Gravitational Wave Observables from Second-Order Tensor Perturbations

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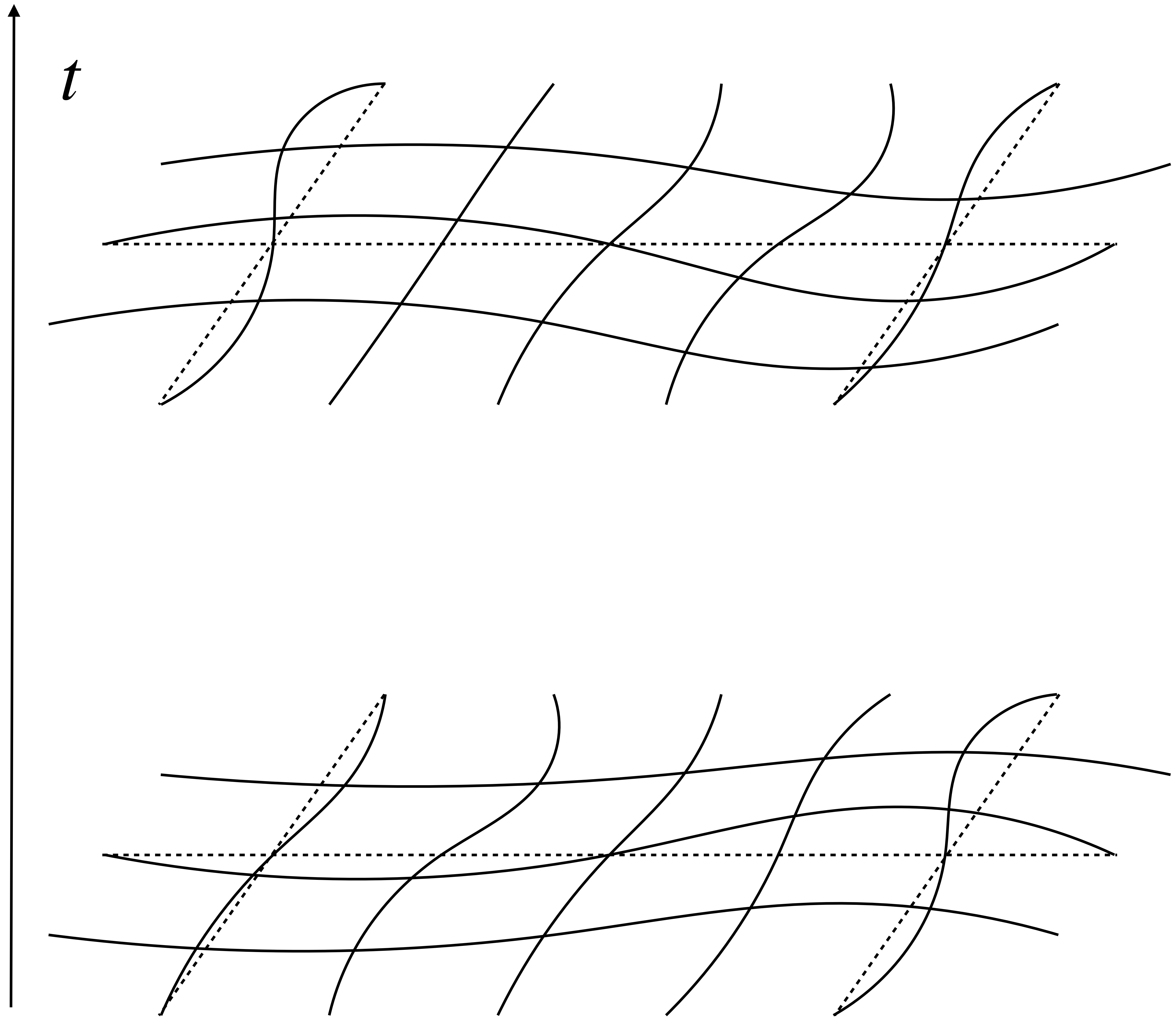
TT components $\quad h_{ij} \rightarrow h_{ij} + \partial_i T \partial_j T + \ldots$

