Inflationary Trispectrum of Gauge Fields from Scalar and Tensor Exchanges

(arXiv:2509.11143)

Haridev S R

Department of Physics

IISc, Bengaluru, India

Collaborators : P. Jishnu Sai and Prof. Rajeev Kumar Jain

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Our Model

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$$S[A_\mu] = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}\, \lambda(\phi)\, F_{\mu\nu}F^{\mu\nu} \quad : \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

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- Using the ADM decomposition to next order one can obtain with scalar fluctuations

$$H_{\zeta AA} = n\int d^3x\,\lambda\zeta\left(A_i'^2 - \frac{1}{2}F_{ij}^2\right) + O(\epsilon)$$

R.K. Jain and M.S. Sloth, Phys. Rev. D 86 (2012) 123528
L. Motta and R.R. Caldwell, Phys. Rev. D 85 (2012) 103532

In-in : Cosmological Diagrams

- In-in master formula

$$\left\langle \Omega \right| \mathcal{O}(\eta_0) \left| \Omega \right\rangle = \langle 0 | \; \bar{T}\left(e^{i \int_{-\infty}^{\eta_0} d\eta \, H_{\text{int}}} \right) \mathcal{O}(\eta_0) \; T\left(e^{-i \int_{-\infty}^{\eta_0} d\eta \, H_{\text{int}}} \right) \; | 0 \rangle$$

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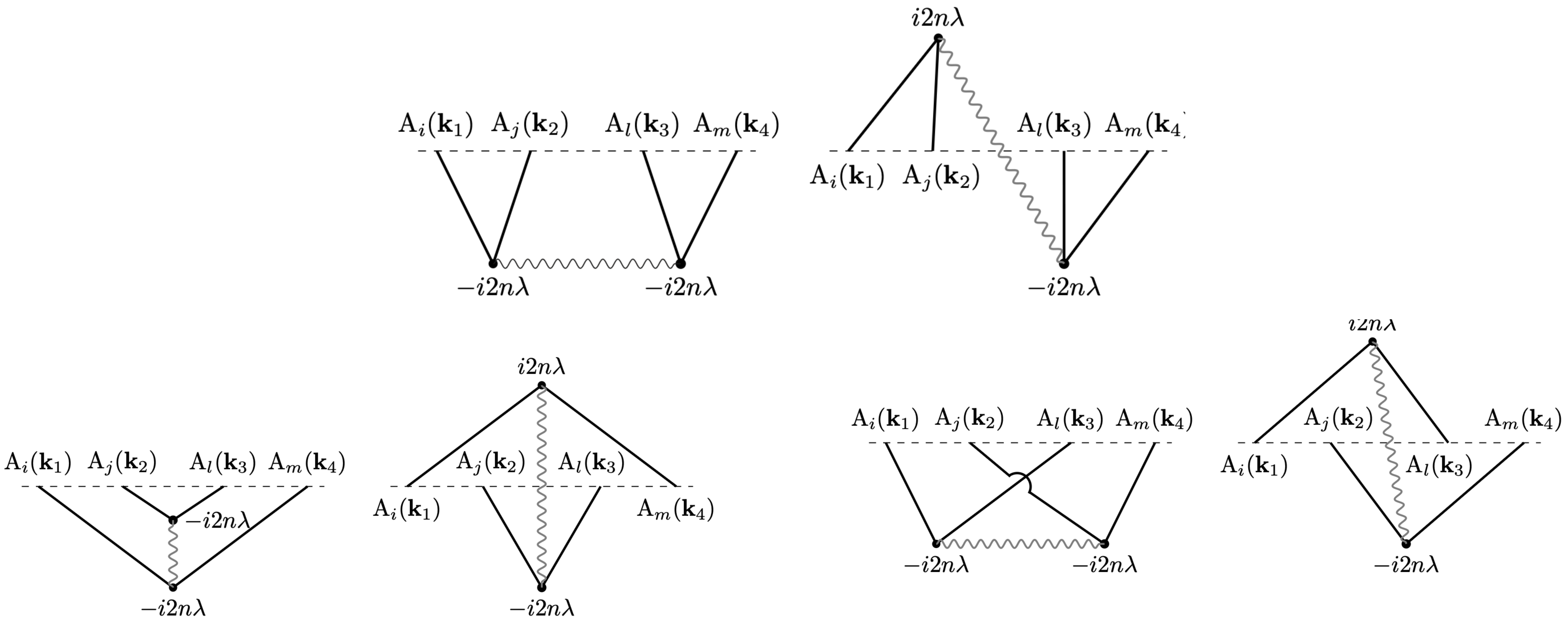

