angular scale
$\Omega_K = 0.0007 \pm 0.0037$
$n_s = 0.9625 \pm 0.0048$
$\frac{dn_s}{d \ln k} = 0.002 \pm 0.010$
$r < 0.036$
$\beta_{\rm iso} \lesssim \mathcal{O}(0.1)$
$|f_{\rm NL}^{\rm local}| \leq 1.25$

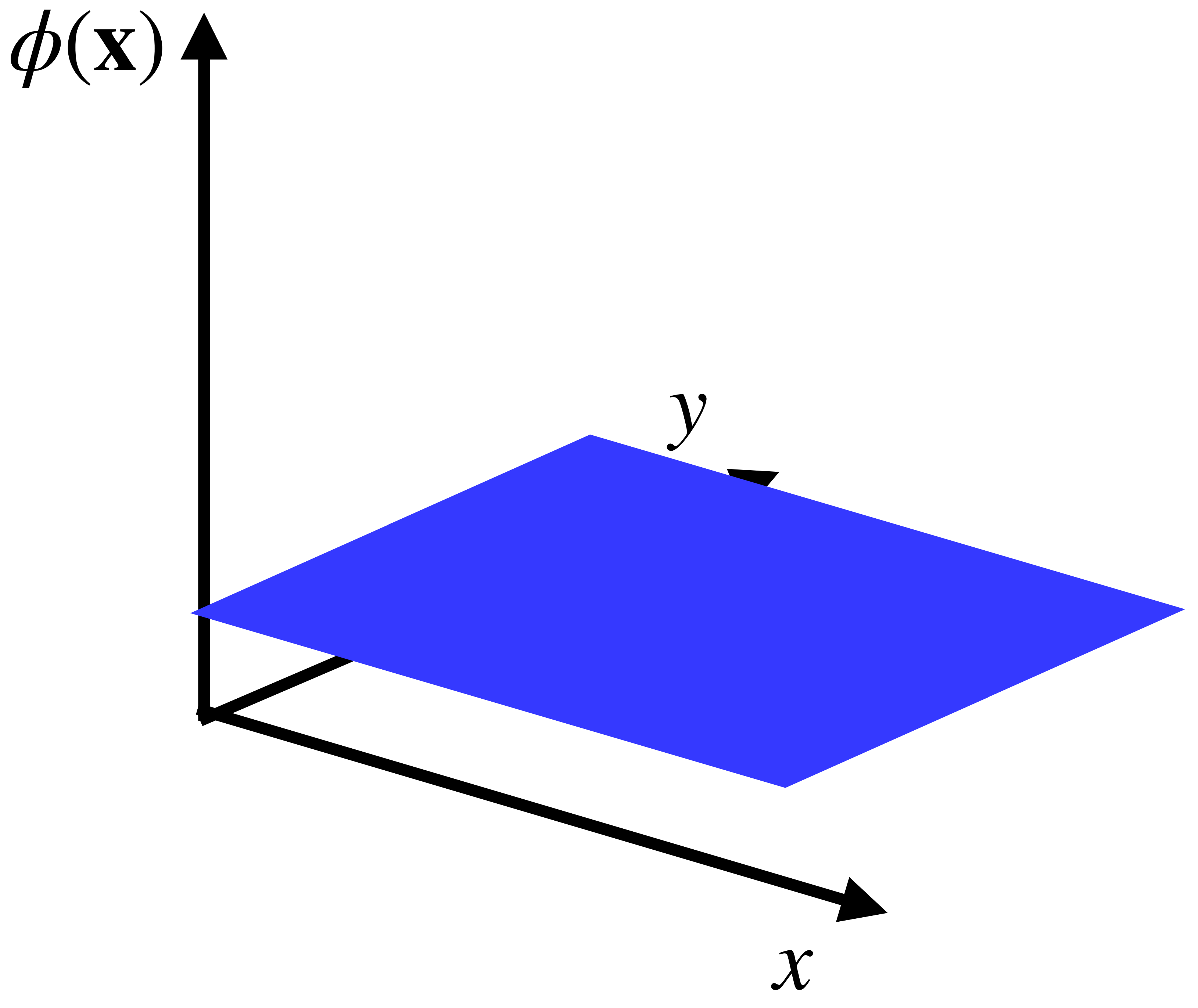
Space of models





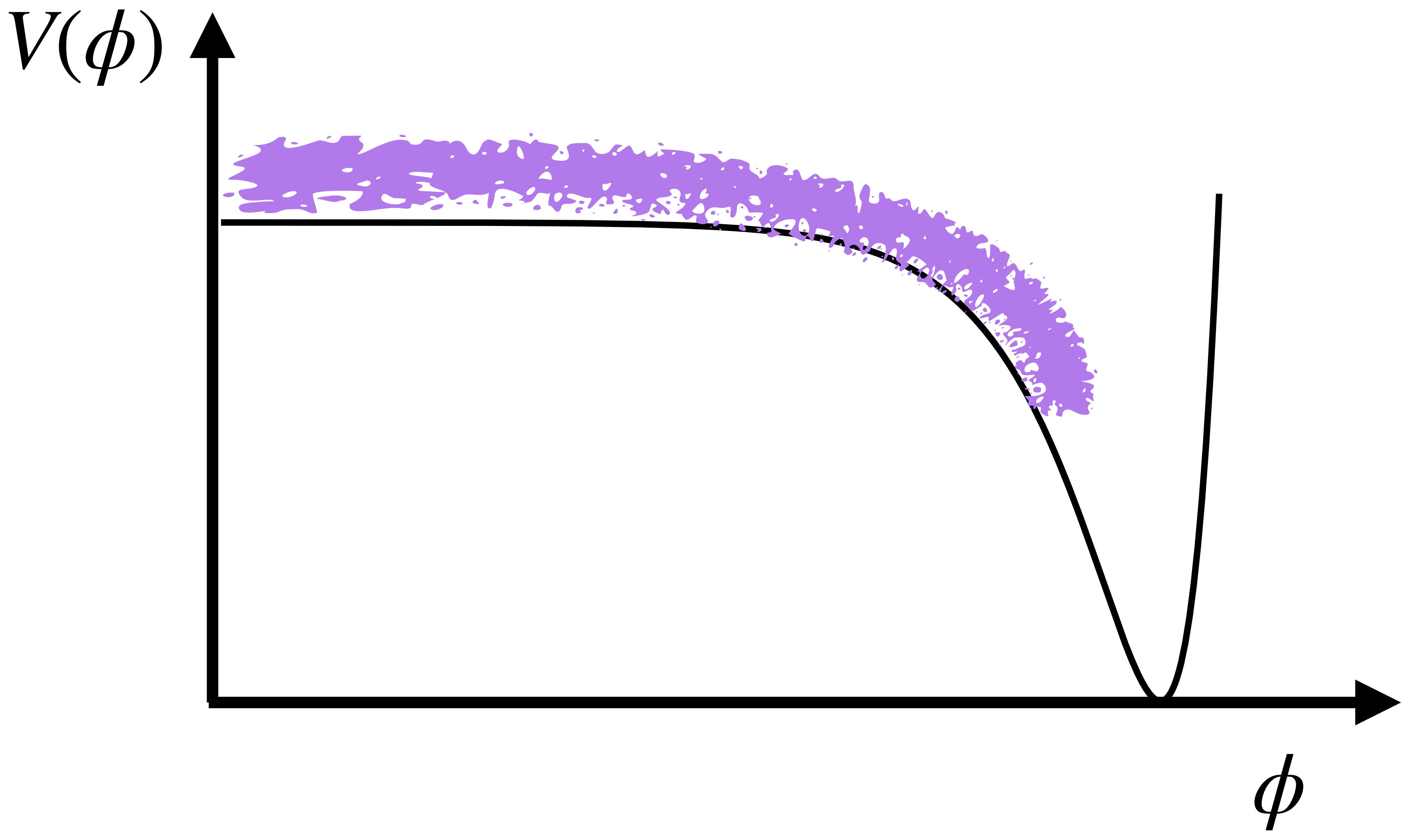
Space of initial conditions

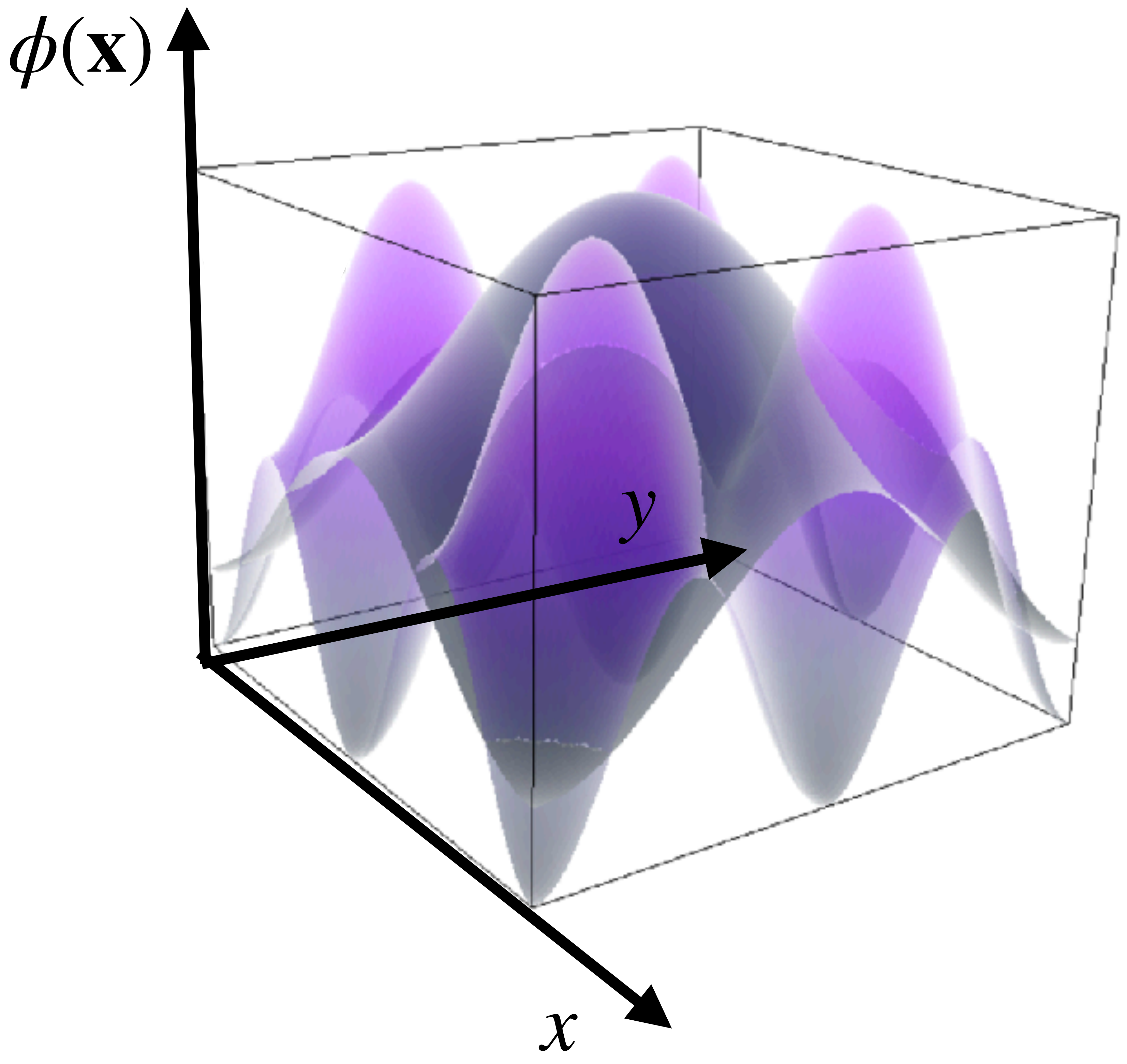




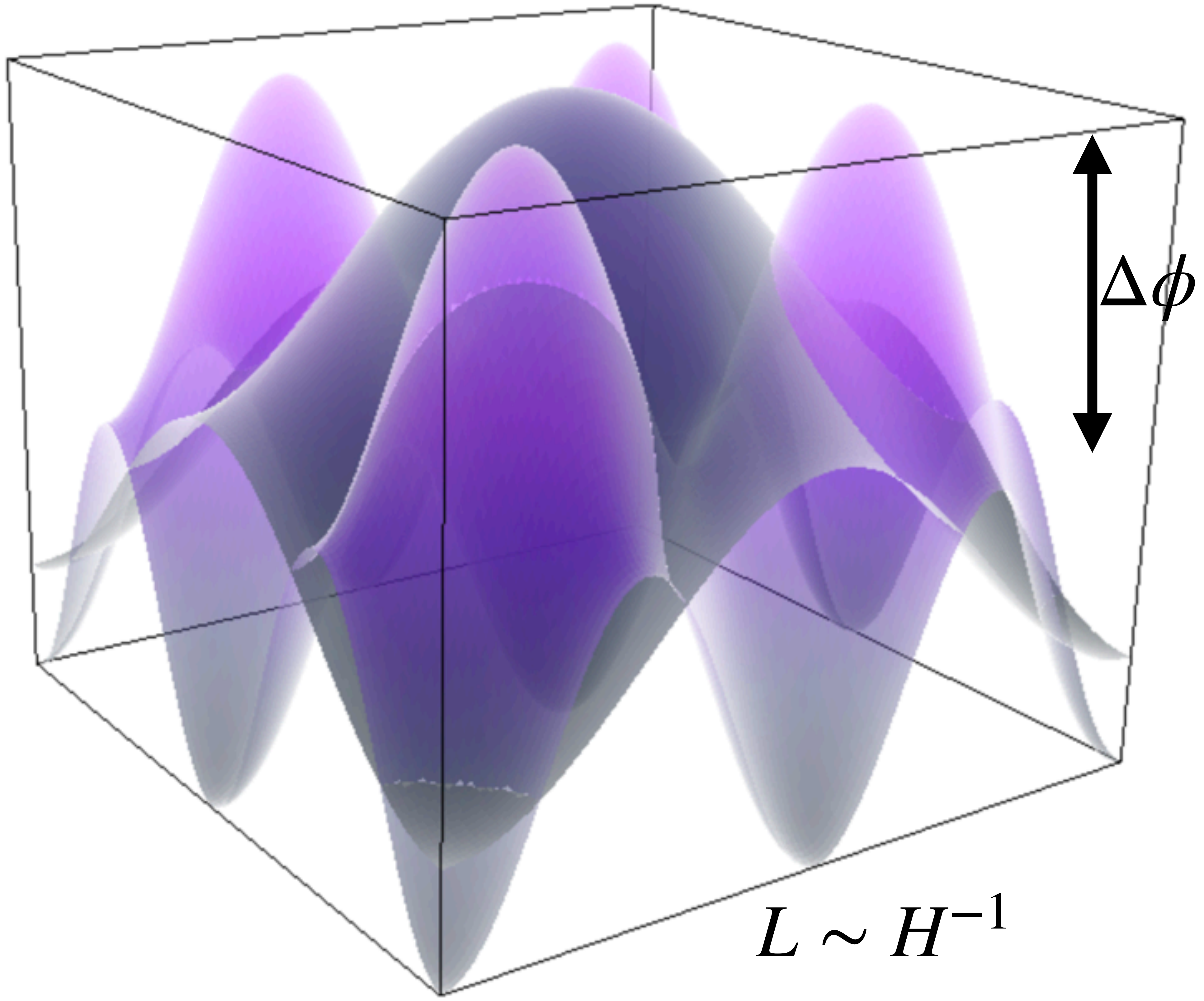
Space of initial conditions

Can inflation start?





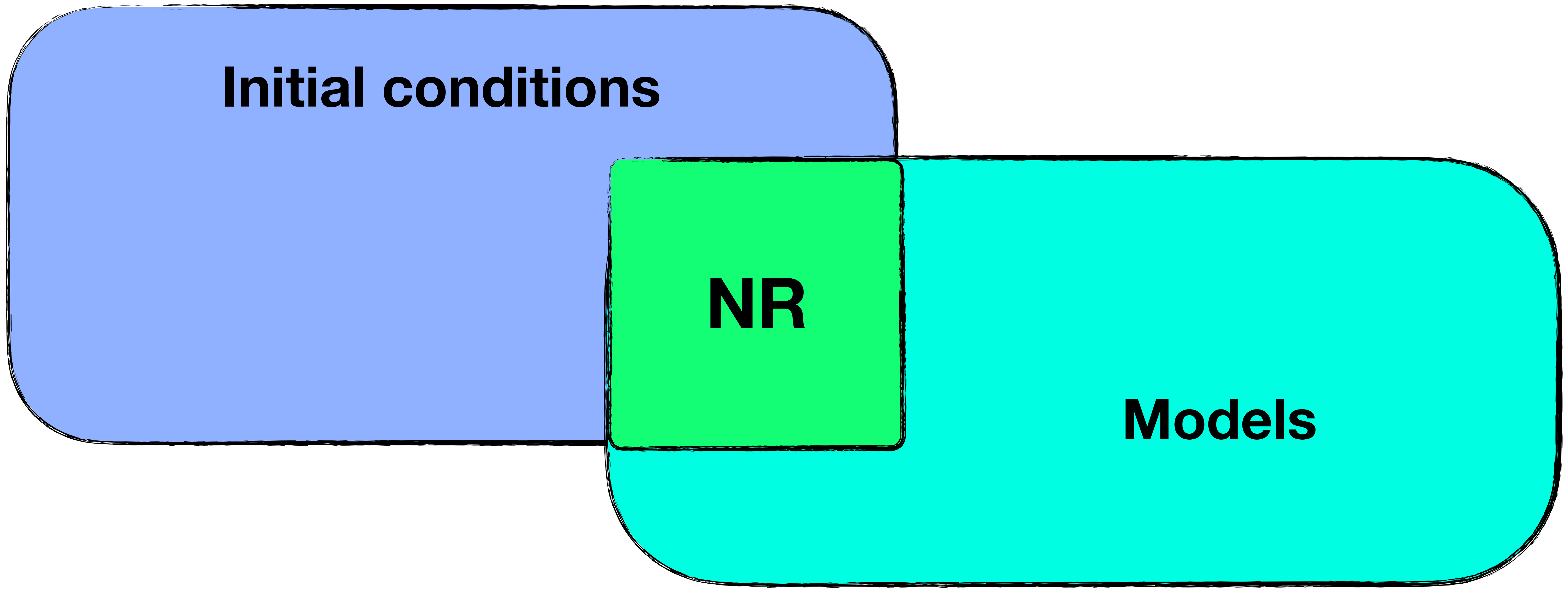
Space of initial conditions



Inflaton field: $\phi \quad \dot{\phi}$

Gravitational field: $\gamma_{ij} \quad \dot{\gamma}_{ij}$

Robustness of inflation



Can inflation start from inhomogeneous initial conditions?

What initial conditions/models of inflation work?