$$x^\mu \rightarrow x^\mu + \zeta^\mu \qquad \zeta^\mu = (T, \partial^i L)$$



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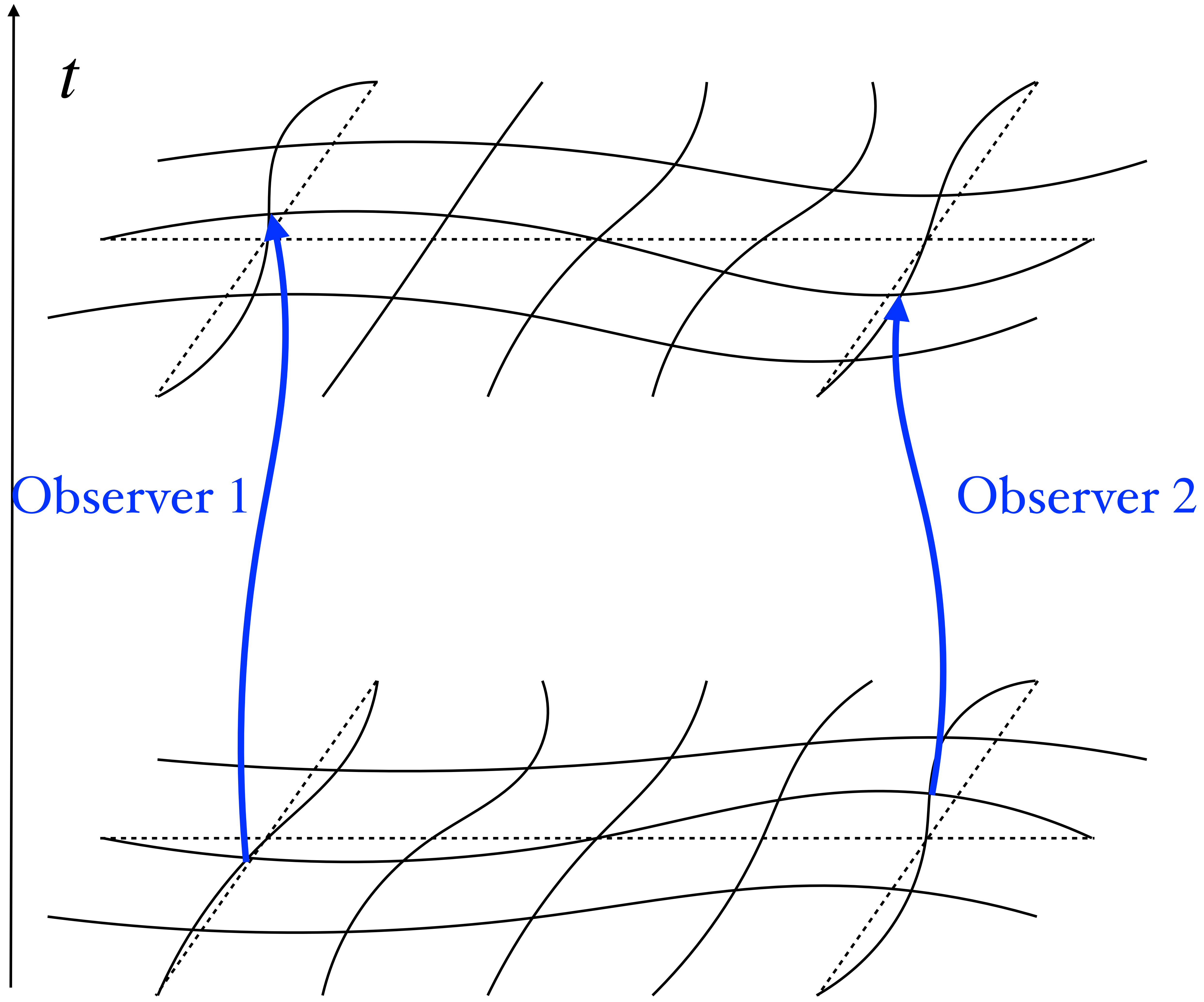

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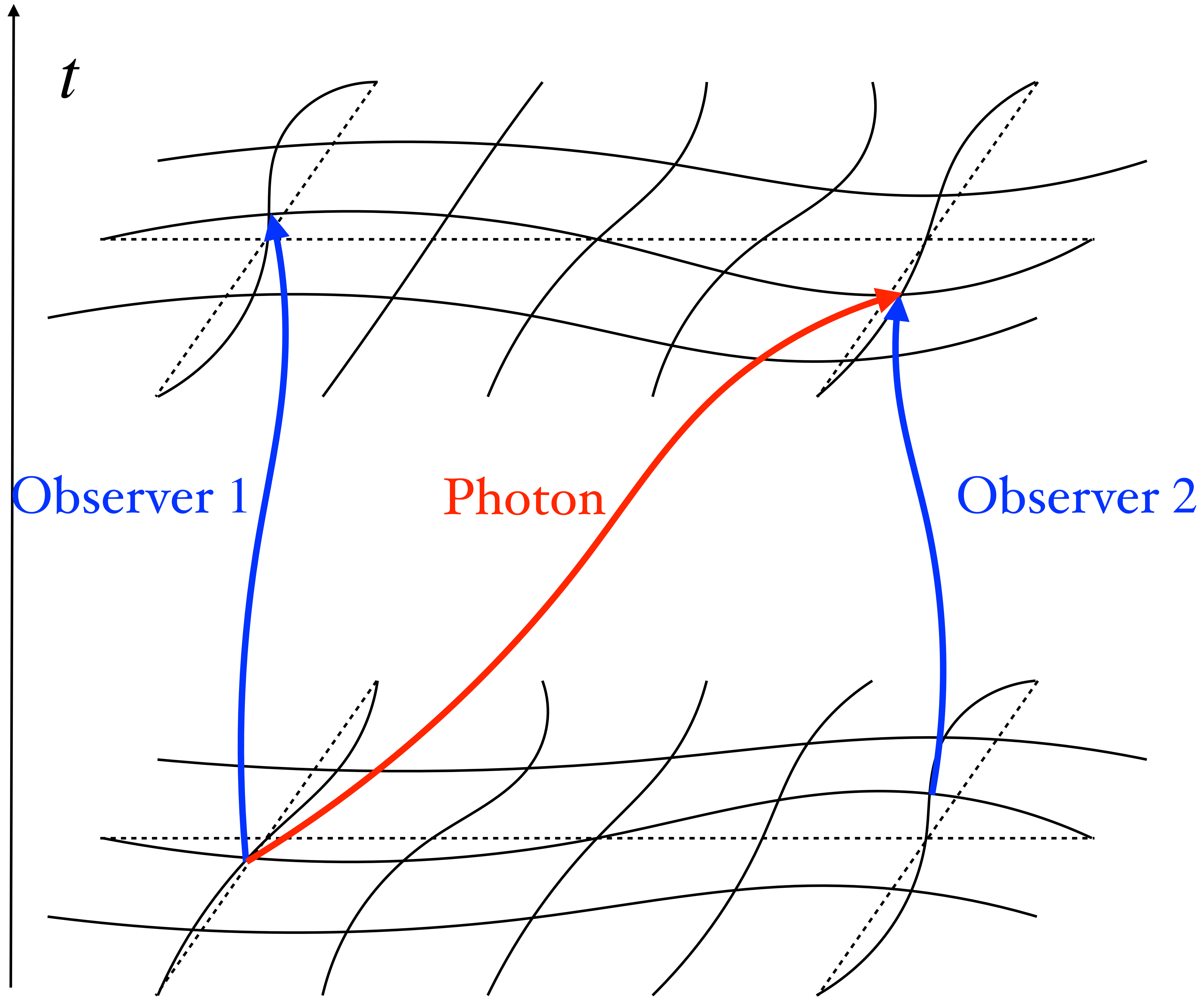

