Parity-odd signal with kurto-spectrum

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based on arXiv: 2509.13207

Sources of parity-odd in LSS

Primordial:

- single-field with non-Bunch-Davies vaccum (ghost inflation)
- cosmological collider with exchange of massive spinning fields
- Inflation with chiral gauge source (Chern-Simons)
- Helical primordial magnetic fields
- etc…

How can constrain these from the late-time Universe?

Trispectrum

Trispectrum (4-pt correlation function) is the leading-order quantity where PO signal appears.

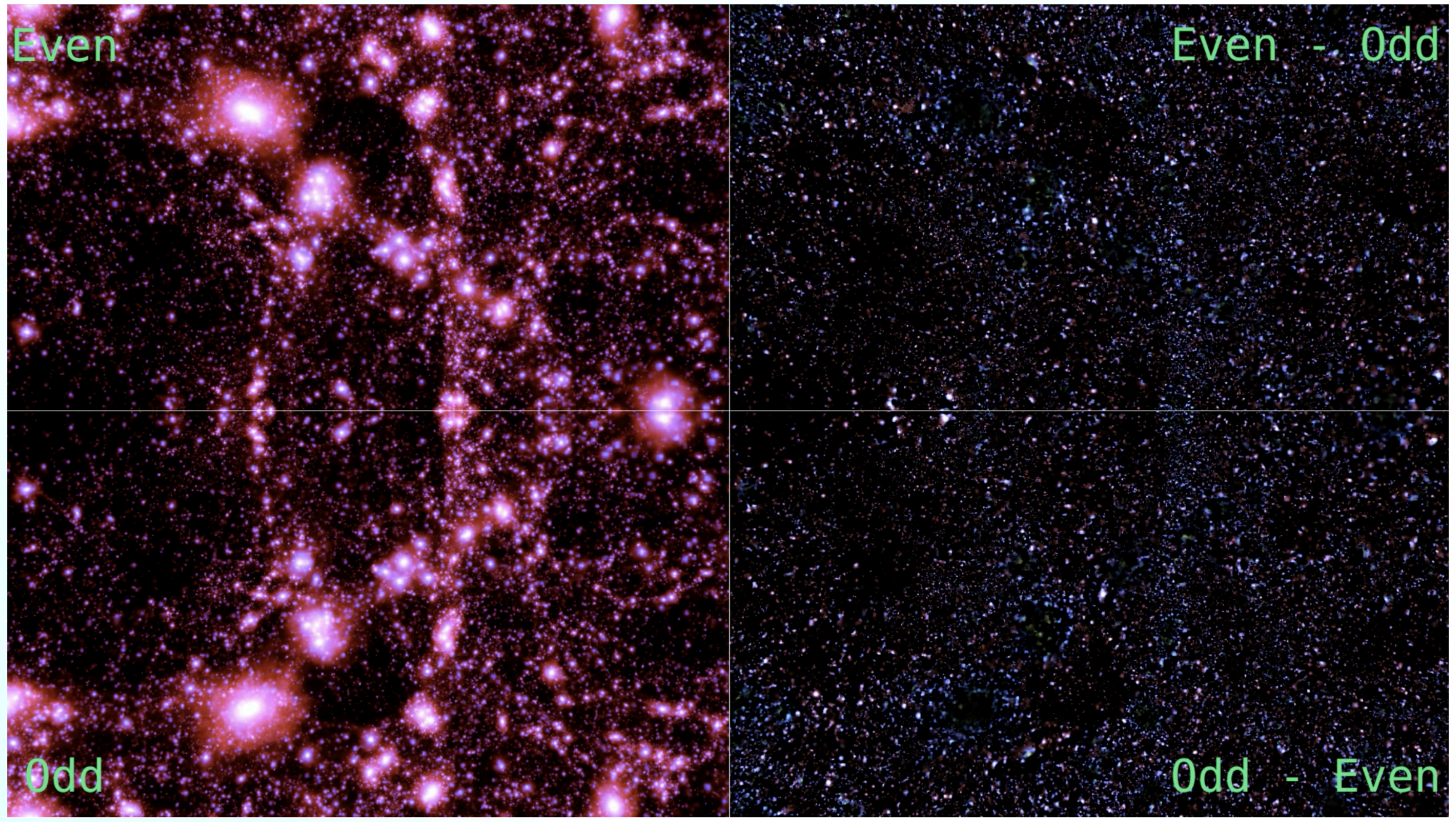
Example model:

$$\phi(\vec{k}) = \phi_{\rm G}(\vec{k}) + i\, g_{\rm NL}^{\rm PO} \int_{\vec{q}_1,\vec{q}_2,\vec{q}_3} (2\pi)^3 \delta_D(\vec{k} - \vec{q}_{123})\, K^{(3)}_{\rm NL,PO}(\vec{q}_1, \vec{q}_2, \vec{q}_3)\, \phi_{\rm G}(\vec{q}_1)\phi_{\rm G}(\vec{q}_2)\phi_{\rm G}(\vec{q}_3),$$

$$K^{(3)}_{\rm NL,PO}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \;=\; \vec{k}_1 \cdot \left(\vec{k}_2 \times \vec{k}_3\right) S(k_1, k_2, k_3)\,.$$

$$T_\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = \; i\, g_{\rm NL}^{\rm PO}\; K^{(3)}_{\rm NL,PO}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\; P_\phi(k_1)\, P_\phi(k_2)\, P_\phi(k_3) \;+\; 23 \text{ perms},$$

$$T_m^{\rm PNG}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4, z) = \prod_{i=1}^{4} \mathcal{M}(k_i, z) T_\phi(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4), \qquad \delta_g(\vec{k}, z) = b_1(z)\delta_m(\vec{k}, z), \qquad T_g^{\rm PNG}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4, z) = b_1^4(z) T_m(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4, z)$$



