IMPERIAL

$\lambda\Phi^4$ as an EFT in de Sitter

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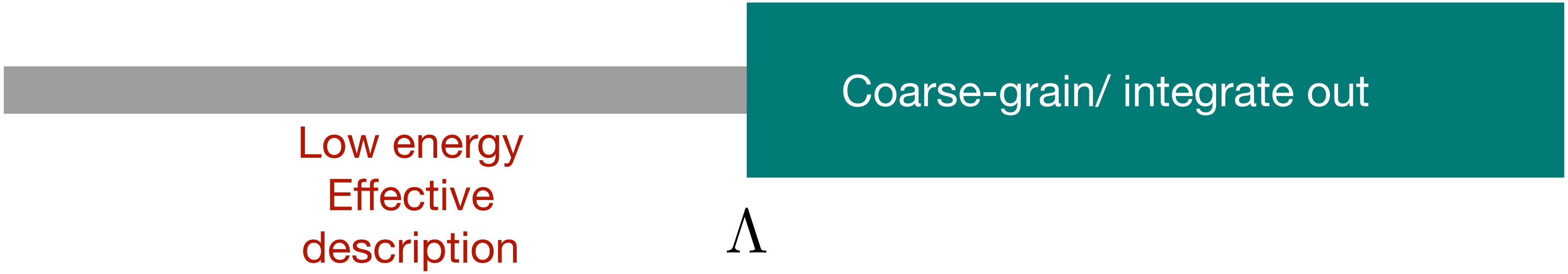
Based on 2510.25826 with Z. Qin and D-G. Wang
2512.XXXX with T. Colas
Also 2311.17990 with A-C. Davis and D-G. Wang

SIMONS FOUNDATION

Effective theories from the top down



Integrate out heavy fields

$$e^{iS_{\rm eff}[\phi]} = \int \mathcal{D}\sigma e^{iS[\phi,\sigma]}$$

- Decoupling with the heavy dof
- Regimes of validity manifest

Coarse-grain short modes

$$e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_S e^{iS[\phi_L,\phi_S]}$$

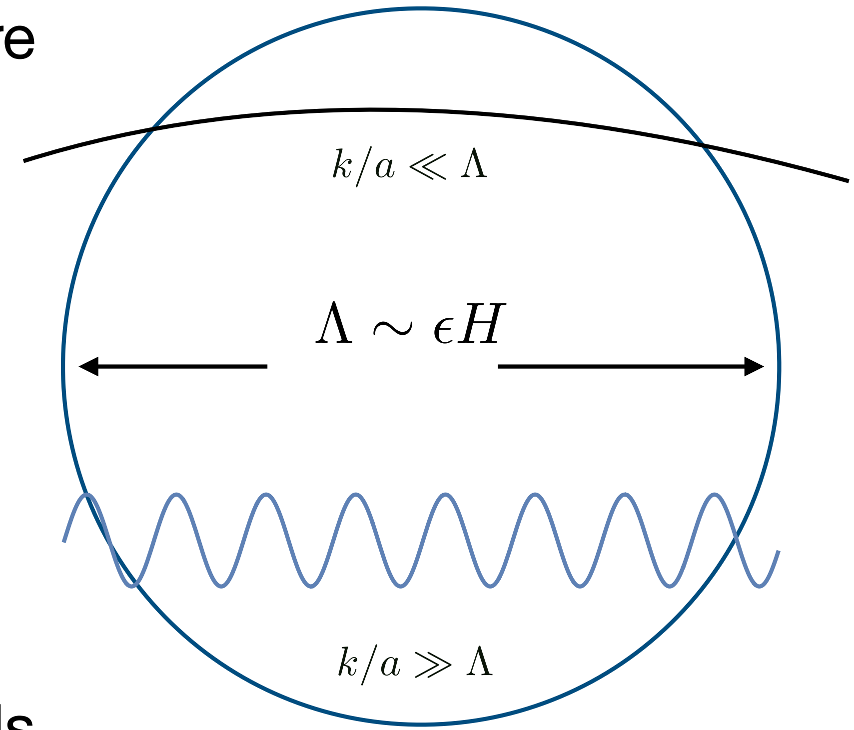
- Wilsonian perspective (RG flows)
- Study how the theory changes

