On the IR behavior of QFTs in de Sitter

(loops, logarithms and stochastic formalism)

Based on:
arXiv:2311.17644 (with Spyros Sypsas)
arXiv:2406.07610 (with Javier Huenupi, Ellie Hughes and Spyros Sypsas)
arXiv:2412.01891 (with Javier Huenupi, Ellie Hughes and Spyros Sypsas)
arXiv:2507.21310 (with Spyros Sypsas and Danilo Tapia)

Gonzalo A. Palma
FCFM, U. de Chile

Inflation 2025, Paris
December 5, 2025

Preamble

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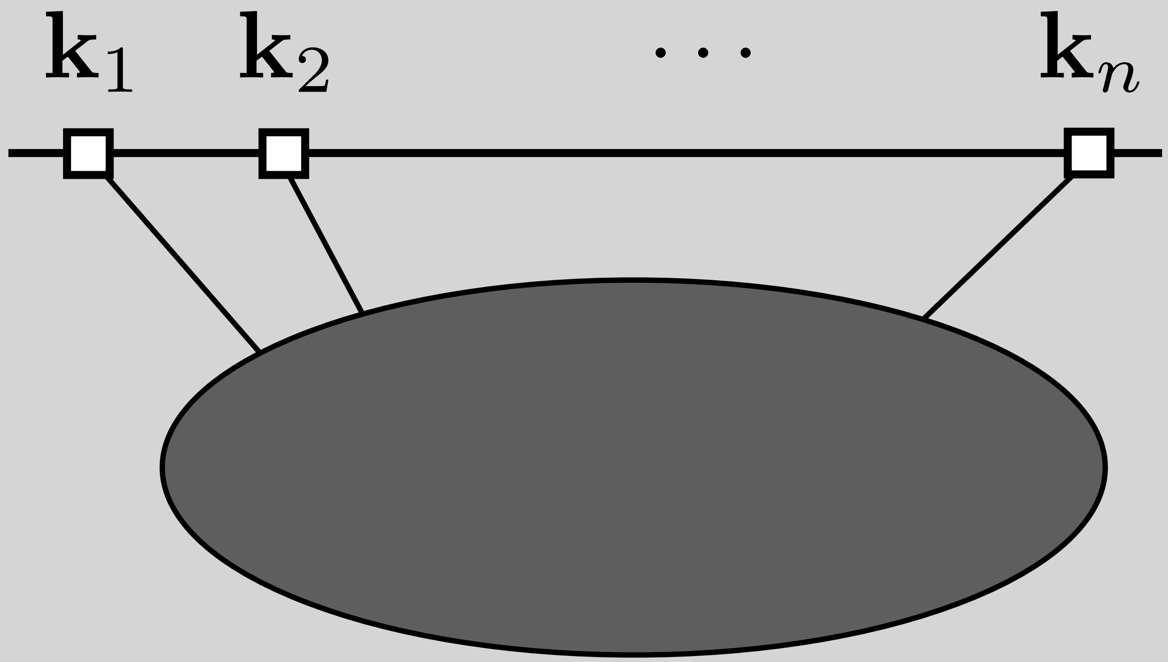
Perturbation theory

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$



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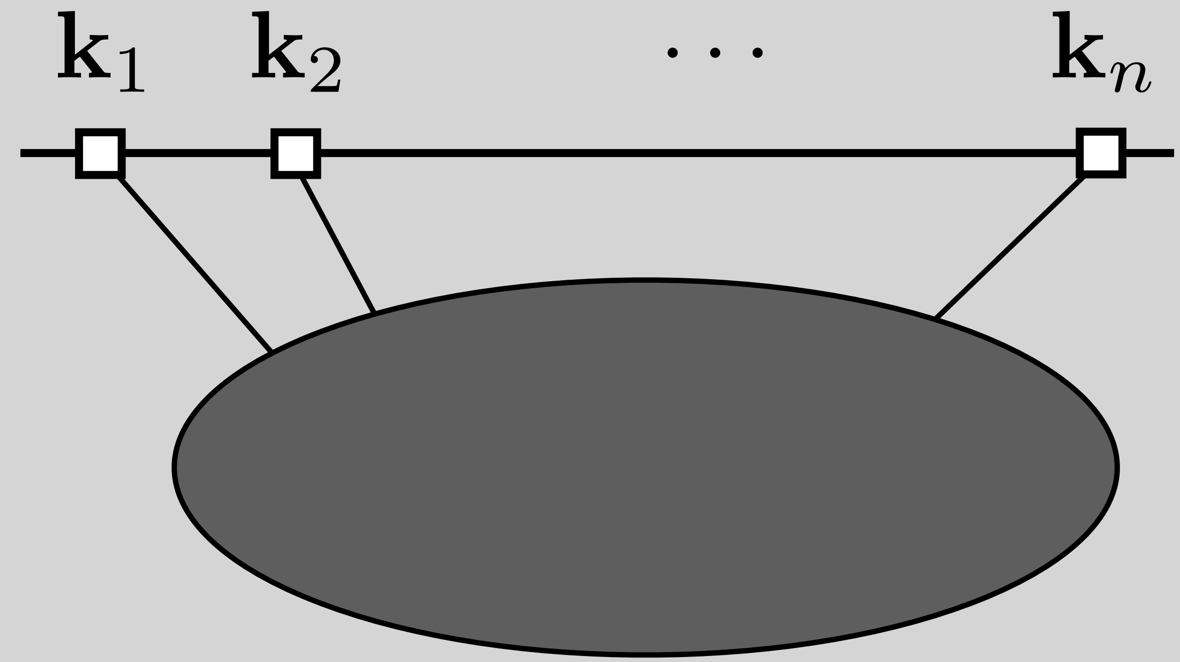
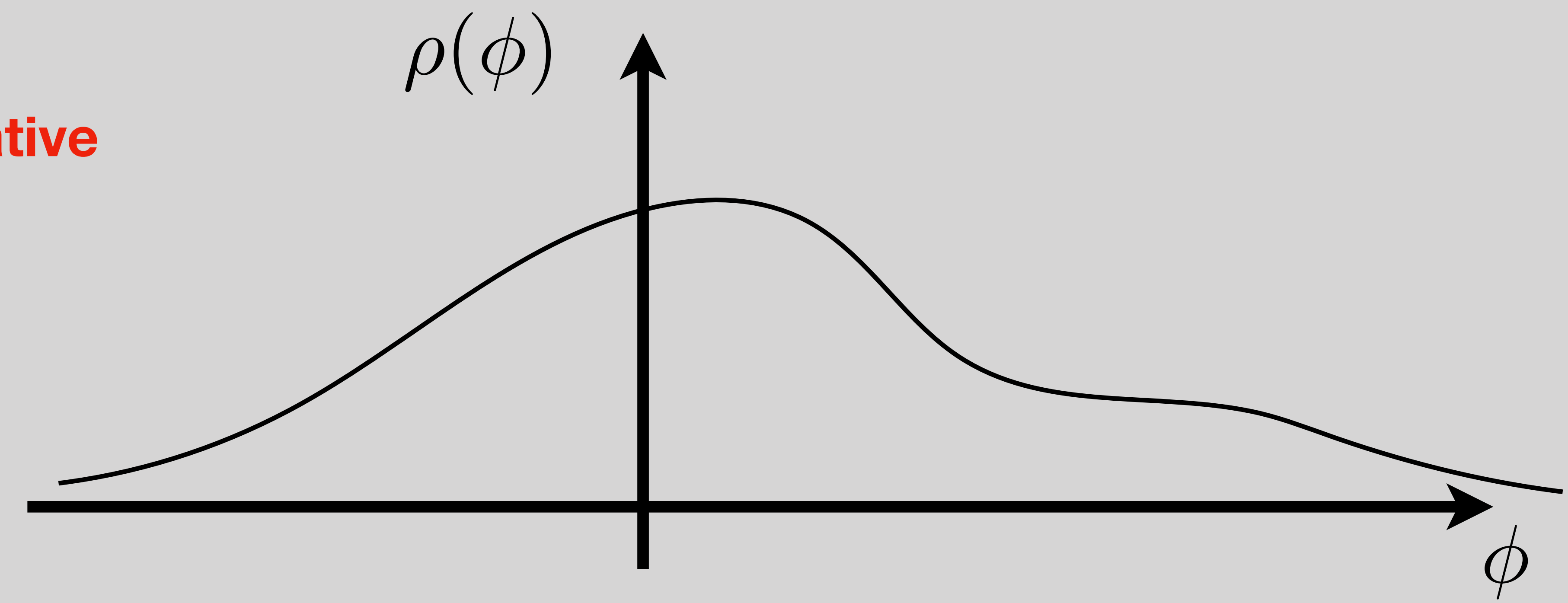


Non-perturbative approach



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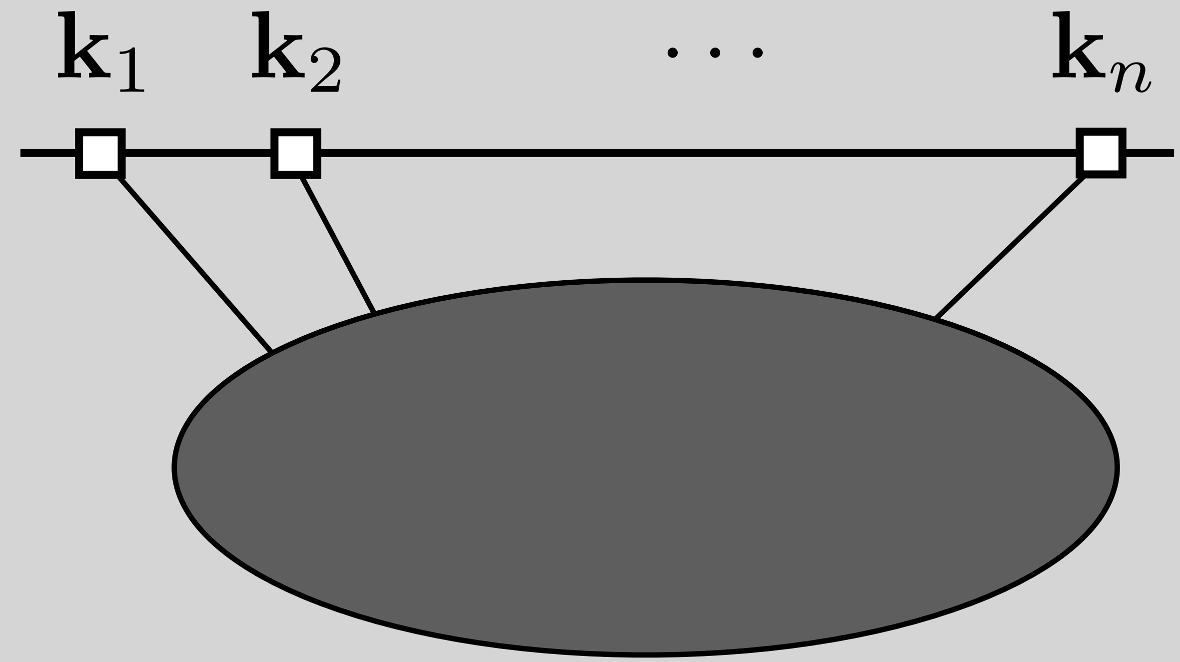
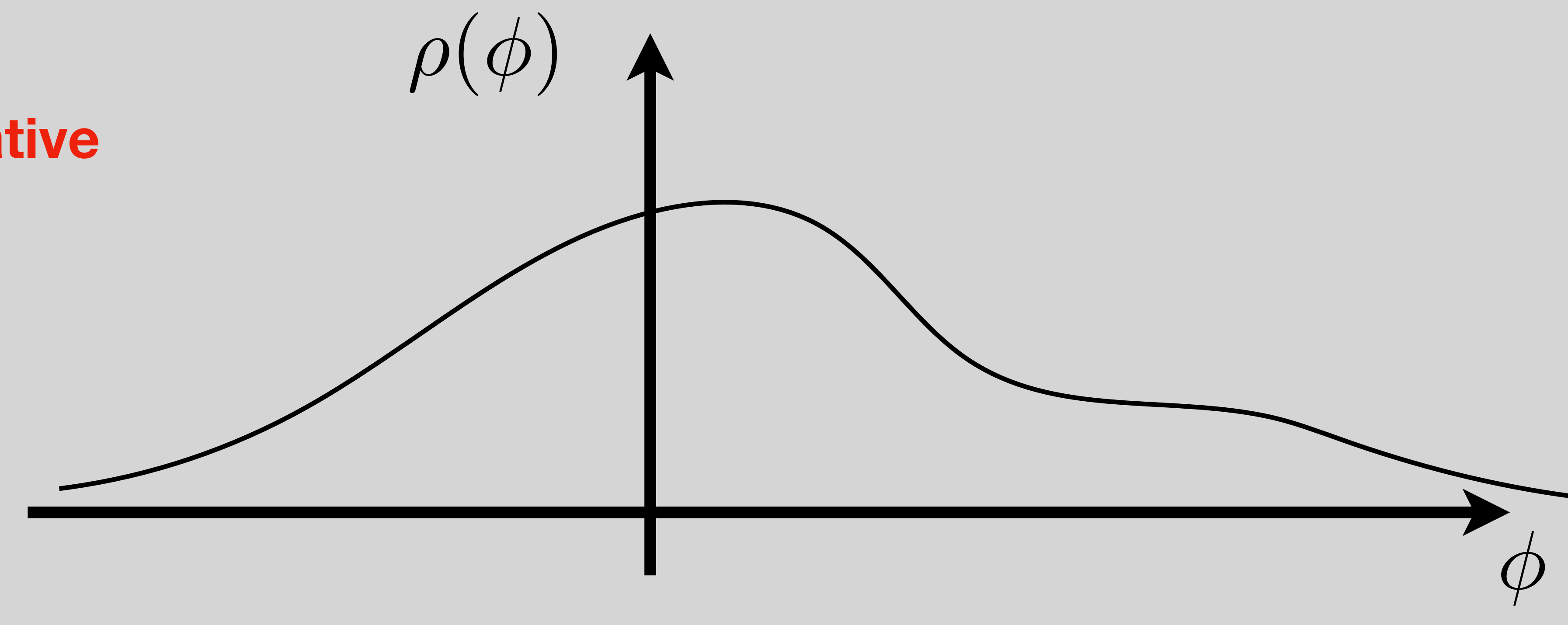


Light fields / massless limit

Non-perturbative approach



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de Sitter breaking by massless scalars (Secular growth)

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Perturbative computation
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(Loops and logarithms)

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(IR resummation)

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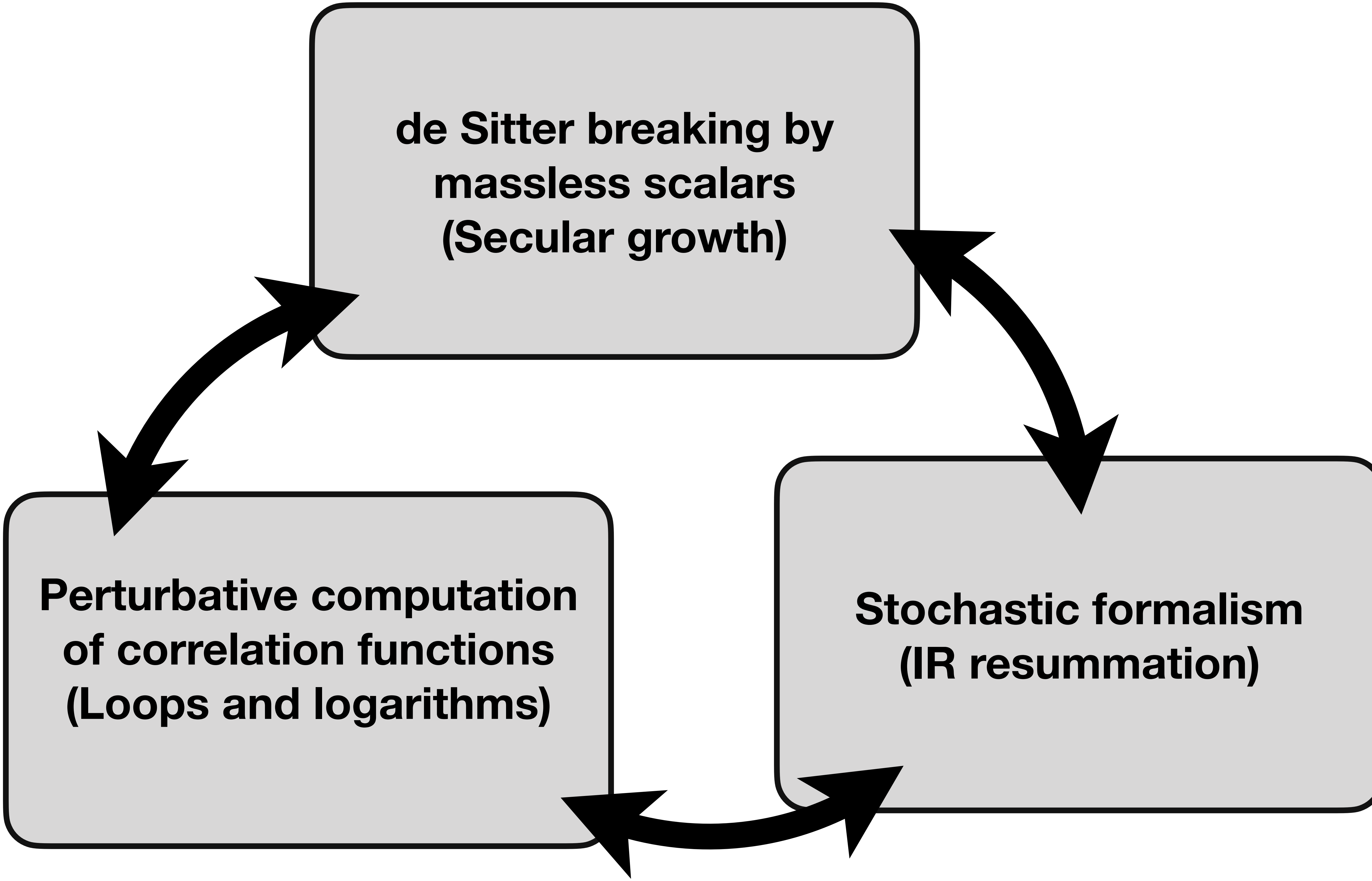
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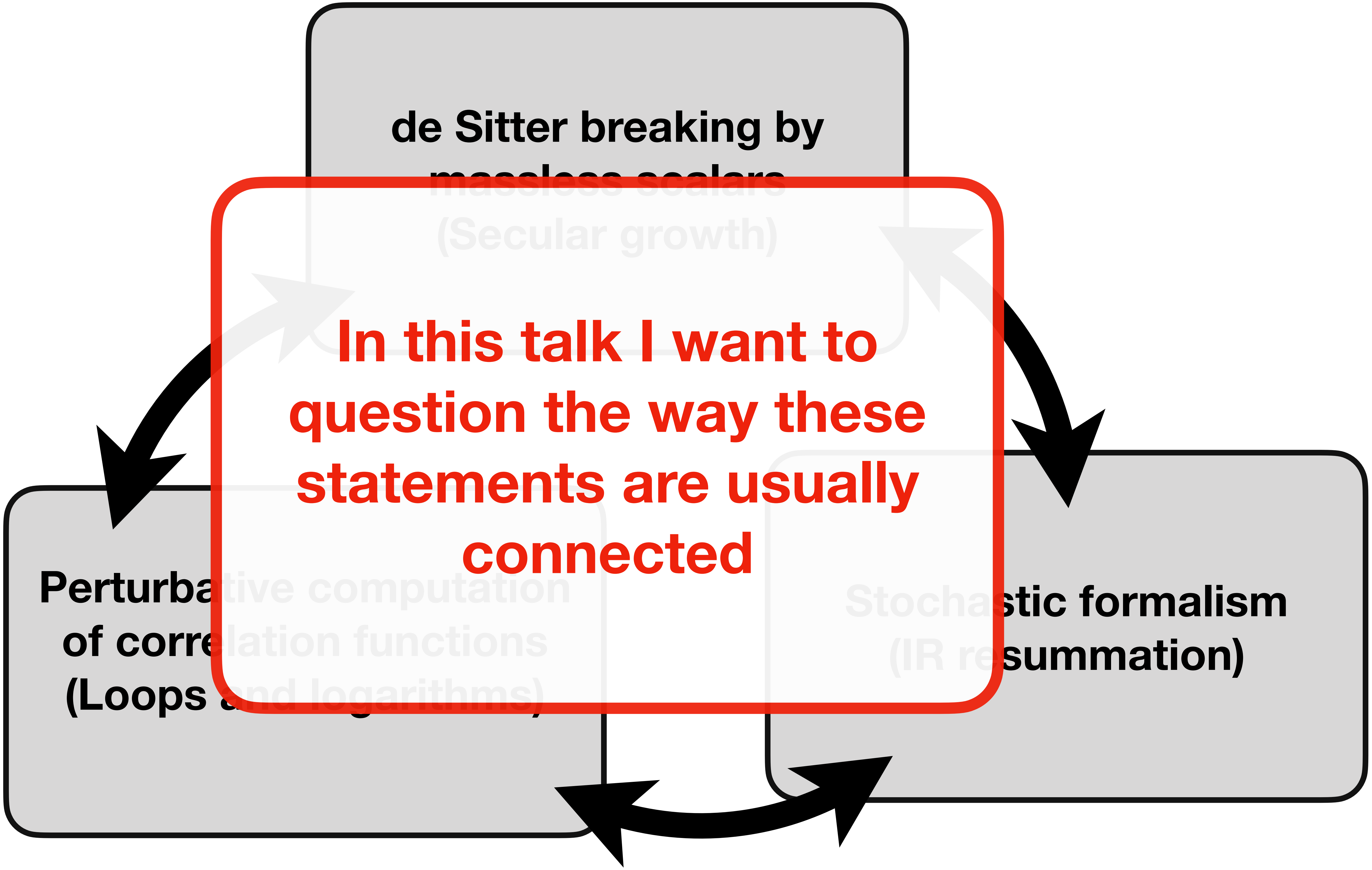
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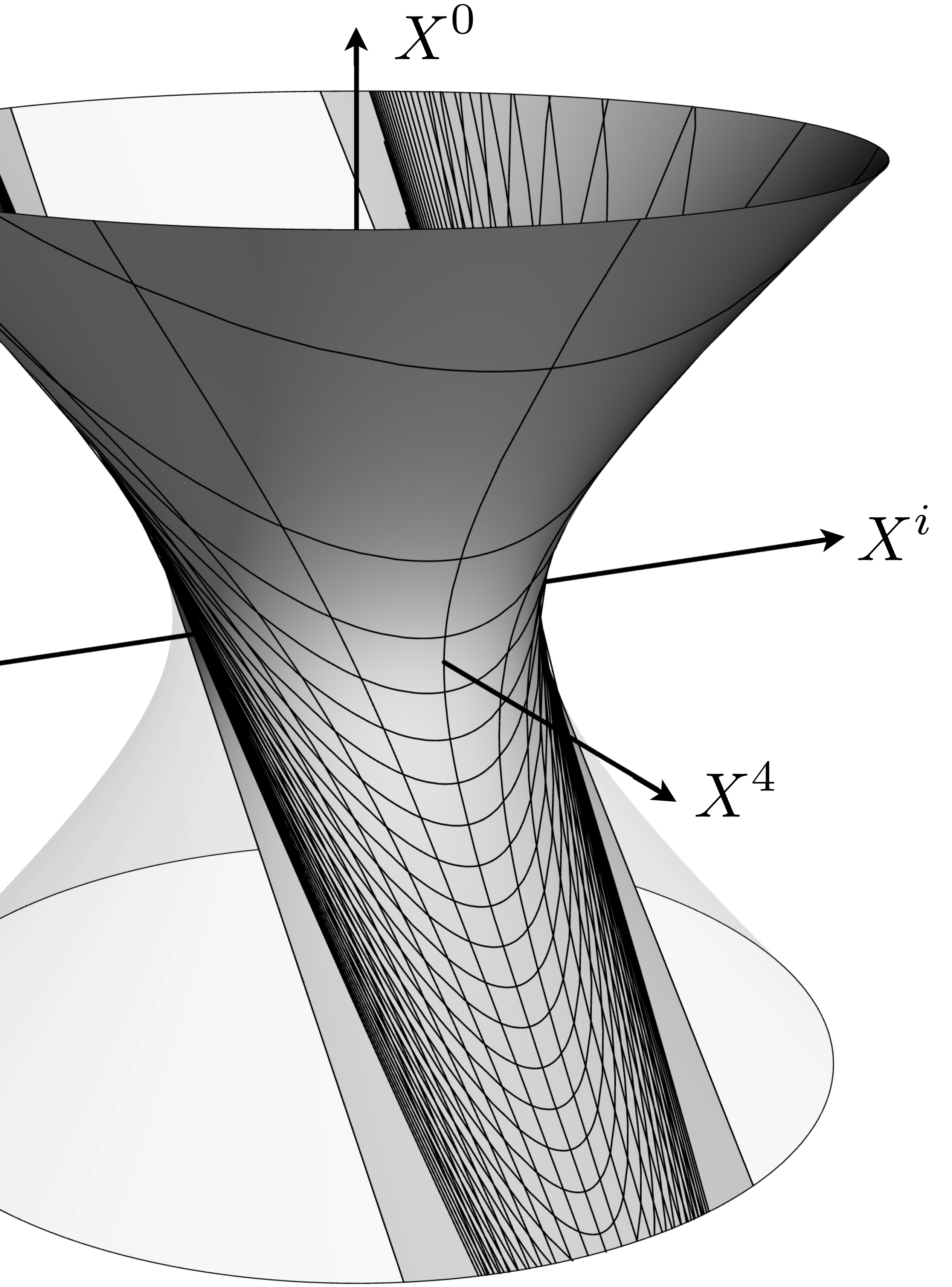
In this talk I want to
question the way these
statements are usually
connected

4D de Sitter



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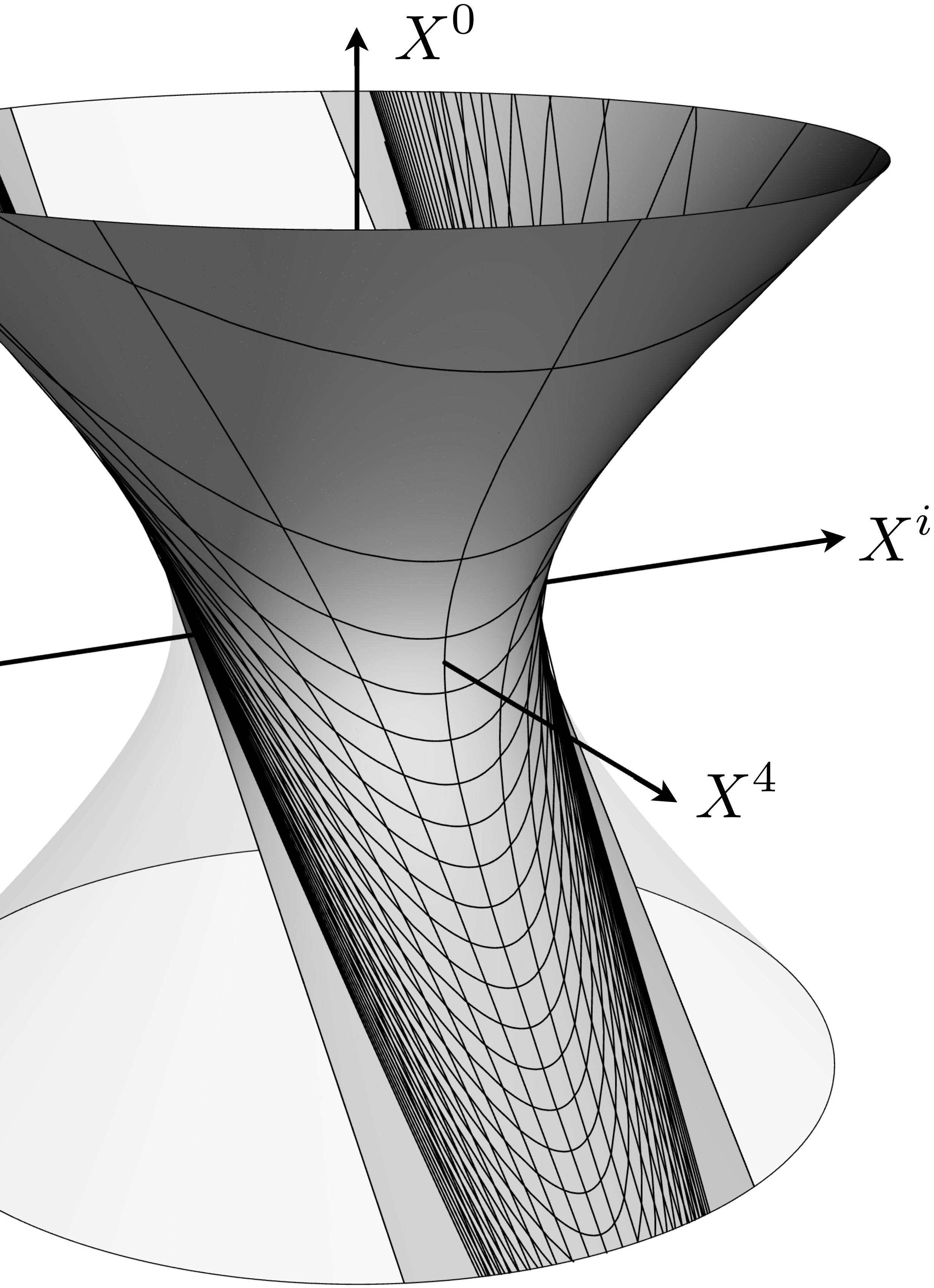

Metric in cosmological coordinates:

$$ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$$

$$a(\tau) = -\frac{1}{H\tau}$$

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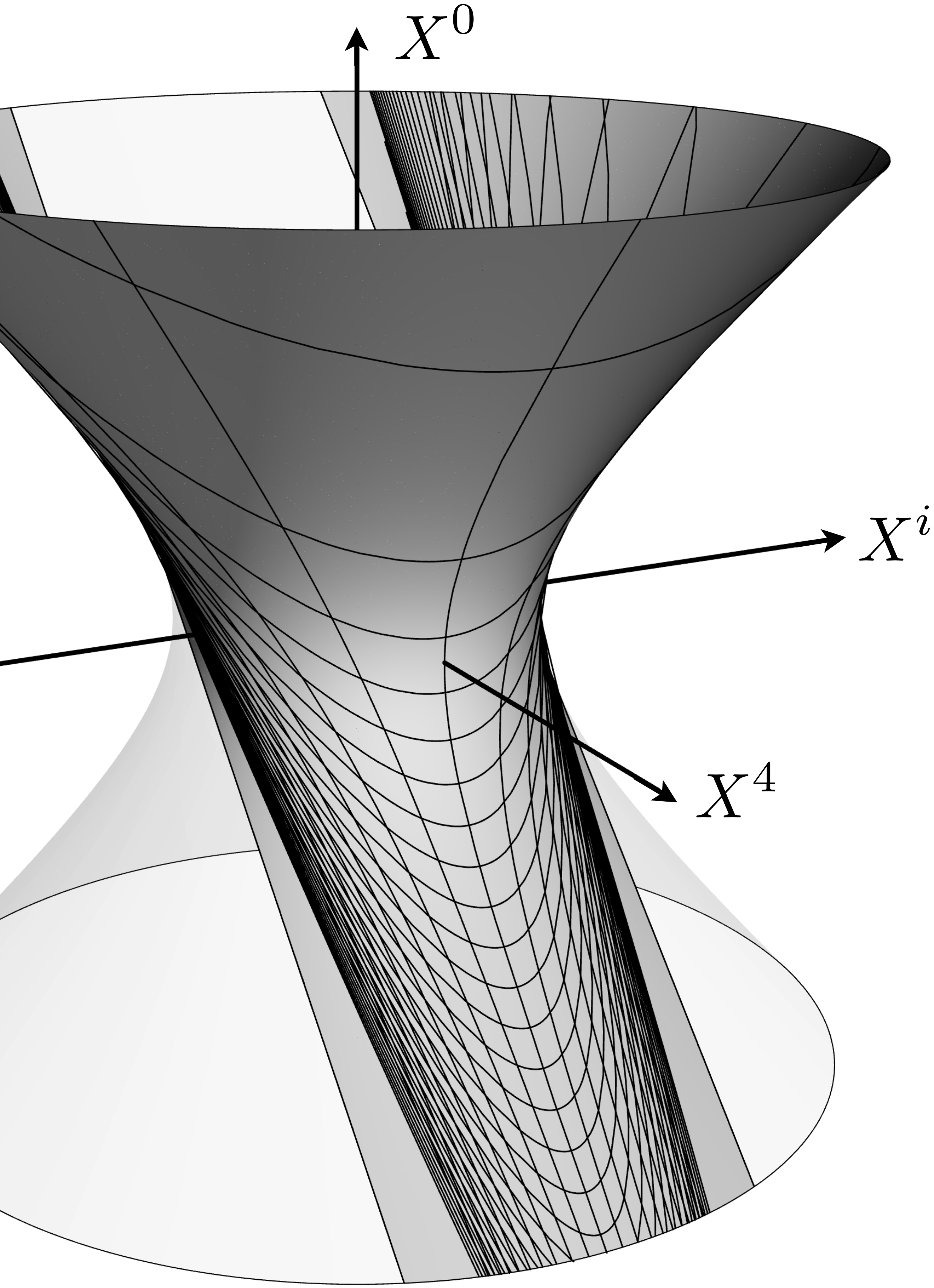

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Invariant under dS transformations:

For instance:

$$\tau \to \bar{\tau} = e^{-\theta}\tau$$

$$x^i \to \bar{x}^i = e^{-\theta}x^i$$

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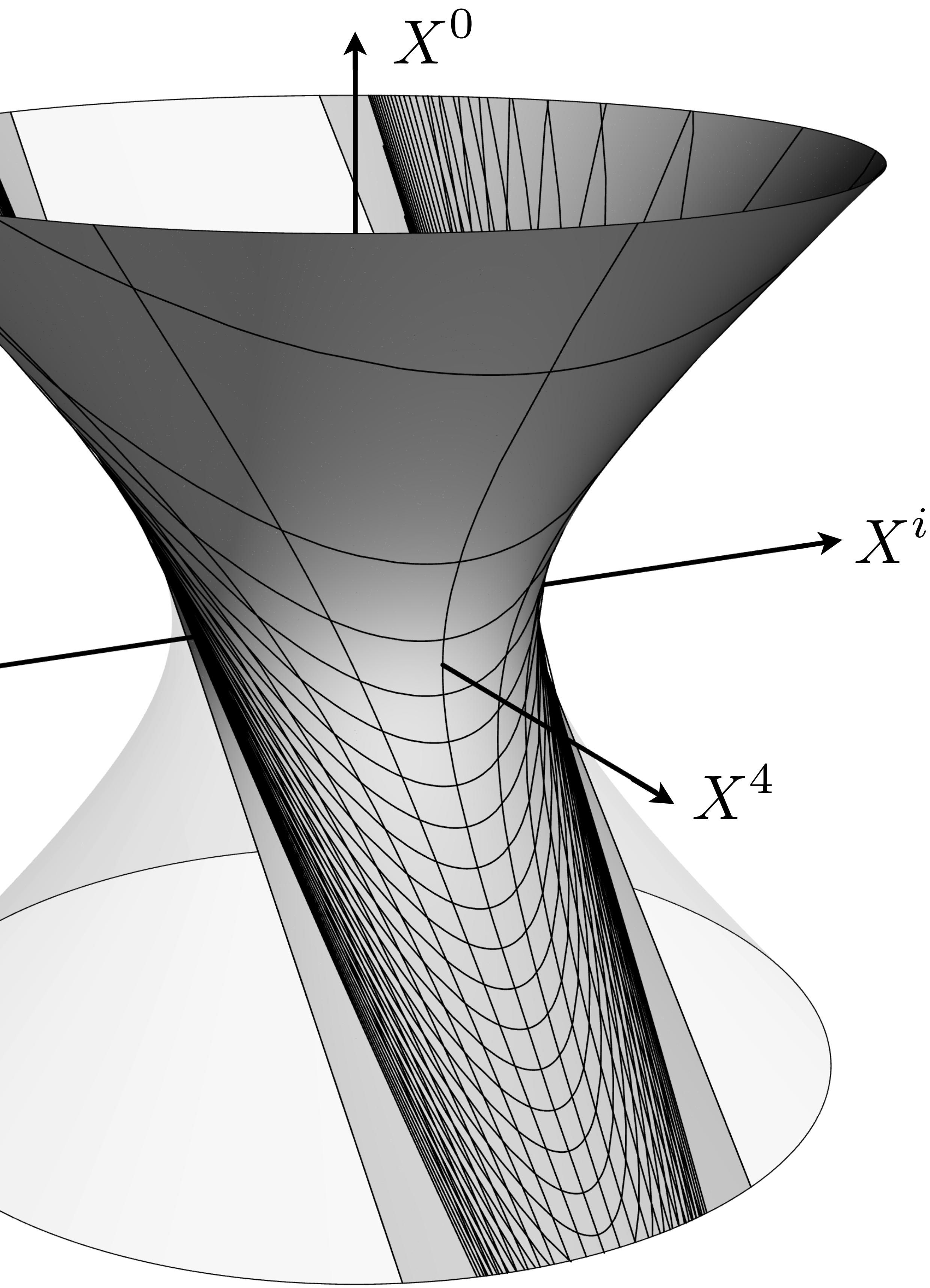

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dS invariant combination

$$Z = 1 - \frac{|\mathbf{x} - \mathbf{x}'|^2 - (\tau - \tau')^2}{2\tau\tau'}$$

SK formalism

Let's consider the computation of correlation functions for a light scalar in dS:

$$S = \int d^3x\, d\tau\, a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 - \mathcal{V}(\varphi) \right]$$

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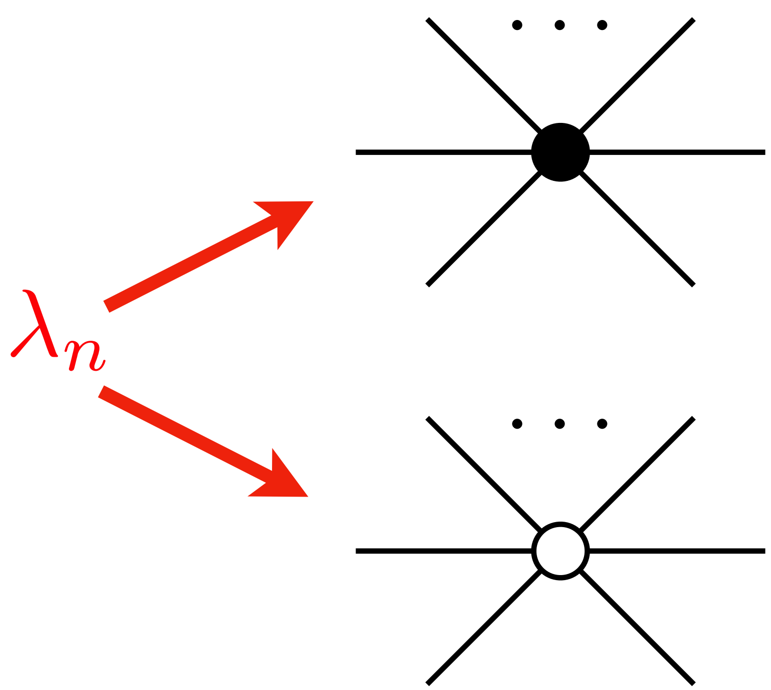
Schwinger-Keldysh formalism:

See: Chen, Wang & Xianyu (2017)