Quijote-ODD used in this work:

1 (Gpc/h)^3 boxes at z=0,1

500 realizations for each g

$g_{\rm NL}^{\rm PO} = \pm\, 10^6 \qquad \bar{n} = 1.38 \times 10^{-5}\ (h/{\rm Mpc})^3$

Quijote-ODD sim: Coulton+2023

Trispectrum

Trispectrum (4-pt correlation function) is the leading-order quantity where PO signal appears.

Previous work on BOSS data:

- About 7-sigma detection of PO trispectrum (Philcox 2022; Hou+2023)
- The large sigma is originated from data mock mismatch (Krolewski+2024)

and obstacles:

- The covariance estimation is hard (using analytical method, due to the large number of degree of freedom)
- mis-evaluation of covariance will result in the false signal detection

Composed-field spectra

power-spectrum-like observable + easier covariance estimation

Schmittfull+2014
Abidi+2018
Moradinezhad+2019
Schmittfull and Moradinezhad+2020

$$\mathscr{D}_n[\delta](\vec{x}) = \prod_{i=1}^{n} \mathscr{D}_i(\vec{x})\delta(\vec{x}),$$

$$\langle D_n[\delta](\vec{k}) D_m[\delta](\vec{k}')\rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')\mathscr{P}_{D_n[\delta],D_m[\delta]}(\vec{k})\,.$$

(n+m spectrum)

In simulation, the field needs to be filtered by W_R(k)

tailored to pick-up certain signal

Composed-field spectra for PO (Kurto spectrum)

A physical construction

$$\phi(\vec{k}) = \phi_{\rm G}(\vec{k}) + i\, g_{\rm NL}^{\rm PO} \int_{\vec{q}_1,\vec{q}_2,\vec{q}_3} (2\pi)^3 \delta_D(\vec{k}-\vec{q}_{123})\, K^{(3)}_{\rm NL,PO}(\vec{q}_1,\vec{q}_2,\vec{q}_3)\, \phi_{\rm G}(\vec{q}_1)\phi_{\rm G}(\vec{q}_2)\phi_{\rm G}(\vec{q}_3),$$

$$K^{(3)}_{\rm NL,PO}(\vec{k}_1,\vec{k}_2,\vec{k}_3) \;=\; \vec{k}_1\cdot\left(\vec{k}_2\times\vec{k}_3\right) S(k_1,k_2,k_3)\,.$$

One Levi-Civita in the PO field

To pick up only scalar parity-odd, we need one additional Levi-Civita, otherwise the angular averaging over k-modes will result zero.

Vector x pseudo-vector

$$\mathscr{P}^{\rm PO}_{2\times2}(k) = \int \frac{d\Omega_k}{4\pi} \langle \vec{V}^{ab}(\vec{k}) \cdot \vec{A}^{cd}(-\vec{k})\rangle,$$

$$V_i^{ab}(\vec{x}) = \delta^a(\vec{x})\partial_i\delta^b(\vec{x})$$

$$A_i^{cd}(\vec{x}) = \epsilon_{ijk}\partial_j\partial^{-2}V_k^{cd}(\vec{x})$$

$$\langle\delta\delta\delta\delta\rangle = \langle\delta_L\delta_L\delta_L\delta_p\rangle + \text{NLO}$$

Jamieson+2024: primordial only verified the effectiveness of the estimator

Scalar x pseudo-scalar

$$\mathscr{P}^{\rm PO}_{3\times1}(k) = \int \frac{d\Omega_k}{4\pi} \langle \delta(\vec{k}_2)\Psi^{abc}(\vec{k}_1)\rangle$$

$$\Psi^{abc}(\vec{x}) = \nabla\delta^a(\vec{x}) \cdot \vec{B}^{bc}(\vec{x})$$

$$\vec{B}^{bc}(\vec{x}) = \nabla\delta^b(\vec{x}) \times \nabla\delta^c(\vec{x})$$