Early work

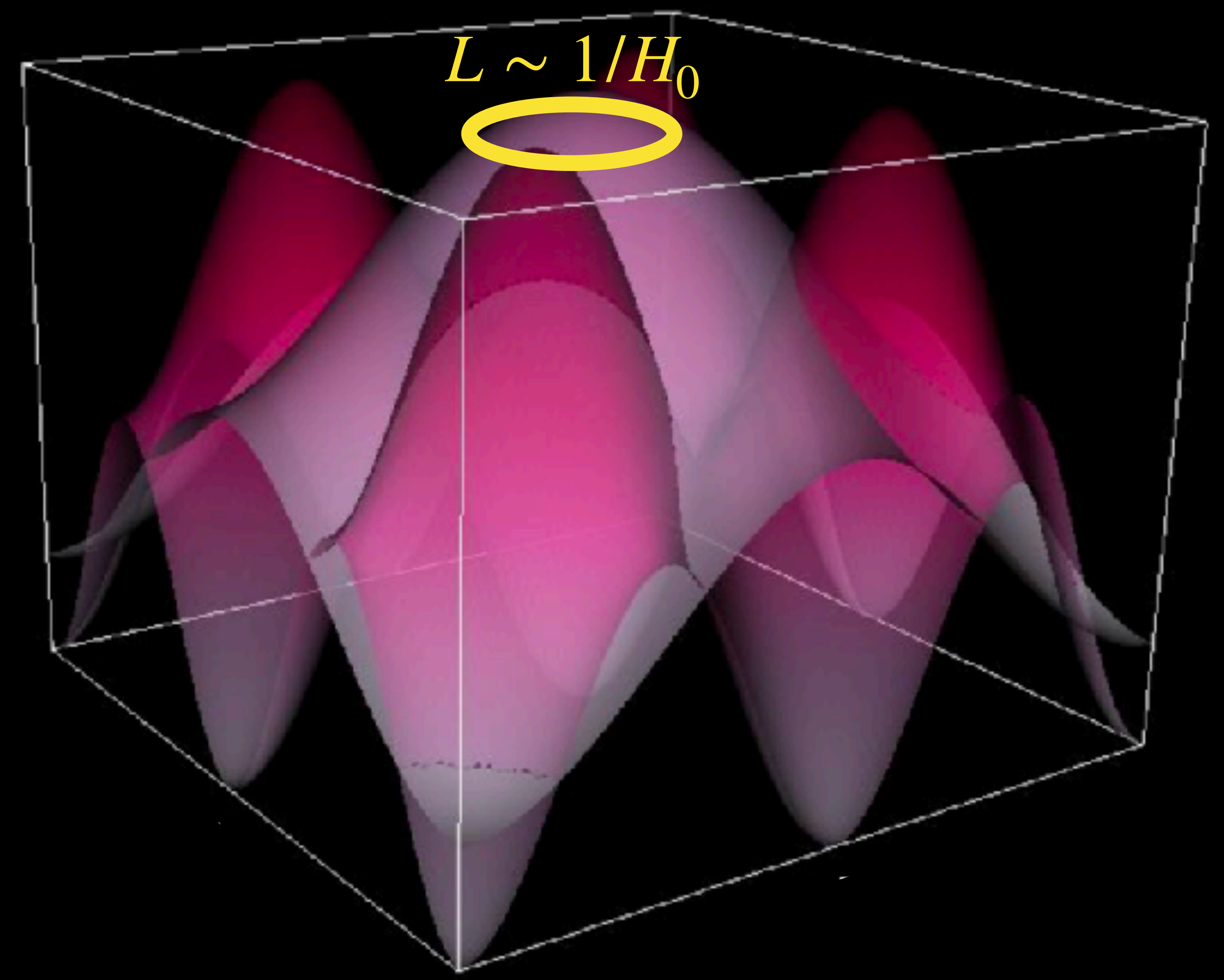
- 1+1 NR simulations: Kurki-Suonio et al (1987)
- 1+1 NR simulations: Goldwirth & Piran (1990)
- 1+1 NR simulations: Shinkai & Maeda (1993)
- First 3+1 NR simulations: Laguna et al (1993)
- Axisymmetry with GWs: Shibata et al (1994)

More recently

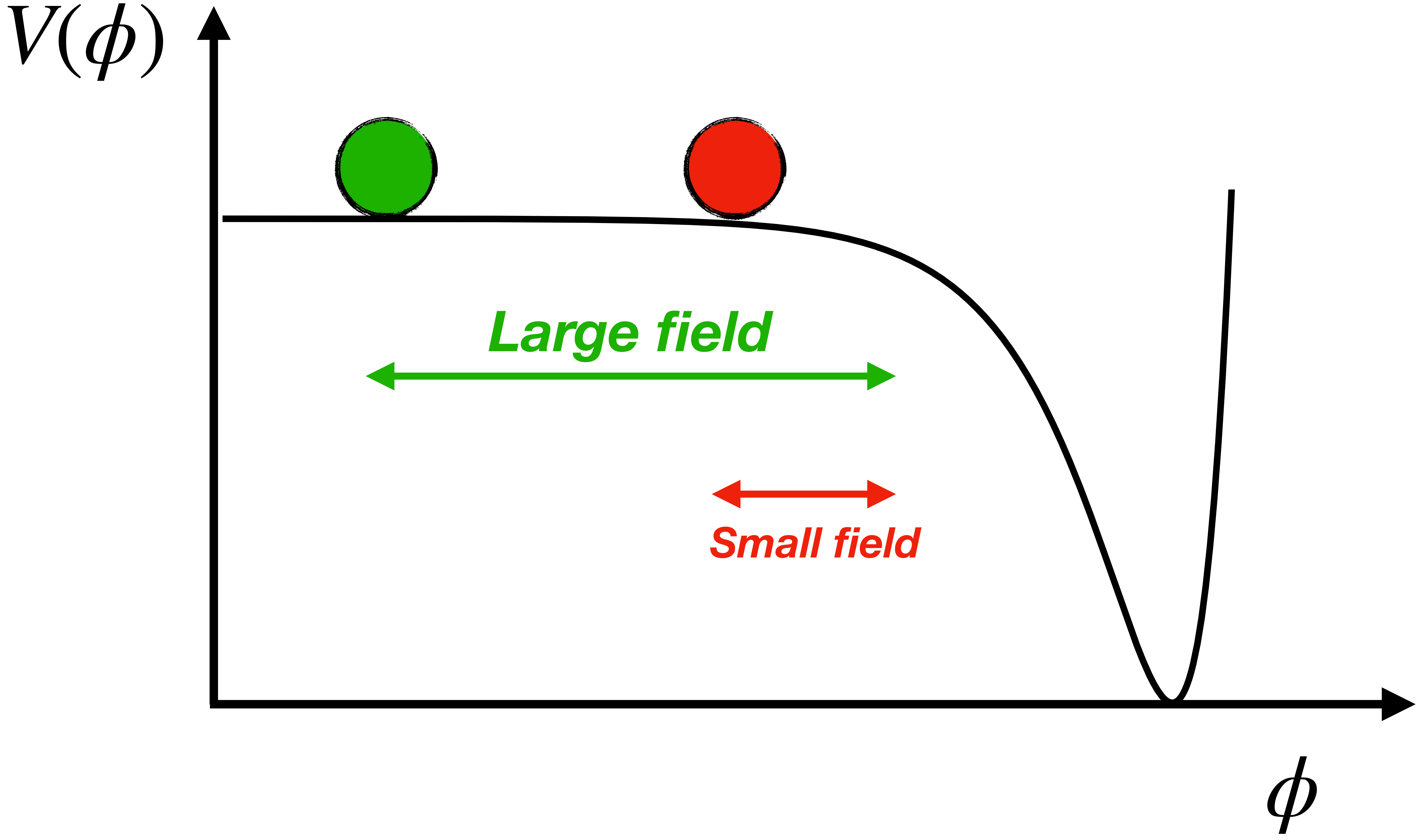
- East et al (2016)
- Clough et al (2017)
- Aurrekoetxea et al (2019)
- Joana & Clesse (2020)
- Corman & East (2022)
- Garfinkle et al (2023)
- Elley et al (2024)
- Joana (2024)

Conclusion:

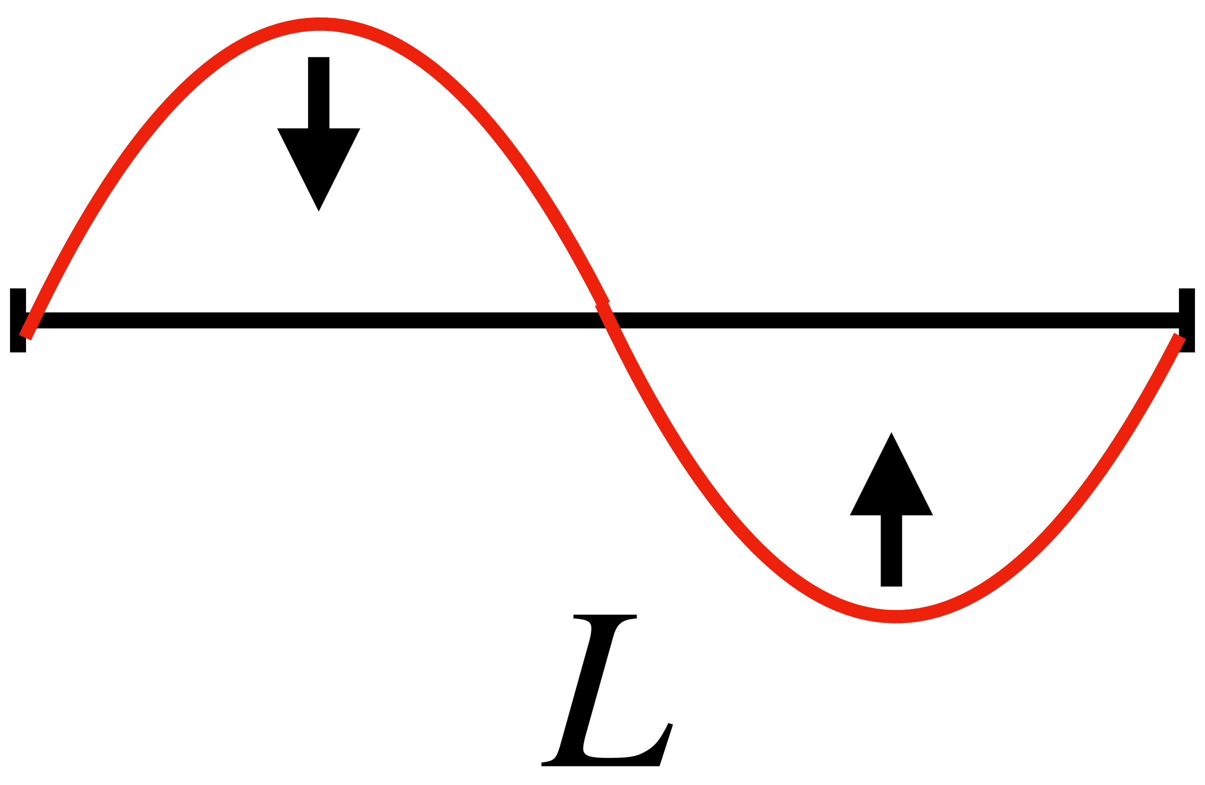
Inflation works as long as there exists a homogeneous Hubble-sized patch

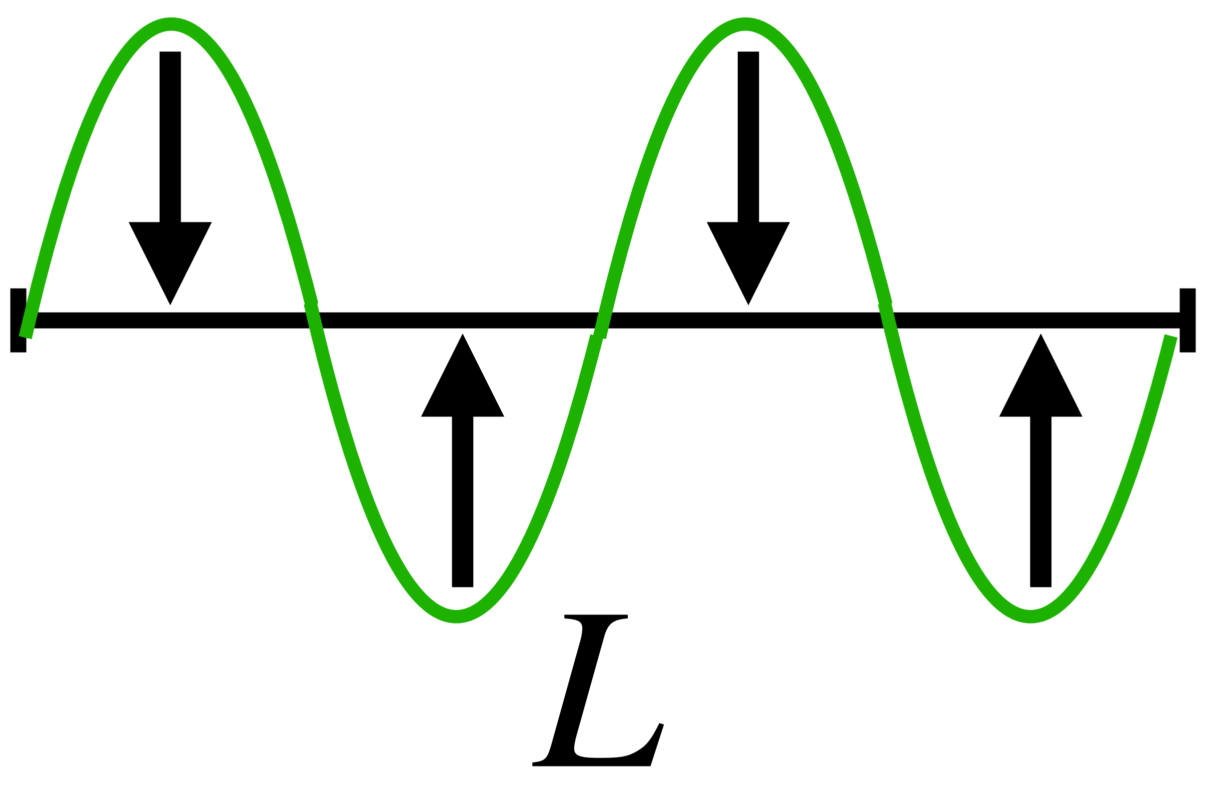


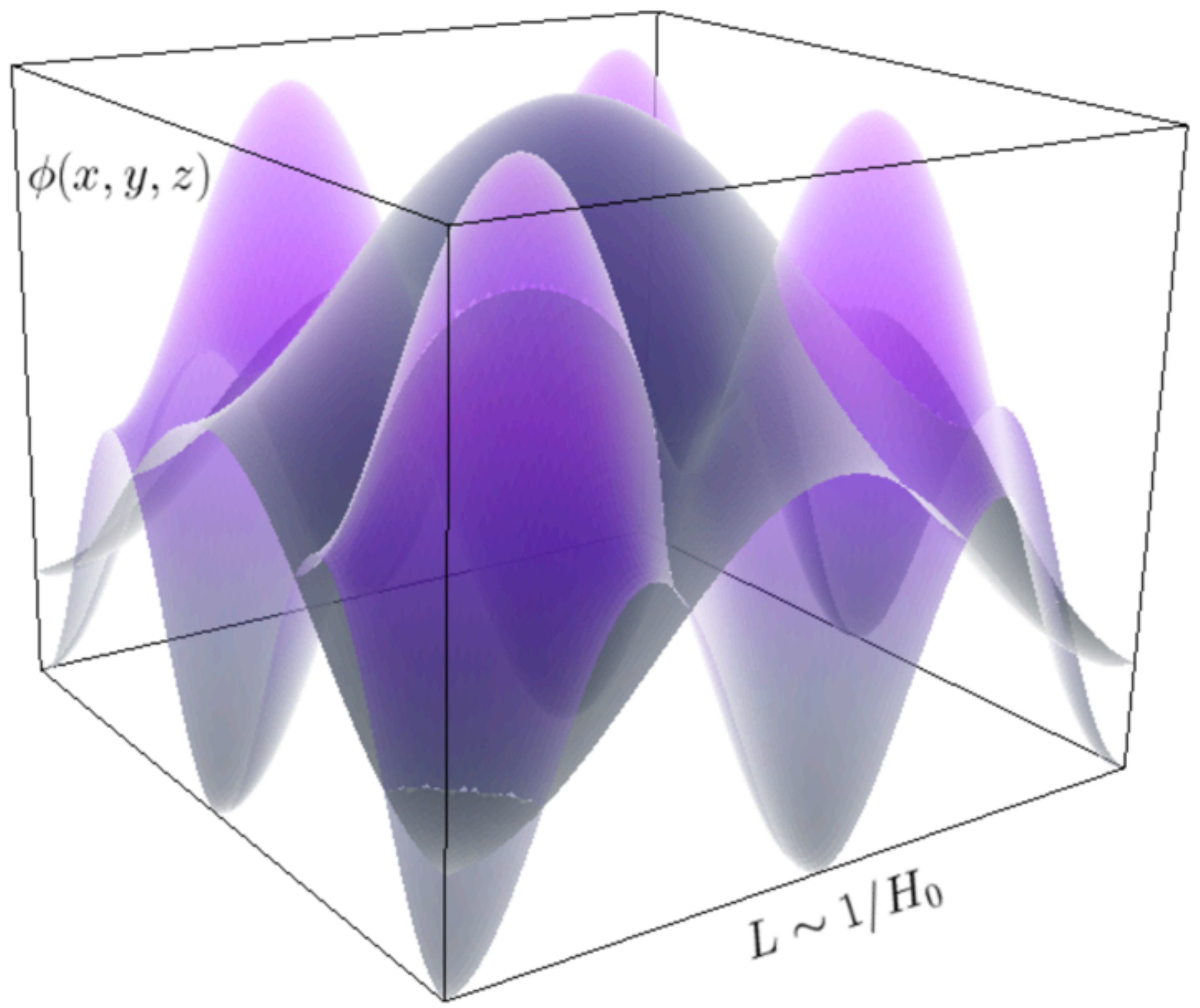
Lesson 1:
Small field more dangerous



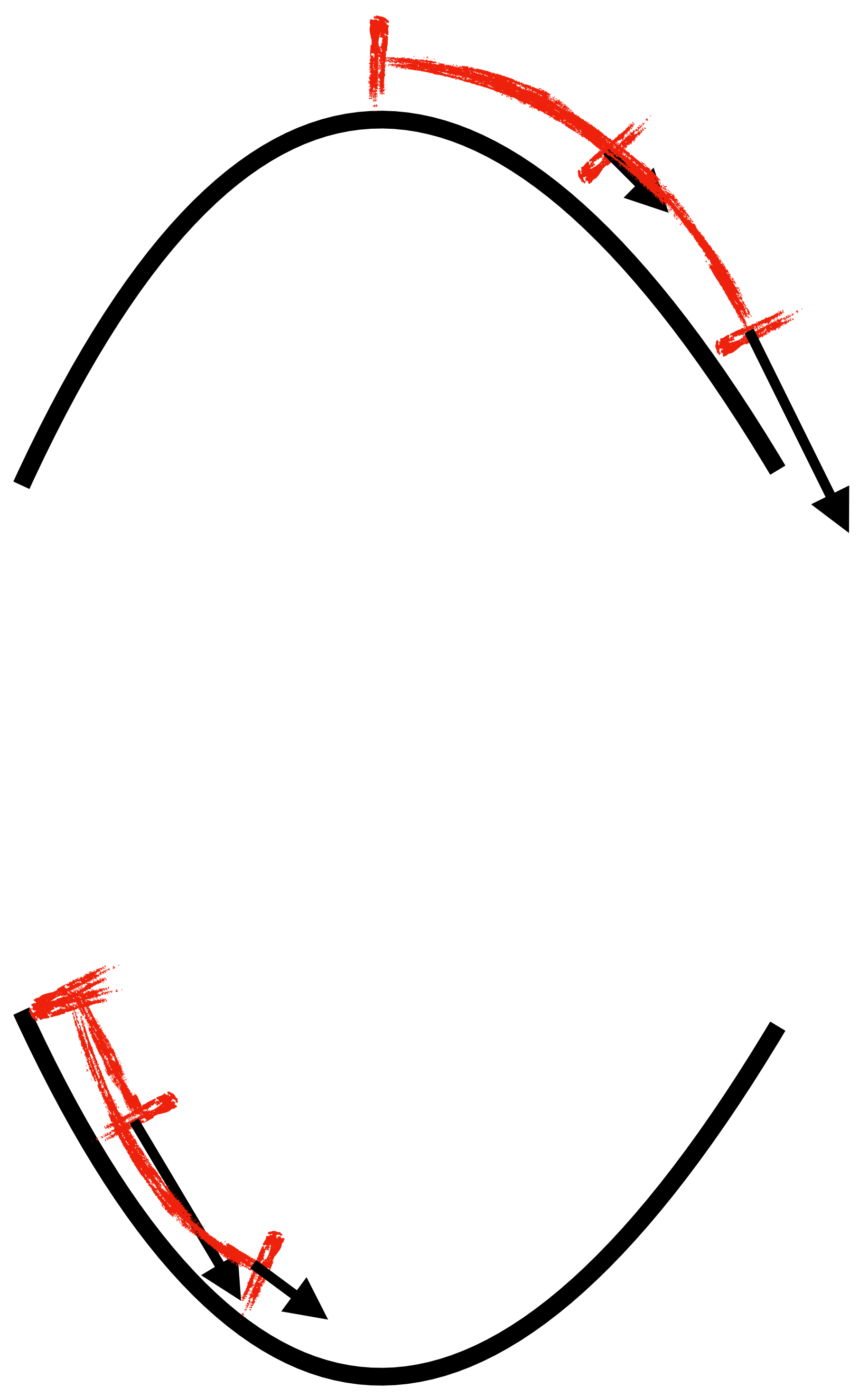
Lesson 2:
Long wavelength more dangerous