Effective Field Theories in Cosmology

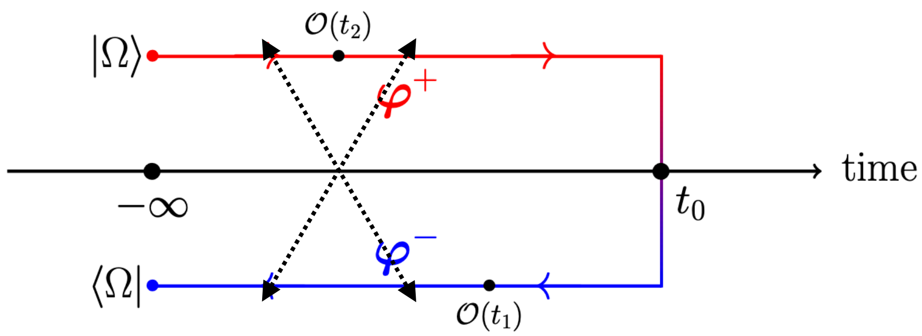
- What are the possible EFTs that are valid on superhorizon scales?

- Stochastic theory as a long wavelength effective action where superhorizon modes are coarse grained

- What is the structure of these kinds of EFTs?



Cosmological EFT and density matrix



T. Colas talk

$$\rho[\varphi_+, \varphi_-] = \Psi[\varphi_+]\Psi^*[\varphi_-] = \int_\Omega^{\phi_+(t_0)=\varphi_+} \mathcal{D}\phi_+ \int_\Omega^{\phi_-(t_0)=\varphi_-} \mathcal{D}\phi_- e^{iS[\phi_+]-iS[\phi_-]}$$

Pure state

$$\begin{aligned}\rho[\varphi_+, \varphi_-] &= \mathrm{Tr}_\varsigma \rho[\varphi_+, \varphi_-, \varsigma_+, \varsigma_-] \\ &= \int d\varsigma \int^{\varphi_+} \mathcal{D}\phi_+ \int^{\varphi_-} \mathcal{D}\phi_- \int^{\varsigma} \mathcal{D}\sigma_+ \int^{\varsigma} \mathcal{D}\sigma_- e^{iS[\phi_+,\sigma_+]-iS[\phi_-,\sigma_-]} \\ &= \mathcal{D}\phi_+ \int^{\varphi_-} \mathcal{D}\phi_- e^{iS_{\mathrm{EFT}}[\varphi_a,\varphi_r]}\end{aligned}$$

Mixed state

Diffusion

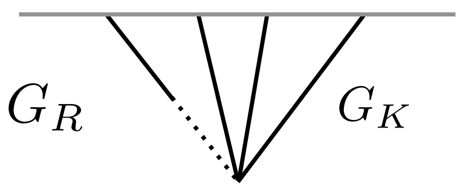
Not necessarily Hamiltonian → Dissipation

Observable 4-pt function

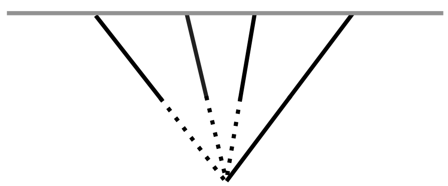
- Let's start by discussing a simpler case $\lambda\phi^4$

$$S[\phi_a,\phi_r] = \int d^4x a^3 \left[-\partial_\mu\phi_a\partial^\mu\phi_r - \lambda\phi_a\phi_r^3 - \frac{\lambda}{4}\phi_a^3\phi_r \right] \qquad \phi_r = \frac{1}{2}(\phi_+ + \phi_-), \qquad \phi_a = \phi_+ - \phi_-$$

