

On the IR behavior of QFTs in de Sitter (loops, logarithms and stochastic formalism)

Based on:

arXiv:2311.17644 (with Spyros Sypsas)

arXiv:2406.07610 (with Javier Huenupi, Ellie Hughes and Spyros Sypsas)

arXiv:2412.01891 (with Javier Huenupi, Ellie Hughes and Spyros Sypsas)

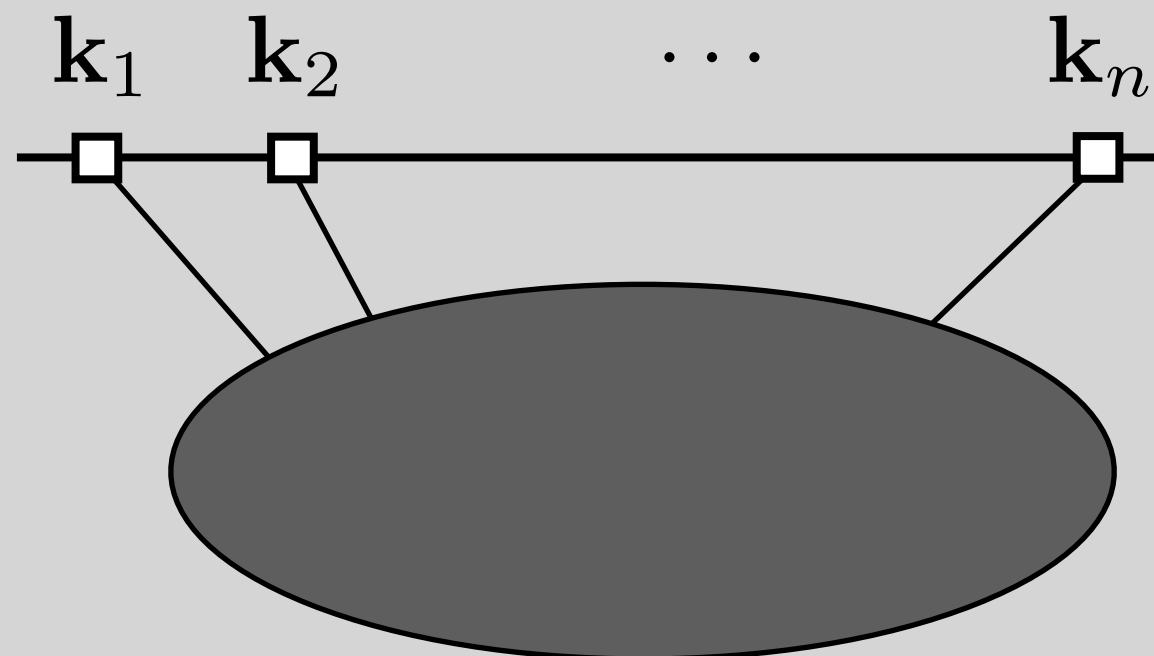
arXiv:2507.21310 (with Spyros Sypsas and Danilo Tapia)

Gonzalo A. Palma
FCFM, U. de Chile

Inflation 2025, Paris
December 5, 2025

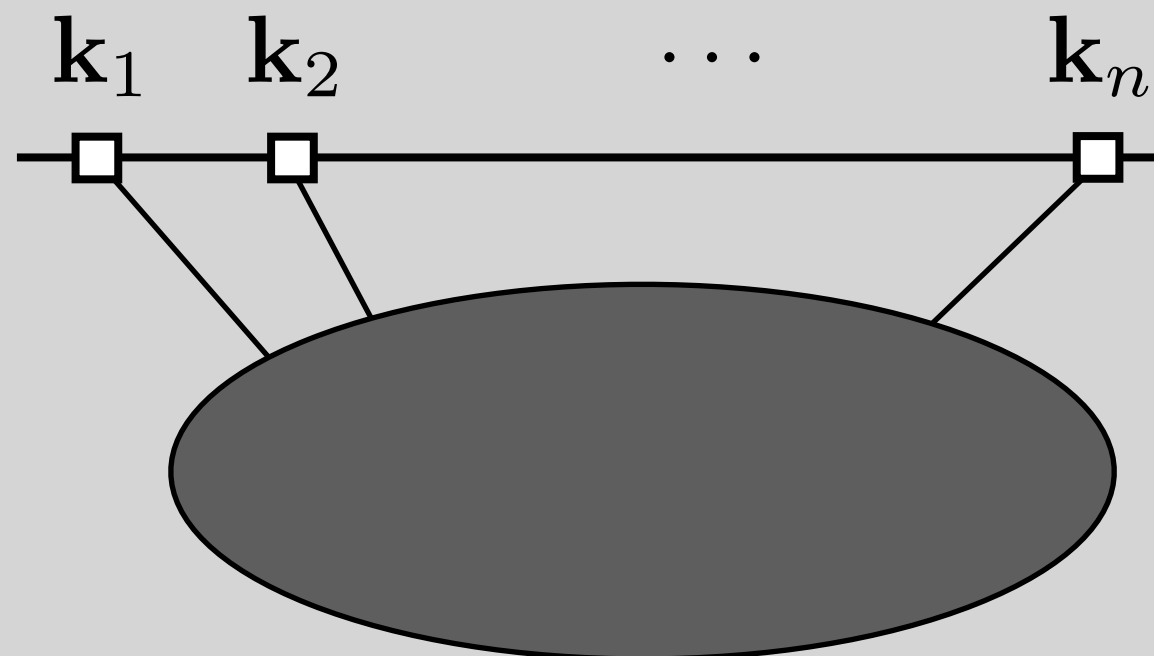
Perturbation theory

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$

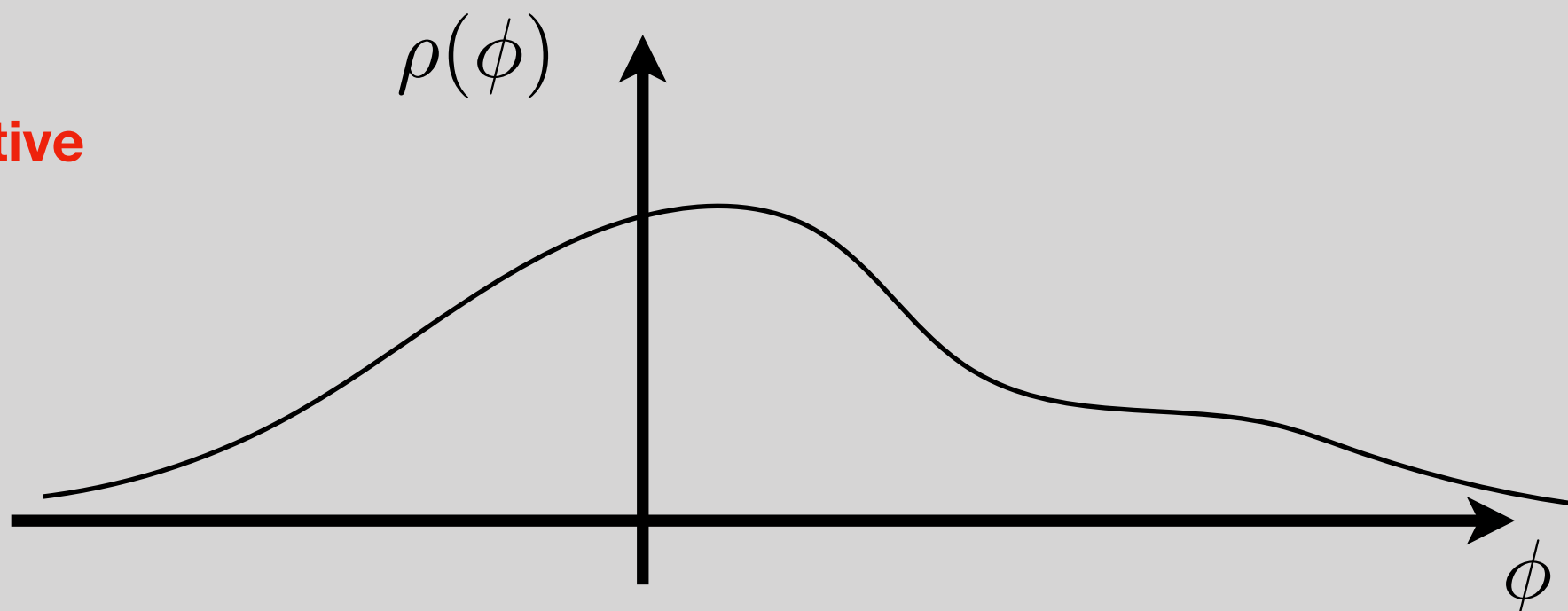


Perturbation theory

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$

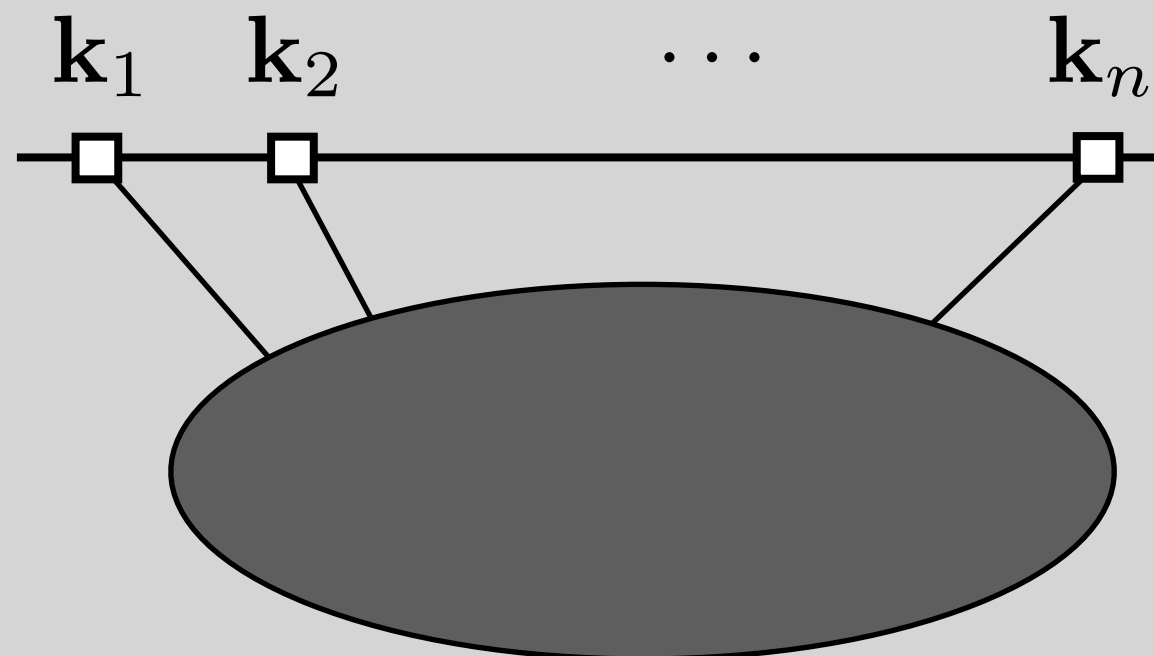


Non-perturbative approach



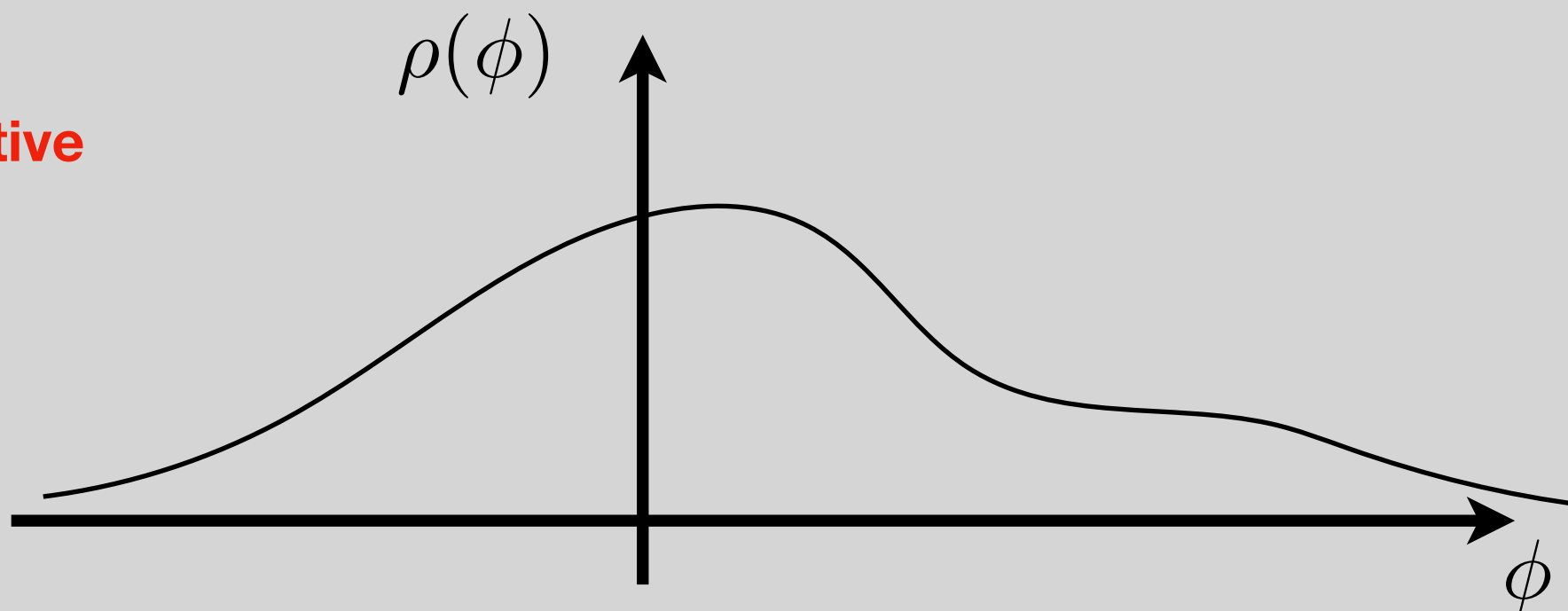
Perturbation theory

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$



Light fields / massless limit

Non-perturbative approach





Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**



Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**



Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**

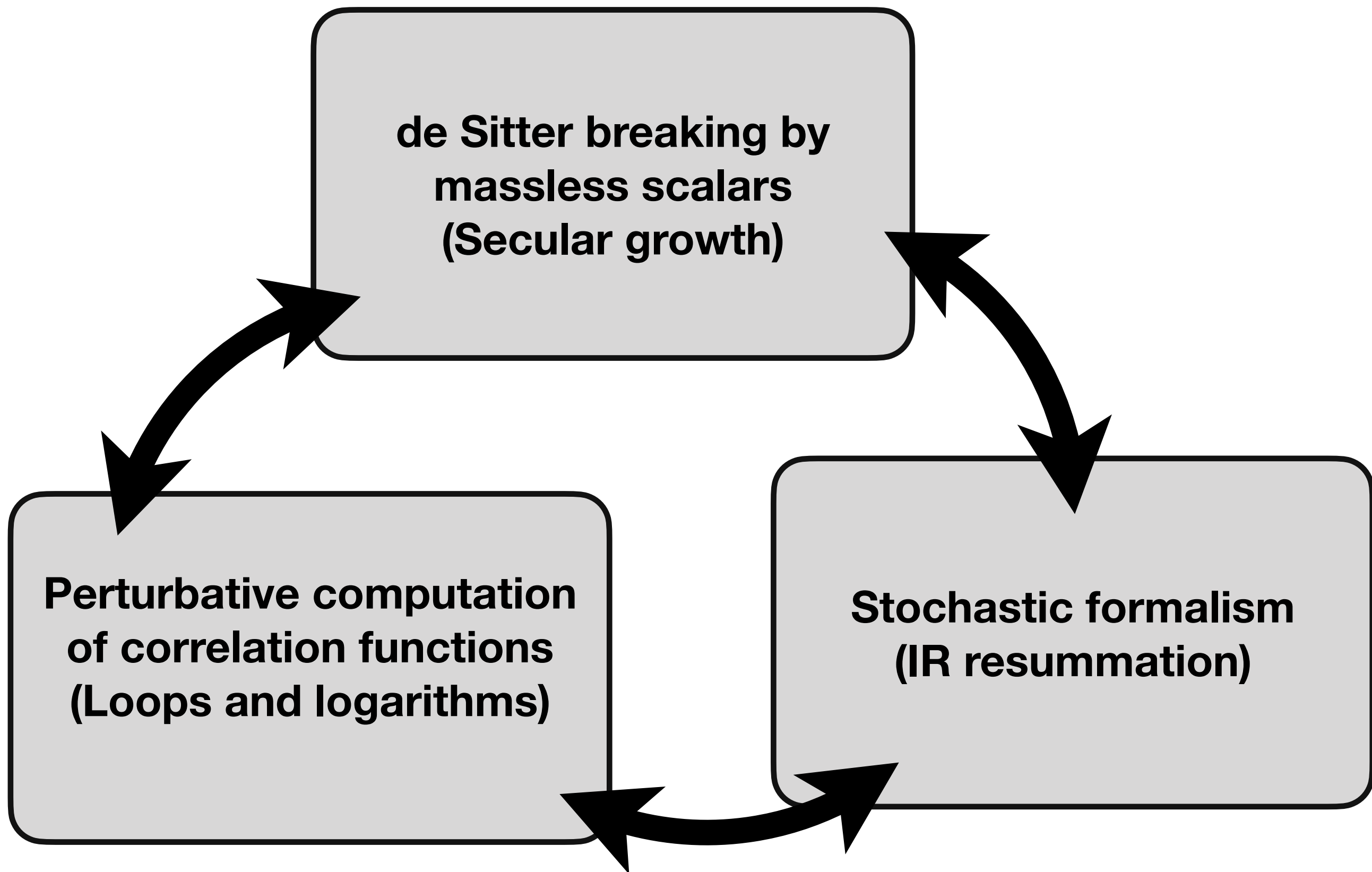
**Stochastic formalism
(IR resummation)**

Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**

**Stochastic formalism
(IR resummation)**



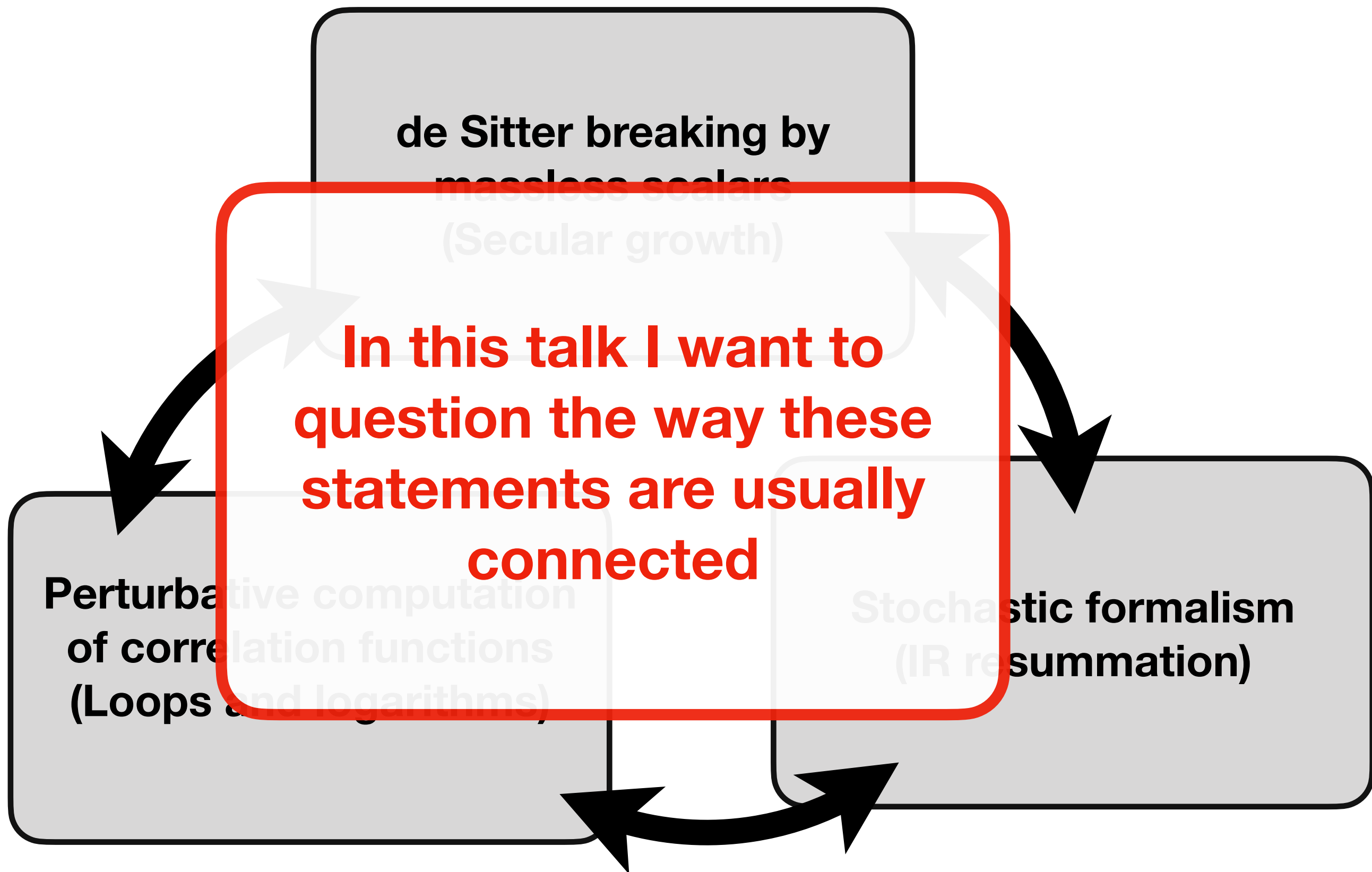
Preamble

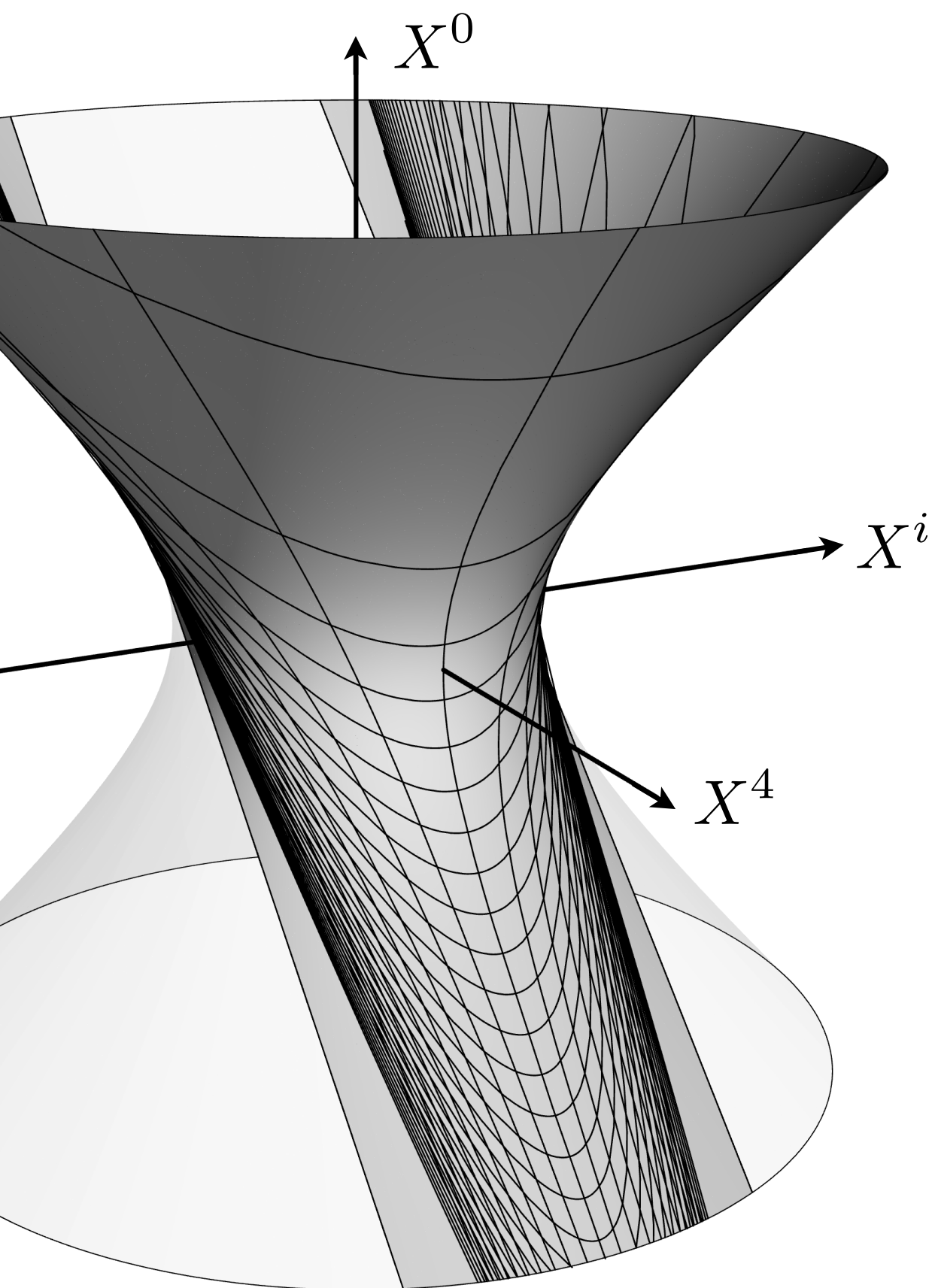
de Sitter breaking by
massless scalars
(Secular growth)

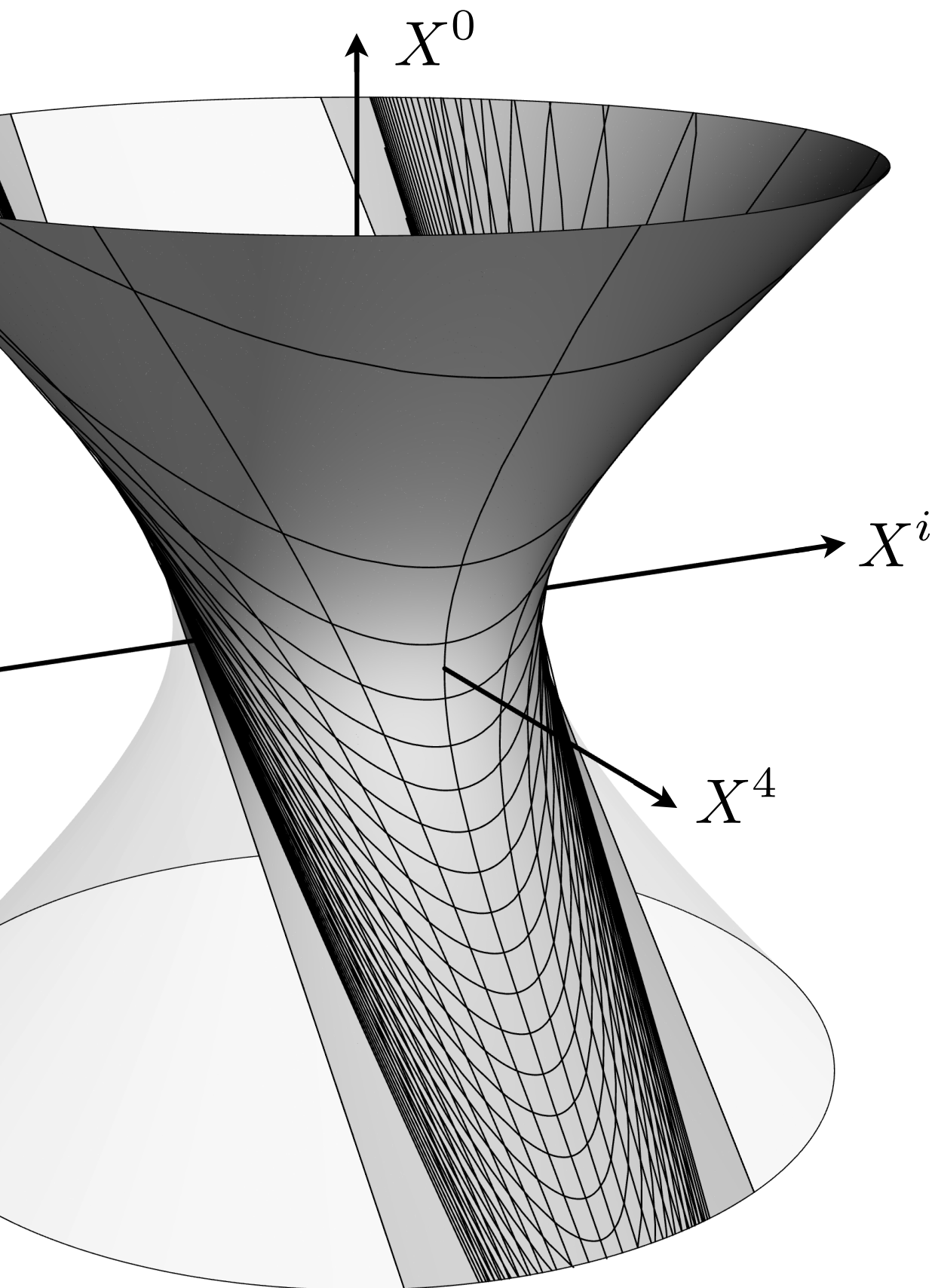
**In this talk I want to
question the way these
statements are usually
connected**

Perturbative computation
of correlation functions
(Loops and logarithms)

Stochastic formalism
(IR resummation)

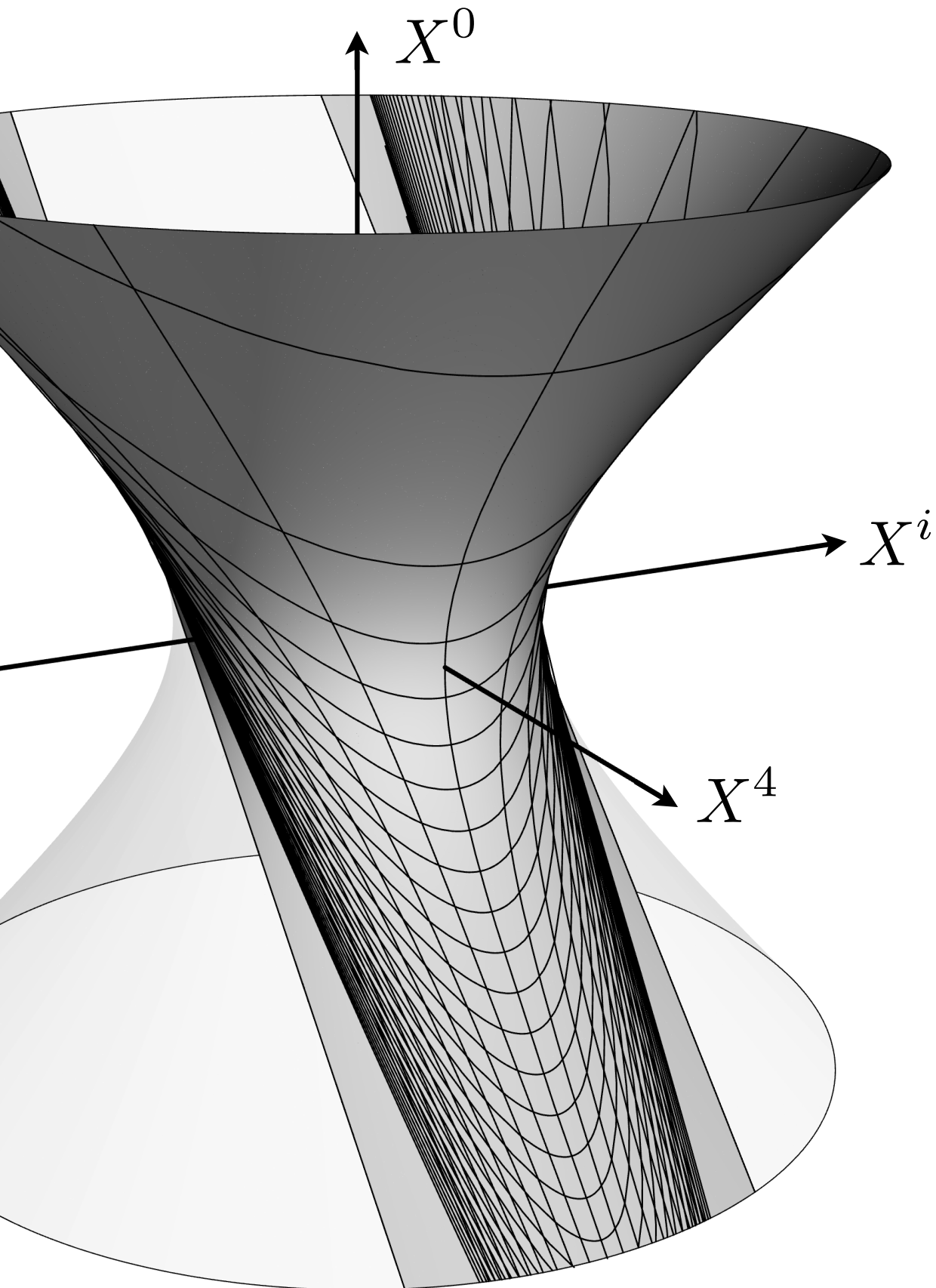






Metric in cosmological coordinates:

$$ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$
$$a(\tau) = -\frac{1}{H\tau}$$



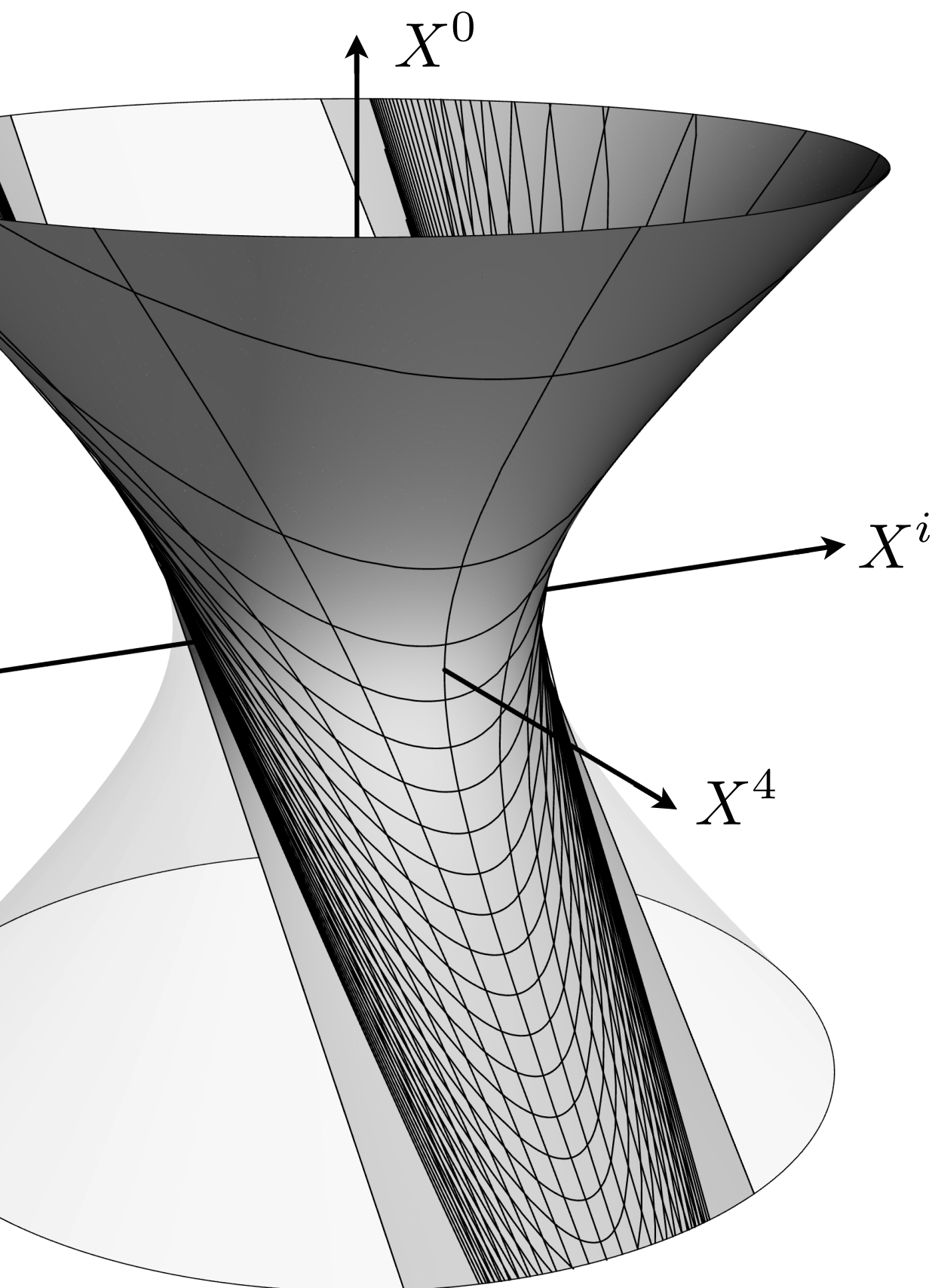
Metric in cosmological coordinates:

$$ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$
$$a(\tau) = -\frac{1}{H\tau}$$

Invariant under dS transformations:

For instance:

$$\tau \rightarrow \bar{\tau} = e^{-\theta} \tau$$
$$x^i \rightarrow \bar{x}^i = e^{-\theta} x^i$$



Metric in cosmological coordinates:

$$ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$

$$a(\tau) = -\frac{1}{H\tau}$$

Invariant under dS transformations:

For instance:

$$\tau \rightarrow \bar{\tau} = e^{-\theta} \tau$$

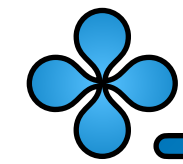
$$x^i \rightarrow \bar{x}^i = e^{-\theta} x^i$$

dS invariant combination

$$Z = 1 - \frac{|\mathbf{x} - \mathbf{x}'|^2 - (\tau - \tau')^2}{2\tau\tau'}$$

Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$



Let's consider the computation of **correlation functions** for a light scalar in dS:

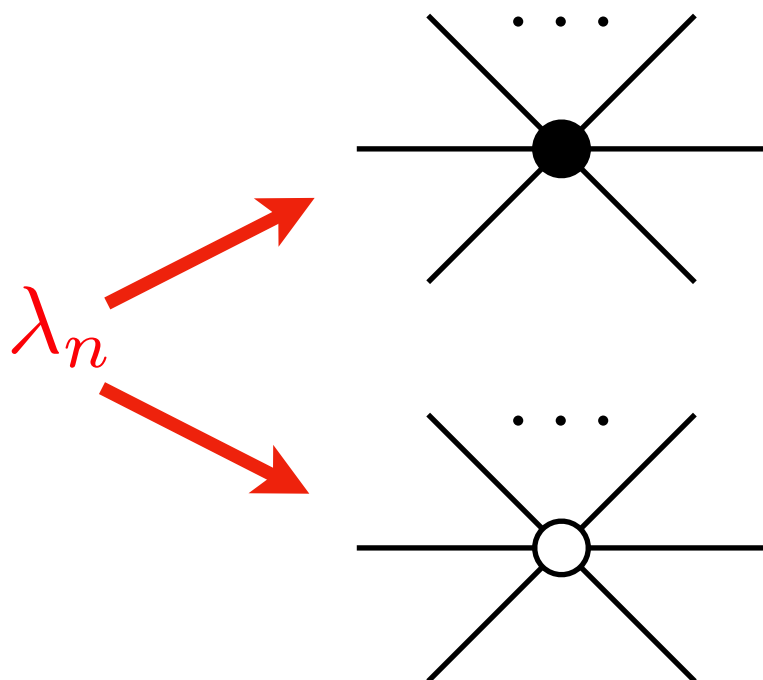
$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Schwinger-Keldysh formalism:

See: Chen, Wang & Xianyu (2017)