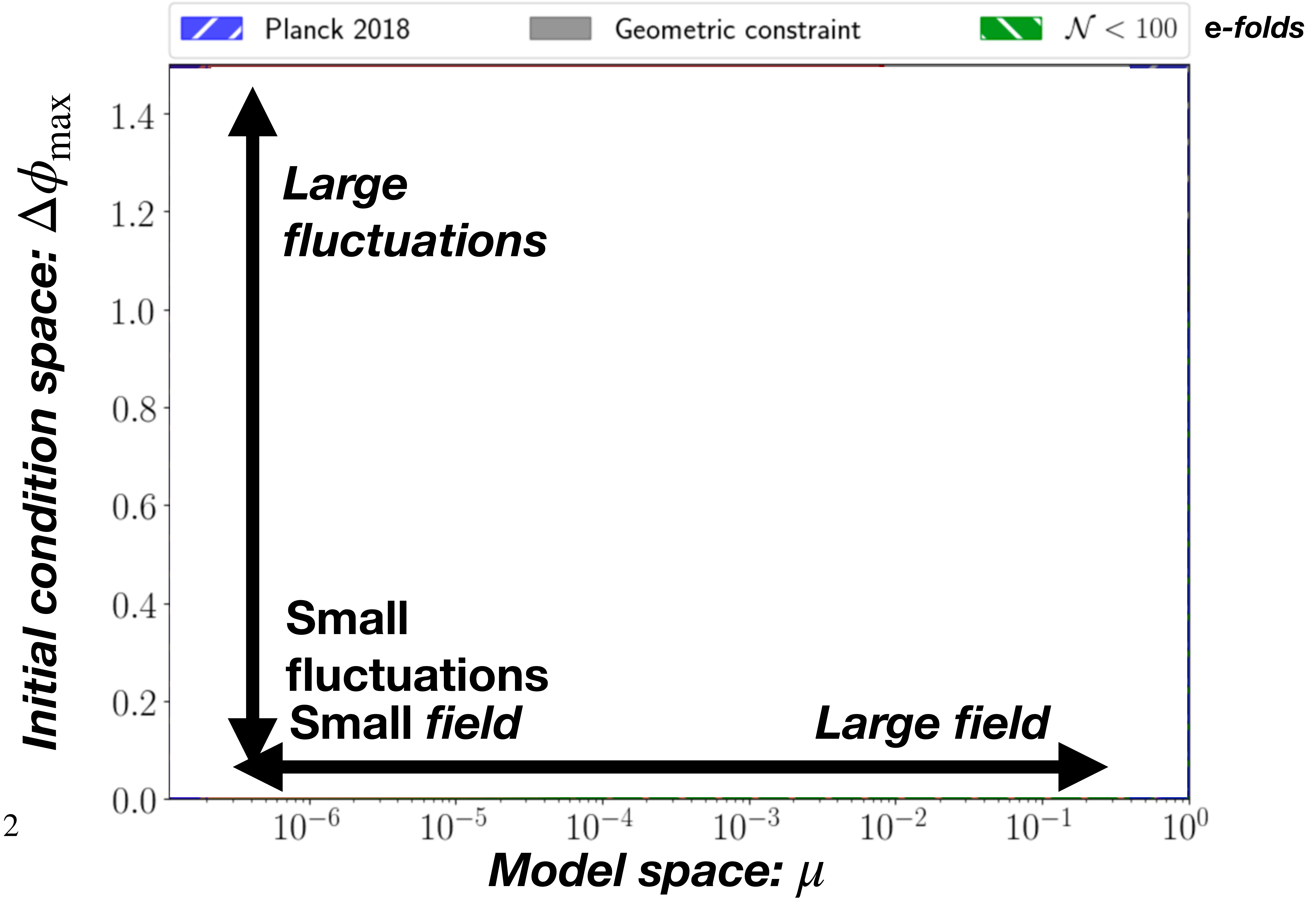




Lesson 3:
Concave models more dangerous

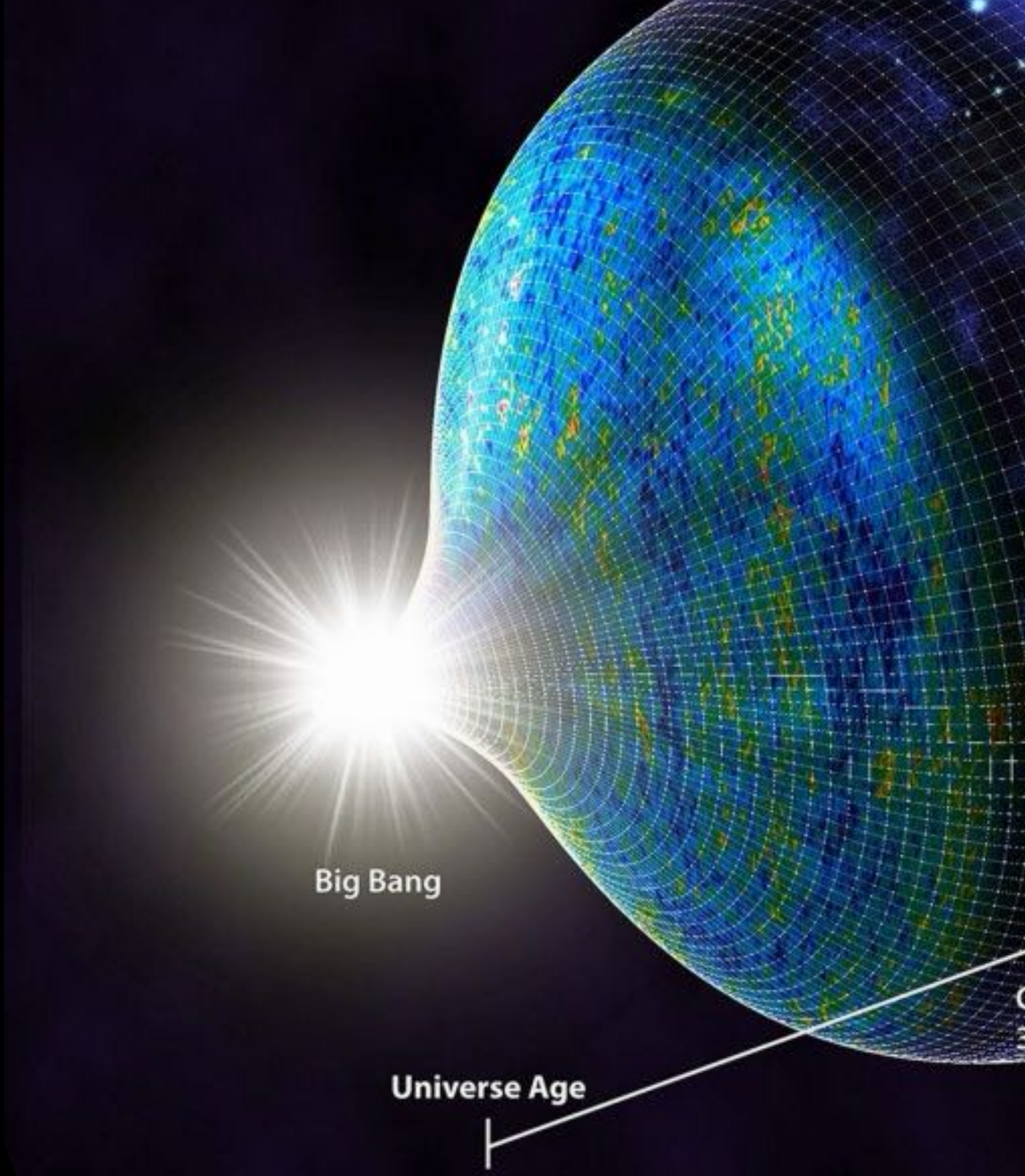


NR + Theory + Observations

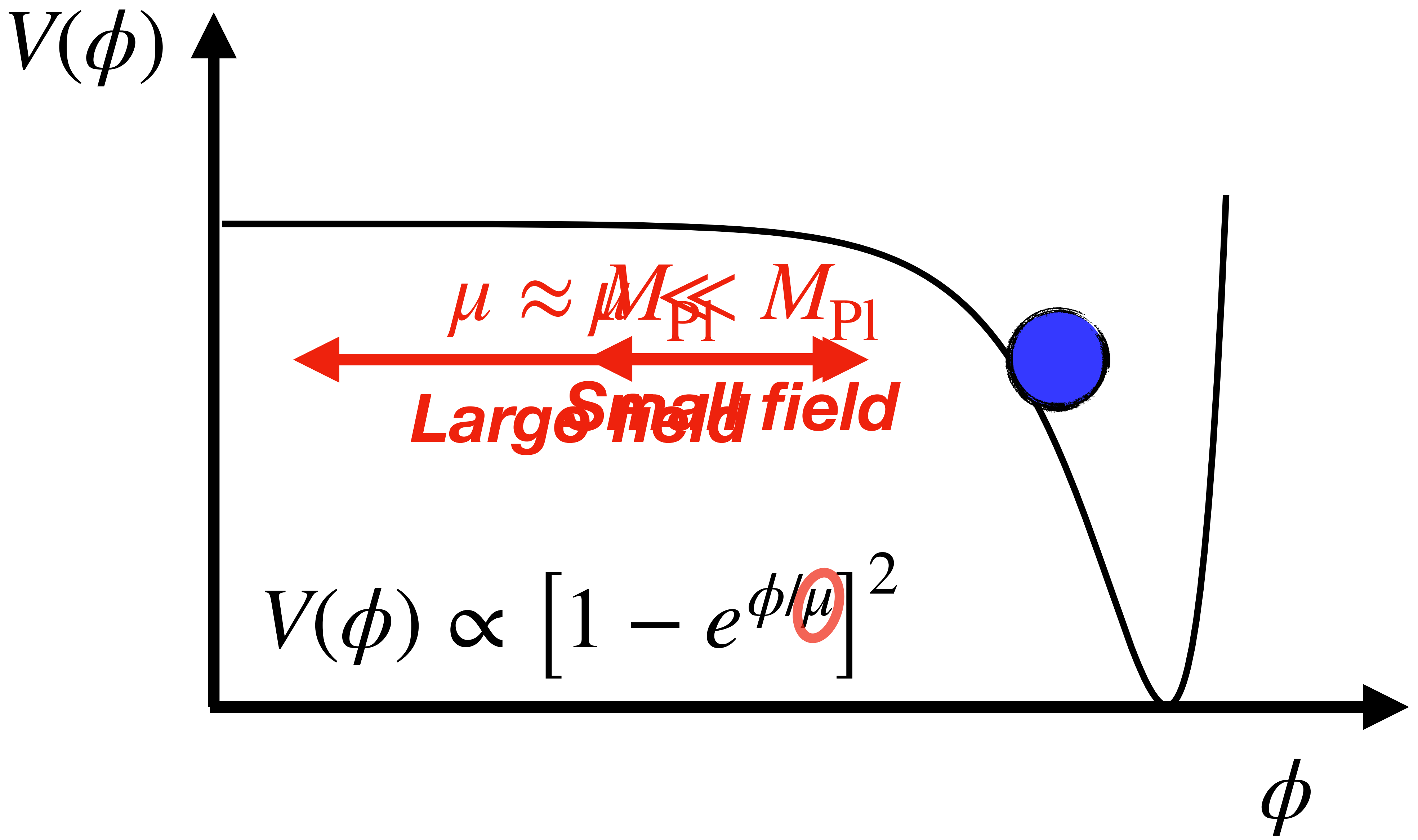


$$V(\phi) \propto \left[1 - e^{\phi/\mu}\right]^2$$

Reheating



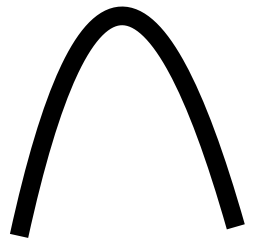
Inflation has to end… **Reheating**



$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$$

Resonances:* $\delta\phi$ *grows

- *Parametric*: if $\bar{\phi}(t)$ periodic
- *Tachyonic*: if $\bar{\phi}(t)$ probes $V'' < 0$



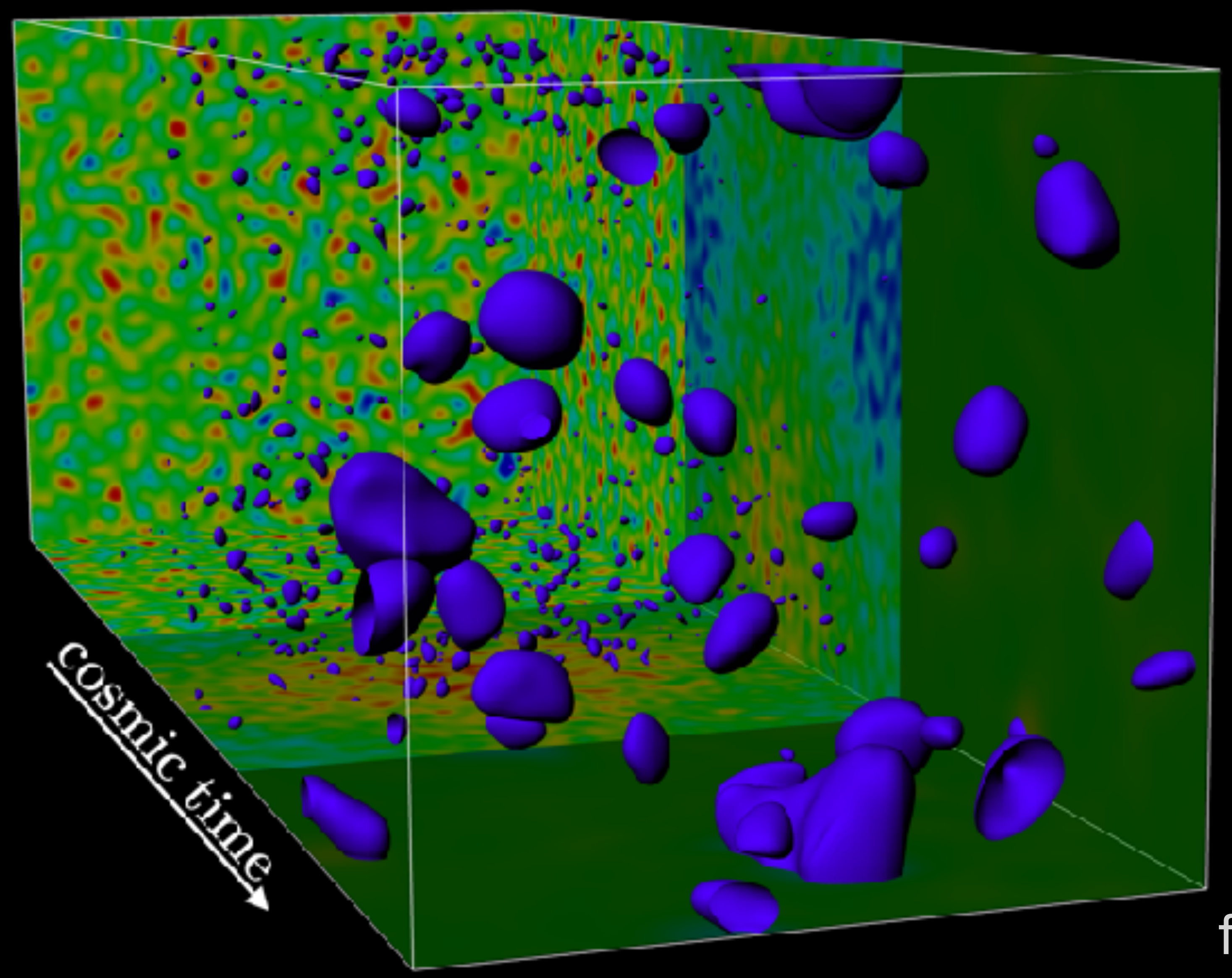
Early work

- Planar 1+1 NR simulations: Parry & Easther (1999, 2000)
- Spherical 1+1 NR: Finelly & Khlebnikov (2001)
- First 3+1 NR simulations: Bastero-Gil et al (2010)

Conclusion:

Fluctuations can grow

'Oscillons' can form



from Aurrekoetxea+ (2025)

More recently

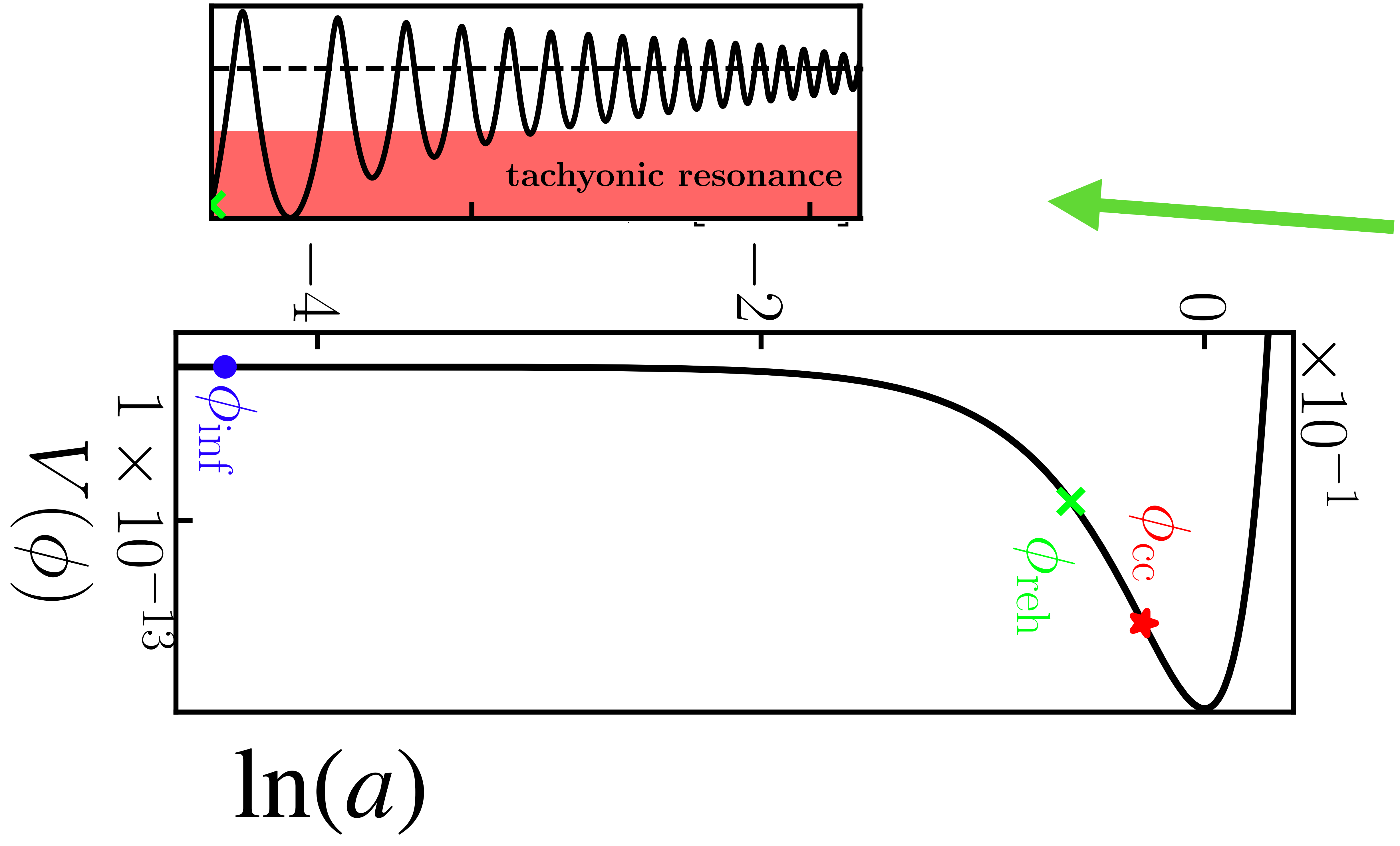
- Giblin & Tishue (2019)
- Kou et al (2021, 2022)
- Aurrekoetxea, Clough, Muia (2023)
- Adshead et al (2024)

Is GR important? Can BHs form?

What is the spectrum of GWs?

