THE INFLATIONARY QCD AXION

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based on: M. Gorghetto, E. Hardy, H. Nicolaescu, A. N., M. Redi, JHEP 02 (2024), 223

The (Minimal) QCD Axion

$$\mathscr{L}_{\rm SM} \supset \theta_{\rm strong} \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Why CP-violation in QCD is tiny $(\bar{\theta}_{\rm strong} \ll 1)$?

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- Why CP-violation in QCD is tiny ($\bar{\theta}_{\rm strong} \ll 1$)?
- QCD Axion solution: promote $\theta_{\rm strong}$ to a dynamical field $\rightarrow \dfrac{a}{f_a}$
- *Axion* potential minimized at $a = \bar{\theta}_{\rm strong} = 0$ (CP conserving)

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- Dynamical explanation of $\bar{\theta}_{\text{strong}} \ll 1$

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- Below Λ_{QCD} a potential is generated by this operator:

$$V_a \approx \Lambda_{QCD}^4 \left[1 - \cos\left(\frac{a}{f_a}\right)\right]$$

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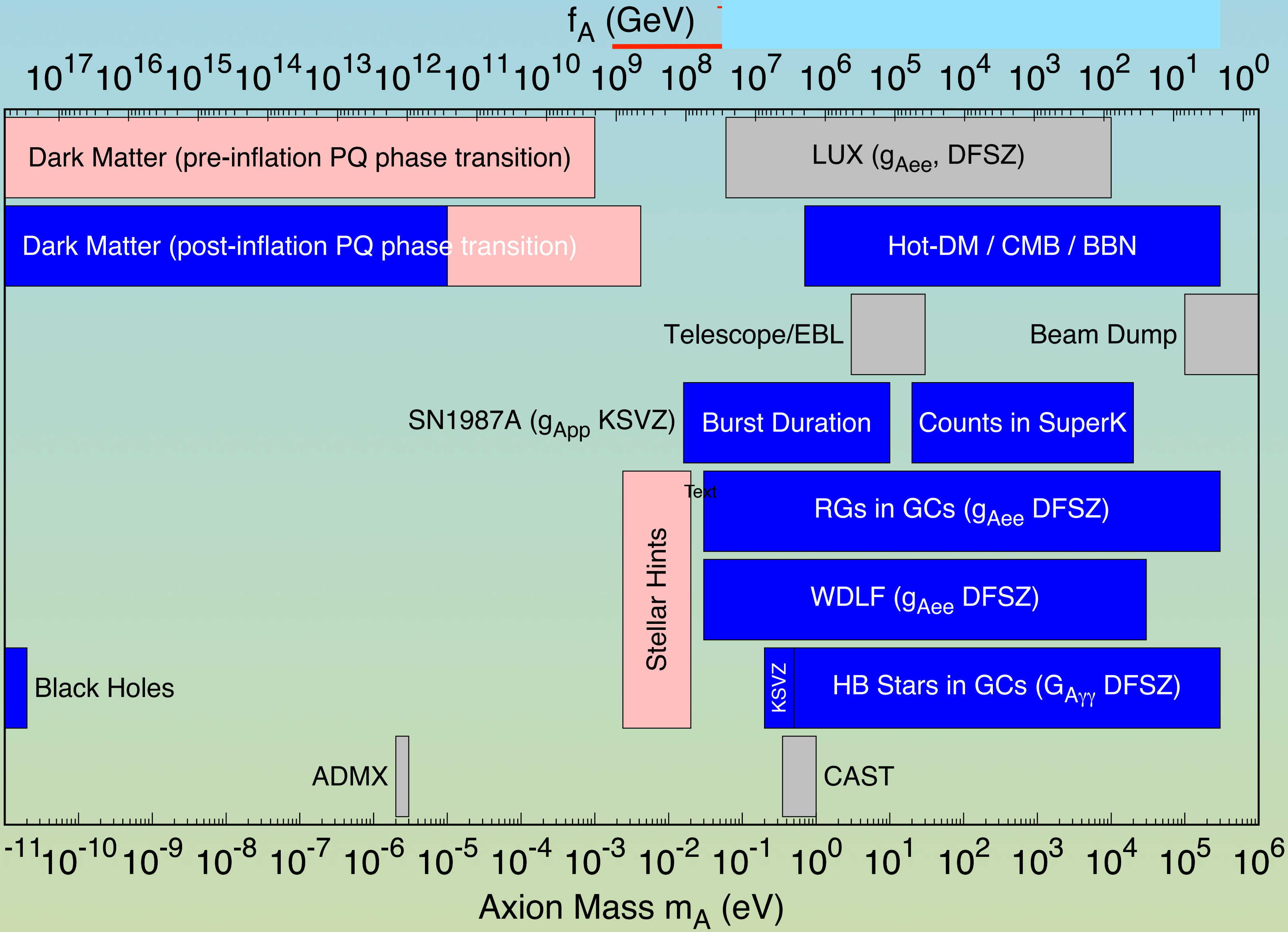
- Light scalar particle, $m_a \approx \Lambda_{QCD}^2 / f_a \approx 0.57eV\left(\frac{10^7 GeV}{f_a}\right)$
- Large f_a required *(Cosmology, Supernovae, star cooling…)*



fA (GeV)
10^17 10^16 10^15 10^14 10^13 10^12 10^11 10^10 10^9 10^8 10^7 10^6 10^5 10^4 10^3 10^2 10^1 10^0
Dark Matter (pre-inflation PQ phase transition)
LUX (gAee, DFSZ)
Dark Matter (post-inflation PQ phase transition)
Hot-DM / CMB / BBN
Telescope/EBL
Beam Dump
SN1987A (gApp KSVZ)
Burst Duration
Counts in SuperK
Text
Stellar Hints
RGs in GCs (gAee DFSZ)
WDLF (gAee DFSZ)
KSVZ
HB Stars in GCs (GAγγ DFSZ)
Black Holes
ADMX
CAST
10^-11 10^-10 10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6
Axion Mass mA (eV)

AXION AS COLD DARK MATTER

- The axion can form a coherent condensate with high occupation numbers
- Can be described by classical fields
- Example: start at 'initial time' with homogenous $a(t_i) = a_0 \neq 0$ over our horizon
- $V(a) = \Lambda_{\rm QCD}^4 \left[1 - \cos\left(\frac{a}{f_a}\right)\right]$ at Temperatures below $\Lambda_{\rm QCD}$

