Inflationary Fossils beyond perturbation theory

R. Impavido in collaboration with N. Bartolo (UniPD)

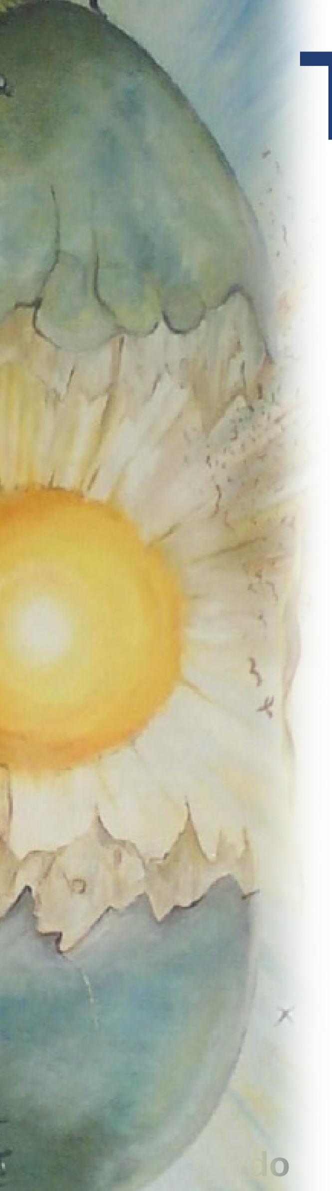
ArXiv 2507.17593

Università degli Studi di Ferrara

INFN

IFIC Institut de Física Corpuscular

Universitat de València



The inflationary scenario

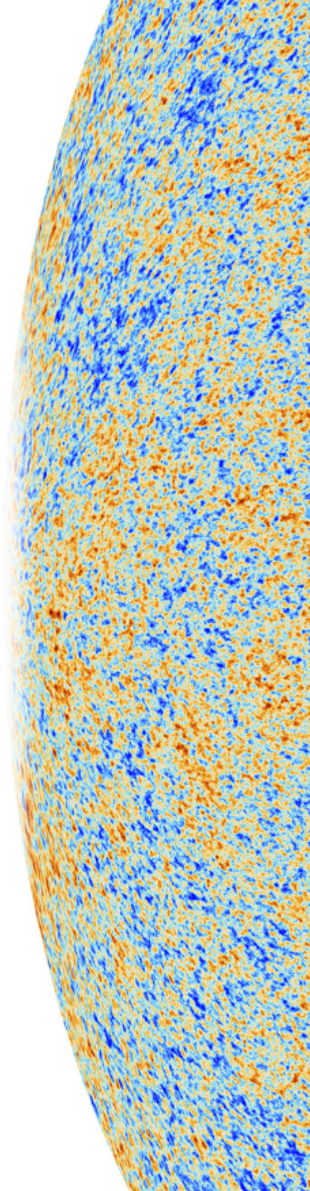
- The early universe is approximately a de Sitter space, expanding at an accelerated rate

- The accelerated expansion washes away inhomogeneities, explaining the great homogeneity of, e.g. the CMB

- Massless fields in de Sitter are frozen, at late times, by the cosmological expansion, at a value $\simeq H$

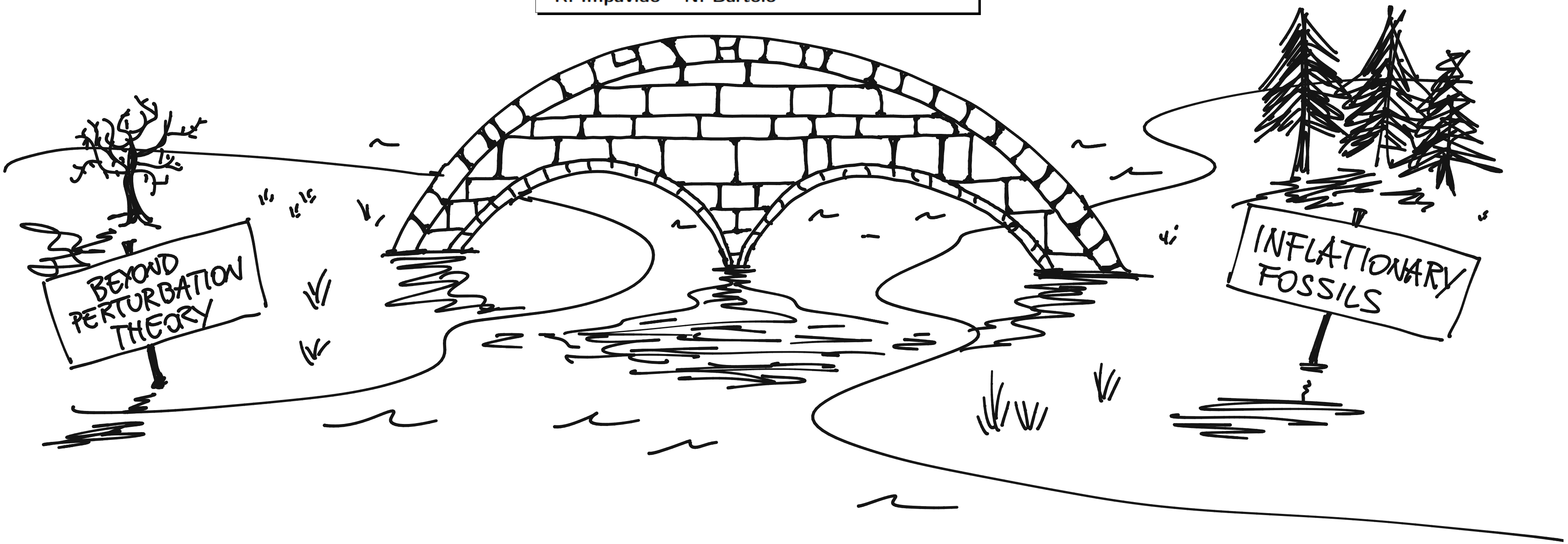
- The accelerated expansion is sustained by the v.e.v. of the inflaton

- Quantum fluctuations of the inflaton are responsible for the inhomogeneities of the CMB



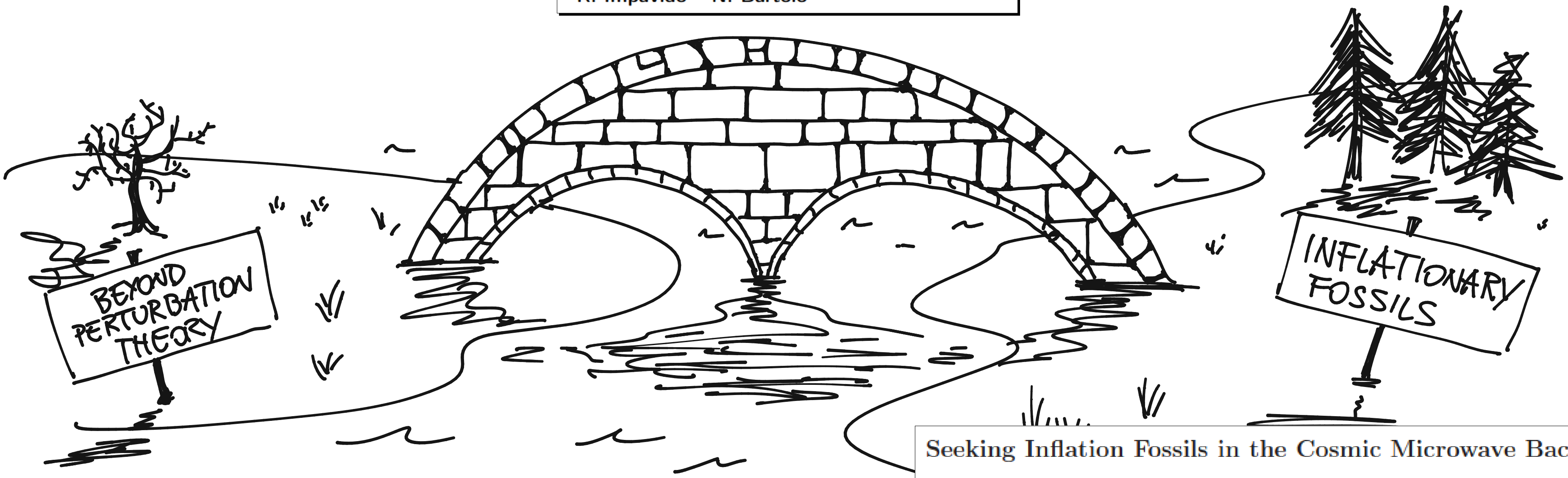
Inflationary Fossils beyond perturbation theory

