Massive Abelian Gauge Field in Axion Inflation

José Jaime Terente Díaz*

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CFisUC

1290 Universidade de Coimbra

*work in progress in collaboration with Ricardo Z. Ferreira & Alessio Notari

For a homogeneous (pseudo-)scalar field $\phi(t)$, the eom displays a source of dissipation in addition to the Hubble friction

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2 V_{,\phi} = \frac{\alpha}{a^2 f}\langle \mathbf{E}\cdot\mathbf{B}\rangle$$

$\mathcal{H} \equiv \dot{a}/a$ is the conformal/comoving Hubble parameter ('universal' friction: $2\mathcal{H}\dot{\phi}$); $a(\eta)$ is the scale factor.

$\langle \mathbf{E}\cdot\mathbf{B}\rangle \Rightarrow$ relies on the interplay between ϕ and A_μ ('interaction-induced' friction).

Gravitational interaction via Friedmann equation, $3\mathcal{H}^2 m_{\mathrm{P}}^2 = a^{-2}\rho$, where ρ is the spatially-averaged, comoving total energy density: $\rho = \frac{1}{2}(a\dot{\phi})^2 + a^4 V + \rho_A$

$$\rho_A \equiv \frac{1}{2}\left[\langle|\mathbf{E}|^2\rangle + \langle|\mathbf{B}|^2\rangle + (am)^2\left(A_0^2 + \langle|\mathbf{A}|^2\rangle\right)\right]$$

We may estimate analytically the two backreaction terms during a regime of weak backreaction

$$\langle \hat{\mathbf{E}} \cdot \hat{\mathbf{B}} \rangle \simeq -\frac{1}{4\pi^2} \int_{\mathrm{k_{IR}}}^{k_{\mathrm{UV}}} \mathrm{d} \ln k \; k^4 \frac{\partial}{\partial \eta} \left(|\mathcal{A}_+|^2 \right)$$

$$\rho_A \simeq \frac{1}{4\pi^2} \int_{k_{\mathrm{IR}}}^{k_{\mathrm{UV}}} \mathrm{d} \ln k \; k^3 \left[|\dot{\mathcal{A}}_+|^2 + \left(k^2 + \frac{\bar{m}^2}{\eta^2} \right) |\mathcal{A}_+|^2 \right]$$

For very heavy gauge fields $\bar{m} \equiv m/H \gg 1$ during inflation

$$\langle \hat{\mathbf{E}} \cdot \hat{\mathbf{B}} \rangle \simeq \frac{16\pi}{3} \bar{m}^7 e^{-2\bar{m}\pi} \; \langle \hat{\mathbf{E}} \cdot \hat{\mathbf{B}} \rangle|_{\bar{m}=0}$$

$$\rho_A \simeq \frac{256\pi}{27} \bar{m}^7 e^{-2\bar{m}\pi} \; \rho_A|_{\bar{m}=0}$$

And determine the onset of moderate/strong backreaction $\Rightarrow$ $\frac{\alpha}{a^4 f} |\langle \hat{\mathbf{E}} \cdot \hat{\mathbf{B}} \rangle| \simeq |V_{,\phi}|$ and $a^{-4} \rho_A \simeq V$ $\Rightarrow$ Mass relaxes contraints on $m_{\mathrm{P}} \frac{|V_{,\phi}|}{V}$ and $\frac{V}{m_{\mathrm{P}}^4}$.

Onset of backreaction results in many inverse decay processes ('id') $\mathcal{A}_+ + \mathcal{A}_+ \to \delta\phi \Rightarrow$ new contribution to the power spectrum $\mathcal{P}_{\mathcal{R}_\phi}$

$$\mathcal{P}_{\mathcal{R}_\phi} \equiv \left(\frac{\mathcal{H}}{\dot{\phi}}\right)^2 \left(\mathcal{P}_{\rm vac} + \mathcal{P}_{\rm id}\right)$$