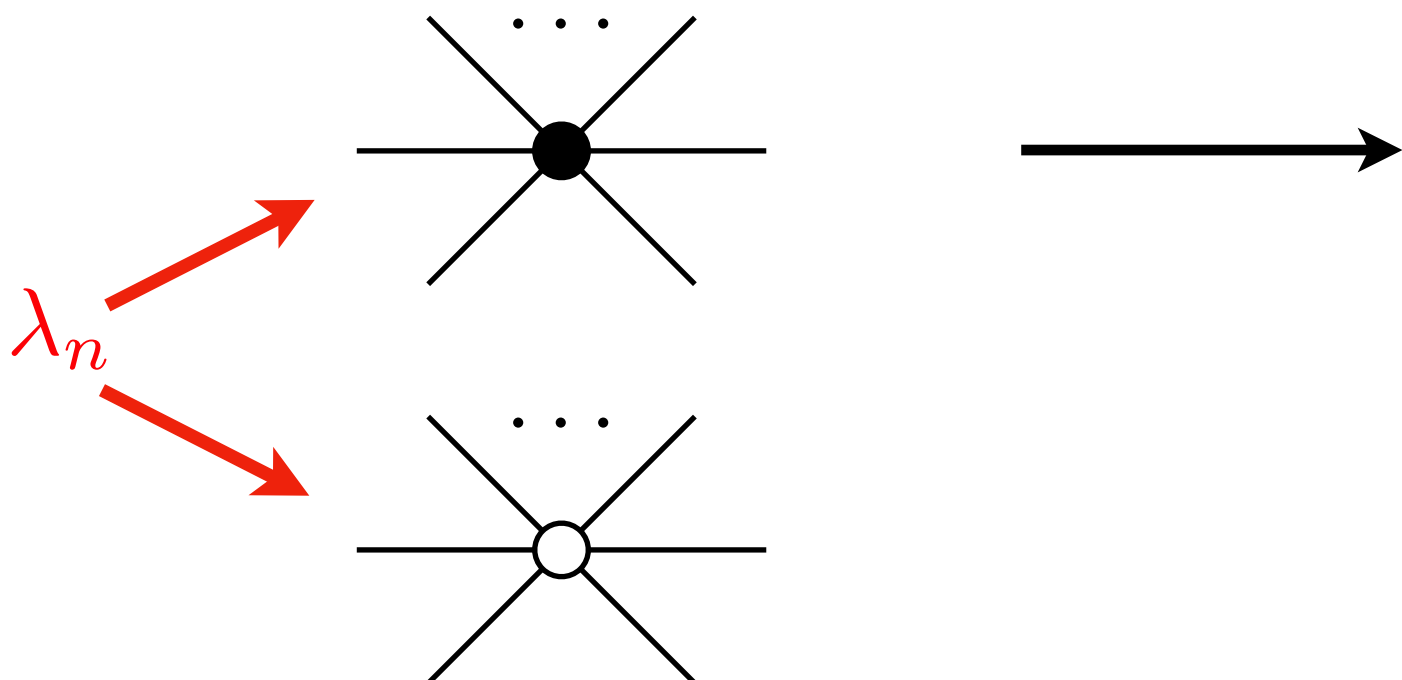


Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Schwinger-Keldysh formalism:

See: Chen, Wang & Xianyu (2017)



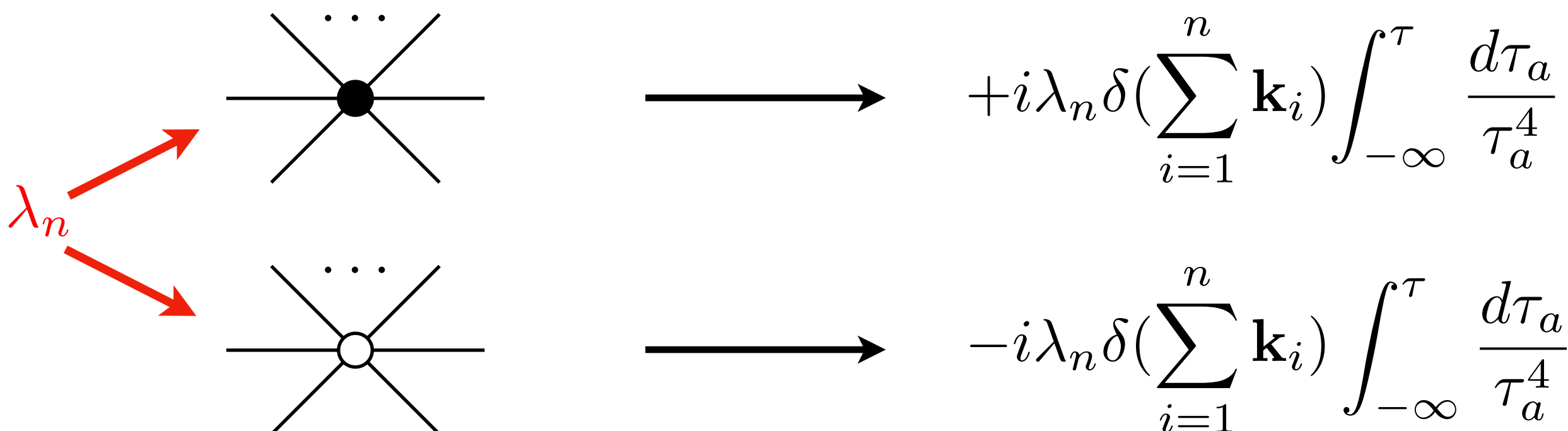
$$+i\lambda_n \delta\left(\sum_{i=1}^n \mathbf{k}_i\right) \int_{-\infty}^{\tau} \frac{d\tau_a}{\tau_a^4}$$

Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Schwinger-Keldysh formalism:

See: Chen, Wang & Xianyu (2017)

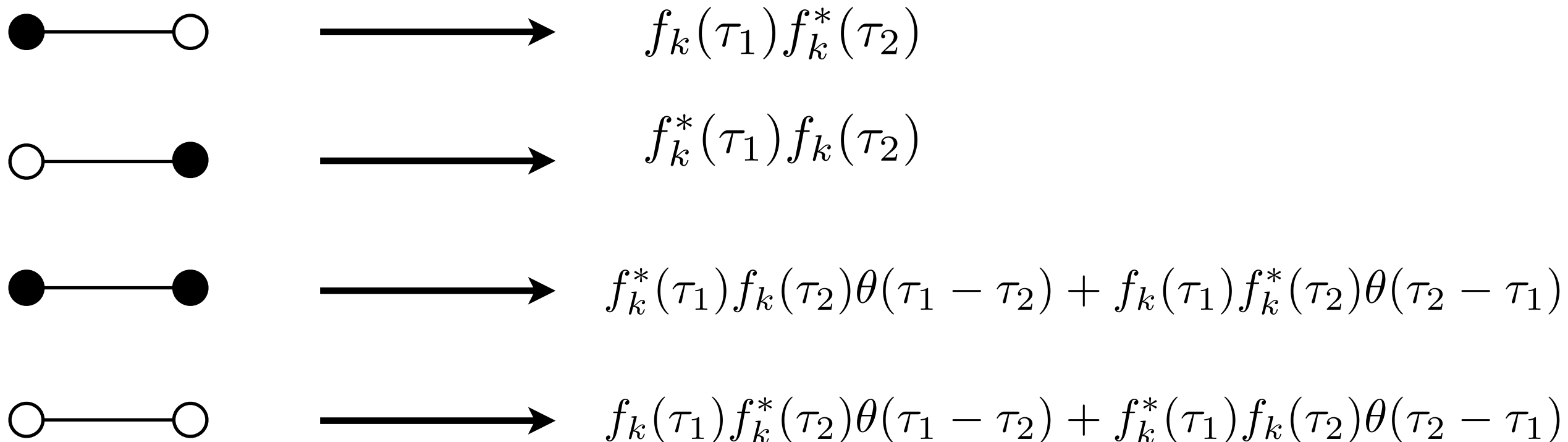


Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Schwinger-Keldysh formalism:

See: Chen, Wang & Xianyu (2017)



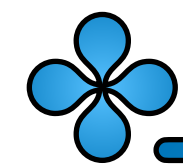
Let's consider the computation of **correlation functions** for a light scalar in dS:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \sum_n \frac{\lambda_n}{n!} \varphi^n \right]$$

Schwinger-Keldysh formalism:

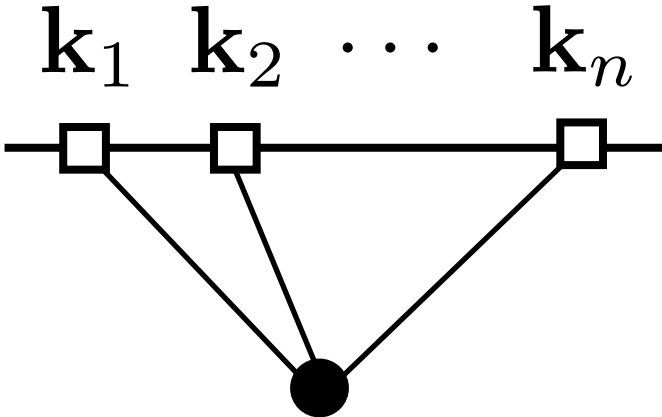
See: Chen, Wang & Xianyu (2017)

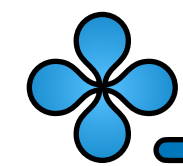
$$\begin{array}{ll} \bullet \text{---} \square & \longrightarrow f_k(\tau_1) f_k^*(\tau) \\ \circ \text{---} \square & \longrightarrow f_k^*(\tau_1) f_k(\tau) \end{array}$$



Correlation functions

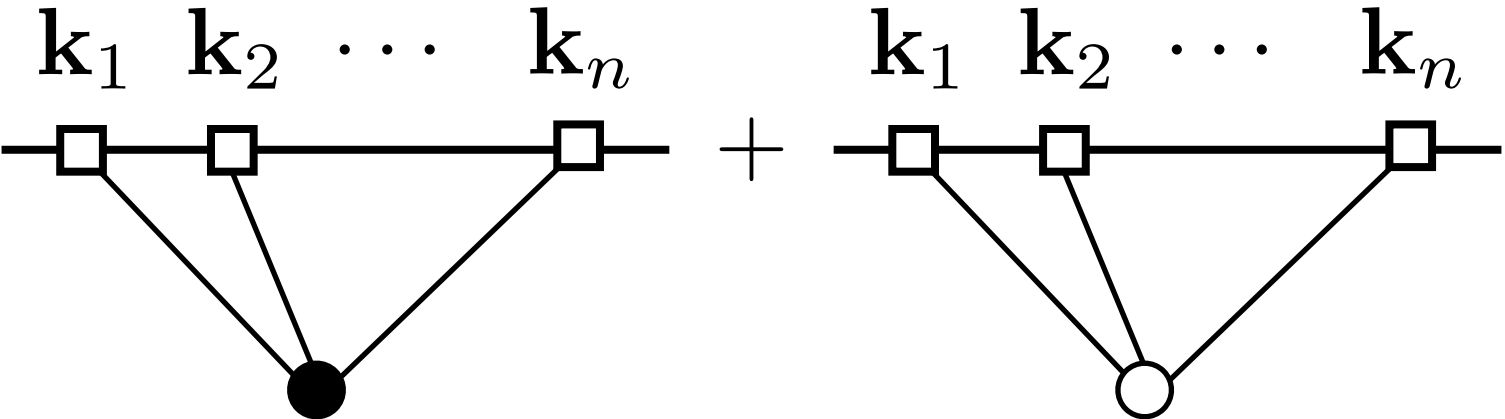
05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$




Correlation functions

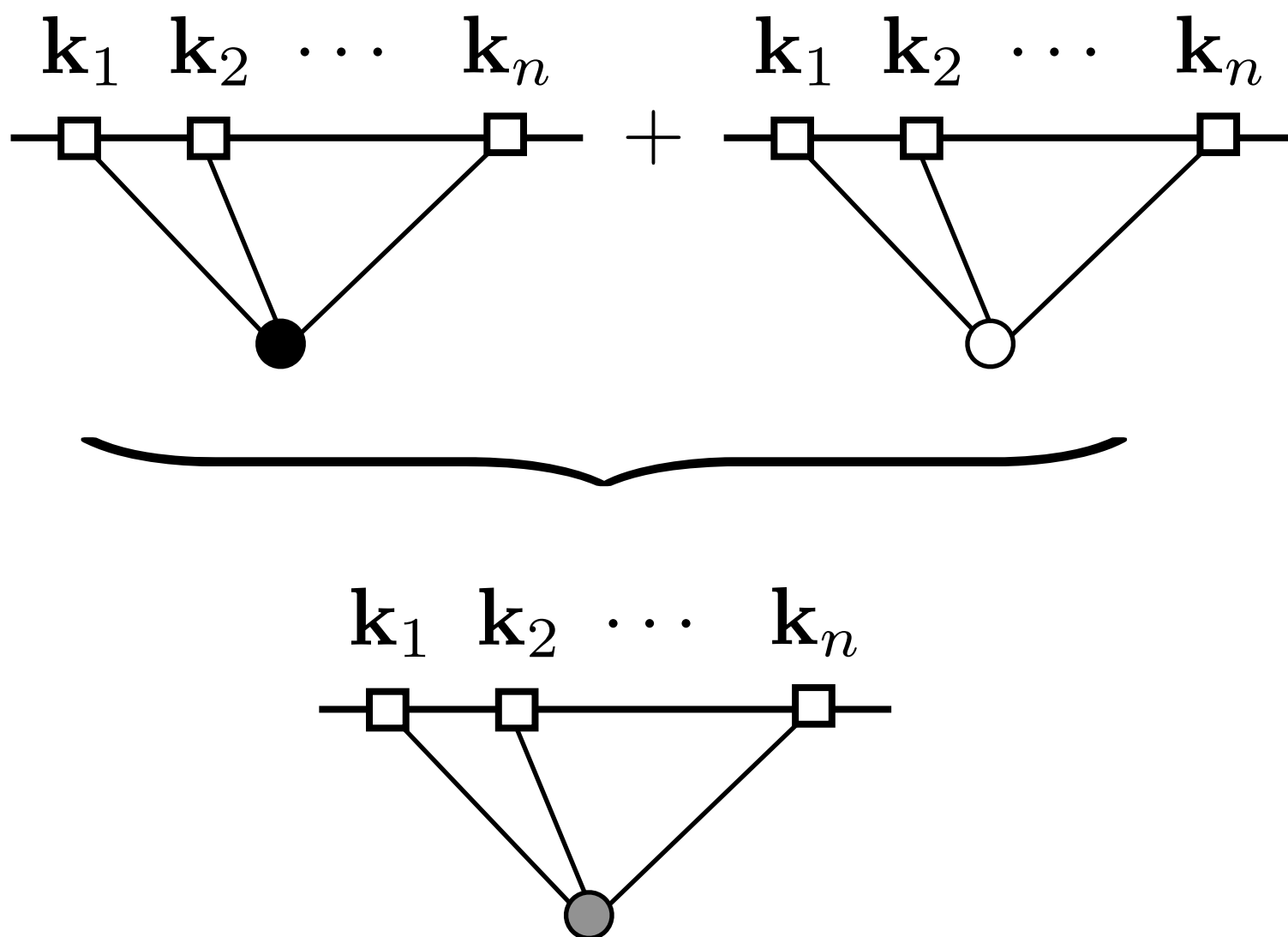
05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The diagram shows two Feynman diagrams separated by a plus sign. Each diagram consists of a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \dots \mathbf{k}_n . From the \mathbf{k}_1 and \mathbf{k}_2 vertices, two lines extend downwards to a common vertex. In the first diagram, this vertex is a solid black circle. In the second diagram, it is an open circle.

Correlation functions

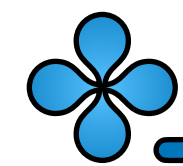
05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The diagram illustrates the calculation of correlation functions using Feynman diagrams. It shows two terms in a sum, which are then grouped by a brace and equated to a single term below.

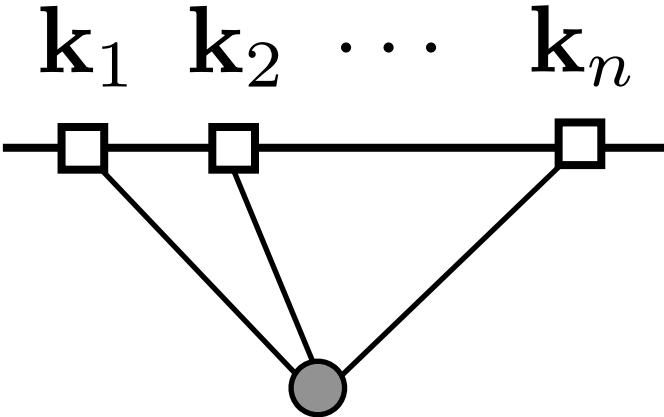
The first term shows a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . These vertices are connected to a solid black circle below the line. The second term is identical but the circle is white.

A large brace groups these two terms, and below it, a single diagram is shown with a gray circle connected to the same three vertices.



Correlation functions

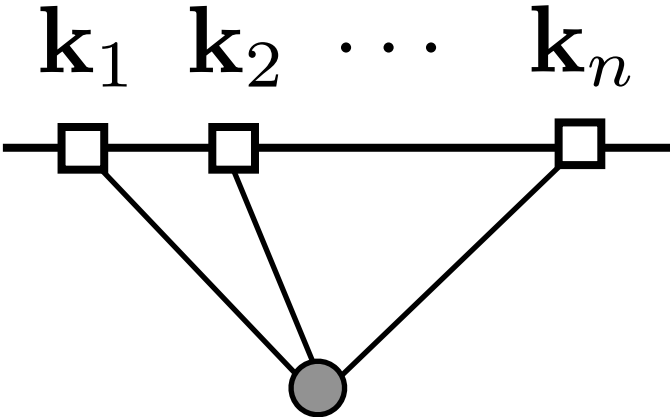
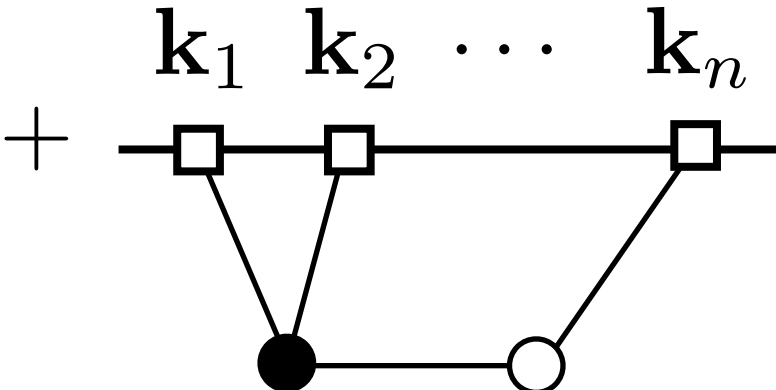
05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The diagram shows a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . These vertices are connected to a single shaded circular vertex below the line, representing a correlation function.

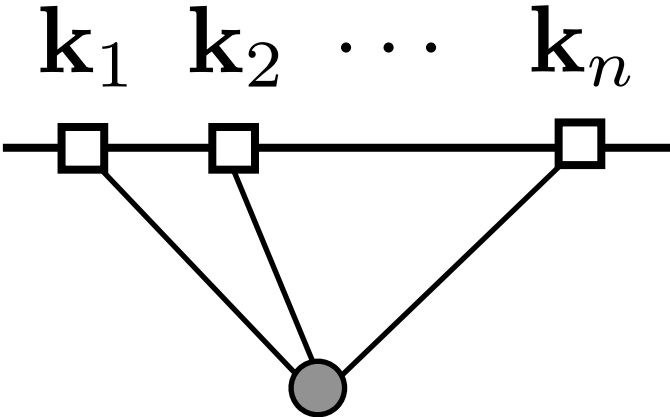
Correlation functions

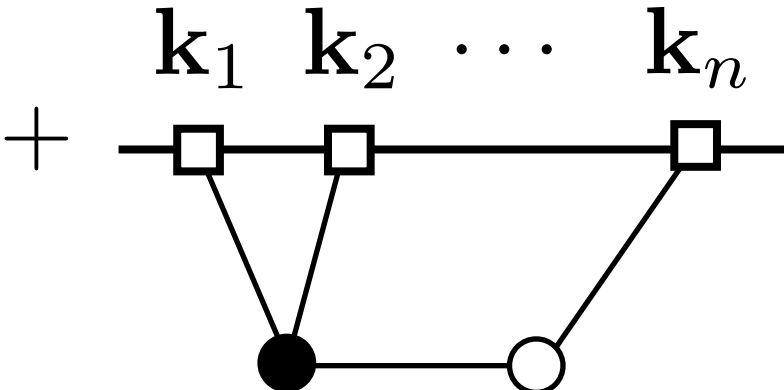
05

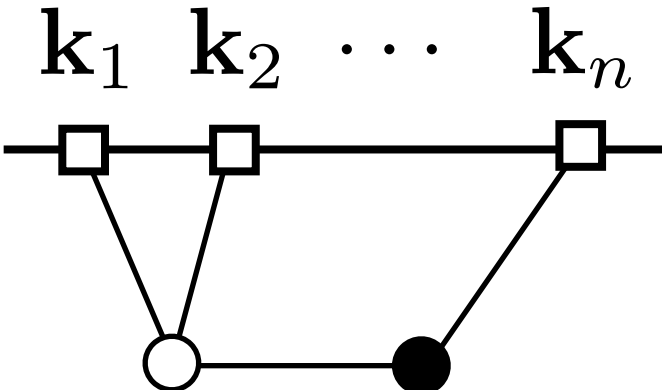
$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$

$$+$$


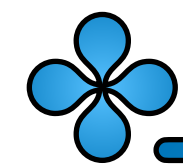
Correlation functions

05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


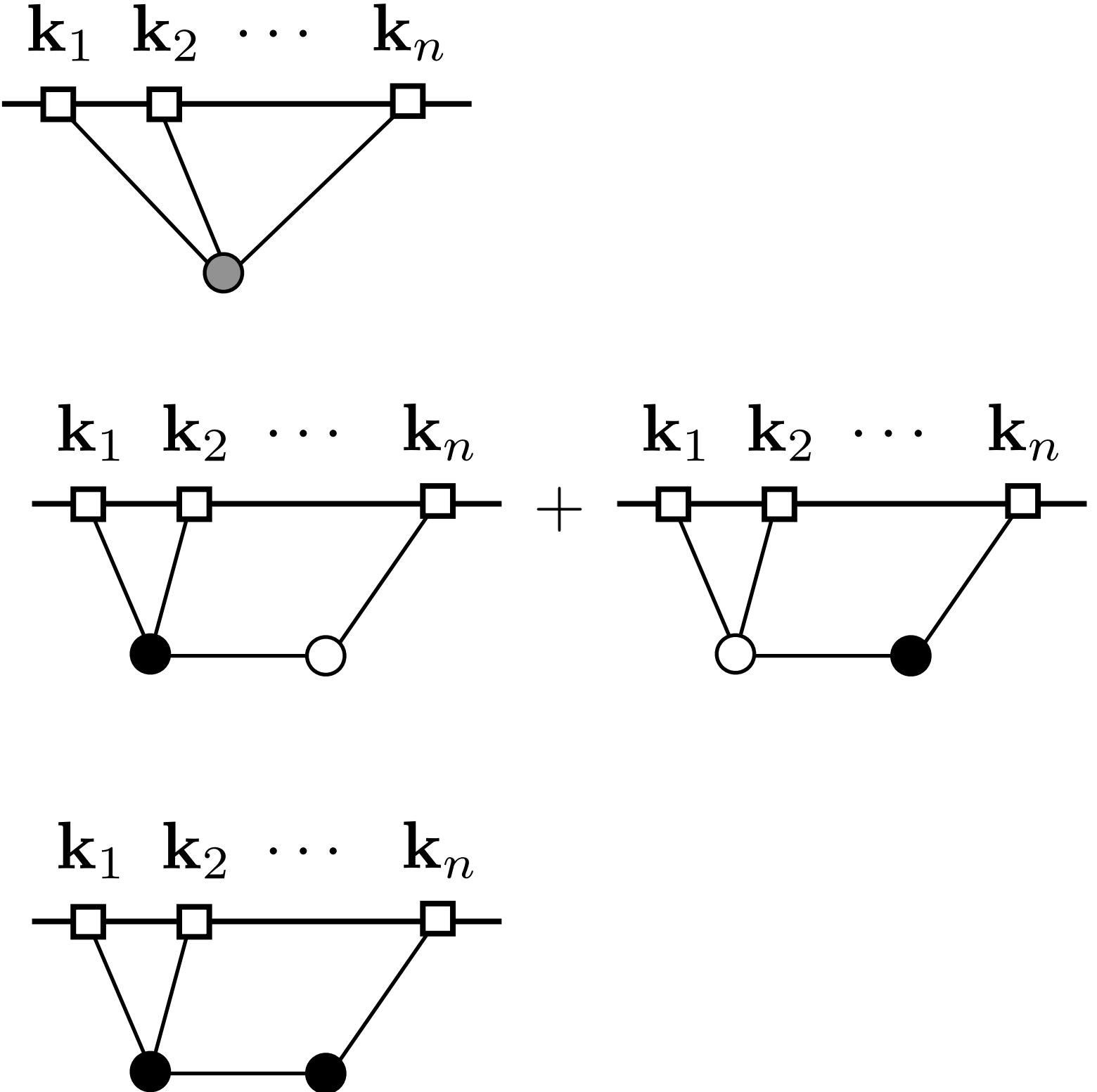
$$+ \quad$$


$$+ \quad$$




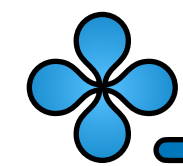
Correlation functions

05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


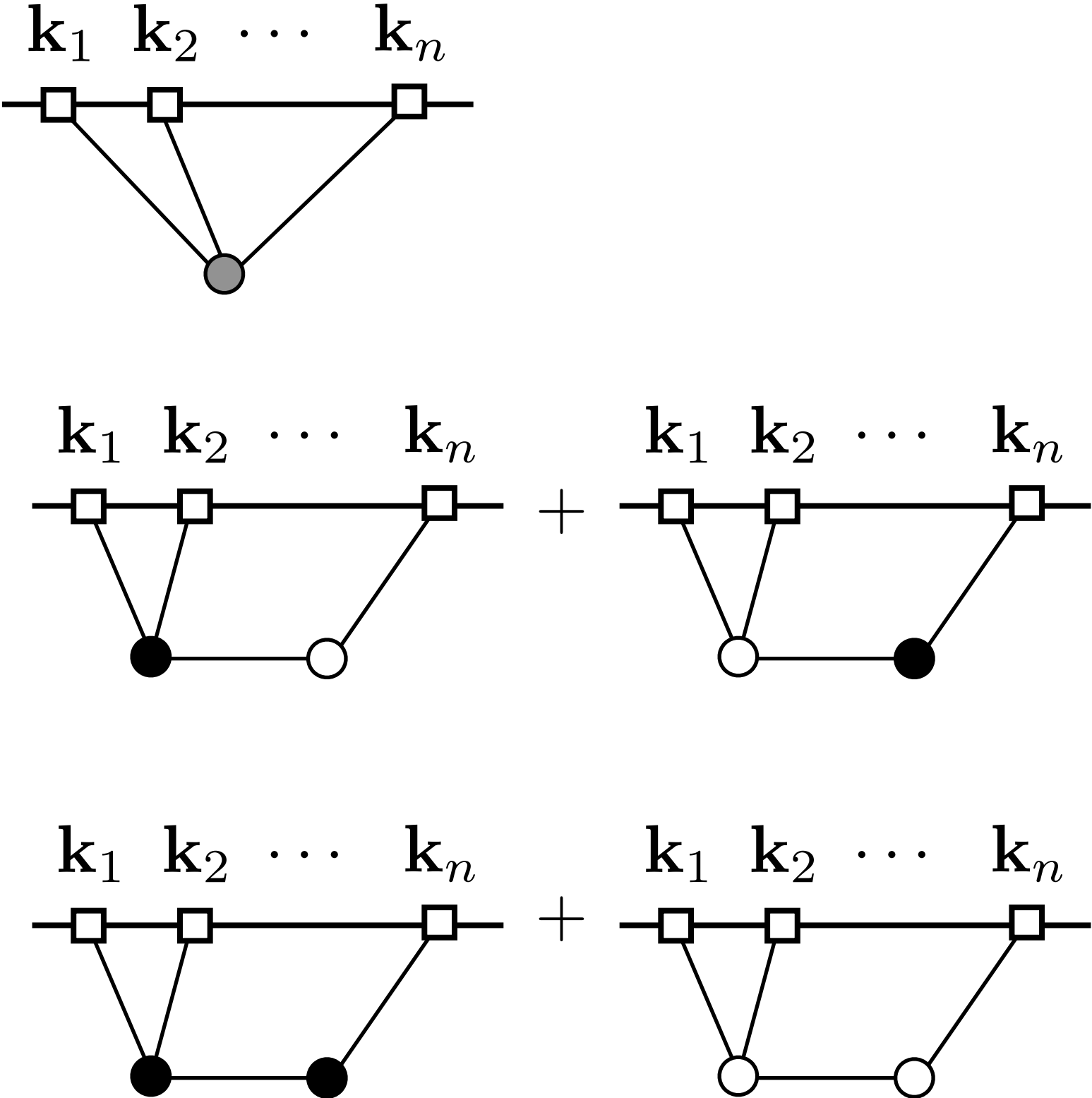
The equation shows the expansion of the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$ into a sum of Feynman diagrams. Each diagram consists of a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . Below the horizontal line, vertices are connected by lines, and each vertex is also connected to the horizontal line by a diagonal line.

- The first diagram has a single gray circular vertex connected to all three square vertices.
- The second diagram has two circular vertices connected by a horizontal line. The left vertex is black and connected to \mathbf{k}_1 and \mathbf{k}_2 . The right vertex is white and connected to \mathbf{k}_n .
- The third diagram is identical to the second, but the left vertex is white and the right vertex is black.
- The fourth diagram has two black circular vertices connected by a horizontal line. The left vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the right vertex is connected to \mathbf{k}_n .



Correlation functions

05

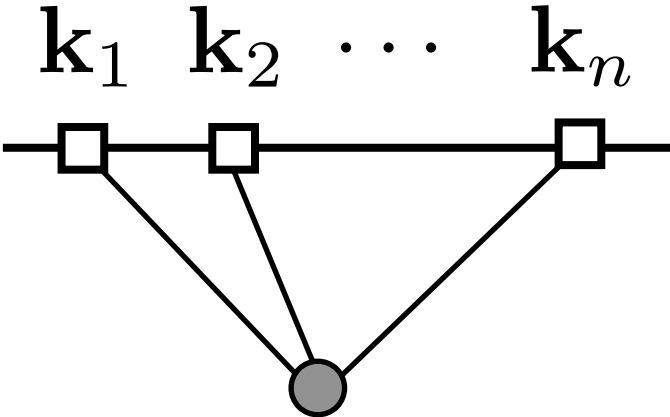
$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows the expansion of the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$ into a sum of five Feynman diagrams. Each diagram consists of a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n from left to right. The diagrams are:

- 1. A single vertex (gray circle) connected to all three square vertices.
- 2. Two vertices (one black, one white) connected to the square vertices: the black vertex connects to \mathbf{k}_1 and \mathbf{k}_2 , and the white vertex connects to \mathbf{k}_2 and \mathbf{k}_n .
- 3. Two vertices (one white, one black) connected to the square vertices: the white vertex connects to \mathbf{k}_1 and \mathbf{k}_2 , and the black vertex connects to \mathbf{k}_2 and \mathbf{k}_n .
- 4. Two vertices (both black) connected to the square vertices: the first black vertex connects to \mathbf{k}_1 and \mathbf{k}_2 , and the second black vertex connects to \mathbf{k}_2 and \mathbf{k}_n .
- 5. Two vertices (both white) connected to the square vertices: the first white vertex connects to \mathbf{k}_1 and \mathbf{k}_2 , and the second white vertex connects to \mathbf{k}_2 and \mathbf{k}_n .

Correlation functions

05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


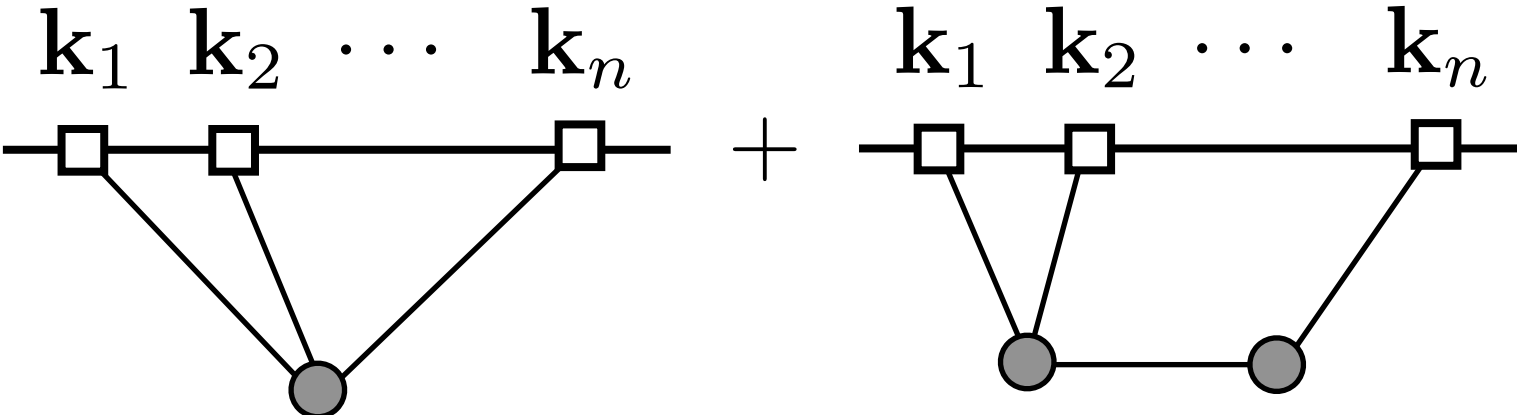
$$\left\{ \begin{array}{l} + \text{ (diagram with one black and one white vertex) } + \text{ (diagram with one white and one black vertex) } \\ + \text{ (diagram with two black vertices) } + \text{ (diagram with two white vertices) } \end{array} \right.$$

The diagrams in the curly braces represent the following structures:

- Diagram 1: A horizontal line with three square vertices labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The first and second squares connect to a single black circular vertex below. The third square connects to a white circular vertex further to the right. These two vertices are connected by a horizontal line.
- Diagram 2: A horizontal line with three square vertices labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The first and second squares connect to a single white circular vertex below. The third square connects to a black circular vertex further to the right. These two vertices are connected by a horizontal line.
- Diagram 3: A horizontal line with three square vertices labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The first and second squares connect to a single black circular vertex below. The third square connects to another black circular vertex further to the right. These two vertices are connected by a horizontal line.
- Diagram 4: A horizontal line with three square vertices labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The first and second squares connect to a single white circular vertex below. The third square connects to another white circular vertex further to the right. These two vertices are connected by a horizontal line.

Correlation functions

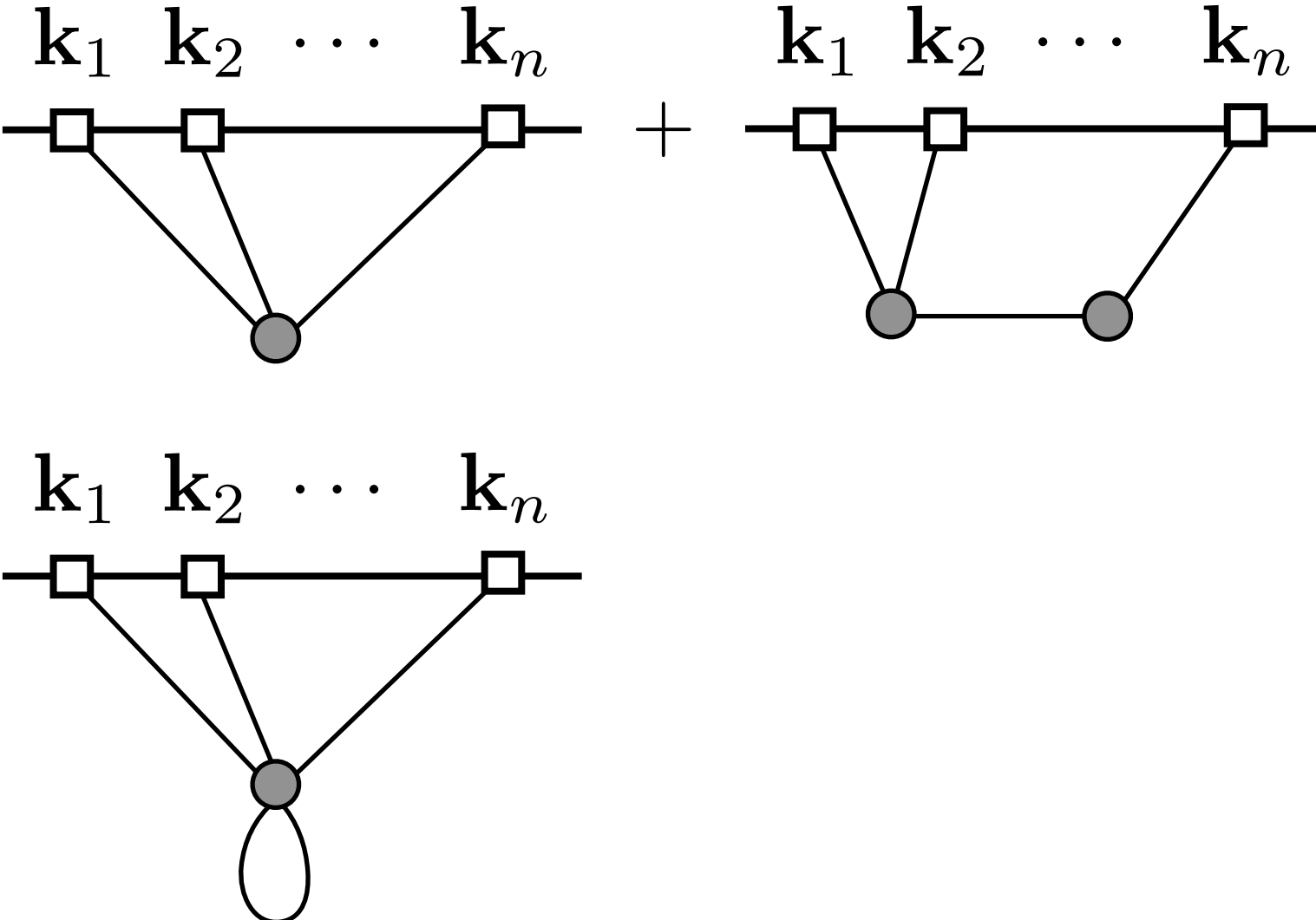
05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The first diagram shows a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . A single circle vertex is connected to all three square vertices. The second diagram shows a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . Two circle vertices are connected to the square vertices: the first circle vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the second circle vertex is connected to \mathbf{k}_2 and \mathbf{k}_n . The two circle vertices are also connected to each other.

✿ Correlation functions

05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows three Feynman diagrams representing the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$. Each diagram consists of a horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . The diagrams are separated by plus signs.

- Diagram 1 (Top Left):** A horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . A single vertex (gray circle) is connected to all three square vertices.
- Diagram 2 (Top Right):** A horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . Two vertices (gray circles) are connected to the square vertices: the first vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the second vertex is connected to \mathbf{k}_2 and \mathbf{k}_n . The two vertices are also connected to each other by a horizontal line.
- Diagram 3 (Bottom):** A horizontal line with three square vertices labeled \mathbf{k}_1 , \mathbf{k}_2 , and \mathbf{k}_n . A single vertex (gray circle) is connected to all three square vertices. This vertex has a self-loop (a line that starts and ends at the same vertex).

Correlation functions

05

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$

The diagrams illustrate the expansion of the correlation function into a sum of terms, each represented by a Feynman diagram. The diagrams show the interaction between the fields $\varphi_{\mathbf{k}_1}(\tau)$ and $\varphi_{\mathbf{k}_n}(\tau)$ through various internal lines and vertices.

Correlation functions

05

$$\begin{aligned}
 \langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle = & \text{Diagram 1} + \text{Diagram 2} \\
 & + \text{Diagram 3} + \text{Diagram 4} \\
 & + \text{Diagram 5}
 \end{aligned}$$

The diagrams represent Feynman diagrams for the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$. Each diagram consists of a horizontal line with n external legs labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The diagrams show different ways to connect these external legs to internal vertices (represented by gray circles) and internal lines (represented by black lines).

- Diagram 1:** A triangle diagram with a single vertex at the bottom connected to the first, second, and n -th external legs.
- Diagram 2:** A triangle diagram with two vertices at the bottom connected to the first, second, and n -th external legs, and connected to each other by a horizontal line.
- Diagram 3:** A triangle diagram with a single vertex at the bottom connected to the first, second, and n -th external legs, and a self-loop on the bottom vertex.
- Diagram 4:** A triangle diagram with a single vertex at the bottom connected to the first, second, and n -th external legs, and two self-loops on the bottom vertex.
- Diagram 5:** A triangle diagram with two vertices at the bottom connected to the first, second, and n -th external legs, and connected to each other by a horizontal line with a self-loop.

Correlation functions

05

$$\begin{aligned}
 \langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle = & \text{Diagram 1} + \text{Diagram 2} \\
 & + \text{Diagram 3} + \text{Diagram 4} \\
 & + \text{Diagram 5} + \text{Diagram 6}
 \end{aligned}$$

The diagrams represent Feynman diagrams for the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$. Each diagram consists of a horizontal line with n external legs labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The diagrams show various internal structures:

- Diagram 1:** A single vertex (gray circle) connected to all n external legs.
- Diagram 2:** Two vertices (gray circles) connected by a horizontal line. The first vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the second vertex is connected to \mathbf{k}_{n-1} and \mathbf{k}_n .
- Diagram 3:** A single vertex (gray circle) connected to all n external legs, with a loop (self-energy) attached to the vertex.
- Diagram 4:** A single vertex (gray circle) connected to all n external legs, with two loops (self-energy) attached to the vertex.
- Diagram 5:** Two vertices (gray circles) connected by a horizontal line. The first vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the second vertex is connected to \mathbf{k}_{n-1} and \mathbf{k}_n . There is a loop (self-energy) on the internal line connecting the two vertices.
- Diagram 6:** Two vertices (gray circles) connected by a horizontal line. The first vertex is connected to \mathbf{k}_1 and \mathbf{k}_2 , and the second vertex is connected to \mathbf{k}_{n-1} and \mathbf{k}_n . There is a loop (self-energy) on the internal line connecting the two vertices, and a loop (self-energy) on the first vertex.

