Light scalars in de Sitter: the stochastic story

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Theoretical challenges

- IR issues in **rigid** de Sitter space for light, self-interacting scalar fields (see previous talks!)

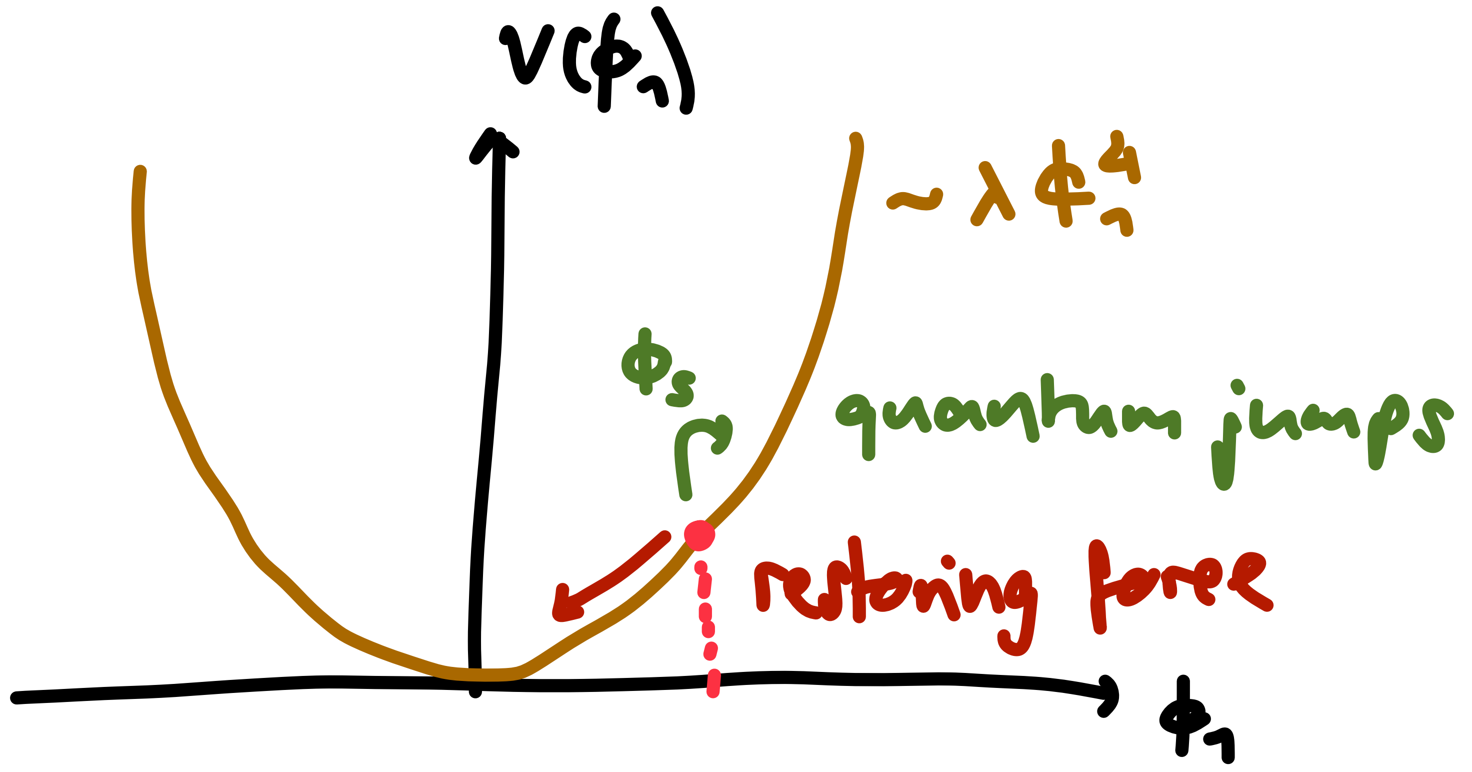
$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4, \qquad \lambda \ll 1, \qquad m^2 < \sqrt{\lambda}H^2$$

$$\boxed{\Lambda(t) \equiv \varepsilon a(t) H}$$

$$\phi^\ell(\vec{x}) \equiv \int_0^{\Lambda(t)} \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \phi_{\vec{k}}$$

