Departures from slow roll inflation and magnetogenesis

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Plan of the talk

1. Observational evidence for magnetic fields
2. Generation of magnetic fields in slow roll inflation
3. Challenges in inflationary models leading to features
4. Circumventing the challenges using two-field models
5. Cross-correlation between scalar perturbations and magnetic fields
6. Summary

Plan of the talk

Strengths of observed magnetic fields

Magnetic fields are observed at different strengths over a range of scales in the universe.

- In galaxies, the strength of the observed magnetic fields is $\mathcal{O}(10^{-6}\,\mathrm{G})$, which is coherent over scales of 1–$10\,\mathrm{Kpc}$[1].
- In clusters of galaxies, the strength of the magnetic fields is $\mathcal{O}(10^{-7}$–$10^{-6}\,\mathrm{G})$ with a coherent length of $10\,\mathrm{Kpc}$–$1\,\mathrm{Mpc}$[2].
- In the intergalactic voids, the strength of the magnetic fields is greater than $10^{-16}\,\mathrm{G}$, which is coherent on scales above $1\,\mathrm{Mpc}$[3].
- The observations of the anisotropies in the cosmic microwave background (CMB) constrain the magnetic fields at the scale of $1\,\mathrm{Mpc}$ to be less than $10^{-9}\,\mathrm{G}$[4].

While astrophysical processes may explain magnetic fields in galaxies and clusters of galaxies, a *cosmological mechanism* may be needed to explain the magnetic fields in voids[5].

[1] R. Beck, Space Sci. Rev. **99**, 243 (2001).

[2] See, for instance, T. E. Clarke, P. P. Kronberg and H. Böhringer, Astrophys. J. **547**, L111 (2001).

[3] A. Neronov and I. Vovk, Science **328**, 73 (2010).

[4] Planck Collaboration (P. A. R. Ade *et al.*), Astron. Astrophys. **594**, A19 (2016).

[5] However, see, D. Garg, R. Durrer and J. Schober, arXiv:2505.14774 [astro-ph.CO].

Plan of the talk

Nonconformally coupled, helical electromagnetic fields

The conformal and parity invariance of the electromagnetic field is broken by considering an action of the form[6]:

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\,\sqrt{-g}\,\left[J^2(\phi)\,F_{\mu\nu}\,F^{\mu\nu} - \frac{\gamma}{2}\,I^2(\phi)\,F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right],$$

where $J(\phi)$ and $I(\phi)$ denote coupling functions, the field tensor $F_{\mu\nu}$ is expressed in terms of the vector potential A_μ as $F_{\mu\nu} = (\partial_\mu\,A_\nu - \partial_\nu\,A_\mu)$ and $\widetilde{F}^{\mu\nu} = (\epsilon^{\mu\nu\alpha\beta}/\sqrt{-g})\,F_{\alpha\beta}$, with $\epsilon^{\mu\nu\alpha\beta}$ being the completely anti-symmetric Levi-Civita tensor, and γ is a constant.

On working in the Coulomb gauge wherein $A_\eta = 0$ and $\partial_i\,A^i = 0$, one finds that the rescaled Fourier mode functions $\mathcal{A}_k^\sigma = J\,\bar{A}_k^\sigma$ satisfy the equation

$$\mathcal{A}_k^{\sigma\,\prime\prime} + \left(k^2 + \frac{2\,\sigma\,\gamma\,k\,I\,I'}{J^2} - \frac{J''}{J}\right)\,\mathcal{A}_k^\sigma = 0,$$

where $\sigma = \pm 1$ correspond to the two helicities.

[6] M. M. Anber and L. Sorbo, JCAP **10**, 018 (2006);
C. Caprini and L. Sorbo, JCAP **10**, 056 (2014);
D. Chowdhury, L. Sriramkumar and M. Kamionkowski, JCAP **10**, 031 (2018).

Scale invariant amplitudes and the non-helical limit

In de Sitter inflation wherein $a(\eta) = -1/(H_{\rm I}\,\eta)$, the function J is often assumed to be[7]

$$J(\eta) = [a(\eta)/a(\eta_{\rm e})]^n = (\eta/\eta_{\rm e})^{-n},$$

where $\eta_{\rm e}$ denotes the conformal time at the end of inflation.

