Institute for Basic Science

Cosmological correlators beyond the de-Sitter lamppost

Yuhang Zhu

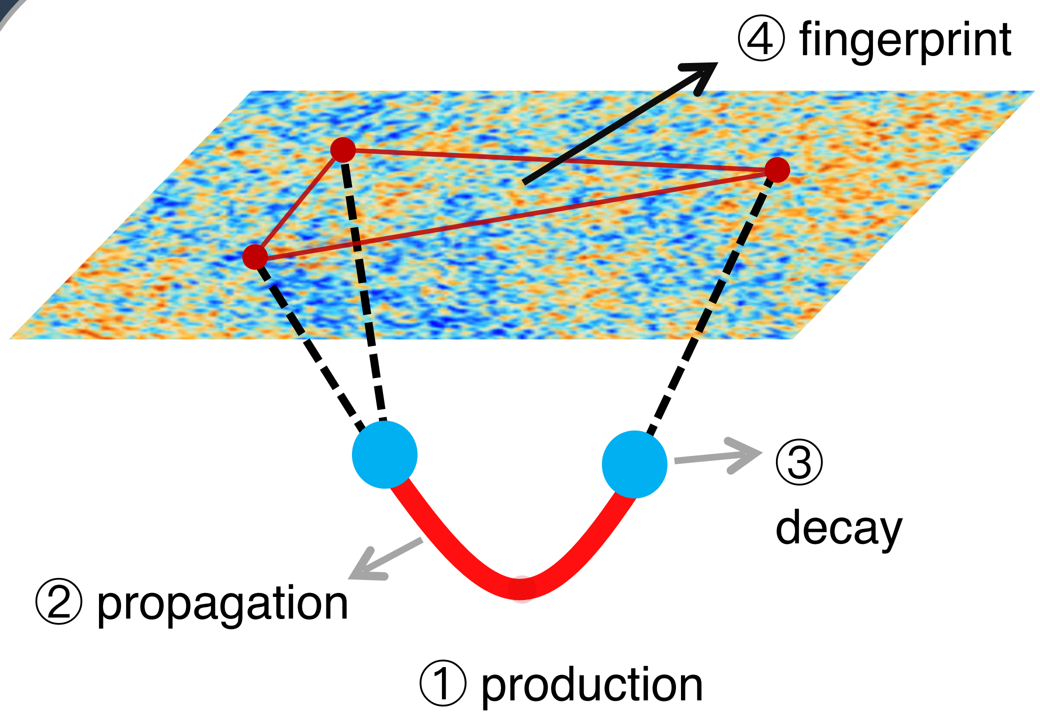
Institute for Basic Science

Dec.04l @ Inflation 2025, IAP

with Sadra Jazayeri, Zhehan Qin, Sébastien Renaux-Petel, Xi Tong, Denis Werth,

arXiv: 2506.01555, 2511.00152

1. *Cosmological correlators probe extremely high energy physics*

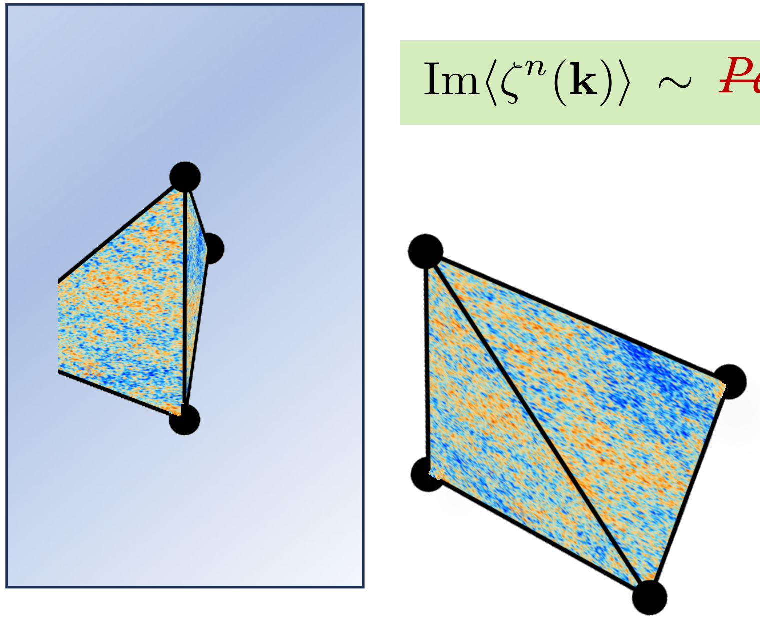


Cosmological collider [see Zhong-Zhi Xianyu's talk]

$$S \sim \mathcal{A}(\lambda, m)\, e^{-\pi\mu} \left(\frac{k_3}{k_1}\right)^{1/2} \sin\left[\mu \log\left(\frac{k_3}{k_1}\right) + \vartheta\right]$$