Split propagators

06

One may split propagators into real and imaginary parts:

$$\text{---} = \text{Re}(\text{---}) + i \text{Im}(\text{---})$$

Split propagators

06

One may split propagators into real and imaginary parts:

$$\text{---} = \underbrace{\text{Re}(\text{---})}_{\text{= } \text{=}} + i \underbrace{\text{Im}(\text{---})}_{\text{= } \text{---}}$$

The diagram illustrates the decomposition of a propagator into its real and imaginary parts. On the left, a single horizontal line connects two gray circular nodes. This is set equal to the sum of two terms. The first term is the real part, $\text{Re}(\text{---})$, which is represented by a horizontal line with two nodes, with a curly brace underneath it pointing to a double horizontal line connecting two nodes. The second term is the imaginary part, $i \text{Im}(\text{---})$, which is represented by a horizontal line with two nodes, with a curly brace underneath it pointing to a dashed horizontal line connecting two nodes.

Split propagators

06

One may split propagators into real and imaginary parts:

$$\text{---} = \underbrace{\text{Re}(\text{---})}_{\text{= } \text{=}} + i \underbrace{\text{Im}(\text{---})}_{\text{= } \text{---}}$$

The diagram illustrates the decomposition of a propagator into its real and imaginary parts. On the left, a single horizontal line with two gray circular endpoints represents the propagator. This is set equal to the sum of two terms. The first term is the real part, $\text{Re}(\text{---})$, which is represented by a horizontal line with two gray circular endpoints where the line is drawn as two parallel solid lines. The second term is the imaginary part, $i \text{Im}(\text{---})$, which is represented by a horizontal line with two gray circular endpoints where the line is drawn as a dashed line. Brackets are used to group the real and imaginary parts of the original propagator, with the real part corresponding to the double solid line and the imaginary part corresponding to the dashed line.

Then one can prove:

Split propagators

06

One may split propagators into real and imaginary parts:

$$\text{---} = \underbrace{\text{Re}(\text{---})}_{\text{---}} + i \underbrace{\text{Im}(\text{---})}_{\text{---}}$$

The diagram illustrates the decomposition of a propagator into its real and imaginary parts. On the left, a single horizontal line connects two gray circular vertices. This is set equal to the sum of two terms. The first term is the real part, $\text{Re}(\text{---})$, which is represented by a horizontal line with two gray vertices, with a curly brace underneath indicating it is the real part. The second term is the imaginary part, $i \text{Im}(\text{---})$, which is represented by a horizontal dashed line with two gray vertices, with a curly brace underneath indicating it is the imaginary part.



Then one can prove:

 Every vertex must have at least one imaginary propagator attached to it

One may split propagators into real and imaginary parts:

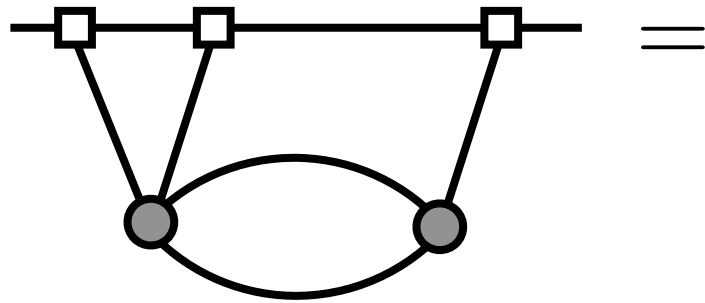
$$\text{---} = \underbrace{\text{Re}(\text{---})}_{\text{= double line}} + i \underbrace{\text{Im}(\text{---})}_{\text{= dashed line}}$$

Then one can prove:

-  Every vertex must have at least one imaginary propagator attached to it
-  It is impossible to form a closed loop only with imaginary propagators

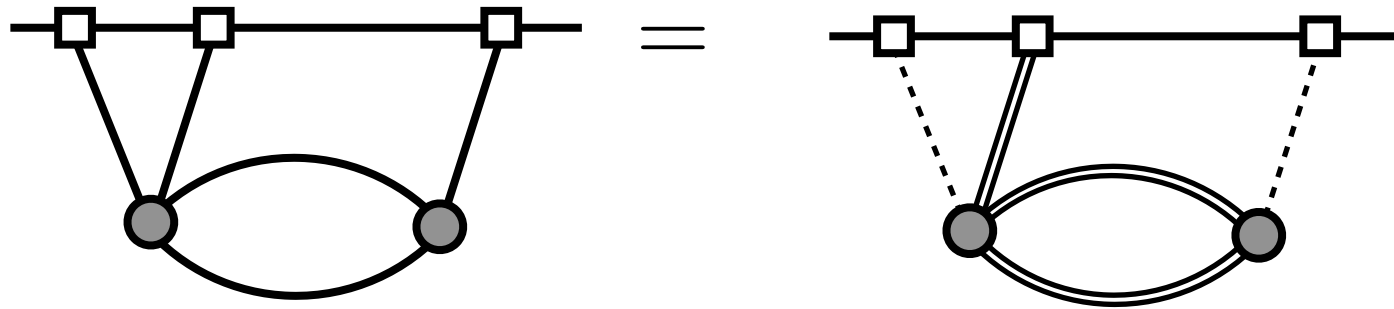
Split propagators

07



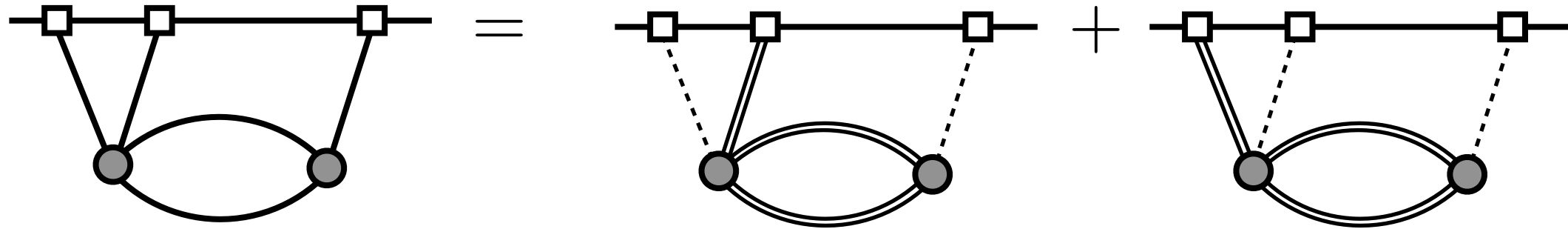
Split propagators

07



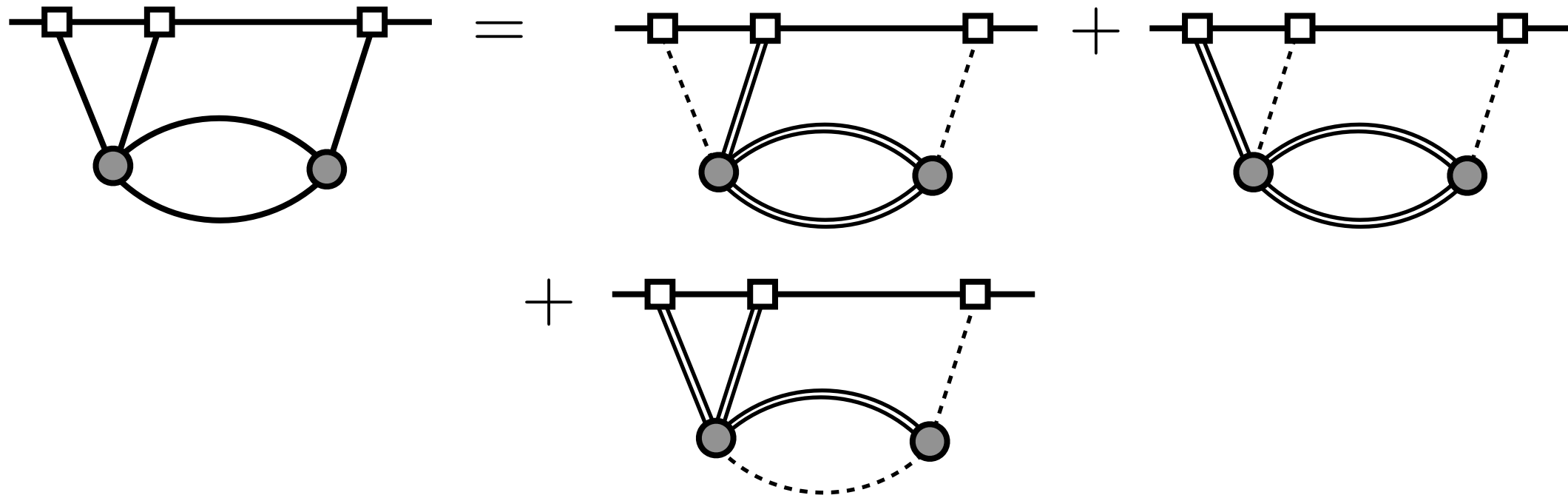
Split propagators

07



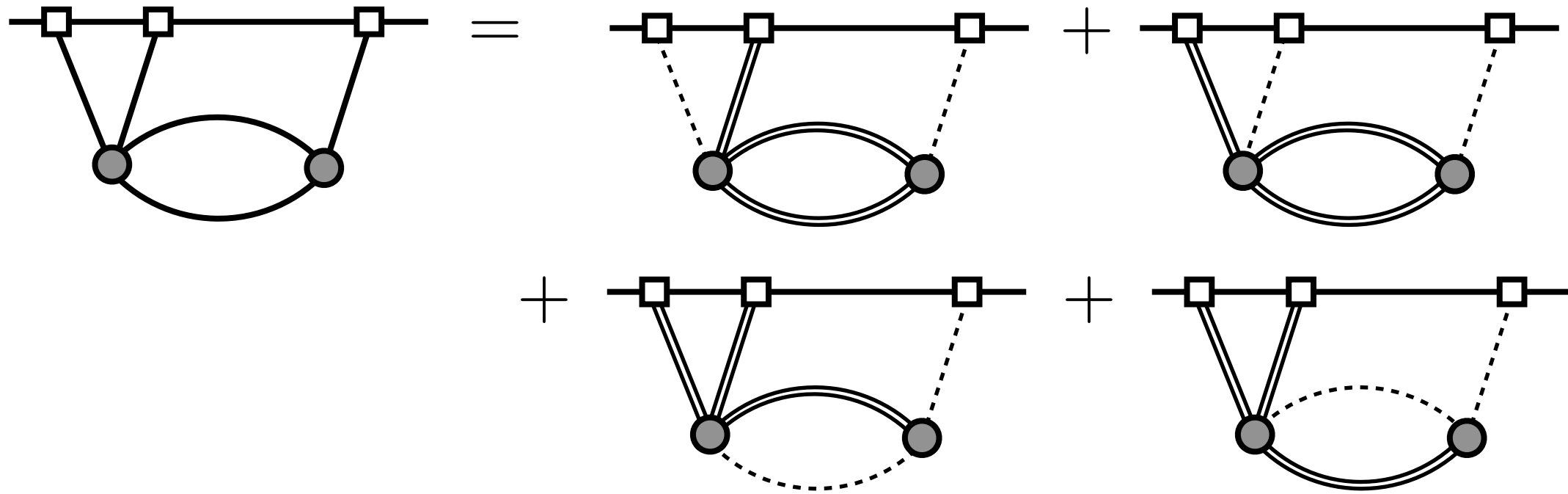
Split propagators

07



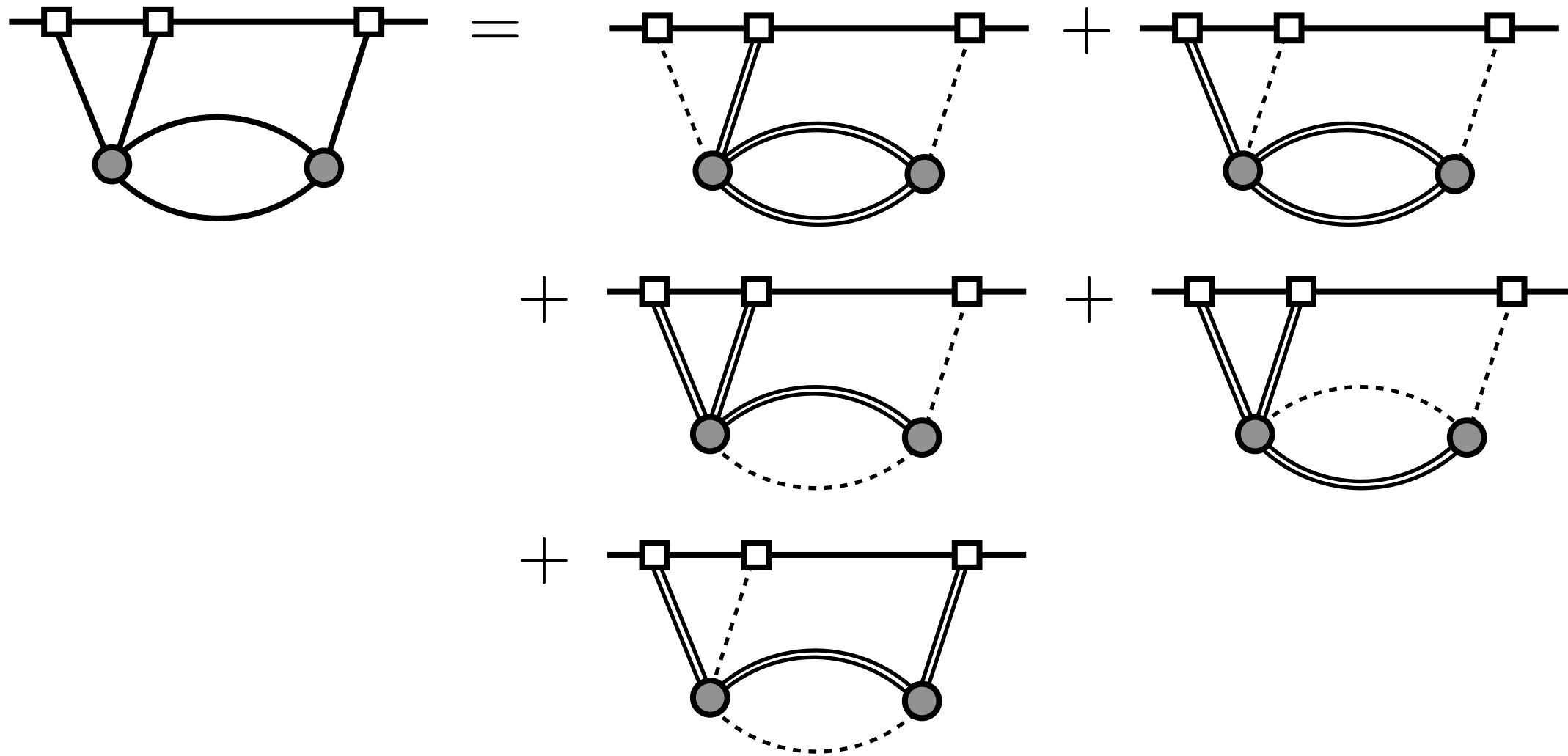
Split propagators

07



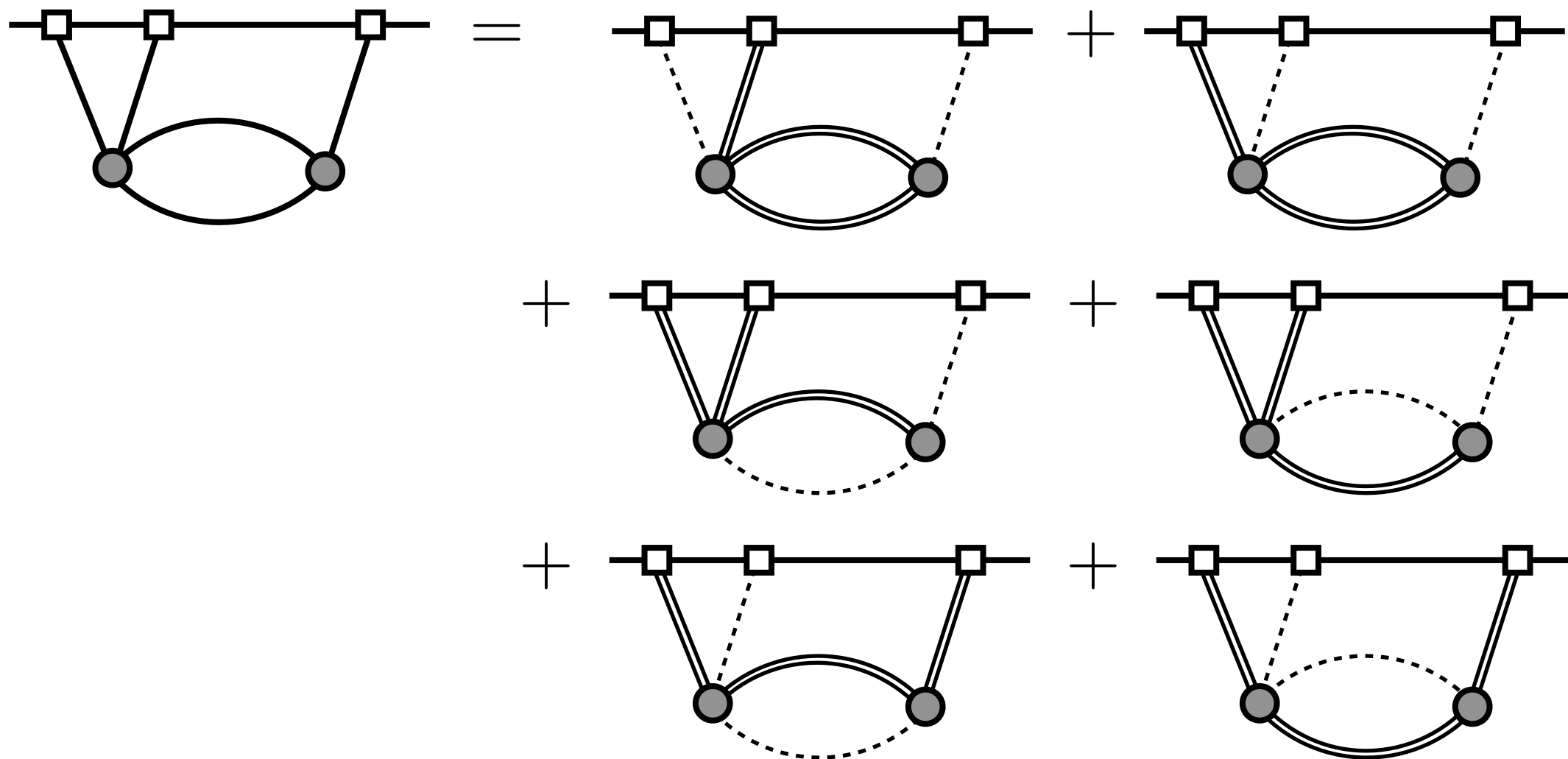
Split propagators

07



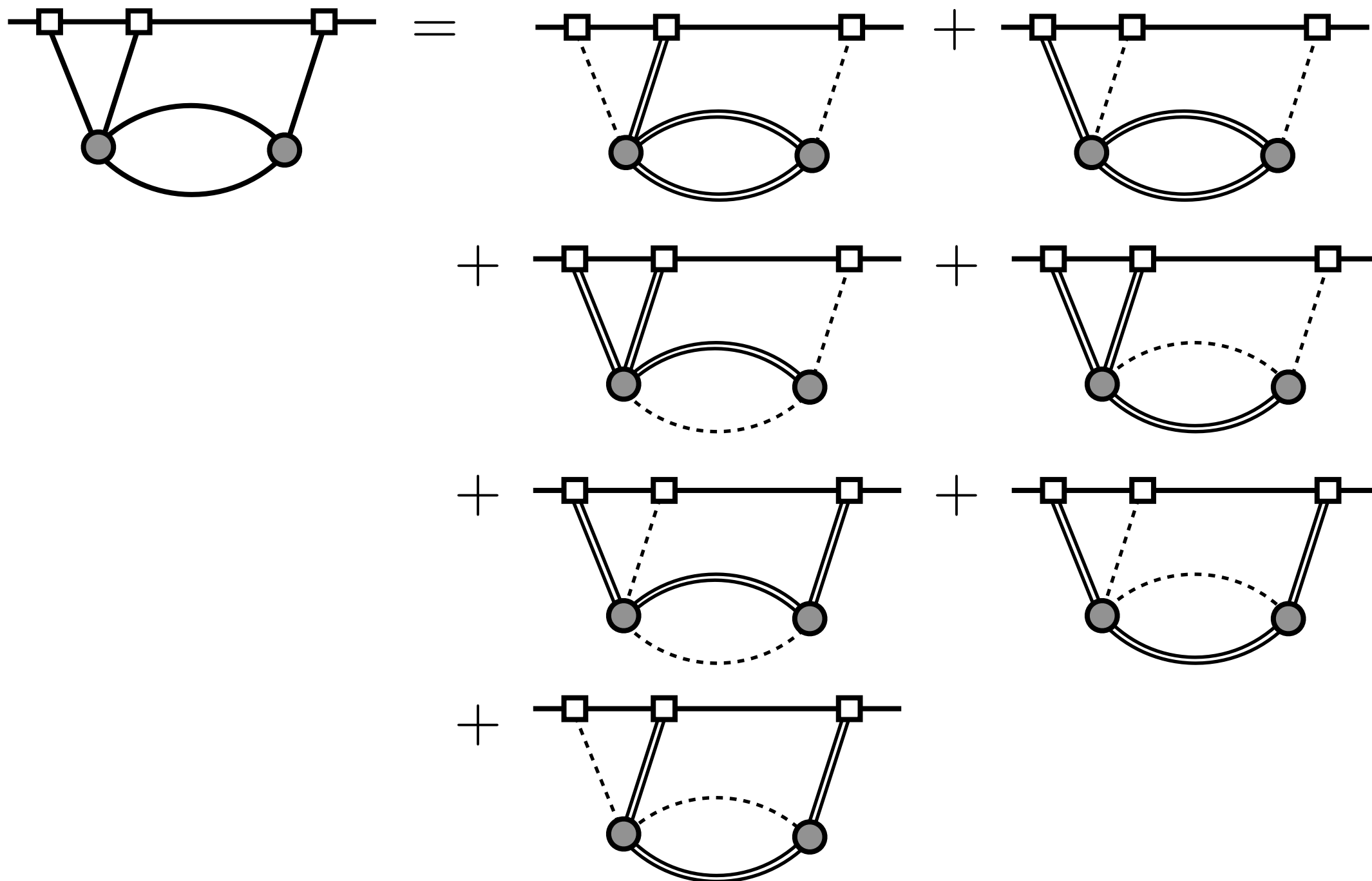
Split propagators

07



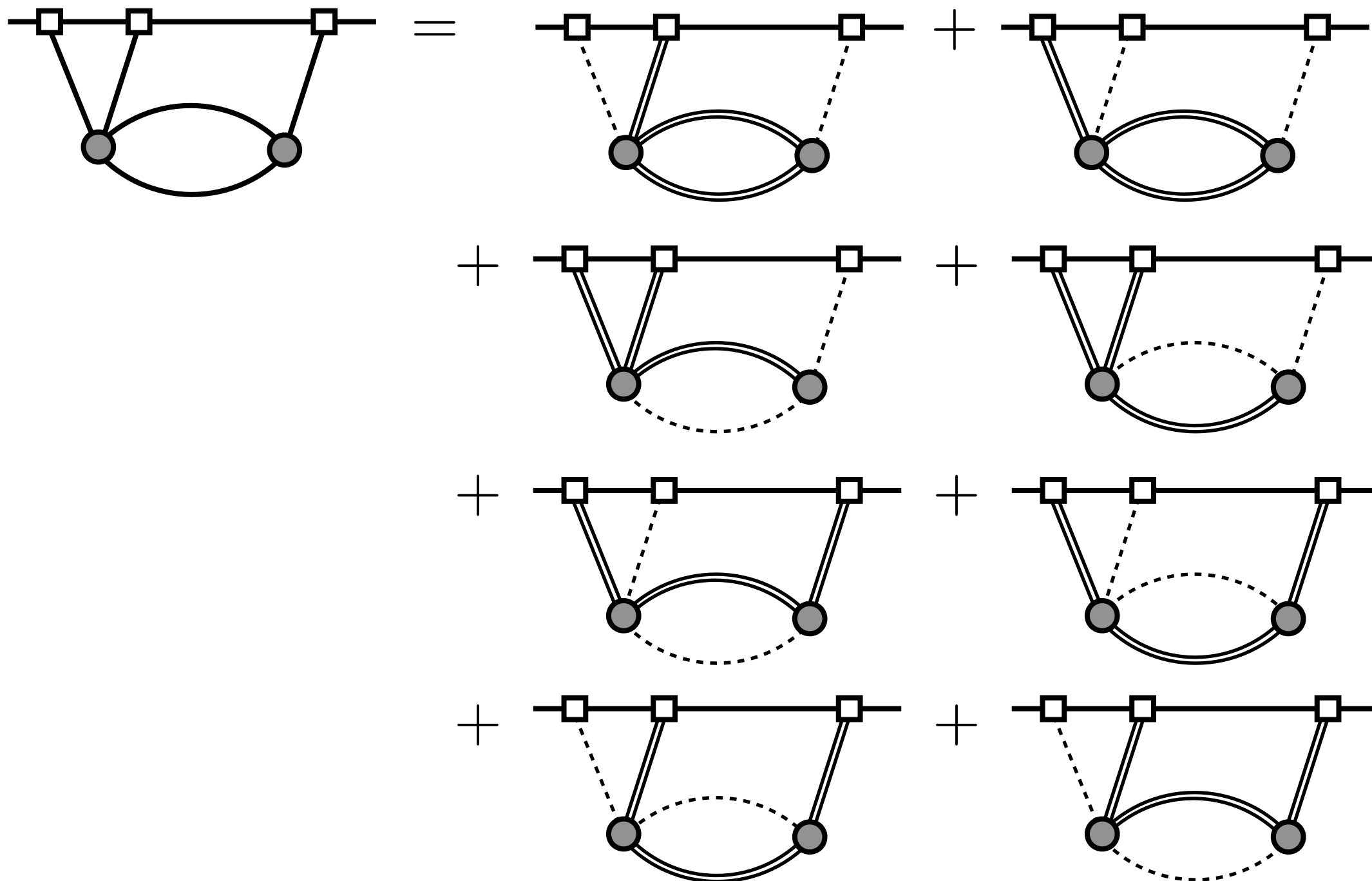
Split propagators

07



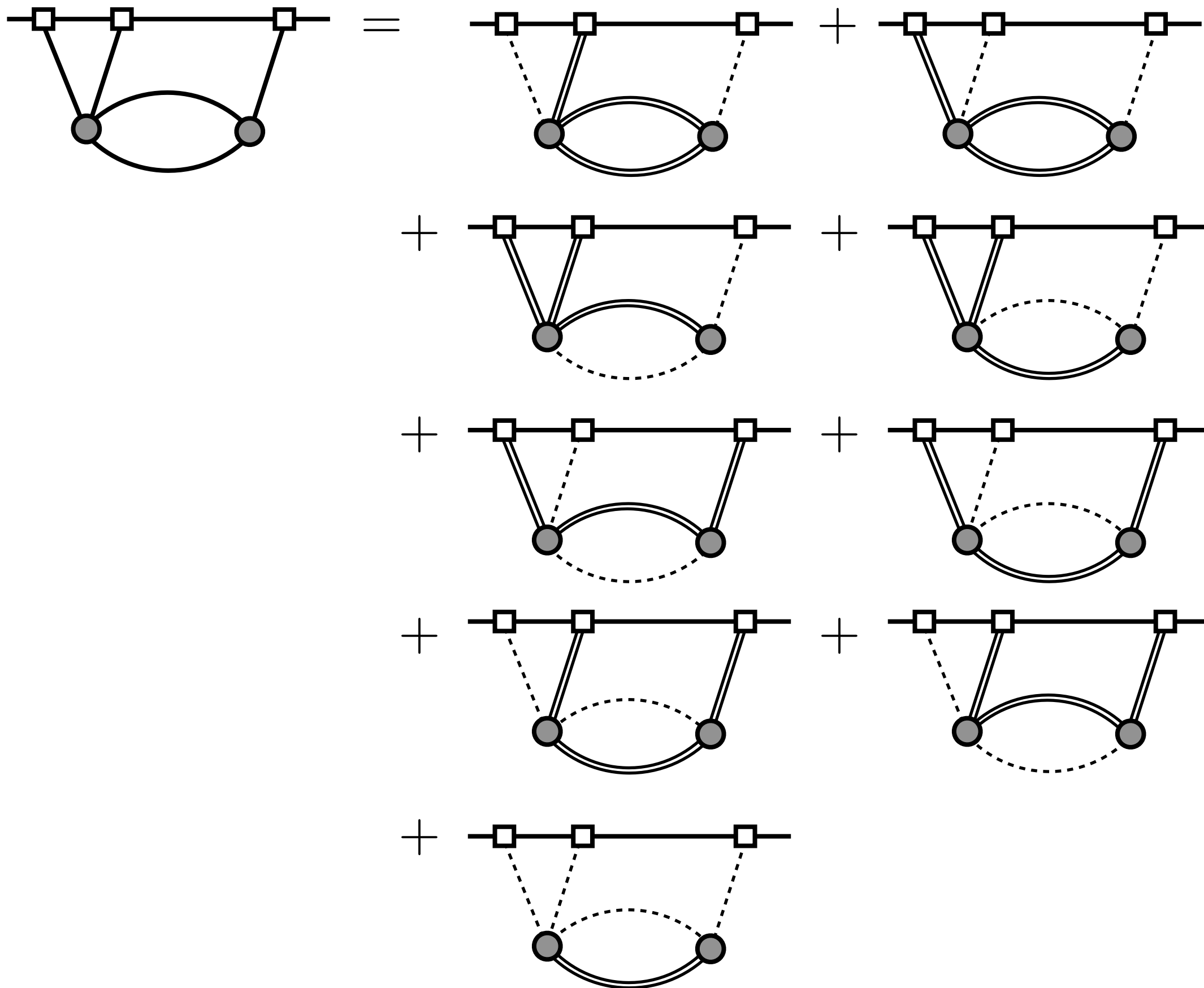
Split propagators

07



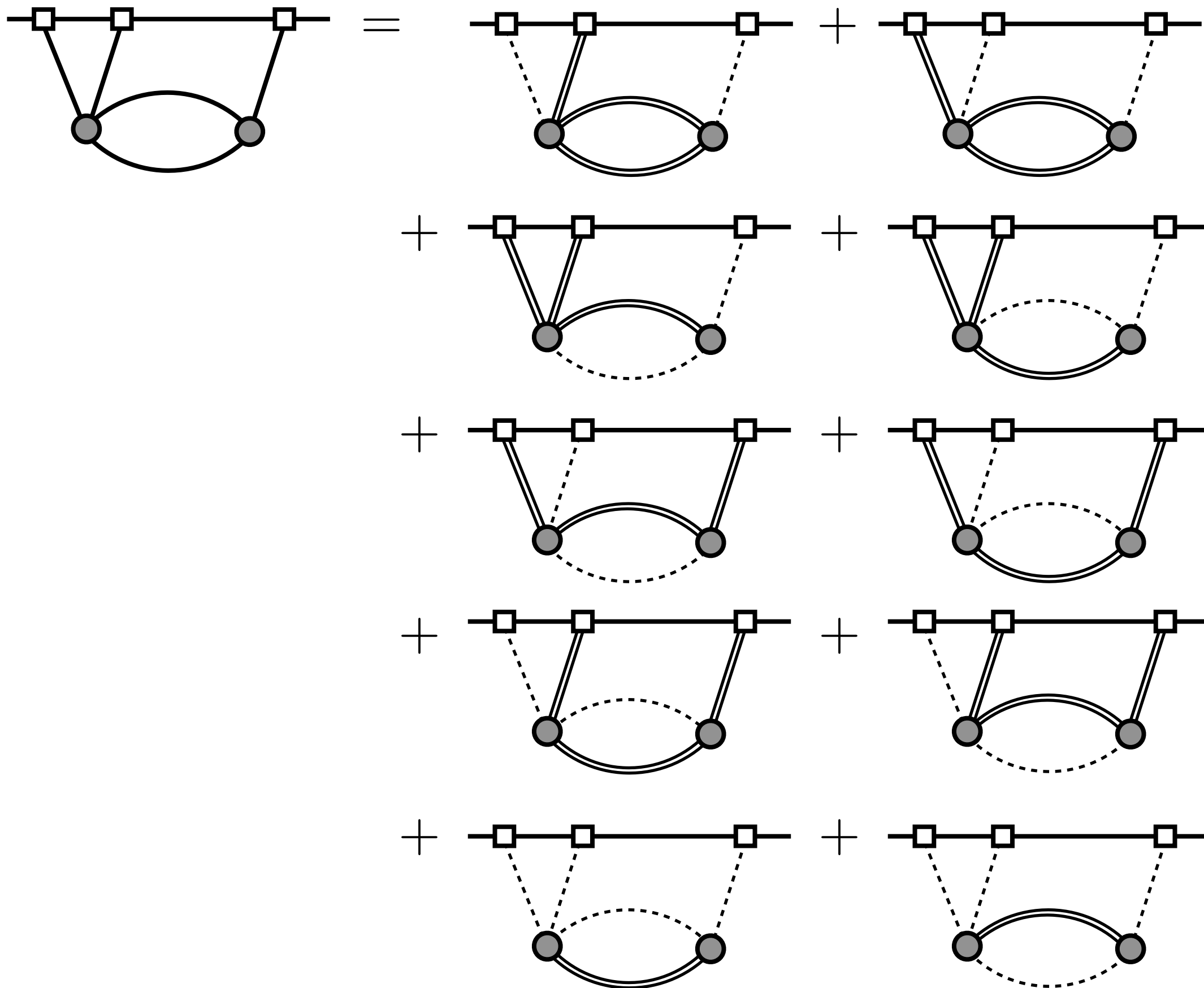
Split propagators

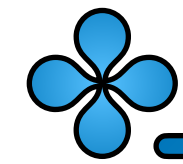
07



Split propagators

07





Logarithms in momentum space

08

Real and imaginary propagators have different time dependences:

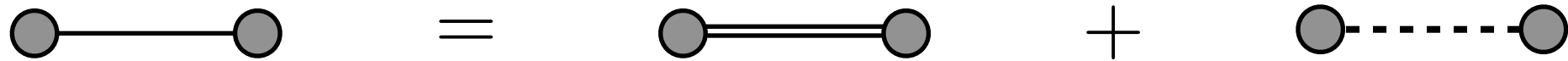
$$\text{---} = \text{=} \text{=} + \text{---}$$

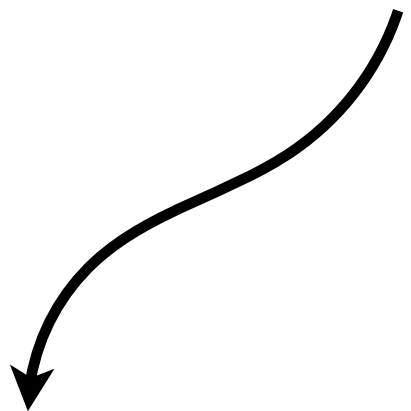
The diagram shows an equation between Feynman propagators. On the left is a single solid line between two gray circular vertices. This is equal to the sum of two terms: a double solid line between two gray circular vertices, and a dashed line between two gray circular vertices.