- Starobinsky's stochastic formalism: outside the horizon, evolution becomes **classical** $\rightarrow$ Fokker-Planck equation for the long-wavelength field $\phi_1^\ell \equiv \phi^\ell(\mathbf{x}_1)$ at position $\mathbf{x}_1$

$$\boxed{\frac{\partial}{\partial t}P[\phi_1, t] = \Big[\underbrace{\frac{\partial}{\partial\phi_1}\frac{V'(\phi_1)}{3H}}_{\text{classical drift}} + \underbrace{\frac{H^3}{8\pi^2}\frac{\partial}{\partial\phi_1^2}}_{\text{stochastic diffusion}}\Big]P[\phi_1, t]}$$



Starobinsky's stochastic approach

- <u>Fokker-Planck equation:</u> captures dynamics of long-wavelength field

$$\frac{\partial}{\partial t}P[\phi_1,t] = \underbrace{\left[\frac{\partial}{\partial\phi_1}\frac{V'(\phi_1)}{3H} + \frac{H^3}{8\pi^2}\frac{\partial}{\partial\phi_1^2}\right]}_{=:\ H_0\ \text{"free Hamiltonian"}}P[\phi_1,t]$$

~ Schrödinger-like

- solution: $P[\phi_1, t] = \sum_{p=0}^{\infty} e^{-\lambda_p t}\,\Phi_p(\phi_1)$ from EV problem $\boxed{H_0\,\Phi_p(\phi_1) = \lambda_p\,\Phi_p(\phi_1)}$

spectrum

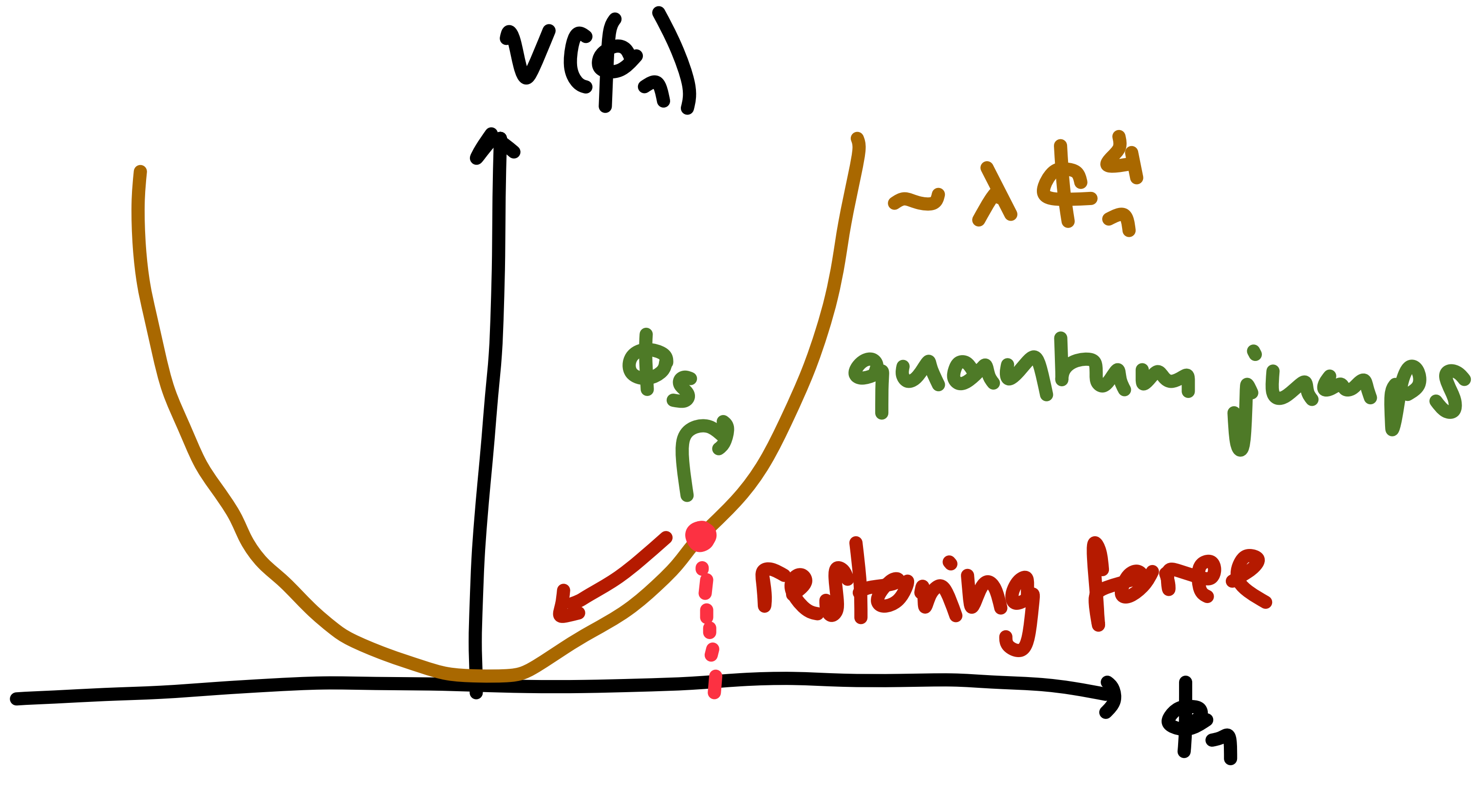
states: composite operators of original scalar ϕ_1

