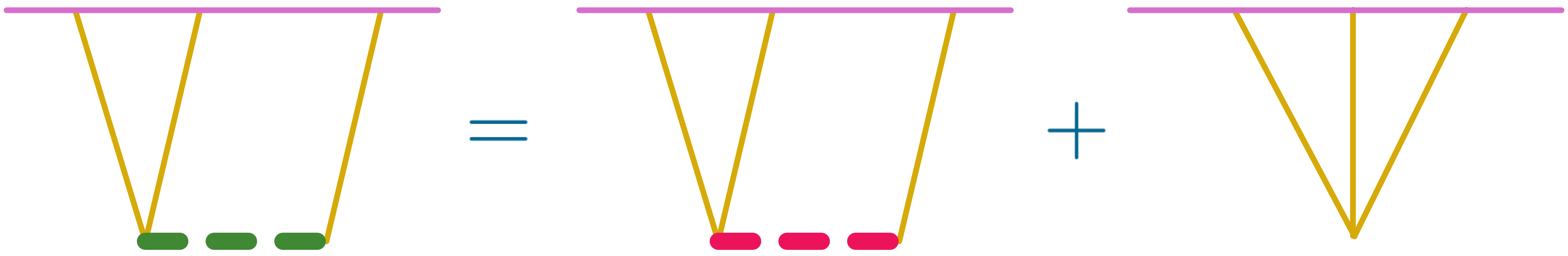
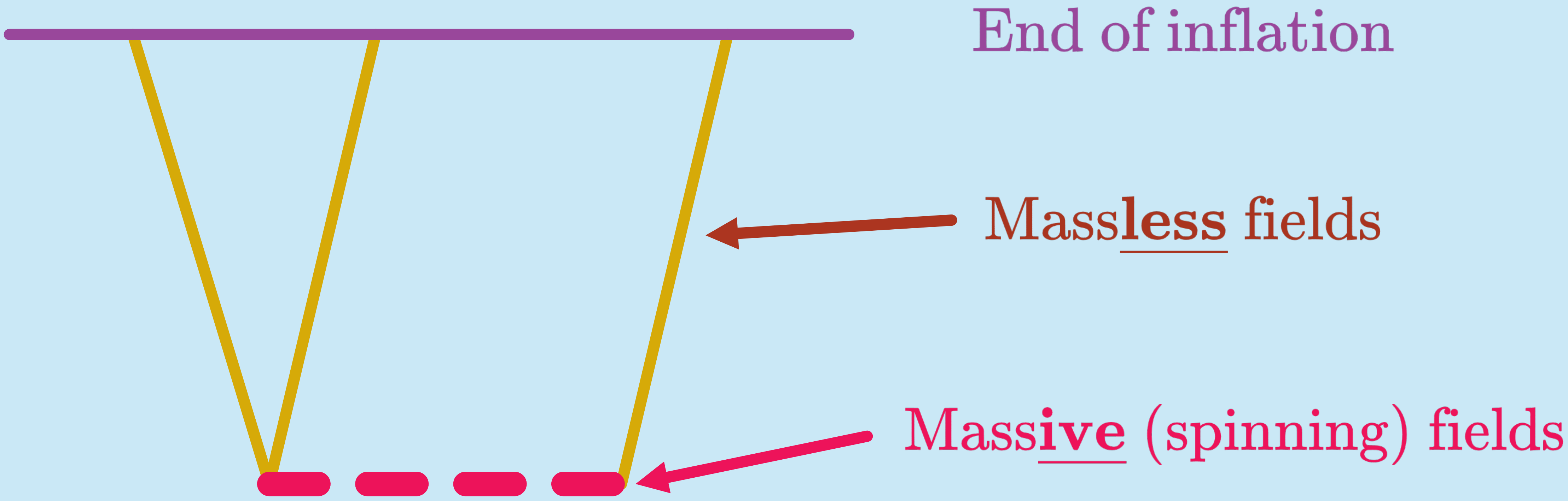
Massive spinning fields during inflation



Trevor Cheung
with David Stefanyszyn
University of Nottingham
Based on arXiv:2509.08888

What we compute



EFT of inflation

Inflation is driven by a scalar field:

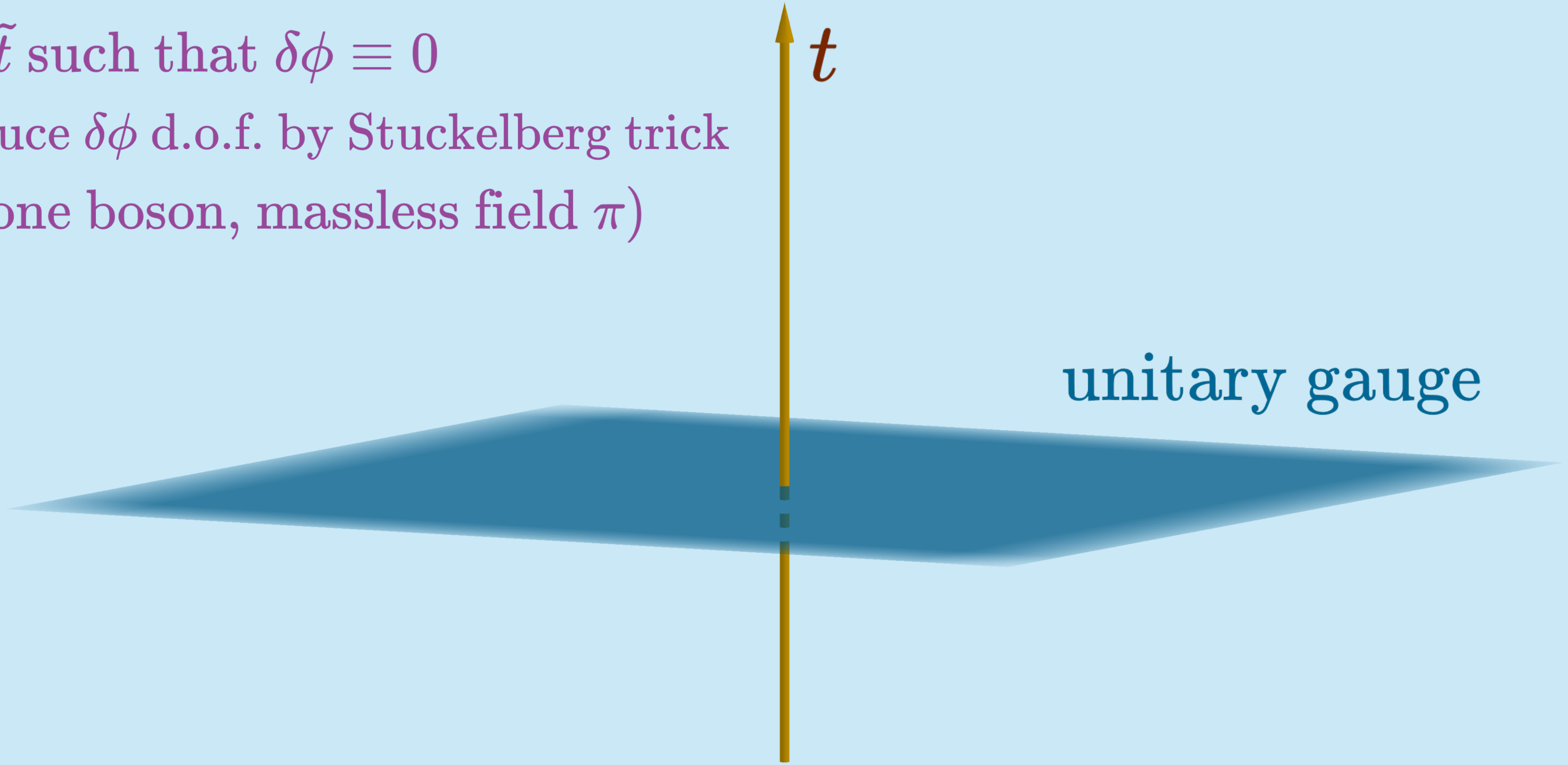
$$\phi(t, \mathbf{x}) = \underbrace{\boxed{\phi_0(t)}}_{\text{time-dependent background}} + \underbrace{\boxed{\delta\phi(t, \mathbf{x})}}_{\text{fluctuations}}$$

$$\boxed{\phi(t,\mathbf{x}) = \phi_0(t) + \delta\phi(t,\mathbf{x})}$$

choose $\tilde{t}$ such that $\delta\phi \equiv 0$

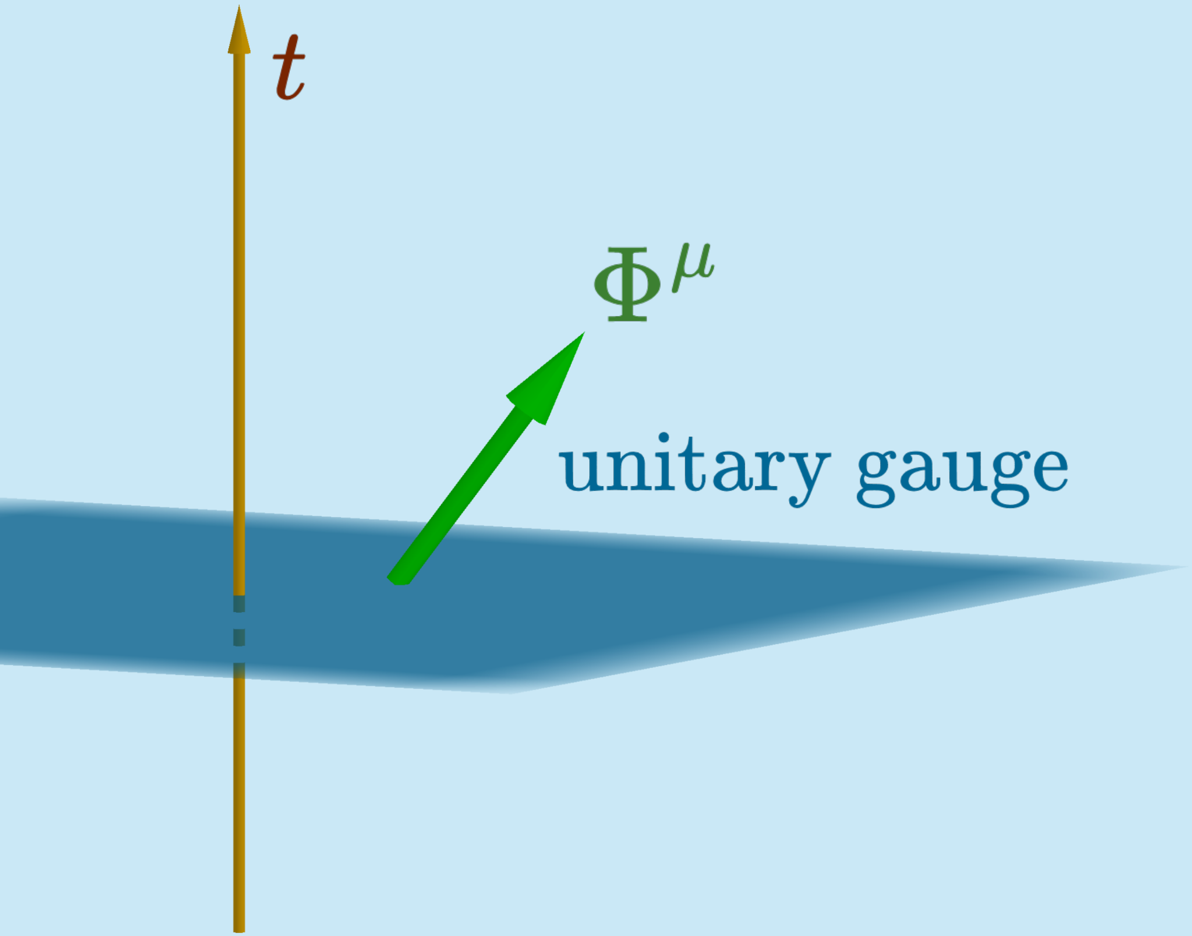
reintroduce $\delta\phi$ d.o.f. by Stuckelberg trick