- Two vertices



$$= \frac{\lambda}{4}\frac{H^4}{\prod k_i^3}\sum k_i^3 \log(-\eta_0) + \mathcal{A}(k_i).$$



$$= \frac{\lambda}{4}H^2\mathcal{B}(k_i).$$

- Amplitude pole only after considering both

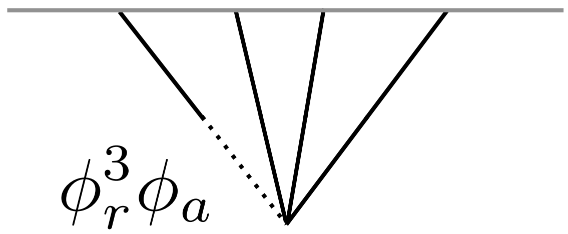
$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\rangle' = \frac{\lambda}{4}\frac{H^4}{\prod k_i^3}\sum k_i^3 \log(-k_T\eta_0) + \mathrm{Poly}(k_i).$$

Superhorizon Limit

- On superhorizon scales $k|\eta| \ll 1$ we have that,

$$\frac{\phi_r}{\phi_a} \sim \frac{\langle \phi_r^2 \rangle}{\langle \phi_a \phi_r \rangle} = \frac{G_K(k, \eta, \eta')}{G_R(k, \eta, \eta')} \sim \frac{1}{(k\eta)^3}.$$

