Decoherence of Primordial Perturbations in the View of a Local Observer

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Contributed talk
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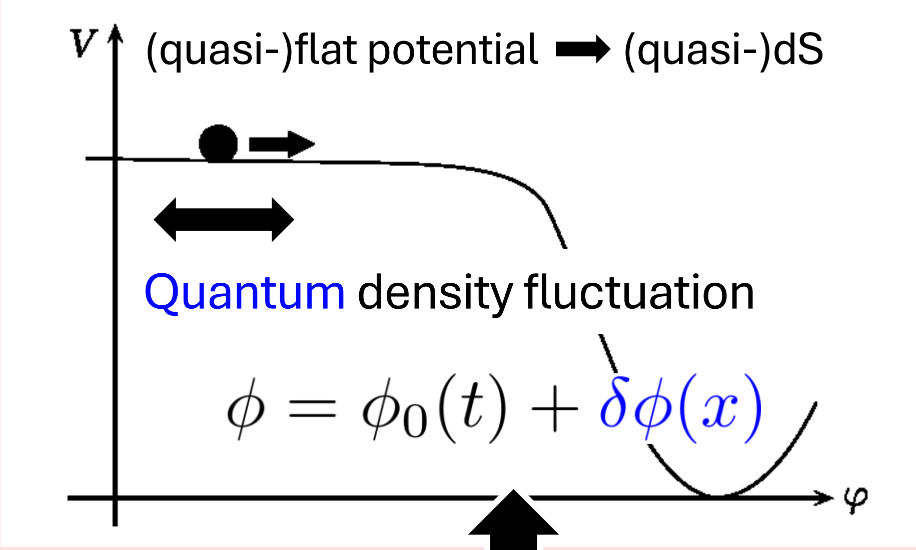
Based on
2504.10472 with Junsei Tokuda (McGill University)

Inflation as a source for cosmological perturbations



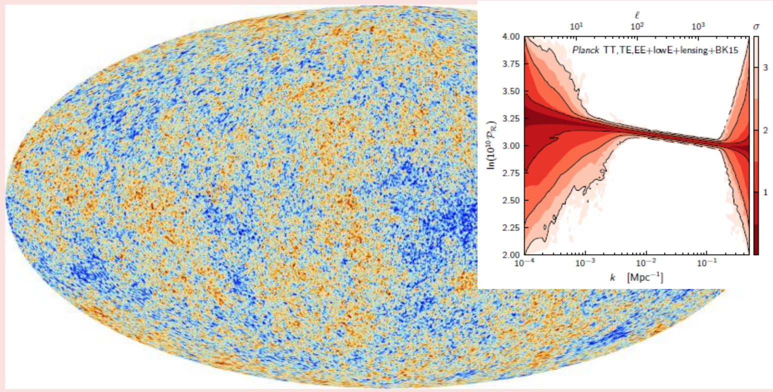
Quantum density fluctuation

$$\phi = \phi_0(t) + \delta\phi(x)$$

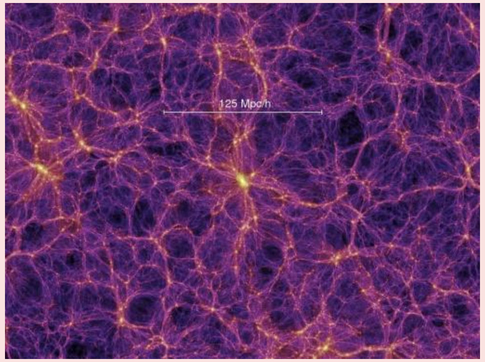
Quantum curvature perturbation

$$h_{ij} = (e^{\zeta(x)}a(t))^2(\delta_{ij} + \gamma_{ij})$$

How (fast) classicalized?



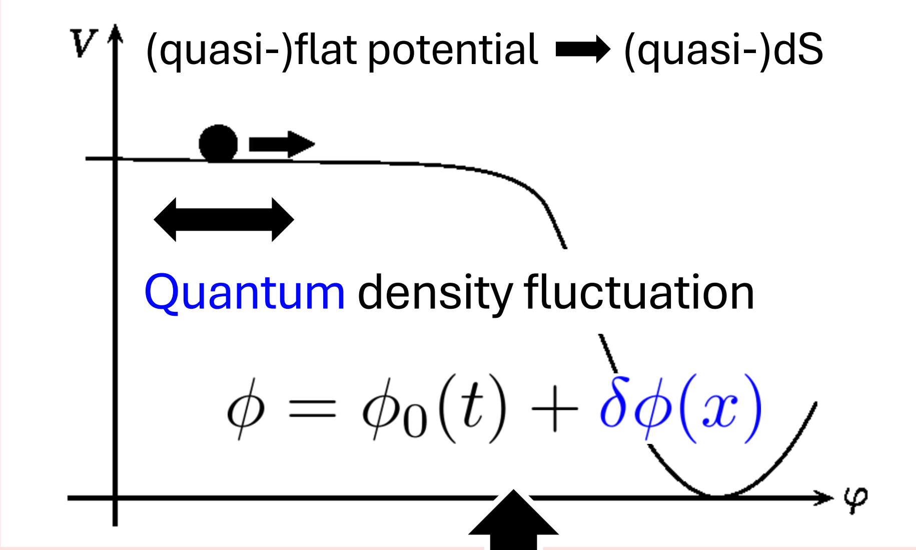
[Planck 1807.06211]



[Millennium Simulation 2005]

