Oscillations and parity violation in GW background: The role of extra tensor modes

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[arXiv: 2302.14080, 2307.13109, 2508.08481]

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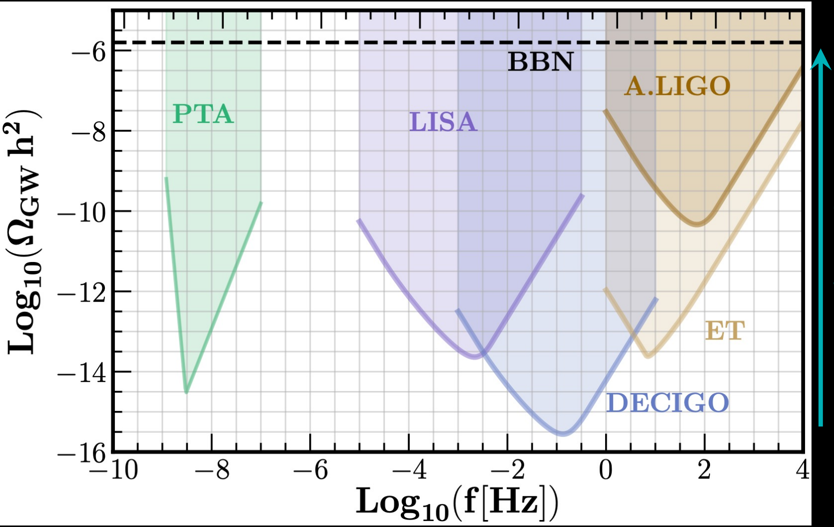
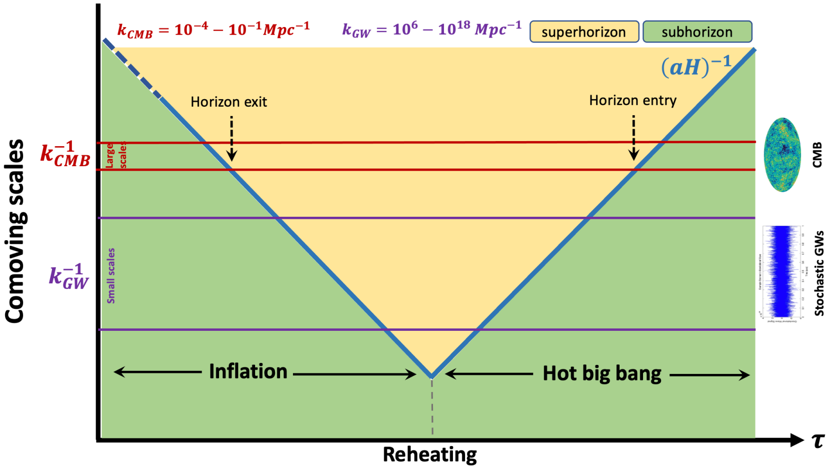
Gravitational wave detectors



The future GW detectors can detect $\Omega_{GW} > 10^{-16}$ in the frequency bounds $10^{-9} \leq f[Hz] \leq 10^{4}$

Credit:G. Domenech, arXiv:2109.01398

GW observations probe the late stage of inflation beyond the CMB!



CMB scales: $10^{-4} \lesssim k_{CMB} \lesssim 10^{-1}\mathrm{Mpc}^{-1} / 10^{-19} \lesssim f \lesssim 10^{-16}\mathrm{Hz}$

GWs scales: $10^{6} \lesssim k_{GW} \lesssim 10^{18}\,\mathrm{Mpc}^{-1} / 10^{-8} \lesssim f \lesssim 10^{3}\ \mathrm{Hz}$

“Secondary scalar-induced” GWs

Scalar perturbations source gravitational waves at the *non-linear* level:

$$\boldsymbol{\Omega}_{\mathrm{GW}}^{\mathrm{induced}} = \mathcal{O}(\mathbf{10^{-5}})\mathcal{P}_{\mathcal{R}}^{\mathbf{2}}$$

