Positive Geometry in Cosmological Correlators

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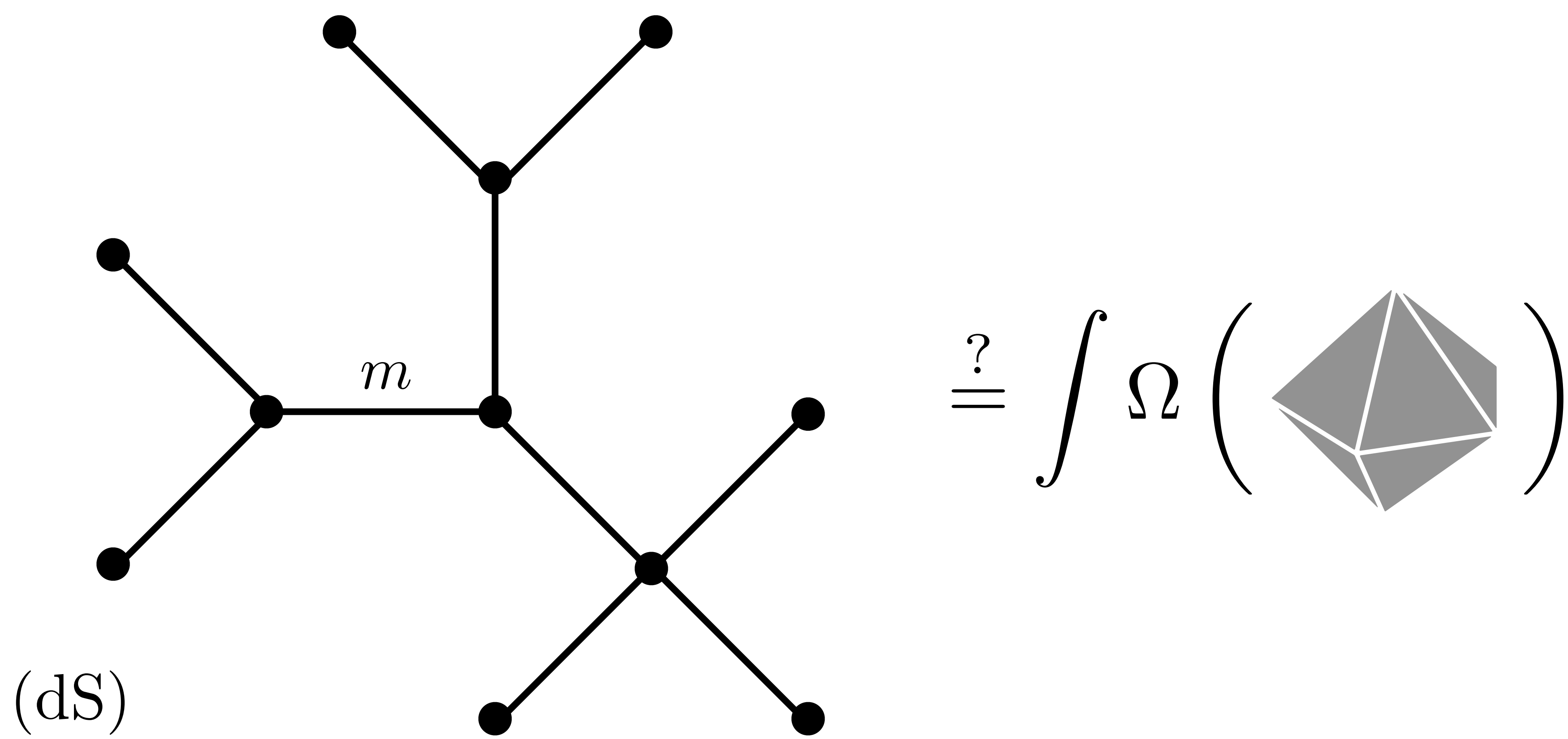
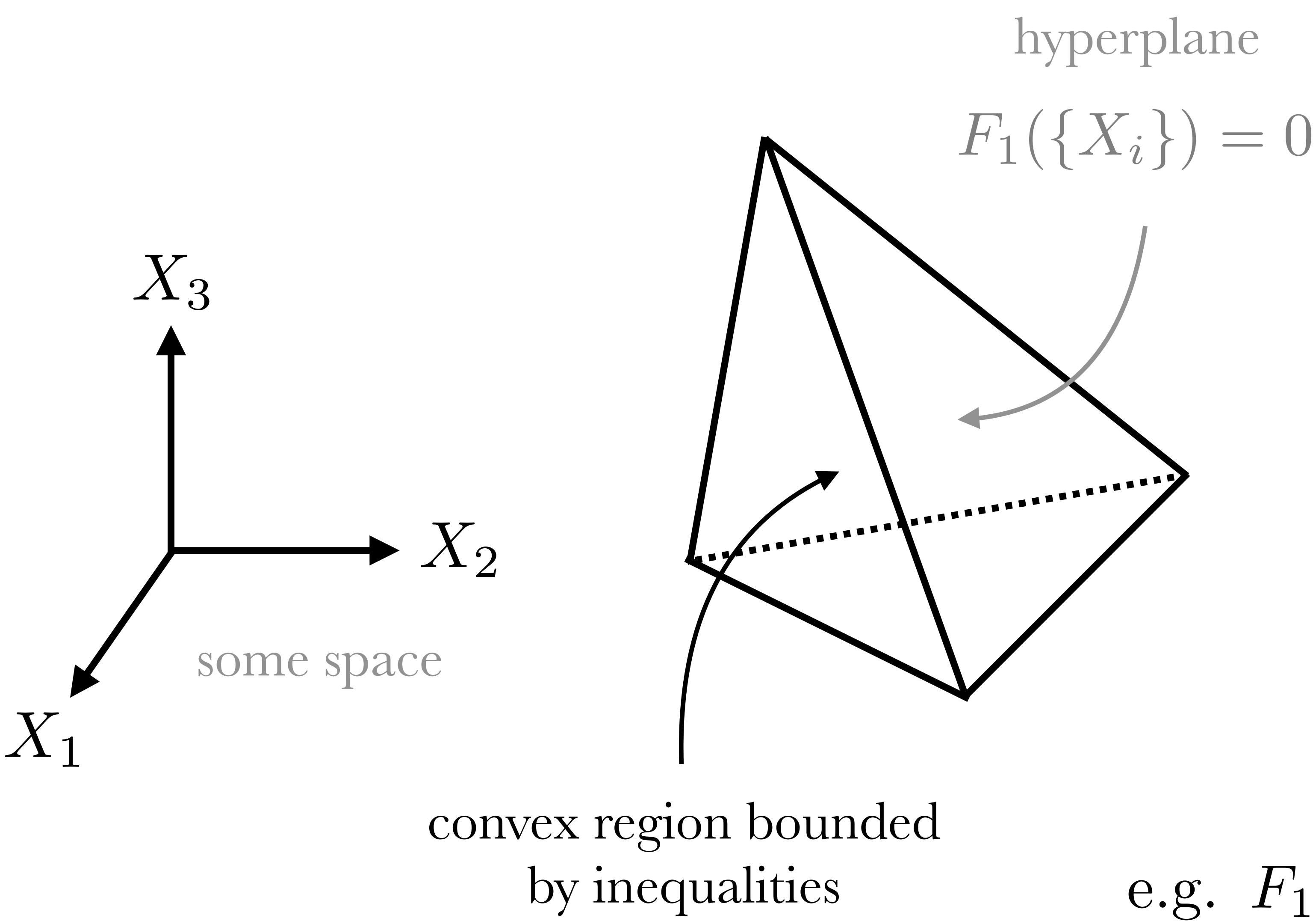
Can massive de Sitter cosmological correlators be described by positive geometries?



