Curvature profiles from stochastic inflation

Inflation 2025

IAP, PARIS, 4 NOVEMBER 2025

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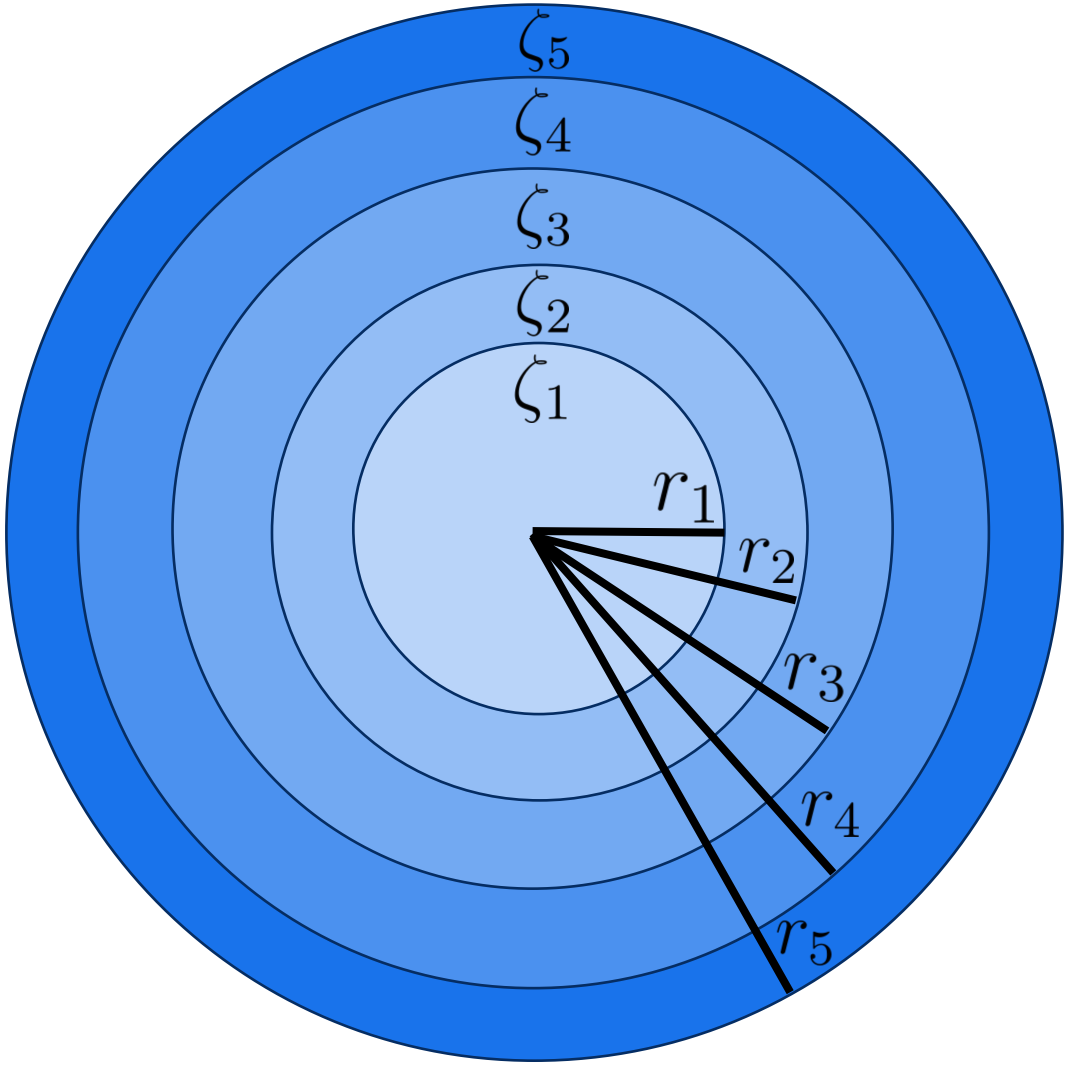
Perturbation profiles



Radial profile
around a random point?

High peaks: primordial black holes

- Dark matter
- GW source

From compaction function:

$$\mathcal{C}(r) = 2\frac{\delta M}{R} = \frac{2}{3}\left(1 - [1 + r\zeta'(r)]^2\right)$$

Non-linear perturbations

Stochastic inflation

Divide field in two:

$$\phi_{\text{tot}} = \phi + \delta\phi$$

Stochastic inflation

Divide field in two:

$$\phi_{\rm tot} = \phi + \delta\phi$$

$$\int_{k<k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

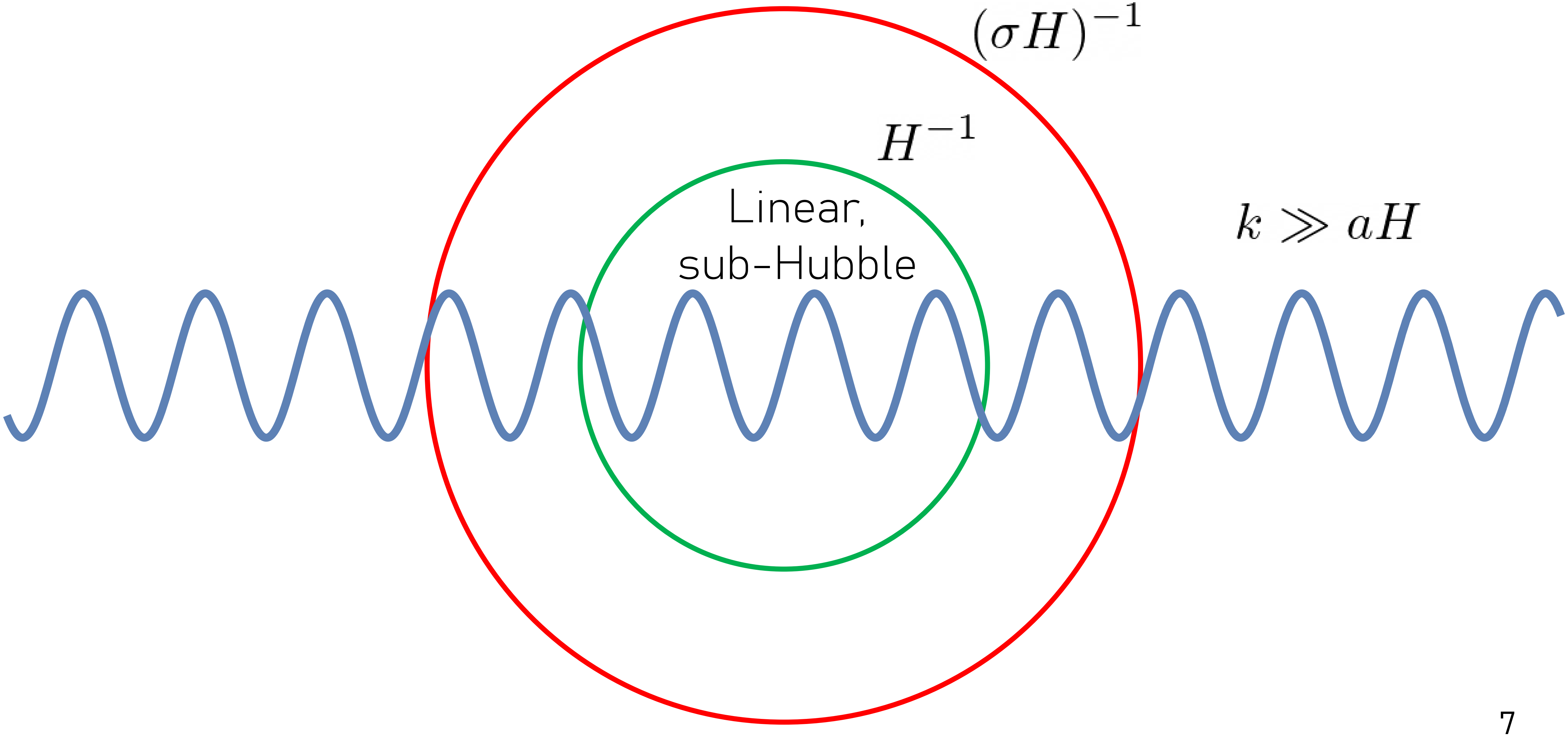
Separate universe

$$\int_{k>k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

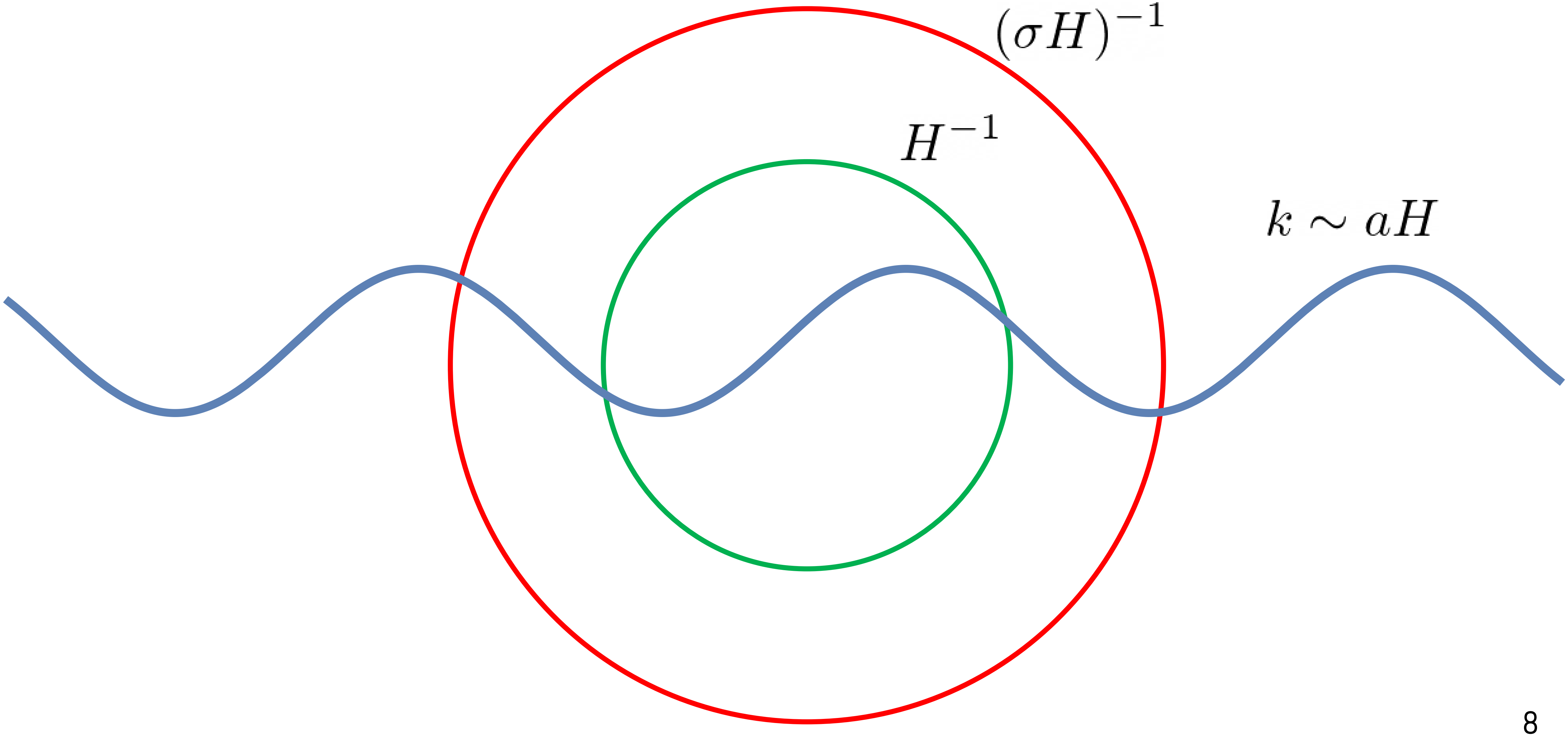
Linear perturbations

Coarse-graining scale: $k = k_\sigma \equiv \sigma a H$

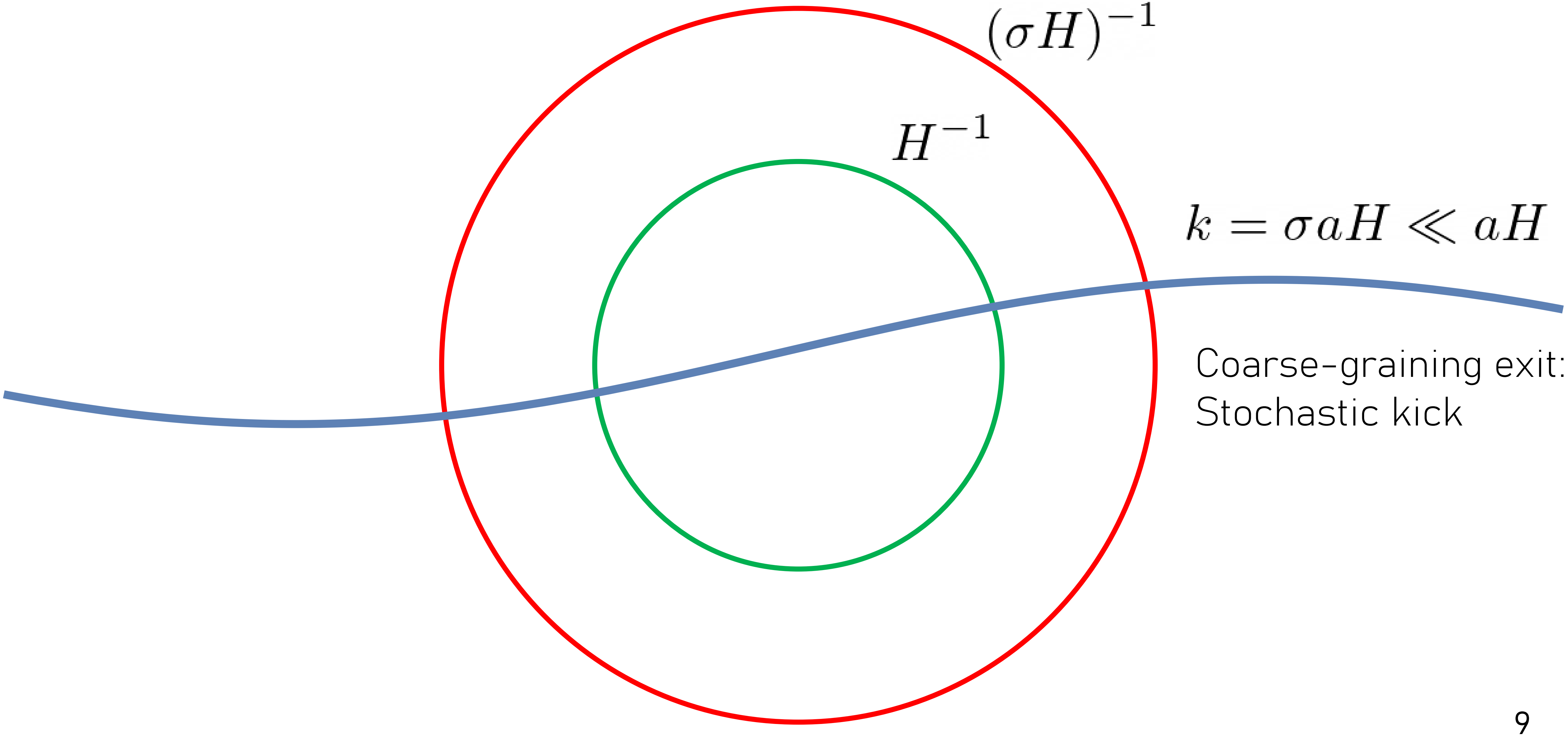
Drifting modes: stochasticity



Drifting modes: stochasticity



Drifting modes: stochasticity



Stochastic inflation

$$\dot{\bar{\phi}} = \bar{\pi} + \xi_\phi \ , \quad \dot{\bar{\pi}} = -\left(3 - \frac{1}{2}\bar{\pi}^2\right)\left(\bar{\pi} + \frac{V_{,\bar{\phi}}(\bar{\phi})}{V(\bar{\phi})}\right) + \xi_\pi$$

e-folds as time variable

$$\langle \xi_X(N)\xi_Y(\tilde{N})\rangle = \mathcal{P}_{XY}(N, k_\sigma)\delta(N - \tilde{N})$$

We use the constant-roll approximation

Curvature profiles

Initial ϕ

stochastic
evolution

Switch noise off

classical
evolution

Final ϕ

$N = 0$

Final coarse-graining scale

$k = \sigma a H$

final $N = N_{<k}$

Delta N formalism

$$
\begin{aligned}
\Delta N_{<k} &= N_{<k} - \langle N_{<k} \rangle \\
&= \zeta_{<k}(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \zeta_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \theta(k-p)
\end{aligned}
$$

Delta N formalism

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$$= \zeta_{<k}(\mathbf{x}) \equiv \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} \zeta_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \theta(k-p)$$