(Goldstone boson, massless field π)



[C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore; 0709.0293]

Cosmological collider (CC) physics



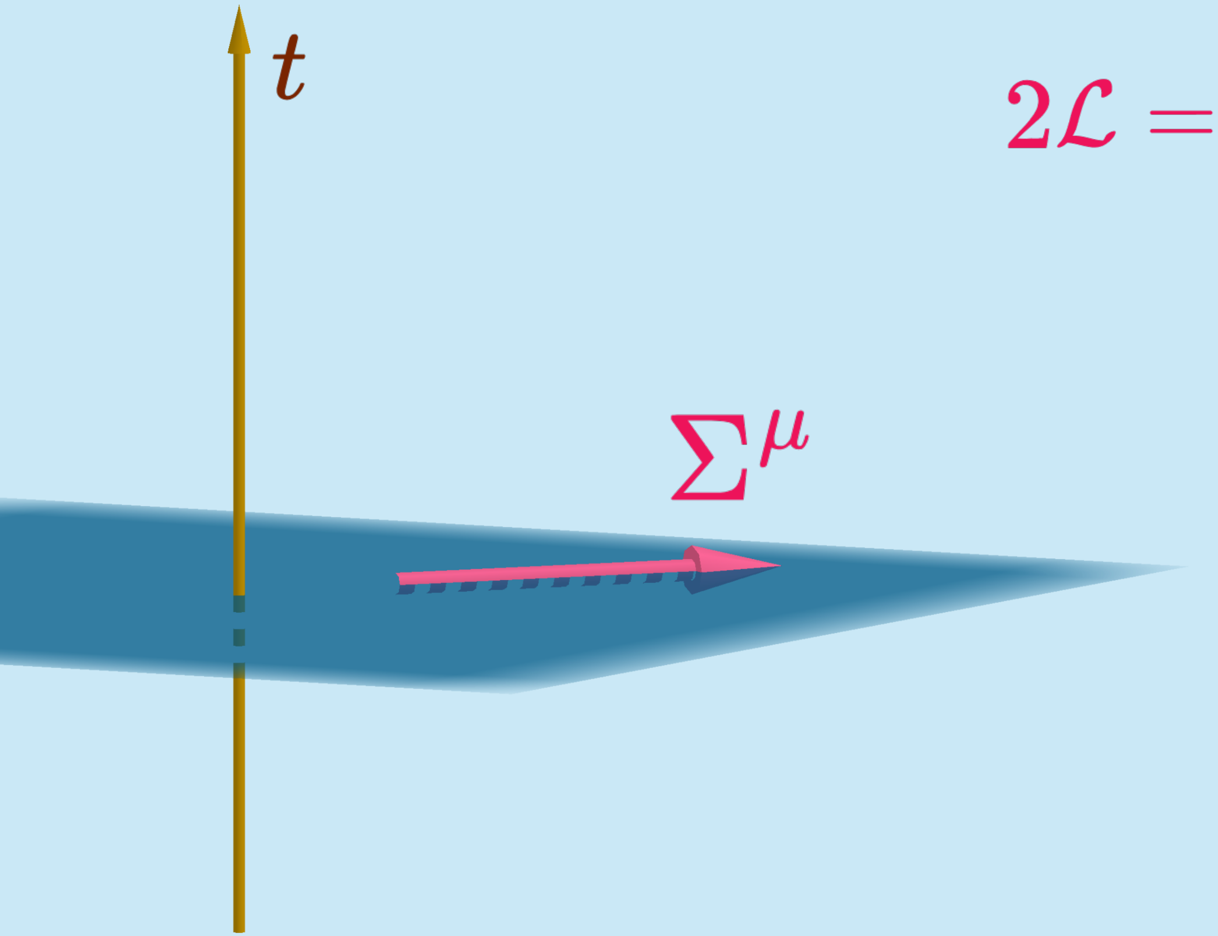
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$$

(spin-1)

only 3 degrees of freedom
(A_0 is not dynamical)

[C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore; 0709.0293]

Cosmological condensed matter (CCM) physics



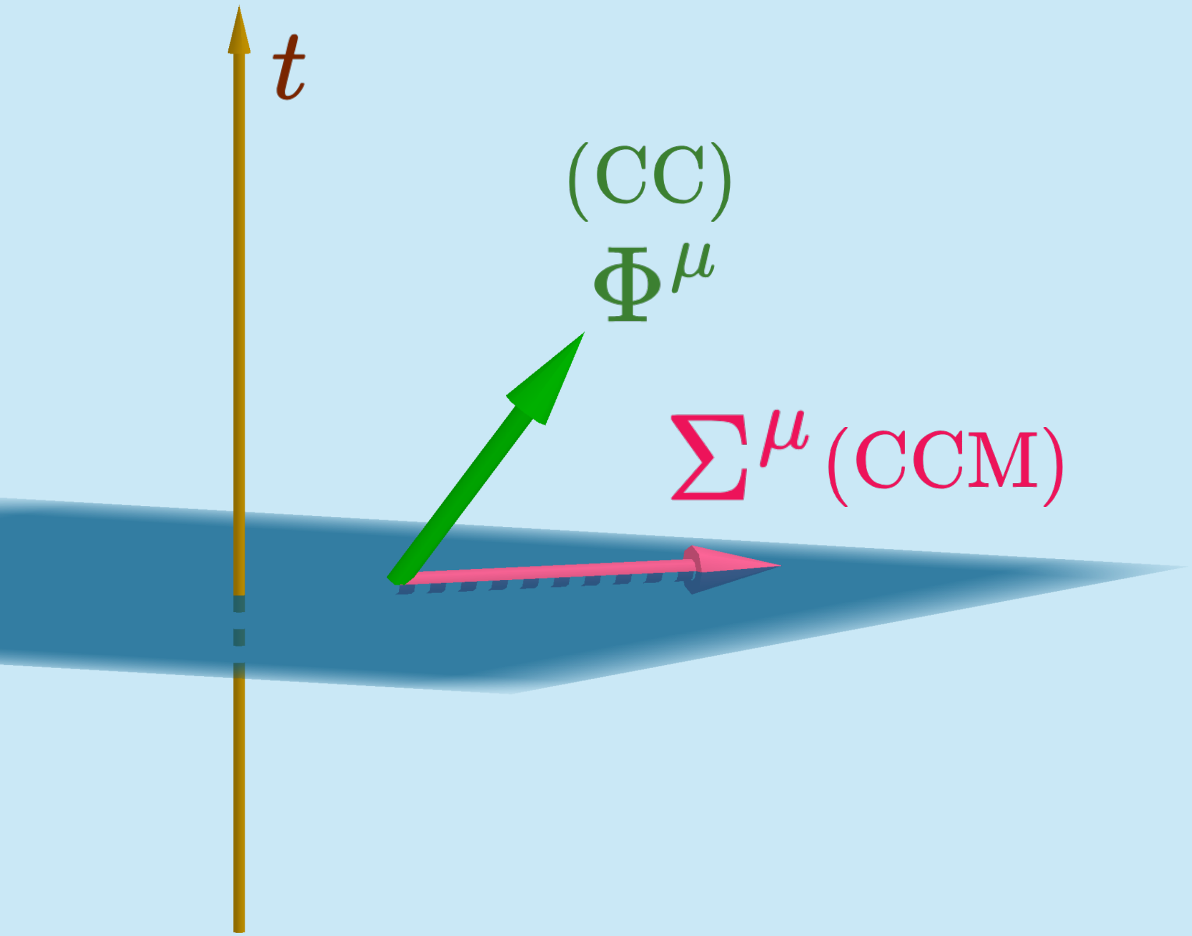
$$2\mathcal{L} = \partial_\eta\Sigma_i\partial_\eta\Sigma^i + c_1^2\partial_i\Sigma^i\partial_j\Sigma^j + c_2^2\partial_i\Sigma^j\partial^i\Sigma_j - M^2a^2\Sigma_i\Sigma^i$$

(spin-1; parity-even)

! only 3 degrees of freedom

[C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore; 0709.0293]

Do they give same π correlators?



[C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore; 0709.0293]