When $I = J$ and $n = 2$, the spectra of the magnetic and electric fields are given by[8]

$$\mathcal{P}_{_{\rm B}}(k) = \frac{9\,H_{_{\rm I}}^4}{4\,\pi^2}\,\frac{\sinh\,(4\,\pi\,\gamma)}{4\,\pi\,\gamma\,\left(1+5\,\gamma^2+4\,\gamma^4\right)},$$

$$\mathcal{P}_{_{\rm E}}(k) = \mathcal{P}_{_{\rm B}}(k)\left[\gamma^2 - \frac{\sinh^2(2\,\pi\,\gamma)}{3\,\pi\,\left(1+\gamma^2\right)\,f(\gamma)}\,(-k\,\eta_{\rm e}) + \frac{1}{9}\,\left(1+23\,\gamma^2+40\,\gamma^4\right)\,(-k\,\eta_{\rm e})^2\right].$$

Given the amplitude of the magnetic field at the end of inflation, the strength of the magnetic field *today*, say, B_0, can be estimated to be

$$B_0 \simeq 4.5\times 10^{-12}\,\left(\frac{H_{_{\rm I}}}{10^{-5}\,M_{_{\rm Pl}}}\right)\,f^{1/2}(\gamma)\;{\rm G}.$$

[7] See, for example, J. Martin and J. Yokoyama, JCAP **01**, 025 (2008).

[8] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Coupling function in the Starobinsky model

Consider the popular Starobinsky model described by the potential

$$V(\phi) = V_0 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right)\right]^2.$$

The evolution of the field in the slow roll approximation is described by the expression

$$N - N_{\mathrm{e}} \simeq -\frac{3}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right],$$

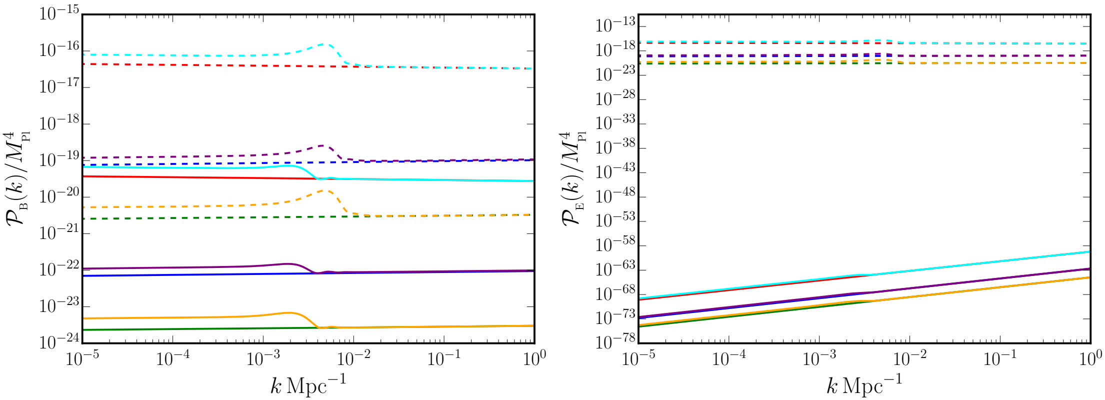
where ϕ_{e} is the value of the field at the end of inflation.

Therefore, to achieve the desired dependence of the coupling function on the scale factor, we can choose $J(\phi)$ in the model to be[9]

$$J(\phi) = \exp\left\{-\frac{3\,n}{4}\left[\exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\mathrm{Pl}}}\right) - \exp\left(\sqrt{\frac{2}{3}}\,\frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right) - \sqrt{\frac{2}{3}}\left(\frac{\phi}{M_{\mathrm{Pl}}} - \frac{\phi_{\mathrm{e}}}{M_{\mathrm{Pl}}}\right)\right]\right\}.$$

[9] See, for example, J. Martin and J. Yokoyama, JCAP **01**, 025 (2008);
S. Kanno, J. Soda and M. Watanabe, JCAP **12**, 009 (2009);
M. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009).