R. Impavido[a,b] N. Bartolo[c,d,e]



Inflationary Fossils beyond perturbation theory

R. Impavido[a,b] N. Bartolo[c,d,e]



Seeking Inflation Fossils in the Cosmic Microwave Background

Liang Dai, Donghui Jeong, and Marc Kamionkowski

Clustering Fossils from the Early Universe

Donghui Jeong[1] and Marc Kamionkowski[1,2]

[1] Department of Physics an

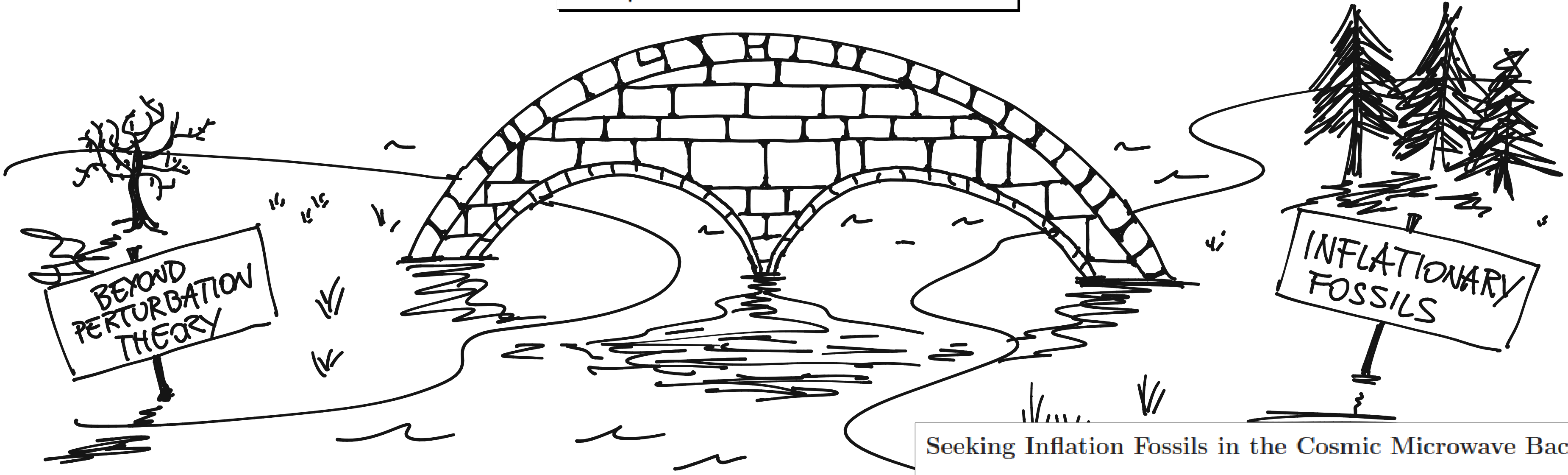
[2] Califor

Inflationary tensor fossils in large-scale structure

Emanuela Dimastrogiovanni,[a] Matteo Fasiello,[b] Donghui Jeong,[c,d] Marc Kamionkowski[e]

Inflationary Fossils beyond perturbation theory

R. Impavido[a,b] N. Bartolo[c,d,e]



Seeking Inflation Fossils in the Cosmic Microwave Background

Liang Dai, Donghui Jeong, and Marc Kamionkowski

Clustering Fossils from the Early Universe

…g[1] and Marc Kamionkowski[1,2]

Primordial Stochastic Gravitational Wave Background Anisotropies: in-in Formalization and Applications

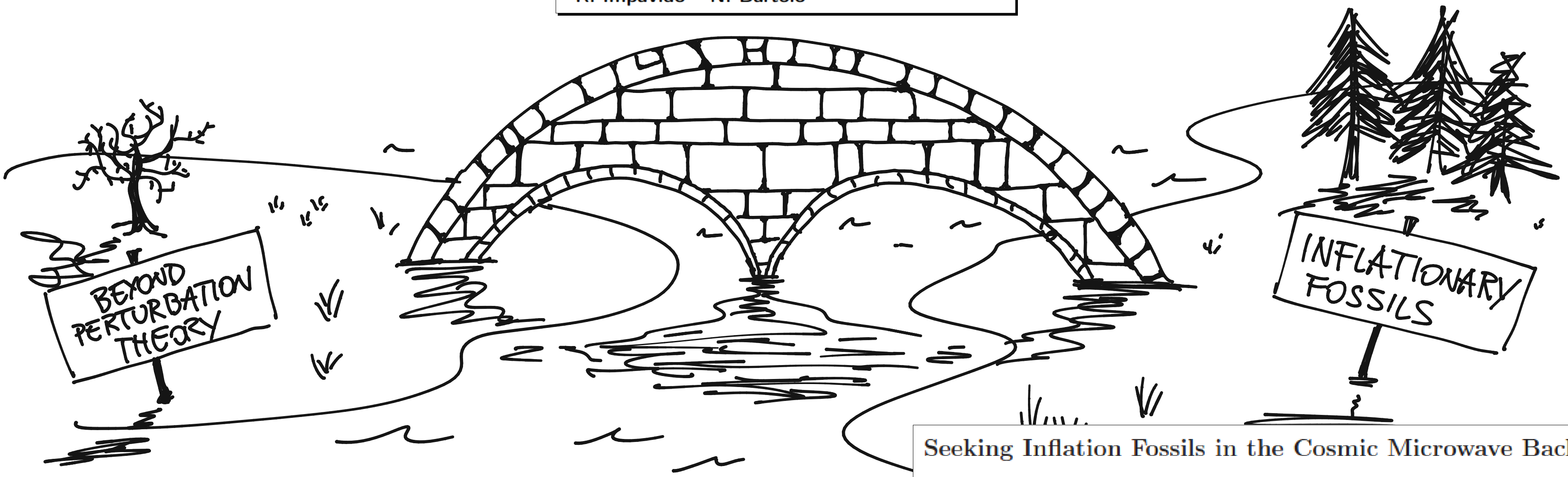
Ema Dimastrogiovanni,[a,b] Matteo Fasiello,[c,d] Lucas Pinol[c]

…ary tensor fossils in large-scale …e

…astrogiovanni,[a] Matteo Fasiello,[b] Donghui Jeong,[c,d] Marc Kamionkowski[e]

Inflationary Fossils beyond perturbation theory

R. Impavido[a,b] N. Bartolo[c,d,e]



Beyond Perturbation Theory in Inflation

Marco Celoria[a,b], Paolo Creminelli[a,b], Giovanni Tambalo[c], and Vicharit Yingcharoenrat[d,e]

Seeking Inflation Fossils in the Cosmic Microwave Background

Liang Dai, Donghui Jeong, and Marc Kamionkowski

Clustering Fossils from the Early Universe

…[1] and Marc Kamionkowski[1,2]

Primordial Stochastic Gravitational Wave Background Anisotropies: in-in Formalization and Applications

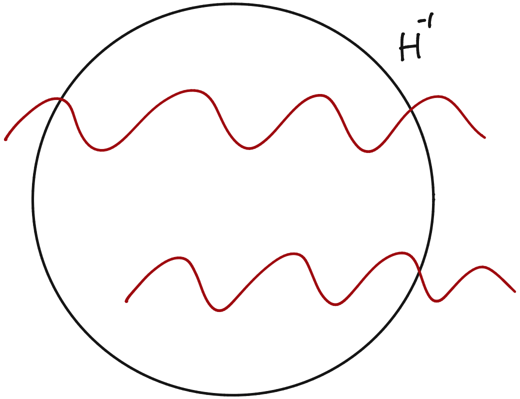
Ema Dimastrogiovanni,[a,b] Matteo Fasiello,[c,d] Lucas Pinol[c]