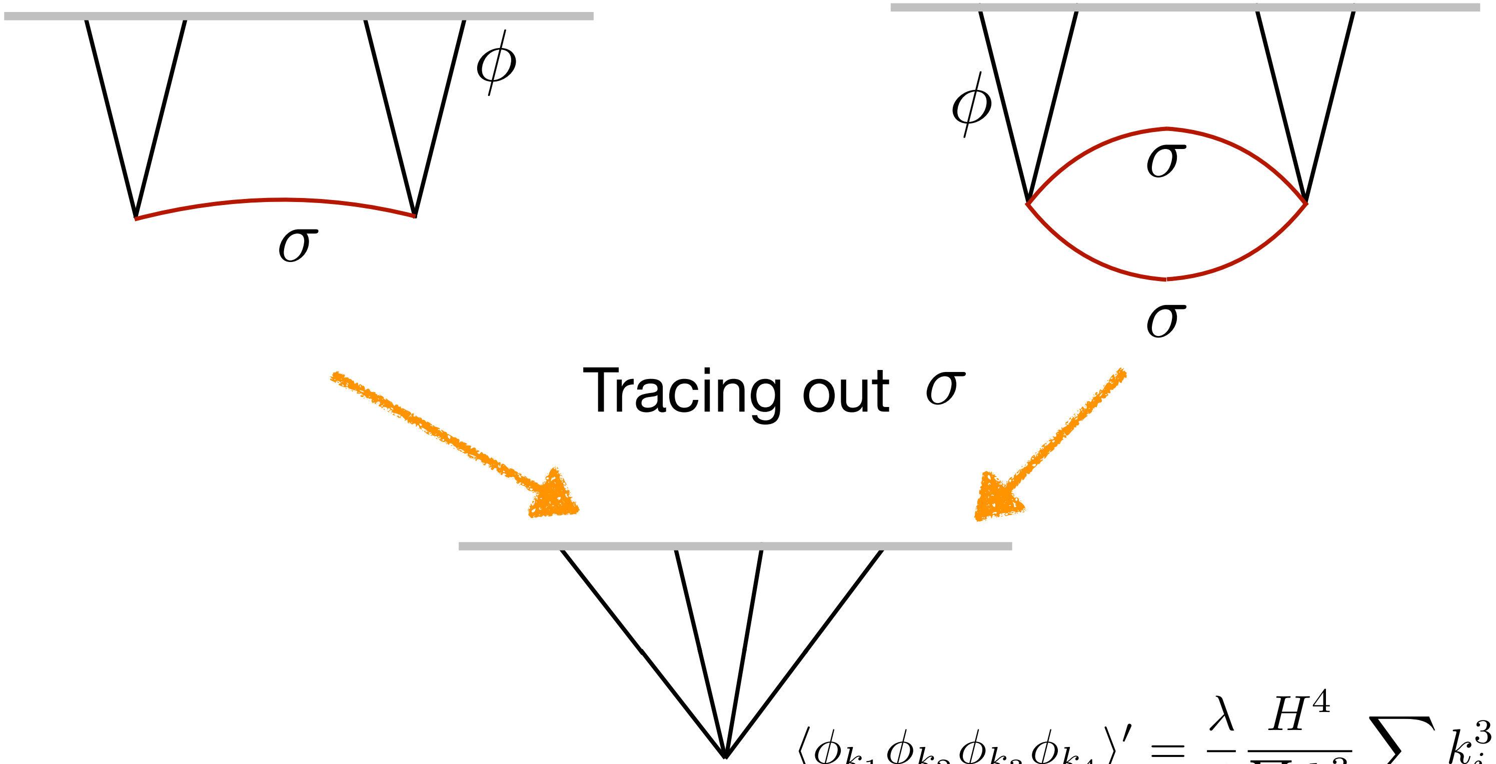
- One diagram dominates



- Same thing happens in stochastic inflation where a semiclassical limit arises

Burges et al '09
Baumgart and Sundrum '19
SC, AC Davis and D-G Wang '23

UV realisations of $\lambda\Phi^4$



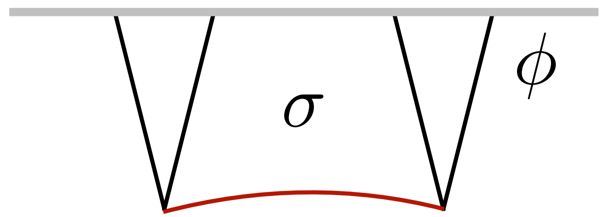
$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\rangle' = \frac{\lambda}{4}\frac{H^4}{\prod k_i^3}\sum k_i^3 \log(-k_T\eta_0) + \dots$$

SC, Qin and Wang '25

Matching cubic vertex

SC, Qin and Wang '25

We perform the matching at different regimes with the 4-point function on a $\alpha\phi^2\sigma$



$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\rangle' = \mathcal{I}_1 + \mathcal{I}_2 + \mathcal{I}_3$$

EFT expansion

Particle production

$$\mathcal{I}_1 = -\frac{\alpha^2 H^4}{8m^2 \prod k_i^3}\sum k_i^3 \log(-k_T\eta_0) \qquad \mathcal{I}_2 \sim \alpha^2 \frac{H^4}{m^4}\mathcal{B}(k_i) \qquad \mathcal{I}_3 \sim -\alpha\frac{H^4}{8m^2} \times e^{-\pi H m}$$

$$m^2 \gg H^2 \qquad \lambda_{\text{eff}} = -\frac{1}{2}\frac{\alpha^2}{m^2}$$

Local and unitary $\lambda\phi^4$

Theory is well approximated by a pure state

$$\gamma = 1$$

$$m^2 \ll H^2 \qquad \lambda_{\text{eff}} = -\frac{1}{2}\frac{\alpha^2}{m^2}$$

$$S_{\text{EFT}} = -\int d^3x \int dt a^3(t)\left[\partial_\mu\phi_a\partial^\mu\phi_r + \lambda_1\phi_a\phi_r^3 + \lambda_2\phi_a^3\phi_r\right]$$

$$+ i\int d^3x \int dt a^3(t) \int d^3y\ \phi_a^2 \lambda_{\text{NL}}(|\vec{x}-\vec{y}|)\phi_r^2$$

$$\gamma \ll 1$$

Diffusion

Stochastic Regime

- For light masses $m^2 \ll H^2$ we still can match to the leading log to a single field EFT provided that