Logarithms in momentum space

08

Real and imaginary propagators have different time dependences:



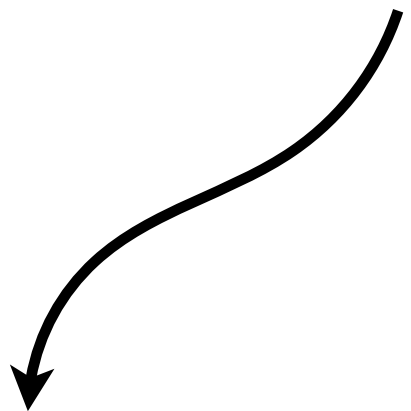

$$\propto \frac{H^2}{2k^3} \left[1 + \dots \right]$$

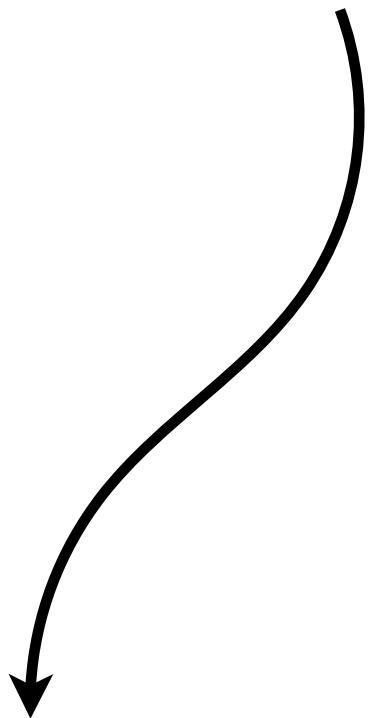
Logarithms in momentum space

08

Real and imaginary propagators have different time dependences:


$$\text{---} = \text{=} + \text{---}$$

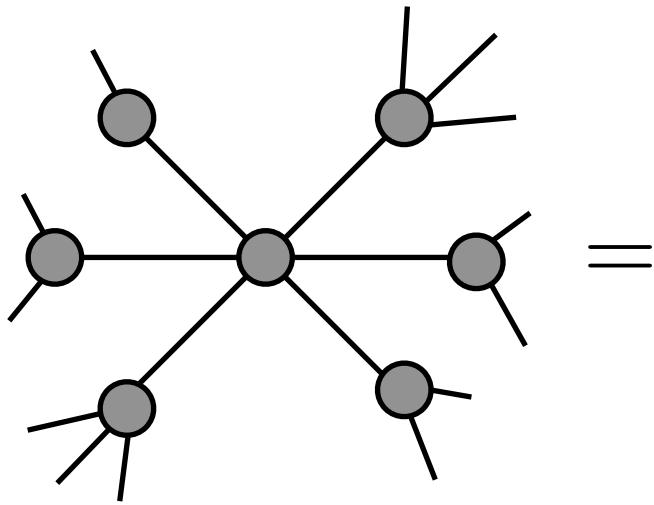

$$\propto \frac{H^2}{2k^3} \left[1 + \dots \right]$$


$$\propto \frac{H^2}{6} \left[(\tau_a^3 - \tau_b^3) + \dots \right]$$

Logarithms in momentum space

09

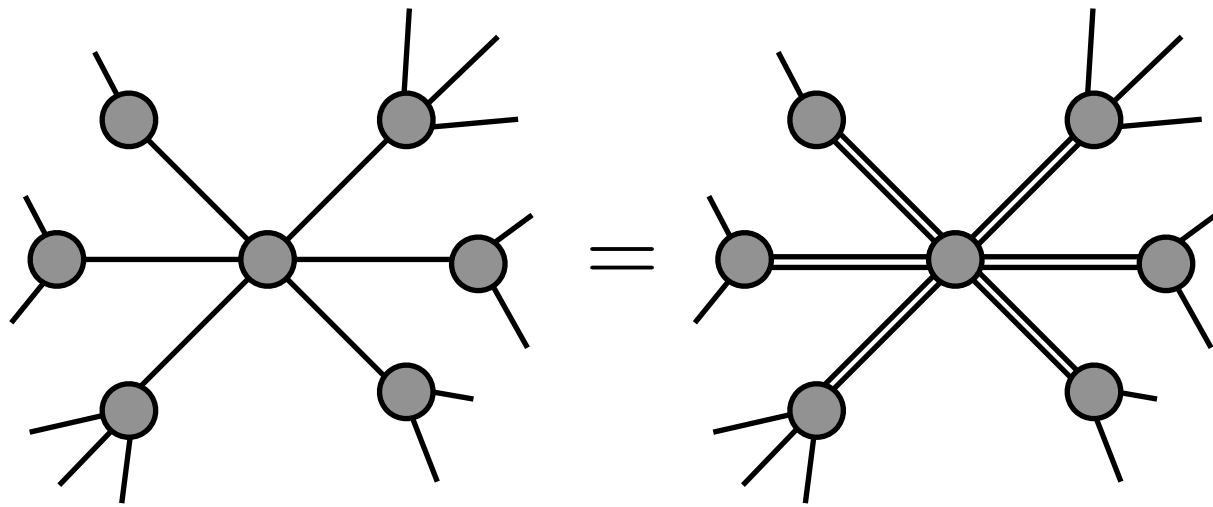
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

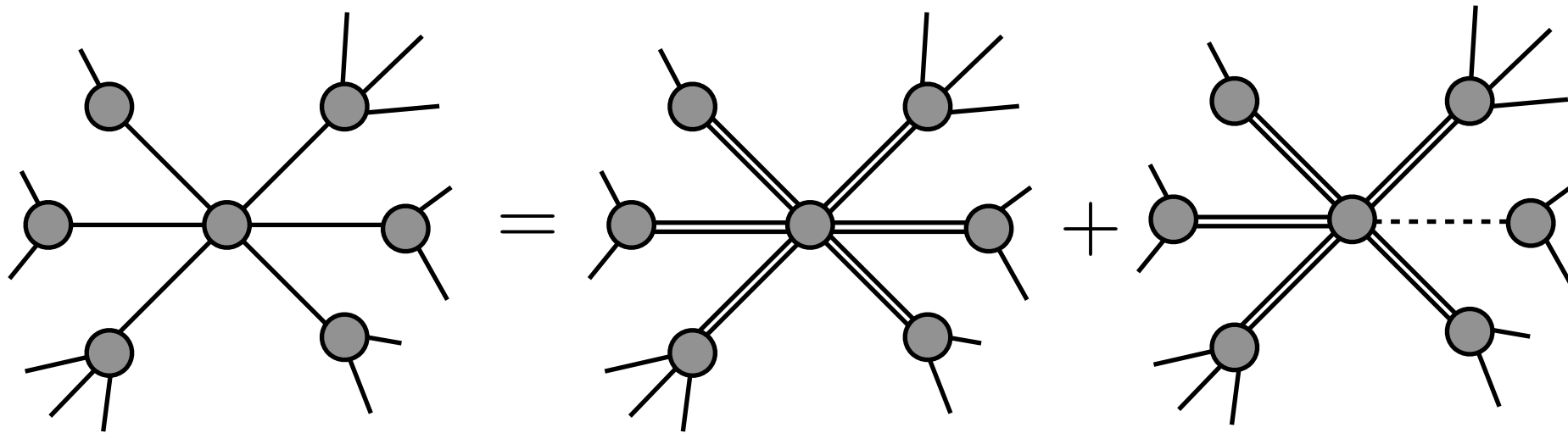
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

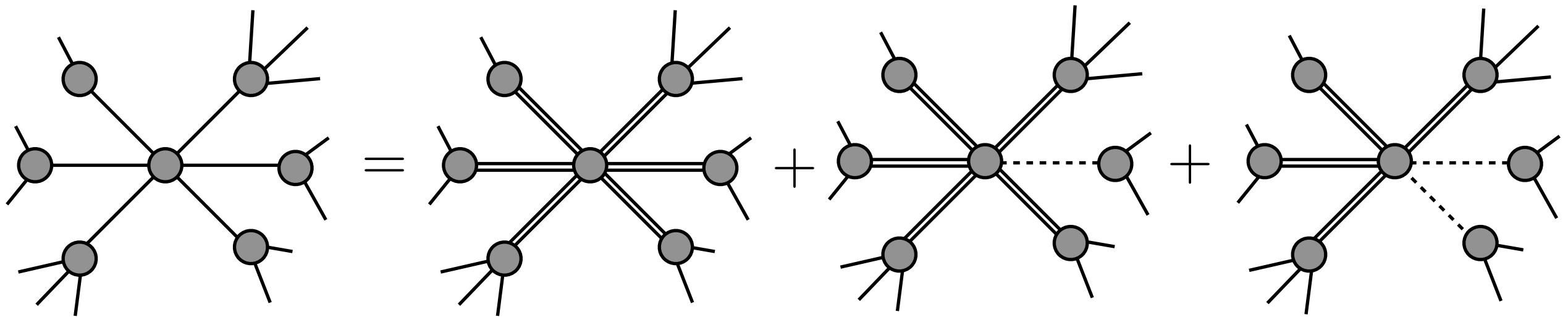
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

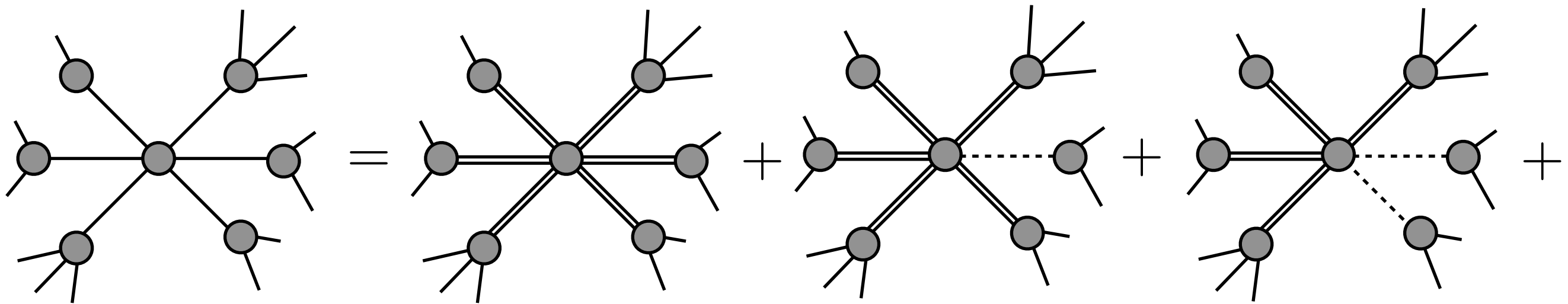
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

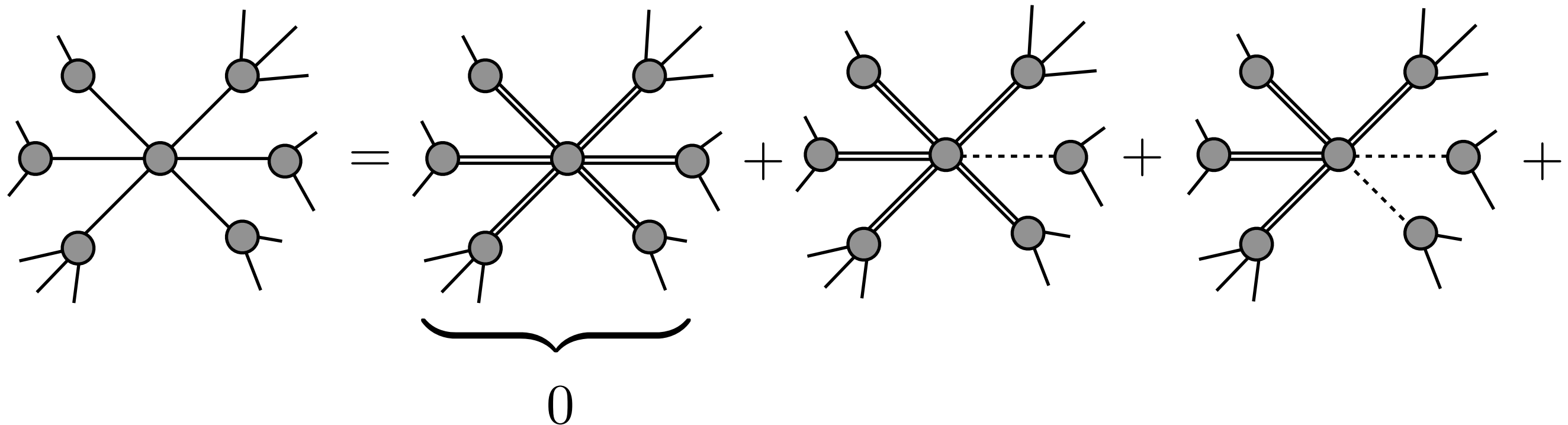
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

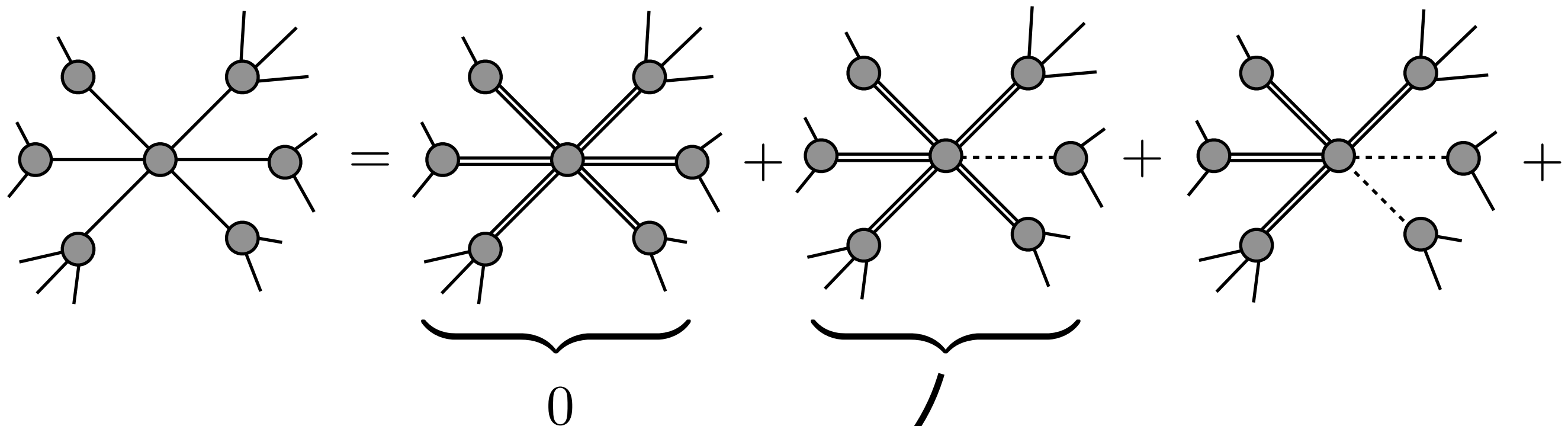
It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



Logarithms in momentum space

09

It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:

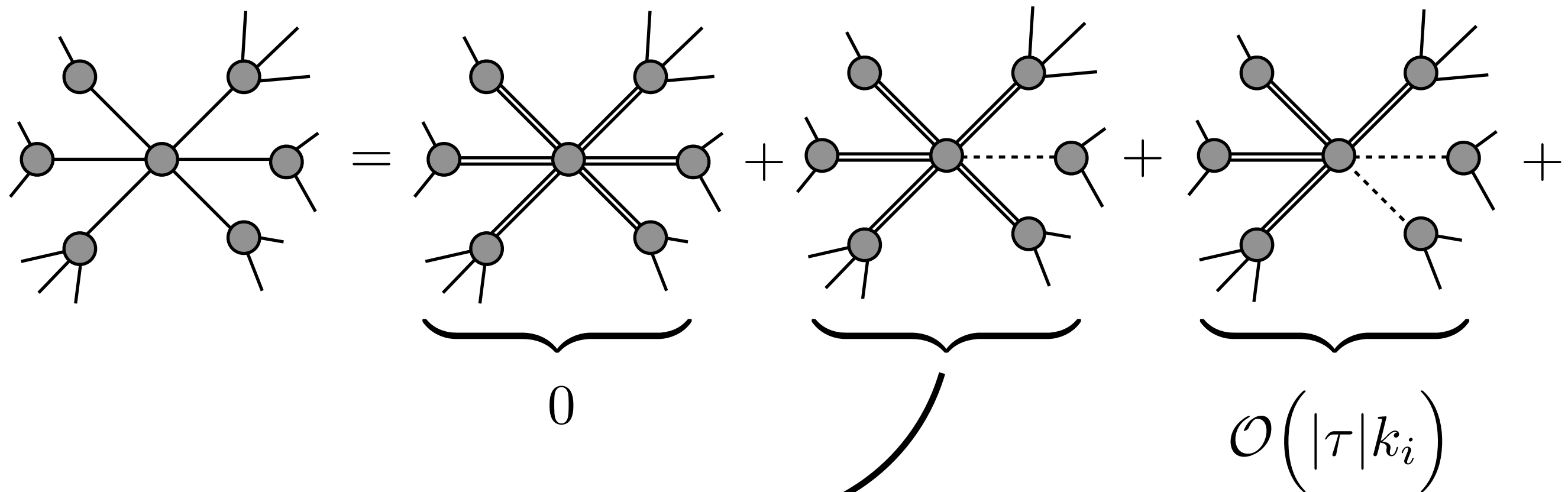


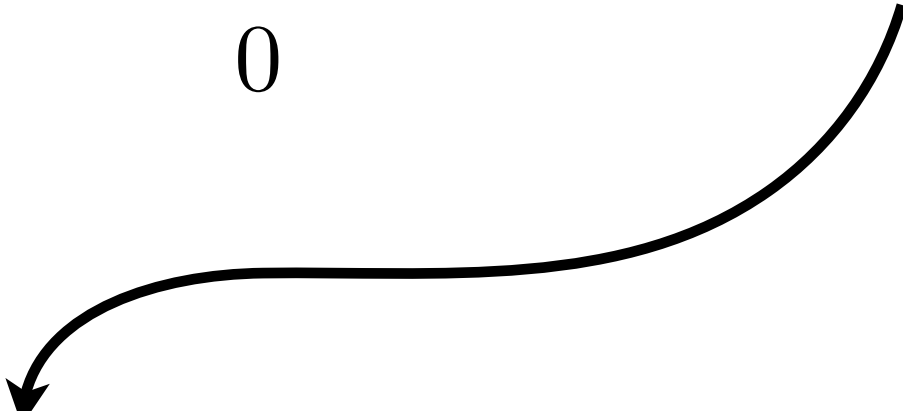
$$\ln \left[-\tau \times f(k_1, \dots, k_n) \right]$$

Logarithms in momentum space

09

It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$:



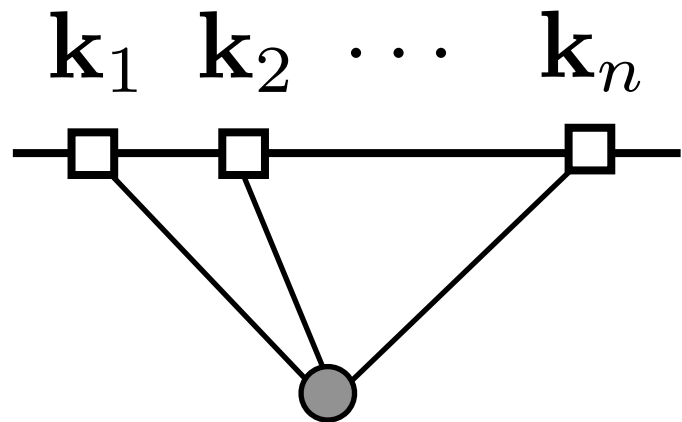


$$\ln \left[-\tau \times f(k_1, \dots, k_n) \right]$$

Logarithms in momentum space

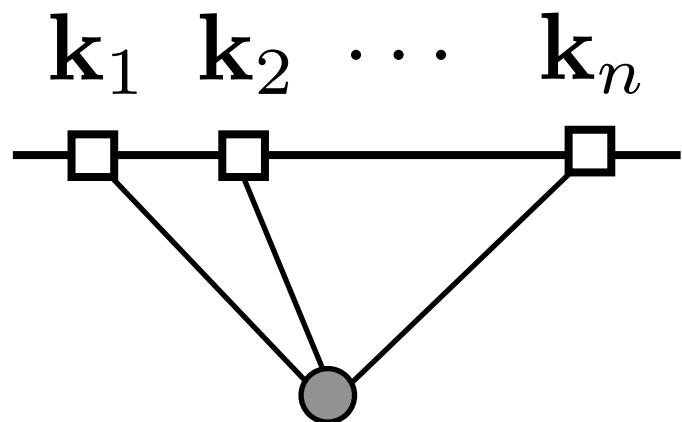
10

Tree-level examples:

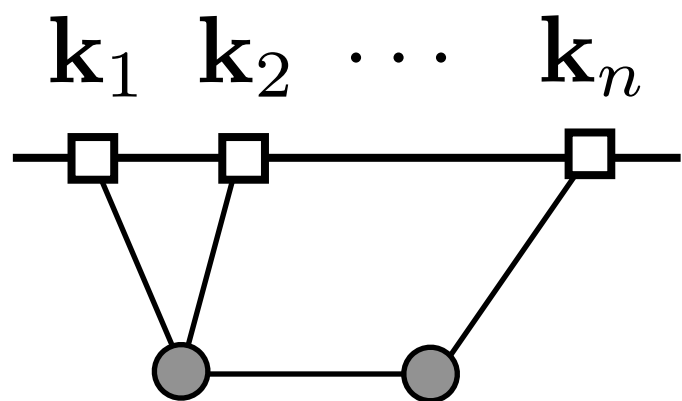


$$\propto \ln \left[-\tau(k_1 + \cdots + k_n) \right]$$

Tree-level examples:



$$\propto \ln \left[-\tau(k_1 + \cdots + k_n) \right]$$



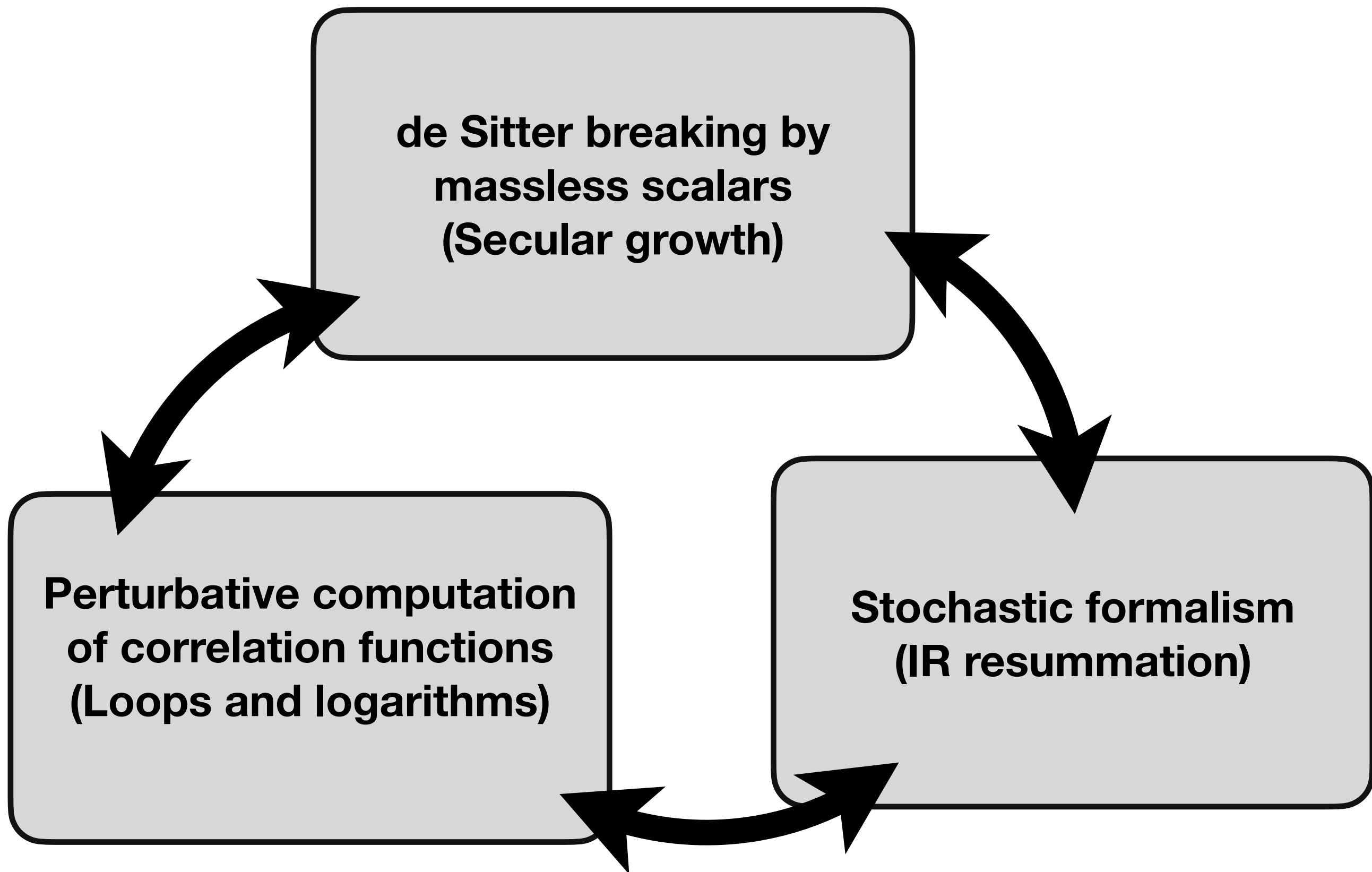
$$\begin{aligned} \propto & \ln \left[-\tau(k_I + K_1) \right] \ln \left[-\tau(k_I + K_2) \right] \\ & + \ln \left[-\tau(k_I + K_1) \right]^2 \\ & + \ln \left[-\tau(k_I + K_2) \right]^2 \end{aligned}$$

Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**

**Stochastic formalism
(IR resummation)**

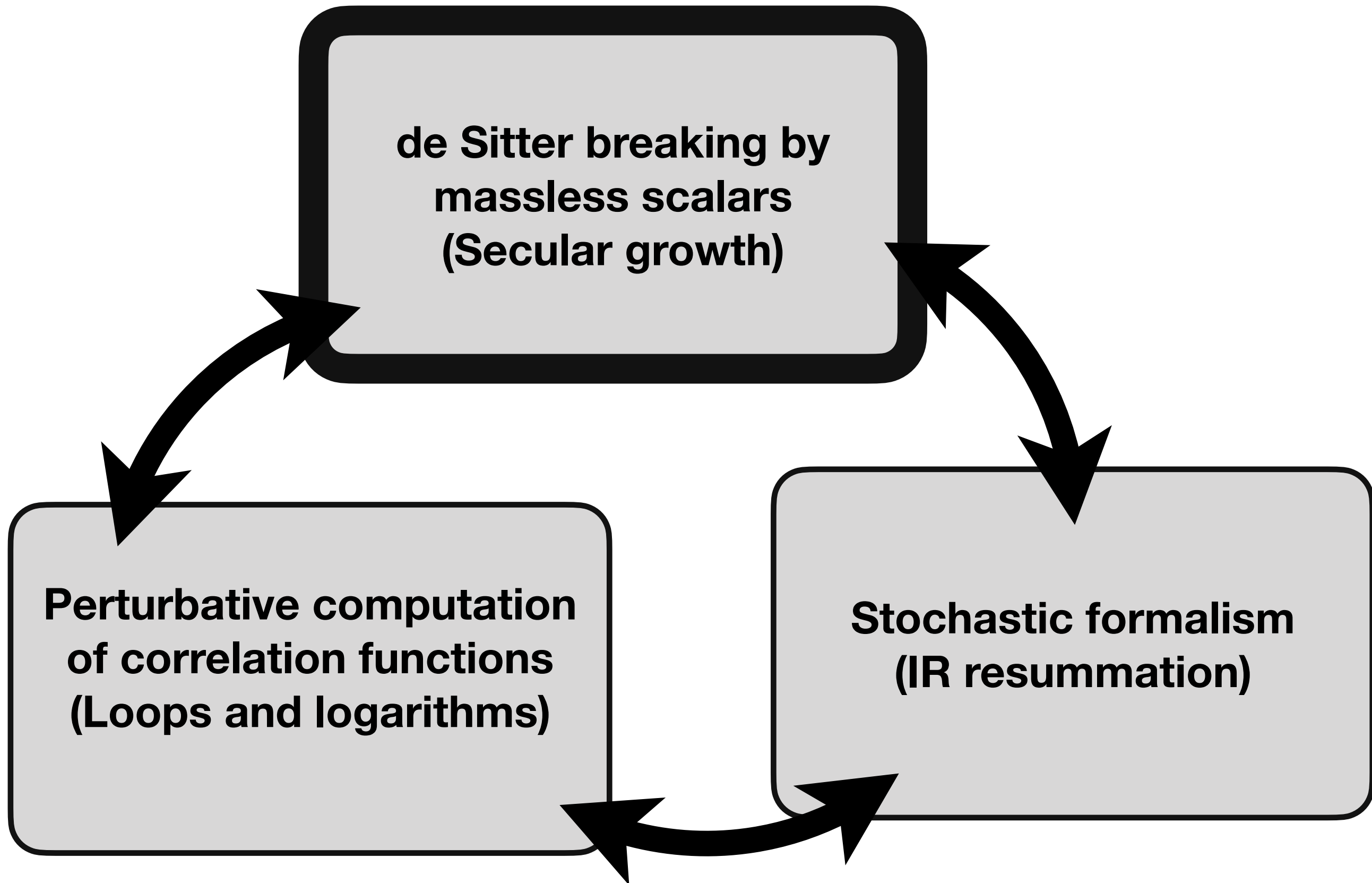


Preamble

**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**

**Stochastic formalism
(IR resummation)**



Massless scalar field theories

11

Consider the following massless free theory

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 \right]$$

Massless scalar field theories

11

Consider the following massless free theory

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 \right]$$

It has a shift symmetry

$$\ddot{\varphi} - \frac{2}{\tau} \dot{\varphi} - \nabla^2 \varphi = 0$$

Consider the following massless free theory

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 \right]$$

It has a shift symmetry

$$\ddot{\varphi} - \frac{2}{\tau} \dot{\varphi} - \nabla^2 \varphi = 0$$

Quantize it:

$$\varphi(\mathbf{x}, \tau) = \int_{\mathbf{k}} \left[f_k(\tau) \hat{a}_{\mathbf{k}} + f_k^*(\tau) \hat{a}_{-\mathbf{k}}^\dagger \right] e^{-i\mathbf{k} \cdot \mathbf{x}}$$

Consider the following massless free theory

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 \right]$$

It has a shift symmetry

$$\ddot{\varphi} - \frac{2}{\tau} \dot{\varphi} - \nabla^2 \varphi = 0$$

Quantize it:

$$\varphi(\mathbf{x}, \tau) = \int_{\mathbf{k}} \left[f_k(\tau) \hat{a}_{\mathbf{k}} + f_k^*(\tau) \hat{a}_{-\mathbf{k}}^\dagger \right] e^{-i\mathbf{k} \cdot \mathbf{x}}$$

BD-vacuum:

$$f_k(\tau) = \frac{iH}{\sqrt{2k^3}} [1 + ik\tau] e^{-ik\tau}$$

Massless scalar field theories

Let's compute the two-point correlation function:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle$$

Let's compute the two-point correlation function:

$$\begin{aligned} G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') &= \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right) \end{aligned}$$

Let's compute the two-point correlation function:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle$$

$$= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \ln a(\tau) + \ln a(\tau') + \ln \frac{1}{0} \right]$$

Let's compute the two-point correlation function:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle$$

$$= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \ln a(\tau) + \ln a(\tau') + \ln \frac{1}{0} \right]$$

IR divergence

Allen (1985)

Let's compute the two-point correlation function:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle$$

$$= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \ln a(\tau) + \ln a(\tau') + \ln \frac{1}{0} \right]$$

Secular growth

IR divergence


Allen (1985)

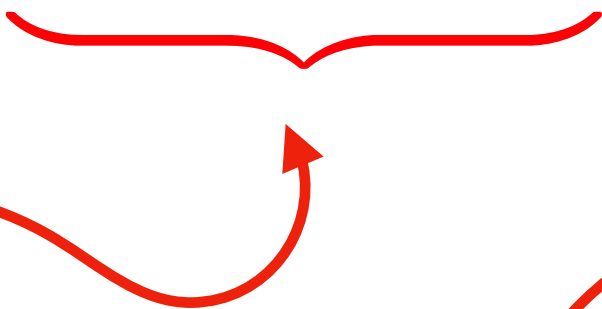
Massless scalar field theories

12

Let's compute the two-point correlation function:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle$$

$$= \frac{1}{2\pi^2} \int_{\emptyset}^{\infty} \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$


$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \ln a(\tau) + \ln a(\tau') + \ln \frac{1}{\emptyset} \right]$$


Secular growth

You would have obtained the same result with a co-moving cutoff !

IR divergence



Allen (1985)

Massless scalar field theories

12

Let's compute the two-point correlation function:

$$\begin{aligned} G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') &= \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right) \end{aligned}$$

Let me give a step back. What is the origin of this growth?

Massless scalar field theories

12

Let's compute the two-point correlation function:

$$\begin{aligned} G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') &= \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right) \end{aligned}$$

Let me give a step back. What is the origin of this growth?

Notice that the integral has the form:

$$I(s) = \int_0^\infty \frac{dk}{k} F(sk)$$

Massless scalar field theories

12

Let's compute the two-point correlation function:

$$\begin{aligned} G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') &= \langle \Omega | \varphi(\mathbf{x}, \tau) \varphi(\mathbf{x}', \tau') | \Omega \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right) \end{aligned}$$

Let me give a step back. What is the origin of this growth?

Notice that the integral has the form:

$$I(s) = \int_0^\infty \frac{dk}{k} F(sk)$$

Shift symmetry

$$F(0) = \frac{H^2}{4\pi^2}$$

Massless scalar field theories

13

Let's examine the invariance of $I(s)$ under dilations:

$$s \rightarrow \bar{s} = e^{-\theta} s$$

Massless scalar field theories

Let's examine the invariance of $I(s)$ under dilations:

$$s \rightarrow \bar{s} = e^{-\theta} s$$

$$I(e^{-\theta} s) = \int_0^\infty \frac{dk}{k} F(e^{-\theta} sk)$$

Massless scalar field theories

13

Let's examine the invariance of $I(s)$ under dilations:

$$s \rightarrow \bar{s} = e^{-\theta} s$$

$$I(e^{-\theta} s) = \int_0^\infty \frac{dk}{k} F(e^{-\theta} s k)$$

Expand in terms of $\epsilon \equiv 1 - e^{-\theta}$ to all orders and integrate by parts

Massless scalar field theories

13

Let's examine the invariance of $I(s)$ under dilations:

$$s \rightarrow \bar{s} = e^{-\theta} s$$

$$I(e^{-\theta} s) = \int_0^\infty \frac{dk}{k} F(e^{-\theta} s k)$$

Expand in terms of $\epsilon \equiv 1 - e^{-\theta}$ to all orders and integrate by parts

$$I(e^{-\theta} s) = I(s) + \theta F(0)$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}')$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}')$$

$$\bar{\mathbf{x}} = e^{-\theta} \mathbf{x}$$

$$\bar{\tau} = e^{-\theta} \tau$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}') = G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau')$$

$$\bar{\mathbf{x}} = e^{-\theta} \mathbf{x}$$

$$\bar{\tau} = e^{-\theta} \tau$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}') = G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') + \theta \frac{H^2}{4\pi^2}$$

$$\bar{\mathbf{x}} = e^{-\theta} \mathbf{x}$$

$$\bar{\tau} = e^{-\theta} \tau$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}') = G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') + \theta \frac{H^2}{4\pi^2}$$

$$\bar{\mathbf{x}} = e^{-\theta} \mathbf{x}$$

$$\bar{\tau} = e^{-\theta} \tau$$

$$F(0)$$

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}') = G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') + \theta \frac{H^2}{4\pi^2}$$

$$\bar{\mathbf{x}} = e^{-\theta} \mathbf{x}$$

$$\bar{\tau} = e^{-\theta} \tau$$

$$F(0)$$

But recall that this feature is due to the shift symmetry

Back to Allen's result:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + \ln a(\tau) + \ln a(\tau') + \ln \frac{1}{0} \right]$$

Allen (1985)

Back to Allen's result:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + \underbrace{\ln a(\tau) + \ln a(\tau')}_{\text{Secular growth}} + \ln \frac{1}{0} \right]$$

Secular growth

Allen (1985)

Back to Allen's result:


$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + \underbrace{\ln a(\tau) + \ln a(\tau')}_{\text{Secular growth}} + \ln \frac{1}{0} \right]$$

Allen (1985)

(due to the shift symmetry)

Back to Allen's result:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + \underbrace{\ln a(\tau) + \ln a(\tau')}_{\text{Allen (1985)}} + \ln \frac{1}{0} \right]$$

Secular growth  **(due to the shift symmetry)**


However, in a shift symmetric theory, observables are also shift symmetric invariant. For instance:

$$G_{\mu\nu'}(x, x') = \partial_\mu \partial_{\nu'} \langle \varphi(x) \varphi(x') \rangle$$

See for instance Tolley & Turok (2001)

Back to Allen's result:

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + \underbrace{\ln a(\tau) + \ln a(\tau')}_{\text{Allen (1985)}} + \ln \frac{1}{0} \right]$$

Secular growth  **(due to the shift symmetry)**

However, in a shift symmetric theory, observables are also shift symmetric invariant. For instance:

$$G_{\mu\nu'}(x, x') = \partial_\mu \partial_{\nu'} \langle \varphi(x) \varphi(x') \rangle$$

(No secular growth here)

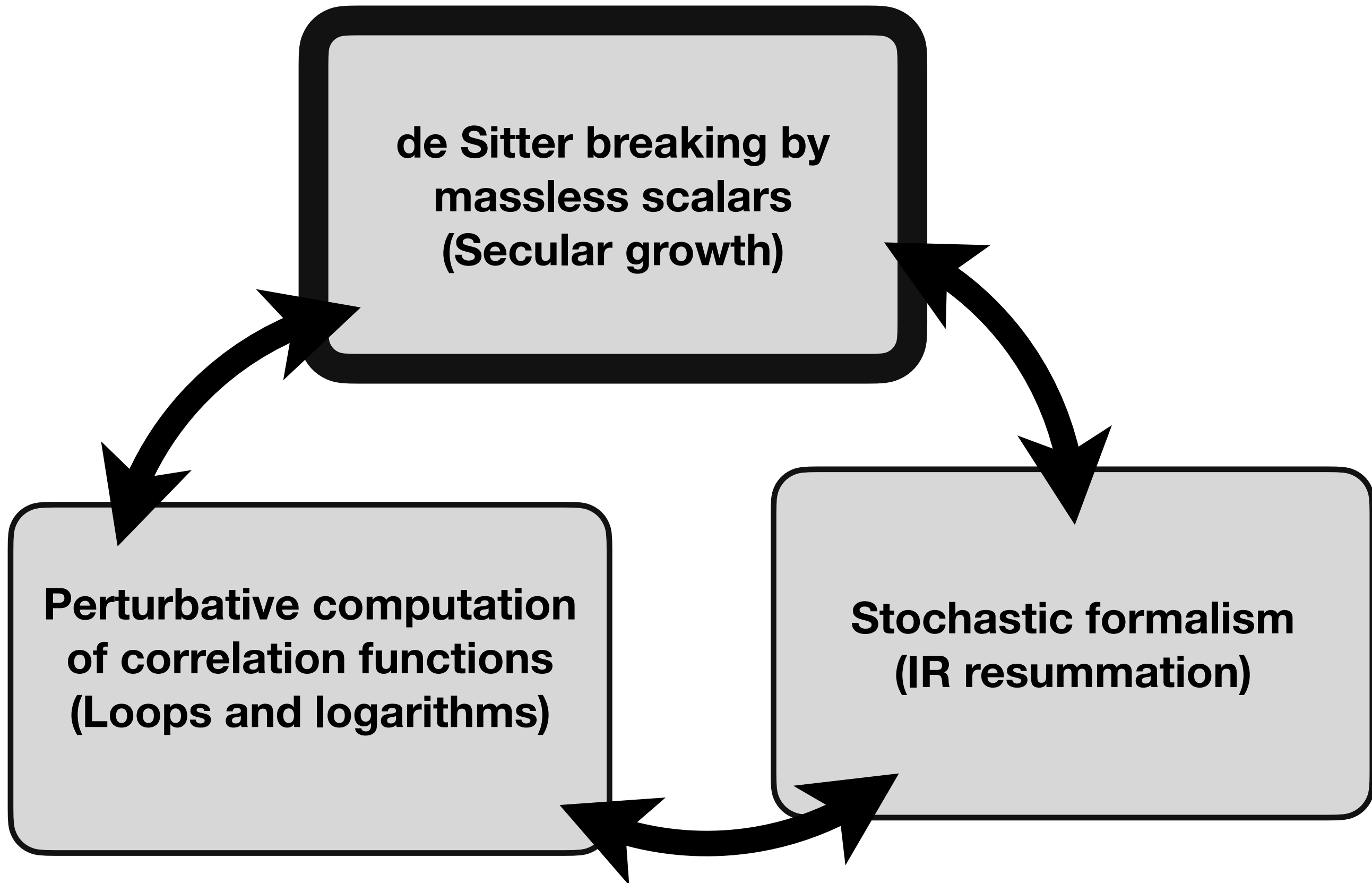
See for instance Tolley & Turok (2001)

Preamble

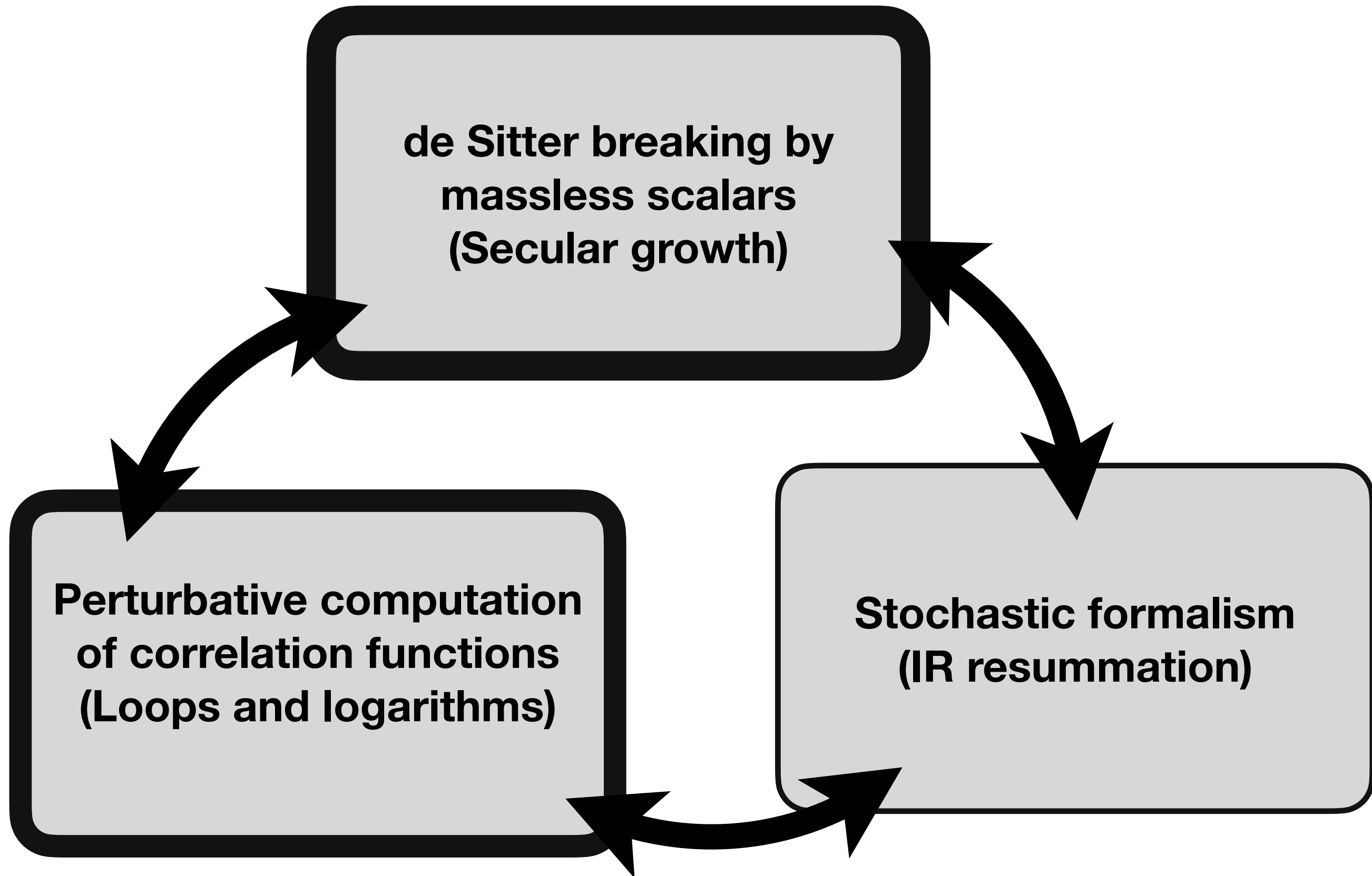
**de Sitter breaking by
massless scalars
(Secular growth)**

**Perturbative computation
of correlation functions
(Loops and logarithms)**

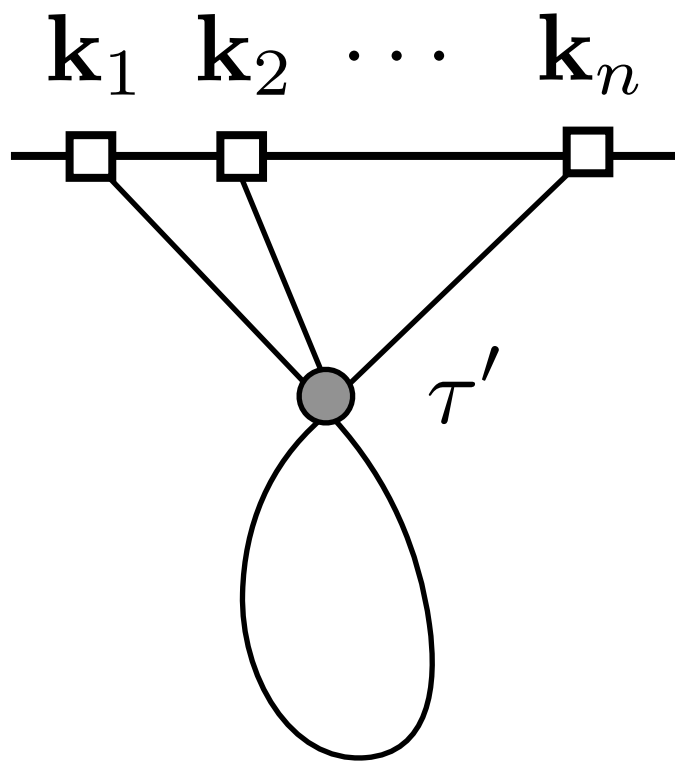
**Stochastic formalism
(IR resummation)**



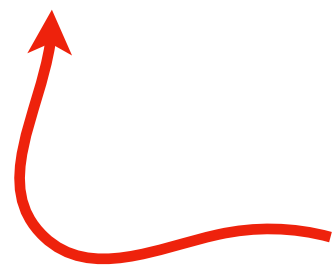
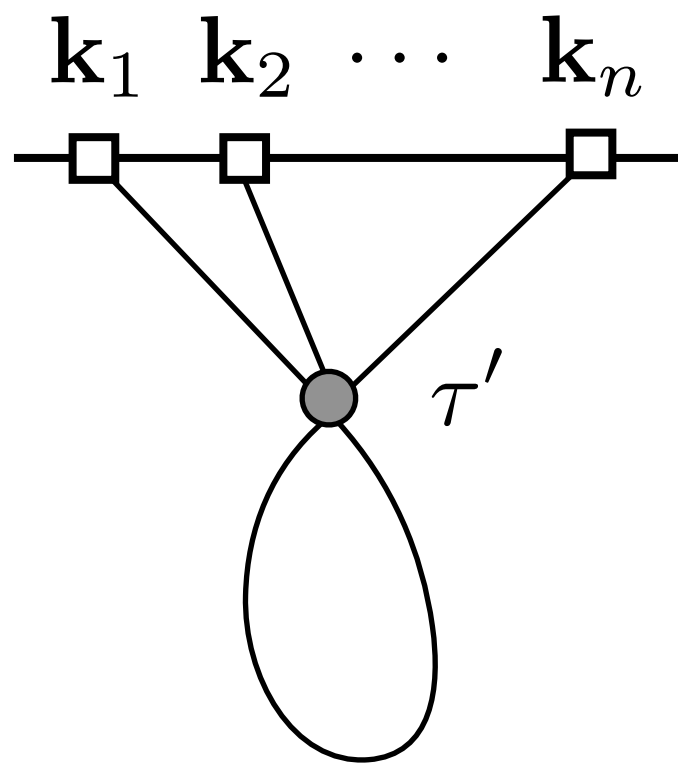
Preamble



Why is Allen's result relevant for correlation functions?