Coherent oscillations $\ddot{a} + 3H\dot{a} + V'(a) = 0$

Approximately $\ddot{a} + 3H\dot{a} + m_a^2 a = 0$

Frozen when $m_a \ll H$ (early times)

Oscillates in time like matter when $m_a \gtrsim H$ (late times)

AXION AS COLD DARK MATTER

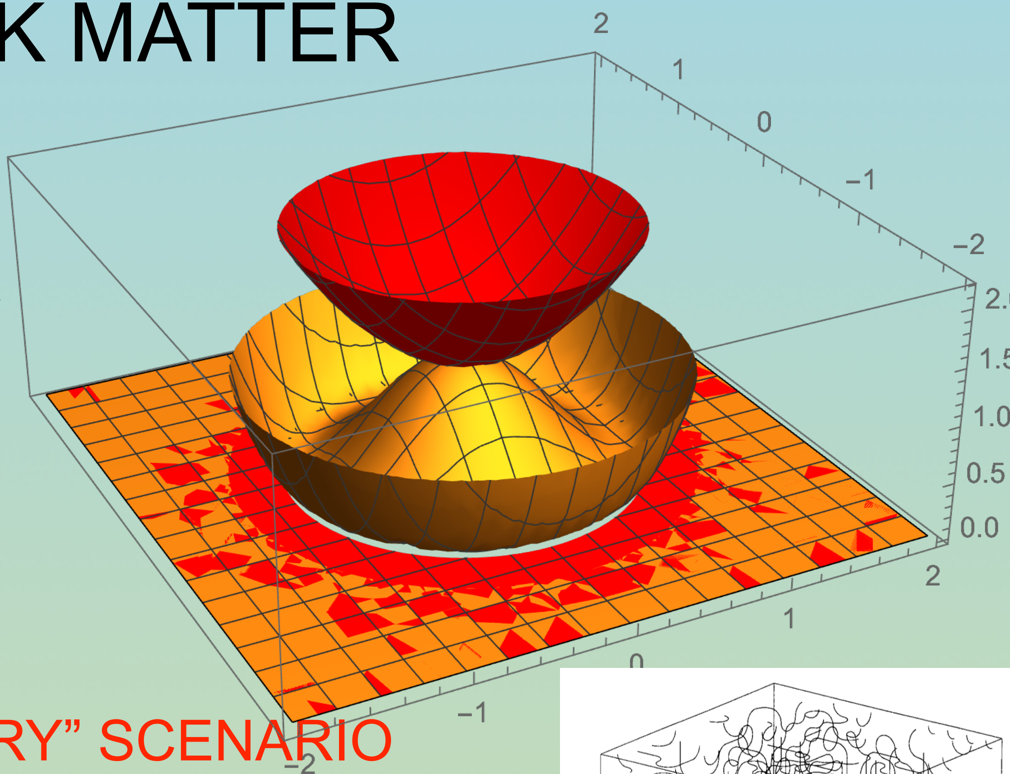
- The axion arises from a Complex Scalar (``KSVZ" models):
- $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$ $v = f_a \ (N_{\rm DW} = 1)$

- The symmetry is broken during inflation $\Phi = v e^{i\theta} = f_a e^{i\frac{a}{v}}$,
- A scalar field in inflation has quantum fluctuations of order H_i
- If very small ($H_i \ll f_a$)

- $\theta(t_i) = a(t_i)/v$ is a random value in $(-\pi, \pi)$, almost homogenous in our horizon

``PRE-INFLATIONARY" SCENARIO

AXION AS COLD DARK MATTER



- Another possible scenario: If the Universe reaches $T = f_a$
- At $T > f_a$ the symmetry is restored $v(\Phi, T) \approx T^2|\Phi|^2$
- At $T \approx f_a$ the symmetry gets broken

