Inflation on small scales: From theoretical progress to GW observatories

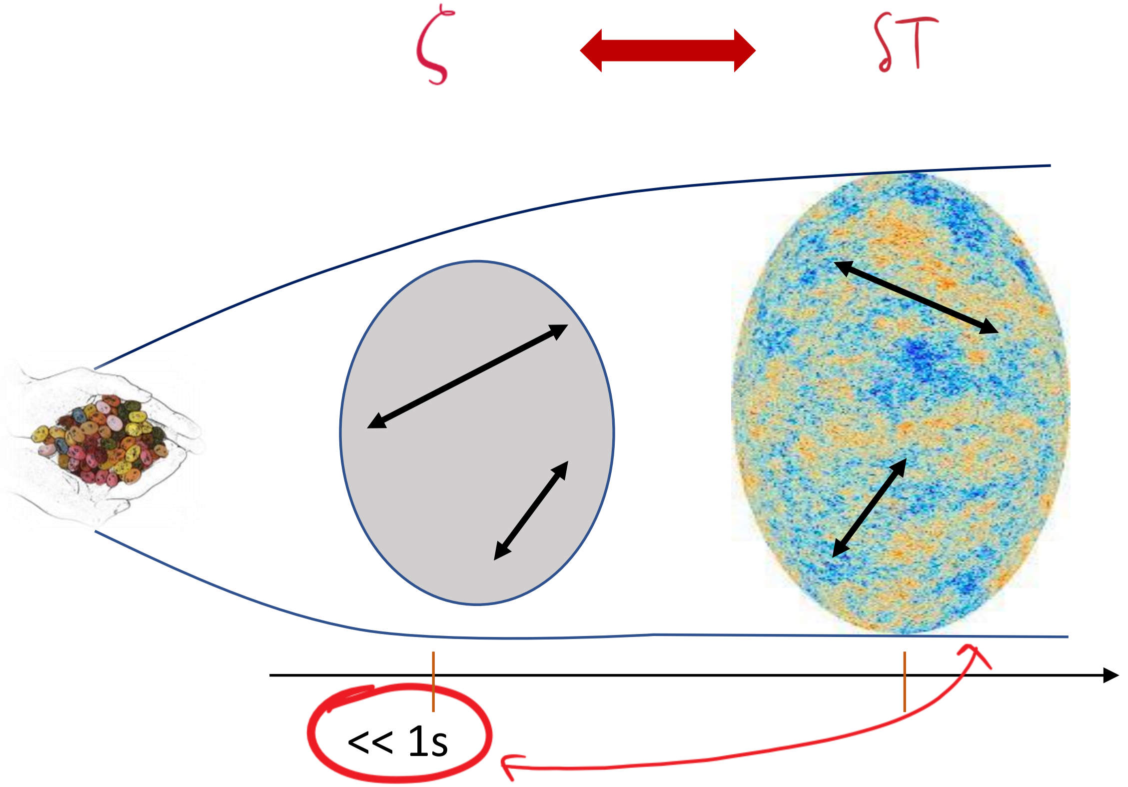
Jacopo Fumagalli

05/12/2025

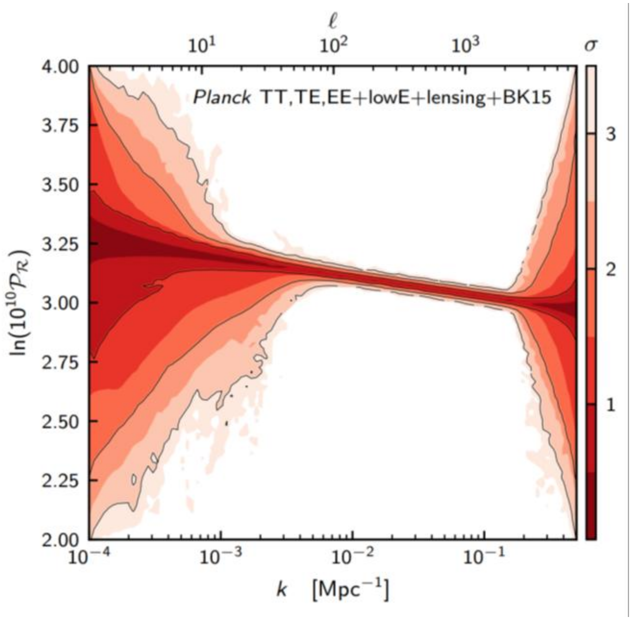
Inflation 2025 - IAP

ICCUB
Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

INFLATION, WHY WE LIKE IT SO MUCH...



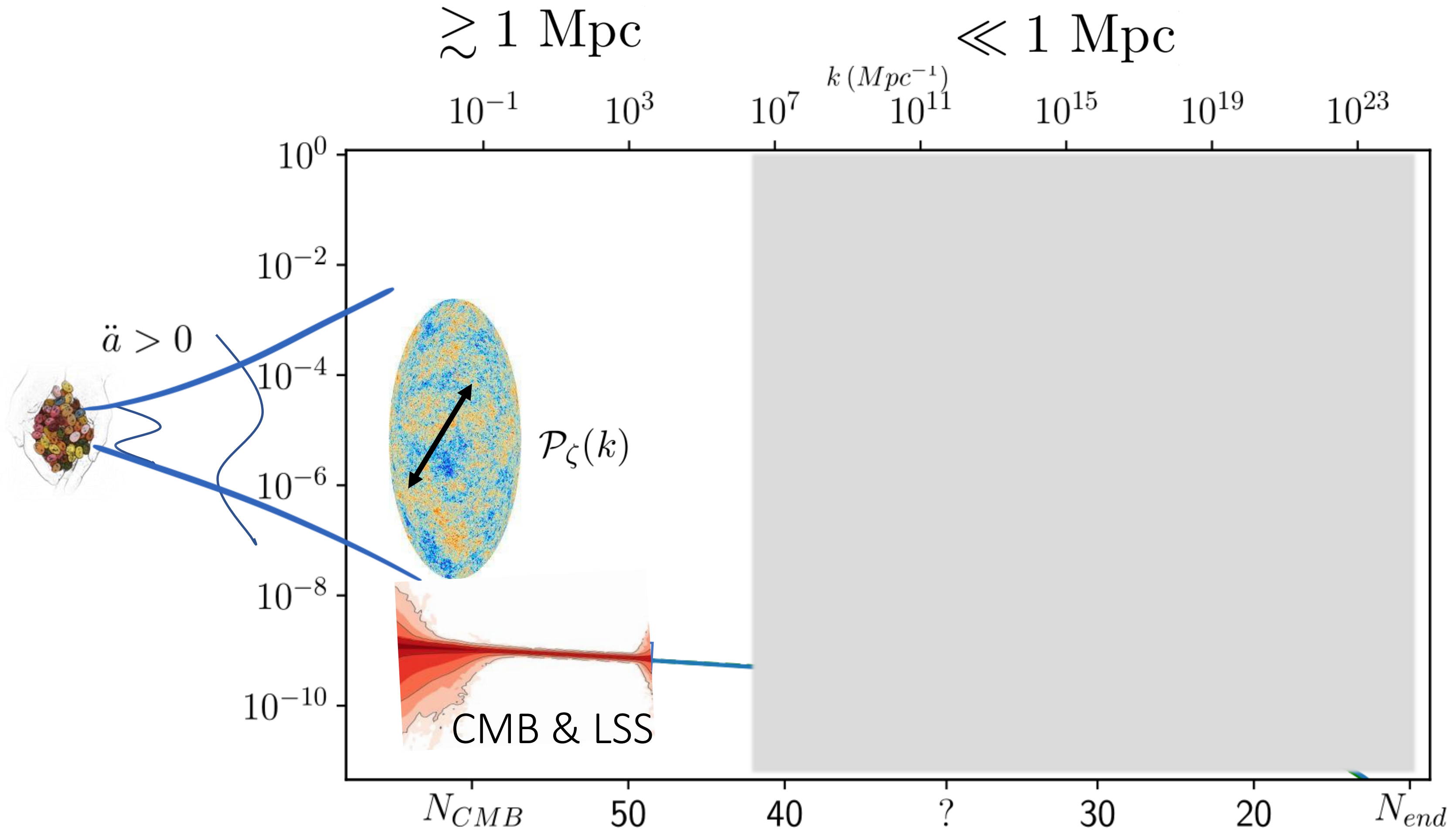
Fluctuations:
Almost scale-invariant,
Gaussian, super-Horizon...



Planck '18

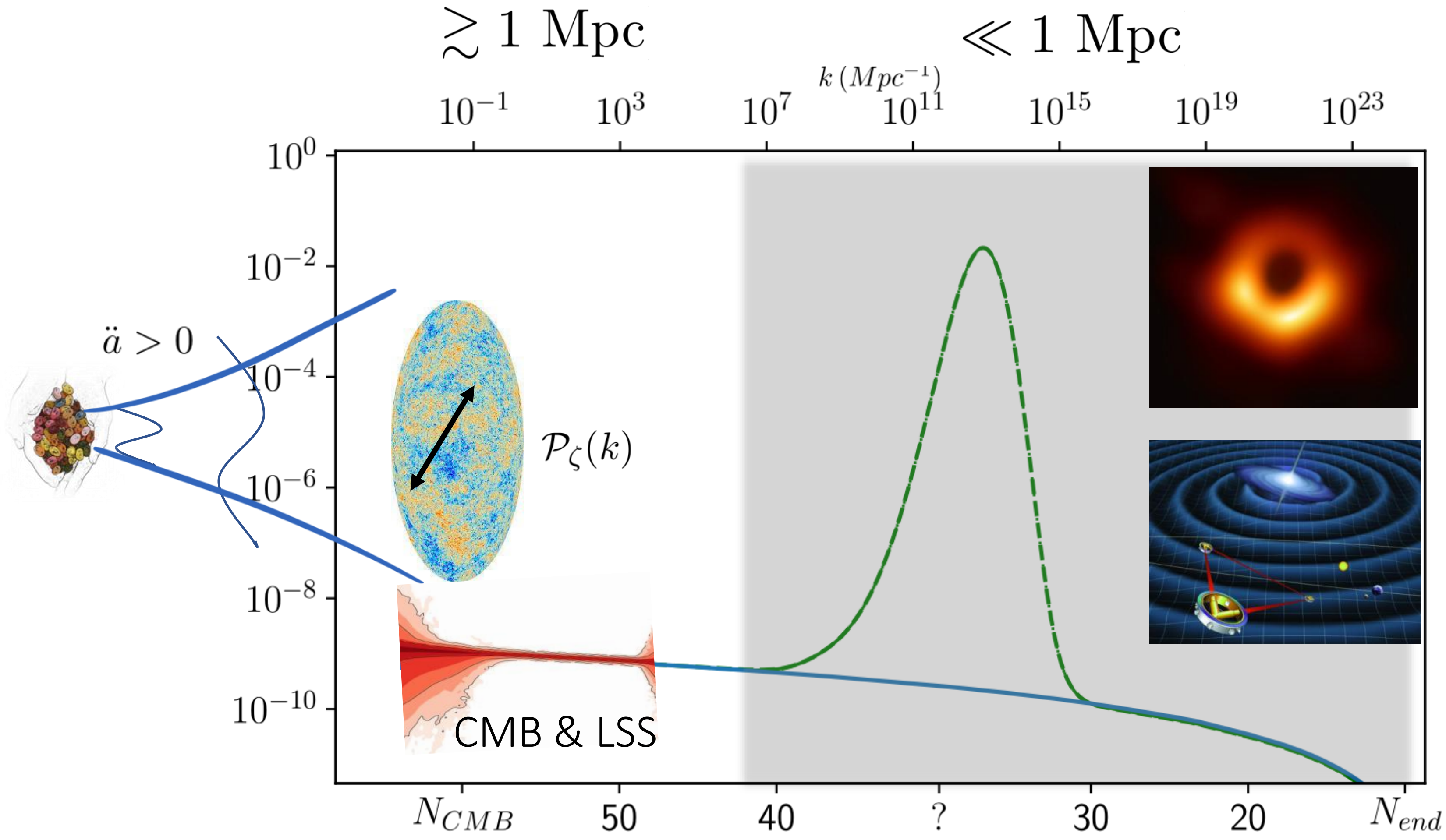
$$\langle\hat{\zeta}(\mathbf{x},\tau)\hat{\zeta}(\mathbf{x},\tau)\rangle = \int d\ln k \cdot \mathcal{P}_\zeta(k,\tau)$$