Vacuum contribution is $\mathcal{P}_{\rm vac} \simeq \frac{H^2|\Gamma(\nu)|^2}{\pi^3(1+\epsilon)^2}\left(\frac{-k\eta}{2}\right)^{3-2\nu}$ (or $\simeq \frac{H^2}{4\pi^2}$ if $\nu = 3/2$ and $\epsilon = 0$ as in exact de Sitter). In the case of id

$$\mathcal{P}_{\rm id} \simeq \frac{\alpha^2\pi}{8f^2}\mathcal{P}_{\rm vac}\int\frac{{\rm d}^3\mathbf{q}}{(2\pi)^3}\left[1-\frac{\mathbf{q}\cdot(\mathbf{k}-\mathbf{q})}{|\mathbf{q}||\mathbf{k}-\mathbf{q}|}\right]^2\left|\int_{-\infty}^0 {\rm d}\tilde{\eta}\,\frac{\sqrt{-\tilde{\eta}}}{a(\tilde{\eta})}{\rm Re}\left[H_\nu^{(1)}(-k\tilde{\eta})\right]\mathsf{A}\left(\tilde{\eta},|\mathbf{q}|,|\mathbf{k}-\mathbf{q}|\right)\right|^2$$

where

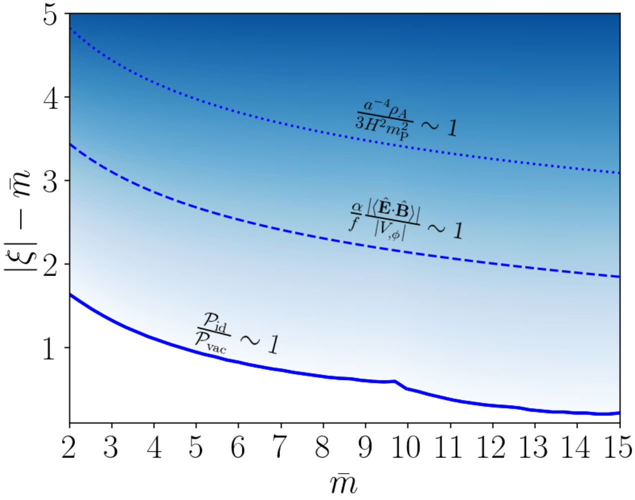
$$\mathsf{A}(\eta,|\mathbf{q}|,|\mathbf{k}-\mathbf{q}|) \equiv |\mathbf{q}|\dot{\mathcal{A}}_+(\eta,|\mathbf{k}-\mathbf{q}|)\mathcal{A}_+(\eta,|\mathbf{q}|) + |\mathbf{k}-\mathbf{q}|\dot{\mathcal{A}}_+(\eta,|\mathbf{q}|)\mathcal{A}_+(\eta,|\mathbf{k}-\mathbf{q}|)$$

In de Sitter ($q_* \equiv |\mathbf{q}|/k$ & $p_* \equiv |\mathbf{k}-\mathbf{q}|/k$)

$$\mathcal{P}_{\rm id} \simeq \frac{\alpha^2\mathcal{P}_{\rm vac}^2}{16f^2}\int_0^\infty\frac{{\rm d}q_*}{q_*}\int_{|1-q_*|}^{1+q_*}\frac{{\rm d}p_*}{p_*}\left[(p_*+q_*)^2-1\right]^2\times$$

$$\times\overbrace{\left|\int_{-\infty}^0 {\rm d}\tilde{\eta}\;\left[\sin(-k\tilde{\eta})+k\tilde{\eta}\cos(-k\tilde{\eta})\right]\mathsf{A}\left(k\tilde{\eta},q_*,p_*\right)\right|^2}^{\mathsf{I}(q_*,p_*)}$$

Taking $H \sim 10^{-5} m_{\mathrm{P}}$, $A_s = \frac{H^4}{4\pi^2 (\mathrm{d}\phi/\mathrm{d}t)^2} \sim 2.2 \times 10^{-9}$...



Estimation for the onset of backreaction for heavy gauge field: domination of the inverse decay contribution (solid line), backreaction on the eom of the homogeneous scalar field (dashed line), backreaction on the Hubble rate (dotted line) $\Rightarrow$ for $\mathcal{P}_{\mathrm{id}} \sim \mathcal{P}_{\mathrm{vac}}$ we have $\frac{\alpha}{f}|\langle \hat{\mathbf{E}} \cdot \hat{\mathbf{B}} \rangle| \sim |V_{,\phi}| \times 10^{-5}$

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