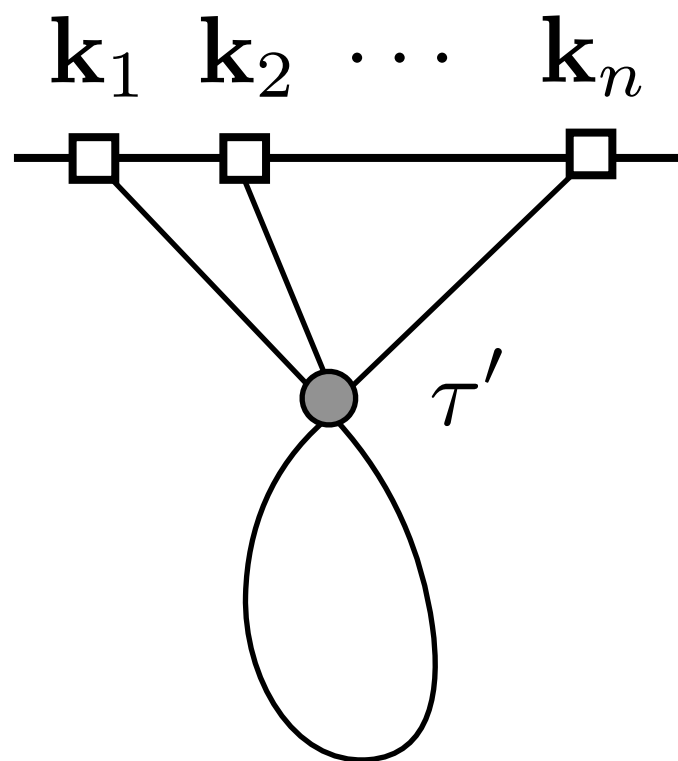


Why is Allen's result relevant for correlation functions?



$$G(0; \tau', \tau') = \frac{H^2}{4\pi^2} \left[\ln a(\tau') + \ln \frac{1}{0} \right]$$

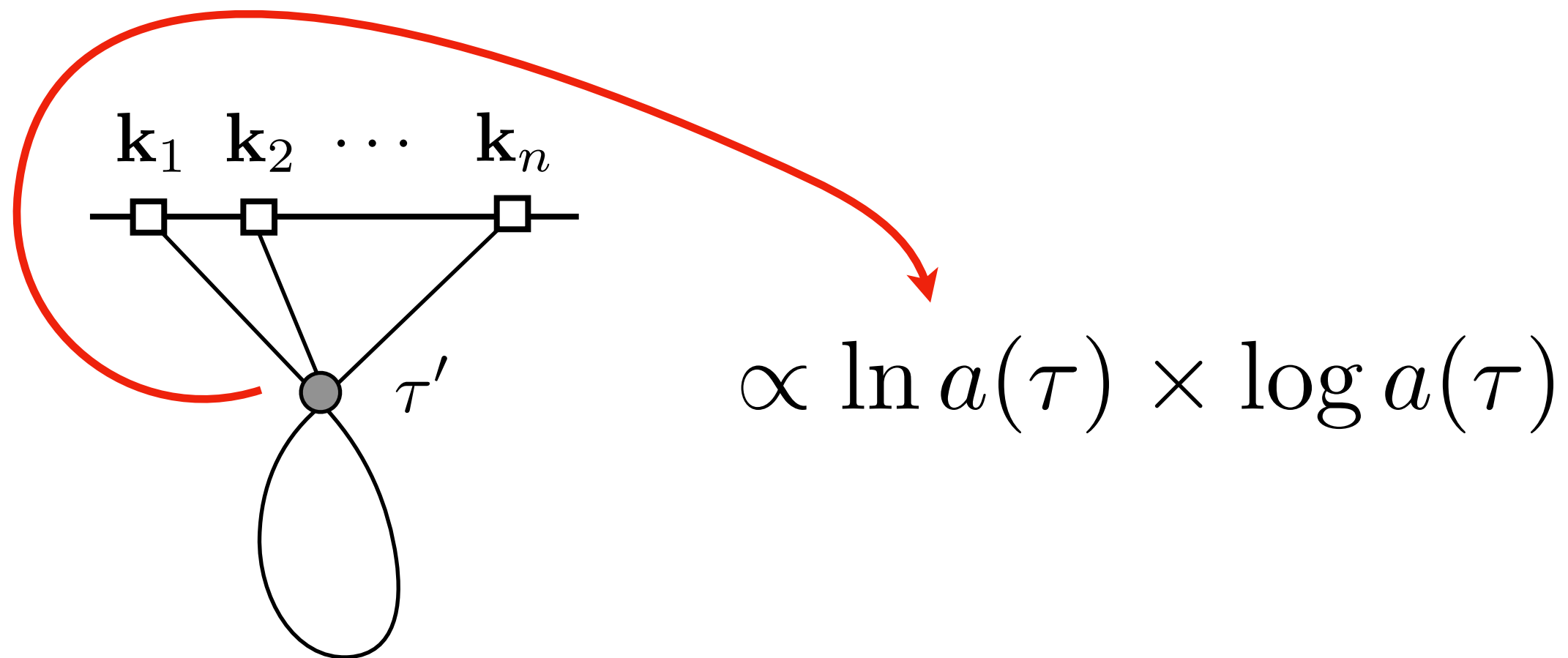
Why is Allen's result relevant for correlation functions?



$$\propto \ln a(\tau) \times \log a(\tau)$$

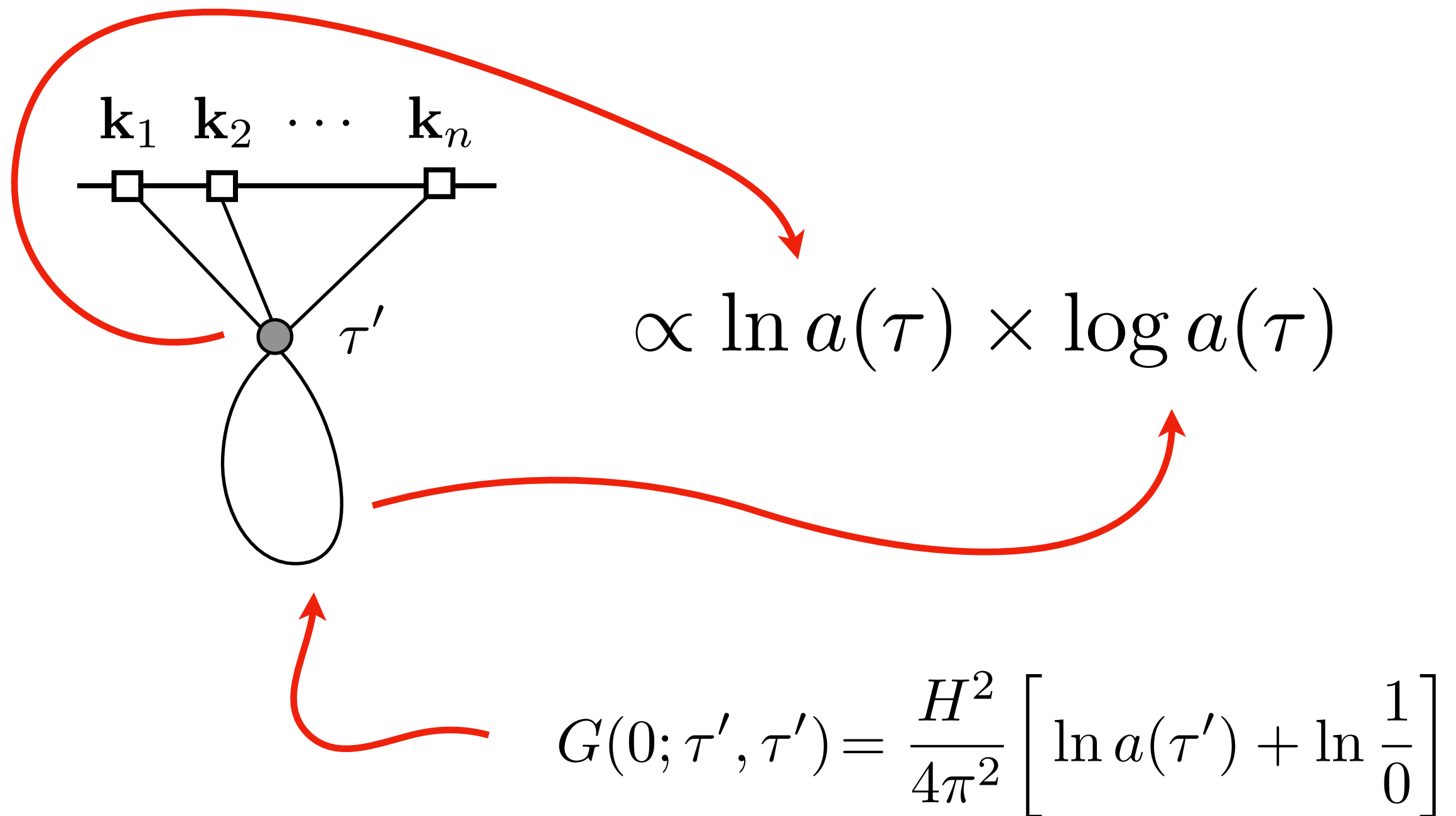
$$G(0; \tau', \tau') = \frac{H^2}{4\pi^2} \left[\ln a(\tau') + \ln \frac{1}{0} \right]$$

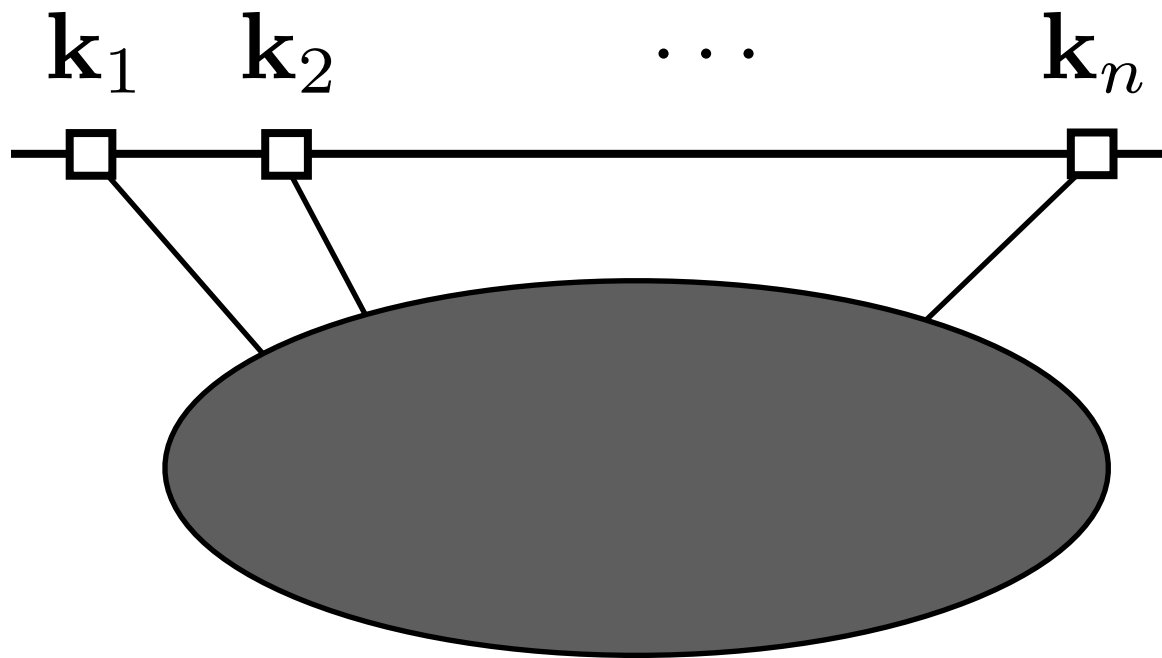
Why is Allen's result relevant for correlation functions?



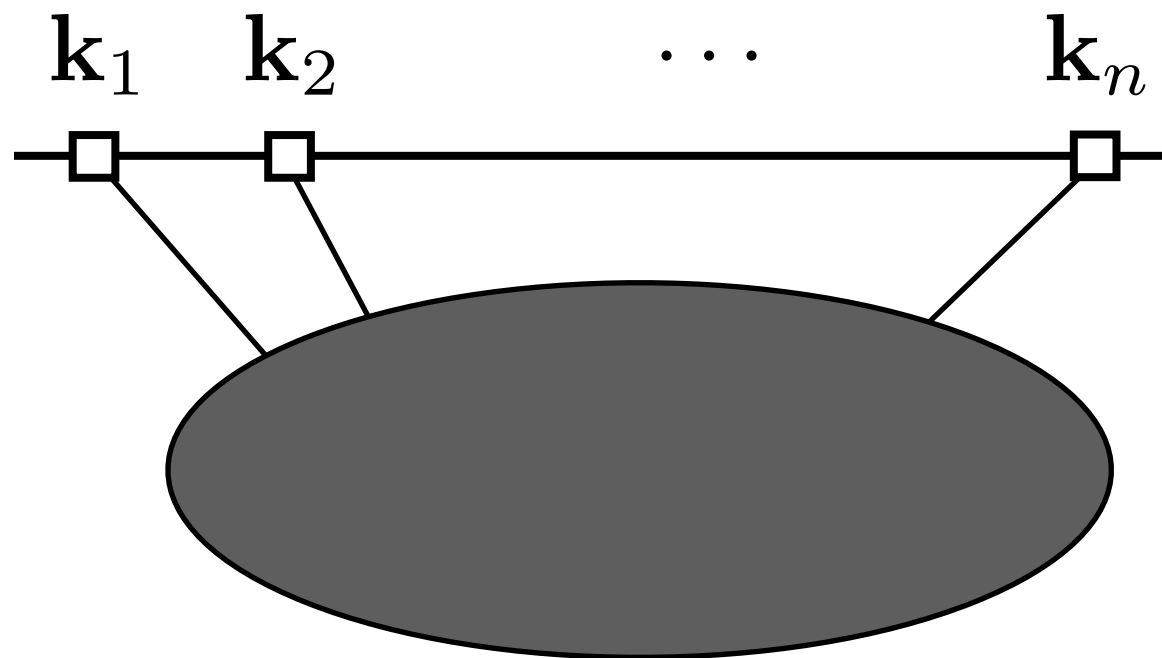
$$G(0; \tau', \tau') = \frac{H^2}{4\pi^2} \left[\ln a(\tau') + \ln \frac{1}{0} \right]$$

Why is Allen's result relevant for correlation functions?



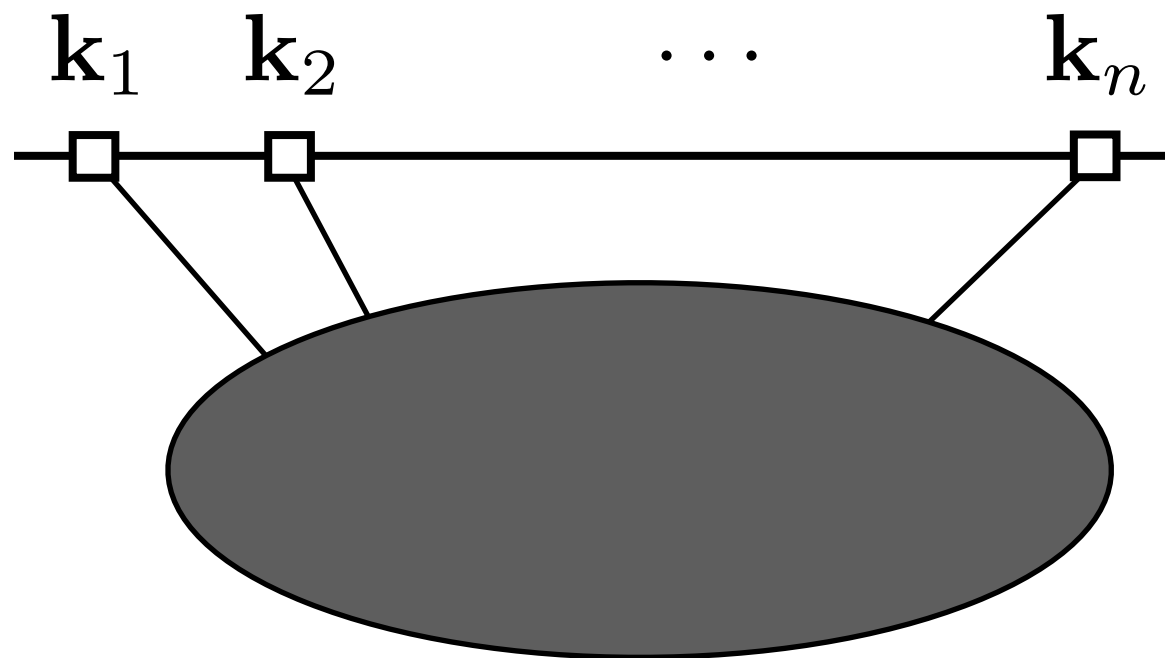


$$\propto [\ln a(\tau)]^V \times [\log a(\tau)]^L$$



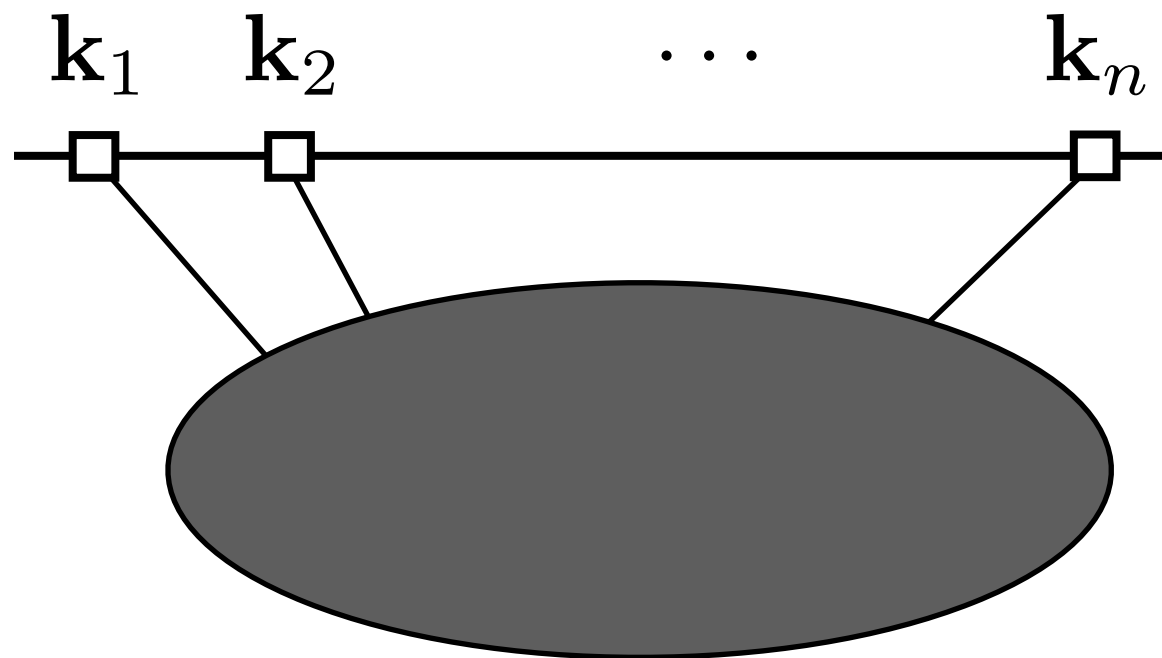
$$\propto [\ln a(\tau)]^V \times [\log a(\tau)]^L$$

- ✿ Now you have de Sitter invariant secular growth (from vertices) and de Sitter breaking secular growth (from loops)



$$\propto [\ln a(\tau)]^V \times [\log a(\tau)]^L$$

- ✿ Now you have de Sitter invariant secular growth (from vertices) and de Sitter breaking secular growth (from loops)
- ✿ However, on top of giving you secular growth, loops continue to imply IR divergences coming from integrals



$$\propto [\ln a(\tau)]^V \times [\log a(\tau)]^L$$

- ✿ Now you have de Sitter invariant secular growth (from vertices) and de Sitter breaking secular growth (from loops)
- ✿ However, on top of giving you secular growth, loops continue to imply IR divergences coming from integrals
- ✿ To trust computations in the limit $k_i |\tau| \rightarrow 0$ you have to find a way to resume all of these IR contributions

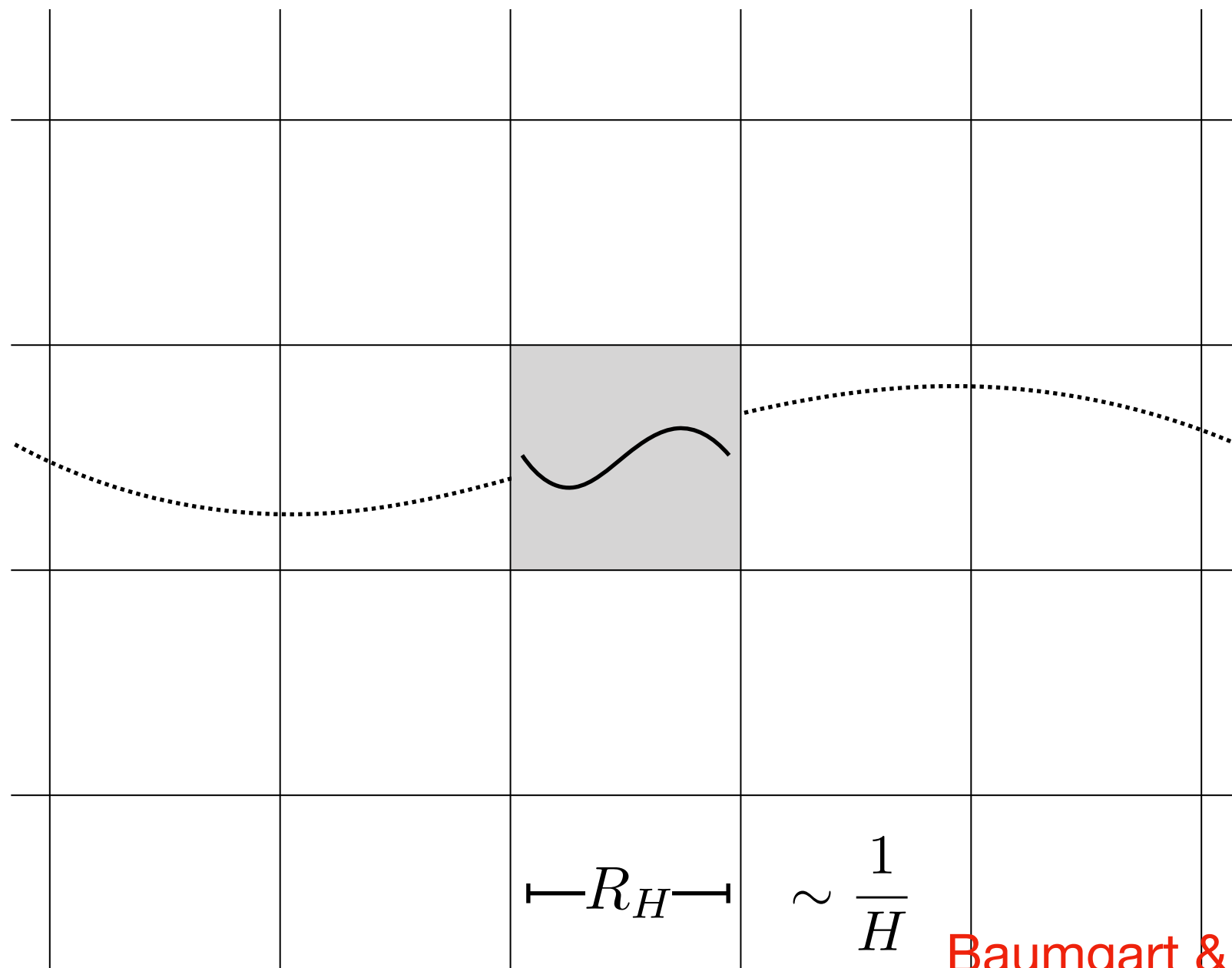
Is it possible to treat the divergences in a controlled way, such that only the secular growth needs to be addressed?

Is it possible to treat the divergences in a controlled way, such that only the secular growth needs to be addressed?

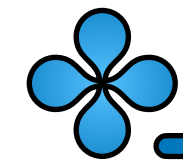
There is a nice classical argument by Starobinsky and many others:

Is it possible to treat the divergences in a controlled way, such that only the secular growth needs to be addressed?

There is a nice classical argument by Starobinsky and many others:



Baumgart & Sundrum (2019)

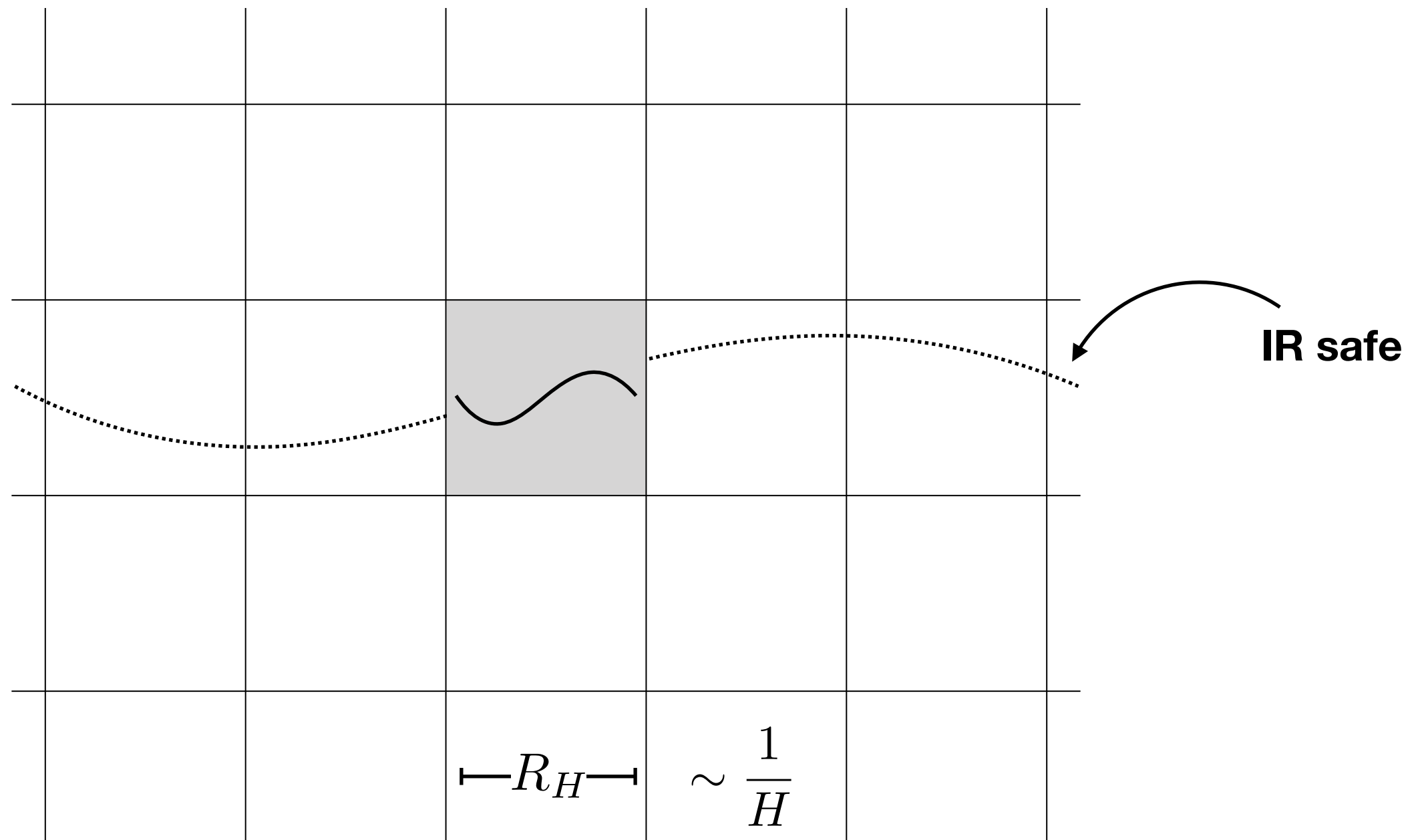


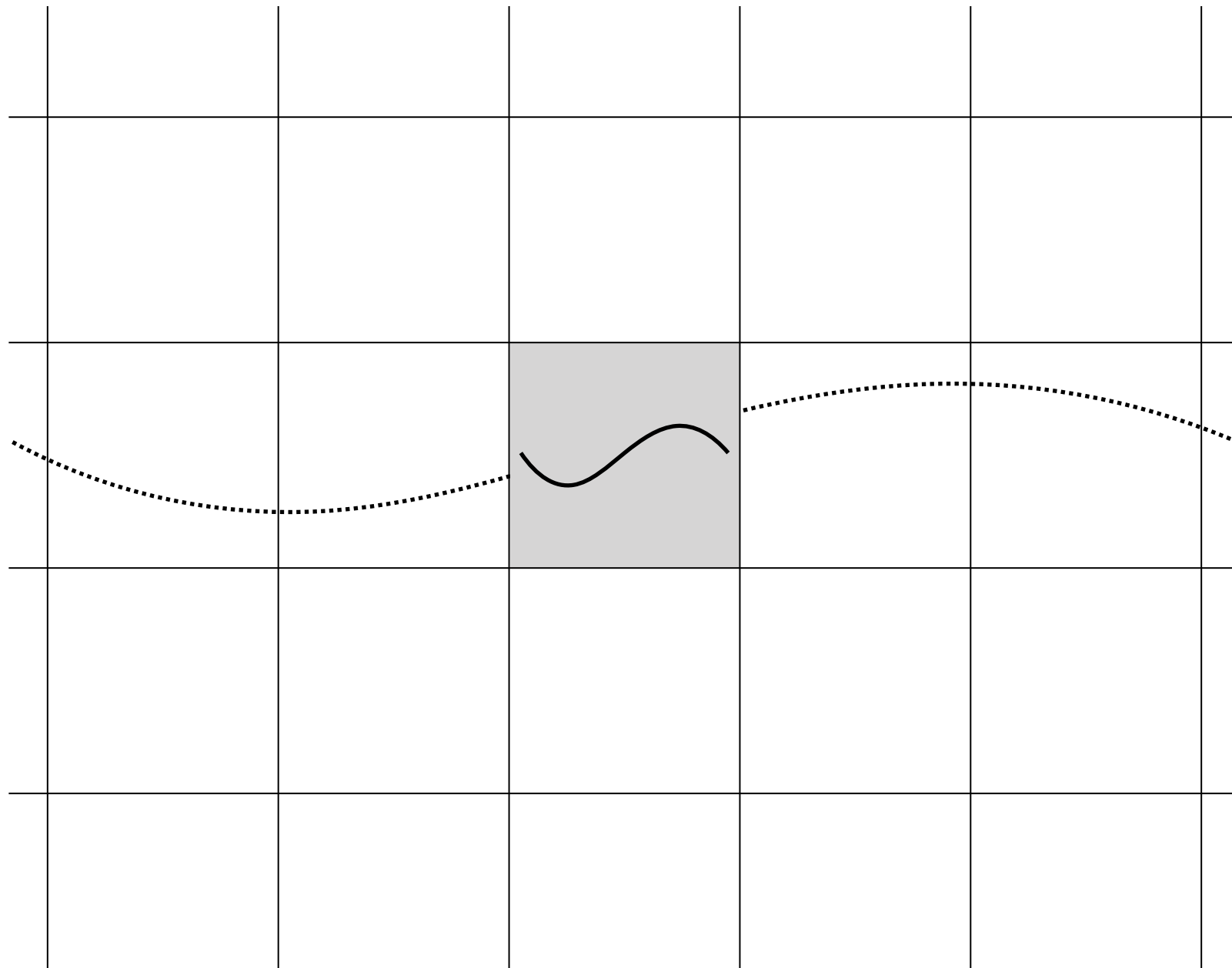
Correlators in dS

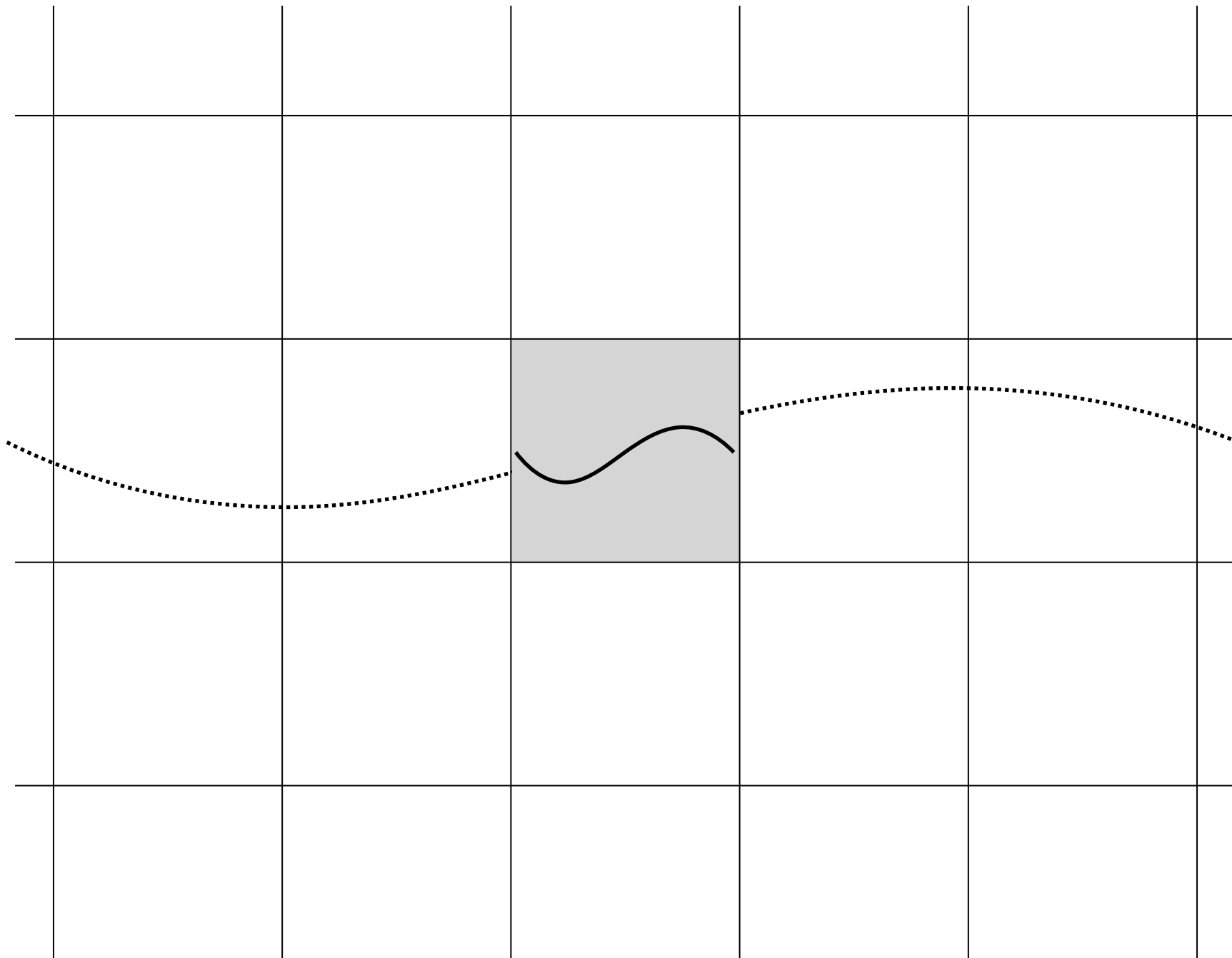
18

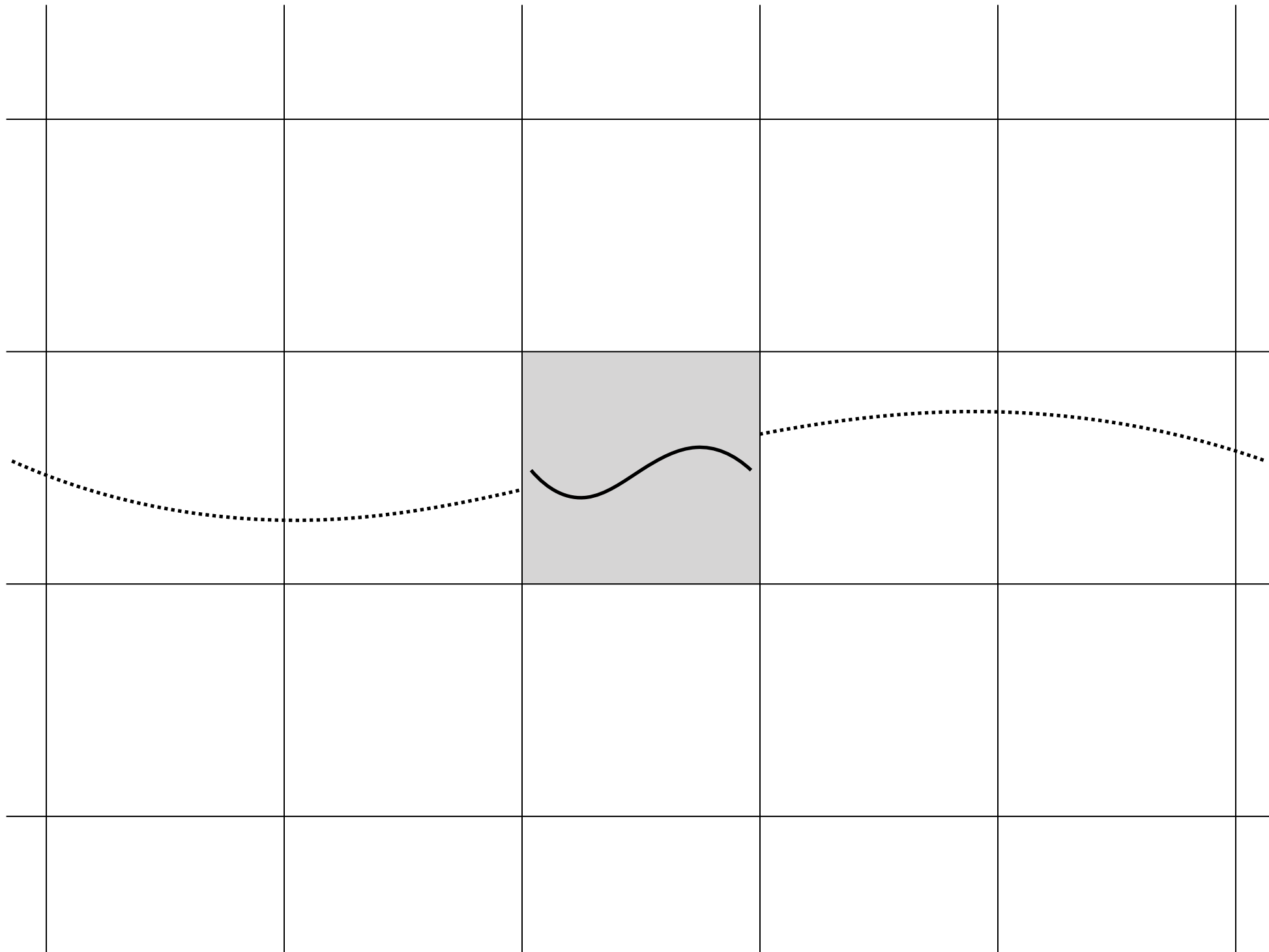
Let's say inflation is preceded by an (IR finite) radiation dominated era