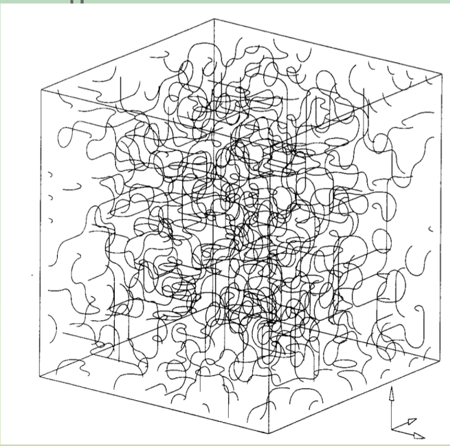
``POST-INFLATIONARY” SCENARIO

The field falls randomly

Strings form when the phase wraps from 0 to 2π

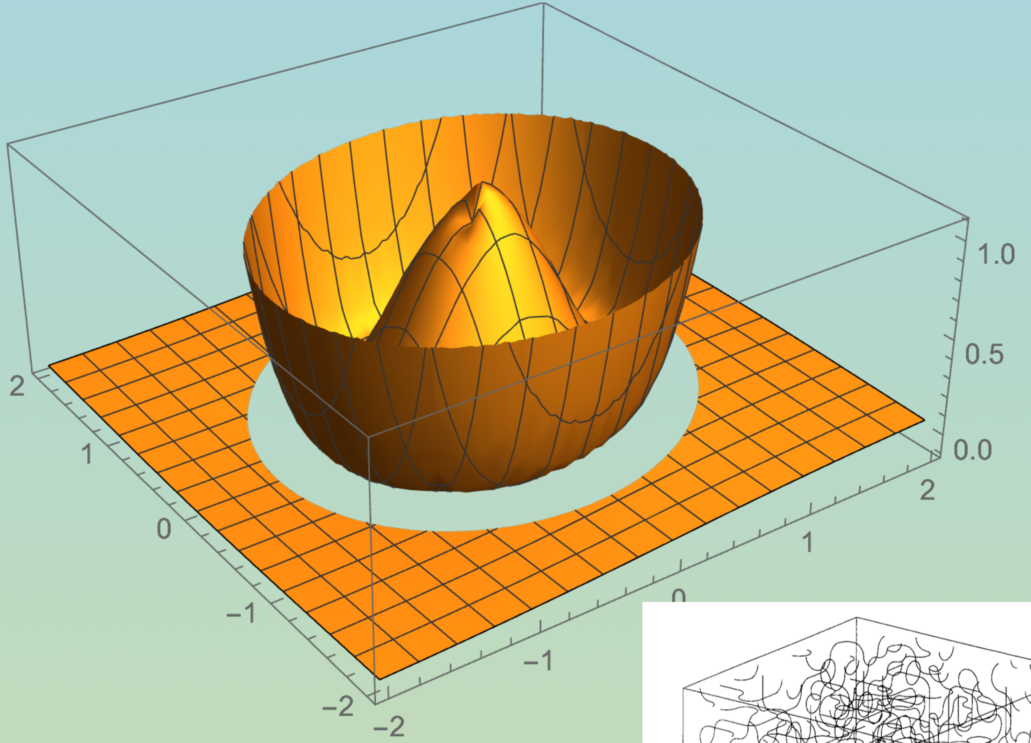
Network of strings forms

After initial transient → “scaling" behavior
O(1) string per Hubble volume



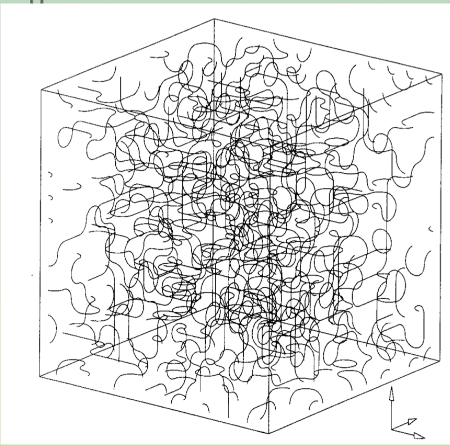
AXION AS COLD DARK MATTER

- Much later the potential gets tilted
- $V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 + \Lambda_{QCD}\left(1 - \cos(\frac{a}{f_a})\right)$



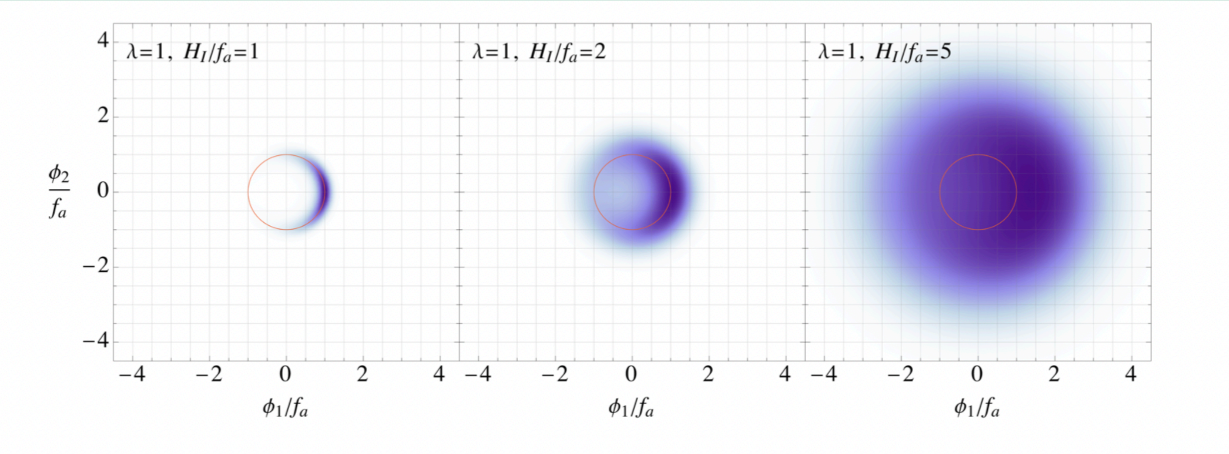
The field goes in the only minimum (after forming domain walls at $a/f_a \approx \pi$)

Strings and walls decay into (cold?) axions, which add to Cold Dark Matter



AXION AS COLD DARK MATTER

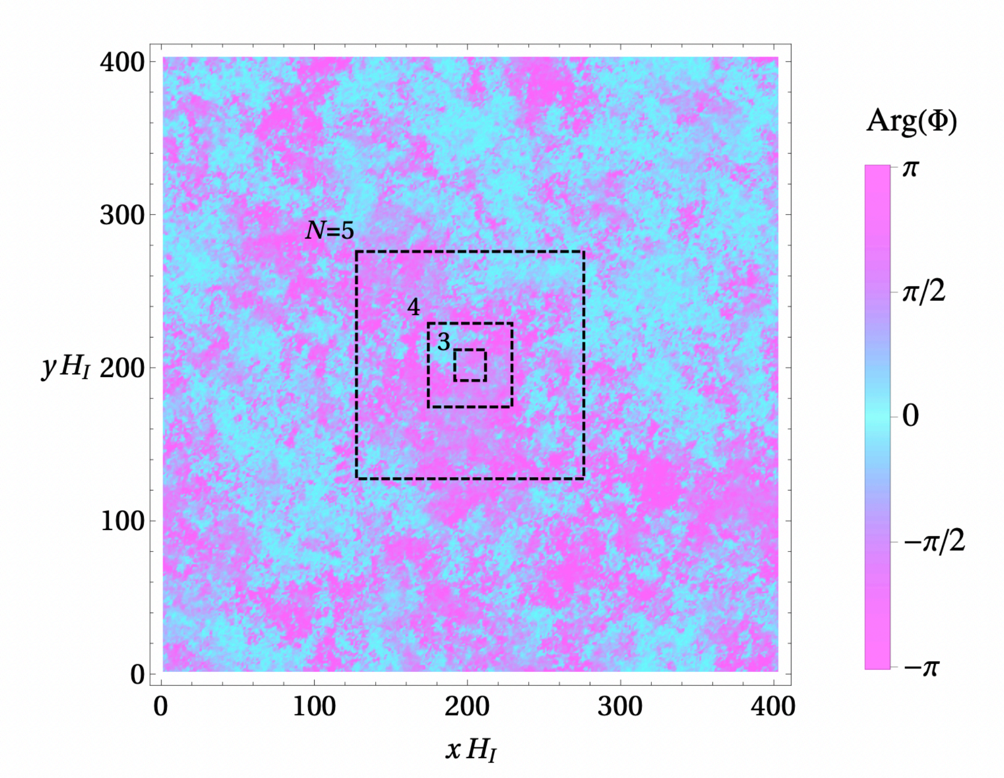
- THIRD POSSIBILITY: "STOCHASTIC INFLATIONARY SCENARIO"
- $H_i \gtrsim f_a$ large fluctuations during inflation (see Lyth 1992, Lyth & Stewart 1992)
- Both the angular and the radial field have large fluctuations



