

Primordial correlators from multi-point propagators



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Based on 2503.15194

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Cosmological inflation

- Period of accelerated expansion in the early universe, solves hot big bang issues and produces seed for the large-scale-structure

$$ds^2 = -dt^2 + a^2[(1 - 2\zeta)\delta_{ij} + \gamma_{ij}]dx^i dx^j$$

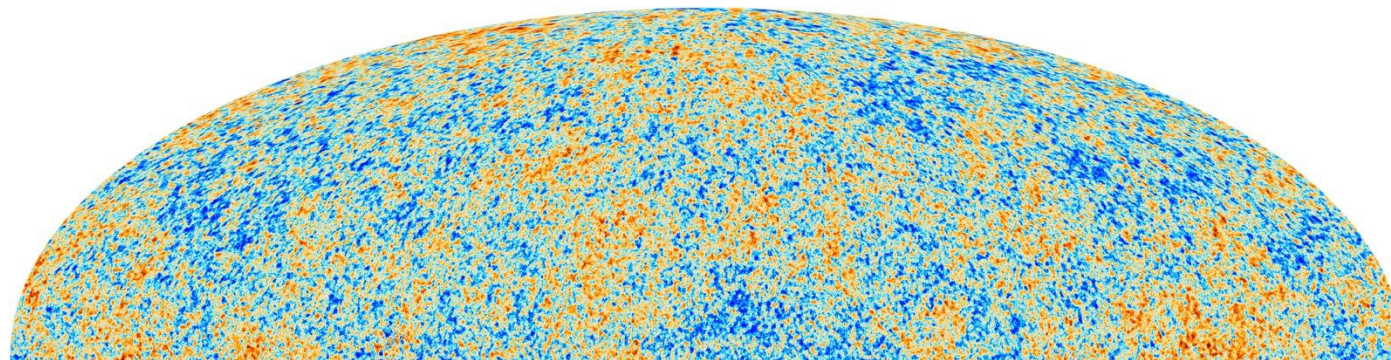
Curvature perturbation

Tensor perturbation

- 2- and 3-point correlators of the curvature perturbation ζ

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(k)$$

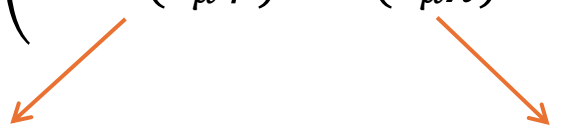
$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$$



Phase-space correlators

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(R - (\partial_\mu \phi)^2 - (\partial_\mu \chi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} m_\chi^2 \chi^2 \right)$$

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$$\begin{cases} \phi = \phi_0 + \delta\phi \\ \pi_\phi = \frac{\delta S}{\delta[\partial_\mu \phi]} = \pi_{\phi,0} + \delta\pi_\phi \end{cases}$$

$$\begin{cases} \chi = \chi_0 + \delta\chi \\ \pi_\chi = \frac{\delta S}{\delta[\partial_\mu \chi]} = \pi_{\chi,0} + \delta\pi_\chi \end{cases}$$

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$$\delta X^a \equiv (\delta\phi, \delta\chi, \delta\pi_\phi, \delta\pi_\chi)$$

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$$\zeta(k) = N_a(k) \delta X^a + \frac{1}{2} N_{ab}(k) \delta X^a \delta X^b + \dots$$

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$$\zeta(k) = N_a(k) \delta X^a + \frac{1}{2} N_{ab}(k) \delta X^a \delta X^b + \dots$$

$$\langle \delta X^a \delta X^b \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_a + \mathbf{k}_b) \Sigma^{ab}(k)$$

$$\langle \delta X^a \delta X^b \delta X^c \rangle \equiv (2\pi)^3 \delta(\mathbf{k}_a + \mathbf{k}_b + \mathbf{k}_c) B^{abc}(k_a, k_b, k_c)$$

How to compute phase-space correlators

In-In formalism²

$$\Sigma^{ab}(k) \equiv \langle 0 | \delta x^a \delta x^b | 0 \rangle,$$

$$B^{abc}(k_a, k_b, k_c) \equiv$$

$$\left\langle 0 \left| \frac{i}{3!} \int_{-\infty}^t dt' H_{def} [\delta x^d \delta x^e \delta x^f, \delta x^a \delta x^b \delta x^c] \right| 0 \right\rangle$$

Transport formalism³

$$\frac{d}{dN} \Sigma^{ab}(k) = u_c^a \Sigma^{cb}(k) + u_c^b \Sigma^{ac}(k)$$

$$\begin{aligned} \frac{d}{dN} B^{abc}(k_a, k_b, k_c) &= u_d^a(k_a) B^{dbc}(k_a, k_b, k_c) + \\ &+ u_{de}^a(k_a, k_b, k_c) \Sigma^{db}(k_b) \Sigma^{ec}(k_c) + 2 \text{ perms} \end{aligned}$$

Multi-point propagators

A different formulation of the Transport formalism can be implemented using the multi-point propagators (**MPPs**)⁴

$$\delta X^a(N, \mathbf{k}) = \Gamma_i^{a(N_0, N)}(k) \delta X^i(N_0, \mathbf{k}) + \frac{1}{2!} \int \frac{d^3 k_1}{(2\pi)^3} \Gamma_{ij}^{a(N_0, N)}(k, k_1, |\mathbf{k} - \mathbf{k}_1|) \delta X^i(N_0, \mathbf{k}_1) \delta X^j(N_0, \mathbf{k} - \mathbf{k}_1) + \dots$$

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$$\frac{d\Gamma_i^{a(N, N_0)}(k)}{dN} = u_b^{a(N)}(k) \Gamma_i^{b(N, N_0)}(k)$$



$$\begin{aligned} \frac{d\Gamma_{ij}^{a(N, N_0)}(k_1, k_2, k_3)}{dN} &= u_b^{a(N)}(k_1) \Gamma_{ij}^{b(N, N_0)}(k_1, k_2, k_3) \\ &+ u_{bc}^{a(N)}(k_1, k_2, k_3) \Gamma_i^{b(N, N_0)}(k_2) \Gamma_j^{c(N, N_0)}(k_3). \end{aligned}$$

Correlators from multi-point propagators

Then, we can use the Γ 's to propagate the two- and three-point correlators

$$\Sigma^{ab(N)}(k) = \Gamma_i^{a(N,N_0)}(k) \Gamma_j^{b(N,N_0)}(k) \Sigma_{ij}^{(N_0)}(k),$$

$$\begin{aligned} B^{abc(N)}(k_1, k_2, k_3) = & \Gamma_i^{a(N,N_0)}(k_1) \Gamma_j^{b(N,N_0)}(k_2) \Gamma_k^{c(N,N_0)}(k_3) B_{ijk}^{(N_0)}(k_1, k_2, k_3) \\ & + \Gamma_{ij}^{a(N,N_0)}(k_1, k_2, k_3) \Gamma_k^{b(N,N_0)}(k_2) \Gamma_l^{c(N,N_0)}(k_3) \Sigma_{ik}^{(N_0)}(k_2) \Sigma_{jl}^{(N_0)}(k_3) \\ & + 2 \text{ perms} \end{aligned}$$

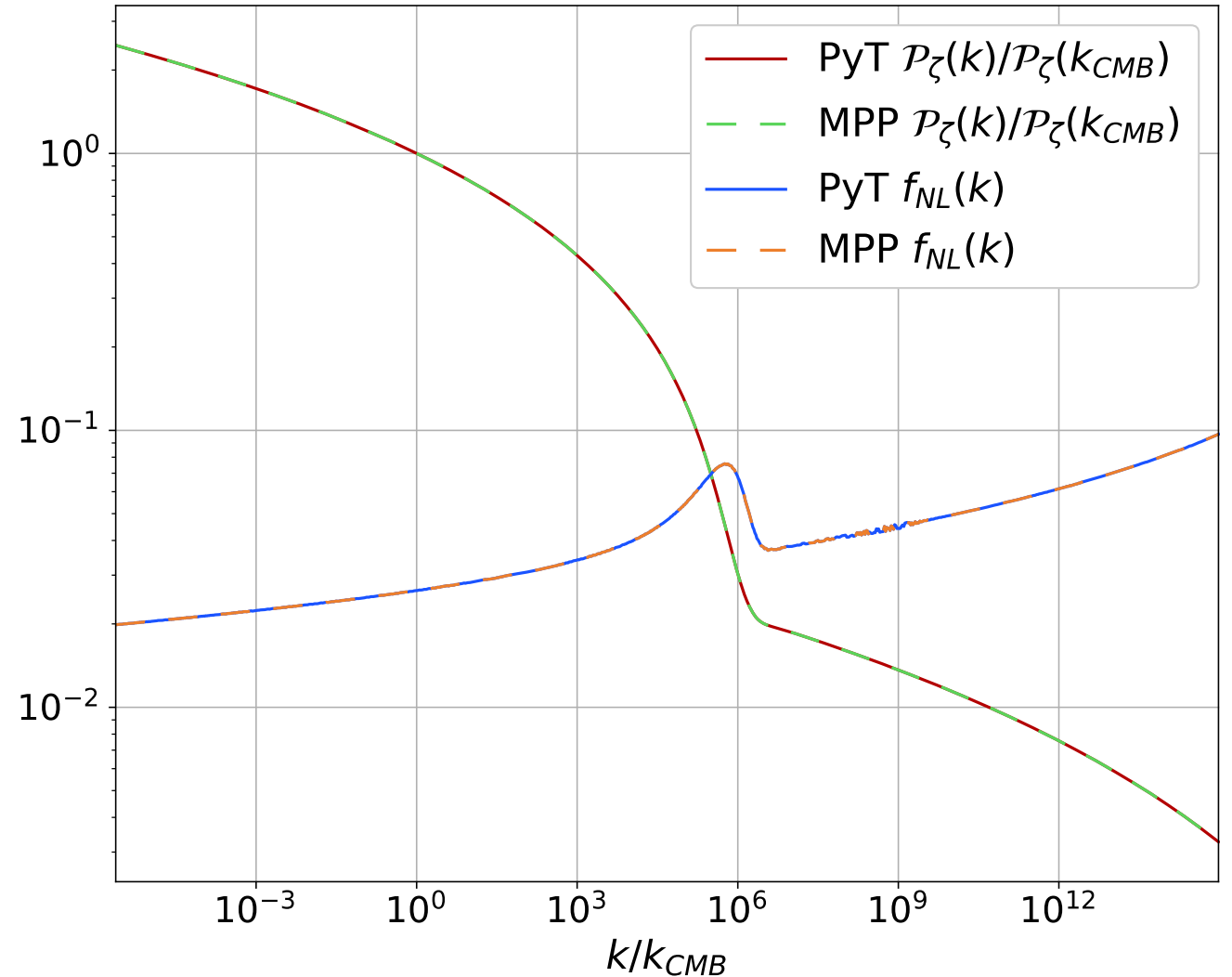
PyTransport 3.0

- Is a package that computes the evolution of the 2- and 3-point correlation functions at tree-level, for Multi-field inflationary models with curved field-space.
- In the last update we included functions to compute the evolution of the correlation functions using the MPPs.