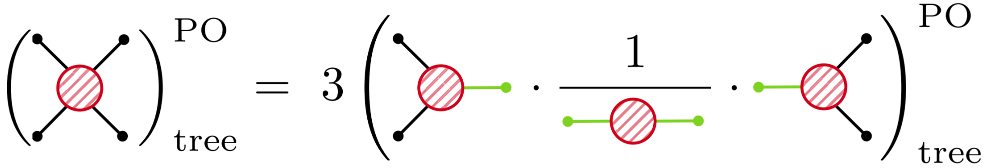
mass

2. *Cosmological correlators probe symmetries of universe*

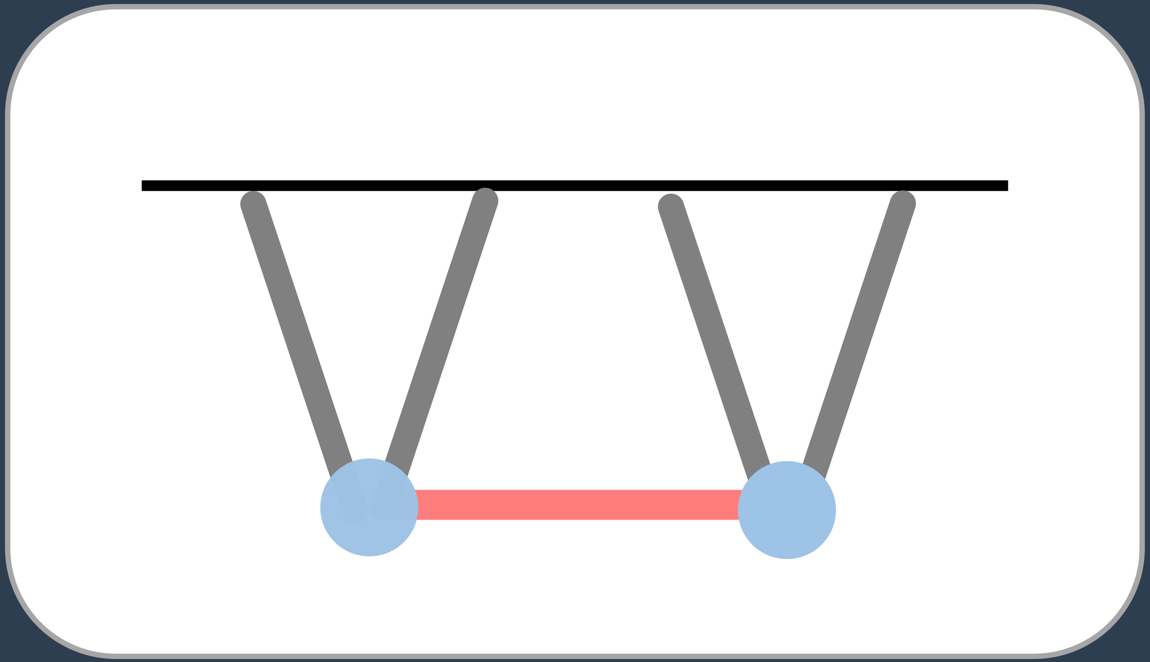


$\mathrm{Im}\langle\zeta^n(\mathbf{k})\rangle \sim$ ~~*Parity*~~

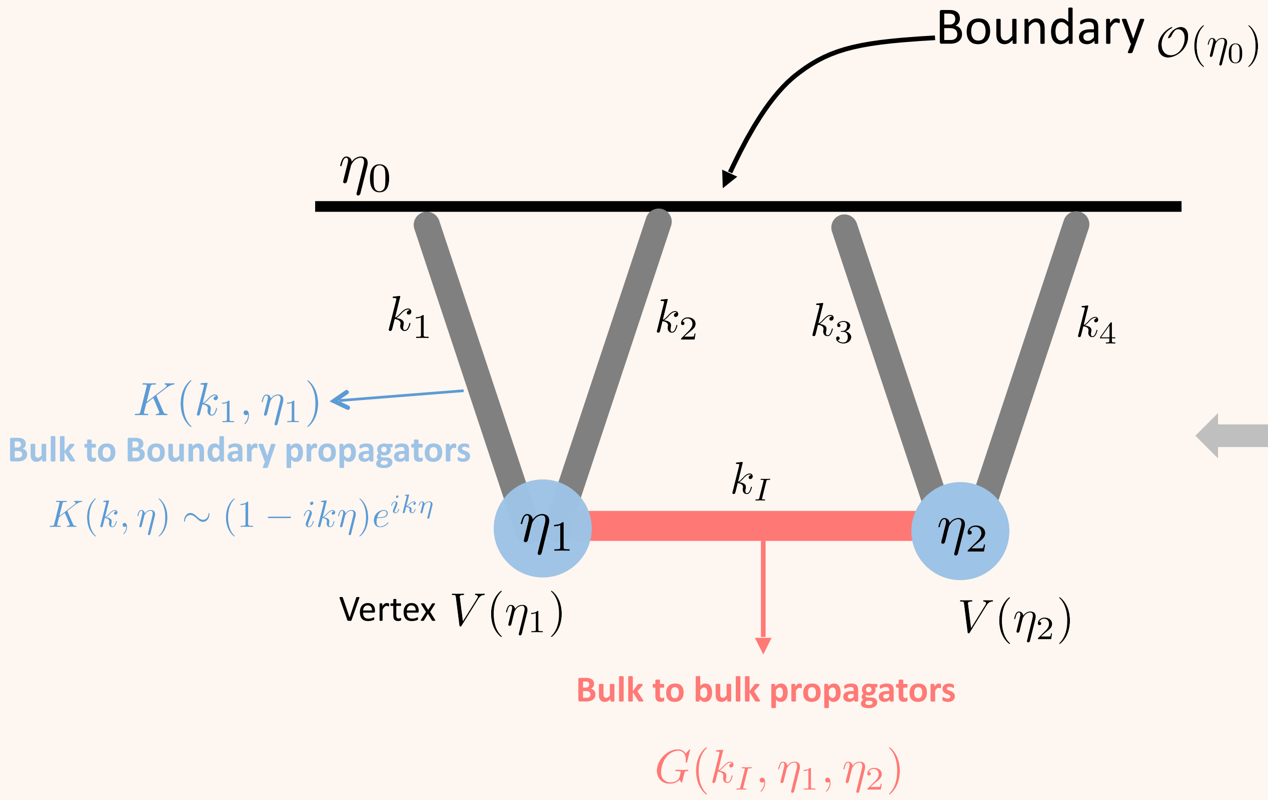
PO correlators [see Xi Tong's talk]

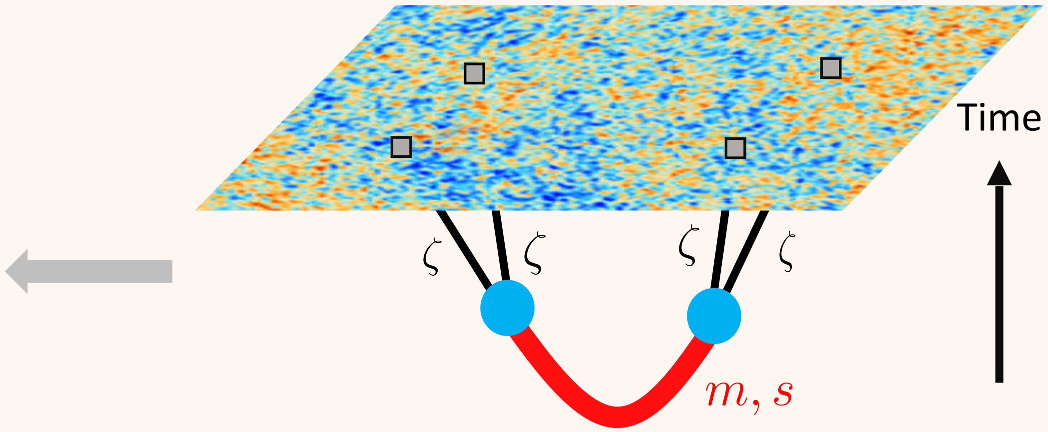


Calculation of Cosmological Correlators



Analytical Calculation of cosmological correlators





$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle' = \int d\eta_1 \int d\eta_2 V(\eta_1) V(\eta_2) G(k_I, \eta_1, \eta_2) K(k_1, \eta_1) K(k_2, \eta_1) K(k_3, \eta_2) K(k_4, \eta_2)$$

Too difficult to solve analytically!

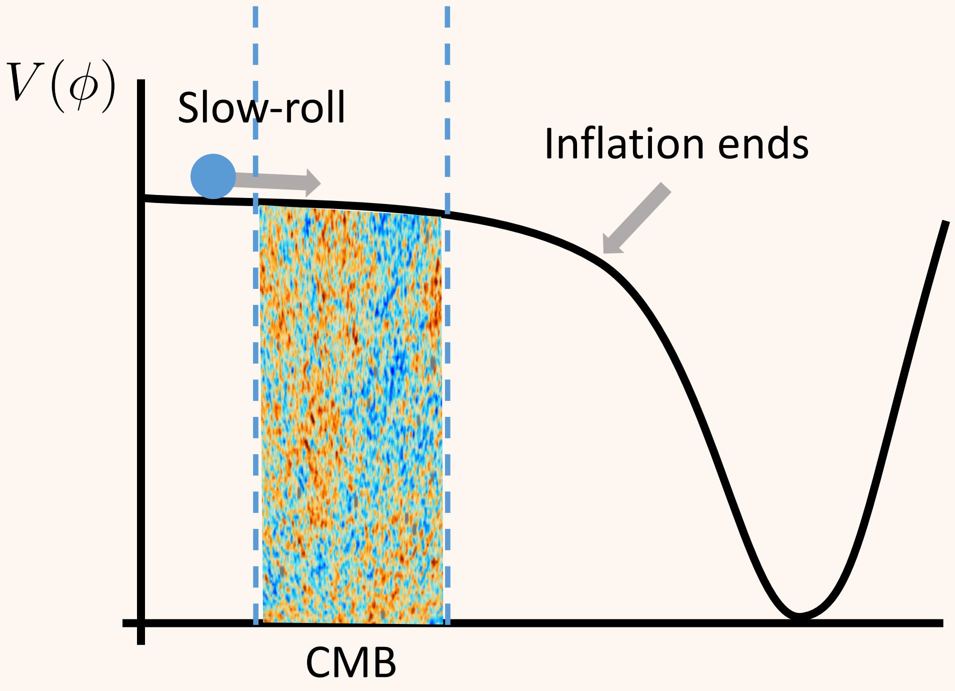
(1) Nested Time integral

$$G \supset \theta(\eta_1 - \eta_2) v_k(\eta_1) v_k^*(\eta_2) + \theta(\eta_2 - \eta_1) v_k^*(\eta_1) v_k(\eta_2)$$