- Strings form due to large inflationary fluctuations
- If Temperature is never large enough after inflation ($T < f_a$) Symmetry is NOT restored after inflation

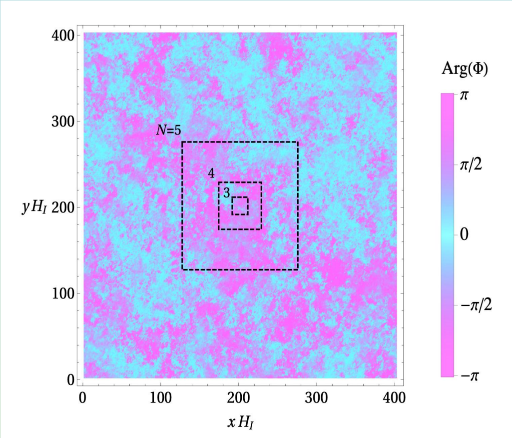
AXION AS COLD DARK MATTER

- On <u>small</u> patches: angle θ almost constant
- On <u>large</u> patches: can wrap from 0 to 2π
- Strings form, separated by a length $d = e^{N_s}/H_I$
- 2 CASES:
- If $N_s \gtrsim 60$ field coherent in our entire horizon
- If $N_s < 60$ Strings separated by macroscopic length d



RANDOMISATION DURING INFLATION

- Once a Fourier mode exits horizon ⇨ becomes classical with amplitude $\frac{H_I}{2\pi}$ ⇨ gives a kick to average field value
- In a coarse-grained region of size H^{-3} : field undergoes random walk, independently in causally disconnected regions, $\mathrm{Var}(\Phi) = \frac{H_I}{2\pi} N$



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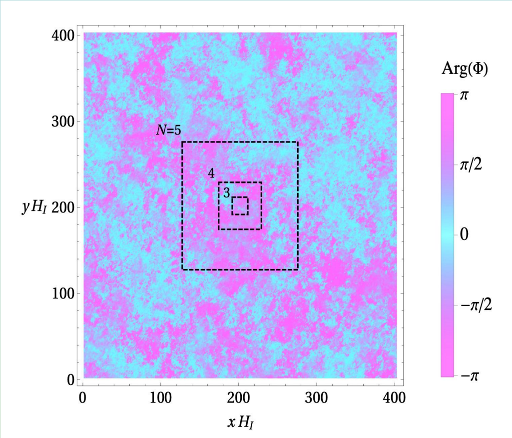
- Φ grows as $\sqrt{N}$ until it reaches quantum vs classical equilibrium:

$$P(\Phi)=\exp\left[-8\pi^2/3V(\Phi)/H^4\right]$$



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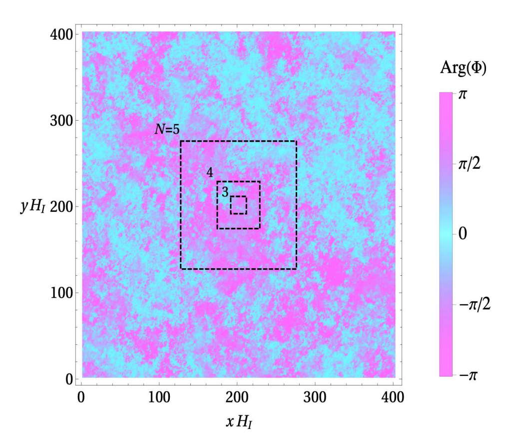

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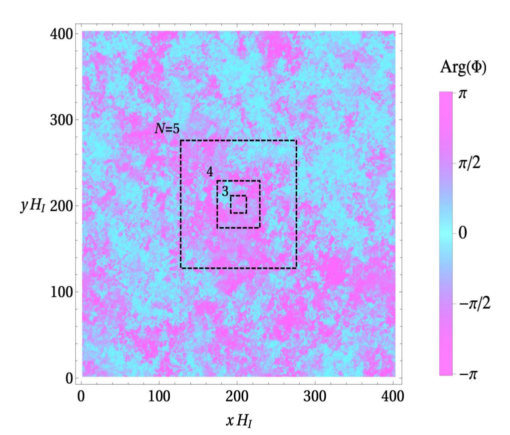

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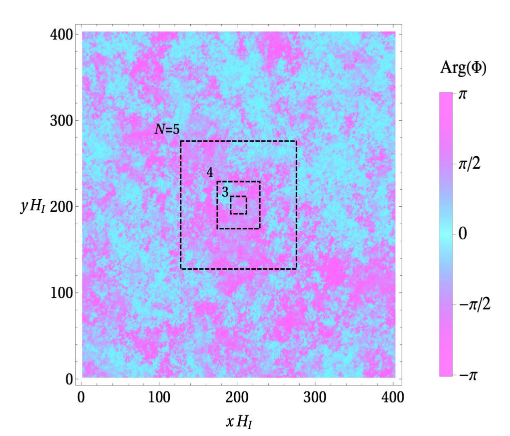
- After N_s Arg(Φ) randomized in $[0, 2\pi]$.