INFLATION AT LARGE SCALES



Constrained at large scales

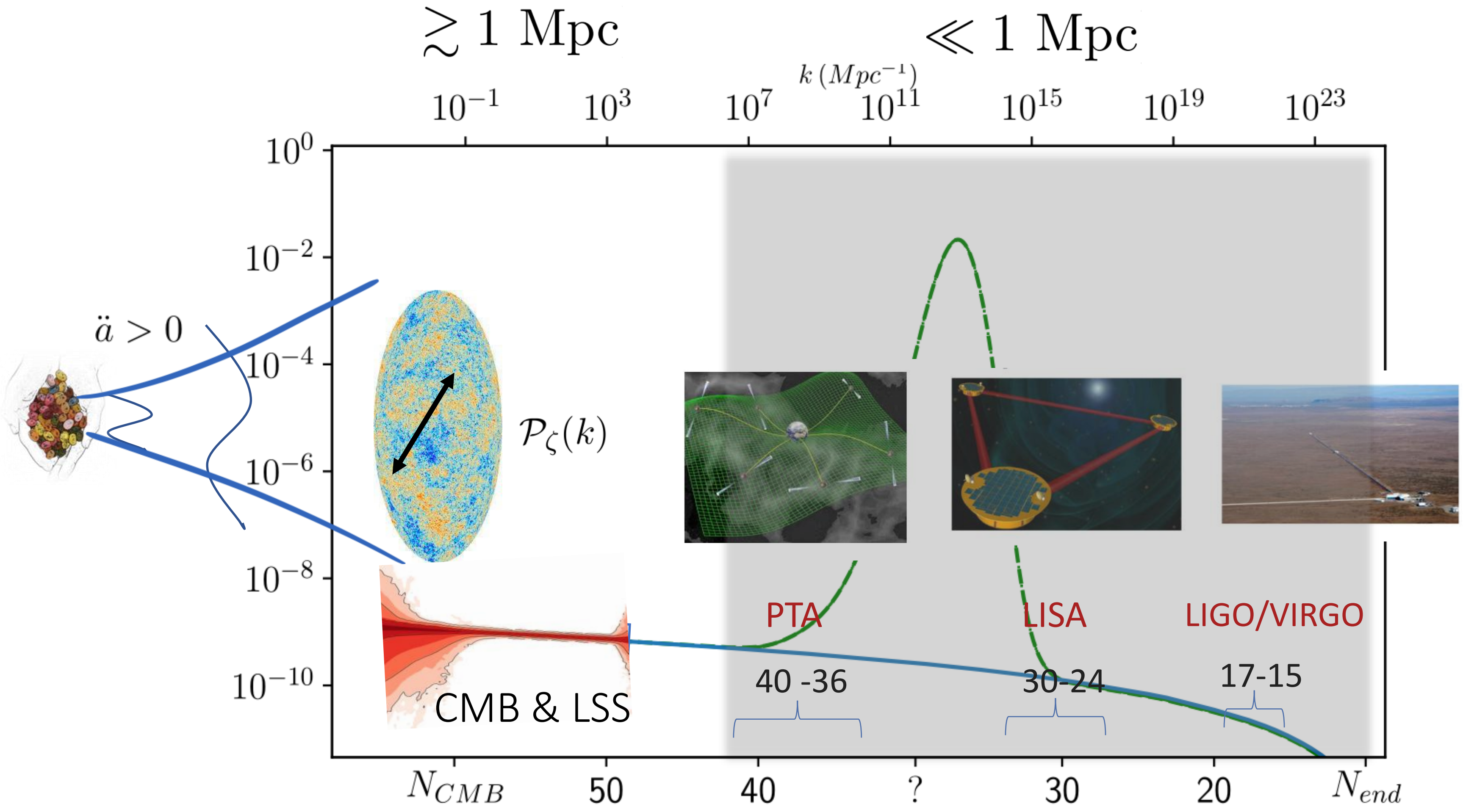
INFLATION ON SMALL SCALES



Constrained at large scales

Unconstrained at small scales

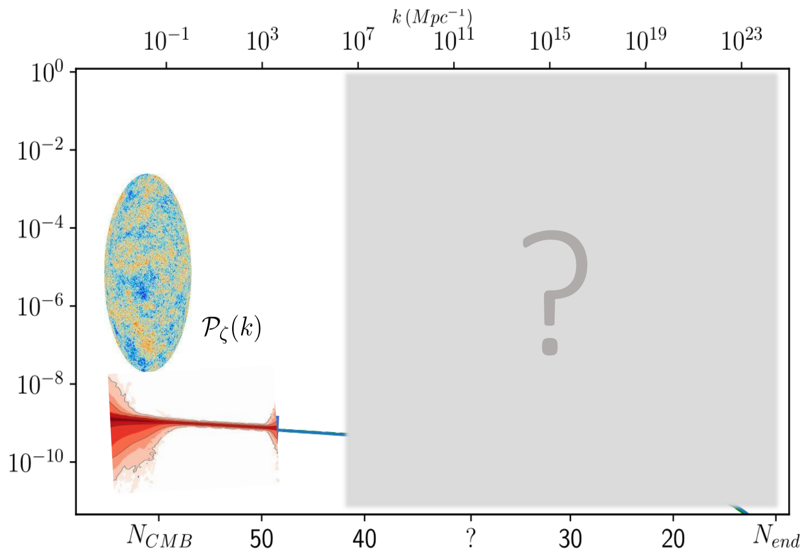
PROBING INFLATION AT SMALL SCALES WITH GWs



Constrained at large scales

Unconstrained at small scales

QUESTIONS



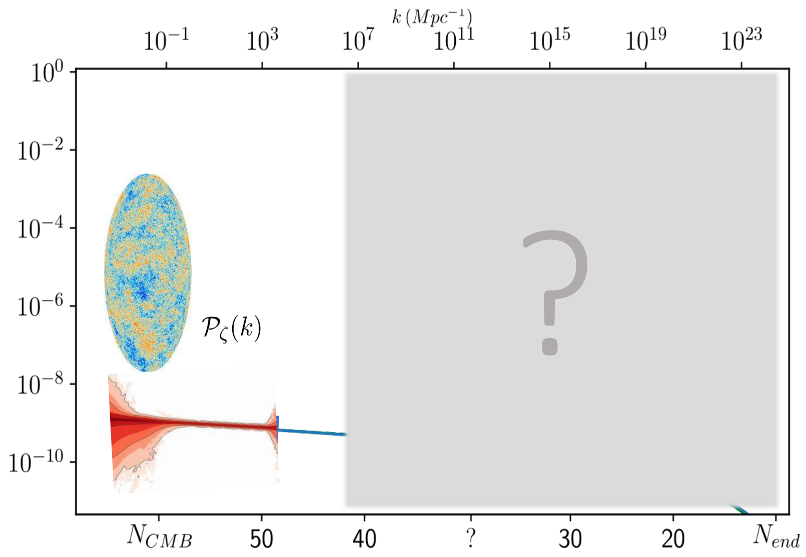
What signatures probe physics at these scales?

Is perturbation theory enough?

Are these scenarios consistent at loop level?

What are the detection prospects for a primordial signal?

QUESTIONS



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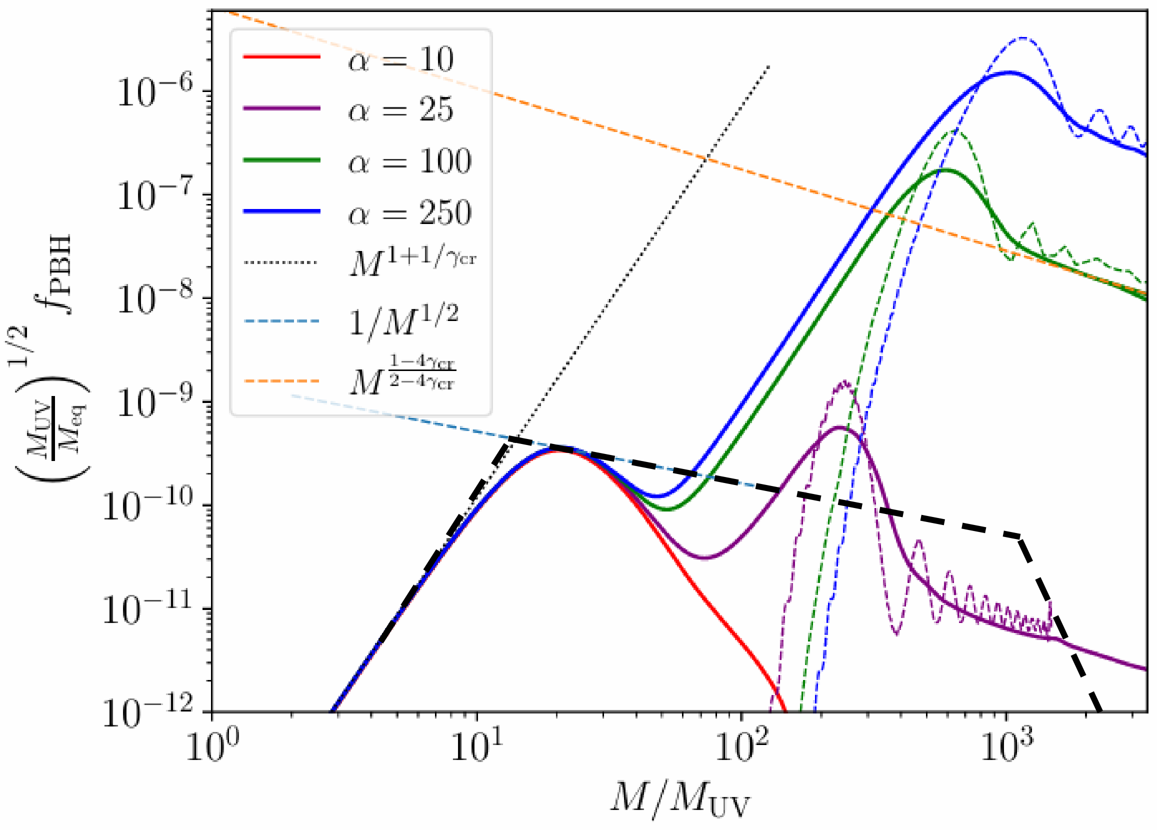
( ) PRIMORDIAL BLACK HOLES STATISTICS

(Standard) Broad spectrum leads to unexpected shape of the PBH mass function

$$\mathcal{P}_\zeta(k) = A_s\,\theta(k - k_{\rm IR})\,\theta(k_{\rm UV} - k), \qquad \alpha \equiv k_{\rm UV}/k_{\rm IR} \gg 1$$



JF, J. Garriga, C. Germani, R. Shet 2412. 07709

SCALAR INDUCED gravitational waves

SCALAR PERTURBATIONS ACT AS A TENSOR SOURCE

$$\boxed{\Box h_{ij} = S_{ij}^{TT}} \sim (\partial\zeta)^2$$

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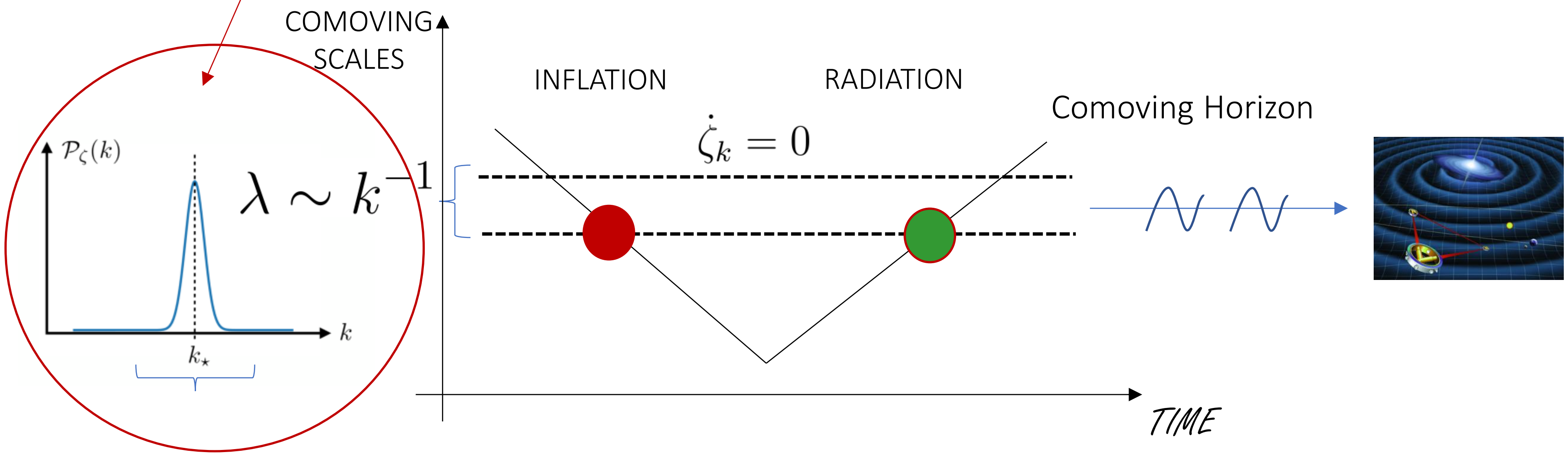

Acquaviva et al. ’02;
Mollerach, Harari, Matarrese ’03;
Ananda, Clarkson, Wands ’06;
Baumann et al. ’07

GWs SENSITIVE TO THE PRIMORDIAL FLUCTUATIONS

SCALAR INDUCED gravitational waves

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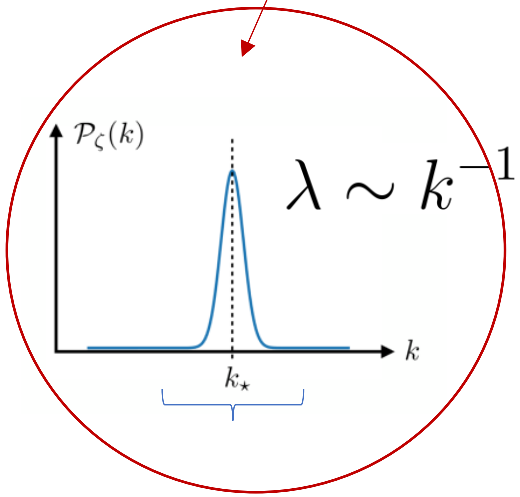
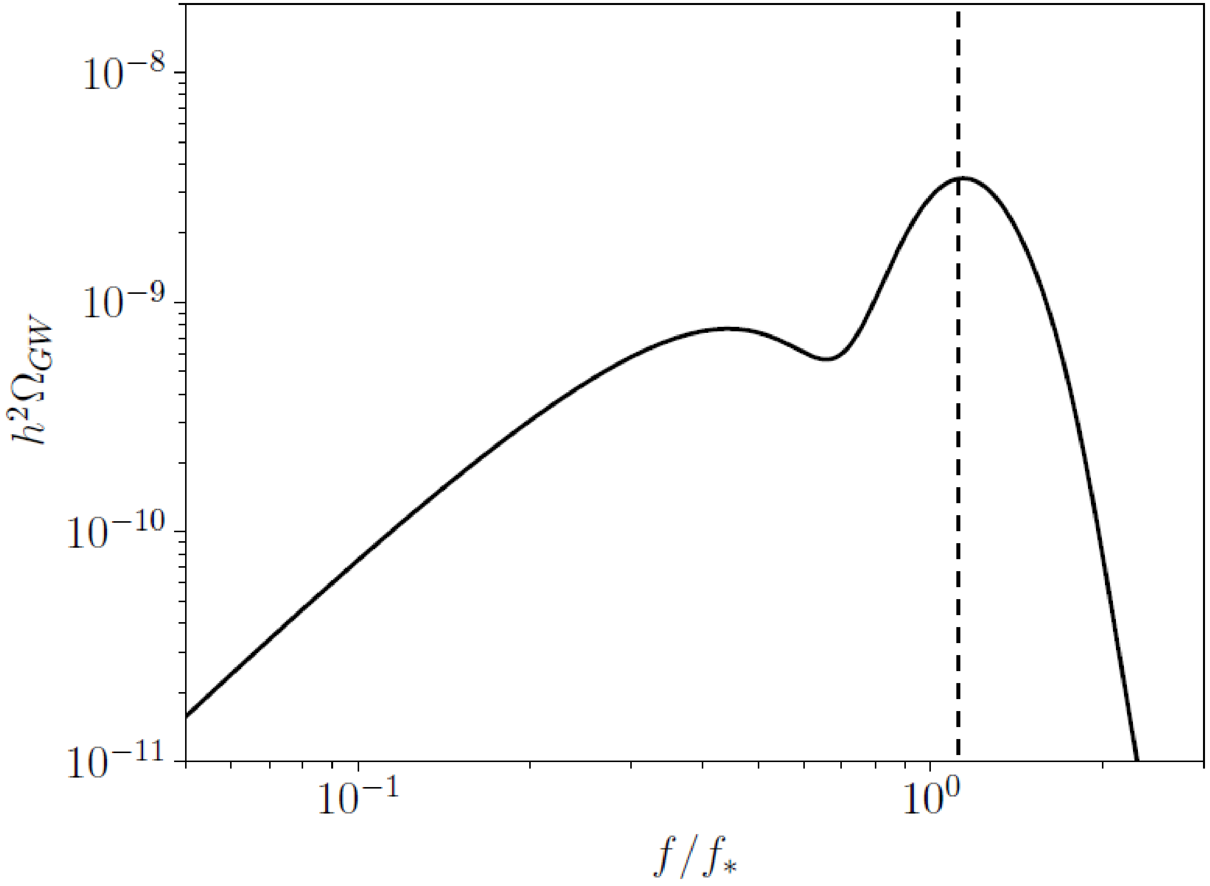
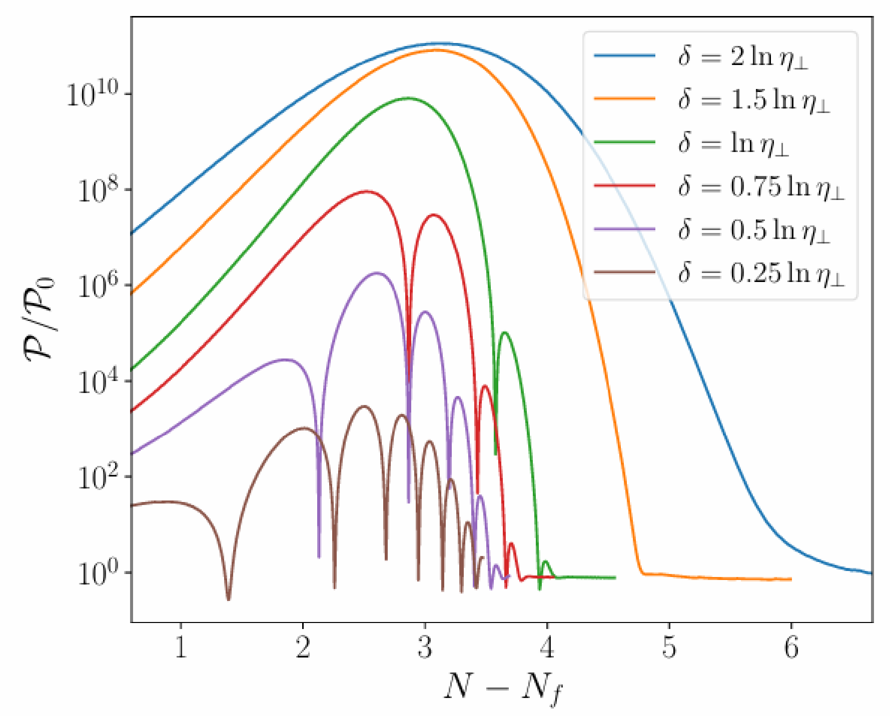