Inflation has to end… Reheating

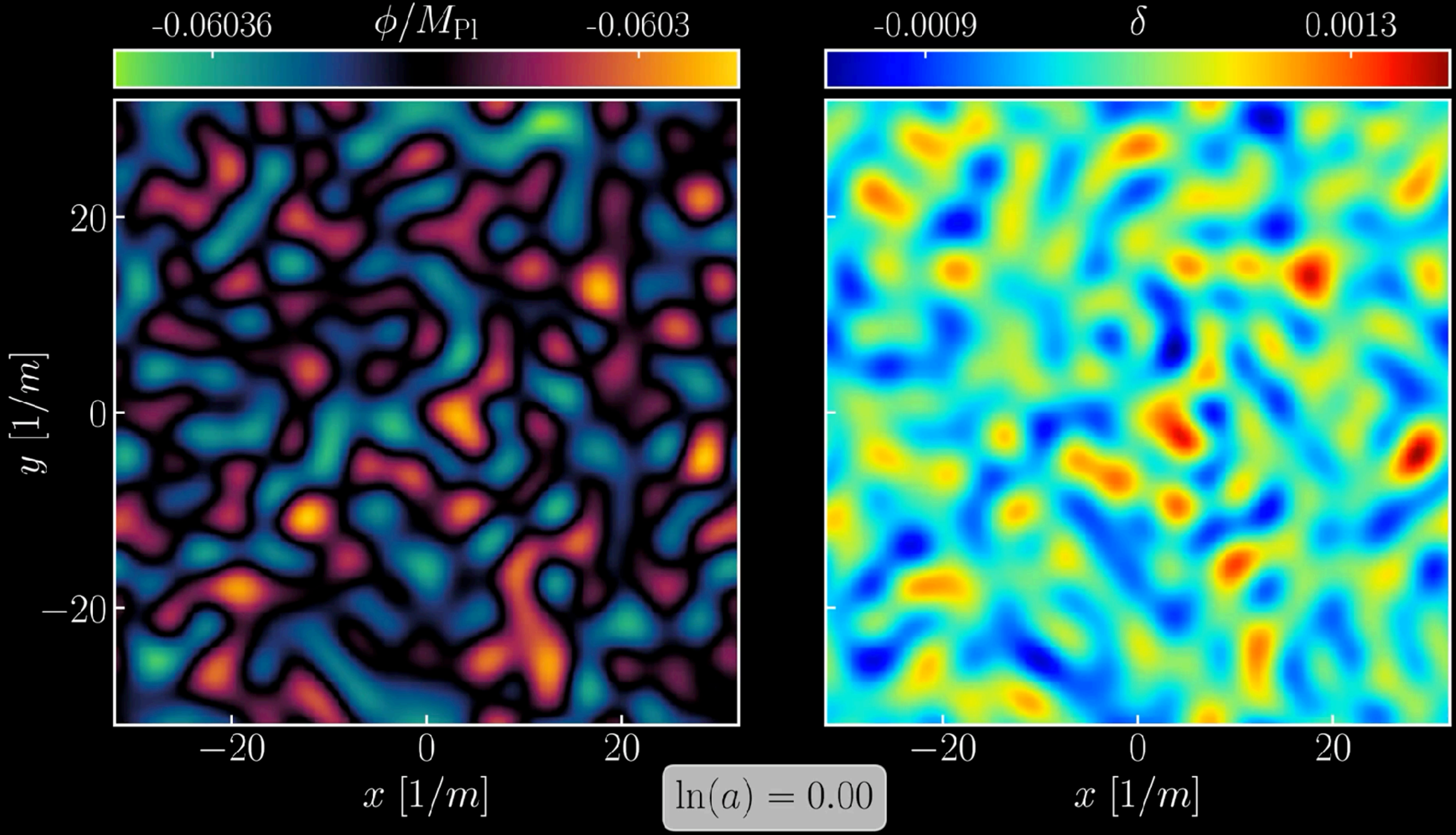
$V(\phi)$



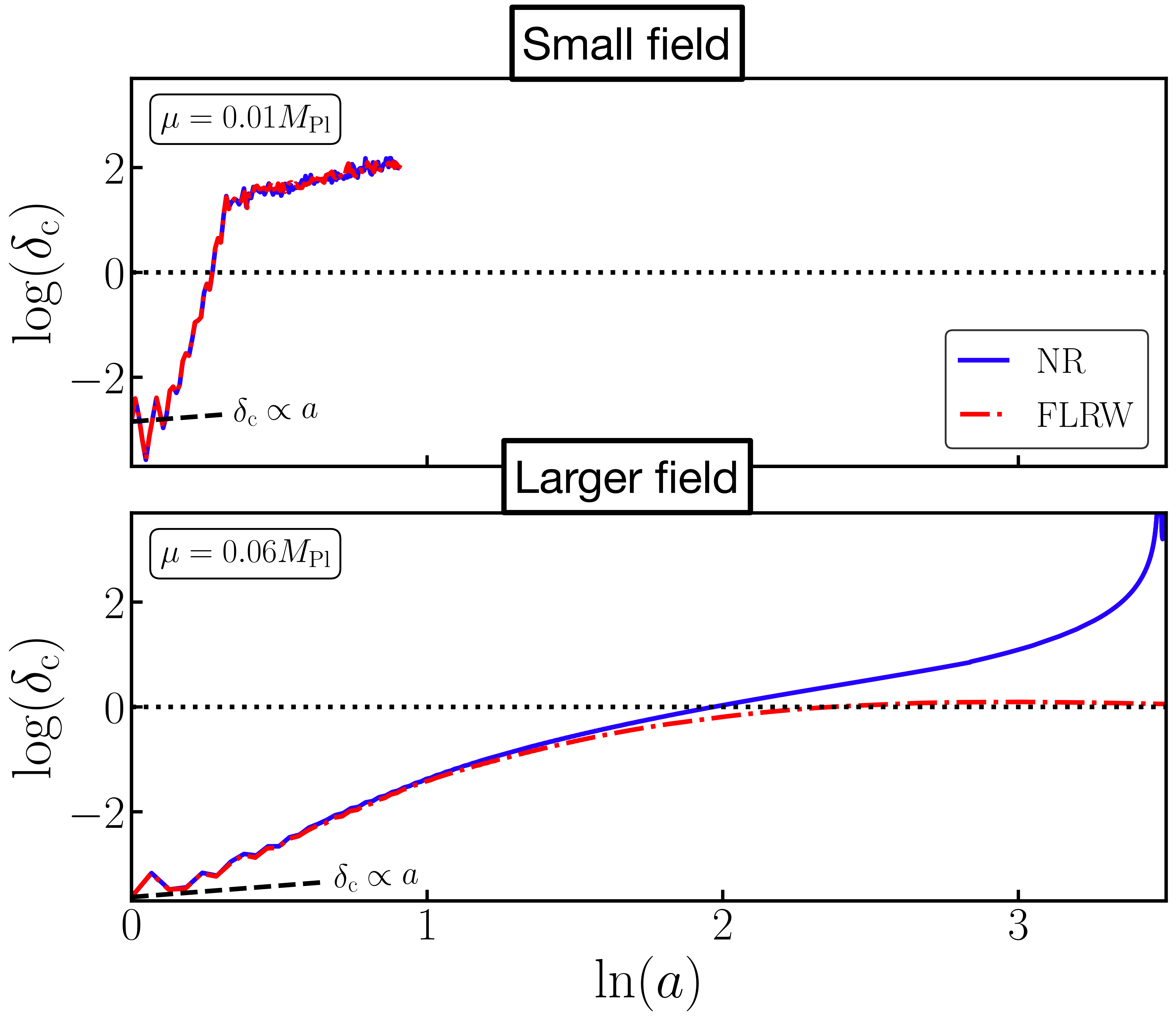
Start simulations here

e-folds (time evolution)

[Aurrekoetxea, Clough, Muia (2023)]



Lesson 1: Do we need backreaction?



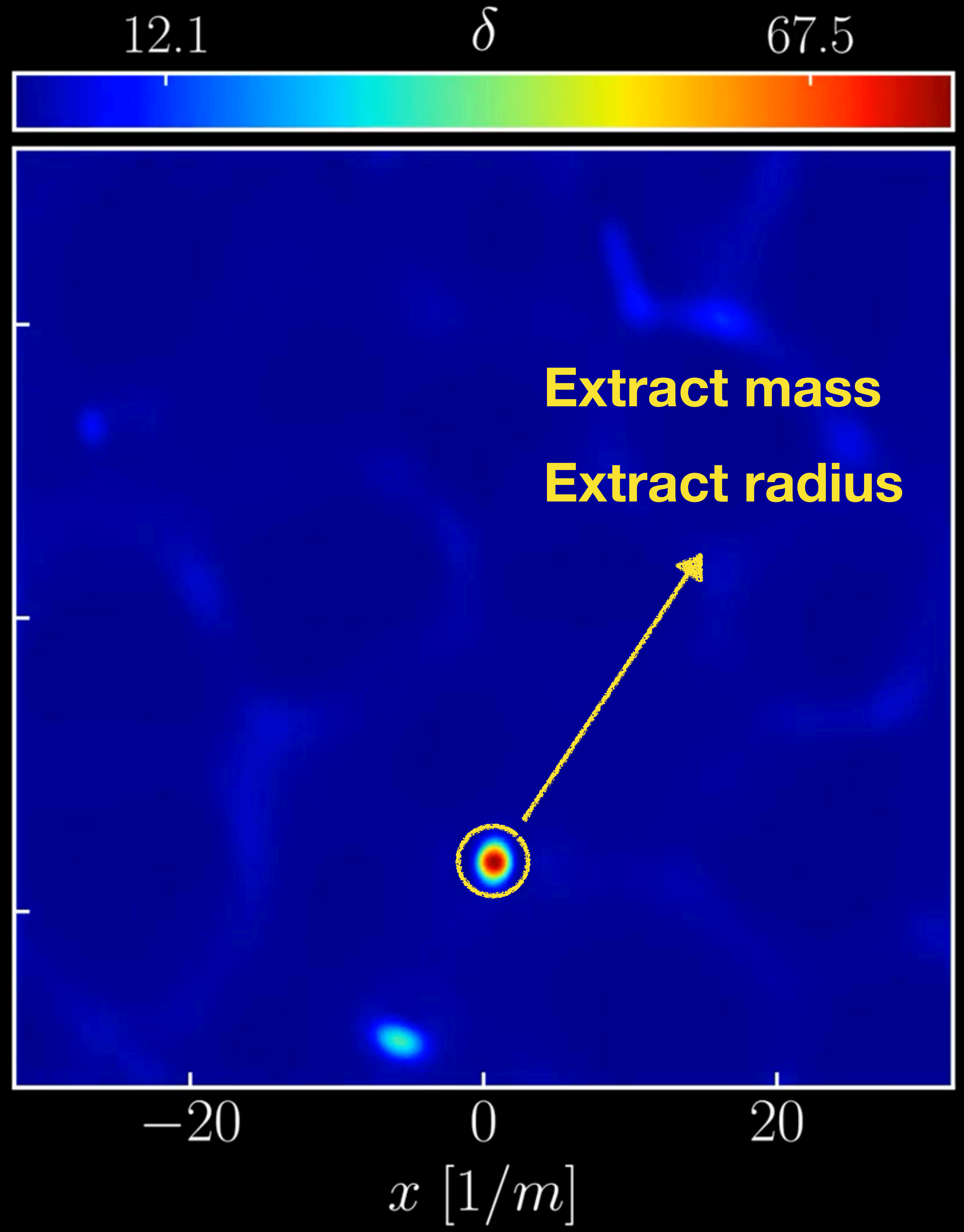
$$\nabla^\mu \nabla_\mu \phi = V'(\phi)$$

FLRW:

$$\frac{\ddot{a}}{a} = -\frac{\bar{\rho} + 3\bar{p}}{6M_{\rm Pl}^2}$$

NumRel:

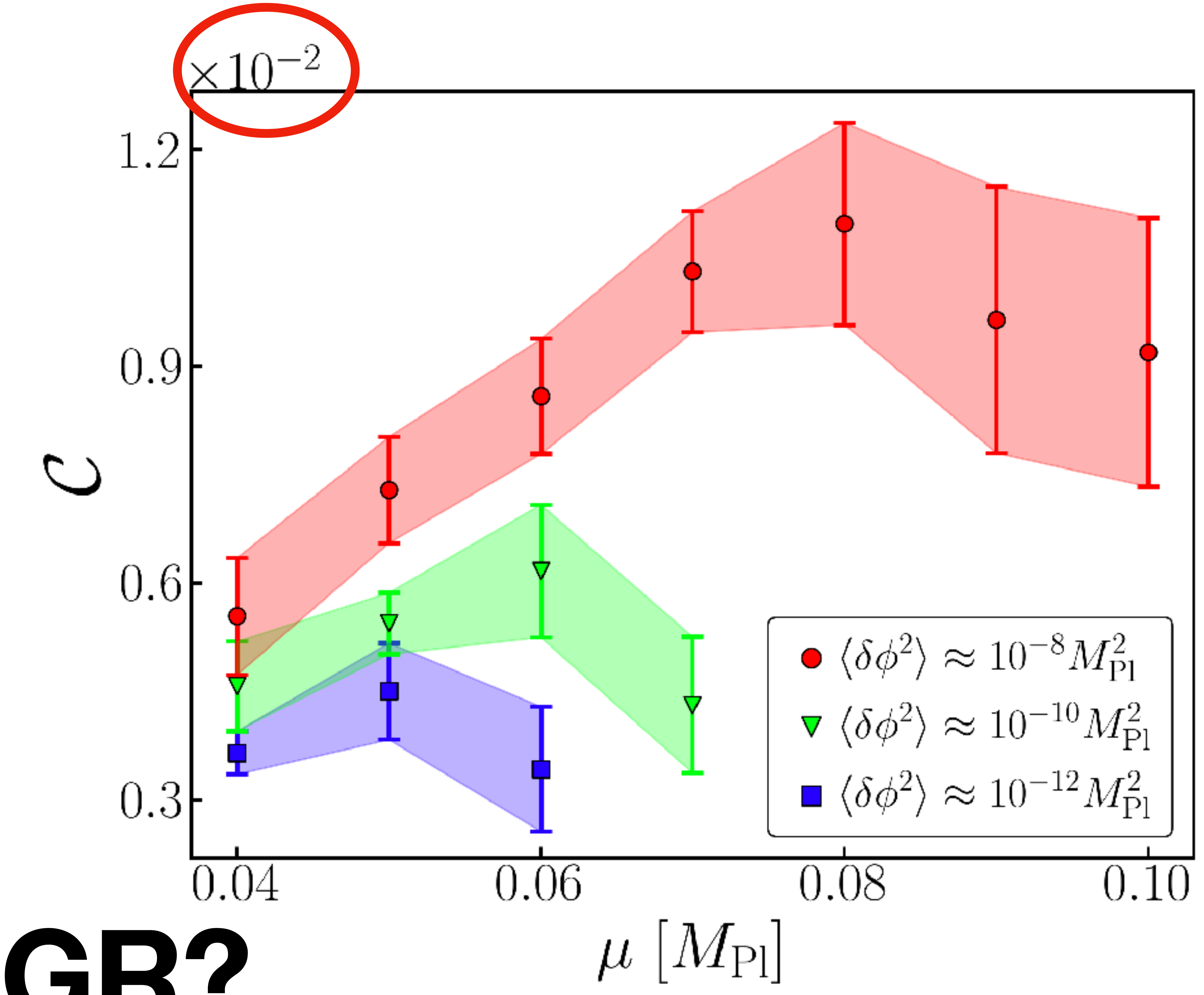
$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



Lesson 3:
Do we form BHs?

$$\mathcal{C} = GM/R$$

Black holes: $\mathcal{C} = 0.5$



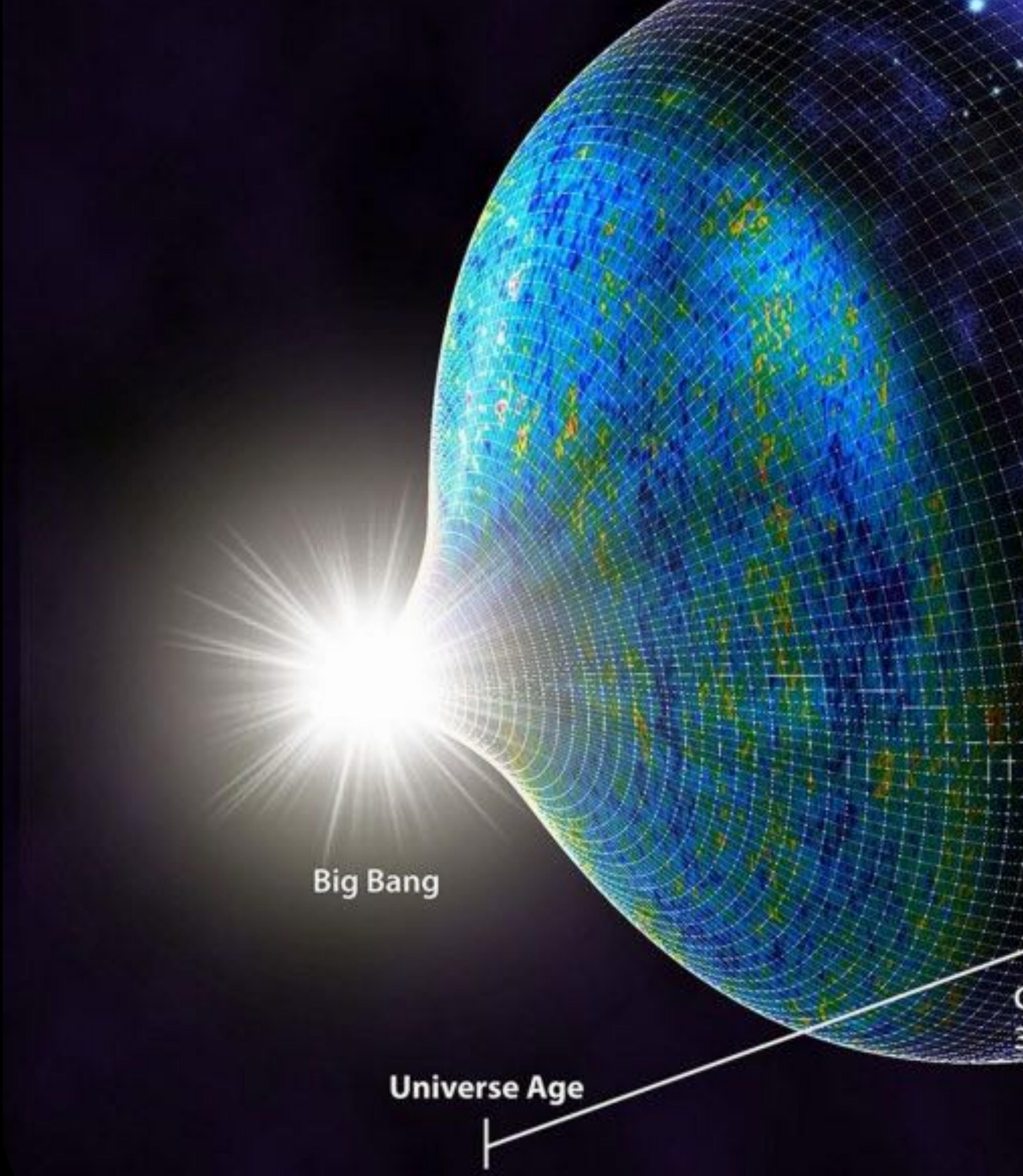
Lesson 1*:
Do we need full GR?

Maybe no…

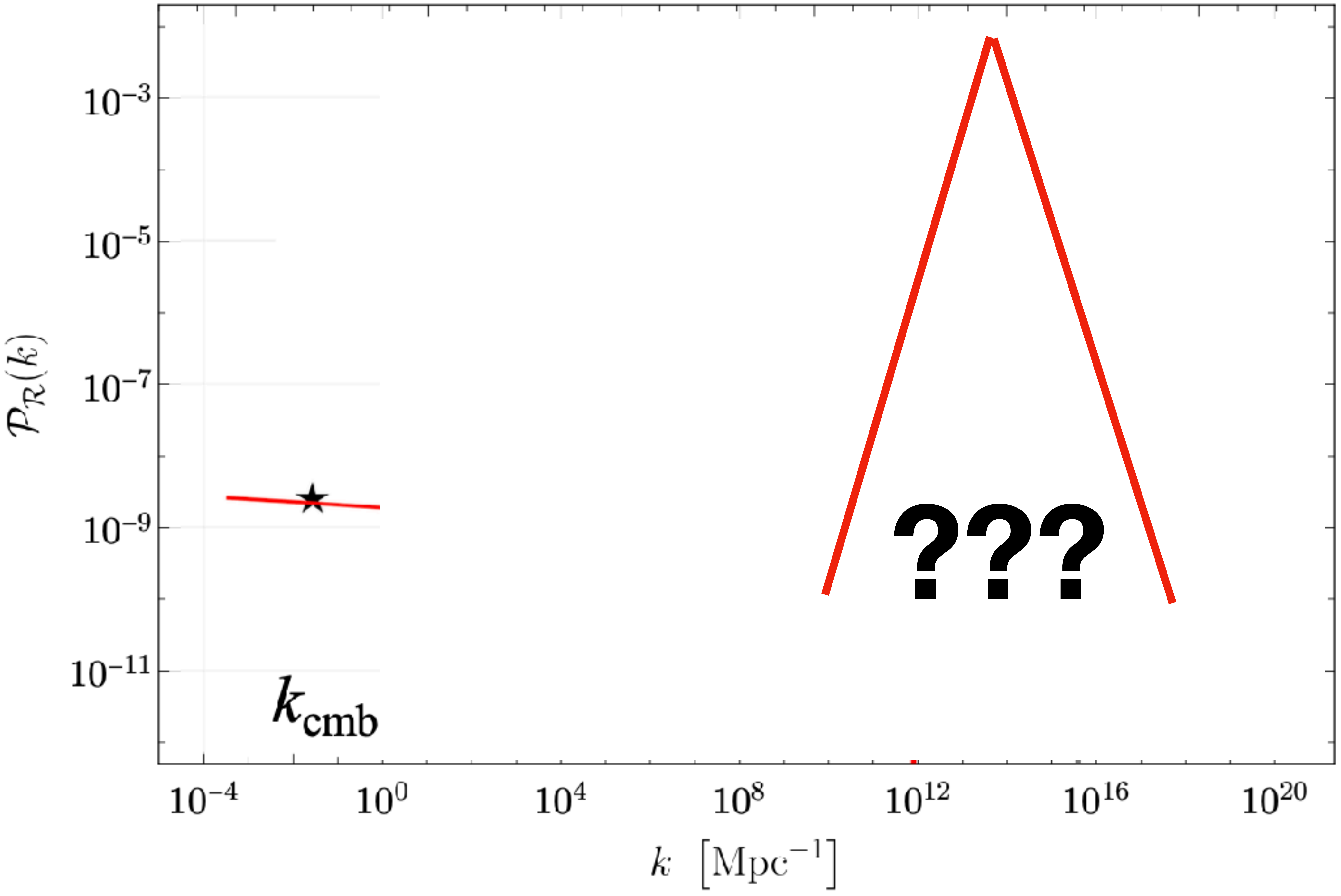
FLRW + Schrodinger-Poisson as Amin et al

$$\nabla^2 \Phi = 4\pi G a^2 (\rho - \bar{\rho})$$

Primordial BHs



PBHs from inflation



From the CMB $\mathscr{P}_{\mathscr{R}}(k_{\rm cmb}) = 10^{-9}$

To produce PBHs, the power spectrum must exceed some critical threshold

$$\mathscr{P}_{\mathscr{R}}(k_{\rm pbh}) \geq 10^{-3}$$

Early work

- Misner & Sharp (1964) and May & White (1966)
- Nadezhin et al (1978)
- Bicknell & Henriksen (1979)
- Baumgarte et al (1995)
- Niemeyer & Jedamzik (1999)
- Hawke & Stewart (2002)
- Green et al (2004)
- Musco, Miller, Rezzola (2005)