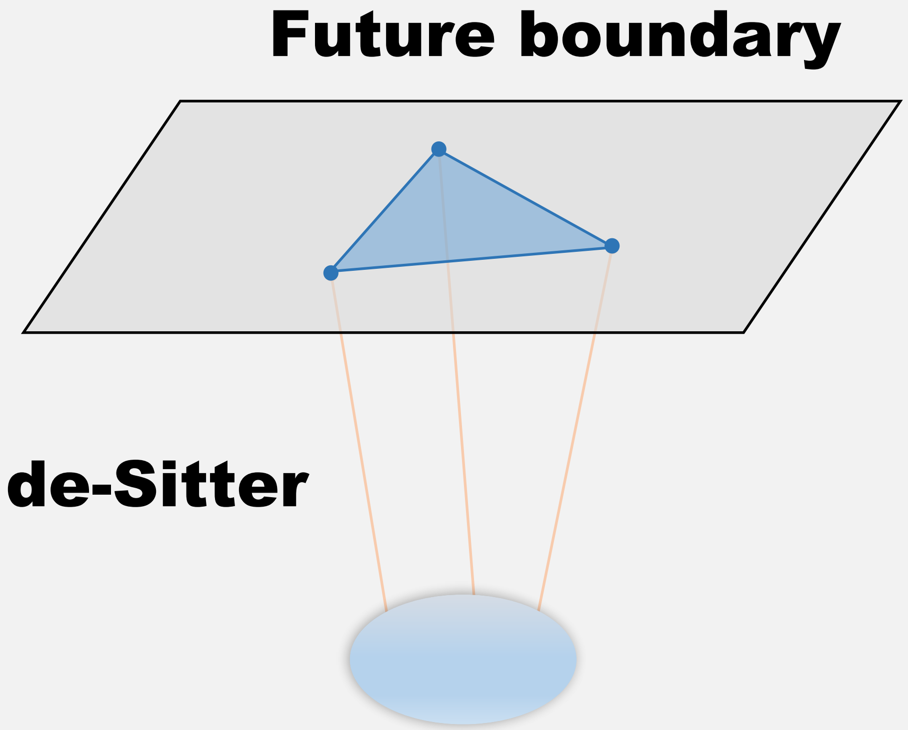
(2) Mode functions are complicate

$$v_k \sim H_{i\mu} \text{ or } W_{i\kappa, i\mu}$$

Inflation



de-Sitter

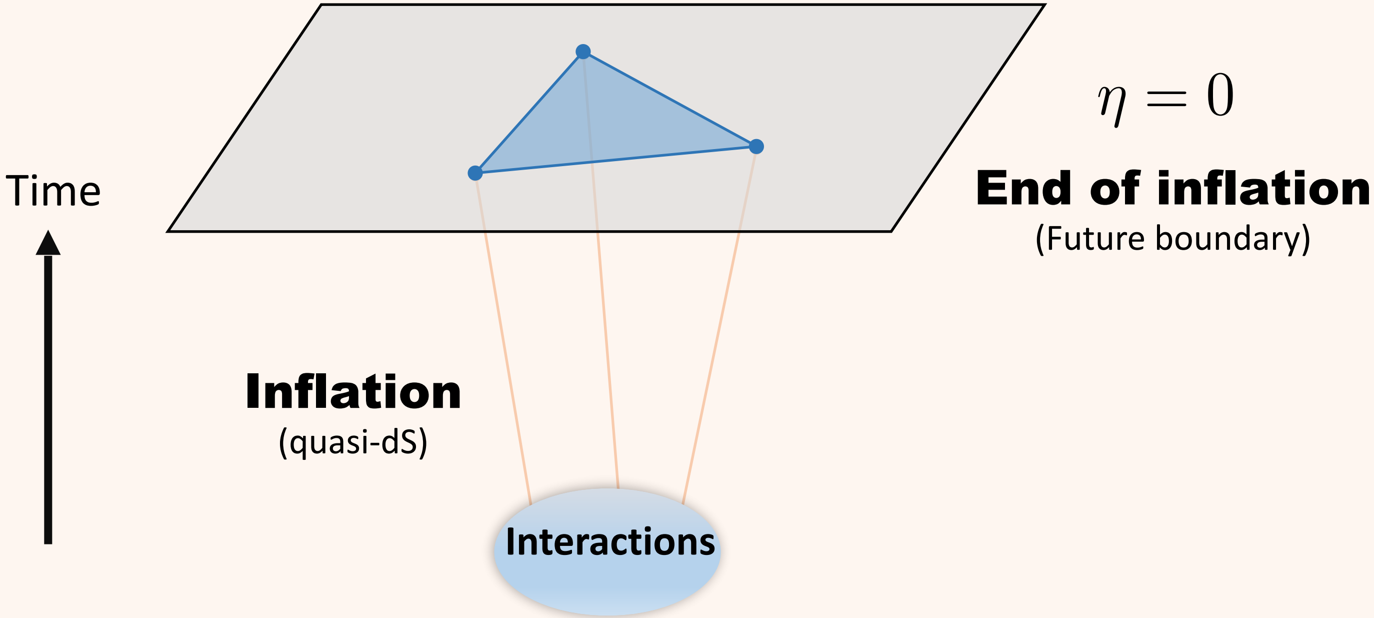


$$V(\phi) \sim \Lambda$$

- Symmetry is powerful

dS Bootstrap

[see Hayden Lee's talk]



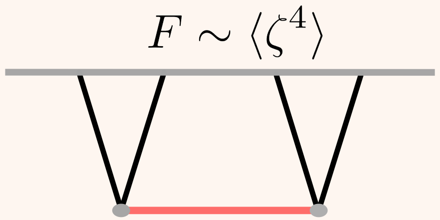
dS Symmetry : $\mathrm{d}s^2 = \dfrac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{H\eta^2}$