Vary coarse-graining scale for same patch

Delta N formalism

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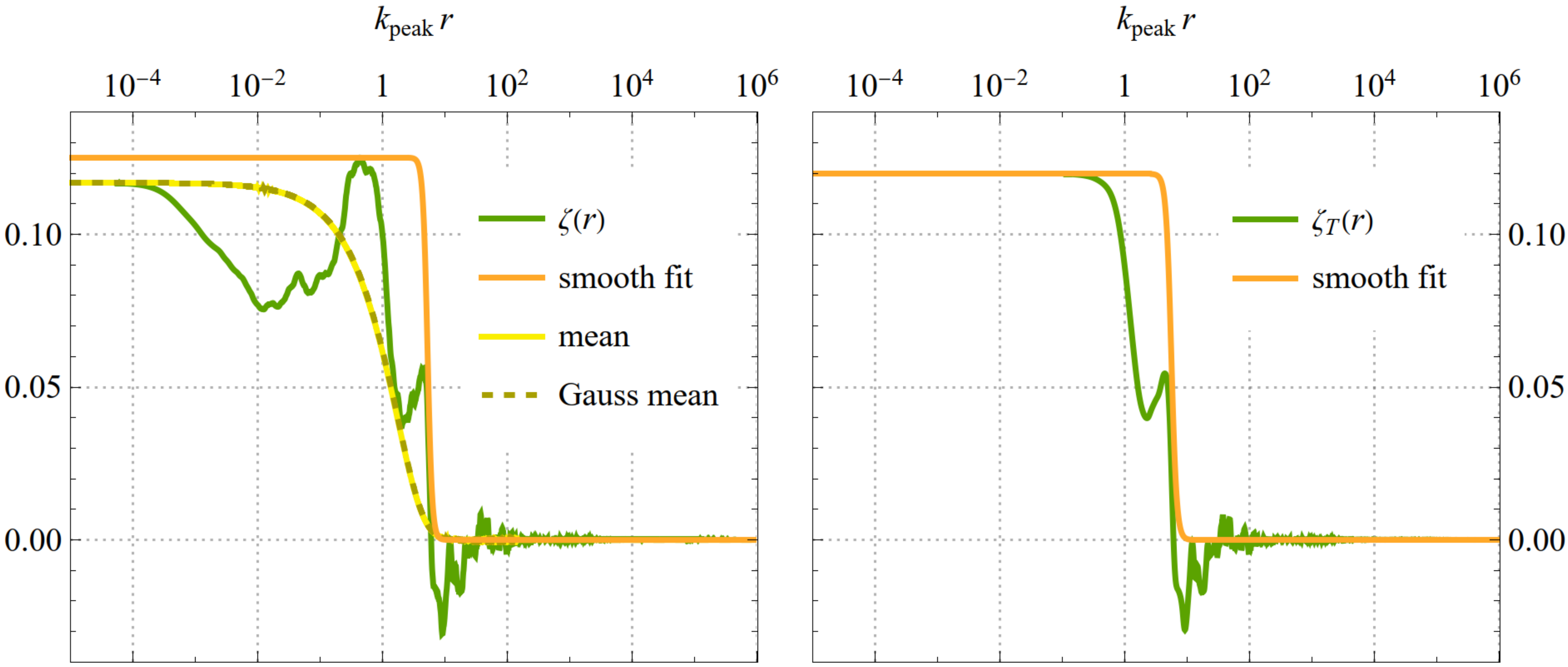
Vary coarse-graining scale for same patch

Obtain: $\zeta_k = \sqrt{\frac{\pi}{2}} \frac{1}{k^3} \frac{\mathrm{d}\zeta_{<k}}{\mathrm{d}\ln k} \rightarrow \zeta(r) = \int_0^\infty \frac{\mathrm{d}k}{k} \frac{\mathrm{d}\zeta_{<k}}{\mathrm{d}\ln k} \frac{\sin(kr)}{kr}$

Example PBH model

Raatikainen:2025gpd



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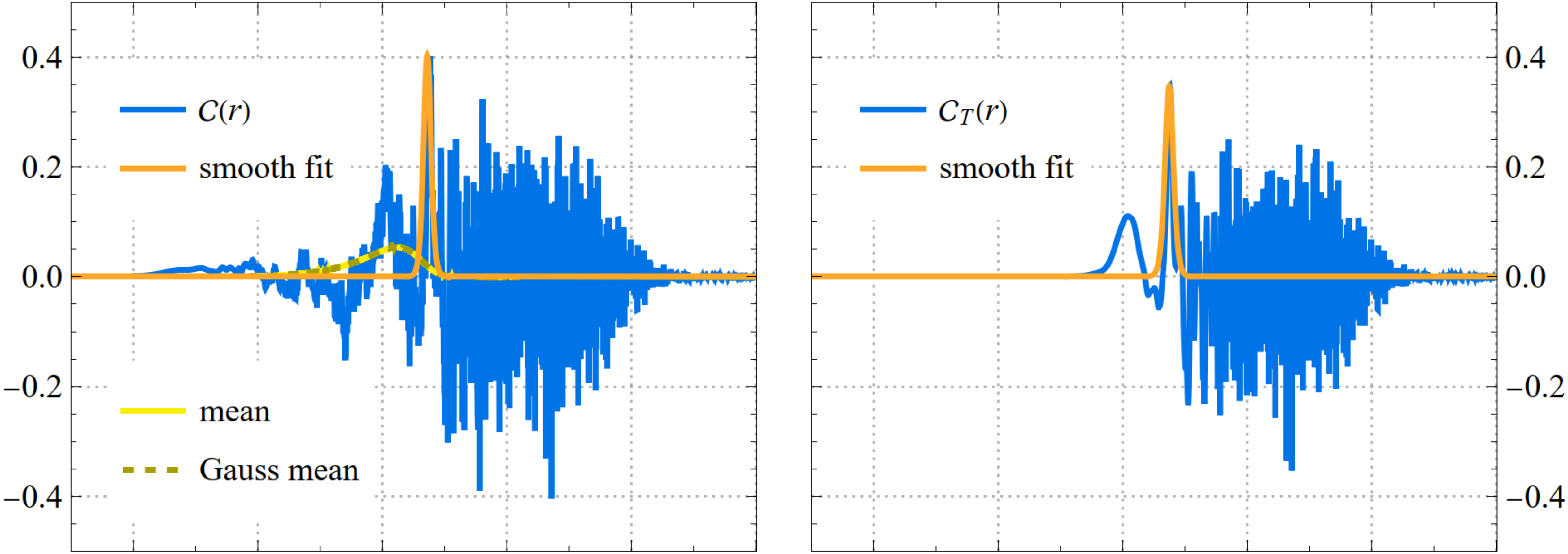