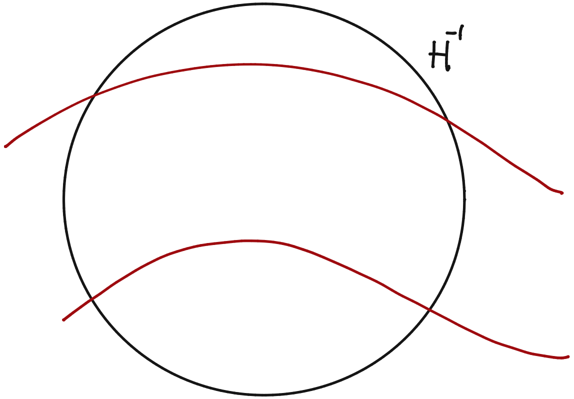
…ary tensor fossils in large-scale …e

…astrogiovanni,[a] Matteo Fasiello,[b] Donghui Jeong,[c,d] Marc Kamionkowski[e]

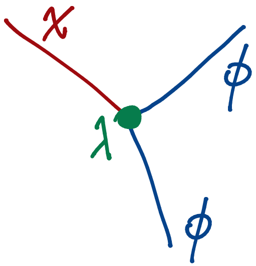
Inflationary fossils

How is it possible to characterize long modes of a field that interacted with the inflaton?





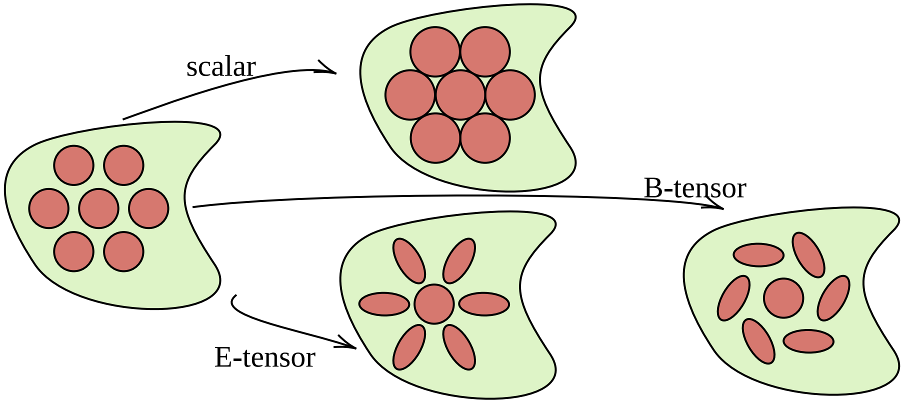
$$\lambda\phi\phi\chi$$



$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle\,|_{\chi_L} = \langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle + \lim_{\mathbf{q}\to 0} \frac{\bar{\chi}\,\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\chi_{\mathbf{q}}\rangle}{\langle \chi_{\mathbf{q}}\chi_{\mathbf{q}}\rangle}$$

Inflationary fossils

How is it possible to characterize long modes of a field that interacted with the inflaton?



[Dai et al 2013]

$$\langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \rangle \, |_{\chi_L} = \langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \rangle + \lim_{\mathbf{q}\to 0} \frac{\bar{\chi} \, \langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \chi_{\mathbf{q}} \rangle}{\langle \chi_{\mathbf{q}} \chi_{\mathbf{q}} \rangle}$$

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i \phi)^2 + \chi'^2 - (\partial_i \chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H \chi \phi^2$$

$\chi :$ **free part +** $\lambda\phi^2 = 0$

$\phi :$ **free part +** $\lambda\chi\phi = 0$

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i\phi)^2 + \chi'^2 - (\partial_i\chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H\chi\phi^2$$

approximation(s)

$\lambda \ll 1$ weak coupling

$\chi :$ **free part +** $\lambda\phi^2 = 0$

$\phi :$ **free part +** $\lambda\chi\phi = 0$

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2} \left(\phi'^2 - (\partial_i \phi)^2 + \chi'^2 - (\partial_i \chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H \chi \phi^2$$

approximation(s)

$\lambda \ll 1$ weak coupling

$\chi :$ **free part +** ~~$\lambda\phi^2$~~ $= 0$

$\phi :$ **free part +** $\lambda \chi \phi = 0$

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i\phi)^2 + \chi'^2 - (\partial_i\chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H\chi\phi^2$$

χ : **free part +** ~~$\lambda\phi^2$~~ $= 0$

ϕ : **free part +** $\lambda\chi\phi = 0$

approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i\phi)^2 + \chi'^2 - (\partial_i\chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H\chi\phi^2$$

$\chi :$ **free part +** ~~$\lambda\phi^2$~~ $= 0$

$\phi :$ **free part +** $\lambda\bar{\chi}\phi = 0$

approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i\phi)^2 + \chi'^2 - (\partial_i\chi)^2\right) - \frac{\lambda}{2\eta^4 H^4} H\chi\phi^2$$

χ : **free part +** ~~$\lambda\phi^2$~~ $= 0$

ϕ : **free part +** $\lambda\bar{\chi}\phi = 0$

approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

$\bar{\chi} \gg H$ large mode of χ

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S_{\text{eff}} = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2} \left(\phi'^2 - (\partial_i \phi)^2 - \frac{m^2}{\eta^2 H^2} \phi^2 \right)$$

$$m^2 = H\bar{\chi}\lambda$$

χ : free part + ~~$\lambda\phi^2$~~ $= 0$

ϕ : free part + $\lambda\bar{\chi}\phi = 0$

approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

$\bar{\chi} \gg H$ large mode of χ

Beyond perturbation theory

Is it possible to solve nonperturbatively, in some approximation, a theory of two interacting fields in (quasi) de Sitter space?

$$S_{\text{eff}} = \int d\eta d^3\mathbf{x} \frac{1}{2\eta^2 H^2}\left(\phi'^2 - (\partial_i \phi)^2 - \frac{m^2}{\eta^2 H^2}\phi^2\right)$$

$$m^2 = H\bar{\chi}\lambda$$

$$\langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle_{\text{eff}} = \langle \phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle (-k\eta)^{3-2\sqrt{9/4-m^2/H^2}}$$

[Celoria et al 2021]