Classical anisotropy and inhomogeneity

Inflation as a source for cosmological perturbations

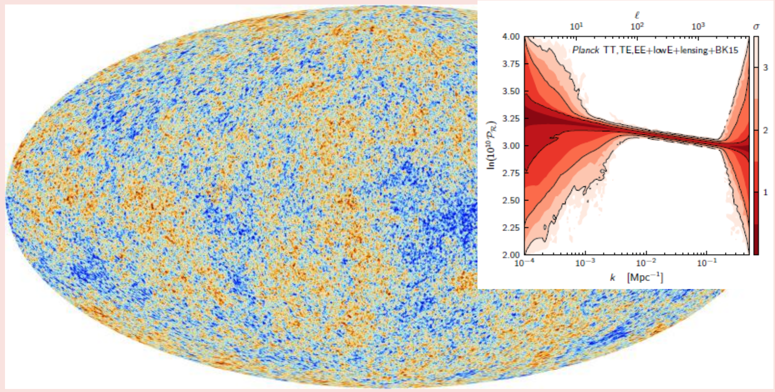


Quantum curvature perturbation

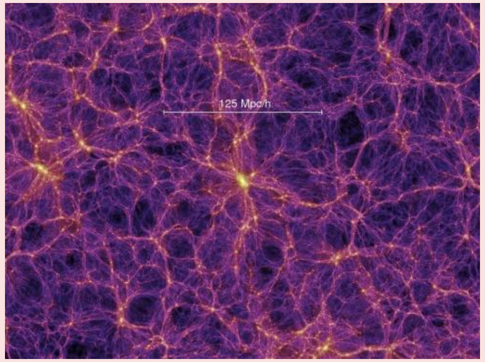
$$h_{ij} = (e^{\zeta(x)} a(t))^2 (\delta_{ij} + \gamma_{ij})$$

How (fast) classicalized?

Can we test quantum gravity from bottom-up?



[Planck 1807.06211]

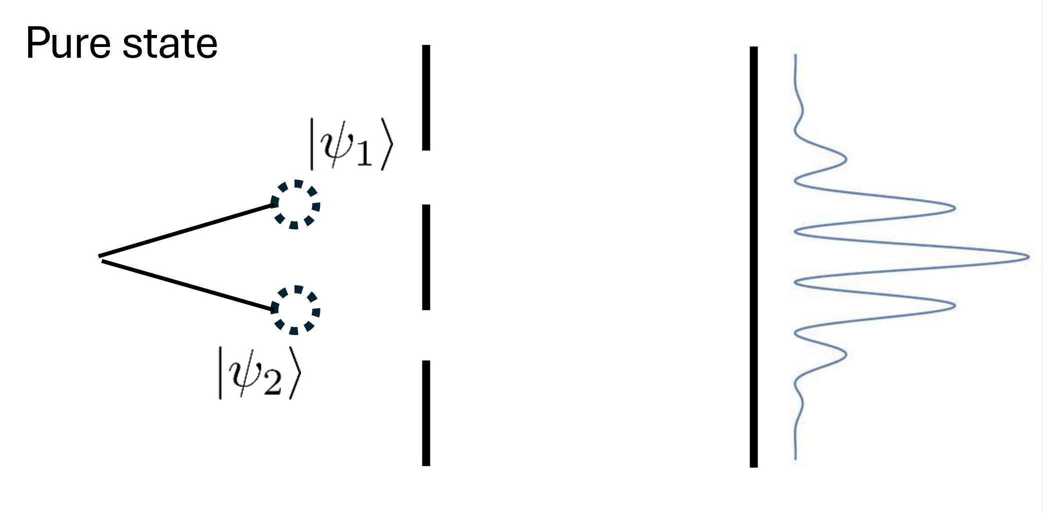


[Millennium Simulation 2005]

Classical anisotropy and inhomogeneity

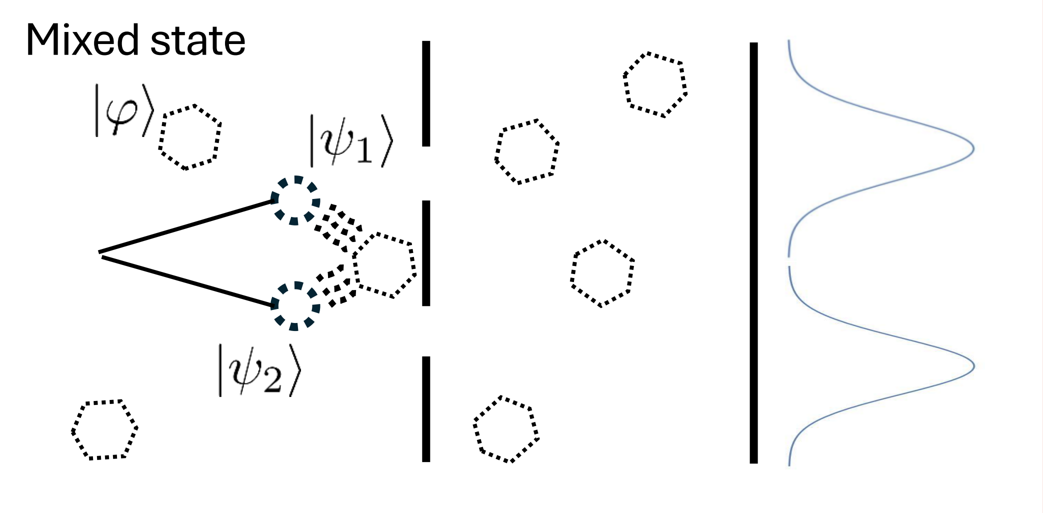
Quantum interference and decoherence

- Example from double slit experiment (Infinite slits at each time step ⇒ path integral)



$$|\Psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$$

$$\langle\Psi|\widehat{A}|\Psi\rangle = |\alpha|^2\,\langle\psi_1|\widehat{A}|\psi_1\rangle + |\beta|^2\,\langle\psi_2|\widehat{A}|\psi_2\rangle + (\alpha\beta^*\,\langle\psi_2|\widehat{A}|\psi_1\rangle + \text{c.c.})$$



$$|\Psi\rangle = \alpha\,|\psi_1\rangle\,|\varphi_1\rangle + \beta\,|\psi_2\rangle\,|\varphi_2\rangle$$

$$\rho_\psi = \mathrm{Tr}_\varphi[|\Psi\rangle\,\langle\Psi|] = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle\varphi_2|\varphi_1\rangle \\ \alpha^*\beta\langle\varphi_1|\varphi_2\rangle & |\beta|^2 \end{pmatrix}$$

$\langle\varphi_2|\varphi_1\rangle \sim 0$ if scattered to independent states.