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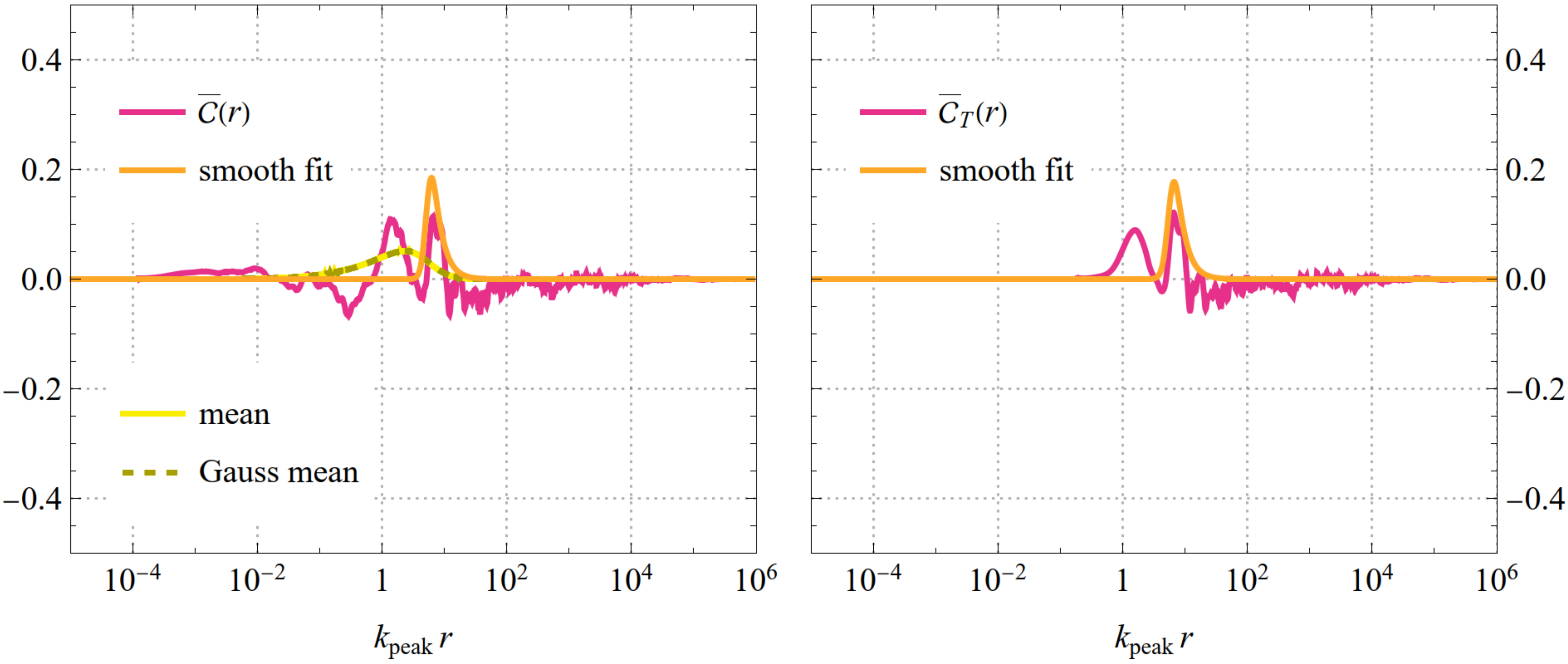