At CMB scales $k \sim k_{CMB}$, $\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$. If power spectrum be scale-dependent such that $\mathcal{P}_{\mathcal{R}} \gg \mathcal{P}_{\mathcal{R}}$ where $\mathcal{P}_{\mathcal{R}}$ is the power spectrum at GW scale $k \sim k_{GW}$, one may achieve

$$\Omega_{\mathrm{GW}}^{\mathrm{induced}} \gtrsim \mathcal{O}(10^{-16})$$

which is large enough to be detected by the GW detectors. Thus, $\mathcal{P}_{\mathcal{R}} \gtrsim 10^{-5}$ is needed which means the initial power spectrum $\mathcal{P}_{\mathcal{R}} \sim 10^{-9}$ should be enhanced at least by “four orders of magnitude” at small scales.

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Why $\Omega_{\mathrm{GW}}^{\mathrm{induced}}$ is proportional to $\mathcal{P}_{\mathcal{R}}^2$?

Because $\mathcal{R}$ gives a source to h which starts at the quadratic order

$$h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m \mathcal{R} \partial_n \mathcal{R}]$$

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Why $\Omega_{\rm GW}^{\rm induced}$ is proportional to $\mathcal{P}_{\mathcal{R}}^2$?

Because $\mathcal{R}$ gives a source to h which starts at the quadratic order

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \partial^2 h_{ij} \propto \mathcal{T}_{ij}^{mn}[\partial_m\mathcal{R}\partial_n\mathcal{R}]$$

It is not possible to have a “linear” source by $\mathcal{R}$

SVT decomposition theorem

$$h''_{ij} + 2\frac{a'}{a}h'_{ij} - \partial^2 h_{ij} = S^{\mathrm{TT}}_{ij}\,, \qquad S^{\mathrm{TT}}_{ij} = \mathcal{O}(\epsilon_t) + \mathcal{O}(\epsilon_s^2) + \mathcal{O}(\epsilon_v^2) + \mathcal{O}(\epsilon_t^2) + \cdots\,,$$

Curvature perturbation $\mathcal{R}$: Perturbation of the dominant (effective) field

$$S^{\mathrm{TT}}_{ij} \propto \mathcal{T}^{mn}_{ij}[\partial_m \mathcal{R} \partial_n \mathcal{R}]; \qquad \Omega^{\mathrm{induced}}_{\mathrm{GW},\mathcal{R}} \propto \mathcal{P}^2_{\mathcal{R}}$$

Spectator fields (s, v, t): Fields with negligible energy density

$$S^{\mathrm{TT}}_{ij} \propto \mathcal{T}^{mn}_{ij}[\partial_m s \partial_n s]; \qquad \Omega^{\mathrm{induced}}_{\mathrm{GW},s} \propto \mathcal{P}^2_{s}$$

$$S^{\mathrm{TT}}_{ij} \propto \mathcal{T}^{mn}_{ij}[v_m v_n]; \qquad \Omega^{\mathrm{induced}}_{\mathrm{GW},v} \propto \mathcal{P}^2_{v}$$

$$S^{\mathrm{TT}}_{ij} \propto \mathcal{T}^{mn}_{ij}[t_{mn}]; \qquad \Omega^{\mathrm{induced}}_{\mathrm{GW},t} \propto \mathcal{P}_{t}$$

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Spectator fields (s, v, t): Fields with negligible energy density

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$$S^{\rm TT}_{ij} \propto \mathcal{T}^{mn}_{ij}[v_m v_n]; \qquad \Omega^{\rm induced}_{{\rm GW},v} \propto \mathcal{P}^2_v$$

$$S^{\rm TT}_{ij} \propto \mathcal{T}^{mn}_{ij}[t_{mn}]; \qquad \Omega^{\rm induced}_{{\rm GW},t} \propto \mathcal{P}_t$$

Typical situation is $\mathcal{P}_{s,v,t} \ll \mathcal{P}_{\mathcal{R}}$. Even if $\mathcal{P}_t \ll \mathcal{P}_{\mathcal{R}}$, $\Omega^{\rm induced}_{{\rm GW},t}$ and $\Omega^{\rm induced}_{{\rm GW},\mathcal{R}}$ may be at the same order since tensor modes can appear at the "linear" level.

