N-body simulations of primordial features with GENGARS

arXiv:2508.01855

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IAP
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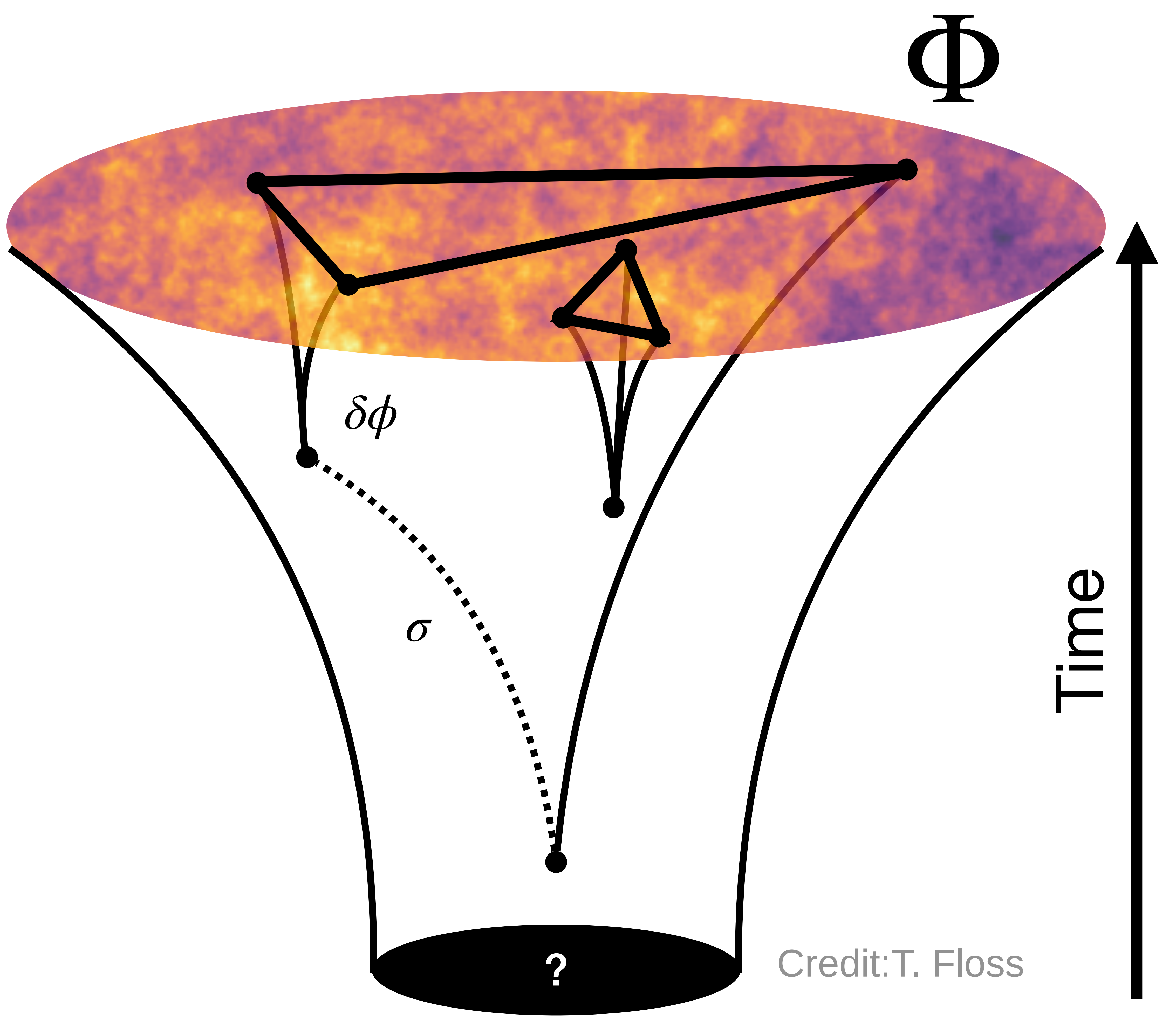
Primordial non-Gaussianity

- **Inflaton fluctuations $\delta\phi$ source primordial gravitational potential perturbations Φ**



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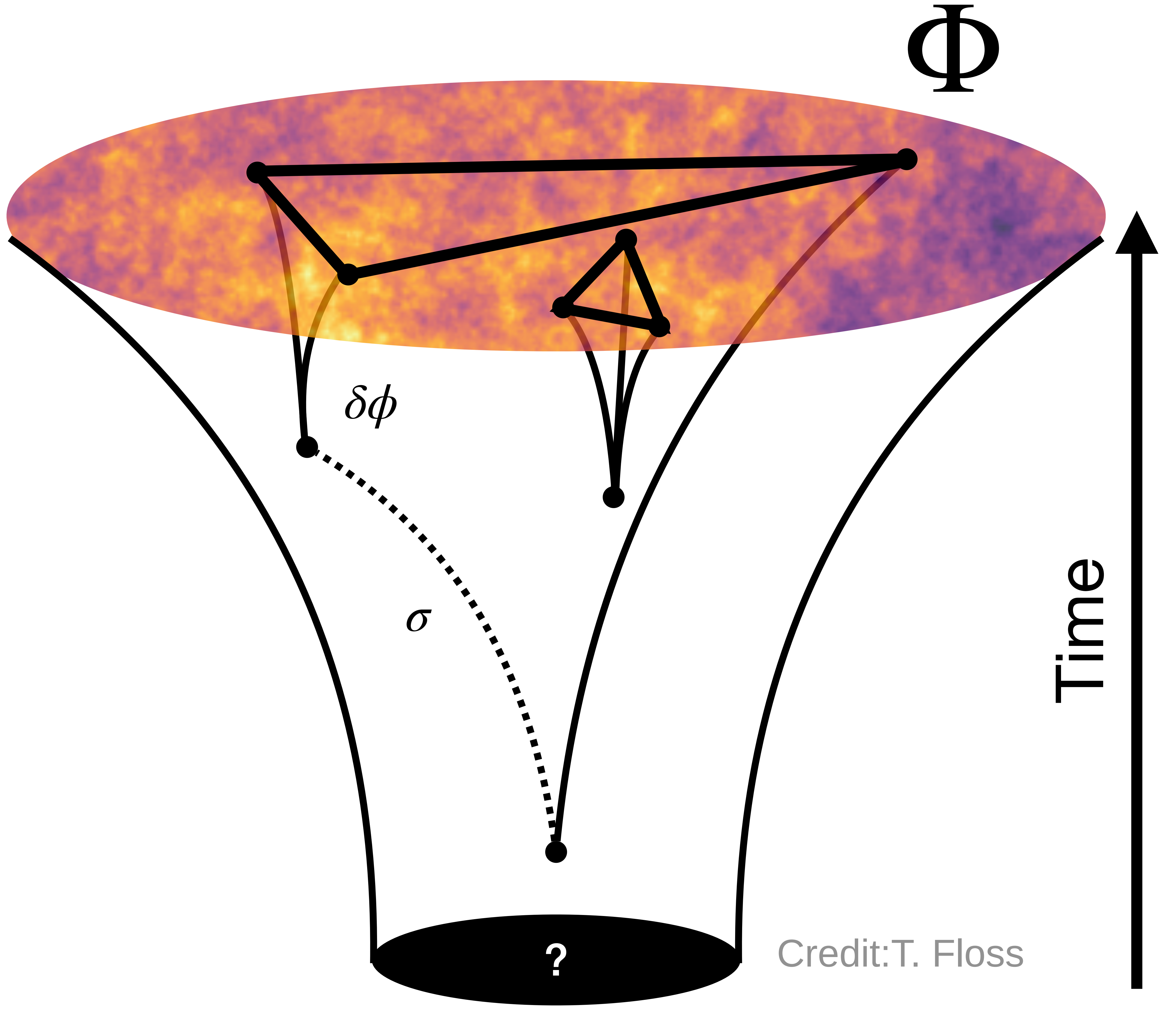
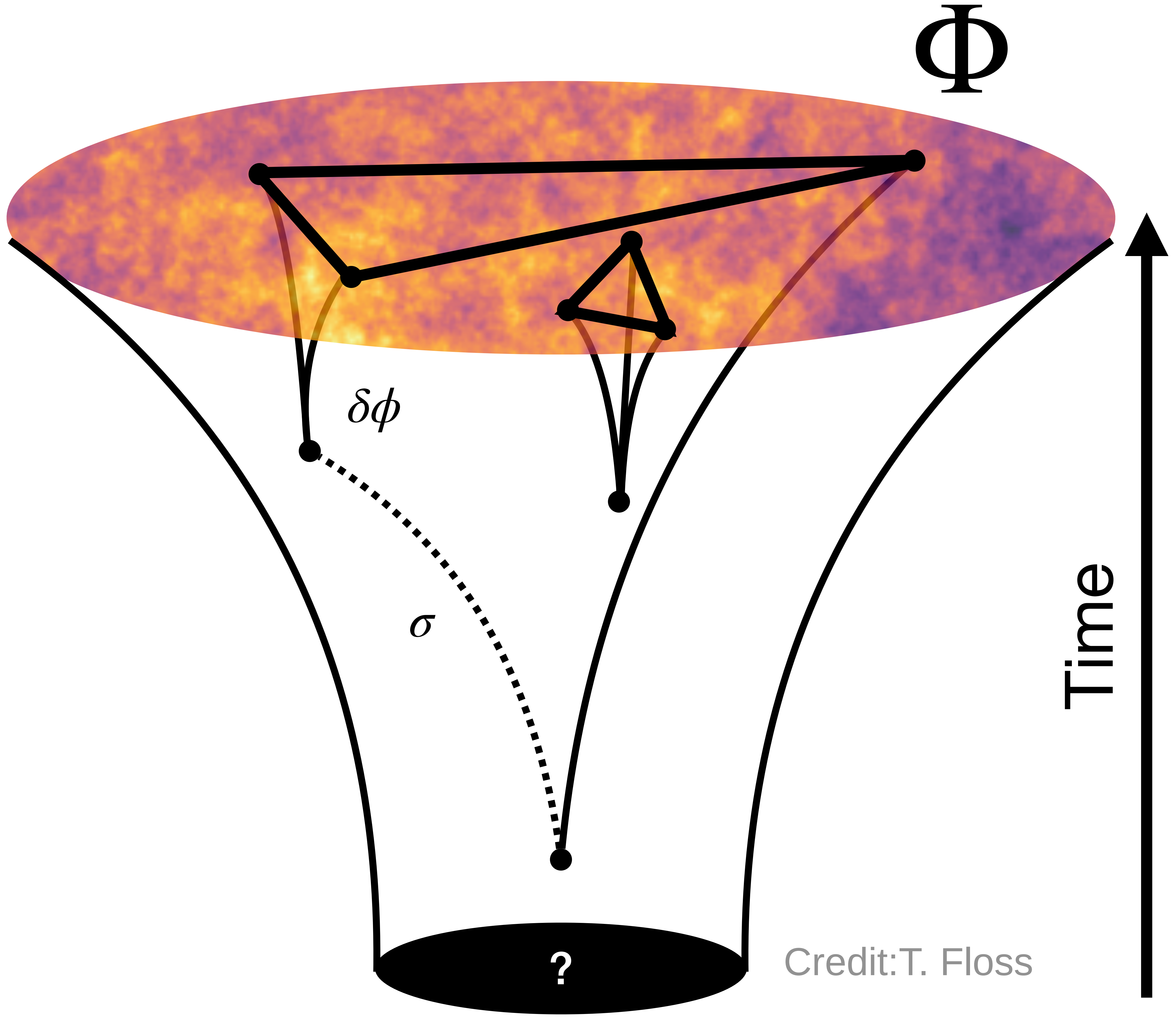


Primordial non-Gaussianity

- Inflaton fluctuations $\delta\phi$ source primordial gravitational potential perturbations Φ

- $$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) + f_{NL}\Phi_{NG}(\mathbf{x})$$
  Primordial non-Gaussianity (PNG)