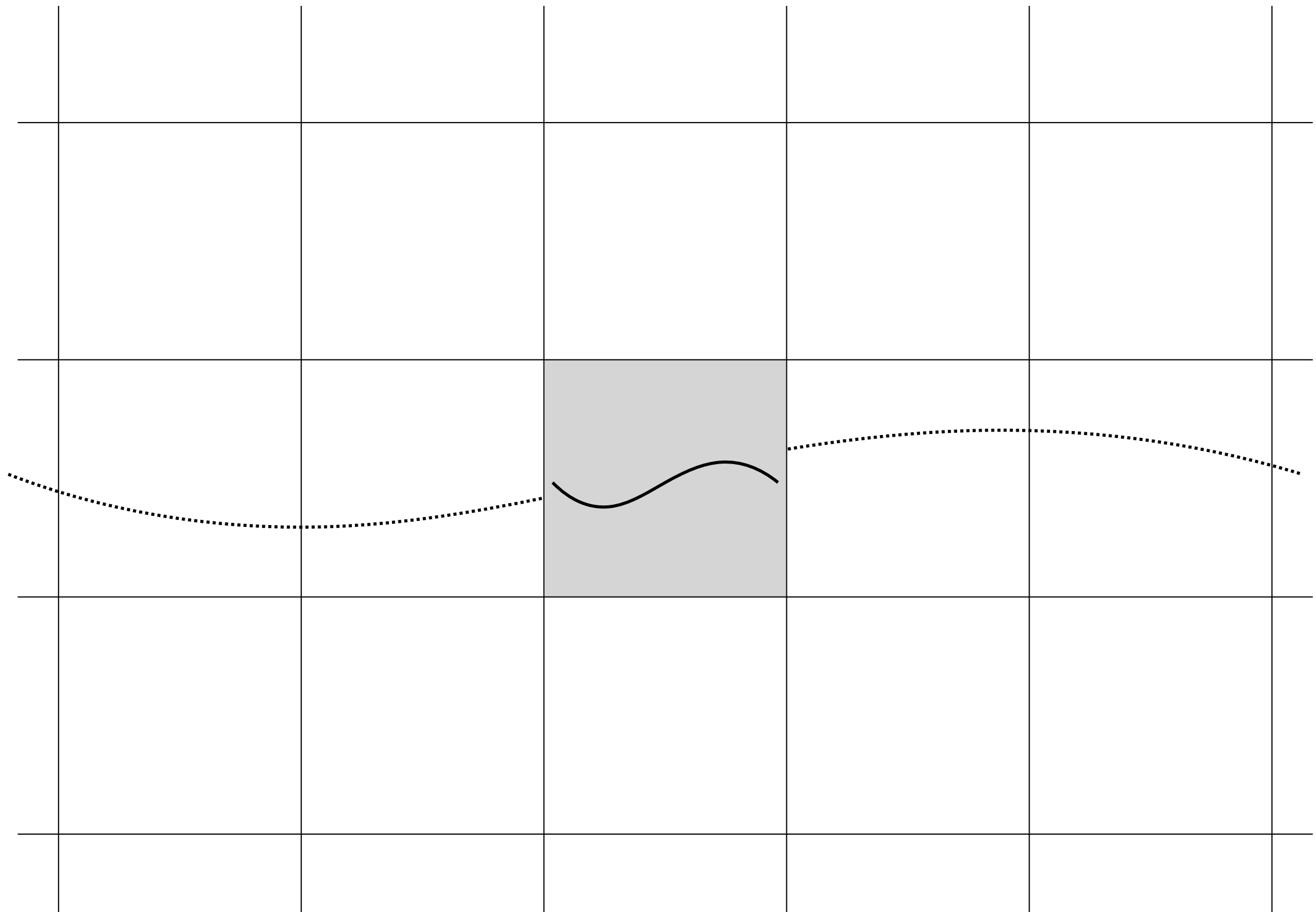
$$\tau = \tau_0$$

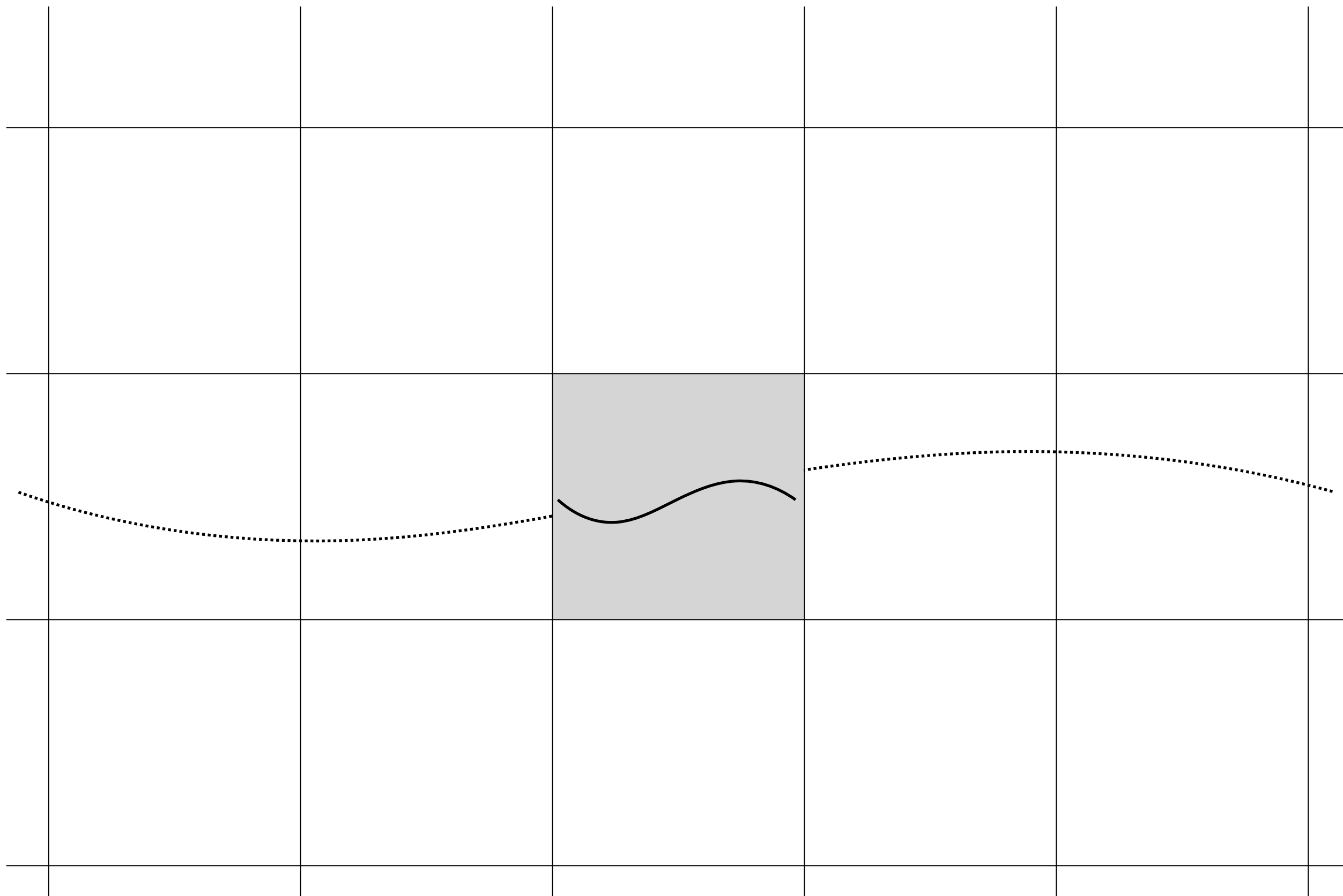


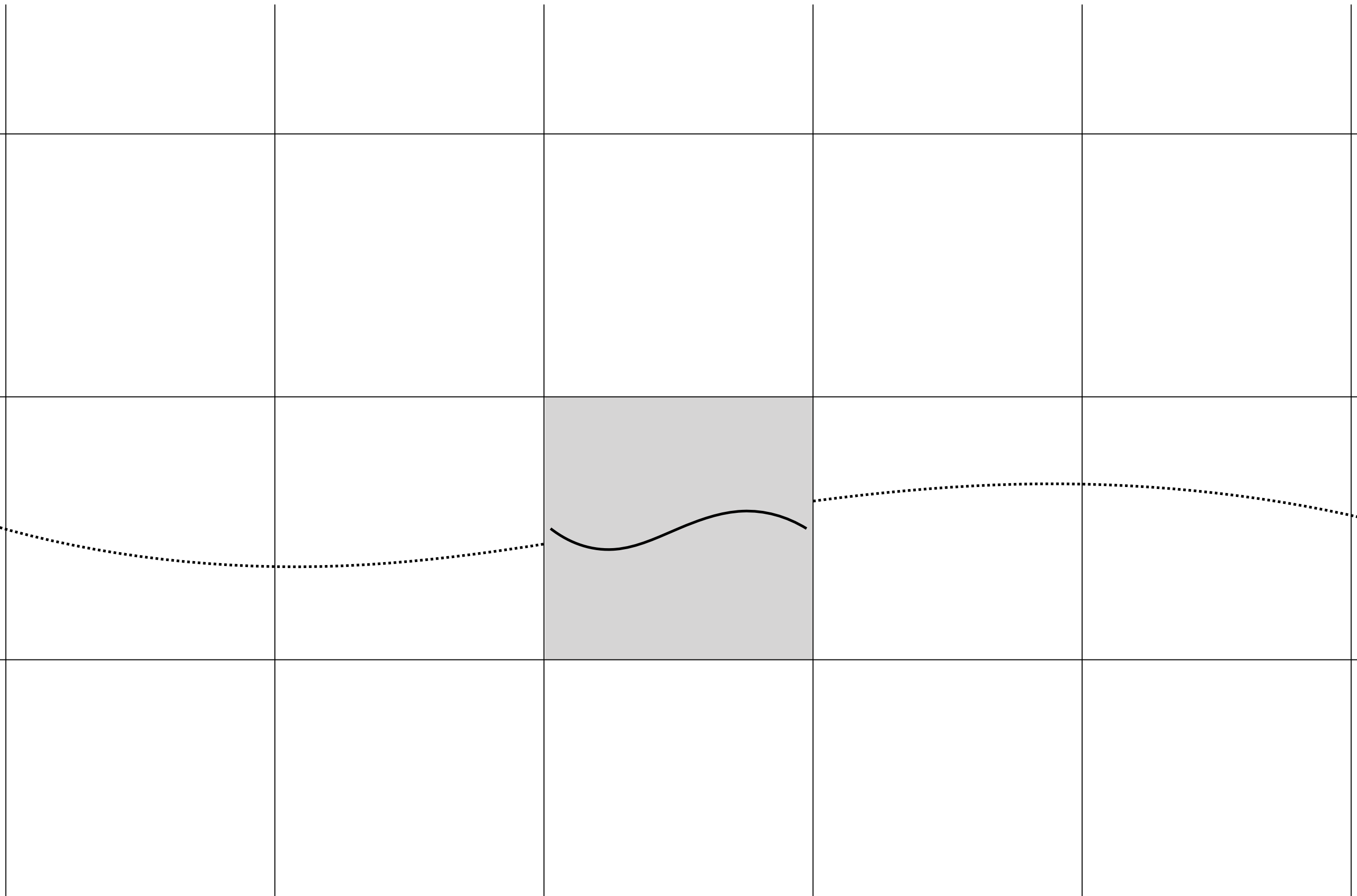


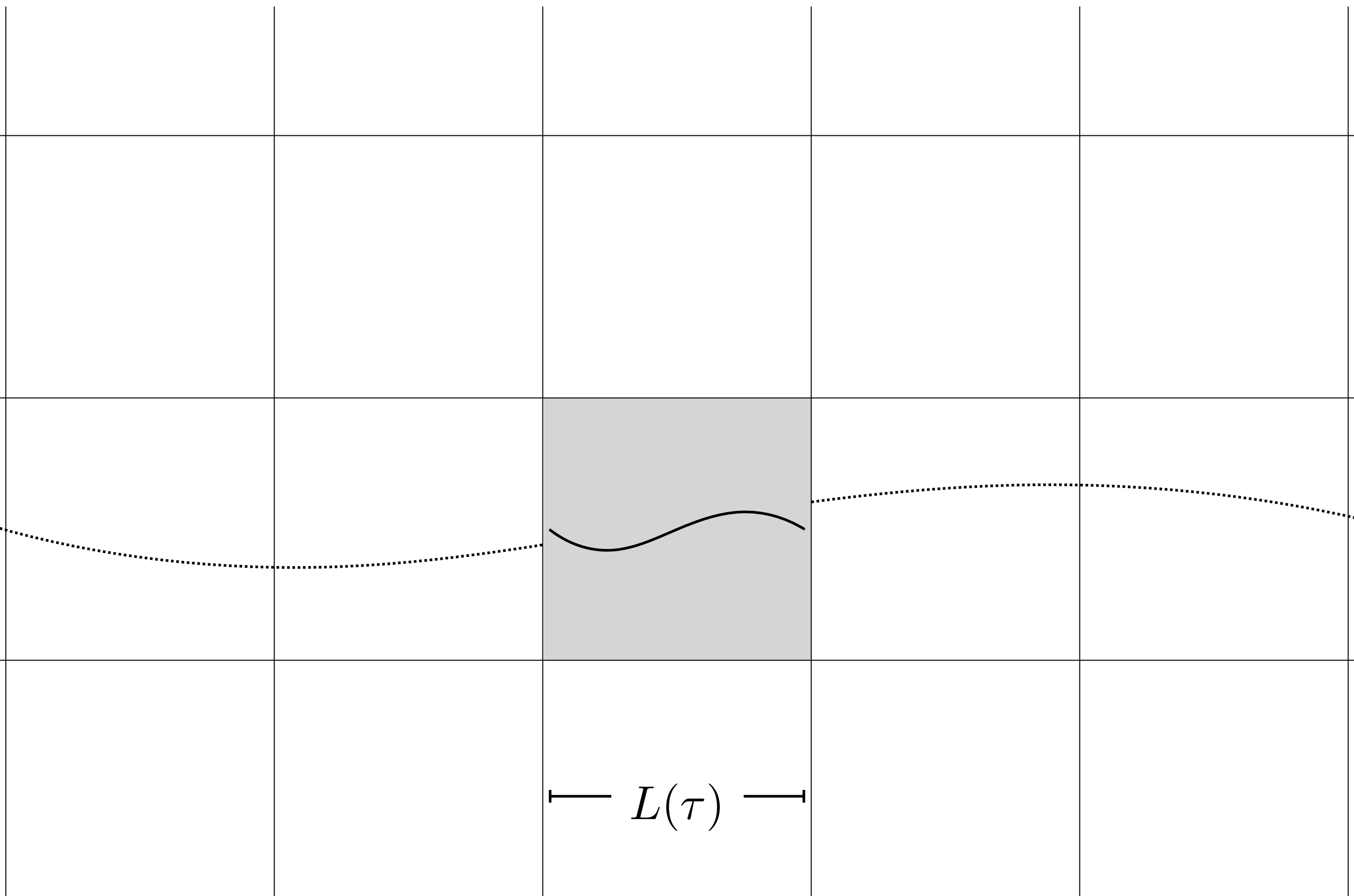


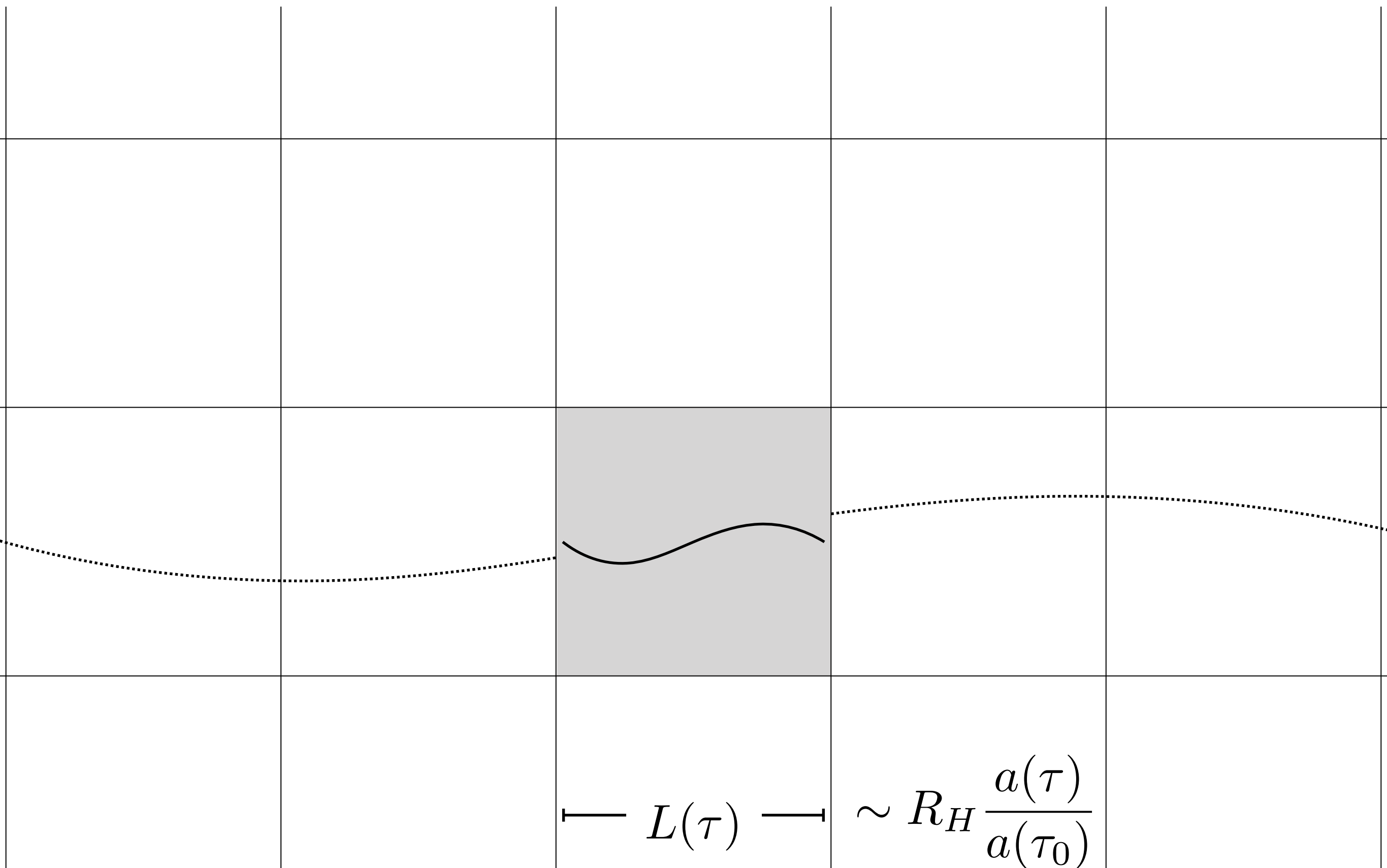


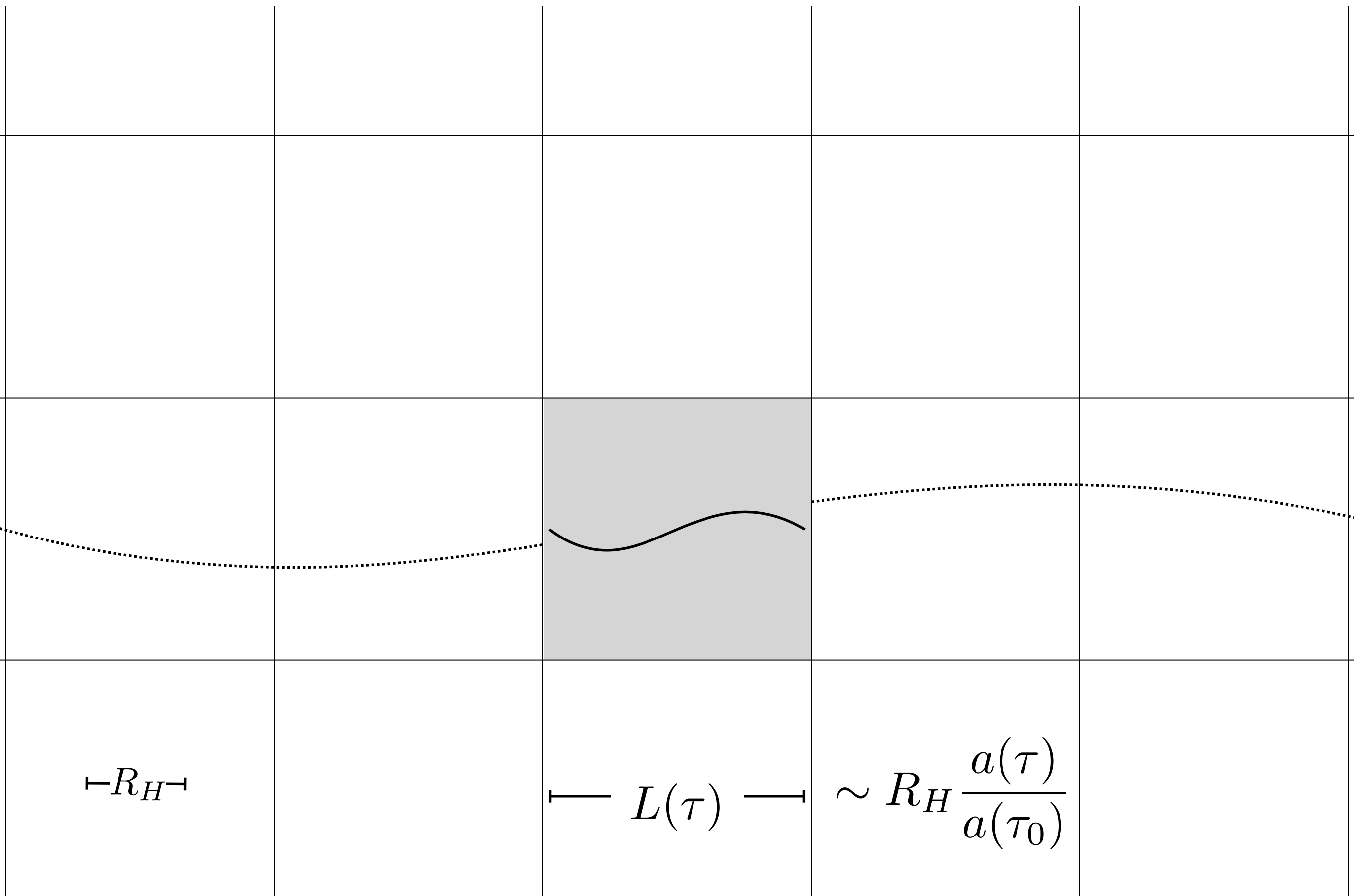


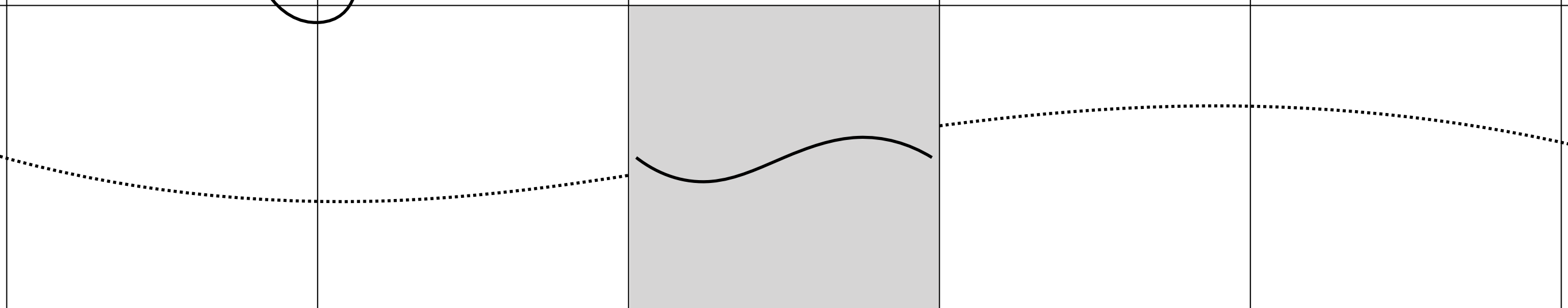
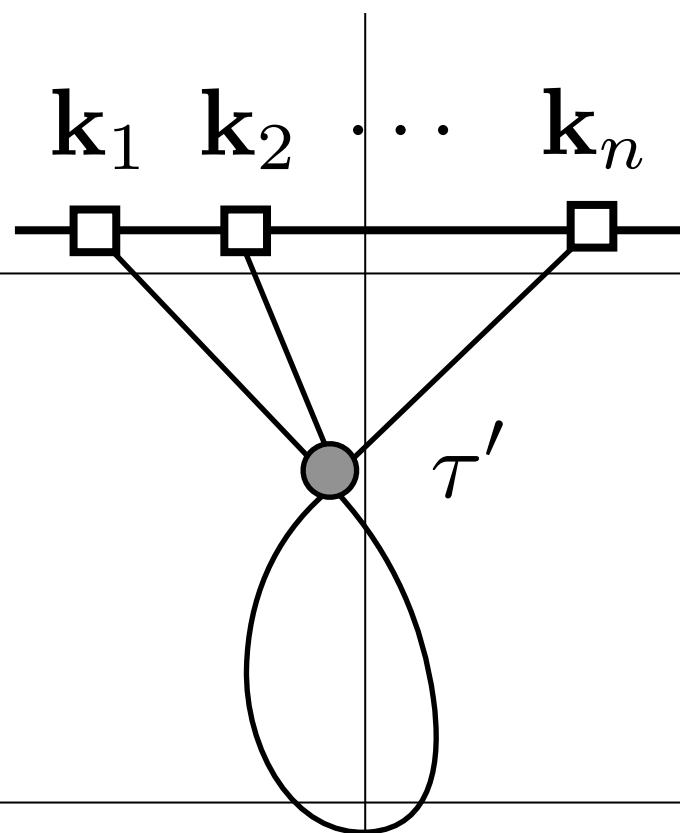






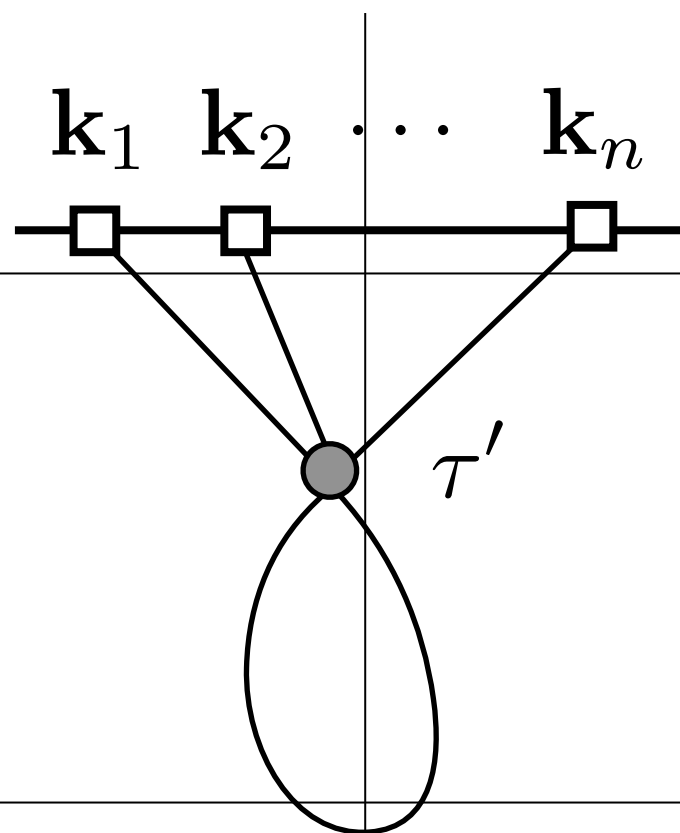




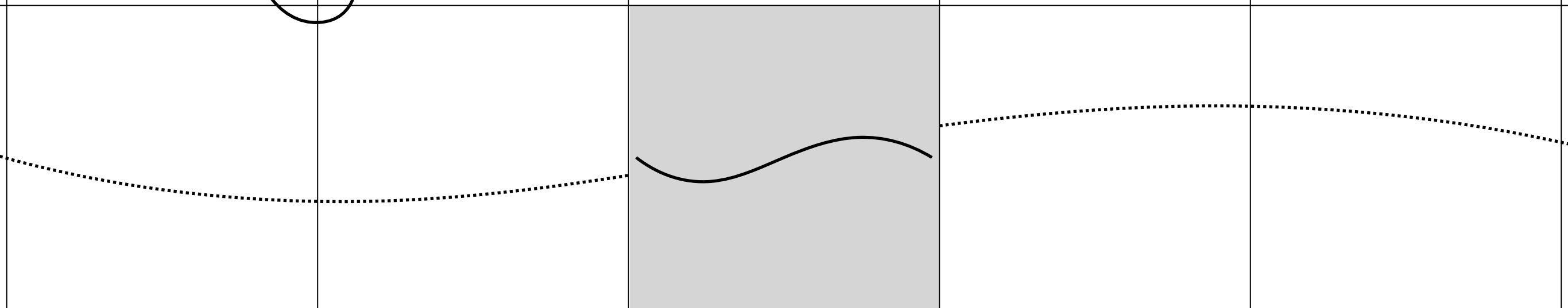


$$\vdash R_H \vdash$$

$$\vdash L(\tau) \vdash \sim R_H \frac{a(\tau)}{a(\tau_0)}$$



$$\propto \ln \left[\frac{k_1 + \dots + k_N}{a(\tau)} \right] \times \ln \frac{a(\tau)}{a(\tau_0)}$$


 $\vdash R_H \vdash$

$$\vdash L(\tau) \vdash \sim R_H \frac{a(\tau)}{a(\tau_0)}$$

What about theories with non-derivative interactions?



$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$

 Here the shift symmetry that led to secular growth is broken



What about theories with non-derivative interactions?

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$

-  Here the shift symmetry that led to secular growth is broken
-  Therefore, one should not trust the free theory all the way down up $k = 0$

What about theories with non-derivative interactions?

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$

- ✿ Here the shift symmetry that led to secular growth is broken
- ✿ Therefore, one should not trust the free theory all the way down up $k = 0$

- ✿ There must exist a lengthscale Λ_{IR}^{-1} beyond which the evolution becomes strongly nonlinear