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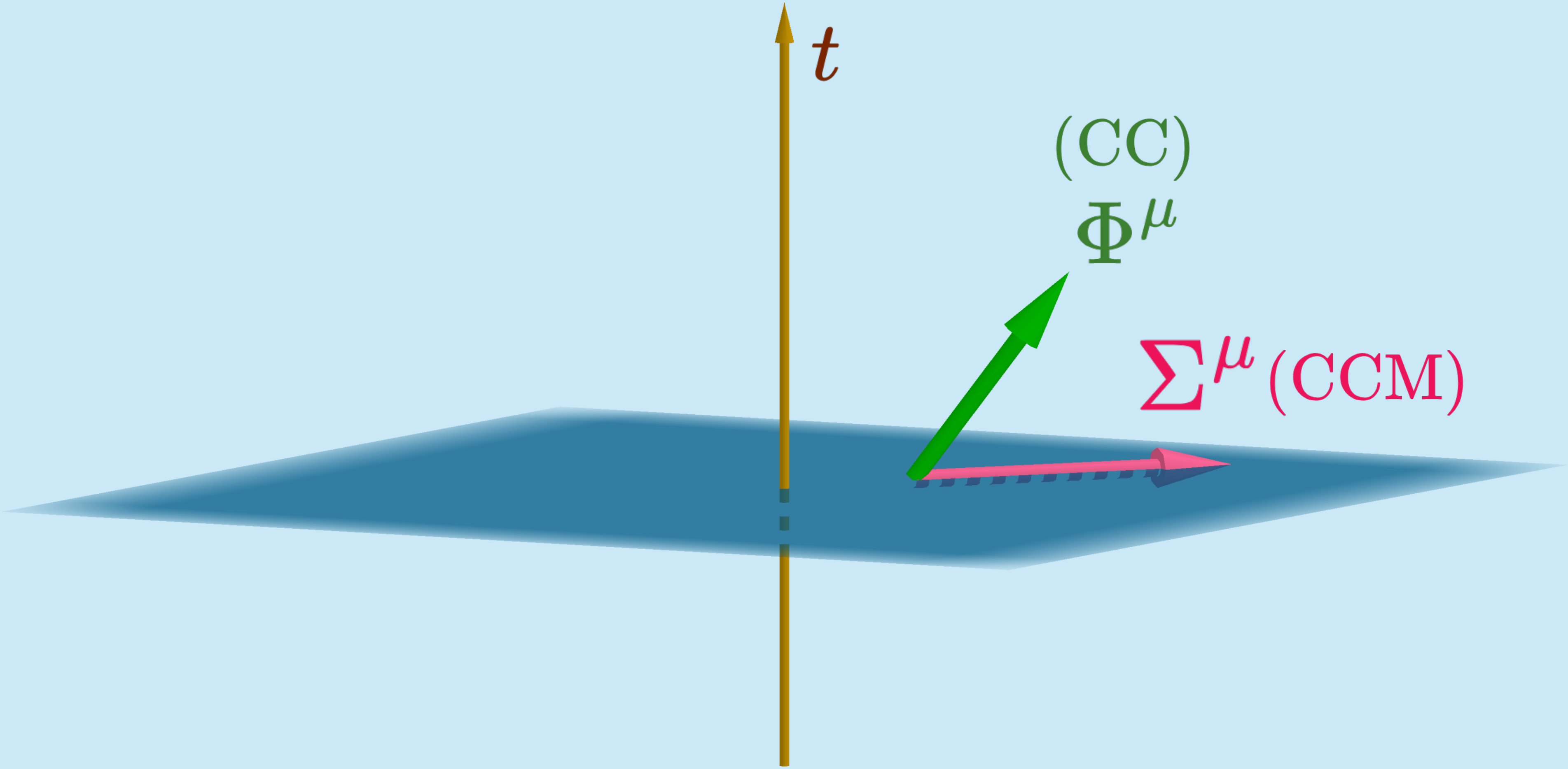
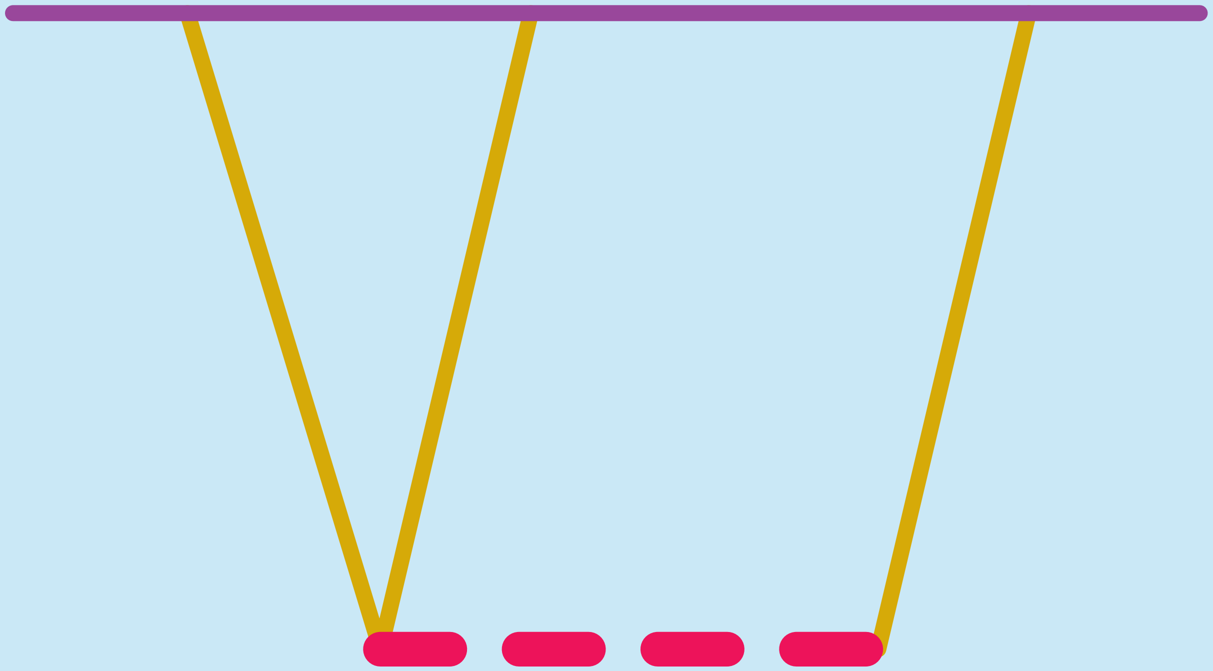


[C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore; 0709.0293]

Feynman rules



Contribution to $\langle \pi\pi\pi \rangle$:

$$(i\lambda_1)(i\lambda_2)\int_{-\infty}^{0} a^4(\eta_1)d\eta_1 \int_{-\infty}^{0} a^4(\eta_2)d\eta_2$$

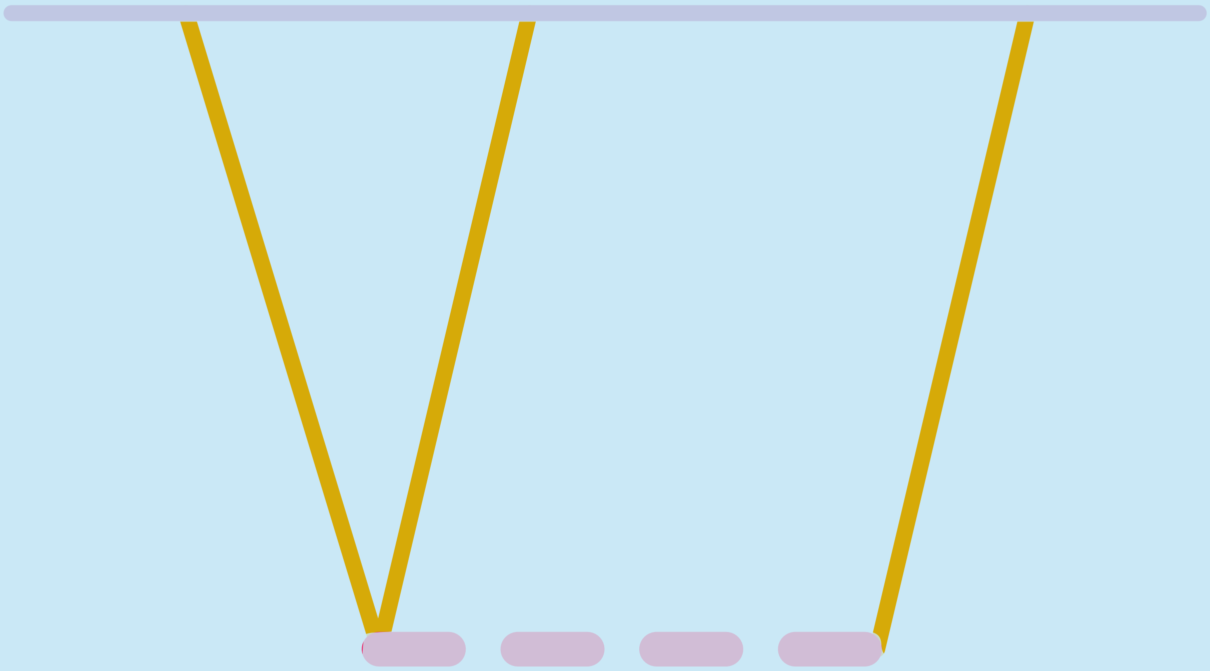
$$G_\pi(\eta_1, \mathbf{k}_1)G_\pi(\eta_1, \mathbf{k}_2)$$

$$G_{\mathrm{CC/CCM}}(\eta_1, \eta_2, \mathbf{k}_3)$$

$$G_\pi(\eta_2, \mathbf{k}_3)$$

[X. Chen, Y. Wang, Z. Xianyu; 1703.10166]

Feynman rules



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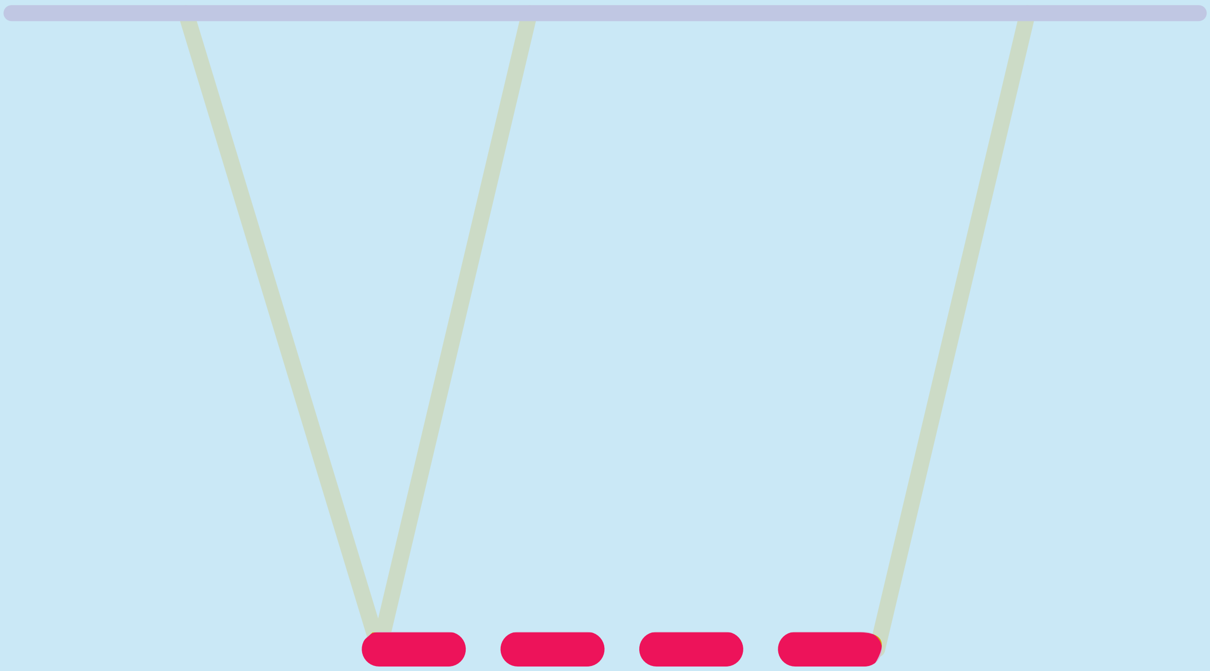
bulk-boundary propagator

[X. Chen, Y. Wang, Z. Xianyu; 1703.10166]

Feynman rules



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bulk-bulk propagator

[X. Chen, Y. Wang, Z. Xianyu; 1703.10166]