Tests: Double Inflation

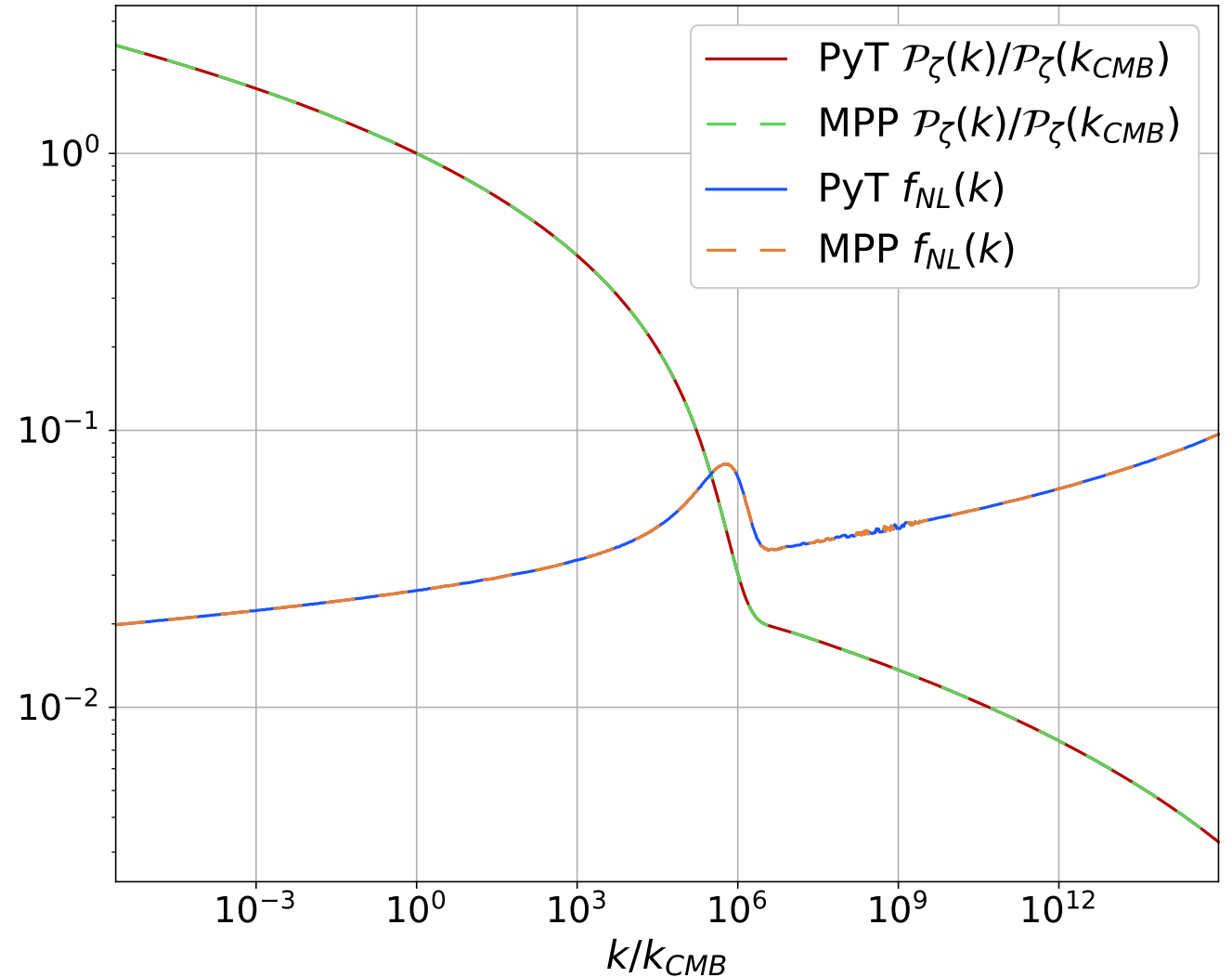
$$V = \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}m_\phi^2\phi^2$$



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$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k)$$