Translation: $P_i = \partial_i$

Rotation: $J_{ij} = x_i \partial_j - x_j \partial_i$

Dilation: $D = -\eta\partial_\eta - x_i\partial_i$

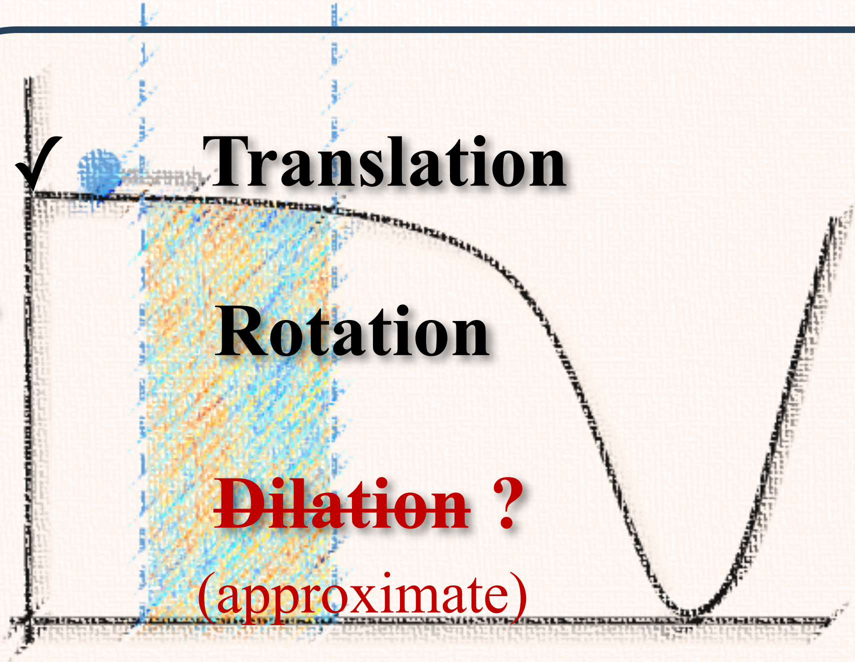
dS boosts: $K_i = 2x_i\eta\partial_\eta + \left(2x^j x_i + (\eta^2 - x^2)\delta_i^j\right)\partial_j$



$$\left[u^2(1-u^2)\partial_u^2 - 2u^3\partial_u + \mu^2 + \frac{1}{4}\right] F = g^2 \frac{uv}{u+v}$$

+ **Boundary conditions** **=** **fix the correlators**

Inflation $\neq$ de-Sitter



✓ **Translation**

✓ **Rotation**

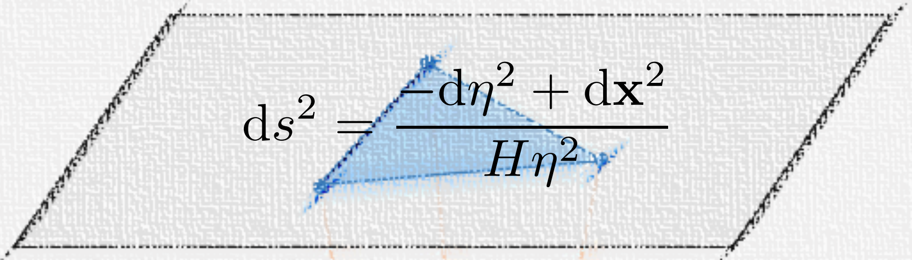
~~Dilation~~ ?

(approximate)

~~Boost~~ ?

(preferred direction of time)

$\phi(t)$



$$ds^2 = \frac{-d\eta^2 + d\mathbf{x}^2}{H\eta^2}$$

- **3 Translation** $P_i = \partial_i$
- **3 Rotation** $J_{ij} = x_i\partial_j - x_j\partial_i$
- **1 Dilation** $D = -\eta\partial_\eta - x_i\partial_i$
- **3 dS boosts**

$$K_i = 2x_i\eta\partial_\eta + \left(2x^j x_i + (\eta^2 - x^2)\delta_i^j\right)\partial_j$$

Cosmological bootstrap beyond dS lamppost

de Sitter

- Much larger signals
- Richer phenomenology

less symmetries

⬇

more difficult !

~~boost~~

~~Scale invariance~~

Boost-Breaking

[Qin, Renaux-Petel, Tong, Werth, **YHZ** 2506.01555]

massive fields in dS:

$$\left[\frac{\partial^2}{\partial\eta^2} - \frac{2}{\eta}\frac{\partial}{\partial\eta} + \left(k^2 + \frac{m^2}{H^2\eta^2}\right)\right]\sigma(k,\eta) = 0$$

Dispersion relation $\omega(k,\eta)$

~~Boost:~~ $\omega^2 = m^2 + c_s^2 k_p^2 + \kappa\, k_p$

Sound speed

- low speed collider
- new shapes

....

e.g.
[Pimentel, Wang, 2022]
[Jazayeri, Renaux-Petel 2022]

\+

Chemical potential

$\phi(t) F\tilde{F}$

- enhanced particle production

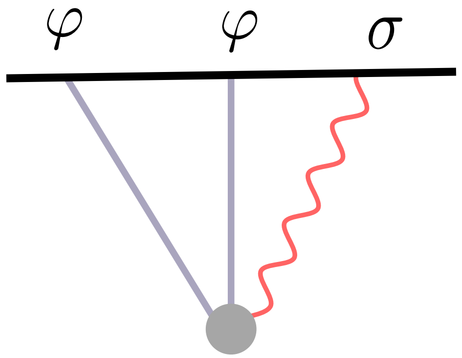
$e^{-\pi\mu} \rightarrow e^{-\pi(\mu-\kappa)}$

e.g. [Qin, Xianyu, 2022]

....

Boost-Breaking

~~Boost~~ EoM: $$\left(\eta^2\partial_\eta^2 - 2\eta\,\partial_\eta + c_s^2 k^2\eta^2 \pm 2\,\kappa\,k\,\eta + m^2\right)\sigma^\pm = 0$$



$$f_3 = \int_{-\infty}^{0} \frac{d\eta}{\eta^2}\, e^{ik_{12}\eta}\,\sigma(s,\eta)$$

$$\int d\eta\, e^{ik_{12}\eta}\,\partial_\eta\,\sigma(s,\eta) \;\Rightarrow\; \partial_\eta \to k_{12}$$

$$\int d\eta\, e^{ik_{12}\eta}\,\eta\,\sigma(s,\eta) \;\Rightarrow\; \eta \to \partial_{k_{12}}$$

$$\int_{-\infty}^{0} \frac{d\eta}{\eta^2}\, e^{ik_{12}\eta}\left[\text{~~Boost~~ EoM}\right]\sigma(s,\eta) = 0$$

$$\Big[\underbrace{(k_{12}^2 - c_s^2 s^2)\partial_{k_{12}}^2 + (2+i\kappa)k_{12}\partial_{k_{12}} + (m^2-2)}_{\hat{\Delta}_{12}}\Big] f_3 = 0$$

Boost-Breaking

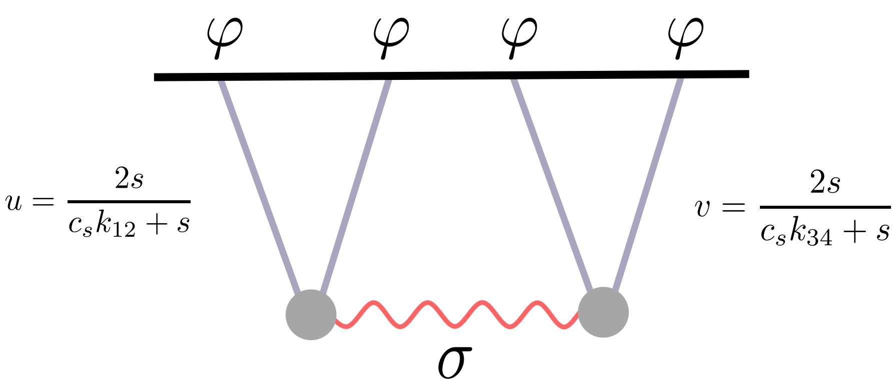
~~Boost~~ EoM:

$$\left(\eta^2\partial_\eta^2 - 2\eta\,\partial_\eta + c_s^2 k^2\eta^2 \pm 2\,\kappa\,k\,\eta + m^2\right)\sigma^\pm = 0$$