Feynman rules

$$G_{\mathrm{CC/CCM}}(\eta_1, \eta_2, \mathbf{k}_3)$$

$$= \boxed{u(\eta_1, \mathbf{k})}\boxed{u^*(\eta_2, \mathbf{k})}\theta(\eta_1 - \eta_2) + (\eta_1 \leftrightarrow \eta_2)$$

Flat-space:

$$\langle T\phi(x^0, \mathbf{p})\phi(y^0, -\mathbf{p})\rangle'$$

$$= \boxed{\frac{e^{iE_{\mathbf{p}}x^0}}{\sqrt{2E_{\mathbf{p}}}}}\boxed{\frac{e^{-iE_{\mathbf{p}}y^0}}{\sqrt{2E_{\mathbf{p}}}}}\theta(x^0 - y^0) + (x^0 \leftrightarrow y^0)$$

[X. Chen, Y. Wang, Z. Xianyu; 1703.10166]

CC/CCM correspondence

$$G_{\mathrm{CC/CCM}}(\eta_1, \eta_2, \mathbf{k}_3)$$
$$= u(\eta_1, \mathbf{k}) u^*(\eta_2, \mathbf{k}) \theta(\eta_1 - \eta_2) + (\eta_1 \leftrightarrow \eta_2)$$

$$\boxed{u_{\mathrm{CC}} = -\frac{1}{m}\left(\partial_\eta - \frac{2}{\eta}\right) u_{\mathrm{CCM}}}$$

(spin-1; longitudinal mode)

CC/CCM correspondence

$$G_{\mathrm{CC/CCM}}(\eta_1, \eta_2, \mathbf{k}_3)$$
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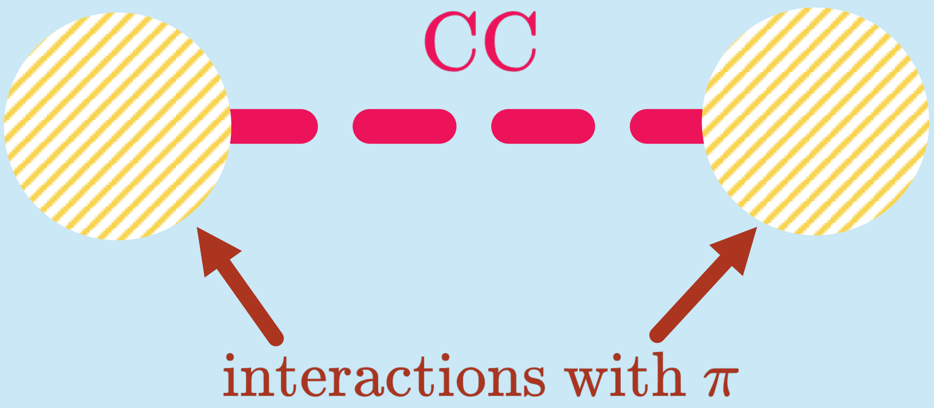
(spin-1; longitudinal mode)

$$G_{\mathrm{CC}} = \frac{1}{m^2} \boxed{\left(\partial_{\eta_1} - \frac{2}{\eta_1}\right)} \boxed{\left(\partial_{\eta_2} - \frac{2}{\eta_2}\right)} G_{\mathrm{CCM}}$$
$$- \frac{i}{m^2 a^2} \delta(\eta_1 - \eta_2)$$

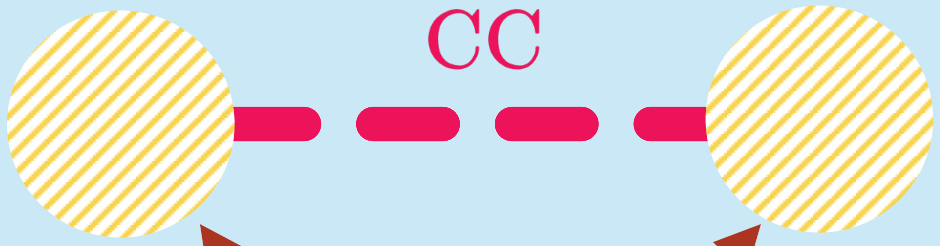
CC/CCM correspondence

$$G_{\mathrm{CC}} = \frac{1}{m^2}\left(\partial_{\eta_1} - \frac{2}{\eta_1}\right)\left(\partial_{\eta_2} - \frac{2}{\eta_2}\right)G_{\mathrm{CCM}} - \frac{i}{m^2a^2}\delta(\eta_1 - \eta_2)$$



$$= \int d\eta_1 d\eta_2 (\mathrm{stuff}(\eta_1))(\mathrm{stuff}(\eta_2)) \times \left[\frac{1}{m^2}\left(\partial_{\eta_1} - \frac{2}{\eta_1}\right)\left(\partial_{\eta_2} - \frac{2}{\eta_2}\right)G_{\mathrm{CCM}} - \frac{i}{m^2a^2}\delta(\eta_1 - \eta_2)\right]$$

CC/CCM correspondence



interactions with π

$$= \int d\eta_1 d\eta_2 (\text{stuff}(\eta_1))(\text{stuff}(\eta_2)) \times$$

*boundary terms vanish

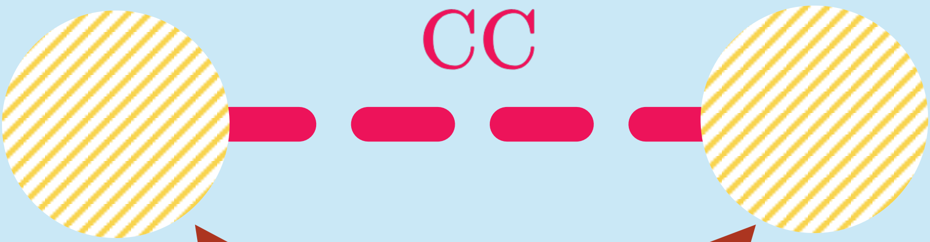
integrate by parts

$$\left[\frac{1}{m^2}\boxed{\left(\partial_{\eta_1} - \frac{2}{\eta_1}\right)}\boxed{\left(\partial_{\eta_2} - \frac{2}{\eta_2}\right)} G_{\text{CCM}} - \frac{i}{m^2 a^2}\delta(\eta_1 - \eta_2)\right]$$

$-\left(\partial_\eta + \frac{2}{\eta}\right)$ interactions with π

$=$

CC/CCM correspondence



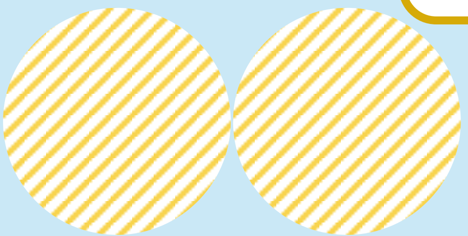
$$= \int d\eta_1 d\eta_2 \boxed{(\text{stuff}(\eta_1))(\text{stuff}(\eta_2))} \times \left[\frac{1}{m^2}\left(\partial_{\eta_1}-\frac{2}{\eta_1}\right)\left(\partial_{\eta_2}-\frac{2}{\eta_2}\right)G_{\text{CCM}} - \boxed{\frac{i}{m^2 a^2}\delta(\eta_1-\eta_2)}\right]$$

contact diagram

$-\left(\partial_\eta+\frac{2}{\eta}\right)$ interactions with π

$=$

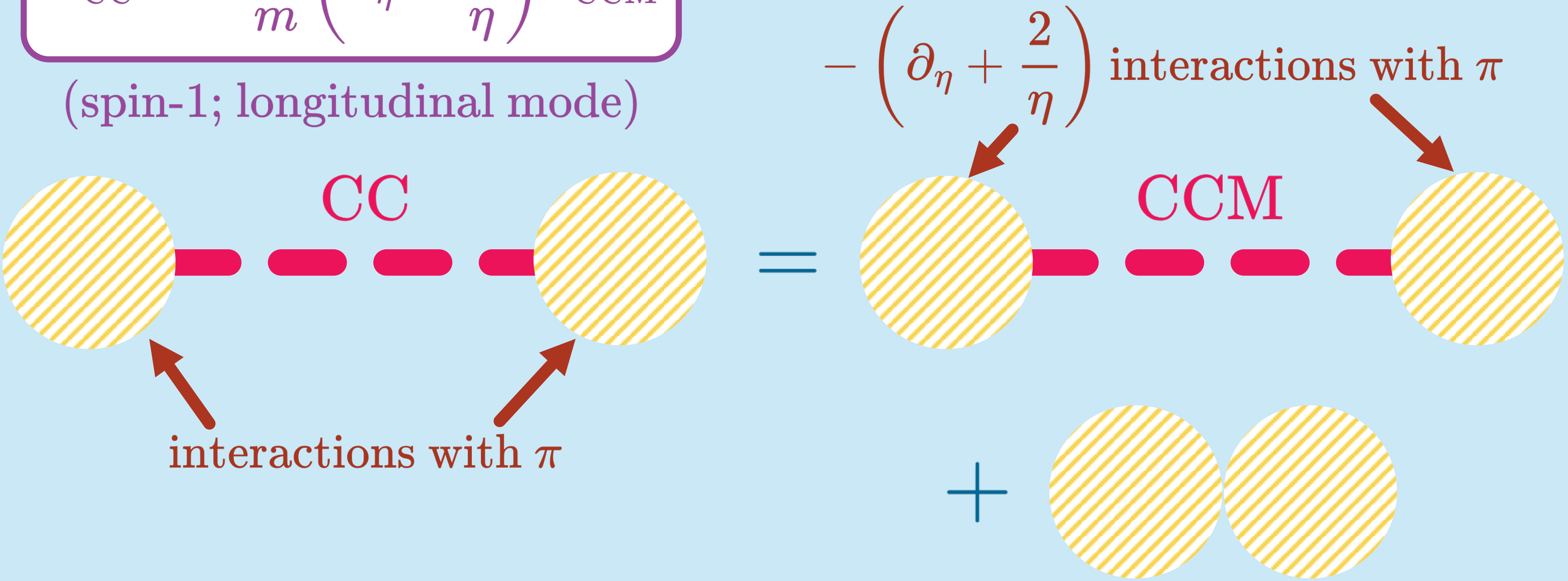
$+$



True if only 1 integral

$$\boxed{u_{\mathrm{CC}} = -\frac{1}{m}\left(\partial_\eta - \frac{2}{\eta}\right) u_{\mathrm{CCM}}}$$

(spin-1; longitudinal mode)