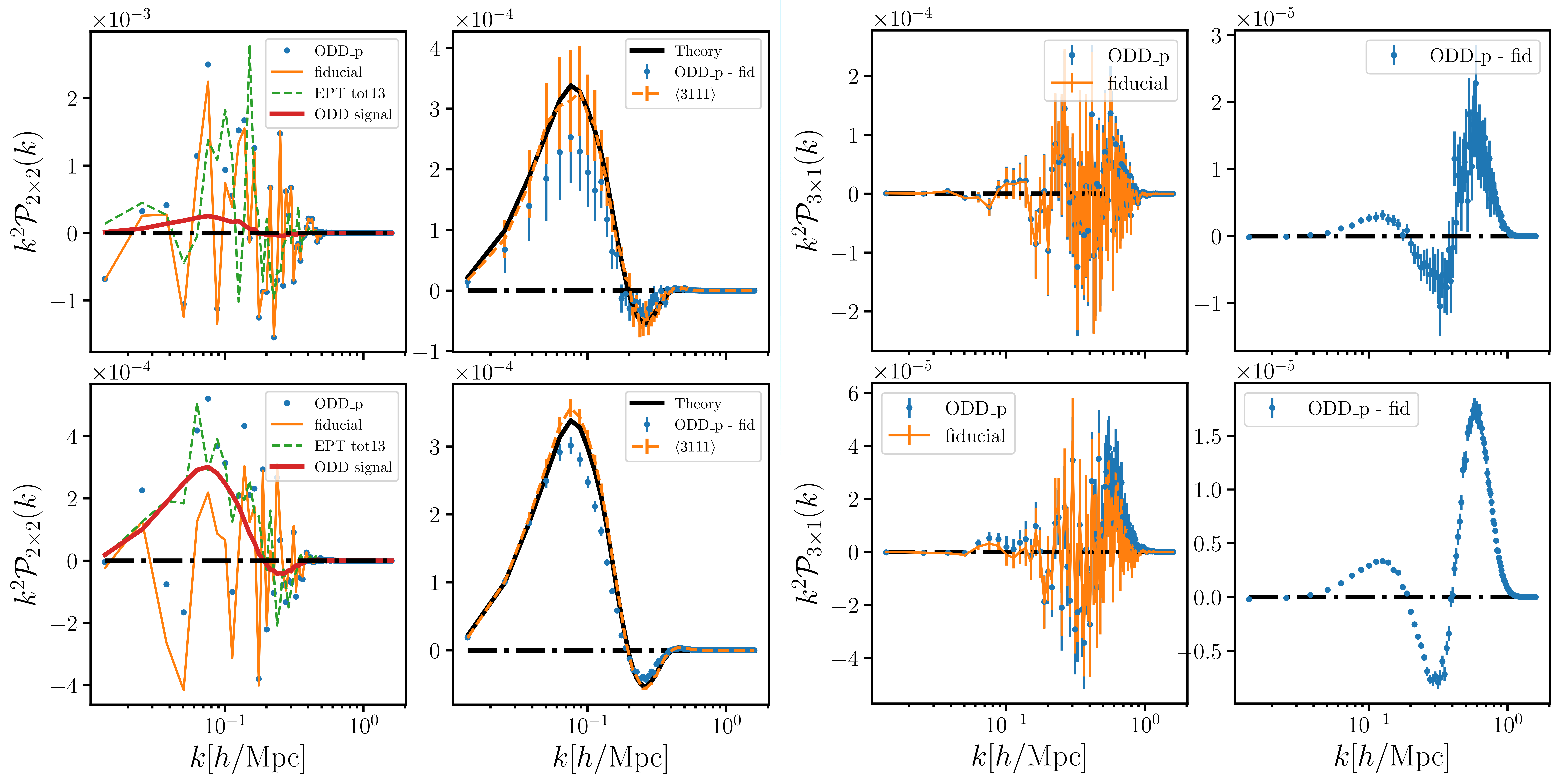
The parity-even part is expected to be zero

Measurement on matter fields:

Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]

1 (Gpc/h)^3

40 (Gpc/h)^3



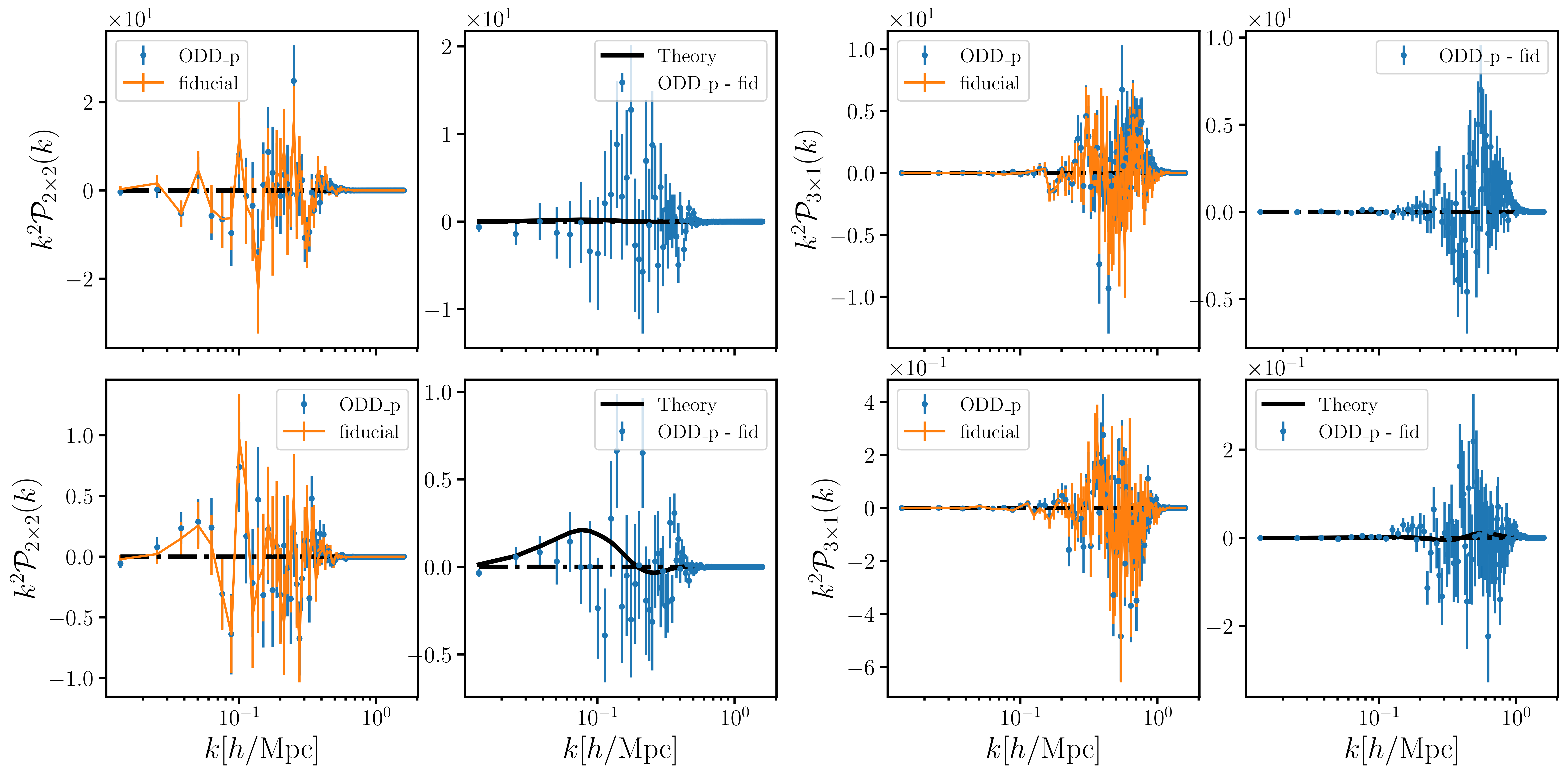
The PE noise dominates over PO signal, which goes to similar level with 40 realisations