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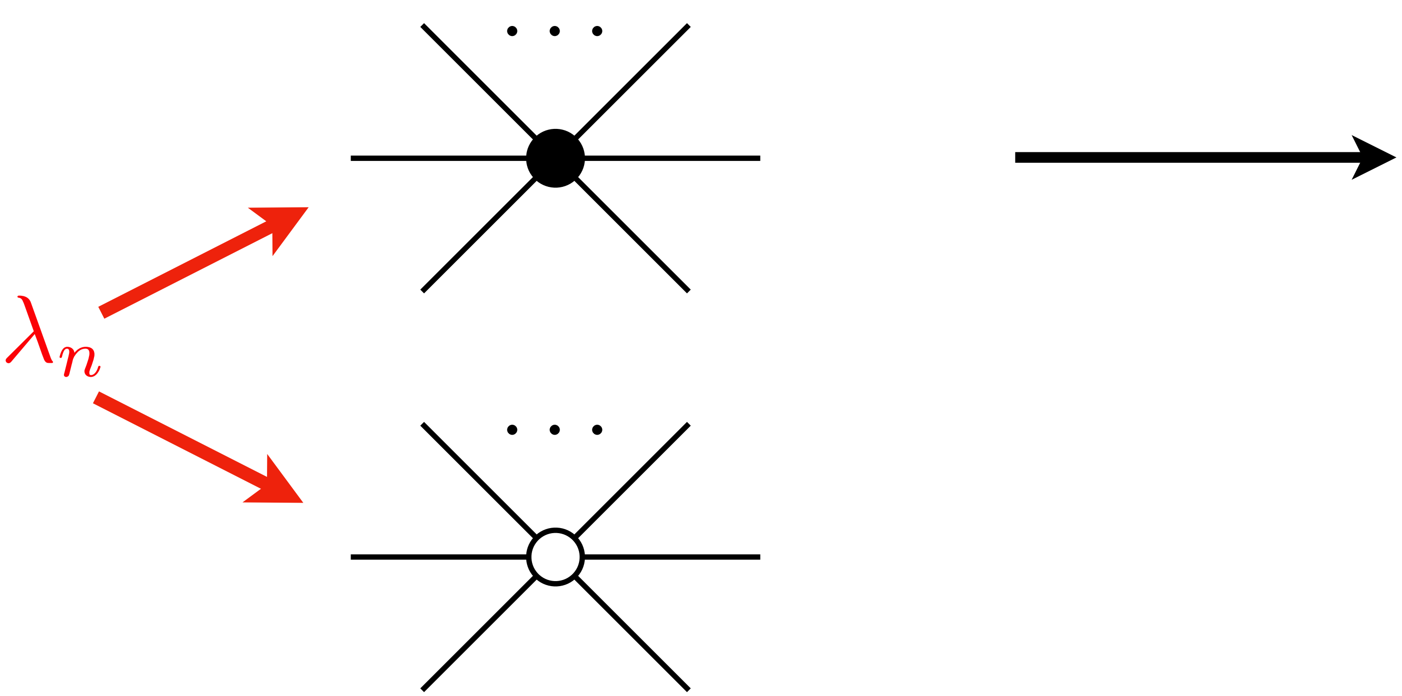

$$+ i\lambda_n \delta\left(\sum_{i=1}^n \mathbf{k}_i\right) \int_{-\infty}^{\tau} \frac{d\tau_a}{\tau_a^4}$$

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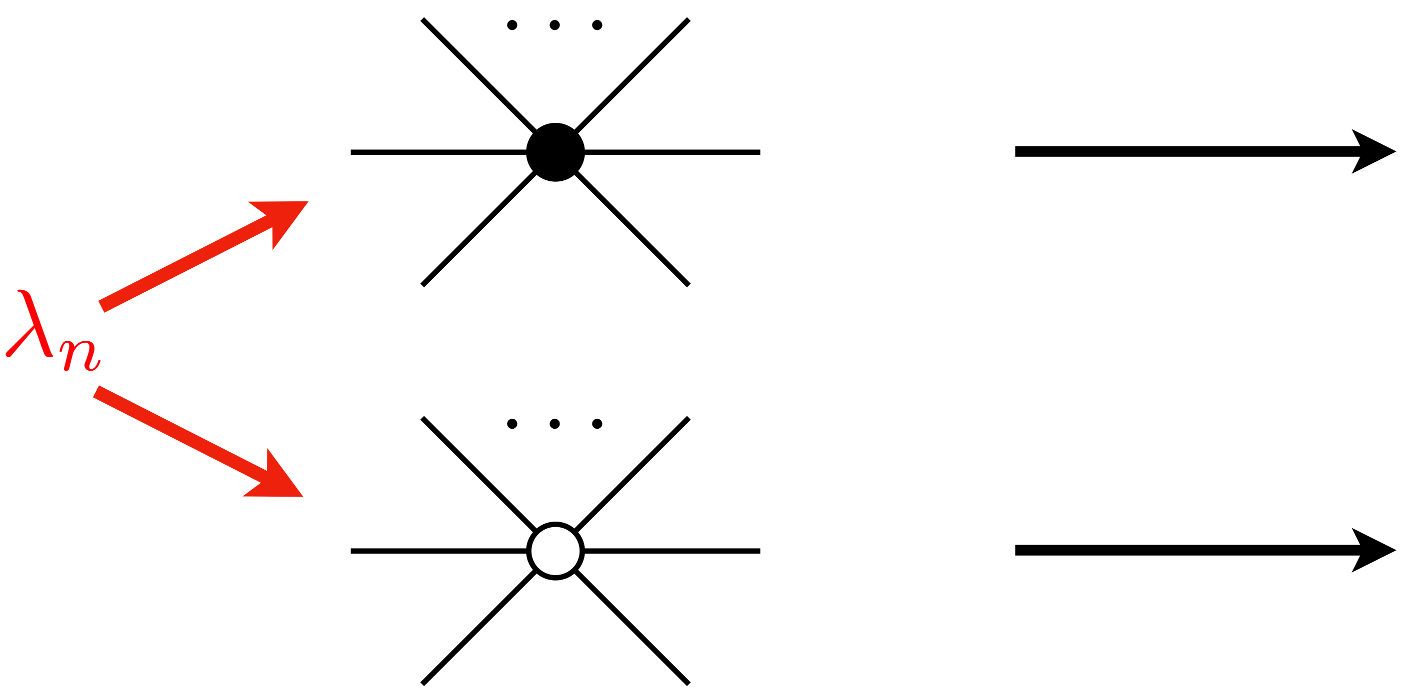

$$+i\lambda_n \delta\Big(\sum_{i=1}^{n}\mathbf{k}_i\Big)\int_{-\infty}^{\tau}\frac{d\tau_a}{\tau_a^4}$$

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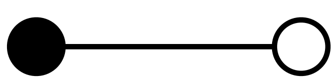
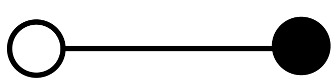
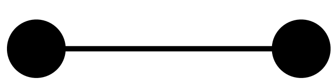
 → $f_k(\tau_1) f_k^*(\tau_2)$

 → $f_k^*(\tau_1) f_k(\tau_2)$

 → $f_k^*(\tau_1) f_k(\tau_2) \theta(\tau_1 - \tau_2) + f_k(\tau_1) f_k^*(\tau_2) \theta(\tau_2 - \tau_1)$

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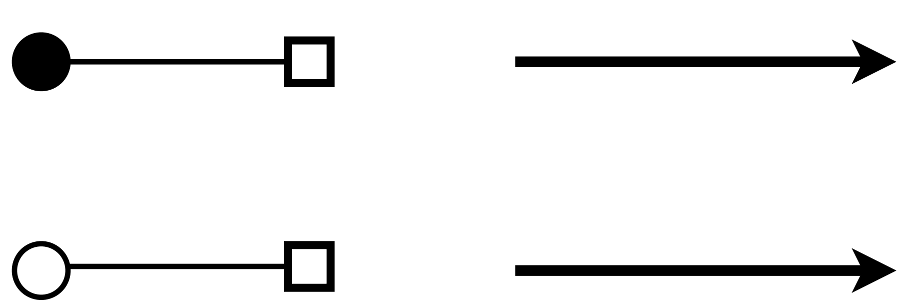

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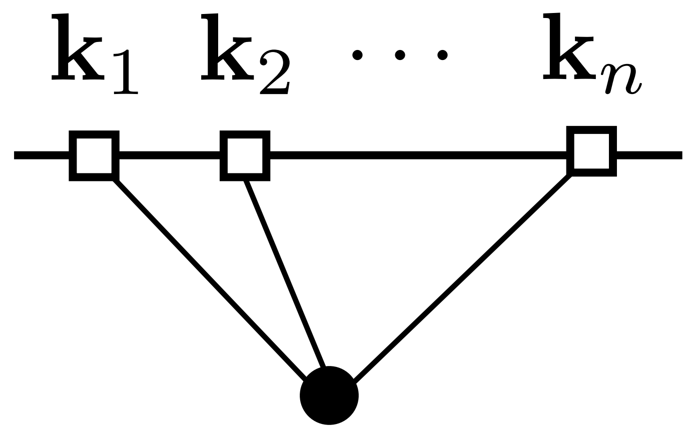

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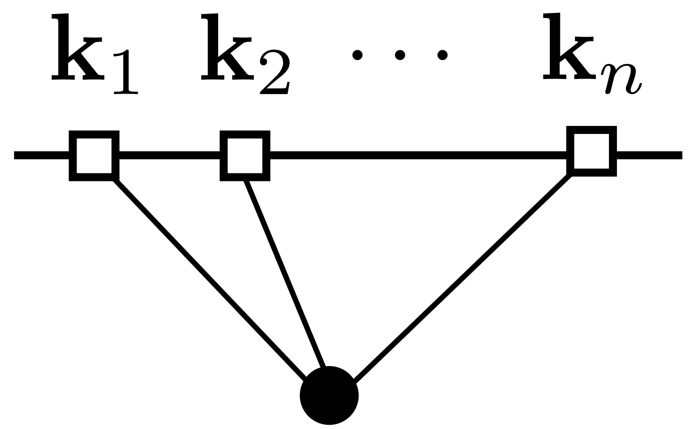

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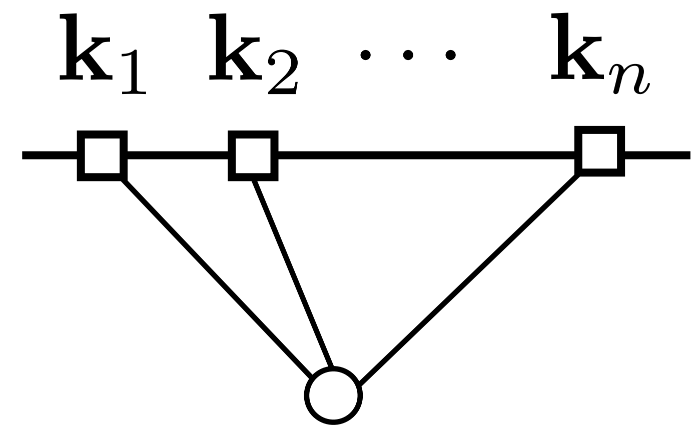

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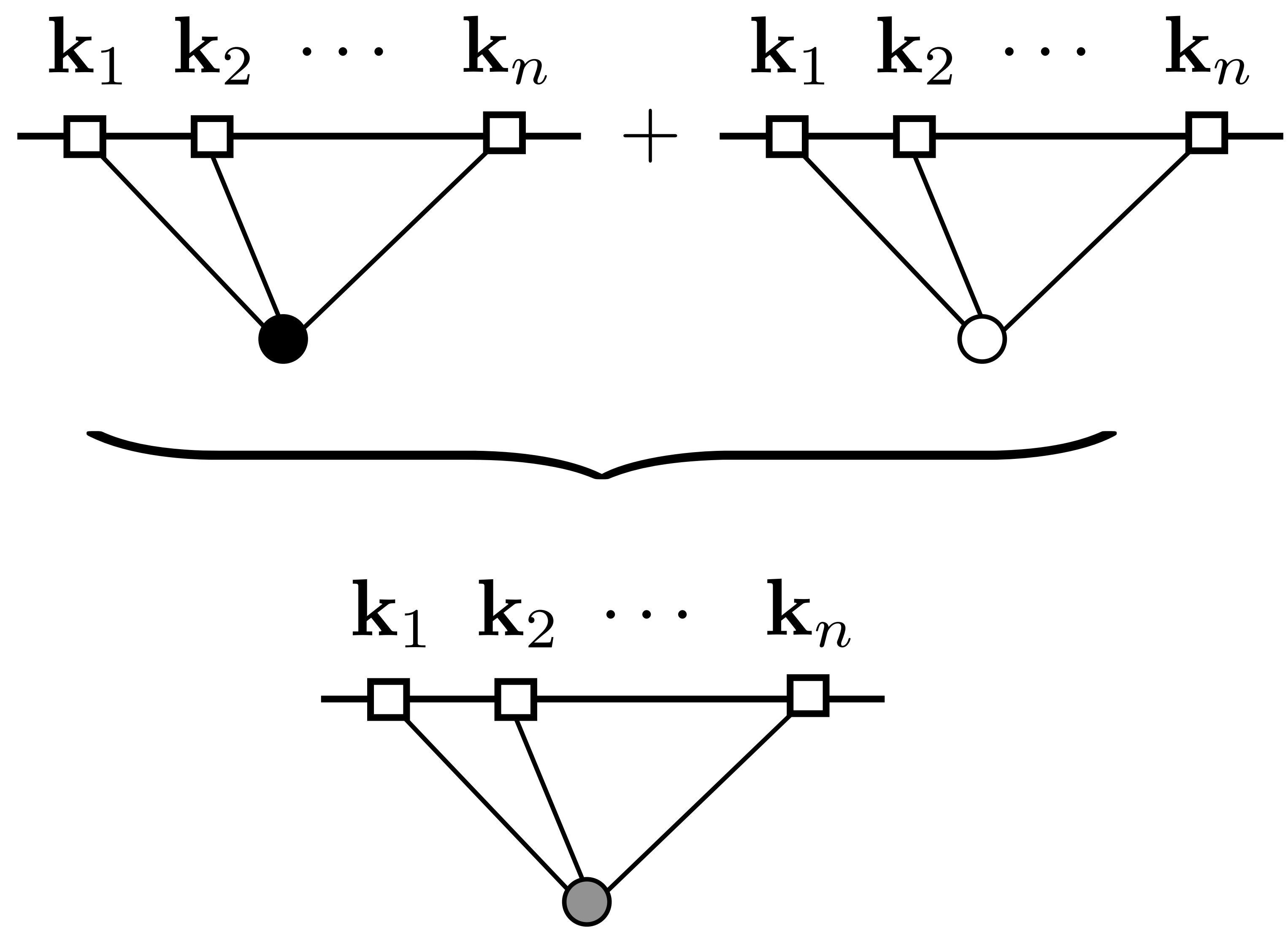

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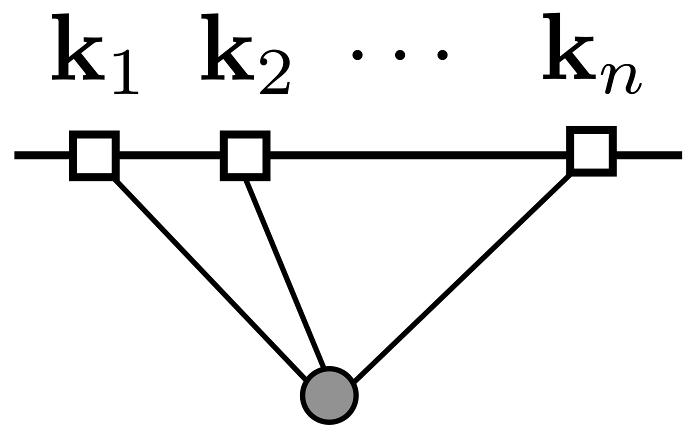

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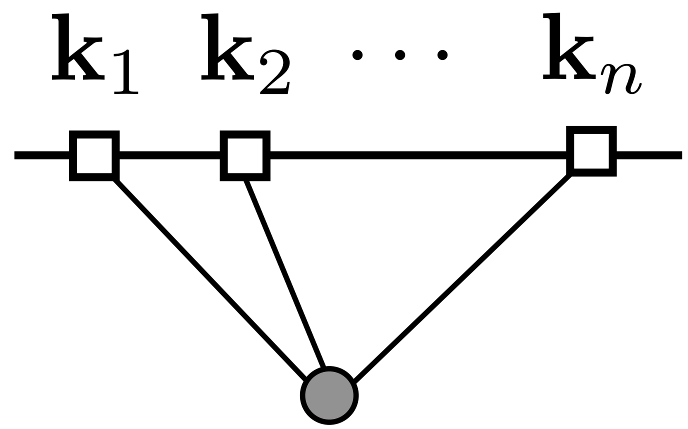

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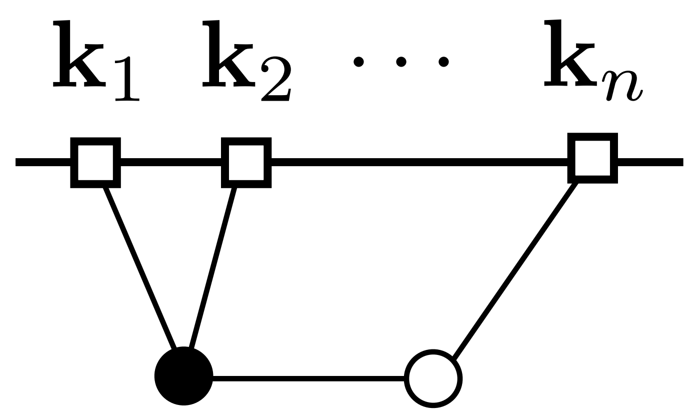

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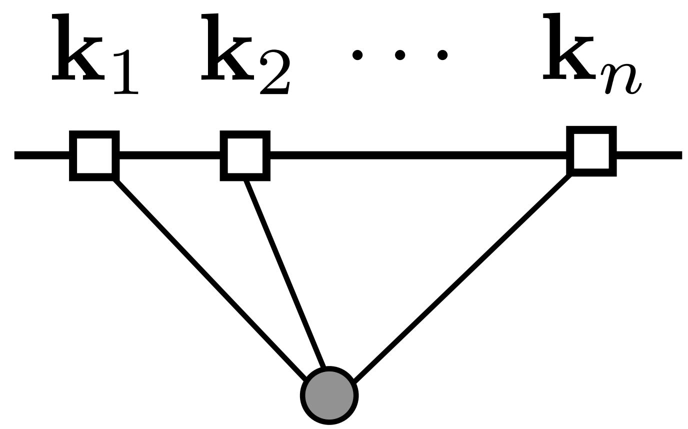

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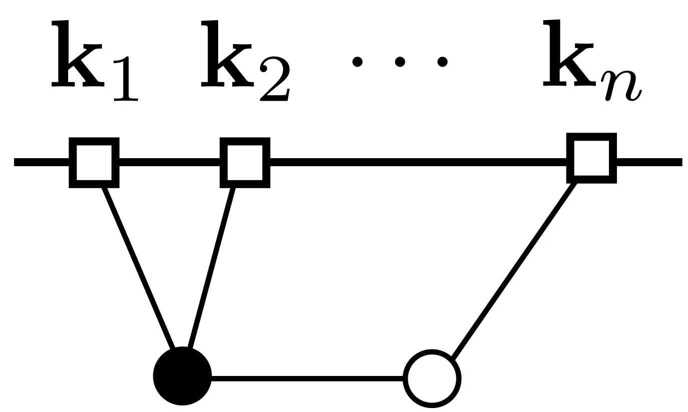

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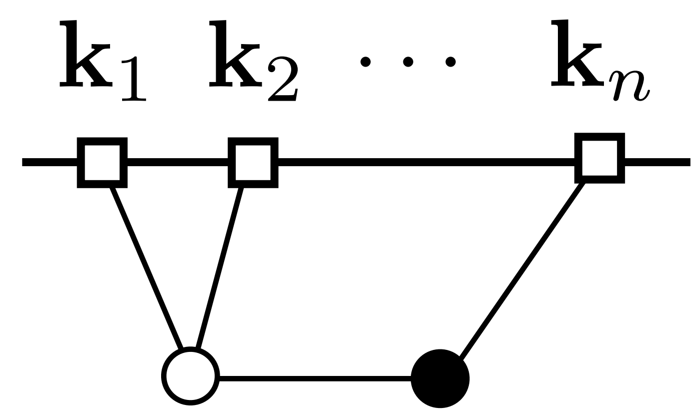

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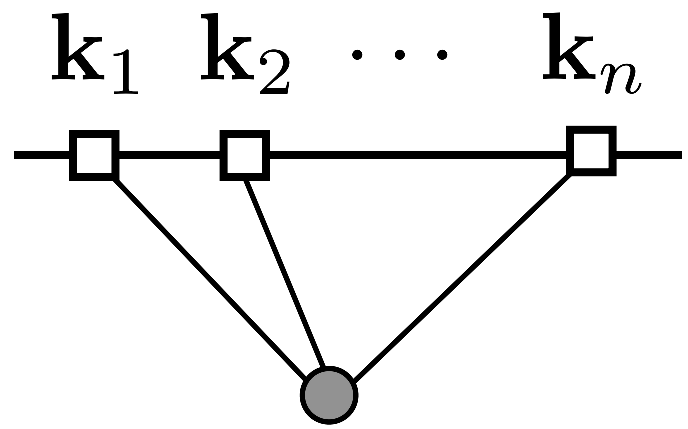

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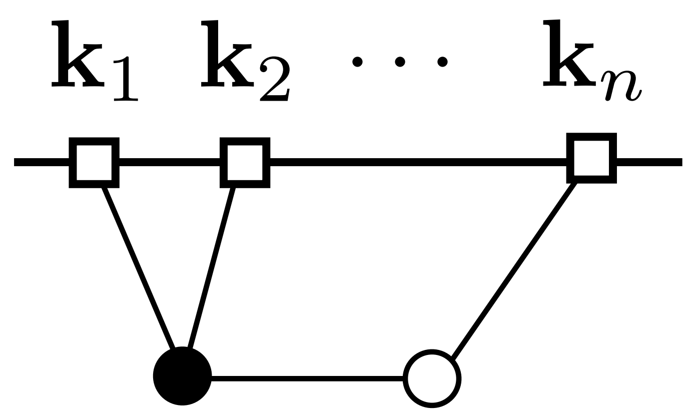

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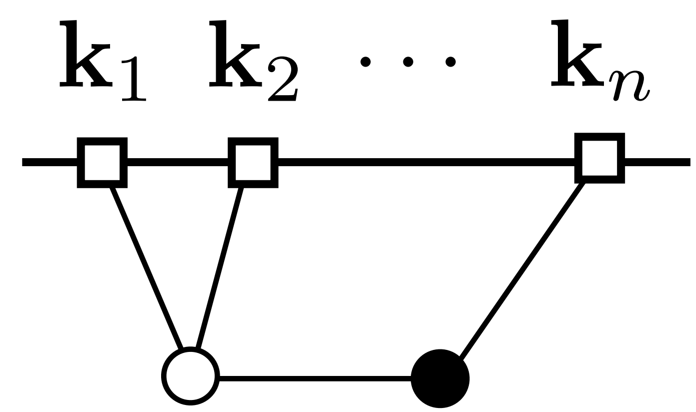

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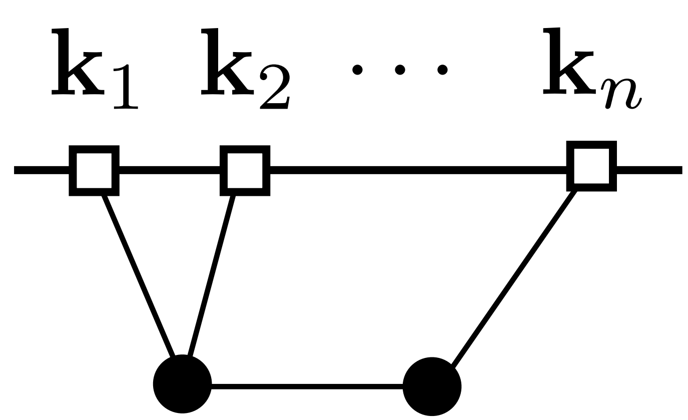

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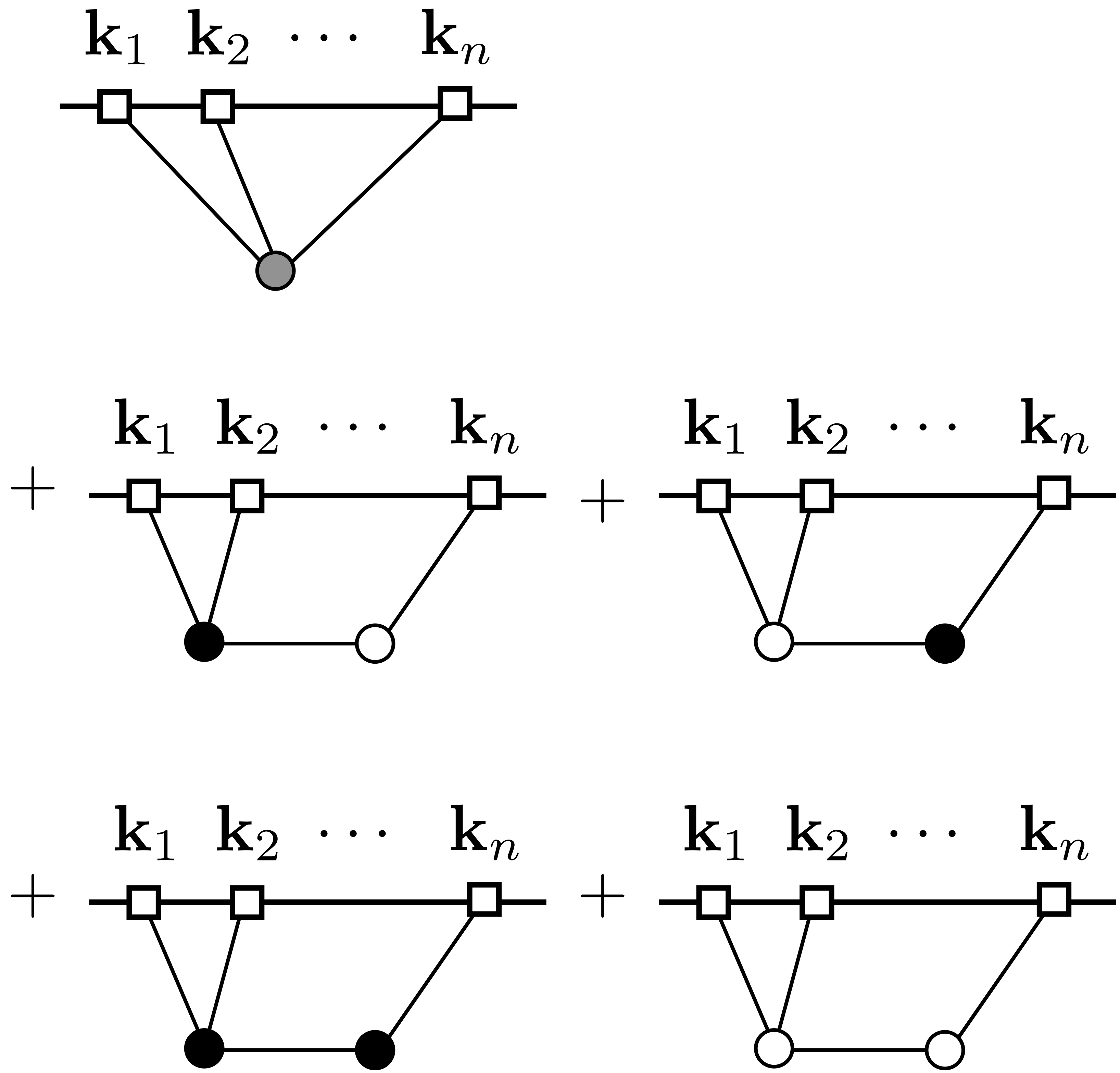

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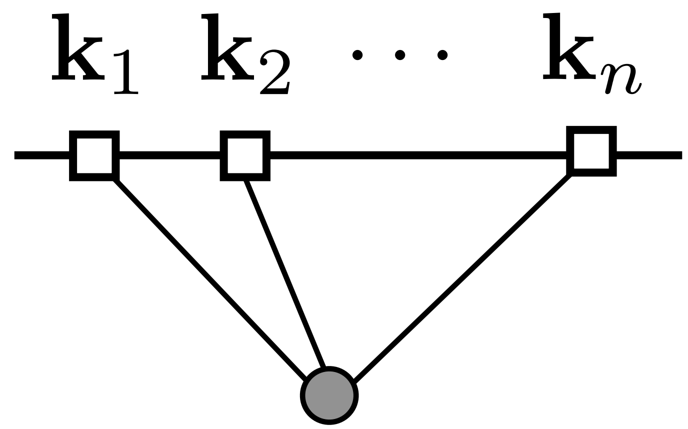

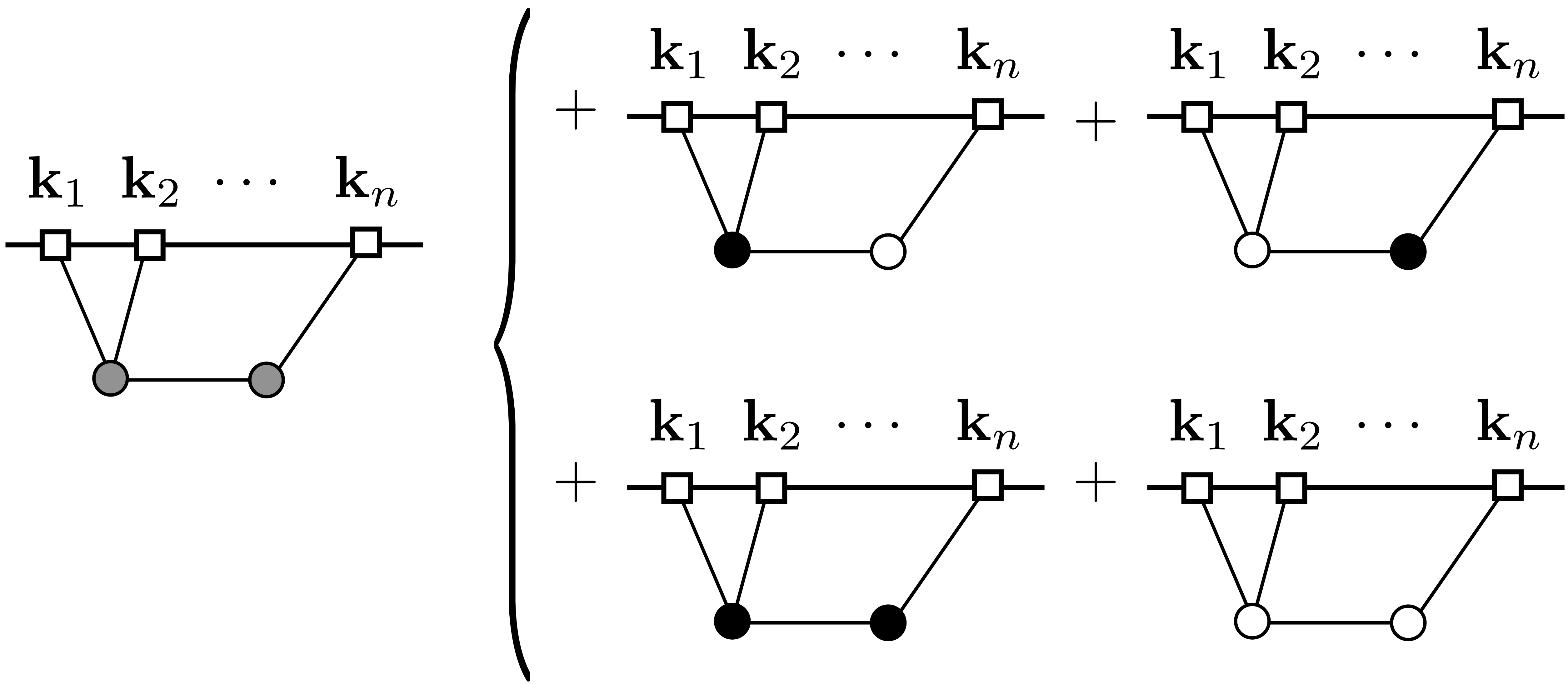

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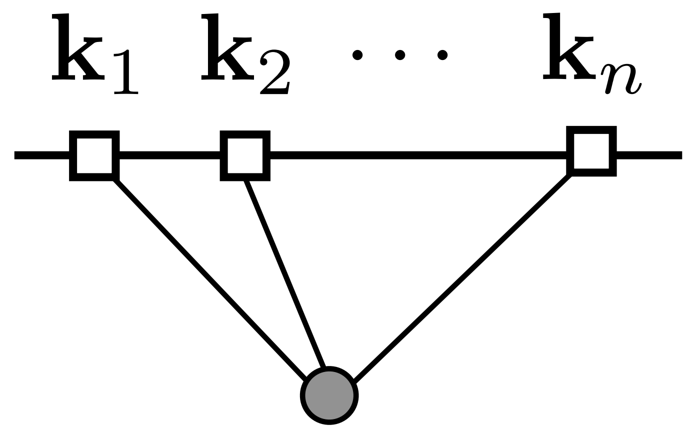

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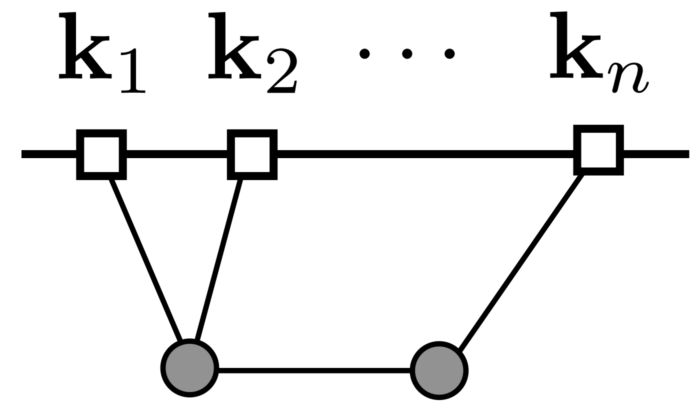

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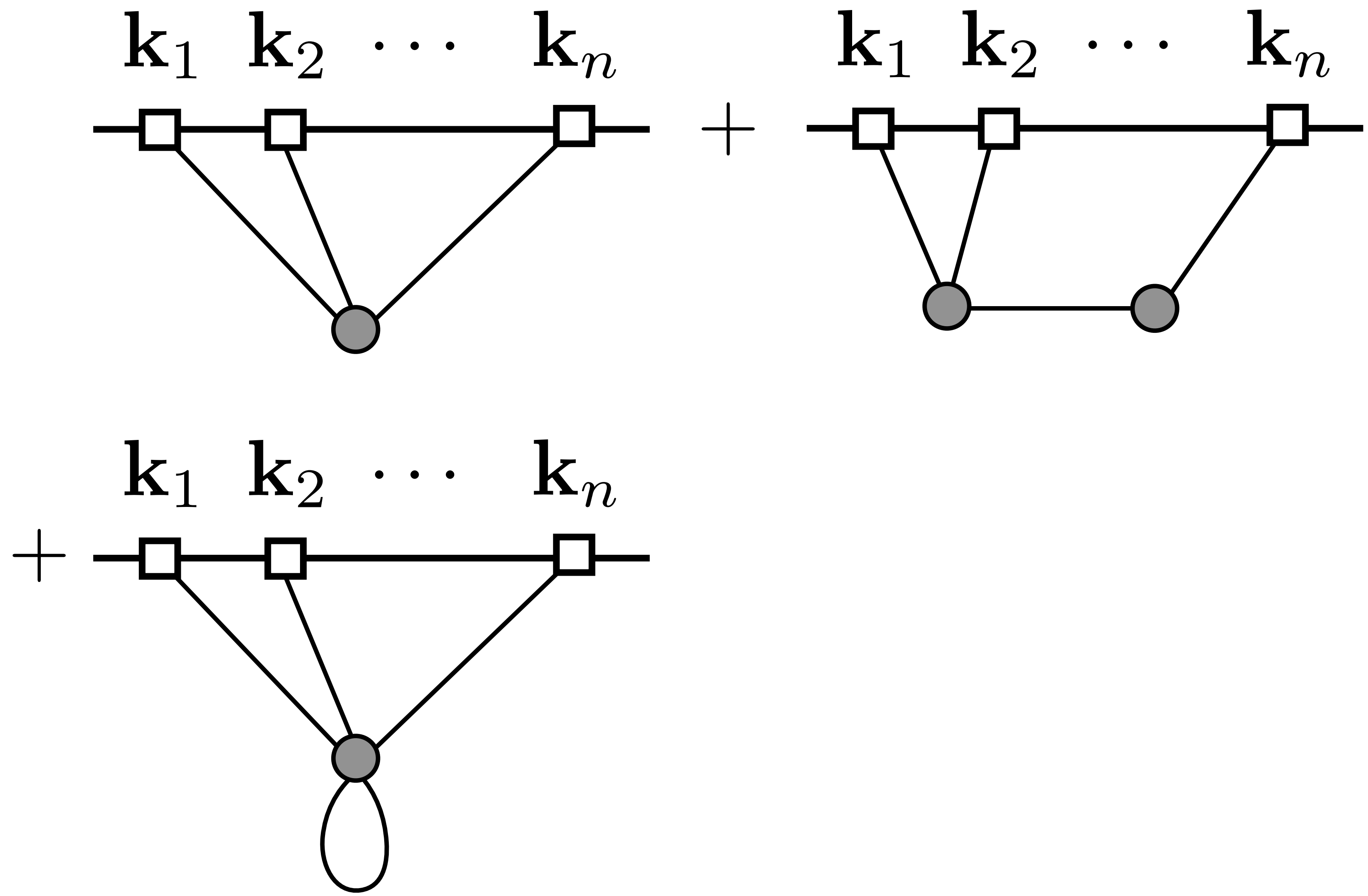

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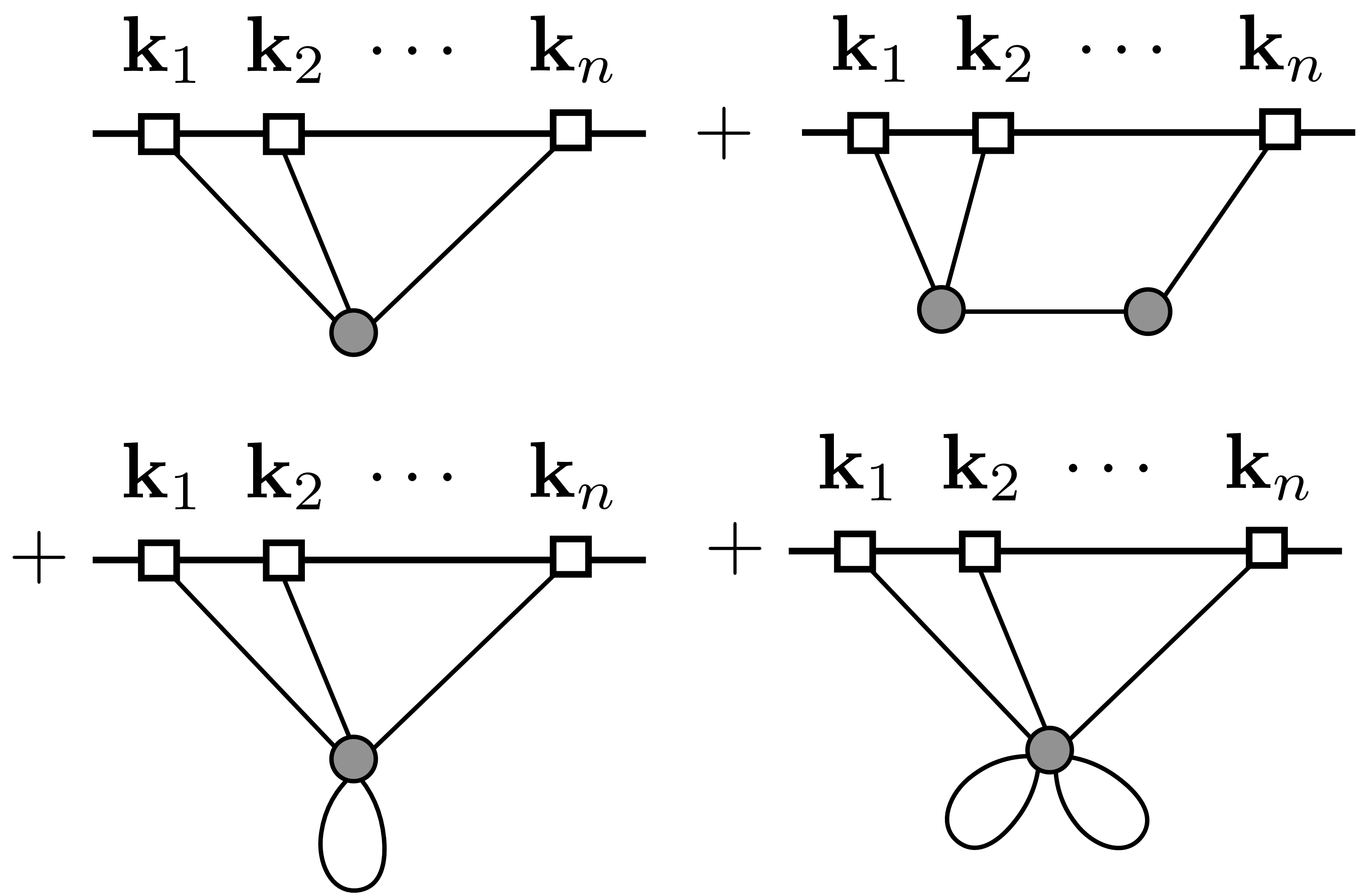