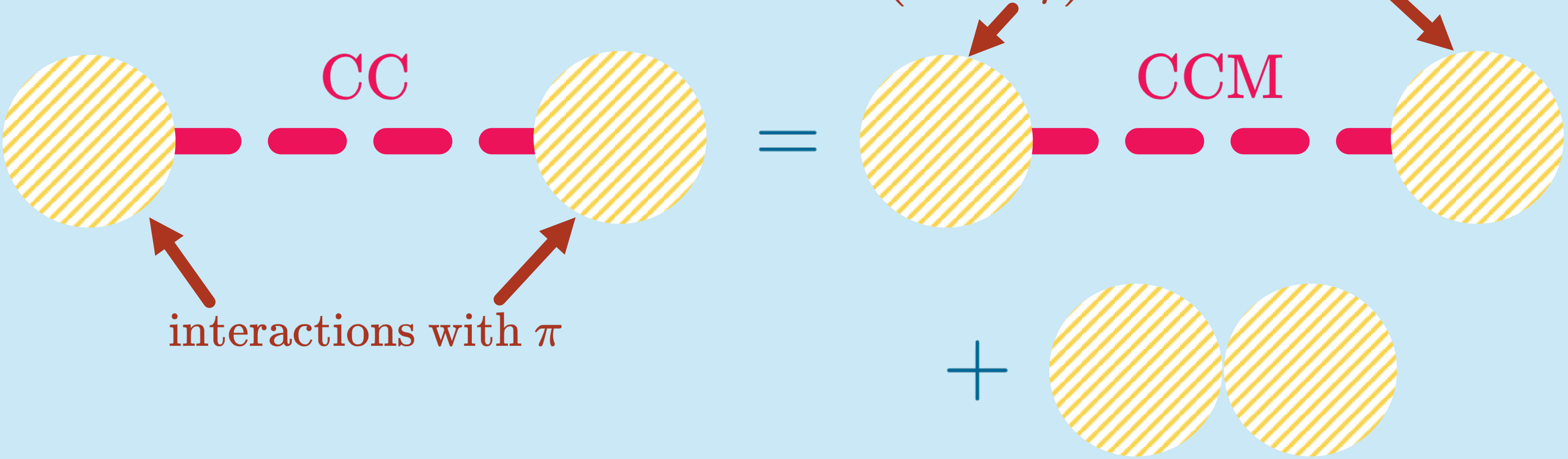


True if only 1 integral

$$\boxed{u_{\rm CC} = a u_{\rm CCM}}$$

(spin-1; transverse modes)

$-\left(\partial_\eta + \frac{2}{\eta}\right)$ interactions with π

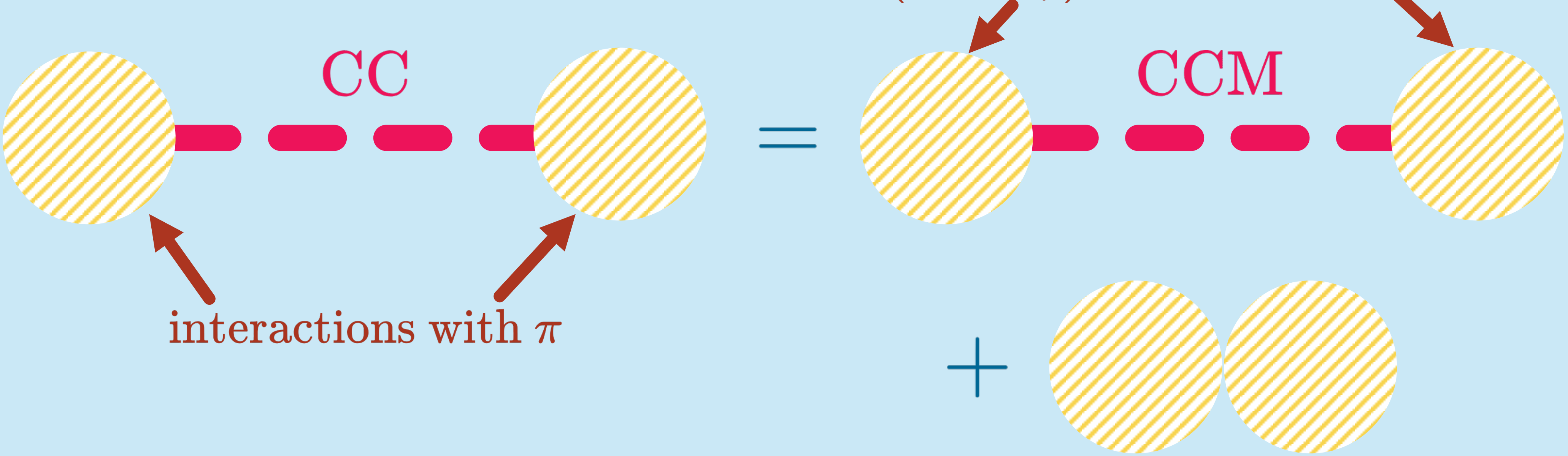


True if only 1 integral

$$\boxed{u_{\rm CC} = a u_{\rm CCM}}$$

(spin-1; transverse modes)

$$-\left(\partial_\eta + \frac{2}{\eta}\right) \text{ interactions with } \pi$$



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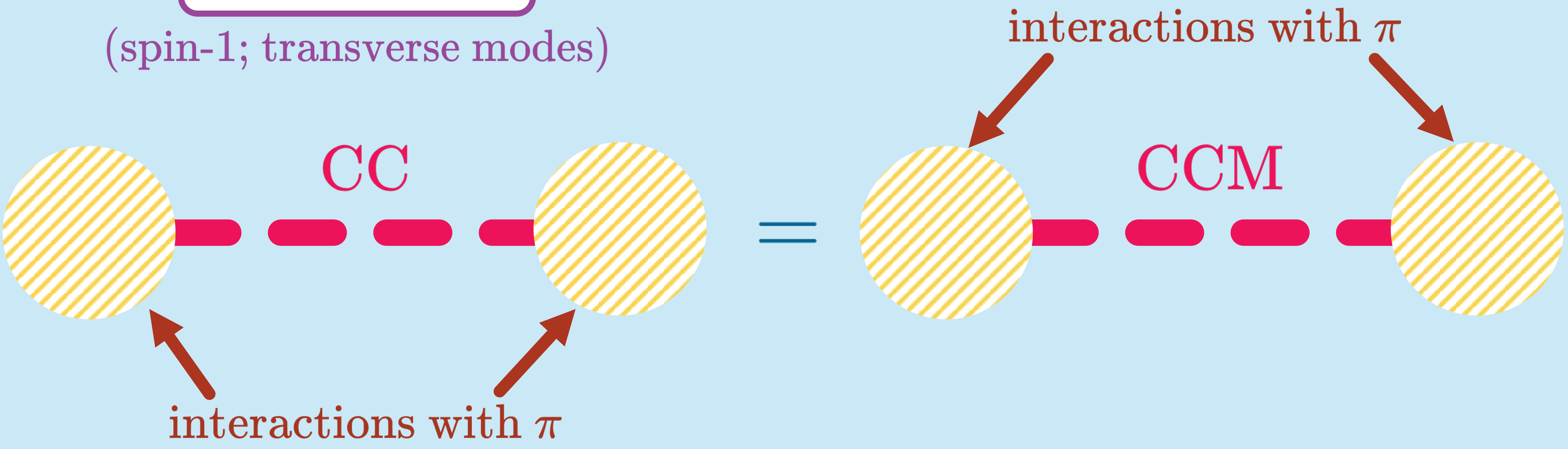
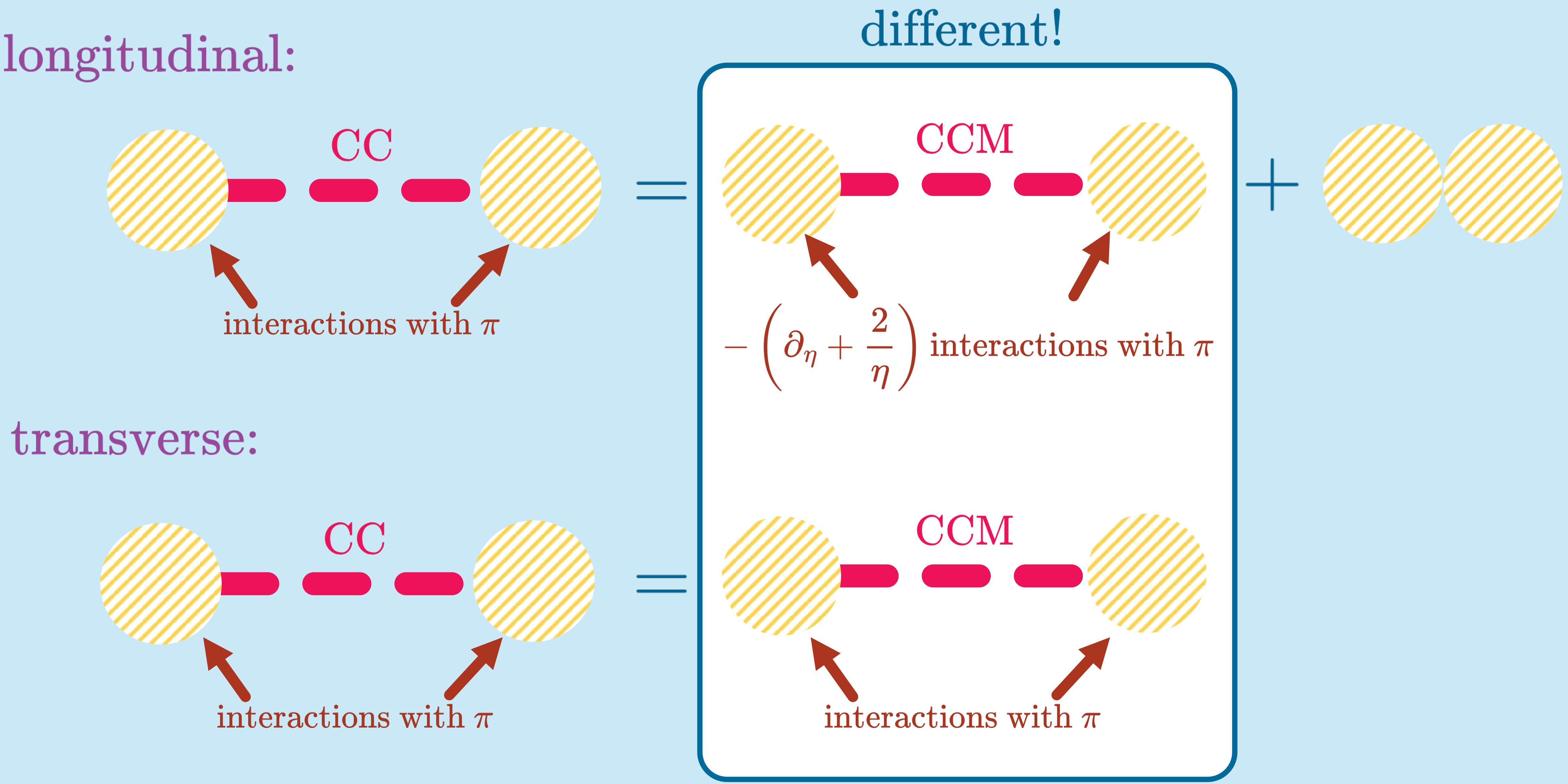
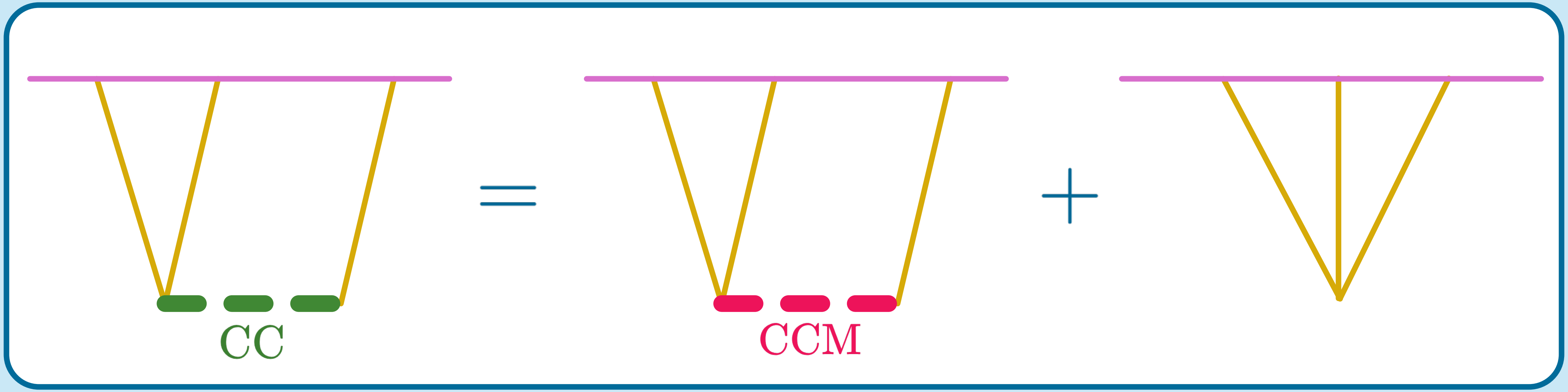