Spectra of electromagnetic fields in slow roll inflation



The spectra of the magnetic (on the left) and electric (on the right) fields in the non-helical (solid curves) and helical (dashed curves) cases, arising in three slow roll inflationary models (in red, blue and green). We have also plotted the corresponding spectra when a step has been introduced in these potentials (in cyan, purple and orange). We have set $\gamma = 1$ for which $f(\gamma) \simeq 10^3$.

Plan of the talk

Generating features on large scales

Given a potential $V(\phi)$ that leads to slow roll inflation, we can introduce a step in the potential as follows[10]:

$$V_{\rm step}(\phi) = V(\phi)\, \left[1 + \alpha \tanh\left(\frac{\phi - \phi_0}{\Delta\phi}\right)\right],$$

which leads to a burst of oscillations in the scalar power spectrum.

A model that leads to sharp drop in power on large scales is the so-called punctuated inflationary model described by the potential[11]

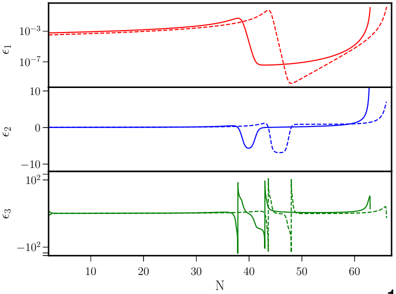
$$V(\phi) = \frac{m^2}{2}\,\phi^2 - \frac{2\,m^2}{3\,\phi_0}\,\phi^3 + \frac{m^2}{4\,\phi_0^2}\,\phi^4,$$

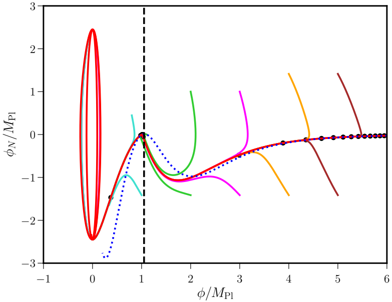
which contains a point of inflection at ϕ_0.

[10] D. K. Hazra, M. Aich, R. K. Jain, L. Sriramkumar and T. Souradeep, JCAP **10**, 008 (2010).

[11] R. K. Jain, P. Chingangbam, J-O. Gong, L. Sriramkumar and T. Souradeep, JCAP **01**, 009 (2009);
R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D **82**, 023509 (2010);
H. V. Ragavendra, D. Chowdhury and L. Sriramkumar, Phys. Rev. D **106**, 043535 (2022).

Enhancing scalar power on small scales





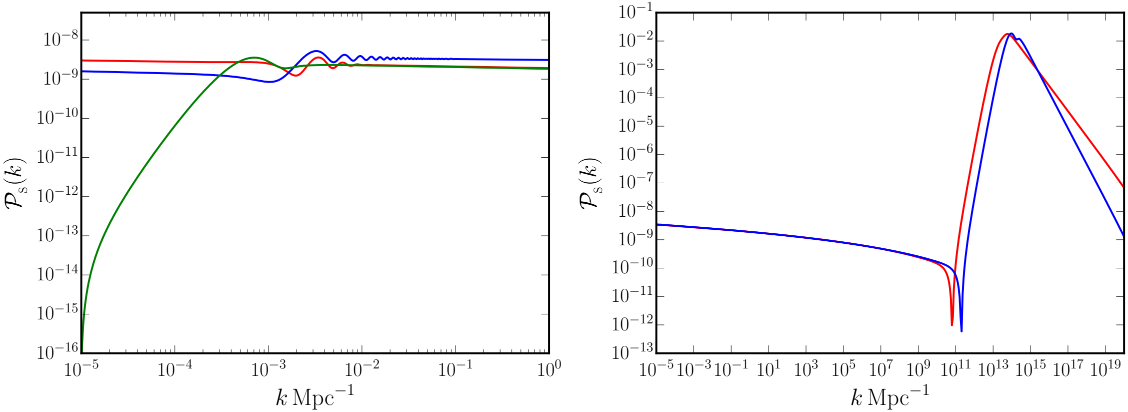
Potentials leading to ultra slow roll inflation[12]:

$$V(\phi) = V_0\, \frac{6\,(\phi/v)^2 - 4\,\alpha\,(\phi/v)^3 + 3\,(\phi/v)^4}{[1+\beta\,(\phi/v)^2]^2},$$

$$V(\phi) = V_0\, \left\{\tanh\left(\frac{\phi}{\sqrt{6}\,M_{_{\rm Pl}}}\right) + A\,\sin\left[\frac{\tanh\left[\phi/\left(\sqrt{6}\,M_{_{\rm Pl}}\right)\right]}{f_\phi}\right]\right\}^2.$$