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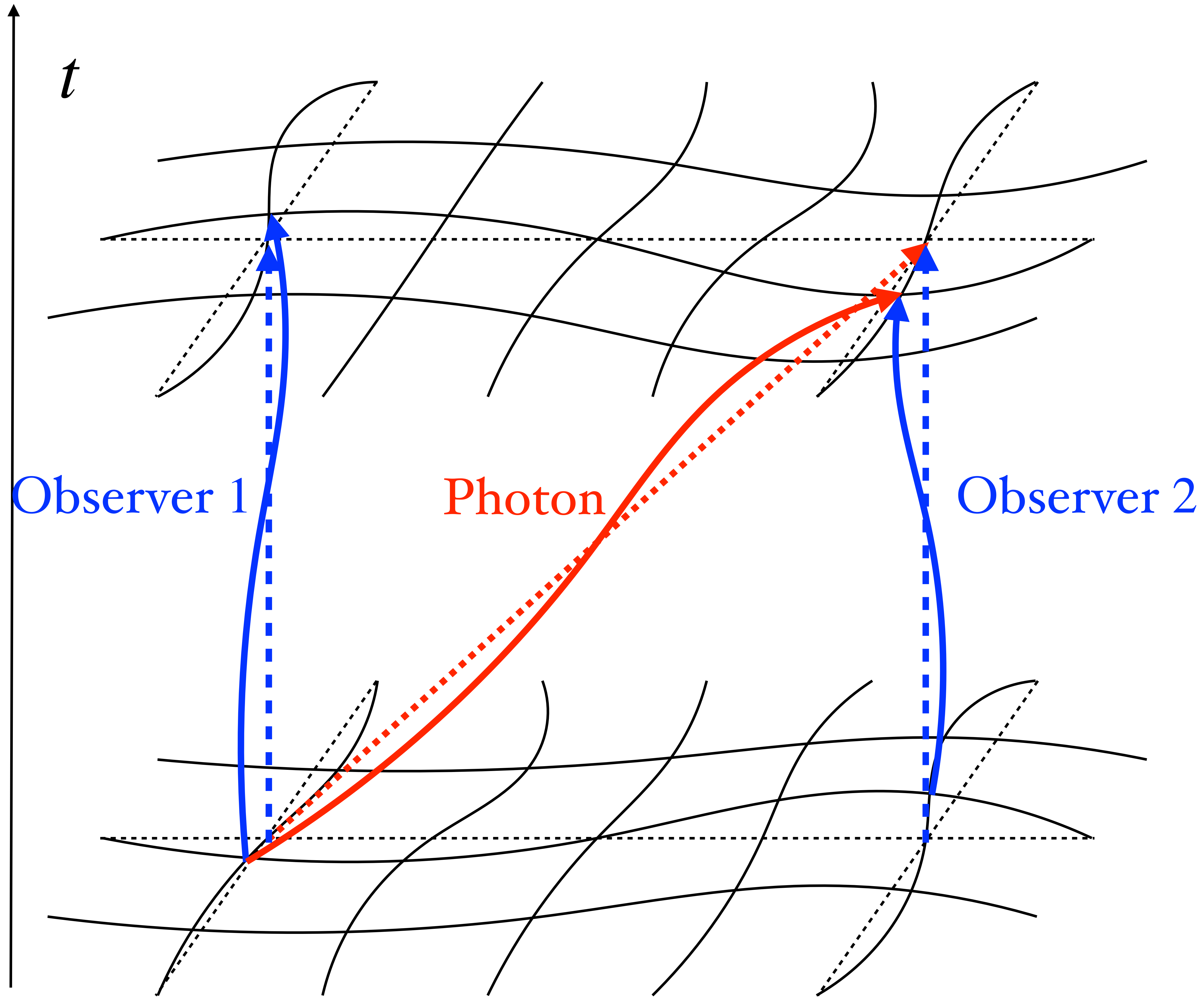
$$\tau_{\mathrm{Df}} = \tau_{\mathrm{Df}}^{(0)} + \sum_{l=0}^{\infty} \tau_{\mathrm{Df}}^{(l)}$$