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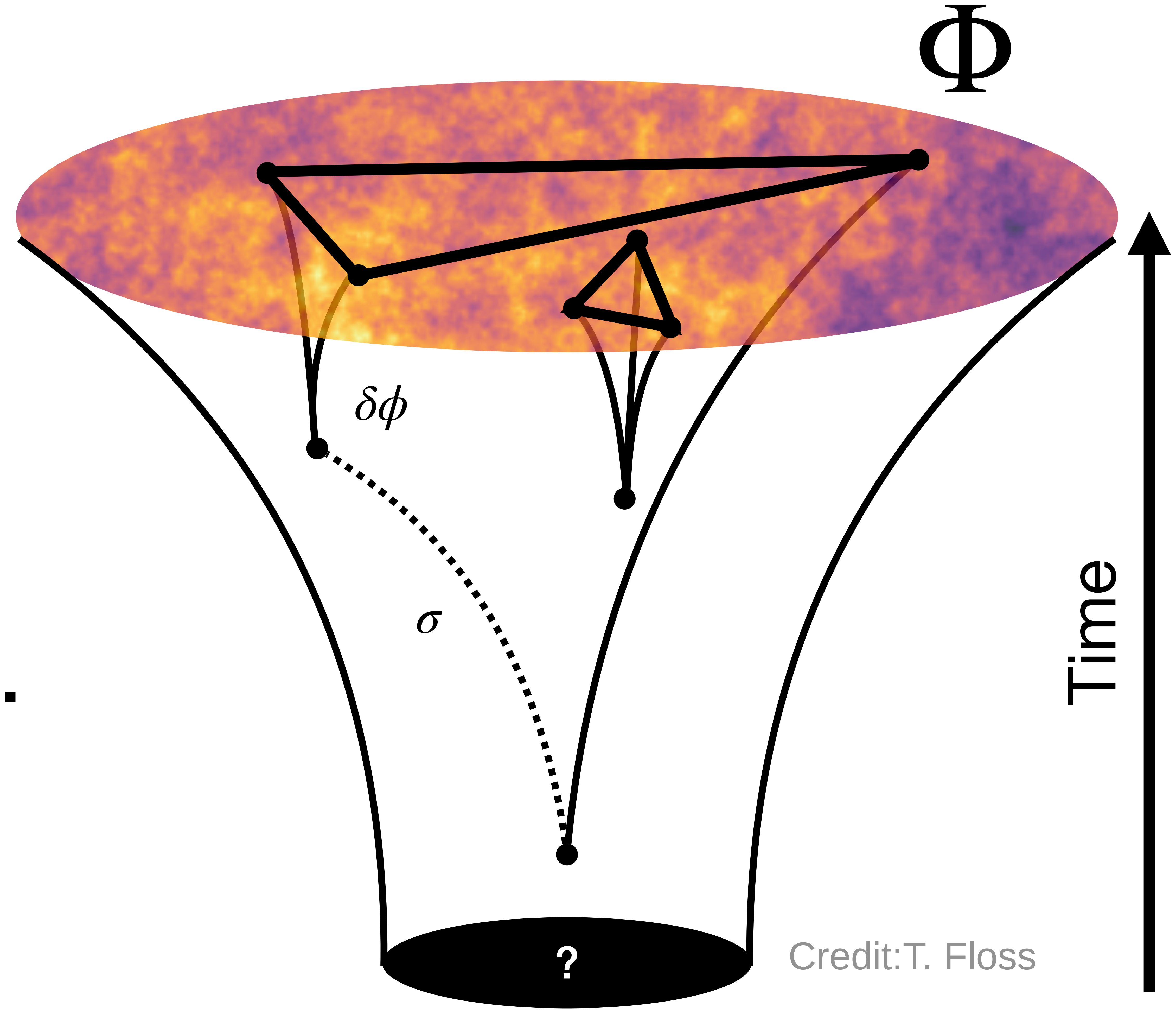
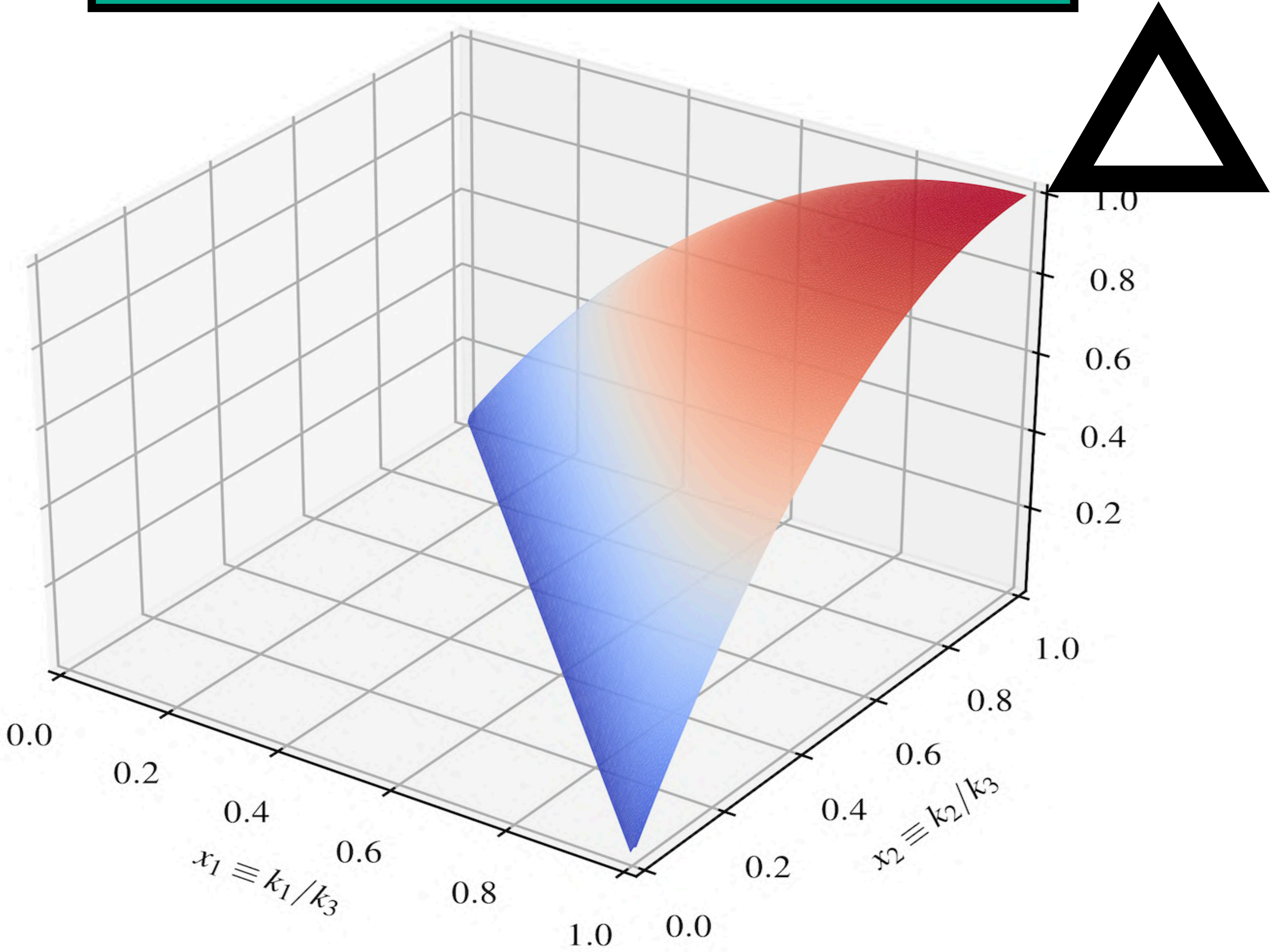
  Primordial non-Gaussianity (PNG)

- **Self-interacting inflaton, multiple fields, …**
  → Different PNG types, specified by
  $$B_\Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3)\rangle \neq 0$$
  and characterized through triangle configurations



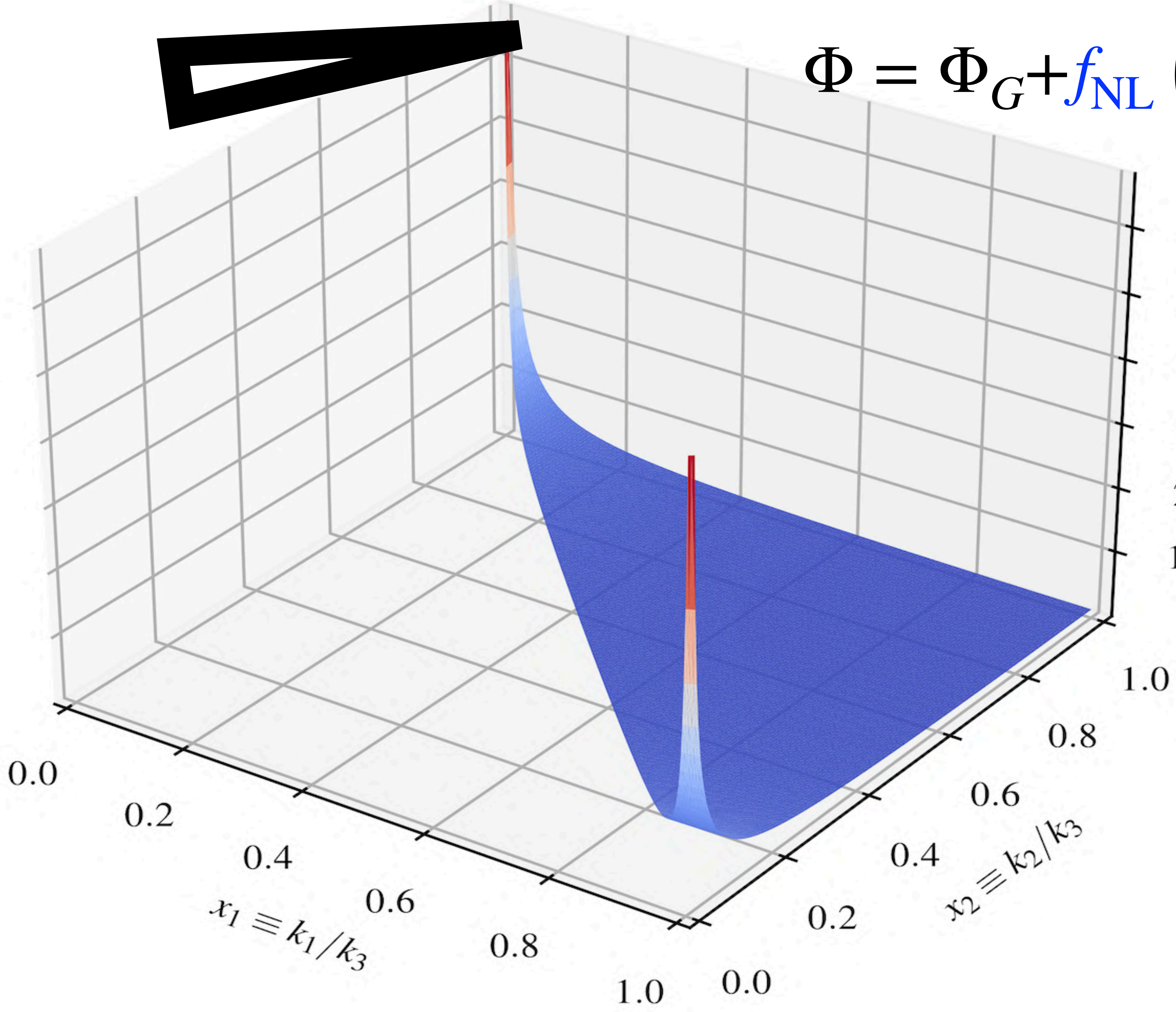
Separable templates

Equilateral PNG template



Local PNG template

$$\Phi = \Phi_G + f_{\rm NL}\left(\Phi_G^2 - \langle \Phi_G^2 \rangle\right)$$



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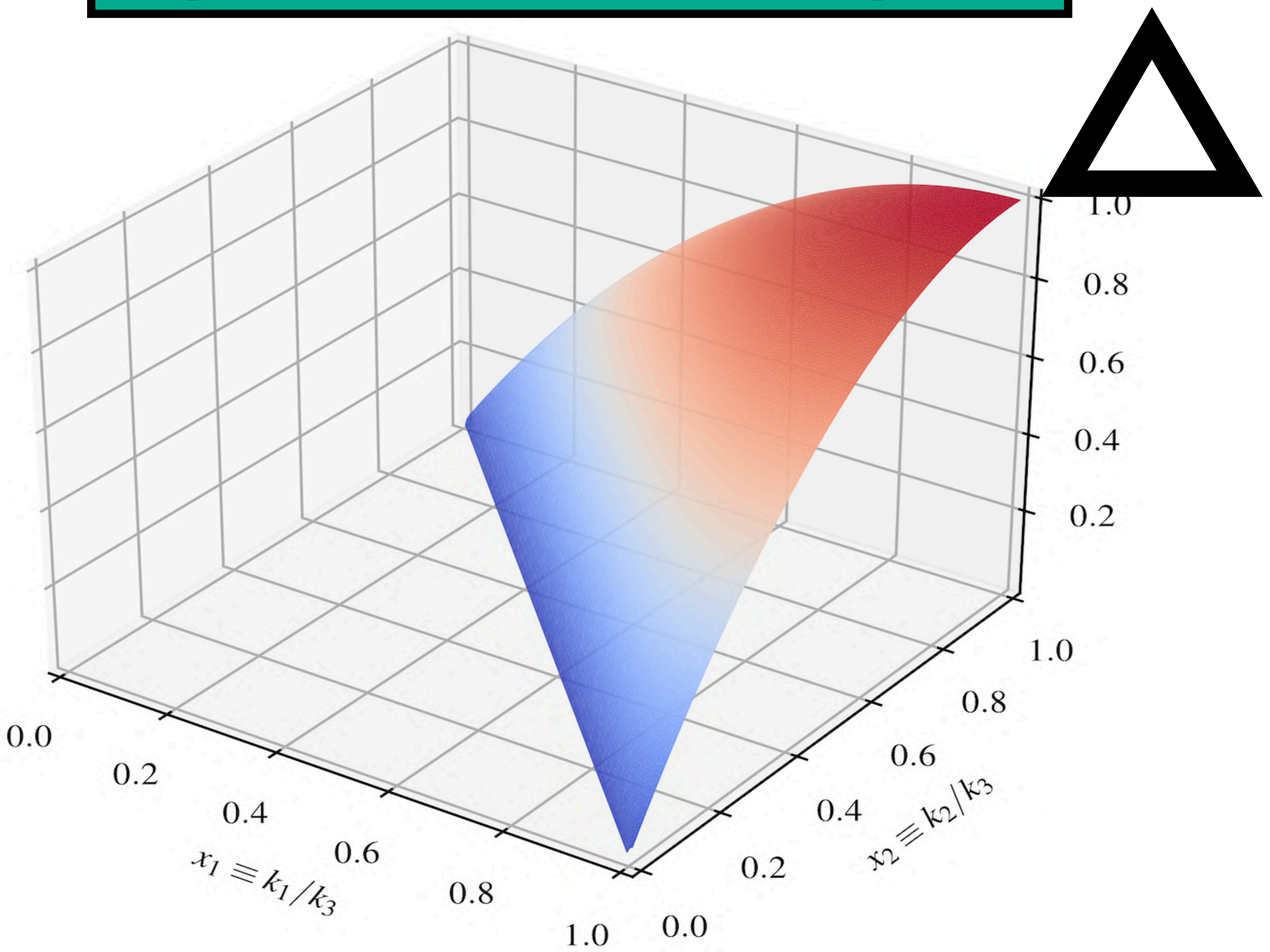