Boundary differential equation:

$$\left[\Delta_{\pm,u} + \left(\mu^2 + \frac{1}{4}\right)\right] F = \frac{1}{2}\frac{uv}{u+v-uv}$$

$$\Delta_{\pm,u} \equiv u^2(1-u)\partial_u^2 - \left[(1 \pm i\lambda\kappa)u^2\right]\partial_u$$



$$F = \int_{-\infty}^{0} \frac{d\eta}{\eta^2}\frac{d\eta'}{\eta'^2}\, e^{ik_{12}\eta}\, G(s,\eta,\eta')\, e^{ik_{34}\eta'}$$

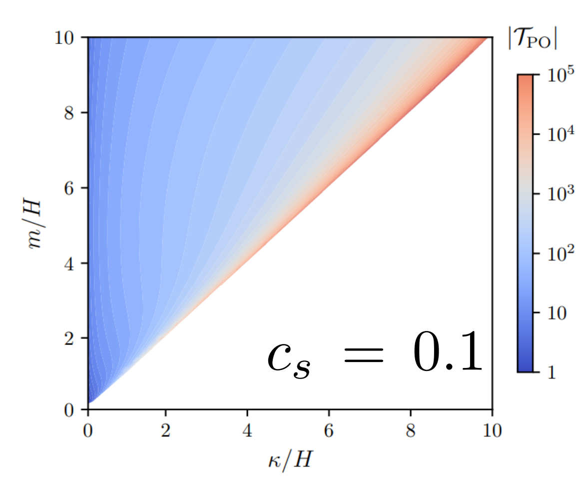
Boundary conditions

- ➢ Folded limit
- ➢ Total-energy limit
- ✓ soft limit

Boost-Breaking Phenomenology

$$\left(\eta^2\partial_\eta^2 - 2\eta\,\partial_\eta + c_s^2 k^2\eta^2 \pm 2\,\kappa\,k\,\eta + m^2\right)\sigma^\pm = 0$$

① *Large PO trispectrum*



"relatively **simple physical template**
"

② *New cosmological collider signal*

$$\exp\left(-c_s\,\mu\,\frac{k_L}{k_S}\right)\cos\left(c_s\,\kappa\,\frac{k_L}{k_S}\right)$$

$\cos(\cdots)$ oscillation instead of $\cos(\log\cdots)$

See [2506.01555] for other applications

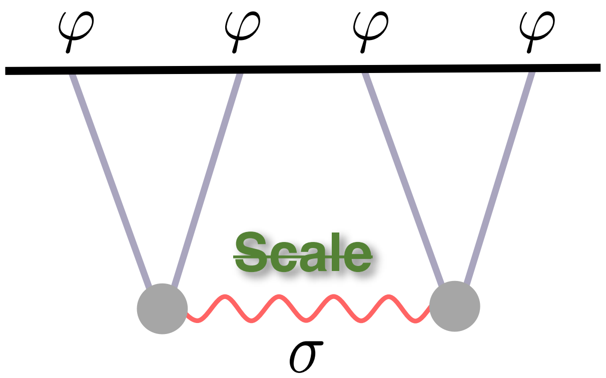
Scale-Breaking

[Jazayeri, Tong, **YHZ** 2511.00152]

$$\omega^2 = m^2(t) + c_s^2\,(t)\,k_p^2 + \cdots$$

e.g. from axion monodromy inflation

$$m^2(t) = m_0^2\left[1 + g^2\cos\omega(t-t_0)\right]$$
$$= m_0^2 + \frac{1}{2}g^2\,m_0^2\left[\left(\frac{\eta}{\eta_0}\right)^{i\omega} + \left(\frac{\eta}{\eta_0}\right)^{-i\omega}\right]$$



$$F = \int_{-\infty}^{0}\frac{d\eta}{\eta^2}\frac{d\eta'}{\eta'^2}\,e^{ik_{12}\eta}\,G(s,\eta,\eta')\,e^{ik_{34}\eta'}$$

$$\int d\eta\, e^{ik_{12}\eta}\,\eta\,\sigma(s,\eta) \;\longrightarrow\; \eta \to \partial_{k_{12}}$$

$$\int d\eta\, e^{ik_{12}\eta}\,\eta^{i\omega}\,\sigma(s,\eta) \;\longrightarrow\; ?$$

Fractional derivatives!

Scale-Breaking

Integro-differential Bootstrap

Fractional derivatives

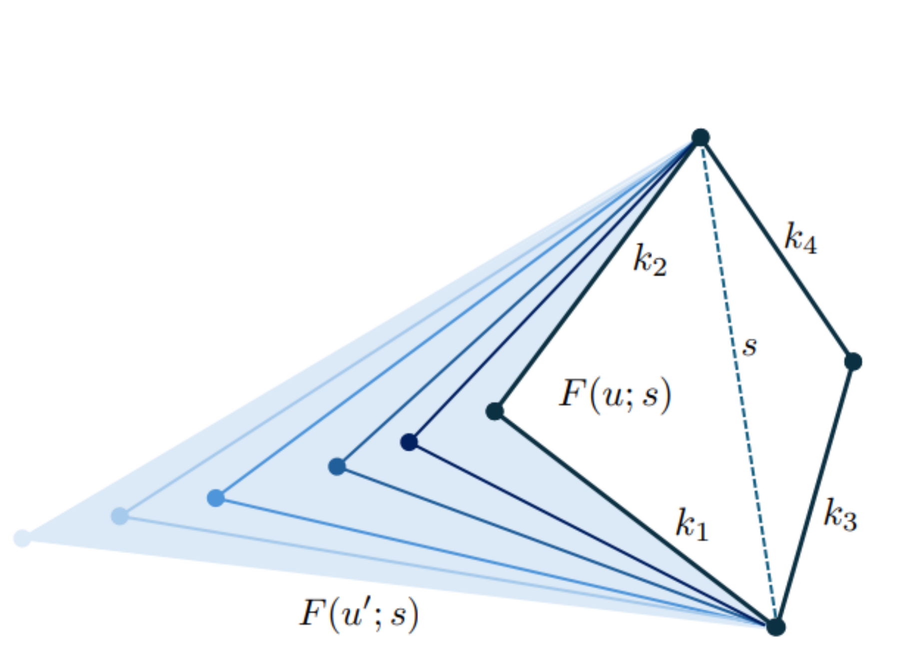
$$\left(\frac{\eta}{\eta_0}\right)^{-i\omega} = \frac{e^{-\frac{\pi\omega}{2}}}{\Gamma(i\omega)} \int_0^\infty \frac{\mathrm{d}q}{q} \, (-q\eta_0)^{i\omega} e^{iq\eta}$$

$$\int d\eta \, e^{ik_{12}\eta} \, \eta^{i\omega} \, \sigma(s,\eta)$$

$$\int dq \, d\eta \, e^{i(k_{12}+q)\eta} \, K(q) \, \sigma(s,\eta)$$

Memory Kernel

$$\hat{\Delta}_{12} \, F(k_{12}, k_{34}, s) = \frac{1}{k_T} + \int_0^\infty \mathrm{d}q \, K(q) \, F(k_{12}+q, k_{34}, s)$$