PRIMORDIAL FEATURES AT SMALL SCALES

If phenomenon triggering the enhancement last short enough..

spectrum develops linear <u>CHARACTERISTIC OSCILLATIONS</u>



J.F., S. Renaux-Petel, L. Witkowski 2012.02761

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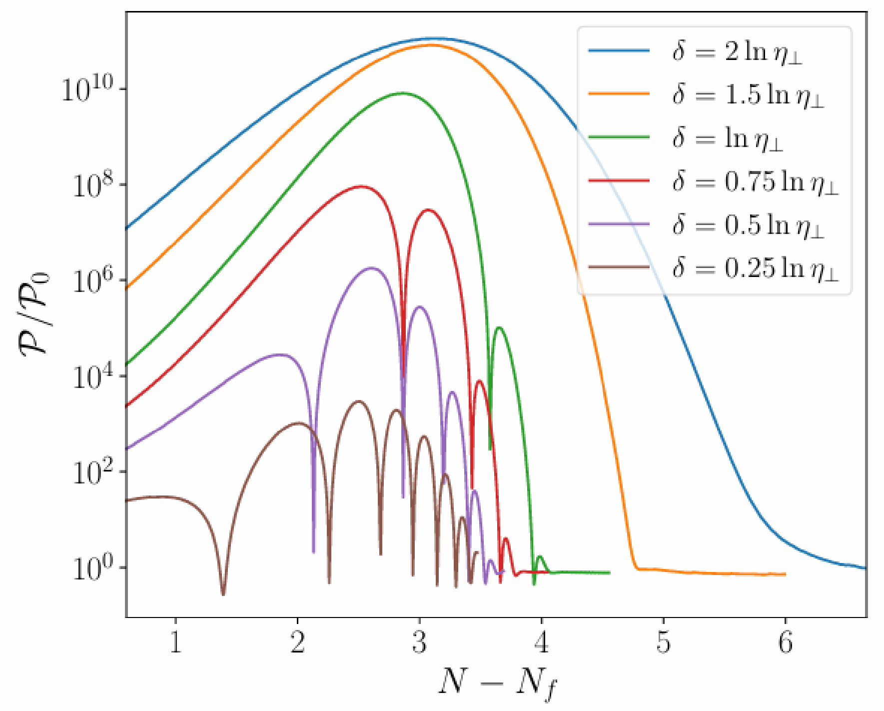
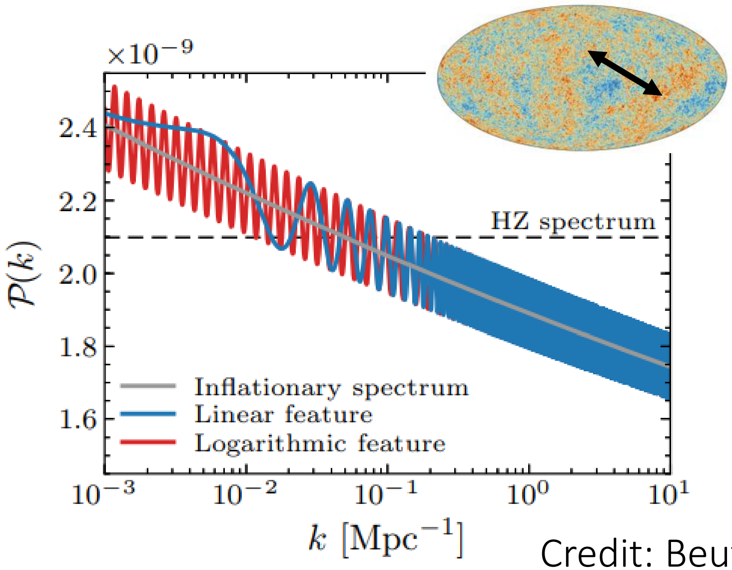


Remember M. Ballardini's talk



Credit: Beutler et al. '19

J.F., S. Renaux-Petel, L. Witkowski 2012.02761

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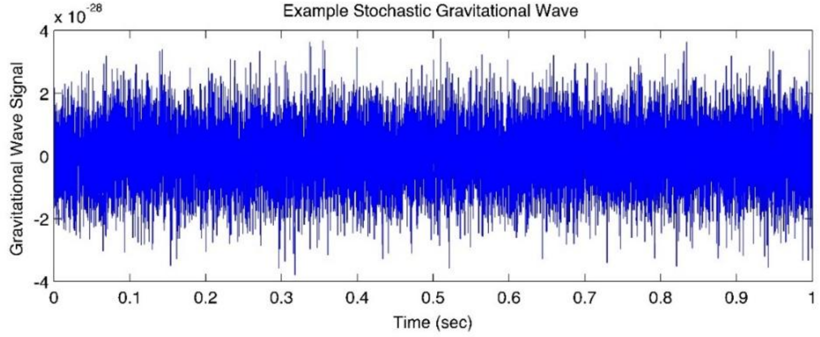
Large particle production

→ O(1) oscillations

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\left(1 + A_{\rm lin}\cos\left(\omega_{\rm lin} k + \phi_{\rm lin}\right)\right)$$

$$\mathcal{P}_\zeta(k) = \overline{\mathcal{P}}(k)\left(1 + A_{\rm log}\cos\left(\omega_{\rm log}\log(k/k_{\rm ref}) + \phi_{\rm log}\right)\right)$$



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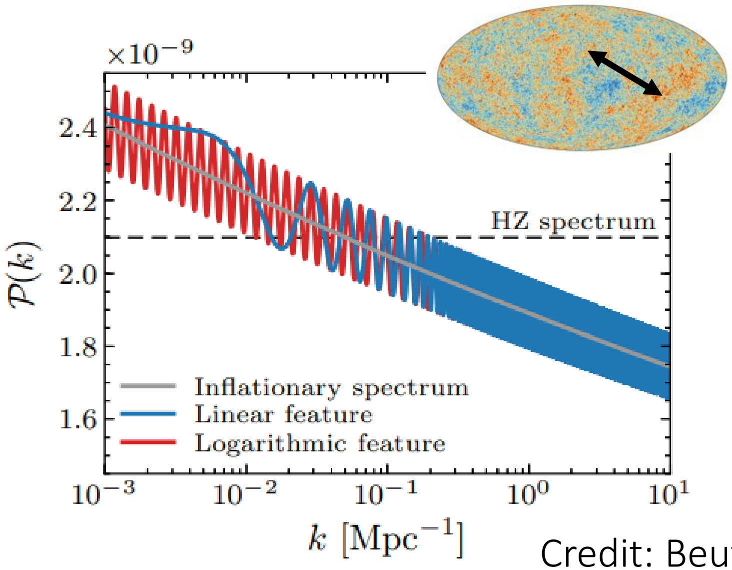

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J.F., S. Renaux-Petel, L. Witkowski 2012.02761

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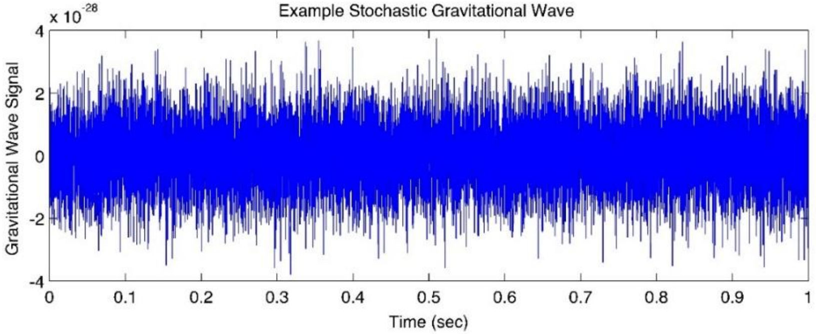

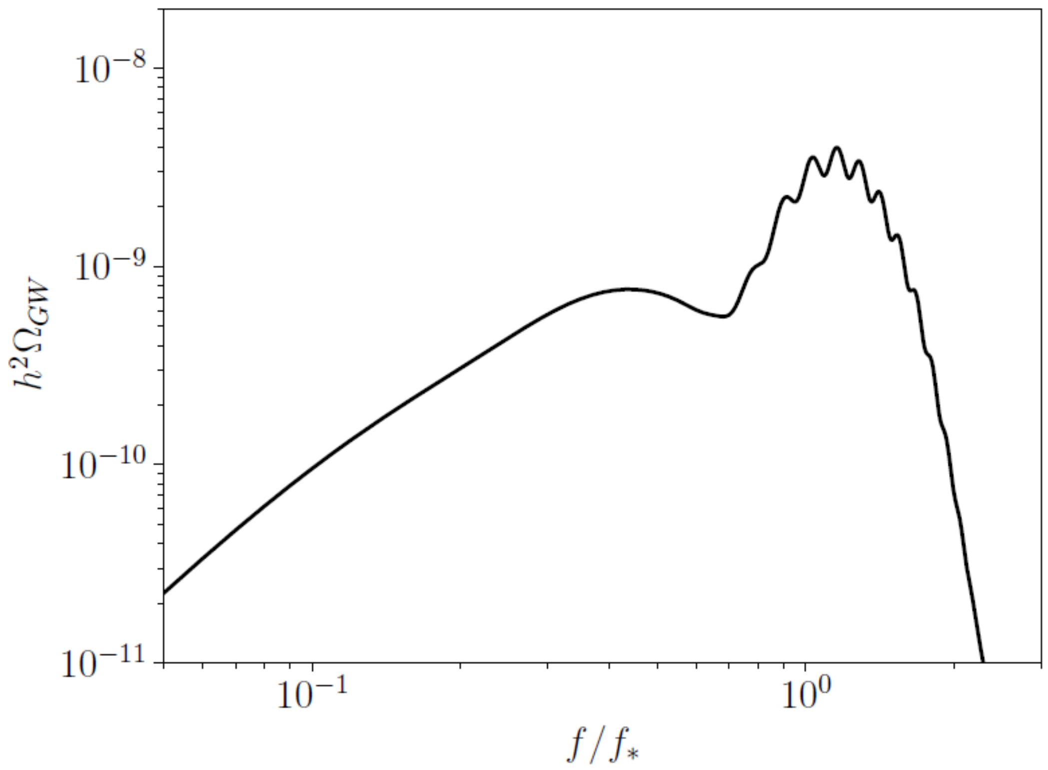

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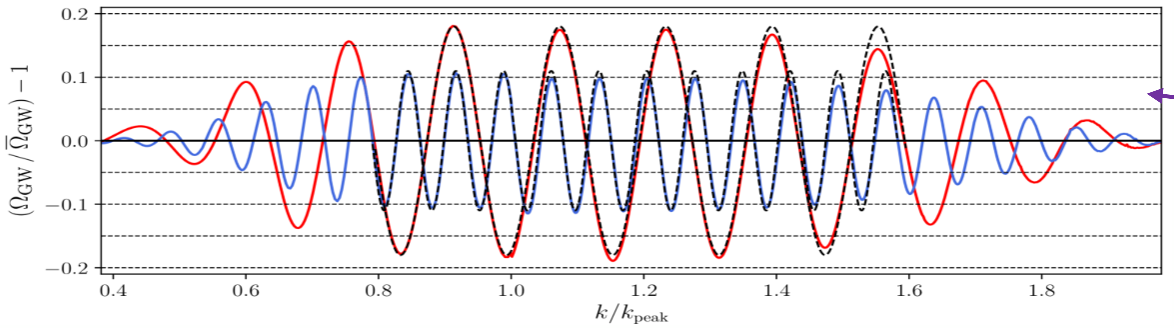
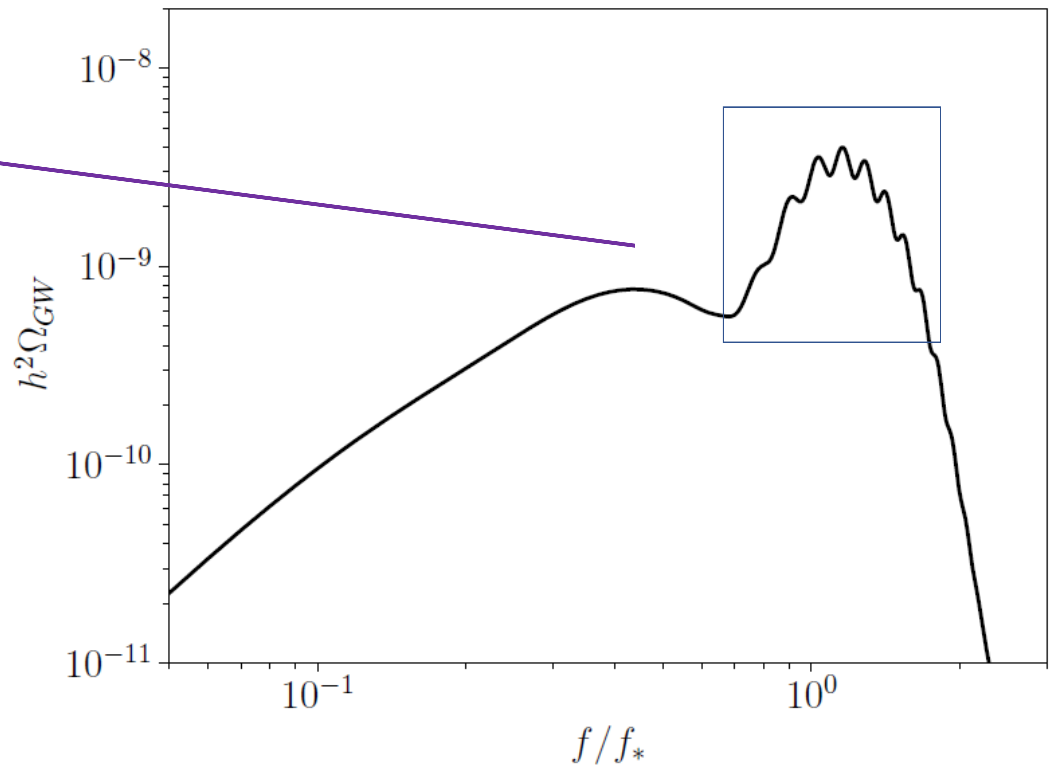