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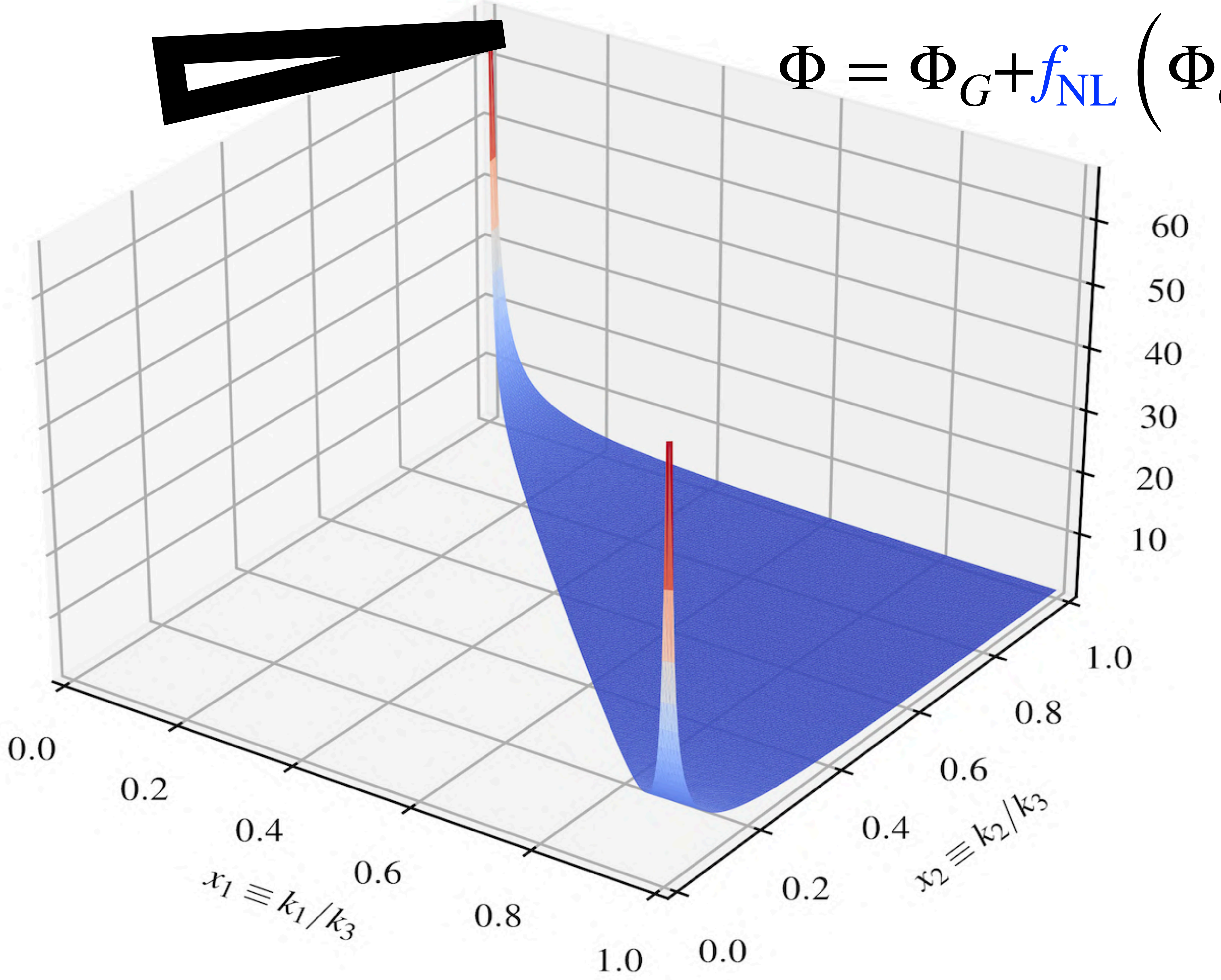
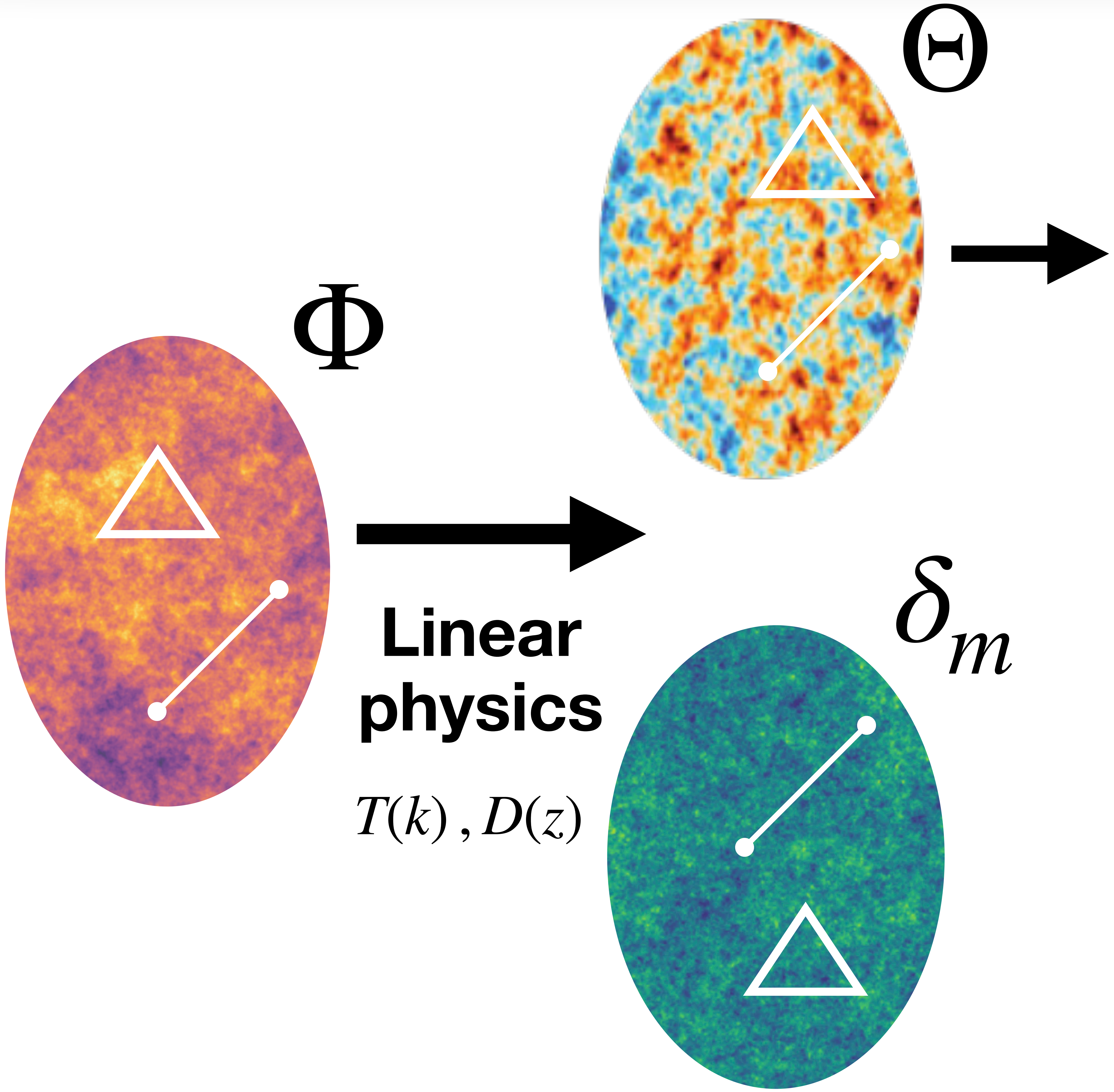


Separability: $$B_\Phi(k_1, k_2, k_3) = \sum_{i=1}^{N_i} b_1^i(k_1) b_2^i(k_2) b_3^i(k_3)$$