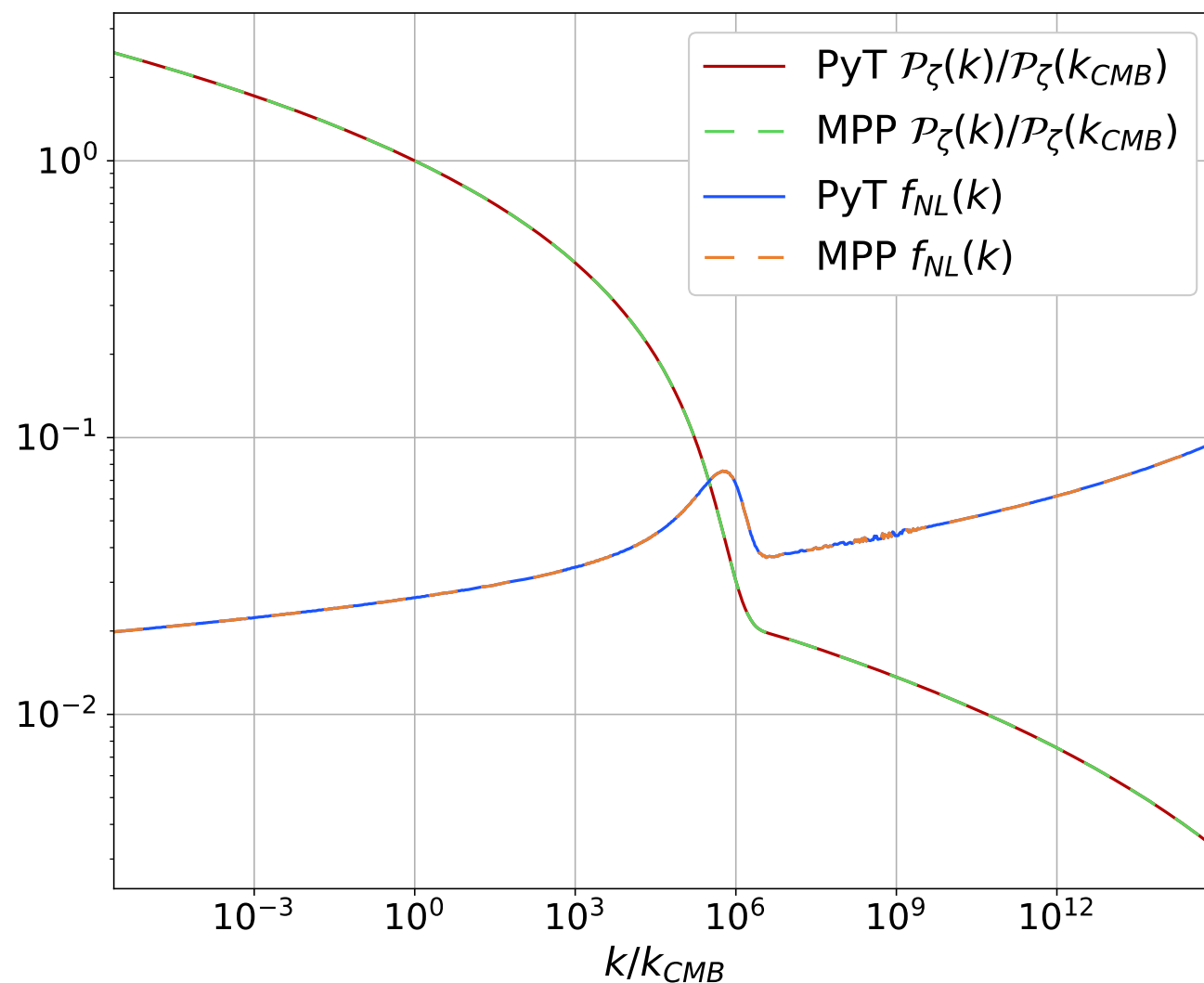


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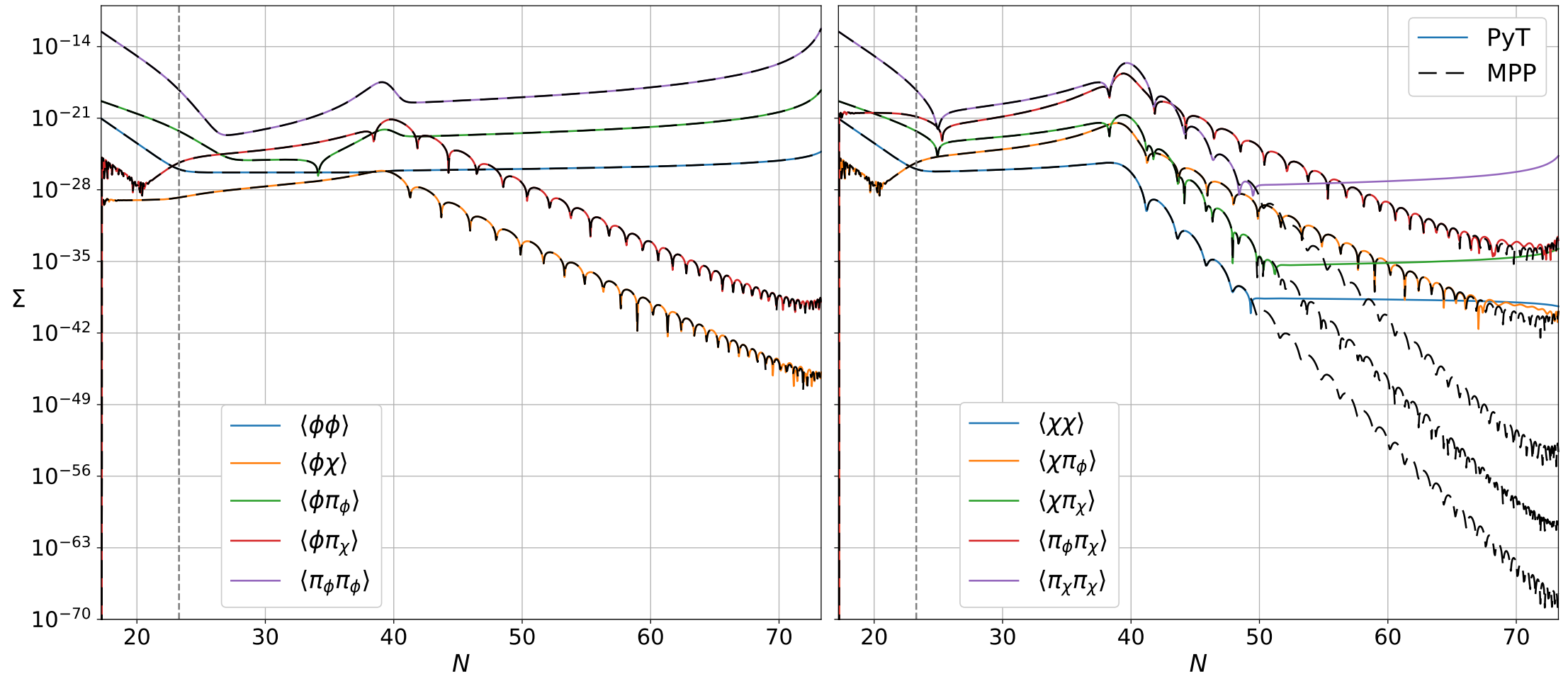
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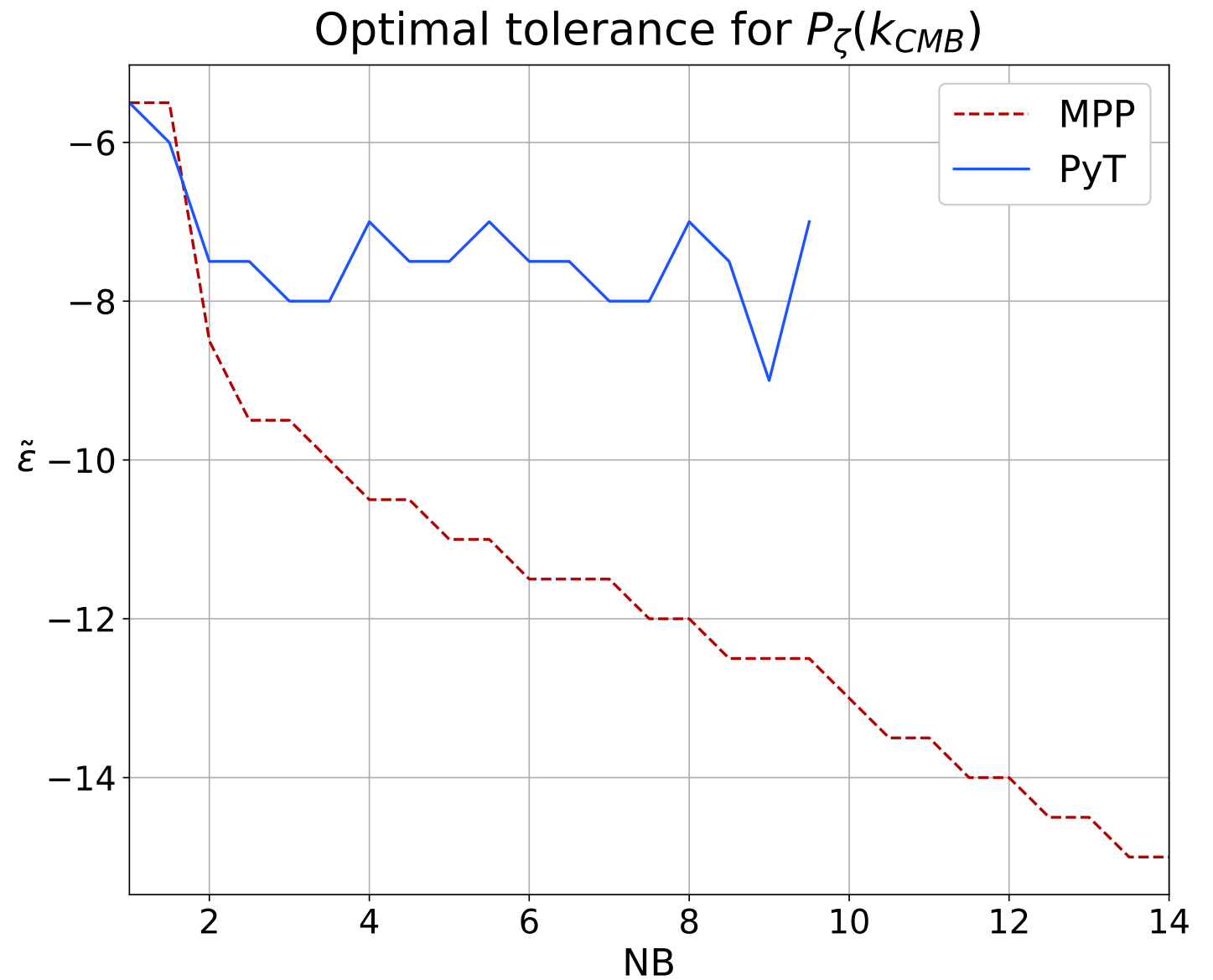
$$f_{\text{NL}} \equiv \frac{5}{6} \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_1)P_\zeta(k_3)}$$