χ : **free part** + ~~$\lambda\phi^2$~~ $= 0$

ϕ : **free part** + $\lambda\bar{\chi}\phi = 0$

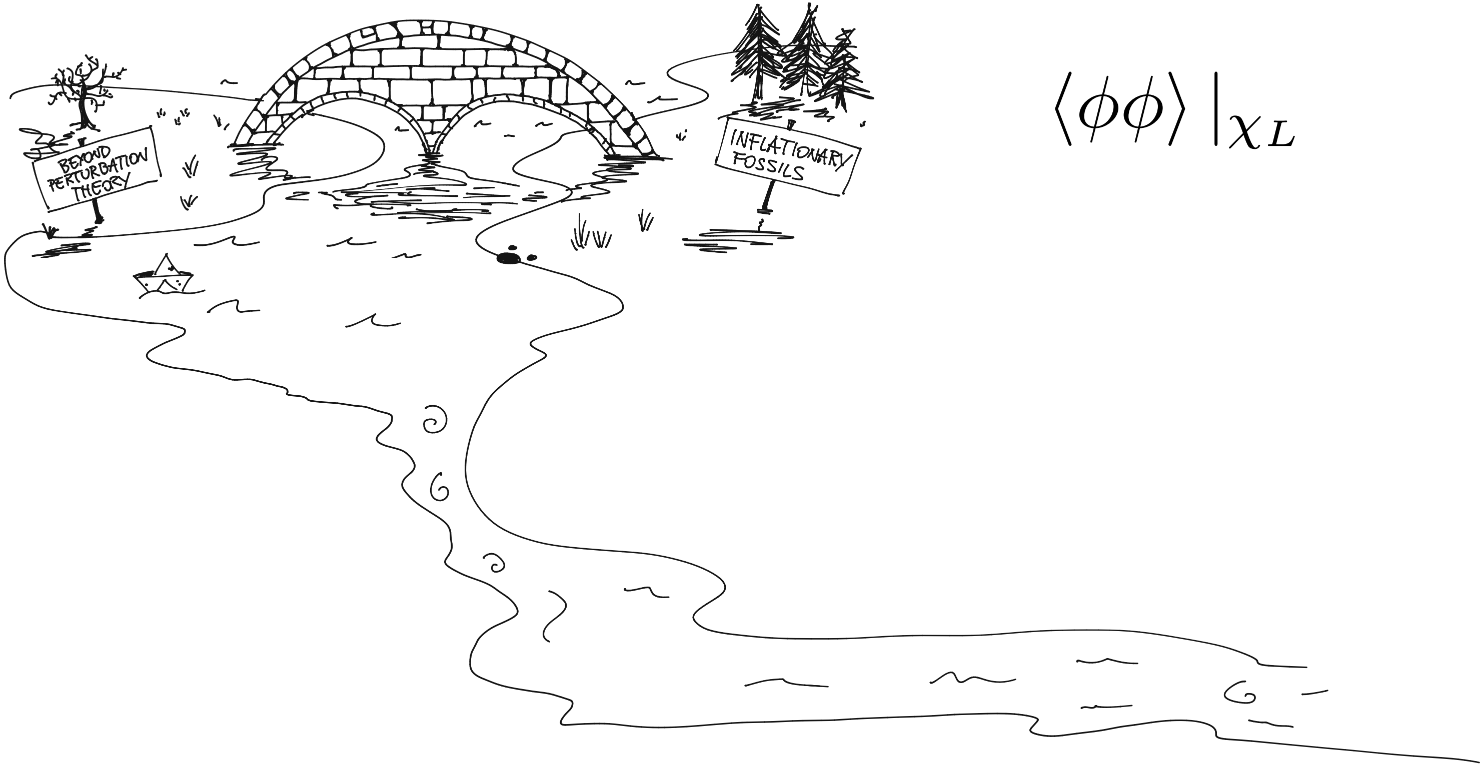
approximation(s)