"Primary tensor-induced" GWs [Gorji & Sasaki (2023)]

$$\Omega_{\mathrm{GW},\mathcal{R}}^{\mathrm{induced}} = \mathcal{O}(10^{-5})\mathcal{P}_{\mathcal{R}}^2$$

$$\Omega_{\mathrm{GW},t}^{\mathrm{induced}} = \mathcal{O}(10^{-5})\mathcal{P}_t$$

"Primary tensor-induced" GWs [Gorji & Sasaki (2023)]

$$\Omega^{\text{induced}}_{\text{GW},\mathcal{R}} = \mathcal{O}(10^{-5})\mathcal{P}^2_{\mathcal{R}} \qquad \Omega^{\text{induced}}_{\text{GW},t} = \mathcal{O}(10^{-5})\mathcal{P}_t$$

Models that can provide extra tensor modes:

- Yang-Mills theories with homogeneous and isotropic vev
  Maleknejad & Sheikh-Jabbari (2013)
  Aoki, Fujita, Kawaguchi, Yanagihara (2025)
- Bi-gravity theories
  de Rham, Gabadadze, Tolley (2011)
  Hassan, Rosen (2012)
- Modified gravity theories with dynamical torsion/nonmetricity
  Aoki, Mukohyama (2020)
  Aoki, Bahamonde, Gigante Valcarcel, Gorji (2024)
- A spin-two (or higher spin) spectator field
  Bordin, Creminelli, Khmelnitsky, Senatore (2018)
- ...

Tensor-induced origin for the PTA signal: No PBH production

[Gorji, Sasaki, Suyama (2023)]

The PTA signal reported by NANOGrav, EPTA/InPTA, PPTA, and CPTA:

$$10^{-9} \lesssim \Omega_{\rm GW}^{\rm PTA} \lesssim 10^{-7} \qquad (10^{-9} \lesssim f^{\rm PTA}/{\rm Hz} \lesssim 10^{-7})$$

- **Secondary scalar-induced GWs** (see, i.e., D. G. Figueroa, *et al*, arXiv:2307.02399 and references therein):

$$\Omega_{{\rm GW},\mathcal{R}}^{\rm induced} = \mathcal{O}(10^{-5})\mathcal{P}_{\mathcal{R}}^2 \Rightarrow 10^{-2} \lesssim \mathcal{P}_{\mathcal{R}}^{\rm PTA} \lesssim 10^{-1}$$

  Large values of $\mathcal{P}_{\mathcal{R}}^{\rm PTA}$ may lead to the overproduction of PBH!

- **Primary tensor-induced GWs**:

$$\Omega_{{\rm GW},t}^{\rm induced} = \mathcal{O}(10^{-5})\mathcal{P}_t \Rightarrow 10^{-4} \lesssim \mathcal{P}_t^{\rm PTA} \lesssim 10^{-2}$$

t_{ij} is a spectator field and since it's energy density is subdominant, it will not lead to any PBH formation.

The model

The effective field theory action for extra traceless-transverse tensor modes t_{ij}, coupled minimally to gravity, in FLRW background is [Bordin, Creminelli, Khmelnitsky, Senatore (2018)]

$$\begin{aligned} S = & \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}\tau\, a^2 \frac{M_{\rm Pl}^2}{4}\left[\left(h'_{ij}\right)^2 - \left(\partial_i h_{jk}\right)^2\right] \\ & + \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}\tau\, a^2 f^2 \left[\left(t'_{ij}\right)^2 - c_t^2\left(\partial_i t_{jk}\right)^2 + 2\theta\frac{a'}{a}\varepsilon^{ijk} t_{im}\partial_j t_k{}^m\right] \\ & + \alpha \int \mathrm{d}^3x\,\mathrm{d}\tau\, aa' \frac{M_{\rm Pl}}{2}\left[h'_{ij} t^{ij}\right] \end{aligned}$$