$$\log(-k_T \eta_0) \gg \frac{H^2}{m^2}$$

- The $\phi_r^3 \phi_a$ vertex dominates. Semiclassical limit

Baumgart and Sundrum '19

- Off-diagonal components directly related to decoherence

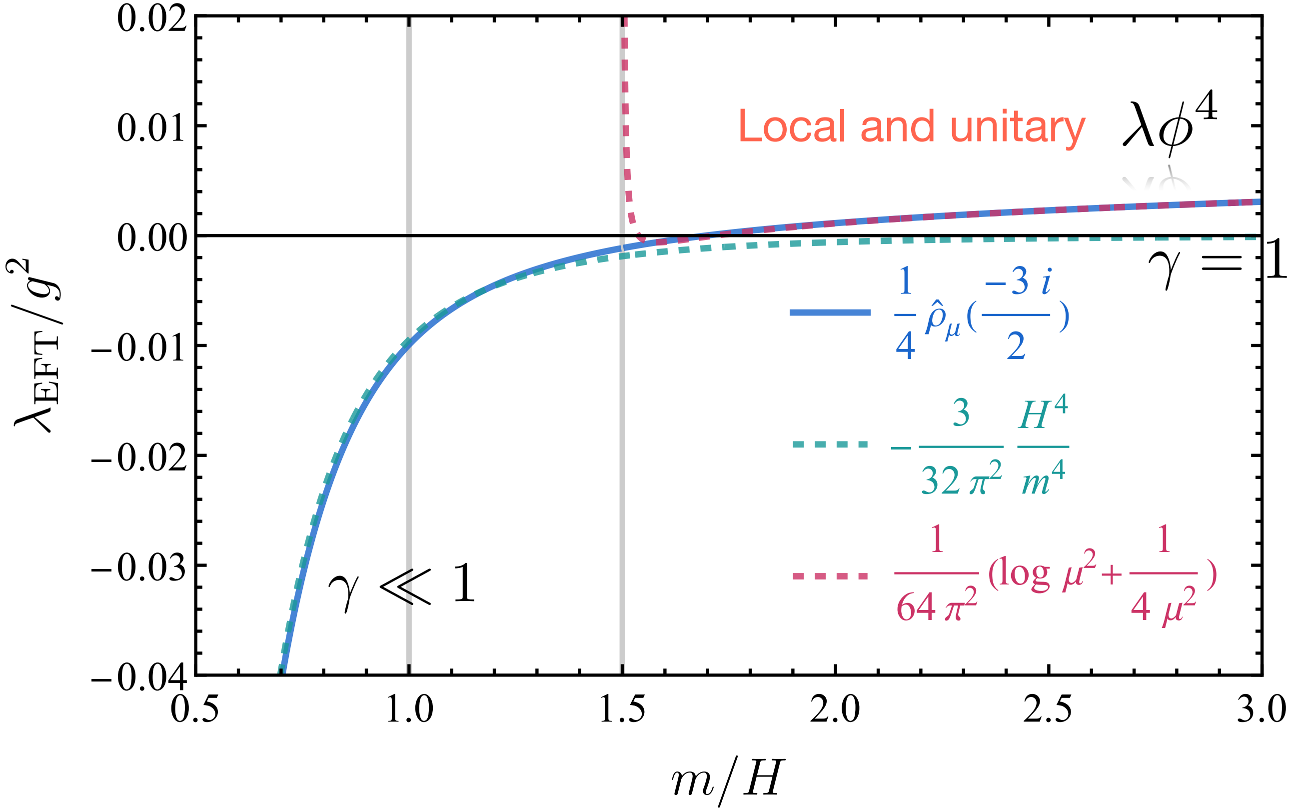
$$\lambda_{\rm NL} \sim \frac{\alpha^2}{H^2} \frac{\Gamma(\nu)^2}{3+2\nu} \left(\frac{s^2 \eta_0 \eta}{4} \right)^{-\nu+3/2}$$

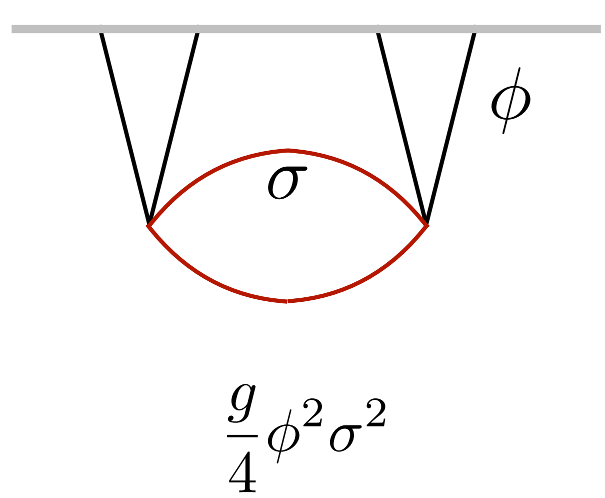
- Similar to the onset of the stochastic theory