Short answer: We don’t know yet!

Goal of the talk: Give you the tools to understand the question & why it’s worth asking

What is a *positive geometry* in simple terms?



<u>Set of inequalities</u>

semi-algebraic set

$$F_j(\{X_i\}) \geq 0$$

e.g. $F_1 = \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3$

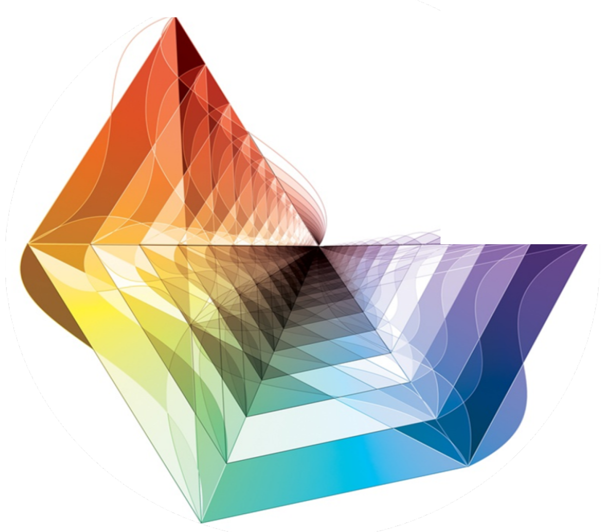
polytope

Positive geometry is a *new emergent field of mathematics* that is related to and inspired by physics

Positive geometries can describe (flat-space) scattering amplitudes in *some* theories

