Interact or Twist: Cosmological Correlators from Field Redefinitions Revisited

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based on 2508.12856 and on–going work, in collaboration with Yanjiao Ma, Dong-Gang Wang, Yi Wang, and Wenqi Yu.

Outline

Cosmological Correlators and Field Redefinition

- S-matrices are field–redefinition invariant.
- Cosmological correlators are not: they are defined on a space–like boundary of the inflationary spacetime, typically the reheating surface, and thus they are affected by field–redefinitions and boundary terms.
- For field redefinitions, within single field scenarios, correlators are highly constrained by scale invariance and locality Maldacena (2003); Pajer (2021). As only massless scalars are involved, these early studies normally led to the local shape in cosmological correlators.

Meanwhile...

More possibilities when we allow more than one degree of freedom!

The Toy Model

The toy model consists of one massless scalar field ϕ which enjoys a shift symmetry and another massive field σ,

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 + \frac{1}{\Lambda}\sigma\partial_\mu\sigma\partial^\mu\phi, \qquad (1)$$

where the interaction term can be removed via a Integration–by–Parts (IBP) or the following field redefinition,

$$\phi = \tilde{\phi} + \frac{1}{2\Lambda}\sigma^2. \qquad (2)$$

Field Redefinition Calculations

It can be noticed that the bispectrum of ϕ is completely determined by the field redefinition,

$$\begin{aligned} \langle \phi(\mathbf{x}_1)\phi(\mathbf{x}_2)\phi(\mathbf{x}_3)\rangle =& \frac{1}{8\Lambda^3}\langle \sigma(\mathbf{x}_1)^2\sigma(\mathbf{x}_2)^2\sigma(\mathbf{x}_3)^2\rangle \\ =& \frac{C_\sigma^3 \eta_0^{6\Delta_\sigma}/8\Lambda^3}{x_{12}^{2\Delta_\sigma} x_{23}^{2\Delta_\sigma} x_{31}^{2\Delta_\sigma}} . \end{aligned} \tag{3}$$

It is interesting to notice that this boundary correlator of a massless scalar takes the same form as the three-point function $\langle \mathcal{O}_\Delta(\mathbf{x}_1)\mathcal{O}_\Delta(\mathbf{x}_2)\mathcal{O}_\Delta(\mathbf{x}_3)\rangle_{\rm CFT}$ in a 3d Euclidean CFT with $\Delta = 2\Delta_\sigma$.

Analyticity in the momentum space

With the "not–that–standard" Schwinger–Keldysh Feynman rule, The diagram in the momentum space is

$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\phi_{\mathbf{k}_3}\rangle' = \left(-\frac{\eta_0}{2}\right)^{9-6\nu} \frac{H^6\Gamma(\nu)^6}{\pi^3\Lambda^3} I_\nu(k_1,k_2,k_3), \tag{4}$$

$$\text{with} \quad I_\nu \equiv \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{|\mathbf{q}|^{2\nu}|\mathbf{q}+\mathbf{k}_2|^{2\nu}|\mathbf{q}-\mathbf{k}_3|^{2\nu}}, \tag{5}$$

and

$$I_\nu = \frac{\pi^{-\frac{3}{2}}2^{-\frac{1}{2}}}{\Gamma\left(\frac{3\Delta-3}{2}\right)\Gamma\left(\frac{3-\Delta}{2}\right)^3}(k_1k_2k_3)^{\Delta-\frac{3}{2}}\int_{-\infty}^{0} d\eta(-\eta)^{1/2} \times K_{\Delta-3/2}(-k_1\eta)K_{\Delta-3/2}(-k_2\eta)K_{\Delta-3/2}(-k_3\eta), \tag{6}$$

where $K_{\Delta-3/2}$ is the Bessel K function Bzowski et al. (2014); Coriano et al. (2013); Bzowski et al. (2016).

Analyticity in the momentum space

For general masses, we always find a logarithmic pole in the $k_T = k_1 + k_2 + k_3 \to 0$ limit ,

$$\lim_{k_T \to 0} I_\nu = -\frac{(k_1 k_2 k_3)^{\Delta-2}}{4\Gamma\left(\frac{3\Delta-3}{2}\right)\Gamma\left(\frac{3-\Delta}{2}\right)^3} \log k_T. \tag{7}$$

It is normally expected that, for cosmological correlators, this total energy singularity is generated by bulk interactions, as the residue of the pole corresponds to flat-space amplitudes Maldacena and Pimentel (2011); Raju (2012), while field redefinitions can only affect parts of the correlators that are regular in the $k_T \to 0$ limit. The computation here serves as a counterexample showing that this singularity can also be a consequence of field redefinitions with composite operators.

Slide with a beautiful figure

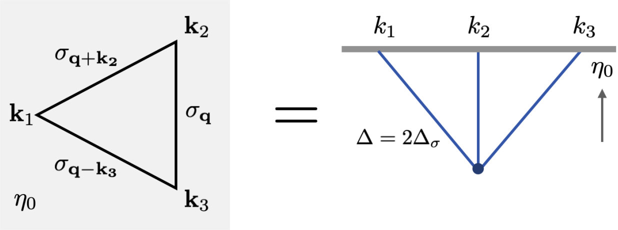

Figure 1: The boundary "one-loop" triangle graph from field redefinition can be re-expressed as a contact interaction of fields with a different mass in the bulk Wang et al. (2025).