Scalar Exchange: Magnetic Trispectra

- We focus on the index free trispectra

$$\langle \boldsymbol{B}(\boldsymbol{k}_1) \cdot \boldsymbol{B}(\boldsymbol{k}_2)\, \boldsymbol{B}(\boldsymbol{k}_3) \cdot \boldsymbol{B}(\boldsymbol{k}_4) \rangle = (2\pi)^3\, \delta^{(3)} \left(\sum_{\alpha=1}^{4} \mathbf{k}_\alpha \right) \mathscr{T}_{\zeta B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$

With $\mathscr{T}_{\zeta B} = \frac{1}{a^8} \left(\frac{\dot{\lambda}}{H\lambda} \right)^2 \sum_{S \in 1,2,3} \sum_{X,Y \in A,A'} Q^{\zeta}_{SXY}\, I^{SXY}$

- Polarisation function is

$$Q^{\zeta}_{1AA} = k_1^2 k_2^2 k_3^2 k_4^2 \left(1 + \cos^2\theta_{12}\right)\left(1 + \cos^2\theta_{34}\right)$$

$$Q^{\zeta}_{1AA'} = 2k_1^2 k_2^2 k_3 k_4 \left(1 + \cos^2\theta_{12}\right)\cos\theta_{34}$$

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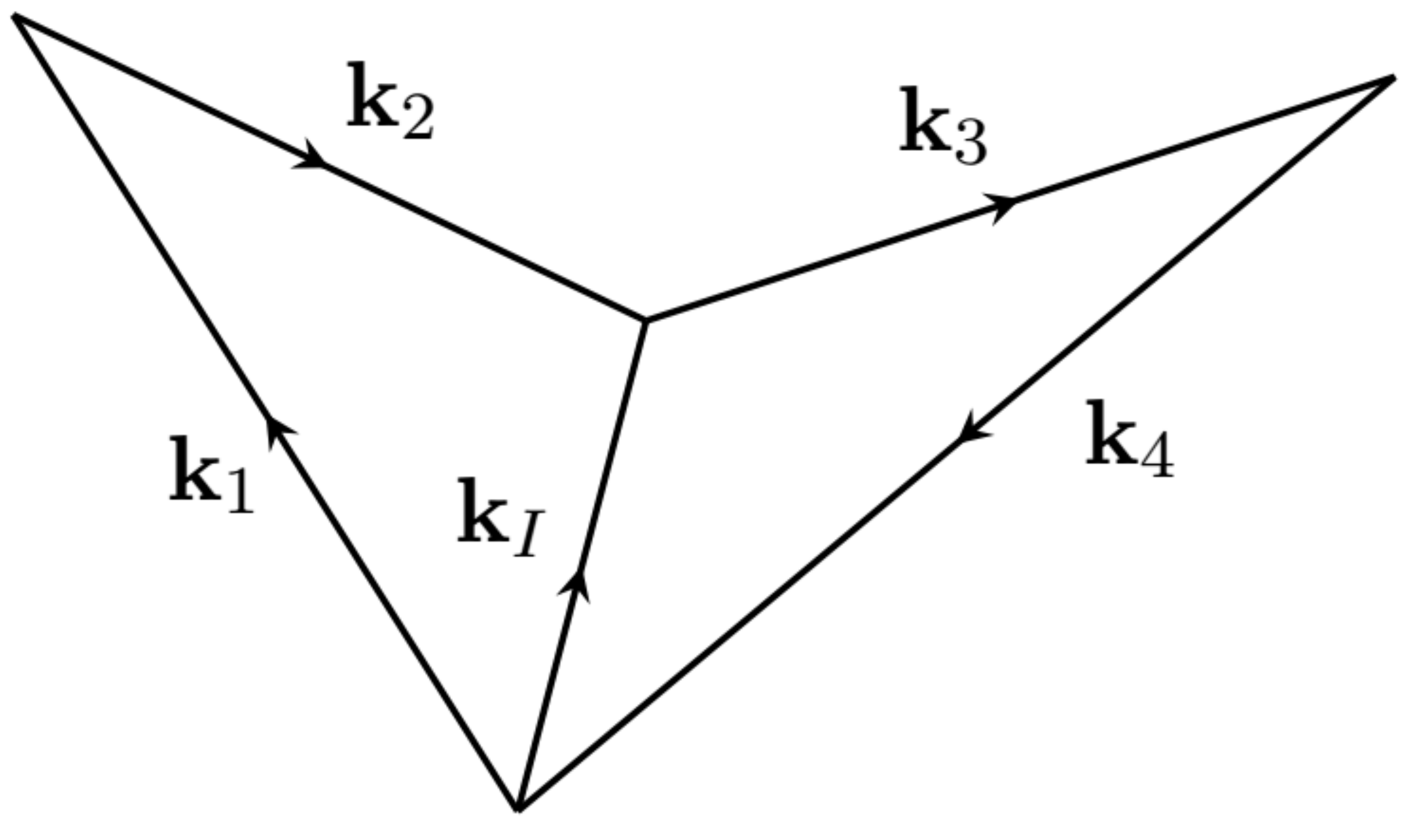
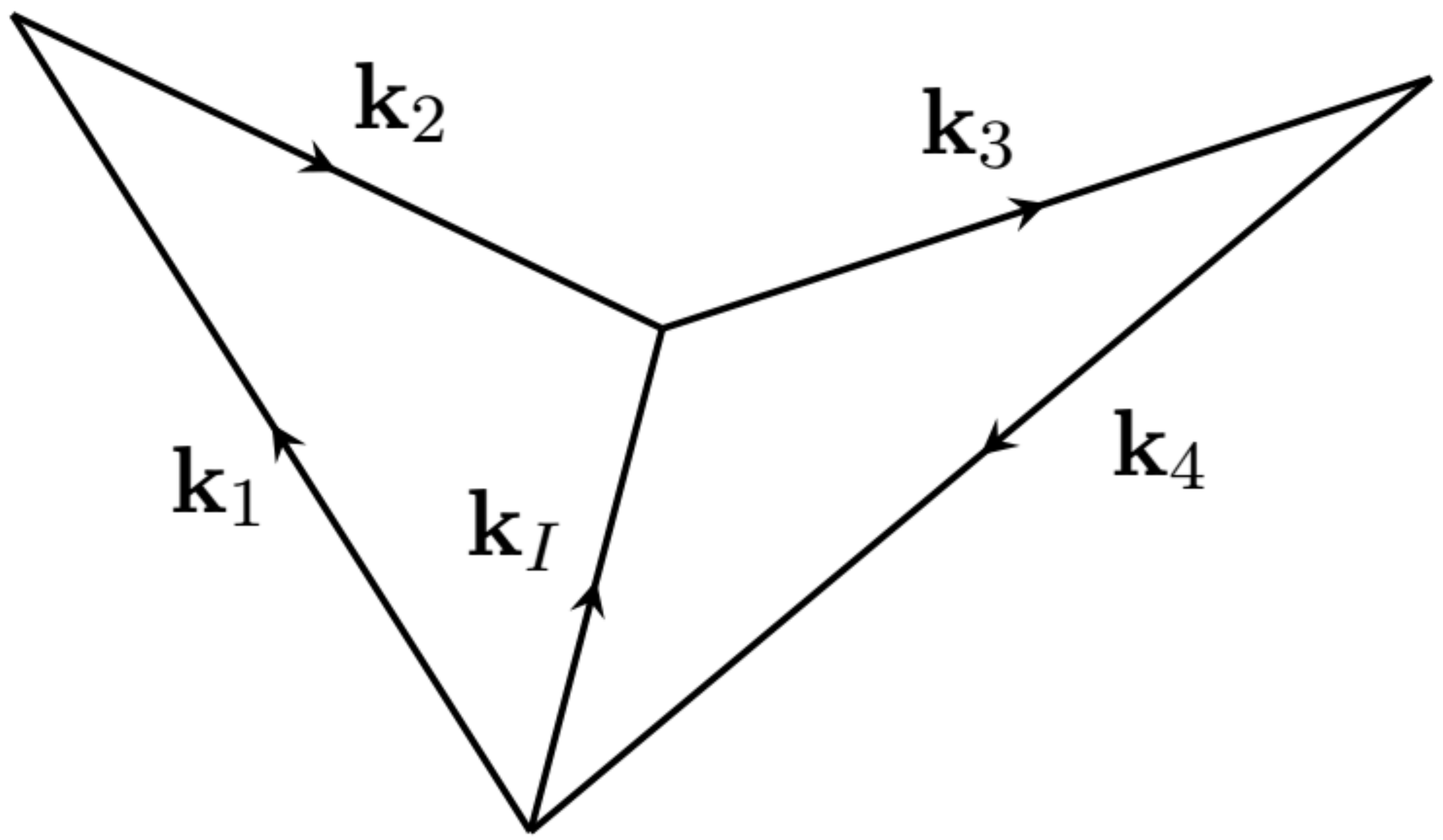
Momentum Quadrilateral