where $c_t(\tau)$ and $f(\tau)$ are functions of time while α and θ are constants. EoMs are given by

$$\frac{M_{\rm Pl}}{2}\left(h_{\mathbf{k}}^{\lambda\,\prime\prime} + 2\frac{a'}{a}h_{\mathbf{k}}^{\lambda\,\prime} + k^2 h_{\mathbf{k}}^{\lambda}\right) = -\alpha\frac{a'}{a}\left[t_{\mathbf{k}}^{\lambda\,\prime} + \frac{(aa')'}{aa'}t_{\mathbf{k}}^{\lambda}\right]$$

$$t_{\mathbf{k}}^{\lambda\,\prime\prime} + 2\frac{(af)'}{af}t_{\mathbf{k}}^{\lambda\,\prime} + \left(c_t^2k^2 + 2\lambda\theta k\frac{a'}{a}\right)t_{\mathbf{k}}^{\lambda} = \frac{\alpha}{f^2}\frac{a'}{a}\left(\frac{M_{\rm Pl}}{2}h_{\mathbf{k}}^{\lambda\,\prime}\right) \qquad \lambda = \pm 1$$

Oscillatory features in frequency

The system can be diagonalized by substituting

$$h_{\mathbf{k}} = \frac{1}{a}\left(\mathrm{c}X_{\mathbf{k}} + \mathrm{d}\,Y_{\mathbf{k}}\right)\,, \qquad t_{\mathbf{k}} = \frac{1}{af}\left(-\mathrm{d}X_{\mathbf{k}} + \mathrm{c}\,Y_{\mathbf{k}}\right)\,,$$

such that

$$X''_{\mathbf{k}} + \omega_X^2 X_{\mathbf{k}} = 0\,, \qquad Y''_{\mathbf{k}} + \omega_Y^2 Y_{\mathbf{k}} = 0\,,$$

For the modes deep inside the horizon $k \gg \mathcal{H}$, the positive frequency WKB solutions are

$$X_{\mathbf{k}} \propto e^{-i\int^{\tau}\omega_X(\tilde{\tau})d\tilde{\tau}}\,, \qquad Y_{\mathbf{k}} \propto e^{-i\int^{\tau}\omega_Y(\tilde{\tau})d\tilde{\tau}}\,,$$

Even if there is no oscillation in the power spectra of $X_{\mathbf{k}}$ and $Y_{\mathbf{k}}$

$$\mathcal{P}_X \propto |X_{\mathbf{k}}|^2\,, \qquad \mathcal{P}_Y \propto |Y_{\mathbf{k}}|^2\,,$$

there will be **oscillatory features** in the power spectra of $h_{\mathbf{k}}$ and $t_{\mathbf{k}}$ for $\alpha \neq 0$

$$\mathcal{P}_h \propto |\mathrm{c}X_{\mathbf{k}} + \mathrm{d}\,Y_{\mathbf{k}}|^2\,, \qquad \mathcal{P}_t \propto |-\mathrm{d}X_{\mathbf{k}} + \mathrm{c}\,Y_{\mathbf{k}}|^2\,,$$

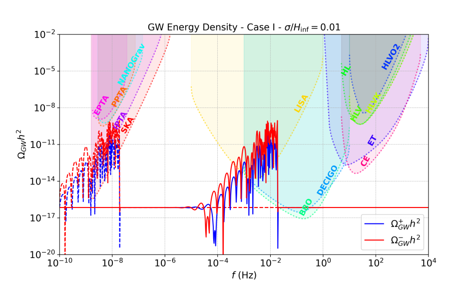
This is analogous to the **neutrino oscillation** when the oscillations are absent in the mass basis while the neutrino is detected in the flavor basis. [Caldwell, Devulder, Maksimova (2016)]

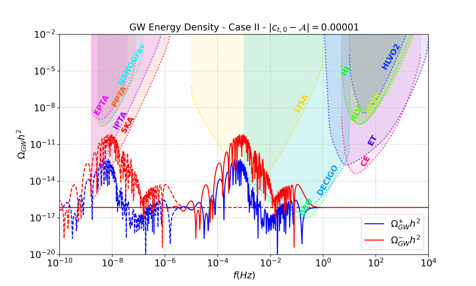
There is always oscillatory features in frequency of GW since the detectors observe $h_{\mathbf{k}}$ and not diagonalized fields $X_{\mathbf{k}}$ and $Y_{\mathbf{k}}$.