Loops and matching

- Matching of the 4 point function to the EFT

$$\langle \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\rangle'_{\rm IR} = \frac{g^2\sum k_i^3}{16\prod k_i^3}\left[\widehat{\rho}_\mu\left(\frac{-3i}{2}\right) - \frac{1}{16\pi^2}\log\mu_R^2\right]\log(-k_T\eta_0)$$





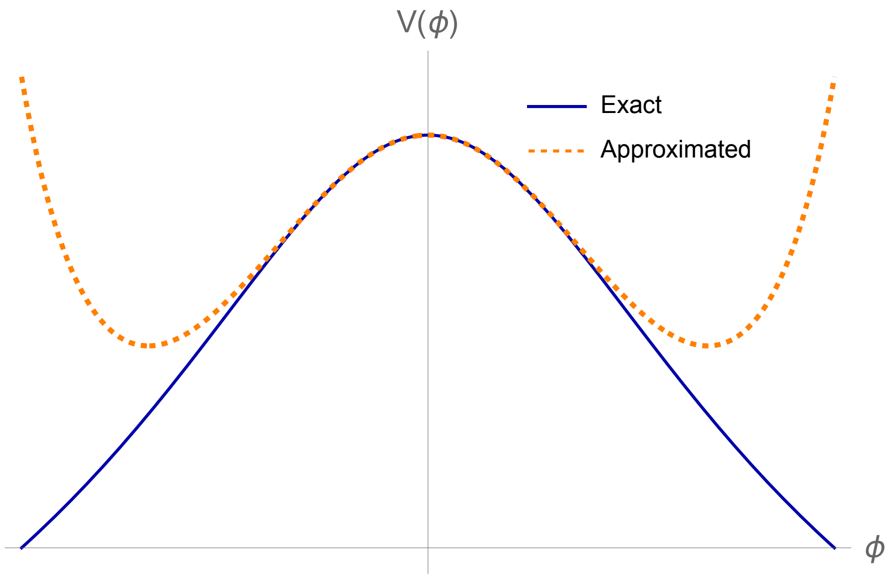
SC, Qin and Wang '25

Stochastic Theory

Stochastic approach leads to

$$P[\phi, \sigma] \sim \exp\left(-\frac{8\pi^2}{3H^4}\left(\frac{1}{2}m^2\sigma^2 + \frac{g}{4}\phi^2\sigma^2\right)\right)$$