Amplituhedron

(Planar $N = 4$ SYM)
tree level & loop integrands

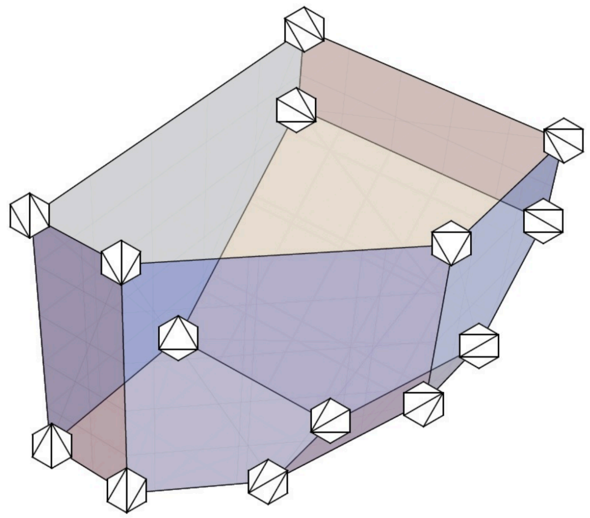


[Arkani-Hamed, Trnka '13]

momentum twistor space/
positive Grassmanian

ABHY Associahedron

($\mathrm{Tr}(\phi^3)$ / bi-adjoint ϕ^3)



[Arkani-Hamed, Bai, He, Yan '17]

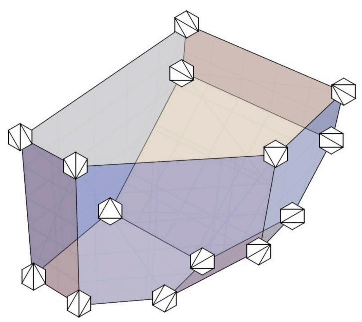
kinematic space of Mandelstam invariants

Surfacehedron generalisation to loop integrands

[Arkani-Hamed,Frost, Salvatori, Plamondon, Thomas '23]

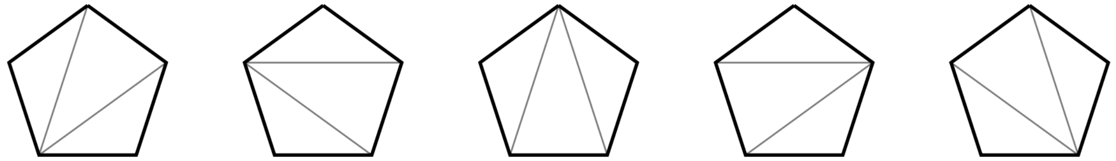
Novel properties emerge from positive geometries

No Feynman diagrams



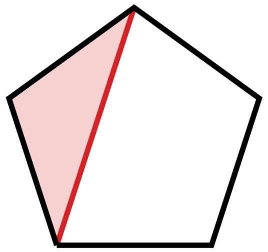
$\supset s, t$

New decompositions from triangulations



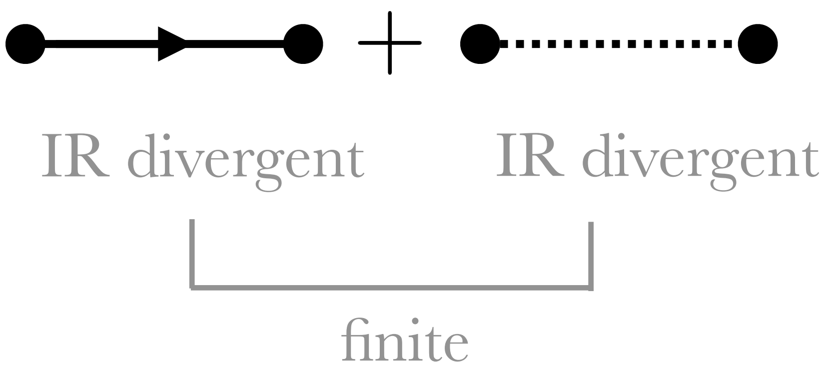
Physical properties made manifest

e.g. singularities encoded in the facet structure of the positive geometry



$\to 0$

No spurious divergences



No computational advantage but rather new conceptual viewpoint!

Positive geometry can bring a *novel fundamental perspective* on cosmological correlators

<u>Geometry</u>
(defined by set of inequalities in physical or auxiliary space)

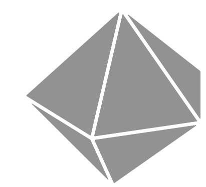
↓

<u>Canonical differential form</u>
(rational integrands)

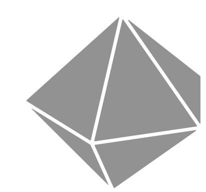
↓

<u>Cosmological correlators</u>
(special functions after integration)

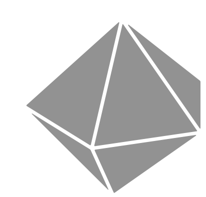
What geometries are relevant?

 (?)

How to construct the forms?

$\Omega\Big($  $\Big)$ (?)

How to perform the integration?

$\int \Omega\Big($  $\Big)$ (?)