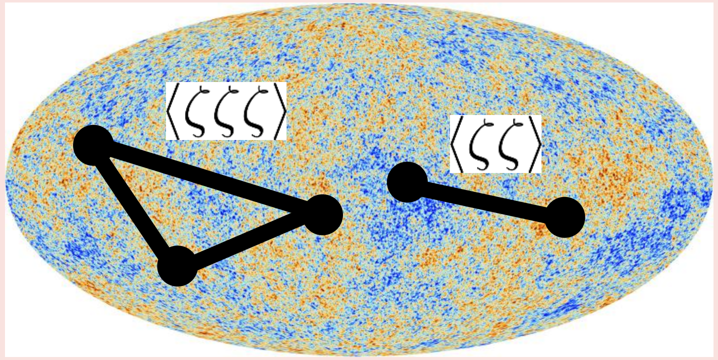
More scattering, more independent, less interference.

Wavefunction formalism

- **Observables: correlation functions**

$$\langle\Omega|\hat{\zeta}^n(t)|\Omega\rangle = \int d\zeta(t)\langle\Omega|\zeta;t\rangle\,\langle\zeta;t|\Omega\rangle\zeta^n \equiv \int d\zeta(t)|\Psi[\zeta(t)]|^2\zeta^n$$

$$\hat{\zeta}(t)\,|\zeta;t\rangle = \zeta(t)\,|\zeta;t\rangle$$



✓ System: single mode $\pm\mathbf{k}_\mathrm{S} \in \{\mathbf{k}_\mathrm{CMB}\}$, $\quad \mathcal{H}_\mathrm{total} = \mathcal{H}_{\mathbf{k}_\mathrm{S}} \otimes \mathcal{H}_{\mathbf{k}_\mathrm{E}}$

- **Wavefunction at a certain time slice**

Gaussian

Gravitational non-linearity

$$\Psi[\zeta(t)] \equiv \langle\zeta;t|\Omega\rangle = \exp\left[-\frac{1}{2}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\psi_2\zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int\frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\frac{d^3\mathbf{k}_3}{(2\pi)^3}\psi_3\zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots\right]$$

$$\left(= \int_\Omega^\zeta \mathscr{D}\zeta' e^{iS[\zeta']}\right)$$

ψ_n : coefficient of the expansion

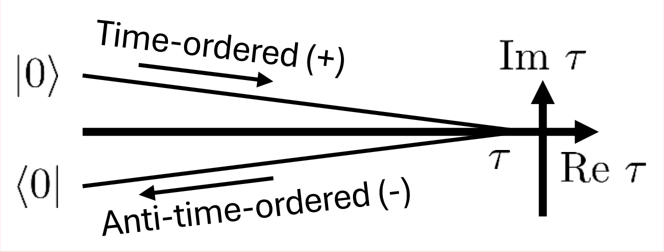
✓ Free propagation: $e^{-\int_\mathbf{k}\psi_2\zeta_\mathbf{k}\zeta_{-\mathbf{k}}}$ ➡ no entanglement between $\mathbf{k}_\mathrm{S}$ and $\mathbf{k}_\mathrm{E}$
(no scattering)

➡ Non-linearities cause decoherence.

Unitarity and Schwinger-Keldysh formalism

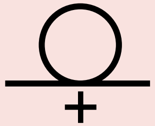
❑ Expectation values at a time slice

✓ Perturbation theory in interaction picture (2 pt.)



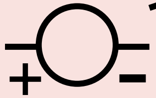
$$\langle\Omega|U^\dagger\zeta^2(\tau_0)U|\Omega\rangle = \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_\mathrm{I}}\right)\zeta_\mathrm{I}^2(\tau)\left(\mathrm{T}e^{-i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_\mathrm{I}}\right)\right|0\right\rangle$$

$$= \langle 0|\zeta_\mathrm{I}^2|0\rangle - i\int_{\tau_0}^{\tau} d\tau'\,\langle 0|\zeta_\mathrm{I}^2(\tau)H_\mathrm{I}(\tau')|0\rangle + \mathrm{c.c.}$$

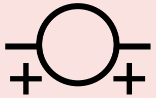


etc.

$$+\int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|H_\mathrm{I}(\tau')\zeta_\mathrm{I}^2(\tau)H_\mathrm{I}(\tau'')|0\rangle - \int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|\zeta_\mathrm{I}^2(\tau)\mathrm{T}[H_\mathrm{I}(\tau')H_\mathrm{I}(\tau'')]|0\rangle + \mathrm{c.c.}$$



etc.



etc.

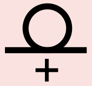
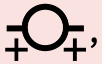
✓ Comparison with density matrix

$$\langle\Omega|U^\dagger\mathcal{O}_{0,\mathrm{S}}U|\Omega\rangle = \mathrm{Tr}[\rho(\tau)\mathcal{O}_{0,\mathrm{S}}] = \mathrm{Tr}_\mathrm{S}[\rho_\mathrm{S}(\tau)\mathcal{O}_{0,\mathrm{S}}]$$

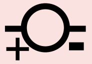
Defined in a subsystem

$$\begin{cases} \rho(\tau) = U\,|\Omega\rangle\langle\Omega|\,U^\dagger \\ \rho_\mathrm{S}(\tau) = \mathrm{Tr}_\mathrm{E}[U\,|\Omega\rangle\langle\Omega|\,U^\dagger] \end{cases}$$

Path integral

$\sim\Psi$. Unitary evolution within "+" contour (corrections in $\langle\zeta^2\rangle$: , , ...)

$$\rho_\mathrm{S}[\zeta,\tilde{\zeta};\tau] = \int_\Omega^{\zeta}\mathscr{D}\zeta_+\int_\Omega^{\tilde{\zeta}}\mathscr{D}\zeta_-\,e^{iS[\zeta_+]-iS[\zeta_-]+iS_\mathrm{IF}[\zeta_+,\zeta_-]}$$

Non-unitarity
(contributions in $\langle\zeta^2\rangle$: , ...)