- Energy scale
- When during inflation?
- For how long?

$$\omega_{\rm lin}^{\rm GW} = \sqrt{3}\,\omega_{\rm lin}$$

JF , S. Renaux-Petel, L. T. Witkowski, 2012.02761



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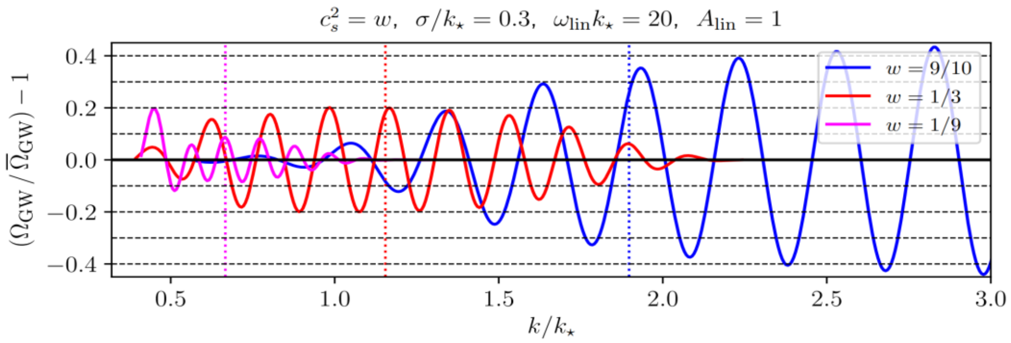

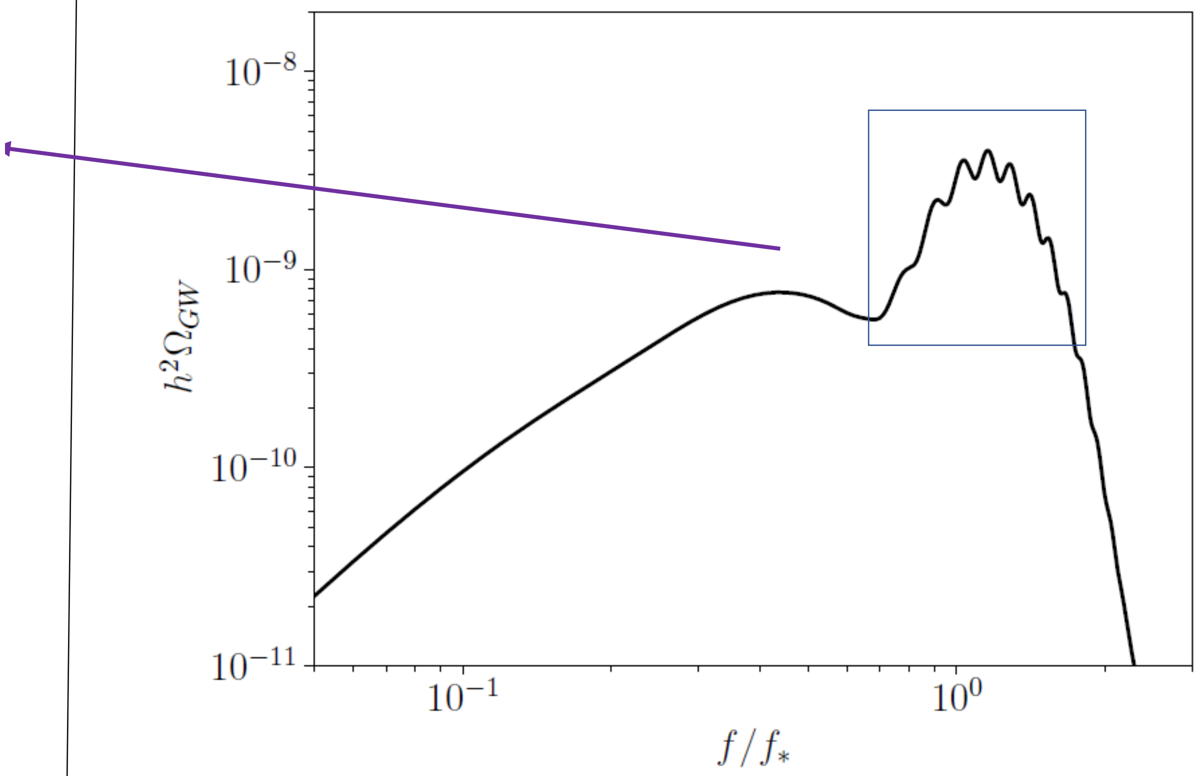

- Energy scale
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$$\omega_{\rm lin}^{\rm GW} = \sqrt{3}\,\omega_{\rm lin}$$

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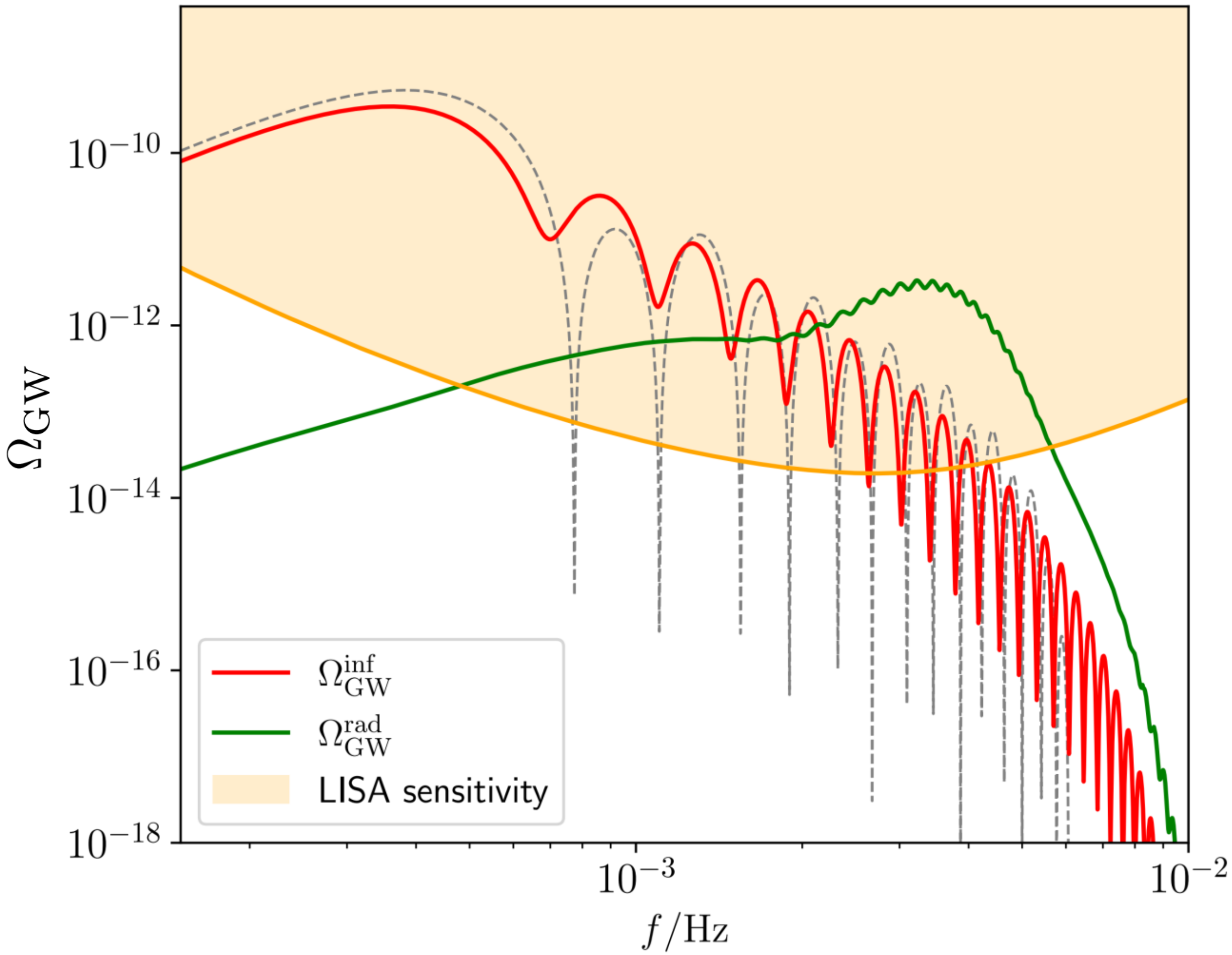
- Cosmic expansion at horizon re-entry

L. T. Witkowski, G. Domenech,
JF, S. Renaux-Petel 2110.09480

J.F., S. Renaux-Petel, L. Witkowski 2012.02761

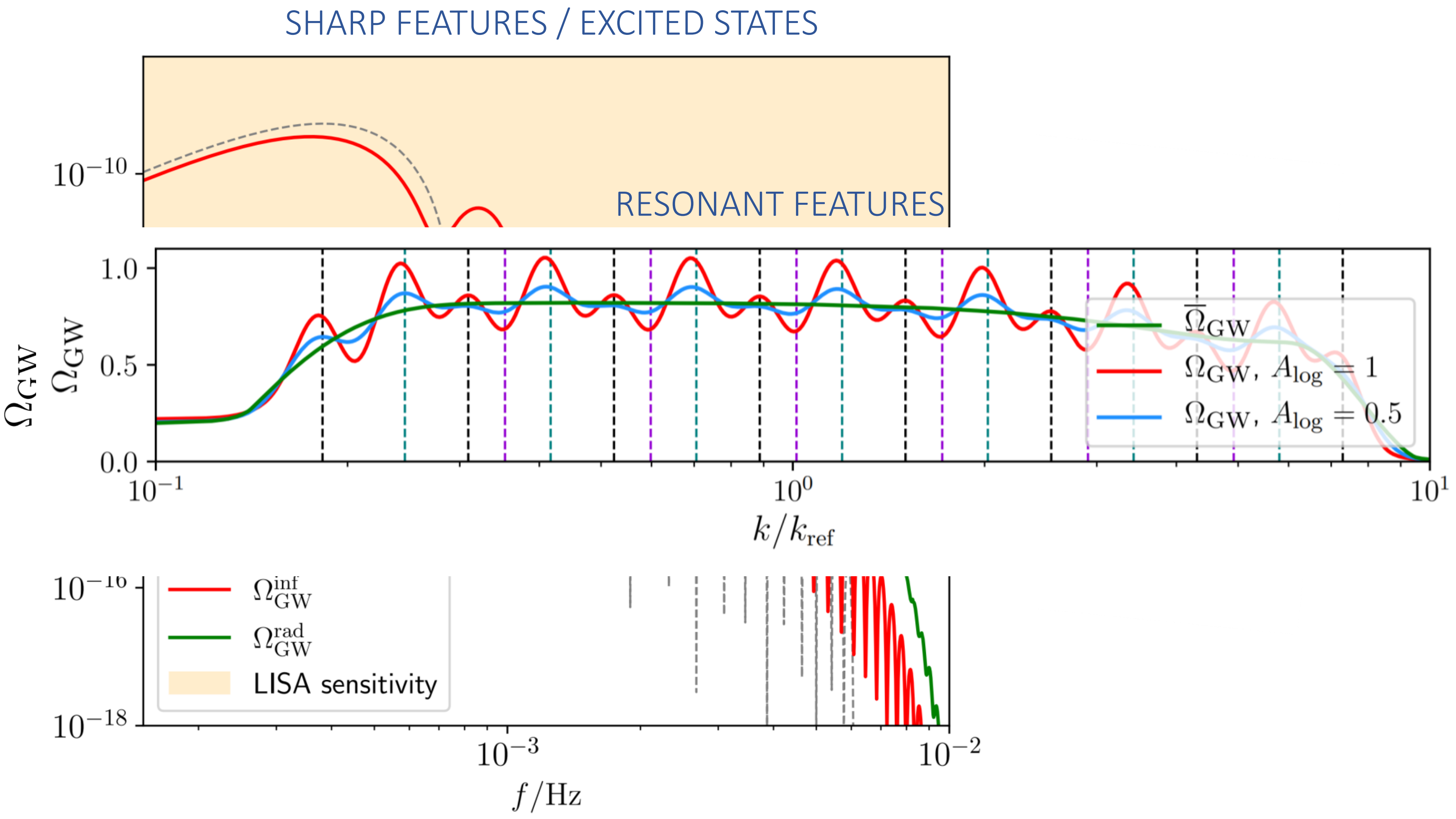
PRIMORDIAL FEATURES AT SMALL SCALES

SHARP FEATURES / EXCITED STATES



JF , S. Renaux-Petel, S.Sypsas, G.Palma, L. T. Witkowski, C. Zenteno 2111.14664

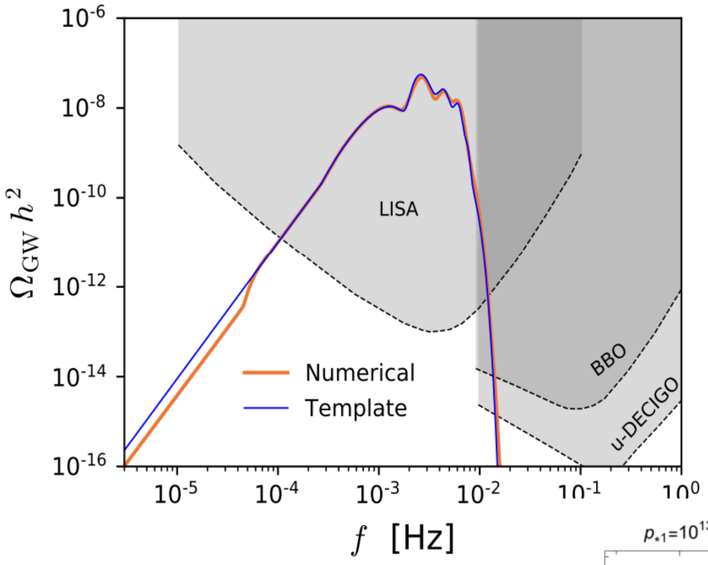
PRIMORDIAL FEATURES AT SMALL SCALES

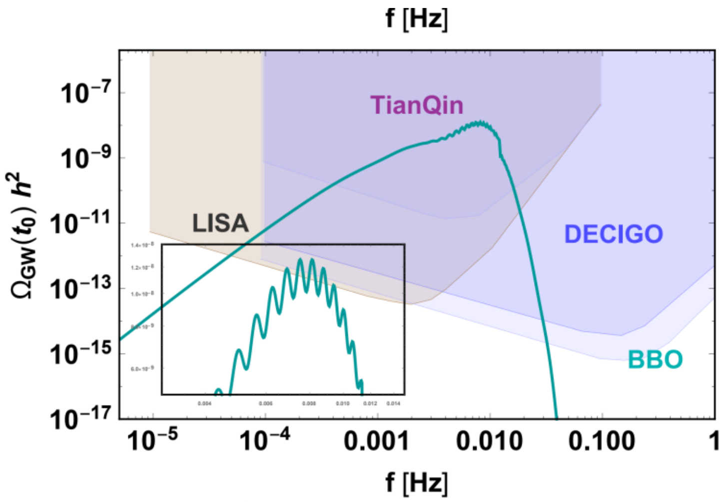


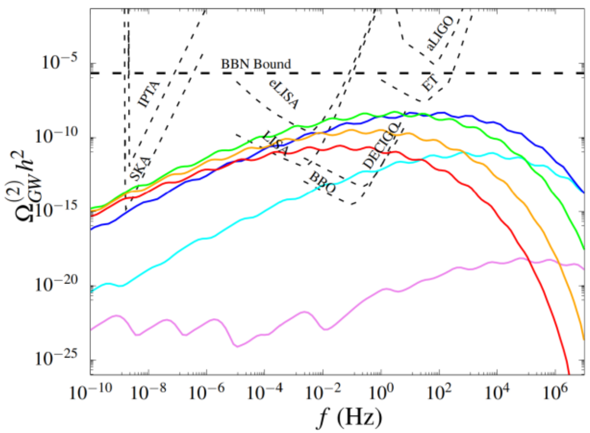
JF , S. Renaux-Petel, L. T. Witkowski 2105.06481