1+1 MISNER-SHARP / HERNANDEZ

- Shibata & Sasaki (1999)

1+1 BSSN

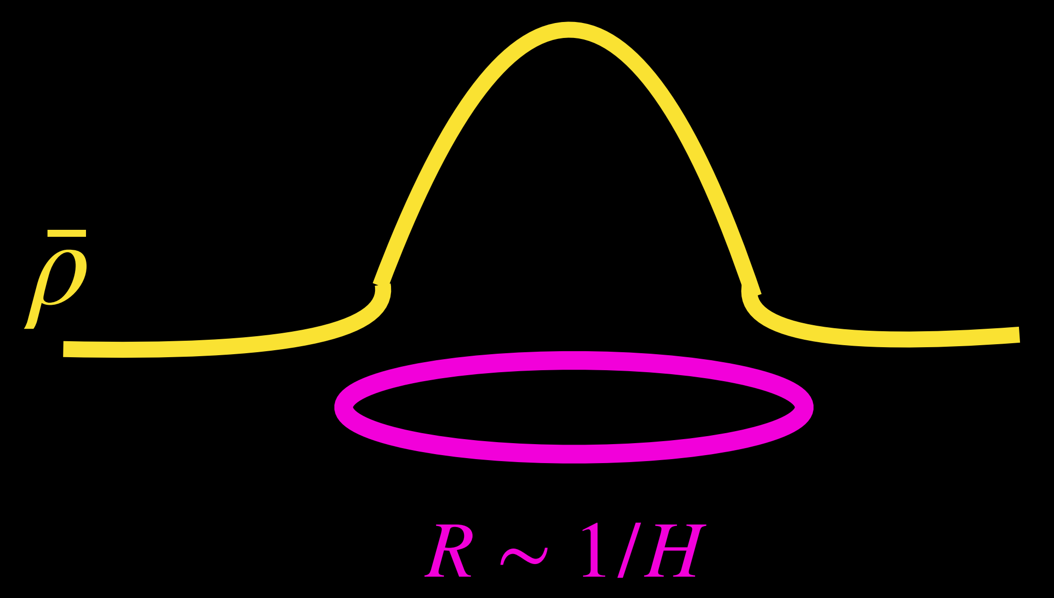
More recently

- More 1+1 Misner-Sharp / Hernandez…

- Yoo, Harada, Okawa (2020)
- De Jong, Aurrekoetxea, Lim (2021)
- De Jong et al (2023)
- Escriva & Yoo (2024)

3+1 BSSN

$$\delta\rho = \rho/\bar{\rho} - 1$$

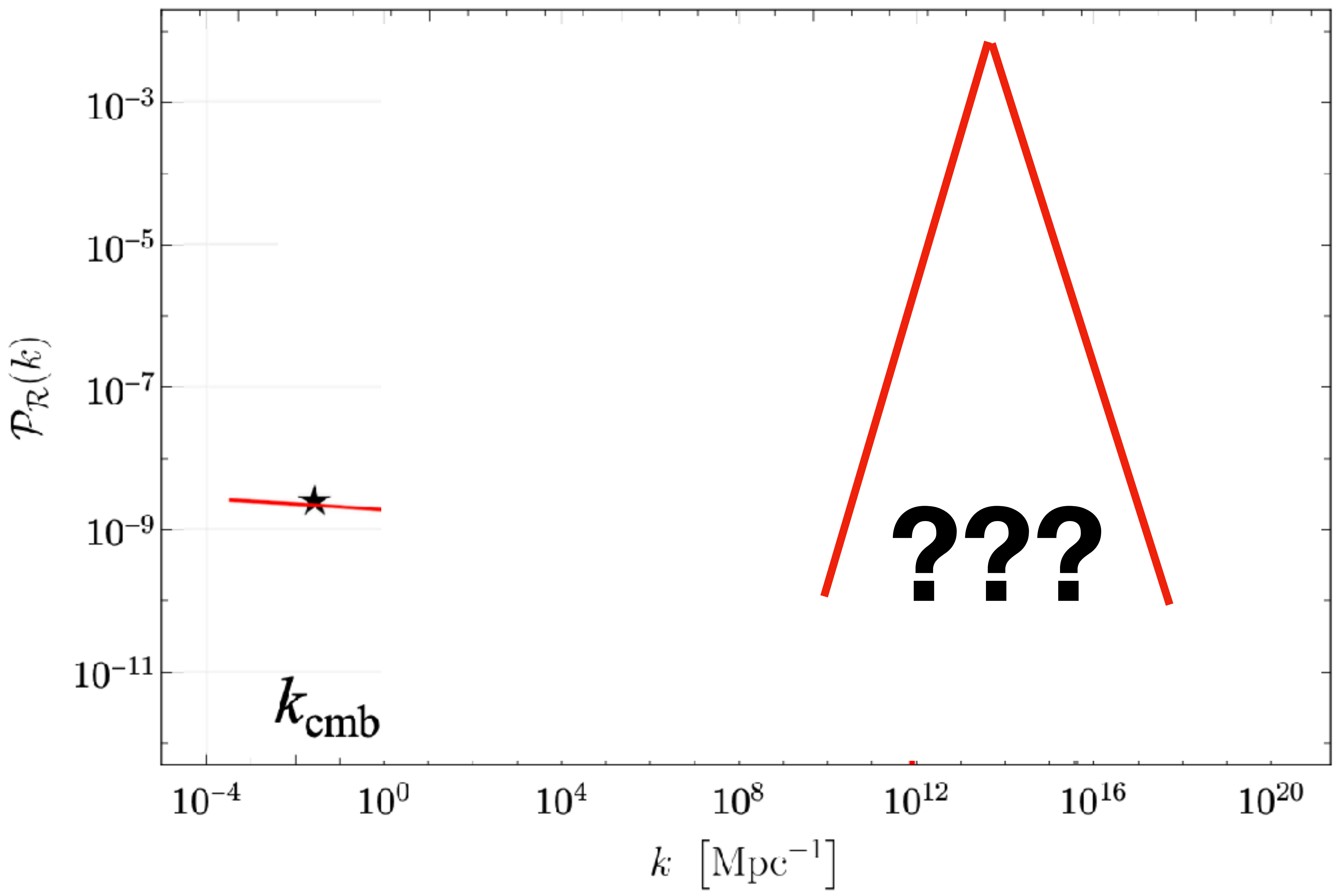


Conclusion:

- $\delta_{\rm crit} \approx 0.3$
- $M_{\rm BH} \approx 0.1 M_H$

- Small spin BHs from non-spherical
- Small modification to the threshold $\delta_{\rm crit}$

PBHs from inflation



From the CMB $\mathscr{P}_{\mathscr{R}}(k_{\rm cmb}) = 10^{-9}$

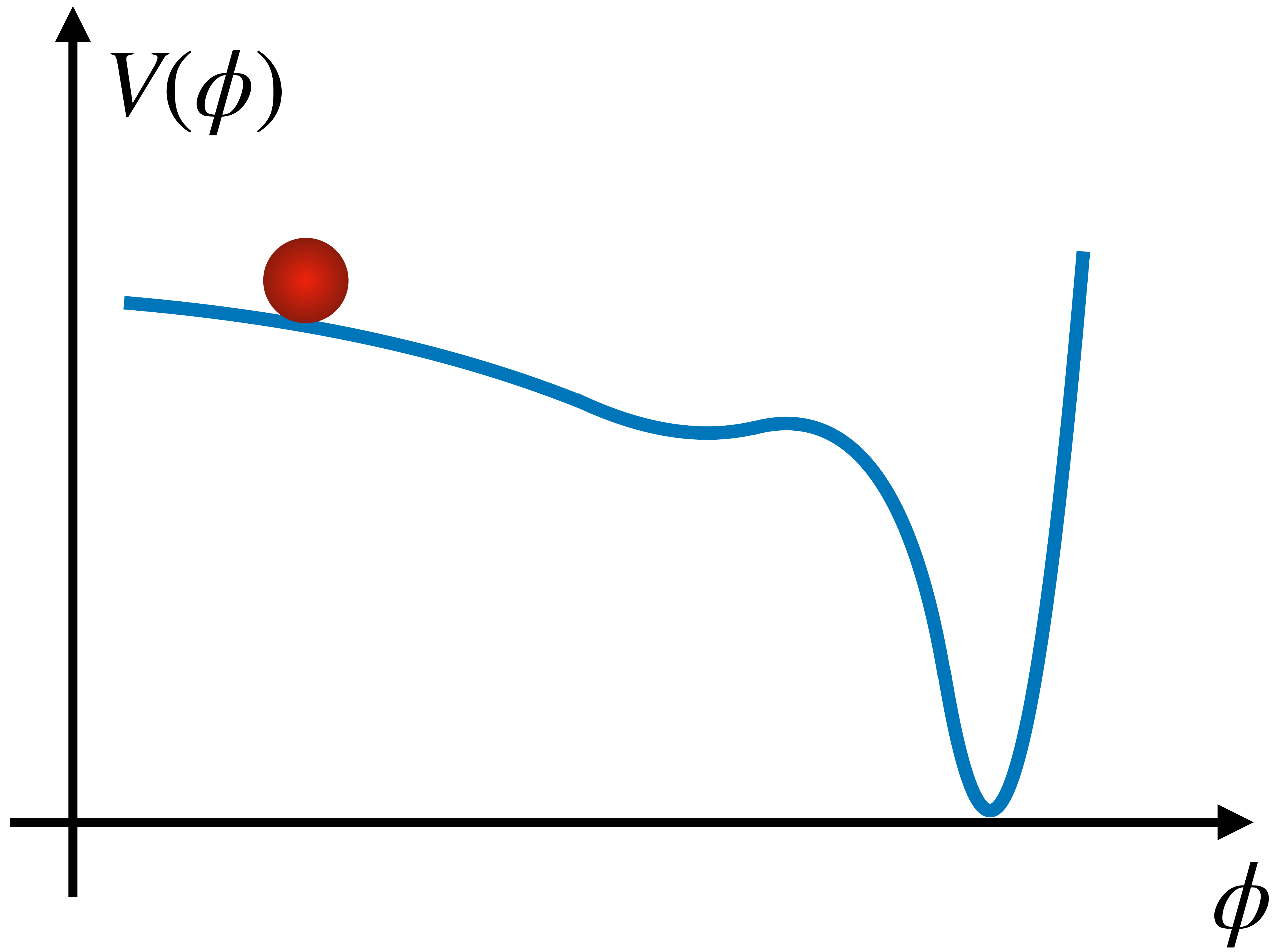
To produce PBHs, the power spectrum must exceed some critical threshold

$$\mathscr{P}_{\mathscr{R}}(k_{\rm pbh}) \geq 10^{-3}$$

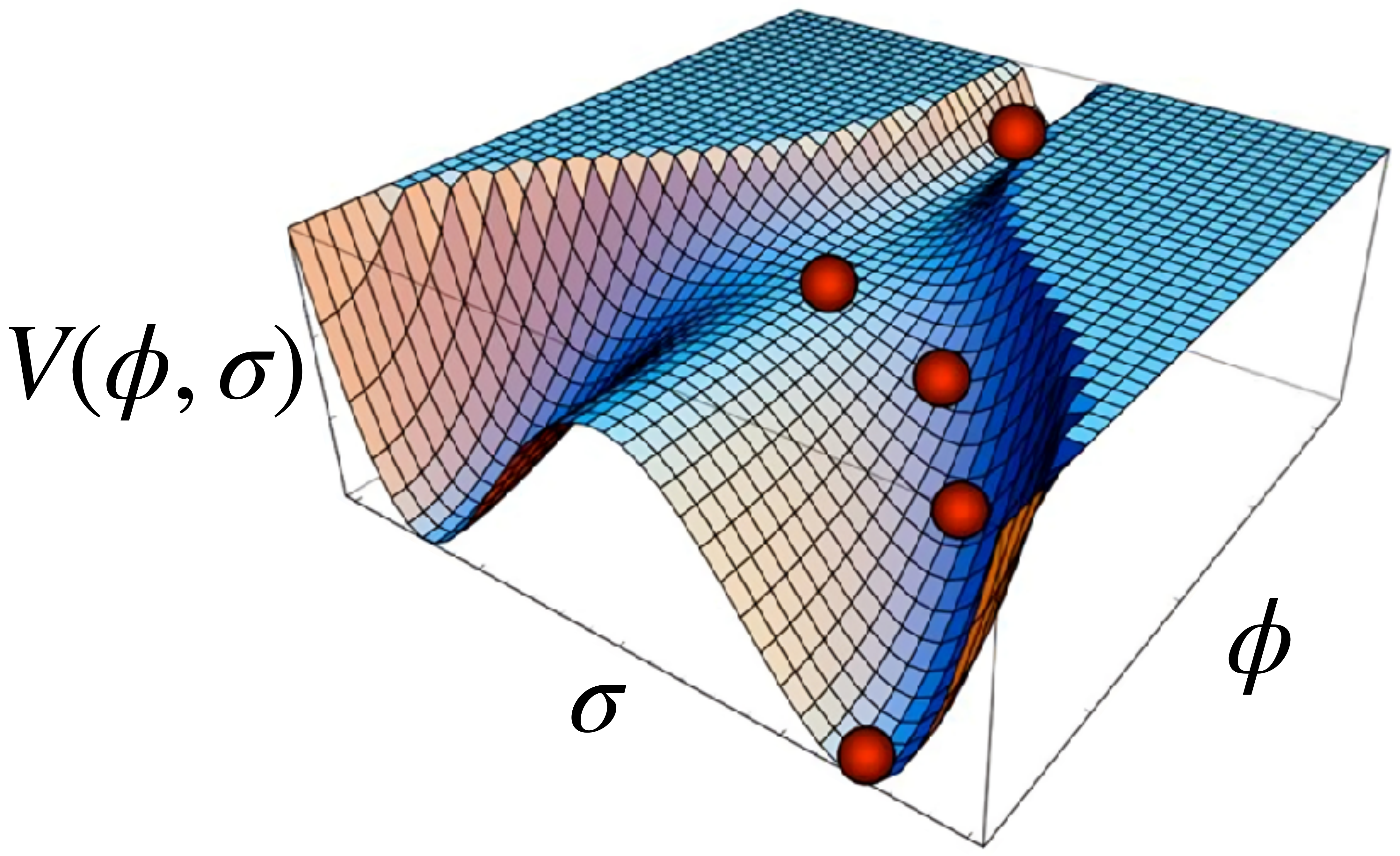
PBHs from inflation

To produce PBHs, the power spectrum must exceed some critical threshold

Ultra-slow roll inflation

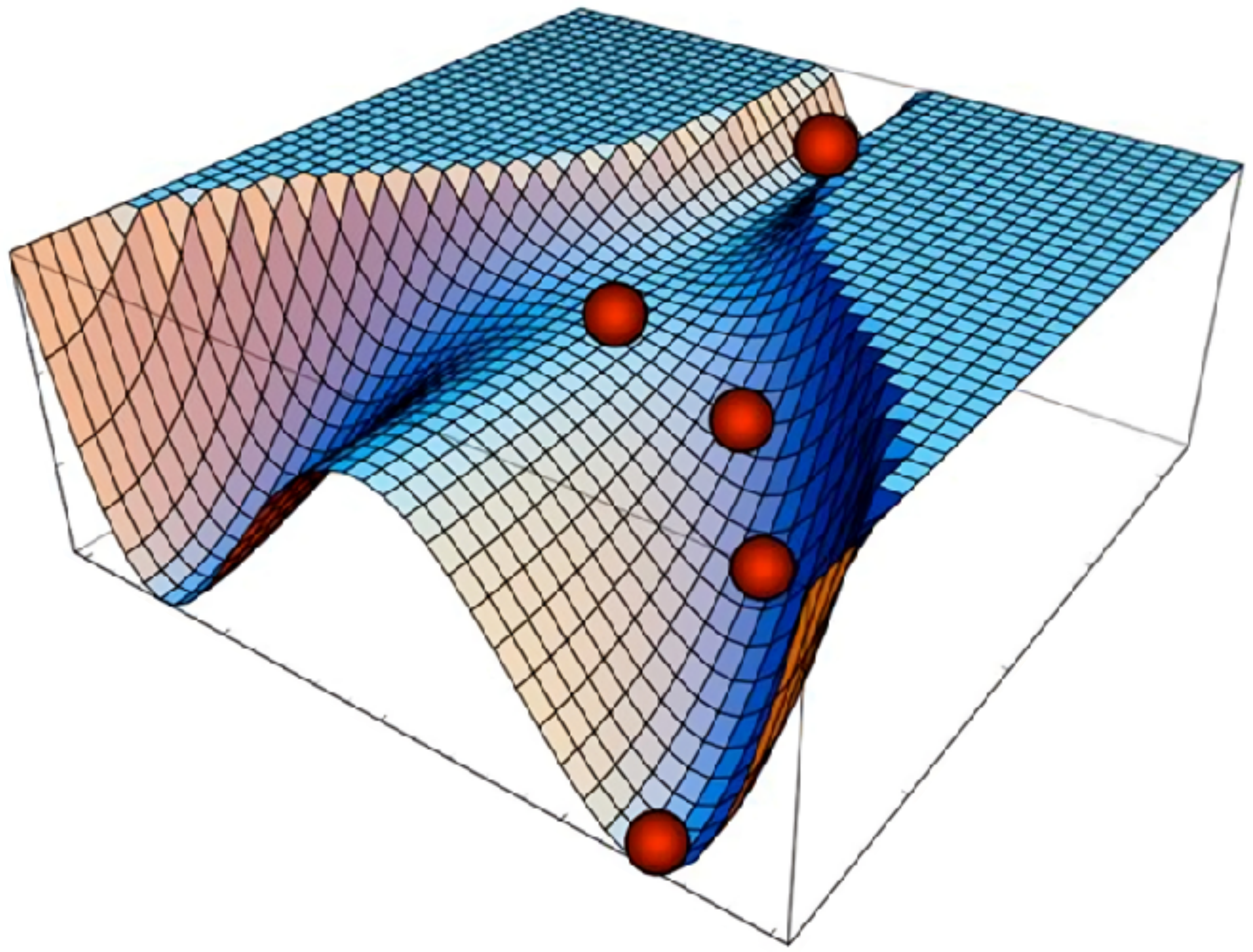


Hybrid inflation



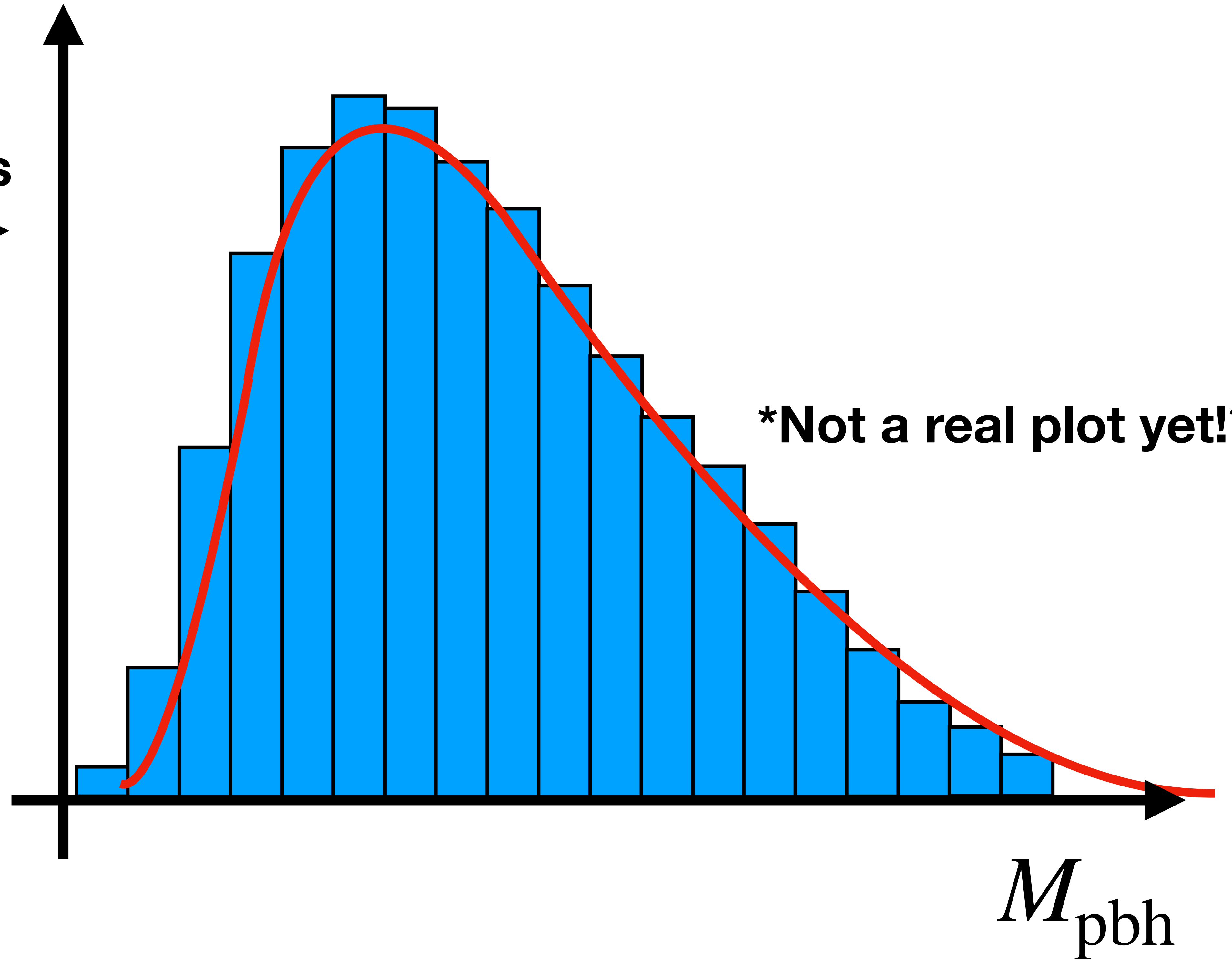
Aurrekoetxea, Barreto, Guth, Kaiser (2026?)