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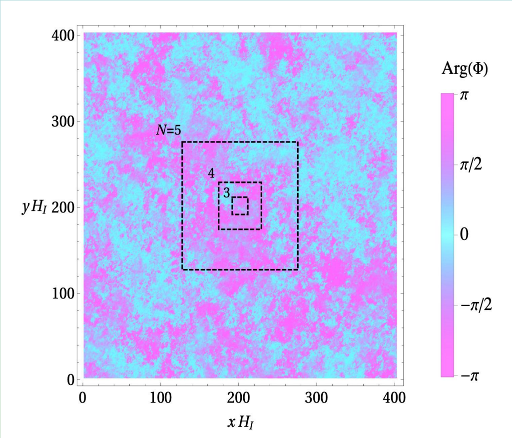

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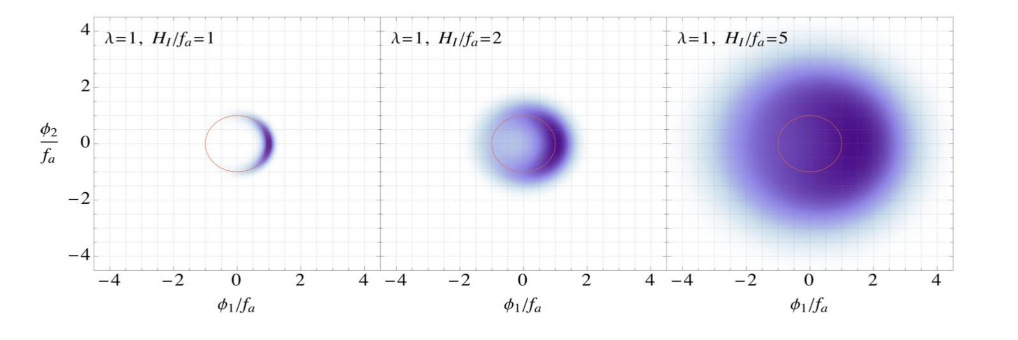
- At equilibrium: $\langle|\Phi|\rangle = H_I/\lambda^{1/4}$
- After N_s Arg(Φ) randomized in $[0, 2\pi]$. $N_s = 60$ realized for $\lambda \simeq 0.05$

RANDOMIZATION DURING INFLATION

- $\lambda \lesssim 1$



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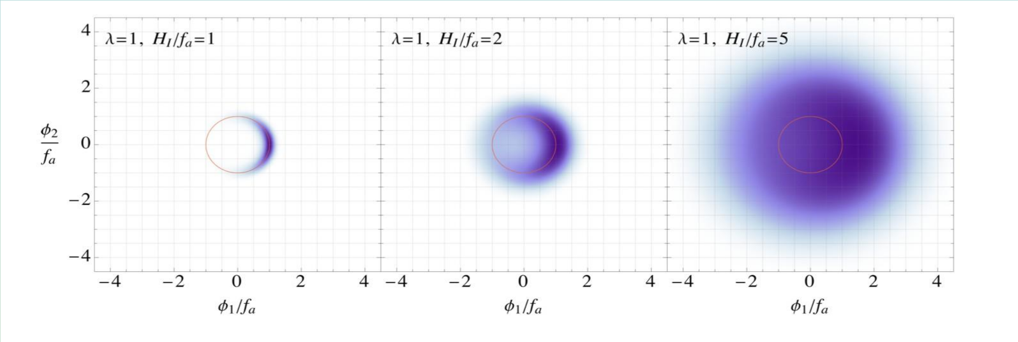
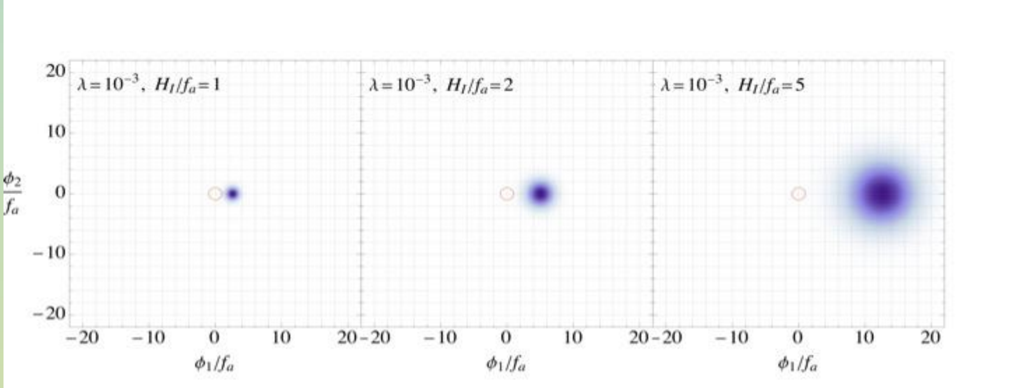


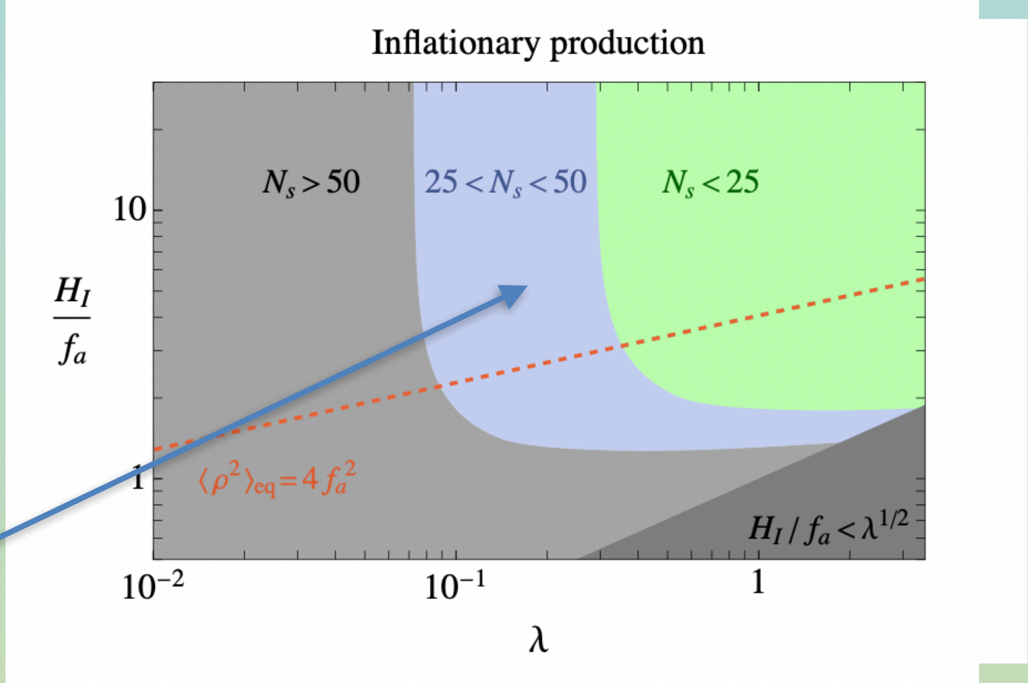
- $\lambda \gg 1$

Radial field: $\langle|\Phi|\rangle = H_I/\lambda^{1/4}$



AXION AS COLD DARK MATTER

- Strings separated by a length $d = e^{N_s}/H_I$
- $N_s \approx 10/\sqrt{\lambda}$
- If $N_s \gtrsim 60$ field coherent in our entire horizon
- If $N_s < 60$ Strings form, separated by a macroscopic length d
-
- If 25< $N_s < 60$ strings reenter the horizon after QCD phase transition: "LATE STRINGS" (NEW phenomenology)