Tracing out environmental modes

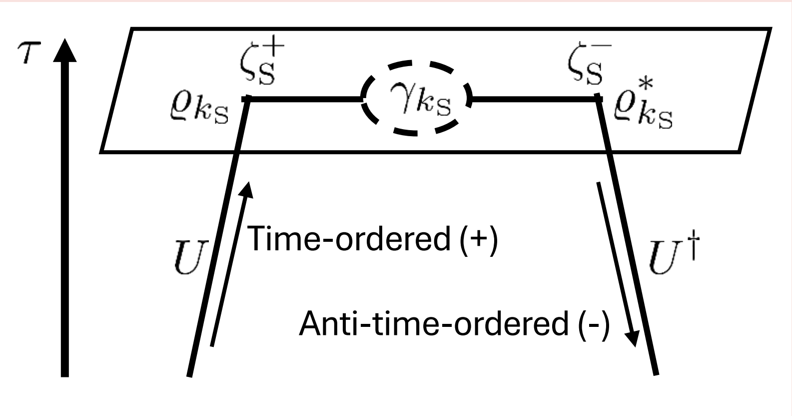
[Nelson 1601.03734]

❑ Gaussian approximation $(\mathcal{H}_{\rm S} = \mathcal{H}_{\pm\mathbf{k}_{\rm S}},\ \pm\mathbf{k}_{\rm S} \in \{\mathbf{k}_{\rm CMB}\})$

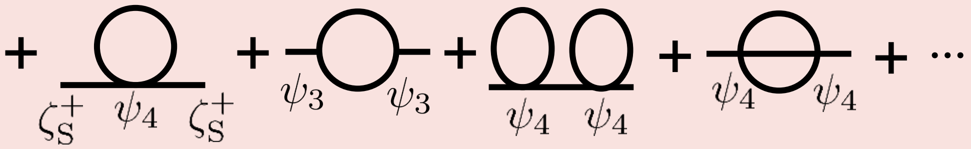
$$\rho_{\rm S}[\zeta_{\rm S}^+,\zeta_{\rm S}^-] = \int d\zeta_{\rm E}(t)\Psi[\zeta_{\rm S}^+,\zeta_{\rm E}]\Psi^*[\zeta_{\rm S}^-,\zeta_{\rm E}]$$

$$\equiv N\exp\Big[\underbrace{-\varrho_{k_{\rm S}}|\zeta_{\mathbf{k}_{\rm S}}^+|^2 - \varrho_{k_{\rm S}}^*|\zeta_{\mathbf{k}_{\rm S}}^-|^2}_{\text{“Pure state” part }\ \Psi\Psi^*} + \underbrace{\frac{\gamma_{k_{\rm S}}}{2}(\zeta_{\mathbf{k}_{\rm S}}^+\zeta_{\mathbf{k}_{\rm S}}^{-*} + \zeta_{\mathbf{k}_{\rm S}}^{+*}\zeta_{\mathbf{k}_{\rm S}}^-)}_{\text{Mixed part due to non-unitarity}} + \cdots\Big]$$

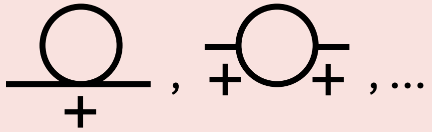
(~influence functional)



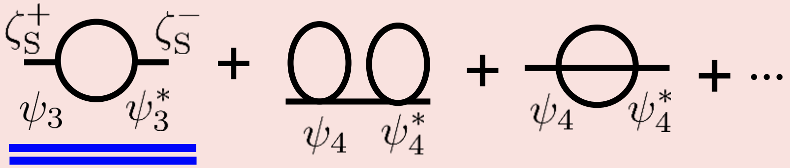
✓ $\varrho_{k_{\rm S}} : \psi_2^{\rm tree} + \psi_2^{\rm loop} +$



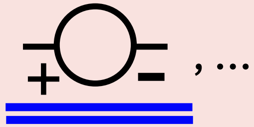
⬅ Time ordered in Schwinger-Keldysh



✓ $\gamma_{k_{\rm S}}$:



⬅ Wightman functions in Schwinger-Keldysh



* Purity: $P = {\rm Tr}[\rho^2] \simeq \dfrac{1}{1+\Gamma}$ where $\Gamma = 4P_{k_{\rm S}}\gamma_{k_{\rm S}} \simeq 2P_{k_{\rm S}}\int_{\mathbf{q}} P_q P_{|\mathbf{k}_{\rm S}-\mathbf{q}|}|\psi_{3,(\mathbf{k}_{\rm S},-\mathbf{q},\mathbf{q}-\mathbf{k}_{\rm S})}|^2$

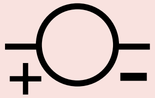
Estimations in previous work

[Nelson 1601.03734, Sou et al. 2207.04435]