$\lambda \ll 1$ weak coupling

$k_\chi \ll k_\phi$ long mode of χ

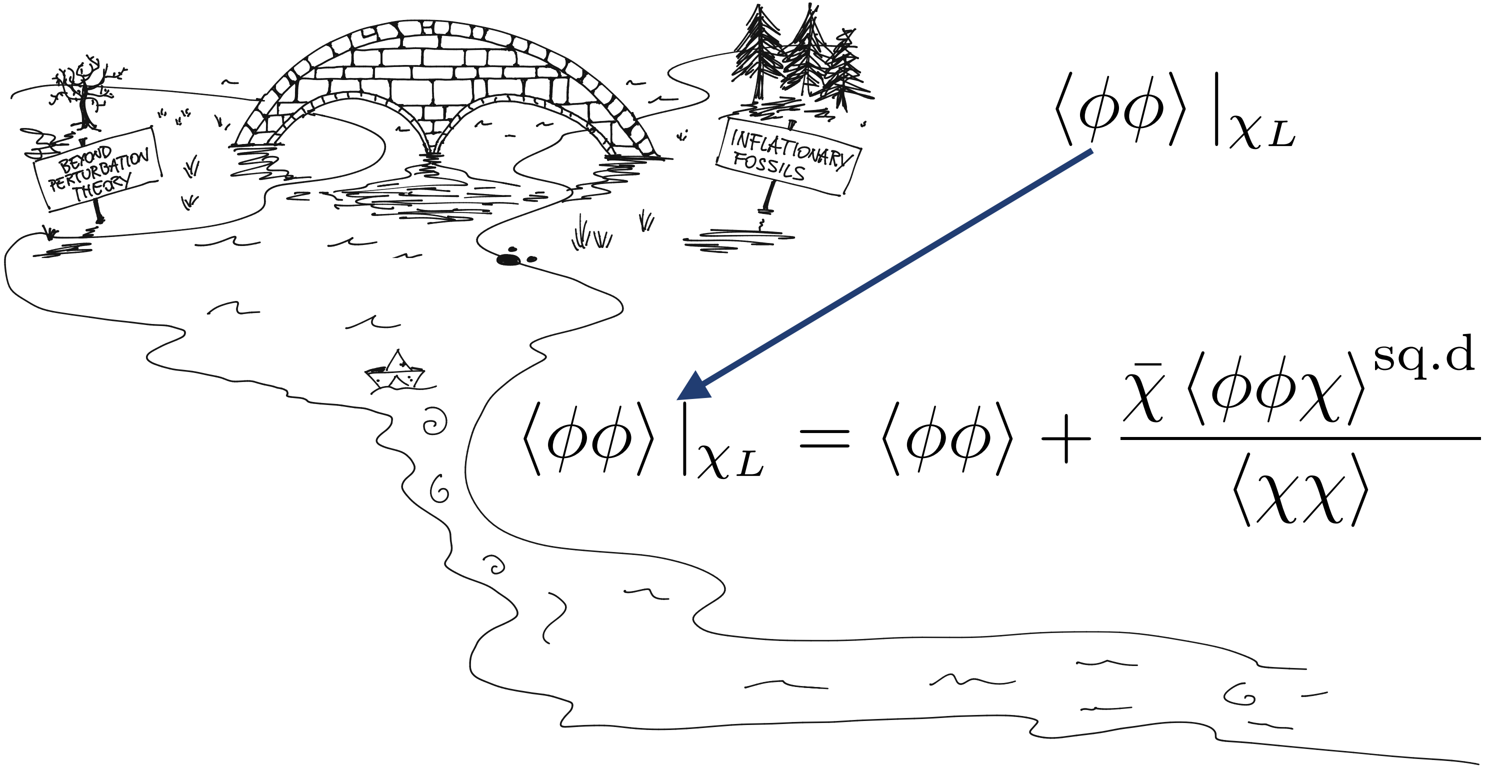
$\bar{\chi} \gg H$ large mode of χ

$$\langle \phi\phi \rangle_{\text{eff}}$$



$$\langle \phi\phi \rangle \big|_{\chi_L}$$

$$\langle\phi\phi\rangle_{\text{eff}}$$

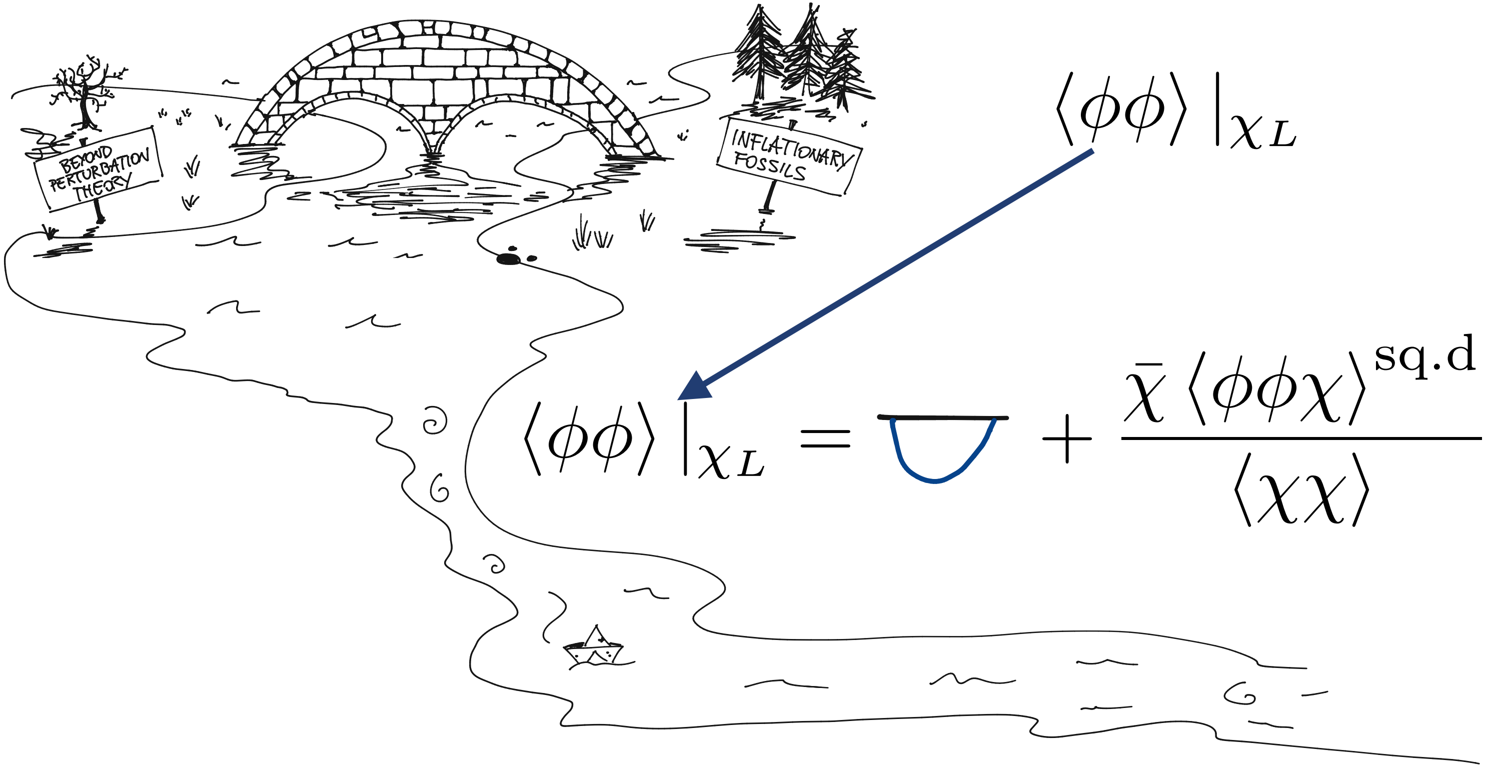


$$\langle\phi\phi\rangle\big|_{\chi_L}$$

$$\langle\phi\phi\rangle\big|_{\chi_L} = \langle\phi\phi\rangle + \frac{\bar{\chi}\,\langle\phi\phi\chi\rangle^{\text{sq.d}}}{\langle\chi\chi\rangle}$$

$$\langle \phi\phi \rangle_{\text{eff}}$$

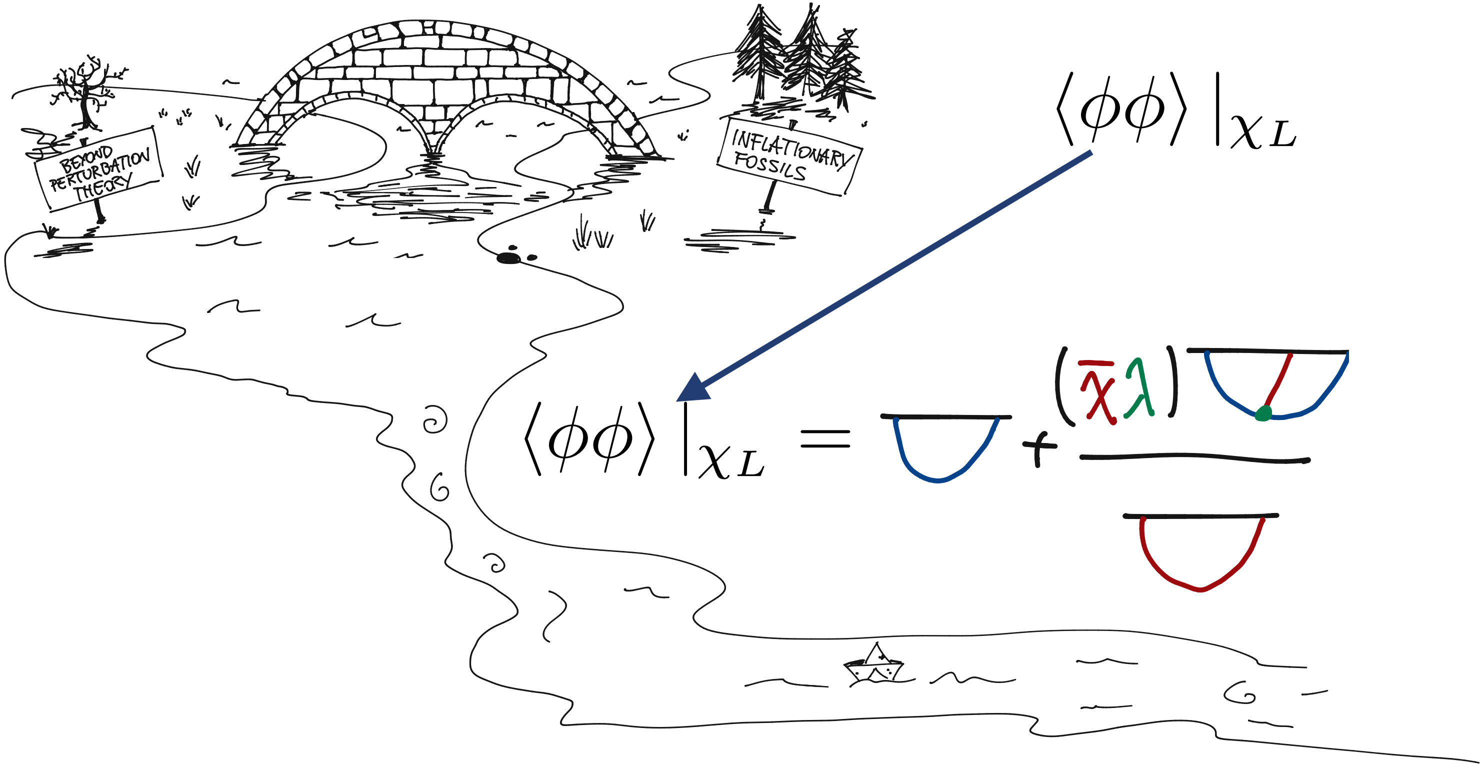
$$\langle \phi\phi \rangle \big|_{\chi_L}$$



$\langle \phi\phi \rangle \big|_{\chi_L} =$ $+ \dfrac{\bar{\chi} \, \langle \phi\phi\chi \rangle^{\text{sq.d}}}{\langle \chi\chi \rangle}$