$$\begin{cases} \text{for } V(\phi_1) \sim m^2\phi_1^2 & \to \text{harmonic oscillator spectrum } \{\lambda_p, \Phi_p(\phi_1)\} \\ \text{for } V(\phi_1) \sim \lambda\phi_1^4 & \to \lambda_p \simeq p^{3/2}\sqrt{\lambda}H, \quad \Phi_p(\phi_1) \text{ numerical} \end{cases}$$

Starobinsky's stochastic approach

- <u>Fokker-Planck equation:</u> captures dynamics of long-wavelength field

$$\frac{\partial}{\partial t}P[\phi_1,t]=\Big[\frac{\partial}{\partial\phi_1}\underbrace{\frac{V'(\phi_1)}{3H}}_{\text{classical drift}}+\underbrace{\frac{H^3}{8\pi^2}}_{\text{stochastic diffusion}}\frac{\partial}{\partial\phi_1^2}\Big]P[\phi_1,t]$$



- simplifying assumptions:

  1. ultra-local evolution → PDE in one, classical variable $\phi_1^\ell\equiv\phi^\ell(\mathbf{x}_1)$
  2. short modes approximated as free and massless

⇒ Starobinsky equation = **leading order** result in "some" expansion (?)

Rigorous stochastic approach

- systematic understanding of corrections to Starobinsky equation: V. Gorbenko and L. Senatore (2019)
- rigorous derivation → **EFT** for **long-wavelength field** in **position space**

→ allows computing n-point correlators via generalized Starobinsky equation:
PDE for n *classical* variables (long-wavelength field points ϕ_i):

mass

gradients

$$\boxed{\frac{\partial}{\partial t} P_n[\phi_1, \dots \phi_n; t] = -\left[H_0^{(n)} + \frac{1}{2}\sum_{i \neq j}^{n} j_0(\varepsilon a(t) H x_{ij}) H'_{ij}\right] P_n[\phi_1, \dots \phi_n; t] \cdot \left(1 + \mathcal{O}(\sqrt{\lambda}, \varepsilon^2)\right)}$$

"free Hamiltonian":

spherical Bessel fct.

"interaction Hamiltonian":

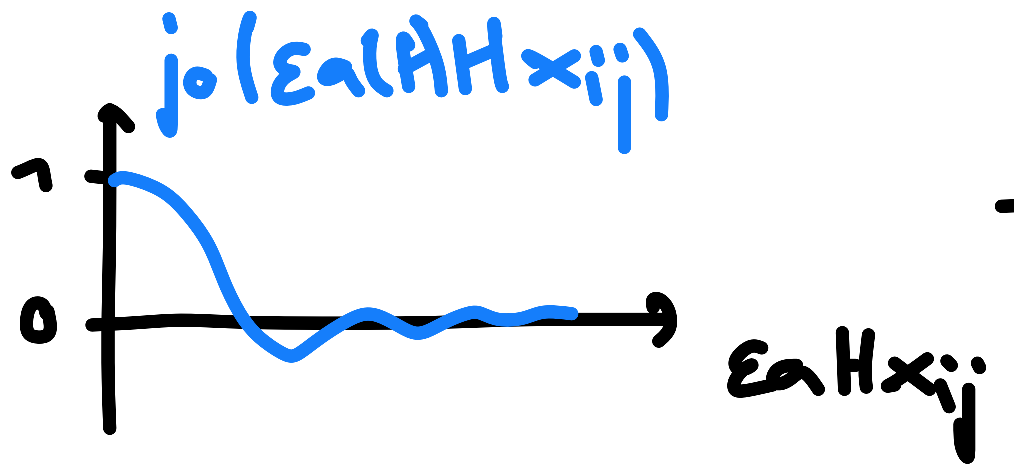
$$H_0^{(n)} := -\sum_{i=1}^{n} \underbrace{\frac{\partial}{\partial \phi_i} \frac{V'(\phi_i)}{3H}}_{\text{drift}} + \underbrace{\frac{H^3}{4\pi^2} \frac{\partial^2}{\partial \phi_i^2}}_{\text{diffusion}}$$