Constraints from cosmological observables



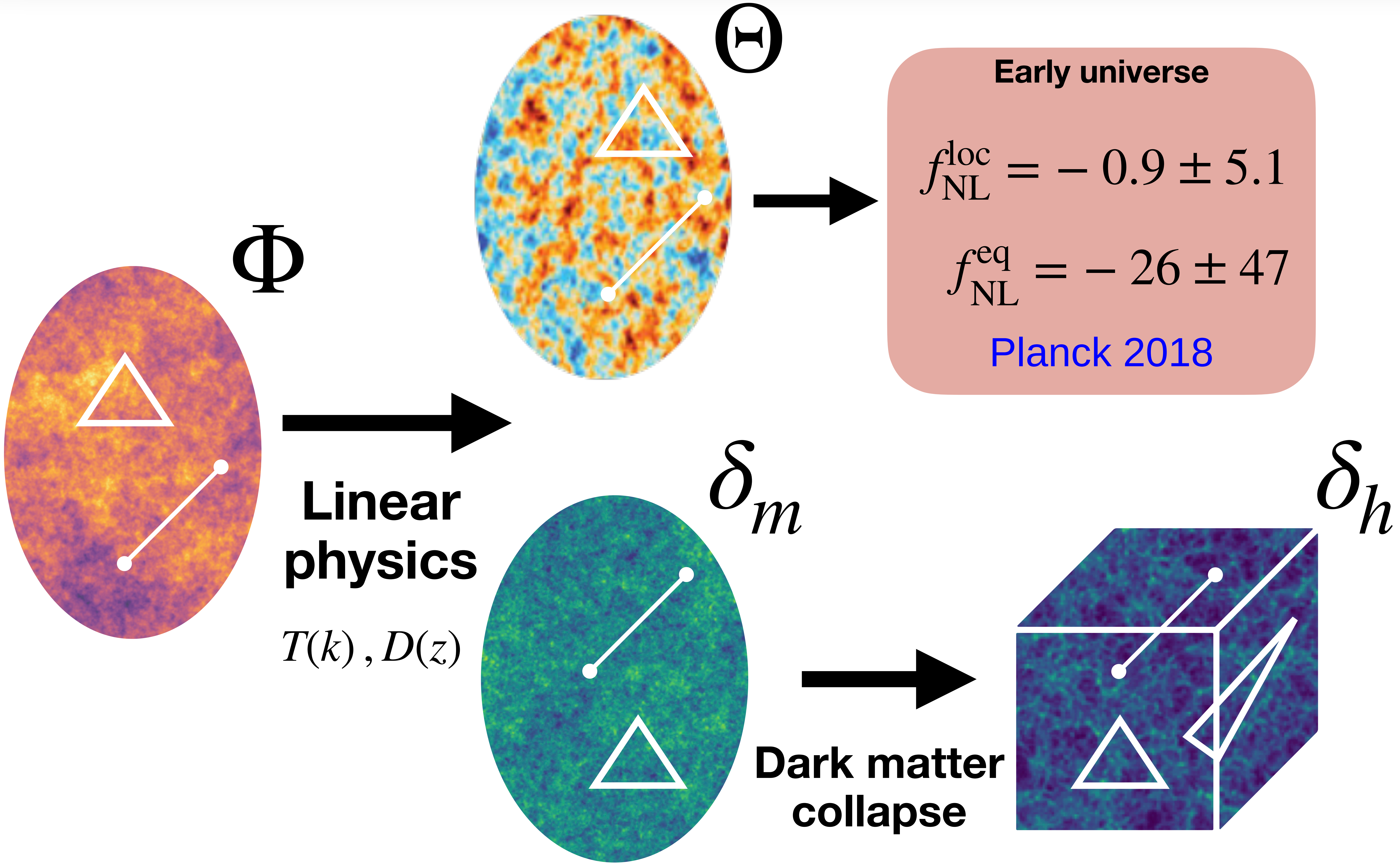
Early universe

$$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$$

$$f_{\rm NL}^{\rm eq} = -26 \pm 47$$

Planck 2018

Constraints from cosmological observables



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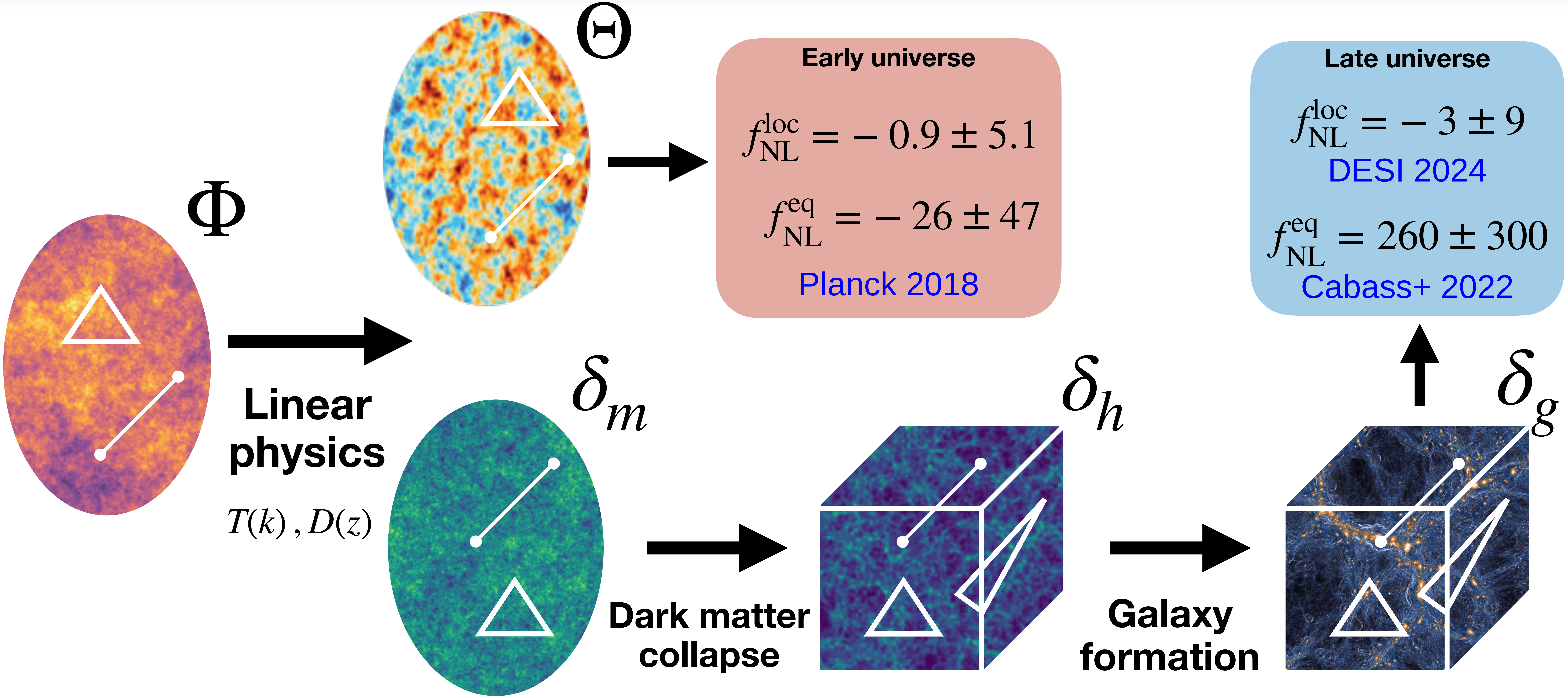
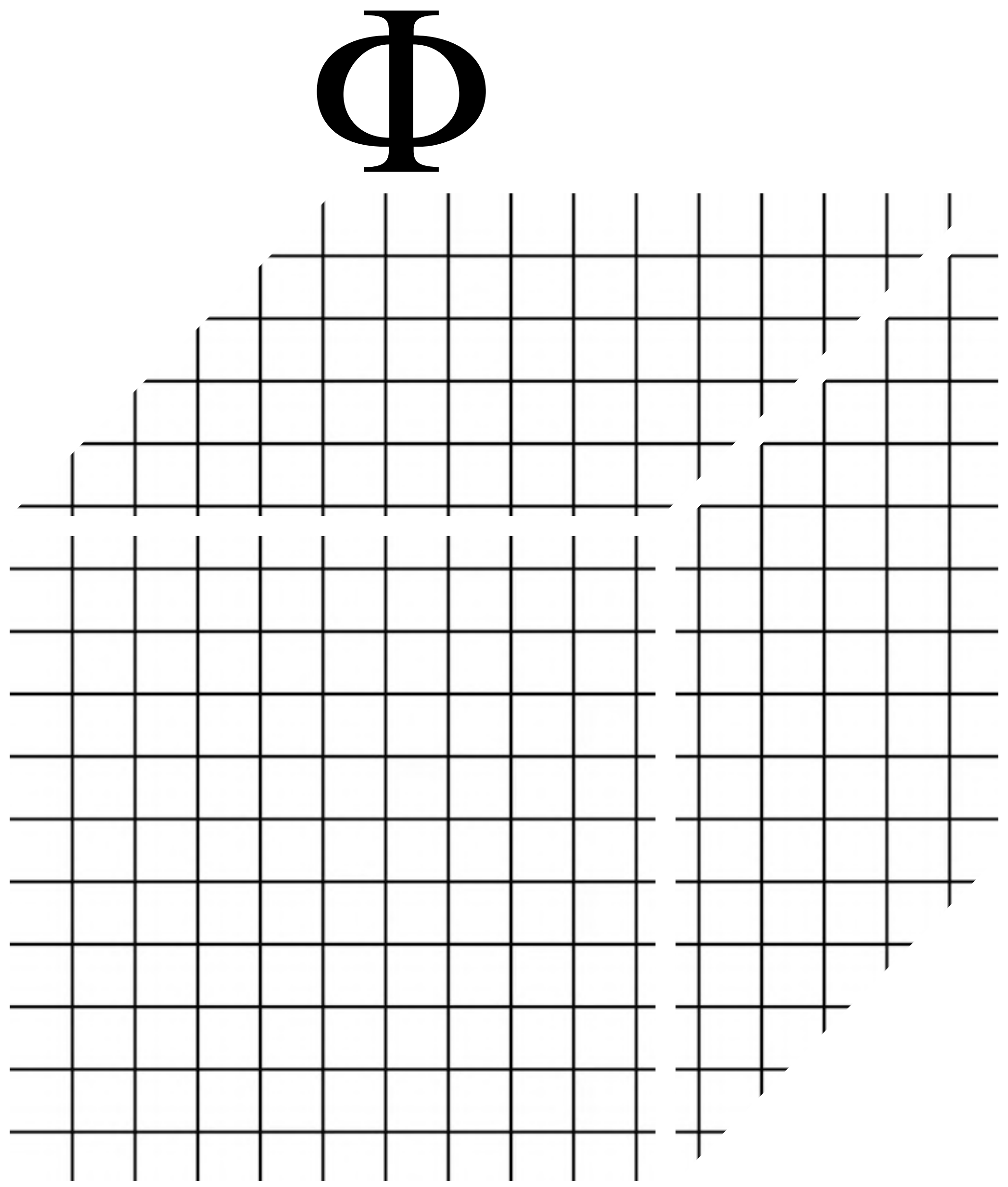
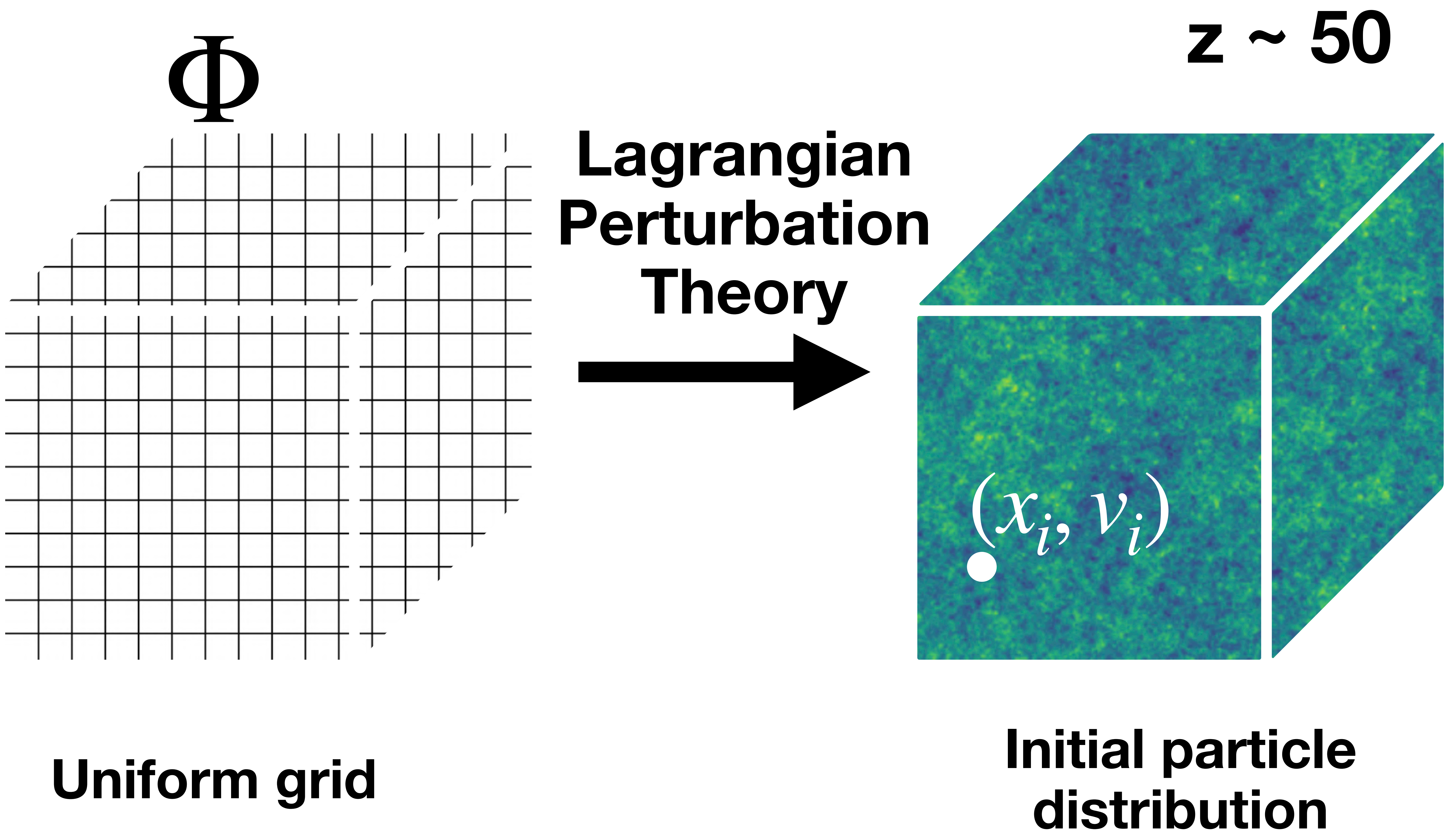


Simulating structure formation



Uniform grid

Simulating structure formation



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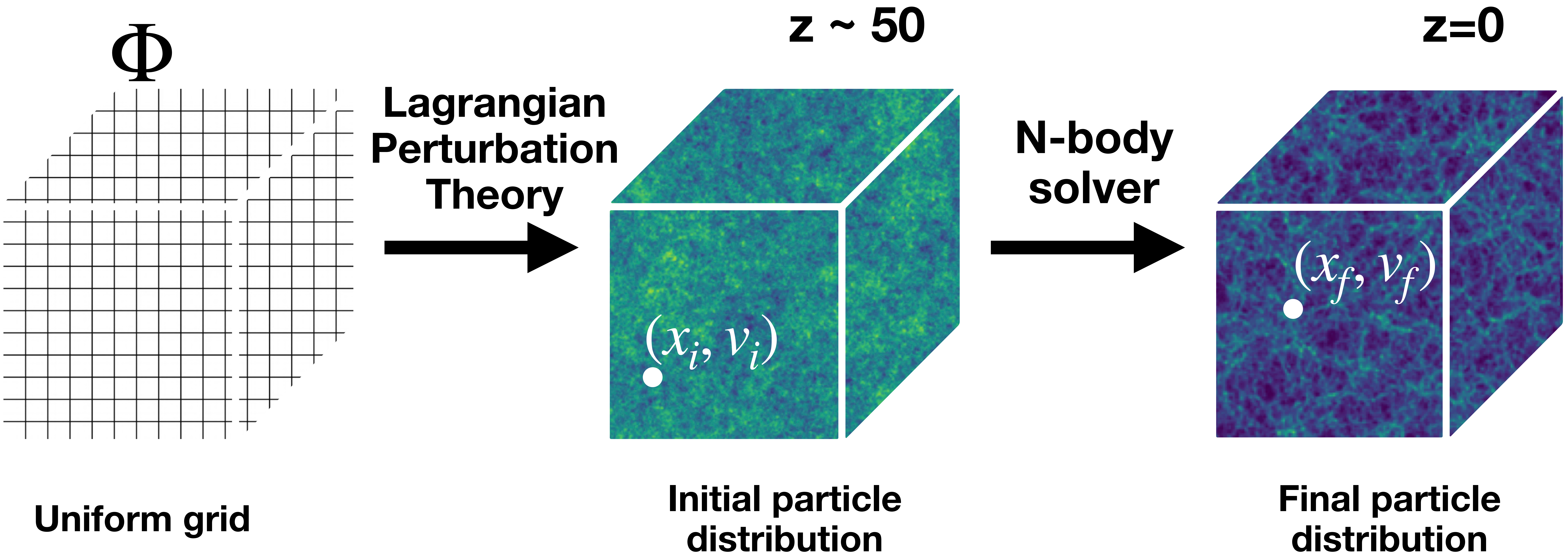
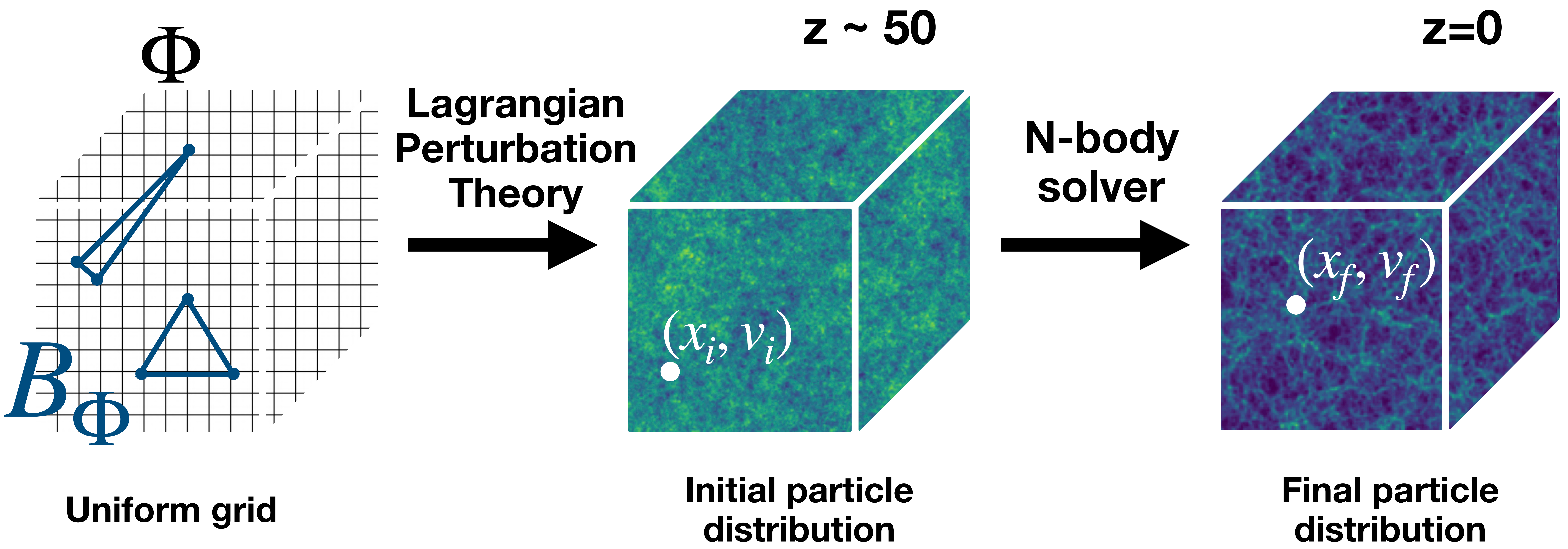


Simulating structure formation with PNG



Initial conditions with arbitrary bispectrum

Local PNG:

$$\Phi(\mathbf{k}) = \Phi^{\mathrm{G}}(\mathbf{k}) + f_{\mathrm{NL}} \int \frac{d^3k'}{(2\pi)^3} \Phi^{\mathrm{G}*}(\mathbf{k}')\Phi^{\mathrm{G}}(\mathbf{k}+\mathbf{k}')$$

$$\Phi = \Phi_G + f_{\mathrm{NL}} \left(\Phi_G^2 - \langle \Phi_G^2 \rangle \right)$$

in real space

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in real space

Generalization:

$$\Phi(\mathbf{k}) = \Phi^{G}(\mathbf{k}) + f_{\rm NL} \int \frac{d^3k'}{(2\pi)^3} \boxed{W(k, k', |\mathbf{k}+\mathbf{k}'|)} \Phi^{G*}(\mathbf{k}')\Phi^{G}(\mathbf{k}+\mathbf{k}')$$

Generating kernel

Schmidt, Kamionkowski 2010

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Generating kernel

Schmidt, Kamionkowski 2010

Its bispectrum reads

$$B_\Phi(k_1, k_2, k_3) = 2f_{\mathrm{NL}}\left[W(k_1, k_2, k_3)P_\Phi(k_1)P_\Phi(k_2) + \mathrm{cyc.}\right].$$