Tests: 2pt evolution

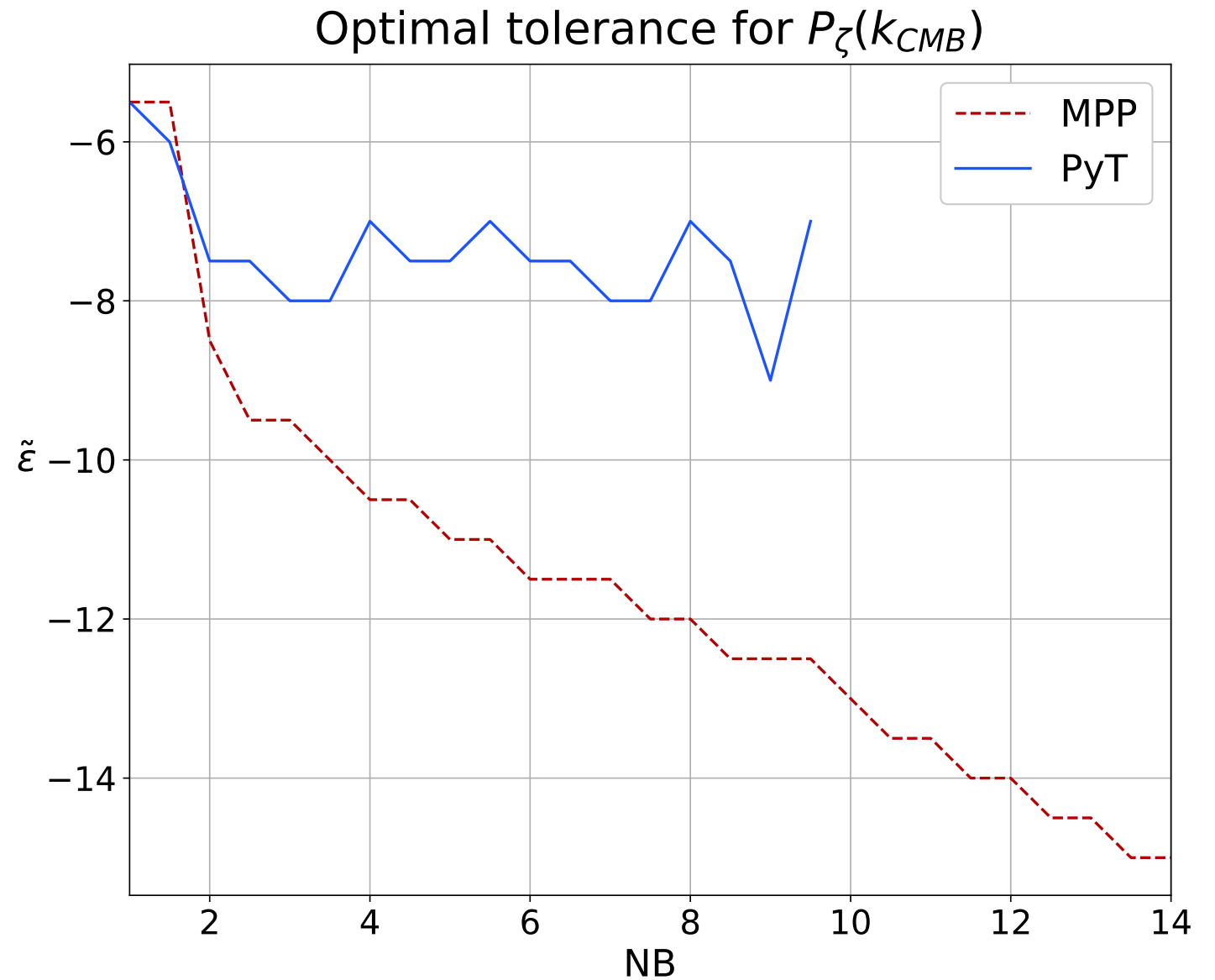


Tests: precision



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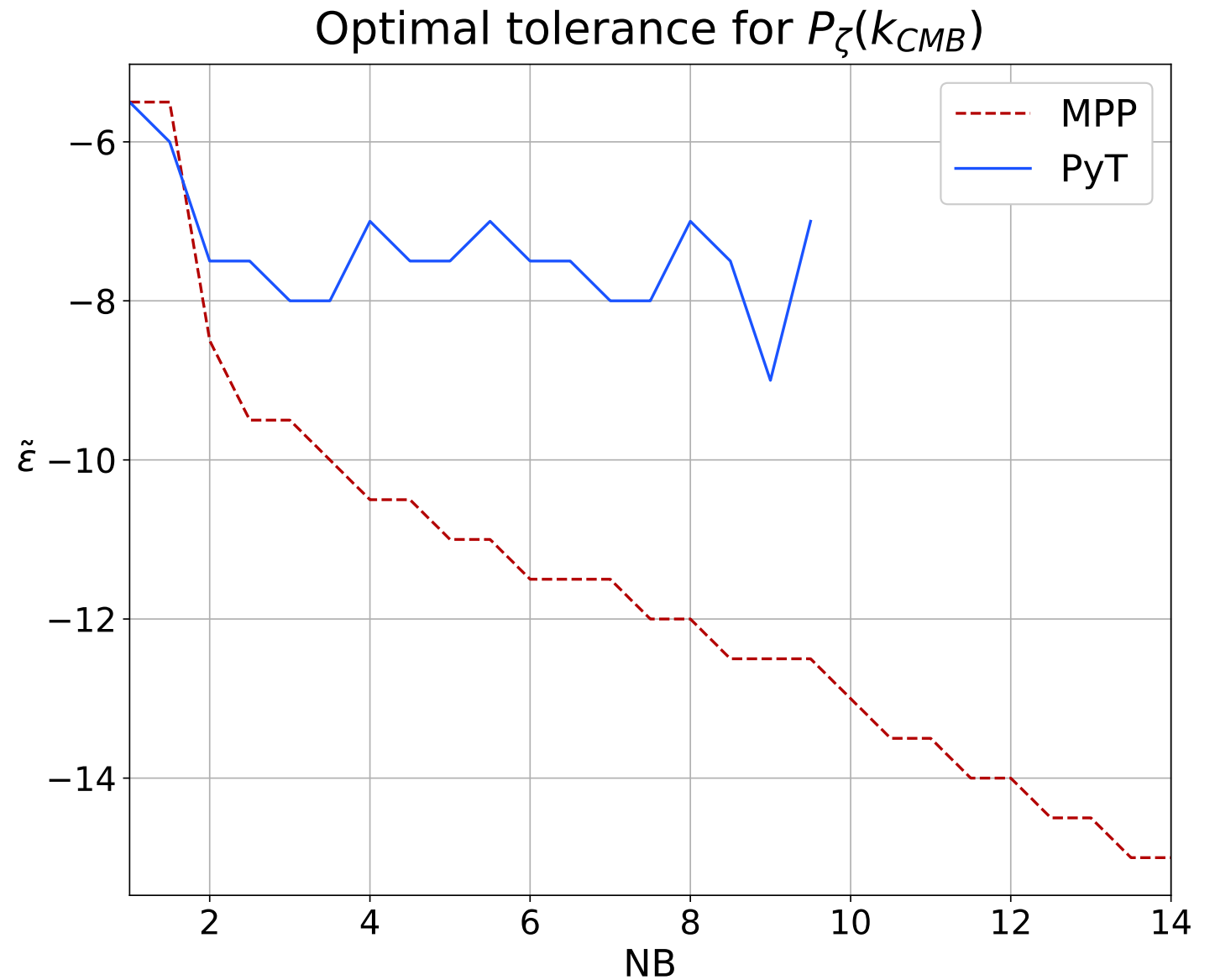
NB $\equiv$ e-folds of sub-horizon evolution



Tests: precision

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$$\epsilon = \log_{10}(\text{tol})$$

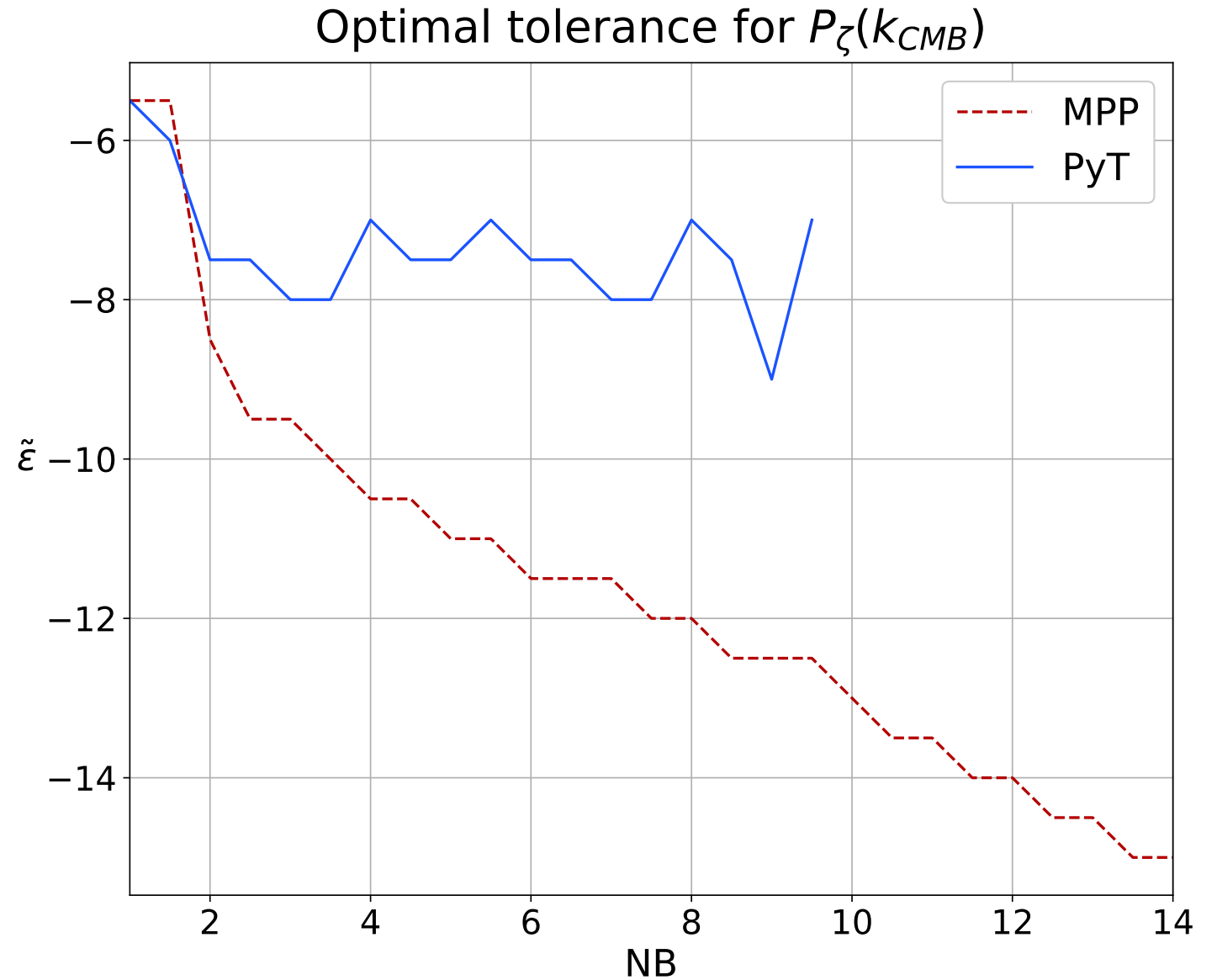


Tests: precision

NB $\equiv$ e-folds of sub-horizon evolution

$$\epsilon = \log_{10}(\text{tol})$$

$$\Delta\mathcal{P}_\zeta = \left| \frac{\mathcal{P}_\zeta^{(n-1)} - \mathcal{P}_\zeta^{(n)}}{\mathcal{P}_\zeta^{(n)}} \right|$$