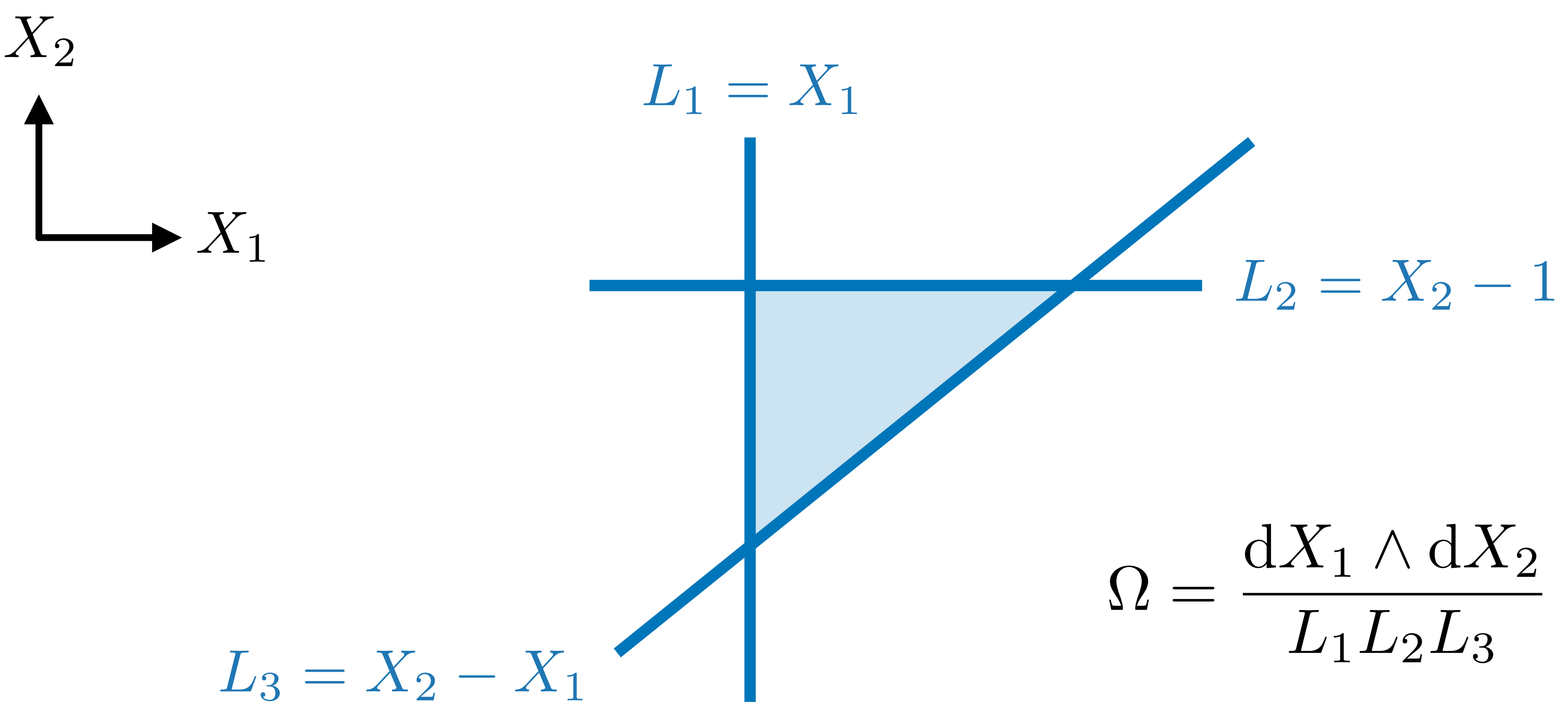
Interval (1D)

The canonical differential form should have logarithmic singularities at the endpoints

$$\begin{cases} X + a = 0 \\ X + b = 0 \end{cases}$$

$$\Omega = \frac{\mathrm{d}X}{X+a} - \frac{\mathrm{d}X}{X+b} = \frac{(b-a)\mathrm{d}X}{(X+a)(X+b)} = \mathrm{d}\log\left(\frac{X+a}{X+b}\right)$$

Triangle (2D)