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Generating kernel

target

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$$B_\Phi(k_1, k_2, k_3) = 2 f_{\mathrm{NL}} \left[W(k_1, k_2, k_3) P_\Phi(k_1) P_\Phi(k_2) + \text{cyc.}\right].$$

There exist different W generating the same target B

Recipe for a good kernel

- **It should be separable** $W(k_1,k_2,k_3)=\sum_{i=1}^{N_i} w_1^i(k_1)w_2^i(k_2)w_3^i(k_3)$

so that $$\Phi(\mathbf{k})=\Phi^{\rm G}(\mathbf{k})+f_{\rm NL}\sum_{i=1}^{N_i} w_1^i(k)\int\frac{d^3k'}{(2\pi)^3}\underbrace{w_2^i(k')\Phi^{\rm G*}(\mathbf{k}')}_{\textbf{FFT}}\,\underbrace{w_3^i(|\mathbf{k}+\mathbf{k}'|)\Phi^{\rm G}(\mathbf{k}+\mathbf{k}')}_{\textbf{FFT}}$$

$\mathcal{O}(N^2)\rightarrow\mathcal{O}(N\log N)$

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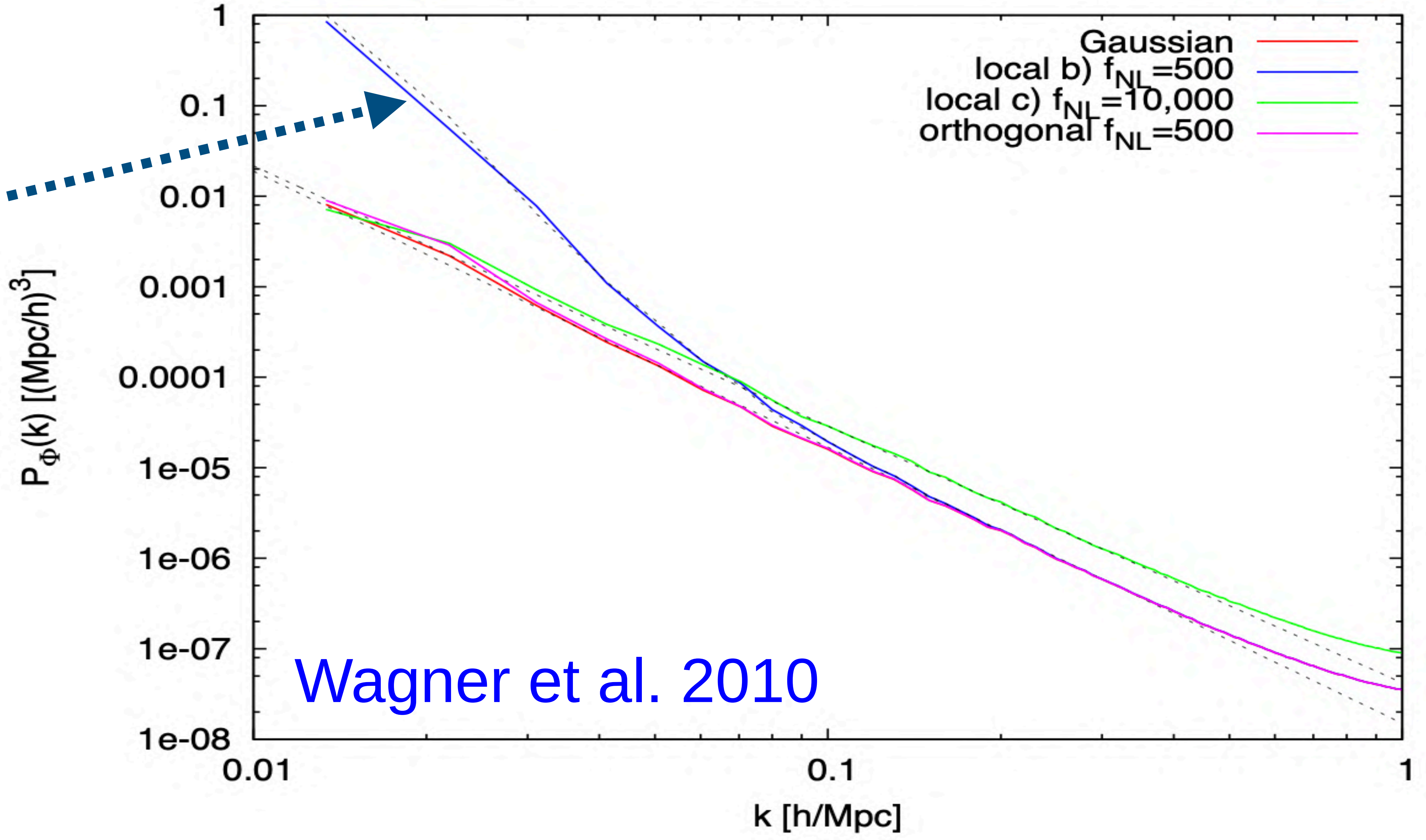
so that $\Phi(\mathbf{k}) = \Phi^{\mathrm{G}}(\mathbf{k}) + f_{\mathrm{NL}} \sum_{i=1}^{N_i} w_1^i(k) \int \frac{d^3k'}{(2\pi)^3} \underbrace{w_2^i(k')\Phi^{\mathrm{G}*}(\mathbf{k}')}_{\mathrm{FFT}} \underbrace{w_3^i(|\mathbf{k}+\mathbf{k}'|)\Phi^{\mathrm{G}}(\mathbf{k}+\mathbf{k}')}_{\mathrm{FFT}}$

$\mathcal{O}(N^2) \to \mathcal{O}(N \log N)$

- It introduces a contribution

$$P_\Phi^{\mathrm{NG}}(k) = 2 f_{\mathrm{NL}}^2 \int \frac{d^3k'}{(2\pi)^3} W^2(k, k', |\mathbf{k}+\mathbf{k}'|) P_\Phi^{\mathrm{G}}(k') P_\Phi^{\mathrm{G}}(|\mathbf{k}+\mathbf{k}'|)$$

to the **primordial power spectrum**, that should be suppressed to avoid spoiling n_s constraints



Can we find a universal kernel?

Reduced bispectrum kernel

Wagner&Verde 2012

$$W(k_1,k_2,k_3) = \frac{B_\Phi(k_1,k_2,k_3)}{2f_{\rm NL}\left[P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_1)P_\Phi(k_3)\right]}$$

- **Universal form: explicitly depends on the target bispectrum**
- **The denominator allows to suppress uncontrolled contributions to $P(k)$**

Can we find a universal kernel?

Reduced bispectrum kernel

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- **Universal form: explicitly depends on the target bispectrum**
- **The denominator allows to suppress uncontrolled contributions to $P(k)$**
- **Issue: even if B is separable, W is not separable !**

  ⟶ **Scales prohibitively with resolution: $\mathcal{O}(N^2)$ (~13 days for $N = 512^3$)**

Separating the reduced bispectrum kernel

- **The key identity is** $$\frac{1}{f(x)} = \int_0^\infty e^{-tf(x)}dt$$

Schwinger parameterization

$$\frac{1}{P_\Phi(k_1)P_\Phi(k_2) + P_\Phi(k_2)P_\Phi(k_3) + P_\Phi(k_1)P_\Phi(k_3)} = \int_0^\infty dt \, \frac{e^{-\frac{t}{P_\Phi(k_1)}}}{P_\Phi(k_1)} \frac{e^{-\frac{t}{P_\Phi(k_2)}}}{P_\Phi(k_2)} \frac{e^{-\frac{t}{P_\Phi(k_3)}}}{P_\Phi(k_3)}$$

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Nt time steps (~300)

- **The denominator becomes the integral of a separable function**

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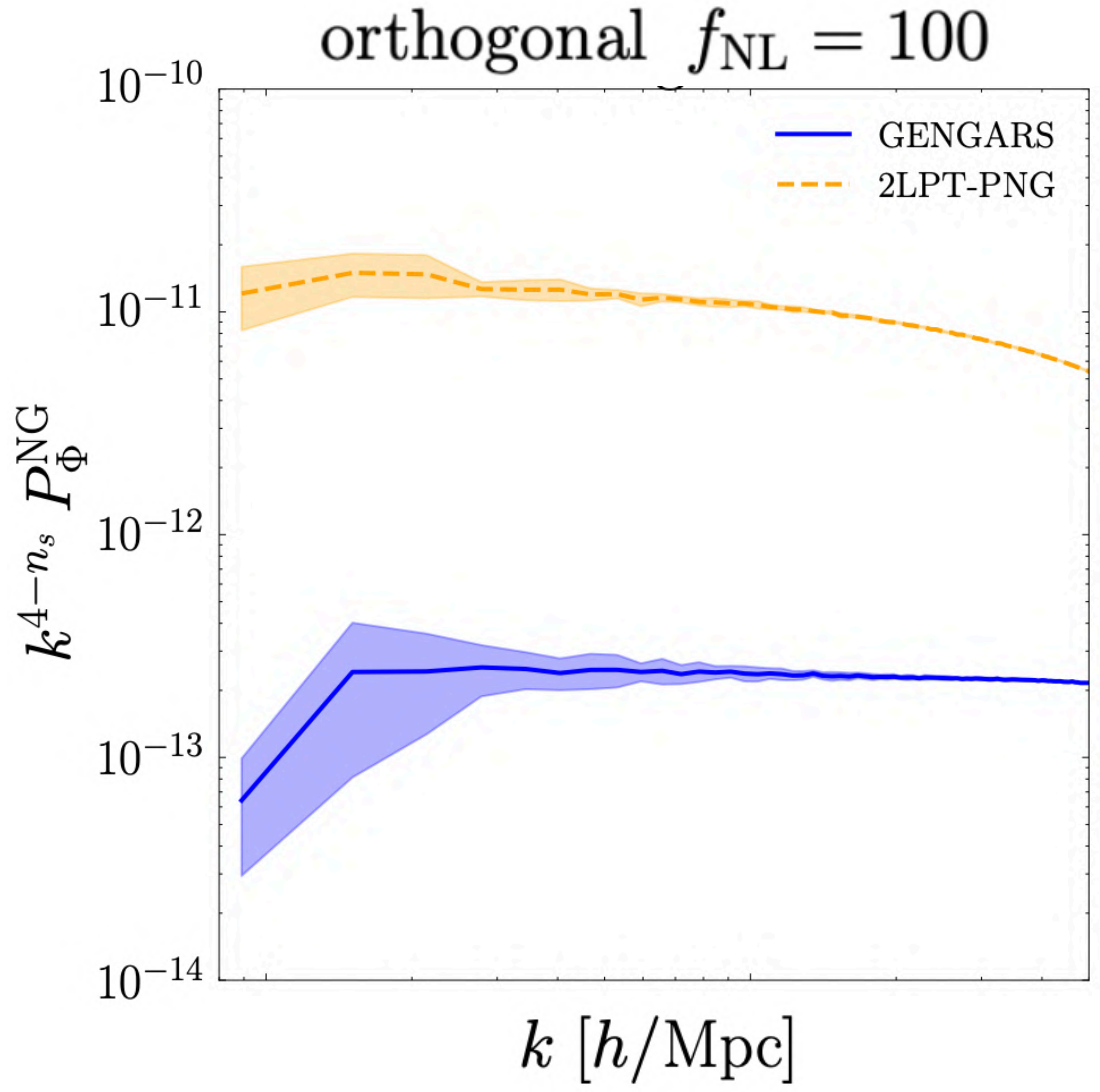
- **GENGARS (GEnerator of Non-Gaussian Arbitrary Shapes)** arXiv:2508.01855
- **Computational scaling** $\mathcal{O}(N^2) \rightarrow \mathcal{O}(N \log N)$ **(~30 min for** $N = 512^3$**)**

Comparison with the state of the art

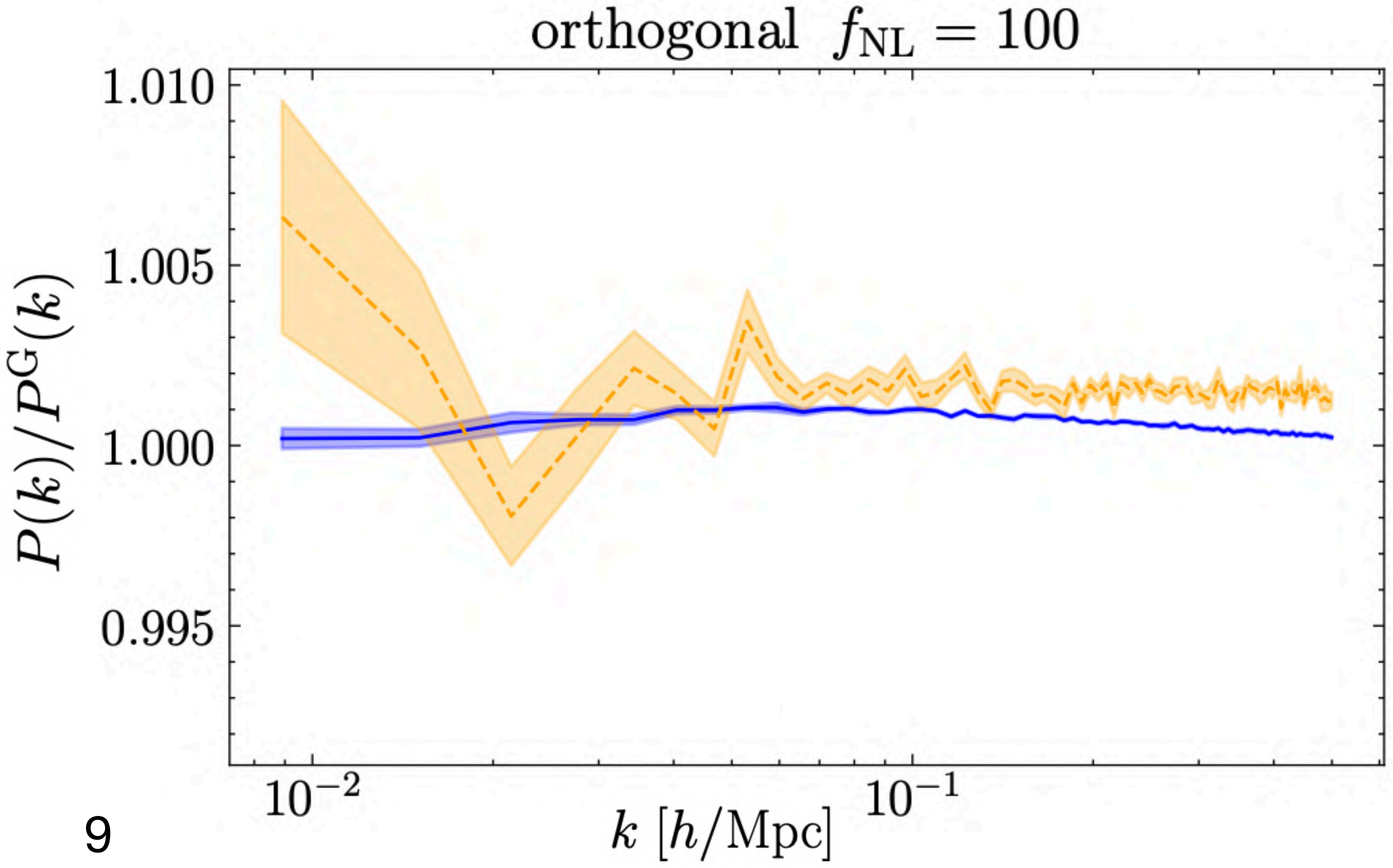
- **Generate ICs with the same parameters as Quijote-PNG simulations**
  ⟶ comparison with 2LPT-PNG Scoccimarro et al. 2012