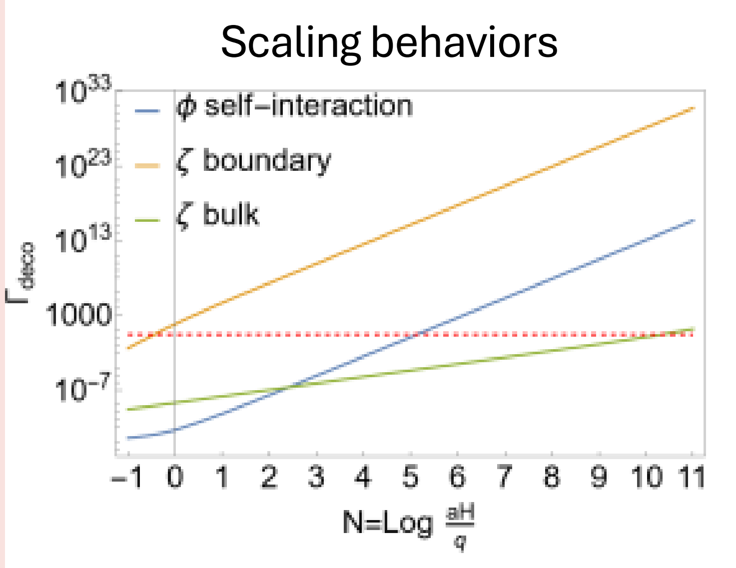
Another method: quantum master equation [Burgess et al. astro-ph/061646, Burgess et al. 2211.11046, etc.]

❑ Dependence on scale factor $(\rho_{\text{off-diag}} \sim e^{-\Gamma})$

$$\Gamma \simeq 2P_{k_{\rm S}} \int_{\mathbf{q}} P_q P_{|\mathbf{k}_{\rm S}-\mathbf{q}|} |\psi_{3,(\mathbf{k}_{\rm S},-\mathbf{q},\mathbf{q}-\mathbf{k}_{\rm S})}|^2$$



$$\sim \frac{H^2}{M_{\rm pl}^2}\left[\left(\underbrace{\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6}_{\partial_t(9aH\zeta^3)} + \underbrace{\epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3}_{a^2\epsilon^2\zeta(\partial\zeta)^2}\right)\underbrace{(1+\log(k_{\rm IR}/k_{\rm S}))}_{\text{IR cutoff}} + \underbrace{\left(\frac{\Lambda_{\rm phys}}{H}\right)^{\#}}_{\text{UV cutoff}}\right]$$



[Sou et al. 2207.04435]

➢ Technical difference from calculations of $\langle\zeta^n\rangle$

$\underbrace{\text{Evaluation at the finite time slice,}}_{\text{UV behavior become worse.}}$ $\underbrace{\text{Effects from boundary terms, Necessity of } \mathrm{Im}\,\psi_n, \ldots}_{\text{IR may not fully be investigated.}}$

✓ IR: inertial observer's coordinate ➡ See our paper [Sano and Tokuda 2504.10472]

✓ UV: time averaged observables as well as renormalization ➡ This talk!

Consistency condition for loop calculations

[Nelson 1601.03734]

$$\psi_3 \quad \Longleftarrow \quad S_3 = \int dt d^3x \Big\{ a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + 2f(\zeta)\left.\frac{\delta\mathcal{L}}{\delta\zeta}\right|_1 + \mathcal{L}_b \Big\}\,, \quad \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

[Sou et al. 2207.04435]

$$\begin{aligned} \mathcal{L}_b = \partial_t\Big[& -9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 \\ & - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 \\ & - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) \\ & - \frac{\eta a}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi)\Big] \end{aligned}$$

Necessary for correlation function

Consistency condition for loop calculations

[Sou et al. 2207.04435]

[Nelson 1601.03734]

$$\psi_3 \longleftarrow$$

$$S_3 = \int dt d^3x \Big\{ a^3\epsilon^2\zeta\dot{\zeta}^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2a\epsilon\dot{\zeta}\partial\zeta\partial\chi + 2f(\zeta)\left.\frac{\delta\mathcal{L}}{\delta\zeta}\right|_1 + \mathcal{L}_b \Big\}\,, \quad \partial^2\chi \equiv a^2\epsilon\dot{\zeta}$$

$$\begin{aligned}\mathcal{L}_b = \partial_t\Big[& -9a^3H\zeta^3 + \frac{a}{H}\zeta(\partial\zeta)^2 \\ & - \frac{1}{4aH^3}(\partial\zeta)^2\partial^2\zeta - \frac{a\epsilon}{H}\zeta(\partial\zeta)^2 \\ & - \frac{\epsilon a^3}{H}\zeta\dot{\zeta}^2 + \frac{1}{2aH^2}\zeta(\partial_i\partial_j\zeta\partial_i\partial_j\chi - \partial^2\zeta\partial^2\chi) \\ & - \frac{\eta a}{2}\zeta^2\partial^2\chi - \frac{1}{2aH}\zeta(\partial_i\partial_j\chi\partial_i\partial_j\chi - \partial^2\chi\partial^2\chi)\Big]\end{aligned}$$

Necessary for correlation function

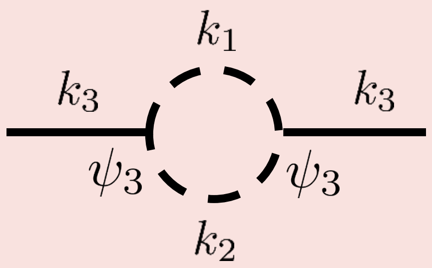
- **Maldacena's consistency condition for wavefunction** [Maldacena astro-ph/0210603, Pimentel 1309.1793]

$$\lim_{k_1\to 0}\psi_3(k_1,k_3) = \left(3 - k_3\frac{d}{dk_3}\right)\psi_2(k_3)$$

Cf.

$$\begin{cases} \lim_{k_1\to 0}\langle\zeta_1\zeta_2\zeta_3\rangle = -\langle\zeta_1\zeta_1\rangle\left(3 + k_3\frac{d}{dk_3}\right)\langle\zeta_3\zeta_3\rangle \\ \langle\zeta_1\zeta_2\rangle = \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle\zeta_1\zeta_2\zeta_3\rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^3 2\,\mathrm{Re}[\psi_2(k_i)]} \end{cases}$$