$$\Omega = \frac{\mathrm{d}X_1 \wedge \mathrm{d}X_2}{L_1 L_2 L_3}$$

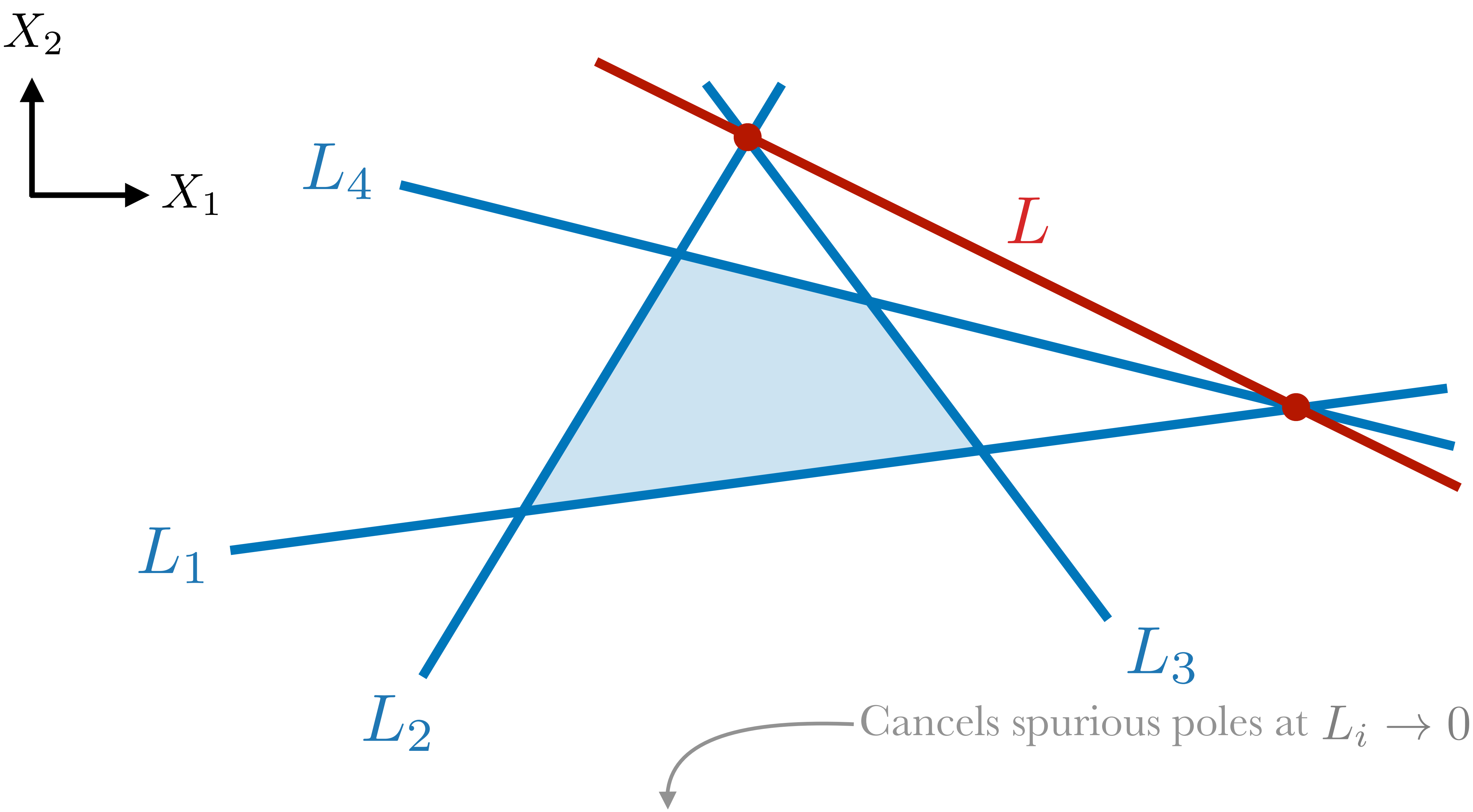
$$\operatorname*{Res}_{L_2 \to 0}[\Omega] = \frac{\mathrm{d}X_1}{X_1(1 - X_1)}$$

← canonical form of the interval

Definition: Unique differential form with logarithmic singularities (only) on all the boundaries of the space, with residues on each boundary given by the canonical form on that boundary

[Arkani-Hamed, Bai, Lam '17]

Quadrilateral (2D)