[12] C. Germani and T. Prokopec, Phys. Dark Univ. **18**, 6 (2017);
J. Garcia-Bellido and E. R. Morales, Phys. Dark Univ. **18**, 47 (2017);
I. Dalianis, A. Kehagias and G. Tringas, JCAP **01**, 037 (2019).

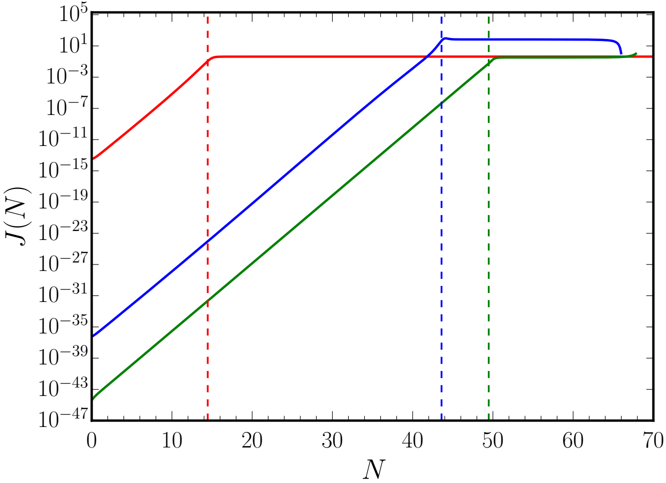
Scalar power spectra with features on large and small scales



Scalar power spectra with features over the CMB (on the left) and small (on the right) scales[13].

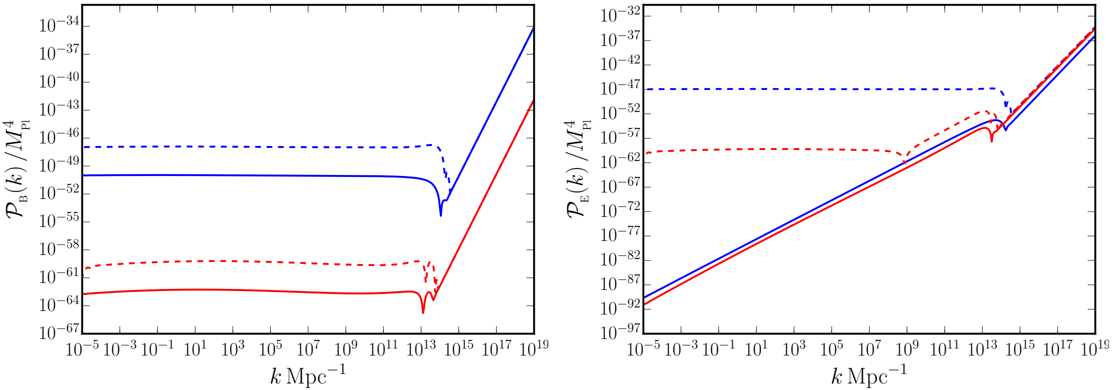
[13] H. V. Ragavendra, P. Saha, L. Sriramkumar and J. Silk, Phys. Rev. D **103**, 083510 (2021).

Behavior of the coupling function in models permitting ultra slow roll



The evolution of the nonconformal coupling function J in inflationary models leading to ultra slow roll. Note that the coupling function does not change appreciably once ultra slow roll sets in (indicated by the vertical lines).

Resulting spectra of the electromagnetic fields



The spectra of the magnetic (on the left) and electric (on the right) fields arising in two inflationary models permitting a period of ultra slow roll (at late times) have been plotted in the non-helical (as solid lines) and helical (as dashed lines) cases[14].