$$k/a(\tau) < \Lambda_{\text{IR}}$$

Light scalar field theories

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$

✿ Light scalar field theories

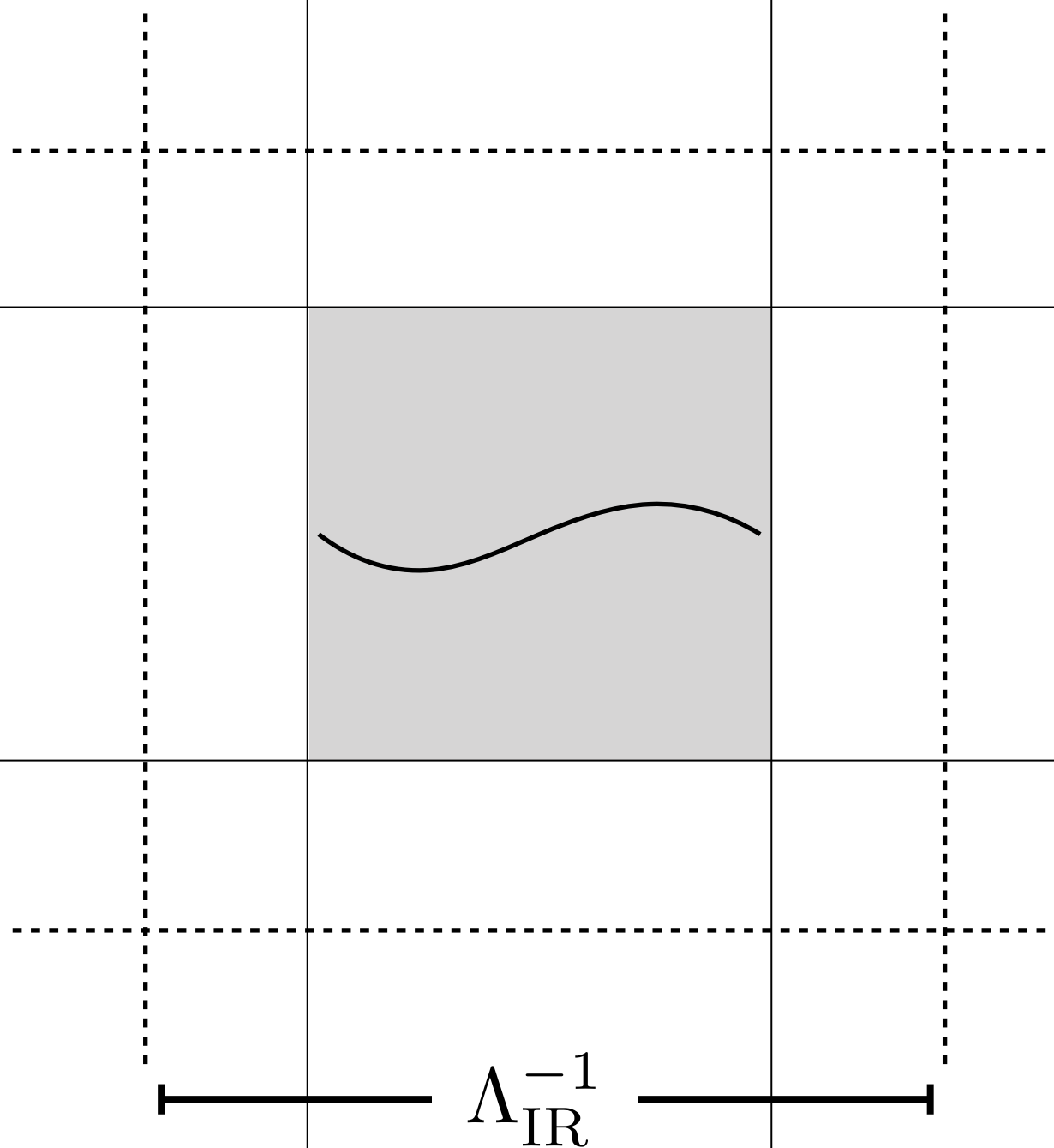
20

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$



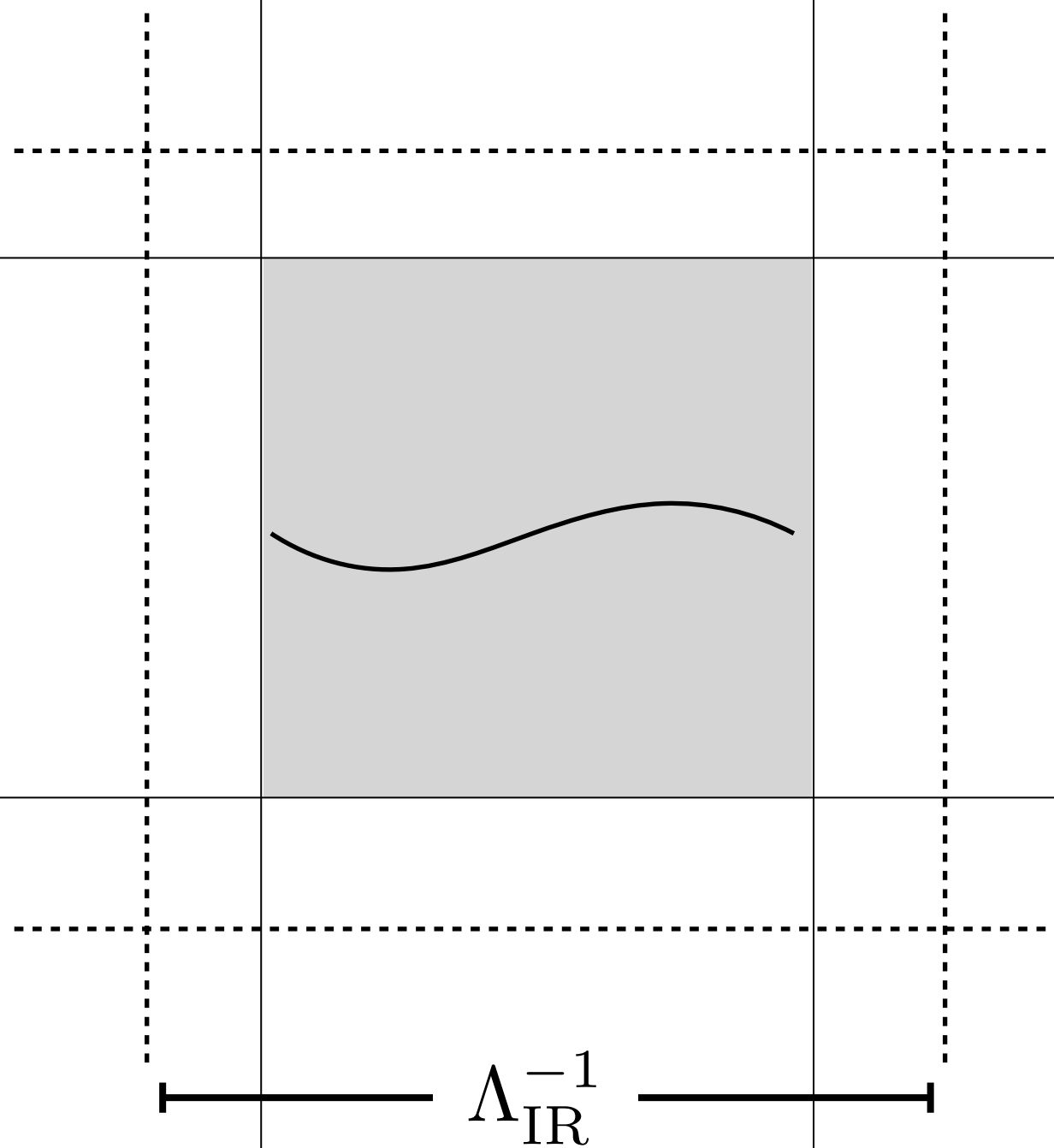
Light scalar field theories

20



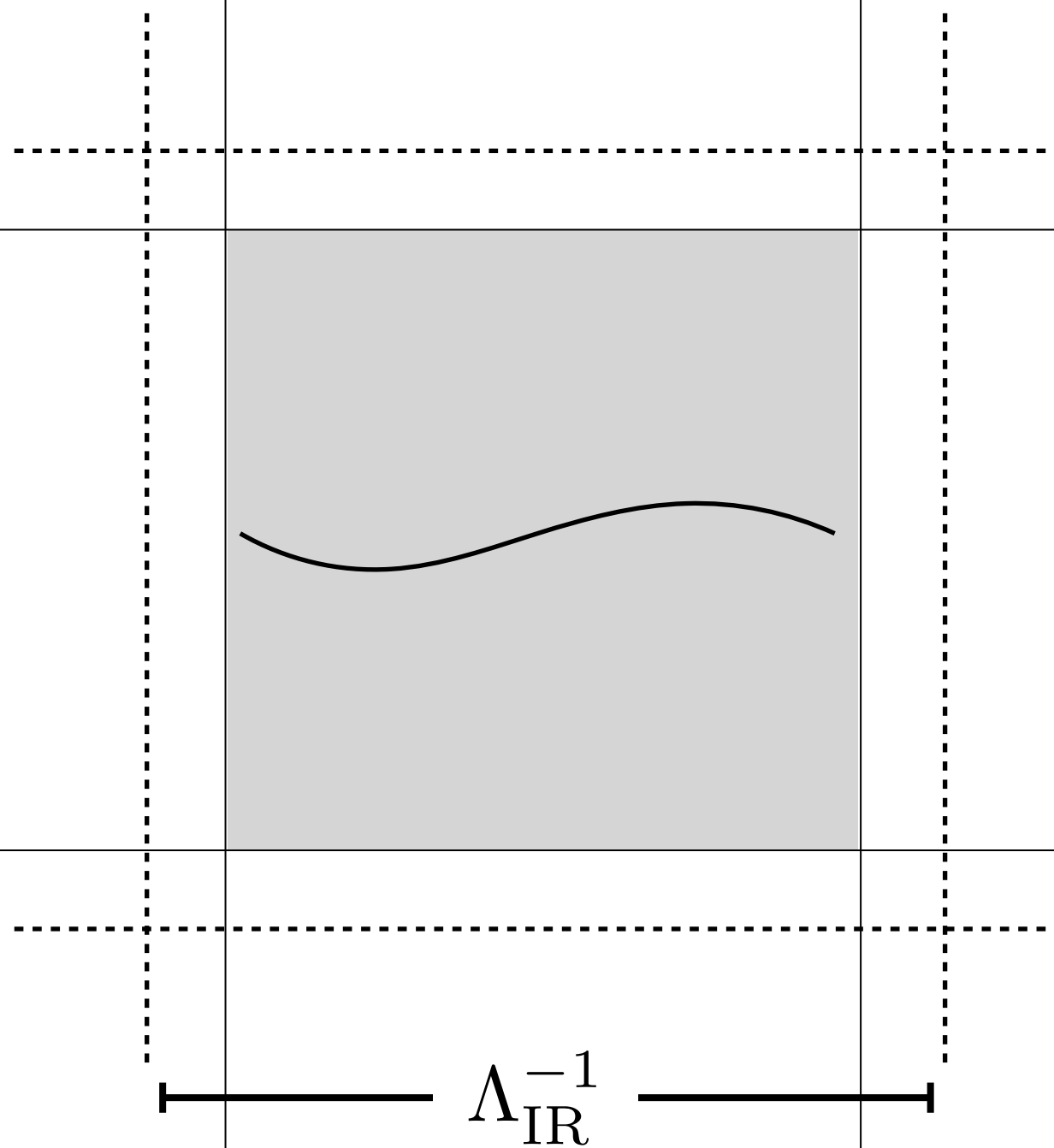
Light scalar field theories

20



Light scalar field theories

20



Light scalar field theories

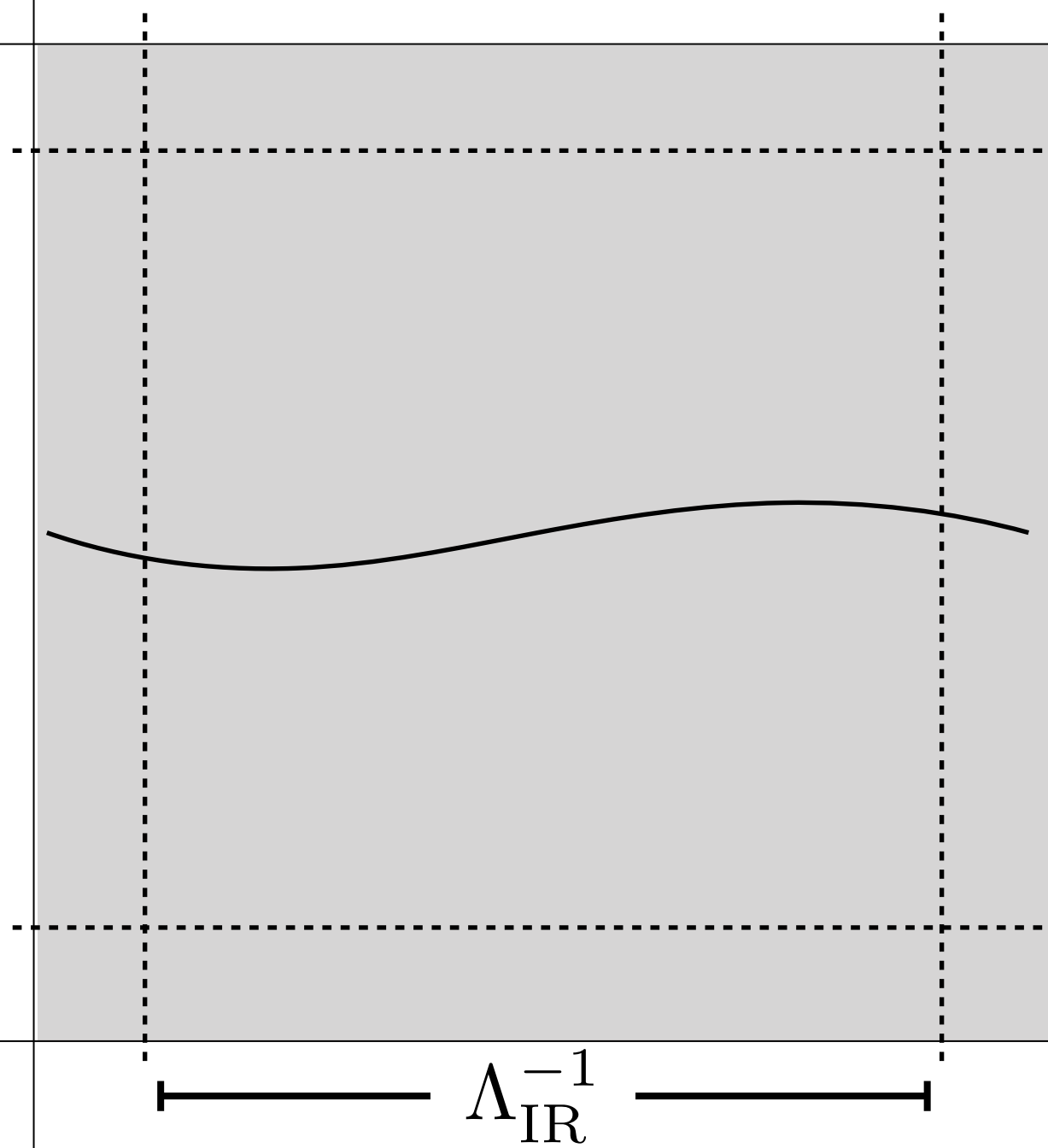
20



$$\text{---} \Lambda_{\text{IR}}^{-1} \text{---}$$

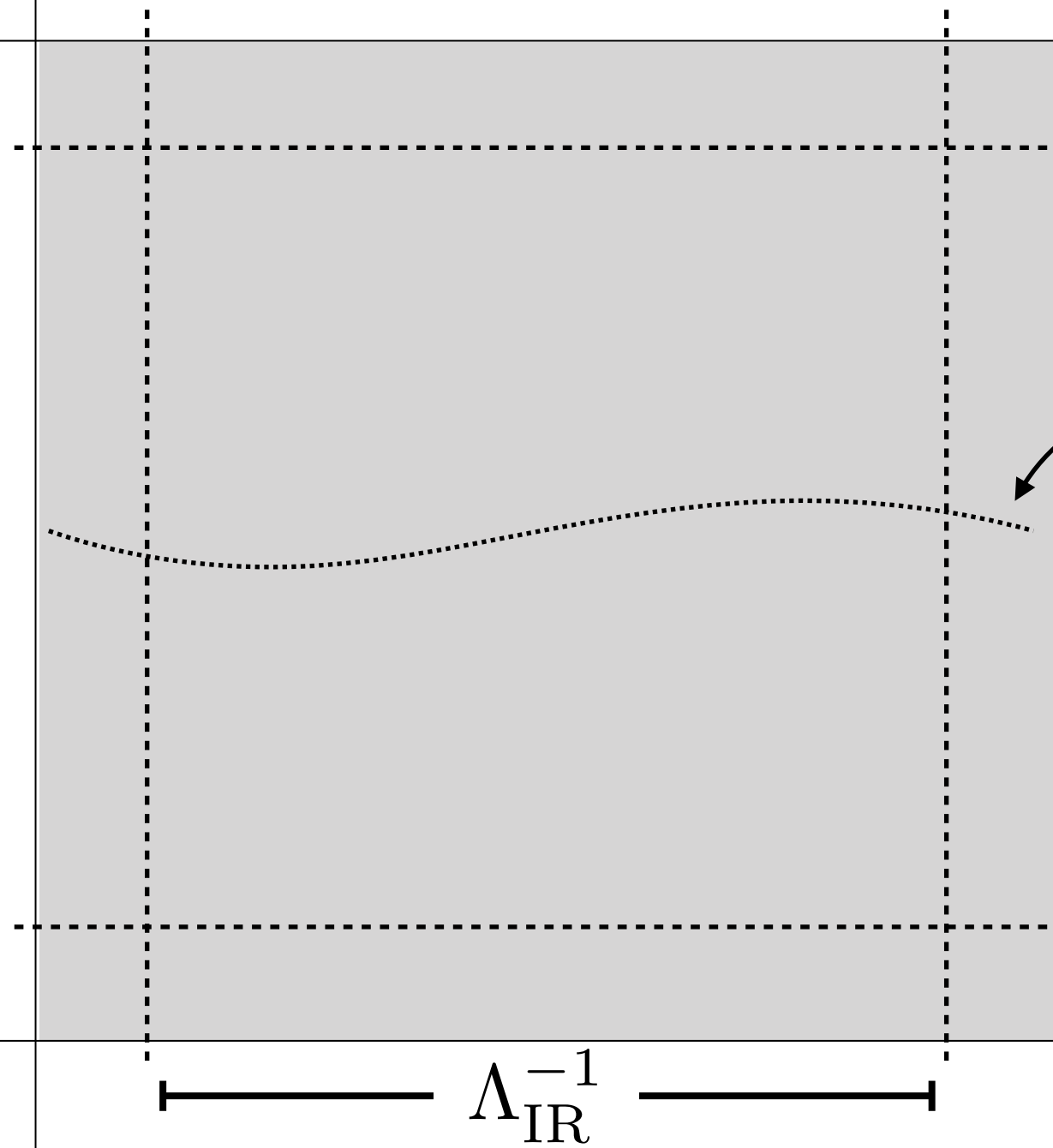
Light scalar field theories

20



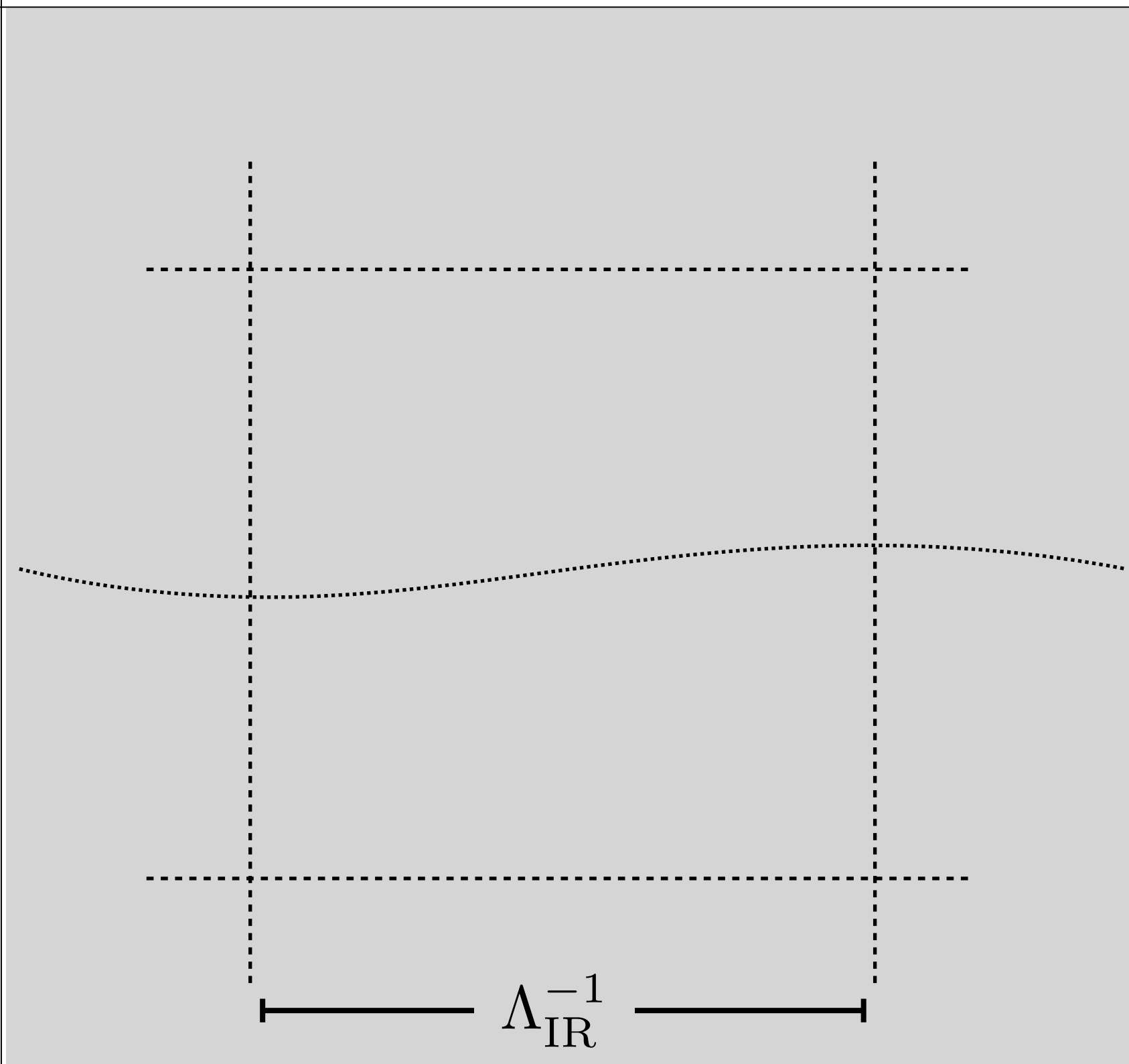
✿ Light scalar field theories

20



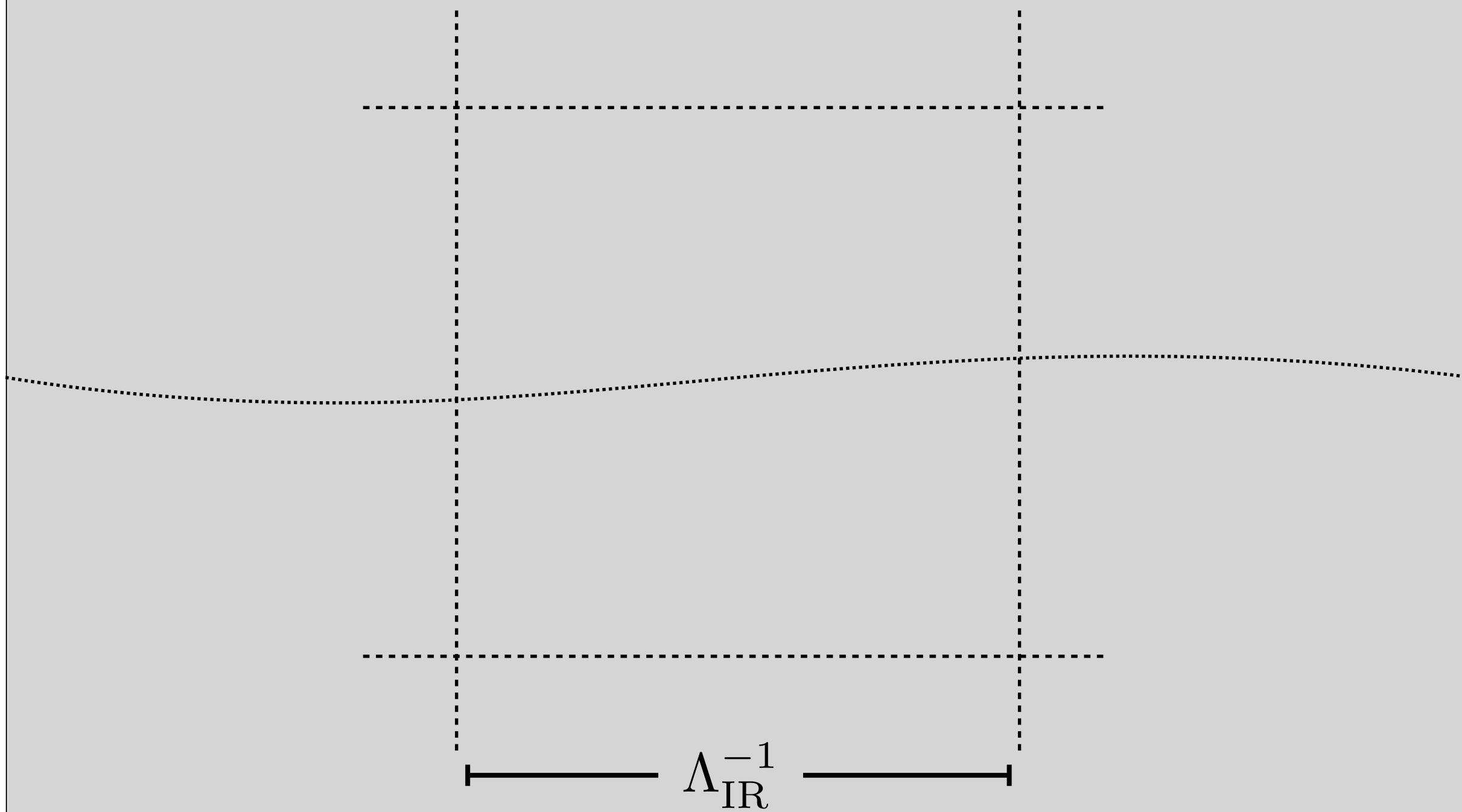
Now this mode
is IR safe

$$\Lambda_{\text{IR}}^{-1}$$



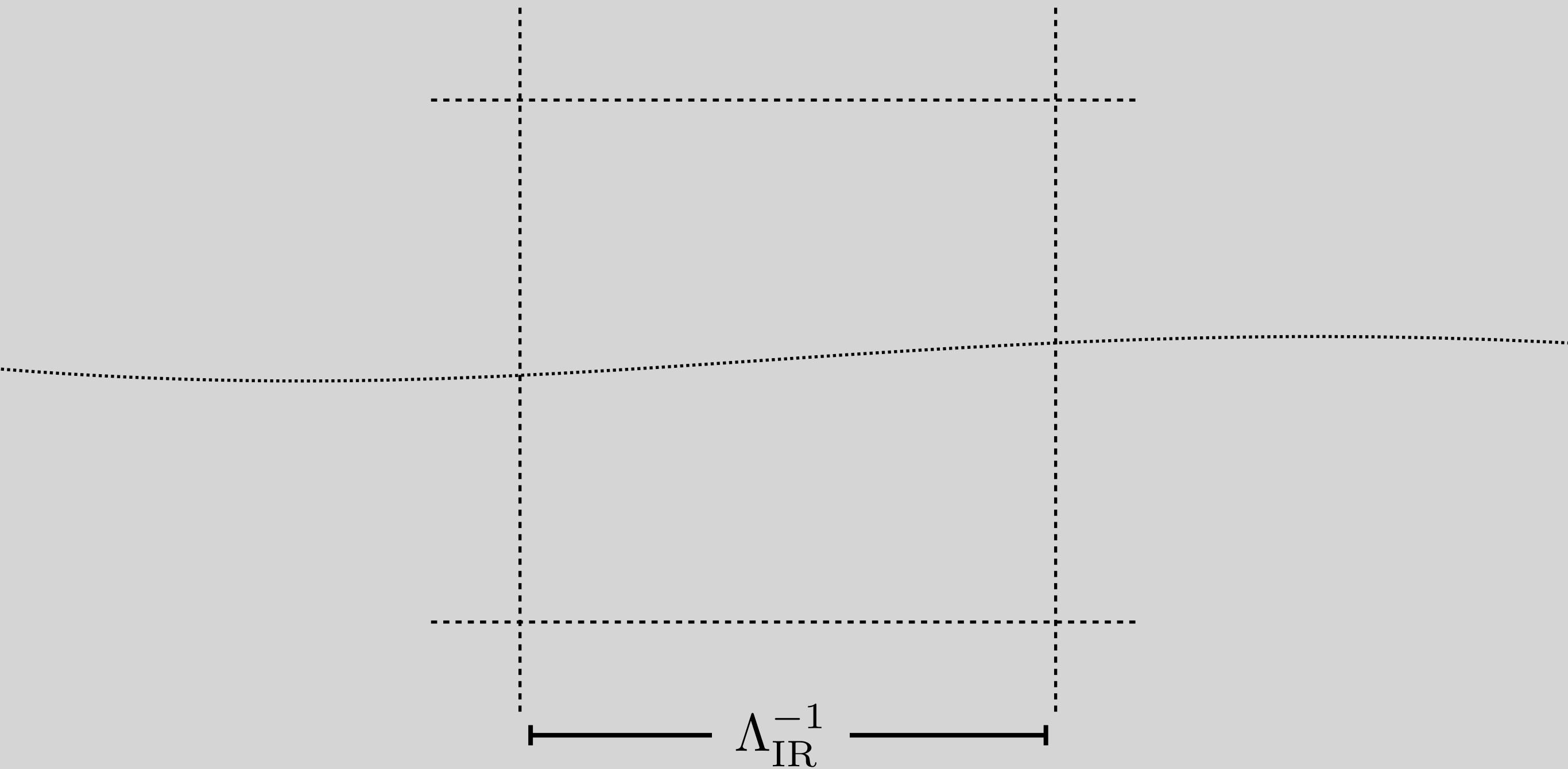
Light scalar field theories

20



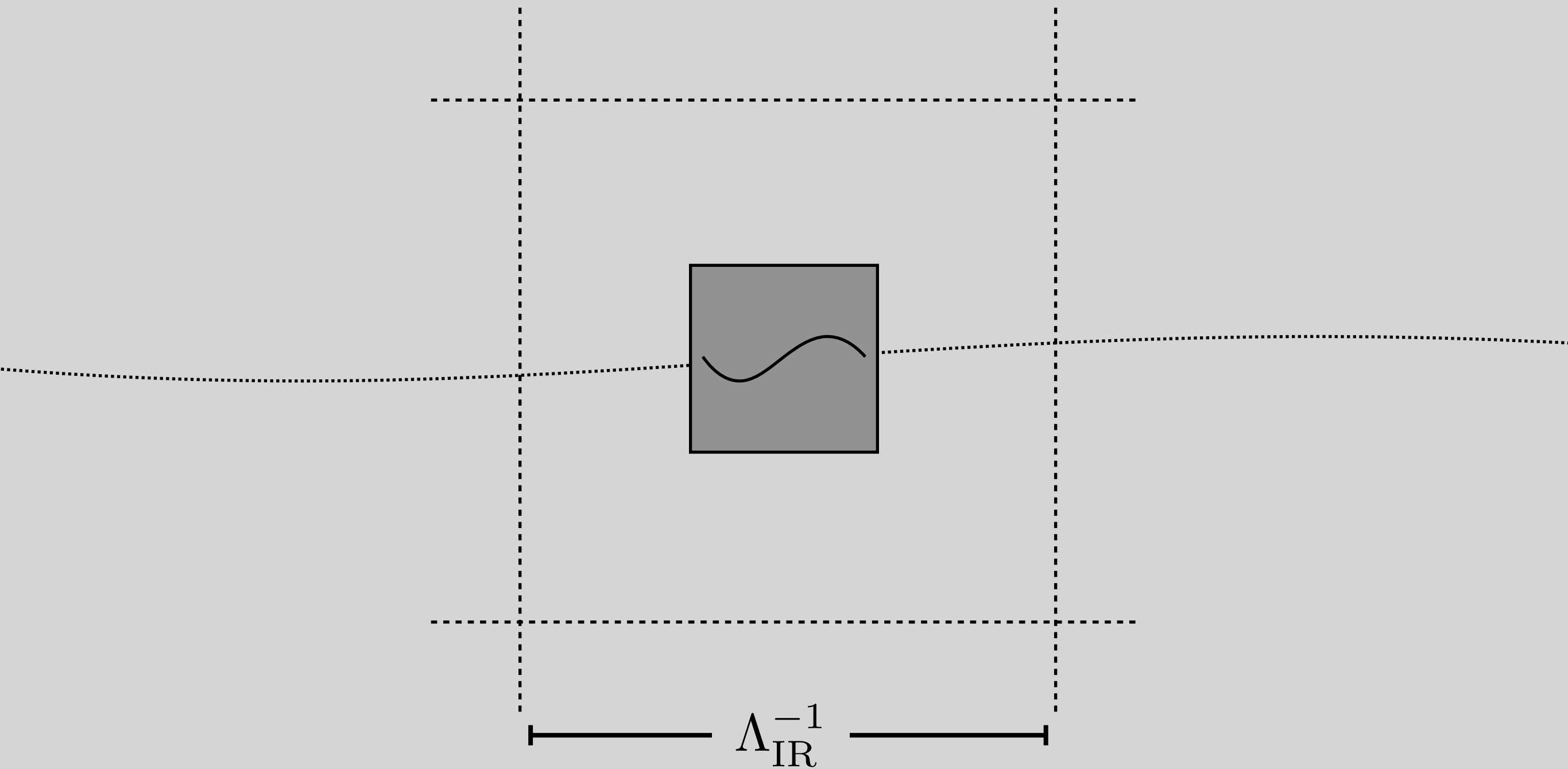
✿ Light scalar field theories

20



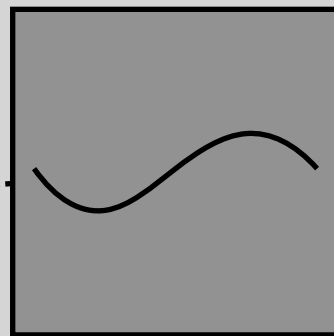
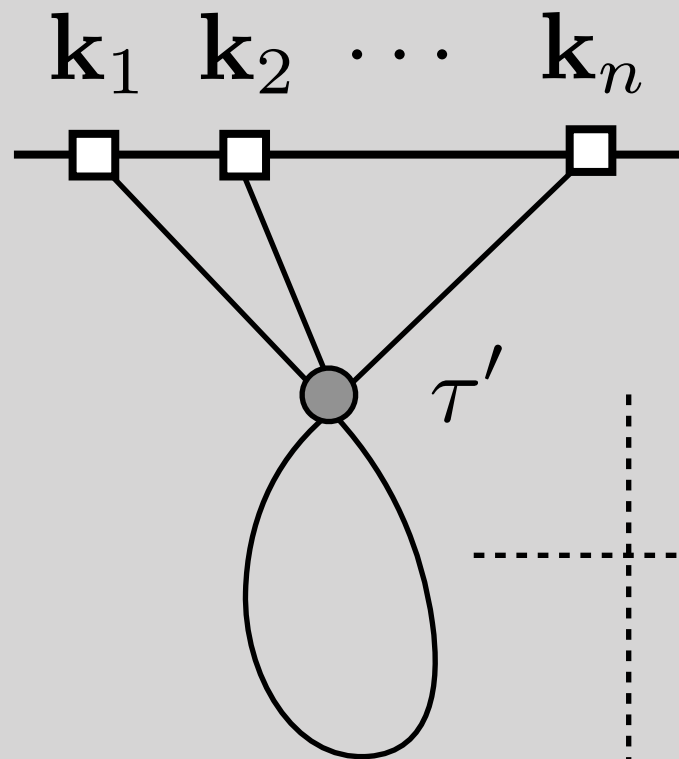
Light scalar field theories

20



Light scalar field theories

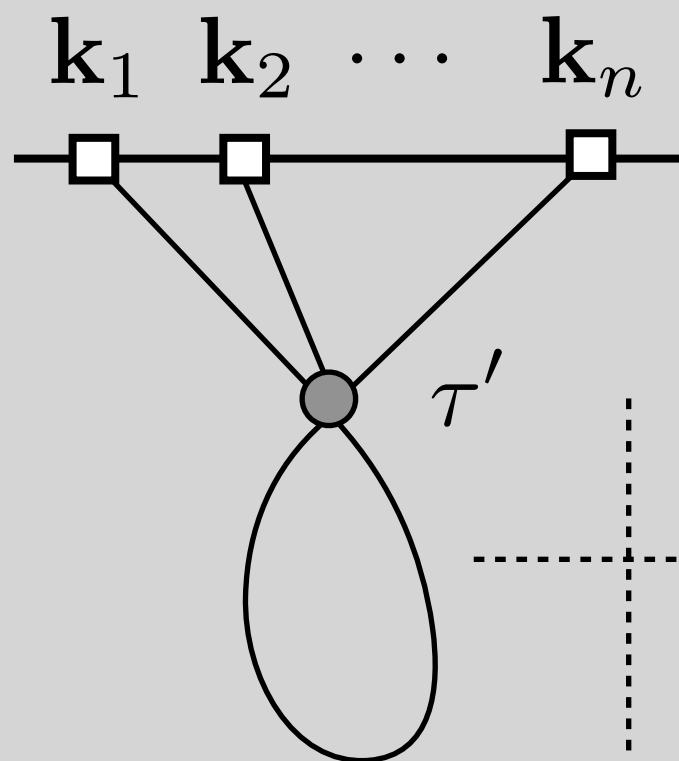
20



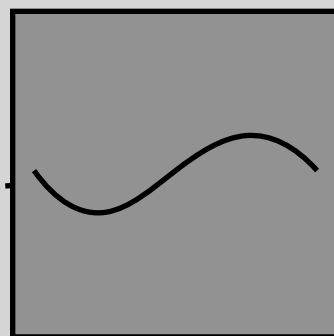
$$\Lambda_{\text{IR}}^{-1}$$

✿ Light scalar field theories

20



$$\propto \ln \left[\frac{k_1 + \dots + k_N}{a(\tau)} \right] \times \ln \frac{H}{\Lambda_{\text{IR}}}$$



$$\Lambda_{\text{IR}}^{-1}$$

Light scalar field theories

21

There is an associated mass scale

$$m_{\text{IR}}^2 = \frac{3H^2}{2 \ln(H/\Lambda_{\text{IR}})}$$

There is an associated mass scale

$$m_{\text{IR}}^2 = \frac{3H^2}{2 \ln(H/\Lambda_{\text{IR}})}$$

Thus, to regulate the IR perform perturbation theory with a mass:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2 \right]$$

There is an associated mass scale

$$m_{\text{IR}}^2 = \frac{3H^2}{2 \ln(H/\Lambda_{\text{IR}})}$$

Thus, to regulate the IR perform perturbation theory with a mass:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2 \right]$$

In the long wavelength regime one has

$$|f_k(\tau)|^2 \sim \frac{1}{k^3} \left(k|\tau| \right)^{2m^2/3H^2}$$

There is an associated mass scale

$$m_{\text{IR}}^2 = \frac{3H^2}{2 \ln(H/\Lambda_{\text{IR}})}$$

Thus, to regulate the IR perform perturbation theory with a mass:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2 \right]$$

In the long wavelength regime one has

$$|f_k(\tau)|^2 \sim \frac{1}{k^3} \left(k|\tau| \right)^{2m^2/3H^2}$$

 If $m^2 \ll m_{\text{IR}}^2$ then external legs are effectively massless

There is an associated mass scale

$$m_{\text{IR}}^2 = \frac{3H^2}{2 \ln(H/\Lambda_{\text{IR}})}$$

Thus, to regulate the IR perform perturbation theory with a mass:

$$S = \int d^3x d\tau a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \frac{1}{2} m^2 \varphi^2 \right]$$

In the long wavelength regime one has

$$|f_k(\tau)|^2 \sim \frac{1}{k^3} \left(k|\tau| \right)^{2m^2/3H^2}$$

 If $m^2 \ll m_{\text{IR}}^2$ then external legs are effectively massless

 However, loops are regulated by m in a dS invariant way

(Recall Zhong-Zhi talk)

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1 - Z} - \ln(1 - Z) + 3 \frac{H^2}{m^2} \right] \quad \text{(With a mass } m \text{)}$$

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 3 \frac{H^2}{m^2} \right] \quad \text{(With a mass } m \text{)}$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 2 \ln \left(\frac{H}{\Lambda} \right) \right] \quad \text{(With a physical cutoff)}$$
$$\Lambda = H e^{-\frac{2H^2}{2m^2}}$$

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 3 \frac{H^2}{m^2} \right] \quad \text{(With a mass } m \text{)}$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 2 \ln \left(\frac{H}{\Lambda} \right) \right] \quad \text{(With a physical cutoff)}$$

$$\Lambda = H e^{-\frac{2H^2}{2m^2}}$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \frac{2}{d-3} \right] \quad \text{(With dim reg employing Melville-Pajer scheme)}$$

$$G(|\mathbf{x} - \mathbf{x}'|; \tau, \tau') = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{k} \left(k^3 f_k(\tau) f_k^*(\tau') \frac{\sin(k|\mathbf{x} - \mathbf{x}'|)}{k|\mathbf{x} - \mathbf{x}'|} \right)$$

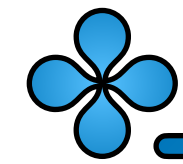
$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 3 \frac{H^2}{m^2} \right] \quad \text{(With a mass } m \text{)}$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + 2 \ln \left(\frac{H}{\Lambda} \right) \right] \quad \text{(With a physical cutoff)}$$

$$\Lambda = H e^{-\frac{2H^2}{2m^2}}$$

$$= \frac{H^2}{8\pi^2} \left[\frac{1}{1-Z} - \ln(1-Z) + \frac{2}{d-3} \right] \quad \text{(With dim reg employing Melville-Pajer scheme)}$$

All of these procedures yield a dS invariant result!



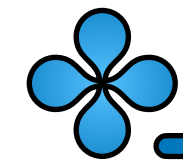
Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

Huenupi, Hughes, GAP & Sypsas (2024)

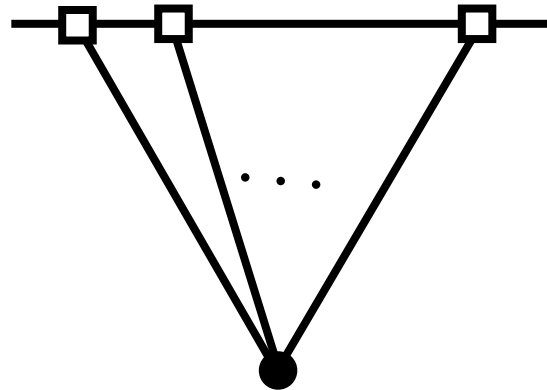
See also: Lee et al. (2023); Creminelli et al. (2024)



Example: Daisy loops

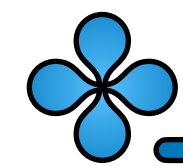
23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


Huenupi, Hughes, GAP & Sypsas (2024)

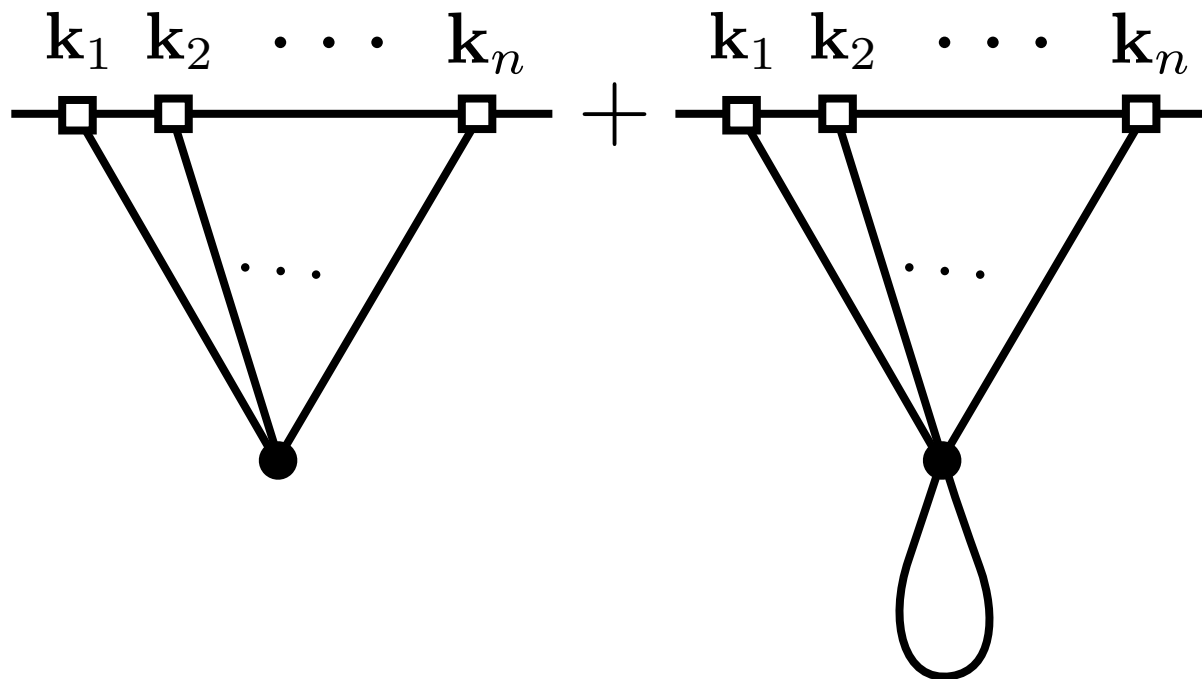
See also: Lee et al. (2023); Creminelli et al. (2024)



Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The diagram shows two Feynman diagrams separated by a plus sign. Each diagram consists of a horizontal line with n external legs labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$ at the top. The first diagram shows these legs meeting at a single vertex, with an ellipsis between \mathbf{k}_2 and \mathbf{k}_n . The second diagram is identical but includes a loop (a teardrop shape) attached to the vertex where the external legs meet.

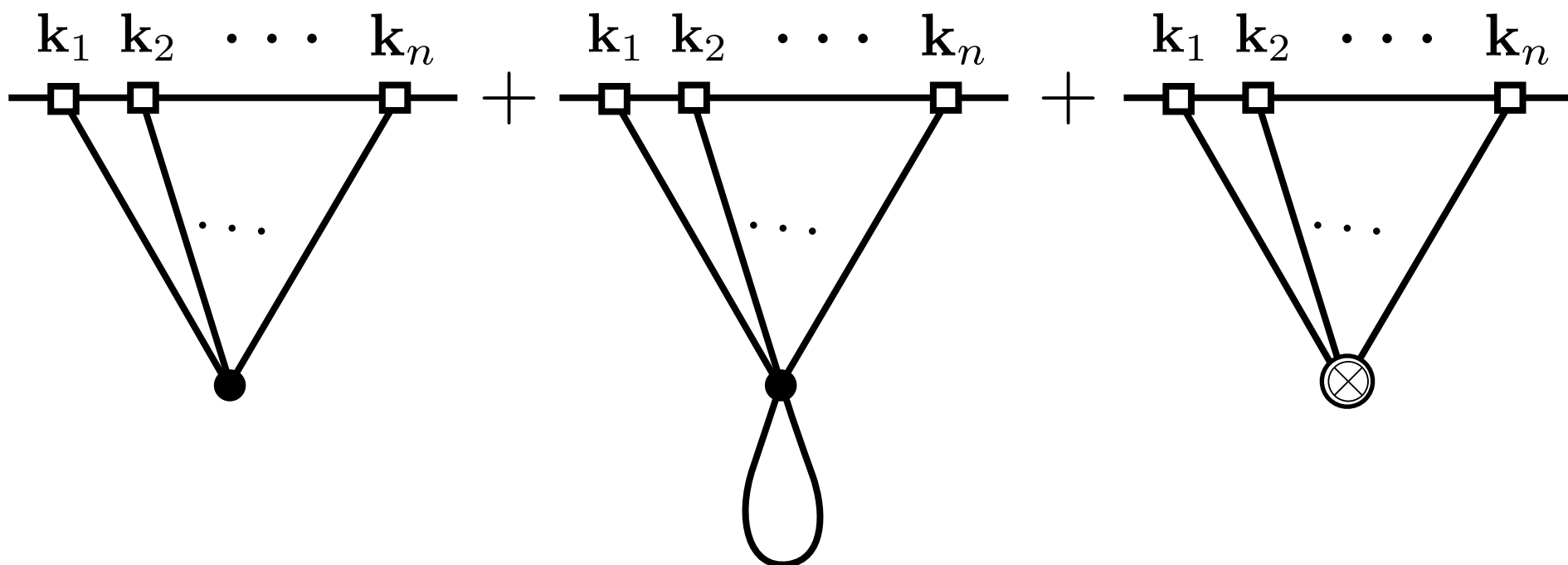
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$ is equal to the sum of three Feynman diagrams. Each diagram represents a different type of loop structure (daisy loop) involving a massive field. The diagrams are separated by plus signs. The first diagram shows a single vertex (black dot) where all external legs meet. The second diagram shows a single vertex (black dot) where all external legs meet, and a loop (teardrop shape) attached to the vertex. The third diagram shows a single vertex (circle with an X) where all external legs meet.