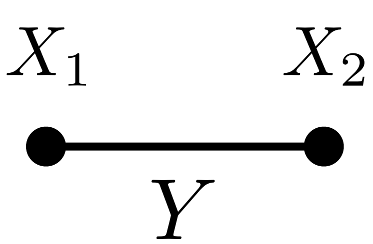


$$\Omega = \frac{L\, \mathrm{d}X_1 \wedge \mathrm{d}X_2}{L_1 L_2 L_3 L_4}$$

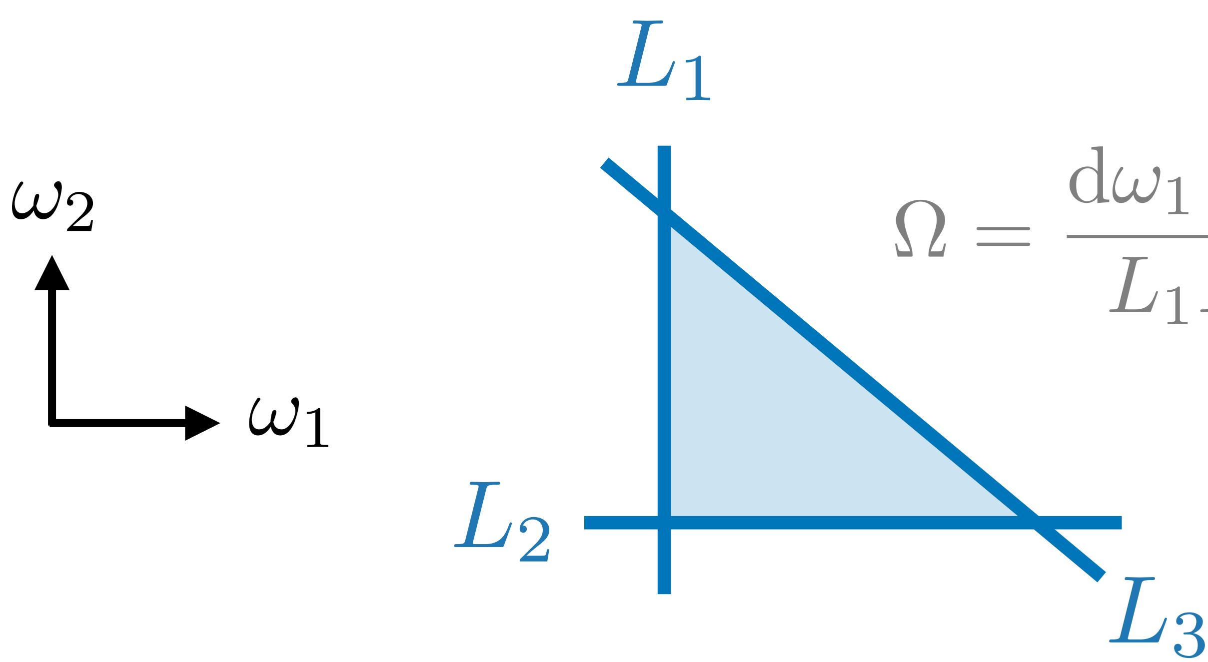
This recursive definition & construction of canonical forms generalises to higher dimensions

De Sitter correlators of a *self-interacting conformally coupled scalar field* emerge from positive geometries

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{R}{12}\phi^2 - \sum_n \frac{\lambda_n}{n!}\phi^n\right]$$



$$= \int_0^\infty \frac{\mathrm{d}\omega_1 \wedge \mathrm{d}\omega_2}{(X_1+\omega_1+Y)(X_2+\omega_2+Y)(X_1+\omega_1+X_2+\omega_2)}$$



$$\Omega = \frac{\mathrm{d}\omega_1 \wedge \mathrm{d}\omega_2}{L_1 L_2 L_3}$$

← Positive geometry in auxiliary "energy" ω-space

[Talk by Hayden Lee]

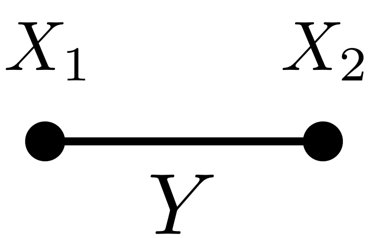
This generalises to arbitrary tree graphs and loop integrands

Massive de Sitter correlators have integral representations with *hypergeometric kernels* & *rational integrands*

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{R}{12}\phi^2 - \tfrac{1}{2}(\partial_\mu\sigma)^2 - \tfrac{1}{2}m^2\sigma^2 - \sum_{m,n}\frac{\lambda_{m,n}}{n!m!}\phi^m\sigma^n\right]$$

Legendre function
(encodes the mass)

rational

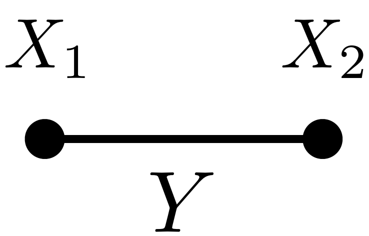


$$= \int_1^\infty P_{i\mu}(\lambda_1)P_{i\mu}(\lambda_2)\mathrm{d}\lambda_1\mathrm{d}\lambda_2\left(\frac{1}{[X_1+X_2+(\lambda_1-\lambda_2)Y][X_1+\lambda_1 Y]} + \frac{1}{[X_1+X_2+(\lambda_2-\lambda_1)Y][X_2+\lambda_2 Y]} - \frac{1}{[X_1+\lambda_1 Y][X_2+\lambda_2 Y]}\right)$$