$$\langle\phi\phi\rangle_{\text{eff}}$$

$$\langle\phi\phi\rangle\big|_{\chi_L}$$



$$\langle\phi\phi\rangle\big|_{\chi_L} =$$

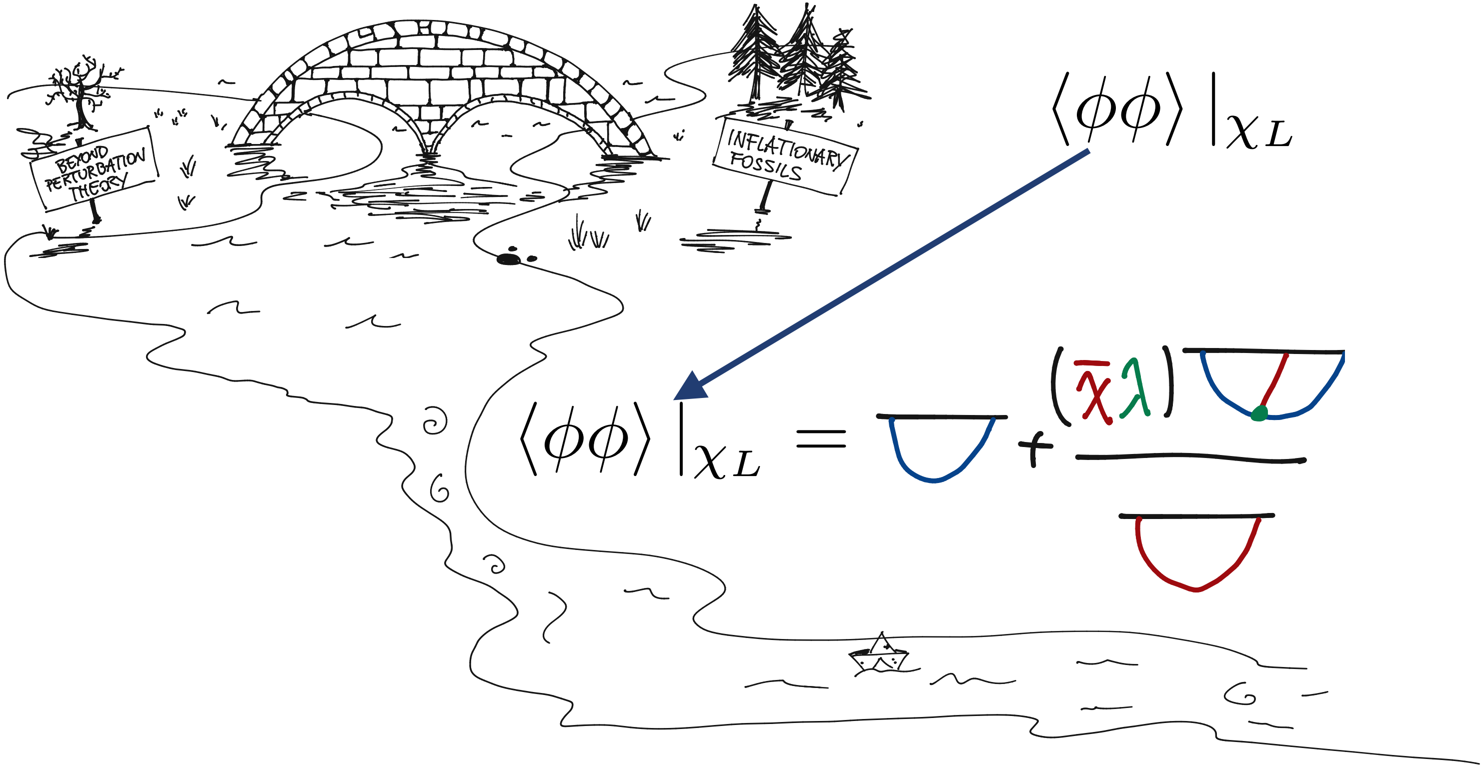
$(\bar{\chi}\lambda)$

$$\langle\phi\phi\rangle_{\text{eff}}$$

$$\downarrow$$

$$\langle\phi\phi\rangle_{\text{eff}}(\lambda\bar{\chi})$$

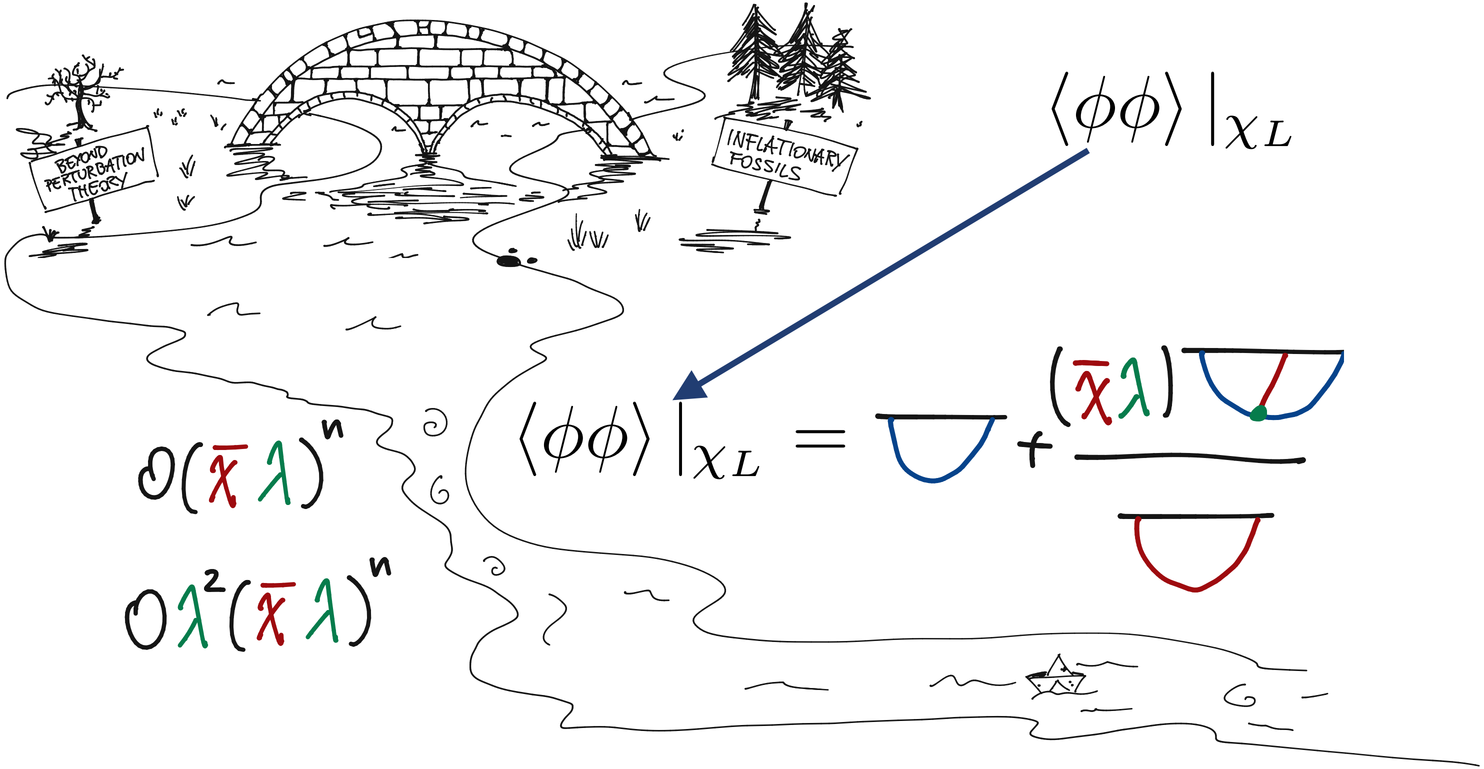
$$\langle\phi\phi\rangle\big|_{\chi_L}$$



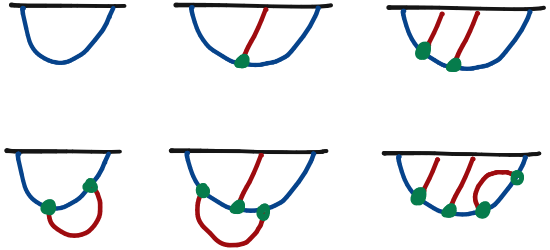
$$\langle \phi\phi \rangle_{\text{eff}}$$