$$H'_{ij} := \underbrace{-\frac{H^3}{4\pi^2} \frac{\partial^2}{\partial \phi_i \partial \phi_j}}_{\text{diffusion}}$$

Rigorous stochastic approach

- solving the generalized Starobinsky equation:

spherical Bessel fct.



→ interaction **switches off** over time!

$$\boxed{\frac{\partial}{\partial t} P_n[\phi_1, \ldots \phi_n; t] = -\left[H_0^{(n)} + \frac{1}{2}\sum_{i \neq j}^{n} j_0(\varepsilon a(t) H x_{ij}) H'_{ij}\right] P_n[\phi_1, \ldots \phi_n; t] \cdot \left(1 + \mathcal{O}(\sqrt{\lambda}, \varepsilon^2)\right)}$$

~ Schrödinger-like

$$H_0^{(n)} := -\sum_{i=1}^{n} \frac{\partial}{\partial \phi_i} \frac{V'(\phi_i)}{3H} + \frac{H^3}{4\pi^2} \frac{\partial^2}{\partial \phi_i^2}$$ (free part)

$$H'_{ij} := -\frac{H^3}{4\pi^2} \frac{\partial^2}{\partial \phi_i \partial \phi_j}$$ (2-pt interaction)

- superposition ansatz: $P_n[\phi_1, \ldots, \phi_n; t] = \sum_{p_1, \ldots, p_n = 1}^{\infty} \underbrace{C_{p_1 \ldots p_n}(t)}_{=\langle \Phi_{p_1}(\phi_1) \ldots \Phi_{p_n}(\phi_n) \rangle} \Phi_{p_1}(\phi_1) \ldots \Phi_{p_n}(\phi_n)$

$\langle \Phi_{p_1}(\phi_1) \ldots \Phi_{p_n}(\phi_n) \rangle$ found perturbatively

Rigorous stochastic approach

- <u>result</u>: correlation functions are **conformally invariant** $\rightarrow$ de Sitter boundary symmetry ✓

$$n=2:\ \langle \Phi_{p_1}(\phi_1)\Phi_{p_2}(\phi_2)\rangle \propto \frac{\delta_{p_1p_2}}{(a(t)Hx_{12})^{2\lambda_{p_1}/H}}$$

$$n=3:\ \langle \Phi_{p_1}(\phi_1)\Phi_{p_2}(\phi_2)\Phi_{p_3}(\phi_3)\rangle \propto \frac{I_{p_1p_2p_3}}{(a(t)Hx_{12})^{(\lambda_{p_1}+\lambda_{p_2}-\lambda_{p_3})/H}(a(t)Hx_{13})^{(\lambda_{p_1}+\lambda_{p_3}-\lambda_{p_2})/H}(a(t)Hx_{23})^{(\lambda_{p_2}+\lambda_{p_3}-\lambda_{p_1})/H}} \qquad (I_{p_1p_2p_3(p_4)}\sim\mathcal{O}(1))$$

$$n=4:\ \langle \Phi_{p_1}(\phi_1)\Phi_{p_2}(\phi_2)\Phi_{p_3}(\phi_3)\Phi_{p_4}(\phi_4)\rangle \propto I_{p_1p_2p_3p_4}\Bigg(\underbrace{\frac{x_{12}x_{34}}{x_{13}x_{24}}}_{=u}\Bigg)^{\alpha}\Bigg(\underbrace{\frac{x_{12}x_{34}}{x_{14}x_{23}}}_{=v}\Bigg)^{\beta}\prod_{j<k}^{4}(a(t)Hx_{jk})^{(\sum_{i=1}^{4}\frac{\lambda_{p_i}}{3}-\lambda_{p_j}-\lambda_{p_k})/H} \qquad (\alpha,\beta\sim\mathcal{O}(\sqrt{\lambda}))$$

(invariant cross ratios)

$\rightarrow$ CFT language: Φ_p's are conformal primaries of weight $\sim \frac{\lambda_p}{H} \sim \mathcal{O}(\sqrt{\lambda})$

Rigorous stochastic approach

- probing the 4-point function in the **collapsed limit** ($k_I \to 0$) in Fourier space:

non-perturbative

$k_I := |\vec{k}_1 + \vec{k}_2|$ exchanged mom.