Tests: precision

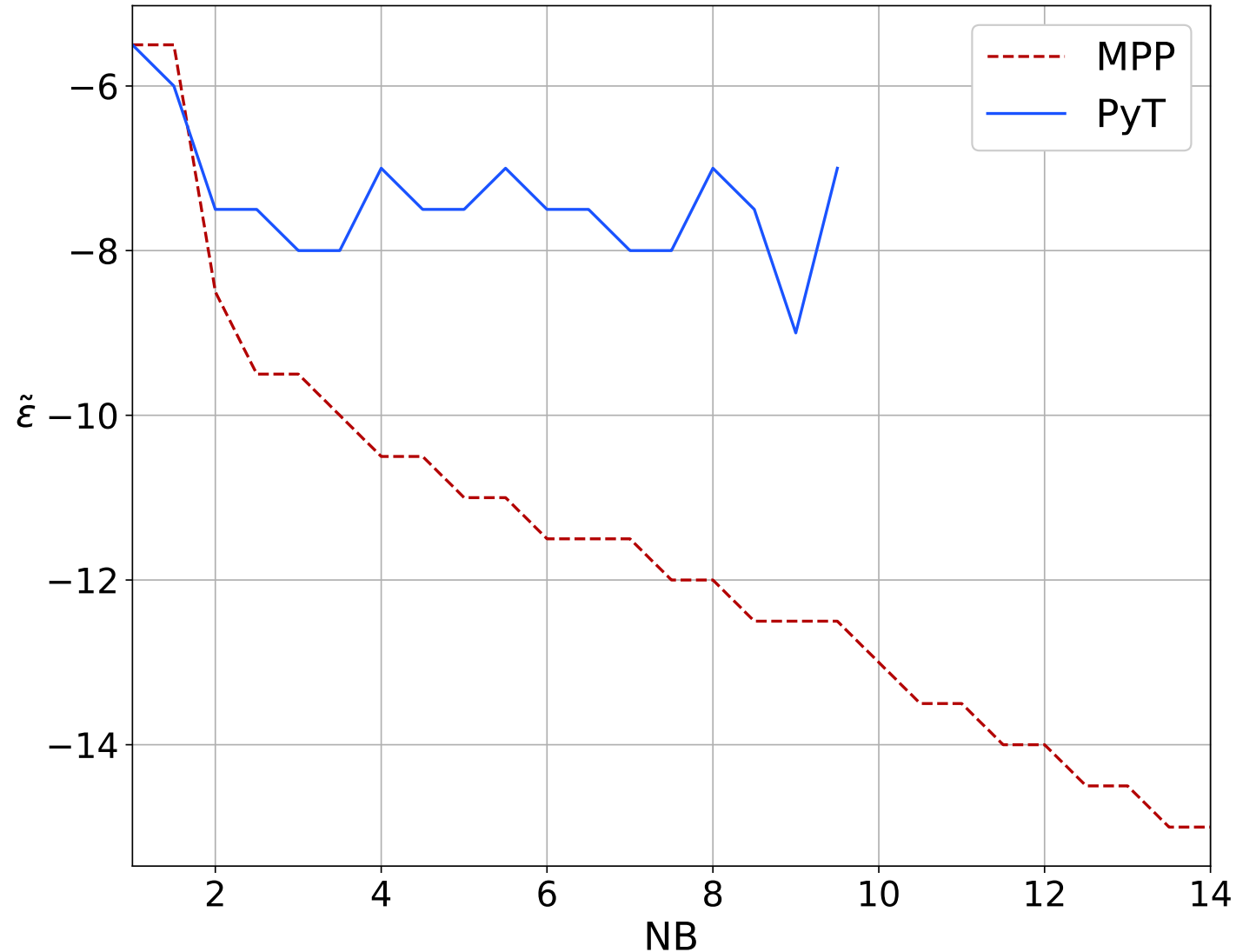
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$$\tilde{\epsilon} \rightarrow \Delta\mathcal{P}_\zeta(\epsilon) < 10^{-8}$$

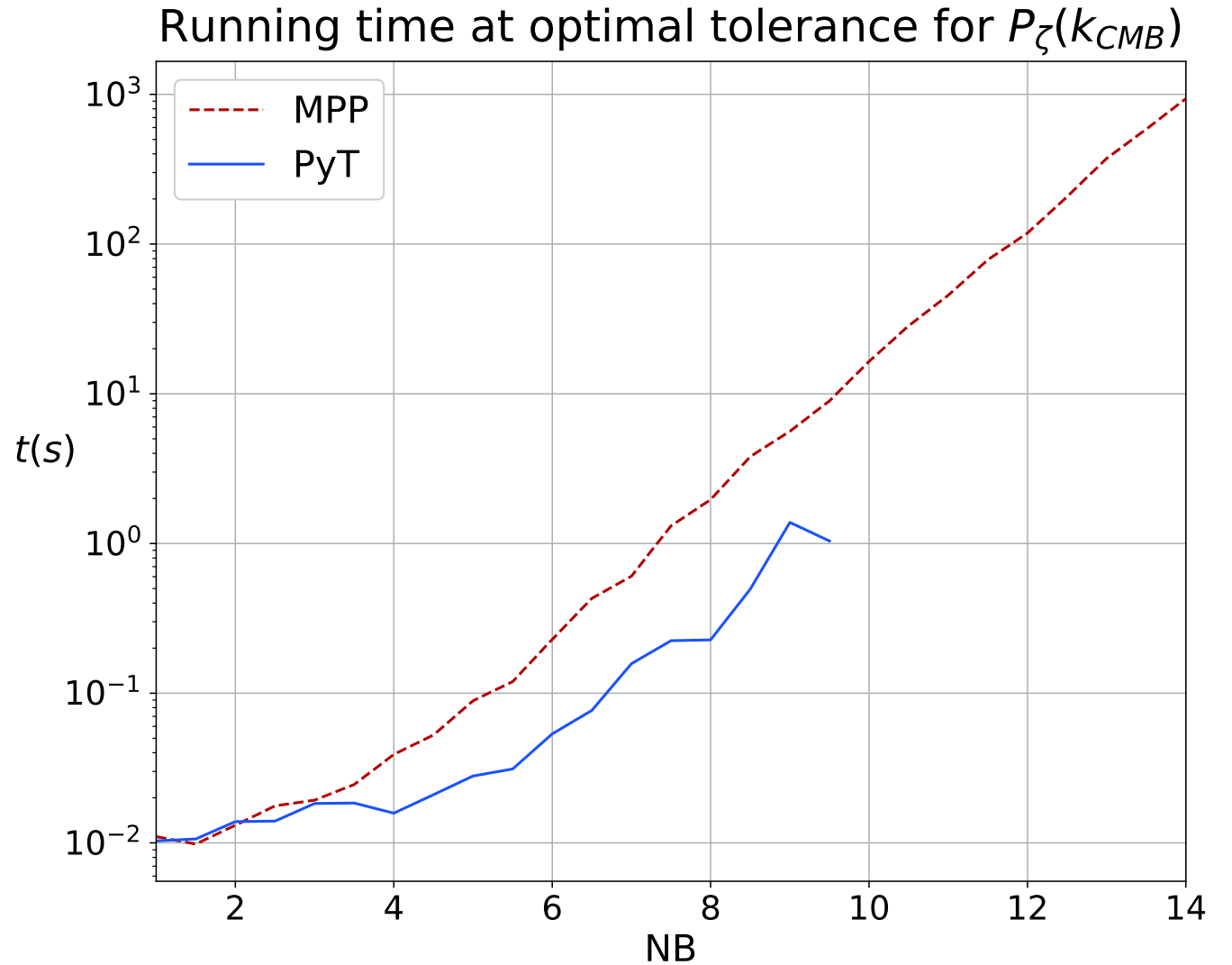
Optimal tolerance for $\mathcal{P}_\zeta(k_{CMB})$