$$\downarrow$$

$$\langle \phi\phi \rangle_{\text{eff}}(\lambda\bar{\chi})$$



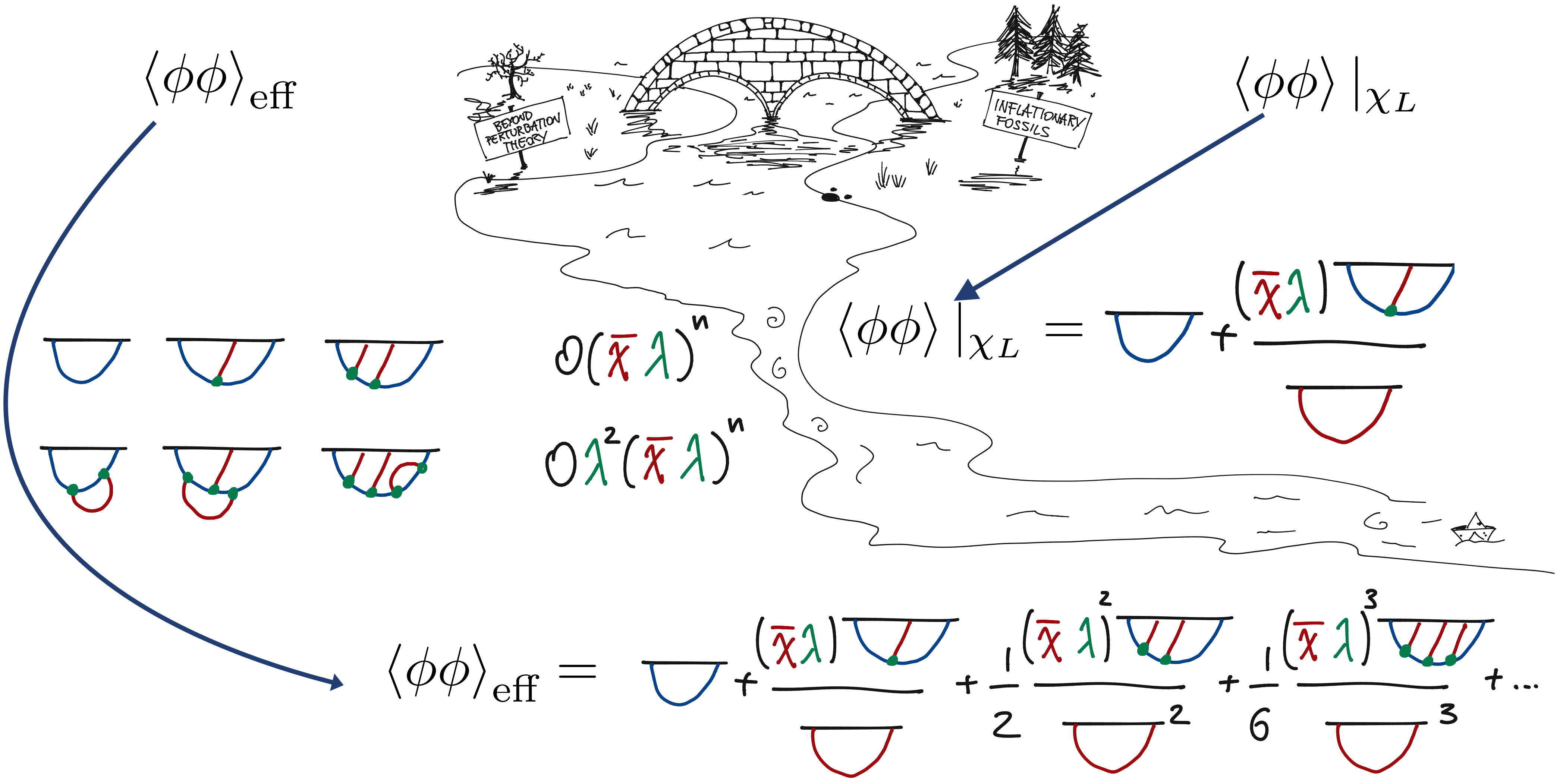
$$\langle \phi\phi \rangle\,|_{\chi_L}$$



$$\mathcal{O}(\bar{\chi}\lambda)^n$$

$$\mathcal{O}\lambda^2(\bar{\chi}\lambda)^n$$

$$\langle \phi\phi \rangle\,|_{\chi_L} = \ldots + (\bar{\chi}\lambda)\ldots$$


$\langle\phi\phi\rangle_{\rm eff}$
BEYOND PERTURBATION THEORY
INFLATIONARY FOSSILS
$\langle\phi\phi\rangle\,|_{\chi_L}$
$\mathcal{O}((\bar\chi\lambda)^n)$
$\mathcal{O}\lambda^2(\bar\chi\lambda)^n$
$\langle\phi\phi\rangle\,|_{\chi_L} =$
$\langle\phi\phi\rangle_{\rm eff} =$

Results: all matchings

Checked the matching between Fossils and BPT to first order

Coupling	Type	Effect BPT
$\lambda\chi\sigma^2$	scalar-scalar, linear	effective mass
$\lambda\chi\sigma'\sigma'$	scalar-scalar, derivative	change of kin. term
$\lambda\chi\partial_i\sigma\partial^i\sigma$	scalar-scalar, derivative	change of kin. term
$\epsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta$	scalar-tensor, derivative	change of kin. term
$\epsilon\zeta\left(\gamma'_{ij}\gamma'_{ij}g^{00}-\partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right)$	scalar-tensor, der.	change of kin. term
CCs-violating	scalar-scalar, derivative	change of kin. term

Results: all matchings

Checked the matching between Fossils and BPT to first order

Coupling	Type	Effect BPT
$\lambda\chi\sigma^2$	scalar-scalar, linear	effective mass
$\lambda\chi\sigma'\sigma'$	scalar-scalar, derivative	change of kin. term
$\lambda\chi\partial_i\sigma\partial^i\sigma$	scalar-scalar, derivative	change of kin. term
$\epsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta$	scalar-tensor, derivative	change of kin. term
$\epsilon\zeta\left(\gamma'_{ij}\gamma'_{ij}g^{00} - \partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right)$	scalar-tensor, der.	change of kin. term
CCs-violating	scalar-scalar, derivative	change of kin. term

Large (& long mode) scalar *fossils* modfy GW's speed of sound

free part for ζ | free part for γ

$$S = \int d^3\mathbf{x} d\eta \frac{-\epsilon}{\eta^4 H^4}\left[\zeta'\zeta' g^{00} + \partial_i\zeta\partial_j\zeta g^{ij}\right] - \frac{1}{8\eta^4 H^4}\left[\gamma'_{ij}\gamma'_{ij} g^{00} + \partial_k\gamma_{ij}\partial_h\gamma_{ij} g^{hk}\right]$$

$$\frac{-\epsilon}{8\eta^4 H^4}\left[\zeta\gamma'_{ij}\gamma'_{ij} g^{00} - \zeta\partial_k\gamma_{ij}\partial_h\gamma_{ij} g^{hk}\right] - \frac{1}{4\eta^4 H^4} g^{hk}\left[\gamma'_{ij}\partial_k\gamma_{ij}\partial_h\chi\right]$$

interaction $\gamma\gamma\zeta$ | auxiliary field

Large (& long mode) scalar *fossils* modfy GW's speed of sound

free part for ζ free part for γ

$$S = \int d^3\mathbf{x}d\eta \frac{-\epsilon}{\eta^4 H^4}\left[\zeta'\zeta' g^{00} + \partial_i\zeta\partial_j\zeta g^{ij}\right] - \frac{1}{8\eta^4 H^4}\left[\gamma'_{ij}\gamma'_{ij}g^{00} + \partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right]$$

$$\frac{-\epsilon}{8\eta^4 H^4}\left[\zeta\gamma'_{ij}\gamma'_{ij}g^{00} - \zeta\partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right] - \frac{1}{4\eta^4 H^4}g^{hk}\left[\gamma'_{ij}\partial_k\gamma_{ij}\partial_h\chi\right]$$

interaction $\gamma\gamma\zeta$ auxiliary field

$$S_{\text{eff}} = \int d\eta d^3\mathbf{x}\frac{-1}{8\eta^4 H^4}\left[\gamma'_{ij}\gamma'_{ij}g^{00}(1+\epsilon\bar{\zeta}) + \partial_h\gamma_{ij}\partial_k\gamma_{ij}g^{hk}(1-\epsilon\bar{\zeta})\right]$$

$$\langle\gamma_{\mathbf{k}_1,ij}\gamma_{\mathbf{k}_2,ij}\rangle' = \frac{4H^2}{k^3}\frac{\sqrt{1+\epsilon\bar{\zeta}}}{(1-\epsilon\bar{\zeta})^{3/2}} \sim \frac{4H^2}{k^3}\left(1+2\epsilon\bar{\zeta}\right)$$

BPT BPT@1st order = *Fossils*

Results: all matchings

Checked the matching between Fossils and BPT to first order

Coupling	Type	Effect BPT
$\lambda\chi\sigma^2$	scalar-scalar, linear	effective mass
$\lambda\chi\sigma'\sigma'$	scalar-scalar, derivative	change of kin. term
$\lambda\chi\partial_i\sigma\partial^i\sigma$	scalar-scalar, derivative	change of kin. term
$\epsilon\gamma_{ij}\partial_i\zeta\partial_j\zeta$	scalar-tensor, derivative	change of kin. term
$\epsilon\zeta\left(\gamma'_{ij}\gamma'_{ij}g^{00} - \partial_k\gamma_{ij}\partial_h\gamma_{ij}g^{hk}\right)$	scalar-tensor, der.	change of kin. term
CCs-violating	scalar-scalar, derivative	change of kin. term

Sum up + Conclusions

- In 2507.17593 we formalized the missing link between the *Inflationary Fossils* and a technique beyond perturbation theory, checking the result at first order in many different cases. Resumming diagrams = skipping nested in-in integrals

- To go beyond pertubation theory we integrated out a light field at late time

- To modify the power spectrum we resummed (∞)many tree level contributions
Cfr. [Bordin et al 2018], [Pinol et al 2023, Werth et al 2024]

- It is now possible to extend the *Fossils* approach to large primordial fluctuations. Interesting phenomenology lies ahead: it could be possible to characterize large primordial GW or large curvature modes beyond perturbation theory

Backup slides

“Large” fluctuation?