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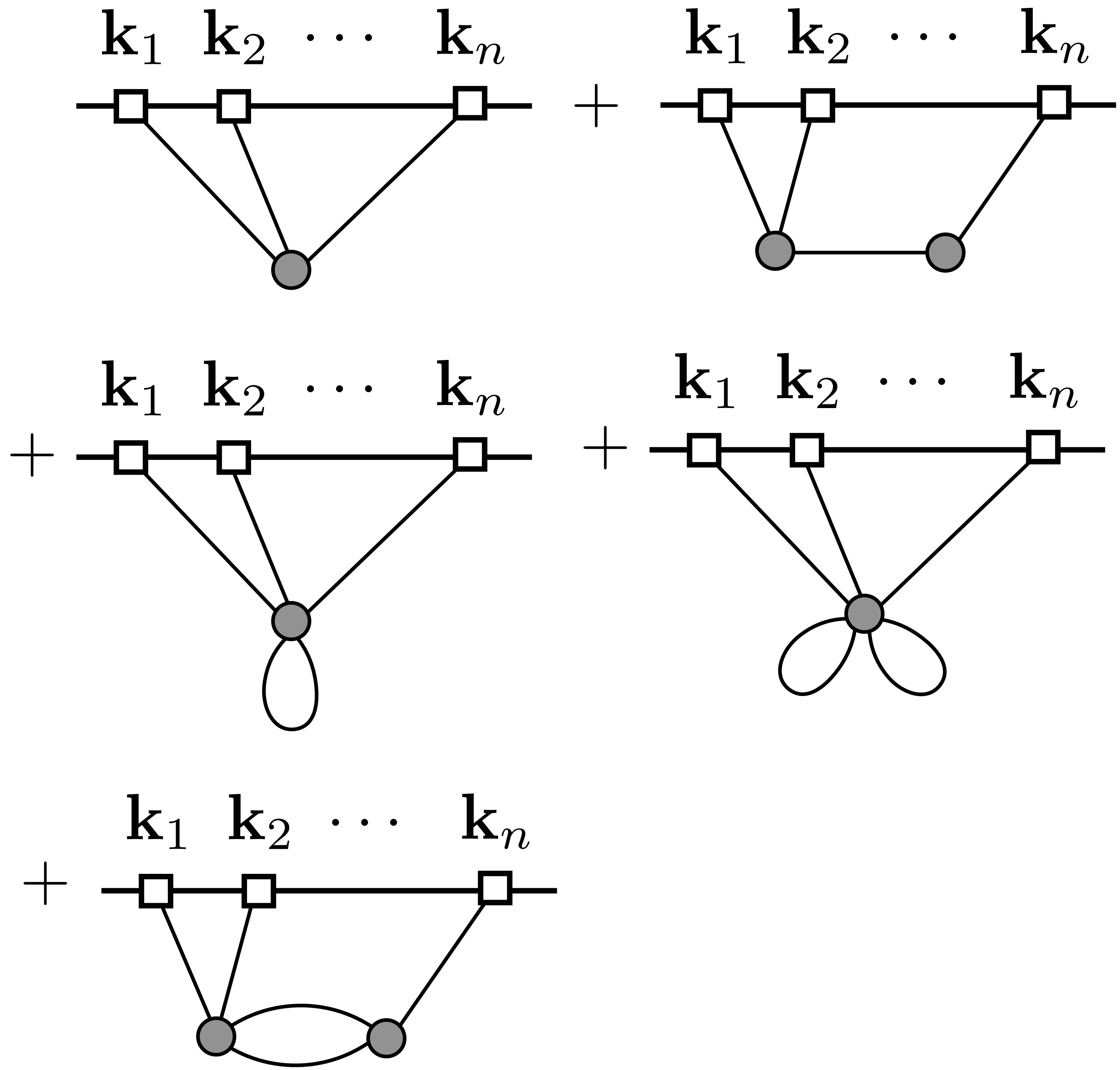

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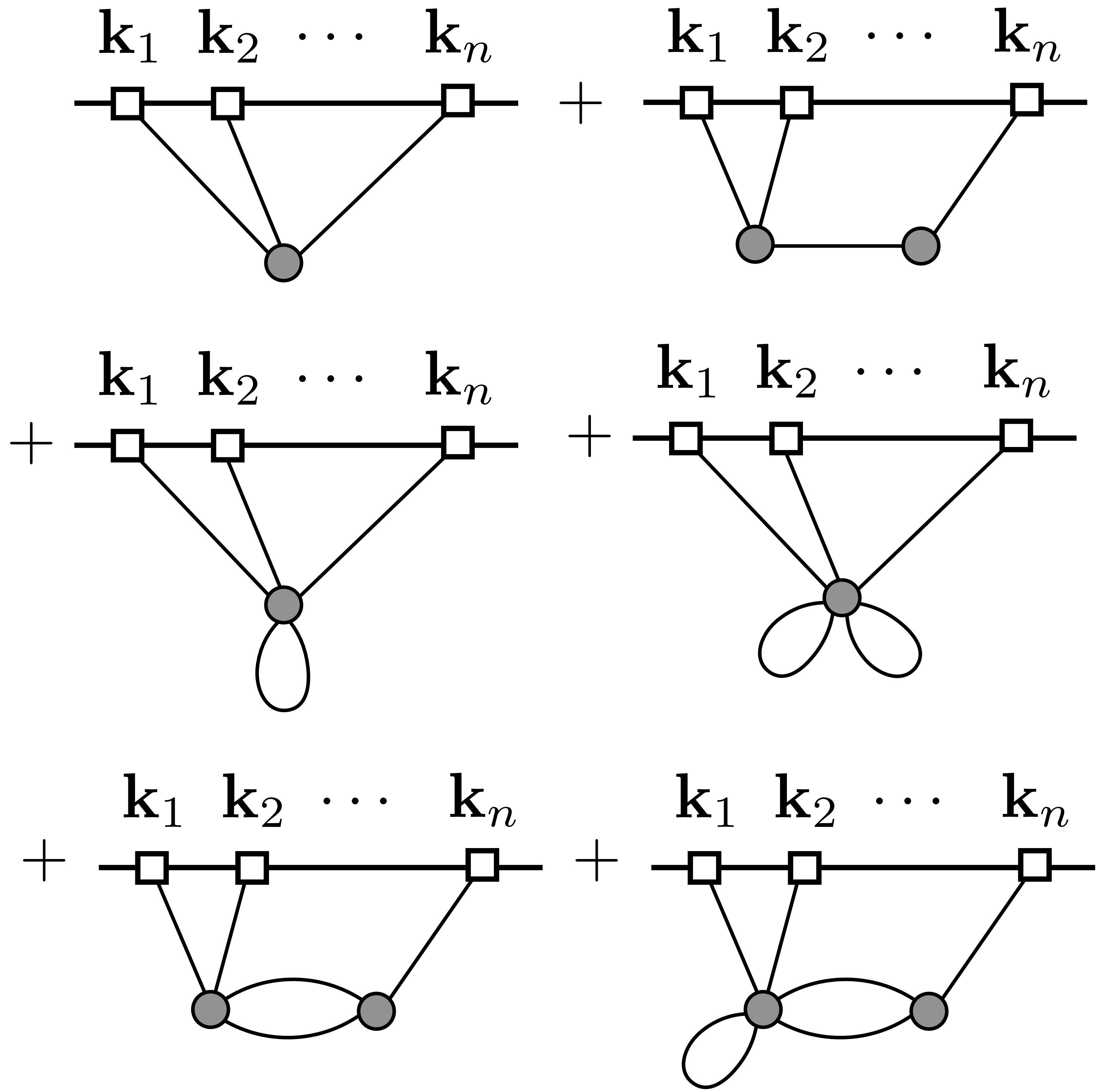

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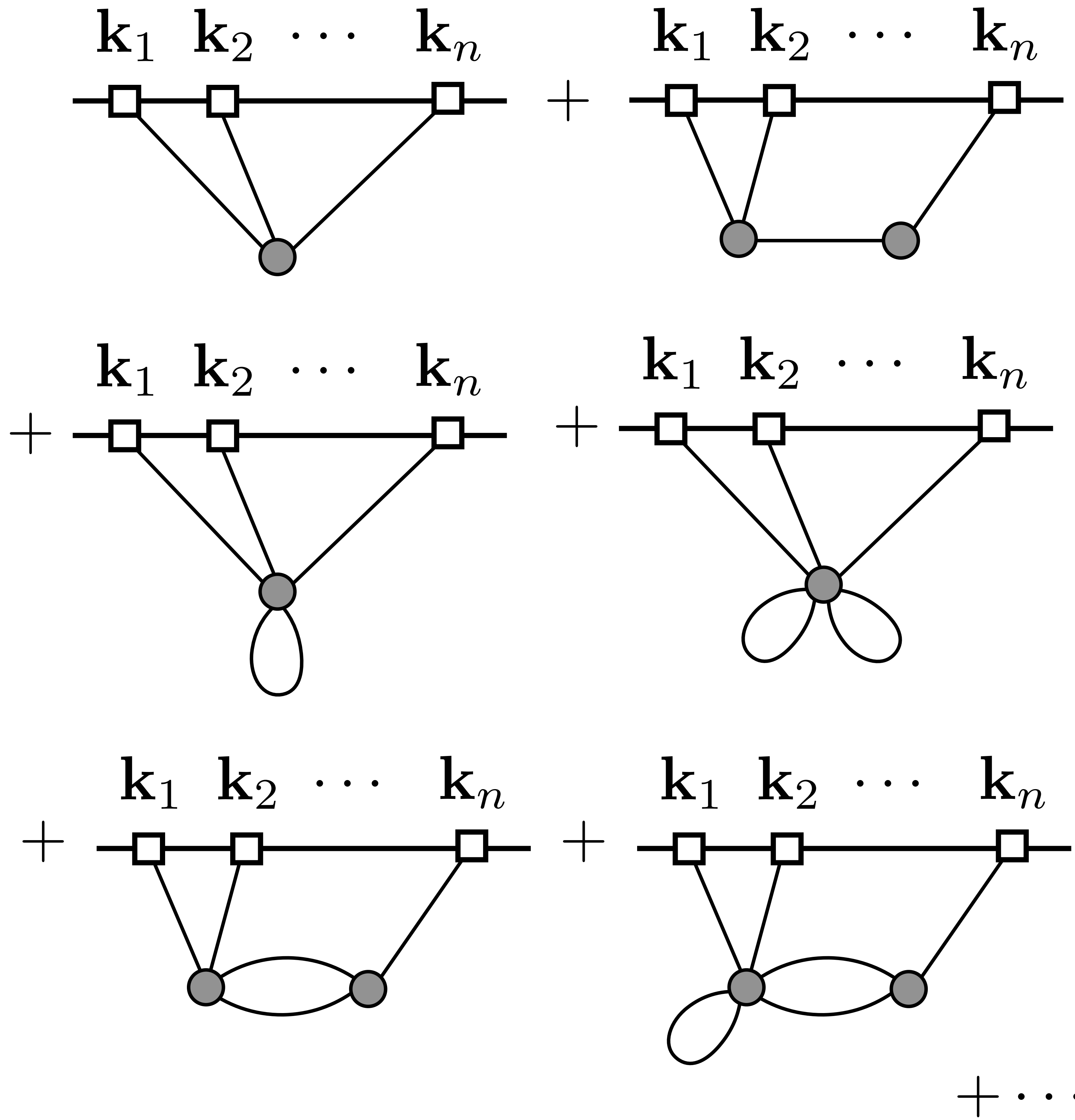

Split propagators

One may split propagators into real and imaginary parts:

 $=\ \mathrm{Re}($  $)\ +\ i\,\mathrm{Im}($  $)$

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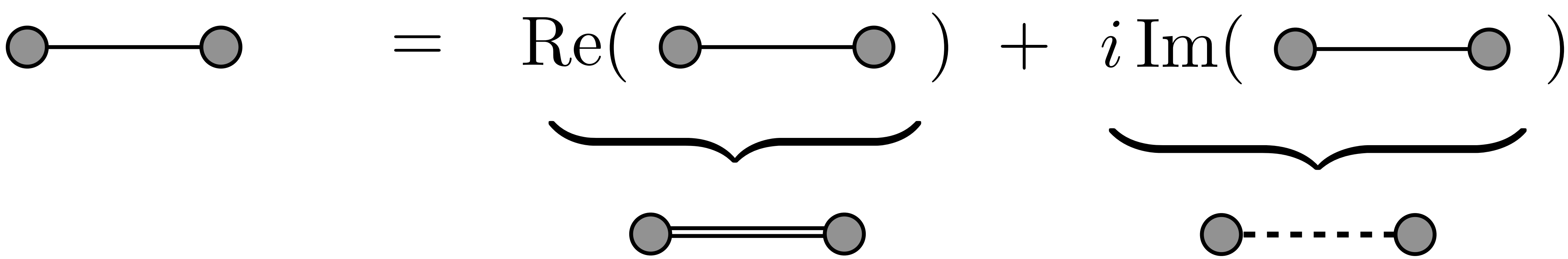

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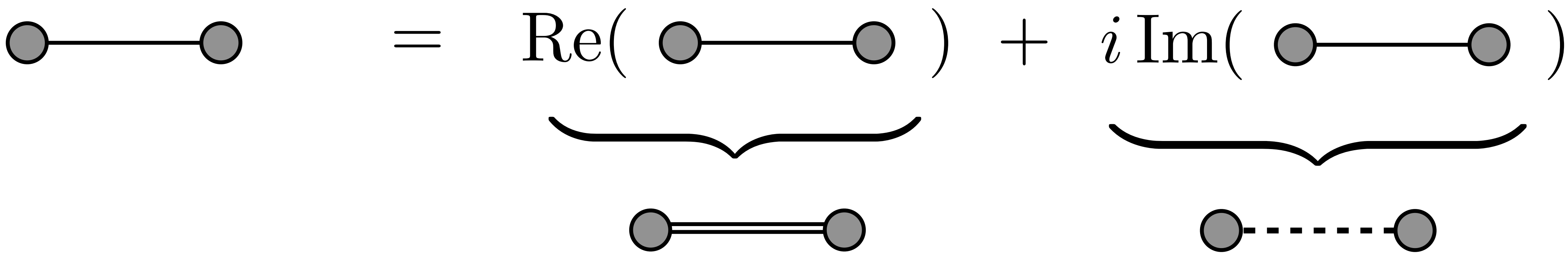

Then one can prove:

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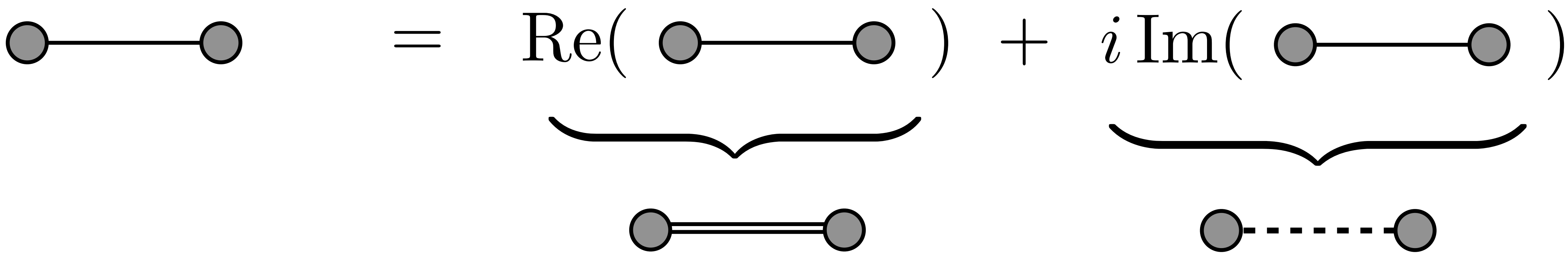

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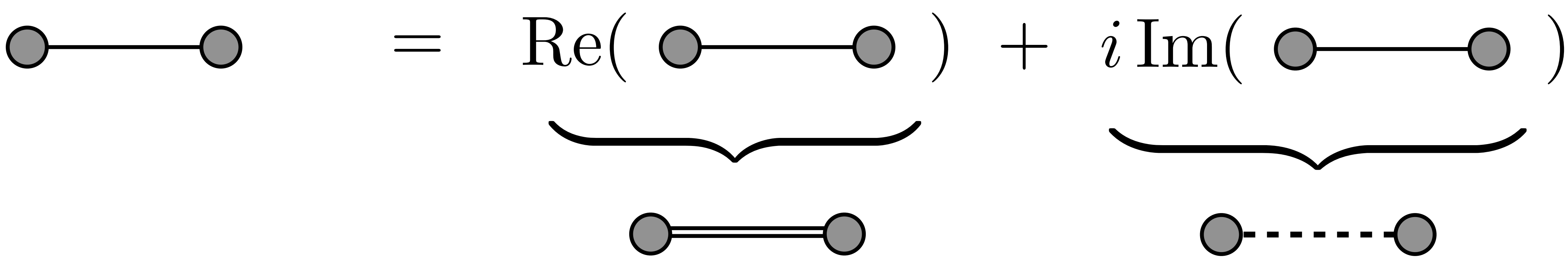
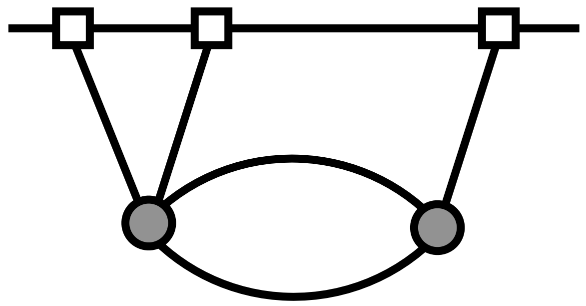


Then one can prove:

- **Every vertex must have at least one imaginary propagator attached to it**
- **It is impossible to form a closed loop only with imaginary propagators**

Split propagators



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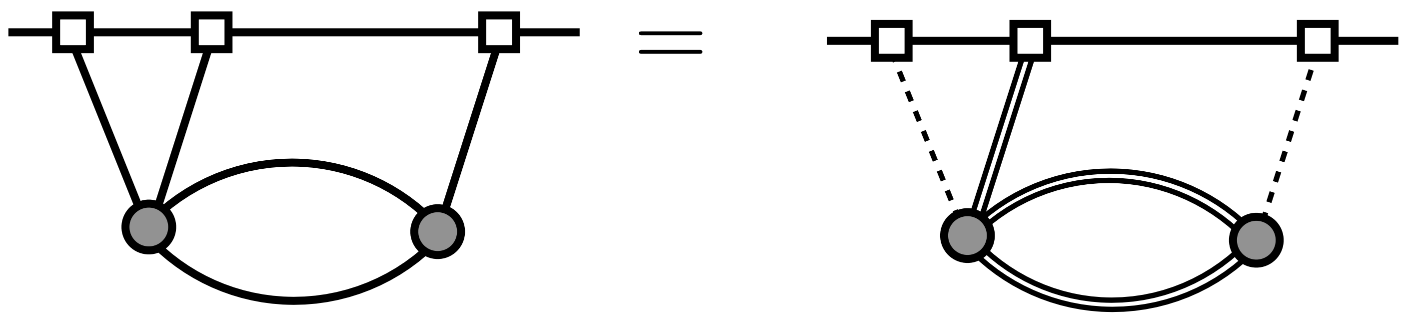

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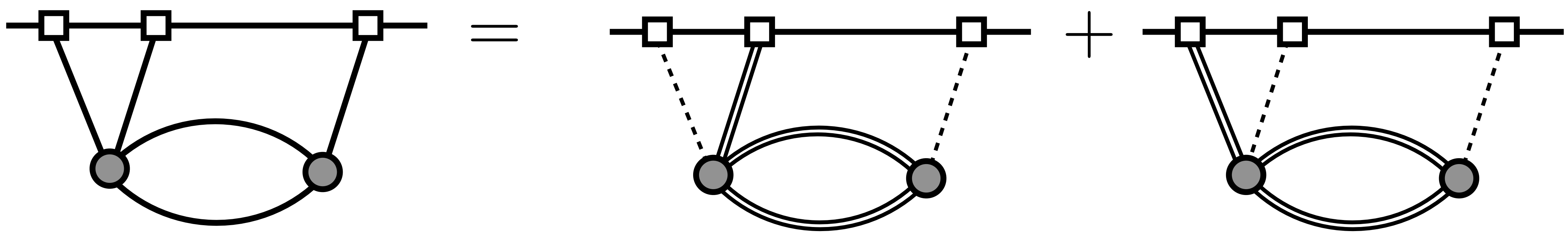

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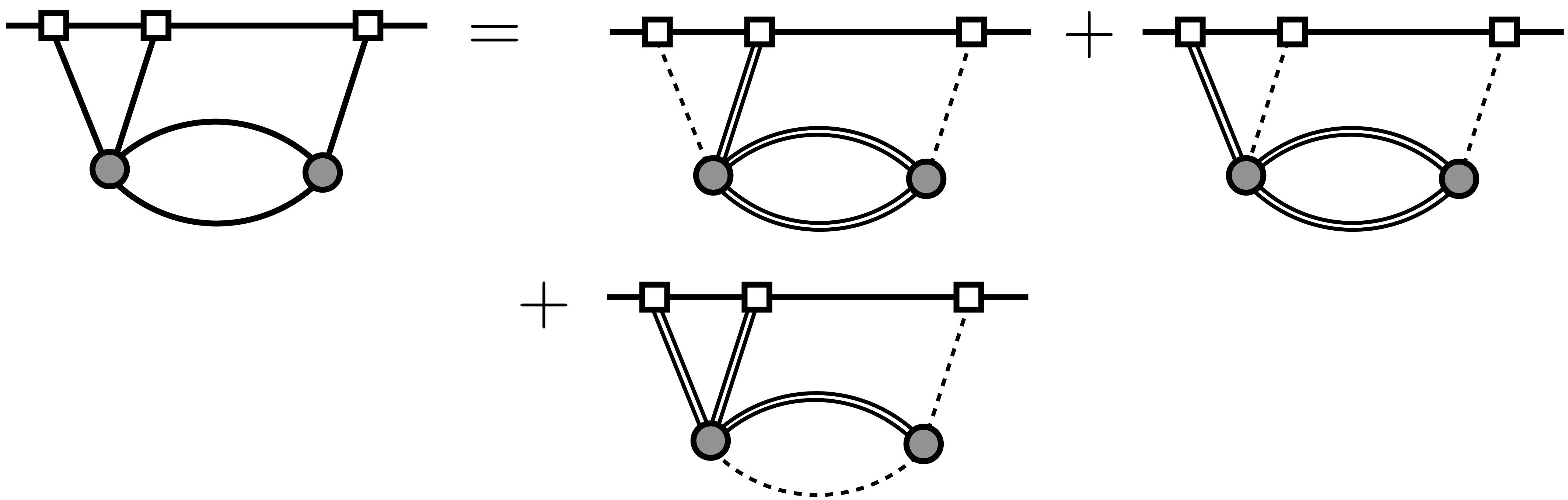
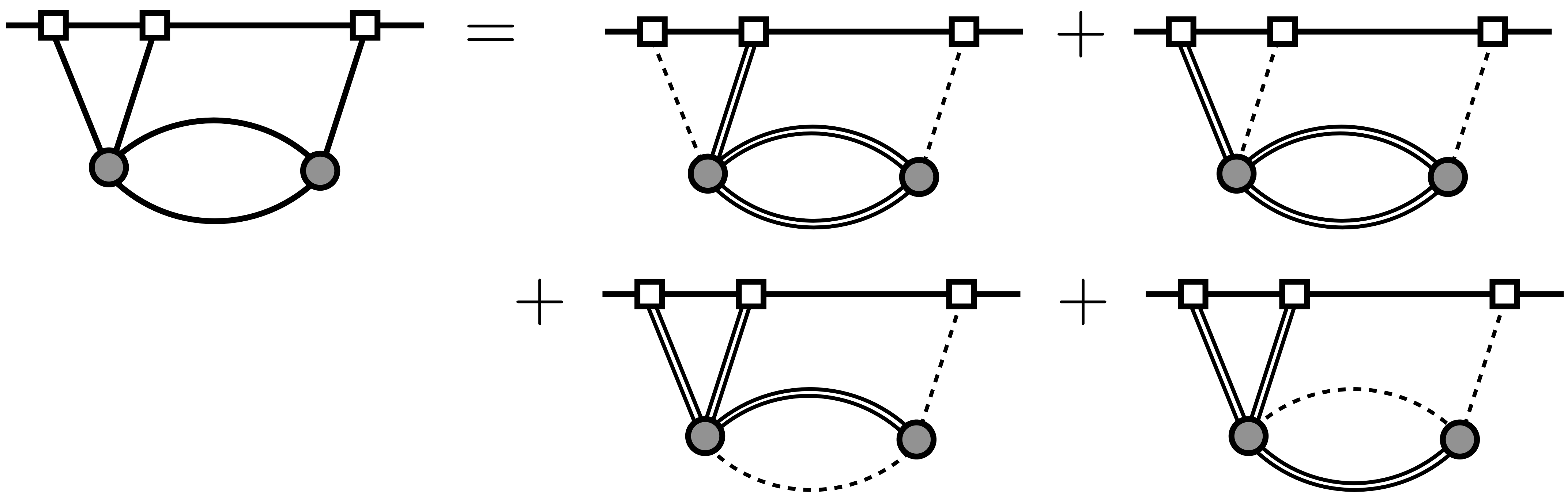


Split propagators



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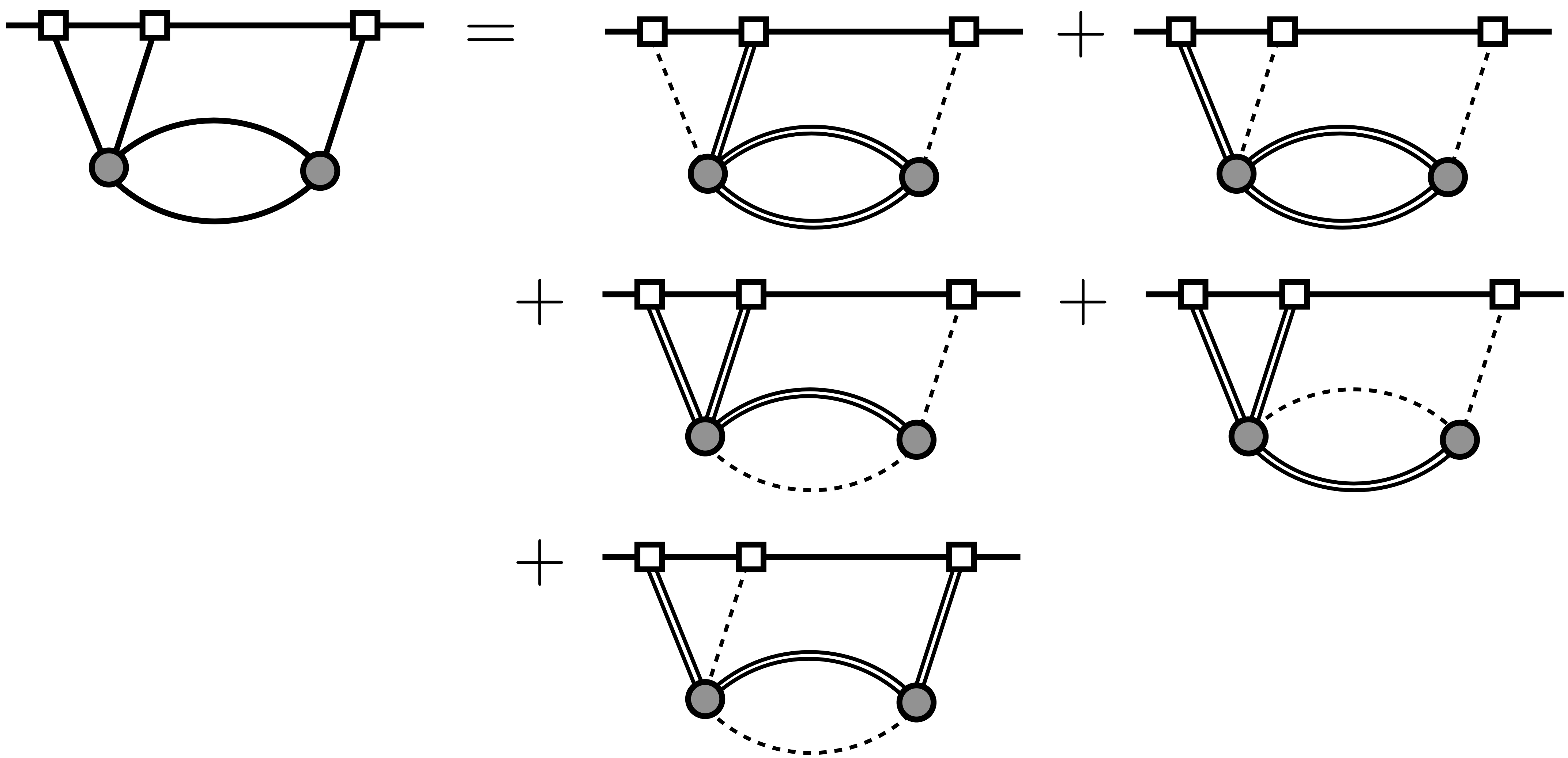

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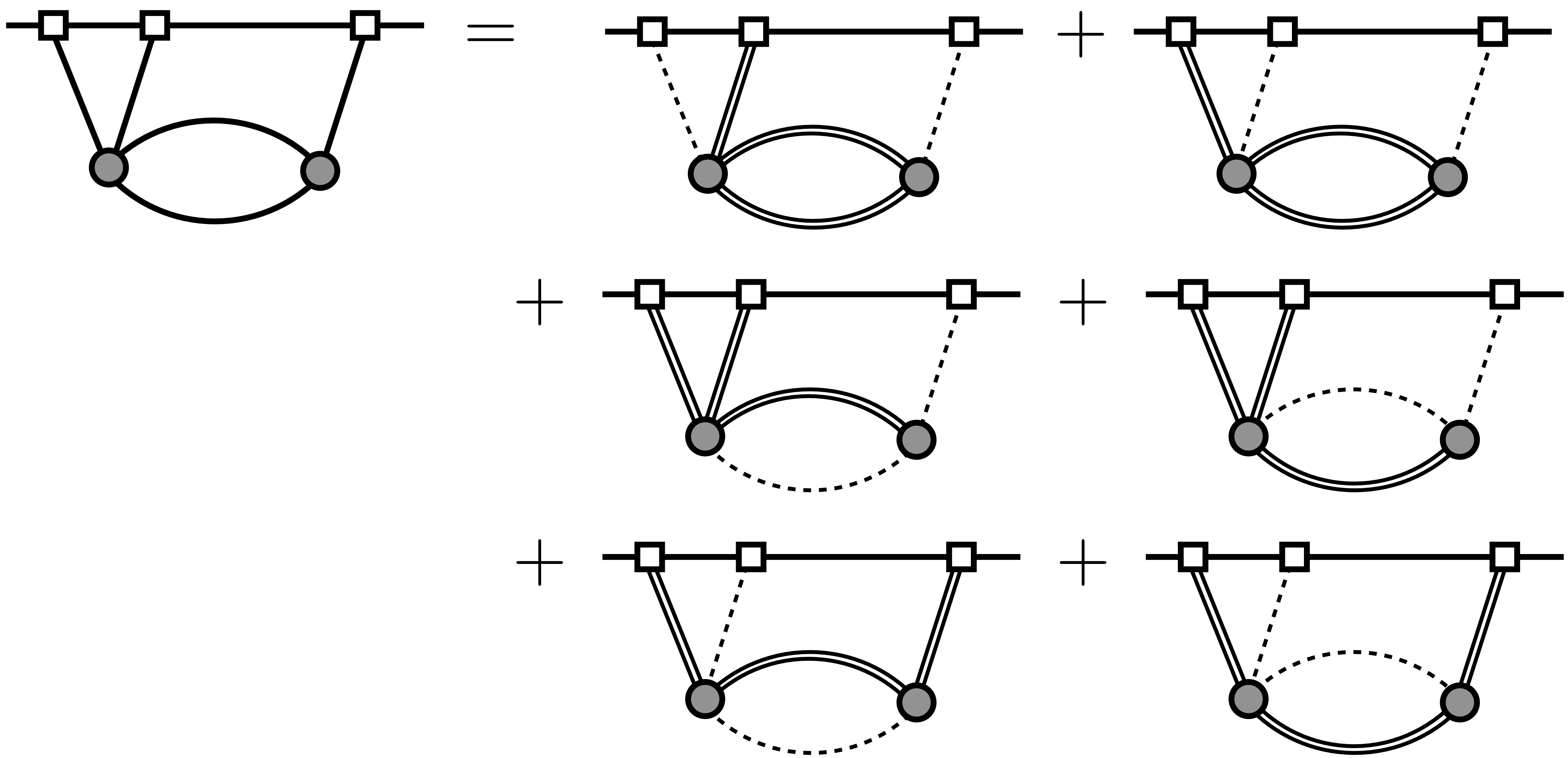

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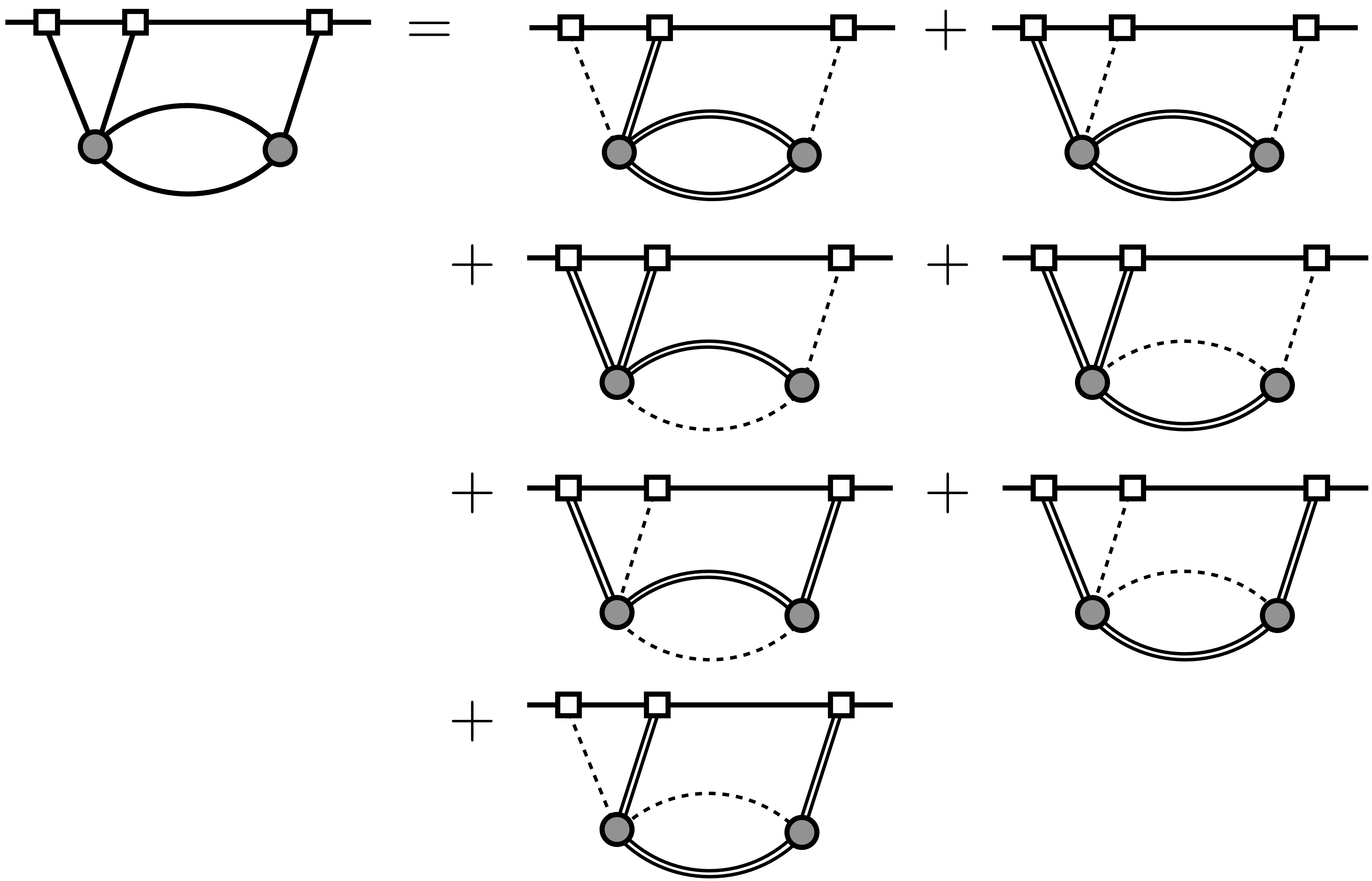

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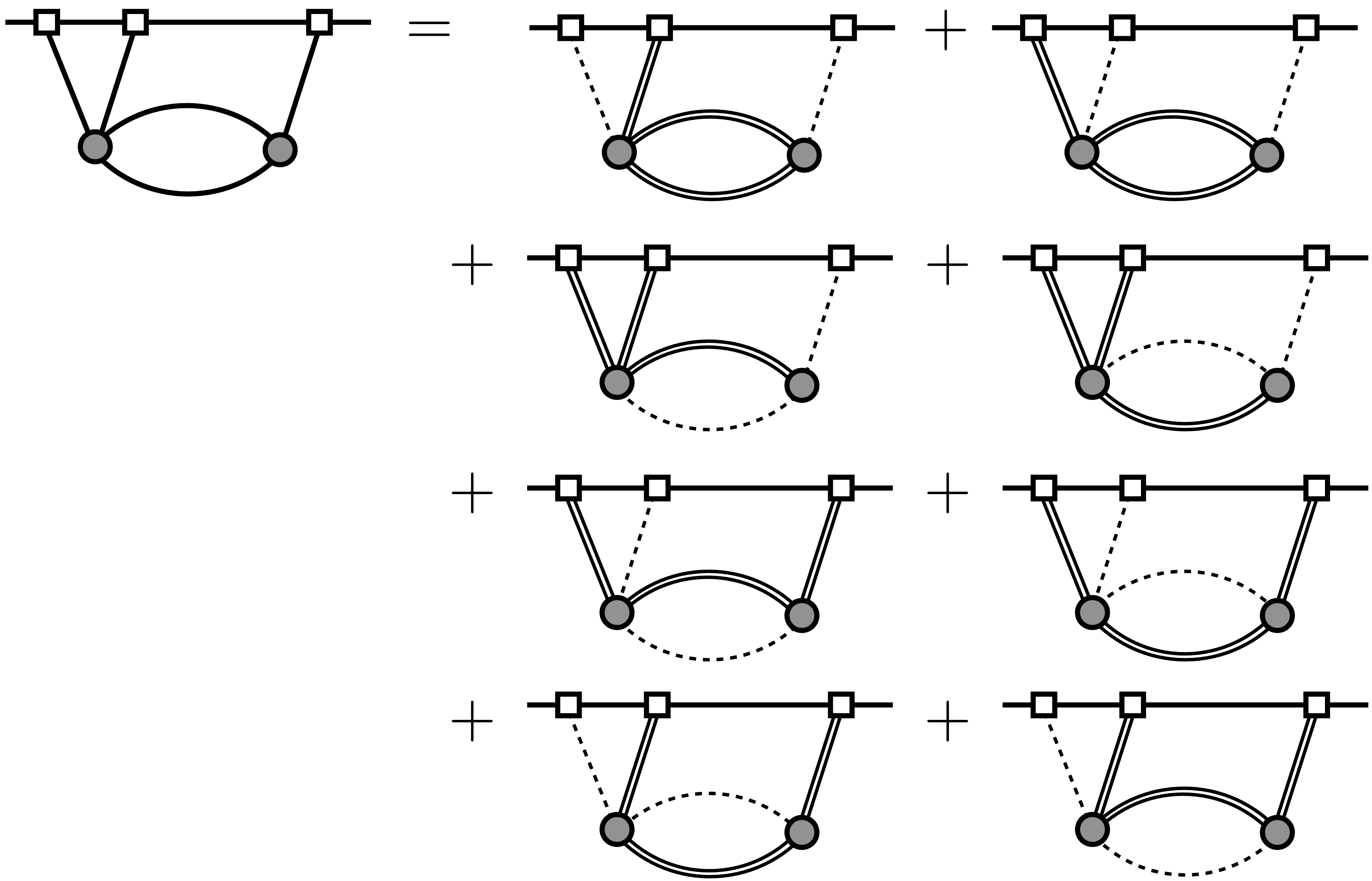