Given an inflationary model



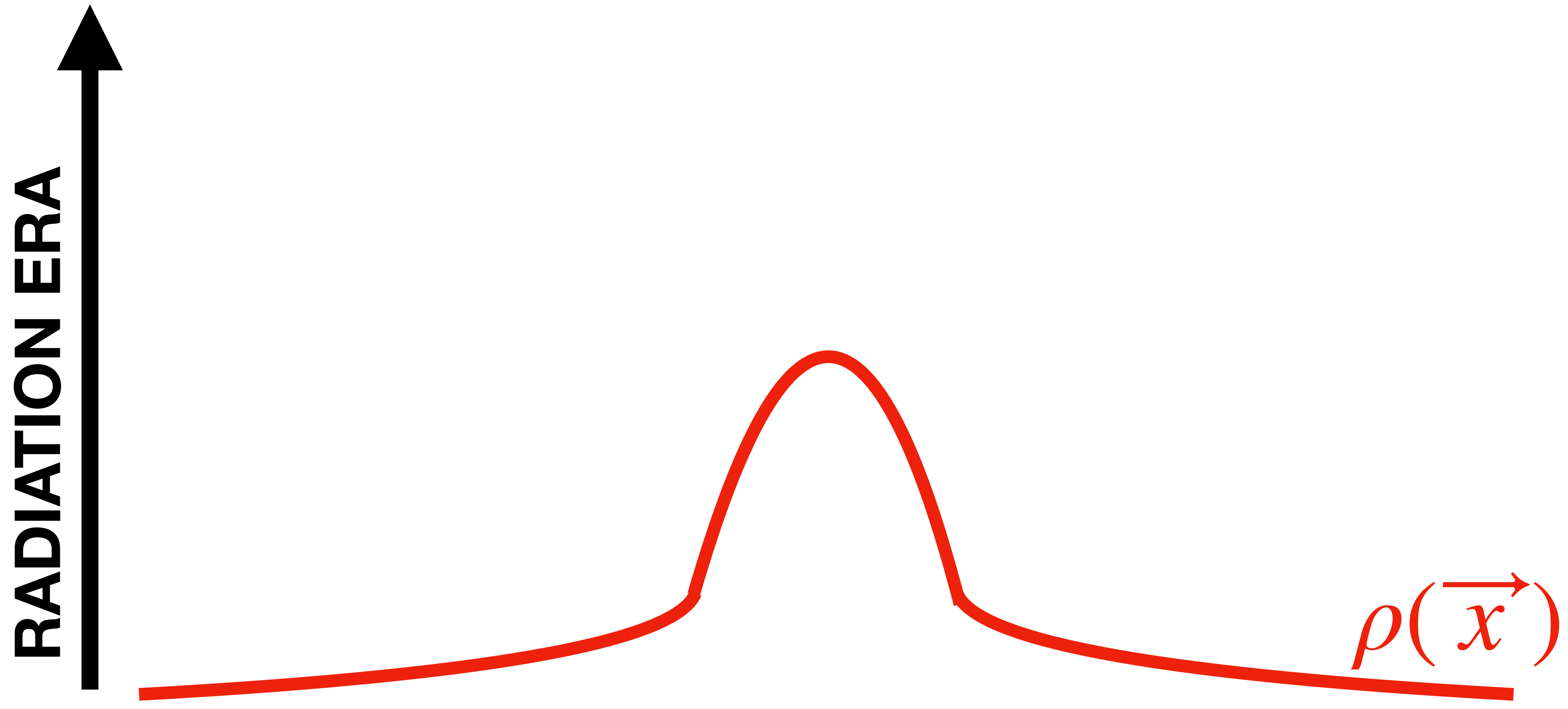
Run $\mathcal{O}(10^5)$ simulations

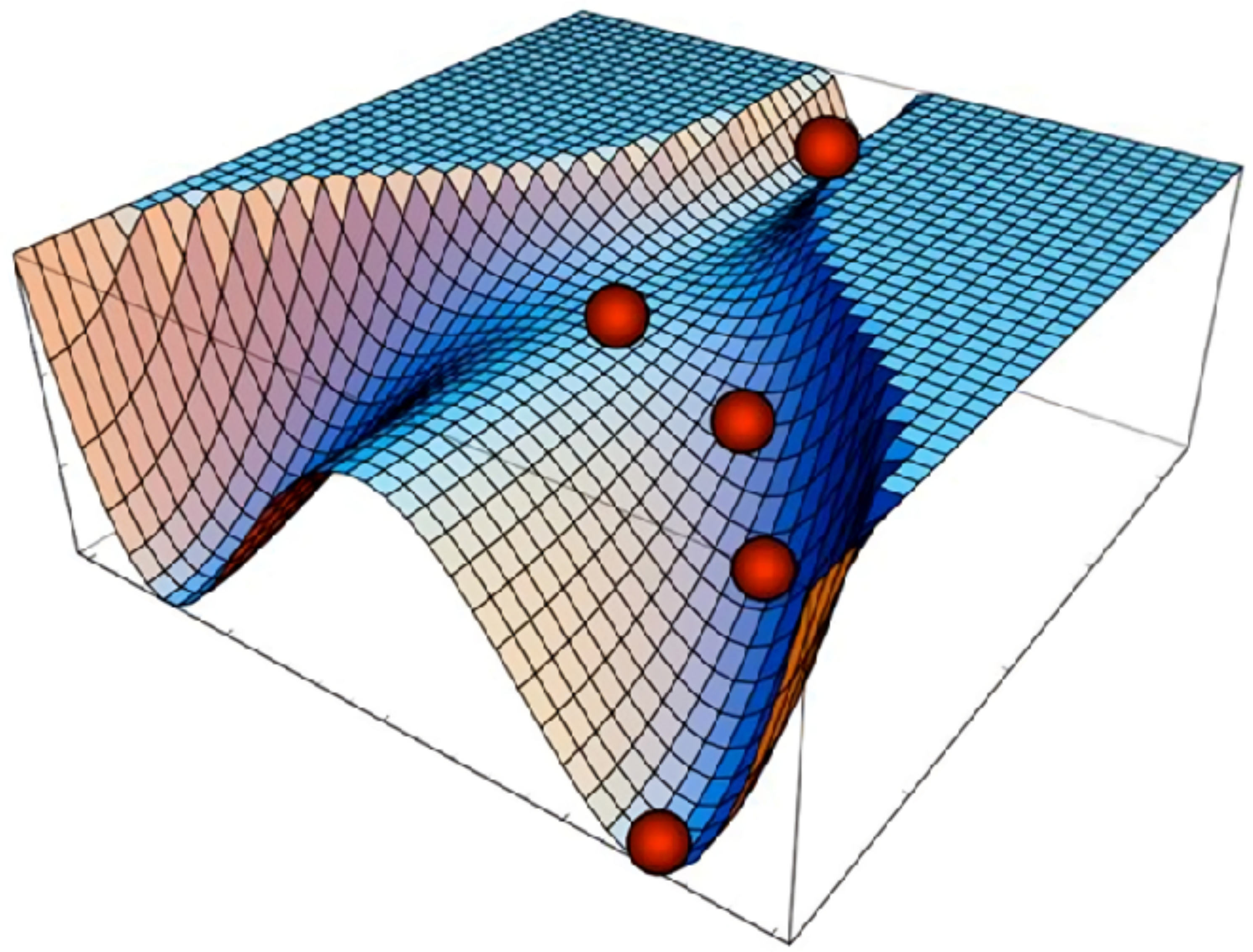
Probability distribution of BH mass



Can we get $M_{\rm pbh} \sim (10^2 - 10^6) M_\odot$ **?**

Initial conditions from inflation

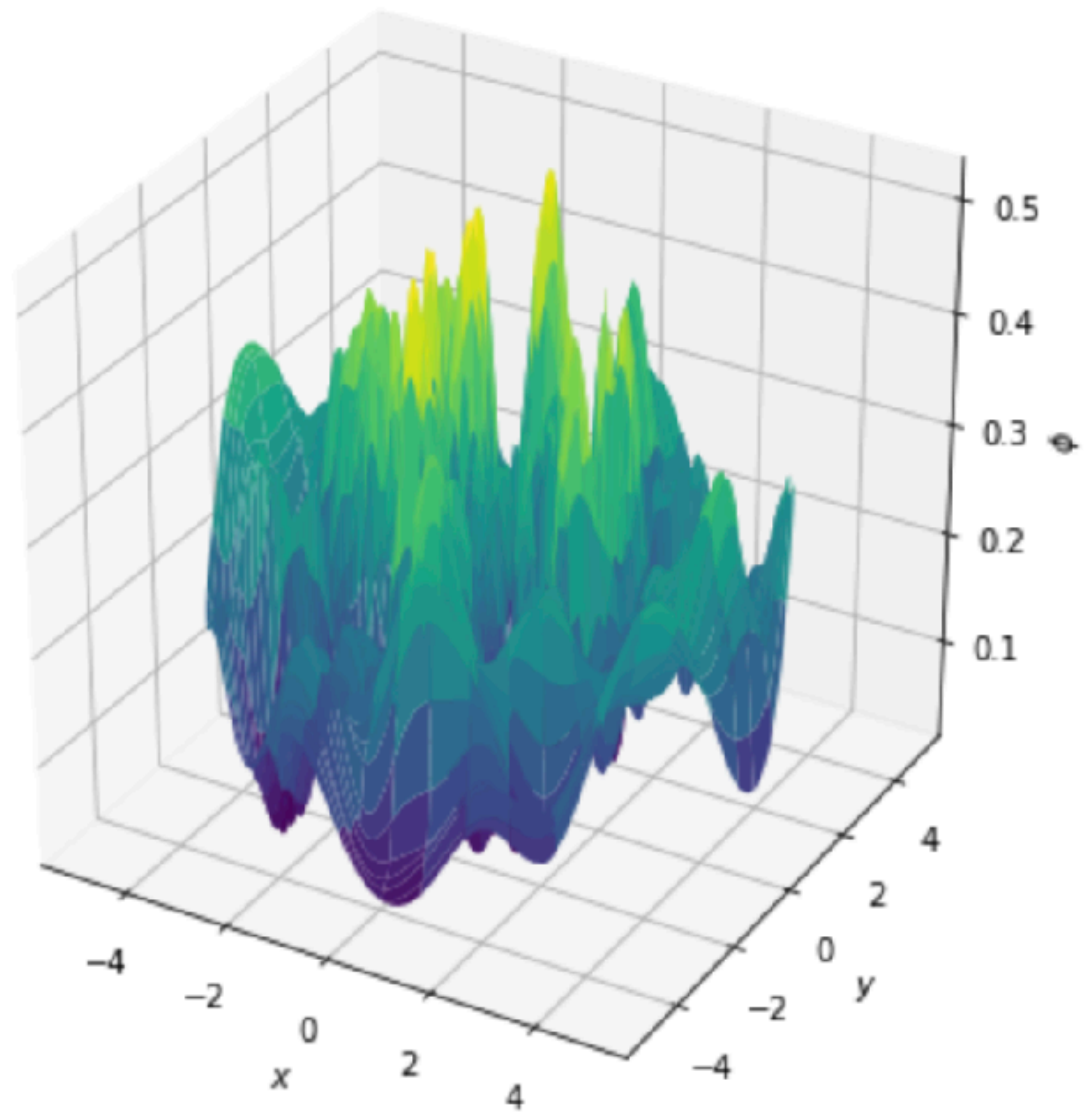


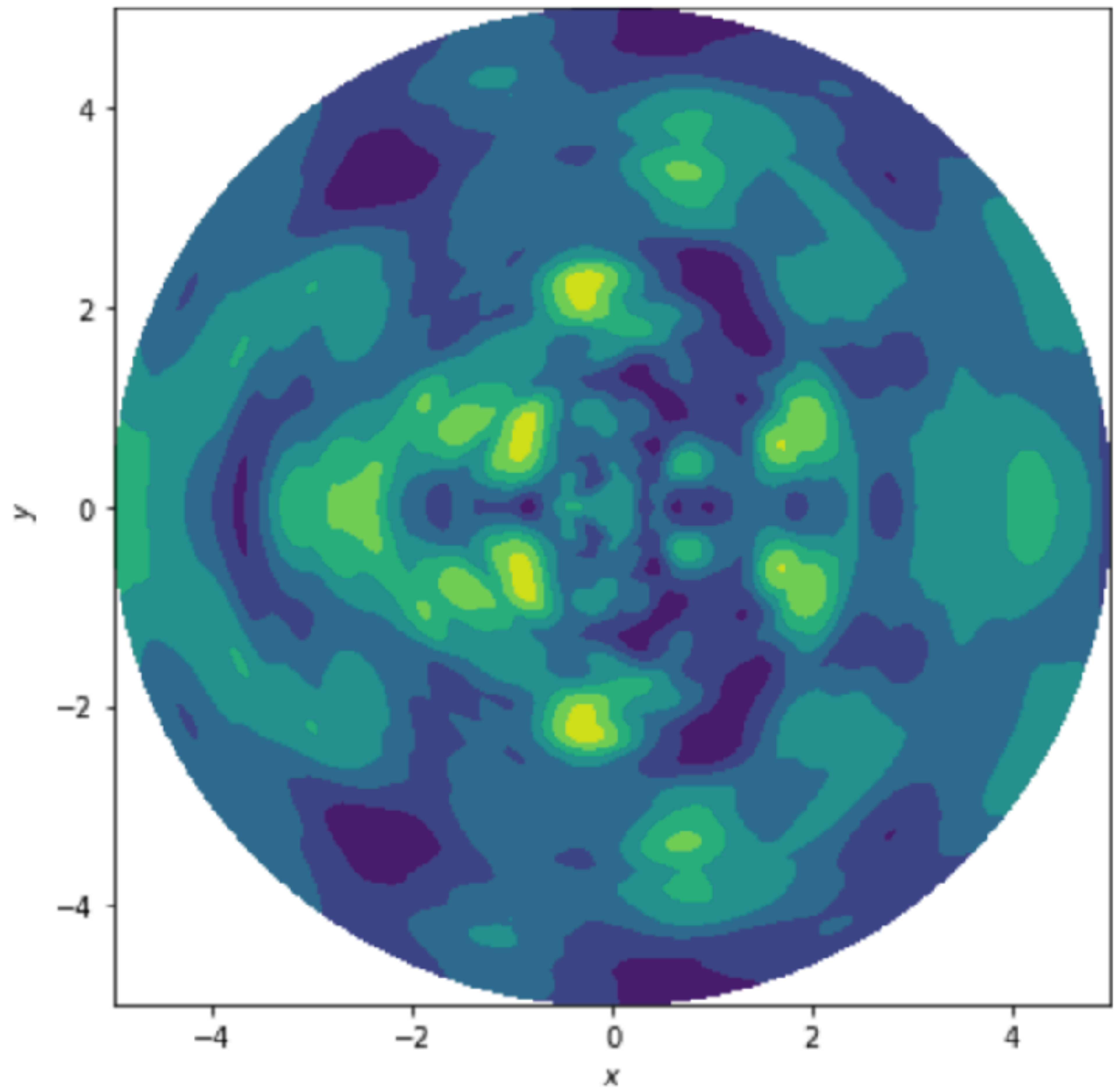


Need GR Hydrodynamics $\nabla_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + P g^{\mu\nu}$$