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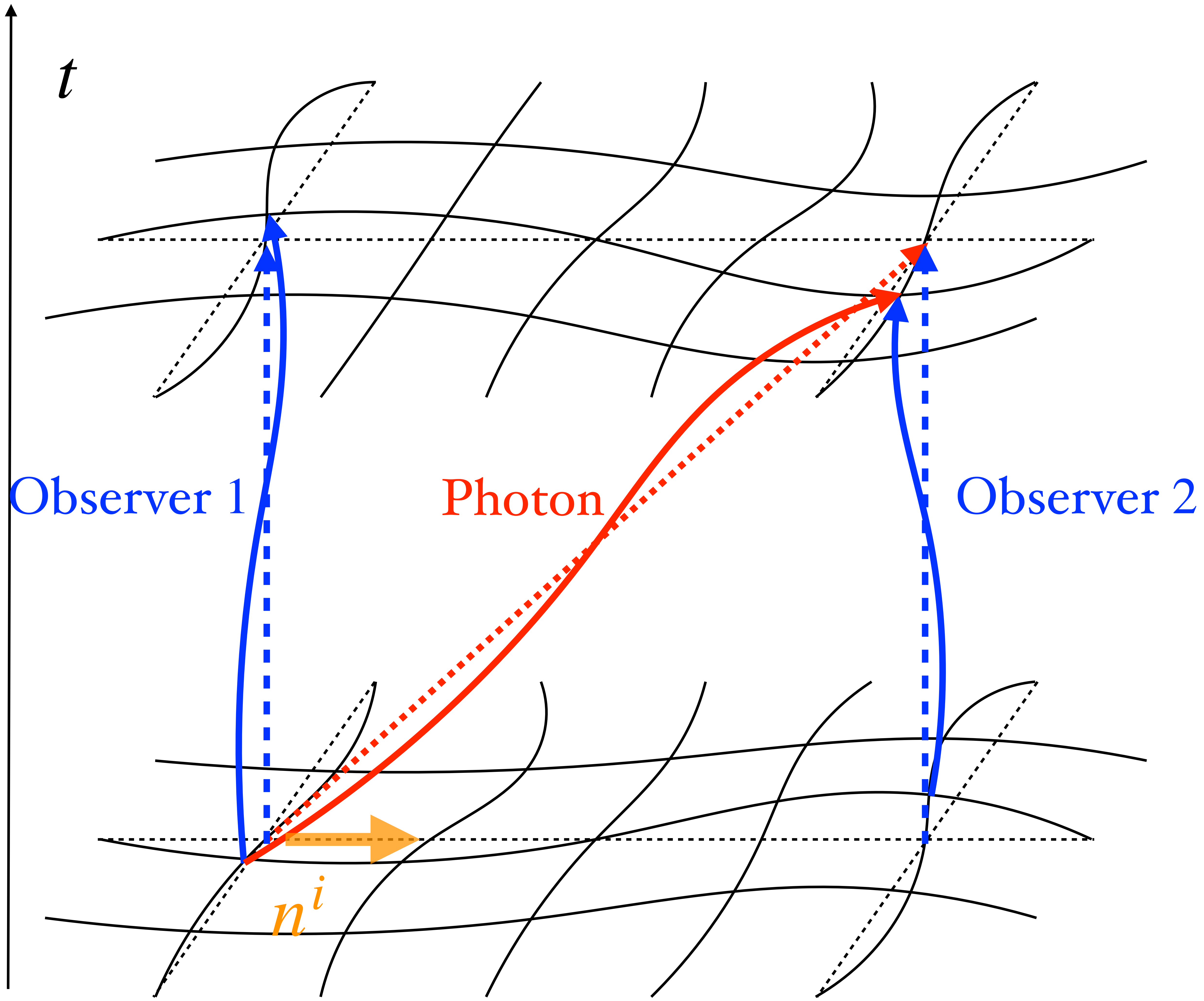

$$\tau_{\mathrm{Df}} = \tau_{\mathrm{Df}}^{(0)} + \sum_{l=0}^{\infty} \tau_{\mathrm{Df}}^{(l)}$$

$$\tau_{\mathrm{D\,f}}^{(1)} = a_{\mathrm{f}} \left[\frac{1}{2} n^i n^j \int_0^L \mathrm{d}\lambda \, h_{ij}^{(1)} \right.$$
$$+ n^i \left(X_{\mathrm{D\,f}}^{(1)i} - X_{\mathrm{E\,i}}^{(1)i} \right)$$
$$\left. - \int_0^L \mathrm{d}\lambda \, (\Phi^{(1)} + \Psi^{(1)}) + X_{\mathrm{E\,i}}^{(1)0} - X_{\mathrm{D\,f}}^{(1)0} \right]$$