Tests: running time

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k)$$

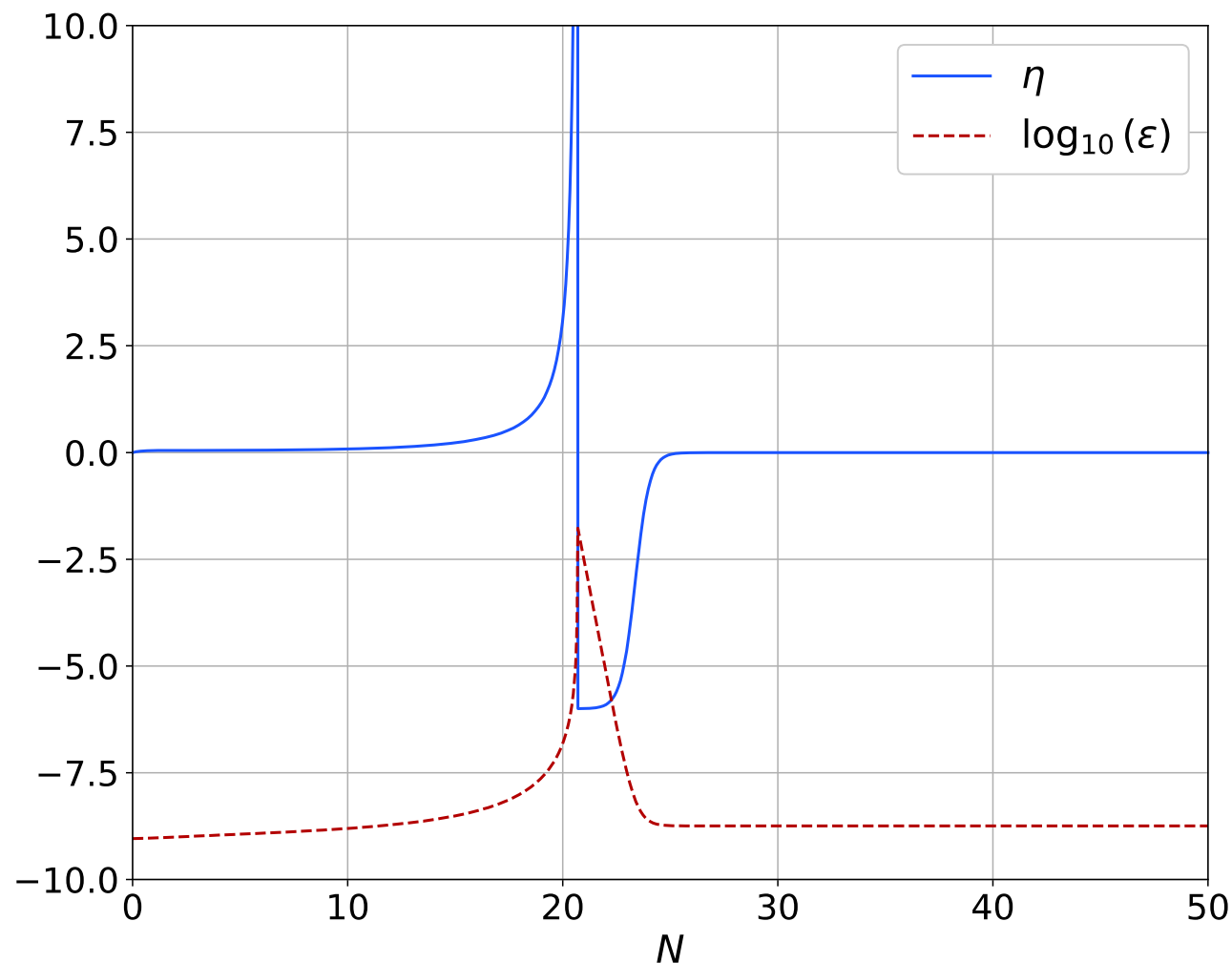
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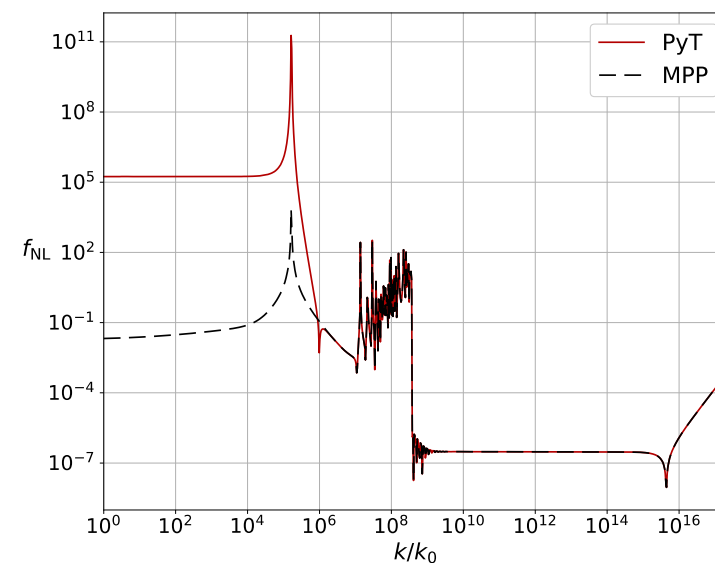
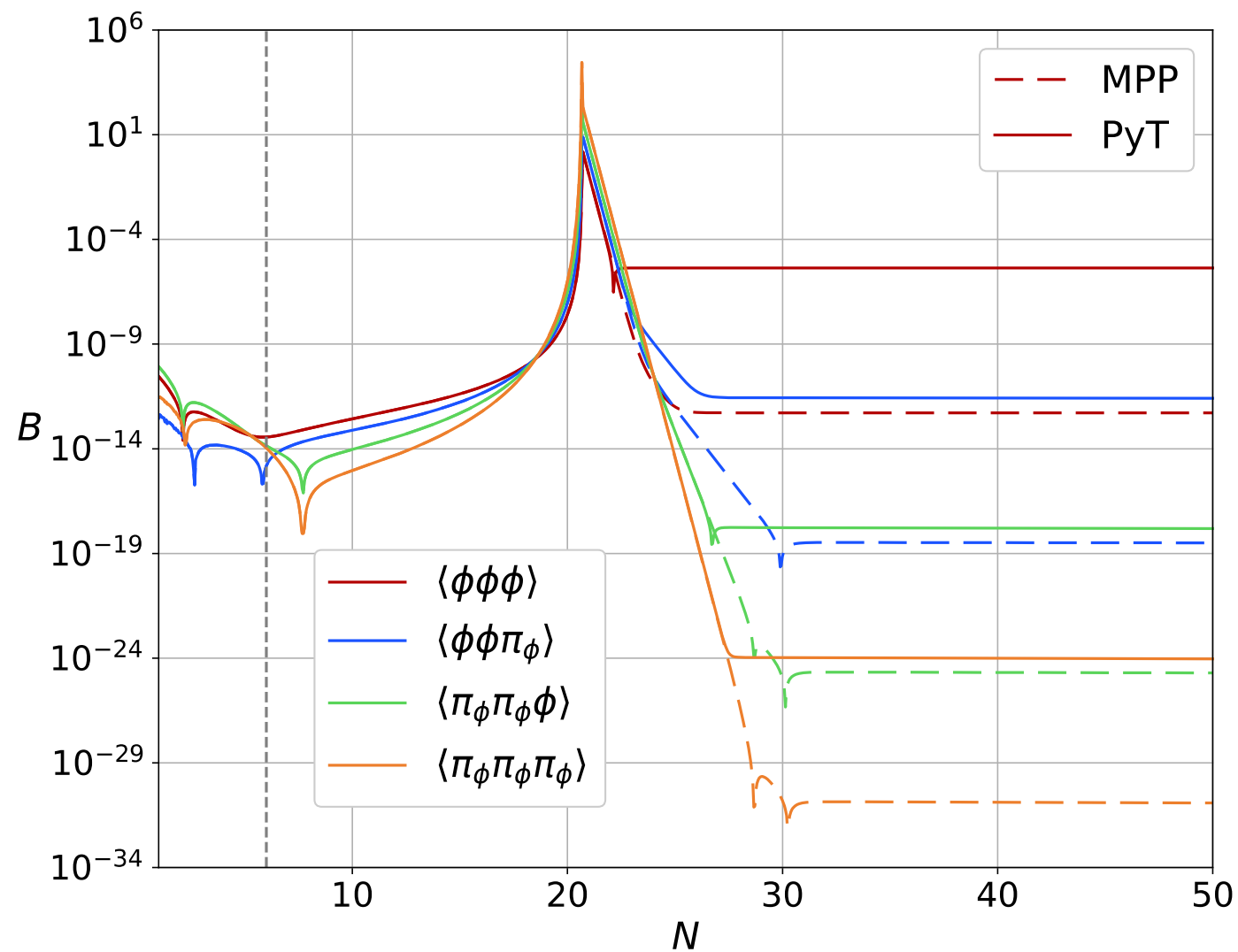
Ultra-slow-roll model

USR phase $\rightarrow \epsilon \ll 1, \eta \sim -6$

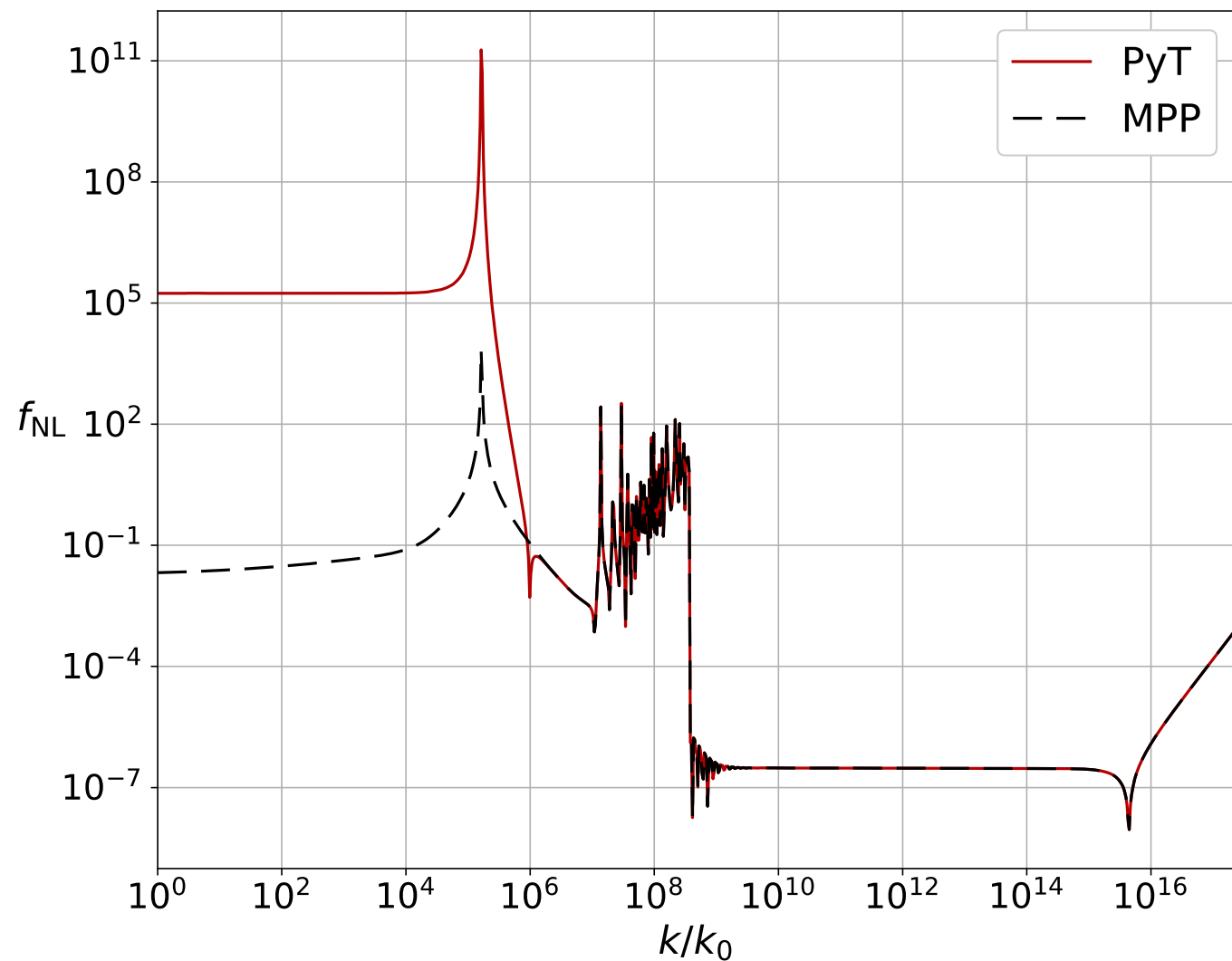
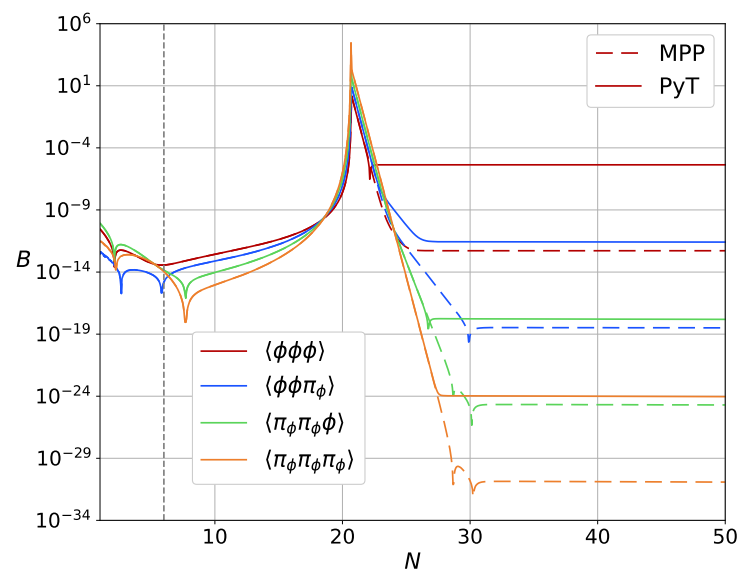
$$\epsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \frac{\ddot{\epsilon}}{\epsilon H}$$