Scale-Breaking

Integro-differential Bootstrap

$$\hat{\Delta}_{12}\, F(k_{12}, k_{34}, s) = \frac{1}{k_T} + \int_0^{\infty} \mathrm{d}q\, K(q)\; F(k_{12}+q, k_{34}, s)$$

Numerical

✓ Boundary

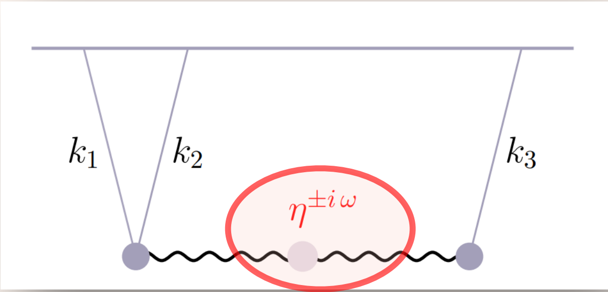
? Bulk

CosmoFlow

[Pinol, Renaux-Petel, Werth, 2023 2024]

Analytical

Perturbatively



Approximate

WKB-type ?

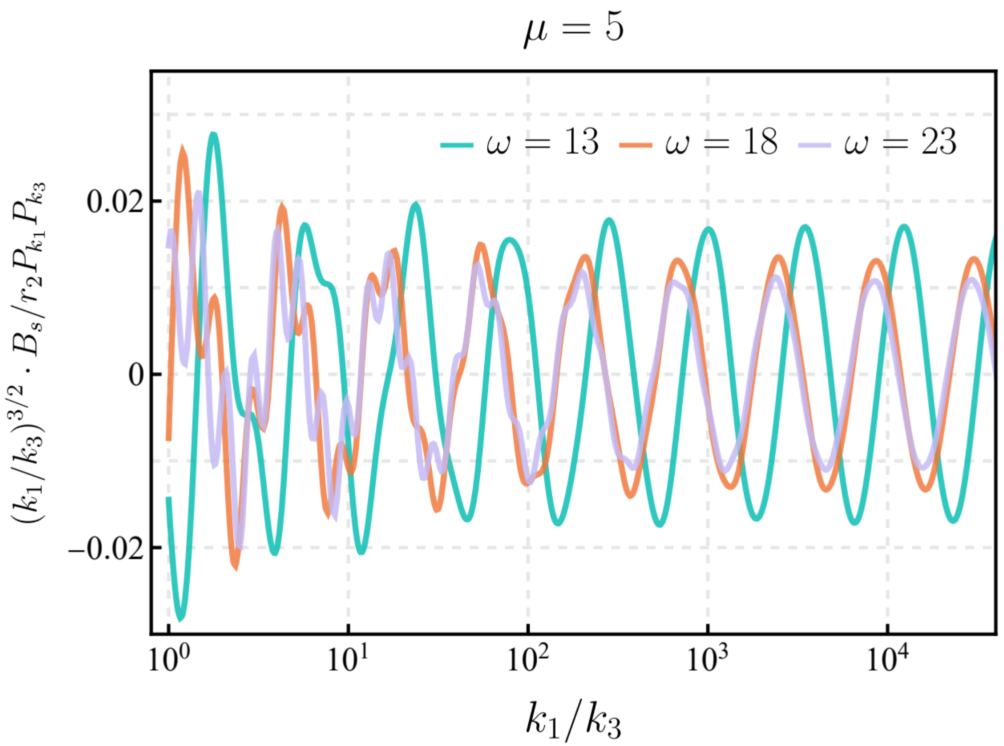
Treat it as interaction "mass insertion"

$$g^2 \cos(\omega t)\sigma^2$$

Scale-Breaking Phenomenology

Cosmological collider signals at $\mathcal{O}(g^2)$

$$B(k_1,k_2,k_3) \approx \left\{ |\mathcal{A}_1(\mu,\omega)| \cos\left[(\mu-\omega)\log\left(\frac{k_3}{2k_1}\right) + \omega\log(-k_3\eta_0) + \vartheta_1(\mu,\omega)\right] + |\mathcal{A}_2(\mu,\omega)| \cos\left[\mu\log\left(\frac{k_3}{2k_1}\right) + \omega\log(-k_3\eta_0) + \vartheta_2(\mu,\omega)\right] \right\}$$



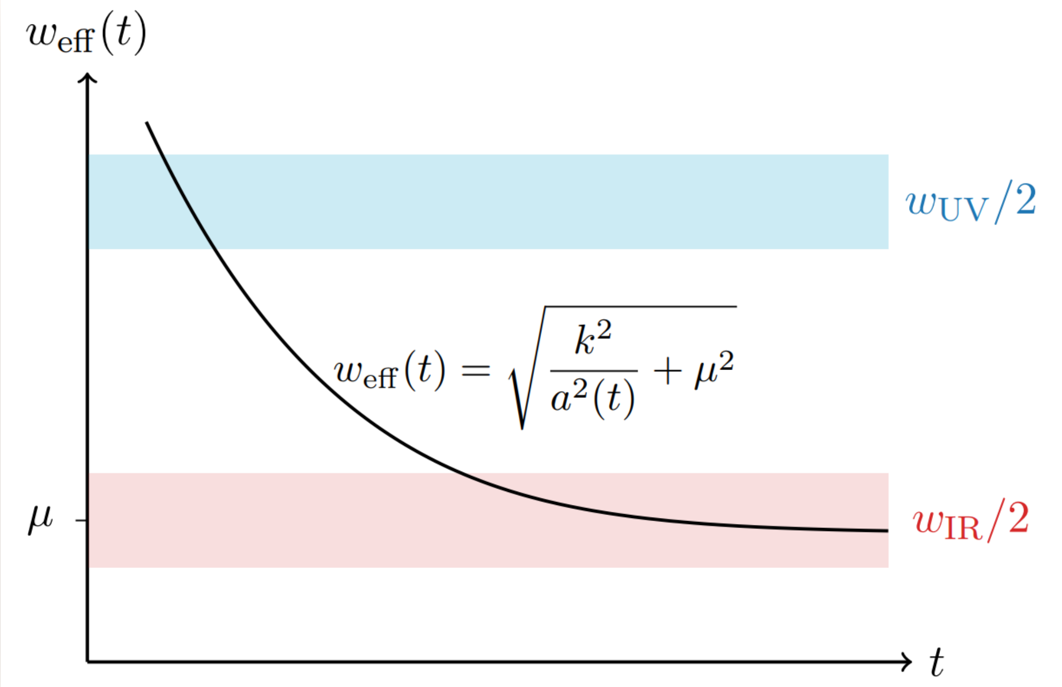
Two types of oscillations

Frequency	**Signal size**
$\mu - \omega$	$\mathcal{A}_1 \sim e^{-\pi\mu}$
μ	$\mathcal{A}_2 \sim \mathcal{O}(1)$

Coefficients have a pole around $\omega \sim 2\mu$

Resonance!