- Momentum conservation $\Longrightarrow$ closed shapes



- Shape is not planar

Equisided Configurartions



- Equisided configuration: $|\mathbf{k}_1| = |\mathbf{k}_2| = |\mathbf{k}_3| = |\mathbf{k}_4|$
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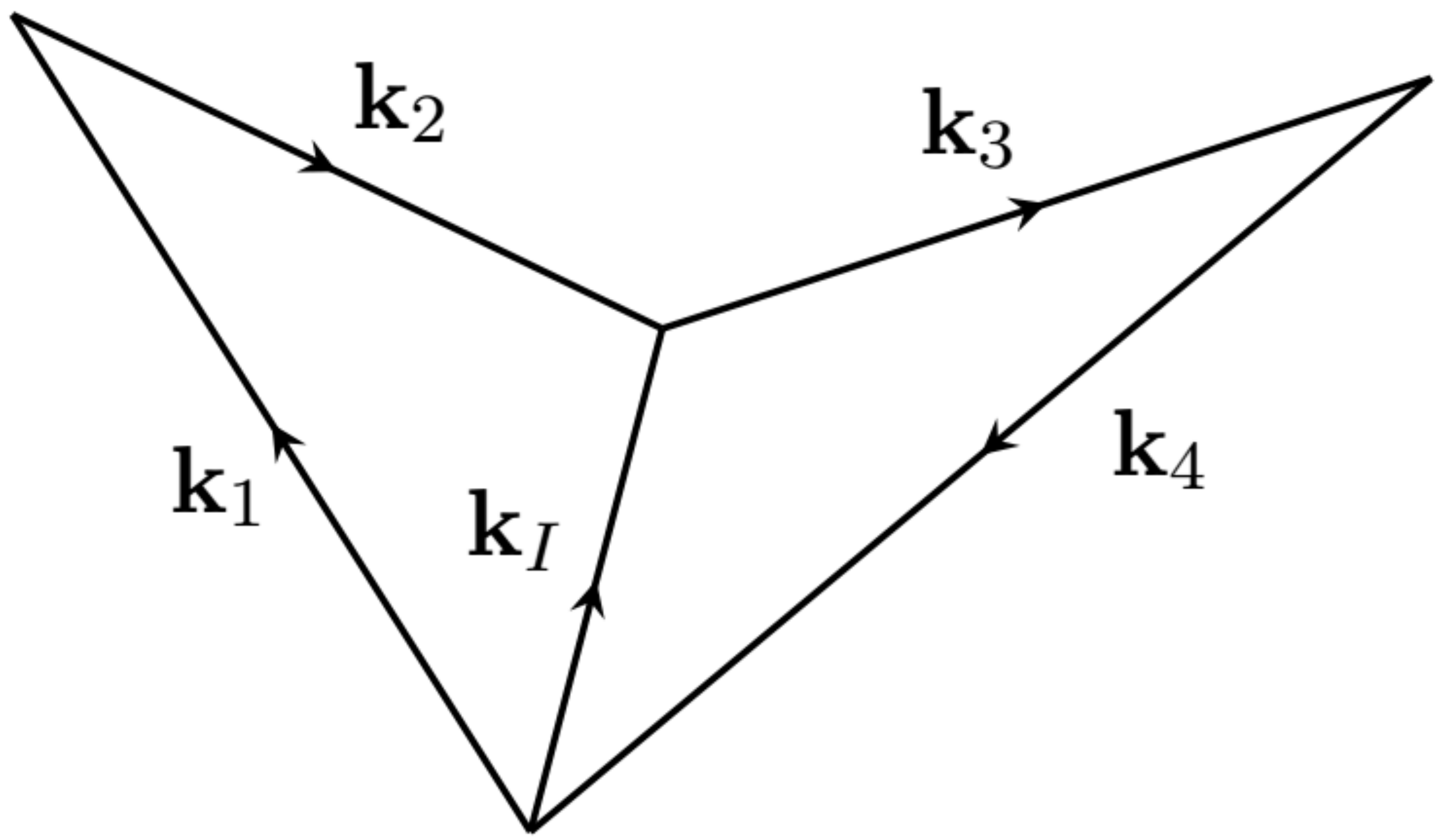