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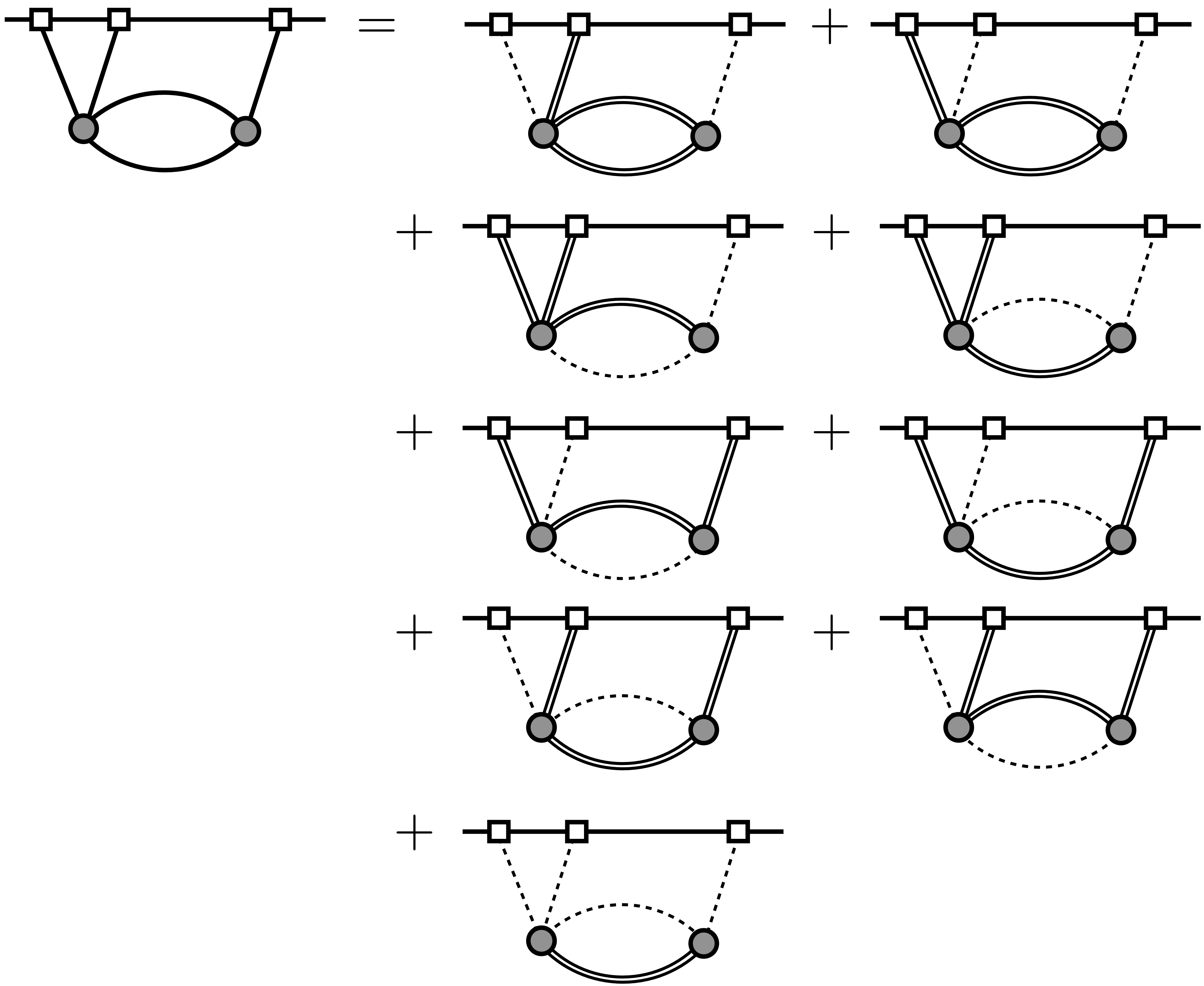

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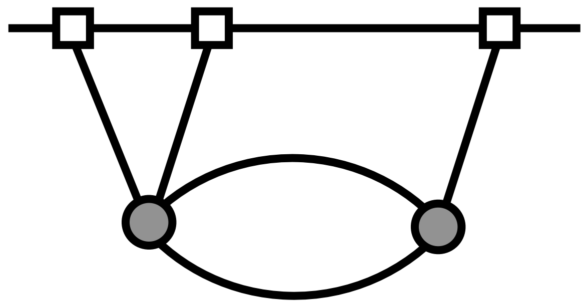

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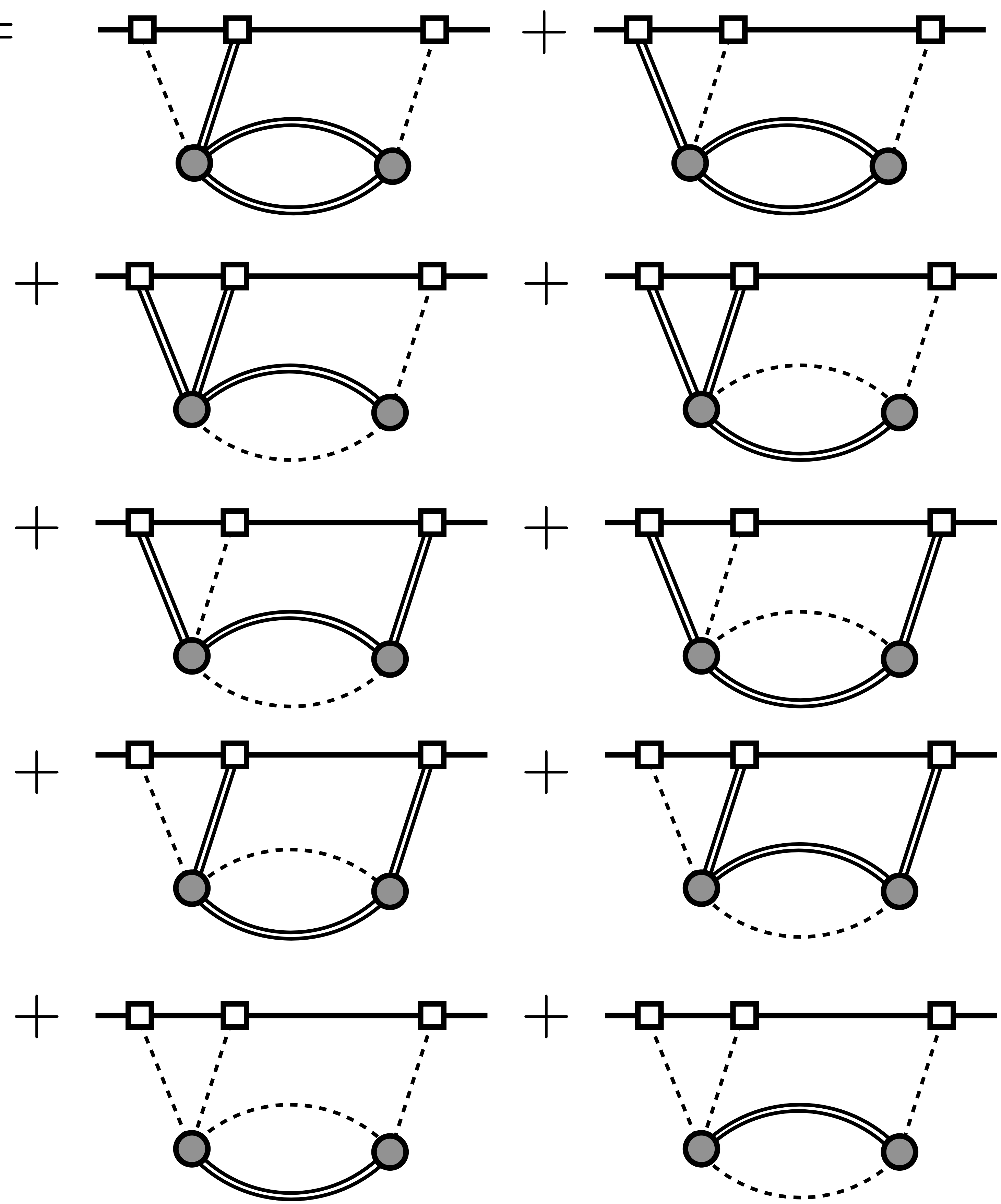
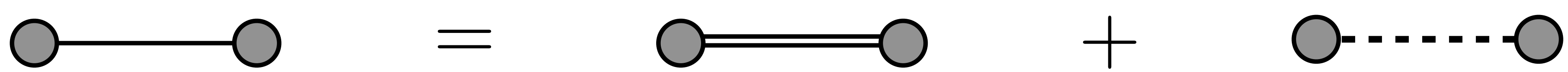


Logarithms in momentum space

Real and imaginary propagators have different time dependences:



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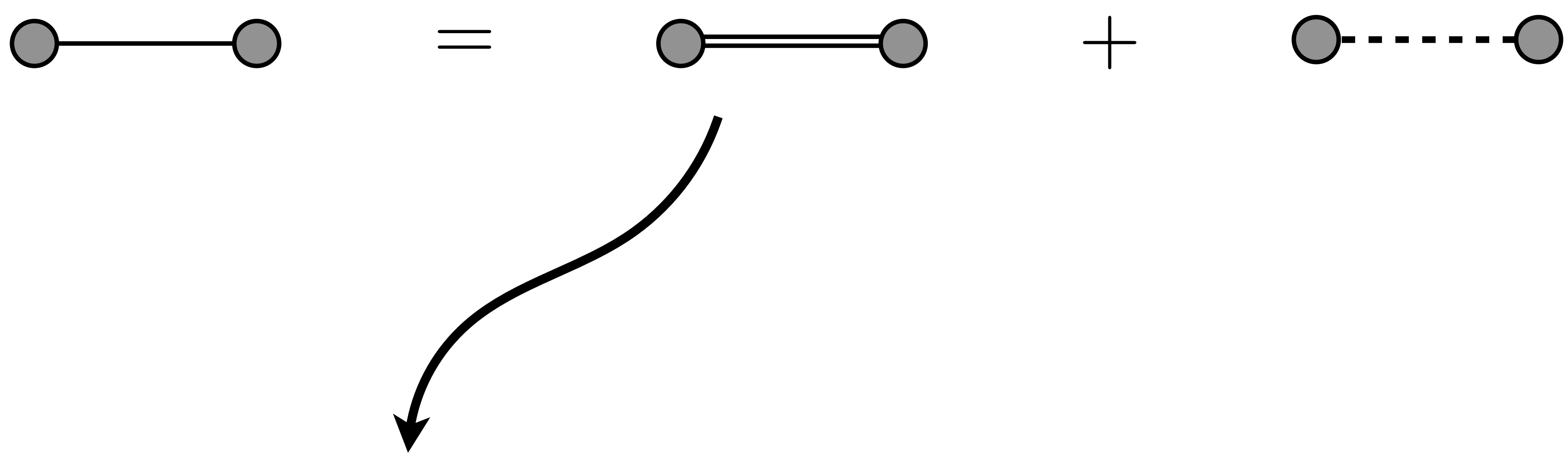

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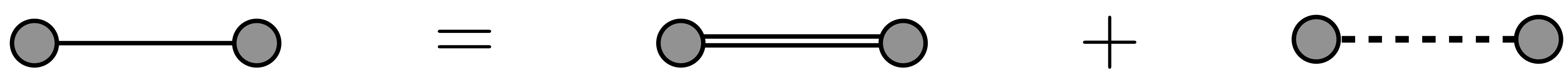
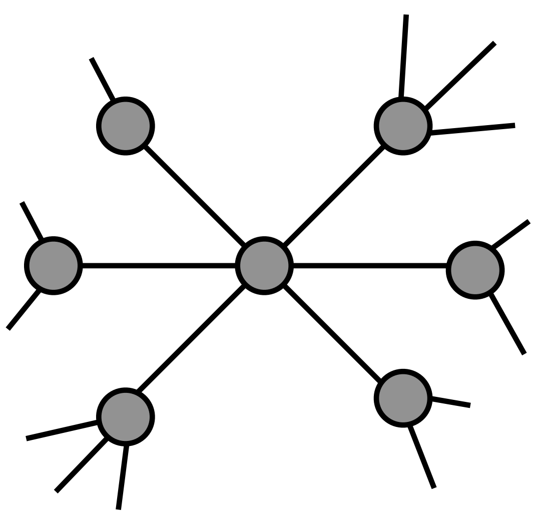


$$\propto \frac{H^2}{2k^3}\left[1 + \cdots\right]$$

$$\propto \frac{H^2}{6}\left[(\tau_a^3 - \tau_b^3) + \cdots\right]$$

Logarithms in momentum space

It is possible to track the time dependence of any diagram in the long wavelength regime $k|\tau| \ll 1$ **:**



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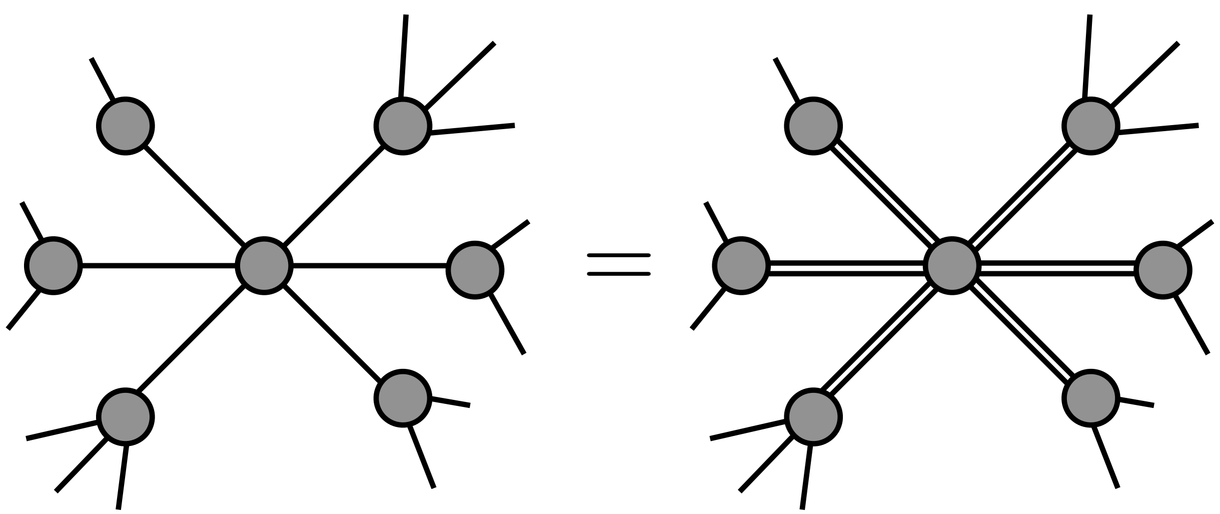

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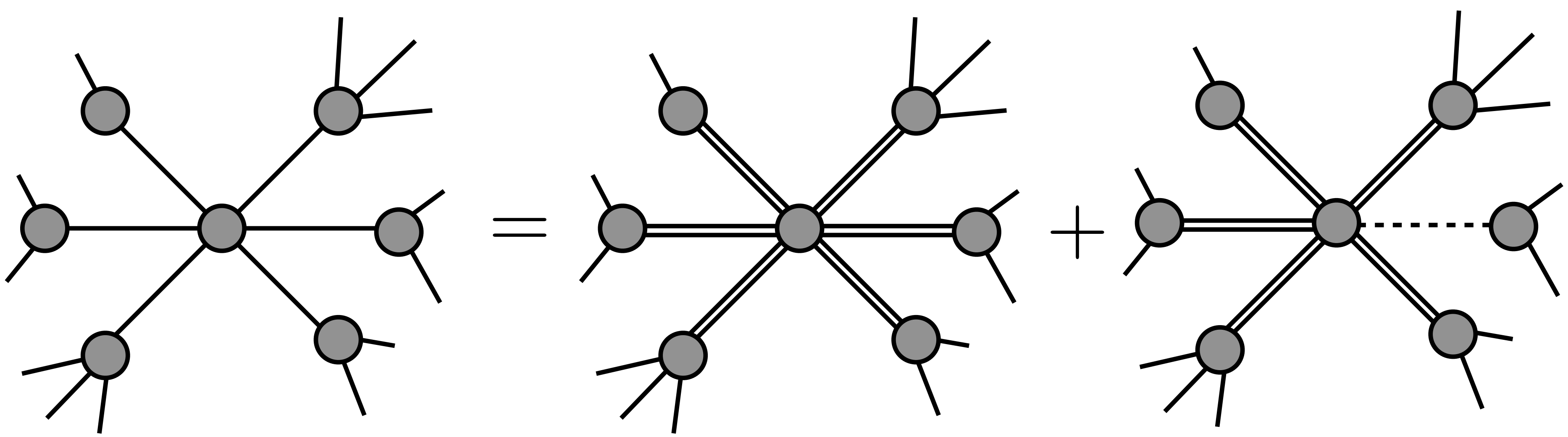

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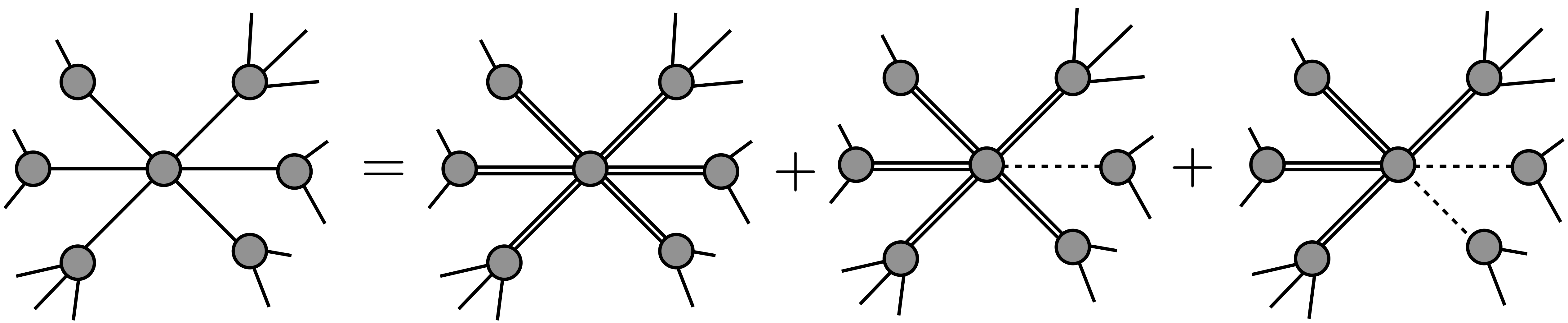

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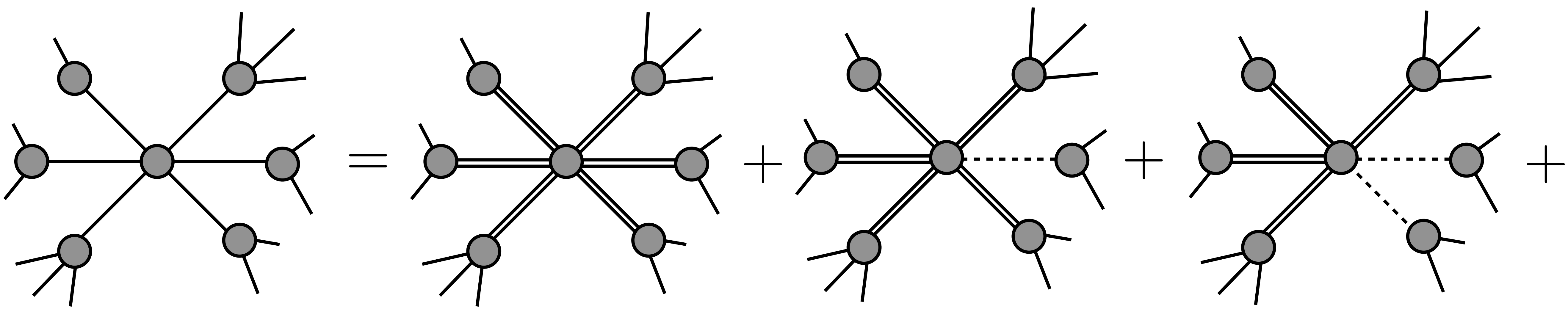

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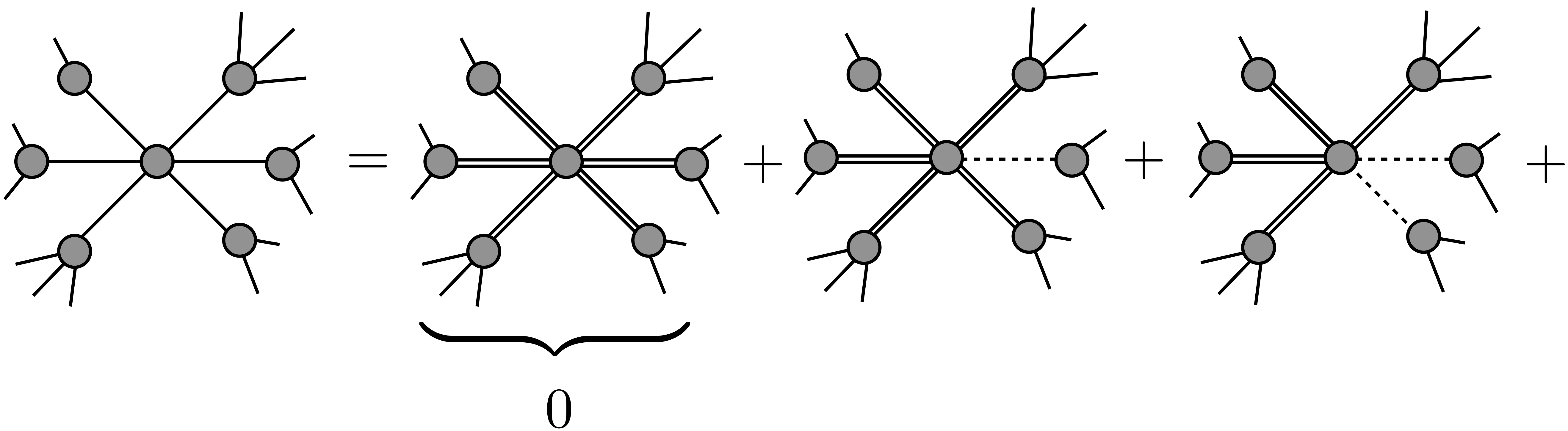

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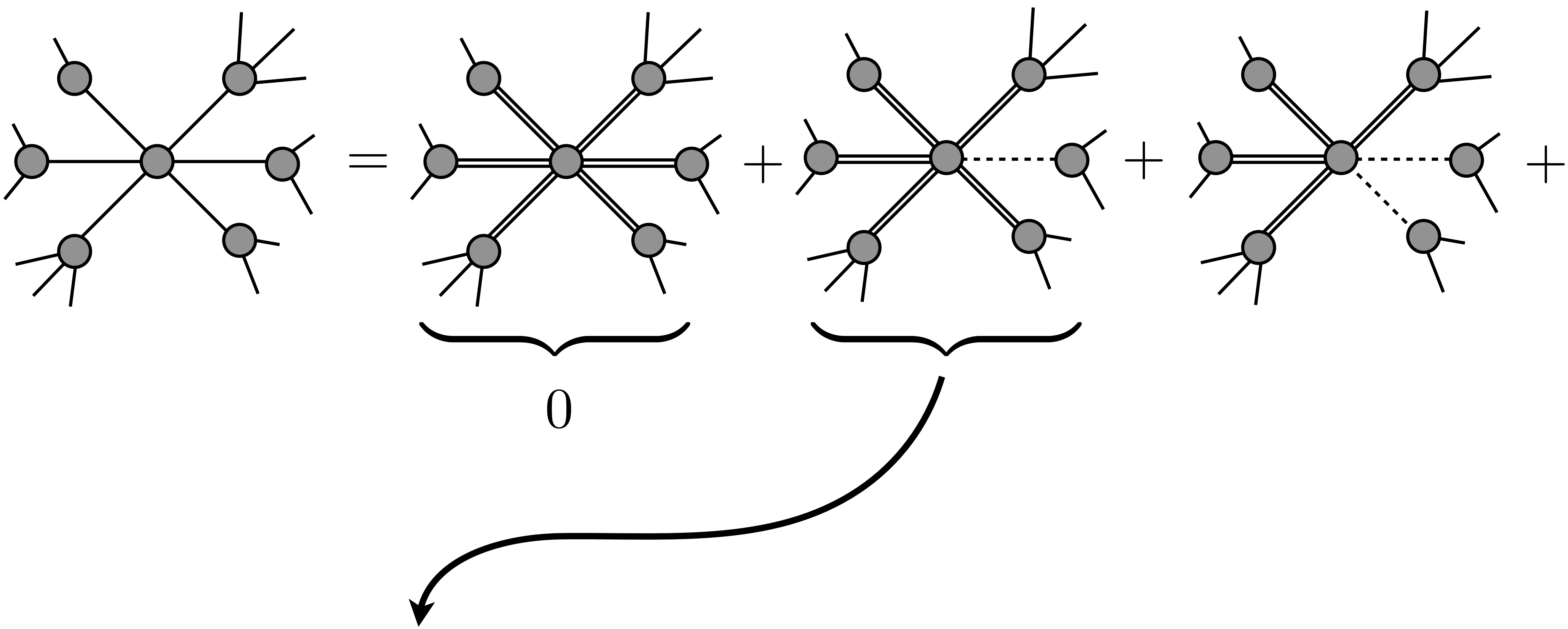

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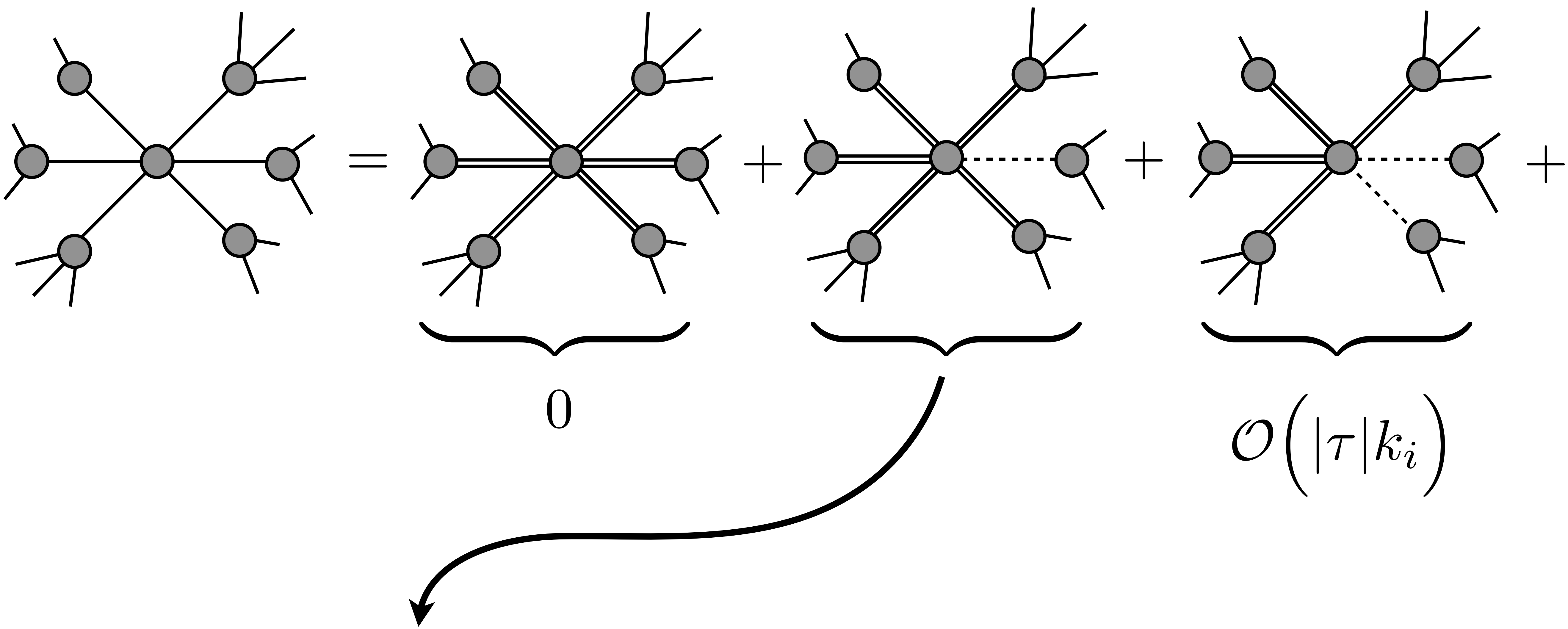

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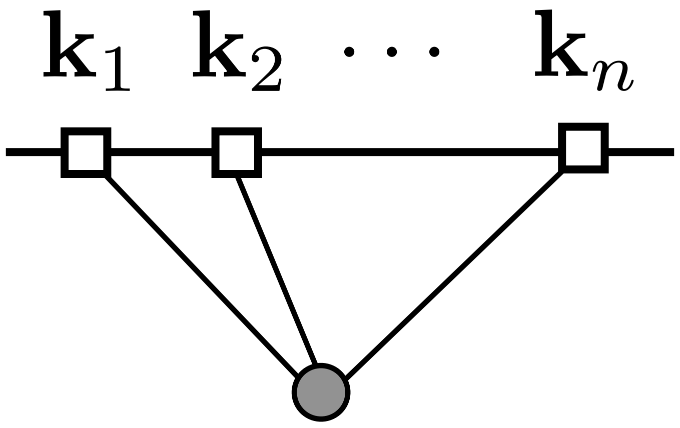
Logarithms in momentum space

Tree-level examples:



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Logarithms in momentum space

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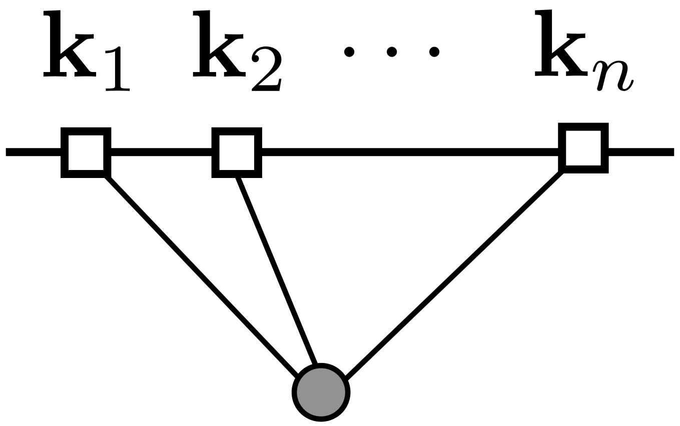

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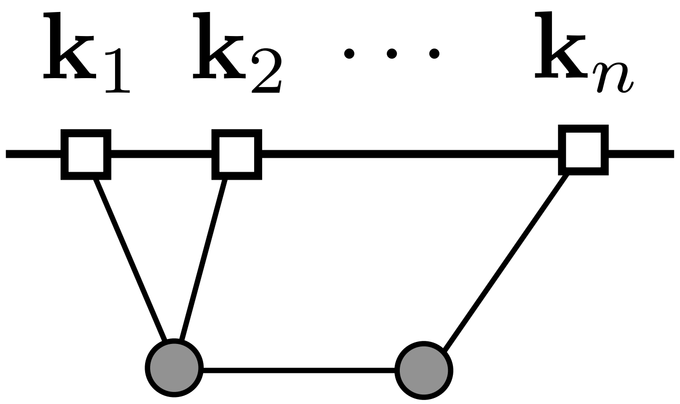

$$\propto \ln\Big[-\tau(k_I+K_1)\Big]\ln\Big[-\tau(k_I+K_2)\Big]$$
$$+\ \ln\Big[-\tau(k_I+K_1)\Big]^2$$
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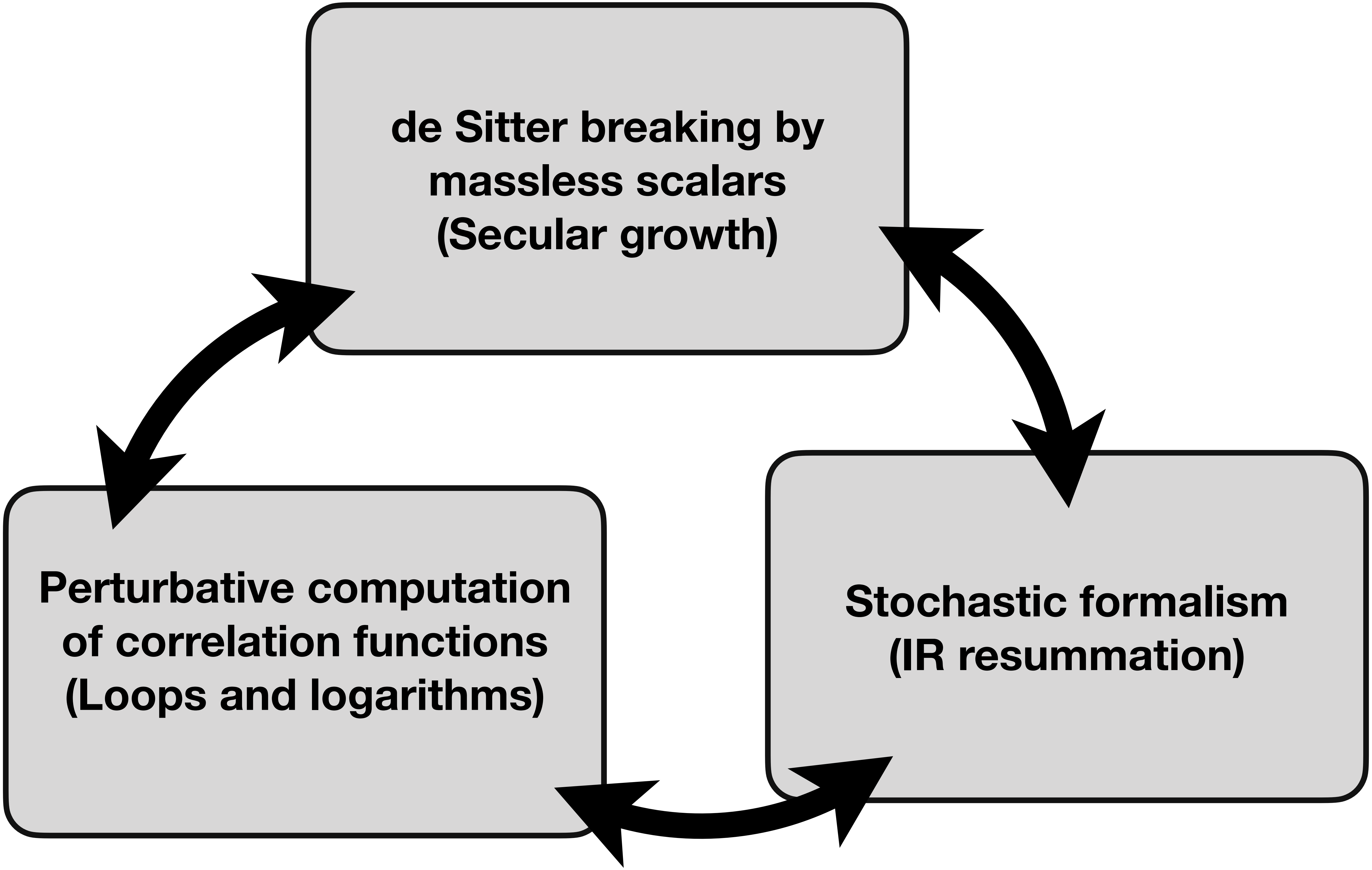
Preamble


de Sitter breaking by
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(Secular growth)
Perturbative computation
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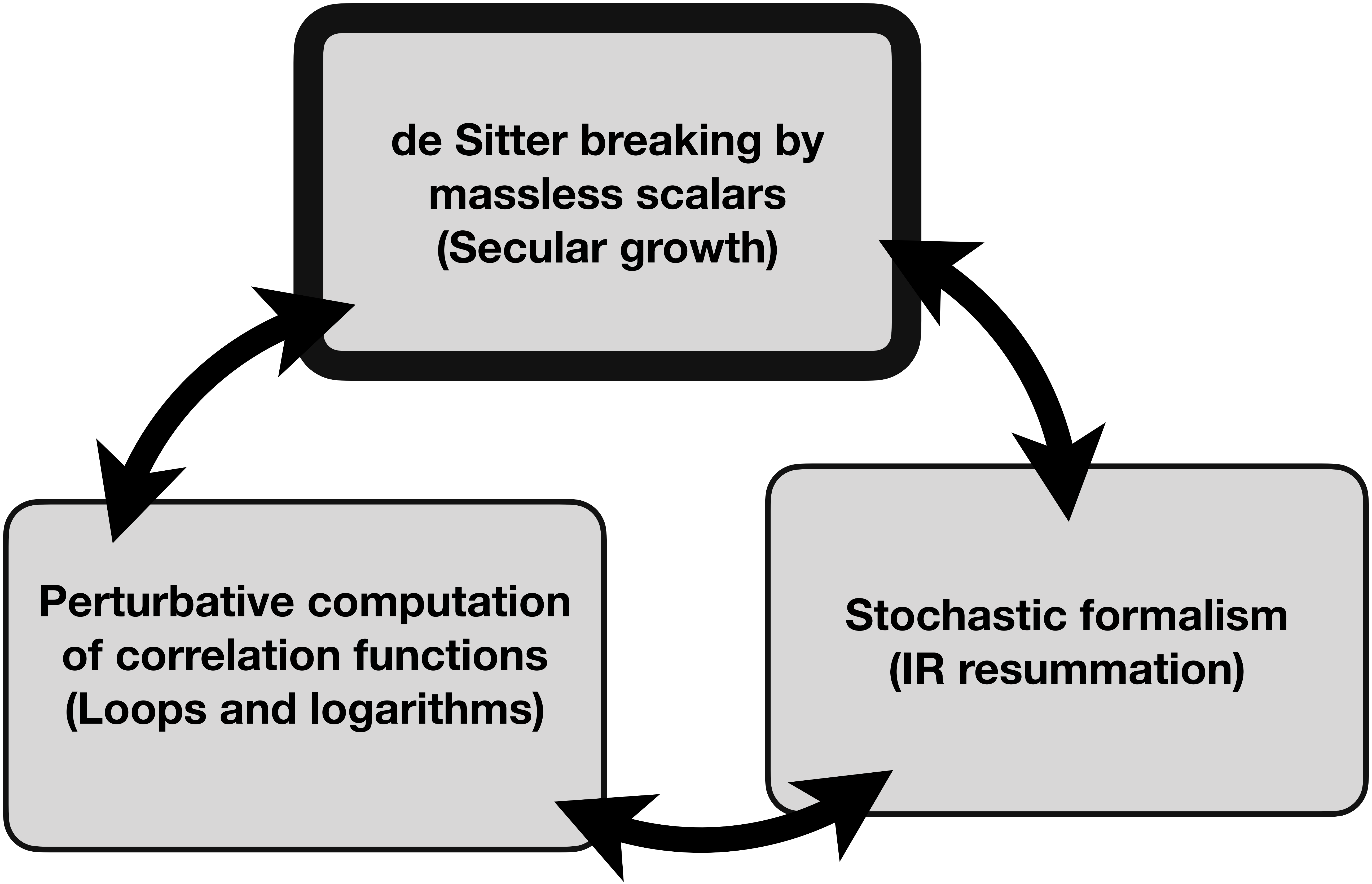
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Massless scalar field theories

Consider the following massless free theory

$$S = \int d^3x d\tau \, a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla \varphi)^2 \right]$$

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BD-vacuum:

$$f_k(\tau) = \frac{iH}{\sqrt{2k^3}} \left[1 + ik\tau \right] e^{-ik\tau}$$

Massless scalar field theories

Let's compute the two-point correlation function:

$$G(|\mathbf{x}-\mathbf{x}'|;\tau,\tau') = \langle\Omega|\varphi(\mathbf{x},\tau)\varphi(\mathbf{x}',\tau')|\Omega\rangle$$

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Allen (1985)

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IR divergence

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Secular growth

IR divergence

Allen (1985)

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Secular growth

You would have obtained the same result with a co-moving cutoff !

IR divergence

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Let me give a step back. What is the origin of this growth?