$$P[\phi]$$



- Integrating over σ matches the QFT result
- Perturbative result valid up to

$$\frac{\phi}{H} \ll \lambda_{\text{eff}}^{-1/4} \sim g^{-1/2}\frac{m}{H}.$$

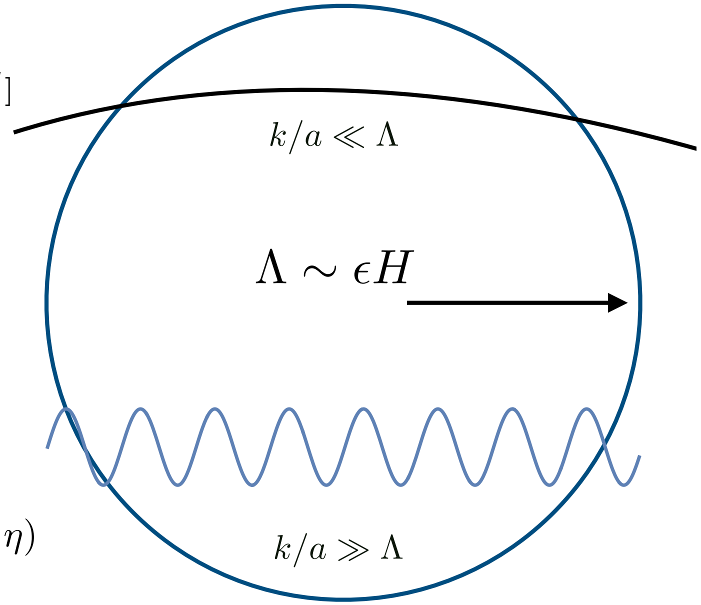
- Theory stable for large ϕ

SC Qin and Wang '25

Coarse-grained dynamics

- Let us go back to the case of a coarse grained fields

$$e^{iS_{\rm IF}[\varphi_r^L,\varphi_a^L]} = \int d\varphi \int_{\rm BD}^{\varphi} \mathcal{D}\varphi_r^S \int_{\rm BD}^{0} \mathcal{D}\varphi_a^S \, e^{iS_{\rm free}[\varphi_r^S,\varphi_a^S]} e^{iS_{\rm cross}[\varphi_r^L,\varphi_a^L;\varphi_r^S,\varphi_a^S]}$$



Diffusion

$$S_{\rm IF}[\varphi_a^L,\varphi_r^L] \supset \begin{array}{l} 4H \displaystyle\int_{\mathbf{k}} \int d\eta \; a^3(\eta)\Lambda(\eta)\delta(k-\Lambda(t))\varphi_r^L(\mathbf{k},\eta)\varphi_a'^L(\mathbf{k},\eta) \\[2ex] \dfrac{iH^3}{\pi^2} \displaystyle\int d^3y \int d^3x \int d\eta a(\eta)^5 j_0(|\mathbf{x}-\mathbf{y}|\Lambda)(1+\Lambda^2\eta^2)\varphi_a'^L(\mathbf{x},\eta)\varphi_a'^L(\mathbf{y},\eta) \end{array}$$

Dissipation

$$\frac{\text{dissipation}}{\text{diffusion}} \sim \frac{\epsilon^3}{1+\epsilon^2}$$

Stochastic regime for small cut-off

In preparation, SC and Colas

RG flow equation

- Imposing that $$\frac{d}{d\log\Lambda} Z_\Lambda[J_r, J_a] = 0$$

SC, AC Davis and D-G Wang '23

Diffusion

- Differential eq $$\frac{\partial P_\Lambda[\phi^\ell]}{\partial \log\Lambda} = -2i\int d^3x\, d^3y\, \frac{\partial G_\Lambda^{rr}(|\mathbf{x}-\mathbf{y}|;t_0,t_0)}{\partial\log\Lambda}\,\frac{\delta^2 P_\Lambda[\phi^\ell]}{\delta\phi^\ell(\mathbf{x})\,\delta\phi^\ell(\mathbf{y})} - 4\int d^3x\, d^3y \int^{t_0} dt'\, \frac{\partial G_\Lambda^{ra}(|\mathbf{x}-\mathbf{y}|;t_0,t')}{\partial\log\Lambda}\,\frac{\delta}{\delta\phi^\ell(\mathbf{x})}\Big(E^\Lambda(t',\mathbf{y};[\phi^\ell])\,P_\Lambda[\phi^\ell]\Big),$$

Dissipation

- In the limit when $\epsilon \to 0$ we recover stochastic inflation and thus semiclassical limit

$$\frac{dP_\Lambda}{dt} = \int d^4x \frac{\delta}{\delta\phi^\ell(\mathbf{x})}\left[\left(\frac{V_\phi^{\rm eff}(\phi^\ell)}{3H}\right)P_\Lambda\right] + \frac{H^2}{8\pi^2}\int d^3x\int d^3y\, j_0(\Lambda|\mathbf{x}-\mathbf{y}|)\frac{\partial P_\Lambda[\phi^l]}{\delta\phi^\ell(\mathbf{x})\,\delta\phi^\ell(\mathbf{y})}$$

In preparation, SC and Colas

Conclussions

- Low energy EFTS of $\lambda\phi^4$ share the same features: diffusion and decoherence. Semiclassical limit arises

- Unitary part dominates in the superhorizon regime

- Diffusion leads to an stochastic description

- Can we use this to go beyond perturbation theory?