Fail if more than 1 helicity exchange



3-point functions guarantee only longitudinal mode exchange

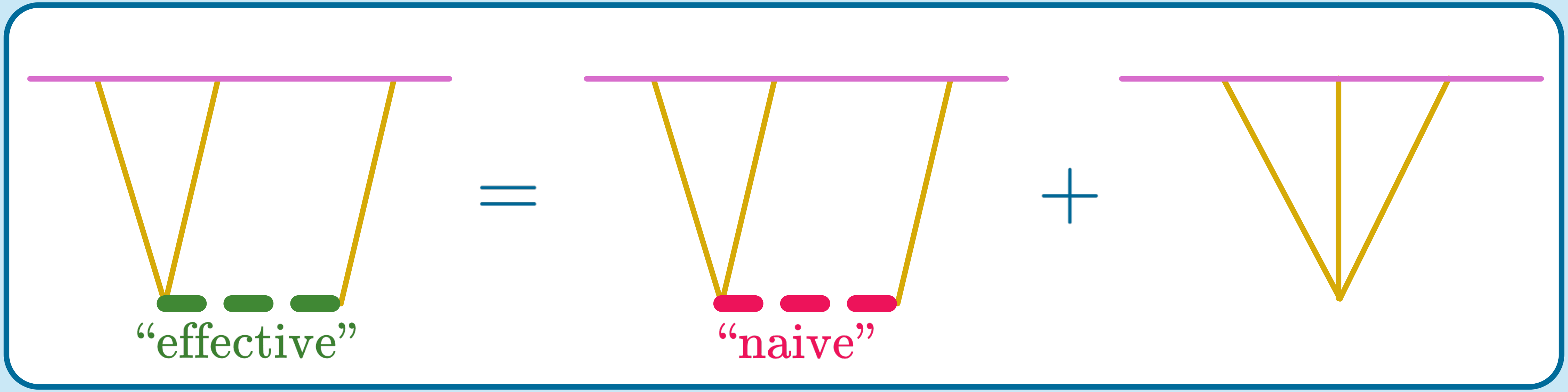
→ can't distinguish between CC and CCM for bispectrum



Another meaning of this diagram:

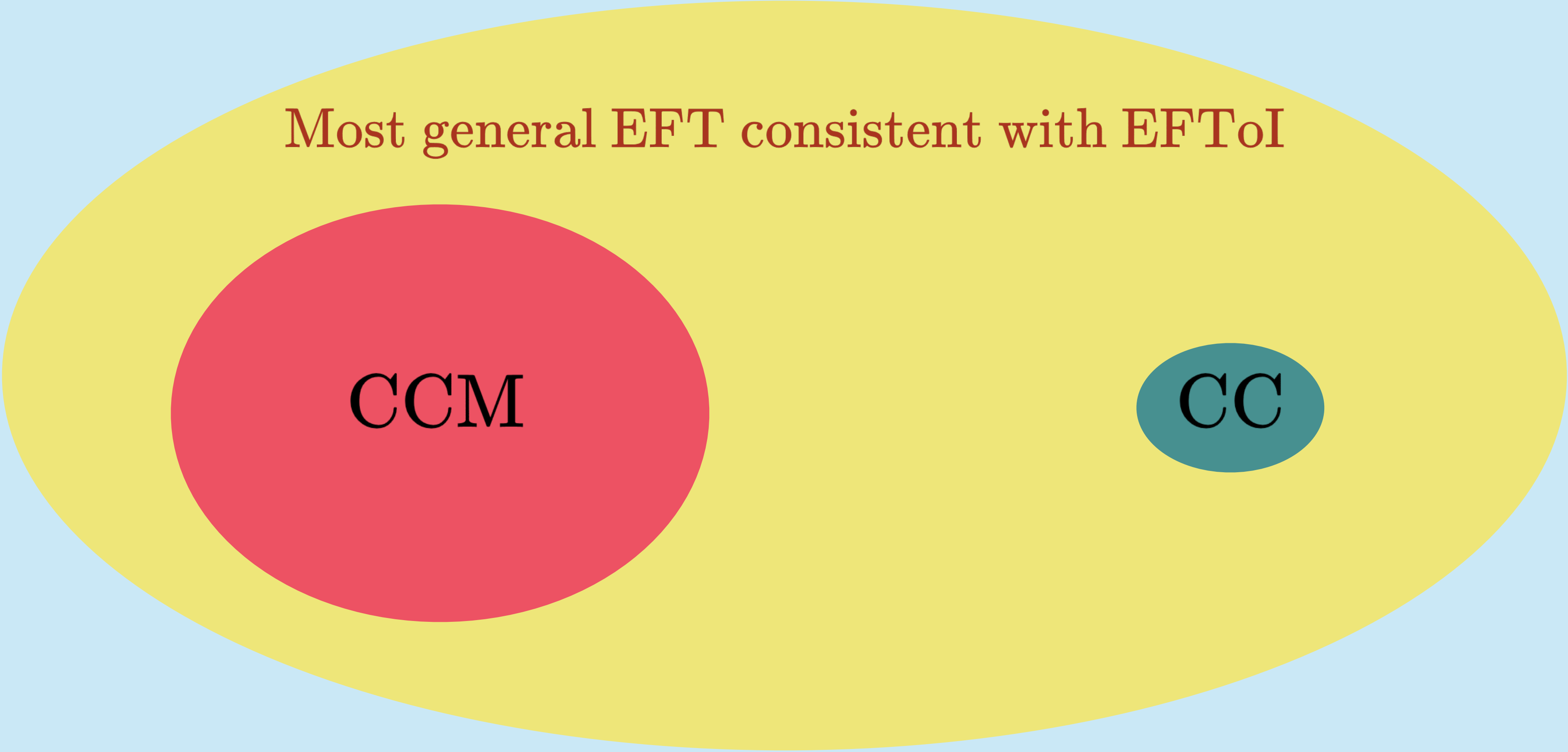
Correction to Feynman rules in CC

the “effective” propagator would give dS invariant correlators*



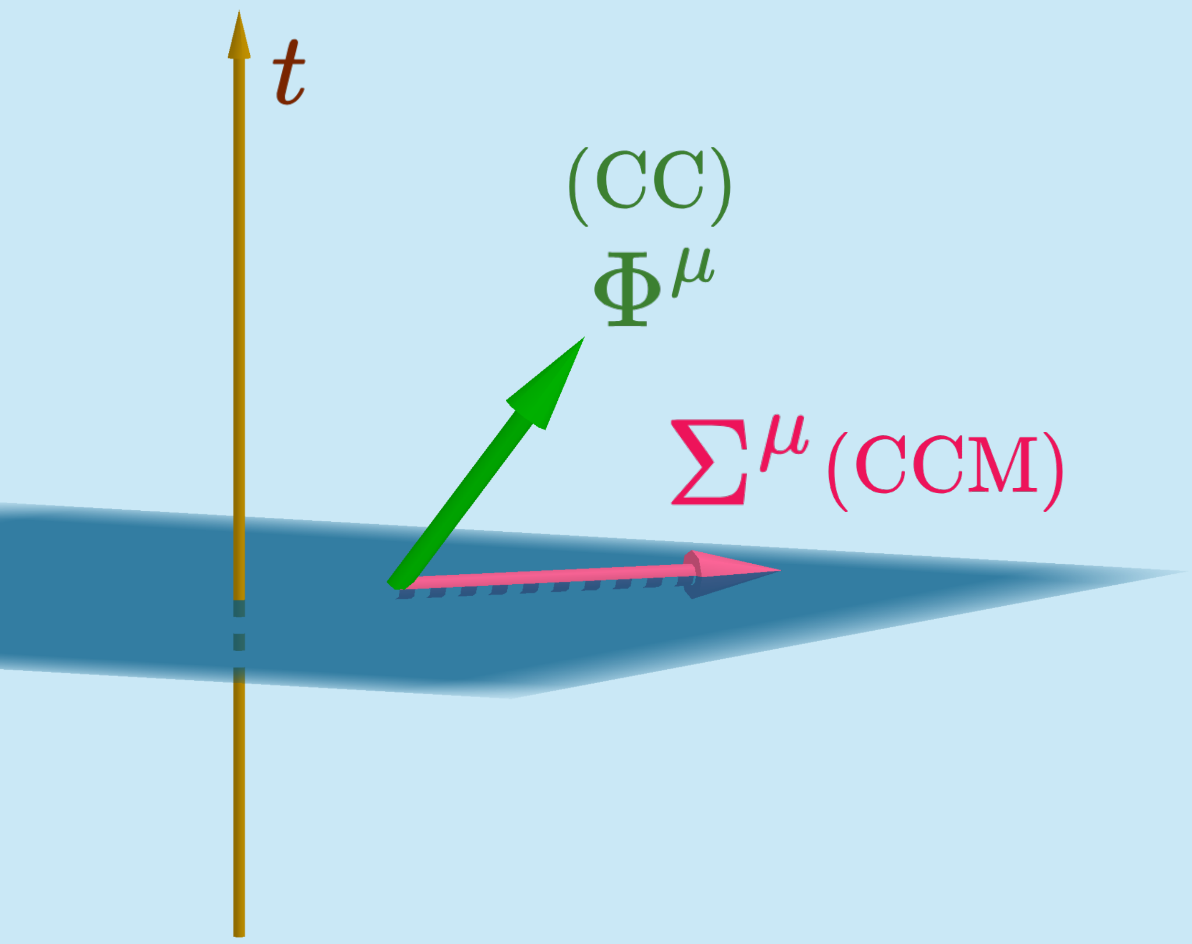
*not directly proved, but the flat space analogue is the actual Lorentz invariant one

More general EFT of massive spinning field?