- **Loop diagram at a time slice**

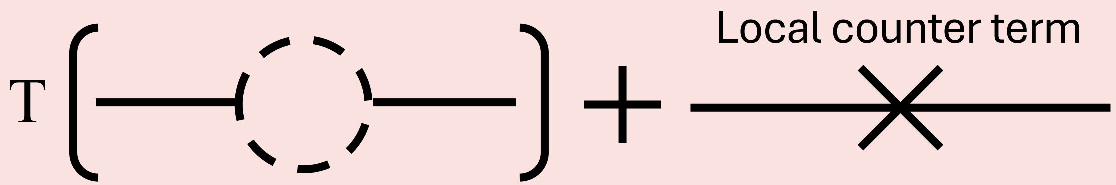


IR: $k_1 \ll k_2 \simeq k_3 \ll aH \longrightarrow \log k_1$ from $\int \langle\zeta_1\zeta_1\rangle\, k_1^2 dk_1$

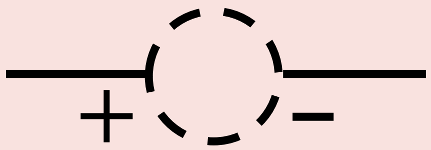
UV: $k_1 \simeq k_2 \gg aH \gg k_3 \longrightarrow k_1^5$ from $\partial_t(a\zeta(\partial_i\zeta)^2/H)$

UV divergence in equal time

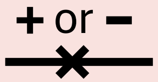
- Unitary evolution



- Non-unitary evolution



No counter term.



How to remove UV div.?

Subtlety of equal time correlators as observables

- ✓ Divergence from infinite external momenta [e.g., Balasubramanian et al. 1108.3568, Bucciotti 2410.01903 for Minkowski spacetime]

E.g., $\langle \mathcal{O}_1^{\mathbf{k}} \mathcal{O}_2^{-\mathbf{k}}(t) \rangle \sim \int d(\mathbf{x}_1 - \mathbf{x}_2) \frac{1}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}}$ diverges even at tree level when $\Delta \geq \frac{3}{2}$.

- ✓ Renormalization of composite operators [e.g., Ch. 6 and 7, “Renormalization”, Collins 2023]

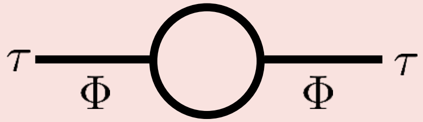
$[\phi^2]_{\mathrm{R}}(x) \equiv Z_a \phi^2(x) + \mu^{d/2-3}(Z_b m^2 + Z_c \Box)\phi(x)$ Defining finite composite operators, leading to modification of observables.

➡ (More intuitive) modification of observables: point-splitting / averaging in time as the resolution of detectors
[Agón et al. 1412.3148, Bucciotti 2410.01903, Burgess et al. 2411.09000 for Minkowski spacetime]

Time averaged observables in de Sitter

[Sano and Tokuda 2504.10472]

- ρ

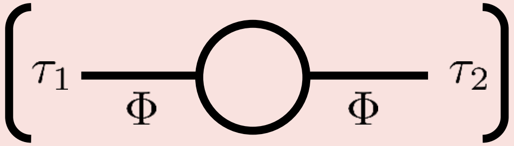
$\langle \Phi_1 \Phi_2(\tau) \rangle \supset$ 

$\Phi = \zeta$ or π

- $\overline{\rho}$

$$\langle \overline{\Phi}_1 \overline{\Phi}_2(\tau) \rangle = \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \langle \Phi_1(\tau_1) \Phi_2(\tau_2) \rangle$$

$$\supset \int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2) \left[\cdots \right]$$



Time averaging

$$\int^\Lambda k^\# dk \longrightarrow \Lambda^\#$$

$$\frac{1}{|\tau_1 - \tau_2|^\#}$$

This is (expected to be) renormalized.

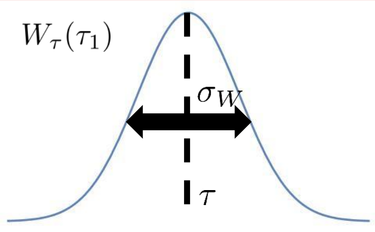
$$\frac{e^{-ik(\tau_1 - \tau_2)}}{|\tau_1 - \tau_2|^\#}$$

Included in Wightman function.
Not renormalized in standard procedure.

❑ Time averaging

$$W_\tau(\tau_1) = \frac{e^{-(\tau_1 - \tau)^2 / 2\sigma_W^2}}{\sqrt{2\pi\sigma_W^2}},$$

$$G(k; \tau_1, \tau_2) \propto e^{-i\underline{k}(\tau_1 - \tau_2)}$$



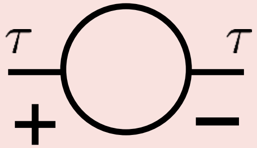
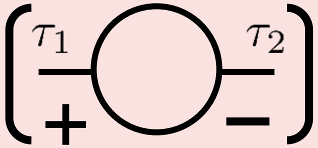
$$\overline{\Gamma}_{\rm UV} \sim \int_{k > aH} dk\ k^\# e^{-\underline{k^2} \sigma_W^2}$$

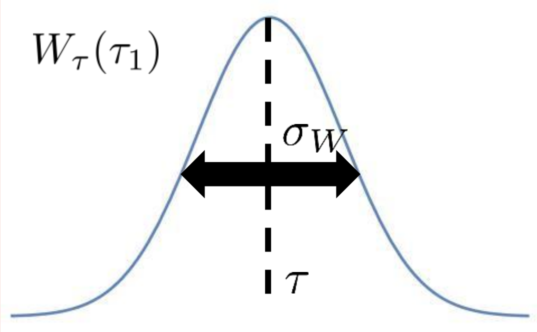
Exponential decay in sub-horizon

Summary and outlook: Averaging scale?