[14] S. Tripathy, D. Chowdhury, R. K. Jain and L. Sriramkumar, Phys. Rev. D **105**, 063519 (2022).

Plan of the talk

Two-field models of interest

It has been shown that two scalar fields ϕ and χ governed by the action

$$S[\phi, \chi] = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2} \partial^\mu \phi \, \partial_\mu \phi - \frac{f(\phi)}{2} \partial^\mu \chi \, \partial_\mu \chi - V(\phi, \chi)\right]$$

described by *separable* potentials and the non-canonical functions $f(\phi) = \exp(2\,b\,\phi)$ or $f(\phi) = \exp(2\,b\,\phi^2)$ can lead to strong features in the scalar power spectrum.

While the potential[15]

$$V(\phi, \chi) = \frac{1}{2} m_\phi^2 \phi^2 + V_0 \frac{\chi^2}{\chi_0^2 + \chi^2}$$

leads to a suppression in power on large scales, the potential[16]

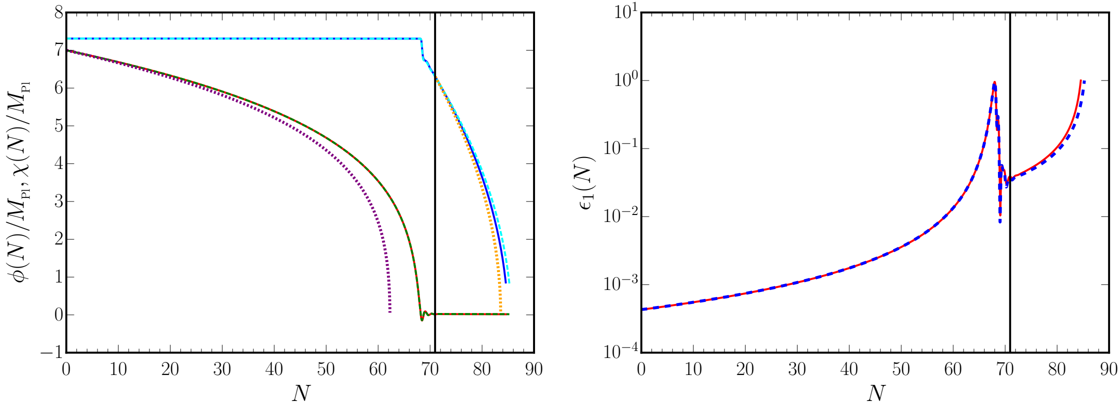
$$V(\phi, \chi) = V_0 \frac{\phi^2}{\phi_0^2 + \phi^2} + \frac{1}{2} m_\chi^2 \chi^2$$

generates enhanced power on small scales.

[15] M. Braglia, D. K. Hazra, L. Sriramkumar and F. Finelli, JCAP **08**, 025 (2020).

[16] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

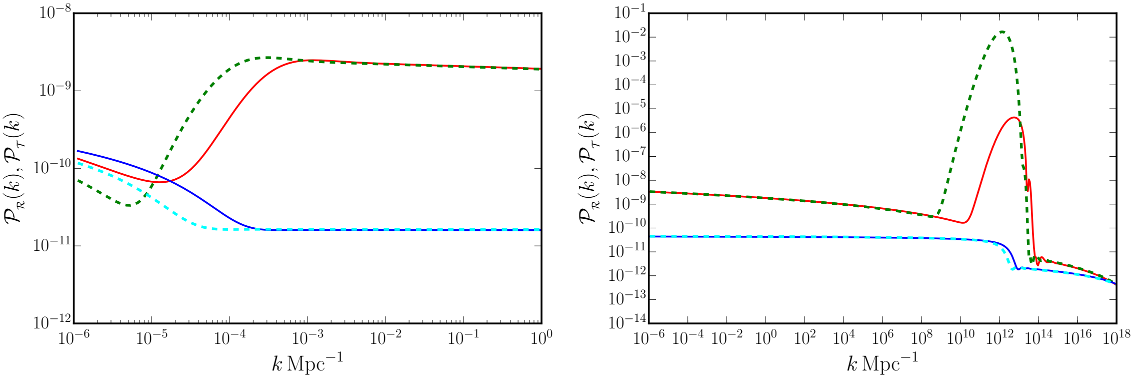
Behavior of the scalar fields and the first slow roll parameter



Behavior of the two scalar fields (on the left) and the first slow roll parameter (on the right) in the second two-field model[17].