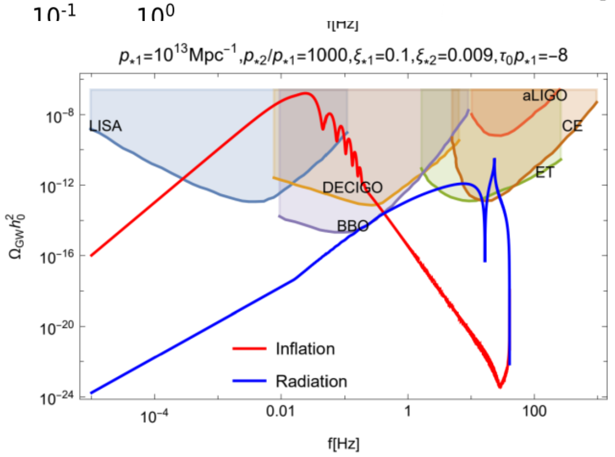
After first proposal 2012.02761

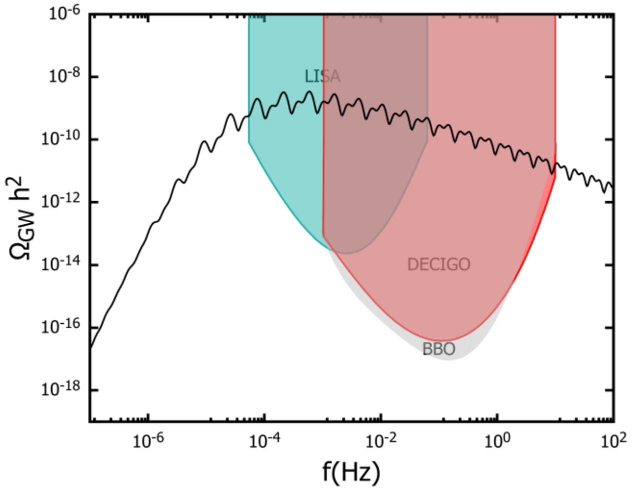
EXPLICIT MODELS LEADING TO FEATURES PROLIFERATE











Braglia, Chen ’20
Dalianis, G.P. Kodaxis, I.D. Stamou,
N. Tetradis and A. Tsigkas-Kouvelis ’21
Battacharya, Zavala ’22 ..

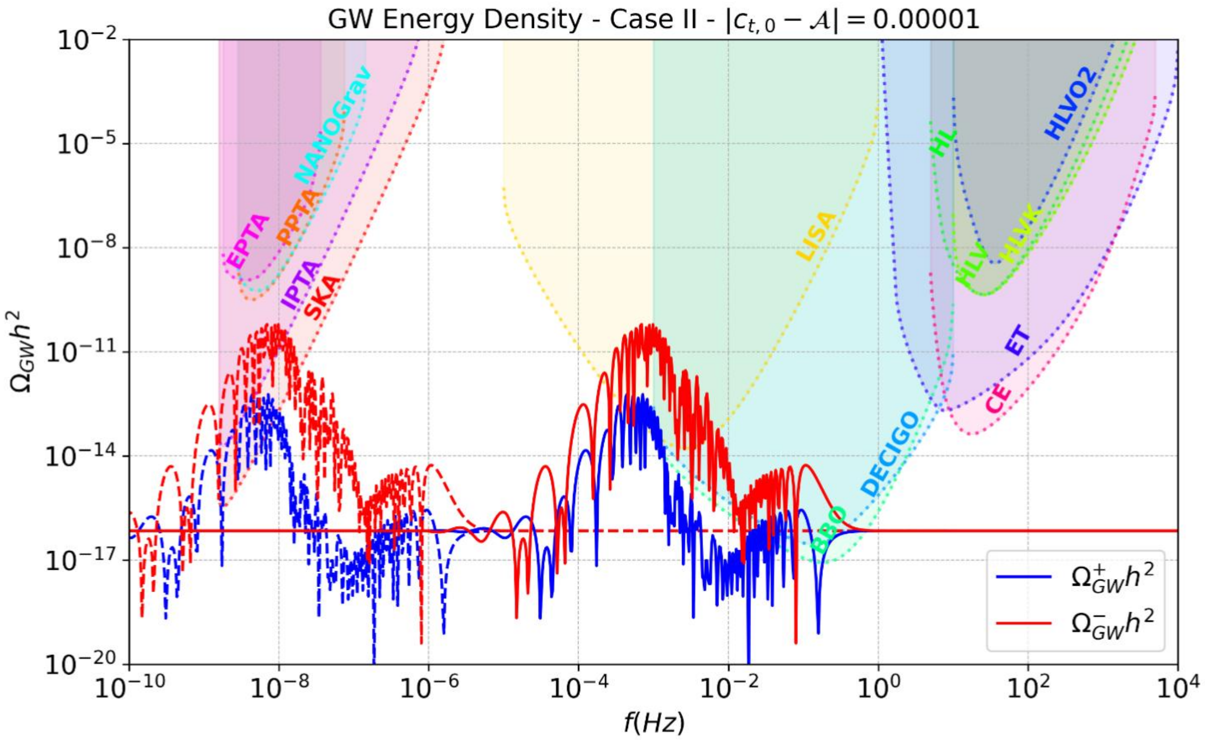
Addazzi, Capoziello, Gan ‘22
N. Mavromotos, V. Spanos, I. Stamou ’22 ..
....

MORE RECENTLY...

Extra spin-2 field:

Inspired/motivated by spin-two spectator field, bi-gravity etc: Bodrin, Creminelli, Khmelnitsky, Senatore ’18, De Rham, Gabadadze, Tolley ‘11, Hassan and Rosen ‘12

$$\Box h_{ij} = S_{ij}^{TT} \sim t_{ij}$$



see M. Ali Gorji’s talk

J. Garriga, M. Ali Gorji, F. Hajkarim and M Sasaki

BEYOND LEADING ORDER ?

$$\boxed{\Box h_{ij} = S_{ij}^{TT}} \sim (\partial\zeta)^2 \qquad \Omega_{\mathrm{GW}} \sim \langle h_{ij} h_{ij} \rangle \sim \int \langle \zeta\,\zeta \rangle \langle \zeta\,\zeta \rangle + \langle \zeta\,\zeta\,\zeta\,\zeta \rangle_c$$

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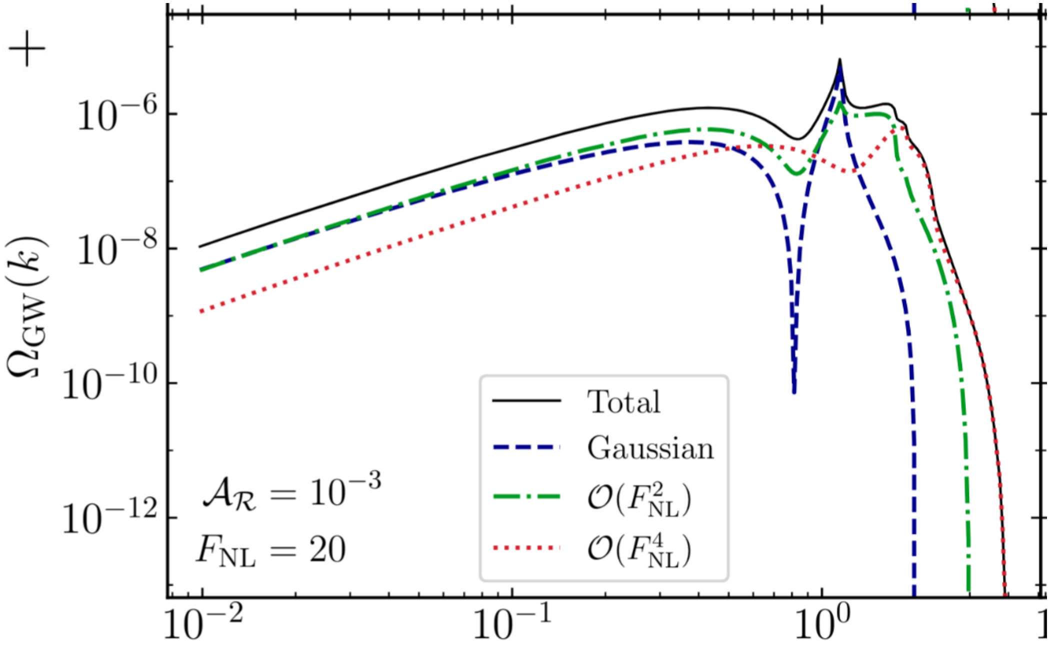
Option 1: Non-Gaussianities

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL}\left(\zeta_g^2(\mathbf{x}) - \langle \zeta_g^2 \rangle\right) + \frac{9}{25} g_{\rm NL} \zeta_g^3(\mathbf{x}) +$$



Next: P. Adshead, K. Lozanov, Z. Weiner '21,
Next-to-Next: G. Perna, C. Testini, A. Ricciardone, S. Matarrese '24
Lattice: X. Zeng, Z. Ning, R.G. Cai, and S.J. Wang '25

BEYOND LEADING ORDER ?

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Option 1: Non-Gaussianities

"No-go theorem for scalar-Trispectrum":
S. Garcia-Saenz, L. Pinol, S. Renaux-Petel, D. Werth '22

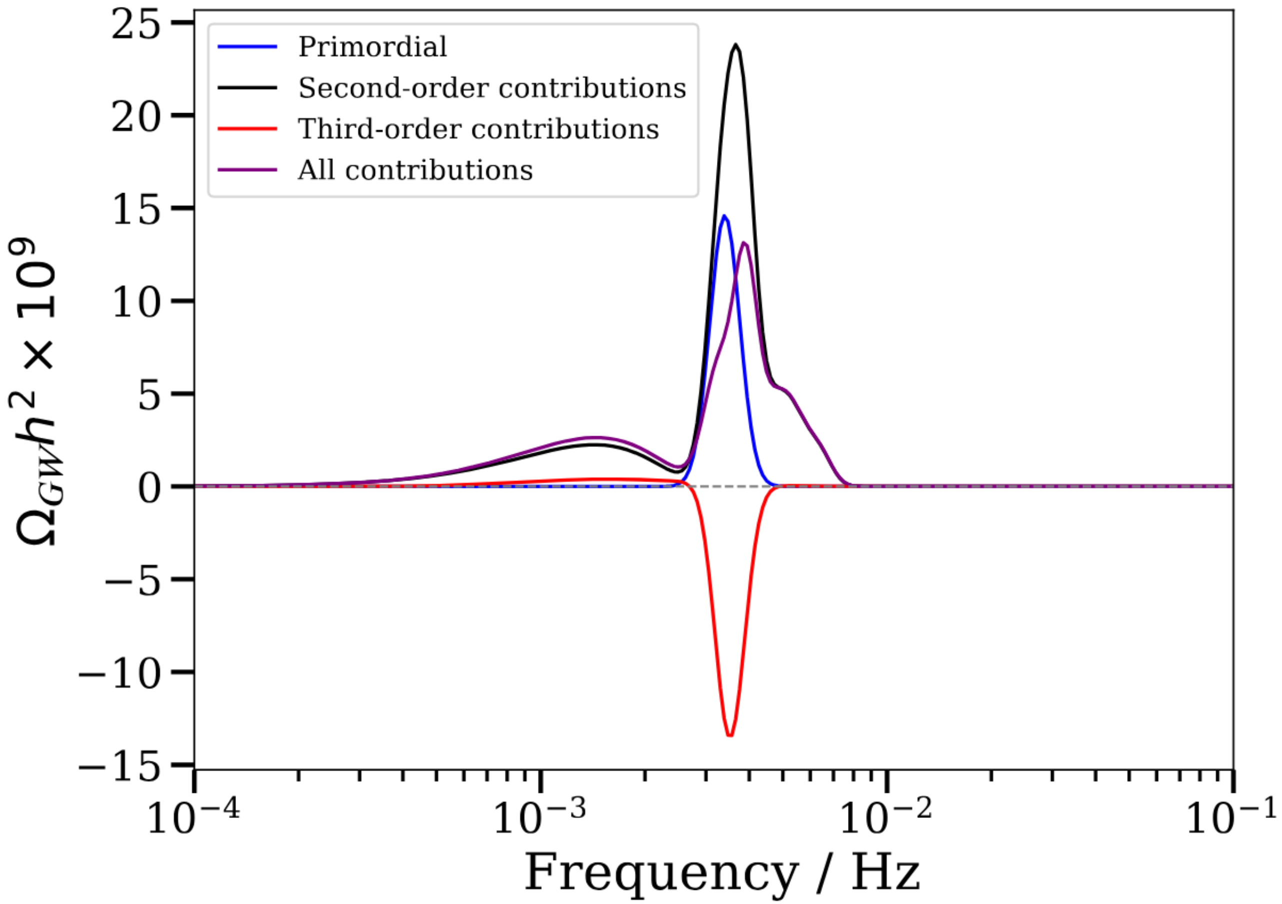
$$P_\zeta^{(1-\text{loop})} / P_\zeta^{(\text{tree})} < 1 \quad \Longrightarrow \quad \frac{\Omega_{\rm GW,c}}{\Omega_{\rm GW,d}} \ll 1$$

BEYOND LEADING ORDER ?

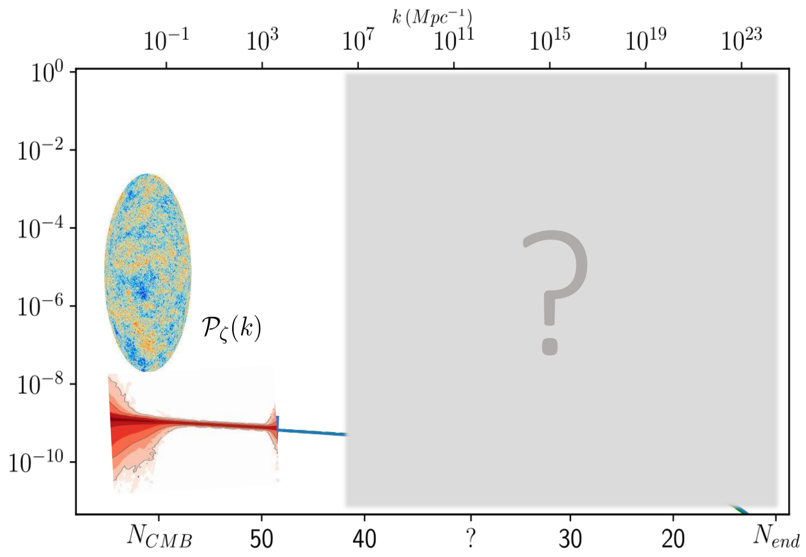
$$\Box h_{ij} = S_{ij}^{TT} \quad \sim h\zeta\zeta + \ldots \qquad \Omega_{\rm GW} \sim \langle h_{ij} h_{ij} \rangle \sim \quad + \langle h^{(3)} h^{(1)} \rangle + \langle h^{(1)} h^{(3)} \rangle$$

Option 2: Higher-order interactions

C. Chen, A. Ota, H. Y. Zhu and Y. Zhu '22
R. Picard, L. E. Padilla,
K. A. Malik, and D. J. Mulryne '25



QUESTIONS



What signatures probe physics at these scales?