[Sano and Tokuda 2504.10472 and ongoing]

- **UV divergence of inflationary decoherence** $(\rho_{\text{off-diag}} \sim e^{-\Gamma})$
  - ✓ Equal time correlators can include divs. which is not removed without redefining observables.

$\Gamma \sim$  $\xrightarrow{\text{Time-averaging}}$ $\int d\tau_1 d\tau_2 W_\tau(\tau_1) W_\tau(\tau_2)$ 



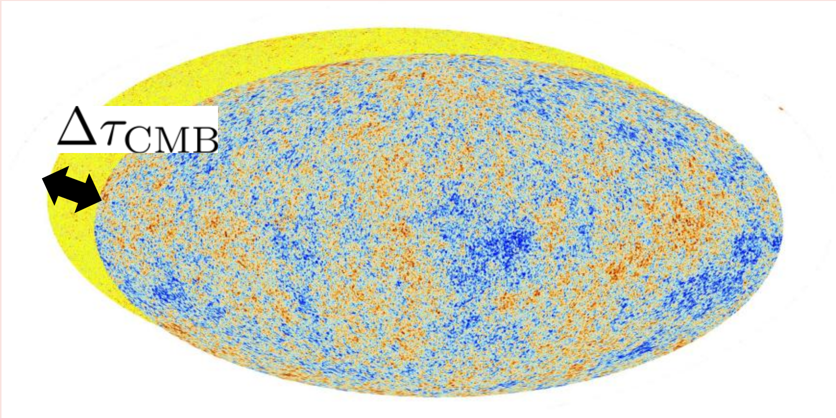
$$\rightarrow \Gamma_{\text{UV}} \sim \int_{k>aH} dk\ k^{\#} e^{-k^2\sigma_W^2}$$

Exp. sup. for decoherence from sub-horizon env.

- **What is σ_W ?**
  - ✓ Theoretical resolution $\sigma_W \gtrsim \frac{1}{a\Lambda_{\text{UV}}}$
  - ✓ Relations to renormalized composite operators?
  - ✓ Phenomenological scale? E.g., $\Delta\tau_{\text{CMB}}$
  - ✓ Observational device's resolution?

  When is ζ "measured"?

  (Quantum history of cosmo. pert. (GW?) including measurement?)



Back up slides

Purity as a quantumness monotone

[Streltsov et al. 1612.07570]

❑ Coherence is basis-dependent

✓ But the maximally mixed state cannot be coherent even when changing basis, $\frac{\hat{1}}{d} = U\frac{\hat{1}}{d}U^\dagger$.

➡ "Maximal coherence" exist for each quantum state.

❑ Coherence monotone

✓ Set of incoherent state $\mathcal{I}$: $\sigma = \sum_i p_i |i\rangle\langle i|$ in the basis $|i\rangle$ ➡ $\sigma \in \mathcal{I}$

✓ Monotone $\mathbb{C}(\rho) = \inf_{\sigma\in\mathcal{I}} D(\rho,\sigma)$ (example: L1 norm, Renyi relative entropy, ...)

(quasi-)distance

$\sum_{i,j,i\neq j} |\rho_{ij}|$

✓ Maximal coherence $\mathbb{C}_{\rm m}(\rho) = \sup_U \mathbb{C}(U\rho U^\dagger)$ (example: $S_\alpha(\rho||\hat{1}/d)$, which is written by **purity** when $\alpha = 2$)

✓ By definition, $\mathbb{C}_{\rm m} \geq \mathbb{C} \geq 0$ when we use the same distance.

❑ Comments on other quantumness

✓ Free states $\begin{cases} \text{Entanglement: separable states } \mathcal{S} \quad \sum_i p_i \rho_{A,i}\otimes\sigma_{B,i} \\ \text{Discord: pointer states } \mathcal{P} \quad \sum_i p_i \rho_{A,i}\otimes|i\rangle\langle i|_B \end{cases}$ ➡ $\mathrm{Tr}_{\rm E}[\mathcal{S}] \supset \mathrm{Tr}_{\rm E}[\mathcal{P}] \supset \mathcal{I}$ ➡ $\mathbb{C}_{\rm m} \geq \mathbb{C} \geq \mathbb{D} \geq \mathbb{E} \geq 0$

Entanglement monotone

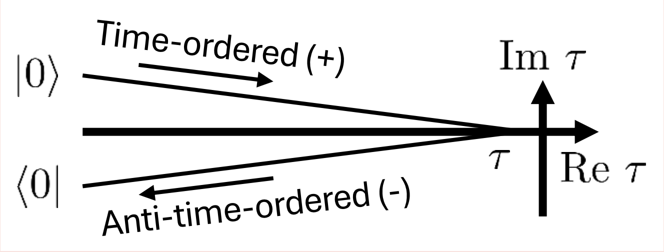
Discord monotone

Distance based

Unitarity and Schwinger-Keldysh formalism

❑ Expectation values at a time slice

✓ Perturbation theory in interaction picture   (2 pt.)



$$\langle\Omega|U^\dagger\zeta^2(\tau_0)U|\Omega\rangle = \left\langle 0\left|\left(\overline{\mathrm{T}}e^{i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\zeta_{\mathrm{I}}^2(\tau)\left(\mathrm{T}e^{-i\int_{\tau_0}^{\tau}\mathrm{d}\tau' H_{\mathrm{I}}}\right)\right|0\right\rangle$$

$$= \langle 0|\zeta_{\mathrm{I}}^2|0\rangle - i\int_{\tau_0}^{\tau} d\tau'\,\langle 0|\zeta_{\mathrm{I}}^2(\tau)H_{\mathrm{I}}(\tau')|0\rangle + \mathrm{c.c.}$$

etc.

$$+\int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|H_{\mathrm{I}}(\tau')\zeta_{\mathrm{I}}^2(\tau)H_{\mathrm{I}}(\tau'')|0\rangle - \int_{\tau_0}^{\tau} d\tau' d\tau''\,\langle 0|\zeta_{\mathrm{I}}^2(\tau)\mathrm{T}[H_{\mathrm{I}}(\tau')H_{\mathrm{I}}(\tau'')]|0\rangle + \mathrm{c.c.}$$

etc.

etc.