Measurement on the halo fields:

Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]

1 (Gpc/h)^3

500 (Gpc/h)^3



PE noise dominates more over PO signal;

PO influence the field phase

Composed field spectra revisit

Optimal estimator construction

Optimal estimator for amplitude of trispectrum:

$$\hat{A} = \frac{1}{N^T_{\rm th}} \int_{\mathbf{k}_1}\int_{\mathbf{k}_2}\int_{\mathbf{k}_3}\int_{\mathbf{k}_4} \langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\delta(\mathbf{k}_4)\rangle_{\rm th} \Big[C^{-1}(\delta^{\rm obs}_{\mathbf{k}_1})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_2})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_3})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_4}) - 6\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_1}\delta^{\rm obs}_{\mathbf{k}_2})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_3})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_4}) + 3\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_1}\delta^{\rm obs}_{\mathbf{k}_2})\, C^{-1}(\delta^{\rm obs}_{\mathbf{k}_3}\delta^{\rm obs}_{\mathbf{k}_4}) \Big]. \tag{3.11}$$

Fergusson+2010

$$K_{\rm NL}(k_1, k_2, k_3) \quad \Big\langle \frac{\delta(k_1)P_L(k_1)}{P_{\rm obs}(k_1)\mathscr{M}(k_1)} \frac{\delta(k_2)P_L(k_2)}{P_{\rm obs}(k_2)\mathscr{M}(k_2)} \frac{\delta(k_3)P_L(k_3)}{P_{\rm obs}(k_3)\mathscr{M}(k_3)} \frac{\delta(k_4)}{P_{\rm obs}(k_4)} \Big\rangle$$

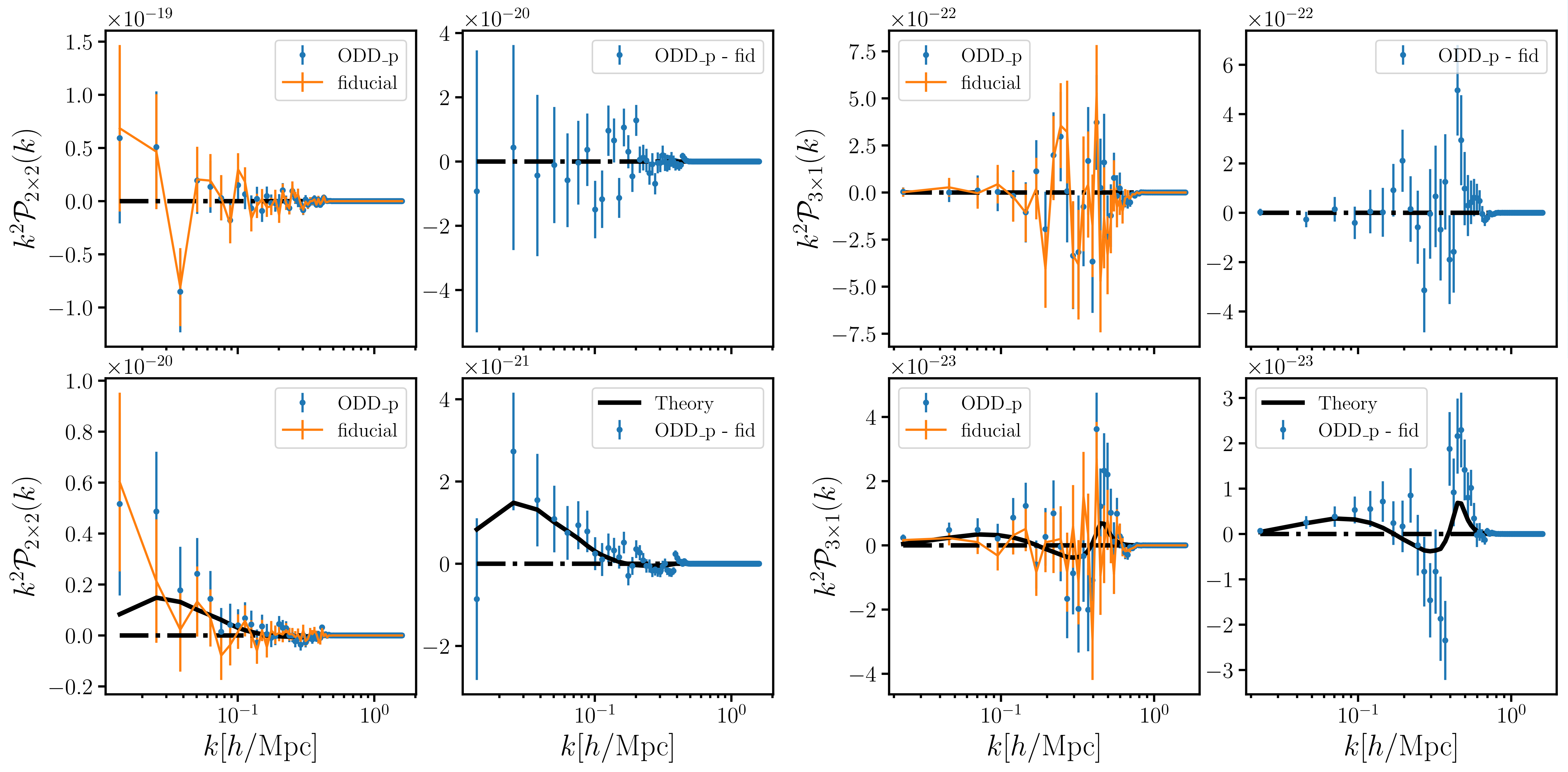
Additional filter on the overdensity field