Most general EFT for a spinning field?

Constraints on coefficients for a healthy theory?



$$
\begin{aligned}
2\mathcal{L} = {} & \sigma_i'\sigma_i' + c_1^2 \partial_i\sigma_i\partial_j\sigma_j \\
& + c_2^2\partial_i\sigma_j\partial_i\sigma_j - M^2a^2\sigma_i\sigma_i \\
& + \alpha\partial_i\sigma_0\partial_i\sigma_0 + m^2a^2\sigma_0\sigma_0 \\
& + 2\gamma\partial_i\sigma_0\sigma_i'
\end{aligned}
$$

CC parameters:

$$\alpha = -\gamma = c_1^2 = -c_2^2 = 1 \qquad m = M$$

CCM parameters:

$$\alpha = \gamma = m = 0$$

Constraints on coefficients for a healthy theory?

1) Bunch-Davies condition

$$\sigma_\mu \sim \exp(i c_s k \eta) e_\mu \qquad \text{as } \eta \to -\infty$$

2) Microcausality (true even when Lorentz invariance is broken)

$$[\sigma_\mu(x), \sigma_\nu(y)] = 0 \qquad \text{for } (x-y)_\mu (x-y)^\mu > 0$$

[S. Dubovsky, A. Nicolis, E. Trincherini, G. Villadoro; 0709.1483]
[L. Hui, A. Nicolis, A. Podo, S. Zhou; 2502.04215]

Constraints on coefficients for a healthy theory?

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$$2\mathcal{L} = \sigma_i' \sigma_i' + c_1^2 \partial_i \sigma_i \partial_j \sigma_j + c_2^2 \partial_i \sigma_j \partial_i \sigma_j - M^2 a^2 \sigma_i \sigma_i + \alpha \partial_i \sigma_0 \partial_i \sigma_0 + m^2 a^2 \sigma_0 \sigma_0 + 2\gamma \partial_i \sigma_0 \sigma_i'$$

$$\alpha = \gamma^2$$
$$c_1^2 + c_2^2 = 0$$

or

CCM

[**TC**, S. Jazayeri, C. D. Pueyo, **DS**, D. G. Wang; upcoming]

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for $(x-y)_\mu (x-y)^\mu > 0$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \boxed{\frac{a_1}{2}} F^{0\mu} F_{0\mu} - \frac{1}{2} m^2 (\sigma_\mu \sigma^\mu - \boxed{a_0} \sigma_0 \sigma^0)$$

(after field redefs)

$$\boxed{\begin{gathered} \alpha = \gamma^2 \\ c_1^2 + c_2^2 = 0 \end{gathered}}$$

or

CCM

[**TC**, S. Jazayeri, C. D. Pueyo, **DS**, D. G. Wang; upcoming]

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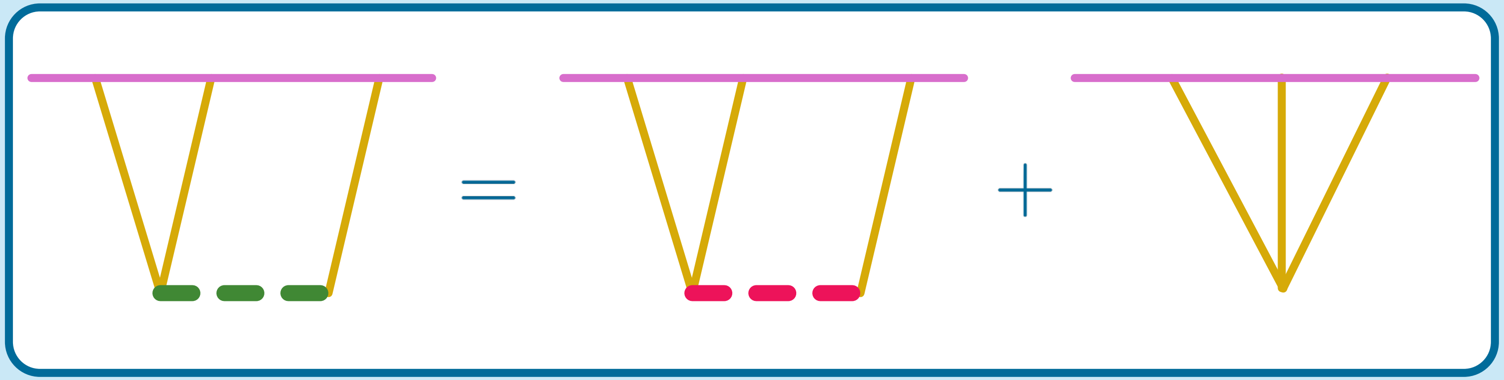

More details here:

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Based on arXiv:2509.08888

Proca field shenanigans

Flat-space:

$$\langle T\phi(x)\phi(y)\rangle$$

$$= \int_{\mathbf{p}} \frac{e^{ip\cdot x}}{\sqrt{2E_{\mathbf{p}}}} \frac{e^{-ip\cdot y}}{\sqrt{2E_{\mathbf{p}}}} \theta(x^0 - y^0) + (x \leftrightarrow y)$$

Proca field shenanigans

Flat-space: $\boxed{\langle T\phi(x)\phi(y)\rangle}$

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Proca field shenanigans

Flat-space: $\boxed{\langle T A_\mu(x) A_\nu(y) \rangle}$ massive Proca

$$= \int_{\mathbf{p}} \frac{e^{ip\cdot x}}{\sqrt{2E_{\mathbf{p}}}} \frac{e^{-ip\cdot y}}{\sqrt{2E_{\mathbf{p}}}} \theta(x^0 - y^0) + (x \leftrightarrow y)$$

Proca field shenanigans

Flat-space:

[S. Weinberg '05]

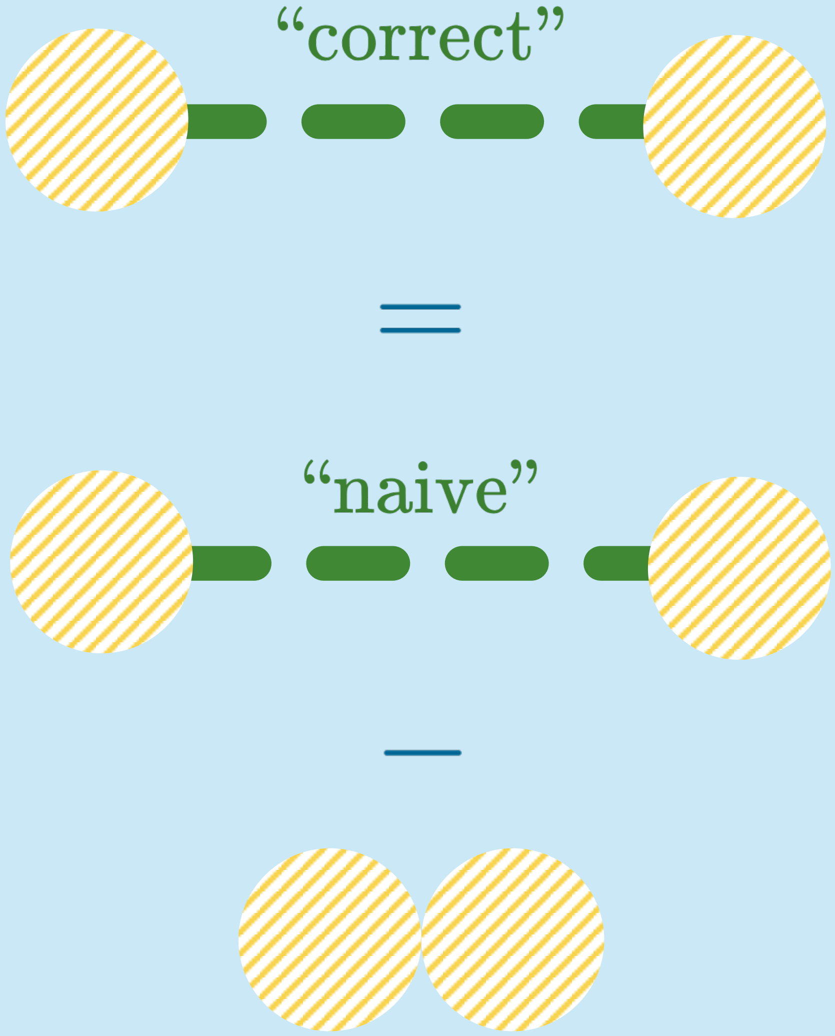
$$\langle T A_\mu(x) A_\nu(y) \rangle$$

$$= \int_{\mathbf{p}} \left(\eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} \right) \frac{e^{ip\cdot x}}{\sqrt{2E_{\mathbf{p}}}} \frac{e^{-ip\cdot y}}{\sqrt{2E_{\mathbf{p}}}} \boxed{\theta(x^0 - y^0)} + (x \leftrightarrow y)$$

$$= \underbrace{\left(\eta_{\mu\nu} - \boxed{\frac{1}{m^2}\partial_\mu \partial_\nu} \right) \langle T\phi(x)\phi(y) \rangle}_{\text{Lorentz covariant}} - \underbrace{\frac{i}{m^2} \delta^0_\mu \delta^0_\nu \delta^{(4)}(x-y)}_{\text{NOT Lorentz covariant}}$$

"naive propagator"

Proca field shenanigans



$$\langle T A_\mu(x) A_\nu(y) \rangle$$

[S. Weinberg ’05]

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$$= \underbrace{\left(\eta_{\mu\nu} - \frac{1}{m^2} \partial_\mu \partial_\nu \right) \langle T \phi(x) \phi(y) \rangle}_{\text{Lorentz covariant}} \underbrace{- \frac{i}{m^2} \delta^0_\mu \delta^0_\nu \delta^{(4)}(x - y)}_{\text{NOT Lorentz covariant}}$$

“naive propagator”

So all diagrams with Proca field exchange are not Lorentz covariant?