Oscillatory spectrum

[Garriga, Gorji, Sasaki, Hajkarim (2025)]

We assume $\alpha \neq 0$ during inflation while $\alpha = 0$ during radiation.





$\alpha = 5$, $\theta = 0.1$, $c_{t,0} = 0.9$. $\mathcal{A} = 0.99$ and $\sigma = H_{\rm inf}^{-1}$ are considered for Case I and Case II.

The existence of the **oscillatory features is model-independent** while the shape and the amplitude are model-dependent.

Summary

- GW observations can give us valuable information about the late stage of inflation which is not accessible by the CMB.
- The contribution of the spectator fields to stochastic GWs is expected to be very small compared with the one from curvature perturbation or the field which dominates the background.
- If a spectator field provides extra tensor perturbation (on top of the metric tensor perturbation), its contribution to the stochastic GWs can be comparable to the one from curvature perturbation.
- The scenario with extra tensor perturbation (primary tensor-induced GWs) is in principle distinguishable from the curvature perturbation case (secondary scalar-induced GWs) (e.g. the PTA signal): PBH will hardly form in the first case.
- Due to the linear mixing between the metric tensor perturbations and extra tensor perturbations, there will be always oscillatory features in the frequency of the GWs.
- Parity violation in the extra tensor sector propagates into the gravitational sector, giving rise to a significant amount of chiral GWs.

References

- MAG, Misao Sasaki
  Primordial-tensor-induced stochastic gravitational waves
  PLB (2023) [arXiv:2302.14080 [gr-qc]]
- MAG, Misao Sasaki, Teruaki Suyama
  Extra-tensor-induced origin for the PTA signal: No primordial black hole production
  PLB (2023) [arXiv:2307.13109 [astro-ph.CO]]
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References

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Thank You!

Backup slides

Spectral density of GWs:

$$\Omega_{\rm GW} = \frac{1}{12}\Omega_{0,r}\mathcal{P}_h = \mathcal{O}(10^{-5})\mathcal{P}_h$$

- At CMB scales $k \sim k_{CMB}$, $\mathcal{P}_h \sim r\,\mathcal{P}_{\mathcal{R}} \lesssim 10^{-11}$. Since the power spectra are almost scale-invariant and red-tilted, $\mathcal{P}_h < 10^{-11}$ at $k \sim k_{GW}$ giving

$$\Omega_{\rm GW}^{\rm pri} < 10^{-16}$$

- Scalar perturbations contribute since $\mathcal{P}_h^{\rm induced} \sim \mathcal{P}_{\mathcal{R}}^2$ (nonlinear interaction) and similarly $\mathcal{P}_{\mathcal{R}} < 10^{-9}$ at $k \sim k_{GW}$ giving

$$\Omega_{\rm GW}^{\rm induced} < 10^{-23}$$

The sensitivity of future GW detectors might reach $\Omega_{\rm GW} = \mathcal{O}(10^{-16})$ (very optimistic). So, $\Omega_{\rm GW}^{\rm pri}$ and $\Omega_{\rm GW}^{\rm induced}$ are too small to be detected by GW detectors.

Contribution of primordial tensor and scalar perturbations to stochastic GWs is too small to be detected by GW detectors.