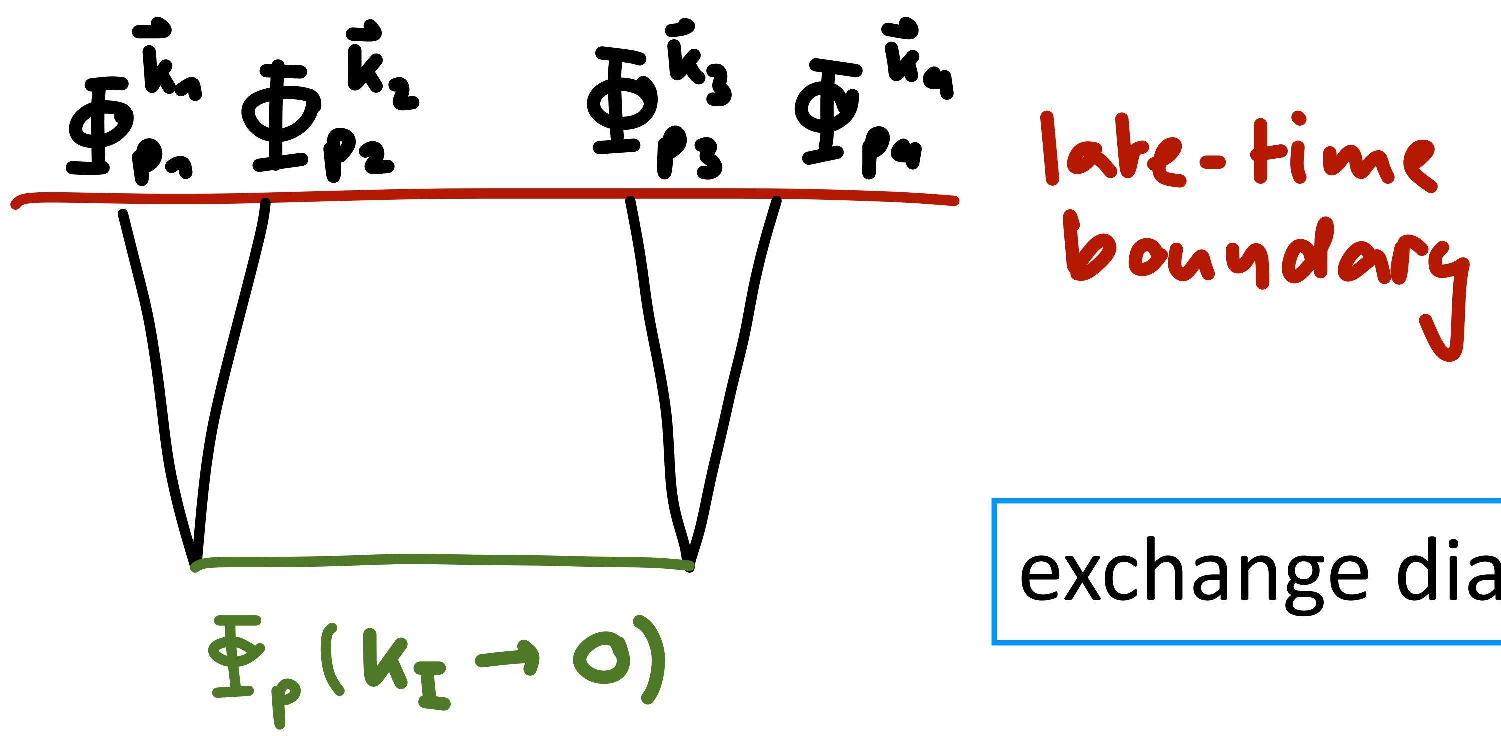
$$\langle \Phi_{p_1}(\mathbf{k}_1)\Phi_{p_2}(\mathbf{k}_2)\Phi_{p_3}(\mathbf{k}_3)\Phi_{p_4}(\mathbf{k}_4)\rangle_{very\,sq.} = (2\pi)^4\delta\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\sum_{p=0}^{\infty}\frac{\langle \Phi_{p_1}(\mathbf{k}_1)\Phi_{p_2}(\mathbf{k}_2)\Phi_p(\mathbf{k}_I)\rangle'_{sq.}\langle \Phi_p(\mathbf{k}_I)\Phi_{p_3}(\mathbf{k}_3)\Phi_{p_4}(\mathbf{k}_4)\rangle'_{sq.}}{\langle \Phi_p(\mathbf{k}_I)\Phi_p(-\mathbf{k}_I)\rangle'}$$