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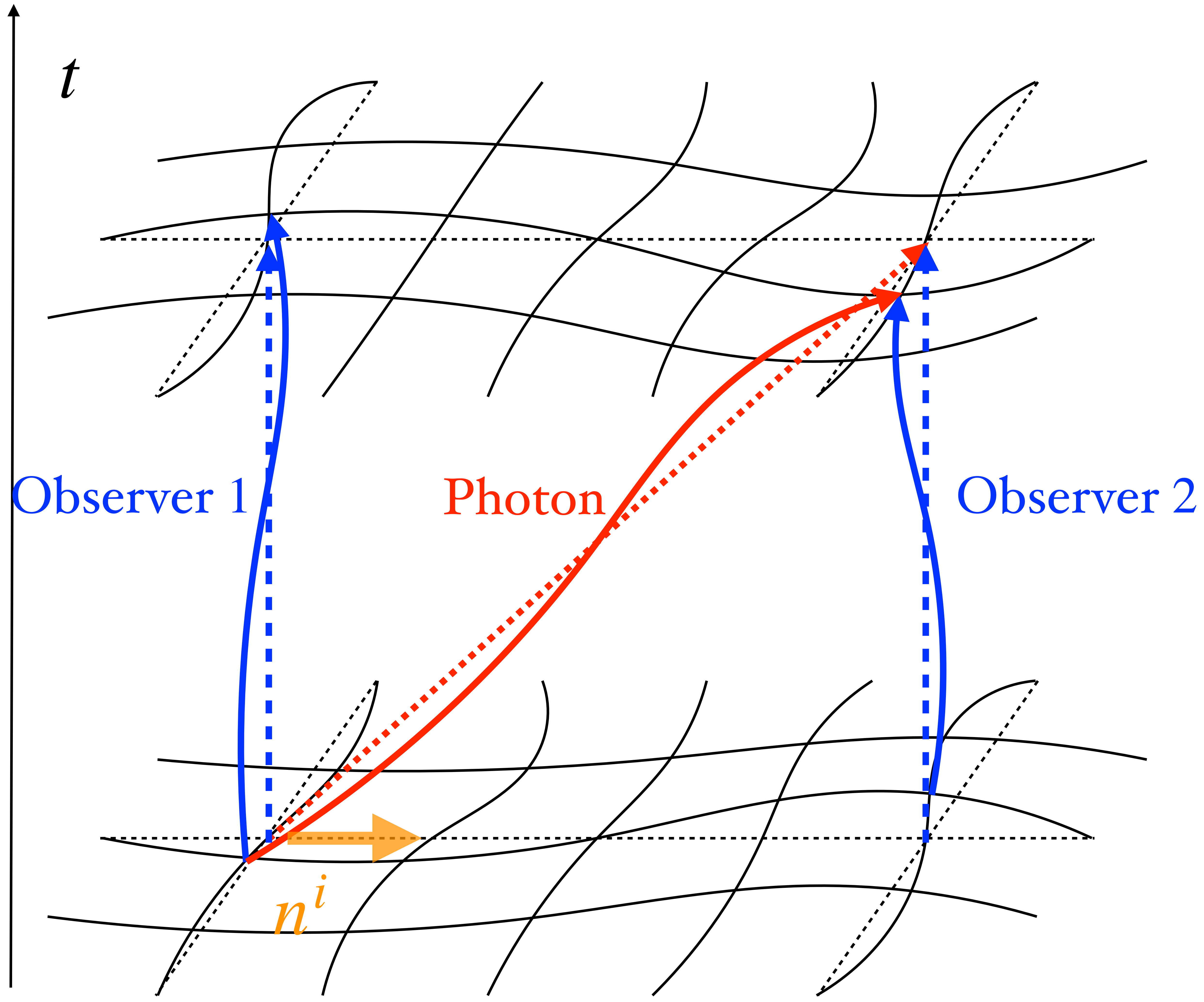