During inflation

$$\frac{M_{\rm Pl}}{2}\left(h_{\mathbf{k}}^{\lambda\prime\prime}+2\frac{a'}{a}h_{\mathbf{k}}^{\lambda\prime}+k^2h_{\mathbf{k}}^{\lambda}\right)=-\alpha\frac{a'}{a}\left[t_{\mathbf{k}}^{\lambda\prime}+\frac{(aa')'}{aa'}t_{\mathbf{k}}^{\lambda}\right]$$

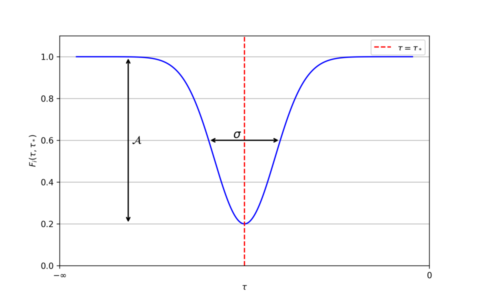
$$t_{\mathbf{k}}^{\lambda\prime\prime}+2\frac{(af)'}{af}t_{\mathbf{k}}^{\lambda\prime}+\left(c_t^2k^2+2\lambda\theta k\frac{a'}{a}\right)t_{\mathbf{k}}^{\lambda}=\frac{\alpha}{f^2}\frac{a'}{a}\left(\frac{M_{\rm Pl}}{2}h_{\mathbf{k}}^{\lambda\prime}\right)\qquad\lambda=\pm1$$

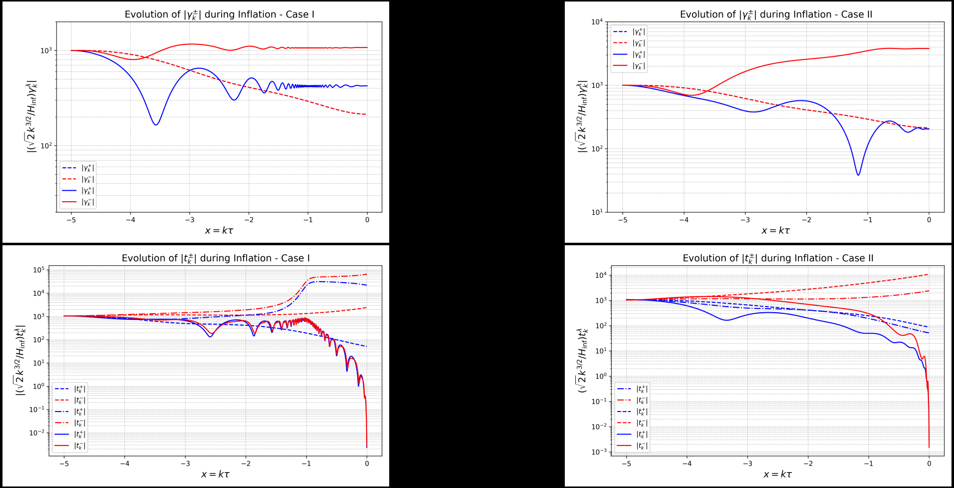
α is linear mixing coupling and θ characterizes parity-violation.

$$F_i(\tau,\tau_*)=c_i-\mathcal{A}\exp\left[-\frac{(\ln[\tau/\tau_*])^2}{2\sigma^2H_{\rm inf}^2}\right]$$

$$F_i=\{f,c_t\}\quad c_i=\{1,c_{t,0}\}$$

$$\begin{cases} c_t=c_{t,0} \;\&\; f=F_1 & \text{Case I}\\ f=1 \;\&\; c_t=F_2 & \text{Case II}\end{cases}$$





Time evolution for the mode k_* ($\alpha = 5$, $\theta = 1$, $\sigma = H_{\rm inf}^{-1}$, $c_{t,0} = 0.9$, $H_{\rm inf}/M_{\rm Pl} = 10^{-5}$). We have considered $|1 - \mathcal{A}| = 10^{-2}/|c_{t,0} - \mathcal{A}| = 0.2$ for Case I/II.

The dashed curves correspond to the standard case while the dotted-dashed curves for $t_k^\pm$ illustrate the case with $\alpha = 0$ but f/c_t given by Case I/II.