$\updownarrow$

→ probes **light states** $\Phi_p(\mathbf{k}_I)$ being exchanged

V. Assassi, D. Baumann, D. Green (2012)

perturbative



exchange diagram

The hadronisation conjecture

Light scalar ϕ in $\lambda\phi^4$-theory in de Sitter space "hadronises" on superhorizon scales into light mesons $\Phi_p(\phi)$ that are late-time conformal primaries of weight λ_{p_i}/H

	QCD:		**stochastic $\lambda\phi^4-$theory on de Sitter**:
small scales:	quarks q	analogy $\longleftrightarrow$	original scalar ϕ
	$\downarrow$		$\downarrow$
large scales:	pions $\pi(q)$ $\hookrightarrow$ local EFT		composite operators $\Phi_p(\phi)$ "light mesons"

The hadronisation conjecture

Light scalar ϕ in $\lambda\phi^4$-theory in de Sitter space "hadronises" on superhorizon scales into light mesons $\Phi_p(\phi)$ that are late-time conformal primaries of weight λ_{p_i}/H

Question: Can we find a weakly-coupled, local bulk EFT description for the mesons $\Phi_p(\phi)$ analogous to QCD?

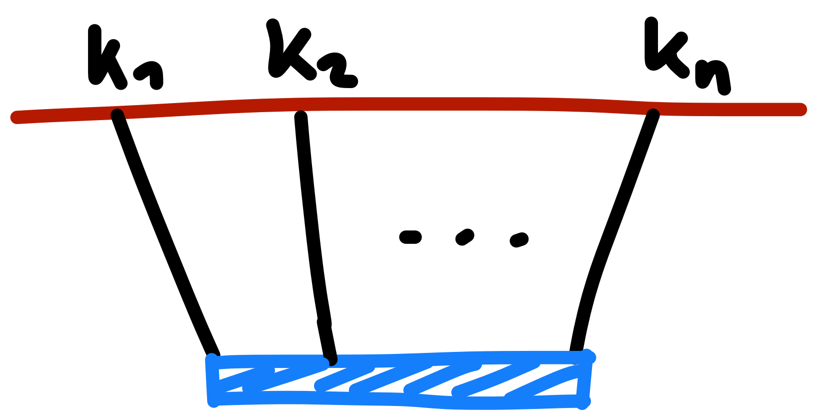
$$\mathscr{L}(\Phi_p) \sim -\frac{1}{2}\sum_{p=0}^{\infty}\left[(\partial\Phi_p)^2 + M_p^2\Phi_p^2\right] - \sum_{p_1,p_2,p_3=0}^{\infty} \boxed{g_{p_1p_2p_3}}\,\Phi_{p_1}\Phi_{p_2}\Phi_{p_3} - \sum_{p_1,p_2,p_3,p_4=0}^{\infty} \boxed{g_{p_1p_2p_3p_4}}\,\Phi_{p_1}\Phi_{p_2}\Phi_{p_3}\Phi_{p_4} + \dots$$

→ find couplings from **matching observables**:

stochastic (non-PT) ? tree-level in-in (PT)

$\langle \prod_i \Phi_{p_i}(\phi_i)\rangle$

↳ Fourier trsf.!