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Shift symmetry

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Expand in terms of $\epsilon \equiv 1 - e^{-\theta}$ **to all orders and integrate by parts**

$$I(e^{-\theta} s) = I(s) + \theta F(0)$$

Massless scalar field theories

Thus, without solving the integral we see that it is not dS invariant

$$G(|\bar{\mathbf{x}} - \bar{\mathbf{x}}'|; \bar{\tau}, \bar{\tau}')$$

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But recall that this feature is due to the shift symmetry

Massless scalar field theories

Back to Allen's result:

$$G(|\mathbf{x}-\mathbf{x}'|;\tau,\tau') = \frac{H^2}{8\pi^2}\left[\frac{1}{1-Z} - \ln(1-Z) + \ln a(\tau) + \ln a(\tau') + \ln\frac{1}{0}\right]$$

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$$G_{\mu\nu'}(x,x') = \partial_\mu \partial_{\nu'} \langle \varphi(x)\varphi(x')\rangle$$

See for instance Tolley & Turok (2001)

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(No secular growth here)

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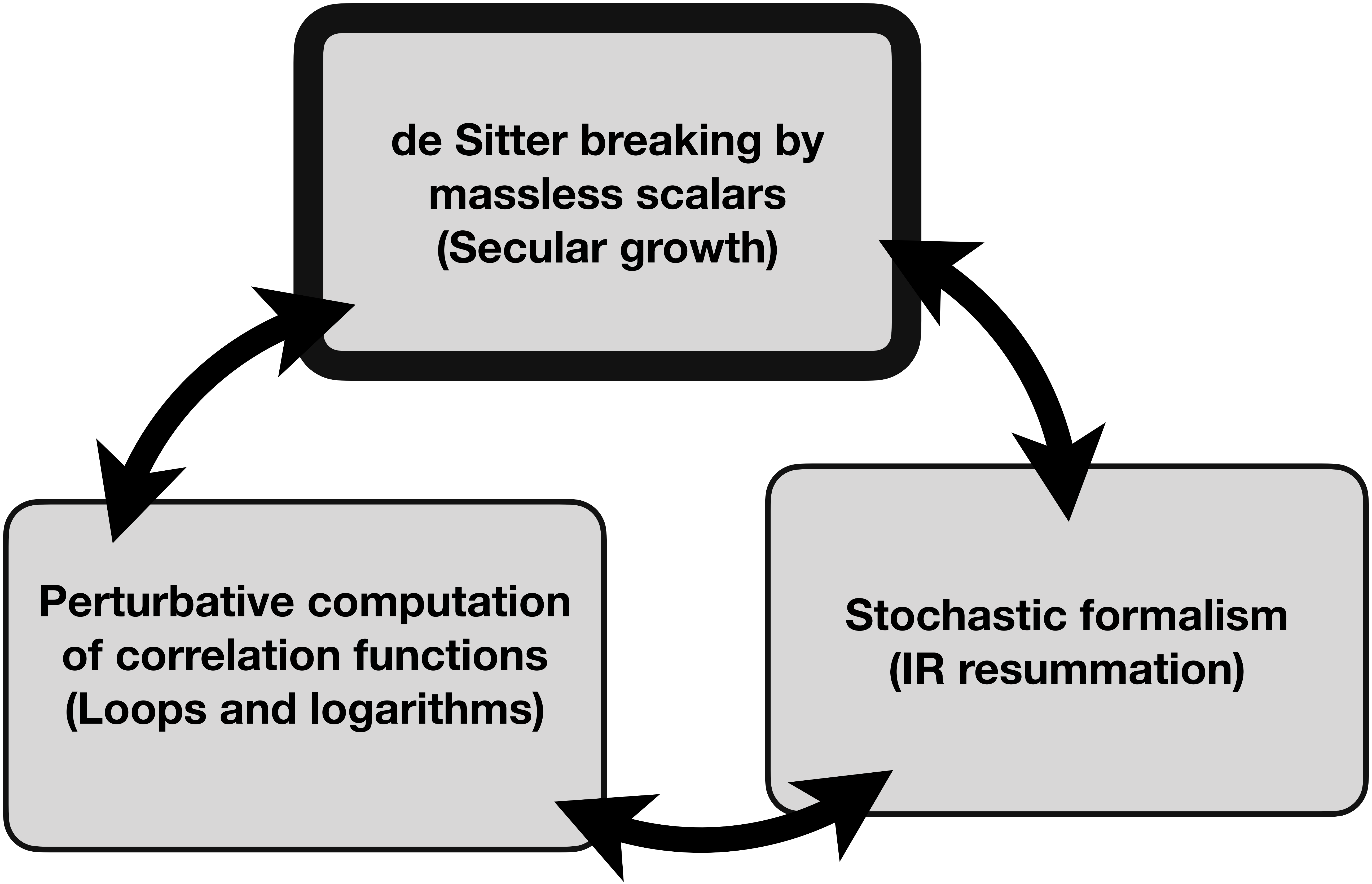

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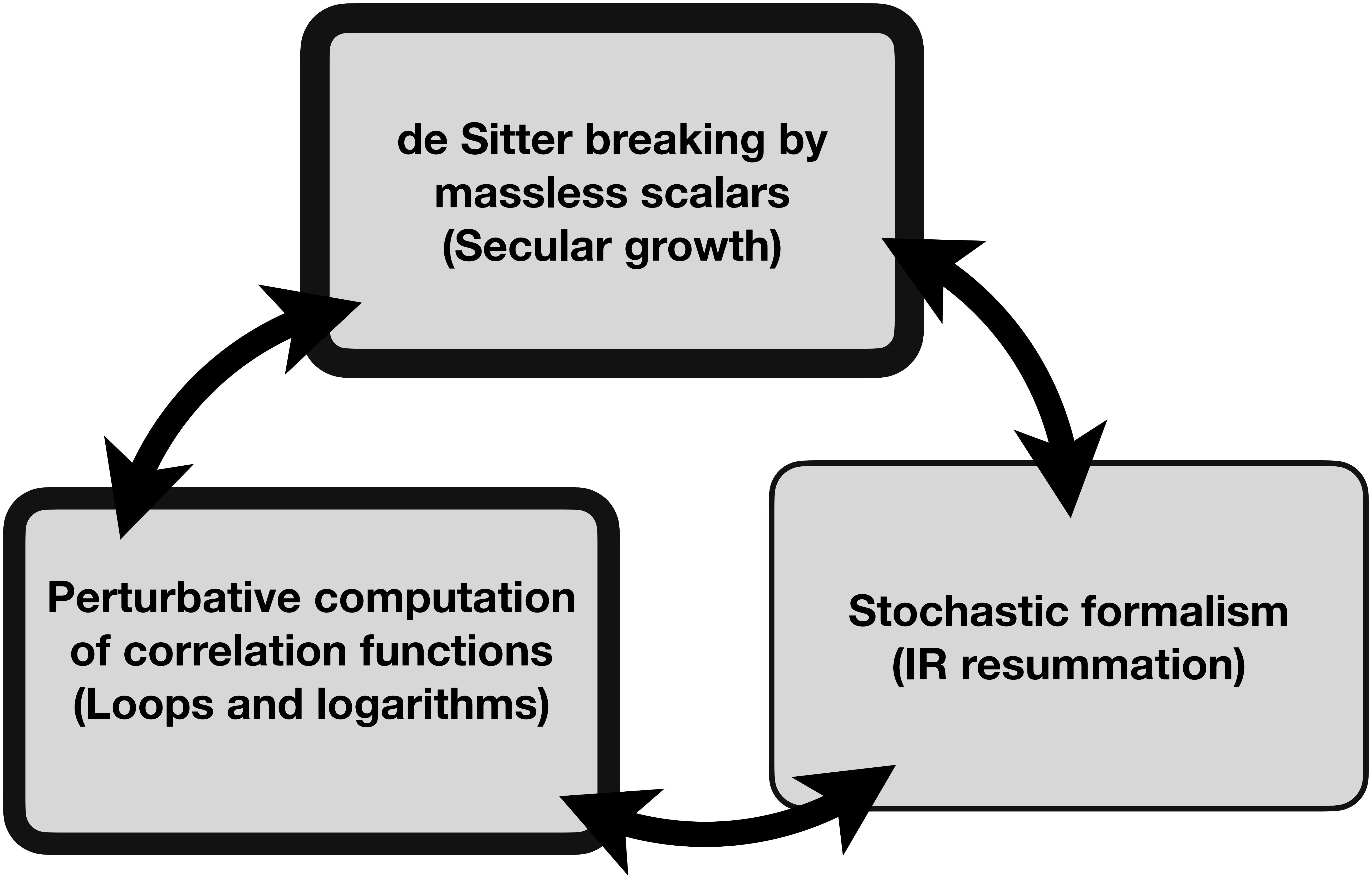
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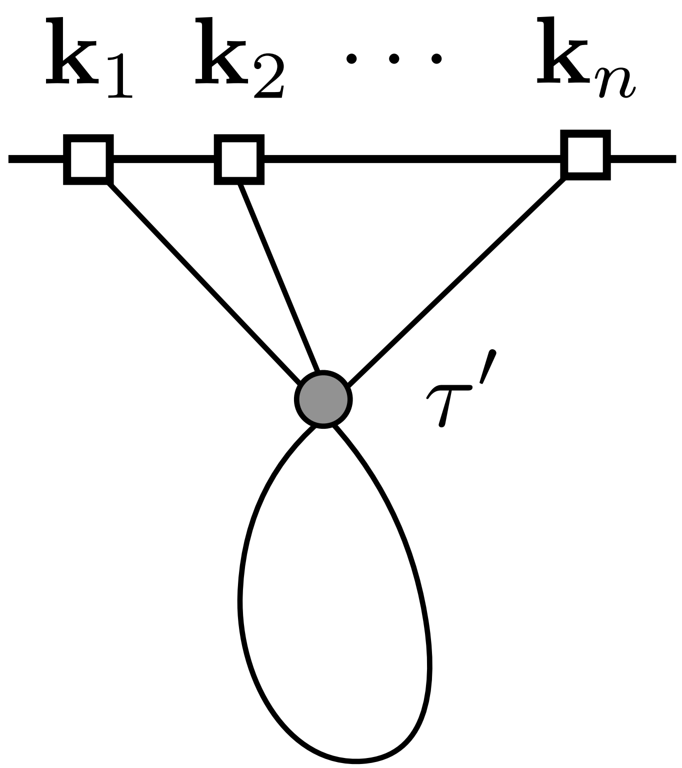

Correlators in dS

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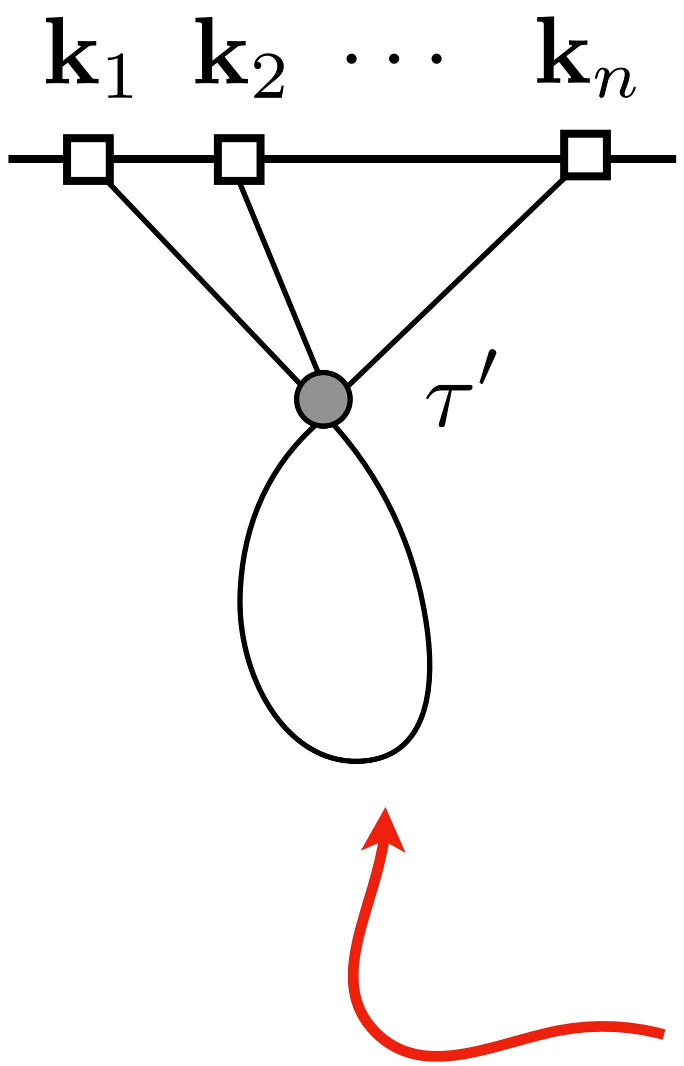

$$G(0;\tau',\tau') = \frac{H^2}{4\pi^2}\left[\ln a(\tau') + \ln\frac{1}{0}\right]$$

Correlators in dS

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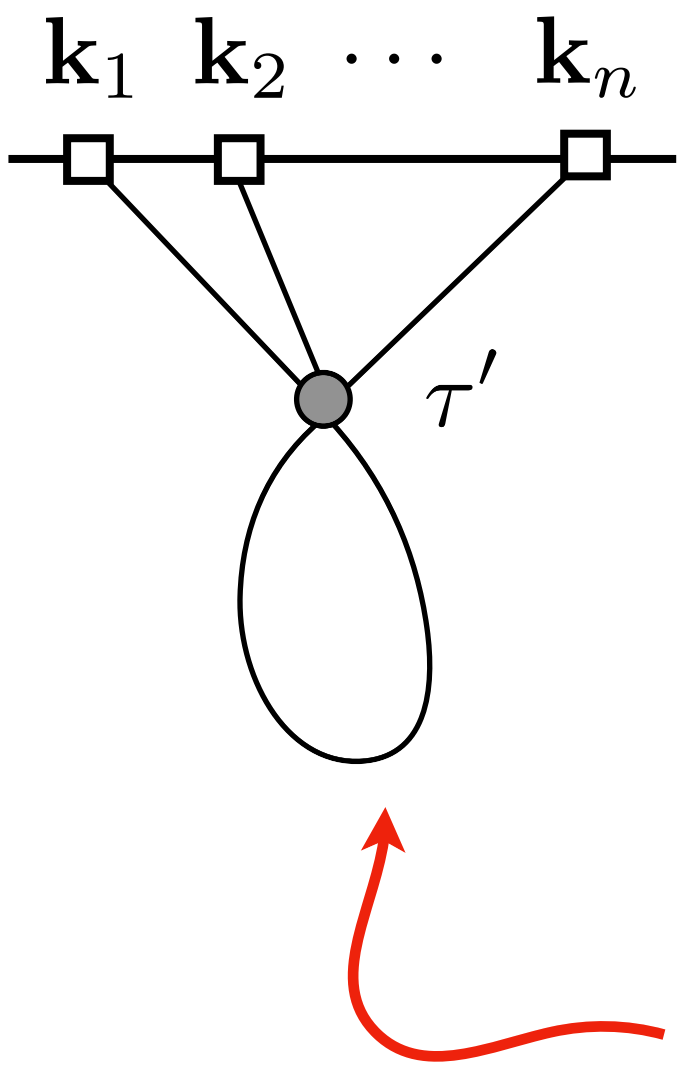

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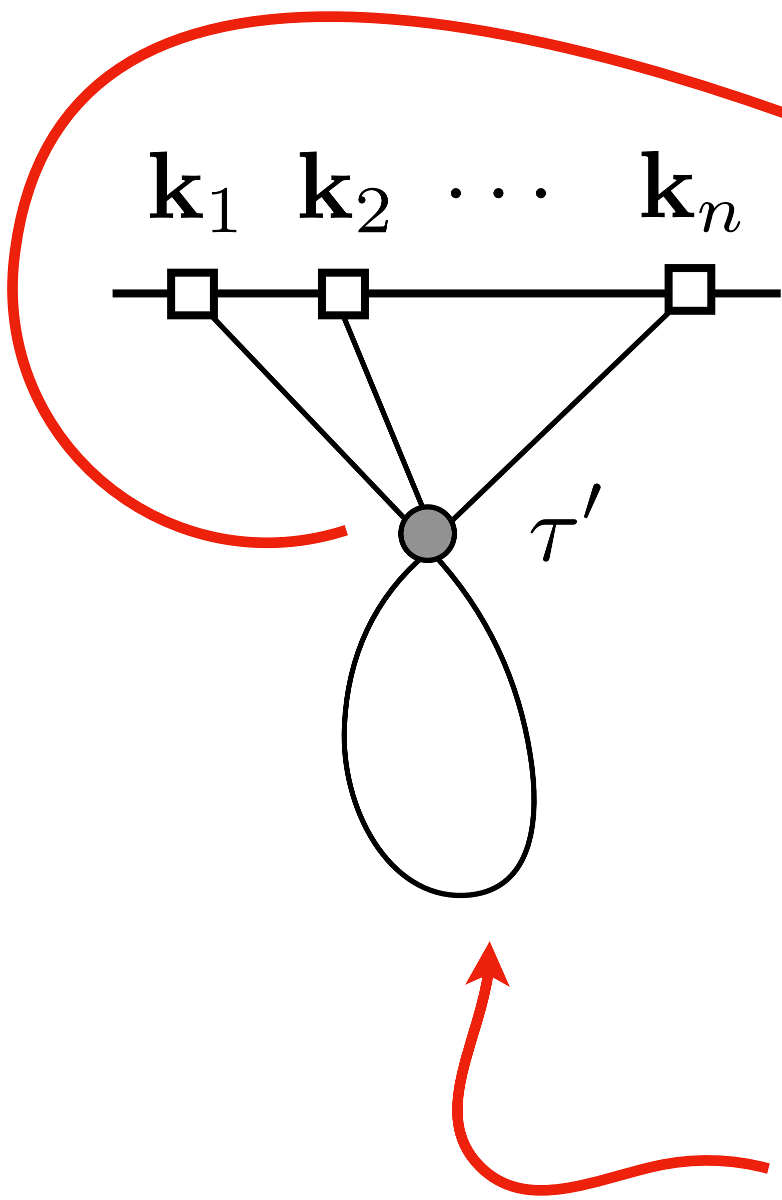

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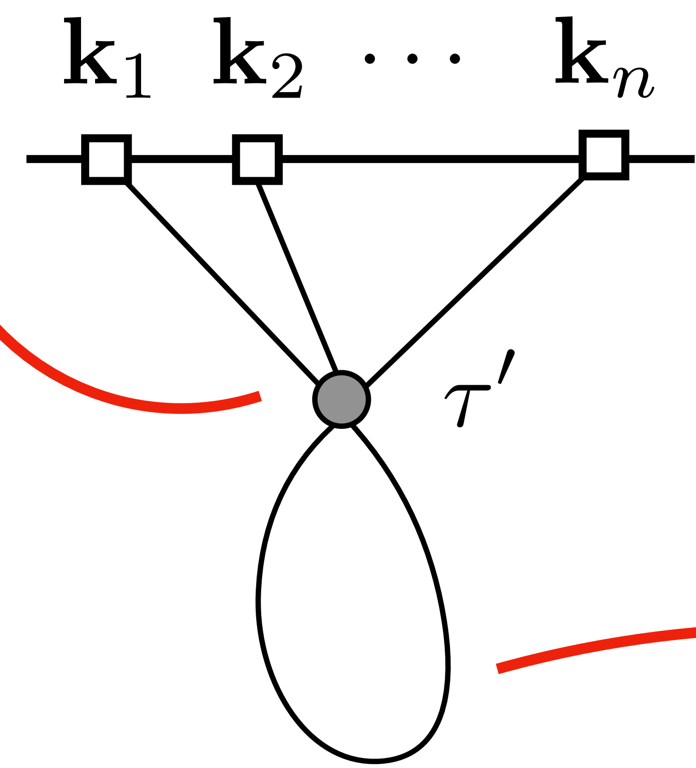

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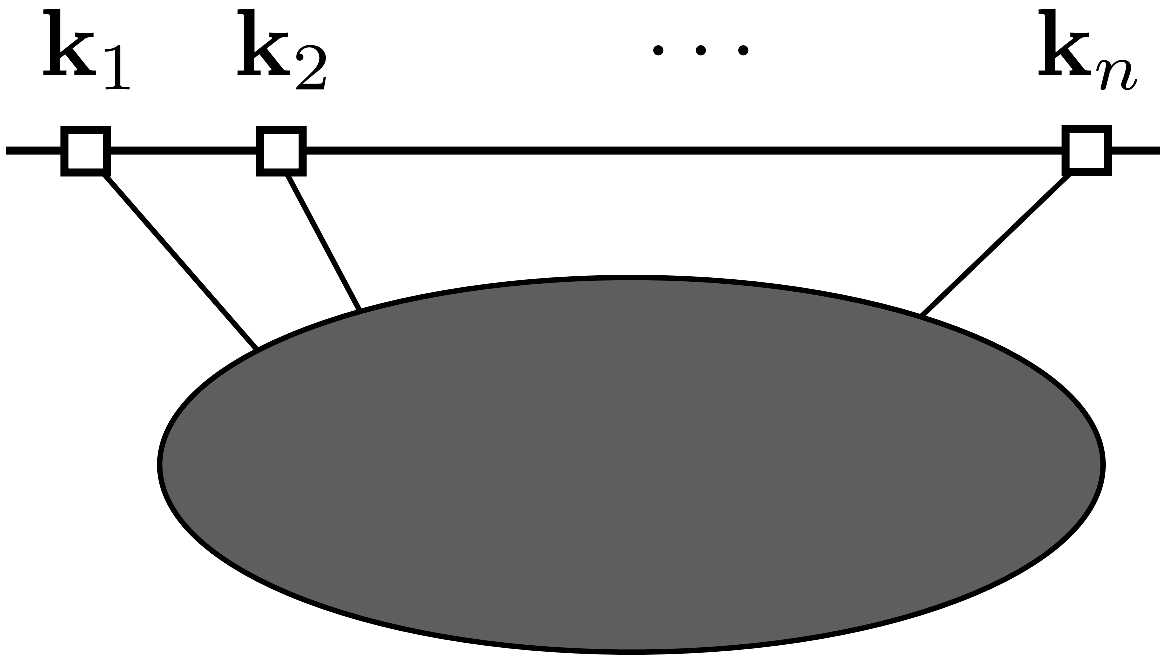

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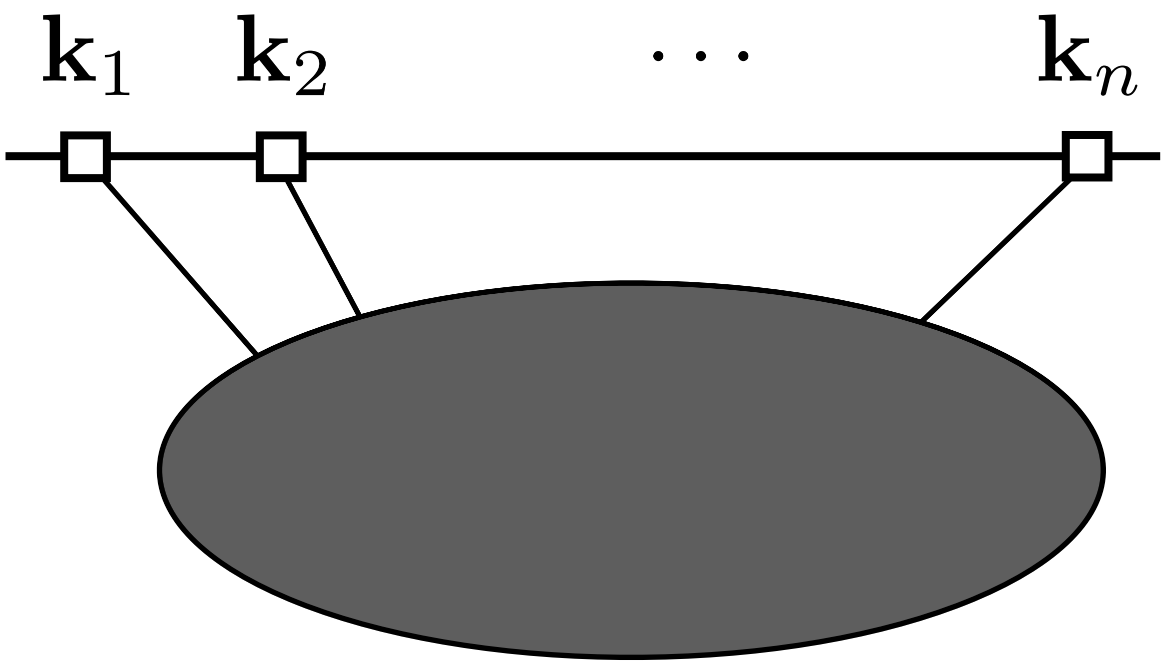

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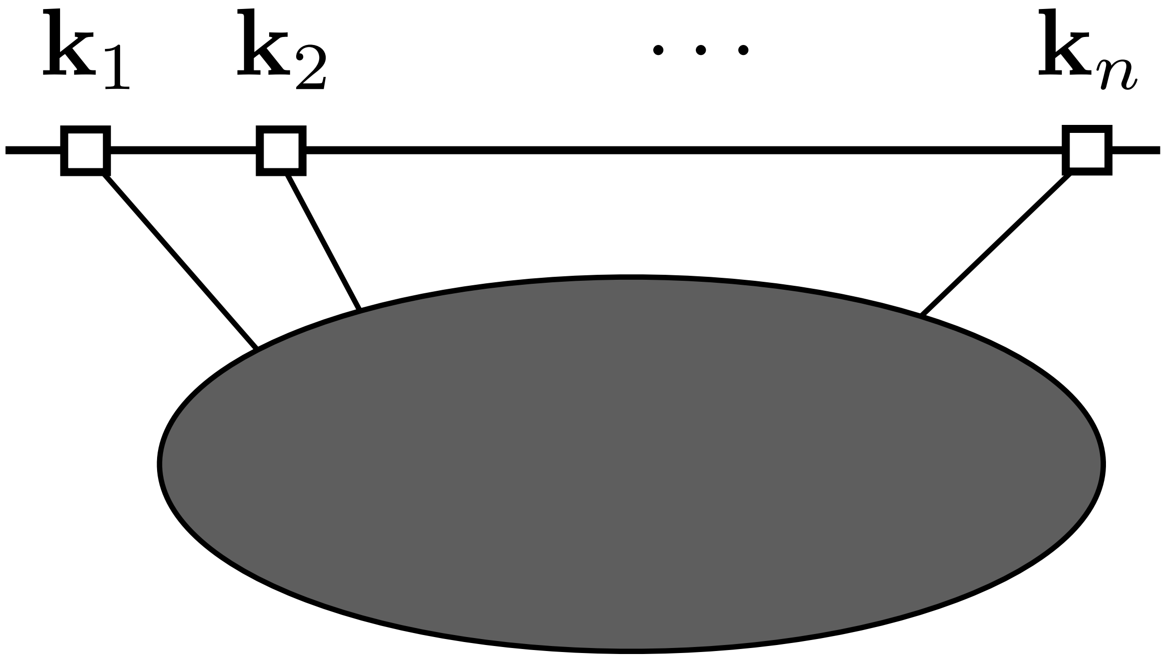

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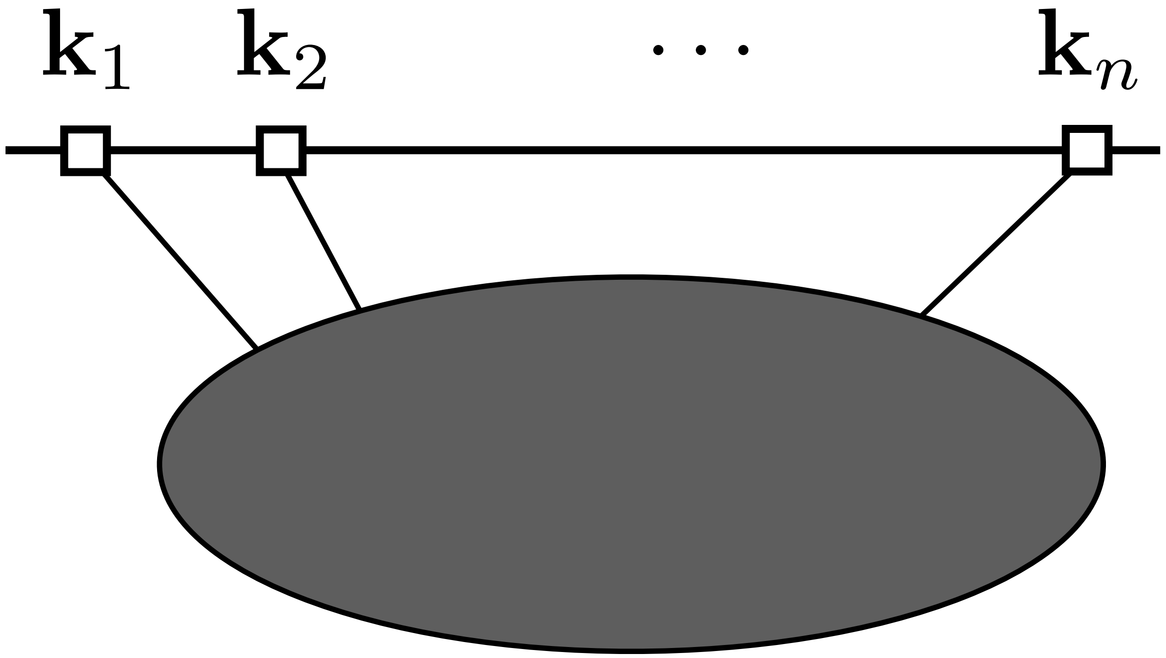

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- Now you have de Sitter invariant secular growth (from vertices) and de Sitter breaking secular growth (from loops)
- However, on top of giving you secular growth, loops continue to imply IR divergences coming from integrals
- To trust computations in the limit $k_i|\tau| \to 0$ you have to find a way to resume all of these IR contributions

Correlators in dS

Is it possible to treat the divergences in a controlled way, such that only the secular growth needs to be addressed?

Baumgart & Sundrum (2019)

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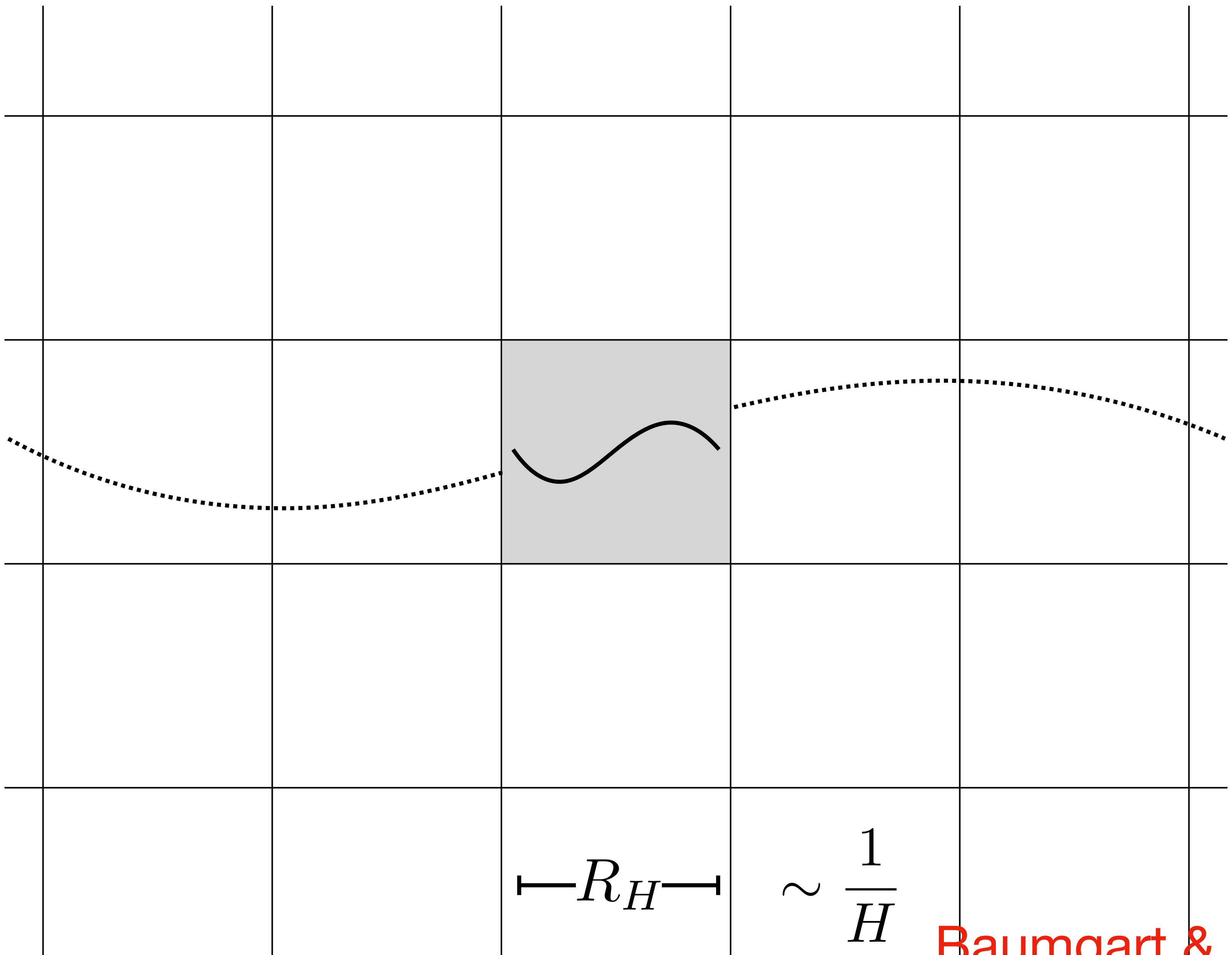

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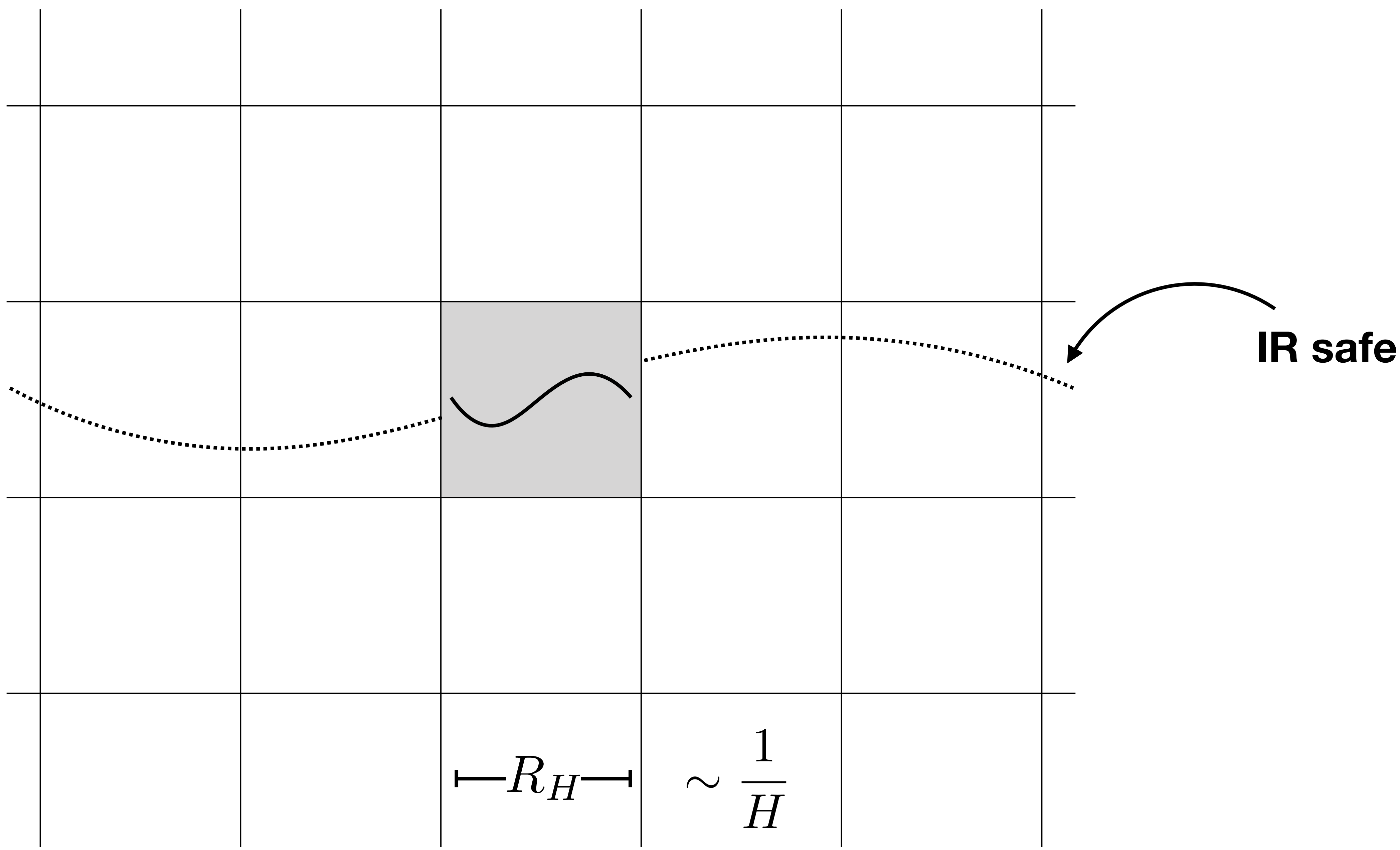
Correlators in dS

Let's say inflation is preceded by an (IR finite) radiation dominated era

$$\tau = \tau_0$$



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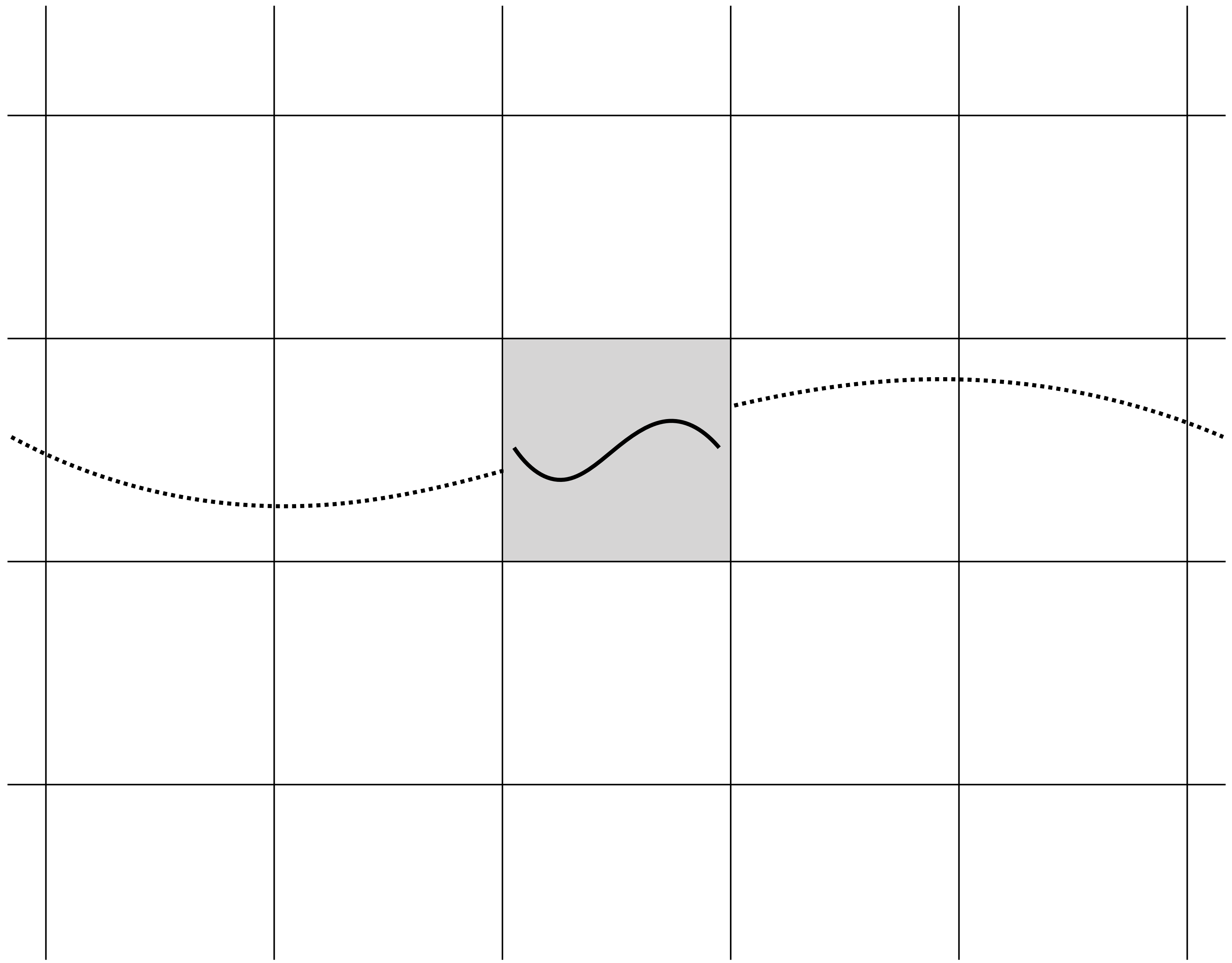

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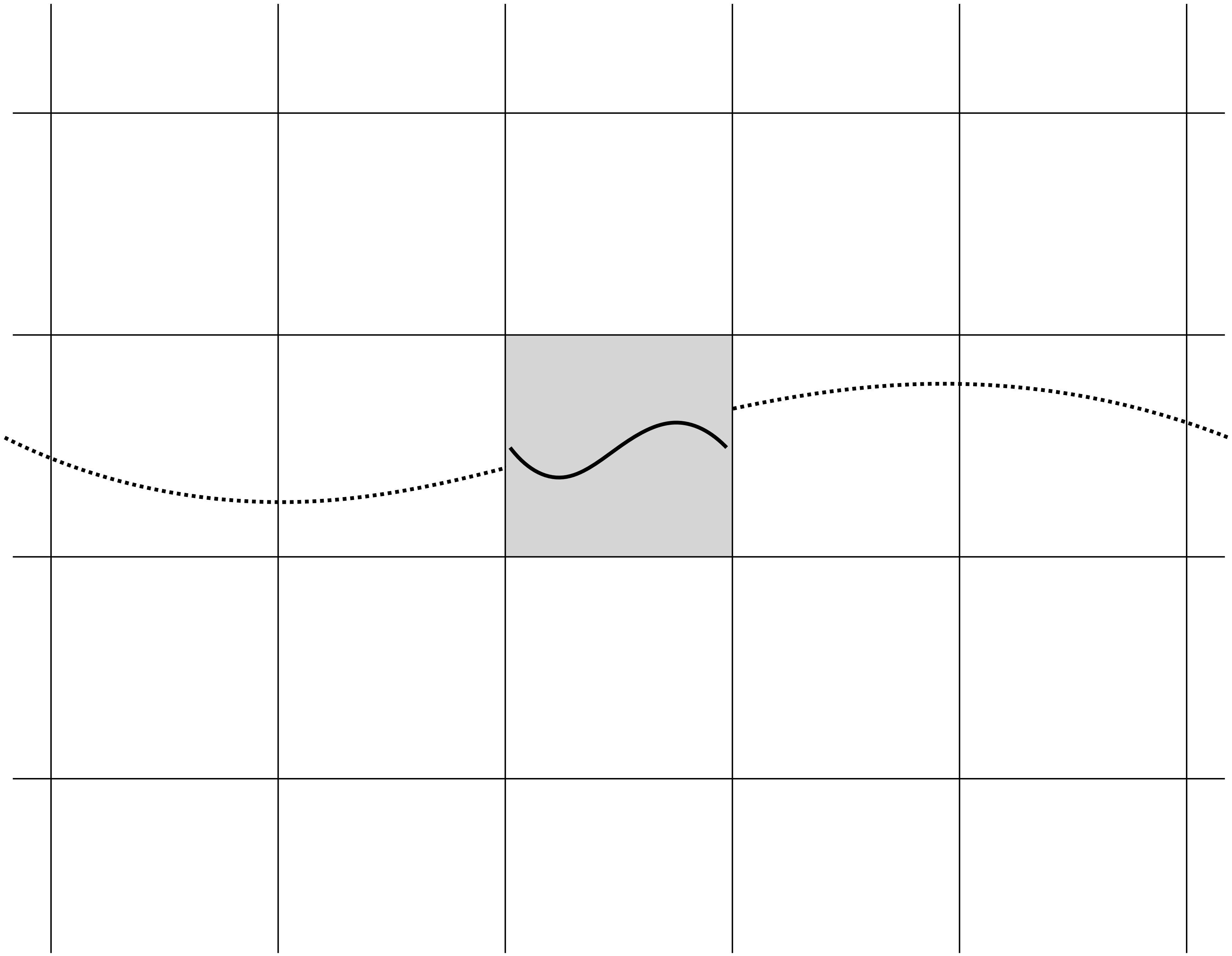