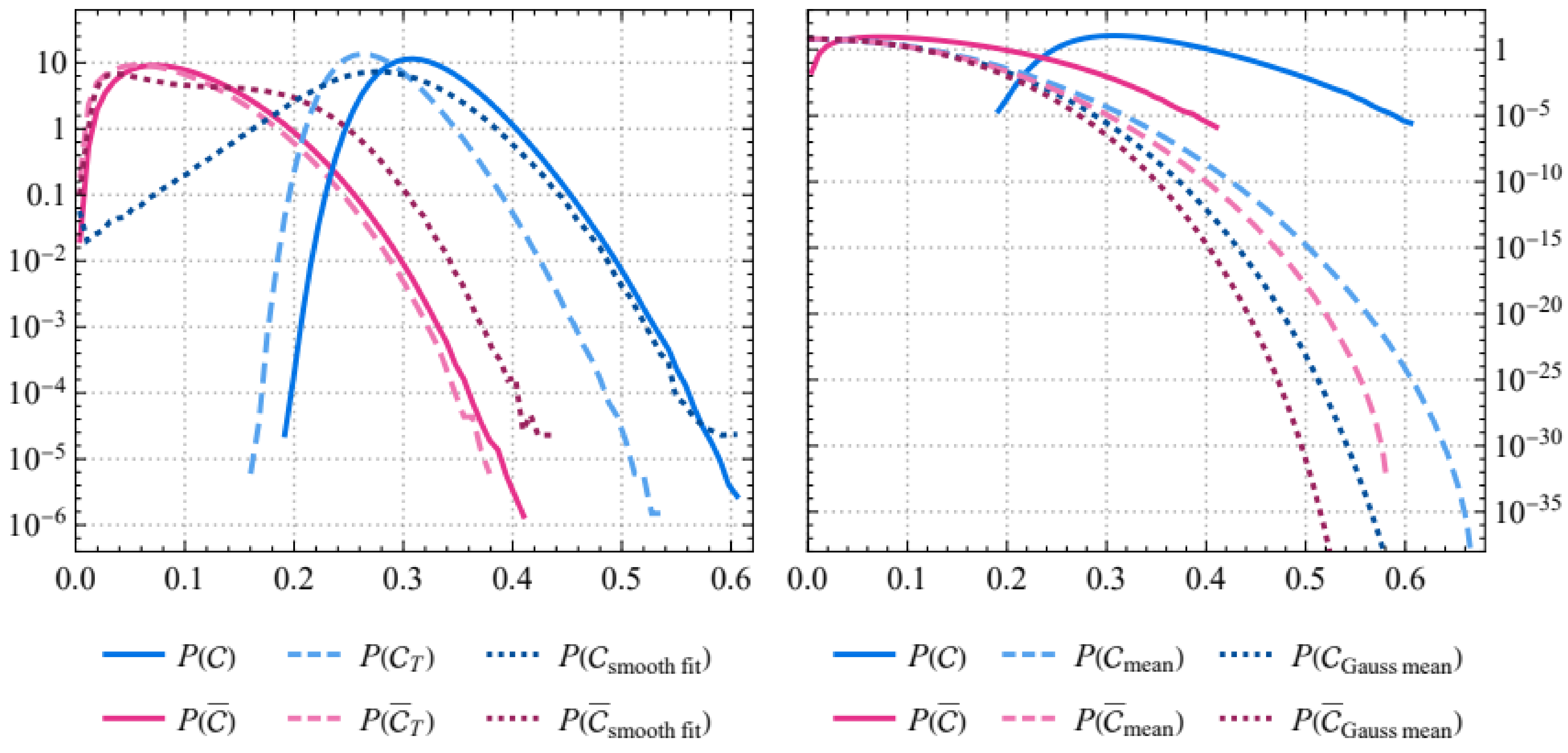
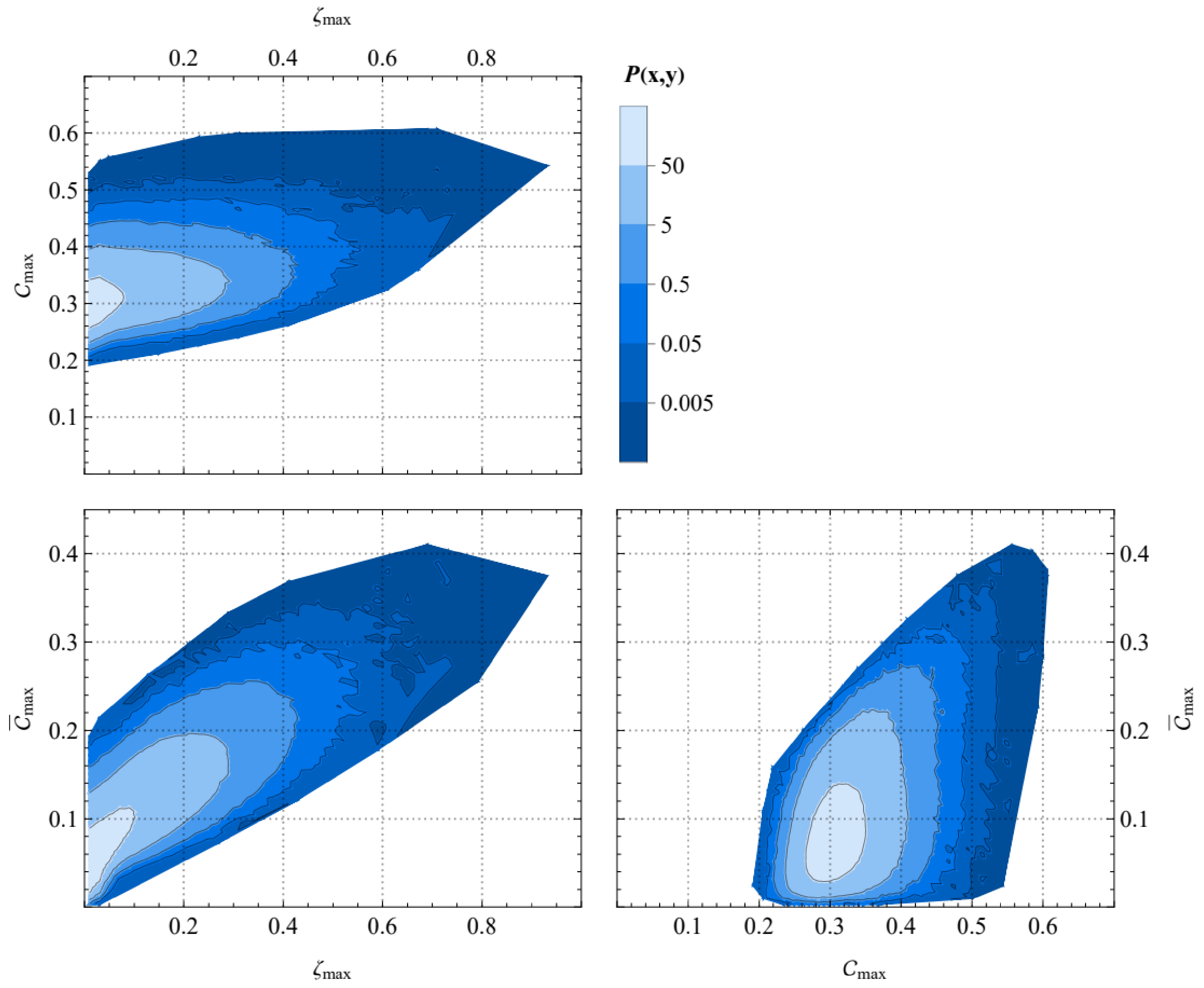
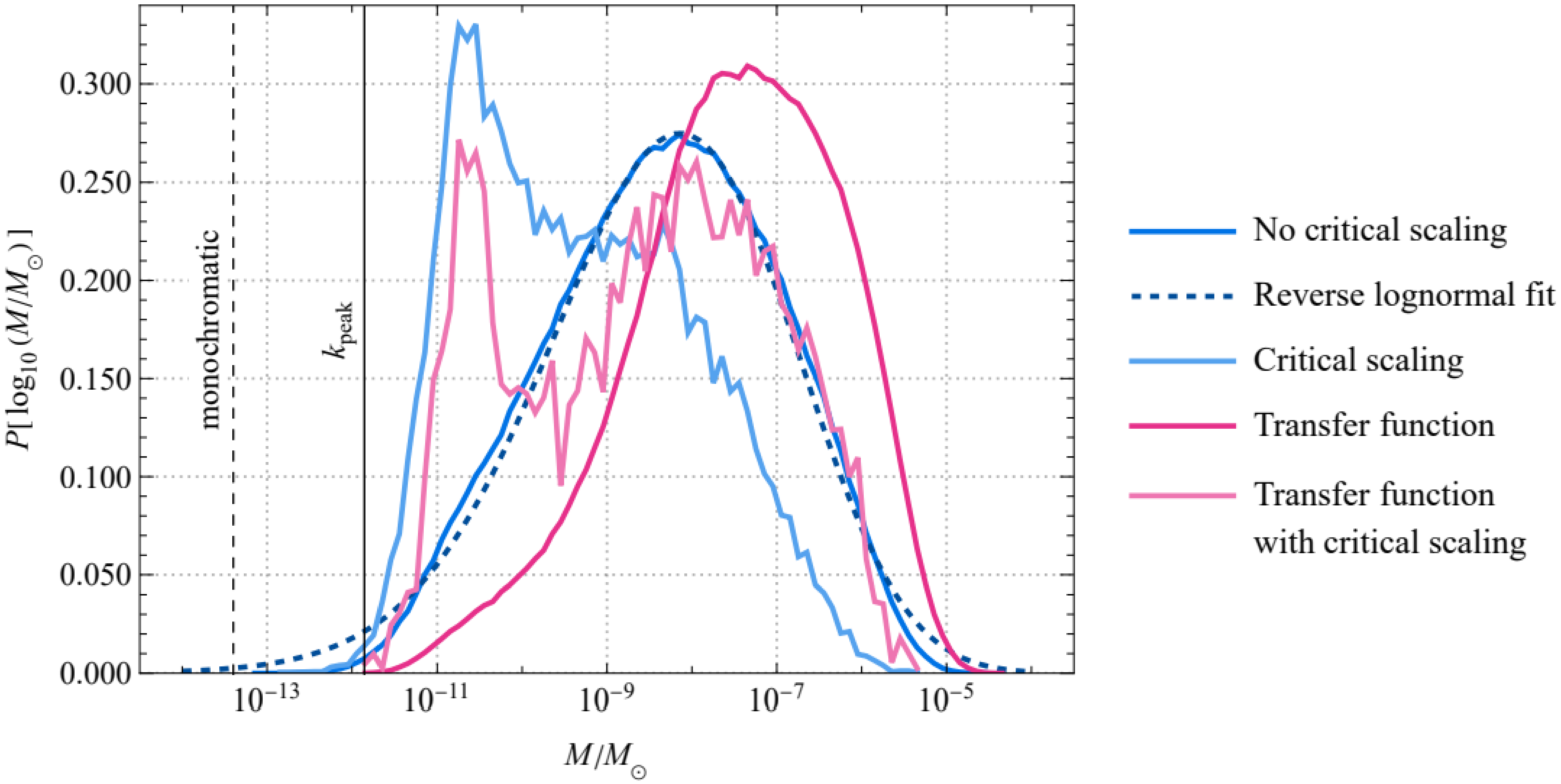