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- The magnetic trispectrum is

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Contains all the informations of three channels and the nested time integrals

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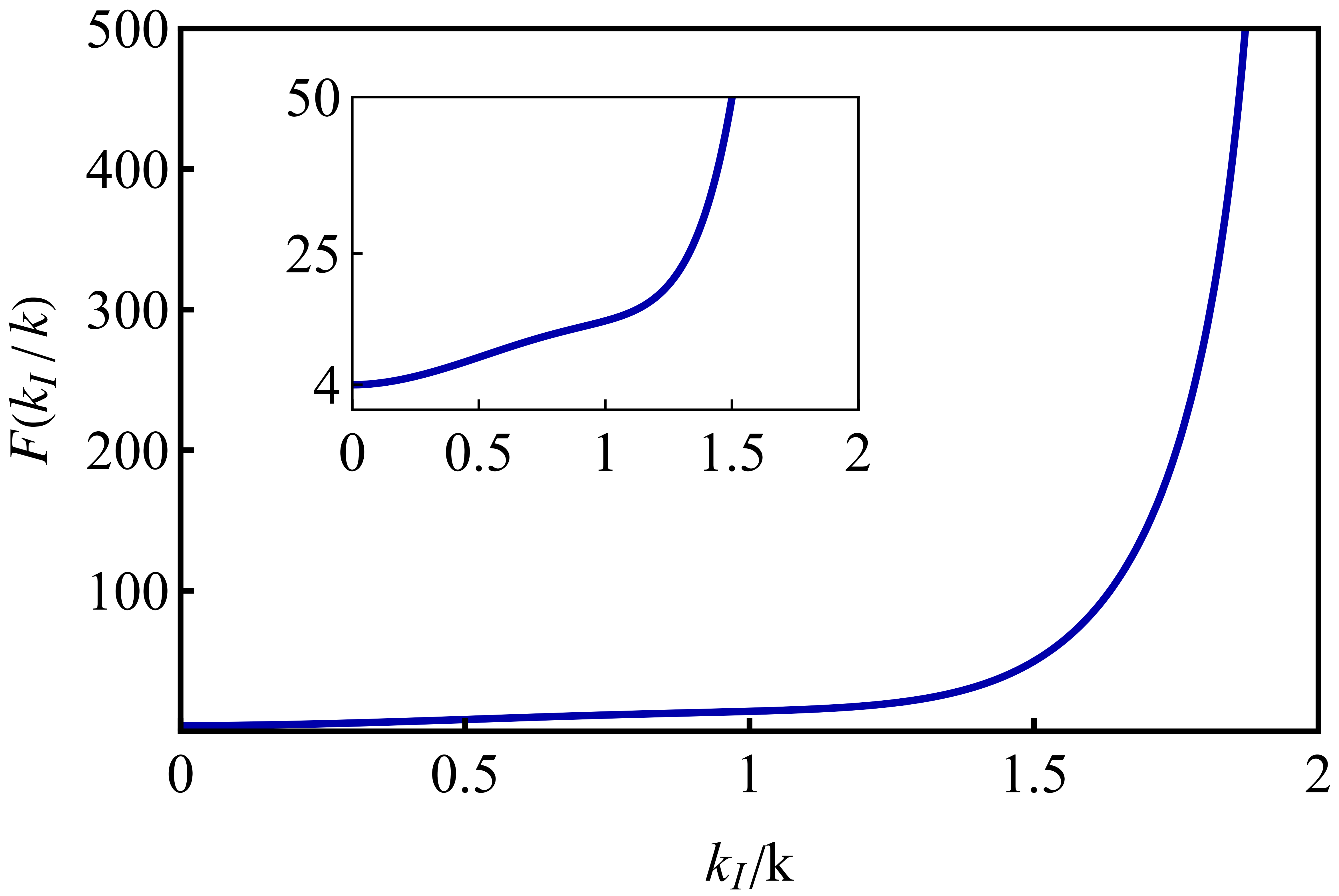