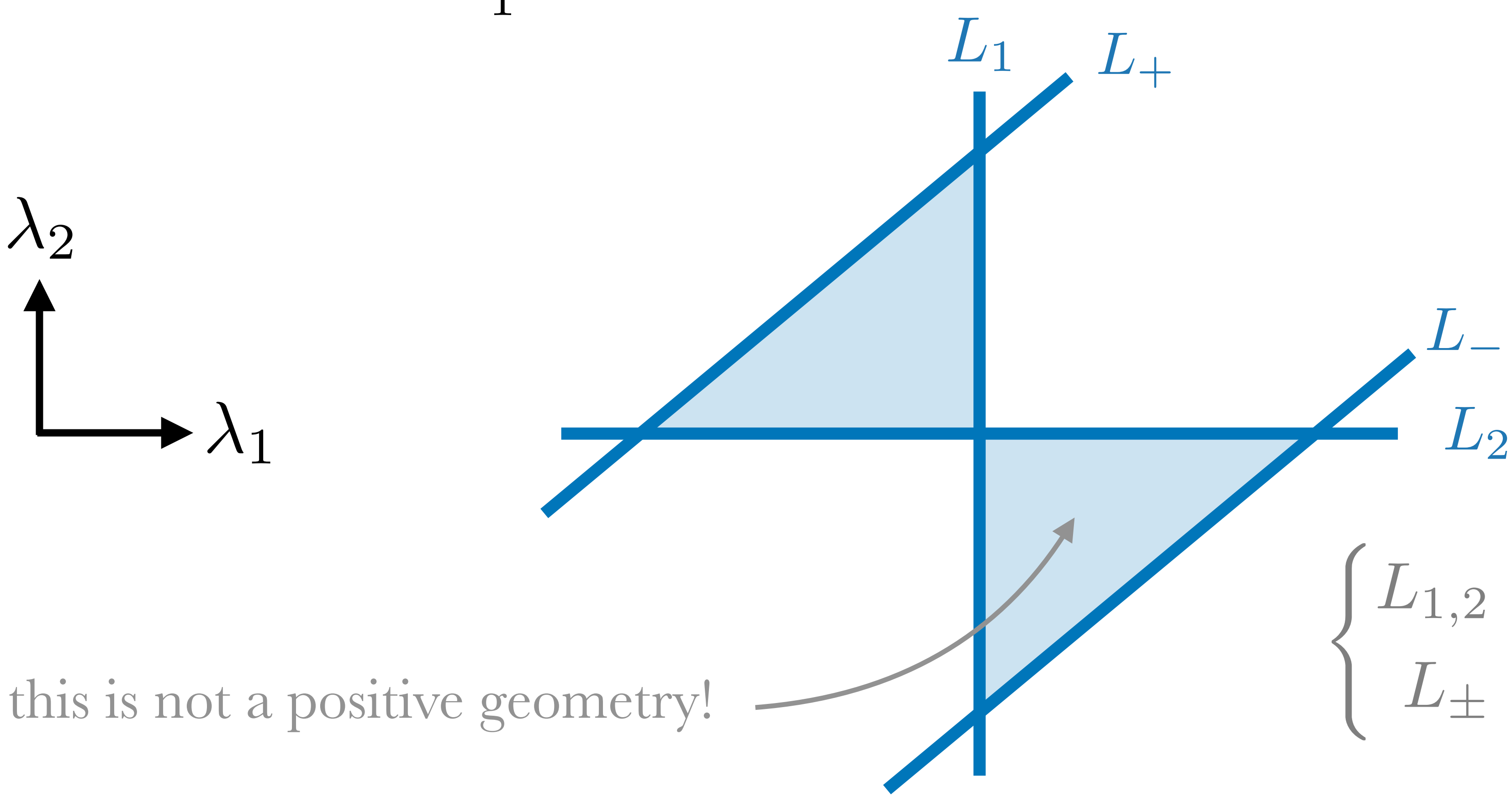
[Talk by Nathan Belrhali]

This generalises to arbitrary tree graphs

… but the rational integrands *are not* canonical differential forms of positive geometries in the λ-space…



$$= \int_1^\infty P_{i\mu}(\lambda_1) P_{i\mu}(\lambda_2) \mathrm{d}\lambda_1 \mathrm{d}\lambda_2 \left(\frac{1}{L_1 L_+} + \frac{1}{L_2 L_-} - \frac{1}{L_1 L_2} \right)$$

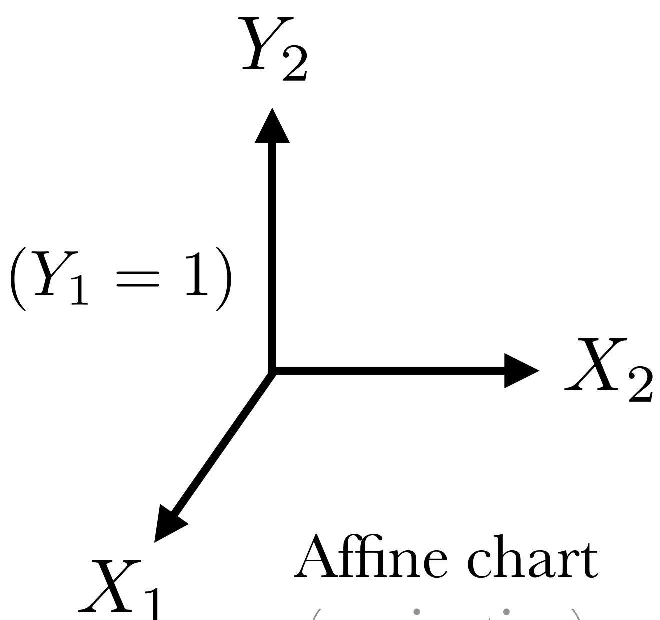


$$\begin{cases} L_{1,2} = X_{1,2} + \lambda_{1,2} Y \\ L_\pm = X_1 + X_2 \pm (\lambda_1 - \lambda_2) Y \end{cases}$$