Scale-Breaking Phenomenology



IR resonance

- IR limit, drop the redshifted momentum

Mathieu equation

$$\left[\frac{\partial^2}{\partial t^2} + \mu^2\left(1 + \tilde{g}^2\cos(\omega t)\right)\right](a^{3/2}\sigma(s,t)) \approx 0$$

- Mode grows exponentially

$$a^{3/2}\sigma(t) \propto e^{\lambda_n t}$$

- Scaling exponent is altered

$$F(k_{12}, s) \sim \left(\frac{k_{12}}{s}\right)^{-1/2+\lambda_1 \pm i\mu}$$

Approximation Method

Qin, Renaux-Petel, Tong, Werth, **YHZ** [2506.01555]

Difficulty: complicated mode functions and time integrals

$$\left[\frac{\partial^2}{\partial\eta^2} + w^2(s,\eta)\right]\sigma(s,\eta) = 0$$

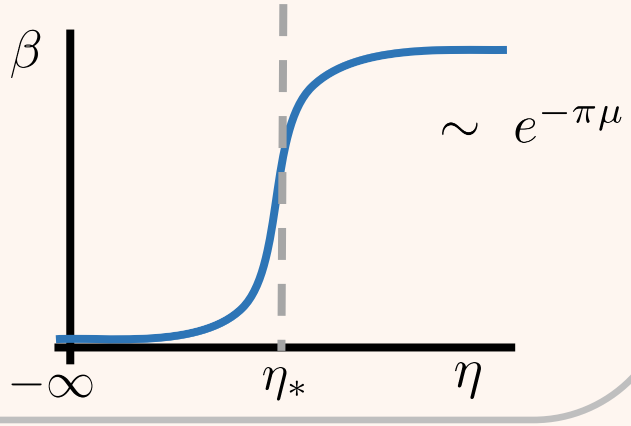
Time dependence.. $v_k \sim H_{i\mu}$ or $W_{i\kappa,i\mu}$

Super-adiabatic WKB

$$\sigma(s,\eta) = \alpha f(\eta) + \beta f^*(\eta)$$

with $\quad f(\eta) \equiv \dfrac{1}{\sqrt{2\omega(\eta)}} e^{-i\int \omega(\eta')d\eta'}$

β is related to production amount



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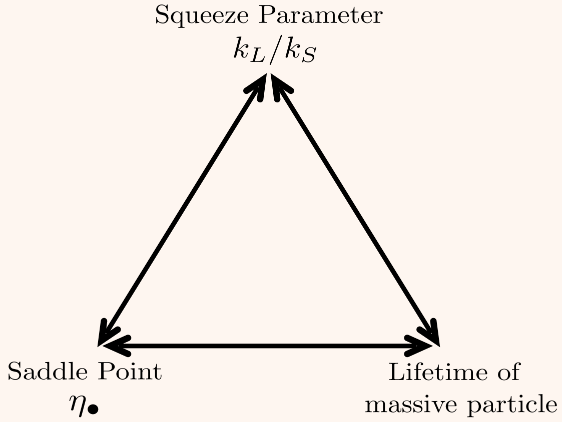
Saddle point method

$$F \approx \int_{-\infty^+}^{0} \frac{d\eta'}{\sqrt{2\omega(\eta')}} \left[e^{i\left(k_{12}\eta' \pm \int_{\eta_i}^{\eta'} d\eta\,\omega(\eta)\right)} \right]$$

$$\frac{d}{d\eta'}\left(k_{12}\eta' \mp \int_{\eta_i}^{\eta'} d\eta\,\omega(\eta)\right) = 0$$

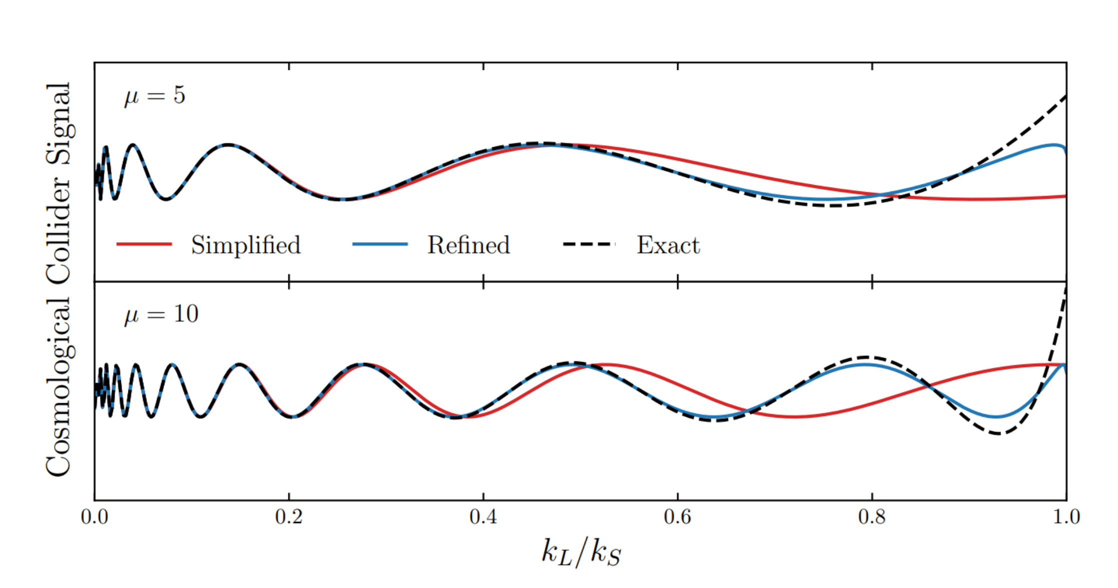


Approximation Method

Qin, Renaux-Petel, Tong, Werth, YHZ [2506.01555]

Simplified: $\cos[\mu \log(k_L/k_S)]$

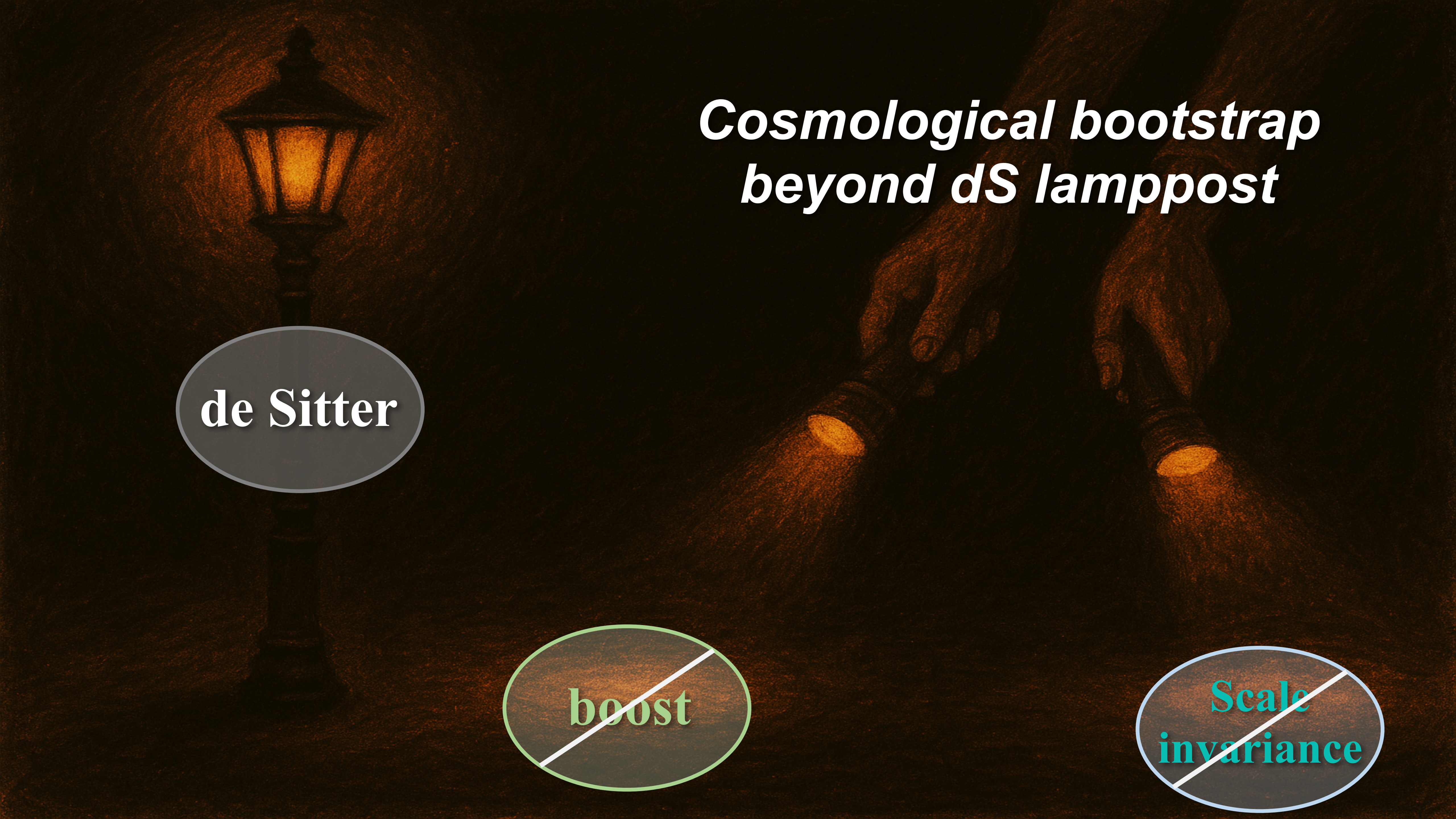
Refined: $\cos[\mu \,\mathrm{arccosh}(k_S/k_L)]$