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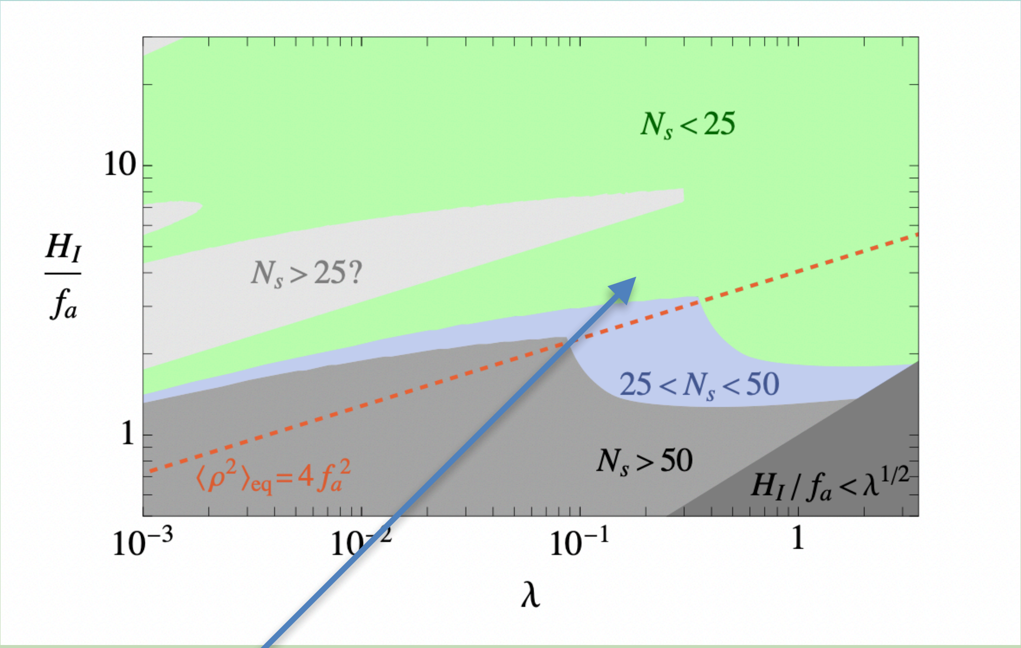
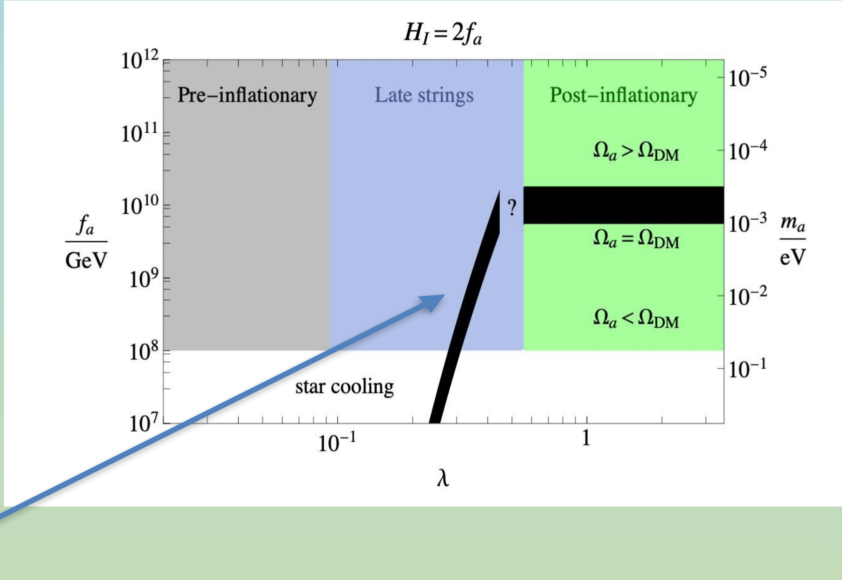


Overshoot mechanism after inflation:
<u>if the field starts high in the potential, it can roll on the opposite side</u>

EARLY STRING FORMATION

AXION DARK MATTER at LARGE m_a

<u>Standard post-inflationary:</u> Uncertainty from string simulations, but close to $f_a \sim 10^{10} - 10^{11}\text{GeV}$ ($m_a \sim 10^{-3} - 10^{-4}\text{eV}$)



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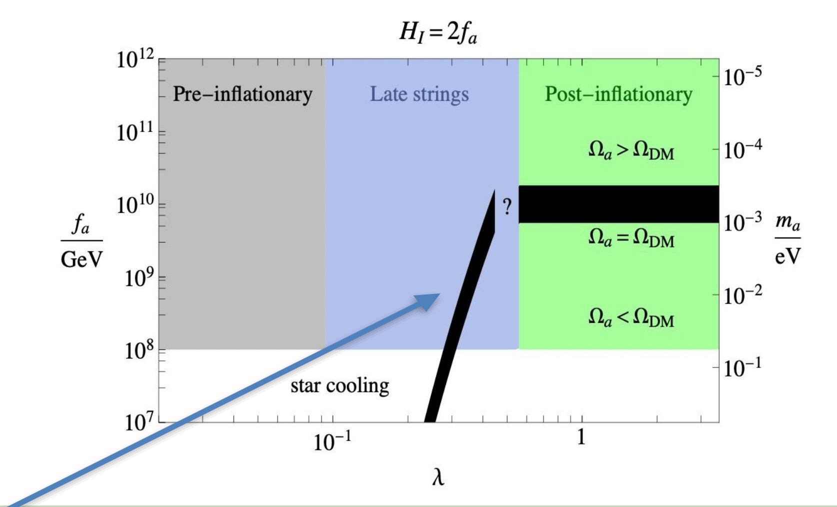
Late strings scenario:

String network born *underdense* (<1 string per Hubble patch)

-Becomes dense (enter horizon) later, even below QCD epoch

-As soon as they enter the horizon: decay into DM axions



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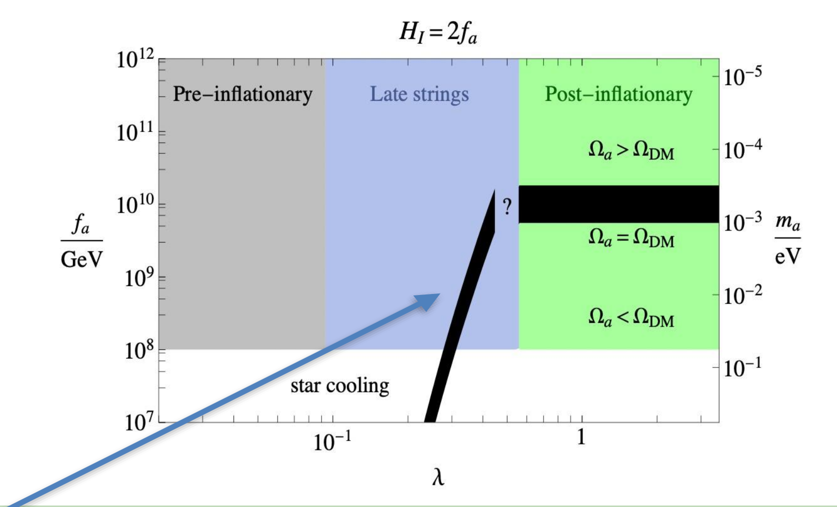

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-As soon as they enter the horizon: decay into DM axions

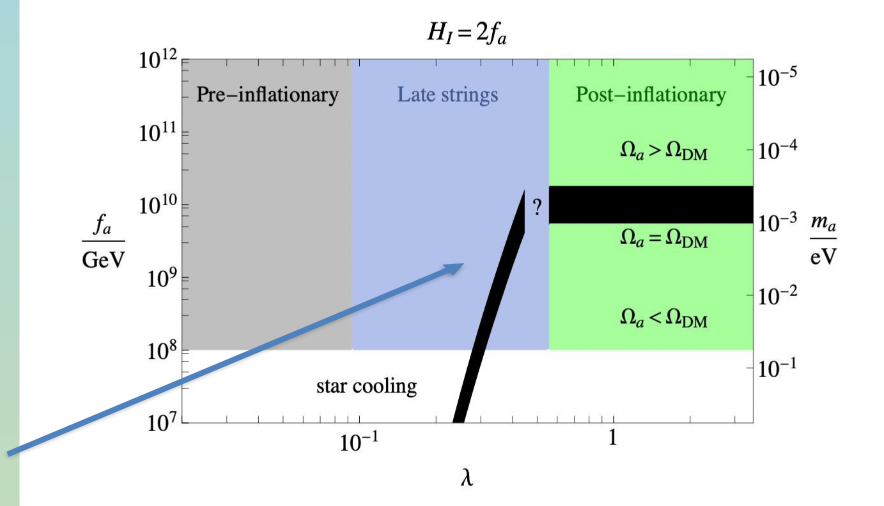
- Smaller f_a can be achieved for correct DM abundance (down to astrophysical bound $f_a \sim 10^{8} - 10^{10}\mathrm{GeV}$ ($m_a \sim 10^{-1} - 10^{-3}\mathrm{eV}$)

AXION MINICLUSTERS + ISOCURVATURE

Late strings scenario:

As in Standard post-inflationary:

String-wall system leaves O(1) inhomogeneities in axion DM at length scales H^{-1} at collapse



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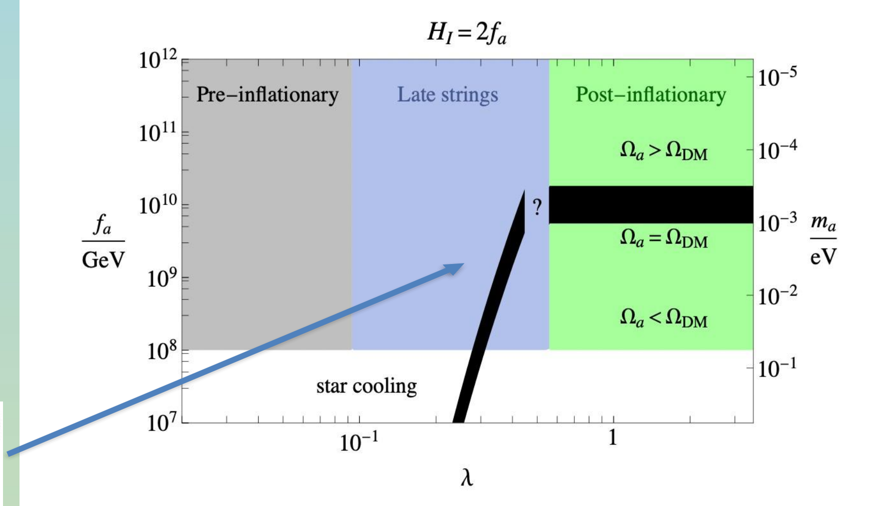
Very large miniclusters, up to

$$M_b \simeq 24 M_\odot \left[\frac{10}{g_*(T_{PQ})}\right]^{\frac{1}{2}} \left[\frac{1\,\mathrm{MeV}}{T_{PQ}}\right]^3$$