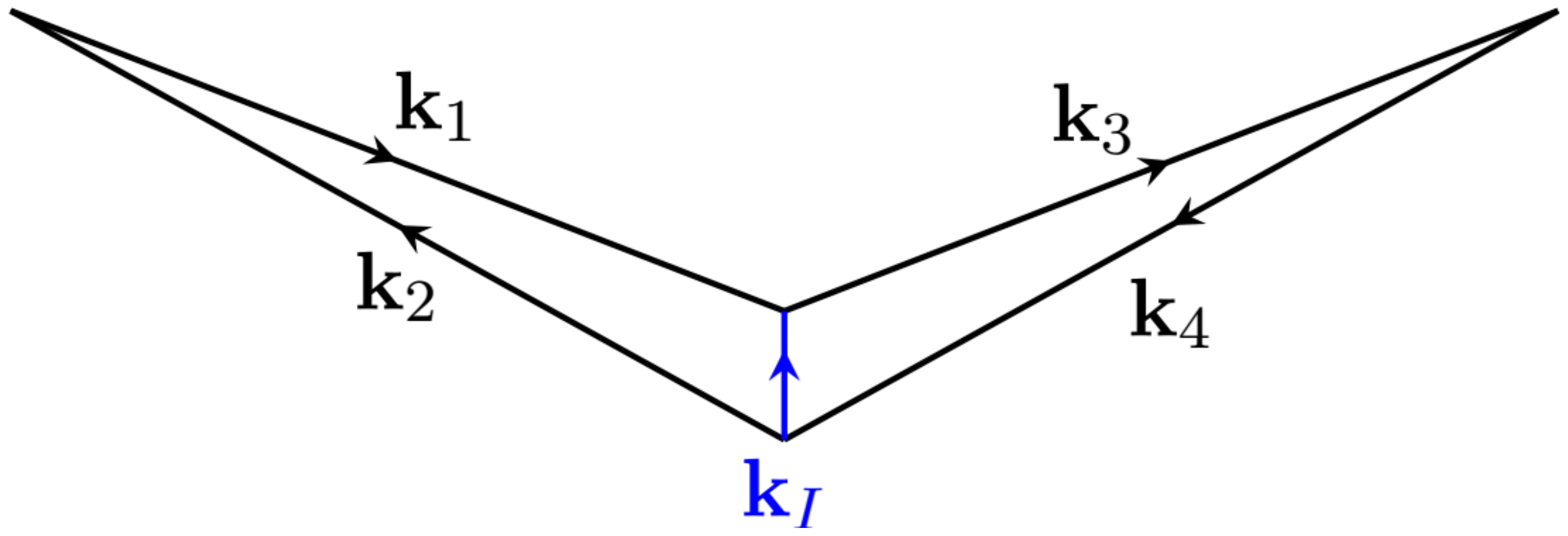
Contains all the informations of three channels and the nested time integrals

- For $\phi = 0$
  - Monotonically increasing function of k_I/k
  - Correlation strength $\propto k_I$
  - $F(0) \neq 0$, is the counter collinear limit