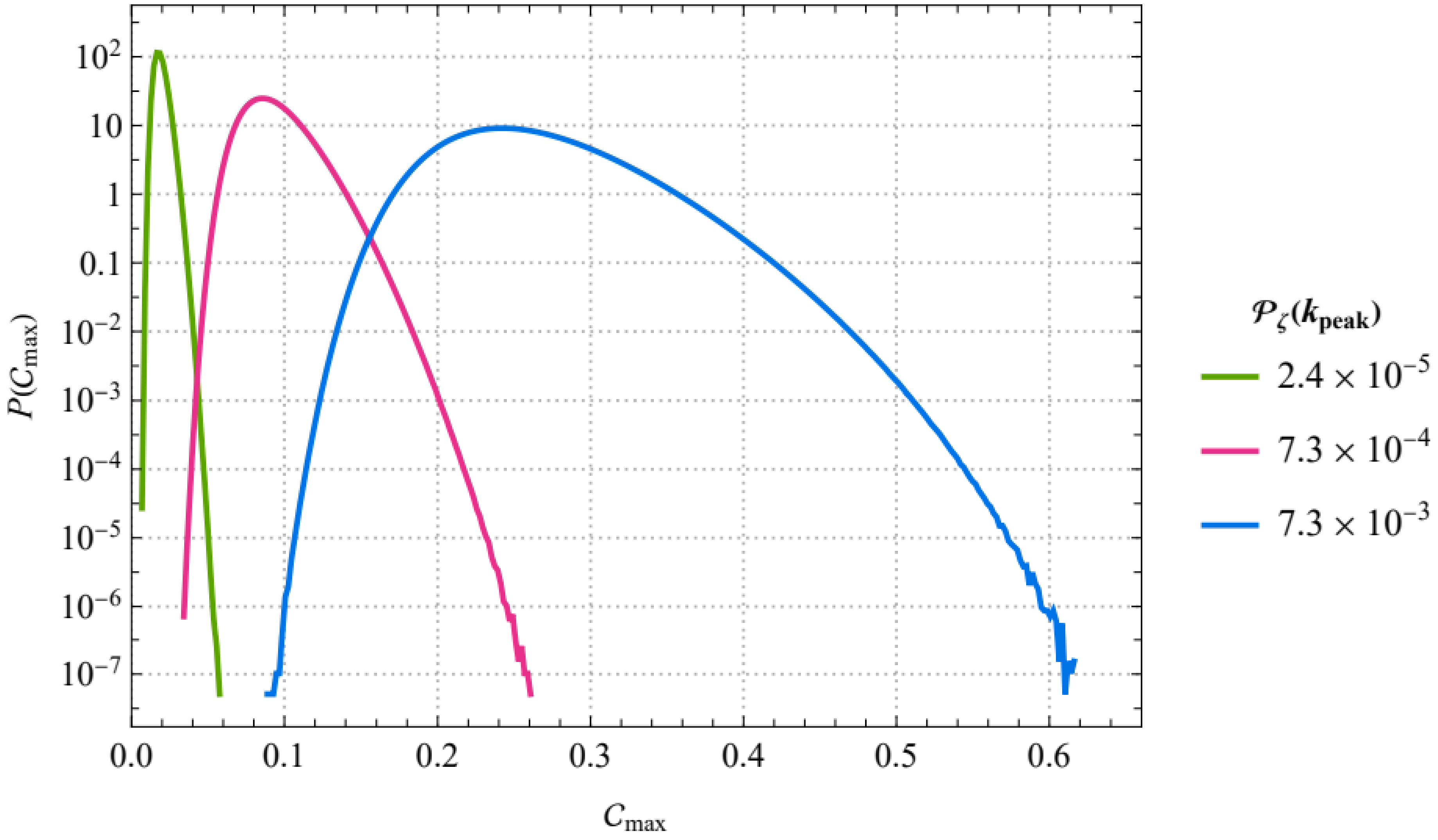
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.0 0.1 0.2 0.3 0.4 0.5 0.6
1
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
10^{-35}
0.0 0.1 0.2 0.3 0.4 0.5 0.6
$P(C)$
$P(C_T)$
$P(C_{\text{smooth fit}})$
$P(\overline{C})$
$P(\overline{C}_T)$
$P(\overline{C}_{\text{smooth fit}})$
$P(C)$
$P(C_{\text{mean}})$
$P(C_{\text{Gauss mean}})$
$P(\overline{C})$
$P(\overline{C}_{\text{mean}})$
$P(\overline{C}_{\text{Gauss mean}})$