Measurement on the halo fields with optimal estimator:

Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]

1 (Gpc/h)^3

500 (Gpc/h)^3



We have more obvious PO detection for 500 realizations

Detectability

Match-filtering amplitude fitting

$$\mathbf{d} = s_0\,\mathbf{f} + \mathbf{n}, \qquad \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{C})$$

$$\hat{s}_0 = \frac{\mathbf{f}^\top \mathbf{C}^{-1} \mathbf{d}}{\mathbf{f}^\top \mathbf{C}^{-1} \mathbf{f}} \qquad \sigma_{s_0} = \left[\mathbf{f}^\top \mathbf{C}^{-1} \mathbf{f}\right]^{-1/2}.$$

$$\nu \equiv \frac{\hat{s}_0}{\sigma_{s_0}} = \frac{\mathbf{f}^\top \mathbf{C}^{-1} \mathbf{d}}{\sqrt{\mathbf{f}^\top \mathbf{C}^{-1} \mathbf{f}}}$$

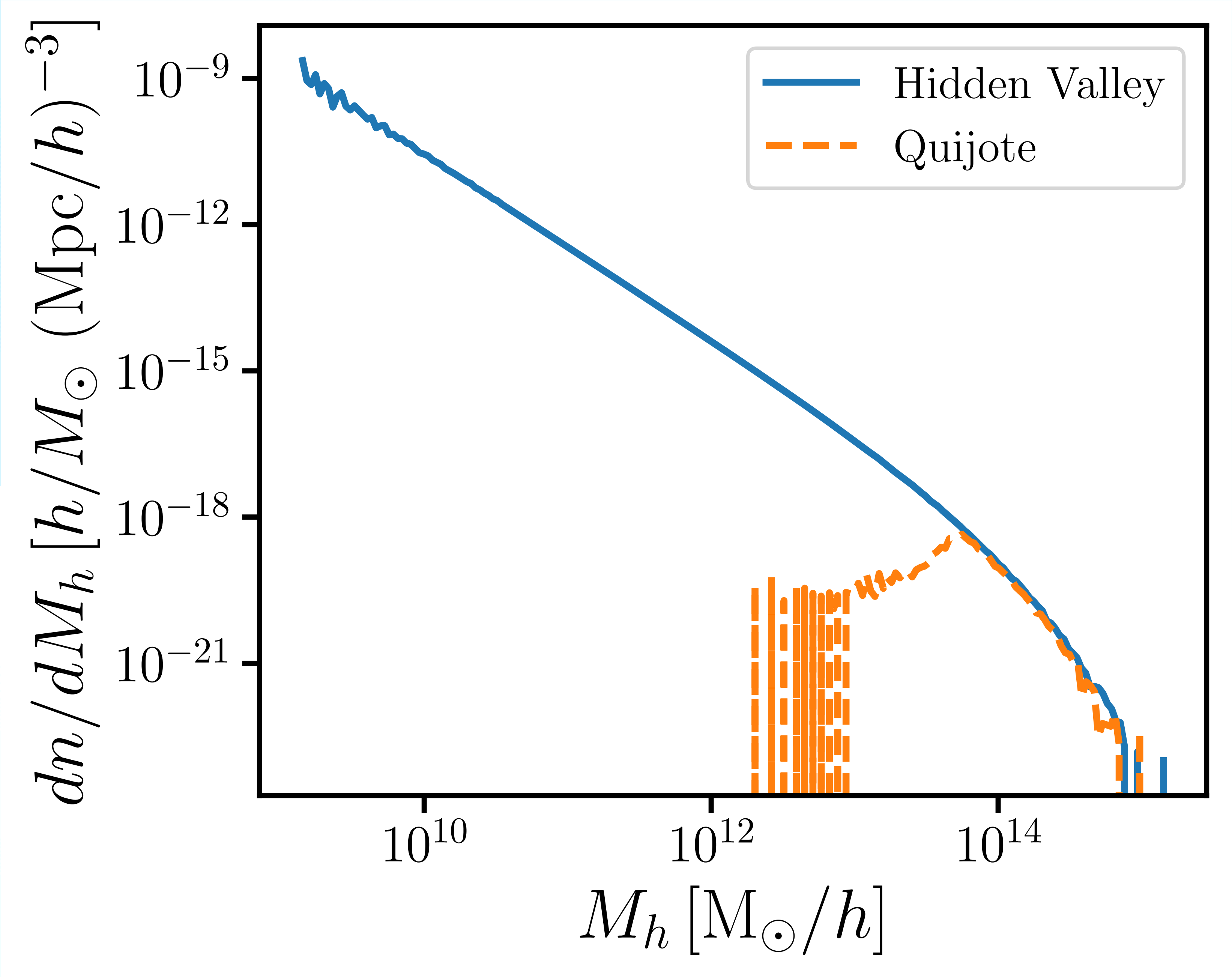
Table 1: Detectability on sigma-level.