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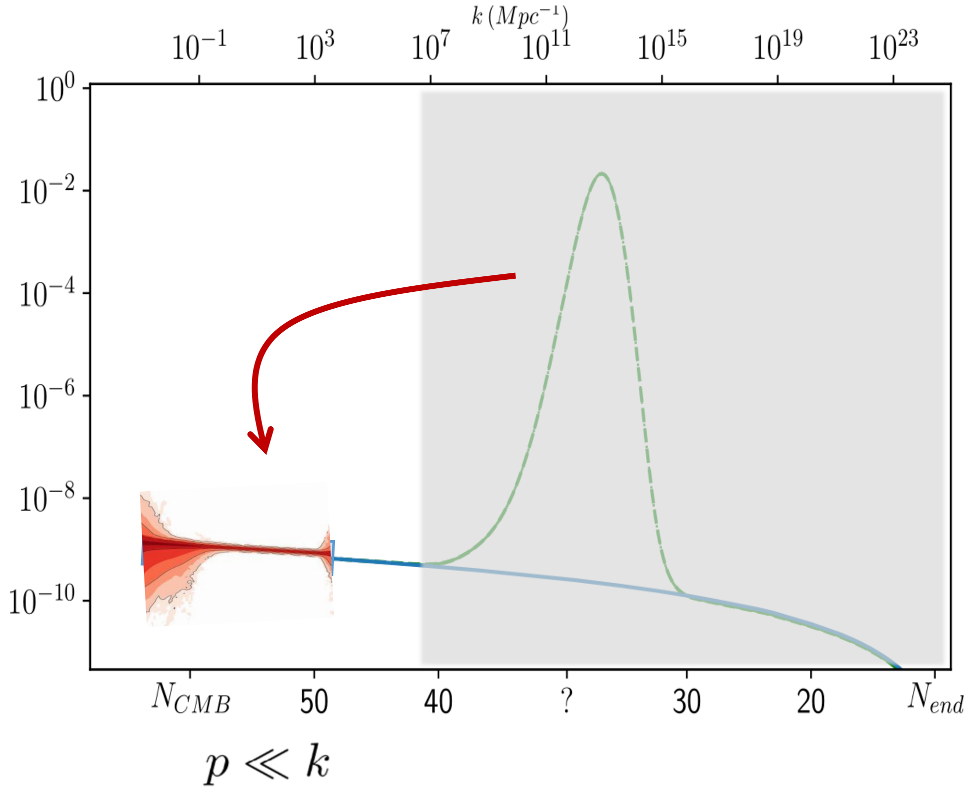
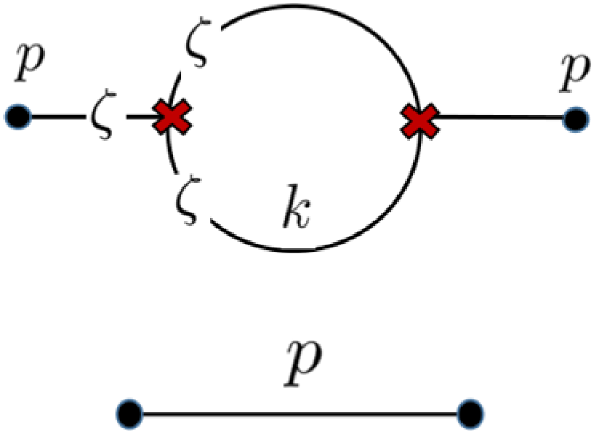
Are these scenarios consistent at loop level?

What are the detection prospects for a primordial signal?

Could small scales perturbations lead to an effect on large scales ?





J. Kristiano, J. Yokoama ‘22,

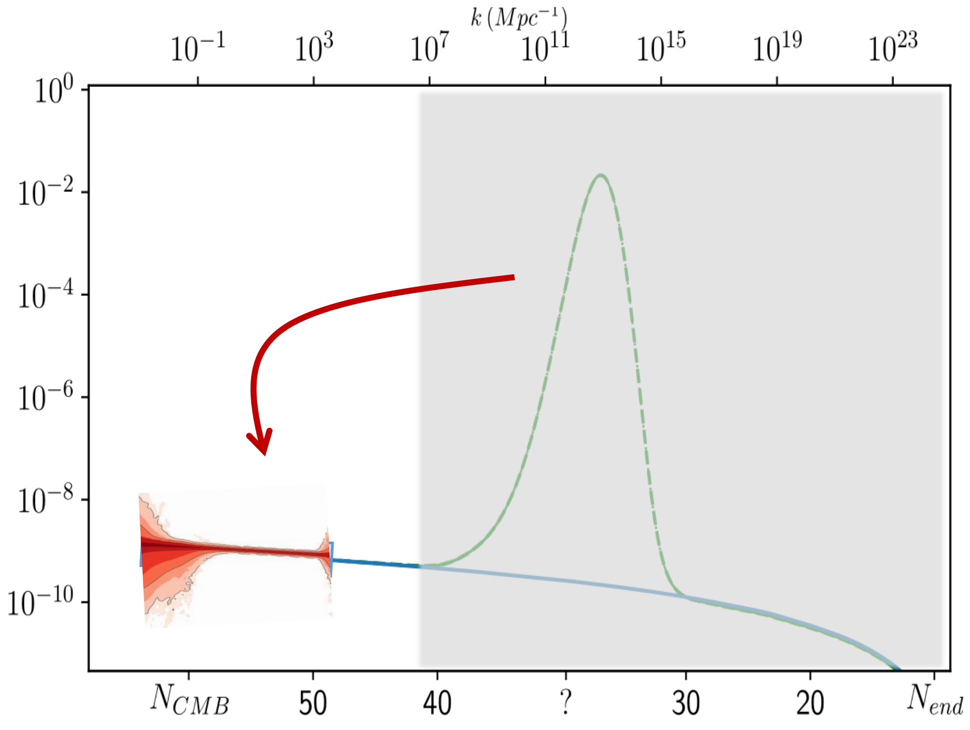
A. Riotto ‘23, H. Fioruzjhai´23, A.Riotto and J. FirouzJahi ´23, G. Franciolini et al. ‘23,

L. Iacconi, D. Seery, D. Mulnryne ‘24, K. Inomata ‘24 +….

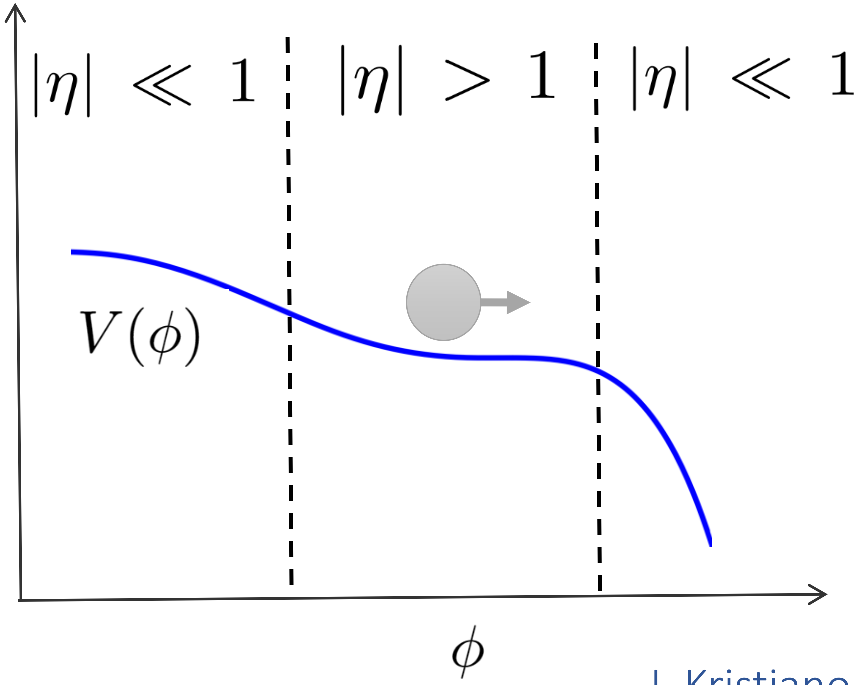
NON-SLOW ROLL INFLATION

Standard way (non-slow-roll) to enhance the power spectrum: $|\eta| > 1$

$$\epsilon \equiv -\dot{H}/H^2 \qquad \eta \equiv \frac{d\ln\epsilon}{dN}, \quad \longrightarrow \quad H^{(3)} \supset -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta'$$



$p \ll k$

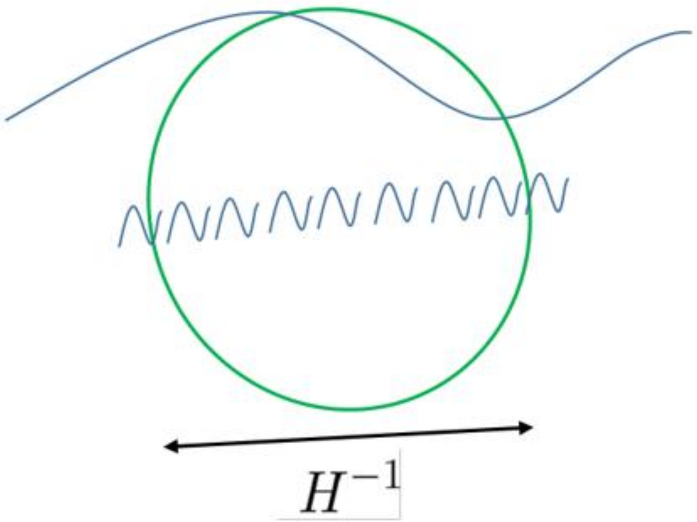


J. Kristiano, J. Yokoama '22,

ONE-LOOP CORRECTIONS

$$\mathcal{P}_\zeta^{1-\text{loop}}(p) = \mathcal{P}^{\text{tree}}(p) \int d\ln k\, C(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

J. Kristiano, J. Yokoama ‘22,
A. Riotto ‘23, H. Fioruzjhai´23,
A.Riotto and J. FirouzJahi ´23, G.
Franciolini et al. ’23…



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A.Riotto and J. FirouzJahi ´23, G.
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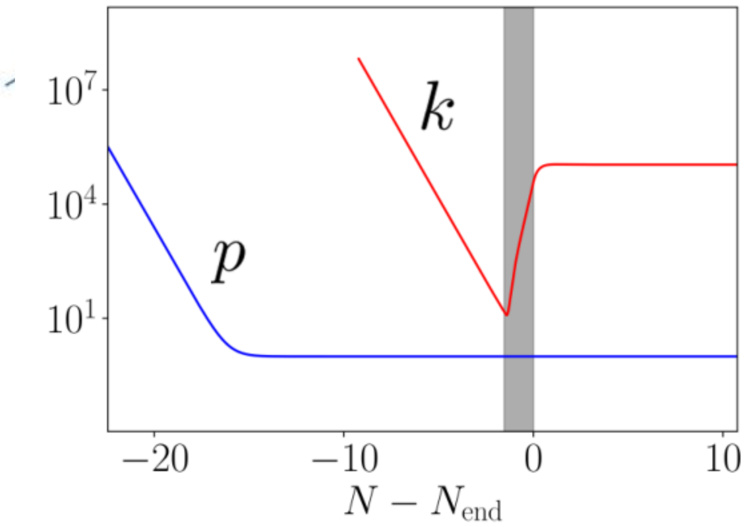
IMPLICATIONS

- Small scales / Large scales effect which is scales independent ?
- Arbitrary super-horizon time evolution of zeta?



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JF, 2305.19263,
JF, 2408.08269

See also Y. Tada, T. Terada and J. Tokuda '23
I. Keisuke '24
R. Kawaguchi, S. Tsujikawa and Y. Yamada '24

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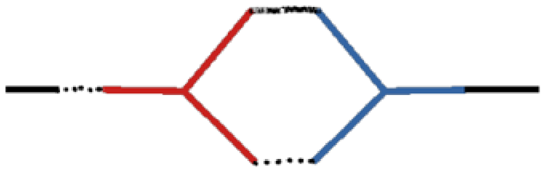
JF, 2305.19263,
JF, 2408.08269

Nothing special about non-slow-roll, many many of these contributions…
…due to the bad memory of the commutators outside the horizon.

RELEVANT TERMS TO SHOW THE CANCELLATION

$$\mathcal{H}_{\text{int}} = -\mathcal{L}^{(3)}$$

$$\mathcal{L} = \mathcal{L}^{(2)}(\zeta, \zeta') + \mathcal{L}^{(3)}(\zeta, \zeta') + \ldots$$



RELEVANT TERMS TO SHOW THE CANCELLATION

$$\mathcal{H}_{\rm int} = -\mathcal{L}^{(3)}$$

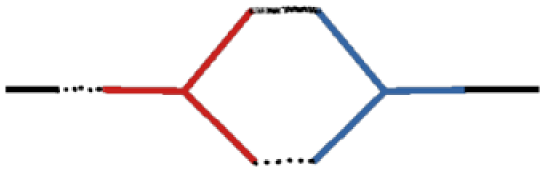
- Cubic interactions – relevance of boundary terms

$$\mathcal{L} = \mathcal{L}^{(2)}(\zeta, \zeta') + \mathcal{L}^{(3)}(\zeta, \zeta') + \dots$$

e.g.

$$\supset \quad -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] \; + \; \dots$$

JF, 2305.19263



RELEVANT TERMS TO SHOW THE CANCELLATION

$$\mathcal{H}_{\rm int} = -\mathcal{L}^{(3)} + \mathcal{H}_3^{(4)} + \mathcal{H}_{\rm diff}^{(4)}$$

- Cubic interactions – relevance of boundary terms

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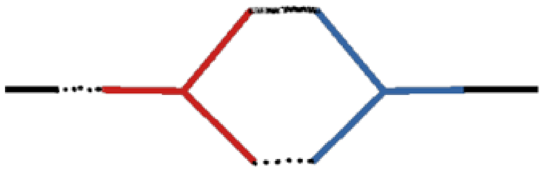
e.g.

$$\supset \quad -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right] + \ldots$$



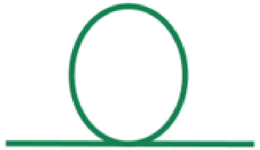
JF, 2305.19263

- Quartic induced interactions

$$P = \frac{\delta\mathcal{L}}{\delta\zeta'} = \frac{\delta\mathcal{L}^{(2)}}{\delta\zeta'} + \frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'} + \ldots \quad \Rightarrow \quad \mathcal{H}_3^{(4)} = \frac{1}{2(2a^2\epsilon)}\left(\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right)^2$$

- Quartic diff. induced

$$\zeta \to \zeta + b, \quad x^i \to x^i e^{-b} + C^i \quad \Rightarrow \quad \mathcal{H}_{\rm diff}^{(4)}$$