Same Story Told in the IBP Way

by performing Integration-by-Part (IBP). The cubic mixing there becomes

$$\frac{1}{\Lambda}\sigma\partial_\mu\sigma\partial^\mu\phi = -\frac{1}{2\Lambda}\sigma^2\Box\phi + \frac{1}{2\Lambda}\nabla_\mu(\sigma^2\partial^\mu\phi). \tag{8}$$

The Feynman rules lead to the following three-point function

$$\langle\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\phi_{\mathbf{k}_3}\rangle' = \frac{1}{8\Lambda^3}\sum_{a_1,a_2,a_3=\pm}\int_{-\infty}^{\eta_0}\prod_{i=1}^{3}\frac{\mathrm{d}\eta_\mathrm{i}}{(-H\eta_i)^2} \tag{9}$$

$$\left[a_i\partial_{\eta_i}K_{a_i}(k_1,\eta_1)\delta(\eta_i-\eta_0)\right]\int\frac{d^3\mathbf{q}}{(2\pi)^3}\prod_{i<j}G_{a_ia_j}(q_{ij};\eta_i,\eta_j)$$

$$=\frac{1}{8\Lambda^3}\int\frac{d^3\mathbf{q}}{(2\pi)^3}|\sigma_q(\eta_0)|^2|\sigma_{|\mathbf{q}+\mathbf{k}_2|}(\eta_0)|^2|\sigma_{|\mathbf{q}-\mathbf{k}_3|}(\eta_0)|^2, \tag{10}$$

For Every Field Redef There is a IBP Boundary Term

When a field redefinition $\phi \to \phi + \delta\phi(\phi, \sigma)$ is used to eliminate a term proportional to the Equation of motion, a corresponding boundary term is generated:

$$\begin{aligned} \delta S =& \int dt \left[\partial_t \left(\frac{\partial L}{\partial \dot{\phi}} \delta\phi \right) - \left(\partial_t \frac{\partial L}{\partial \dot{\phi}} - \frac{\partial L}{\partial \phi} \right) \delta\phi \right] \\ =& (\text{boundary terms})\delta\phi - \int \text{EoM}\delta\phi. \end{aligned} \tag{11}$$

The massive field as the Higgs

We treat the Higgs mass squared as a free parameter, since it receives corrections from Higgs self-coupling, coupling to the inflaton and coupling $\sigma^2 R$ between the Higgs and the Ricci scalar. As a result, during inflation, the Higgs may: (i) have vanishing VEV, where the discussion is parallel to the previous section, so we will not repeat it here; or (ii) have nonzero VEV $\bar{\sigma} \neq 0$ and the electroweak symmetry is spontaneously broken.

The leading Primodial Non–Gaussianity

Then the bispectrum has two components

$$\langle\zeta_1\zeta_2\zeta_3\rangle = \frac{\langle\sigma_1^2\sigma_2^2\sigma_3^2\rangle}{(2\sqrt{2\epsilon}\Lambda M_{\rm pl})^3} + \frac{\bar{\sigma}^2}{2}\frac{\sum_{\rm perms}\langle\sigma_1^2\sigma_2\sigma_3\rangle}{(\sqrt{2\epsilon}\Lambda M_{\rm pl})^3}\ , \qquad (12)$$

Modulated Reheating

From the perspective of multi-field inflation, the above model corresponds to a simple scenario that the inflaton trajectory is along a straight line in a flat 2d field space. Normally we would expect that this just reduces to single field models. Then how come we still find non–trivial PNG from field redefinitions?

Modulated Reheating

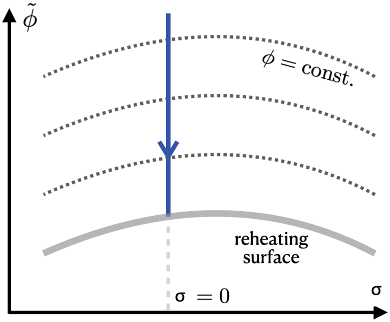

Figure 2: Field redefinitions twist the field space and lead to a nontrivial reheating surface in the new $(\tilde{\phi}, \sigma)$ basis. For our example, the dotted lines are constant ϕ surfaces, while the inflaton trajectory (blue) is along $\sigma = 0$ in both field bases Wang et al. (2025).

The Boundary Effective Field Theory

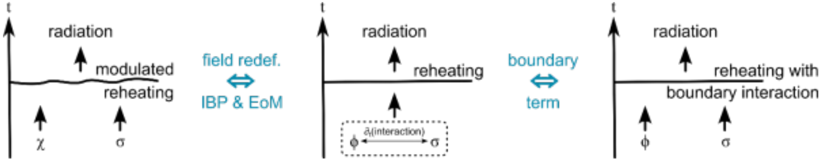

Figure 3: A model of modulated reheating can be described by a term in a BEFT

Summary

1. Boundary contributions from field redefinitions can produced non–Gaussianities similar to bulk interactions, with a total energy pole.
2. For each field redefinition that removes an EoM term, there is a correponding boundary term.
3. The field redefinition can be interpreted as a modulation of the reheating surface.
4. A much larger set of theories with boundary terms, the so–called boundary EFTs, may be useful in studying physics happening with in a few Hubble time, e.g. the reheating.

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