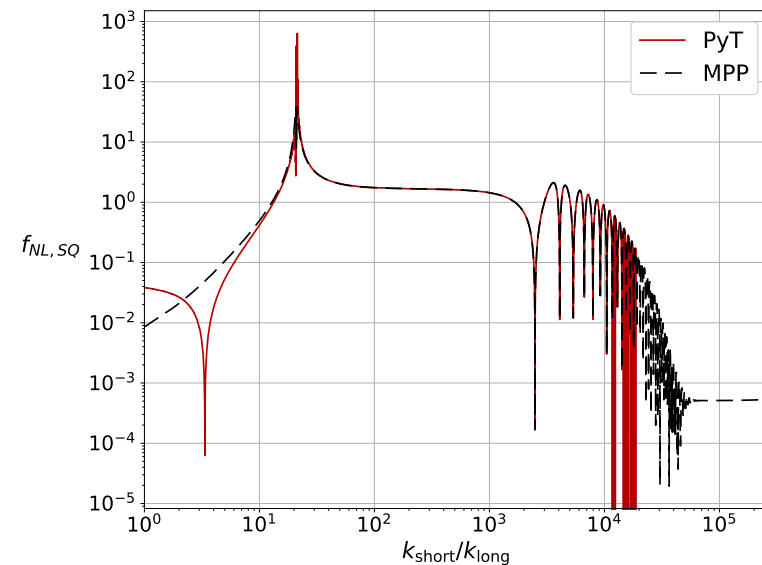
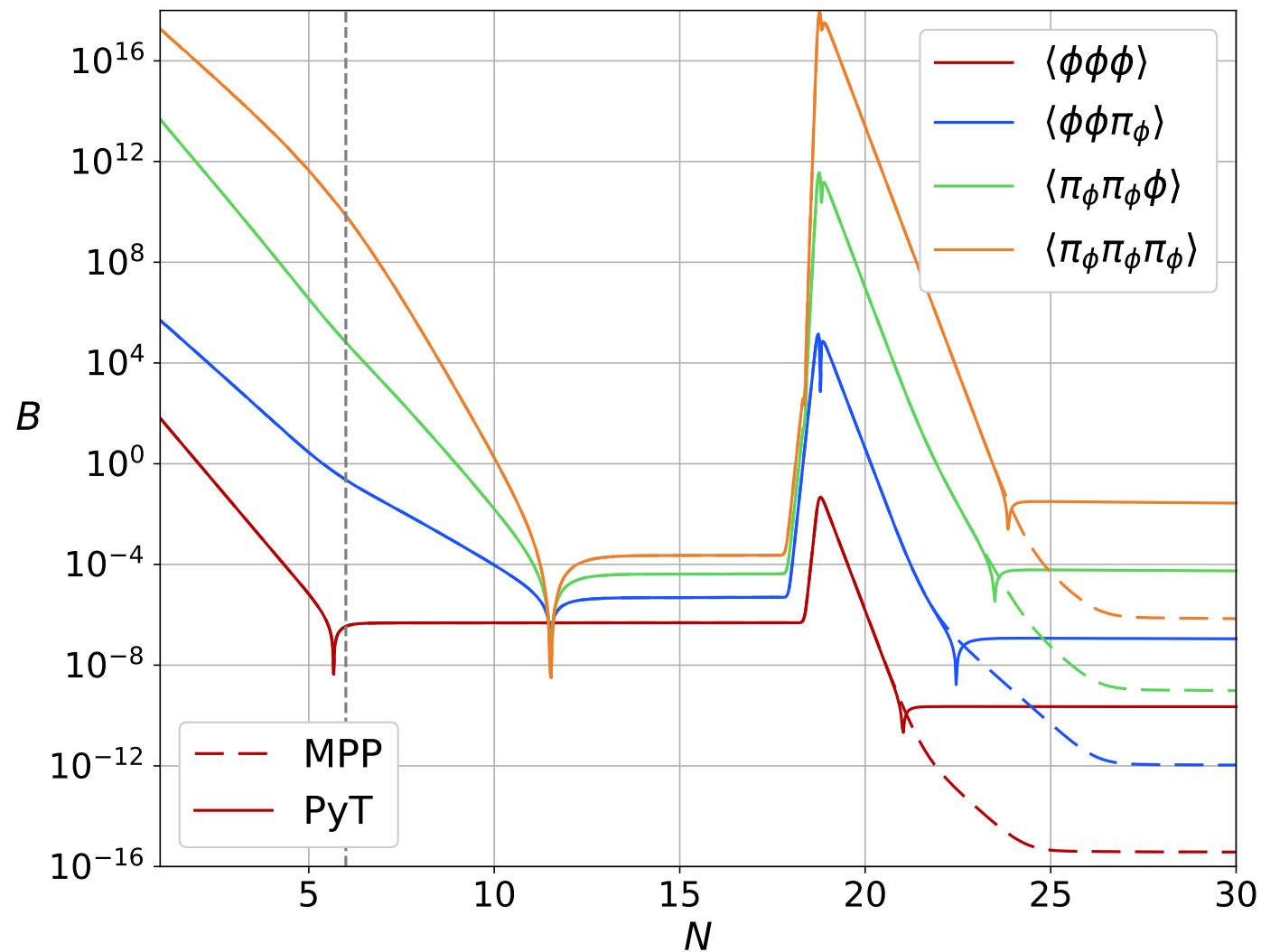
USR model: 3-point correlators



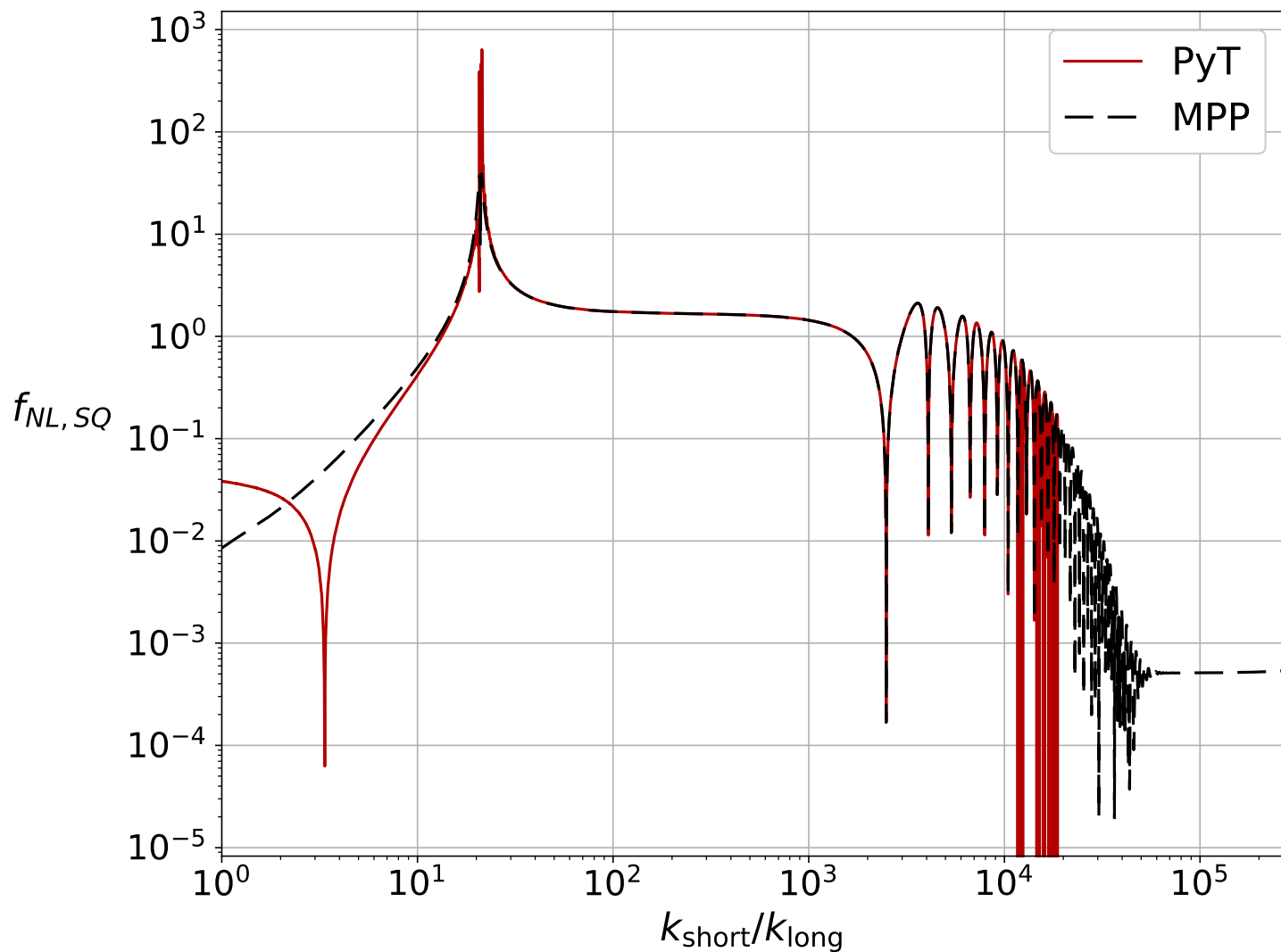
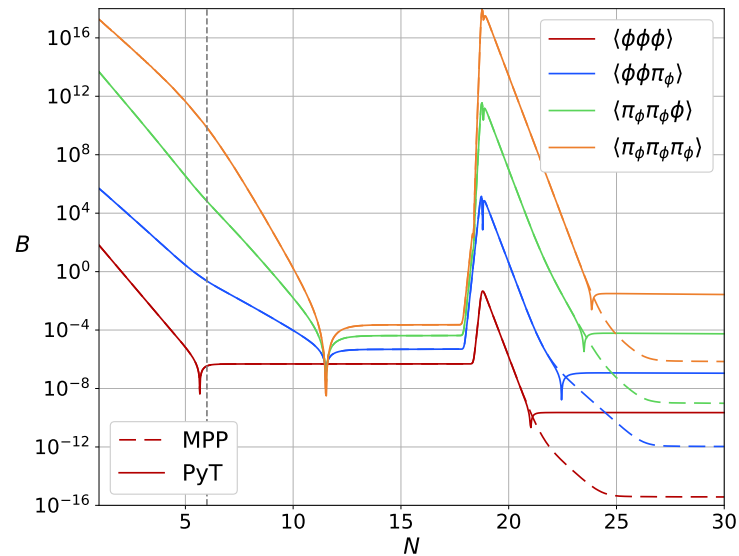
USR model : 3-point correlators