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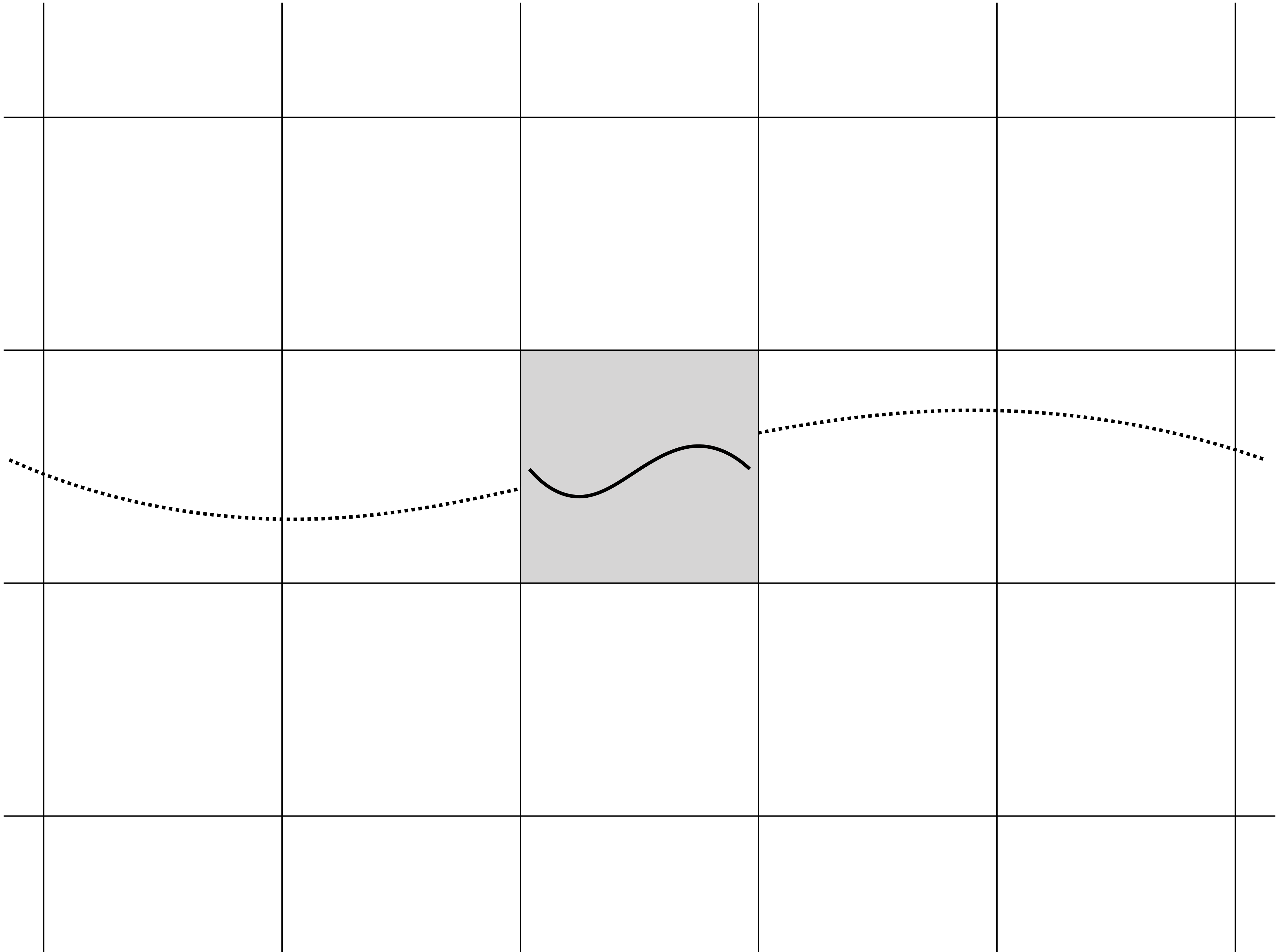

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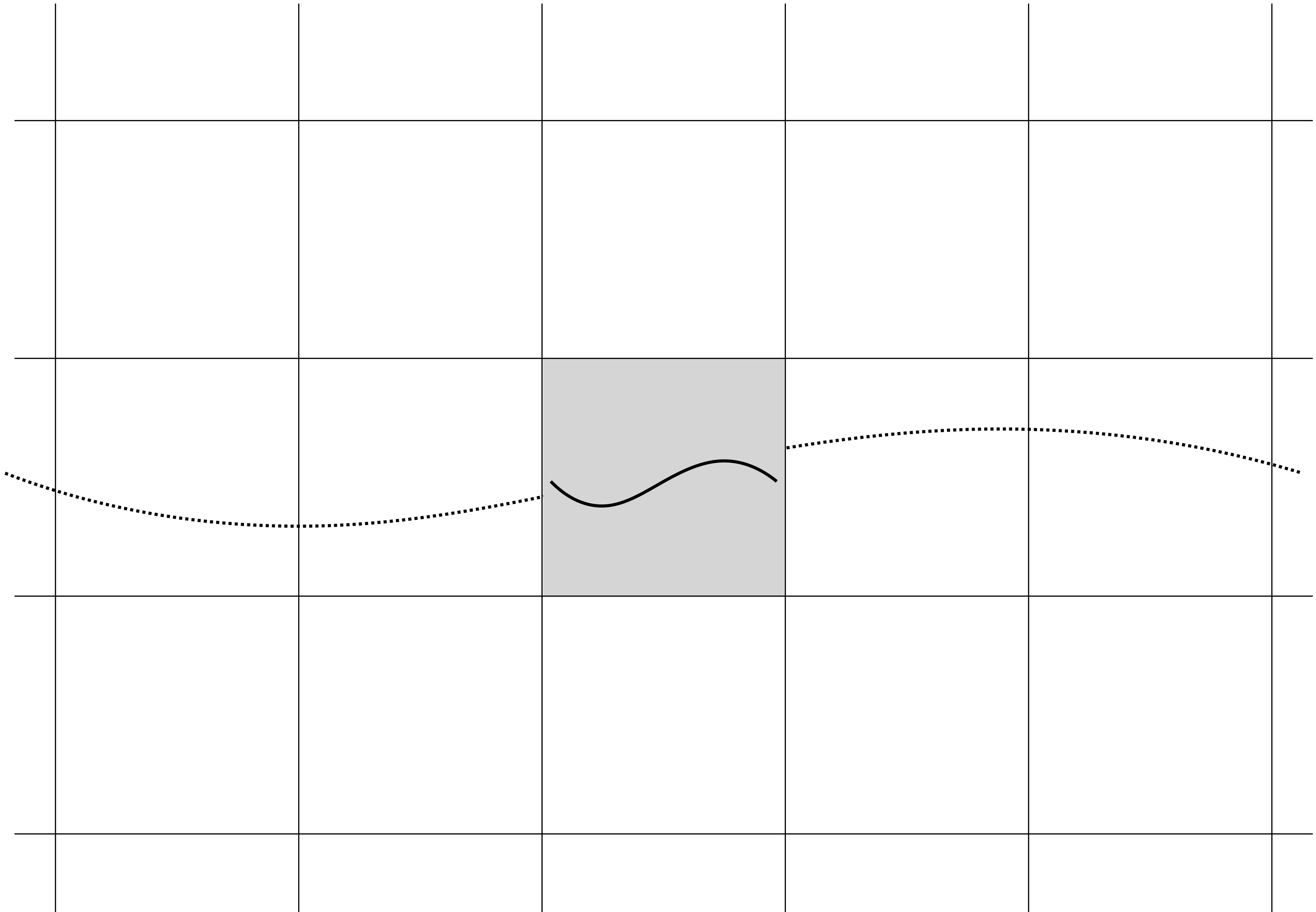

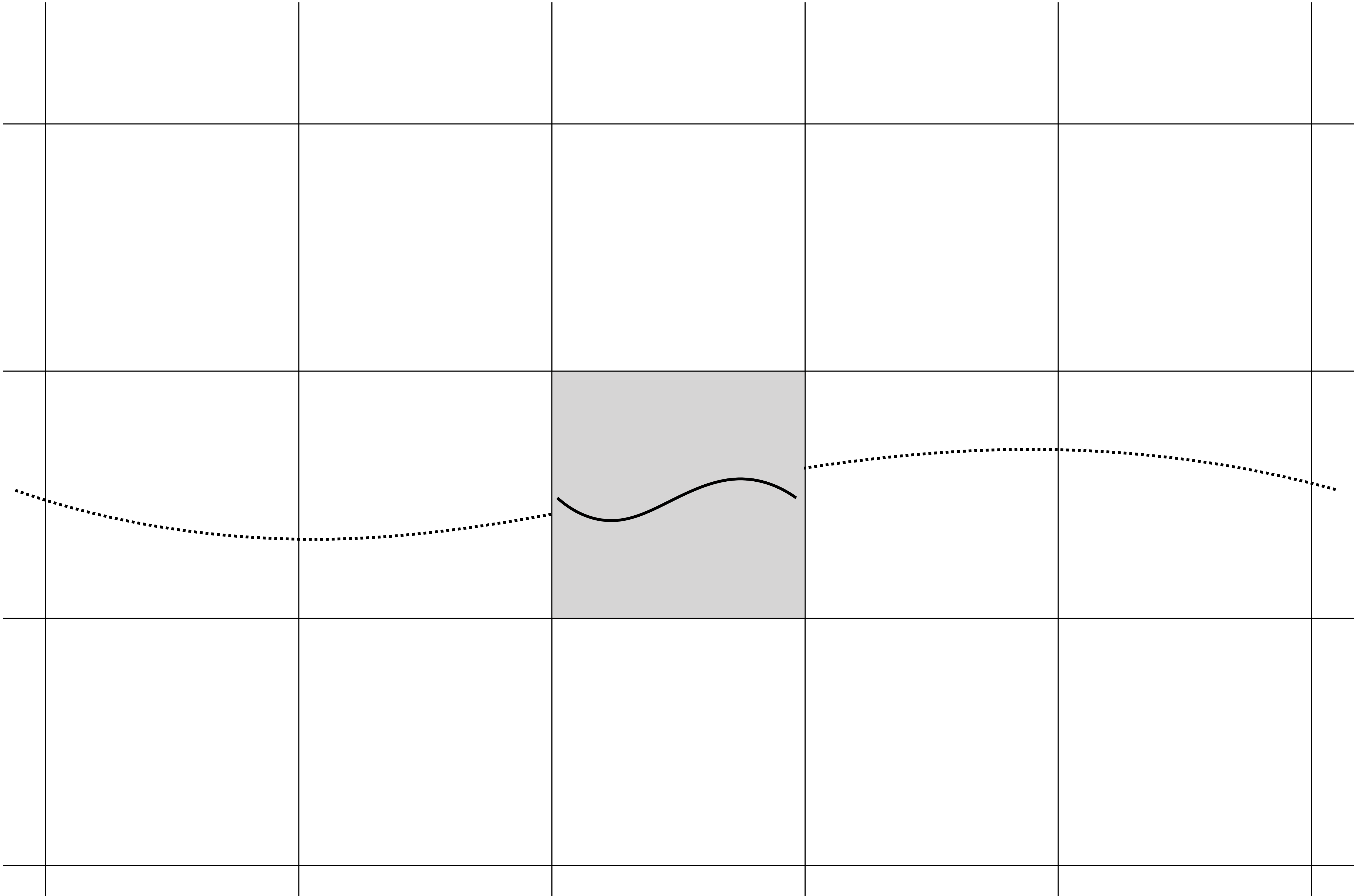

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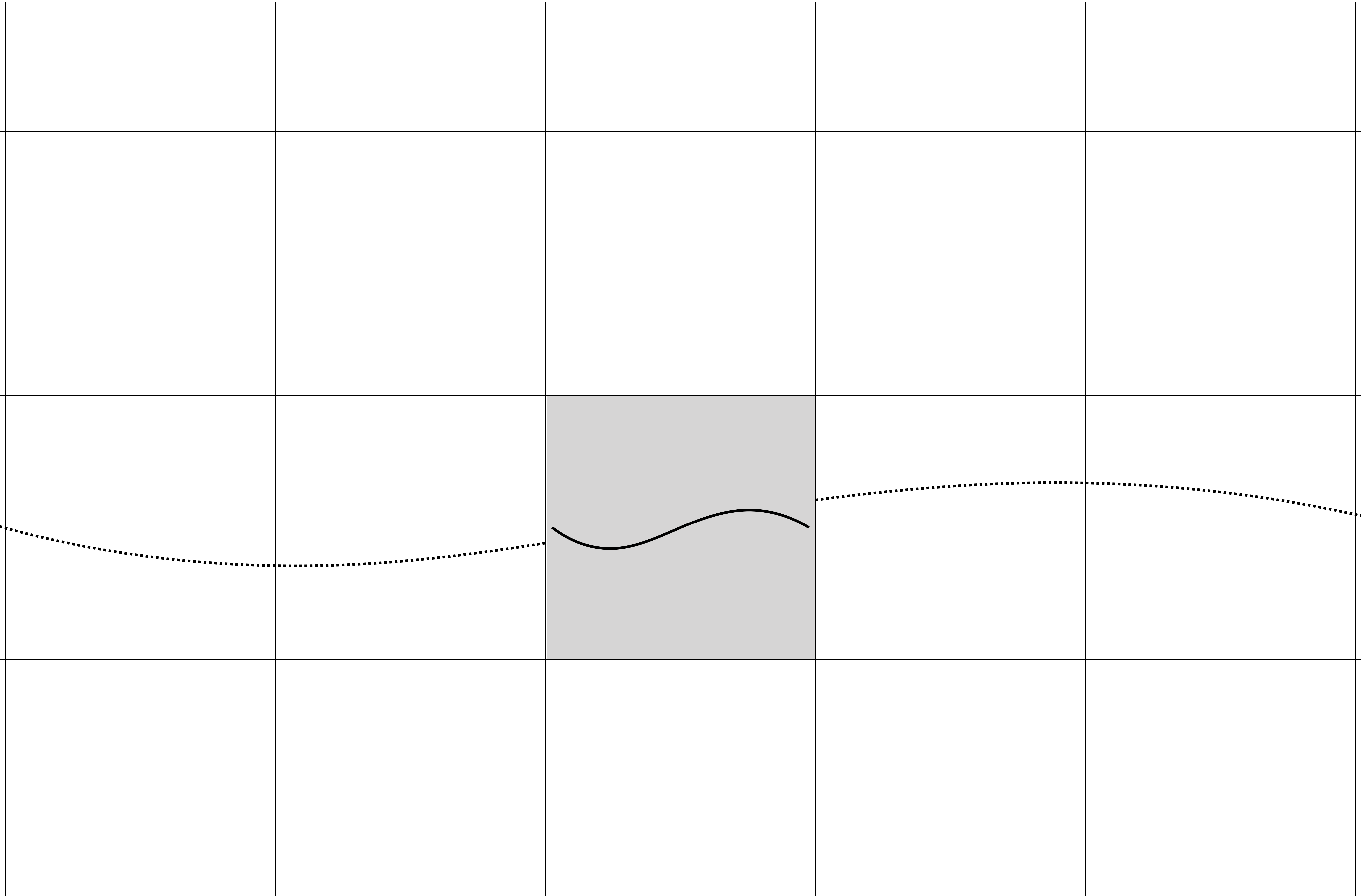

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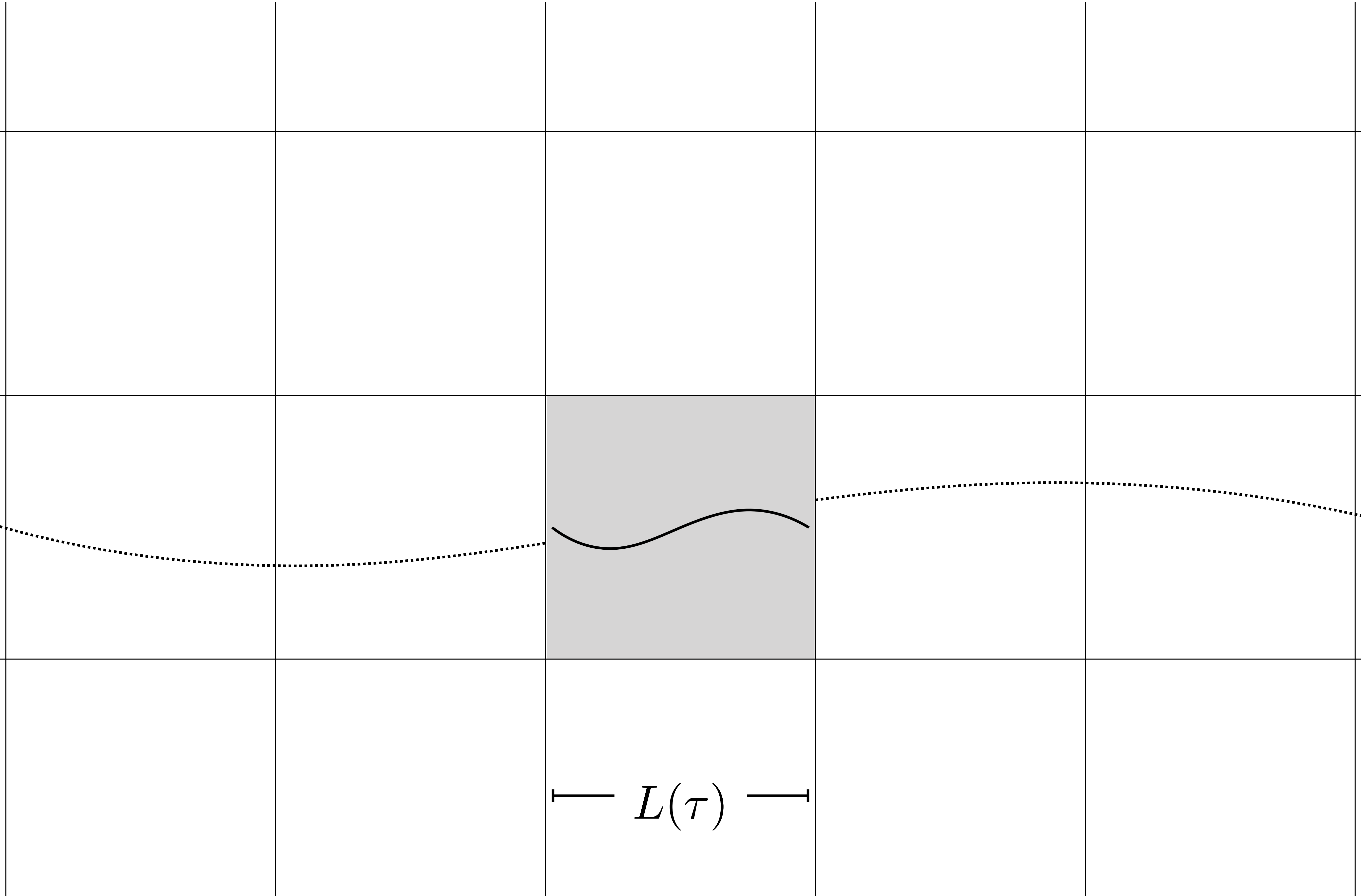

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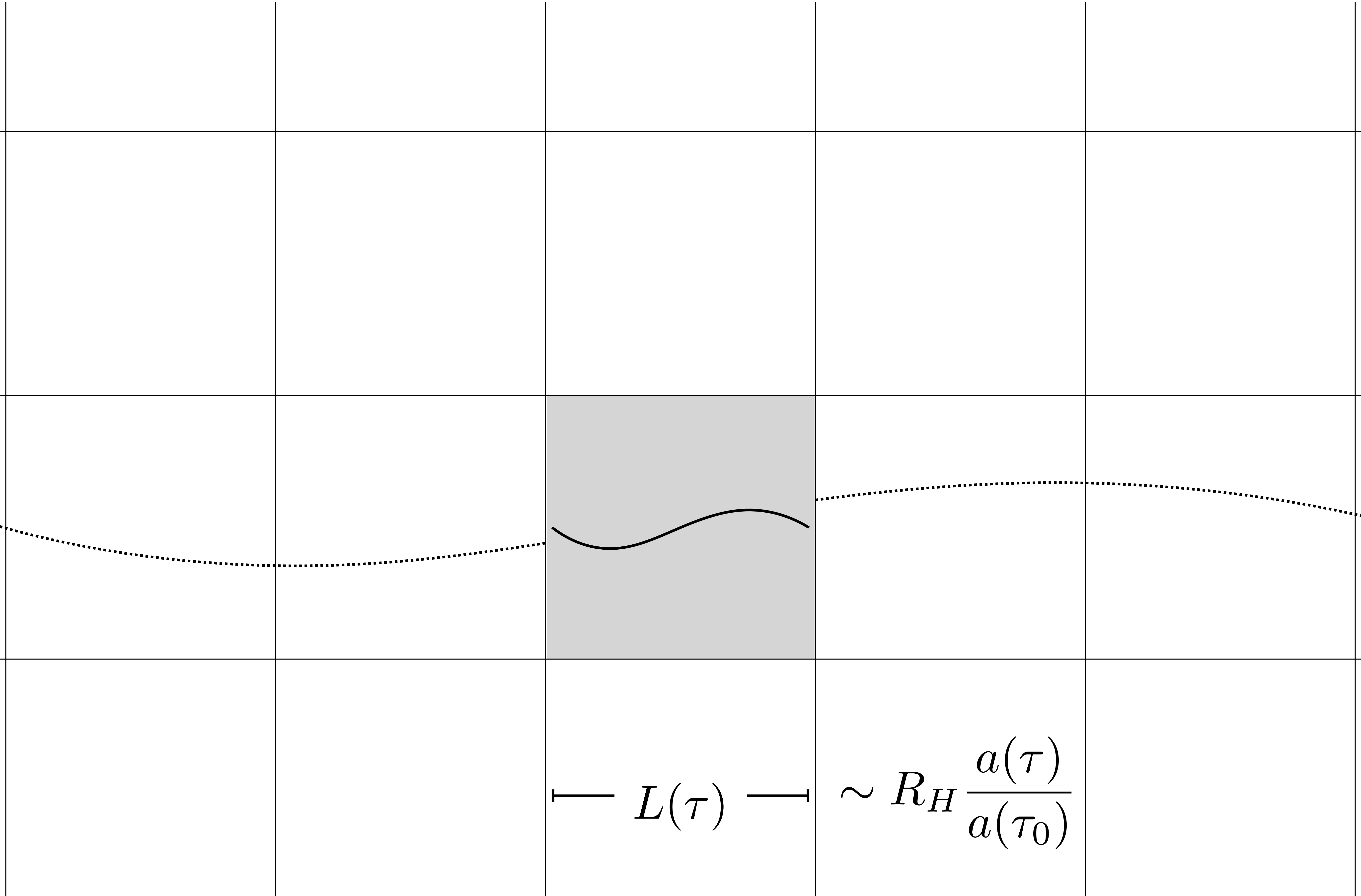

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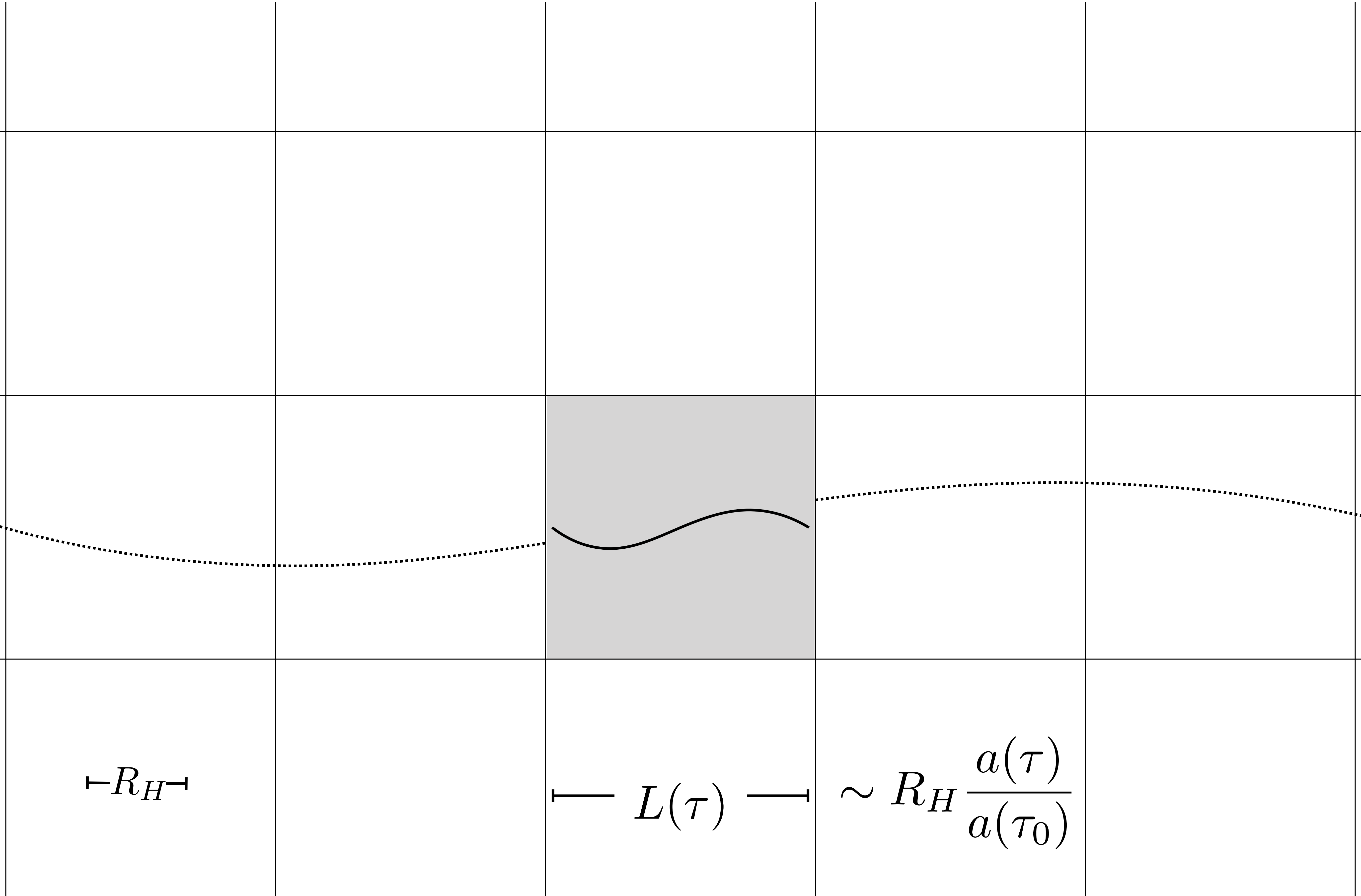

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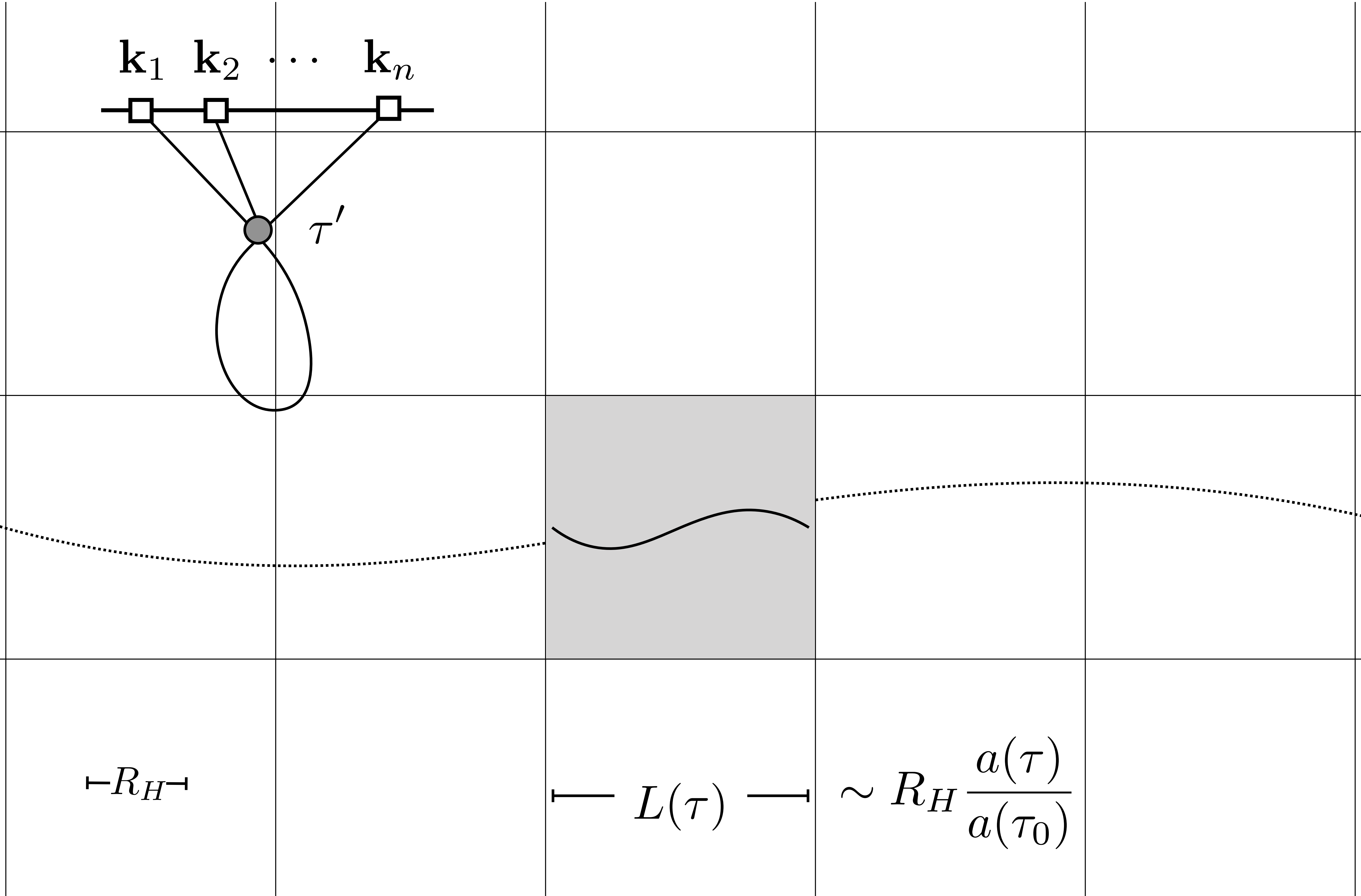

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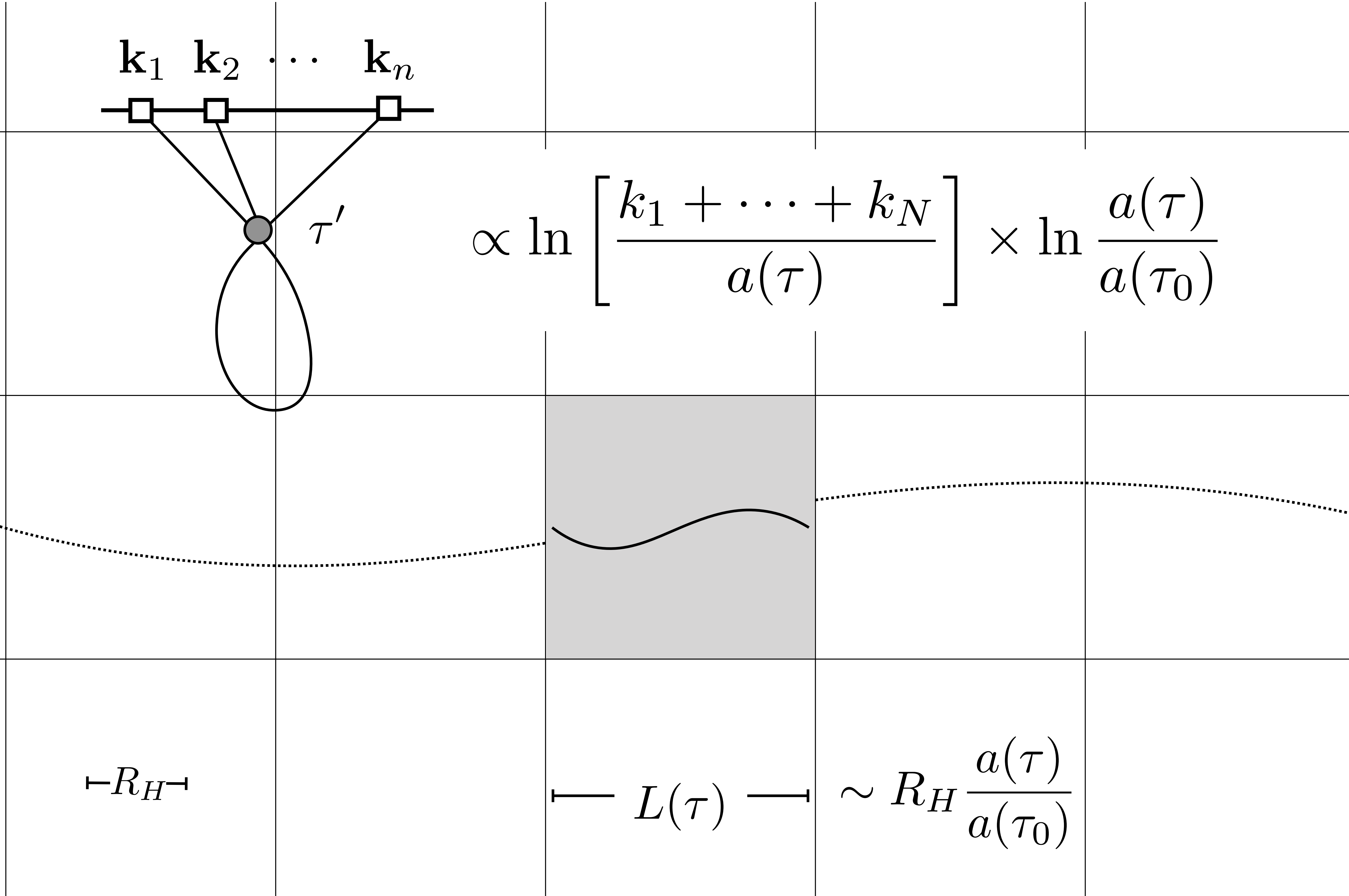

Light scalar field theories

What about theories with non-derivative interactions?

$$S = \int d^3x d\tau\, a^4(\tau) \left[\frac{1}{2a^2(\tau)} \dot{\varphi}^2 - \frac{1}{2a^2(\tau)} (\nabla\varphi)^2 - \mathcal{V}(\varphi) \right]$$

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- **Therefore, one should not trust the free theory all the down up** $k = 0$

- **There must exist a lengthscale** $\Lambda_{\rm IR}^{-1}$ **beyond which the evolution becomes strongly nonlinear**

$$k/a(\tau) \quad < \quad \Lambda_{\rm IR}$$

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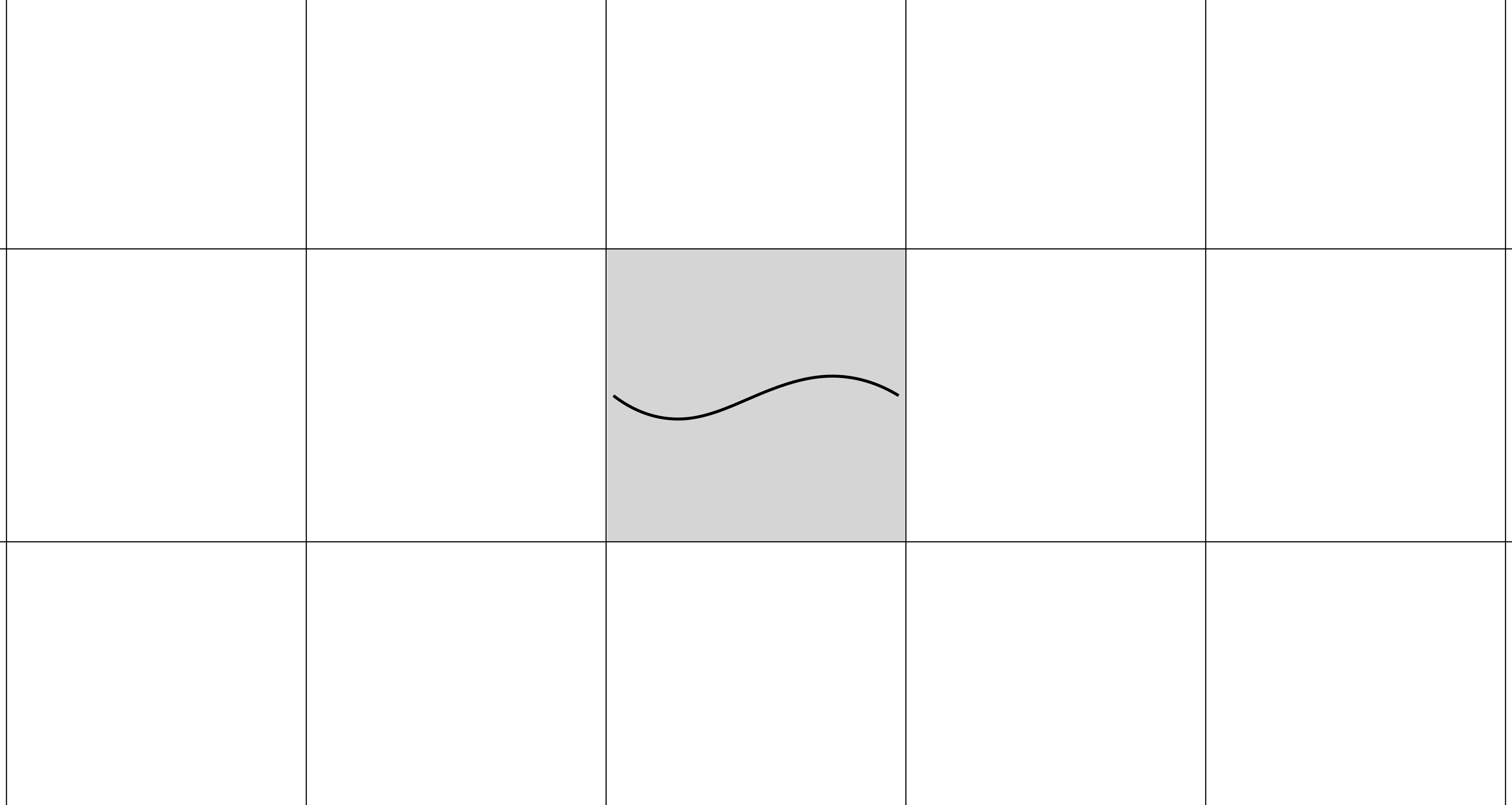