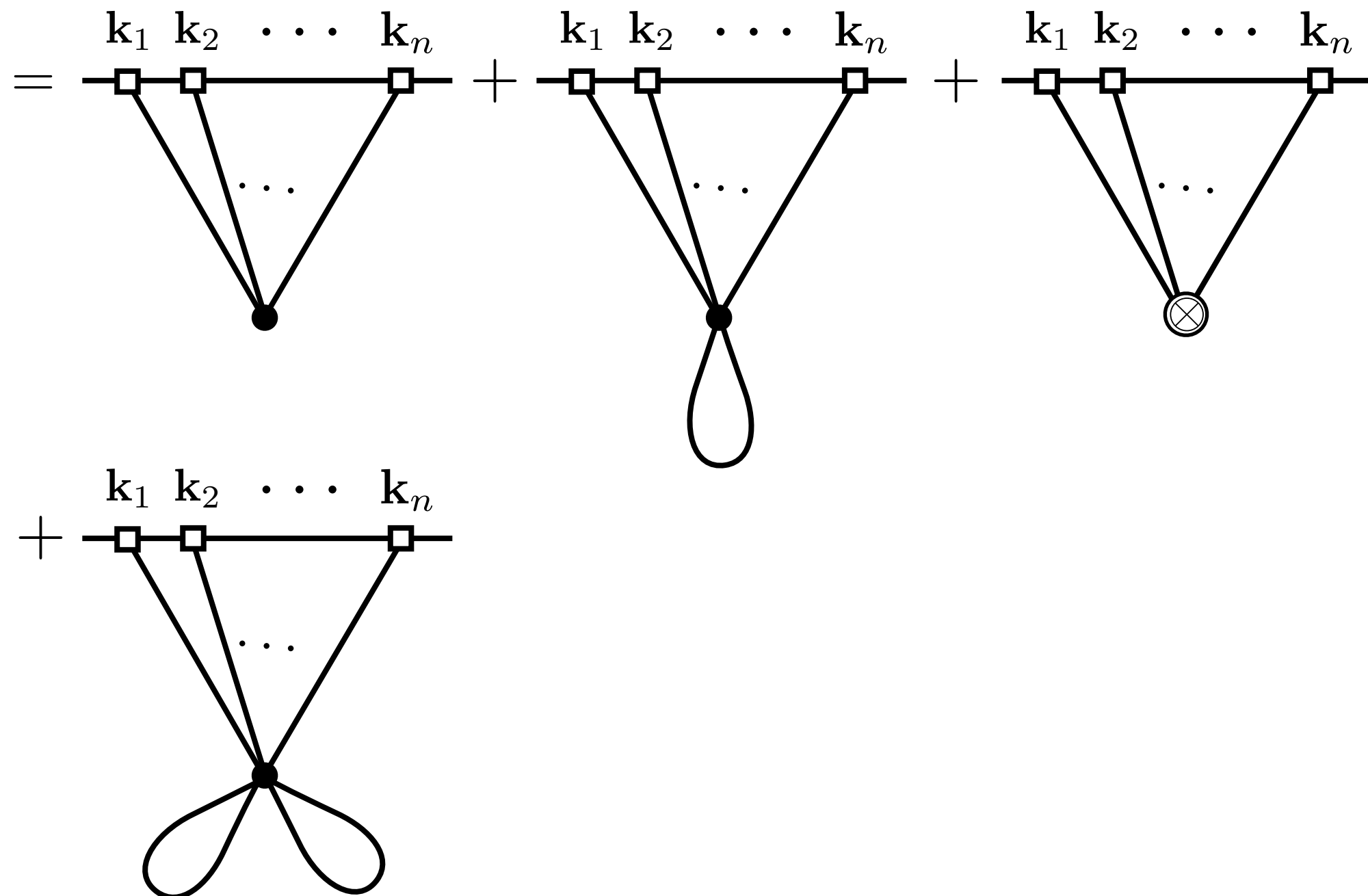
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows four Feynman diagrams representing the correlation function $\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle$. Each diagram consists of a horizontal line with n external legs labeled $\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n$. The diagrams are separated by plus signs. The first three diagrams are connected by plus signs in the top row, and the fourth is connected by a plus sign in the bottom row. The first diagram shows a vertex connected to the external legs. The second diagram shows a vertex connected to the external legs, with a loop attached to the vertex. The third diagram shows a vertex connected to the external legs, with a loop attached to the vertex, and the vertex is marked with a cross. The fourth diagram shows a vertex connected to the external legs, with two loops attached to the vertex.

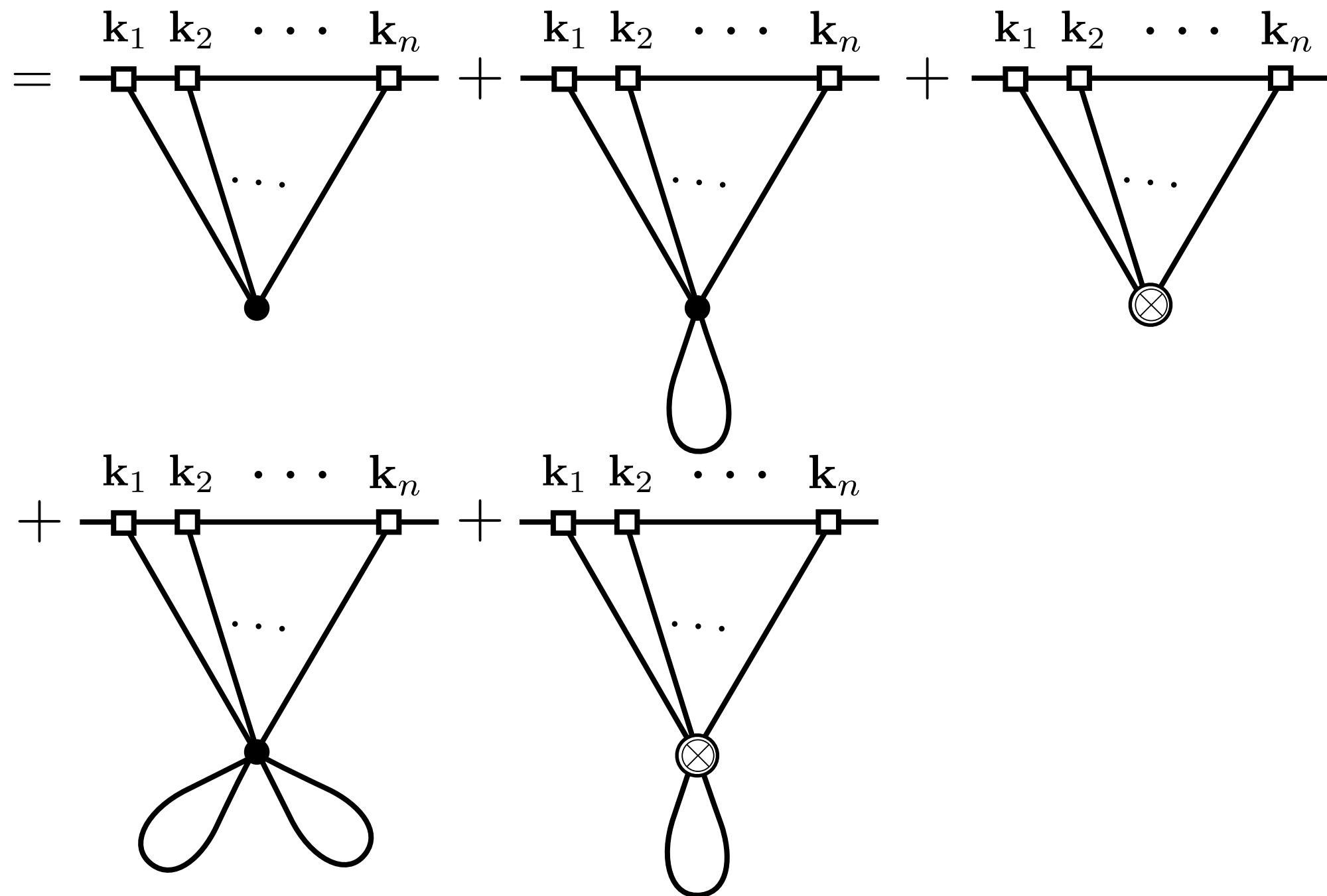
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows five Feynman diagrams representing daisy loops, separated by plus signs. Each diagram consists of a horizontal line with \$n\$ external legs labeled \$\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\$. The legs are connected to a central vertex. The diagrams are:

- Diagram 1: A central black dot vertex connected to the external legs.
- Diagram 2: A central black dot vertex connected to the external legs, with a single loop (bubble) attached to the vertex.
- Diagram 3: A central white circle vertex with a cross inside, connected to the external legs.
- Diagram 4: A central black dot vertex connected to the external legs, with two loops (bubbles) attached to the vertex.
- Diagram 5: A central white circle vertex with a cross inside, connected to the external legs, with a single loop (bubble) attached to the vertex.

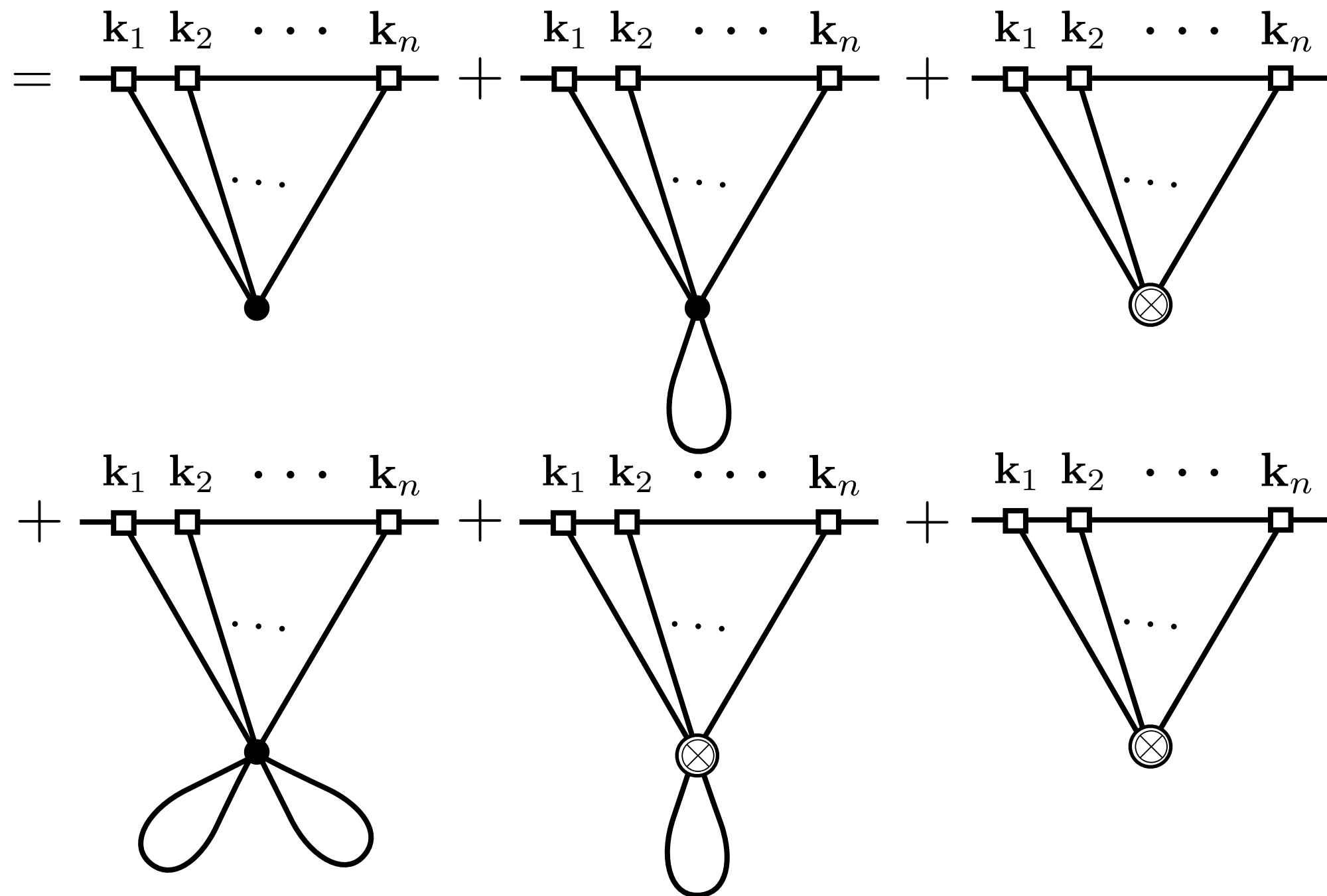
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows a sum of six Feynman diagrams representing daisy loops. Each diagram consists of a horizontal line with \$n\$ external legs labeled \$\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\$. The legs are connected to a central vertex. The diagrams are arranged in two rows of three, separated by plus signs. The first row shows a triangle loop (solid dot), a tadpole loop (solid dot with a loop), and a tadpole loop (crossed circle). The second row shows a bubble loop (solid dot with two loops), a tadpole loop (crossed circle with a loop), and a tadpole loop (crossed circle).

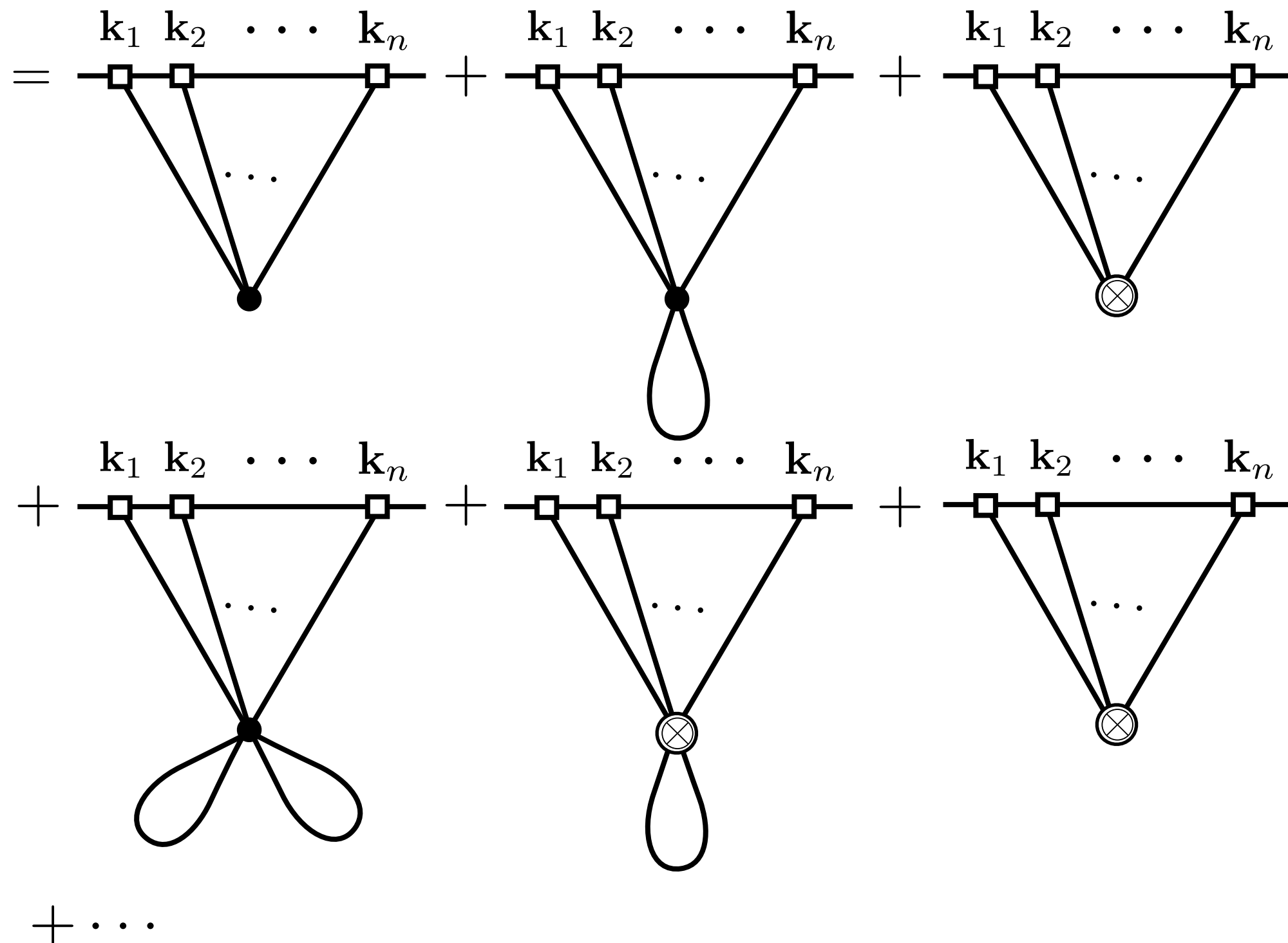
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


The equation shows a sum of Feynman diagrams representing daisy loops. Each diagram consists of a horizontal line with \$n\$ external legs labeled \$\mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n\$. The legs are connected to a central vertex. The diagrams are summed together, with the last term followed by \$+\dots\$.

- Diagram 1: A central black dot vertex connected to \$n\$ external legs.
- Diagram 2: A central black dot vertex connected to \$n\$ external legs, with a single loop (bubble) attached to the vertex.
- Diagram 3: A central white circle vertex with a cross inside, connected to \$n\$ external legs.
- Diagram 4: A central black dot vertex connected to \$n\$ external legs, with two loops (bubbles) attached to the vertex.
- Diagram 5: A central white circle vertex with a cross inside, connected to \$n\$ external legs, with a single loop (bubble) attached to the vertex.
- Diagram 6: A central white circle vertex with a cross inside, connected to \$n\$ external legs.

Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

23

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle = \text{[diagram of a horizontal line with } n \text{ external legs labeled } \mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n \text{]} + \dots$$

The summation gives you back an exact result
valid to all orders in loops proportional to a tree-level
diagram

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle = \text{[diagram of a triangle loop with } n \text{ external legs labeled } \mathbf{k}_1, \mathbf{k}_2, \dots, \mathbf{k}_n \text{ and a shaded vertex labeled } \lambda_n^{\text{eff}} \text{]} + \dots$$

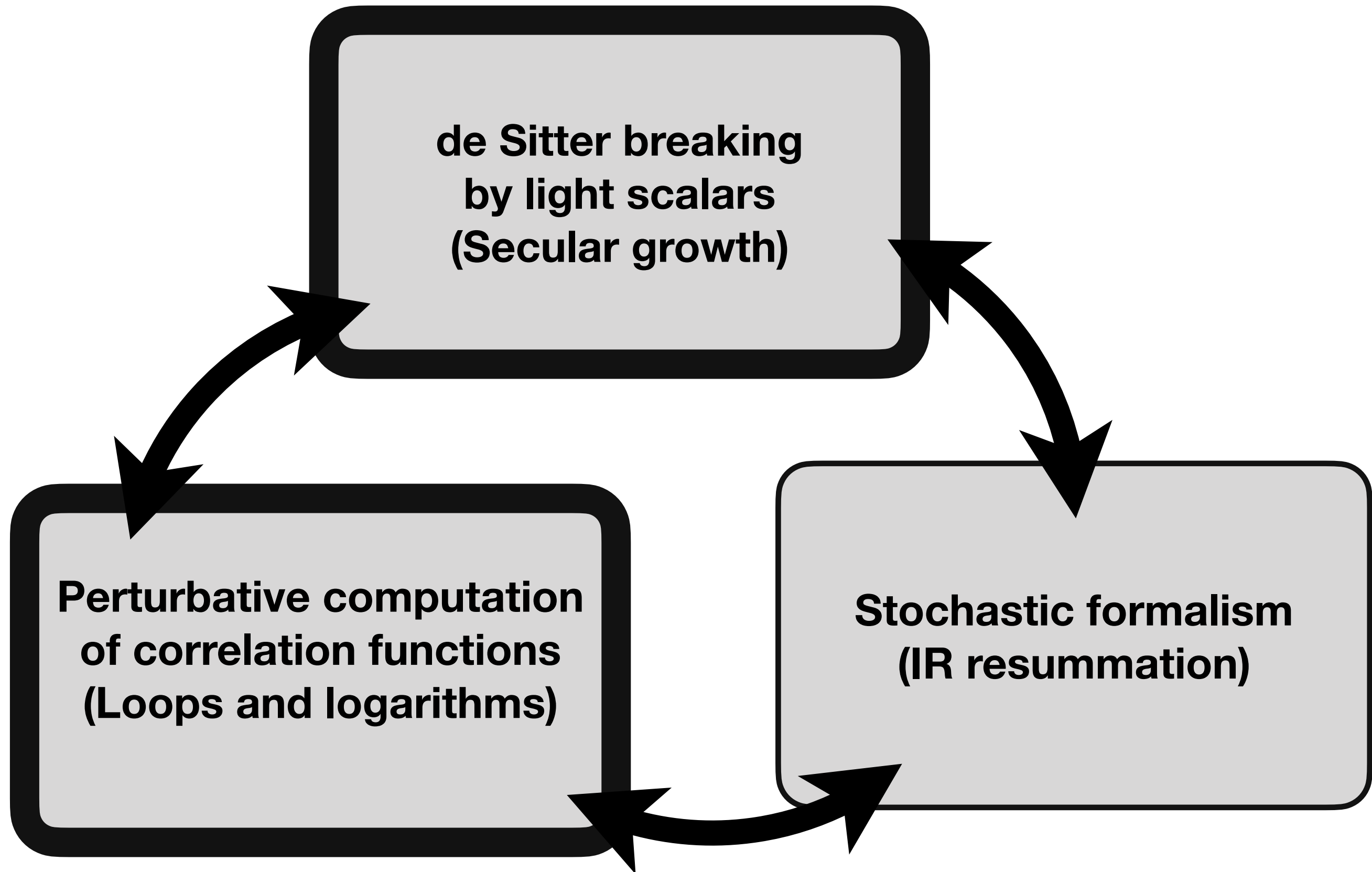
Now you can go back to $m = 0$
with no secular growth to be found!!!

+ ...

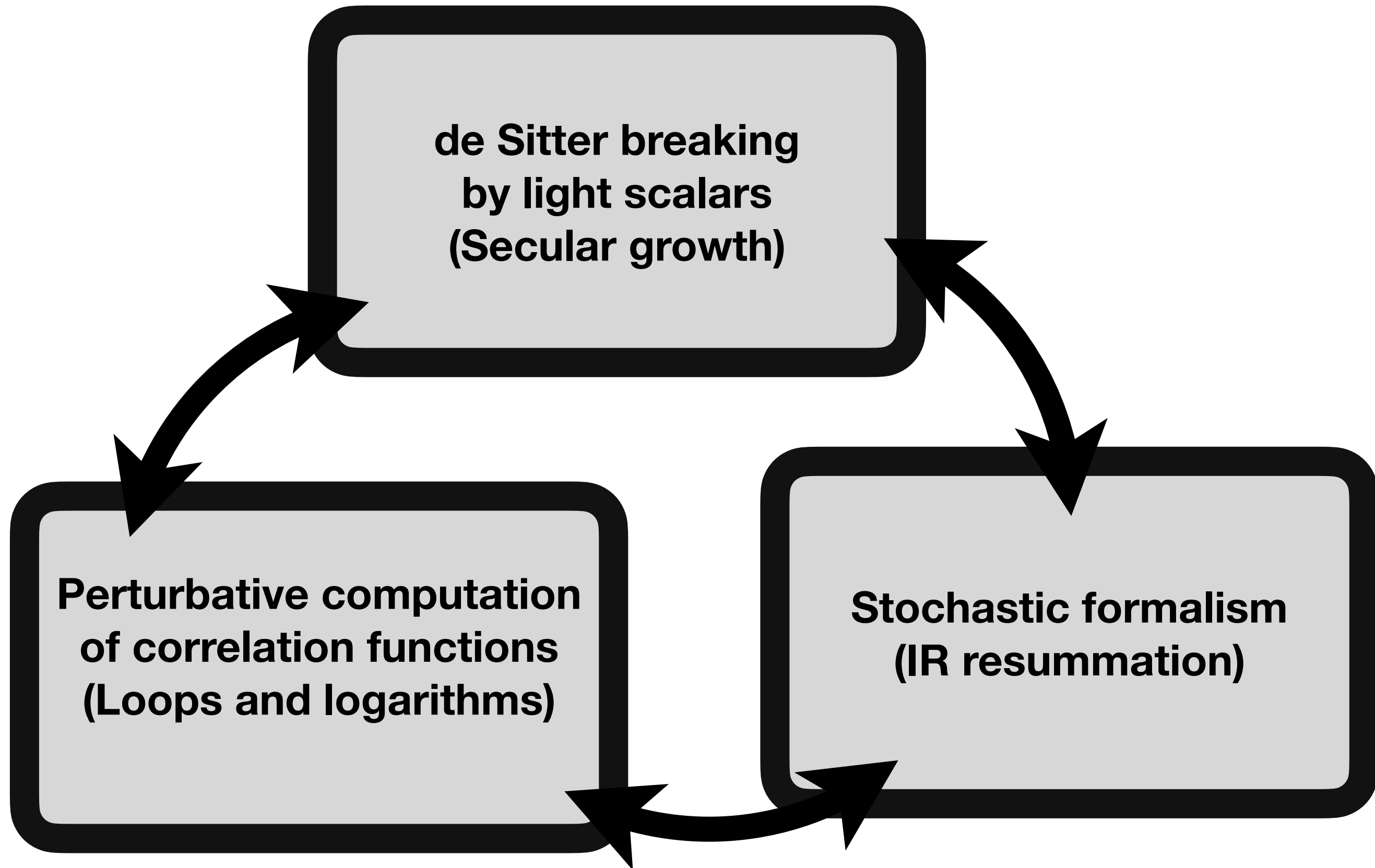
Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

Preamble



Preamble



There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi(\mathbf{x}) \cdots \varphi(\mathbf{x}) \rangle_c$$

There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi_w(\mathbf{x}) \cdots \varphi_w(\mathbf{x}) \rangle_c$$

Where:

$$\varphi_w(\mathbf{x}) = \hat{W} \varphi(\mathbf{x})$$

There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi_w(\mathbf{x}) \cdots \varphi_w(\mathbf{x}) \rangle_c$$

Where:

$$\varphi_w(\mathbf{x}) = \hat{W} \varphi(\mathbf{x}) = \int_{\mathbf{k}} W(k, t) \tilde{\varphi}(\mathbf{k})$$

There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi_w(\mathbf{x}) \cdots \varphi_w(\mathbf{x}) \rangle_c$$

Where:

$$\varphi_w(\mathbf{x}) = \hat{W} \varphi(\mathbf{x}) = \int_{\mathbf{k}} W(k, t) \tilde{\varphi}(\mathbf{k}) = \int_{L^{-1}}^{Ha(t)} dk \tilde{\varphi}(\mathbf{k})$$


There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi_w(\mathbf{x}) \cdots \varphi_w(\mathbf{x}) \rangle_c$$

Where:

Physical


$$\varphi_w(\mathbf{x}) = \hat{W} \varphi(\mathbf{x}) = \int_{\mathbf{k}} W(k, t) \tilde{\varphi}(\mathbf{k}) = \int_{L^{-1}}^{H a(t)} dk \tilde{\varphi}(\mathbf{k})$$

There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \langle \varphi_w(\mathbf{x}) \cdots \varphi_w(\mathbf{x}) \rangle_c$$

Where:

Physical

$$\varphi_w(\mathbf{x}) = \hat{W} \varphi(\mathbf{x}) = \int_{\mathbf{k}} W(k, t) \tilde{\varphi}(\mathbf{k}) = \int_{L^{-1}}^{Ha(t)} dk \tilde{\varphi}(\mathbf{k})$$

Co-moving

Stochastic formalism and loops

There must exist a probability density function (PDF) allowing to compute cumulants

$$\langle \varphi^n \rangle = \int d\varphi \, \rho(\varphi) \, \varphi^n$$

There must exist a probability density function (PDF) allowing to compute cumulants

$$\langle \varphi^n \rangle = \int d\varphi \, \rho(\varphi) \, \varphi^n$$

According to the stochastic formalism that PDF satisfies the following Fokker-Planck equation

(Starobinsky & Yokoyama)

$$\frac{d}{dt} \rho = \left[\frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \varphi^2} \rho + \frac{1}{3H} \frac{\partial}{\partial \varphi} \left(\rho \mathcal{V}' \right) \right]$$

There must exist a probability density function (PDF) allowing to compute cumulants

$$\langle \varphi^n \rangle = \int d\varphi \, \rho(\varphi) \, \varphi^n$$

According to the stochastic formalism that PDF satisfies the following Fokker-Planck equation

(Starobinsky & Yokoyama)

$$\int d\varphi \, \varphi^n \frac{d}{dt} \rho = \int \left[\frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \varphi^2} \rho + \frac{1}{3H} \frac{\partial}{\partial \varphi} (\rho \mathcal{V}') \right] \varphi^n d\varphi$$

(Tsamis & Woodard)

There must exist a probability density function (PDF) allowing to compute cumulants

$$\langle \varphi^n \rangle = \int d\varphi \, \rho(\varphi) \, \varphi^n$$

According to the stochastic formalism that PDF satisfies the following Fokker-Planck equation

(Starobinsky & Yokoyama)

$$\int d\varphi \, \varphi^n \frac{d}{dt} \rho = \int \left[\frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \varphi^2} \rho + \frac{1}{3H} \frac{\partial}{\partial \varphi} \left(\rho \mathcal{V}' \right) \right] \varphi^n d\varphi$$

$$\mathcal{V}(\varphi) = \sum_m \frac{\lambda_m}{m!} \varphi^m$$

(Tsamis & Woodard)

Tsamis & Woodard:

$$\frac{d}{dt} \langle \varphi^n \rangle = n(n-1) \frac{H^3}{8\pi^2} \langle \varphi^{n-2} \rangle - \frac{n}{3H} \sum_{m=2}^{\infty} \frac{\lambda_m}{(m-1)!} \langle \varphi^{m+n-2} \rangle$$

Tsamis & Woodard:

$$\frac{d}{dt} \langle \varphi^n \rangle = n(n-1) \frac{H^3}{8\pi^2} \langle \varphi^{n-2} \rangle - \frac{n}{3H} \sum_{m=2}^{\infty} \frac{\lambda_m}{(m-1)!} \langle \varphi^{m+n-2} \rangle$$

Then Tsamis & Woodard noticed that this is solved by

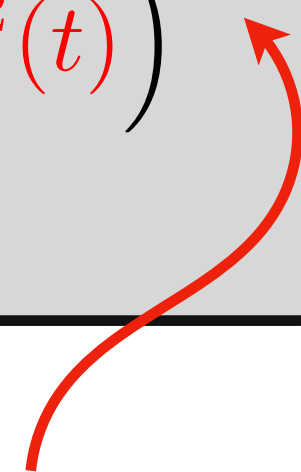
$$\langle \varphi^n \rangle = -\frac{4\pi^2 n}{3H^4} \left(\sigma^2(t) \right)^n \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \frac{1}{n+L} \left(\frac{1}{2} \sigma^2(t) \right)^L$$

Where $\sigma^2(t) = \frac{H^2}{4\pi^2} \ln a(\tau)$

Tsamis & Woodard:

$$\frac{d}{dt} \langle \varphi^n \rangle = n(n-1) \frac{H^3}{8\pi^2} \langle \varphi^{n-2} \rangle - \frac{n}{3H} \sum_{m=2}^{\infty} \frac{\lambda_m}{(m-1)!} \langle \varphi^{m+n-2} \rangle$$

Then Tsamis & Woodard noticed that this is solved by

$$\langle \varphi^n \rangle = -\frac{4\pi^2 n}{3H^4} \left(\sigma^2(t) \right)^n \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \frac{1}{n+L} \left(\frac{1}{2} \sigma^2(t) \right)^L$$


Where $\sigma^2(t) = \frac{H^2}{4\pi^2} \ln a(\tau)$

Loop corrections employing a co-moving IR cutoff !!!

To understand Tsamis and Woodard's result, let me offer a quick derivation of the Fokker-Planck equation found by Satoribinsky and Yokoyama

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{a^2}\nabla^2\varphi + \mathcal{V}' = 0$$

To understand Tsamis and Woodard's result, let me offer a quick derivation of the Fokker-Planck equation found by Satoribinsky and Yokoyama

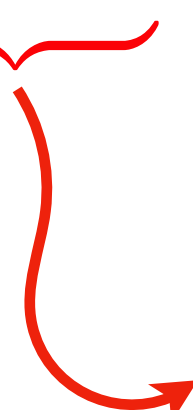
$$w \left\{ \ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{a^2} \nabla^2 \varphi + \mathcal{V}' \right\} = 0$$

To understand Tsamis and Woodard's result, let me offer a quick derivation of the Fokker-Planck equation found by Satoribinsky and Yokoyama

$$w \left\{ \cancel{\ddot{\varphi}} + 3H\dot{\varphi} - \frac{1}{a^2} \cancel{\nabla^2} \varphi + \mathcal{V}' \right\} = 0$$

To understand Tsamis and Woodard's result, let me offer a quick derivation of the Fokker-Planck equation found by Satoribinsky and Yokoyama

$$W \left\{ \cancel{\ddot{\varphi}} + 3H\dot{\varphi} - \cancel{\frac{1}{a^2}\nabla^2\varphi} + \mathcal{V}' \right\} = 0$$



$$3H\dot{\varphi}_w - 3H^2\hat{\xi}(t)$$

Where the noise is

$$\hat{\xi} \equiv H^{-1} \int_{\mathbf{k}} \left(\frac{d}{dt} W(k) \right) \tilde{\varphi}_{\mathbf{k}}(t)$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \nu'(\varphi) \right\} = H \hat{\xi}(t)$$

The next assumption is:

$$W \left\{ \nu'(\varphi) \right\} \simeq \nu'(W\varphi)$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

The next assumption is:

$$W \left\{ \mathcal{V}'(\varphi) \right\} \simeq \mathcal{V}'(W\varphi)$$

This assumption leads to the Langevin equation:

$$\dot{\varphi}_w + \frac{1}{3H} \mathcal{V}'(\varphi_w) = H \hat{\xi}(t)$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

The next assumption is:

$$W \left\{ \mathcal{V}'(\varphi) \right\} \simeq \mathcal{V}'(W\varphi)$$

This assumption leads to the Langevin equation:

$$\dot{\varphi}_w + \frac{1}{3H} \mathcal{V}'(\varphi_w) = H \hat{\xi}(t)$$

This leads to the Fokker-Planck eq. found by Starobinsky

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

The next assumption is:

$$W \left\{ \mathcal{V}'(\varphi) \right\} \simeq \mathcal{V}'(W\varphi)$$

This assumption leads to the Langevin equation: **But wait a second.
This assumption is invalid!**

$$\dot{\varphi}_w + \frac{1}{3H} \mathcal{V}'(\varphi_w) = H \hat{\xi}(t)$$

This leads to the Fokker-Planck eq. found by Starobinsky

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

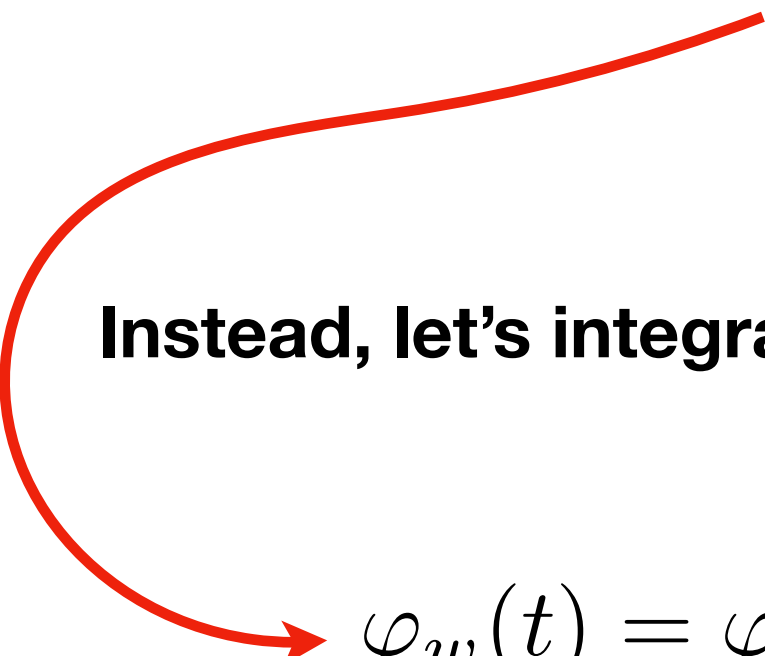
Instead, let's integrate

$$\varphi_w(t) = \varphi_G(t) - \frac{1}{3H} \int_{-\infty}^t dt' W \left\{ \frac{d\mathcal{V}}{d\varphi} [\varphi(t', x)] \right\}$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Instead, let's integrate


$$\varphi_w(t) = \varphi_G(t) - \frac{1}{3H} \int_{-\infty}^t dt' W \left\{ \frac{d\mathcal{V}}{d\varphi} [\varphi(t', x)] \right\}$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Instead, let's integrate

$$\varphi_w(t) = \varphi_G(t) - \frac{1}{3H} \int_{-\infty}^t dt' W \left\{ \frac{d\mathcal{V}}{d\varphi} [\varphi(t', x)] \right\}$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Instead, let's integrate

$$\varphi_w(t) = \varphi_G(t) - \frac{1}{3H} \int_{-\infty}^t dt' W \left\{ \frac{d\mathcal{V}}{d\varphi} [\varphi(t', x)] \right\}$$

Now you have:

$$\dot{\varphi}_w + \frac{1}{3H} W \left\{ \mathcal{V}'(\varphi) \right\} = H \hat{\xi}(t)$$

Instead, let's integrate

$$\varphi_w(t) = \varphi_G(t) - \frac{1}{3H} \int_{-\infty}^t dt' W \left\{ \frac{d\mathcal{V}}{d\varphi} [\varphi(t', x)] \right\}$$

Now we can compute cumulants:

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi} [\varphi(t')] \right\rangle$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left(\langle \varphi_G(t) \varphi(t') \rangle \right)^{n-1} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \left(\frac{1}{2} \langle \varphi(t') \varphi(t') \rangle \right)^L$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left(\left\langle \varphi_G(t) \varphi(t') \right\rangle \right)^{n-1} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \left(\frac{1}{2} \left\langle \varphi(t') \varphi(t') \right\rangle \right)^L$$

External legs

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \underbrace{\left(\langle \varphi_G(t) \varphi(t') \rangle \right)^{n-1}}_{\text{External legs}} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \underbrace{\left(\frac{1}{2} \langle \varphi(t') \varphi(t') \rangle \right)^L}_{\text{Daisy loops}}$$

$$\langle \varphi^n(t) \rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\langle \varphi^n(t) \rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left(\underbrace{\langle \varphi_G(t) \varphi(t') \rangle}_{\text{External legs}} \right)^{n-1} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \left(\frac{1}{2} \underbrace{\langle \varphi(t') \varphi(t') \rangle}_{\text{Daisy loops}} \right)^L$$

Now you have to decide how to cutoff loops



$$\langle \varphi^n(t) \rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

$$\langle \varphi^n(t) \rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left(\underbrace{\langle \varphi_G(t) \varphi(t') \rangle}_{\text{External legs}} \right)^{n-1} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \left(\frac{1}{2} \underbrace{\langle \varphi(t') \varphi(t') \rangle}_{\text{Daisy loops}} \right)^L$$

Now you have to decide how to cutoff loops

If you choose comoving cutoffs, then you recover Starobinsky's result

$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \left\langle \varphi_G^{n-1}(t) \frac{d\mathcal{V}}{d\varphi}[\varphi(t')] \right\rangle$$

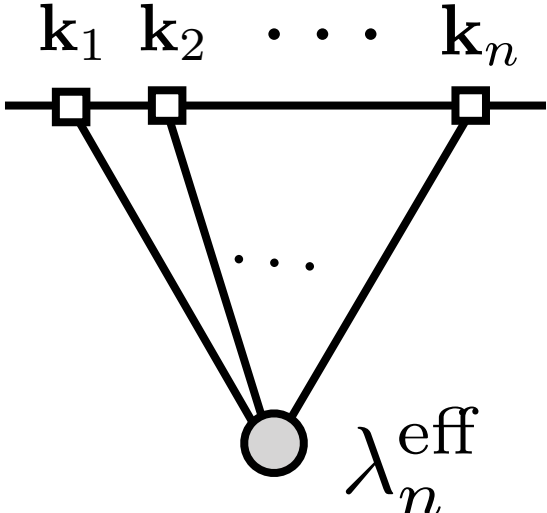
$$\left\langle \varphi^n(t) \right\rangle = -\frac{n}{3H} \int_{t_i}^t dt' \underbrace{\left(\langle \varphi_G(t) \varphi(t') \rangle \right)^{n-1}}_{\text{External legs}} \sum_{L=0}^{\infty} \frac{\lambda_{n+2L}}{L!} \underbrace{\left(\frac{1}{2} \langle \varphi(t') \varphi(t') \rangle \right)^L}_{\text{Daisy loops}}$$

Now you have to decide how to cutoff loops

If you choose comoving cutoffs, then you recover Starobinsky's result

If you choose a physical cutoff, you recover a different Fokker-Planck eq.
(See Spyros talk)

If you choose a physical cutoff you recover cumulants computed out of the exact result

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$


Recall that this result is free of dS breaking secular growth

Now you have to decide how to cutoff loops

If you choose comoving cutoffs, then you recover Starobinsky's result

If you choose a physical cutoff, you recover a different Fokker-Planck eq.
(See Spyros talk)



Conclusions

Summary:

-  **There is a systematic way to connect the stochastic formalism with perturbation theory**




Conclusions

Summary:

-  **There is a systematic way to connect the stochastic formalism with perturbation theory**
-  **This can be done exactly to first order with respect to the potential (where loops appear in the form of daisy loops)**

Conclusions

Summary:

-  **There is a systematic way to connect the stochastic formalism with perturbation theory**
-  **This can be done exactly to first order with respect to the potential (where loops appear in the form of daisy loops)**
-  **The Fokker-Planck equation turns out to have relevant corrections (see Spyros talk)**

Thanks!