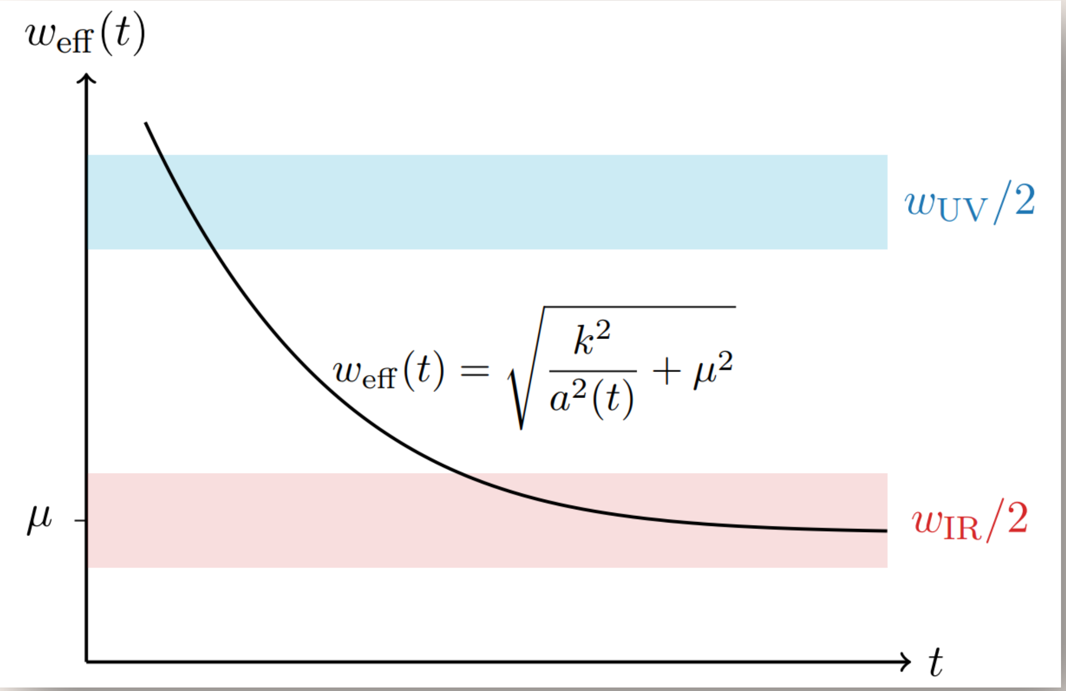
- Good way to understand physics
- Perfect balance between precision, efficiency

We provide nice templates for upcoming cosmological surveys

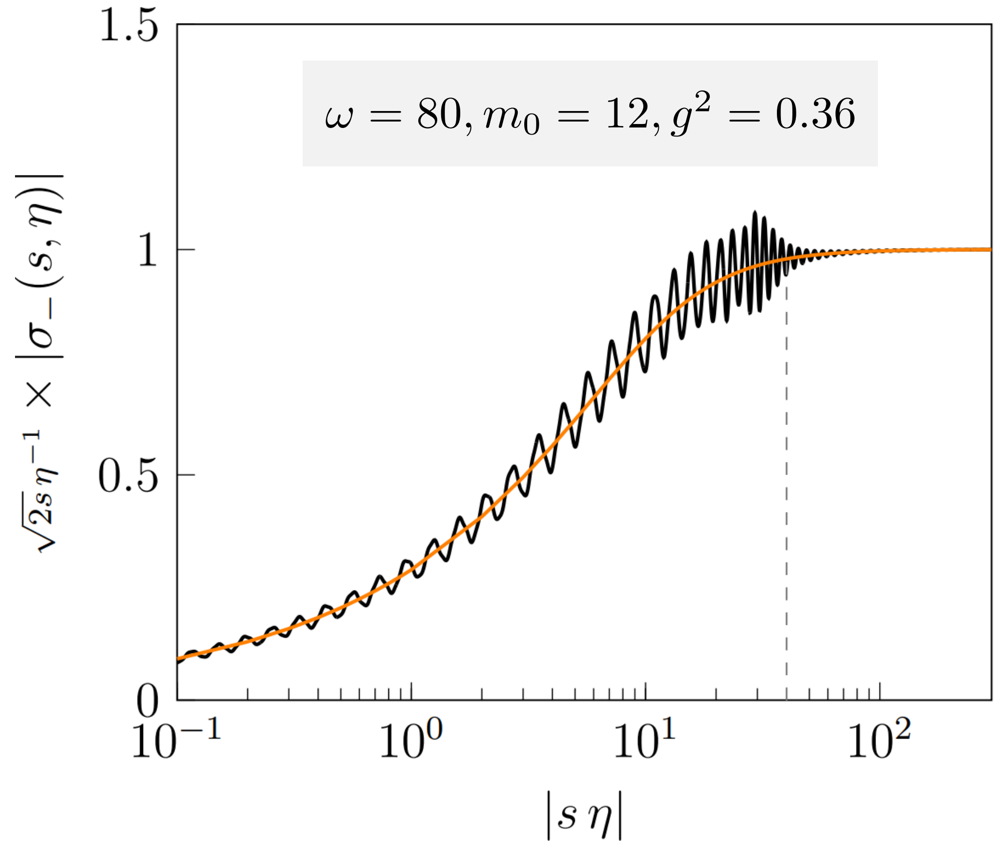
Cosmological bootstrap beyond dS lamppost



Scale-Breaking Phenomenology



UV resonance



- Enhanced (resonant) cosmological collider signals when $\omega \gg m_0$