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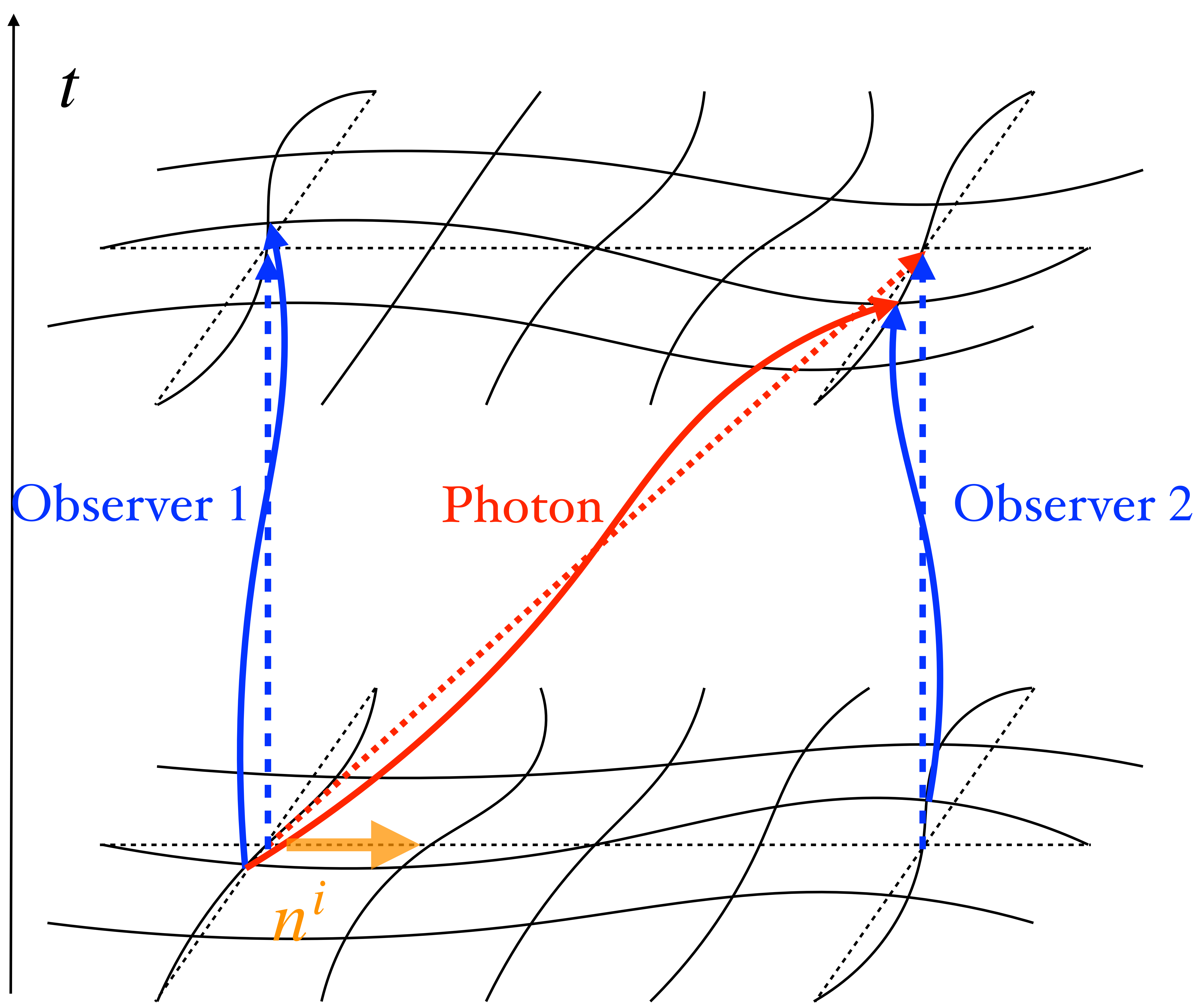

$$\tau_{\mathrm{Df}} = \tau^{(0)}_{\mathrm{Df}} + \sum_{l=0}^{\infty} \tau^{(l)}_{\mathrm{Df}}$$

$$\tau^{(1)}_{\mathrm{D\,f}} = a_{\mathrm{f}} \Bigg[\boxed{\frac{1}{2} n^i n^j \int_0^L \mathrm{d}\lambda\, h^{(1)}_{ij}} \underline{+ n^i \left(X^{(1)i}_{\mathrm{D\,f}} - X^{(1)i}_{\mathrm{E\,i}} \right)} - \boxed{\int_0^L \mathrm{d}\lambda \left(\Phi^{(1)} + \Psi^{(1)} \right)} + \underline{X^{(1)0}_{\mathrm{E\,i}} - X^{(1)0}_{\mathrm{D\,f}}} \Bigg]$$

GW

Shaprio

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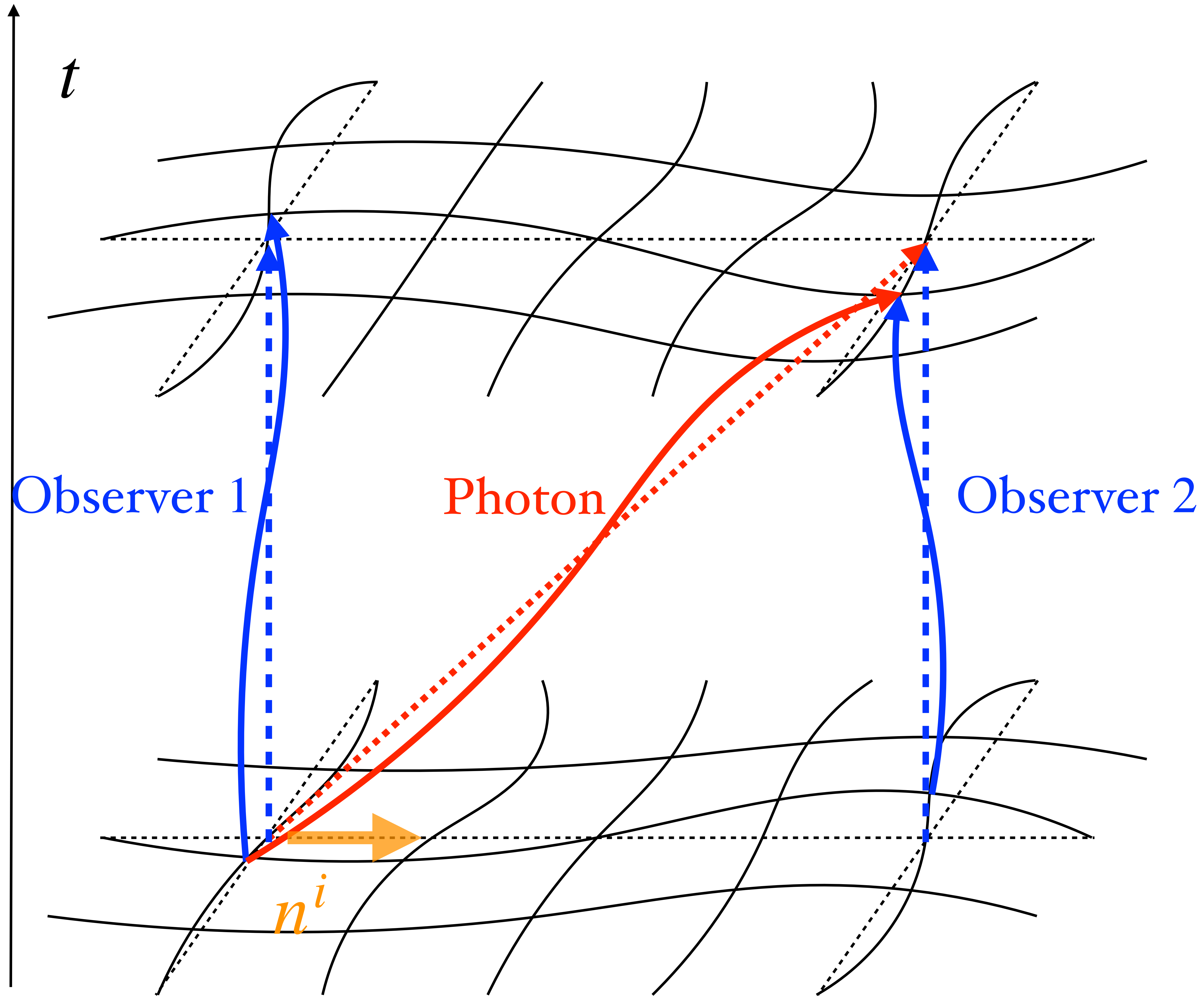