Counter Collinear Limit



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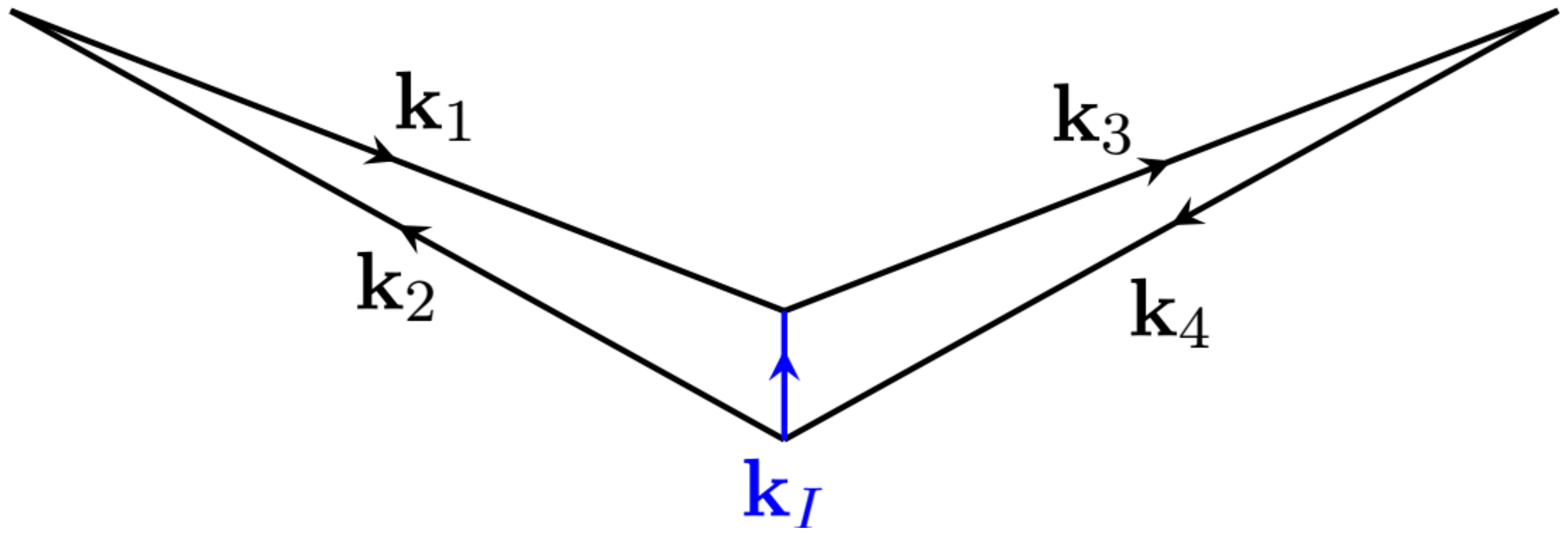

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- Novel consistency relation similar to Suyama-Yamaguchi relation

$$\beta_{NL}^{\zeta} = \left(\frac{\dot{\lambda}}{H\lambda}\right)^2 = \left(b_{NL}^{\zeta}\right)^2$$

T. Suyama and M. Yamaguchi, Phys.Rev. D 77 (2008) 023505 [0709.2545]

- The equality is a lower bound, in general $\beta_{NL}^{\zeta} \geq \left(b_{NL}^{\zeta}\right)^2$

  - The presence of γ exchange validate this inequality

V. Assassi, D. Baumann and D. JCAP 11 (2012) 047 [1204.4207]

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- Correlating the azimuthal orientation of $\boldsymbol{k}_I$ with $\boldsymbol{k}$
- Higher order angular functions

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- Correlating the azimuthal orientation of $\boldsymbol{k}_I$ with $\boldsymbol{k}$
- Higher order angular functions
- In counter collinear limit : enhanced correlation along $\boldsymbol{k}_I$
- Equisided configuration : oscillatory angular dependence (spin -2 nature)

Summary and conclusion

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- We did the systematic computation of four point autocorrelation function of $U(1)$ gauge field with full in-in formalism upto cubic order interation with both tensor and scalar perturbation.
- From the auxiliary gauge field we also obtain the trispectrum for $\boldsymbol{B}$ and $\boldsymbol{E}$, in the contracted form which contributes to energy density.
- In the equisided configurations the non-Gaussian signal is monotonically increasing as the shape approaches the flattened configurations.
- In the counter collinear limit, we showed that the magnetic field trispectrum follows a novel consistency relation.

$$\beta_{NL}^{\zeta} = \left(b_{NL}^{\zeta}\right)^2$$

- Magnetic trispectrum sourced by tensor exchange, inherits a richer angular dependence leads to distinctive signatures. But suppressed by tensor to scale ratio

Thank You

(haridevsr@iisc.ac.in)