The hadronisation conjecture

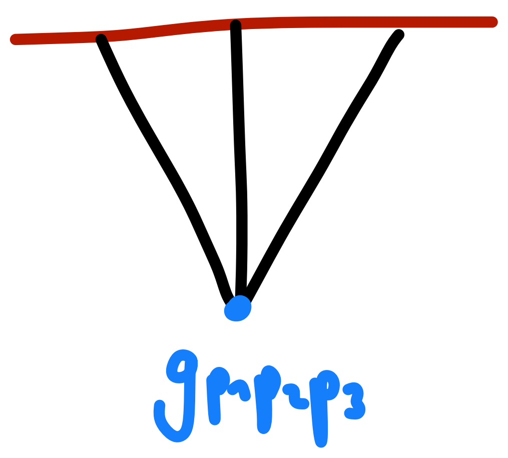
<u>Cubic coupling</u> $g_{p_1p_2p_3}$: match conformal stochastic correlator onto **massive 3-point scalar contact** diagram

stochastic: $\langle \Phi_{p_1}(\phi_1)\, \Phi_{p_2}(\phi_2)\, \Phi_{p_3}(\phi_3) \rangle$

$$\Big\downarrow \text{ Fourier trsf.}$$

$$\langle \Phi_{p_1}(\vec{k}_1)\, \Phi_{p_2}(\vec{k}_2)\, \Phi_{p_3}(\vec{k}_3) \rangle \quad \longleftrightarrow$$

in-in PT for $\sim g_{p_1p_2p_3}\, \Phi_{p_1} \Phi_{p_2} \Phi_{p_3}$:



$$\Rightarrow \boxed{g_{p_1p_2p_3} = I_{p_1p_2p_3} \frac{64\pi^5}{H^2} \frac{\Gamma\left(\frac{1}{2} - \frac{\Sigma_{j=1}^3 \lambda_{p_j}}{2H}\right)}{\prod_{j=1}^3 \Gamma\left(\frac{3}{2} - \frac{\lambda_{p_j}}{H}\right)} \cdot \frac{\left(\frac{\Sigma_{j=1}^3 \lambda_{p_j}}{2H} - \frac{1}{2}\right)\left(\frac{\Sigma_{j=1}^3 \lambda_{p_j}}{2H} - \frac{3}{2}\right)}{\prod_{i<j\neq k}^3 \Gamma\left(\frac{\lambda_{p_i} + \lambda_{p_j} - \lambda_{p_k}}{2H}\right)} \sim \mathcal{O}(\sqrt{\lambda}^3)}$$

$$\uparrow$$

$\lambda_p \simeq p^{3/2} \sqrt{\lambda}\, H$

mesons $\Phi_p(\phi)$ more weakly coupled than original $\lambda\phi^4-$theory!

The hadronisation conjecture

Quartic coupling $g_{p_1p_2p_3p_4}$: interesting testing ground for the EFT conjecture → conformal symmetry less constraining:

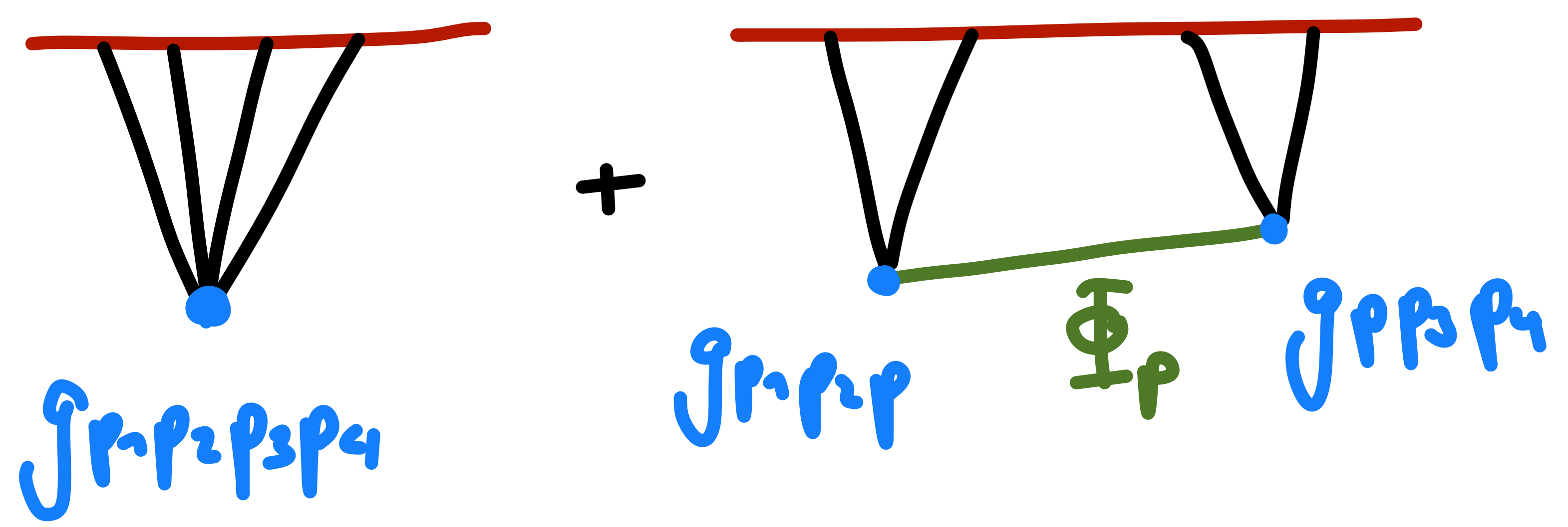
- for primary operators $\mathcal{O}_\Delta$: $\boxed{\langle \mathcal{O}_{\Delta_1}(\mathbf{x}_1)\mathcal{O}_{\Delta_2}(\mathbf{x}_2)\mathcal{O}_{\Delta_3}(\mathbf{x}_3)\mathcal{O}_{\Delta_4}(\mathbf{x}_4)\rangle = f(u,v)\prod_{i<j} x_{ij}^{\sum_i \frac{\Delta_i}{3}-\Delta_j-\Delta_k},}$ $\quad u = \frac{x_{12}x_{34}}{x_{13}x_{24}},\ \ v = \frac{x_{12}x_{34}}{x_{14}x_{23}}$