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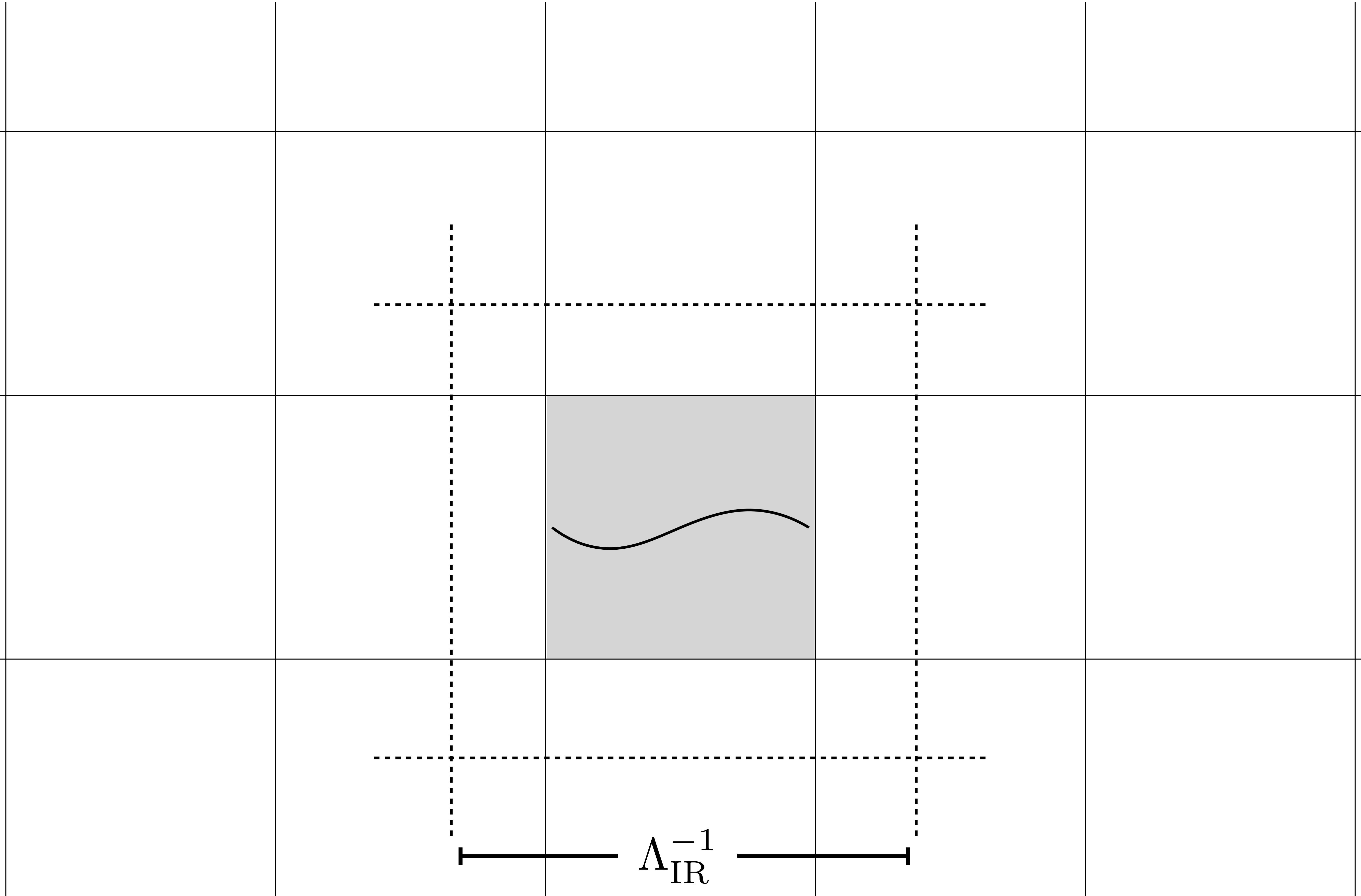

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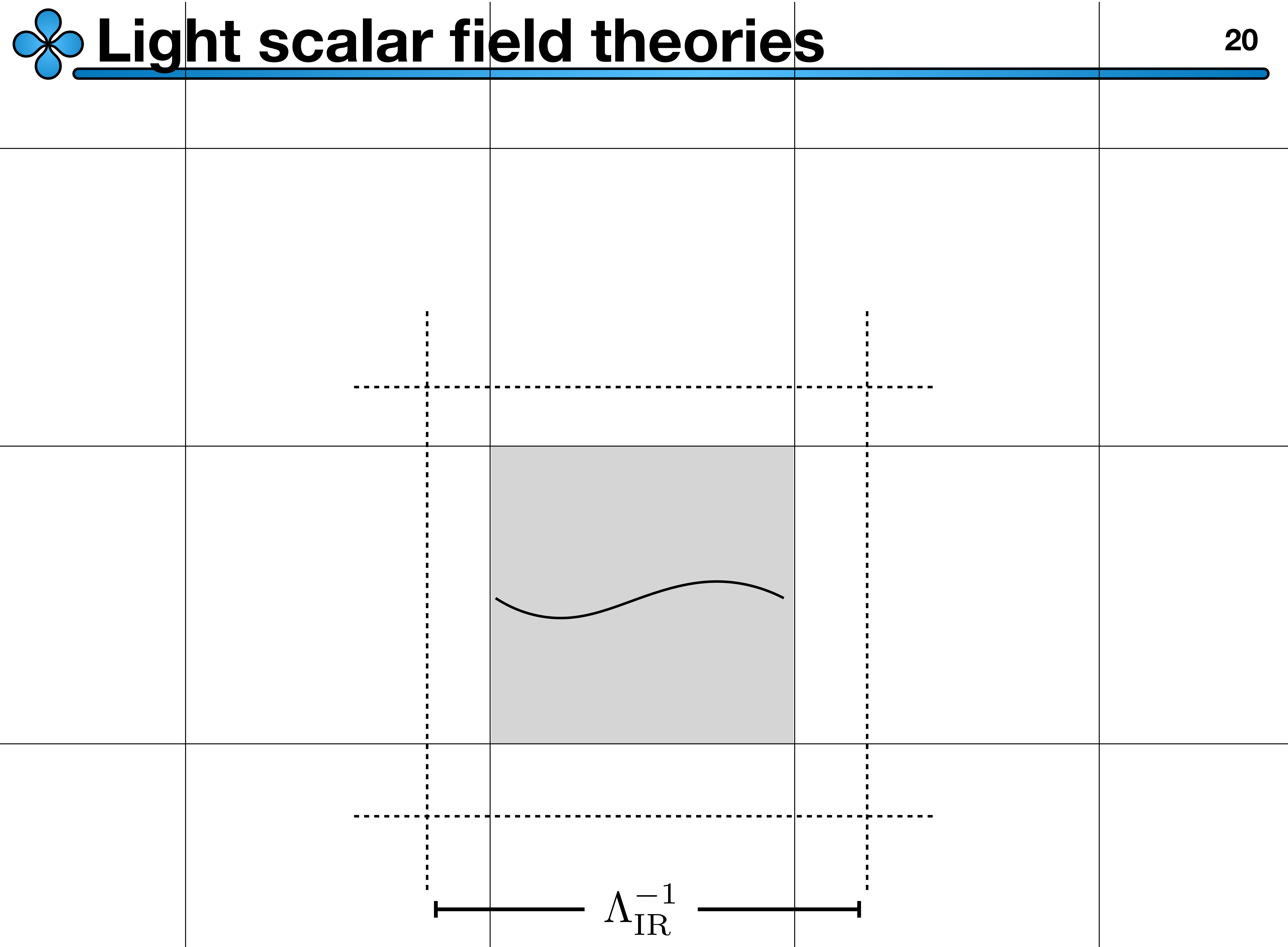

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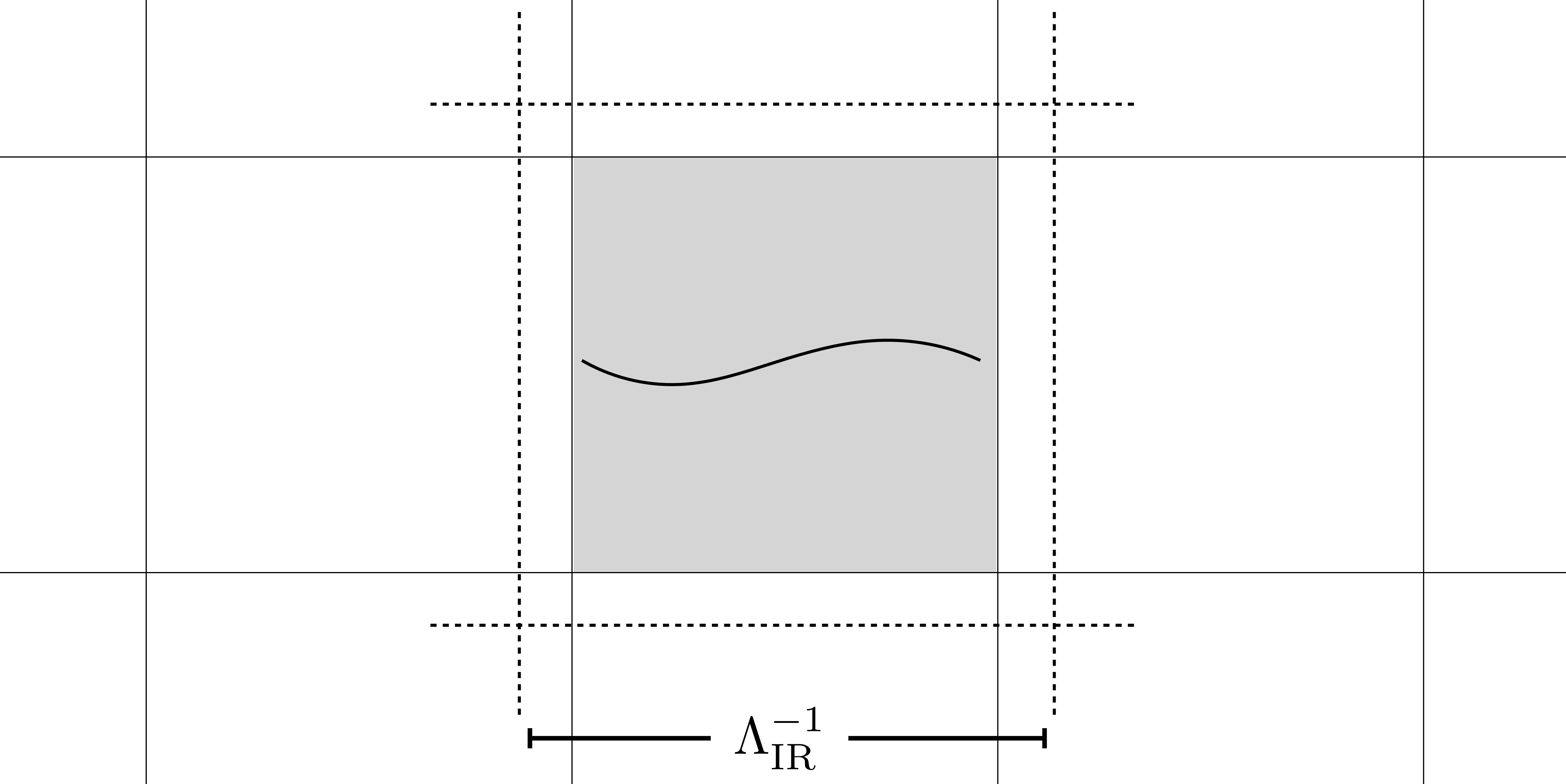

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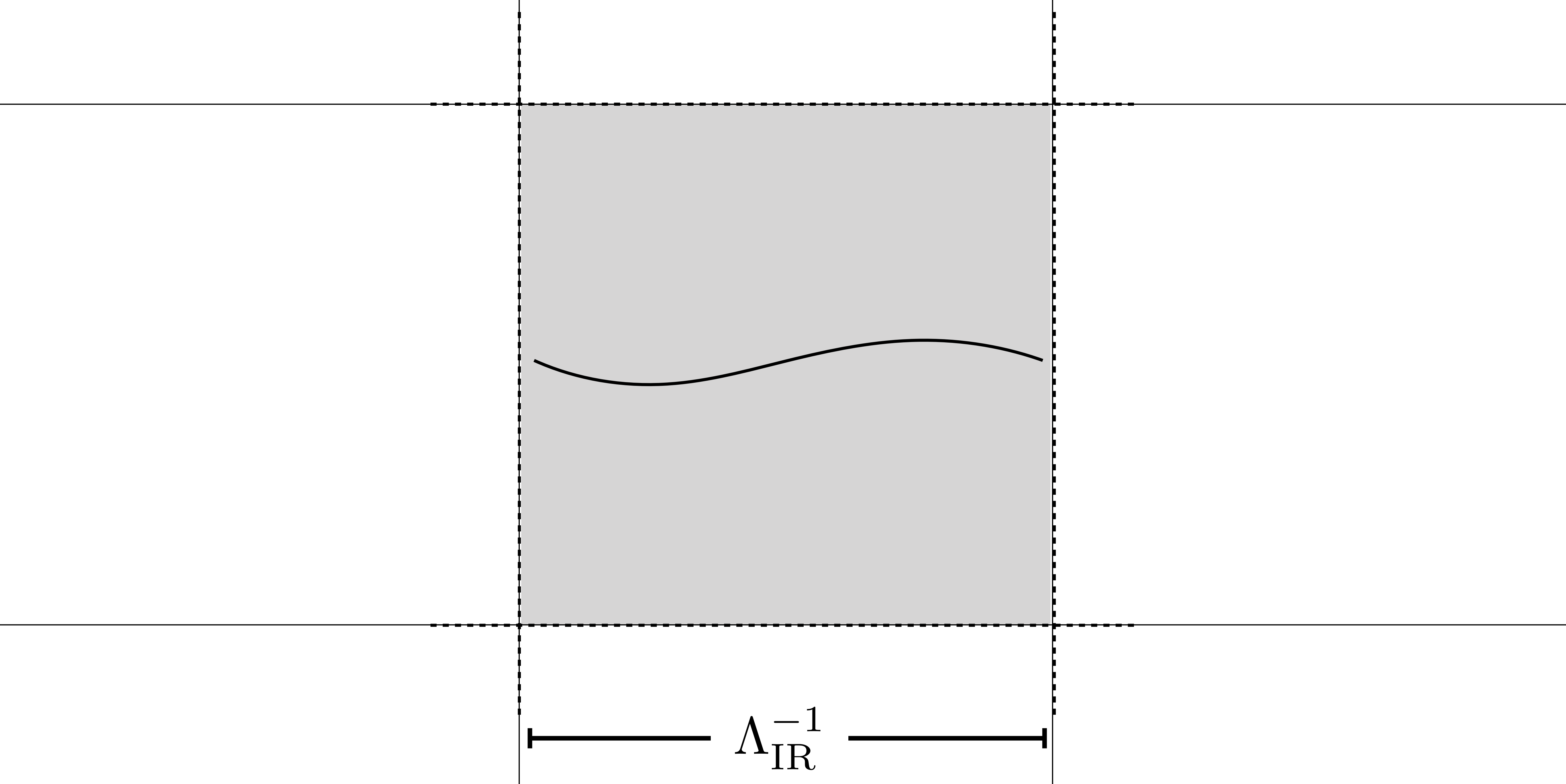

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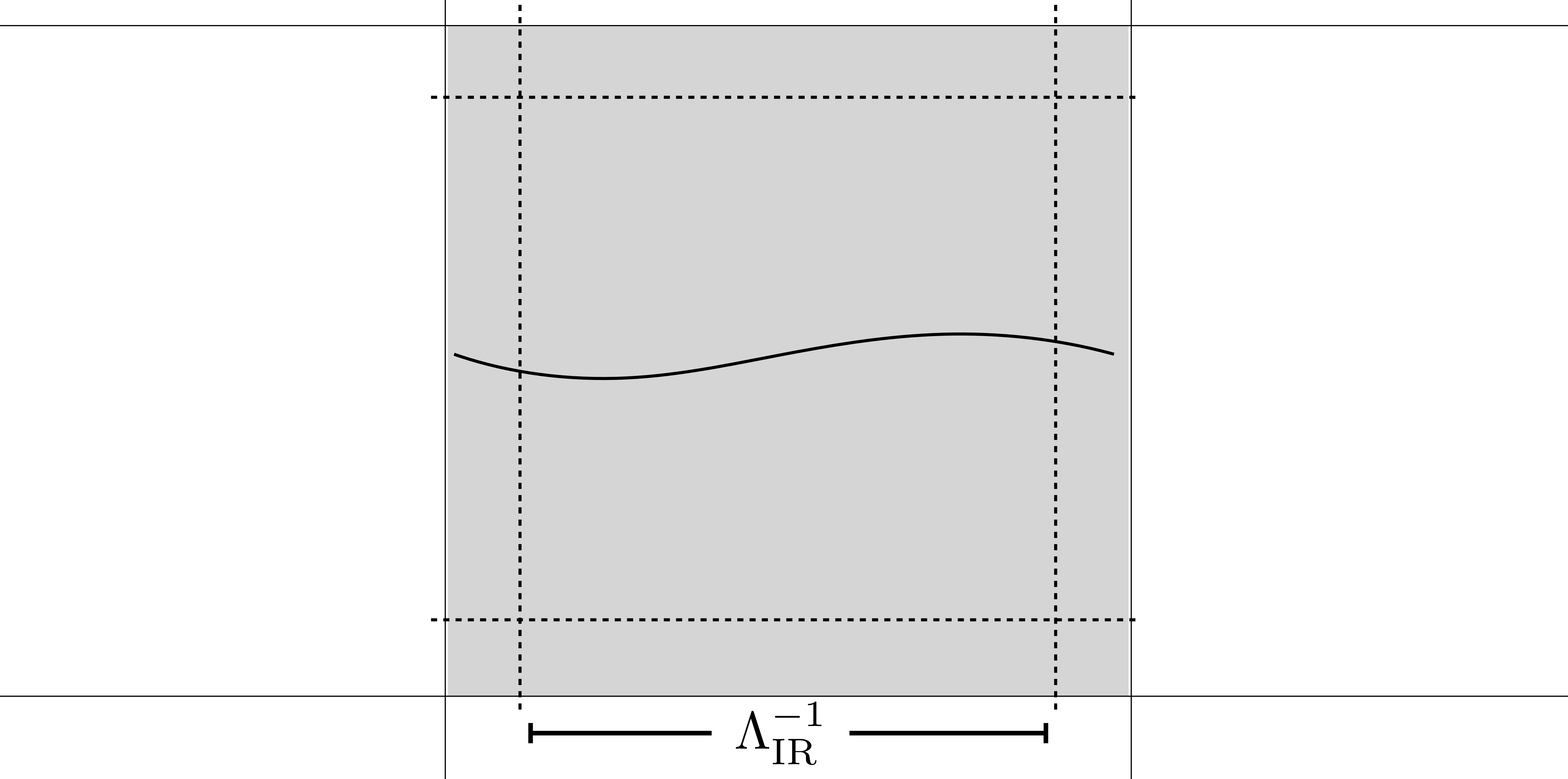

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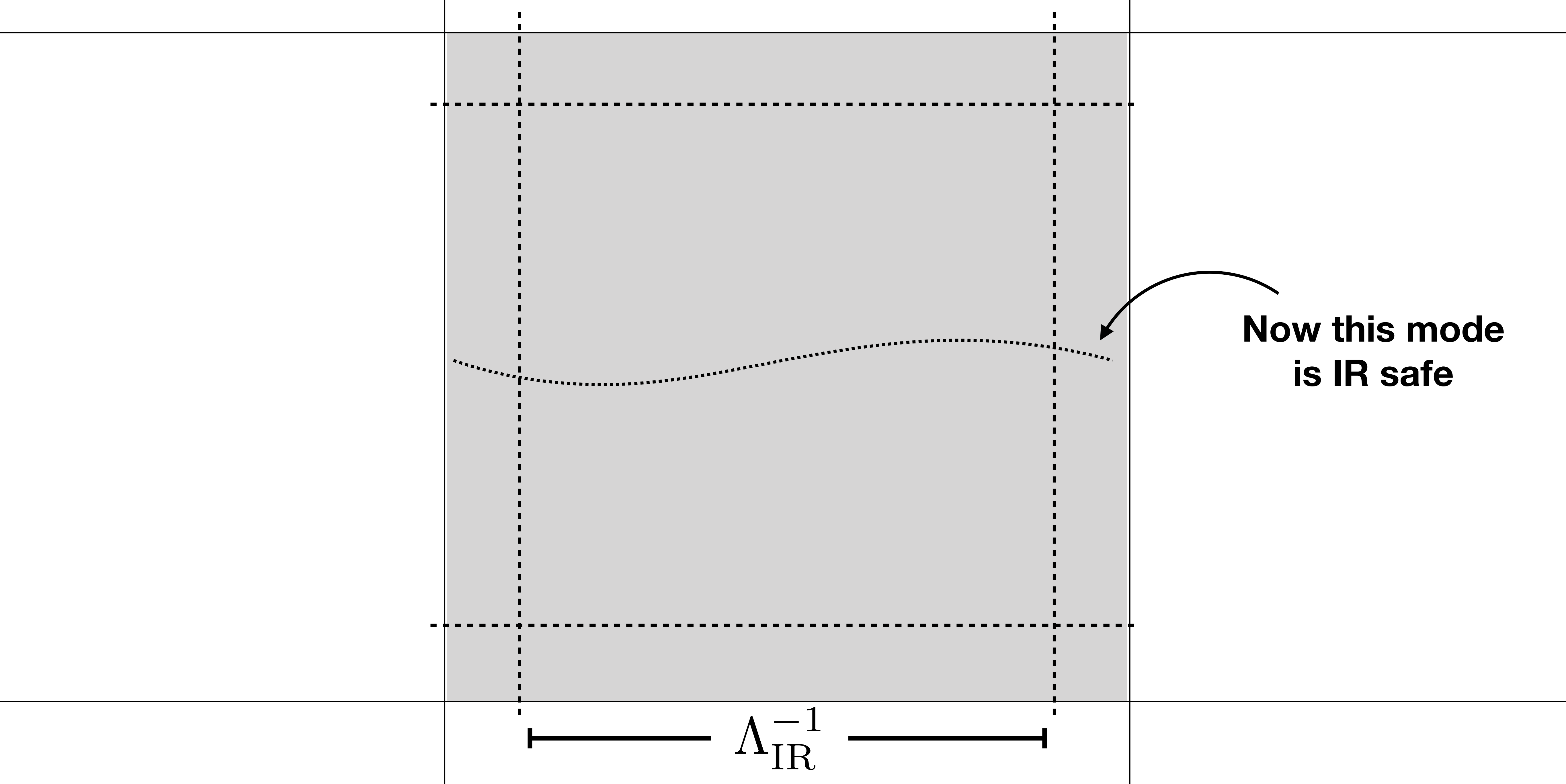

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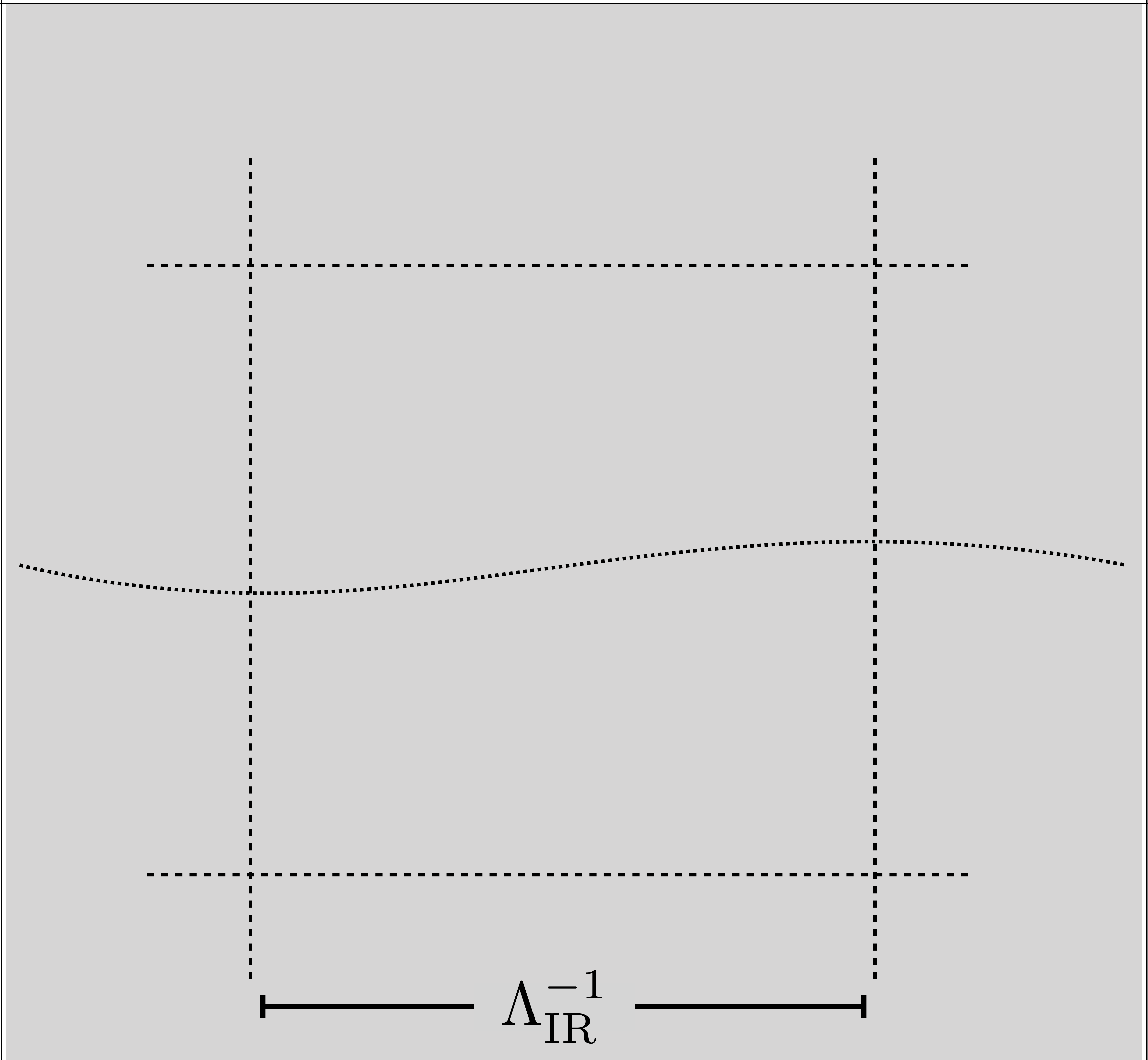

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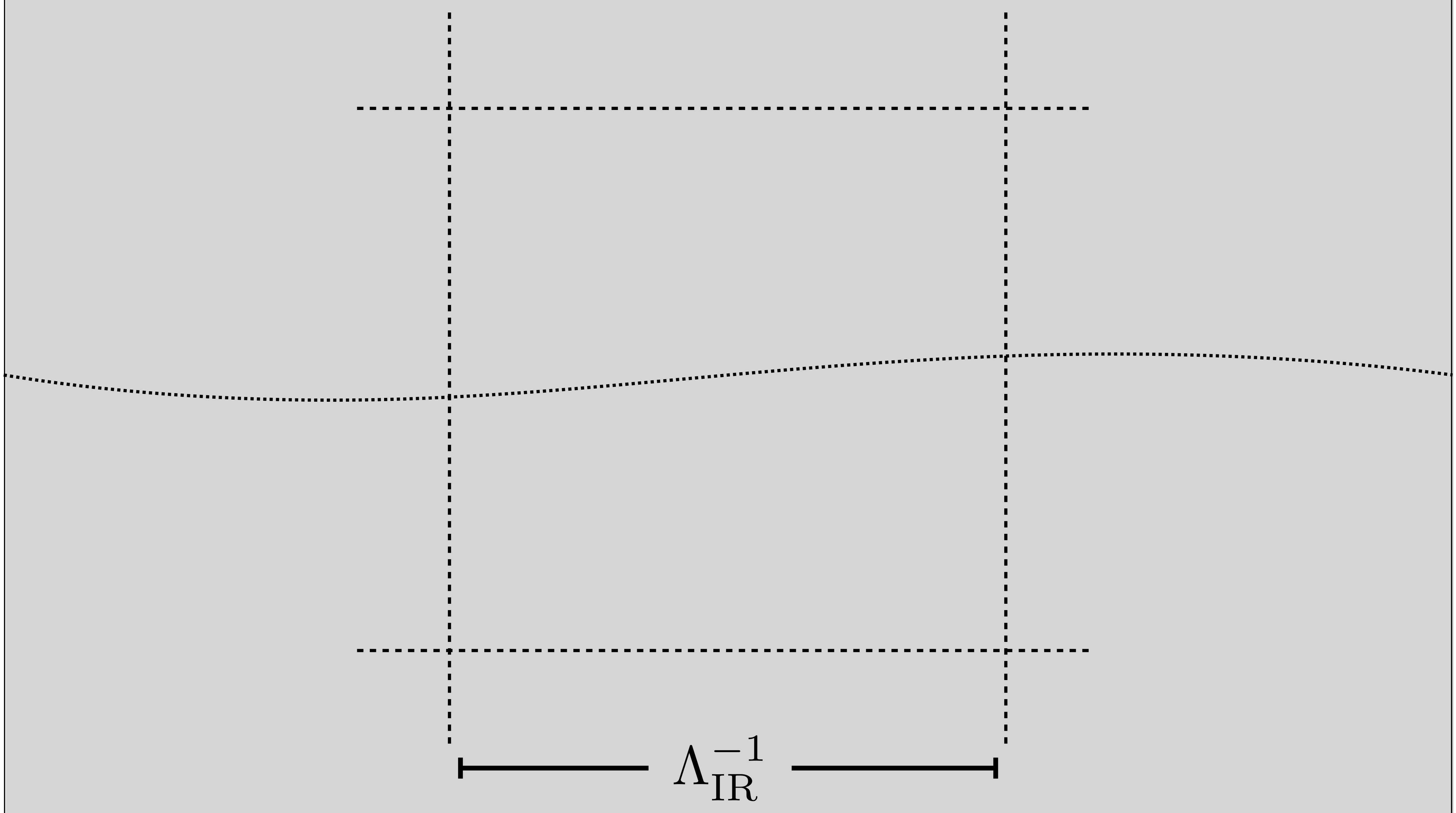

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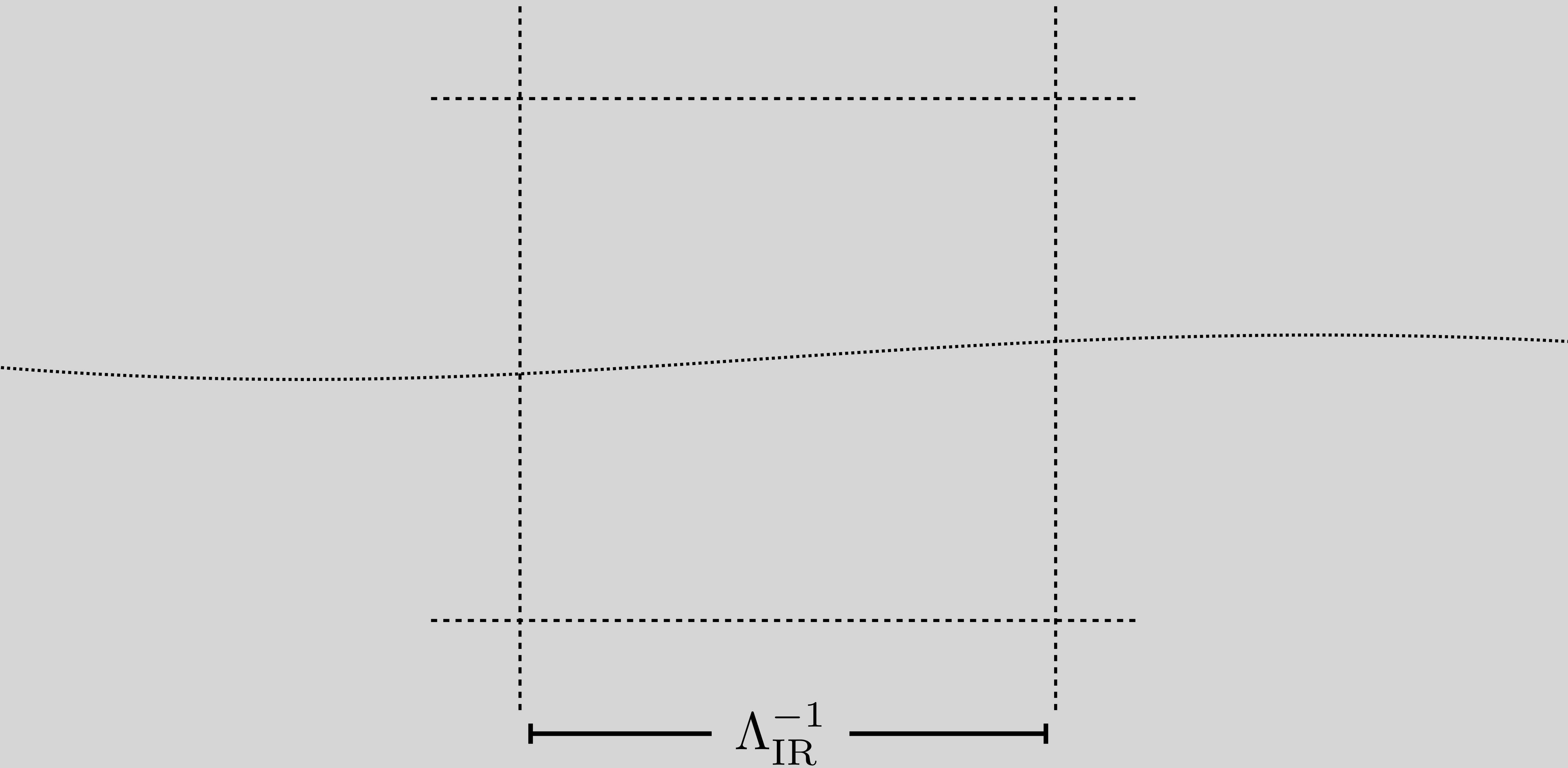

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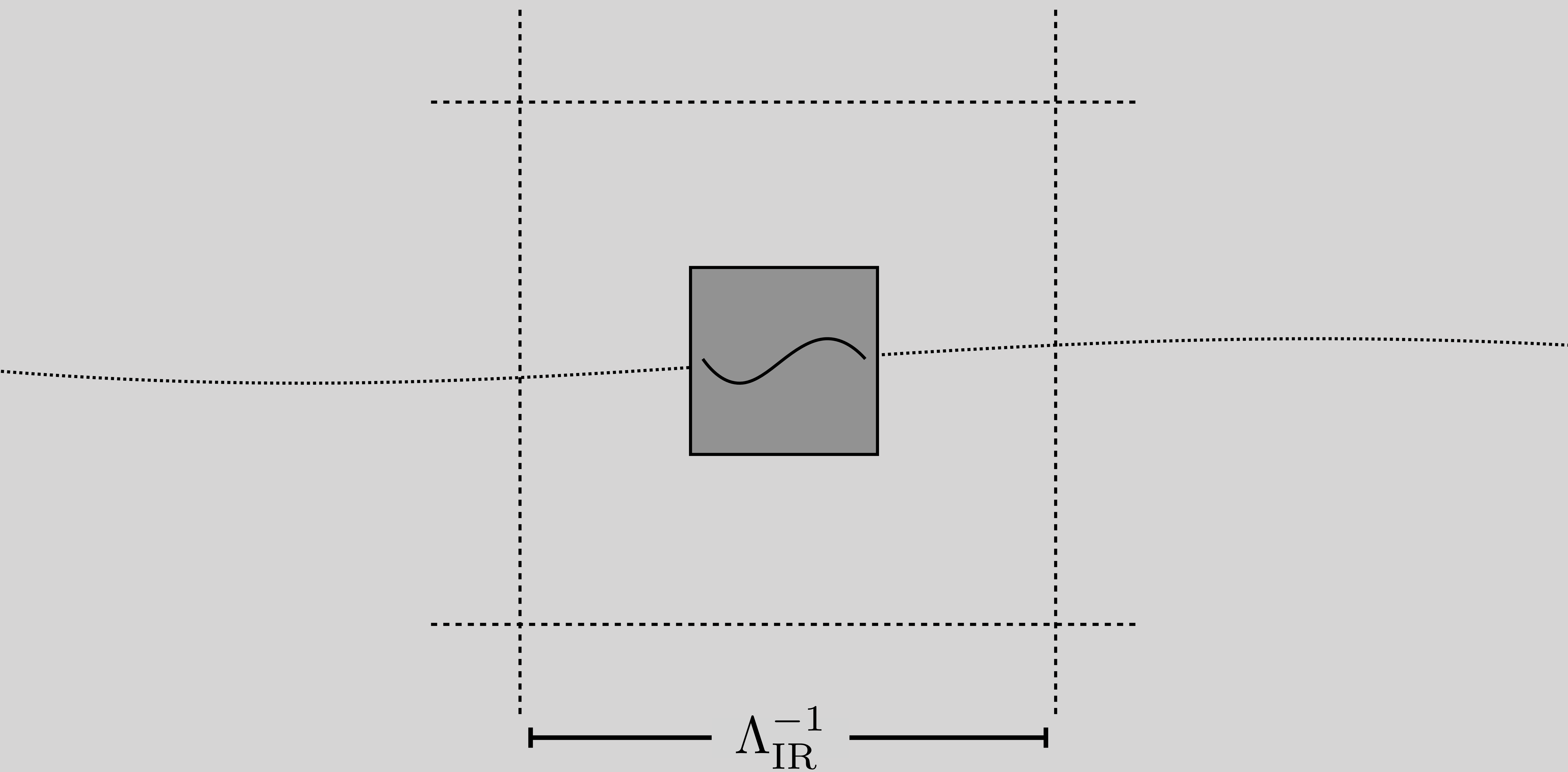

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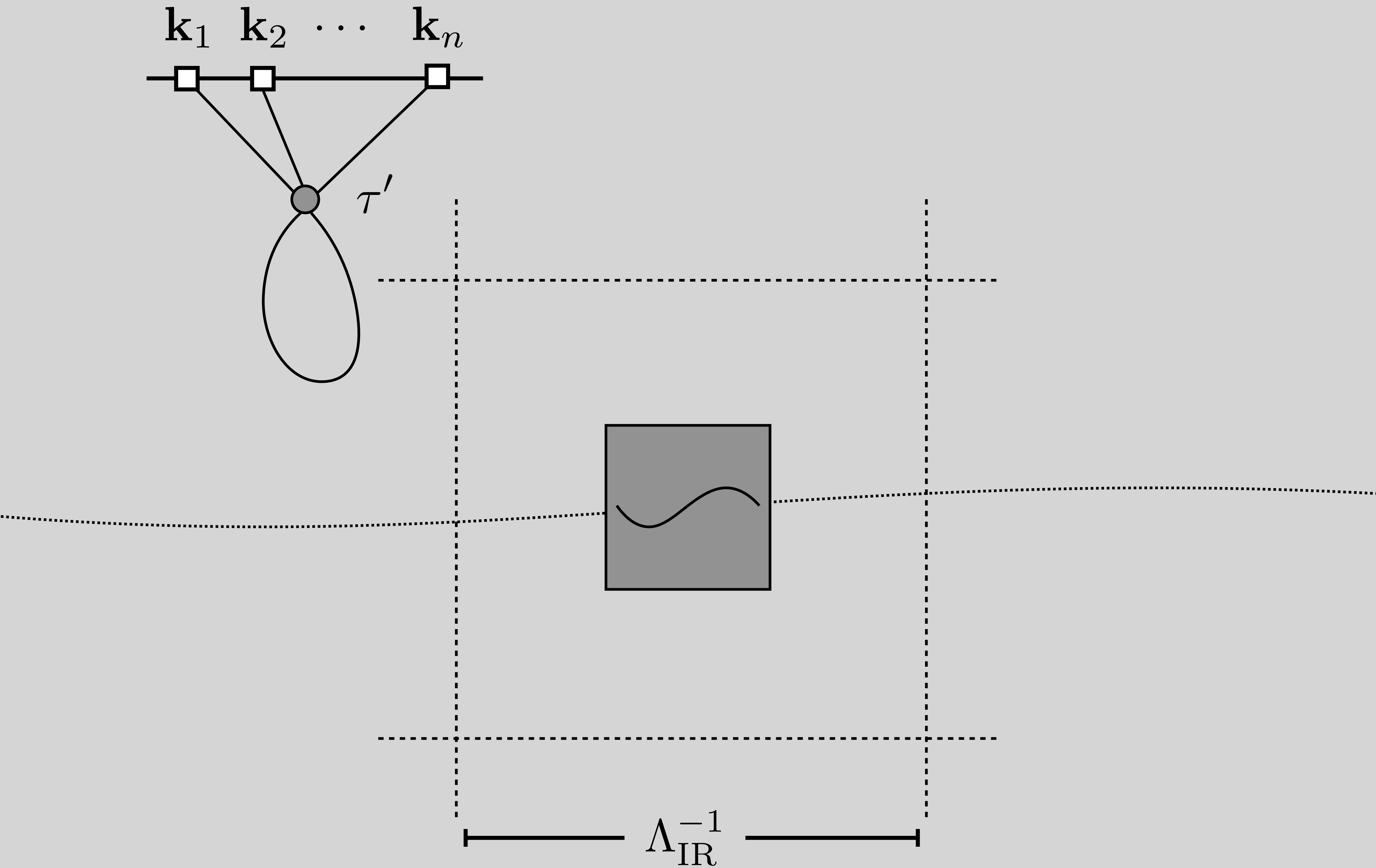

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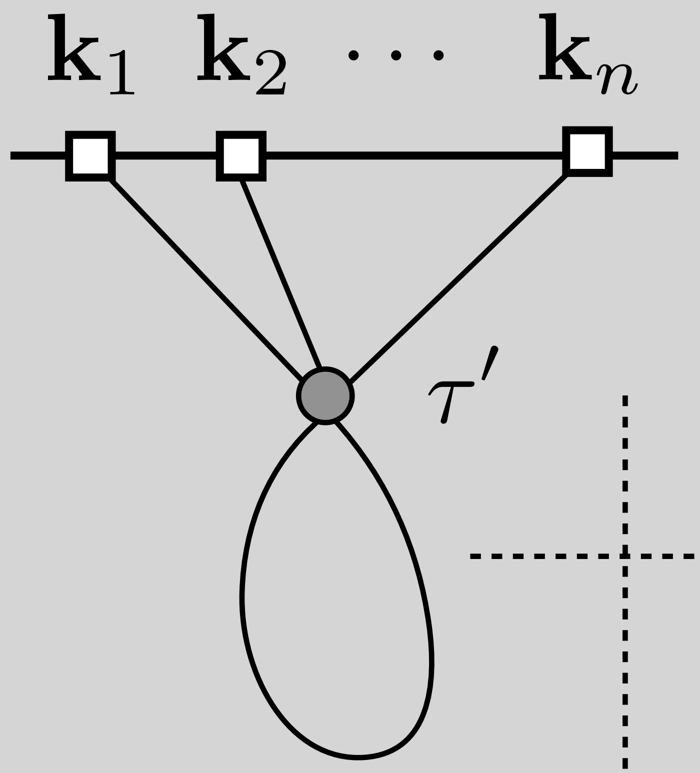
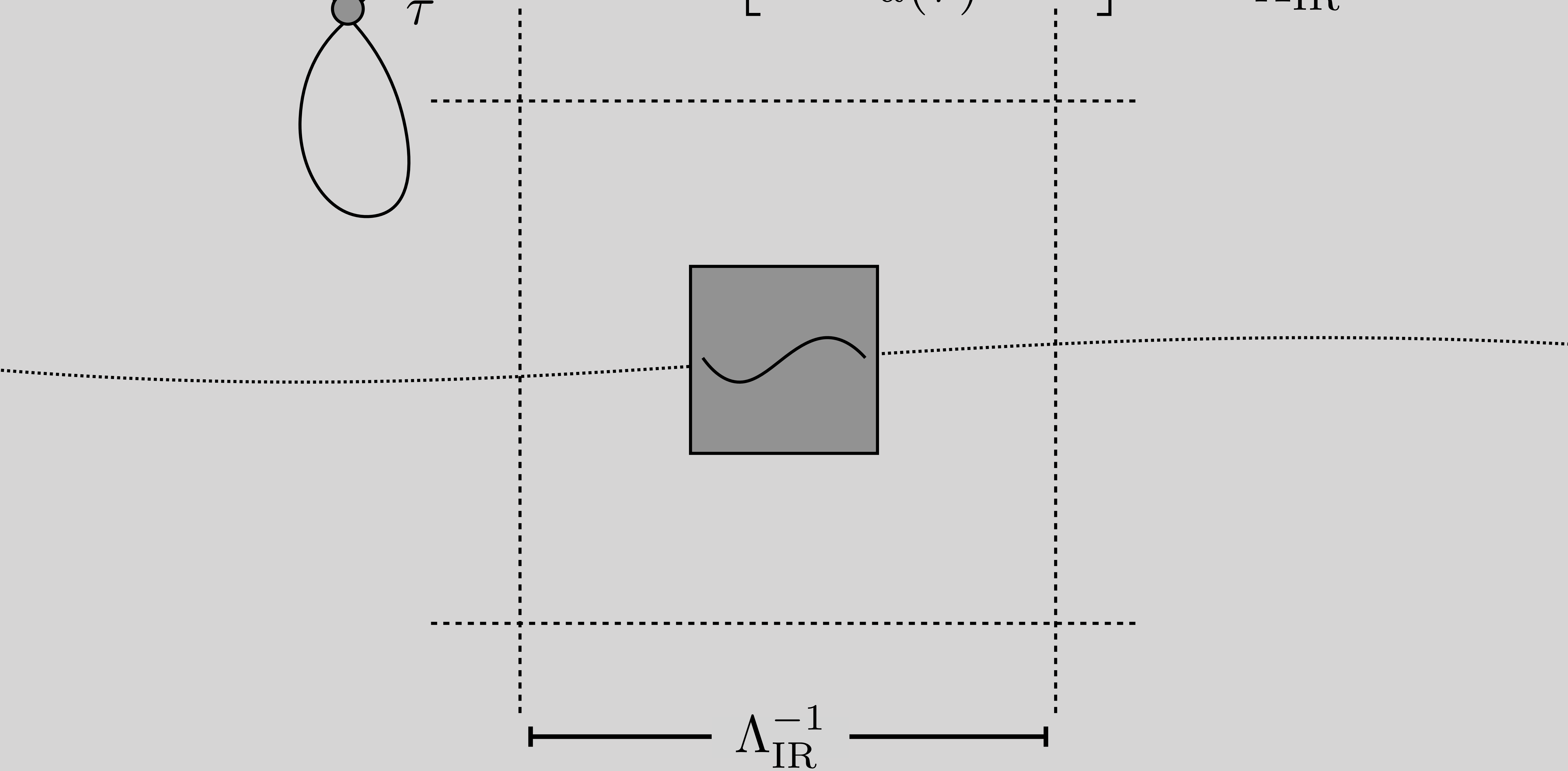


$$\propto \ln\left[\frac{k_1 + \cdots + k_N}{a(\tau)}\right] \times \ln\frac{H}{\Lambda_{\mathrm{IR}}}$$



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- **If** $m^2 \ll m_{\rm IR}^2$ **then external legs are effectively massless**
- **However, loops are regulated by** m **in a dS invariant way**

(Recall Zhong-Zhi talk)

Light scalar field theories

$$G(|\mathbf{x}-\mathbf{x}'|;\tau,\tau') = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}\left(k^3 f_k(\tau) f_k^*(\tau')\frac{\sin(k|\mathbf{x}-\mathbf{x}'|)}{k|\mathbf{x}-\mathbf{x}'|}\right)$$

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All of these procedures yield a dS invariant result!