USR model: squeezed bispectrum



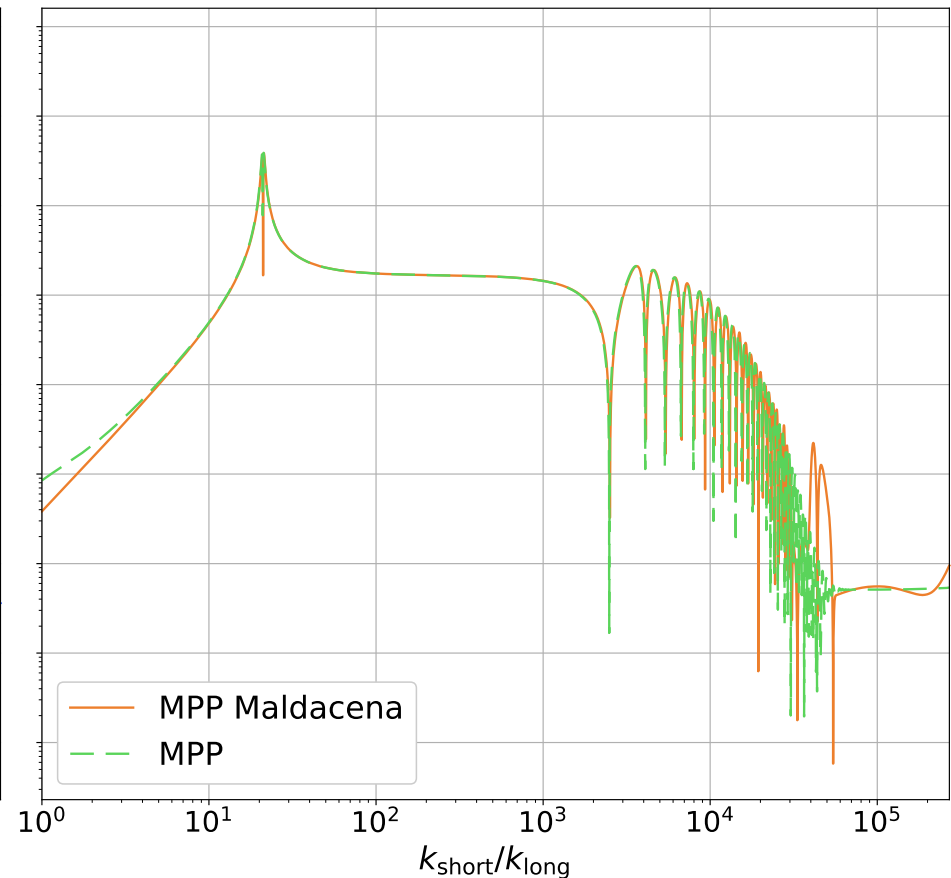
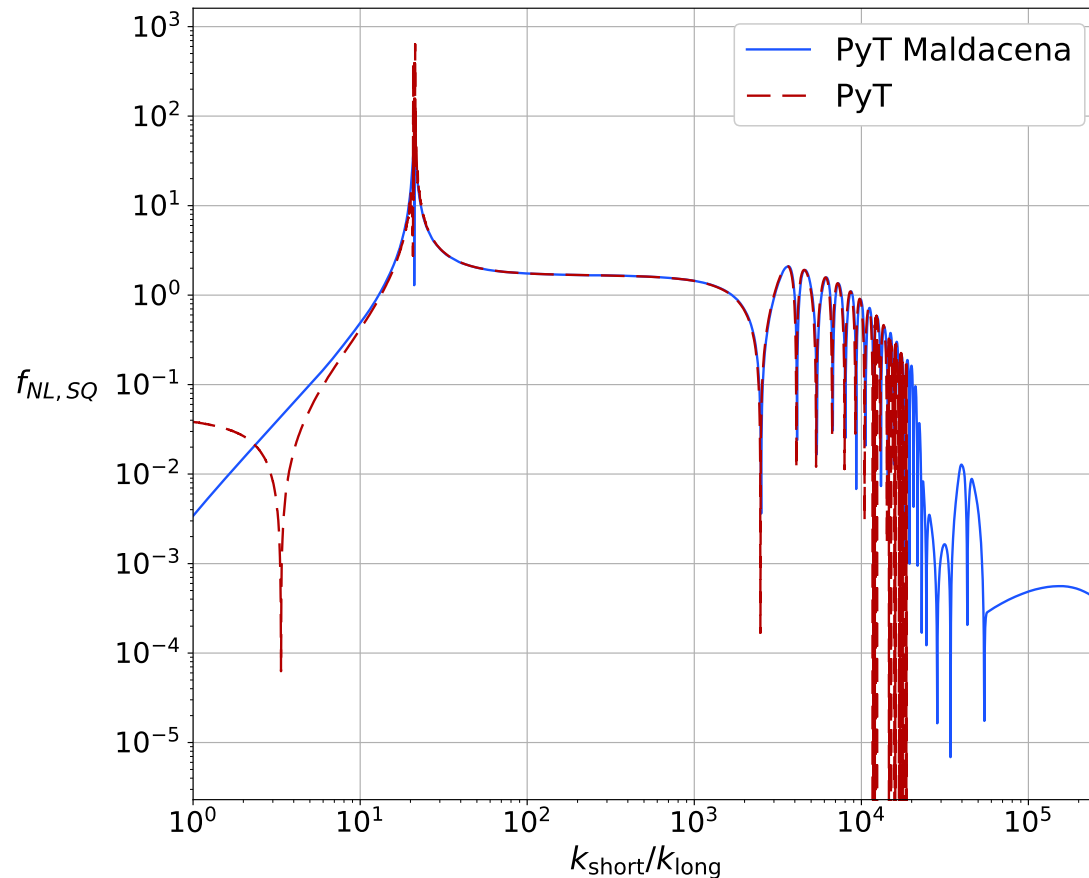
USR model: squeezed bispectrum



USR squeezed bispectrum: Maldacena consistency relation

$$f_{\text{NL},\text{sq}}(k_{\text{long}}, k_{\text{short}}) = \frac{5}{12} [1 - n_s(k_{\text{short}})] \quad \rightarrow$$

$$n_s = \frac{d \log \mathcal{P}_\zeta(k)}{d \log k}$$



Conclusion

- We implemented into PyTransport the MPP formalism
- Tested the accuracy and the performance of the MPPs vs PyTransport
- Verified that the MPP approach can track better the decay of correlators
- Verified that the MPP can correctly compute the bispectrum in cases where PyTransport fails

Future perspectives

- Use the MPPs to compute the power-spectrum and the bispectrum at **1-loop**
- Use the MPPs to compute the trispectrum at tree-level

$$\begin{aligned}\delta X^a(N, \mathbf{k}) = & \Gamma_i^{a(N_0, N)}(k) \delta X^i(N_0, \mathbf{k}) + \frac{1}{2!} \int \frac{d^3 k_1}{(2\pi)^3} \Gamma_{ij}^{a(N_0, N)}(k, k_1, |\mathbf{k} - \mathbf{k}_1|) \delta X^i(N_0, \mathbf{k}_1) \delta X^j(N_0, \mathbf{k} - \mathbf{k}_1) \\ & + \frac{1}{3!} \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \Gamma_{ijk}^{a(N_0, N)}(k, k_1, k_2, |\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2|) \delta X^i(N_0, \mathbf{k}_1) \delta X^j(N_0, \mathbf{k}_2) \delta X^k(N_0, \mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2) + \dots\end{aligned}$$