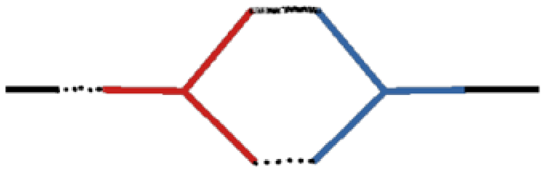
RELEVANT TERMS TO SHOW THE CANCELLATION

$$\mathcal{H}_{\rm int} = \underline{-\mathcal{L}^{(3)}} + \underline{\mathcal{H}_3^{(4)}} \;+\; \underline{\mathcal{H}_{\rm diff}^{(4)}} \;+\; \underline{\mathcal{H}_1^{(2)}}$$

- Cubic interactions – relevance of boundary terms
  e.g.

$$\mathcal{L} = \mathcal{L}^{(2)}(\zeta,\zeta') + \mathcal{L}^{(3)}(\zeta,\zeta') + \ldots$$

$$\supset \quad -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' + \underline{\frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right]} \;+\; \ldots$$



JF, 2305.19263

- Quartic induced interactions

$$P = \frac{\delta\mathcal{L}}{\delta\zeta'} = \frac{\delta\mathcal{L}^{(2)}}{\delta\zeta'} + \frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'} + \ldots \;\Longrightarrow\; \mathcal{H}_3^{(4)} = \frac{1}{2(2a^2\epsilon)}\left(\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right)^2$$

- Quartic diff. induced

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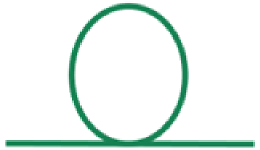


- Quadratic tadpoles induced interactions

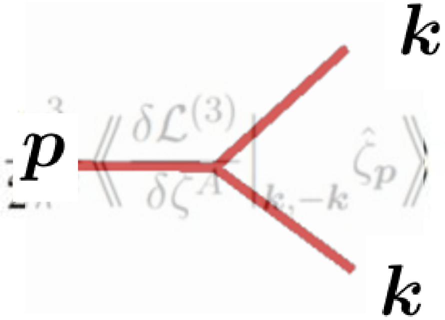
JF, 2408.08296

$$\mathcal{L}_{\rm tad}^{(1)} = c\,\zeta' \quad c = -\left\langle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right\rangle, \;\Longrightarrow\; \mathcal{H}_1^{(2)} = \frac{c}{2a^2\epsilon}\,\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'},$$

Roadmap

1. One-loop as three-point function

$$\mathcal{P}^{1-\text{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \ldots$$



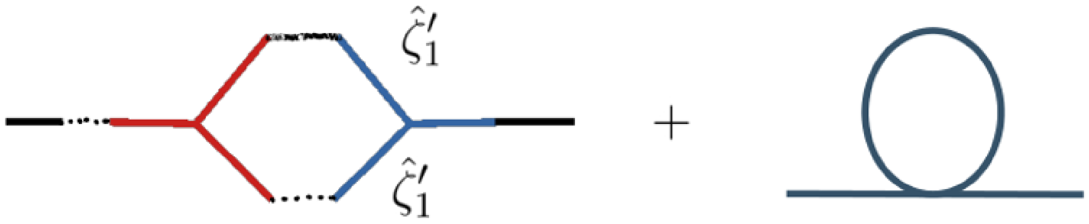
$\boldsymbol{p} \ll \boldsymbol{k}$

1-LOOP AS 3-POINT FUNCTIONS AND QUARTIC INTERACTIONS

- **MIRACLE #1**: Quartic induced Hamiltonian to build 3-point functions

$$\mathcal{H}_{\rm int} = -\mathcal{L}^{(3)} + \underline{\mathcal{H}_3^{(4)}}$$

G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651



E.g.

$$= -\int d\tau_1 \, \langle \hat{\zeta}_1' \hat{\zeta}_1' \hat{\zeta}_{\boldsymbol{p}} \rangle$$

1-LOOP AS 3-POINT FUNCTIONS AND QUARTIC INTERACTIONS

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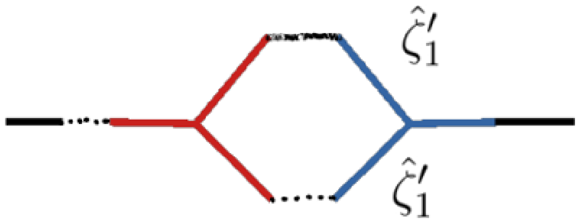

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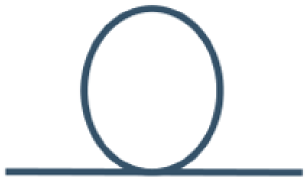


$$= -\int d\tau_1 \quad \langle \hat{\zeta}_1' \hat{\zeta}_1 \hat{\zeta}_{\boldsymbol{p}} \rangle$$

- Caveat Spurious contribution from $\mathcal{H}_3^{(4)}$ when building

MIRACLE #2: They cancel exactly from the "tadpole induced Hamiltonian"

$$\mathcal{H}_{\rm int} = -\mathcal{L}^{(1)}_{\rm tad} + \mathcal{H}_1^{(2)}$$

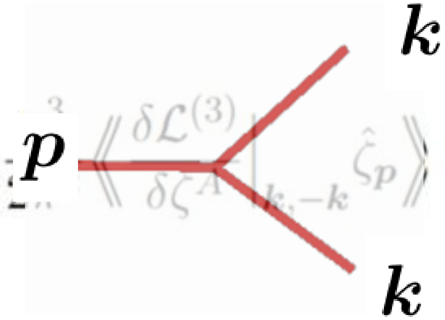
$+$

$= 0$ JF, 2408.08296

Roadmap

1. One-loop as three-point function

$$\mathcal{P}^{1-\mathrm{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \cdots \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{k,-k} \hat{\zeta}_p \right\rangle\!\!\right\rangle$$



$\boldsymbol{p} \ll \boldsymbol{k}$

Roadmap

1. One-loop as three-point function

$$\mathcal{P}^{1-\text{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{p^3}{2\pi^2} \left\langle\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{\boldsymbol{k},-\boldsymbol{k}} \hat{\zeta}_{\boldsymbol{p}} \right\rangle\!\right\rangle$$

2. Consistency relations

$$\mathcal{P}^{\text{tree}}(p) \frac{d}{d \ln k} \left\langle\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{\boldsymbol{k}} \right\rangle\!\right\rangle,$$

Roadmap

1. One-loop as three-point function

$$\mathcal{P}^{1-\text{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{p^3}{2\pi^2} \left\langle\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\Big|_{\boldsymbol{k},-\boldsymbol{k}} \hat{\zeta}_{\boldsymbol{p}} \right\rangle\!\right\rangle$$

2. Consistency relations

Unless $(\partial_i\zeta)^2$

MIRACLE #3: Include quartic interactions implied by residual diff. invariance

$$g_{ij} = a^2 e^{2\zeta} \qquad \zeta \to \zeta + b, \quad x^i \to x^i e^{-b} + C^i$$

Invariant building block: $e^{-\zeta}\partial_i\zeta$

E.g. $\mathcal{L}^{(3)} \supset -c_1\zeta'(\partial_i\zeta)^2 \Longrightarrow \mathcal{L}^{(4)}_{\text{diff}} \supset 2c_1\zeta'\zeta(\partial_i\zeta)^2$

Y. Urakawa and T. Tanaka 0902.3209, 1007.0468
G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651

Roadmap

1. One-loop as three-point function

$$\mathcal{P}^{1-\mathrm{loop}}(p) \propto \int_{\tau_0}^{\tau} d\tau_1 \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{p^3}{2\pi^2} \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{\boldsymbol{k},-\boldsymbol{k}} \hat{\zeta}_{\boldsymbol{p}} \right\rangle\!\!\right\rangle$$

2. Consistency relations

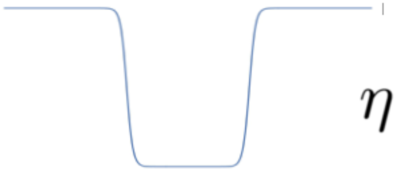
$$\mathcal{P}^{\mathrm{tree}}(p) \frac{d}{d\ln k} \left\langle\!\!\left\langle \frac{\delta \mathcal{L}^{(3)}}{\delta \zeta^A}\bigg|_{\boldsymbol{k}} \right\rangle\!\!\right\rangle,$$

3. Full one-loop computation

SUMMARY

$$\mathcal{L}^{(3)} = \mathcal{L}^{(3)}_{\rm bulk} + \mathcal{L}^{(3)}_{\partial} + \mathcal{L}^{(3)}_{\rm eom}$$

J. Fumagalli, 2408.08296

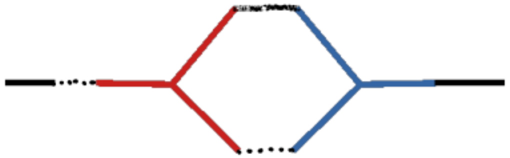


η

$$\mathcal{L}^{(3)} = \frac{a^2\epsilon}{2}\eta'\zeta^2\zeta' - \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta' + \frac{\epsilon a^2}{aH}\zeta'^2\zeta\right] + \mathcal{E}_\zeta\left(\frac{\eta}{2}\zeta^2 + \frac{2}{aH}\zeta'\zeta\right).$$

- Cubic Hamiltonian

$$\mathcal{H}^{(3)}_a = -\frac{a^2\epsilon}{2}\eta'\zeta^2\zeta', \quad \mathcal{H}^{(3)}_b = \frac{d}{d\tau}\left[\frac{a^2\epsilon\eta}{2}\zeta^2\zeta'\right], \quad \mathcal{H}^{(3)}_e = \frac{d}{d\tau}\left[\frac{a\epsilon}{H}\zeta\zeta'^2\right]$$



- Quartic induced Hamiltonian

$$\mathcal{H}^{(4)}_A = 9\epsilon a^2\zeta'^2\zeta^2, \quad \mathcal{H}^{(4)}_B = \frac{\epsilon a^2}{(aH)^2}\zeta^2(\partial^2\zeta)^2, \quad \mathcal{H}^{(4)}_C = -\frac{9\epsilon a^2}{aH}\zeta'^3\zeta,$$

$$\mathcal{H}^{(4)}_D = -\frac{6\epsilon a^2}{aH}\zeta^2\zeta'\partial^2\zeta, \quad \mathcal{H}^{(4)}_E = \frac{3\epsilon a^2}{(aH)^2}\zeta\zeta'^2\partial^2\zeta, \quad \mathcal{H}^{(4)}_F = \frac{9\epsilon a^2}{4(aH)^2}\zeta'^4$$

- Diff. deduced Hamiltonian

$$\mathcal{H}^{(4)}_{{\rm diff},\,A} = a^2\epsilon\eta\,\zeta^2(\partial\zeta)^2, \quad \mathcal{H}^{(4)}_{{\rm diff},\,B} = \frac{4a^2\epsilon}{aH}\zeta\zeta'(\partial\zeta)^2, \quad \mathcal{H}^{(4)}_{{\rm diff},\,C} = \frac{2a^2\epsilon}{aH}\zeta^2\partial\zeta\partial\zeta'.$$

- Tadpole induced Hamiltonian

$$\mathcal{H}^{(2)}_1 = -\frac{1}{2a^2\epsilon}\left\langle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'}\right\rangle\frac{\delta\mathcal{L}^{(3)}}{\delta\zeta'} = -18\epsilon a^2\langle\zeta'\zeta\rangle\zeta'\zeta + \frac{9\epsilon a^2}{aH}\langle\zeta'^2\rangle\zeta'\zeta + \frac{6\epsilon a^2}{aH}\langle\zeta\,\partial^2\zeta\rangle\zeta'\zeta.$$

RESULT

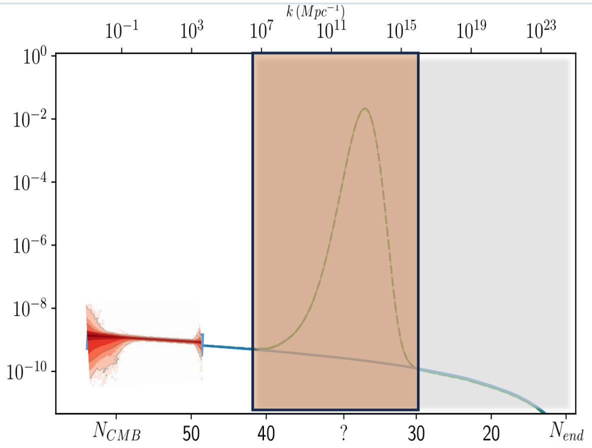
J. Fumagalli, 2408.08296

Full one-loop result from small scale to large scale in non-slow-roll dynamics

$$\mathcal{P}_\zeta^{1-\text{loop}}(p,\tau) = \mathcal{P}_\zeta^{\text{tree}}(p,\tau) \int_{\tau_0}^{\tau} d\tau_1 \int dk\, C(k,\tau_1),$$

$$C(k,\tau_1) = \frac{d}{dk}\left(3\partial_{\tau_1}\mathcal{P}_\zeta(k,\tau_1) - \frac{3}{aH}\mathcal{P}_{\zeta'}(k,\tau_1) + \frac{2}{aH}k^2\mathcal{P}_\zeta(k,\tau_1)\right) + \frac{d}{dk}\left(g_p(\tau,\tau_1)\left(-3\mathcal{P}_{\zeta'}(k,\tau_1) - \eta k^2\mathcal{P}_\zeta(k,\tau_1) - \frac{1}{aH}\partial_{\tau_1}(k^2\mathcal{P}_\zeta(k,\tau_1))\right)\right).$$

NO DEPENDENCE ON THE ENHANCED SHORT MODES!



$p \ll k$

RESULT

J. Fumagalli, 2408.08296

Full one-loop result from small scale to large scale in non-slow-roll dynamics

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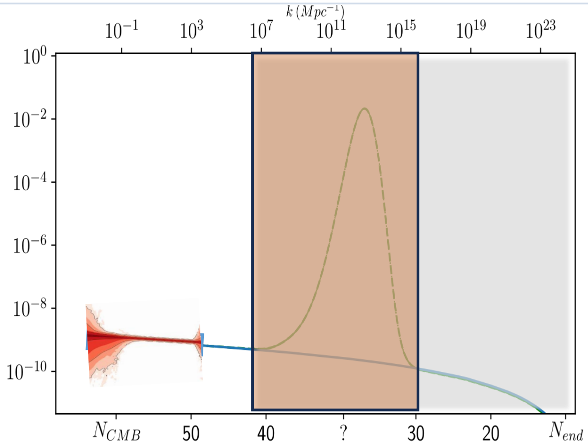
$$+\frac{d}{dk}\left(g_p(\tau,\tau_1)\left(-3\mathcal{P}_{\zeta'}(k,\tau_1) - \eta k^2\mathcal{P}_\zeta(k,\tau_1) - \frac{1}{aH}\partial_{\tau_1}(k^2\mathcal{P}_\zeta(k,\tau_1))\right)\right).$$

NO DEPENDENCE ON THE ENHANCED SHORT MODES!