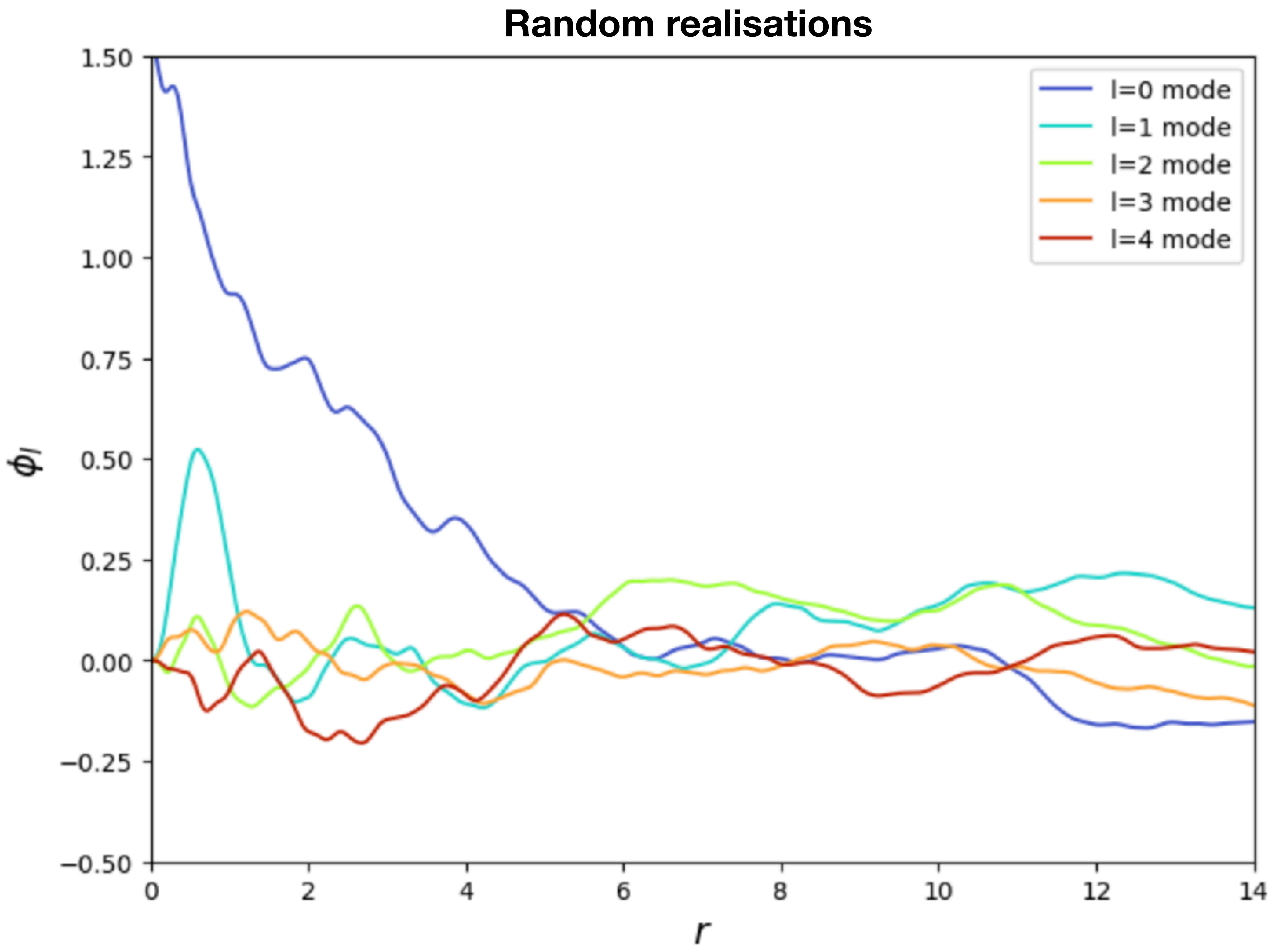
Initial conditions from inflation





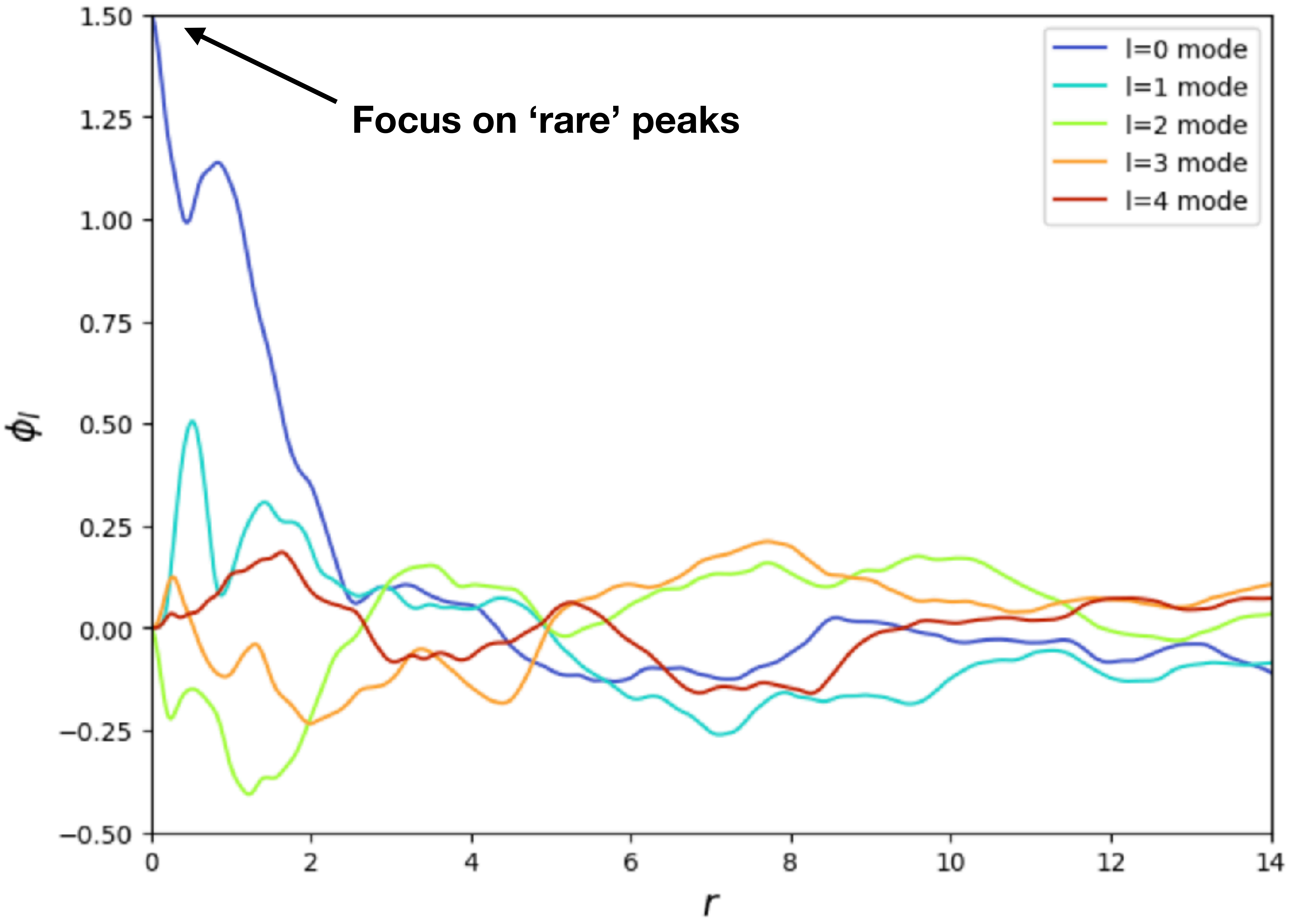
Initial conditions from inflation

Spherical harmonic modes



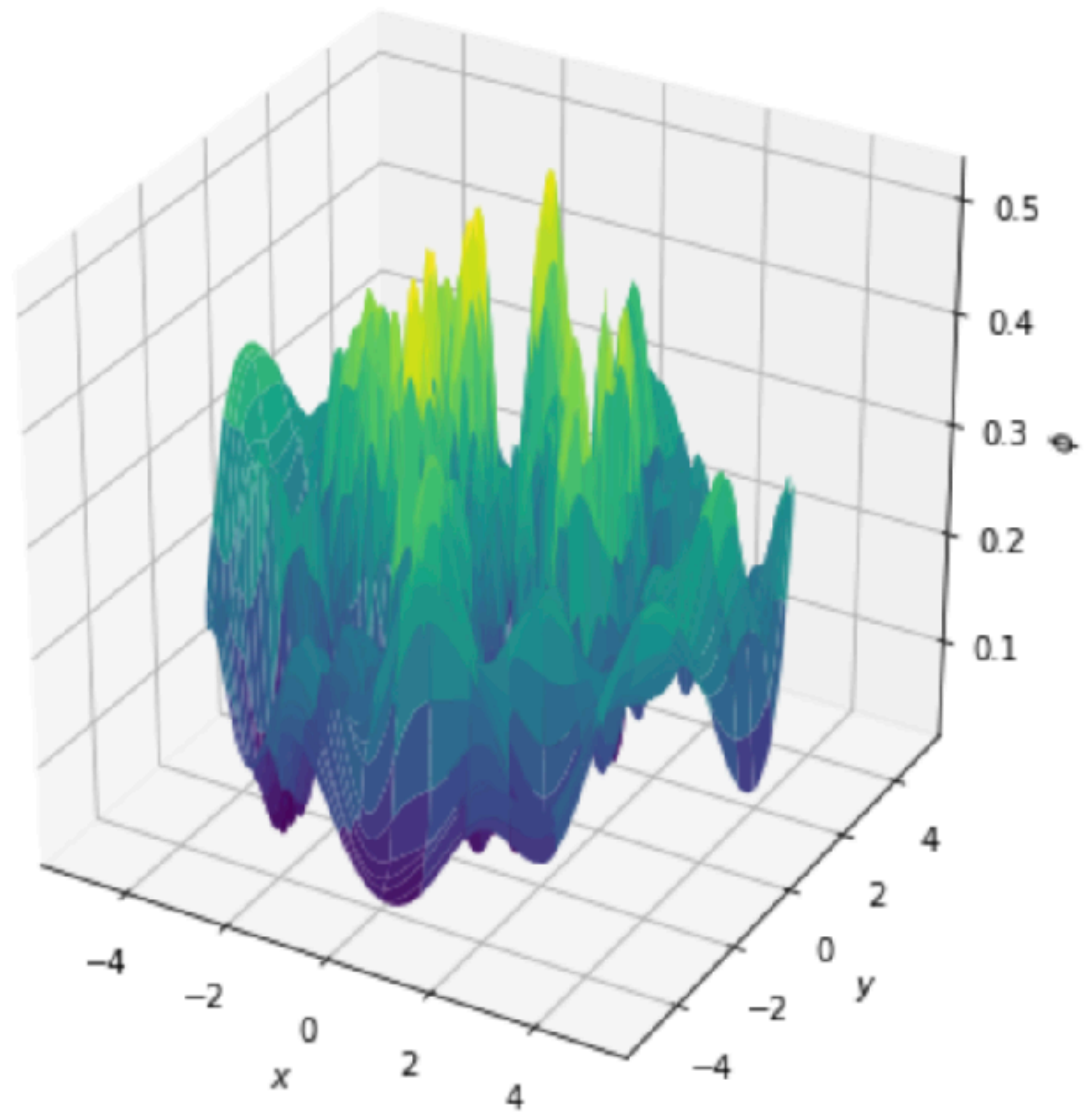
Initial conditions from inflation

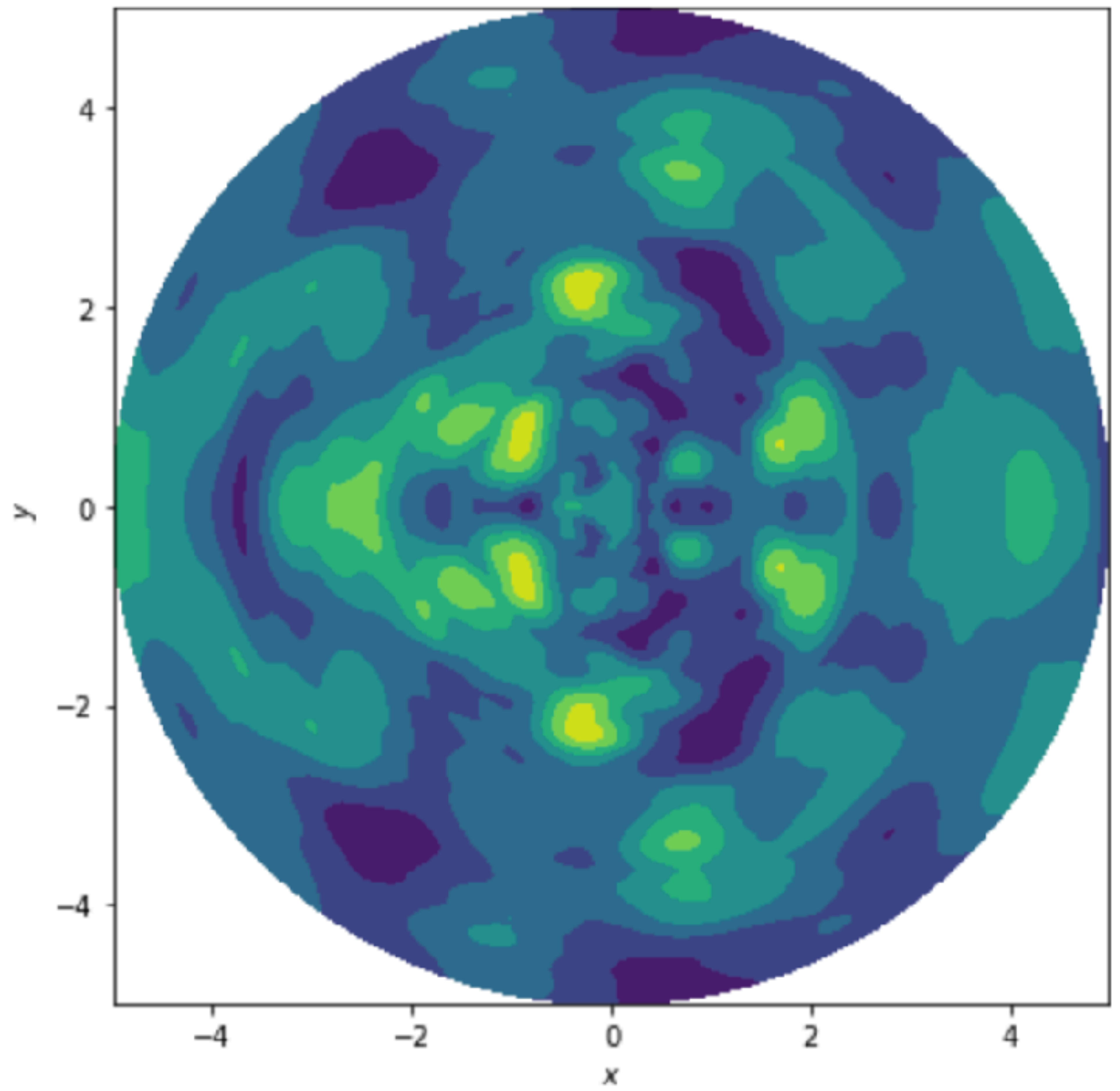
Spherical harmonic modes



Initial conditions from inflation

Random realisation

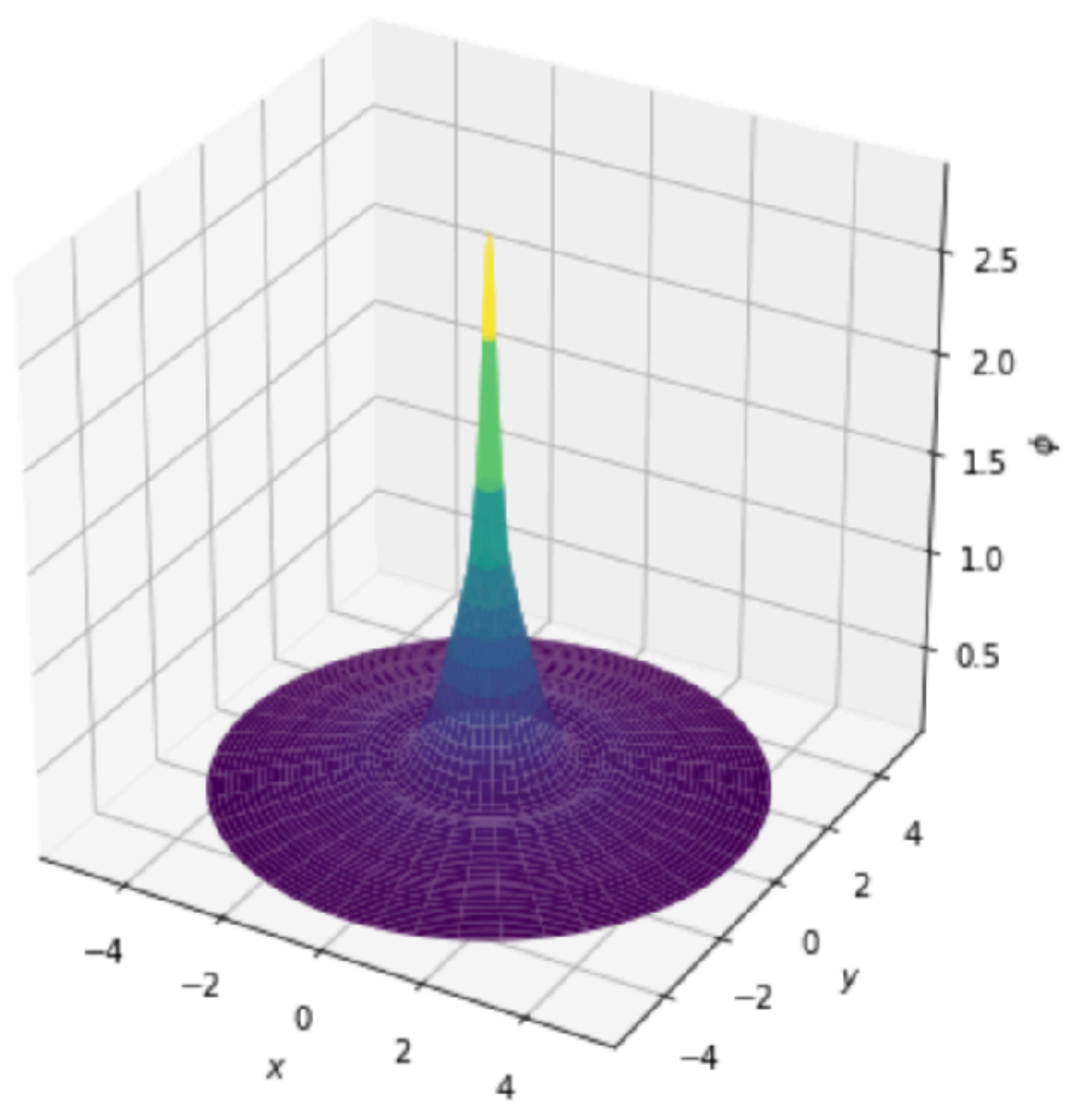




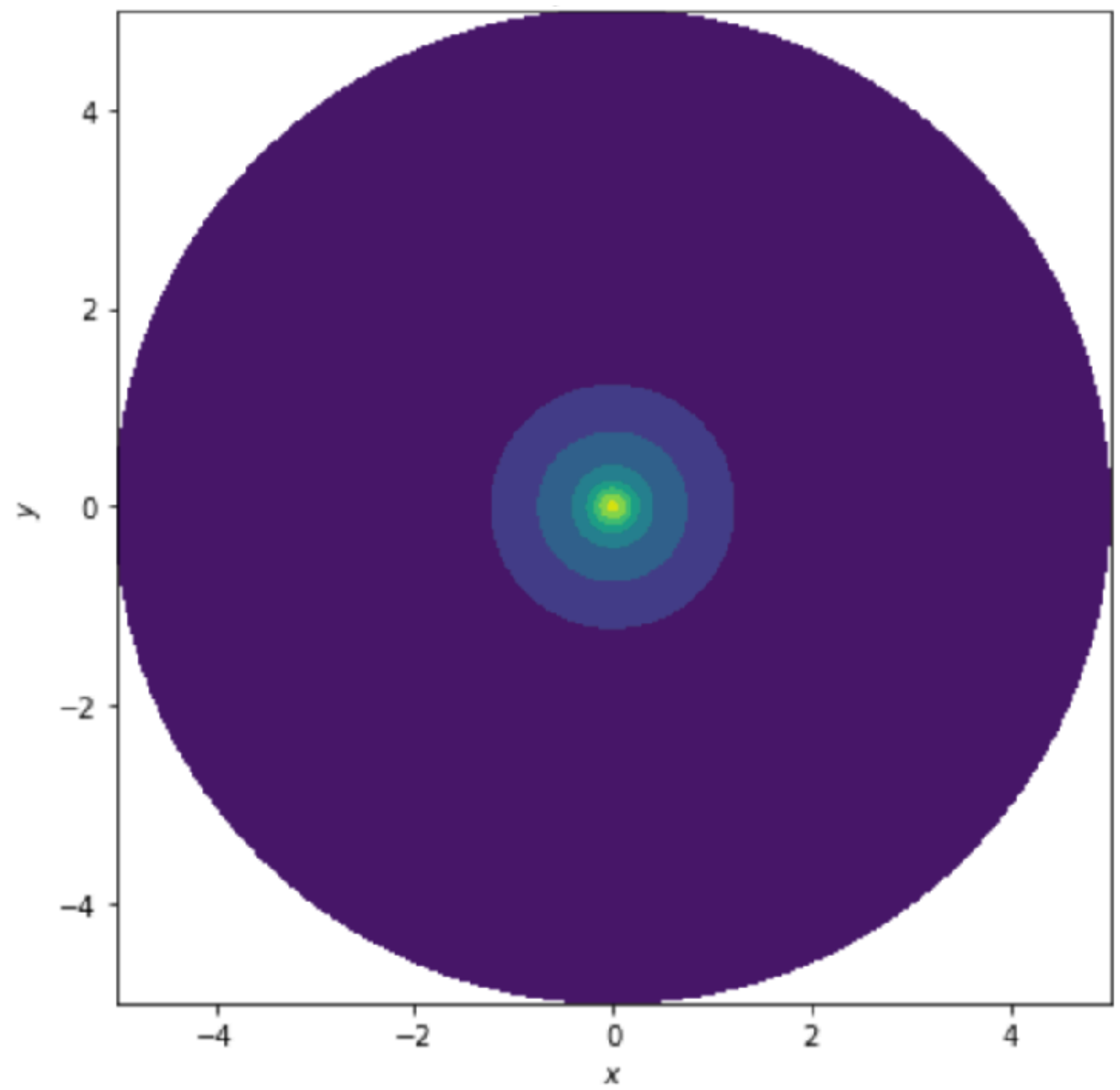
Initial conditions from inflation

Biased realisation

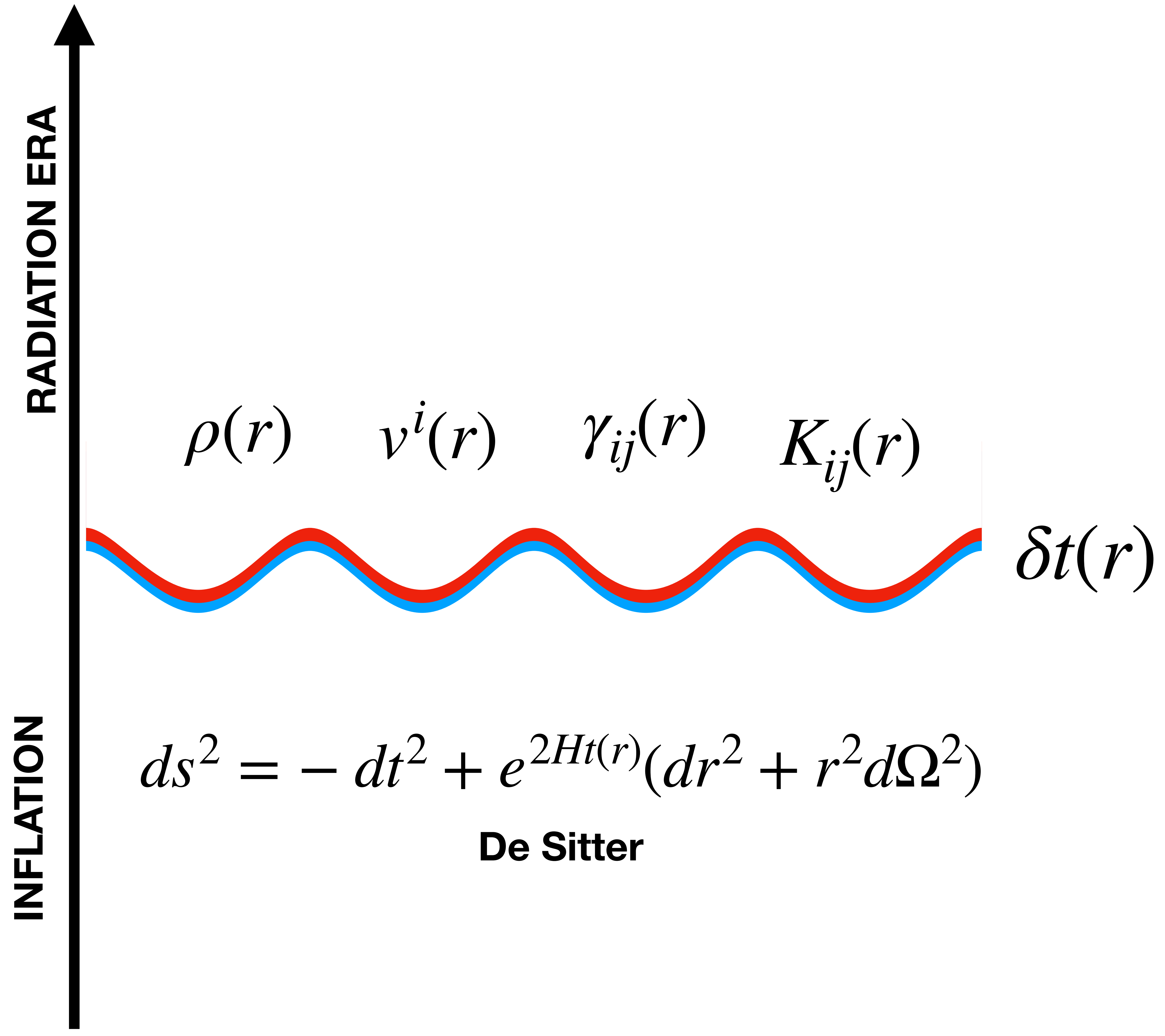
sphericalized

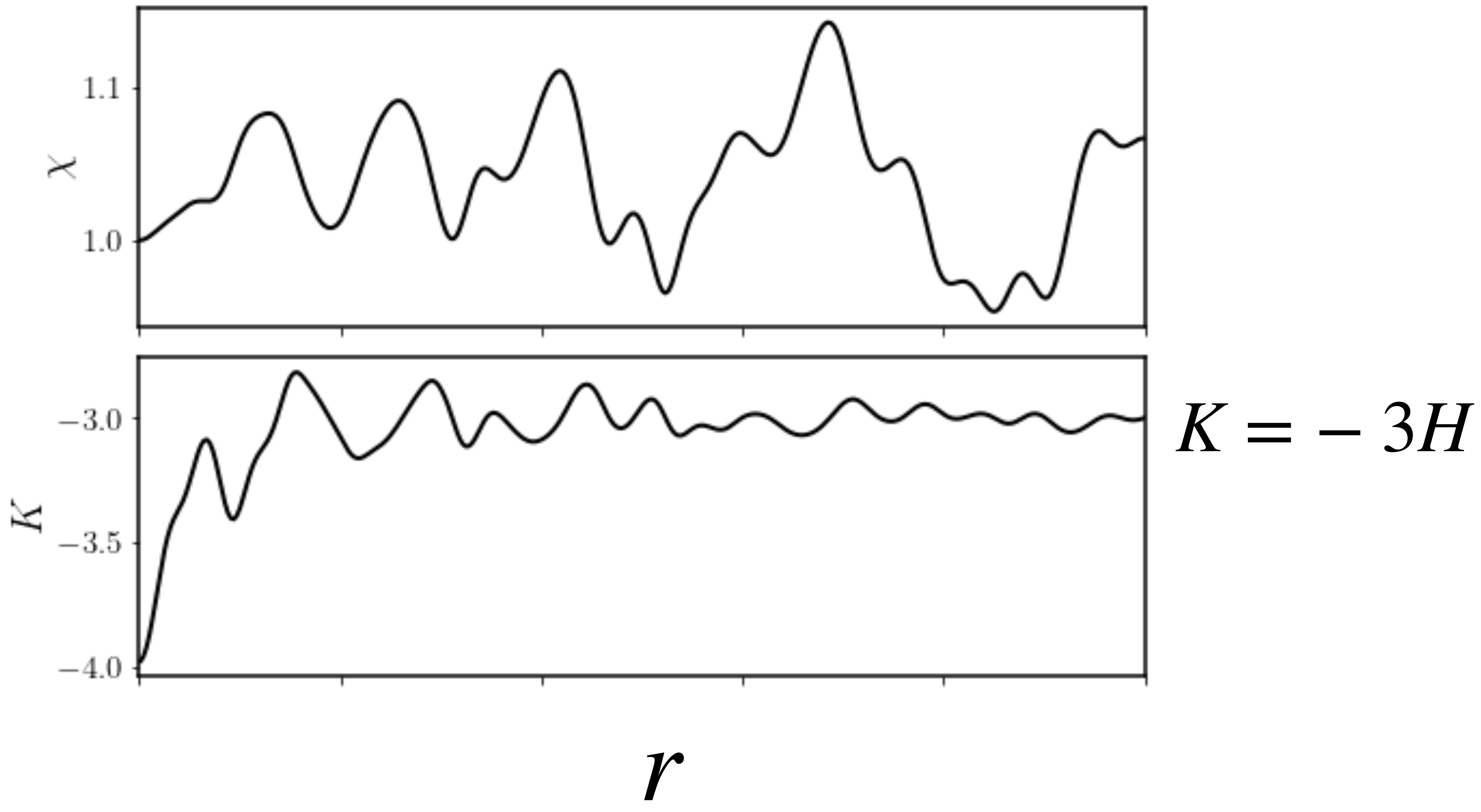