Example: Daisy loops

There is a case that you can resolve exactly with a massive field:

Huenupi, Hughes, GAP & Sypsas (2024)

See also: Lee et al. (2023); Creminelli et al. (2024)

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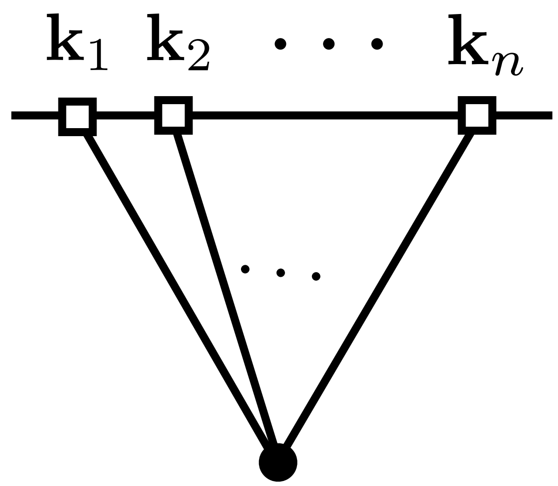

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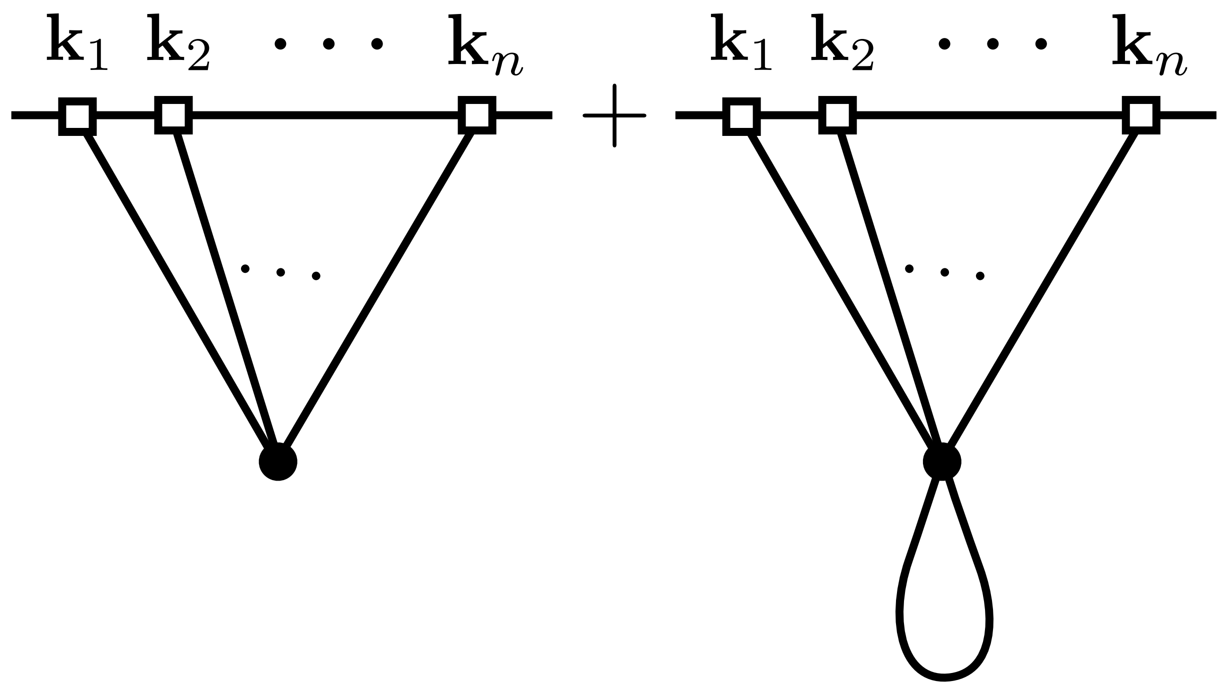

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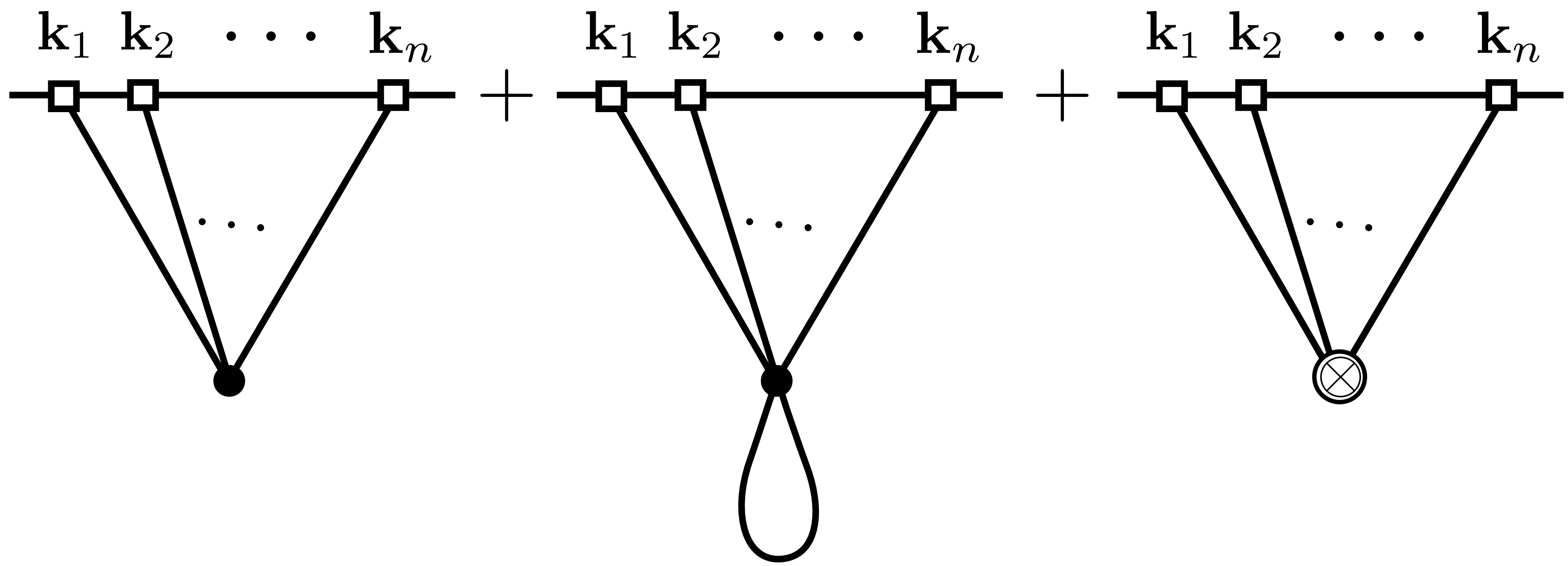

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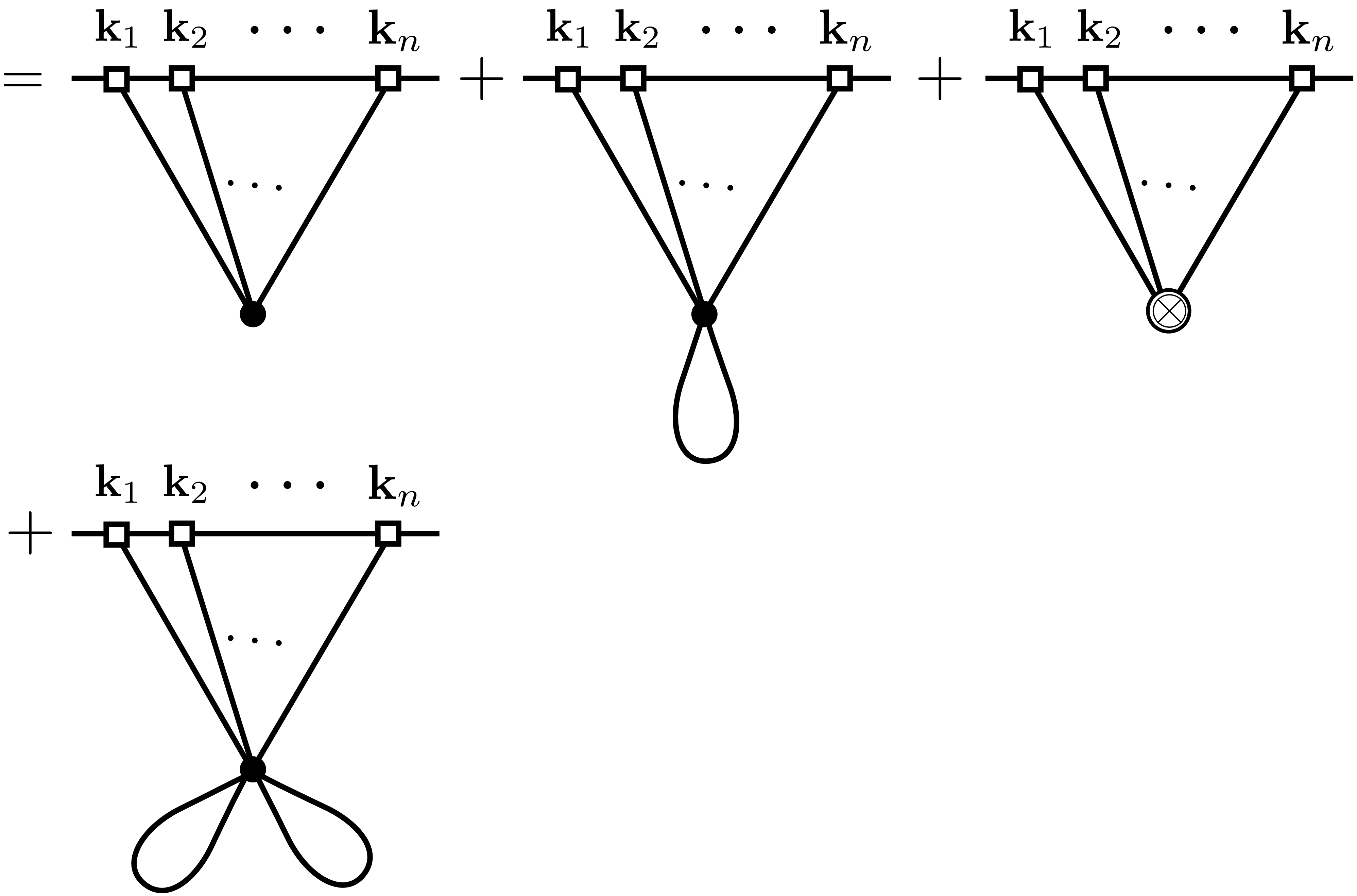

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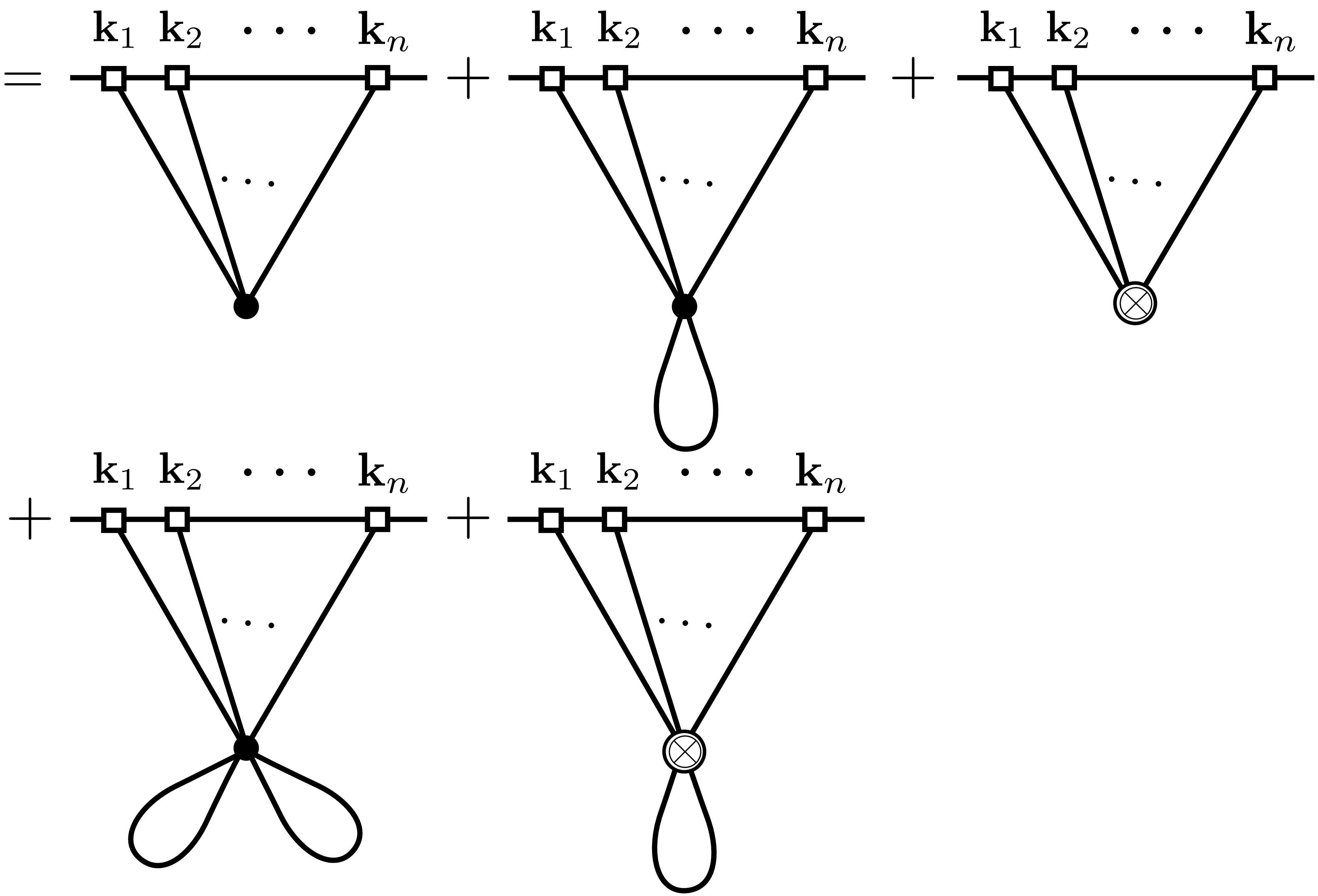

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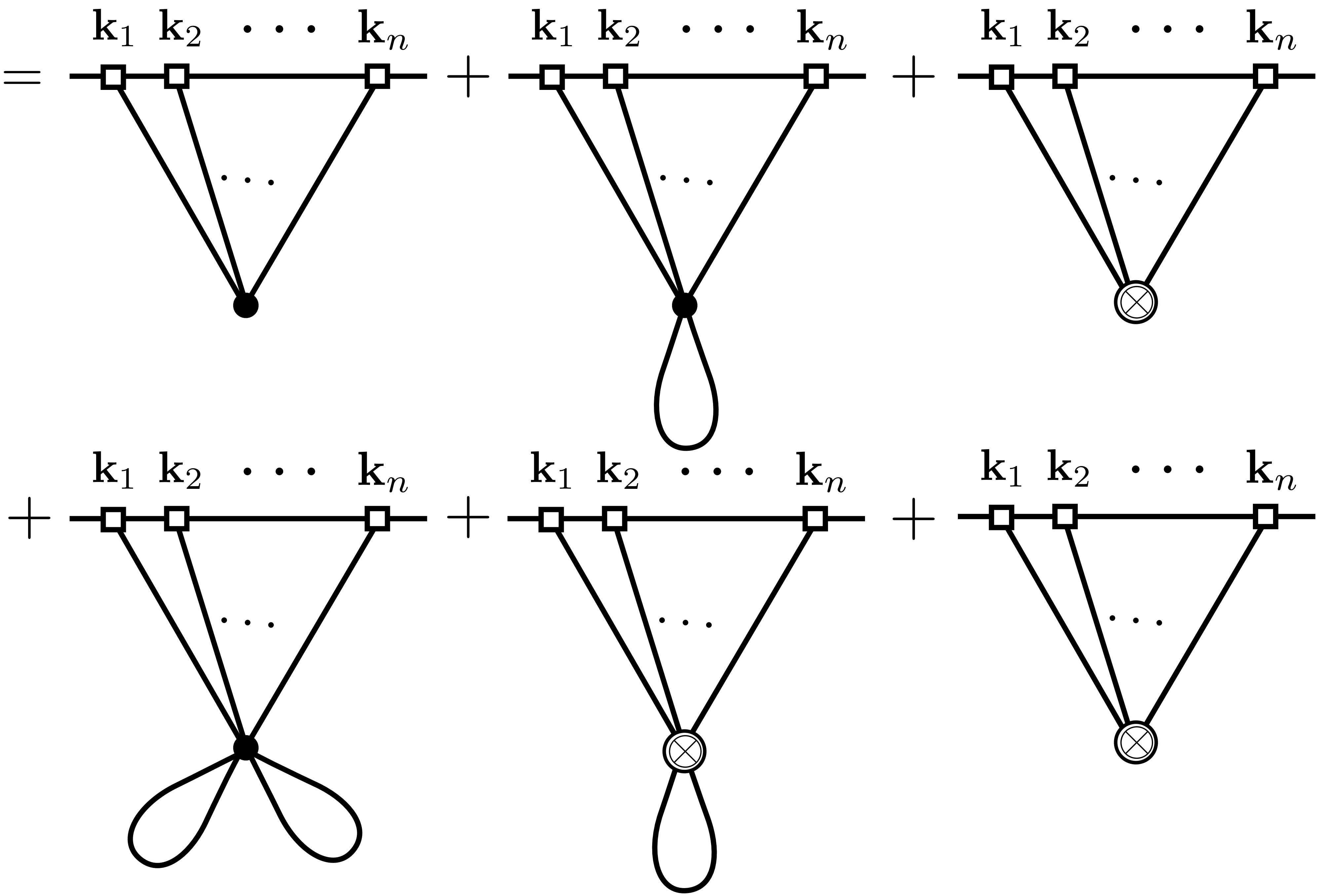

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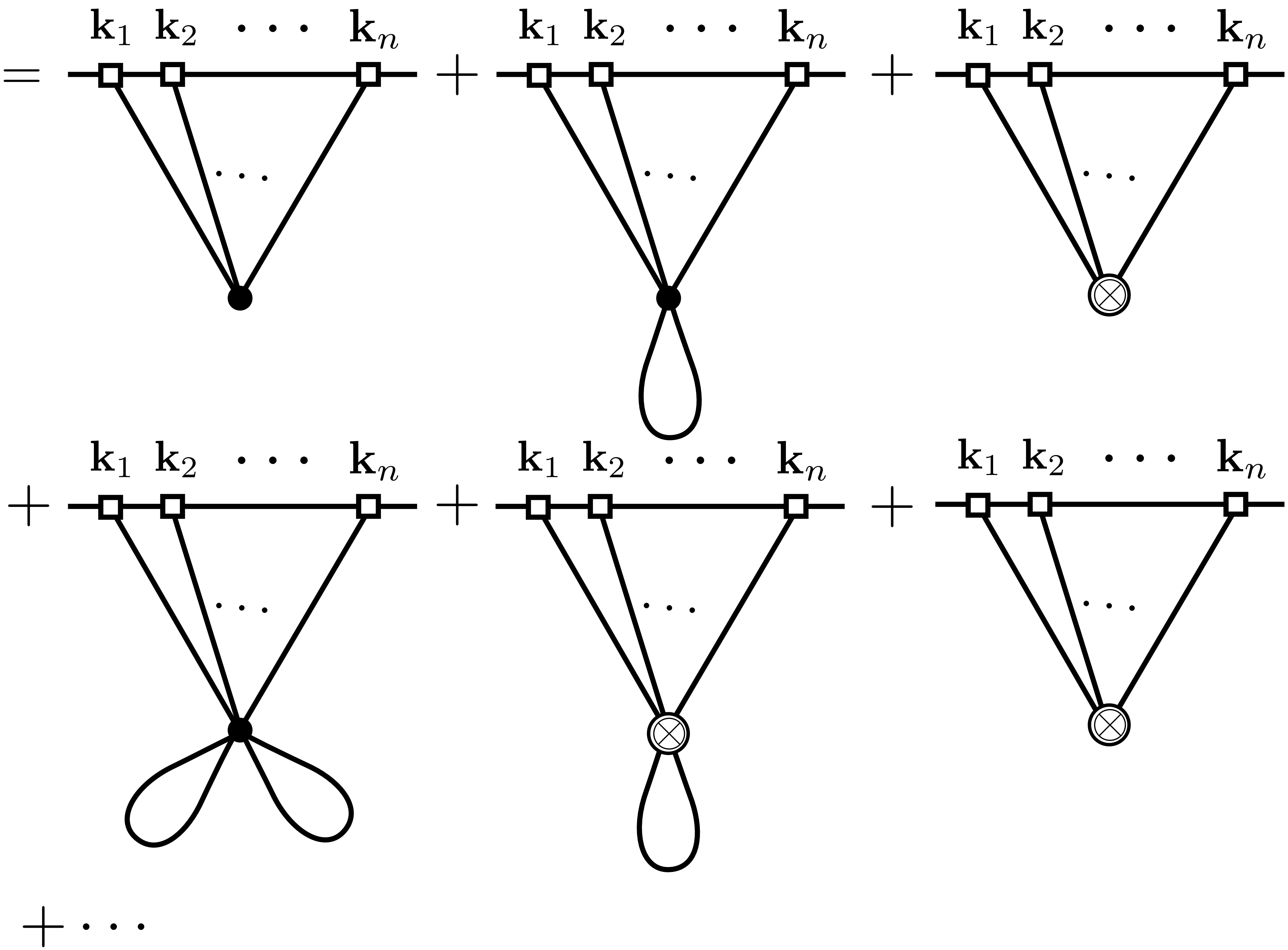
See also: Lee et al. (2023); Creminelli et al. (2024)

Example: Daisy loops

There is a case that you can resolve exactly with a massive field:

$$\langle \varphi_{\mathbf{k}_1}(\tau) \cdots \varphi_{\mathbf{k}_n}(\tau) \rangle =$$



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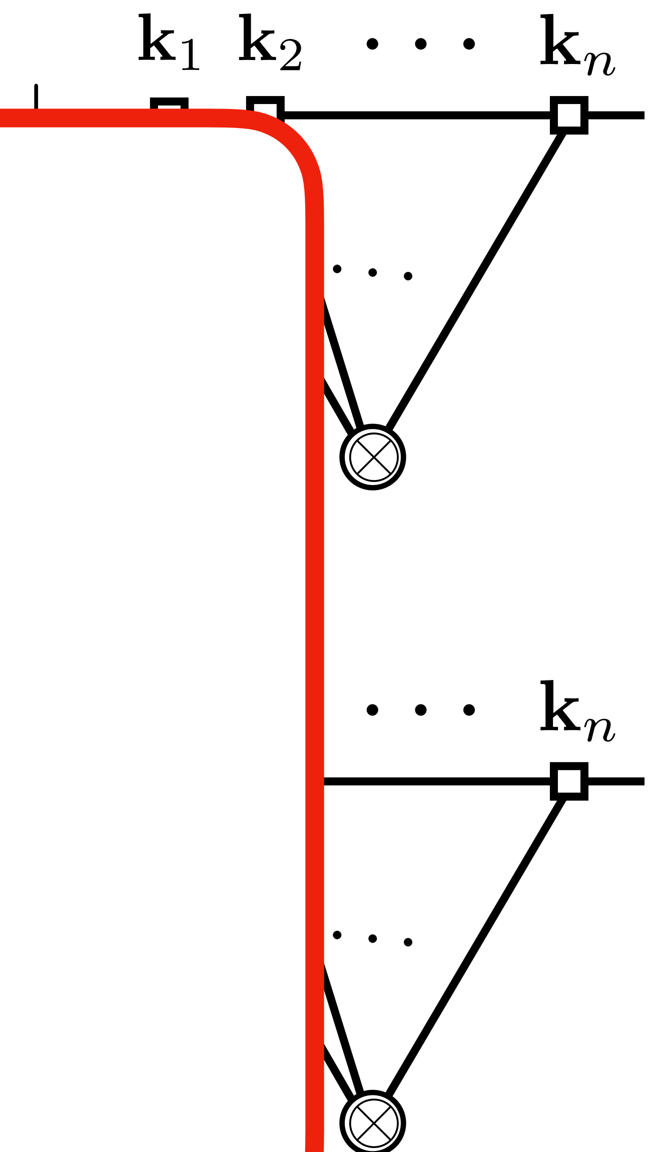

The summation gives you back an exact result valid to all orders in loops proportional to a tree-level diagram

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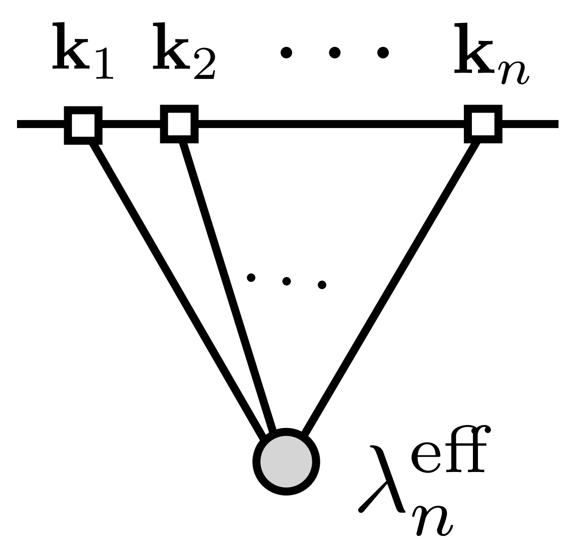

Now you can go back to $m = 0$

with no secular growth to be found!!!

$$+ \cdots$$

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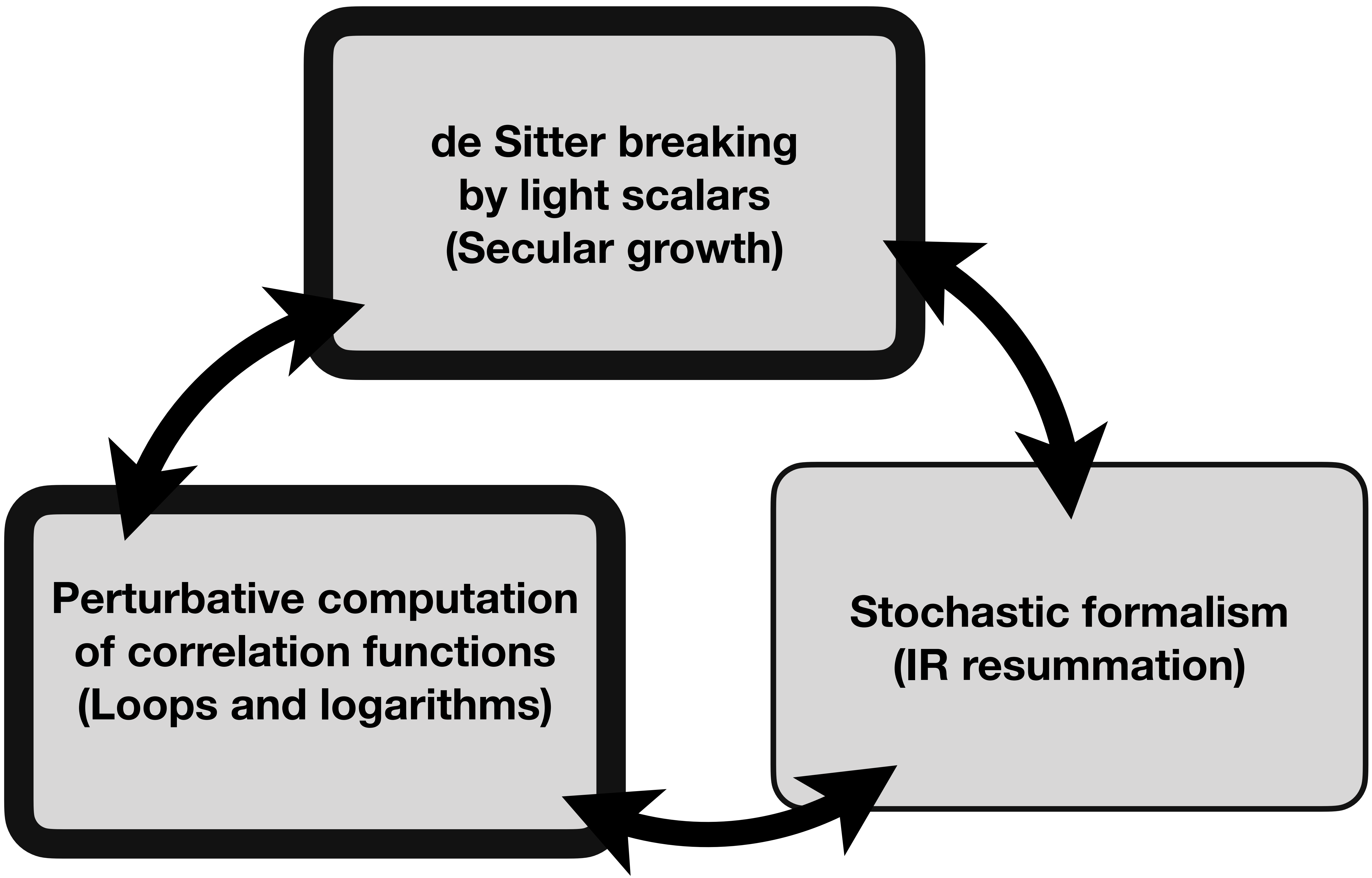
Preamble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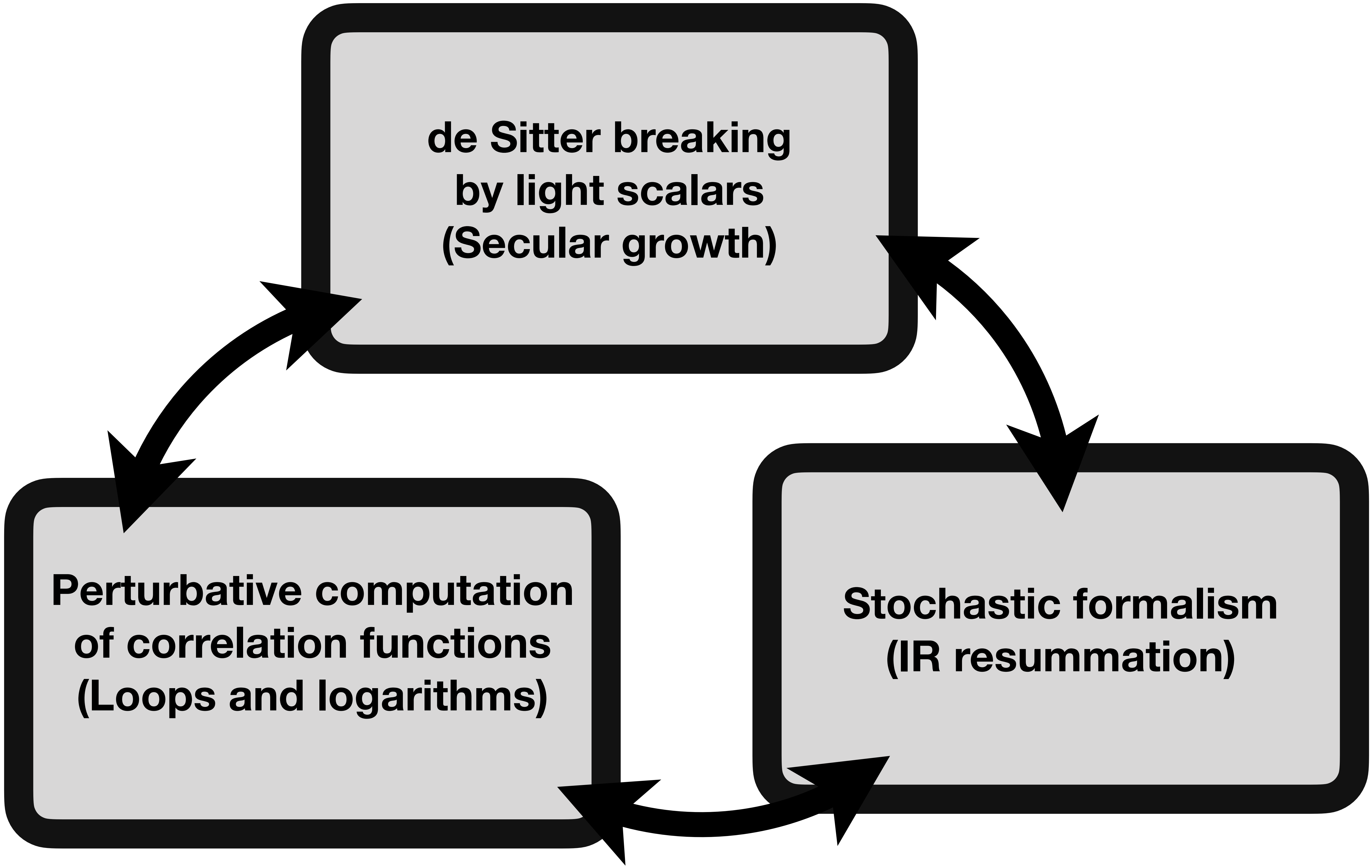
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Stochastic formalism and loops

There is a connection between the stochastic formalism and loop corrections to correlation functions (established by Woodard and Tsamis)

A cumulant is a connected n-point function evaluated at coincident point

$$\langle \varphi^n \rangle = \quad \langle \varphi(\mathbf{x}) \cdots \varphi(\mathbf{x}) \rangle_c$$

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There must exists a probability density function (PDF) allowing to compute cumulants

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$$\mathcal{V}(\varphi) = \sum_m \frac{\lambda_m}{m!} \varphi^m$$

(Tsamis & Woodard)

Stochastic formalism and loops

Tsamis & Woodard:

$$\frac{d}{dt}\left\langle \varphi^n \right\rangle = n(n-1)\frac{H^3}{8\pi^2}\left\langle \varphi^{n-2} \right\rangle - \frac{n}{3H}\sum_{m=2}^{\infty}\frac{\lambda_m}{(m-1)!}\left\langle \varphi^{m+n-2} \right\rangle$$

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Loop corrections employing a co-moving IR cutoff !!!

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To understand Tsamis and Woodard's result, let me offer a quick derivation of the Fokker-Planck equation found by Satoribinsky and Yokoyama

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$$3H\dot{\varphi}_w - 3H^2\hat{\xi}(t)$$

Where the noise is $$\hat{\xi} \equiv H^{-1}\int_{\mathbf{k}} \left(\frac{d}{dt}W(k)\right) \tilde{\varphi}_{\mathbf{k}}(t)$$

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If you choose a physical cutoff, you recover a different Fokker-Planck eq. (See Spyros talk)

Stochastic formalism and loops

If you choose a physical cutoff you recover cumulants computed out of the exact result

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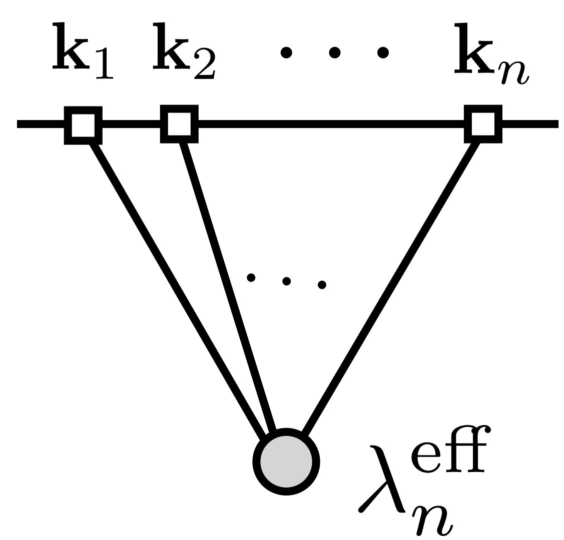

Recall that this result is free of dS breaking secular growth

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- The Fokker-Planck equation turns out to have relevant corrections (see Spyros talk)

Thanks!