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$$\left. - \boxed{\int_0^L \mathrm{d}\lambda\, (\Phi^{(1)} + \Psi^{(1)})} + \underline{X^{(1)0}_{\mathrm{E\,i}} - X^{(1)0}_{\mathrm{D\,f}}} \right]$$

$$\tau^{(2)}_{\mathrm{Df}} \supset a_{\mathrm{f}} n^i n^j \left[\int_0^L \mathrm{d}\lambda\, h^{N(2)}_{ij} + \mathcal{O}\left((X^{(1)}, V^{(1)})^2 \right) \right]$$

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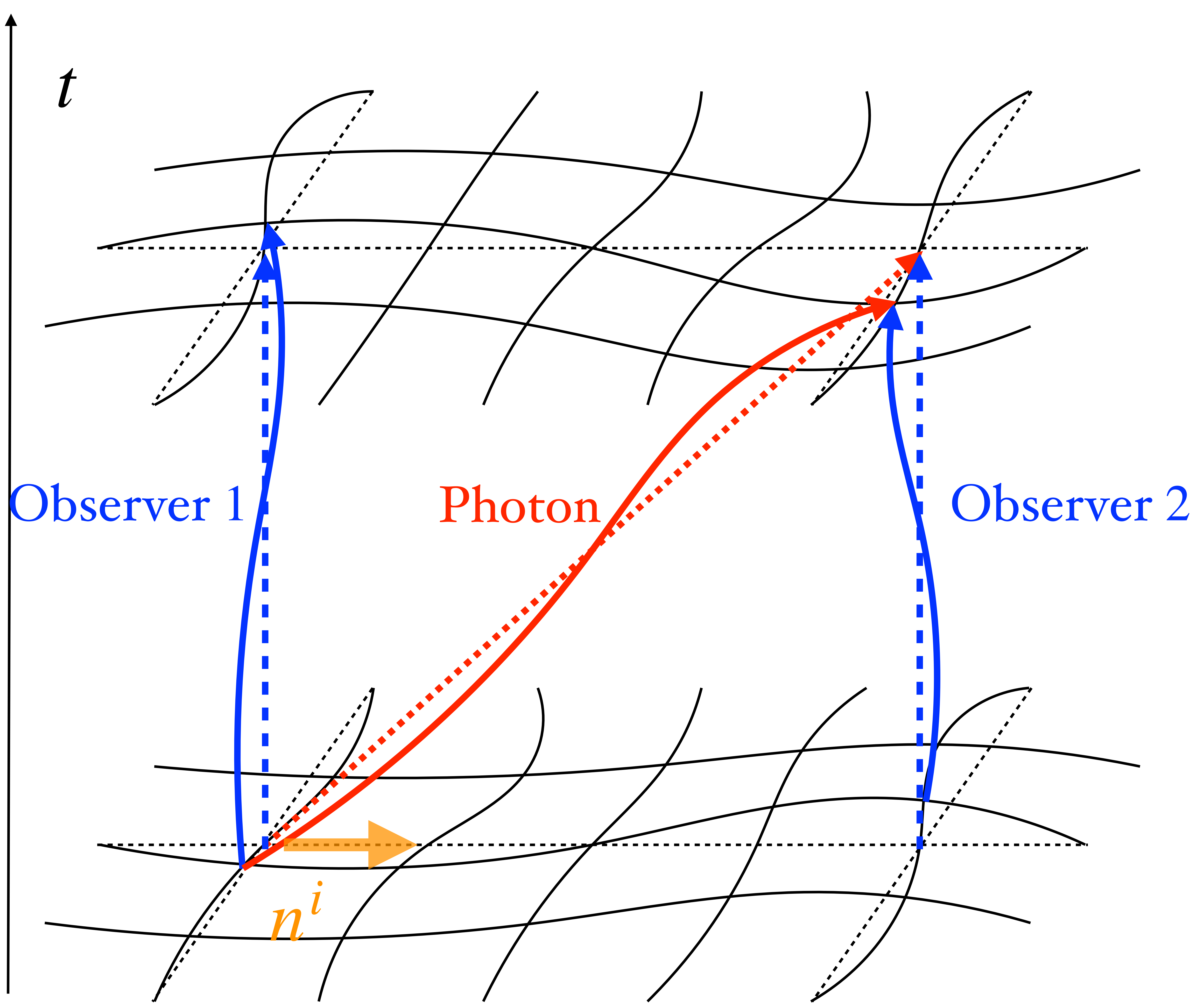