Proca field shenanigans

[S. Weinberg '05]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - J_\mu A^\mu$$

Legendre transform $\longrightarrow$

free Hamiltonian

$$\mathcal{H} = \boxed{\frac{1}{2}\vec{\mathbf{\Pi}}^2 + \frac{1}{2m^2}(\nabla\cdot\vec{\mathbf{\Pi}})^2 + \frac{1}{2}(\nabla\times\vec{\mathbf{A}})^2 + \frac{m^2}{2}\vec{\mathbf{A}}^2}$$

$$+\vec{\mathbf{J}}\cdot\vec{\mathbf{A}} - \frac{1}{m^2}J^0\nabla\cdot\vec{\pi} + \frac{1}{2m^2}(J^0)^2$$

Proca field shenanigans

[S. Weinberg '05]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu \boxed{- J_\mu A^\mu}$$

free Hamiltonian

Legendre transform

$$\mathcal{H} = \boxed{\frac{1}{2}\vec{\mathbf{\Pi}}^2 + \frac{1}{2m^2}(\nabla\cdot\vec{\mathbf{\Pi}})^2 + \frac{1}{2}(\nabla\times\vec{\mathbf{A}})^2 + \frac{m^2}{2}\vec{\mathbf{A}}^2}$$

$$+\boxed{\vec{\mathbf{J}}\cdot\vec{\mathbf{A}} - \frac{1}{m^2}J^0\nabla\cdot\vec{\pi}} + \frac{1}{2m^2}(J^0)^2$$

normal interactions you expect

Proca field shenanigans

[S. Weinberg '05]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu - J_\mu A^\mu$$

Legendre transform $\longrightarrow$

free Hamiltonian

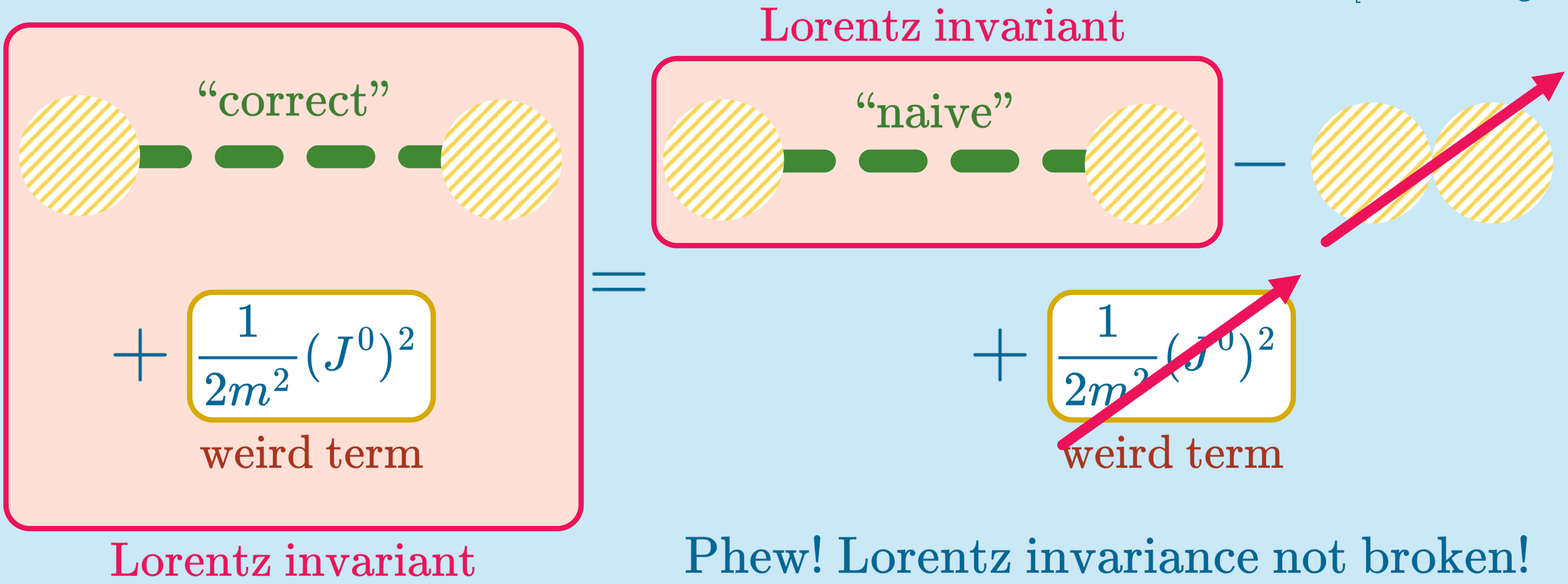
$$\mathcal{H} = \boxed{\frac{1}{2}\vec{\mathbf{\Pi}}^2 + \frac{1}{2m^2}(\nabla\cdot\vec{\mathbf{\Pi}})^2 + \frac{1}{2}(\nabla\times\vec{\mathbf{A}})^2 + \frac{m^2}{2}\vec{\mathbf{A}}^2}$$

$$+\vec{\mathbf{J}}\cdot\vec{\mathbf{A}} - \frac{1}{m^2}J^0\nabla\cdot\vec{\pi} + \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

Proca field shenanigans

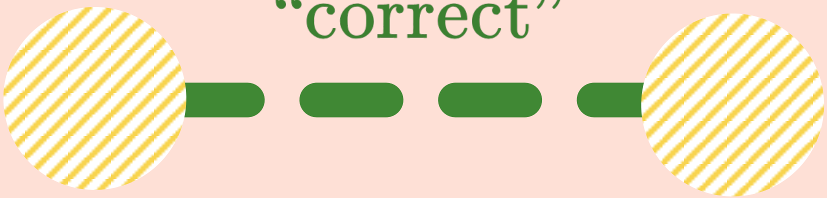
[S. Weinberg '05]



Phew! Lorentz invariance not broken!

dS massive spinning field

“correct”

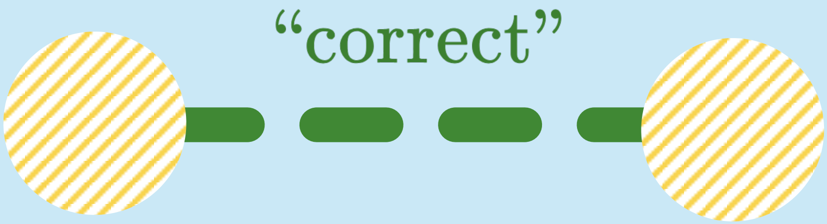


$$+ \boxed{\frac{1}{2m^2}(J^0)^2} =$$

weird term

dS invariant

dS massive spinning field



$$+ \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

$$= u(\eta_1, \mathbf{k})u^*(\eta_2, \mathbf{k})\theta(\eta_1 - \eta_2) + (\eta_1 \leftrightarrow \eta_2)$$

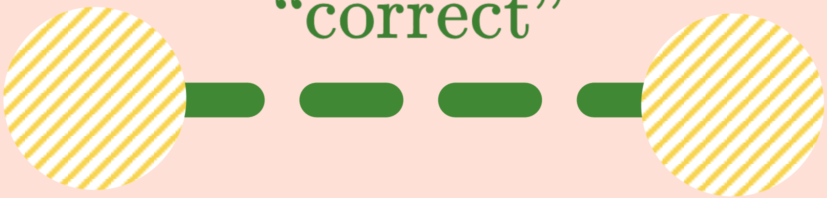
in $(x^0, \mathbf{k})$ domain

$$+ \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

dS massive spinning field

“correct”



$$+ \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

dS invariant

$$=$$

$$u(\eta_1, \mathbf{k})u^*(\eta_2, \mathbf{k})\theta(\eta_1 - \eta_2) + (\eta_1 \leftrightarrow \eta_2)$$

in $(x^0, \mathbf{k})$ domain

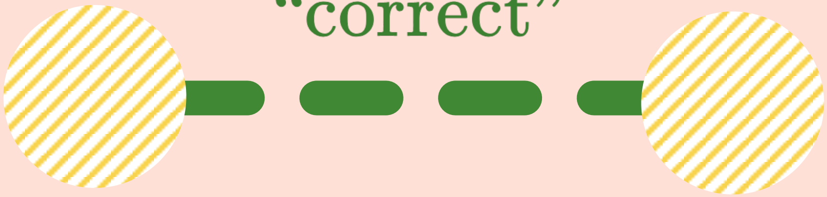
$$+ \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

dS invariant

dS massive spinning field

“correct”



$$+\ \boxed{\frac{1}{2m^2}(J^0)^2}$$

weird term

dS invariant

$$=$$

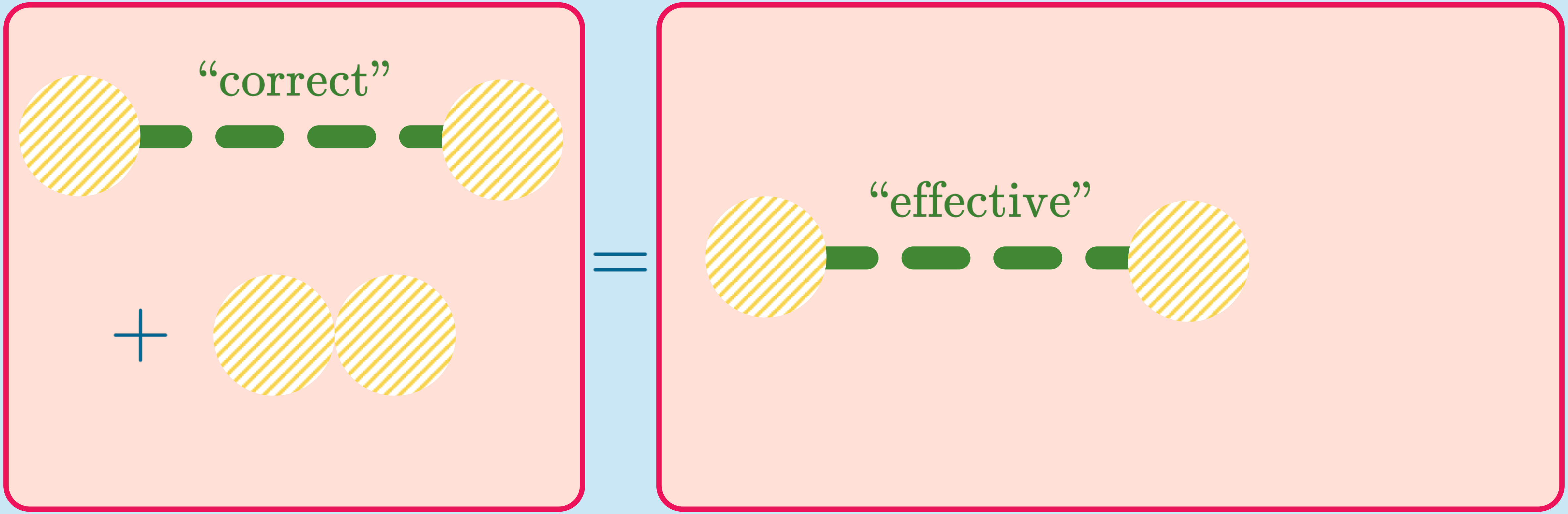
$$u(\eta_1,\mathbf{k})u^*(\eta_2,\mathbf{k})\theta(\eta_1-\eta_2)+(\eta_1\leftrightarrow\eta_2)$$

$$+\ \frac{i}{m^2a^2}\delta(\eta_1-\eta_2)$$

in $(x^0,\mathbf{k})$ domain

dS invariant

dS massive spinning field



dS invariant

dS invariant