- **Orthogonal template should not contribute to the large-scale $P(k)$**

Initial conditions



Dark matter field at $z = 0$

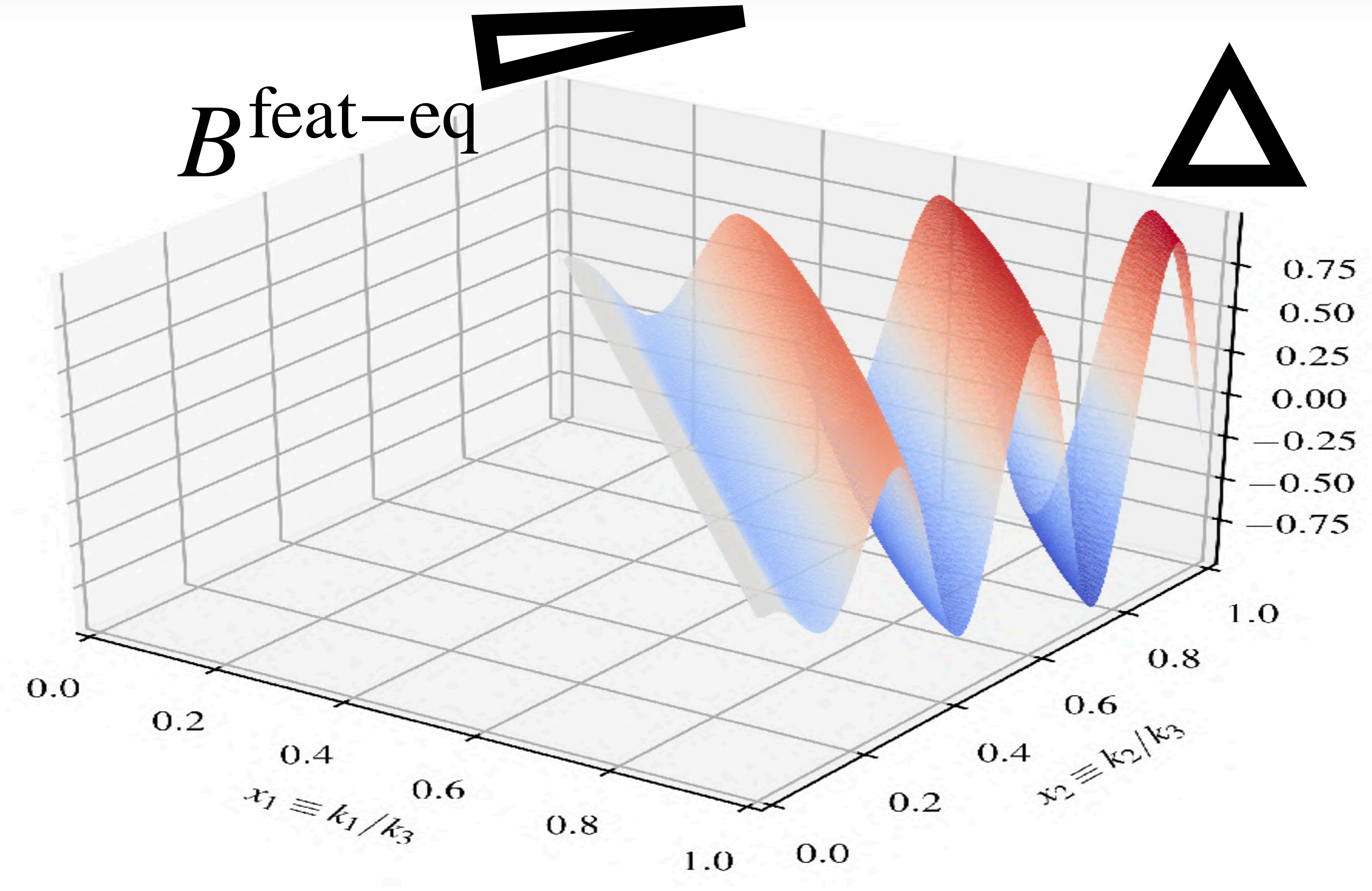


Oscillatory feature in the initial conditions

Equilateral oscillatory feature

$$B^{\rm feat-eq}(k_1,k_2,k_3) \equiv S^{\rm eq}(k_1,k_2,k_3) \times B^{\rm feat}(k_1,k_2,k_3)$$

where $B^{\rm feat}(k_1,k_2,k_3) = \dfrac{6A^2 f_{\rm NL}^{\rm feat}}{(k_1k_2k_3)^2} \sin\left[\omega(k_1+k_2+k_3)+\phi\right]$



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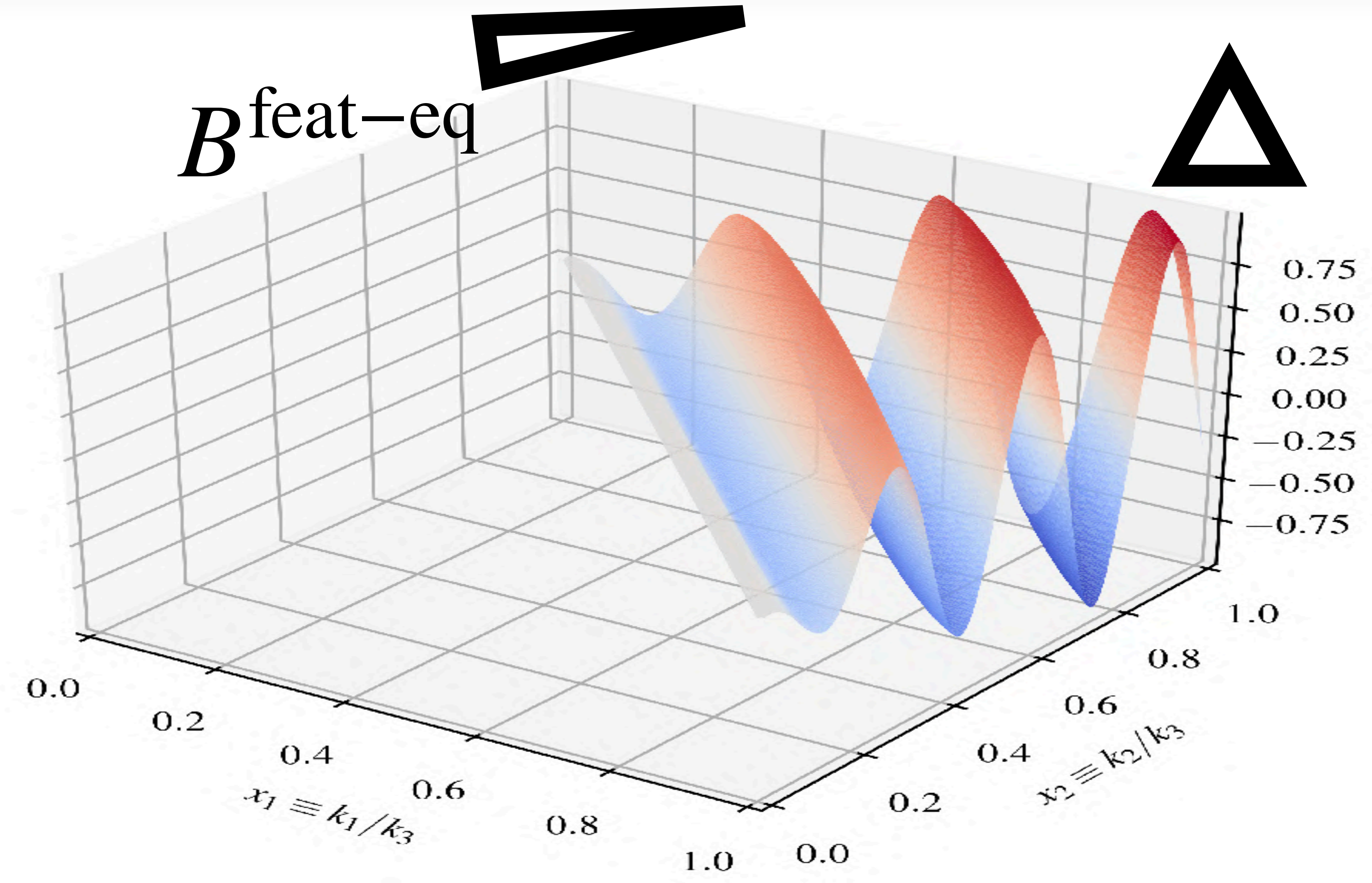

- **Template closely related with models with sharp features in the inflaton potential** Chen et al. 2007

- **Can be written in a separable form using trigonometric identities**

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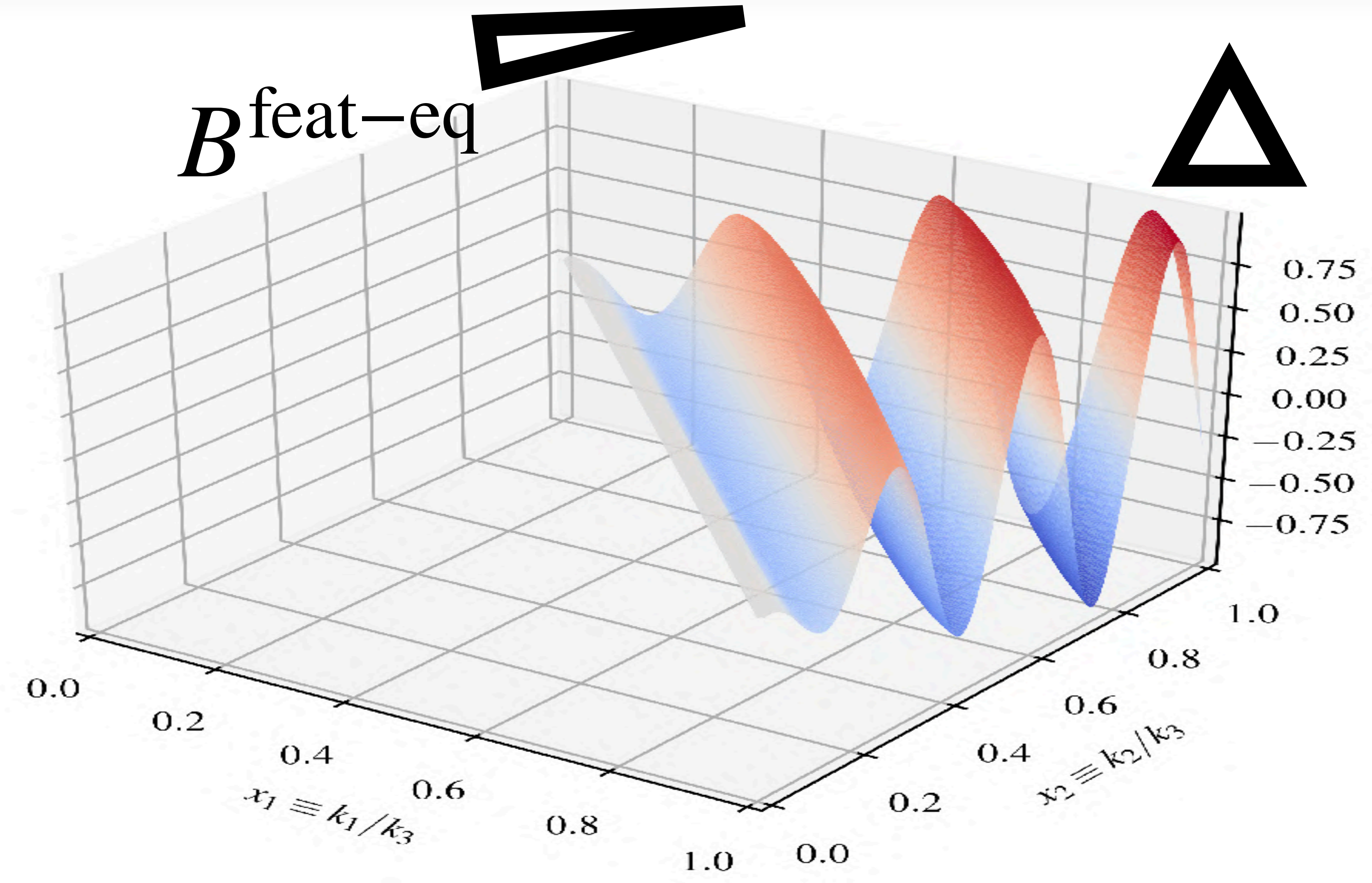
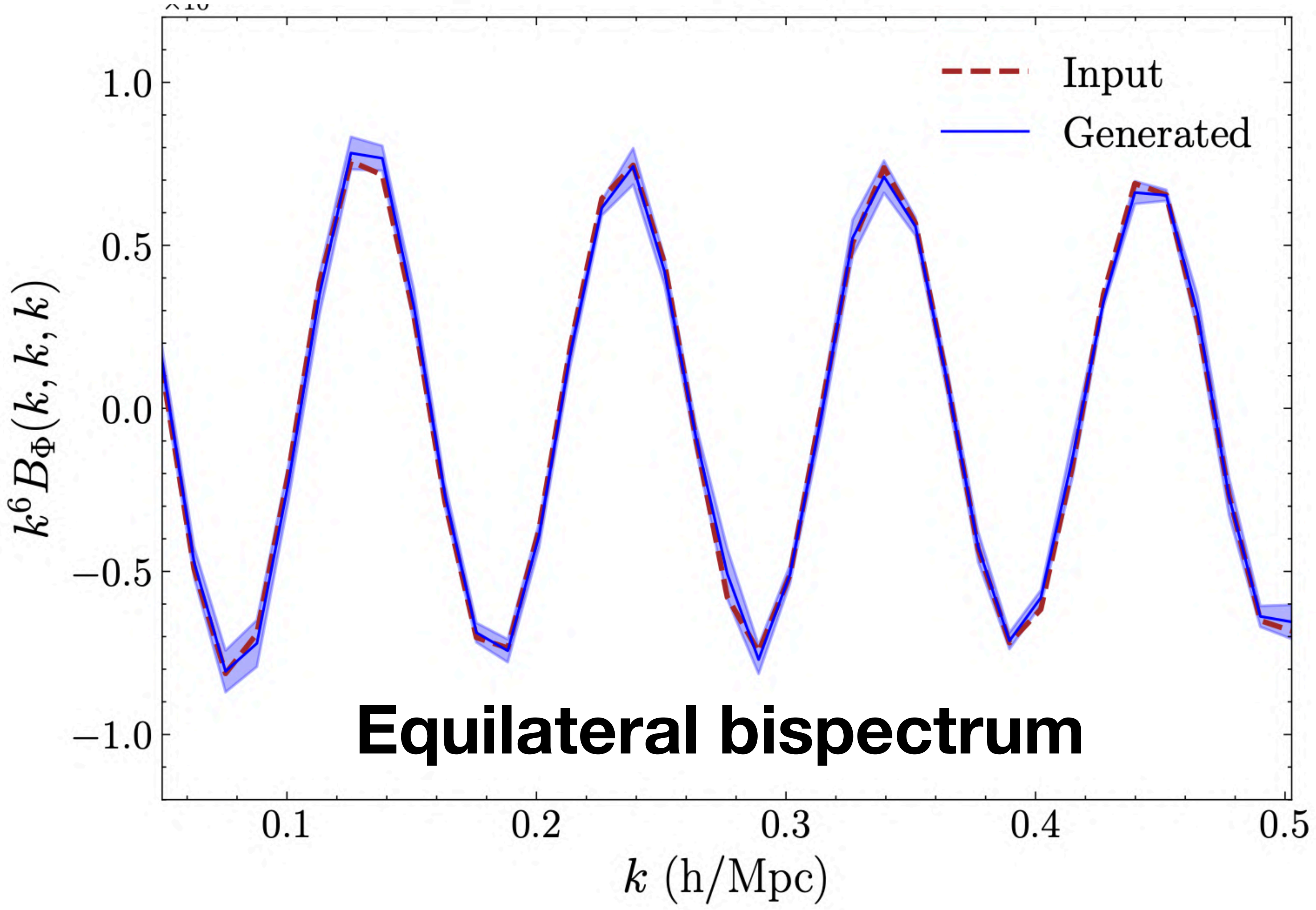