- BPT approach is **semiclassical**: **no problem** in considering a large field

- *Fossils* approach is **quantum field theoretic** (IN IN formalism), in which we are used to have fluctuations of order H if we wish to match with Bunch-Davies in the UV.
We can however apply the field operator to states that are **coherent** states with a **very large number of particles**: we are therefore considering not a high energy limit, but a **high occupation number limit**.

Separate Universe?

The “separate Universe”: gradient expansion → equivalent to (quantized) cosmological P.T. if expanded [Lyth et al 2005]

↓

equivalent to the in-in formalism [Seery et al 2008] → Equivakent to BPT if expanded in the coupling, at the same order

However...

The δN expansion is not an expansion in the coupling, so the resummed results (in the coupling) has no reason to agree with δN

Single field violating the consistency conditions

Chen et al 2013
EPL 102 59001

$$S=\int d\eta d^3\mathbf{x}\Bigg[\sqrt{-g}\frac{-1}{2}\Bigg(\left(\left(\frac{\epsilon}{c_s^2}\right)\zeta_n'\zeta_n' + \epsilon\partial_i\zeta_n\partial_i\zeta_n\right.$$

$$-\left(\frac{\epsilon}{c_s^2}\right)(1-c_s^2)\zeta_n\partial_i\zeta_n\partial_i\zeta_n + \left(\frac{\epsilon}{c_s^4}(-3+3c_s^2)\right)\zeta_n\zeta_n'\zeta_n' - \left(\Sigma\left(1-\frac{1}{c_s^2}\right)+2\lambda\right)\frac{\zeta_n'^3}{H^3}$$

$$+\frac{\epsilon}{c_s^2}\left(\frac{\eta}{c_s^2}\zeta_n'^2\zeta_n + \frac{2}{c_s^2 H}(\zeta_n\zeta_n'\zeta_n'' + \zeta_n'^3)\right) + \epsilon\left(\frac{\eta}{c_s^2}\zeta_n(\partial_i\zeta_n)^2 + \frac{2}{c_s^2 H}(\zeta_n\partial_i\zeta_n\partial_i\zeta_n' + \zeta_n'(\partial_i\zeta_n)^2\right)\Bigg)\Bigg]$$

$$\zeta = \zeta_n + \frac{\eta}{4c_s^2}\zeta_n^2 + \frac{1}{c_s^2 H}\zeta_n\zeta_n' + \dots$$

Cubic Lagrangian for general single field inflation

Single field violating the consistency conditions

Chen et al 2013
EPL 102 59001

$$S = \int d\eta d^3\mathbf{x}\Bigg[\sqrt{-g}\frac{-1}{2}\left(\left(\frac{\epsilon}{c_s^2}\right)\zeta_n'\zeta_n' + \epsilon\partial_i\zeta_n\partial_i\zeta_n\right.$$

$$-\left(\frac{\epsilon}{c_s^2}\right)(1-c_s^2)\zeta_n\partial_i\zeta_n\partial_i\zeta_n + \left(\frac{\epsilon}{c_s^4}(-3+3c_s^2)\right)\zeta_n\zeta_n'\zeta_n' - \left(\Sigma\left(1-\frac{1}{c_s^2}\right)+2\lambda\right)\frac{\zeta_n'^3}{H^3}$$

$$\left.+\frac{\epsilon}{c_s^2}\left(\frac{\eta}{c_s^2}\zeta_n'^2\zeta_n + \frac{2}{c_s^2 H}(\zeta_n\zeta_n'\zeta_n'' + \zeta_n'^3)\right) + \epsilon\left(\frac{\eta}{c_s^2}\zeta_n(\partial_i\zeta_n)^2 + \frac{2}{c_s^2 H}(\zeta_n\partial_i\zeta_n\partial_i\zeta_n' + \zeta_n'(\partial_i\zeta_n)^2)\right)\right)\Bigg]$$

$$\zeta = \zeta_n + \frac{\eta}{4c_s^2}\zeta_n^2 + \frac{1}{c_s^2 H}\zeta_n\zeta_n' + \dots$$

Cubic Lagrangian for general single field inflation

$$f_{\rm NL}^{\rm loc} \sim \frac{5}{4c_s^2}$$

Violating the CCs

$$f_{\rm NL}^{\rm loc} \sim 1 - n_s \sim O(\epsilon)$$

Single field violating the consistency conditions

$$S_{\rm eff} = \int d\eta d^3\mathbf{x}\left[\sqrt{-g}\frac{-1}{2}\left(\frac{\epsilon}{c_s^2}\zeta_n'\zeta_n'\left(1+\zeta_L\left(\frac{-3+3c_s^2}{c_s^2}+\frac{\eta}{c_s^2}\right)\right)\right.\right.$$
$$\left.\left.+\epsilon\partial_i\zeta_n\partial_i\zeta_n\left(1+\zeta_L\left(\frac{c_s^2-1}{c_s^2}+\frac{\eta}{c_s^2}\right)\right)\right)\right]$$

Single field violating the consistency conditions

$$S_{\rm eff} = \int d\eta d^3\mathbf{x}\Bigg[\sqrt{-g}\frac{-1}{2}\left(\frac{\epsilon}{c_s^2}\zeta_n'\zeta_n'\left(1+\zeta_L\left(\frac{-3+3c_s^2}{c_s^2}+\frac{\eta}{c_s^2}\right)\right)\right.$$

$$\left.+\epsilon\partial_i\zeta_n\partial_i\zeta_n\left(1+\zeta_L\left(\frac{c_s^2-1}{c_s^2}+\frac{\eta}{c_s^2}\right)\right)\right)\Bigg]$$

$$\langle\zeta_n\zeta_n\rangle_{\rm eff} = \langle\zeta_n\zeta_n\rangle(1-\eta\zeta_L/c_s^2+\dots) = \langle\zeta_n\zeta_n\rangle\left(1+\frac{\zeta_L\langle\zeta_n\zeta_n\zeta_n\rangle}{\langle\zeta_n\zeta_n\rangle^2}+\dots\right)$$

$$\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = \langle\zeta_{n,\mathbf{k}_1}\zeta_{n,\mathbf{k}_2}\zeta_{n,\mathbf{k}_3}\rangle + \frac{\eta}{4c_s^2}[\langle\zeta_{n,\mathbf{k}_1}\zeta_{n,\mathbf{k}_2}\rangle\langle\zeta_{n,\mathbf{k}_1}\zeta_{n,\mathbf{k}_3}\rangle + \mathrm{perm}]$$

$$f_{\rm NL}^{\rm loc} \sim \frac{5}{4c_s^2}$$

auxiliary field?

See e.g. Maldacena 2003. GR+scalar field using the ADM formalism.

$$S = \frac{1}{2}\int \sqrt{g}[R - (\nabla\phi)^2 - 2V(\phi)] \qquad ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$S = \frac{1}{2}\int \sqrt{h}\left[NR^{(3)} - 2NV + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 - Nh^{ij}\partial_i\phi\partial_j\phi\right]$$

Solve for the constraints in the $\delta\phi = 0$ gauge

$$\nabla_i[N^{-1}(E^i_j - \delta^i_j E)] = 0$$

$$R^{(3)} - 2V - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0$$

$$N^i = \partial_i\psi + N^i_T \text{ where } \partial_i N^i_T = 0 \text{ and } N = 1 + N_1.$$

$$N_1 = \frac{\dot{\zeta}}{\dot{\rho}}\ , \qquad N^i_T = 0\ , \qquad \psi = -e^{-2\rho}\frac{\zeta}{\dot{\rho}} + \chi\ , \qquad \partial^2\chi = \frac{\dot{\phi}^2}{2\dot{\rho}^2}\dot{\zeta}$$

Expanding the action at cubic order and selecting $\gamma\gamma\zeta$ terms leaves some auxialiary χ dependence. However there is no influence for the BPT procedure