[17] M. Braglia, D. K. Hazra, F. Finelli, G. F. Smoot, L. Sriramkumar and A. A. Starobinsky, JCAP **08**, 001 (2020).

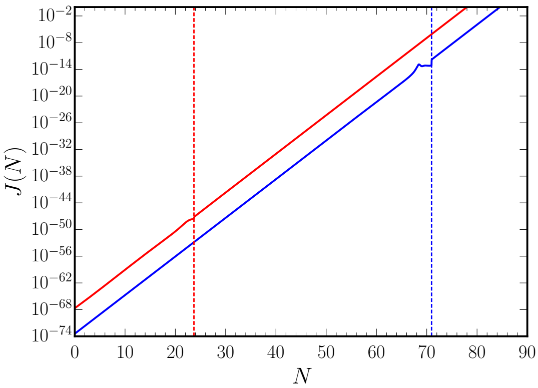
Scalar and tensor power spectra in the two-field models

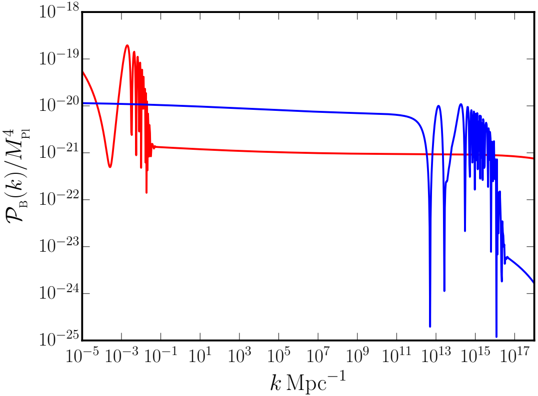


The spectra of curvature (in solid red and dashed green) and tensor (in solid blue and dashed cyan) perturbations, viz. $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_{\mathcal{T}}(k)$, have been plotted for the two-field inflationary models that we have considered[18].

[18] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Circumventing the challenge in two-field models





The evolution of the nonconformal coupling function J (on the left) and the corresponding spectrum of magnetic field (on the right) arising in the two-field inflationary models leading to features on large (in red) and on small (in blue) scales[19]. The vertical lines (on the left) indicate the time when the turn in the field space takes place.

[19] S. Tripathy, D. Chowdhury, H. V. Ragavendra, R. K. Jain and L. Sriramkumar, Phys. Rev. D **107**, 043501 (2023).

Plan of the talk

Coupling to the derivative of the scalar field

To circumvent the difficulty in ultra slow roll inflation, we rewrite the action governing the electromagnetic field in the following form:

$$S[A^\mu] = -\frac{1}{16\,\pi}\int \mathrm{d}^4x\sqrt{-g}\, J^2(X)\, F_{\mu\nu}\, F^{\mu\nu}.$$

Moreover, we assume that the coupling function is given by[20]

$$J(X) = J_0\left(-\frac{\partial_\sigma\phi\,\partial^\sigma\phi}{2}\right)^{n/2},$$

where J_0 is a constant chosen such that J reduces to unity at the end of inflation. In the ultra slow roll inflationary scenarios, we can write the coupling function as

$$J(\eta) \simeq J_0\,\left[H_{\mathrm{I}}^2\, M_{\mathrm{Pl}}^2\,\epsilon_1(\eta)\right]^{n/2},$$

and we shall choose $J_0 = \left[H_{\mathrm{I}}^2\, M_{\mathrm{Pl}}^2\,\epsilon_1(\eta_{\mathrm{e}})\right]^{-n/2}$ so that $J(\eta_{\mathrm{e}}) = 1$.

[20] S. Tripathy, D. Chowdhury, H. V. Ragavendra, and L. Sriramkumar, Phys. Rev. D **111**, 063550 (2025).