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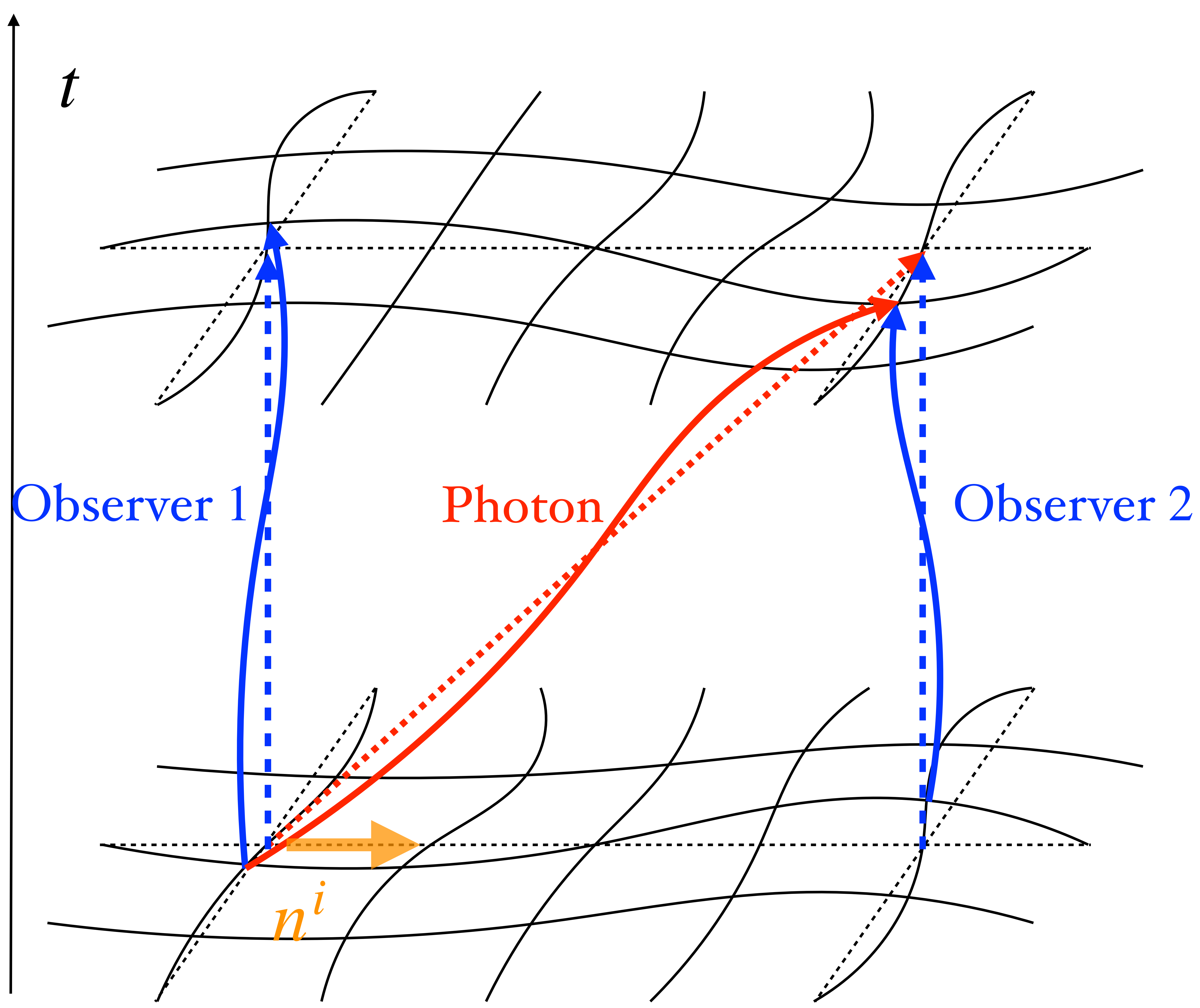

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$$\left. - \boxed{\int_0^L \mathrm{d}\lambda \left(\Phi^{(1)} + \Psi^{(1)}\right)} + \underline{X^{(1)0}_{\rm E\,i} - X^{(1)0}_{\rm D\,f}} \right]$$

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TT components in Newton Gauge