stochastic:

$$\left\langle \prod_{i=1}^{4} \Phi_{p_i}(\phi_i) \right\rangle \sim I_{p_1p_2p_3p_4}\, u^\alpha v^\beta \prod_{j<k}^{4} (a(t)Hx_{jk})^{\left(\sum_{i=1}^{4}\frac{\lambda_{p_i}}{3}-\lambda_{p_j}-\lambda_{p_k}\right)/H}$$

- difficulty: $\boxed{f(u,v) = u^\alpha v^\beta}$ **simple polynomial** $(\alpha, \beta \sim \mathcal{O}(\sqrt{\lambda}))$

in-in PT:



two competing diagrams at the same order, hard to compute in in-in PT

The hadronisation conjecture

Quartic coupling $g_{p_1 p_2 p_3 p_4}$: interesting testing ground for the EFT conjecture

Mellin amplitude (↔ scatt. ampl.)

- Mellin space representation: $\left\langle \prod_{i=1}^{4} \mathcal{O}_{\Delta_i}(\mathbf{x}_i) \right\rangle \sim f(u,v) = \int \frac{ds_{12} ds_{14}}{(2\pi i)^2} \mathcal{M}(s_{12}, s_{14})\, u^{-s_{12}} v^{-s_{14}}$

J. Penedones (2010),
M. Paulos (2011)

- **stochastic correlator:** $f(u,v) = u^{\alpha} v^{\beta} \quad \rightarrow \quad \boxed{\mathcal{M}(s_{12}, s_{14}) = \delta(s_{12}+\alpha)\, \delta(s_{14}+\beta)}$

mismatch with both!

- **exchange** term: $$\boxed{\mathcal{M}_{ex}(s_{12}, s_{14}) = -g^2 \sum_{n=0}^{\infty} \left[\frac{V_{n,\Delta}^2}{s_{12} - \Delta - 2n} + \frac{V_{n,\Delta}^2}{s_{13} - \Delta - 2n} + \frac{V_{n,\Delta}^2}{s_{14} - \underline{\Delta - 2n}} \right]}$$

$s_{12} + s_{13} + s_{14} = \sum_{i=1}^{4} \Delta_i$

$(s + t + u = 4m^2)$

poles at weights of exchanged op. and all its descendants

- **contact** term: $\boxed{\mathcal{M}_{con}(s_{12}, s_{14}) = g \cdot const\,.}$

Summary

- **generalization of Starobinsky's stochastic formalism to higher-point functions** and with systematic understanding of corrections → basis for non-Gaussianities in inflation

- studied higher-point correlation functions of the composite operators $\langle \Phi_{p_1}(\phi_1)\Phi_{p_2}(\phi_2)\Phi_{p_3}(\phi_3)\ldots\rangle$:

  ✓ verified de Sitter invariance

  ✓ provided physical interpretation for composite operators → **hadronization** picture

  X Can these correlators also be found from a weakly coupled effective field theory (EFT)?

→ *the stochastic story is not so simple after all …*