✓ Comparison with density matrix

$$\langle\Omega|U^\dagger\mathcal{O}_{0,\mathrm{S}}U|\Omega\rangle = \mathrm{Tr}[\rho(\tau)\mathcal{O}_{0,\mathrm{S}}] = \mathrm{Tr}_{\mathrm{S}}[\rho_{\mathrm{S}}(\tau)\mathcal{O}_{0,\mathrm{S}}]$$

Defined in a subsystem

$$\begin{cases}\rho(\tau) = U\,|\Omega\rangle\langle\Omega|\,U^\dagger \\ \rho_{\mathrm{S}}(\tau) = \mathrm{Tr}_{\mathrm{E}}[U\,|\Omega\rangle\langle\Omega|\,U^\dagger]\end{cases}$$

Path integral

$\sim\Psi$. Unitary evolution within “+” contour   (corrections in $\langle\zeta^2\rangle$: , , …)

$$\rho_{\mathrm{S}}[\bar{\zeta}_+,\bar{\zeta}_-;\tau] = \int_{\Omega}^{\bar{\zeta}_+}\mathscr{D}\zeta_+\int_{\Omega}^{\bar{\zeta}_-}\mathscr{D}\zeta_- e^{iS[\zeta_+]-iS[\zeta_-]+iS_{\mathrm{IF}}[\zeta_+,\zeta_-]}$$

Non-unitarity   (contributions in $\langle\zeta^2\rangle$: , …)

Tomographic approach to quantum state

[Sano and Tokuda 2504.10472]

- Wavefunction $\Psi[\zeta(t)] = \langle \zeta(t) | \psi \rangle$: defined in equal time. How to consider time averaging?

- **Quantum state tomography**

$$\langle \zeta_1 \zeta_2 \rangle = \frac{1}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \zeta_1 \zeta_2 \zeta_3 \rangle = -\frac{2\,\mathrm{Re}[\psi_3]}{\prod_{i=1}^{3} 2\,\mathrm{Re}[\psi_2(k_i)]}$$

$$\langle \pi_1 \zeta_2 \rangle = -\frac{\mathrm{Im}[\psi_2(k_1)]}{2\,\mathrm{Re}[\psi_2(k_1)]}, \quad \langle \pi_1 \zeta_2 \zeta_3 \rangle = \frac{2\,\mathrm{Im}[\psi_2(k_1)\psi_3^*]}{\prod_{i=1}^{3} 2\,\mathrm{Re}[\psi_2(k_i)]}$$

$$\Psi[\zeta] = \exp\left[-\frac{1}{2}\int_{k_1,k_2} \psi_2 \zeta_{k_1}\zeta_{k_2} - \frac{1}{3!}\int_{k_1,k_2,k_3} \psi_3 \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} - \cdots \right]$$

Quantum state is reconstructed from observables

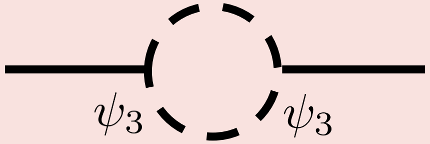
➡ Quantum state is identified as a probability distribution of canonical variables.

✓ E.g., tree-level of averaged quantum fields

$$\langle \bar{\zeta}_1 \bar{\zeta}_2 \rangle \equiv \frac{1}{2\mathrm{Re}[\bar{\psi}_2(k_1)]}, \quad \langle \bar{\pi}_1 \bar{\zeta}_2 \rangle \equiv \frac{\mathrm{Im}[\bar{\psi}_2(k_1)]}{2\,\mathrm{Re}[\bar{\psi}_2(k_1)]}, \ \cdots \quad \Longleftrightarrow \quad \Psi[\bar{\zeta}] \equiv \exp\left[-\frac{1}{2}\int_{k_1,k_2} \bar{\psi}_2 \bar{\zeta}_{k_1}\bar{\zeta}_{k_2} - \cdots \right]$$

with $[\bar{\zeta}_{\mathbf{k}}, \bar{\pi}_{\mathbf{k}'}] \approx i\hbar(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$

Mathematical identity

➡ 

is included in one-loop corrections of correlation functions

Outlook: Importance of late time evolutions

- **Boundary terms in late time** [Sano and Tokuda 2504.10472]
  - ✓ During inflation
  - ✓ Late time universe (but before re-entry)

$$\Gamma_{\rm inf} \sim \frac{H^2}{M_{\rm pl}^2}\left[\frac{1}{\epsilon^2}\left(\frac{aH}{k_{\rm S}}\right)^6 + \epsilon^2\left(\frac{aH}{k_{\rm S}}\right)^3\right] \;\Rightarrow\; \Gamma_{\rm rad.\ dom.} \sim \frac{H^2}{M_{\rm pl}^2}\left[\frac{1}{\epsilon^2}\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^6\left(\frac{a}{a_{\rm f}}\right)^2 + \epsilon^2\left(\frac{a_{\rm f}H_{\rm f}}{k_{\rm S}}\right)^3\left(\frac{a}{a_{\rm f}}\right)^5\right]$$

- **Time averaging scale?**

- **High-frequency gravitational wave** [Takeda and Tanaka 2502.18560]
  - ✓ GW with frequency $f_{\rm GW} \gtrsim 100\ {\rm Hz}$ (?) may be quantum even today!
    * Estimation of thermal decoherence by a scalar field, keeping reheating in mind.

- **Outlook**
  - ✓ Systematic approaches to sub-horizon evolution for more realistic models?
  - ✓ Entanglement harvesting through detectors? Graviton-photon conversion?
  - ✓ What is more than proving quantumness of gravity? QG from bottom up.