Further

IR $\quad \propto (\ldots)|_{k_{\text{IR}}} \propto k_{\text{IR}}\tau_{\text{int}} \quad \ll 1$

UV – dim reg. $\propto \int d^{3+\delta}k \frac{1}{k^{3+\delta}} \frac{d}{d\ln k}\left(k^{3+\delta}\left\langle\!\left\langle \frac{\delta\mathcal{L}^{(3)}}{\delta\zeta^A}\right\rangle\!\right\rangle\right) \longrightarrow 0$



$p \ll k$

G.L. Pimentel, L. Senatore and M. Zaldarriaga, 1203.6651
R. Kawaguchi, S. Tsujikawa and Y. Yamada 2407.19742

RESULT

J. Fumagalli, 2408.08296

Full one-loop result from small scale to large scale in non-slow-roll dynamics

$$\mathcal{P}_\zeta^{1-\mathrm{loop}}(p,\tau) = \mathcal{P}_\zeta^{\mathrm{tree}}(p,\tau) \int_{\tau_0}^{\tau} d\tau_1 \int dk\, C(k,\tau_1),$$

$$C(k,\tau_1) = \frac{d}{dk}\left(3\partial_{\tau_1}\mathcal{P}_\zeta(k,\tau_1) - \frac{3}{aH}\mathcal{P}_{\zeta'}(k,\tau_1) + \frac{2}{aH}k^2\mathcal{P}_\zeta(k,\tau_1)\right)$$
$$+ \frac{d}{dk}\left(g_p(\tau,\tau_1)\left(-3\mathcal{P}_{\zeta'}(k,\tau_1) - \eta k^2\mathcal{P}_\zeta(k,\tau_1) - \frac{1}{aH}\partial_{\tau_1}(k^2\mathcal{P}_\zeta(k,\tau_1))\right)\right).$$

NO DEPENDENCE ON THE ENHANCED SHORT MODES!

Conclusions: In non-slow roll / any single-field dynamics:

$$\mathcal{P}_\zeta^{1-\mathrm{loop}}(p) = \mathcal{P}^{\mathrm{tree}}(p) \int d\ln k\, C(k) + O\left(\frac{p^3}{k^3}\right), \qquad p \ll k$$

DEJA-VU: TENSORS IR EFFECTS

Similar issue for tensors:

A. Ota, M. Sasaki, Y. Wang '22 *"Scale-invariant enhancement of GWs during inflation"*

Similar solutions:

Y. Ema, M. Hong, R Jinno, K. Mukaida '25 "*Cancellation of one-loop corrections to soft tensor power spectrum"*

C.J. Fang, H.W. Hu, Z.K. Guo '25 "One-loop corrections to GWs is forbidden by symmetries"

MORE WORKS..

deltaN: L. Iacconi, D. Mulryne, D. Seery ‘23 ’24

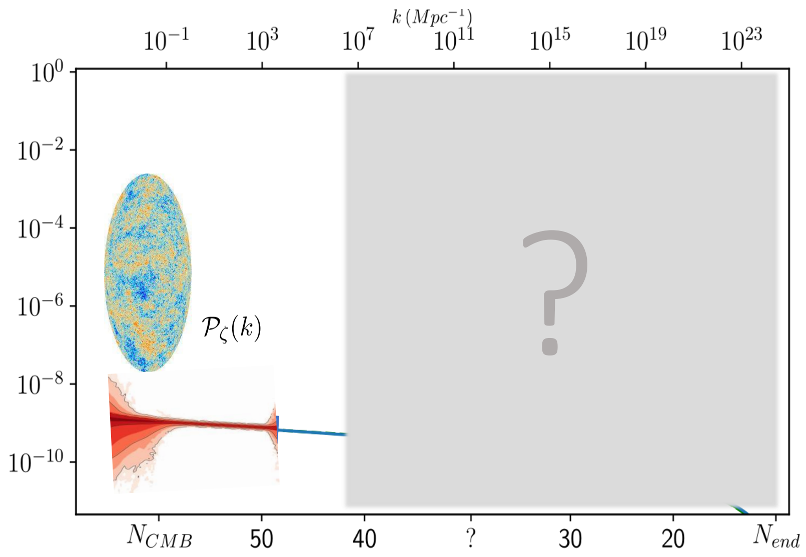
Other gauges: K. Inomata ’24, ‘25, ‘25, G. Ballesteros and J. Egea ’24

total derivatives: M. Braglia, L. Pinol ’24

Renormalization: G. Ballesteros, J. Egea, F. Riccardi ’24, M. Braglia, L. Pinol ’25

.....

QUESTIONS



What signatures probe physics at these scales?

Is perturbation theory enough?

Are these scenarios consistent at loop level?

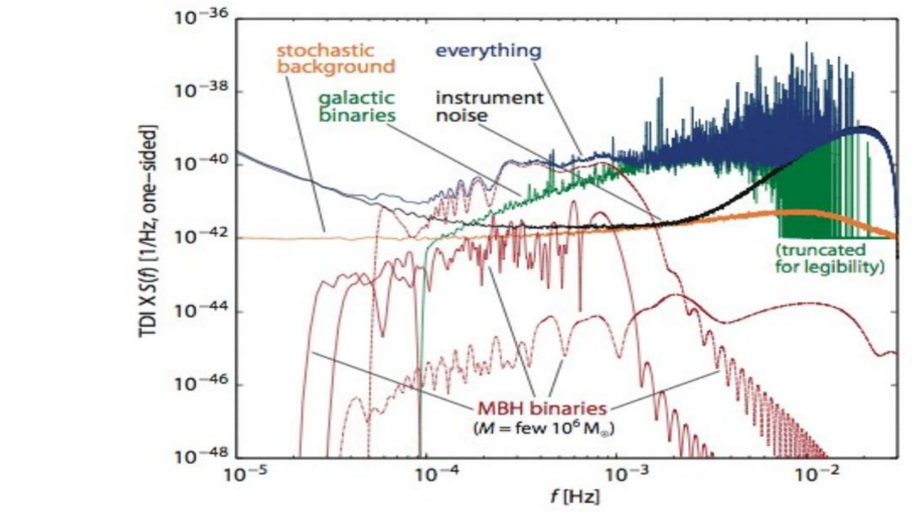
What are the detection prospects for a primordial signal?

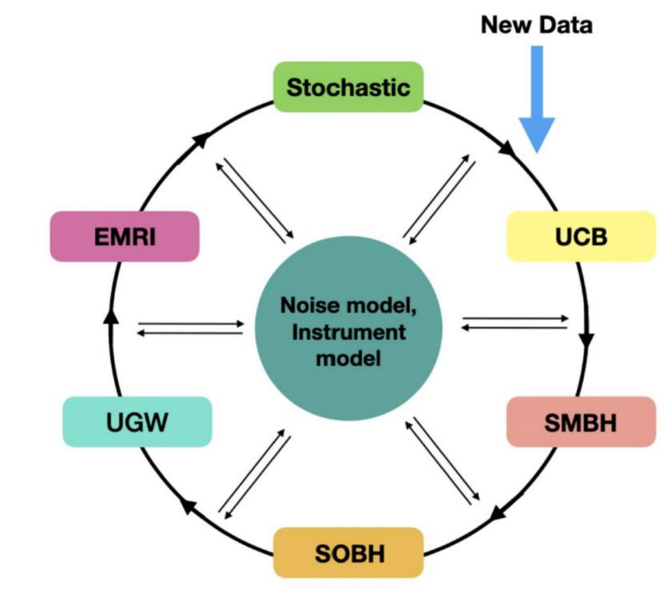
DIGGING UNDER MANY SIGNALS

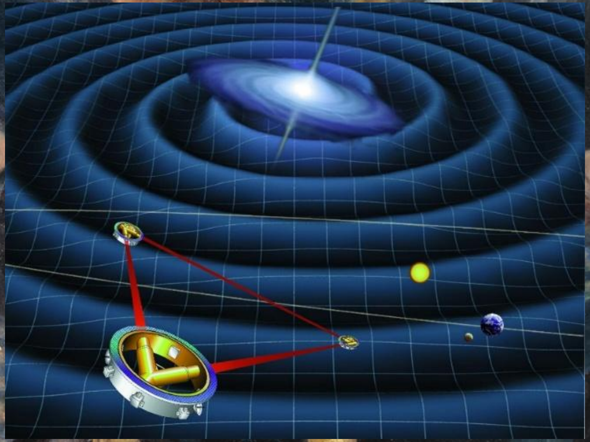
Over the Noise..

<u>Over the signals</u>

- O(10^4) galactic binaries
- O(10/y) MBHBs
- O(1-10) EMRI
- O(10) SBBHs
- Astrophysical foregrounds...







DIGGING UNDER MANY SIGNALS

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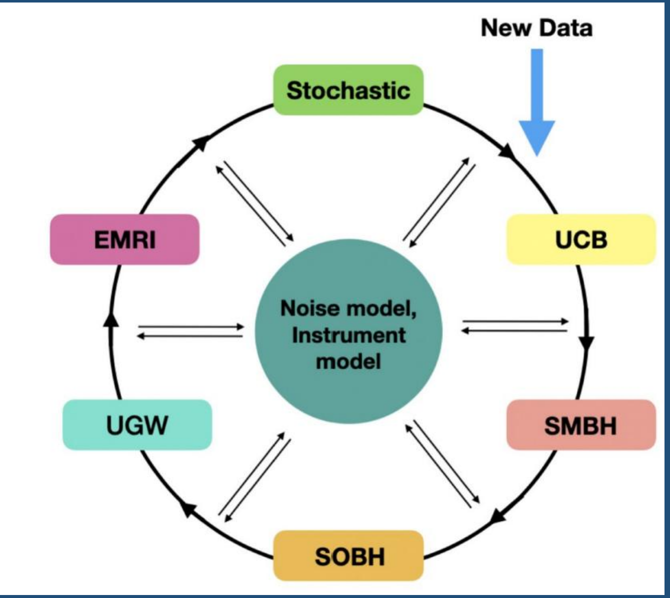


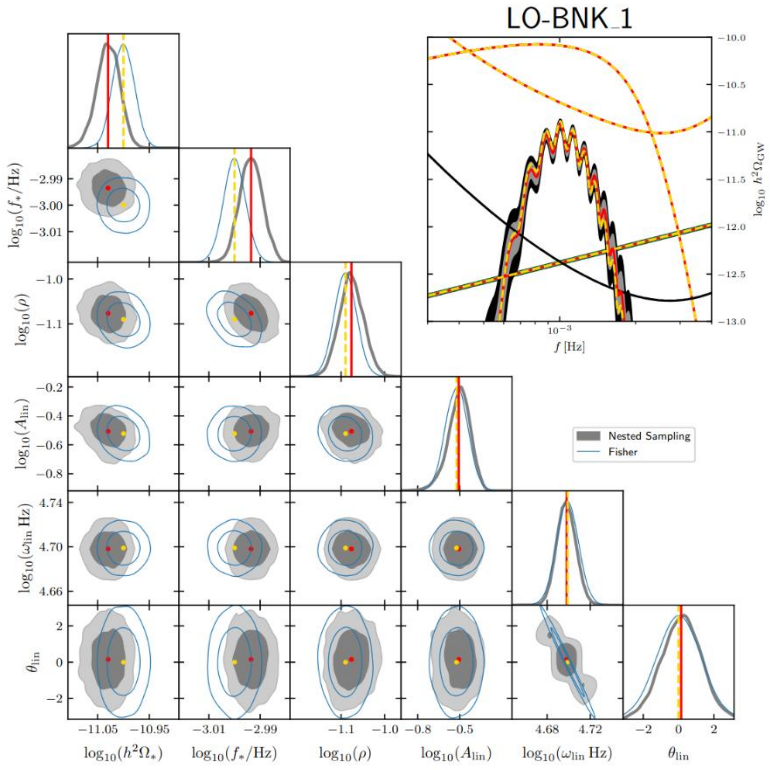
Template-based analysis of primordials inflationary SGWB

Estimate LISA accuracy and parameter reconstruction IF

- Good noise mode from ESA
- Astro foregrounds modelled
- Residual from binary waveform that not mimic a SGWB

JF + LISA CosWG
2407.04356

DIGGING A PRIMORDIAL SIGNALS



Frequency of the oscillations can be reconstruced up to amplitude of order 0.01

If the IF are met, excellent signal reconstruction

Template-based analysis of primordials inflationary SGWB

Estimate LISA accuracy and parameter reconstruction IF

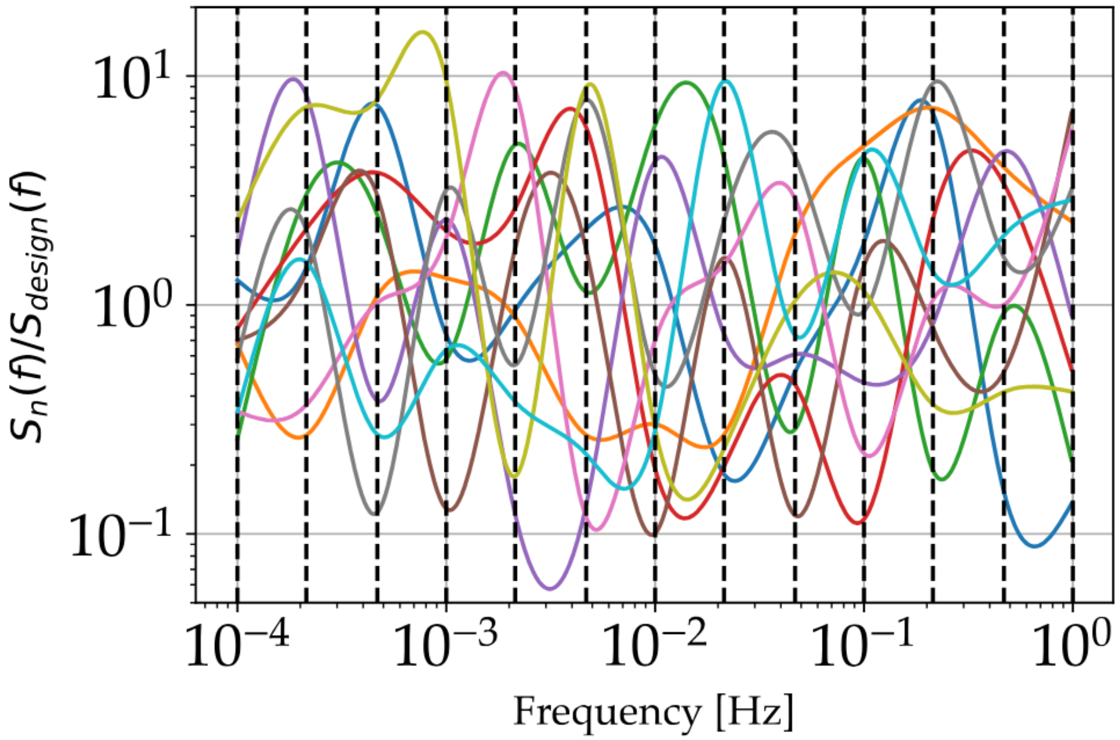
- Good noise mode from ESA
- Astro foregrounds modelled
- Residual from binary waveform that not mimic a SGWB

JF + LISA CosWG 2407.04356

BUT...

Uncertainties in noise modeling can degrade the ability to constrain SGWBs by orders of magnitude.

M. Muratore, J. Gair, L. Speri '23



work in progress...

Conclusions

- The small-scale inflation window opens up a rich variety of **phenomenological, theoretical**, and **observational** opportunities;
  (features in SGWB, loops, PBHs, detection challenges..)

- This is already advancing our understanding of the theory.

- We must be prepared with robust theoretical predictions, as future GW detectors may have the required sensitivity to captures features and higher-order effects in the stochastic background.