ζ_{max}
0.2
0.4
0.6
0.8
C_{max}
0.1
0.2
0.3
0.4
0.5
0.6
P(x,y)
50
5
0.5
0.05
0.005
$\overline{C}_{max}$
0.1
0.2
0.3
0.4
ζ_{max}
C_{max}
$\overline{C}_{max}$



0.300
0.250
0.200
0.150
0.100
0.050
0.000
$P[\log_{10}(M/M_\odot)]$
monochromatic
k_{peak}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
$M/M_\odot$
No critical scaling
Reverse lognormal fit
Critical scaling
Transfer function
Transfer function with critical scaling


10^2
10
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$P(C_{\max})$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
$C_{\max}$
$\mathcal{P}_\zeta(k_{\text{peak}})$
2.4×10^{-5}
7.3×10^{-4}
7.3×10^{-3}

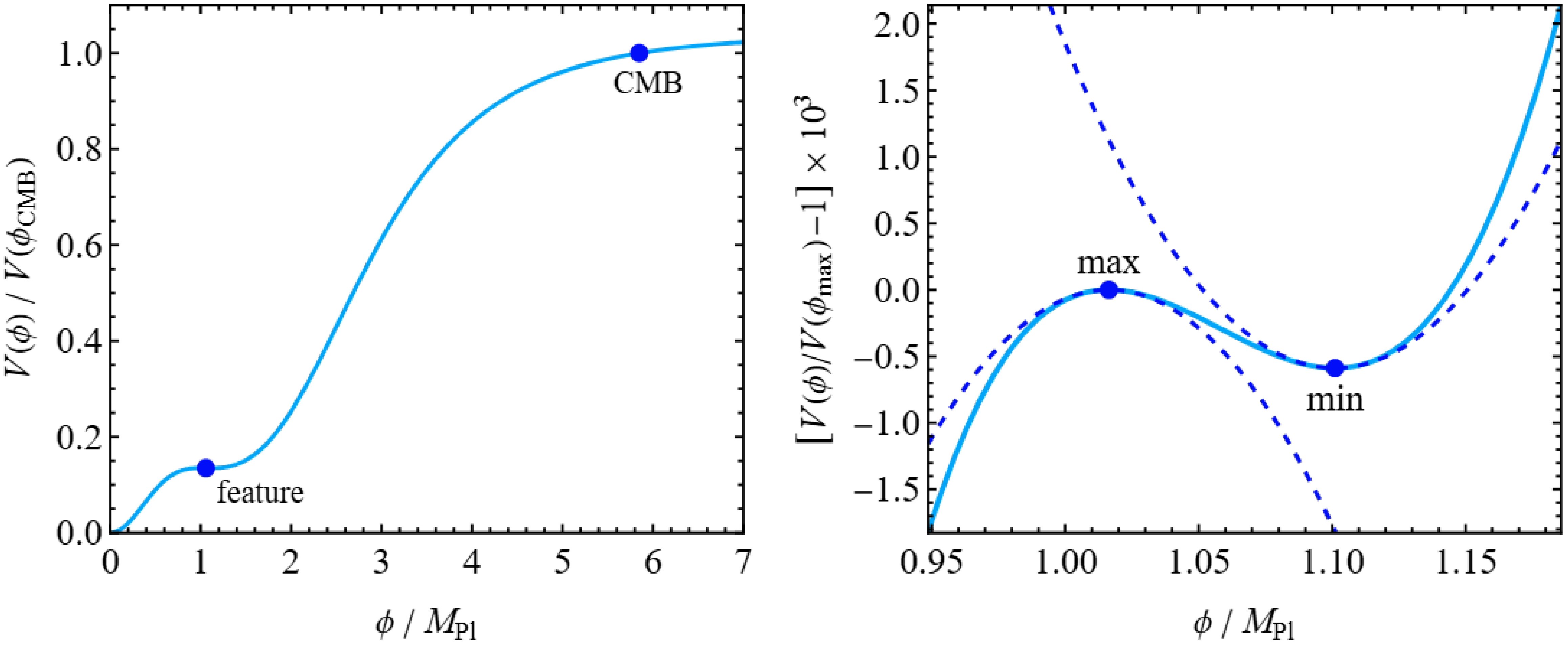
SUMMARY

Stochastic inflation provides
radial curvature profiles

Curvature profiles
give compaction function

Profiles very spiky:
effect on PBH statistics?

Inflection point models



Inflection point models