Cross-correlation between scalar perturbations and magnetic fields

In real space, the cross-correlation between the curvature perturbation and magnetic fields is defined as

$$\langle\hat{\mathcal{R}}(\eta,\boldsymbol{x})\,\hat{B}^i(\eta,\boldsymbol{x})\,\hat{B}_i(\eta,\boldsymbol{x})\rangle = \int\frac{\mathrm{d}^3\boldsymbol{k}_1}{(2\,\pi)^{3/2}}\int\frac{\mathrm{d}^3\boldsymbol{k}_2}{(2\,\pi)^{3/2}}\int\frac{\mathrm{d}^3\boldsymbol{k}_3}{(2\,\pi)^{3/2}}\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta)\,\hat{B}^i_{\boldsymbol{k}_2}(\eta)\,\hat{B}_{i\,\boldsymbol{k}_3}(\eta)\rangle \times \mathrm{e}^{i\,(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{x}},$$

where $B_i = (1/a)\,\epsilon_{ijl}\,\partial_j A_l$. Given the cubic order Hamiltonian H_{int}, the three-point cross-correlation in Fourier space, evaluated at the conformal time η_{e}, is given by

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{B}^i_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{B}_{i\,\boldsymbol{k}_3}(\eta_{\mathrm{e}})\rangle = -i\int_{\eta_{\mathrm{i}}}^{\eta_{\mathrm{e}}}\mathrm{d}\eta\,\langle[\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{B}^i_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{B}_{i\,\boldsymbol{k}_3}(\eta_{\mathrm{e}}),\hat{H}_{\mathrm{int}}(\eta)]\rangle,$$

To describe the cross-correlation between the modes of the curvature perturbation and the magnetic field, we shall introduce the function $\mathcal{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ that is defined through the relation

$$\langle\hat{\mathcal{R}}_{\boldsymbol{k}_1}(\eta_{\mathrm{e}})\,\hat{B}^i_{\boldsymbol{k}_2}(\eta_{\mathrm{e}})\,\hat{B}_{i\,\boldsymbol{k}_3}(\eta_{\mathrm{e}})\rangle = \frac{4\,\pi}{(2\,\pi)^{3/2}}\,\mathcal{B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\,\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3).$$

Behavior of the non-Gaussianity parameter in the squeezed limit

In slow roll inflation, the consistency condition leads to the following value for the non-Gaussianity parameter $b_{\mathrm{NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ in the squeezed limit (i.e. when $k_1 \to 0$ and $\boldsymbol{k}_2 = -\boldsymbol{k}_3$):

$$b_{\mathrm{NL}}^{\mathrm{cr}} = (4 - n_{\mathrm{B}})/2,$$

where n_{B} is the magnetic spectral index[21].

α	n_{B}	$b_{\mathrm{NL}}^{\mathrm{sq}}$ in slow roll	$b_{\mathrm{NL}}^{\mathrm{sq}}$ in ultra slow roll	$b_{\mathrm{NL}}^{\mathrm{cr}}$
2	0	2	0	2
1	2	1	0	1

Value of $b_{\mathrm{NL}}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ in the squeezed limit for $J \propto \eta^{-\alpha}$ in slow/ultra-slow roll inflation[22].

This is equivalent to a similar violation encountered in the case of the scalar bispectrum in pure ultra slow roll inflation[23].

[21] R. K. Jain and M. S. Sloth, Phys. Rev. D **86**, 123528 (2012).

[22] S. Tripathy, D. Chowdhury, H. V. Ragavendra, and L. Sriramkumar, Phys. Rev. D **111**, 063550 (2025).

[23] M. H. Namjoo, H. Firouzjahi, and M. Sasaki, EPL **101**, 39001 (2013);
J. Martin, H. Motohashi, and T. Suyama, Phys. Rev. D 87, 023514 (2013).

Plan of the talk

Summary

- In single-field models of inflation, substantial departures from slow roll inflation can lead to strong features in the spectra of magnetic fields and also suppress their strengths on large scales.
- Some of these challenges can be overcome in two-field models of inflation. However, there always seems to be fine tuning involved.
- It seems necessary to examine the behavior of additional quantities such as the three-point functions and the corresponding observables to arrive at constraints on the nature and form of the nonconformal coupling function.

Collaborators

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Thank you for your attention