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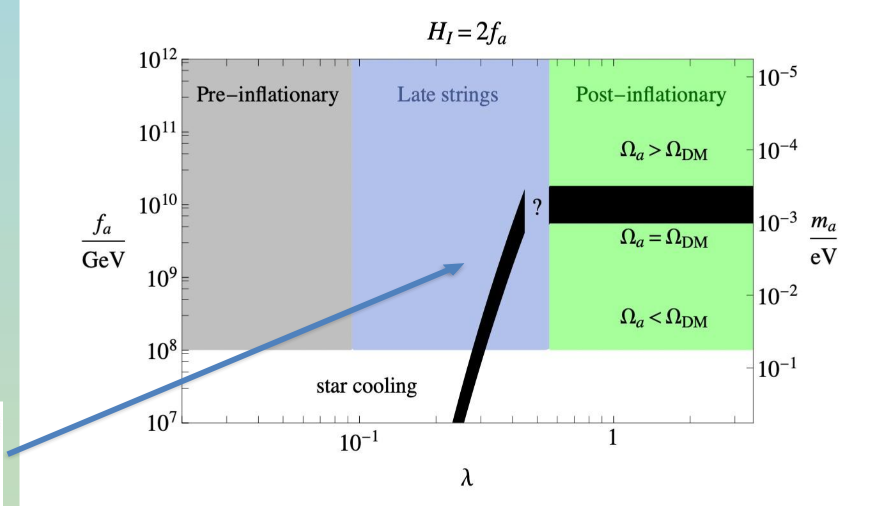


Isocurvature?

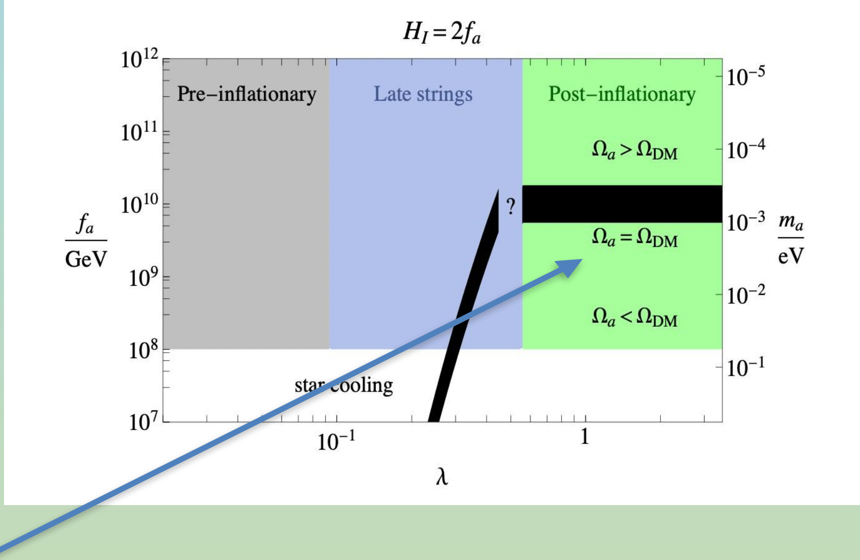
DM axions from misalignment carry large isocurvature (nearly flat), while String-Wall network should have a k^3 IR tail. What happens when DW collapse?

CONCLUSIONS

Late strings scenario:

If $H_I \gtrsim f_a$ and $f_a > T_{max}$

underdense string network could arise from Inflation



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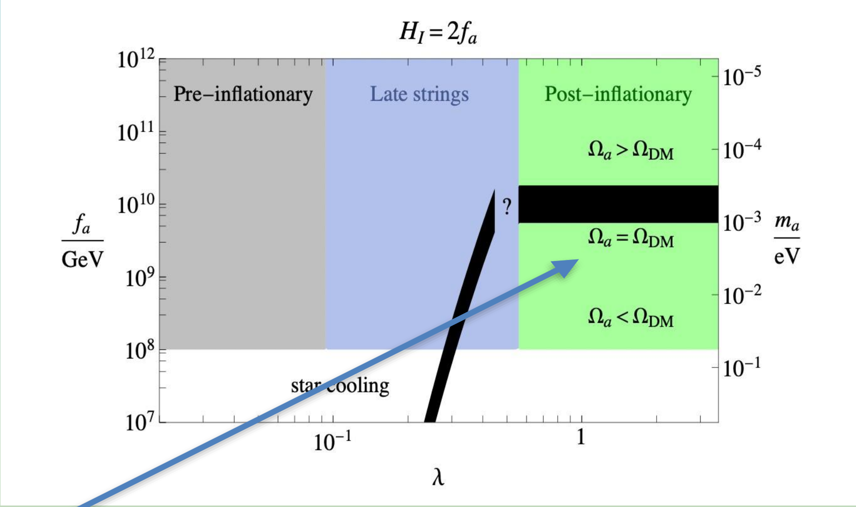
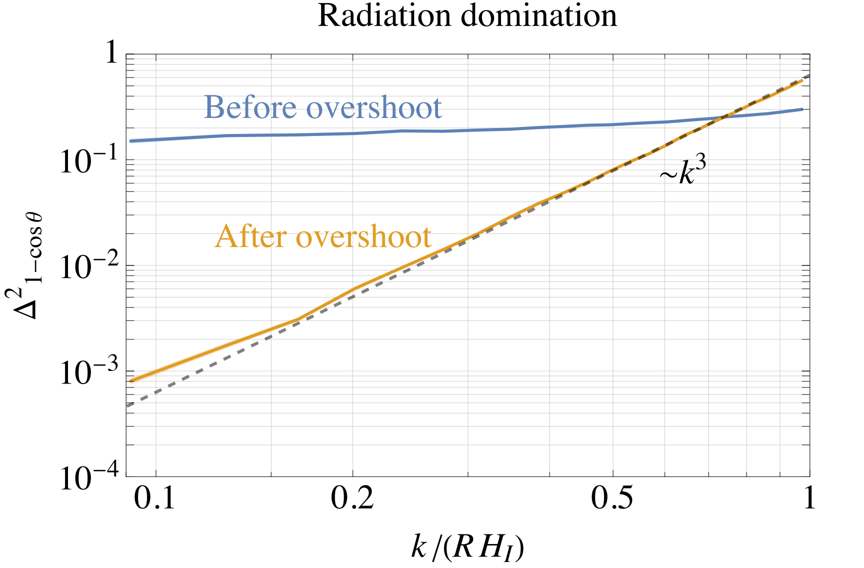


Phenomenological constraints:

- Large miniclusters
- Need to understand better isocurvature constraints



Radiation domination
Before overshoot
After overshoot
~k^3
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.1
0.2
0.5
1
$\Delta^2_{1-\cos\theta}$
$k/(R H_I)$