$$\Omega \stackrel{!}{=} \frac{\mathrm{d}\lambda_1 \wedge \mathrm{d}\lambda_2}{L_1 L_2 L_+} + \frac{\mathrm{d}\lambda_1 \wedge \mathrm{d}\lambda_2}{L_1 L_2 L_+} = \frac{(L_+ + L_-)\, \mathrm{d}\lambda_1 \wedge \mathrm{d}\lambda_2}{L_1 L_2 L_+ L_-}$$

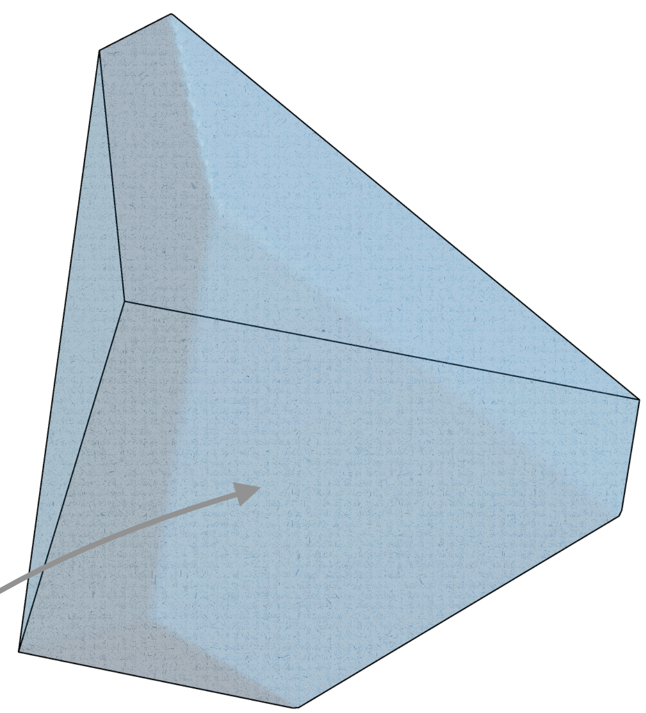
… however we obtain a positive geometry in the

projective kinematic space



Affine chart

(projective)



$(Y_{1,2} = \lambda_{1,2} Y)$

$$\begin{cases} L_{1,2} = X_{1,2} + Y_{1,2} \\ L_{\pm} = X_1 + X_2 \pm (Y_1 - Y_2) \end{cases}$$

$\{L_i \geq 0\}$

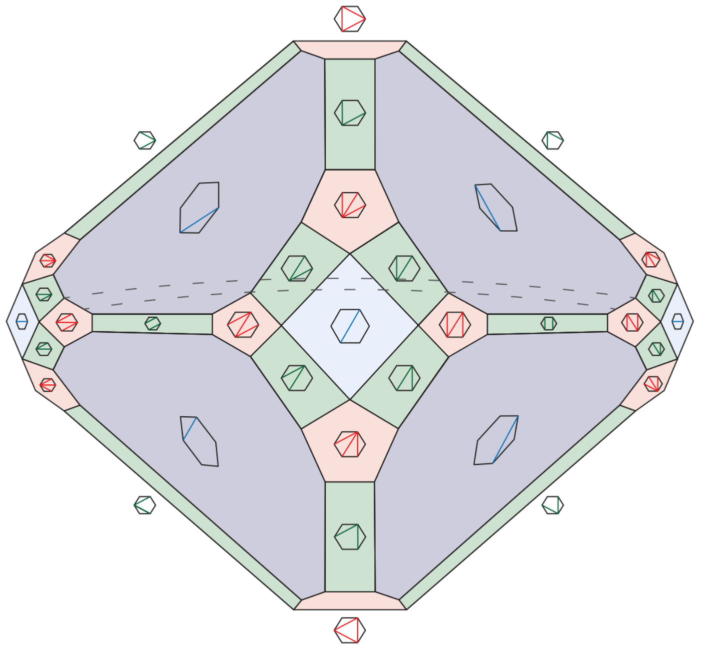
we need a slight deformation of the definition to interpret the integrand as the canonical differential form associated to this positive geometry

Work in progress!

Many open questions…

- Is there a simple & systematic canonical form (deformation) for massive dS correlators?
- Can we read off the complexity (transcendental weight) from a suitable integral representation of massive dS correlators?
- How to evaluate the integrals? (Landau analysis, canonical differential equations, …)
- Can we go beyond individual graphs? (towards dS massive cosmohedron)
- …

$$\Omega \to \tilde{\Omega}$$



Work in progress!


Thank you