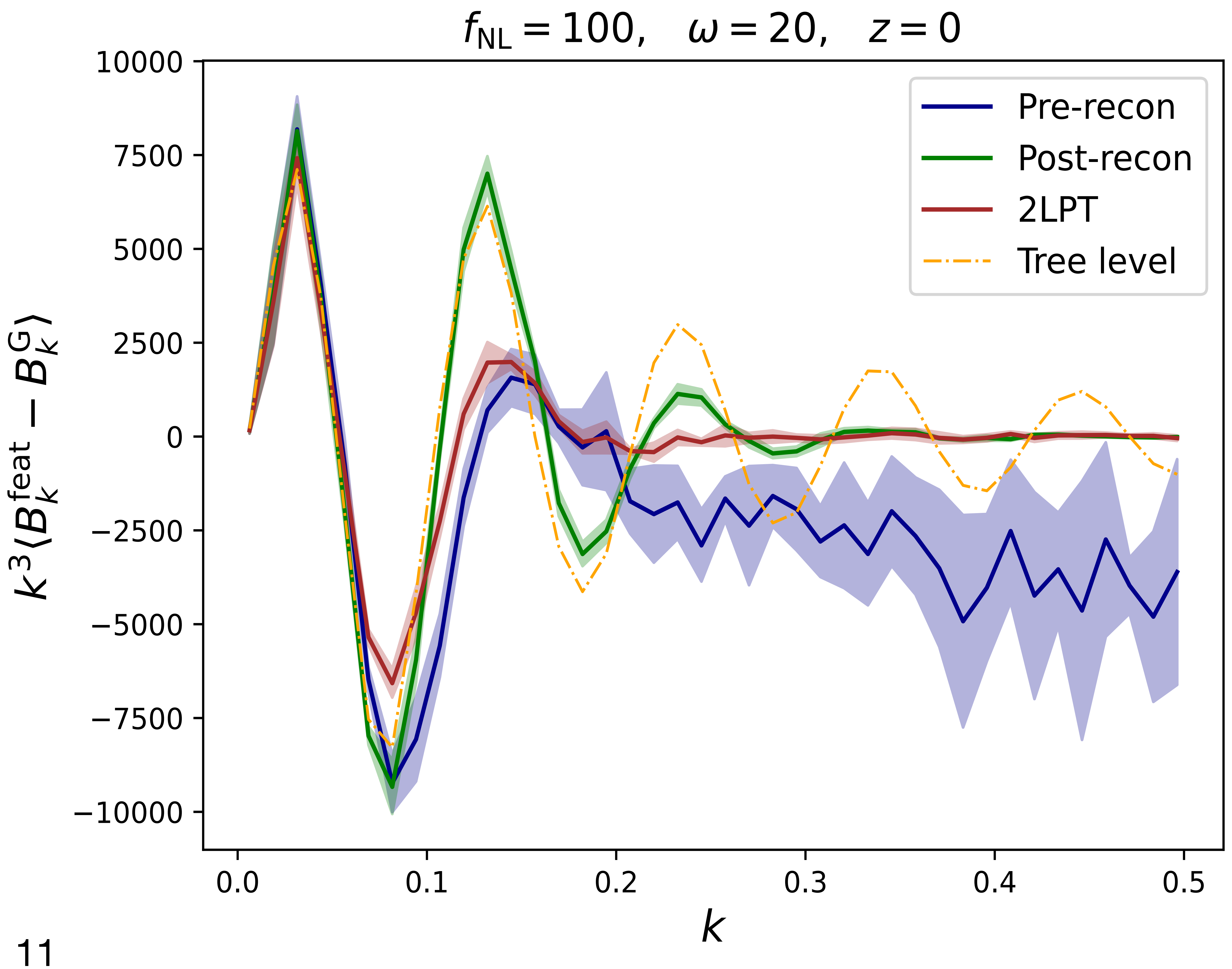


Nonlinear evolution of primordial oscillations

- **Main contribution is the gravitational bispectrum**
  ⟶ Evaluate it from the paired Gaussian simulation and subtract

- **Oscillations are damped in the non-linear regime by large-scale bulk flows**

Equilateral matter bispectrum



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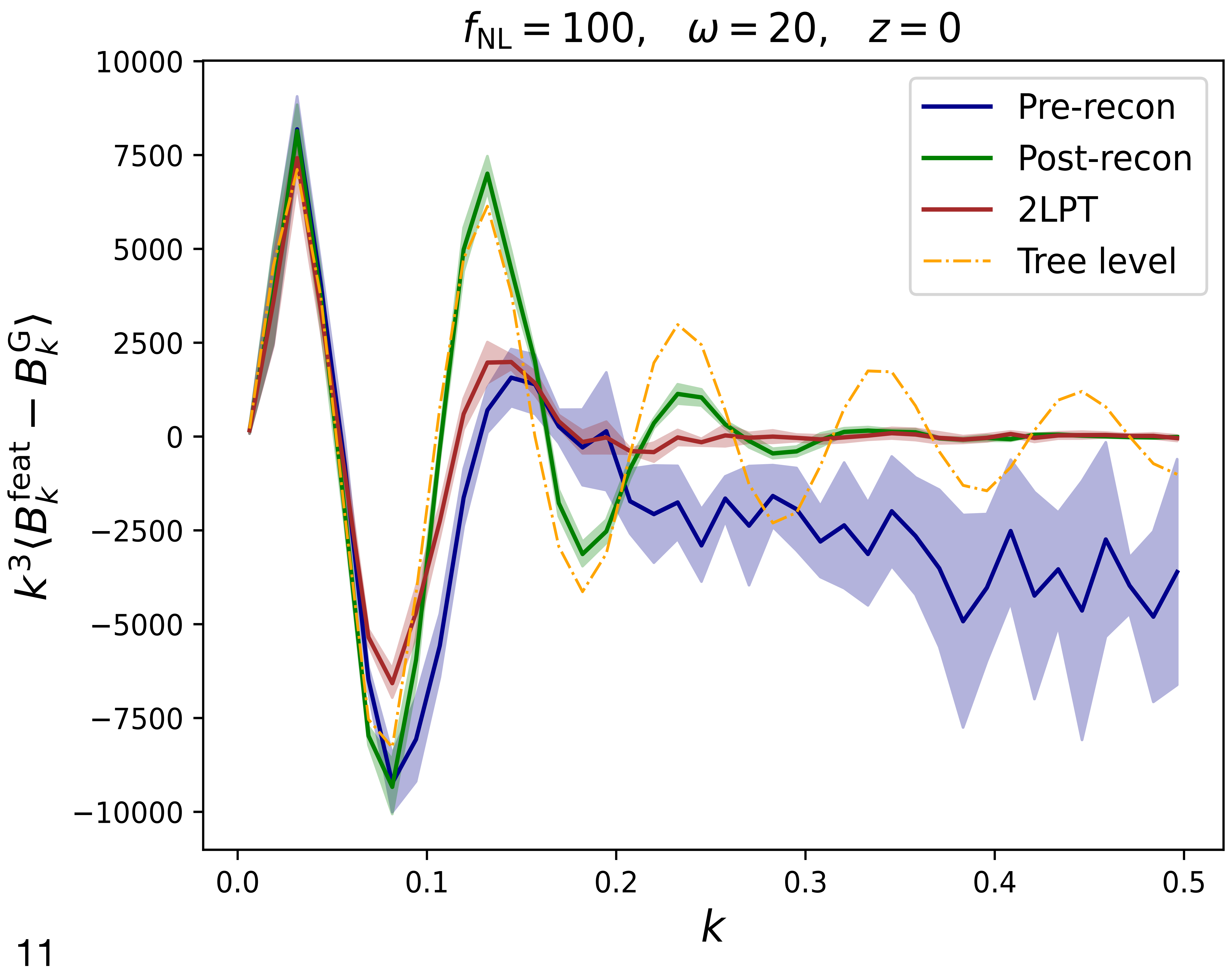

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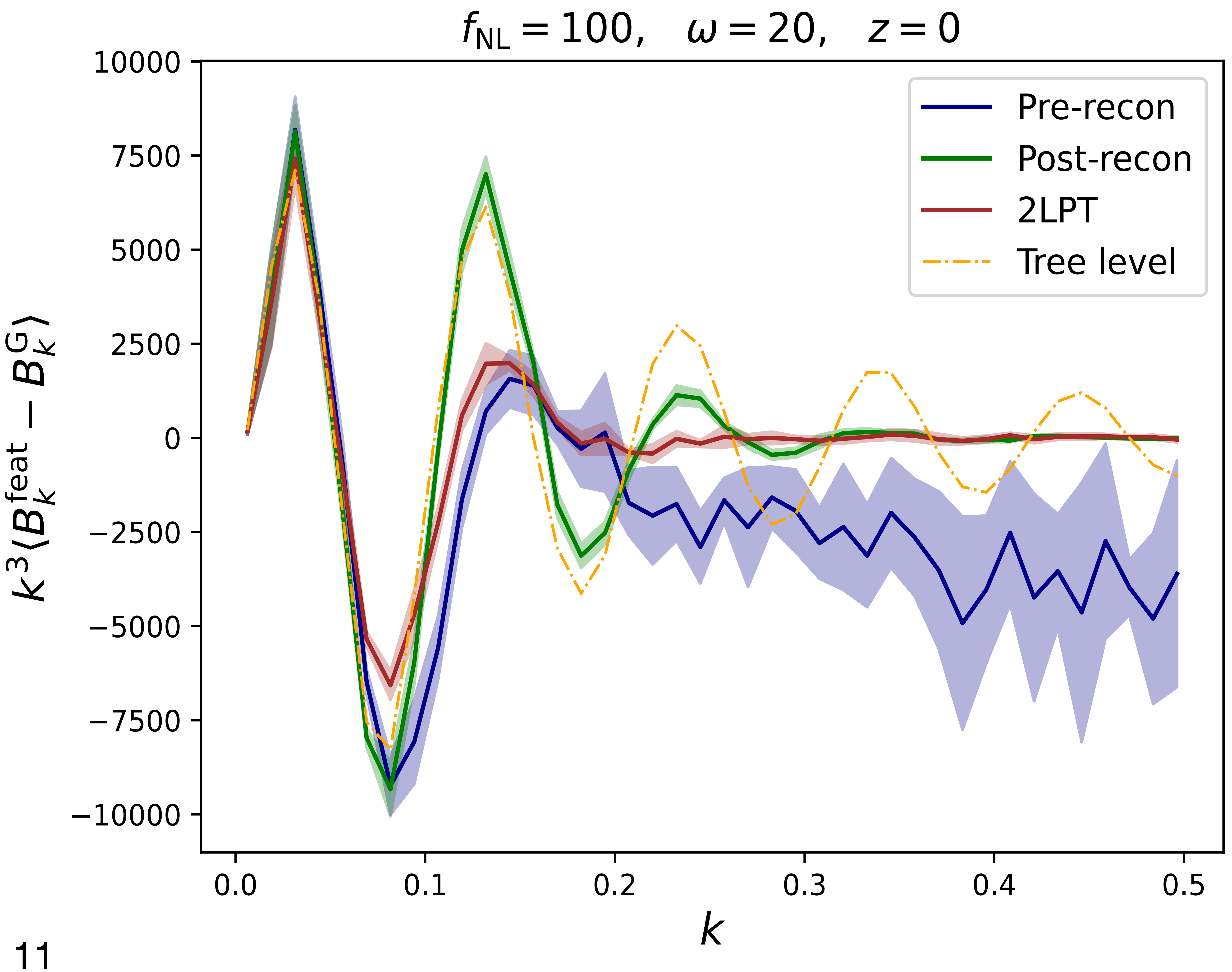
- **Reconstruction allows to recover much of the oscillatory signal (see also** Goldstein, Philcox, EF, Coulton 2025**)**

Equilateral matter bispectrum



Conclusions

- Primordial non-Gaussianity represents a window into the physics of inflation
- Cosmological simulations help us identifying signatures of PNG on LSS and assess their detectability

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- **GENGARS exploits a universal kernel W to generate accurate initial conditions for arbitrary separable templates, improving the implementation of** Wagner&Verde 2012 **by orders of magnitude**

Conclusions

- **Primordial non-Gaussianity represents a window into the physics of inflation**
- **Cosmological simulations help us identifying signatures of PNG on LSS and assess their detectability**
- **GENGARS exploits a universal kernel W to generate accurate initial conditions for arbitrary separable templates, improving the implementation of** Wagner&Verde 2012 **by orders of magnitude**
- **Further improvements (GPU-porting, JAX implementation) would allow to reduce runtime and natural integration with simulation-based inference pipelines**

Thank you!