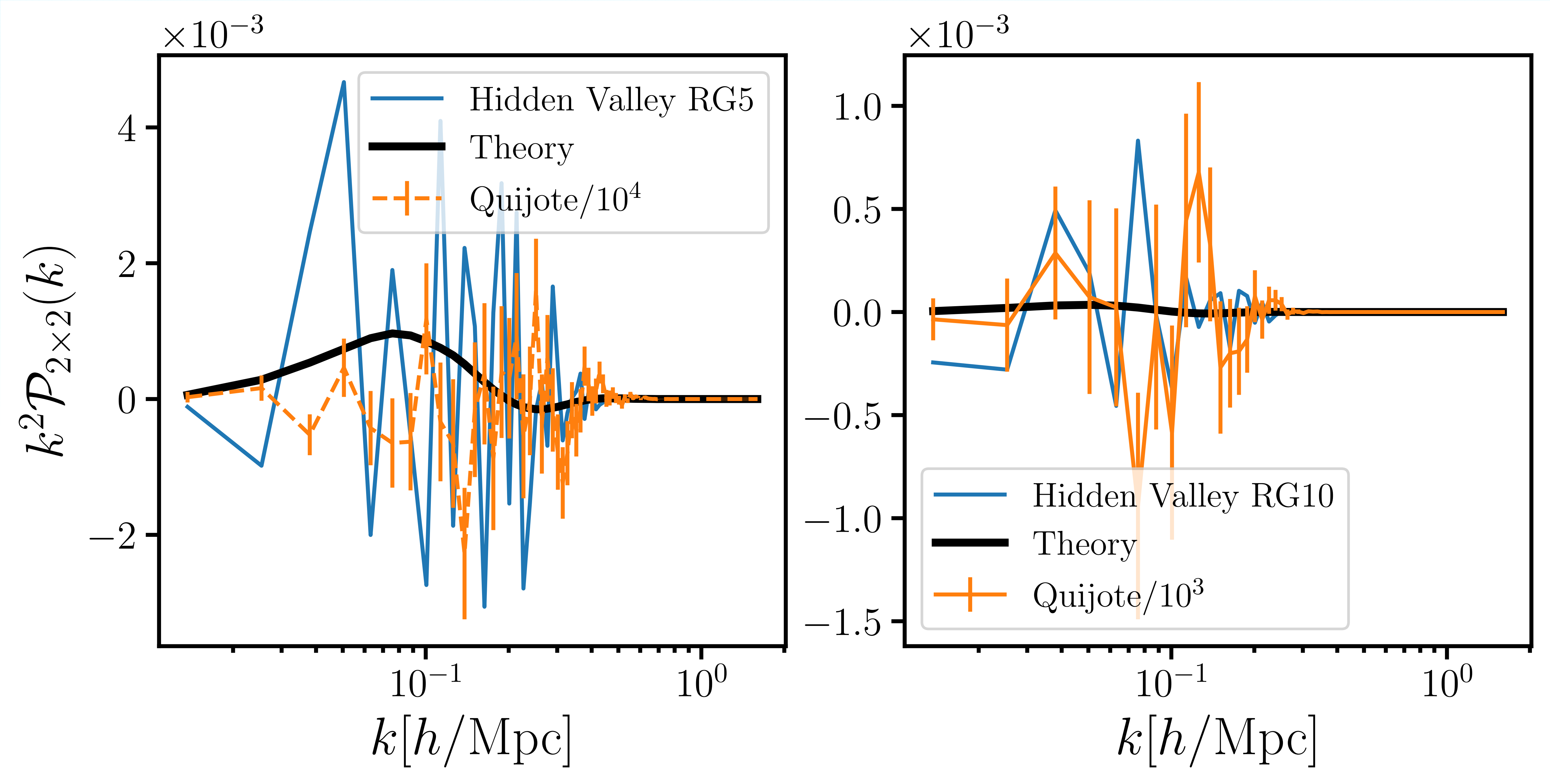
	po	po-fid	po (opt.)	po-fid (opt.)	po (opt.+tanh)	po-fid (opt.+tanh)
			$\mathcal{P}_{2\times 2}$			
m RG5 (40)	4.2	-	-	-	-	-
m RSD RG5 (40)	2.5	-	-	-	-	-
m RG10 (40)	0.0	-	-	-	-	-
m RSD RG10 (40)	1.5	-	-	-	-	-
h RG5 (500)	1.6	0.4	1.5	2.9	3.0	3.6
h RG10 (500)	0.5	2.6	-	-	-	-
h RSD RG10 (500)	0.0	1.9	-	-	-	-
h x m RG5 (500)	2.5	4.6	-	-	-	-
h x m RSD RG5 (500)	1.2	3.0	-	-	-	-
h x m RG10 (500)	2.5	4.2	-	-	-	-
h x m RSD RG10 (500)	2.2	4.2	-	-	-	-
			$\mathcal{P}_{3\times 1}$			
m RG5 (40)	8.0	-	-	-	-	-
m RSD RG5 (40)	4.0	-	-	-	-	-
m RG10 (40)	5.6	-	-	-	-	-
m RSD RG10 (40)	3.2	-	-	-	-	-
h RG5 (500)	1.7	2.3	2.0	3.6	3.5	4.2
h RG10 (500)	0.6	2.5	-	-	-	-
h RSD RG10 (500)	0.2	2.8	-	-	-	-