sphericalized

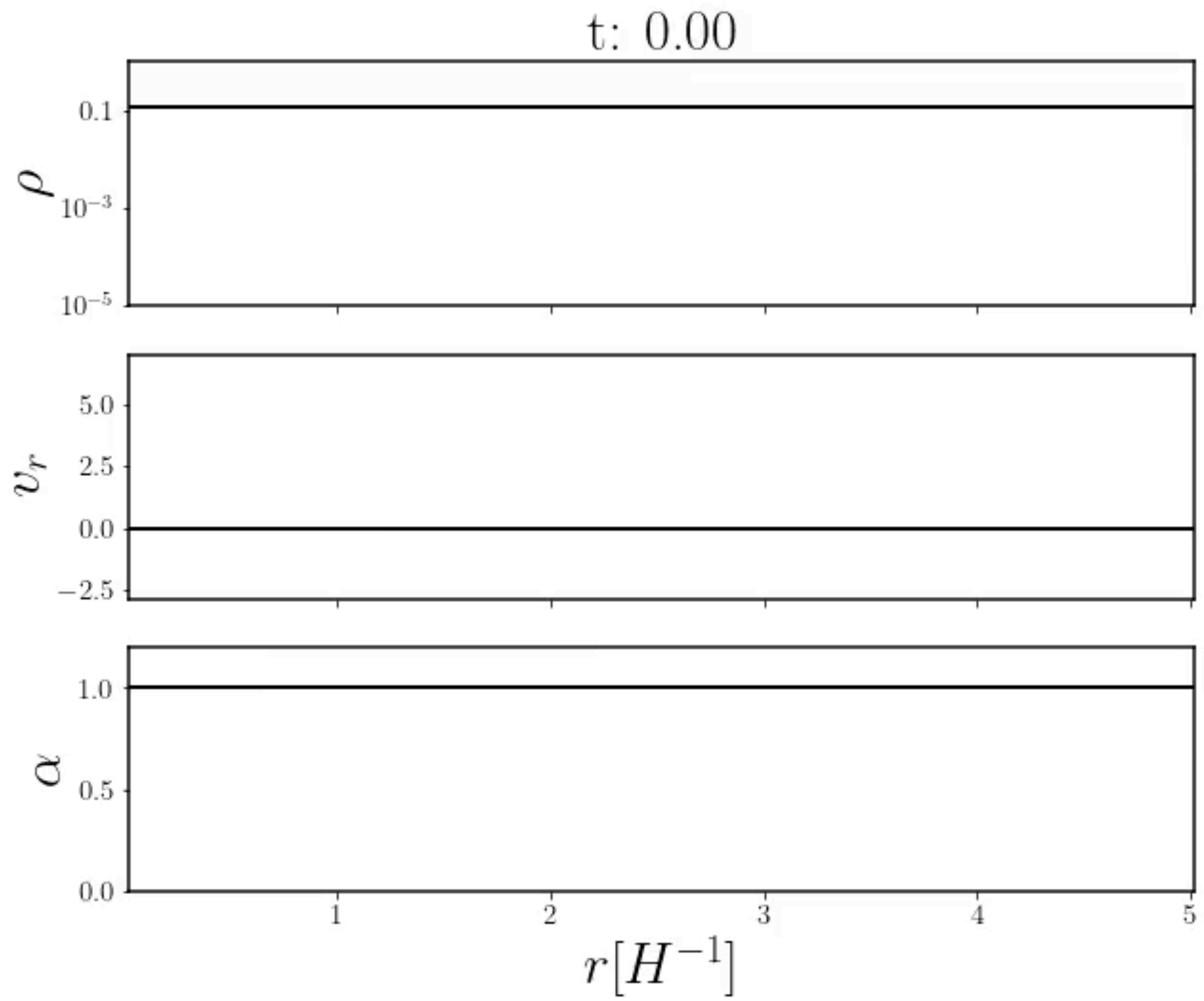


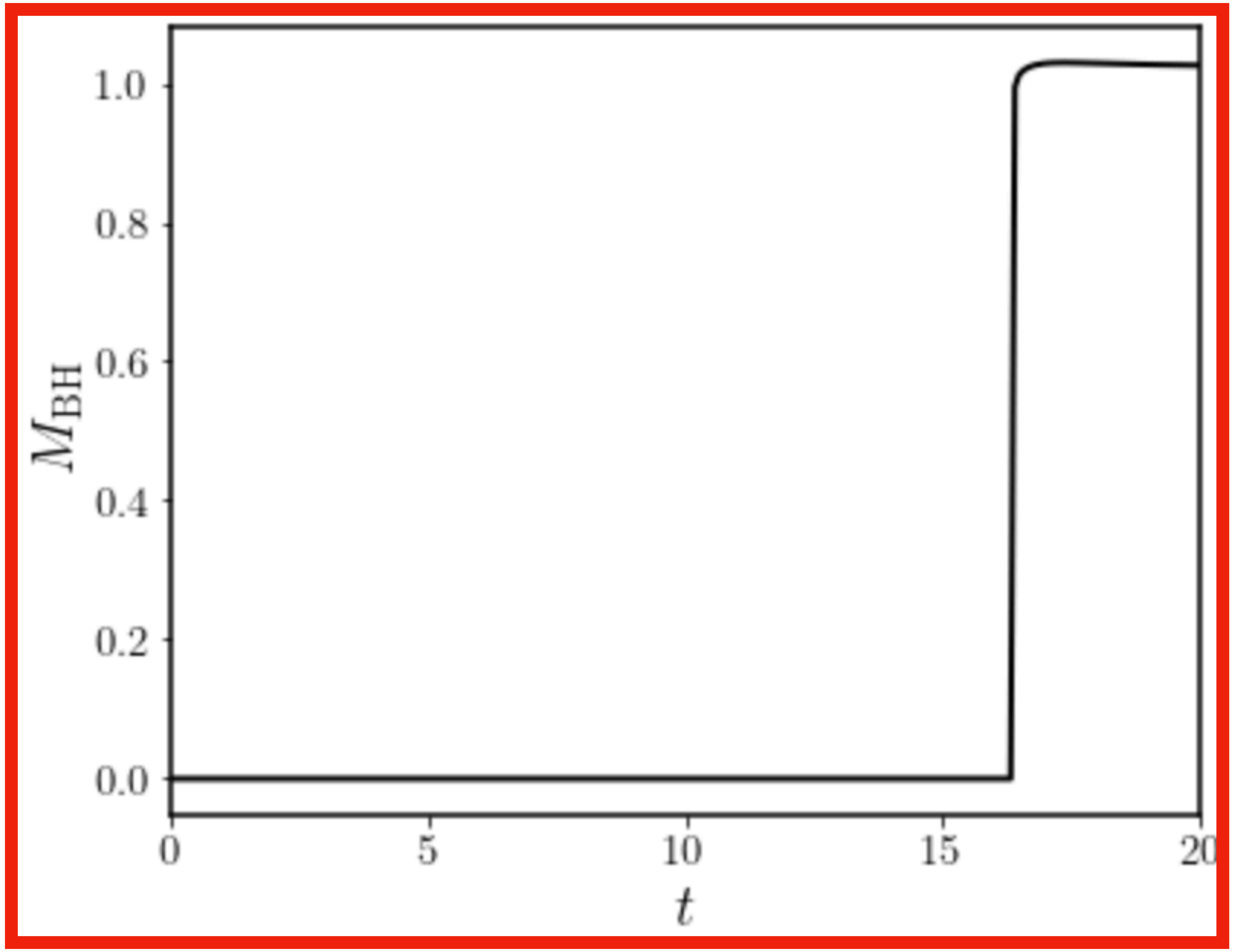
Initial conditions from inflation





Small ‘bias’ peak

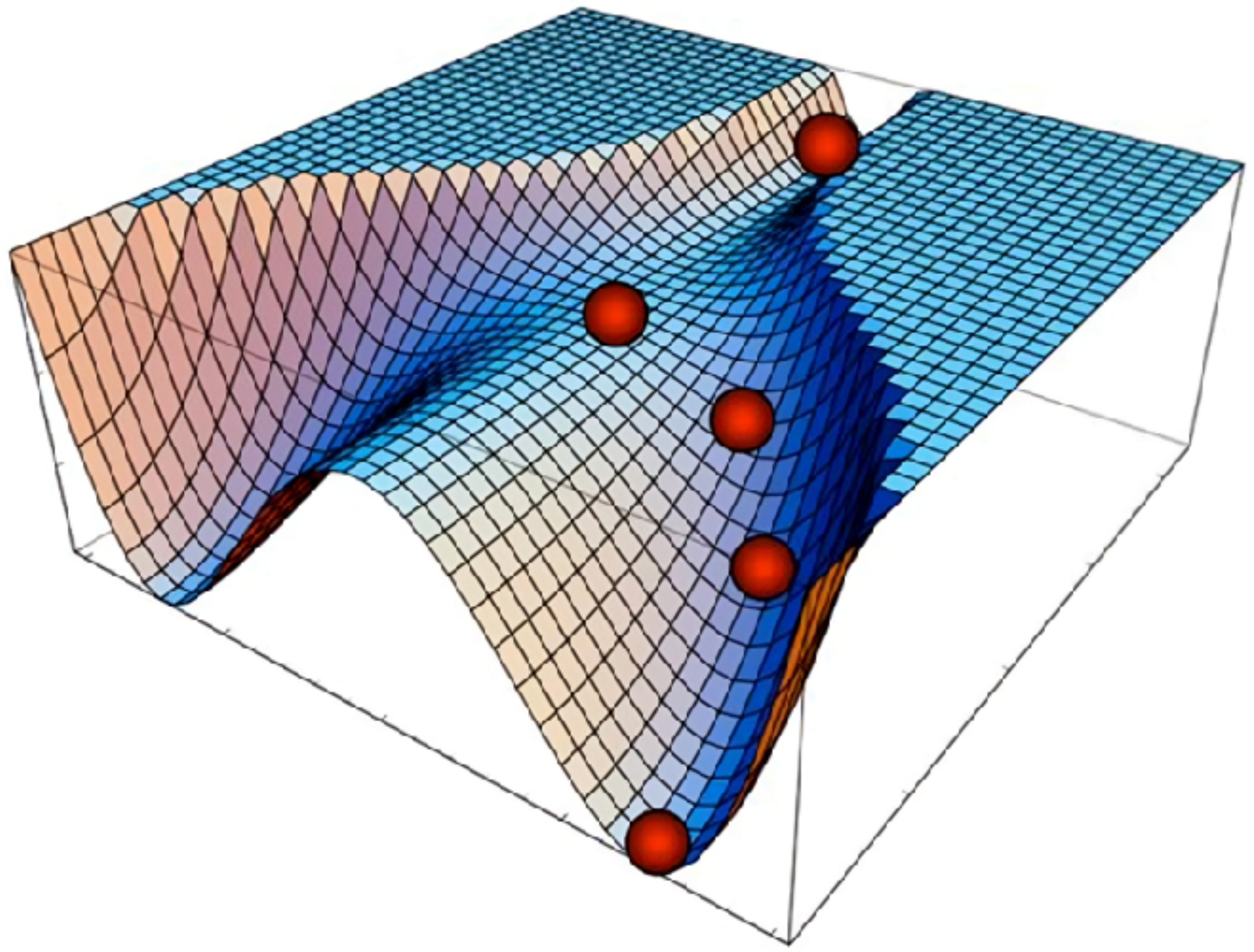




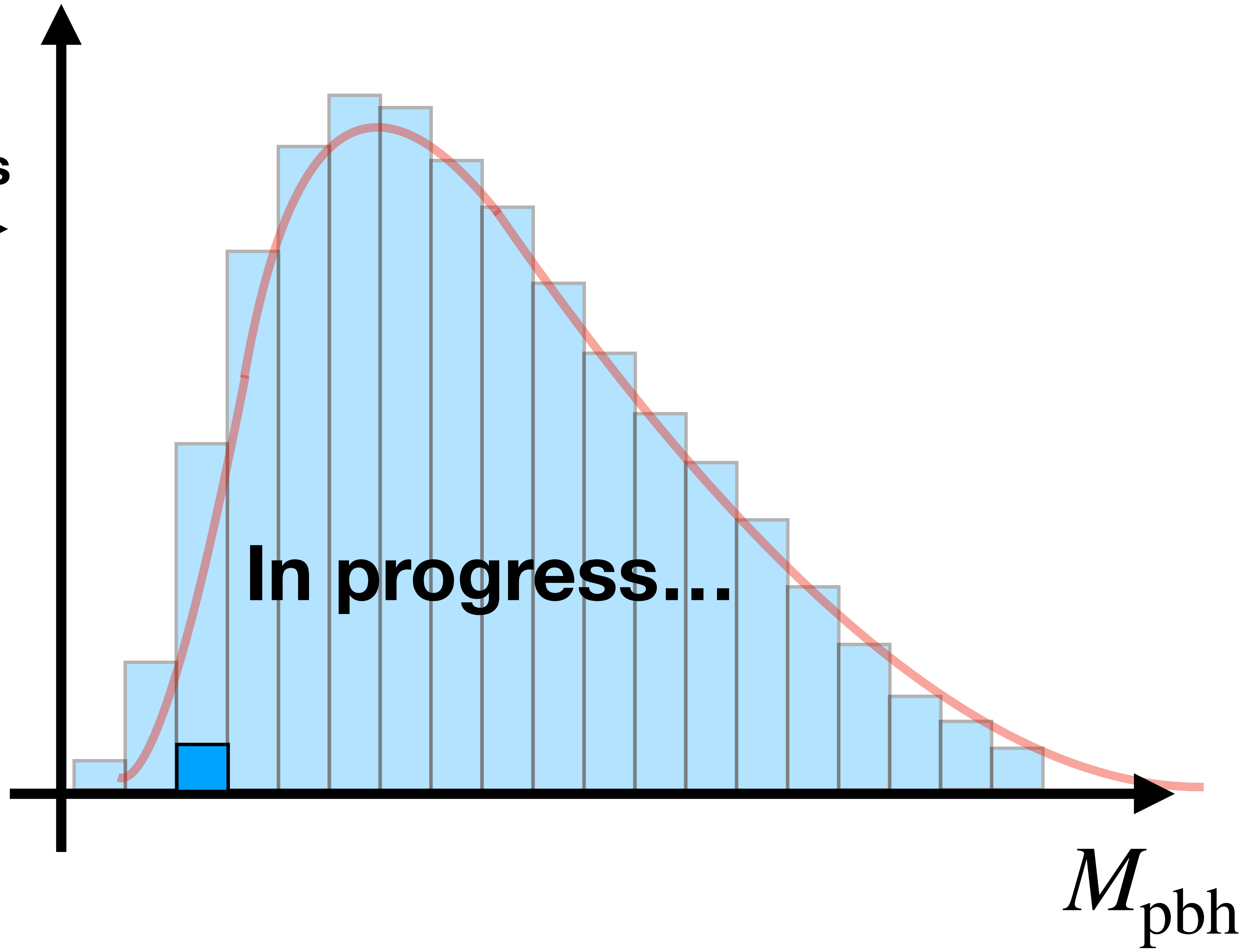
Stay tuned

Given an inflationary model

Obtain the BH mass distribution



Run $\mathcal{O}(10^5)$ simulations



For more see:
Cosmology using numerical relativity [2409.01939]

Cosmological singularities,
bouncing models,
bubble collisions,
phase transitions,
topological defects…

Thank you!

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