Additional slides

Schmidt, Kamionkowski 2010

$$\Phi(\mathbf{k}) = \Phi^{\mathrm{G}}(\mathbf{k}) + f_{\mathrm{NL}} \int \frac{d^3k'}{(2\pi)^3} W(k, k', |\mathbf{k}+\mathbf{k}'|) \Phi^{\mathrm{G}*}(\mathbf{k}') \Phi^{\mathrm{G}}(\mathbf{k}+\mathbf{k}')$$

Example: local shape $W \equiv 1$

Its bispectrum reads $B_\Phi(k_1, k_2, k_3) = 2 f_{\mathrm{NL}} [W(k_1, k_2, k_3) P_\Phi(k_1) P_\Phi(k_2) + \mathrm{cyc.}]$.

Inverse problem for W ⟶ not a unique solution

Assumption:
separable kernel

$$W(k_1, k_2, k_3) = \sum_{i=1}^{N_i} w_1^i(k_1) w_2^i(k_2) w_3^i(k_3)$$

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Its bispectrum reads

$$B_\Phi(k_1, k_2, k_3) = 2 f_{NL} [W(k_1, k_2, k_3)] P_\Phi(k_1) P_\Phi(k_2) + \text{cyc.}].$$

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$\mathcal{O}(N^2) \to \mathcal{O}(N \log N)$

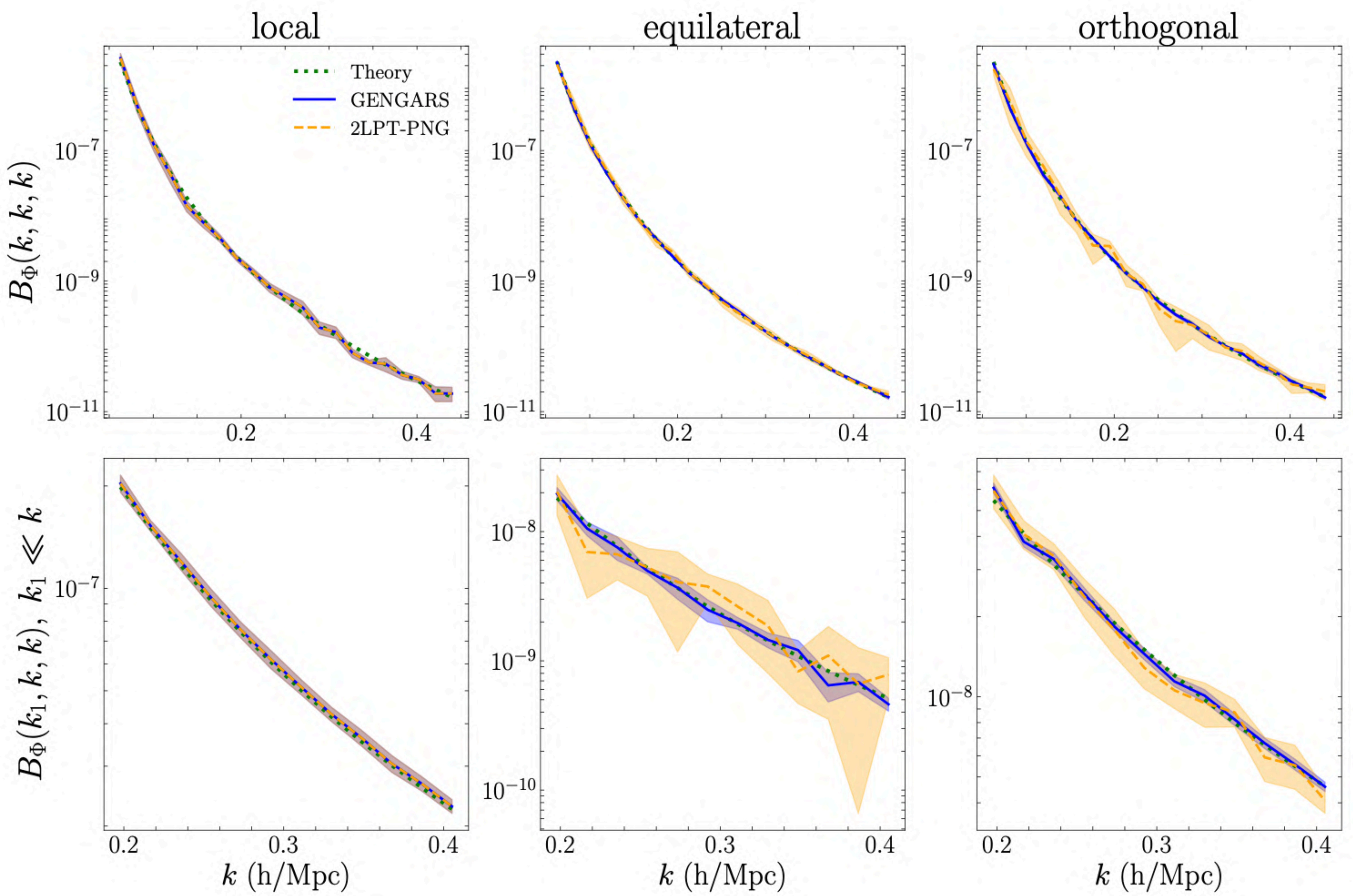
Additional slides

Equilateral template

$$B_\Phi^{\rm eq}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm eq}\Big[-\left(P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perm.}\right) - 2\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + \left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5 \text{ perm.}\right)\Big]$$

Orthogonal template

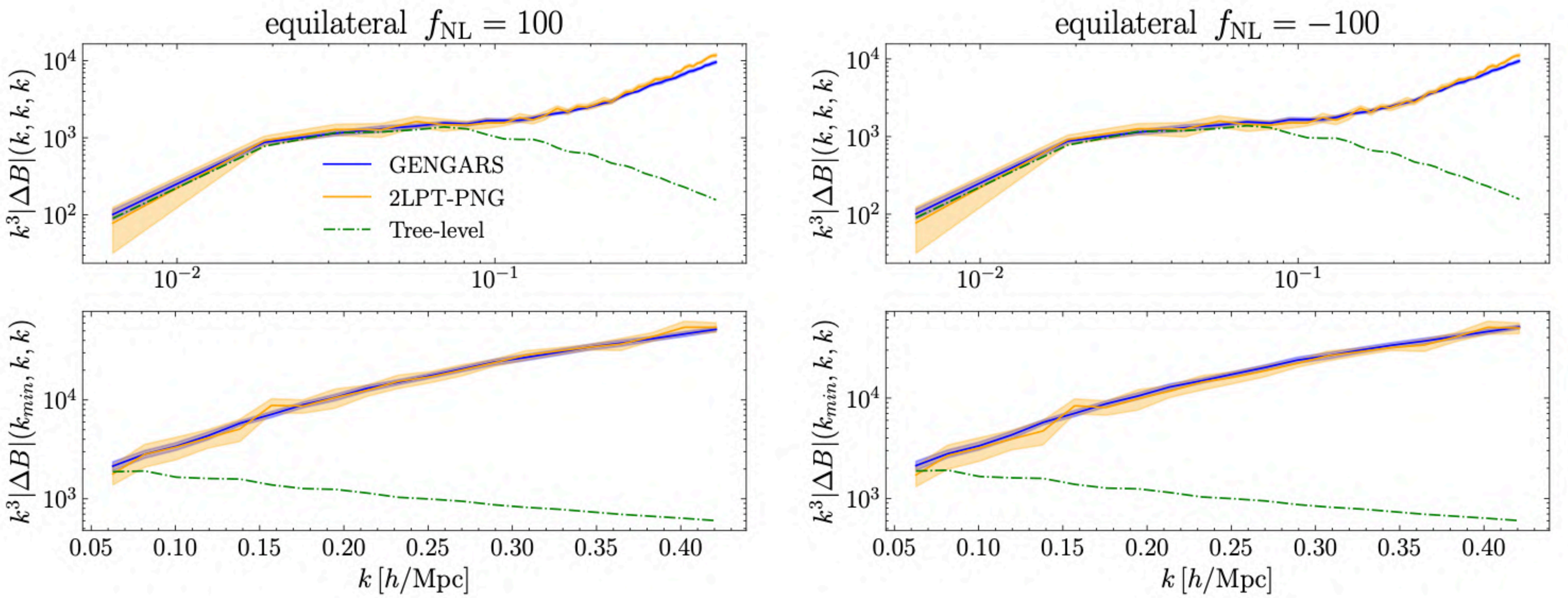
$$B_\Phi^{\rm ort}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm ort}\Big[-3\left(P_\Phi(k_1)P_\Phi(k_2) + 2 \text{ perm.}\right) - 8\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + 3\left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5 \text{ perm.}\right)\Big]$$



Additional slides

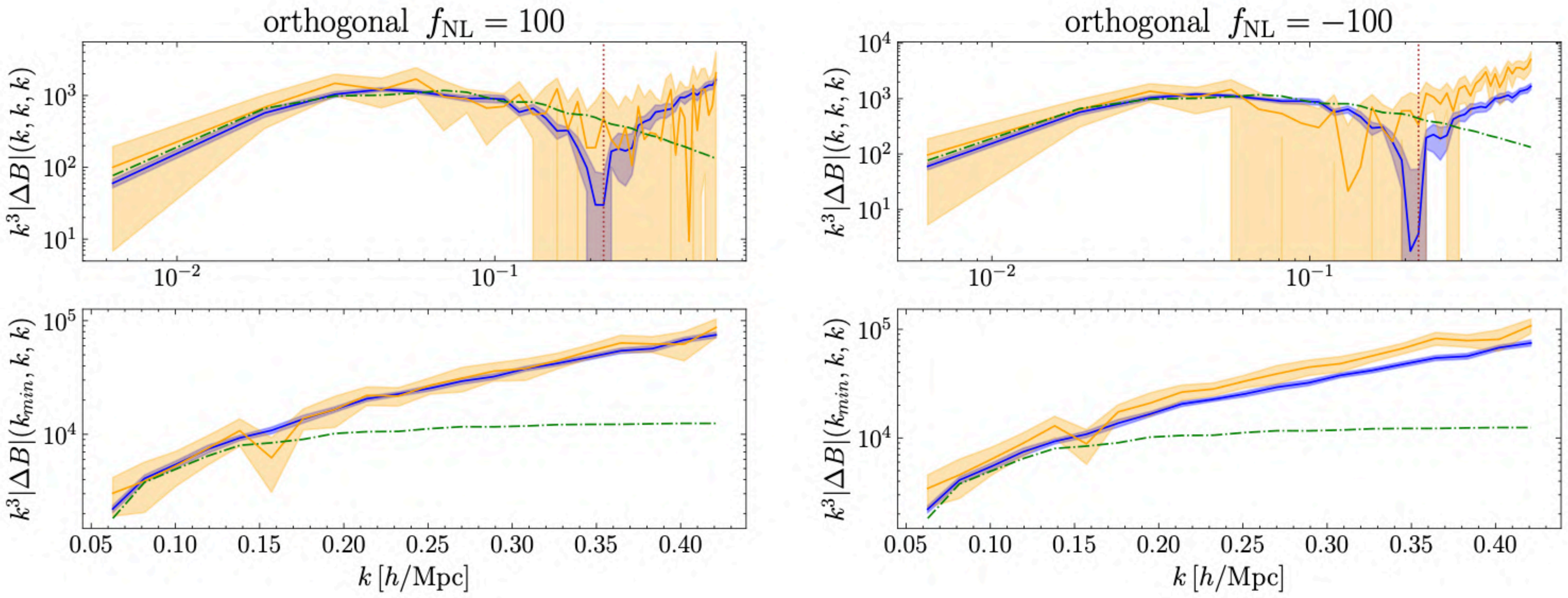
Equilateral template

$$B_\Phi^{\rm eq}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm eq}\Big[-\left(P_\Phi(k_1)P_\Phi(k_2) + 2\ \text{perm.}\right) - 2\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + \left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ \text{perm.}\right)\Big]$$



Orthogonal template

$$B_\Phi^{\rm ort}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm ort}\Big[-3\left(P_\Phi(k_1)P_\Phi(k_2) + 2\ \text{perm.}\right) - 8\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + 3\left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ \text{perm.}\right)\Big]$$



Additional slides

Equilateral template

$$B_\Phi^{\rm eq}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm eq}\Big[-\left(P_\Phi(k_1)P_\Phi(k_2) + 2\ \text{perm.}\right) - 2\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + \left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ \text{perm.}\right)\Big]$$

Orthogonal template

$$B_\Phi^{\rm ort}(k_1,k_2,k_3) = 6 f_{\rm NL}^{\rm ort}\Big[-3\left(P_\Phi(k_1)P_\Phi(k_2) + 2\ \text{perm.}\right) - 8\left(P_\Phi(k_1)P_\Phi(k_2)P_\Phi(k_3)\right)^{2/3} + 3\left(P_\Phi(k_1)^{1/3}P_\Phi(k_2)^{2/3}P_\Phi(k_3) + 5\ \text{perm.}\right)\Big]$$