Source of PE noise

Null measurement on Hidden Valley simulation (Modi+ 2019)

Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]





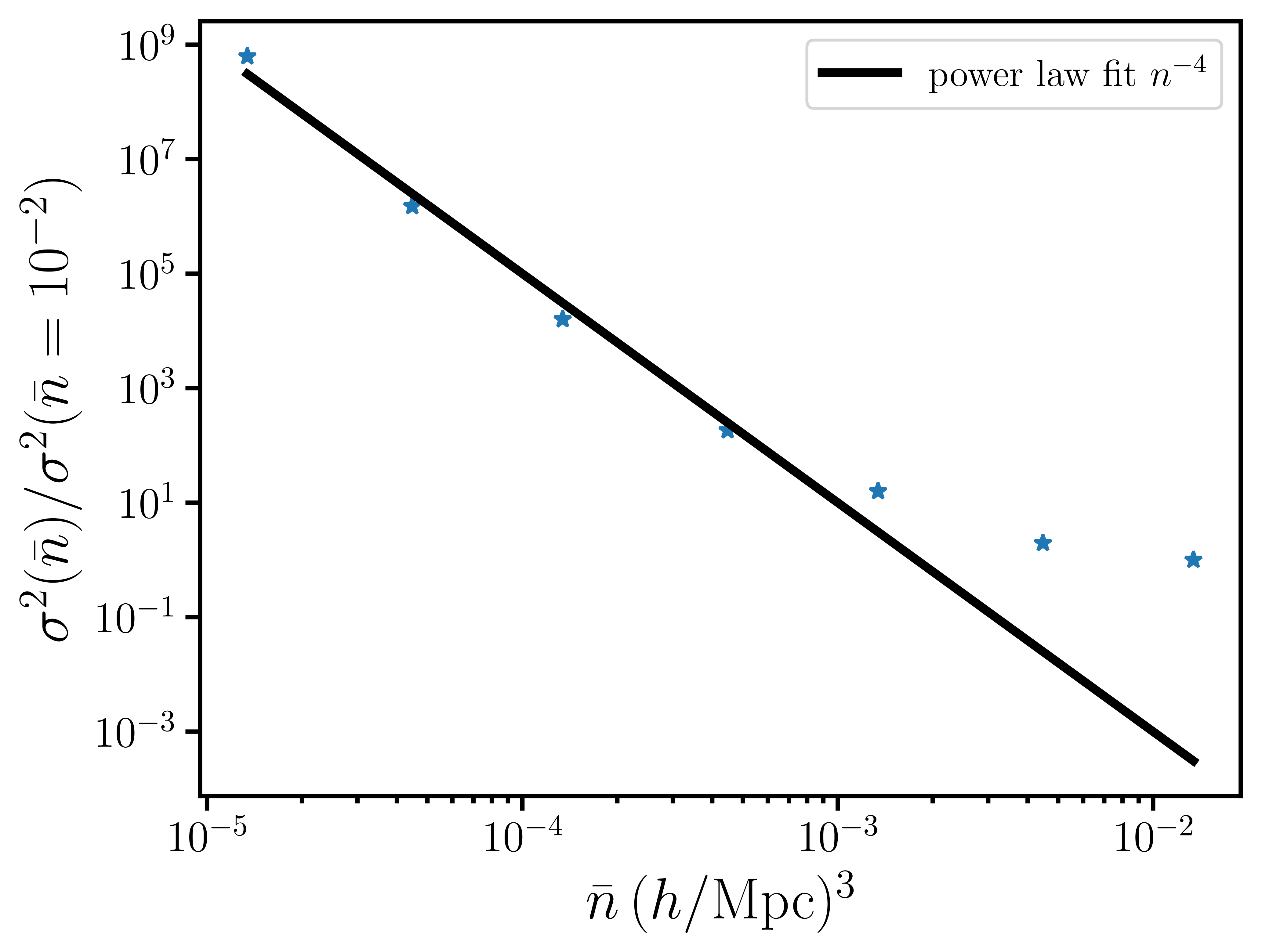
Increasing the mean number density, reduces the PE noise

Implication: PE noise comes from stochasticity

Forecast for upcoming survey

PE noise estimation by down-sampling Quijote matter field

Randomly select DM particles to match certain mean number density



$$\sigma^{\mathrm{fid}}_{\mathrm{survey}} = \sigma^{\mathrm{fid}}_{\mathrm{sim}} \frac{\sqrt{\bar{n}^{\alpha}_{\mathrm{sim}} V_{\mathrm{sim}}}}{\sqrt{\bar{n}^{\alpha}_{\mathrm{survey}} V_{\mathrm{survey}}}}$$

$$\alpha = 4$$

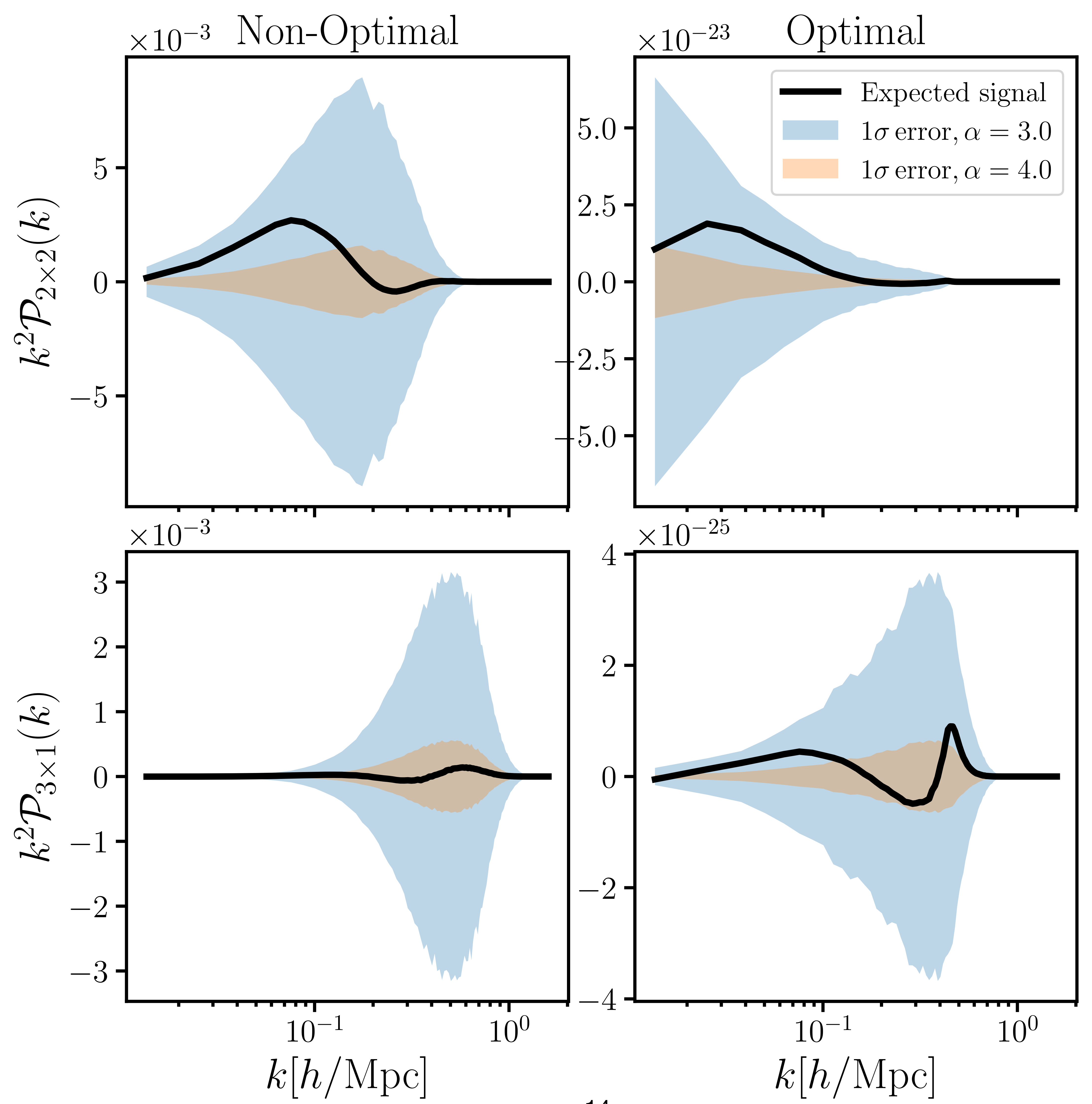
$$\mathscr{P}^{\mathrm{sig}}_{\mathrm{survey}} = \left(\frac{b_{\mathrm{survey}}}{b_{\mathrm{sim}}}\right)^4 \mathscr{P}^{\mathrm{sig}}_{\mathrm{sim}},$$

Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]

Forecast for Euclid

$$\langle D_n[\delta](\vec{k}) D_m[\delta](\vec{k}')\rangle = (2\pi)^3 \delta_D(\vec{k}+\vec{k}')\mathscr{P}_{D_n[\delta],D_m[\delta]}(\vec{k})\,.$$

Full spectroscopy survey



Gao, Moradinezhad, Vlah, [arXiv: 2509.13207]

Conclusion

The parity-even non-linear field introduces large noise in kurto spectrum

The stochasticity in halo field of Quijote simulation is the largest source of noise of kurto spectra

The sharp cut window function and optimal estimator improves the detectability

The stochasticity reduces as number density and survey volume enlarges

Measurement on SDSS BOSS DR12 and DR16 coming soon!